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Stochastic Dominance and Information Value*

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1. INTRODUCTION

Incentives for producing and disseminating information have been analysed in many different contexts. Kihlstrom [6], for example, develops a Bayesian framework to analyze the properties of demand functions for information about product quality. From an entirely different perspective, Spence [8, 9] demonstrates the private value of "signalling" information. Similarly, Wilson [10] considers the value to the firm of producing technological information and the effect of such production on equilibrium theory.

This paper is concerned with the incentives for and social value of information production and distribution where ownership and management of a firm are separated, and control over information decisions is placed with management. In the spirit of Jensen and Meckling [5], we consider two agents, a consuming-investing (CI) agent who supplies all inputs for the firm's production process, and a producing-managing (PM) agent who determines the actual production level. In such a framework, of course, the PM-agent may retain some of the investment of the CI-agent for consumption as perquisites.

The wage contract of the PM-agent is represented as a state-independent sharing rule such that wages are proportional to output. Such an arrangement is typically a suboptimal but realistic representation, and thus all results are constrained by this form of contract.

Information is characterized as having potential value in three dimensions. First, information regarding states of nature may have productive value if the production function is state-dependent. This may be called "internal information" which the PM-agent uses to improve his decision regarding the optimal level of production. Second, the same state information may be delivered to the CI-agent at the option of the PM-agent, reducing informational asymmetry between the two, thus eliminating or reducing some of the uncertainty of the consumer-investor. Finally, information may serve a

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control function, reducing the ability of the PM-agent to consume perquisites at the expense of the CI-agent. As we show, the PM-agent may have incentives to distribute such "monitoring" information if he can thereby induce the CI-agent to invest additional resources in the firm.

In Section 2 the model is developed. Section 3 contains a description of the family of probability density functions considered and the definition of stochastic dominance. In Section 4, we derive conditions under which information has positive private and social value. Conclusions follow in Section 5.

2. Description of the Economy

The economy consists of two economic agents (CI and PM), two periods, and a single commodity. At the beginning of the first period, the CI-agent holds a positive amount of the commodity which can be either consumed during the period or invested in the production process for future (period 2) consumption. The production technology is assumed to be available only to the PM-agent. Assuming absence of futures markets and the inability to store the commodity for future consumption, the CI-agent must invest in the production process of the PM-agent to insure future consumption.

Let α represent the ownership fraction or contractual right of the CI-agent to claim the produced output, and let z denote the value of the production process of the PM-agent, or the amount of the current resource the CI-agent is willing to transfer in exchange for the fractional ownership α. The investment level z is, in general, a function of the ownership share held by the CI-agent. The quantity α in this formulation ultimately depends on the relative bargaining power of the economic agents involved.

The production output, y, is assumed to be a function of the input x and the state of world: $y = y(x, s)$. Thus, economic decisions are made under uncertainty, and we assume that the agents are expected utility maximizers.

The decision model of the PM-agent is postulated by the following stochastic optimization problem:

$$\begin{aligned} V^* &= V^*(\alpha, z) \\ &= \max_{0 \leqslant x \leqslant z} \int_S (\gamma(s)\, v(z - x) + v((1 - \alpha)\, y(x, s)))\, d\nu(s), \end{aligned} \tag{2.1}$$

where

(1) $v' \gg 0$ and $v'' \leqslant 0$,[1]

(2) $S = \{s \mid s \text{ is a possible state of the world}\}$,

[1] For a real-valued function $f = f(t)$, we shall use the notation: (1) $f \geqslant 0$ means that $f(t) \geqslant 0$ for all t, (2) $f > 0$ is equivalent to the conditions that $f \geqslant 0$ and $f(t) > 0$ for some t, and (3) $f \gg 0$ requires that $f(t) > 0$ for all t.

(3) ν is the subjective probability measure on S,

(4) $y: R^{+} \times S \to R^{+}$ is the (technologically efficient) production function (where $R^{+} = \{x \mid x \geqslant 0\}$), with $y' \gg 0$, $y'' \leqslant 0$, where the prime indicates differentiation with respect to x,

(5) $\gamma: S \to [0, 1]$ is increasing and concave,

(6) α is the fractional ownership claim of the CI-agent against the produced output $y(x, s)$.

Throughout our discussion we shall freely make use of the differential calculus, taking for granted that the functions involved are differentiable up to the desired orders.

The optimization formulation (2.1) represents expected utility maximization, which is an atypical representation of the firm's objective function.[2] However, the formulation is quite general, depending on the specific values assigned to the parameters. If $\gamma = 0$, (2.1) is reduced to maximization of expected value of the reward (wage), $(1 - \alpha)y(x, s)$, to the PM-agent. In such a case, the PM-agent assigns no utility to the consumption of perquisites. Suppose, on the other hand, that (1) $v(t) = t$ for all $t \geqslant 0$ (linear utility), (2) s represents the future price relative to the current price, (3) $y(x, s) = y(x) \cdot s$ is the revenue of the firm, and (4) $\gamma = (1 - \alpha)$. Under this set of conditions, (2.1) is parallel with expected profit maximization.

