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Peer learning in an undergraduate linear Algebra course - A social network analysis

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Peer Learning in an Undergraduate Linear Algebra Course - A Social Network Analysis

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***Abstract*—This study employs Social Network Analysis (SNA) to explore peer learning behaviors among undergraduate Linear Algebra students. By examining the relational dynamics within the classroom, SNA unveils patterns of interaction, information flow, and collaboration among students. Our analysis identifies the prevalence and evolution of peer learning, and how it influences the students' academic performance. It also unveils the attributes of the students who engage in peer helping and the formation of small communities through such interactions. The findings of the study can provide valuable insights for educators aiming to enhance peer learning and improve educational practices in Linear Algebra and other fundamental subjects, thereby contributing to curriculum development and educational reforms.**

***Index Terms*—linear algebra, computer science, peer learning, affective skills, social network analysis**

I. Introduction

Linear Algebra is a critical subject for undergraduate computer science (CS) students [1] and plays a crucial role in various aspects of modern technology. It equips students with the necessary foundations for advanced numerical methods in computing, particularly in machine learning and data analytics, making it highly relevant for CS education.

Traditionally, linear algebra courses are taught in a classroom setting using direct instruction. This lecture-based approach, where instructors guide students through the course material [2], [3], remains prevalent in universities. It emphasizes the direct transmission of knowledge from instructor to passive learners. Assessment methods primarily rely on tests and exams, encouraging students to memorize information rather than fostering creative and critical thinking.

In contrast, peer learning, also known as collaborative learning or group work, involves active engagement among students to solve problems, discuss concepts, and learn from each other. Peer learning has emerged as an effective strategy in mathematics courses, promoting active learning, deeper understanding, and the development of essential skills [4]–[6].

Several larger-scale sociometric studies [7]–[9] find that homophily, a tendency of people to develop affect for those who are similar to themselves, is also relevant in academic contexts: students of similar levels of academic performance have been shown to be more likely to develop friendships.

This research employs Social Network Analysis (SNA) and correlation analysis techniques to investigate and gain insights into patterns of peer learning behaviors among students enrolled in an undergraduate Linear Algebra course. By utilizing SNA, we aim to enhance our understanding of how student networks are formed within classrooms and the impact of these networks on students. These theoretical and methodological approaches provide valuable insights into pedagogy, equity, learning, and educational policy, addressing relevant questions in these areas.

In the remainder of the paper, we provide an overview of related work and outline our research questions. We then describe the current design of our course, contextual information about the cohort, the course itself, and the collected data. Next, we detail the methodology employed to analyze the data. Finally, we summarize the key takeaways and lessons learned from this research process.

II. Related Work

Peer learning, where students learn from their interactions with their classmates, has become a central theme [10] in several successful course designs. There is evidence to suggest that peer learning in mathematics enhances student interest in the subject [4]–[6], [11]. Peer learning is an aspect of the broader theme of affective skills development, which has been a recurring theme in pedagogical research [12].

Affect, in the context of affective learning or computing, refers to emotion or desire as an influencing behavior or action. Affective development is the process of developing emotional capacity, which involves the ability to experience, recognize, and express a wide range of emotions, as well as appropriately respond to emotional cues from others. Developing affective skills is crucial for Computer Science students as it has a significant correlation with their learning performance, alongside the development of cognitive skills [12]. Active interactions involving idea sharing and responsive feedback enhance students' cognitive skills, whereas supportive interactions that encourage and respect peers contribute to the development of affective skills [13]. In the classroom setting, it can be measured by collecting student responses as well as details about class participation. The former tells us about the students' state of mind [14], [15] and potential correlations with their performance, while the latter can give us indications about the development of their affective skills [15].

In our Linear Algebra course, which followed a flipped-classroom model (where peer learning may play a bigger role [16] than traditional, direct-instruction pedagogy in understanding the concepts), we set out to quantify the effect of peer learning on the development of the affective skills of our students. Our measurement is performed based on the self-reported class participation entries from the students.

In this paper, we use the self-reported entries of how the students participated in the class activities to measure the development of their affective skills, especially in terms of peer-learning. We will present our findings as answers to a few research questions. We expect our findings to give valuable insights to instructors designing courses with a view to developing the student affective skills, especially in mathematics courses for computer science.