Equation (2.1) may be interpreted as the utility of the PM-agent achieved from consumption of perquisites (state-dependent), $\gamma(s)v(z - x)$, and wages, $v[(1 - \alpha)y(x, s)]$. Since the PM-agent, after having received z, may choose to consume or produce according to (2.1), he may fail to act in the best interest of the CI-agent.

Assuming an interior solution to (2.1), the unique optimal production input x^* that the PM-agent will choose must satisfy

$$\bar{\gamma} v'(z - x) = \int_S v'((1 - \alpha)\, y(x, s))(1 - \alpha)\, y'(x, s)\, d\nu(s) \tag{2.2}$$

where the quantity

$$\bar{\gamma} = \int_S \gamma(s)\, d\nu(s)$$

is assumed to be positive.

We can now summarize the properties of the PM-agent's behavior, demonstrating that (2.1) is a reasonable representation.

THEOREM 2.1. *Given* (α, z), *let* $x^* = x^*(\alpha, z)$ *be the optimal production input that the PM-agent selects.*

[2] The firm's decision problem under a regime of incomplete markets is unresolved. See, for example, Ekern and Wilson [1].

(1) $$0 \leqslant \frac{\partial x^*}{\partial z} \leqslant 1,$$

(2) $$\frac{\partial V^*}{\partial z} > 0 \text{ and } \frac{\partial^2 V^*}{\partial z^2} \leqslant 0,$$

(3) $$\frac{\partial V^*}{\partial \alpha} < 0 \text{ if } \frac{\partial z}{\partial \alpha} = 0,$$

(4) $\partial V^*/\partial\alpha > 0$ for all expectations ν on the uncertain states S if and only if the inequality $(1-\alpha)\, y'(x^*, s)\, \partial z/\partial\alpha > y(x^*, s)$ holds in S.

Proof. For the proof of (1), take the partial derivative of the optimality condition (2.2) with respect to the variable z:

$$\bar{\gamma} v''(z - x^*)\Big(1 - \frac{\partial x^*}{\partial z}\Big) = \int_S \{v''((1-\alpha)\, y)(1-\alpha)^2\, (y')^2 + v'((1-\alpha)\, y)(1-\alpha)\, y''\} \frac{\partial x^*}{\partial z}\, d\nu(s),$$

where the arguments (x^*, s) of the functions y, y', and y'' are suppressed for notational ease. Simplifying the above equation, we obtain:

$$\frac{\partial x^*}{\partial z} = \frac{\bar{\gamma} v''}{\bar{\gamma} v'' + (1-\alpha) \int_S \{v''(1-\alpha)(y')^2 + v'y''\}\, d\nu(s)}\,.$$

The assertion (1) now follows from our assumptions that $v' \gg 0$, $v'' \leqslant 0$, $y' \gg 0$, $y'' \leqslant 0$, and $0 \leqslant \alpha \leqslant 1$.

To prove assertion (2), we differentiate equation (2.1) with respect to z:

$$\frac{\partial V^*}{\partial z} = \int_S \Big\{\gamma(s)\, v'(z - x^*)\Big(1 - \frac{\partial x^*}{\partial z}\Big) + v'((1-\alpha)\, y)(1-\alpha)\, y' \frac{\partial x^*}{\partial z}\Big\}\, d\nu(s).$$

Since x^* must satisfy equation (2.2), the above expression is reduced to:

$$\frac{\partial V^*}{\partial z} = \bar{\gamma} v'(z - x^*) \gg 0.$$

Differentiating the above with respect to z again, we deduce:

$$\frac{\partial^2 V^*}{\partial z^2} = \bar{\gamma} v''(z - x^*)\Big(1 - \frac{\partial x^*}{\partial z}\Big) \leqslant 0.$$

In order to see (3) and (4), we take the partial derivative of (2.1) with respect to α:

$$\frac{\partial V^*}{\partial \alpha} = \int_S \gamma(s)\, v'(z - x^*)\left(\frac{\partial z}{\partial \alpha} - \frac{dx^*}{d\alpha}\right) dv + \int_S v'((1-\alpha)\, y(x^*, s)) \times \left((1-\alpha)\, y'(x^*, s)\frac{dx^*}{d\alpha} - y(x^*, s)\right) dv.$$

Using the optimality condition (2.2), we can rewrite this expression in the form:

$$\frac{\partial V^*}{\partial \alpha} = \int_S v'((1-\alpha)\, y(x^*, s))\left\{(1-\alpha)\, y'(x^*, s)\frac{\partial z}{\partial \alpha} - y(x^*, s)\right\} dv(s).$$

Therefore, the proof of Theorem 2.1 is completed.

Assertion (1) simply says that, as the CI-agent increases his investment z, the PM-agent increases both his production input x^* and his perquisite consumption $(z - x^*)$. Since the PM-agent draws utility from both his perquisite and wage, at his optimum he increases both as the CI-agent invests more in the firm. Assertion (2) confirms that the PM-agent's utility increases as the investment level increases. Assertion (3) demonstrates that, if the CI-agent's share of the output increases, ceteris paribus, the PM-agent's wage decreases, and he is made worse off. However, this is typically not the case, since z is a function of α; hence assertion (4) shows that as α increases, the PM-agent's utility increases as long as the effect of the increase in investment outweighs the loss due to wage reduction.

The consumer-investor holds an initial endowment $e > 0$. His economic decision is to determine an optimal allocation of this resource between current consumption and investment for future consumption. This allocation decision is assumed to be made in terms of the additive utility function $u(c_1) + \beta u(c_2)$, where $u(c_1)$ represents utility on his current consumption c_1 and $u(c_2)$ designates utility on his future consumption which depends on his commodity holding c_2 at the end of the period. The quantity β is assumed to be constant with $0 < \beta < 1$, so the CI-agent prefers to consume a positive amount of the commodity in the future ($\beta > 0$), but his preference on current consumption is greater than on future consumption ($\beta < 1$).

The CI-agent invests $z \geqslant 0$ for future consumption and retains $(e - z) \geqslant 0$ for current consumption. If the PM-agent places the entire investment z into the production process, the resulting output will be $y(z, s)$ depending on the state s that occurs. Since the PM-agent may consume a positive amount of the investment in the form of perquisites, the actual output will lie between zero and the maximum possible output $y(z, s)$, that is, $y = y(z, s) \cdot q$ where $0 \leqslant q \leqslant 1$. The CI-agent is uncertain about both s and q, characterized by the random vector $(s, q) \in S \times I$, where I is the unit interval $[0, 1]$. The implication of this formulation is that since the PM-agent strives to maximize his own utility, he produces additional risk (via a random q) for the CI-agent beyond state uncertainty.

Formally, the consumption-investment decision model of the CI-agent is postulated by the stochastic optimization model:

$$\begin{aligned} U^* &= U^*(\alpha) \\ &= \max_{0 \leqslant z \leqslant e} \left\{ u(e - z) + \beta \iint_{SxI} u(\alpha y(z, s)\, q)\, d\mu(s, q) \right\}. \end{aligned} \tag{2.3}$$

where

(1) $u' \geqslant 0$ and $u'' \leqslant 0$, and

(2) μ represents the CI-agent's probability assessment on (s, q).

As in the case of the PM-agent, we assume an interior optimum and deduce the following necessary and sufficient condition: the optimal investment level z^* satisfies

$$u'(e - z) = \beta \iint_{SxI} u'(\alpha y(z, s)\, q)\, \alpha y'(z, s)\, q\, d\mu(s, q). \tag{2.4}$$

Since α denotes the fractional ownership of the output held by the CI-agent, a larger α should be desirable from his point of view, all else held constant. This assertion is formally demonstrated in the following theorem.

THEOREM 2.2. *As his ownership fraction α is increased, so is the expected utility of the CI-agent*: $\partial U^*/\partial\alpha > 0$.

Proof. In equation (2.3), substitute for z the optimal investment level z^* determined by equation (2.4) and then take the partial derivative of the resulting expression with respect to the variable α:

$$\begin{aligned} \frac{\partial U^*}{\partial \alpha} &= -u'(e - z^*) \frac{\partial z^*}{\partial \alpha} \\ &\quad + \beta \iint_{SxI} u'(\alpha y(z^*, s)\, q) \left\{ y(z^*, s)\, q + \alpha y'(z^*, s)\, q \frac{\partial z^*}{\partial \alpha} \right\} d\mu(s, q). \end{aligned}$$

Since z^* satisfies equation (2.4), we can further simplify the above and obtain:

$$\frac{\partial U^*}{\partial \alpha} = \beta \iint_{SxI} u'(\alpha y(z^*, s)\, q)\, y(z^*, s)\, q\, d\mu(s, q).$$

which is easily seen to be positive.

Although Theorem 2.2 demonstrates that a larger α is preferred by the CI-agent, it is not clear what his response for various values of α will be in terms of current consumption and investment. We now establish necessary and sufficient conditions under which the CI-agent increases his investment as his ownerhsip fraction α increases. Since the outcome of the investment decision is uncertain, whereas current consumption is not, such conditions must depend on the risk-taking behavior of the agent.

THEOREM 2.3. *The investment level chosen by the CI-agent is an increasing function of his ownership fraction α for all his expectations regarding the uncertain investment opportunity if and only if the relative risk-aversion of the CI-agent is less than one.*

Proof. Take the partial derivative of the optimality condition (2.4) with respect to the variable α:

$$-u''(e - z^*)\frac{\partial z^*}{\partial \alpha} = \beta \iint_{SxI} u''(\alpha yq)\Big(yq + \alpha y'q\frac{\partial z^*}{\partial \alpha}\Big)\,\alpha y'q\,d\mu(s, q)$$

$$+ \beta \iint_{SxI} u'(\alpha yq)\Big(y'q + \alpha y''q\frac{\partial z^*}{\partial \alpha}\Big)\,d\mu(s, q),$$

where we have omitted the arguments for $y(z^*, s)$, $y'(z^*, s)$, and $y''(z^*, s)$.
Solving the above in $\partial z^*/\partial\alpha$, we obtain:

$$\frac{\partial z^*}{\partial \alpha} = -\frac{\beta \iint_{SxI} y'q\{u'(\alpha yq) + u''(\alpha yq)\,\alpha yq\}\,d\mu(s, q)}{u''(e - z^*) + \alpha\beta \iint_{SxI}\{u'(\alpha yq)\,y''q + \alpha(y'q)^2\,u''(\alpha yq)\}\,d\mu(s, q)}.$$

Since the denominator is negative, it is seen that $\partial z^*/\partial\alpha > 0$ for all μ if and only if:

$$\iint_{SxI} y'q\{u'(\alpha yq) + u''(\alpha yq)\,\alpha yq\}\,d\mu(s, q) > 0$$

for all μ. A simple mathematical proposition yields that the latter holds if and only if the relative risk-aversion is less than one: $u'(t) + tu''(t) > 0$. This completes the proof.