III. Research Questions

Previous studies have explored the analysis of textual student reflections using text mining and machine learning techniques [17]–[19]. In contrast, our research takes a distinct approach by examining peer learning behaviors within students' reflections. Through the application of social network analysis (SNA) techniques, our study aims to identify instances of peer learning interactions among students and examine their correlation with overall course performance. To address these objectives, we pose the following research questions.

- **RQ1**: To what extent does peer learning occur in Linear Algebra classes?
- **RQ2**: How does the level of peer assistance evolve over the course of a semester?
- **RQ3**: What are the attributes commonly observed among students who actively engage in peer helping as donors?
- **RQ4**: What is the relationship between peer helping and the donors' performance in the course?
- **RQ5**: Do students who receive help typically rely on the same donors?

IV. Course Design and Context

The data collected to answer the research questions above come from an undergraduate course in Linear Algebra offered to the students of Computer Science at our school. It is considered a difficult course because Linear Algebra uses multiple systems of representations, such as algebraic and geometric views as well as abstract multi-dimensional spaces. For these reasons, Linear Algebra is challenging at the undergraduate level [20], [21], requiring a high degree of "cognitive flexibility."

A. Design Philosophy

In our school, Linear Algebra for Computer Science (LA4CS) is a mandatory course for undergraduate CS students. Currently, the course follows the flipped-classroom mode of instruction [22]. In this approach, students come to the classroom prepared, using resources such as the textbook and lecture videos provided, and actively engage in solving exercises to deepen their understanding, with guidance from instructors and teaching assistants. When implemented carefully, the flipped-classroom approach has the potential to enhance student learning [23], [24]. Studies have shown that most students prefer courses that implement this teaching method [25], and demonstrate improved learning outcomes [26] in the flipped classroom.

In a recent study, Liao et al. [27] emphasize the importance of active learning, such as peer instruction, problem-based or project-based learning, and flipped-classroom pedagogy, to enhance student retention rates in undergraduate CS programs. In our course design, we incorporated elements of active and collaborative learning [28] by introducing a mini-project aimed at fostering the development of affective skills.

B. Learning Outcomes

The LA4CS course aims to equip students with the necessary Linear Algebra skills for a successful career in computer science. Upon the successful completion of this course, students should be able to perform the following:

- Solve linear systems and find solutions using effective methods like Gaussian elimination or factorization.
- Test for linear independence, orthogonality, and compute matrix properties such as rank, determinant, and inverse.
- Visualize and compute fundamental spaces of matrices, understand their relation to linear equations, and find their dimensions and bases.
- Identify special matrix properties and compute matrix characteristics and their usage in computing applications.
- Compute eigenvalues, eigenvectors, and use them for diagonalization and solving advanced problems.
- Describe Singular Value Decomposition and Principal Component Analysis in data science algorithms.

C. Contextual Information

We had over 100 students in three sections for this LA4CS course. Each classroom session is three hours long and students have one class per week for a total of 12 classes over a 13-week term, with no instruction in Week 8. In the context of the flipped-classroom model, students are expected to come to class fully prepared by watching instructional videos and reviewing textbook chapter, which are made available approximately one week prior to the class session. The classroom time is then utilized for a quick recap, interactive quizzes, and in-class exercises (ICE).

To support students' learning, a teaching team consisting of the professor, an additional instructor, and a teaching assistant is available to offer assistance whenever required. After two hours of interactive work and guided practice, the remaining hour is dedicated to demonstrating solutions and addressing any remaining questions. The structure outlined above is followed for the initial ten weekly classes. The 11^{th} class is devoted to a mock-final exam, while the 12^{th} class is designated for project presentations.

D. Weekly Reflections & Class Participation

At the end of each weekly session, the students take a quiz as part of their continuous assessment. They also provide weekly reflections to share their learning experience with the instructors. These reflections are summarized and reviewed in the subsequent class. The suggestions are sentiments in the reflections provide valuable insights into how to improve or fine-tune the course delivery.