We will assume that the sharing rule α is Pareto optimal in the analysis to follow. Of course, as has been noted, such optimality is constrained by the fact that only state-independent contracts are available.[3]

[3] The sharing rule α is not necessarily Pareto-optimal. The following condition is necessary and sufficient to guarantee Pareto-optimality of α:

$$(1 - \alpha)y'(x^*, s)\frac{\partial z^*}{\partial \alpha} \leqslant y(x^*, s) \quad \text{in} \quad S.$$

Since $\partial U^*/\partial\alpha > 0$ for all α by Theorem 2.2, a state-independent sharing rule α is Pareto-optimal if and only if $\partial V^*/\partial\alpha \leqslant 0$. Theorem 2.1 yields the conclusion.

3. Orderings on Probability Density Functions

It is important to fix the ranking of probability distributions to be considered if any positive statement regarding information value is to be obtained. Consider the family M of probability density functions $f(z)$ for $z \geqslant 0$, where z is some random prospect. In order to reduce the mathematical arguments involved, we assume that every density $f(z) \in M$ has the following properties: (1) $f(z)$ is piecewise continuous in the sense that it is continuous for all $z > 0$, except possibly at finitely many points, and (2) $f(z)$ has a compact support, that is, $f(z) = 0$ for all $z \geqslant n$, where n is a positive real number which may depend on the density function.

Following Hadar and Russell [3], we define an ordering on the family M.

Definition. Given $f, g \in M, f \gtrsim g$ if and only if:

$$\int_0^y (f(z) - g(z))\, dz \leqslant 0 \text{ for all } y \geqslant 0.$$

If $f \gtrsim g$, we say that the density $f(z)$ stochastically *dominates* the density $g(z)$.

It can be shown that this ordering has all the properties of a partial ordering.

Proposition 3.1. *The ordering $\gtrsim$ defines a partial order on the family M of probability densities*:

(1) *Reflexive Property*: $f \gtrsim f$,

(2) *Symmetric Property*: $f \gtrsim g$ *and* $g \gtrsim f$ *imply that* $f = g$ *with probability one*,

(3) *Transitive Property*: $f \gtrsim g$ *and* $g \gtrsim h$ *imply that* $f \gtrsim h$.

To provide an intuitive interpretation of the ordering, we state without proof two propositions defining its relationship to the preference ordering on M made by an expected utility maximizer.

Proposition 3.2. *Let $f, g \in M$. Then $f \gtrsim g$ if and only if*

$$\int_0^\infty u(z) f(z)\, dz \geqslant \int_0^\infty u(z) g(z)\, dz$$

for all nondecreasing functions u.

Proposition 3.3. *Let $u(z)$ be a real-valued continuous function for $z \geqslant 0$.*

Then the function $u(z)$ is nondecreasing if and only if

$$\int_0^\infty u(z) f(z)\, dz \geqslant \int_0^\infty u(z)\, g(z)\, dz$$

for all $f, g \in M$ with $f \gtrsim g$.

For the proof of these propositions, see Hadar and Russell [3] and Fishburn [2].

Propositions 3.2 and 3.3 are useful representations of stochastic dominance. Expected utility given density function f is greater than or equal to that for density function g given any arbitrary increasing utility function if and only if the dominance relation strictly holds. Similarly, the utility function is nondecreasing if and only if its expected utility is not less for f over g for all $f \gtrsim g$. These propositions will, in some settings, allow an ordering over information production alternatives, as is demonstrated in Section 4.

4. Information Production and Distribution

As noted in the introductory section, information may have value in at least three capacities in the model developed. First, it may improve productive decisions of the PM-agent. Second, it may reduce the risk faced by the CI-agent by reducing the informational asymmetry between producing and investing agents. Finally, a monitoring function may be served by information, reducing the range of alternatives available to the PM-agent. Each alternative will be examined in this section.