In addition, the students also report on their class participation during the session. As class participation is a graded component, carrying 7.5% of the overall course grade, we expect a high rate of reporting. It is the answers to the questions in this self-reported class participation that contain the data we analyze to quantify peer learning and affective skills development.

V. Methodology

The data used in our study originates from the students' self-reported entries of their class participation (CP). It is important to note that CP is a graded component in the LA4CS course, accounting for 7.5% of the overall course grade. The students enter their CP as responses to a series of questions in an untimed quiz on our e-learning platform. The questions relevant to our study are as follows:

- **Q2**: Did you help a friend? Response: Yes / No (Required)
- **Q3**: If you helped a friend, please enter the friend's name and the question/issue you helped with. Response: Textual (Optional)

In addition to the responses, the e-learning platform also records the name of the student entering the responses. In the following description of peer learning, we will refer to this student as the *donor*. The *recipient* of the peer-learning experience is identified in the response to the second question, **Q3**.

To quantify the effect of peer learning on student performance, we will use the score of the students in their final examination. In order to ensure the statistical validity of our quantification, we will be using the z-score of the final exam score instead of the raw score. This involves computing the mean and standard deviation for the entire cohort and using them to calculate the z-scores.

A. Data Preparation

Based on the responses to questions **Q2** and **Q3** above, we identify the *donor* and the *recipient* of peer help for each student, aggregated over the 10 weekly sessions. We then create a social network that represents peer help, resulting in a weighted directed graph. In this graph, each node represents a student, anonymized as a number in this paper. A student *donor* i is connected to a student *recipient* j with a directed edge from i to j, and the weight w_{ij} represents the number of instances of peer help over the 10 weeks (ranging from 0 to 10). We can build networks at both the cohort level and the section level.

One challenge we face is that donors may not always fully identify the recipient in **Q3**. Students often address their friends by their first names, occasionally including surnames or nicknames, or even making spelling mistakes. To address this issue, we create a list of the first names of all students and search each self-reported entry (referred to as the response) for **Q3** against this list. If no name is identified, we manually label that particular response.

We removed four students from the social network: two with the same first name and no specified surnames in the responses, and two who did not attempt the final exam. The donors of the removed students have a directed edge mapped to a dummy node, representing their donor activity. The dummy node serves as the recipient when not specified.

Out of the 981 responses submitted over the 10 weeks, we identify 841 instances of peer helping and 809 recipients (96%). However, 26 recipients could not be identified as the donor neglected to record their names, referring to them as "him" or "her". Additionally, six recipients were not identified due to naming conflicts with other recipients in the same class with the same first name.

During the analysis phase, for degree centrality analysis, we retain the dummy node in the graph as removing it and its edges would impact the degree centrality of the connected nodes. However, during community detection, we remove the dummy node as its presence would affect the optimal partitioning of nodes to maximize graph modularity. We refer to this node as "DUMMY" for the remainder of the analysis.

B. Analysis Methods

Using the social network graph depicted in Fig 1, we can analyze our research questions. Each node is represented by a circle, and its size corresponds to the level of peer helping provided as a donor, measured by the in-degree centrality of the node. The sizes of the arrows indicate the weighted in-degree and weighted out-degree centralities. To calculate the metrics in this study, we utilize both Gephi [29] and Python NetworkX [30].

1) Centrality Measures: Donor and Recipient Behavior: The weighted out-degree centrality of a student indicates the amount of peer help they have provided, while the weighted in-degree centrality represents the amount of peer help they have received. On the other hand, the unweighted out-degree centrality reflects the number of recipients of a student's help, and the unweighted in-degree centrality represents the number of donors from whom a student has received help. These centrality measures provide insights into the patterns of donor and recipient behavior within the social network.

2) Directed Network Reachability R_e: R_e measures the extent of information flow within the peer helping network. It represents the fraction of connected nodes in the directed social network of peer helping [31]. This metric provides insights into the efficiency of information dissemination and communication among students within the network.

$$R_e = \frac{\sum_{i,j \in V \text{ and } i \neq j} I(i,j)}{|V|^2 - 1}$$

where $I(i,j) = 1$ if there is a path from i to j and 0 otherwise; and V denotes the set of vertices in the graph.