The extent of an agent's information will be evaluated in terms of stochastic dominance.[4] We realistically assume that the PM-agent is "more informed" than the CI-agent regarding the probability distribution over states of nature in S. Denoting the respective subjective beliefs of the CI-agent and the PM-

[4] Another more intuitive way of considering the productive value of information is to suppose that it is technological in nature. Letting λ represent the "amount of information," we suppose:

$$y_\lambda(x, s) \geqslant y_{\lambda'}(x, s)$$

for any fixed x and all s. Thus, we can suppose that the *PM*-agent's technology is superior to that of the *CI*-agent, which is correspondingly superior to some arbitrary technology. Similarly, by producing technological information, the *PM*-agent can improve output in every state of nature for a fixed input, by increasing the efficiency of the production process. It is easy to see that if (1) $y(x, s)$ is increasing in s and (2) $f_\lambda(s)$, $f_{\lambda'}(s)$ are two probability density functions with $f_\lambda \gtrsim f_{\lambda'}$, then

$$\iint_S y(x, s) f_\lambda(s)\, ds \geqslant \iint_S y(x, s) f_{\lambda'}(s)\, ds.$$

agent by $g(s)$ and $f(s)$, we say that the PM-agent is "more informed" than the CI-agent if and only if $f(s)$ strictly dominates $g(s)$ in sense of the stochastic dominance relation: $f(s) \gtrsim g(s)$ and $g(s) \not\gtrsim f(s)$. To illustrate, assume that the random variable s is distributed by the probability density function $\pi_1(s) \in M$. If the PM-agent has perfect information, i.e., that which is free of noise, we must require $f = \pi_1$. A totally uninformed CI-agent would imply that his belief $g(s)$ is identical to the uniform distribution $\pi_0(s)$. A typical situation would be the ordering $\pi_1 \gtrsim f \gtrsim g \gtrsim \pi_0$, which we assume to hold before additional information is produced.

The CI-agent is also uncertain regarding the PM-agent's production plan, with the uncertainty represented by the density function $h(q)$. Information then may be produced for the following purposes: (1) to improve the subjective belief $f(s)$ of the PM-agent toward $\pi_1(s)$; (2) to shift the CI-agent's density $g(s)$ toward $f(s)$ (and, thereby, toward $\pi_1(s)$); and (3) to reduce the risk perceived by the CI-agent via revision of $h(q)$.

Before analyzing the incentives for information production, we utilize several simplifications on the decision models of the CI and PM-agents. As stated, let $g(s)$ denote the CI-agent's subjective probability density function of the productive uncertainty $s \in S$, and let $h(q)$ represent his belief regarding the uncertain production decision of the PM-agent. Assuming that the random variables s and q are independently distributed over the interval $[0, 1]$ and the production function has the form $y(x, s) = y(x)s$, we can restate the decision molel (2.3) as follows:

$$U^* = \max_{0 \leq z \leq e} \int_0^1 \int_0^1 \{u(e - z) + \beta u(\alpha y(z)\, sq)\}\, g(s)\, h(q)\, ds\, dq. \tag{4.1}$$

The corresponding optimality condition (from (2.4)) is:

$$u'(e - z) = \alpha\beta \int_0^1 \int_0^1 u'(\alpha y(z)\, sq)\, y'(z)\, sqg(s)\, h(q)\, ds\, dq. \tag{4.2}$$

Assuming $f(s)$ designates the PM-agent's subjective probability density function of the random variable s, the decision model of the PM-agent is postulated by:

$$V^* = \max_{0 \leq x \leq z^*} \int_0^1 \{\gamma v(z^* - x) + v((1 - \alpha)\, y(x)\, s)\} f(s)\, ds, \tag{4.3}$$

where z^* is assumed to have been selected by the CI-agent by use of his optimality condition (4.2), and his preference γ on perquisites is assumed to be state independent. The optimal input x^* is determined by the equation:

$$\gamma v'(z^* - x) = (1 - \alpha) \int_0^1 v'((1 - \alpha)\, y(x)\, s)\, y'(x)\, sf(s)\, ds. \tag{4.4}$$

A. Information Production

Given this framework, we examine first the case of information production for the purpose of improving the PM-agent's probability assessments. Recall that all information-related decisions are made by the PM-agent. Assume that internal information systems can be characterized in terms of the parameter λ_1: $0 \leqslant \lambda_1 \leqslant 1$. If the PM-agent utilizes system λ_1, his subjective probability density is assumed to be revised to:

$$f_{\lambda_1}(s) = f(s) + \lambda_1(\pi_1(s) - f(s))$$

where $f \lesssim \pi_1$. Of course, $\lambda_1 = 0$ means that the system provides no new information since the prior and posterior densities of the PM-agent coincide with each other. But $\lambda_1 = 1$ implies that the PM-agent can obtain perfect information from the system.

Based on this revision process, an inquiry can be made into the incentives and effects of producing such internal information. We do so in the following theorems.

THEOREM 4.1. *The PM-agent's expected utility increases with an increase in λ_1 if he possesses imperfect information.*

Proof. Since the utility function of the PM-agent, $\gamma v(z - x) + v((1 - \alpha) y(x)s)$ is an increasing function of the random variable s, and $f_{\lambda_1} \gtrsim f$, Proposition 3.3 assures us that expected utility increases as λ_1 increases.

THEOREM 4.2. *Given the investment decision z made by the C1-agent, the PM-agent increases his production input x as he becomes more informed if and only if the relative risk-aversion of the PM-agent is less than one.*

Proof. Differentiating the optimality condition (4.4) with respect to the parameter λ_1, we deduce:

$$\begin{aligned} -\gamma v''(z - x^*) \frac{\partial x^*}{\partial \lambda_1} = & \int_0^1 v''\{(1 - \alpha) y'(x^*) s\}^2 \frac{\partial x^*}{\partial \lambda_1} f_{\lambda_1}(s)\, ds \\ & + \int_0^1 v'(1 - \alpha) y''(x^*) s \frac{\partial x^*}{\partial \lambda_1} f_{\lambda_1}(s)\, ds \\ & + \int_0^1 v'(1 - \alpha) y'(x^*) s\{\pi_1(s) - f(s)\}\, ds. \end{aligned}$$

Given that $v' \gg 0$, $y' \gg 0$, $v'' \leqslant 0$, and $y'' \leqslant 0$, it is not difficult to see that $\partial x^*/\partial \lambda_1 > 0$ if and only if:

$$\int_0^1 v'((1 - \alpha) y(x^*) s) s\{\pi_1(s) - f(s)\}\, ds > 0$$

for $\pi_1 \gtrsim f$.

Now let us multiply the above inequality by the expression $(1 - \alpha)y(x^*) > 0$. We then have:

$$\int_0^1 v'((1 - \alpha)\, y(x^*)\, s)(1 - \alpha)\, y(x^*)\, s\{\pi_1(s) - f(s)\}\, ds > 0.$$

This is seen, from Proposition 3.3, to be equivalent to the condition that the function $tv'(t)$ is increasing; that is, the relative risk-aversion of the PM-agent is less than one.

Theorem 4.1 assures us that the PM-agent has incentives to privately produce information for productive purposes, when the stochastic dominance criterion holds. The optimal level of information chosen is dependent on its cost, which we ignore for ease of presentation. Theorem 4.2 establishes conditions under which the PM-agent increases production levels as he becomes more informed. Since $\partial U^*/\partial x^* > 0$ always (the CI-agent is always made better off given a fixed α and z if production is increased), we deduce from Theorem 4.2 that $\partial U^*/\partial \lambda_1 > 0$ if and only if the relative risk aversion of the PM-agent is less than one. Under such conditions, information has unequivocal social value since everyone prefers its production.

B. Information Dissemination

The PM-agent privately holds the internal information generated. Conditions are now established under which he has private incentives to reduce the informational asymmetry existing between himself and the CI-agent by publishing the information. Since we have assumed that $f \gtrsim g$, the PM-agent is more informed than the CI-agent. The value z^* of the production output, determined by the CI-agent, depends on his subjective beliefs. By disseminating information to the CI-agent, the PM-agent may be able to influence the investment decision of the CI-agent. If the information reveals that the risk involved in the production process is less than previously perceived, the CI-agent may choose to increase his investment rather than consuming more during the current period.

Assume that the level of information released by the PM-agent can be characterized in terms of the parameter λ_2, where $0 \leqslant \lambda_2 \leqslant 1$. As λ_2 becomes larger, the corresponding level of information disclosed is greater. Upon receiving information through the system λ_2, the CI-agent is assumed to reassess his subjective beliefs on s in the form:

$$g_{\lambda_2}(s) = g(s) + \lambda_2[f(s) - g(s)]. \tag{4.4}$$

If $\lambda_2 = 0$, then $g_{\lambda_2} = g$ and the subjective beliefs of the CI-agent remain unchanged. As λ_2 approaches one, the subjective beliefs $g_{\lambda_2}(s)$ of the CI-agent move closer to that of the more informed PM-agent.

The decision of the PM-agent regarding information dissemination depends on the response to such information by the CI-agent. We establish the desirability of information from the CI-agent's viewpoint in Theorem 4.3 and establish conditions under which his investment level will increase in Theorem 4.4. Recall that the PM-agent is assumed to be more informed than the CI-agent.

THEOREM 4.3. *The utility of the CI-agent increases as additional information is received.*

Proof. Since $g_{\lambda_2} \gtrsim g$ for all $\lambda_2 > 0$, Proposition 3.3 insures that the CI-agent prefers more information to less.

THEOREM 4.4. *Upon receiving information regarding S, the CI-agent will increase his investment for all expectations $g \lesssim f$ and h if and only if his relative risk-aversion is less than one.*

Proof. The CI-agent determines his optimal investment level z^* according to the condition:

$$u'(e - z^*) = \alpha\beta \int_0^1 \int_0^1 u'(\alpha y(z^*)\, sq)\, y'(z^*)\, sq\, g_{\lambda_2}(s)\, h(q)\, ds\, dq$$

where the density of s, given by equation (4.4), represents the posterior assessment of the CI-agent after he has received information through the system λ_2.