R_e is initially calculated at the section level after removing the DUMMY node and its edges. The removal of the DUMMY node helps prevent overestimation as it tends to be better connected than other nodes. However, computing reachability solely at the cohort level would underestimate the information

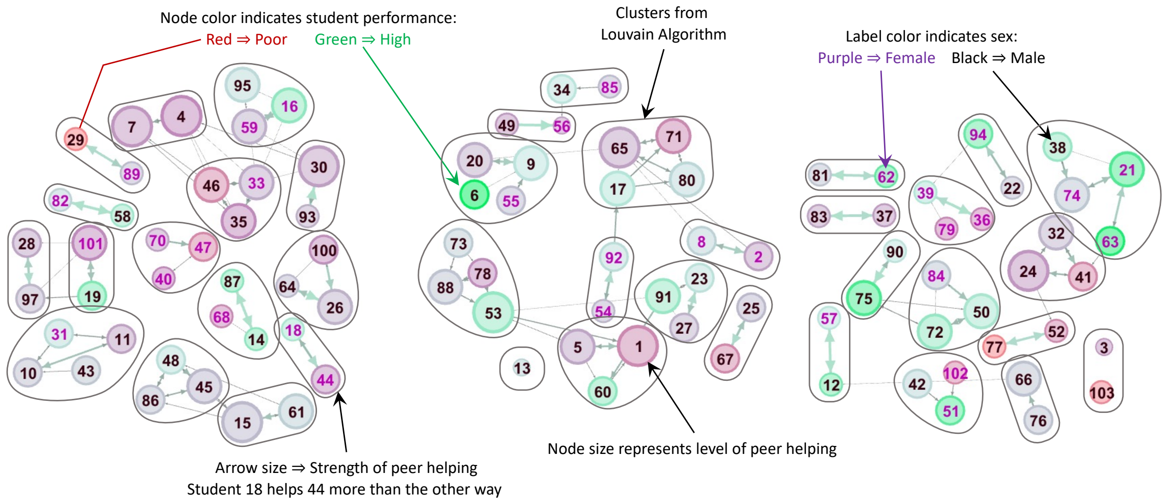

Figure 1: Cohort-level Social Network Representing Peer Helping: This is the state of the social network in Week 11, the last week of class.

flow since the class is divided into sections of approximately 30 students. To address this, the *Combined Reachability* for each week is obtained by taking the simple average of the section-level R_e values. This approach allows for a more accurate assessment of the overall information flow within the cohort.

3) Reciprocity in Peer Helping: At the cohort level, we examined the *unweighted reciprocity*, which measures the proportion of donor-recipient pairs that switch roles at least once in the recorded responses. In this analysis, the DUMMY node was retained since network reciprocity can occur between a removed student and a student currently included in the analysis. The difference in edge weights between a donor-recipient pair quantifies the *peer helping asymmetry*. These measures provide insights into the reciprocity patterns and the presence of true donors within the peer helping network.

4) Community Detection: Louvain's Algorithm [32] is a community detection algorithm that aims to identify clusters within a network. It achieves this by promoting strong connections within communities and discouraging connections between different communities [33]. The algorithm maximizes a measure called *modularity*, which quantifies the extent of community structure in a network [34]. A modularity value greater than or equal to 0.3 is often considered indicative of a significant community structure. The DUMMY node is excluded during the cluster generation process.

By examining the resulting clusters, we can distinguish between *inter-cluster peer-helping*, which occurs when a student from one cluster helps a student from a different cluster, and *intra-cluster peer-helping*, which involves interactions between students within the same cluster. This distinction allows us to gain insights into the patterns of peer-helping behavior within and between communities.

5) Gender and Outsider Analysis: In our analysis, we consider two control variables: *Gender* and *Outsider Status*. Among the students in our cohort, there are 67 male students and 32 female students. Since the LA4CS course is a mandatory course in the Computer Science curriculum, the majority of students taking this course belong to the Computer Science degree program and are in their second year of study. We refer to this group of students (82 in total) as the *CS Majority*. These students are likely to be more familiar with each other due to shared coursework and academic experiences.