Differentiating the above condition with respect to λ_2, we obtain:

$$-u''(e - z^*) \frac{\partial z^*}{\partial \lambda_2} = \alpha\beta \int_0^1 \int_0^1 u''(\alpha y(z^*)\, sq)\, \alpha \{y'(z^*)\, sq\}^2 \frac{\partial z^*}{\partial \lambda_2} g_{\lambda_2}(s)\, h(q)\, ds\, dq$$
$$+ \alpha\beta \int_0^1 \int_0^1 u'(\alpha y(z^*)\, sq)\, y''(z^*) \frac{\partial z^*}{\partial \lambda_2} sq\, g_{\lambda_2}(s)\, h(q)\, ds\, dq$$
$$+ \alpha\beta \int_0^1 \int_0^1 u'(\alpha y(z^*)\, sq)\, y'(z^*)\, sq\{f(s) - g(s)\}\, h(q)\, ds\, dq.$$

Therefore, it is seen that $\partial z^* / \partial \lambda_2 > 0$ if and only if the inequality holds:

$$\int_0^1 \int_0^1 u'(\alpha y(z^*)\, sq)\, y'(z^*)\, sq\{f(s) - g(s)\}\, h(q)\, ds\, dq > 0.$$

Since this inequality can be written as:

$$\int_0^1 \int_0^1 u'(\alpha y(z^*)\, sq)\, \alpha y(z^*)\, sq\{f(s) - g(s)\}\, h(q)\, ds\, dq > 0,$$

Proposition 3.3 yields that $\partial z^*/\partial\lambda_2 > 0$ for all expectations $g \lesssim f$ and h if and only if the function $tu'(t)$ is increasing, that is, the relative risk-aversion of the CI-agent is less than one. The proof of Theorem 4.4 is now completed.

From Theorem 4.4 and Theorem 2.1, we know that $\partial z^*/\partial\lambda_2 > 0$ if and only if the R.R.A. of CI < 1 and $\partial V^*/\partial z > 0$. It follows that the expected utility of the PM-agent increases with information released if and only if the relative risk-aversion of the CI-agent is less than one. Under such circumstances a reduction of information asymmetry has positive social value (subject to cost) in that everyone's position is improved.

C. Information as a Monitoring Device

In addition to the productive uncertainty S, the CI-agent faces another source of uncertainty resulting from the utility maximizing behavior of the PM-agent. The PM-agent is assumed to draw utility not only from his contractual share $(1 - \alpha)$ of the produced output, but also from his consumption of the invested resource in the form of perquisites. To the extent that this behavior is expected by the CI-agent, he may discount such activity in terms of reduced valuation of the production process. The PM-agent may choose to produce information which effectively restricts his range of consumption alternatives if, by doing so, he can influence the valuation of the CI-agent.

Assume that such a control system is indexed by the parameter ρ, where $0 \leqslant \rho \leqslant 1$. Here, ρ is meant to indicate the effectiveness of monitoring. Assume that, as a result of employing control system ρ, the posterior assessment of the CI-agent regarding utilization of the invested resource by the PM-agent has the form:

$$h_\rho(q) = \begin{cases} \left\{\int_\rho^1 h(q)\,dq\right\}^{-1} h(q) & \text{for } \rho \leqslant q \leqslant 1 \\ 0 & \text{otherwise.} \end{cases} \tag{4.5}$$

If $\rho = 0$, the system is totally ineffective. As the value of ρ approaches one, the support of $h_\rho(q)$ shrinks, and the CI-agent is more certain that the PM-agent will allocate more of the invested resource to the production process as input. The term $[\int_\rho^1 h(q)\,dq]^{-1}$ serves to insure that $h_\rho(q)$ is a properly defined density function.

Given this revision process, we now inquire as to the effect of monitoring on the behavior of the CI-agent.

THEOREM 4.5. *The expected utility of the CI-agent is an increasing function of effectiveness, ρ, of the monitoring system.*

Proof. As we have postulated, the expected utility of the CI-agent is given by:

$$U^* = \int_0^1 \int_0^1 \{u(e - z^*) + \beta u(\alpha y(z^*)\, sq)\}\, g(s)\, h_\rho(q)\, ds\, dq.$$

Since the function $u(e - z^*) + \beta u(\alpha y(z^*)\, sq)$ is increasing with q, it suffices to show from Proposition 3.3 that $h_{\rho_1} \gtrsim h_{\rho_2}$ if $0 \leqslant \rho_2 \leqslant \rho_1 \leqslant 1$.

Observe that:

$$h_{\rho_1}(q) - h_{\rho_2}(q) = \begin{cases} -h_{\rho_2}(q) & \text{for } 0 \leqslant q \leqslant \rho_1 \\ (A_1^{-1} - A_2^{-1})\, h(q) & \text{for } \rho_1 \leqslant q \leqslant 1 \end{cases}$$

where

$$A_i = \int_{\rho_i}^1 h(q)\, dq$$

for $i = 1, 2$. Since $\rho_1 \geqslant \rho_2$ implies $A_2 \geqslant A_1$, it is seen that:

$$\int_0^r \{h_{\rho_1}(q) - h_{\rho_2}(q)\}\, dq \leqslant 0$$

for all $0 \leqslant r \leqslant 1$. Therefoıe, we conclude that $h_{\rho_1} \gtrsim h_{\rho_2}$ for $\rho_1 \geqslant \rho_2$. This completes the proof.

THEOREM 4.6. *As the monitoring system becomes more effective, the CI-agent invests more into the production process for all expectations $g(s)$ and $h(q)$ if and only if the relative risk-aversion of the CI-agent is less than one.*

Proof. In order to avoid unnecessary complications, we shall provide a proof under the simplifying assumptions that the CI-agent perceives no uncertainty regarding the random variable s and his belief $h(q)$ on the sole uncertainty q is continuous everywhere.