In addition to the *CS Majority*, we have 17 students who do not belong to the Computer Science degree program or are not in their second year of study. These students are considered *outsiders*. They may have less prior interaction with the majority group and could be perceived as outsiders within the cohort.

By including these control variables in our analysis, we can explore potential variations in peer-helping behaviors based on gender and insider-outsider dynamics within the cohort.

6) Final-Exam Scores: To measure the students' performance in the course, we utilize their final-exam scores as the dependent variable in our analysis ($\mu = 58.7$, $\sigma = 13.9$). In order to compare and analyze the scores on a standardized scale, we normalize the raw final-exam scores to obtain the corresponding z-scores. After normalization, students fall within the range $-3 < z_{\text{finals}} < 3$, as expected.

VI. Findings & Discussions

A. ***RQ1****: To what extent is peer learning occurring in the Linear Algebra class?*

Out of the 981 responses collected over the 10-week duration of the class, 841 responses (86%) indicate instances of peer helping. This high percentage suggests that our approach to promoting peer learning in a math course, which is typically focused on individual knowledge acquisition through lecture-based instruction, has been successful. Several factors contribute to this success:

- Students sit close to their peers in face-to-face classes, promoting efficient communication.

- Interactive technology tools like WooClap keep students engaged, encouraging them to learn from each other.
- The flipped-classroom approach encourages preparedness and group effort before the classroom session.
- In-class exercises require students to work together, and solve problems, creating opportunities for peer learning.

B. ***RQ2***: *How does the level of peer assistance evolve over the course of the semester?*

We analyze **RQ2** by examining unweighted network reciprocity and reachability over time. Upon examining the responses over a 10-week period, we find that cohort-level unweighted network reciprocity experiences an increase from 63.0% to 75.5% over the course of 10 weeks. This increase occurs rapidly from Week 1 to Week 6 (slope $\beta = 0.025$), and then remains relatively stable from Week 6 to Week 11 ($\beta = -0.002$), as depicted in Fig 2.

The effectiveness of the class social network is evaluated based on the measure of directed network reachability. As illustrated in Fig 2, the reachability initially increases until week 5 ($\beta = 0.017$) before stabilizing ($\beta = 0.002$). The combined reachability shows an increment from 0.004 in week 1 to 0.124 at the conclusion of the 10-week class.

At the cohort level, we monitor the modularity of the network and the average cluster size each week. The modularity consistently remains high throughout the weeks of the class ($\beta = 0.001$, $\mu = 0.870$, $\sigma = 0.006$). There is no significant increase in modularity from Week 2 (0.866) to Week 11 (0.870)[1]. The average cluster size also demonstrates relative stability, increasing from 2.10 in Week 1 to 2.45 in Week 11 ($\beta = 0.037$). These findings align with our main observations of high network reciprocity and low combined reachability, indicating a high modularity and small average cluster size.

Reciprocity is a critical factor in maintaining stable social networks as it involves a mutual alignment of feelings and perspectives among interconnected nodes [35]. One-sided relationships are prone to fade away or transform into mutual connections [36]. In the context of the Linear Algebra course, approximately 75% of the connections exhibit reciprocity, indicating that students who receive assistance are highly likely to contribute back. This reciprocal interaction plays a significant role in fostering network stability.

Reachability, on the other hand, refers to the extent to which nodes in a network can connect with each other. In the Linear Algebra course, the overall network reachability is relatively low, indicating limited connectivity between nodes. The analysis reveals that students tend to interact within specific groups when seeking or providing help, resulting in disconnected communities. To promote diversity and inclusivity, the instructor can leverage weak ties, which have higher reachability, to facilitate connections and expand information dissemination and influence within the network.

C. ***RQ3***: *What are the attributes commonly observed among students who actively engage in peer helping as donors?*

In **RQ3**, we explore the influence of gender and outsider status on peer-helping behavior. We also analyze the differences between intra-cluster and inter-cluster instances of peer helping using the Louvain Algorithm. The summarized results are presented in Table I.