Effectively, the decision model (4.1) of the CI-agent is reduced to:

$$U^* = \max_{0 \leqslant z \leqslant e} \int_0^1 \{u(e - z) + \beta u(\alpha y(z)\, q)\}\, h_\rho(q)\, dq$$

and its optimality condition will now have the form:

$$u'(e - z^*) = \alpha\beta \int_0^1 u'(\alpha y(z^*)\, q)\, y'(z^*)\, q h_\rho(q)\, dq,$$

where the density function $h_\rho(q)$ is defined by equation (4.5). Let us rewrite this optimality condition in the form:

$$u'(e - z^*) \int_\rho^1 h(q)\, dq = \alpha\beta \int_\rho^1 u'(\alpha y(z^*)\, q)\, y'(z^*)\, q h(q)\, dq \qquad (4.6)$$

Differentiating the above with respect to the parameter ρ, we obtain:

$$-u''(e - z^*) \frac{\partial z^*}{\partial \rho} \int_\rho^1 h(q)\, dq - u'(e - z^*)\, h(\rho)$$
$$= \alpha\beta \int_\rho^1 u''(\alpha y(z^*)\, q)\, \alpha\{ y'(z^*)\, q\}^2 \frac{\partial z^*}{\partial \rho} h(q)\, dq$$
$$+ \alpha\beta \int_\rho^1 u'(\alpha y(z^*)\, q)\, y''(z^*)\, q \frac{\partial z^*}{\partial \rho} h(q)\, dq$$
$$- \alpha\beta\, u'(\alpha y(z^*)\, \rho)\, y'(z^*)\, \rho h(\rho).$$

One can now show that $\partial z^*/\partial \rho > 0$ for all h if and only if:

$$u'(e - z^*) - \alpha\beta u'(a y(z^*)\rho) y'(z^*)\rho > 0 \tag{4.7}$$

for all h. Combining expressions (4.6) and (4.7), we then conclude that $\partial z^*/\partial \rho > 0$ for all h if and only if:

$$\int_\rho^1 u'(\alpha y(z^*)\, q)\, q h(q)\, dq - u'(\alpha y(z^*)\, \rho)\, \rho \int_\rho^1 h(q)\, dq > 0 \tag{4.8}$$

for all h.

On the other hand, the integration by parts yields that:

$$\int_\rho^1 u'(\alpha y(z^*)\, q)\, q h(q)\, dq = \left[-u'(\alpha y(z^*)\, q)\, q \int_q^1 h(q)\, dq \right]_\rho^1$$
$$+ \int_\rho^1 \{u'(\alpha y(z^*)\, q) + u''(\alpha y(z^*)\, q)\, \alpha y(z^*)\, q\} \left\{ \int_q^1 h(q)\, dq \right\} dq$$
$$= u'(\alpha y(z^*)\, \rho)\, \rho \int_\rho^1 h(q)\, dq + \int_\rho^1 \{u'(\alpha y(z^*)\, q) + u''(\alpha y(z^*)\, q)\, \alpha y(z^*)\, q\}$$
$$\times \left\{ \int_q^1 h(q)\, dq \right\} dq.$$

This equation together with (4.8) shows that $\partial z^*/\partial \rho > 0$ for all h if and only if:

$$\int_\rho^1 \{u'(\alpha y(z^*)\, q) + u''(\alpha y(z^*)\, q)\, \alpha y(z^*)\, q\} \left\{ \int_q^1 h(q)\, dq \right\} dq > 0$$

for all h. Since the latter is seen to be equivalent to $u'(t) + t u''(t) > 0$, we have completed the proof.

It is clear that the CI-agent prefers monitoring information. However, it is never to the PM-agent's advantage to agree to such a device when the relative risk aversion of the CI-agent is greater than one. Since the decision is made by the PM-agent, this is an important result. Of course, the CI-agent would prefer $\rho = 1$, in which case the PM-agent would consume no perquisites. However, the PM-agent would never allow such an effective monitoring device to be used. He is now subject to the additional constraint:

$$x \in \{x \mid y(x) \geqslant \rho y(z)\}$$

due to the monitoring device. He will increase ρ only to the point at which the marginal utility of income [due to increased z (by Theorem 4.6)] is equal to the marginal utility derived from perquisite consumption (due to the restrictions on his input prerogatives). In the case where the CI-agent's relative risk-aversion is greater than one, the PM-agent will choose $\rho = 0$, since any increase in monitoring will decrease investment.

5. CONCLUSIONS

The private and social value of information production and distribution has been examined in many settings. In this paper, three distinct issues regarding information utilization have been analyzed, where the effect of information can be characterized in terms of stochastic dominance of the resulting probability density functions. In general, if the utility functions of the agents in the economy are characterized by appropriate risk-taking characteristics (in particular, relative risk-aversion less than one), information choice by the producer-manager is seen to benefit all agents. Such benefits may derive from (1) the production implications of the information, (2) reduction of informational asymmetry between agents, or (3) reduction of perquisite consumption by the producer-manager. Of course, the ultimate selection of an information production level depends crucially on the costs of the information.

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