Overall, male students engage in more peer helping compared to female students, with a mean difference of $\Delta = 1.26$ instances. The Cohen's d-statistic is $d = 0.423$, and the significance level is $p = 0.021$. However, when considering inter-cluster and intra-cluster peer helping separately, the gender difference in peer helping behavior becomes non-significant. Furthermore, we observe that male students receive help from other clusters more frequently than female students. They also receive more intra-cluster help, although this difference is not statistically significant. In summary, male students are more frequent recipients of peer helping, but the difference is not statistically significant. These findings are presented in Table I, specifically in sections A to C under Gender Comparisons.

As shown in the distributions in Fig 3, we find that outsiders on average have significantly fewer distinct donors ($\Delta = 1.002$, $d = 1.009$, $p = 0.0008$), and recipients ($\Delta = 0.913$, $d = 1.037$, $p = 0.0002$) of their peer help in comparison to the CS students, with both findings statistically significant (better than 1%). Additionally, we also find that outsiders have a significantly smaller total instances of peer help given ($\Delta = 1.97$, $d = 0.581$, $p = 0.005$) and received ($\Delta = 2.52$, $d = 0.584$, $p = 0.011$), compared to their CS counterparts. (See Table I: Section A, under Outsider

[1]In week 1, communities could not be identified due to insufficient edges between nodes

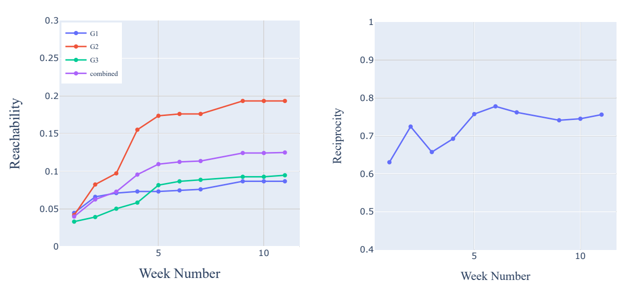

Figure 2: [**RQ2**] Network Reachability (Left) and Unweighted Network Reciprocity (Right) over time

Figure 3: [**RQ3**] Distributions of in-degree and out-degree of student outsiders in comparison to the CS majority

Table I: [**RQ3**, **RQ4**] Comparisons and Correlation Analyses of Peer Learning Behavior

		Overall	Gender Comparison [**RQ3**]			Outsider Status Comparison [**RQ3**]				Correlation Analysis [**RQ4**]				
			Male	Female	$d\,(p)$	Outsider	CS	$d\,(p)$		All	Male	Female	CS	Outsiders
Section A:	**Analysis of Overall Peer Helping – Average Numbers of Instances, Distinct Donors and Recipients**													
Given	μ	8.101	8.507	7.250	0.423	6.471	8.439	0.581	r	0.224	0.095	0.485	0.200	0.291
	σ	2.873	2.543	3.350	(0.021)	4.125	2.440	**(0.005)**	p	0.0256	0.444	**0.005**	0.071	0.257
Received	μ	7.848	8.179	7.156	0.248	5.765	8.280	0.584	r	0.078	-0.013	0.307	-0.006	0.360
	σ	4.168	4.239	3.993	(0.128)	4.63	3.961	(0.011)	p	0.444	0.916	*0.088*	0.956	0.156
# Donors	μ	1.888	2.030	1.594	0.369	1.294	2.207	1.037	r	0.031	0.035	0.064	-0.054	0.639
	σ	1.220	1.255	1.103	(0.048)	0.772	0.978	**(0.000)**	p	0.757	0.780	0.727	0.632	**0.006**
# Recipients	μ	2.051	2.224	1.688	0.572	1.059	2.061	1.009	r	0.063	0.029	0.227	-0.025	0.490
	σ	1.004	1.042	0.821	**(0.006)**	0.659	1.241	**(0.001)**	p	0.535	0.815	0.211	0.824	0.045
Section B:	**Inter-Cluster Peer Helping Analysis – Average Number of Instances**													
Given	μ	0.758	0.866	0.531	0.246	0.235	0.866	0.494	r	0.149	0.158	0.169		
	σ	1.457	1.526	0.859	(0.144)	0.970	1.521	*(0.052)*	p	0.141	0.202	0.355		
Received	μ	0.758	0.971	0.313	0.531	0.294	0.853	0.471	r	-0.020	0.040	-0.173		
	σ	1.457	1.526	0.859	(0.013)	0.848	1.450	*(0.064)*	p	0.844	0.748	0.343		
Section C:	**Intra-Cluster Peer Helping Analysis – Average Number of Instances**													
Given	μ	6.879	7.045	6.531	0.166	5.824	7.098	0.370	r	0.084	-0.099	0.476		
	σ	3.071	3.057	3.121	(0.220)	3.957	2.835	*(0.060)*	p	0.407	0.425	**0.006**		
Received	μ	6.879	7.060	6.500	0.145	5.471	7.171	0.425	r	0.060	-0.070	0.361		
	σ	3.071	3.946	3.767	(0.250)	4.230	3.764	*(0.050)*	p	0.555	0.572	0.043		

Note: The amount of peer help given [received] is the weighted out-degree [in-degree] centrality. The number of distinct donors [recipients] is the unweighted in-degree [out-degree] centrality. Each comparison shows the difference as the Cohen's d-statistic d (and the p-value derived from a one-tailed independent samples Student's T-test). The correlations are calculated from Pearson correlations of the following degrees of freedom (df): $df_{(\text{All})}$ = 97, $df_{(\text{Male})}$ = 65, $df_{(\text{Female})}$ = 30, $df_{(\text{Majority})}$ = 80, $df_{\text{Outsiders}}$ = 15. The p-values are annotated with statistical significance stronger than *10% in italics*, 5% underlined, and **1% in bold**.

Status Comparison.) These findings highlight the disparities in peer-helping behavior between outsiders and CS students, indicating that outsiders are less involved in the peer-helping network.

*D. **RQ4**: What is the relationship between peer helping and the donors' performance in the course?*

In addressing **RQ4**, we examine the relationship between peer-helping behavior and final exam performance, considering gender and outsider status. As shown in Fig 4, at a cohort level, we find a positive correlation between amount of peer help given and final-exam performance with Pearson correlation $r = 0.224$, slope $\beta = 0.078$ and p-value $= 0.0256$.

The correlation between the amount of peer help given and final-exam performance varies based on gender. For female students, there is a strong positive correlation between the frequency of peer helping and final-exam performance

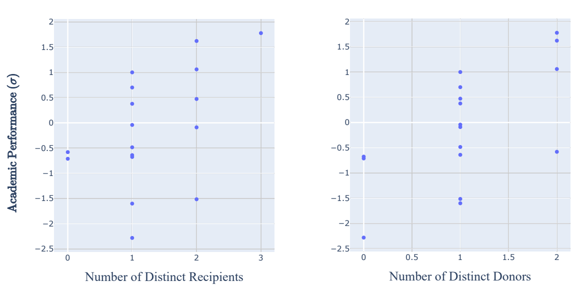

Figure 4: [**RQ4**] Outsiders Analysis: Relationship between the number of unique donors (left) and number of unique recipients (right) with final-exam performance

($r = 0.485$, $\beta = 0.147$, $p = 0.005$). However, for male students, no significant correlation is observed ($r = 0.095$, $\beta = 0.037$, $p = 0.444$). It is worth noting that, as shown in **RQ3**, males generally provide more help than females (Table I: Gender Comparison), but their peer-helping frequency has no observable relationship with their academic performance.

In summary, our results suggest that the amount of peer help given is positively associated with final exam performance at the cohort level. Furthermore, the relationship between peer-helping frequency and academic performance differs between male and female students. In terms of final-exam performance, female students tend to help more only if they are better at the subject.

Examining the correlation between the number of distinct recipients assisted by students and their final-exam performance reveals no significant relationship ($r = 0.078$, $\beta = 0.019$, $p = 0.444$). Analyzing by gender, female students exhibit a slightly positive yet insignificant correlation ($r = 0.227$, $\beta = 0.282$, $p = 0.211$), whereas male students show no correlation ($r = 0.029$, $\beta = 0.028$, $p = 0.815$). Controlling for the outsider status, a statistically significant positive correlation emerges between the number of distinct recipients and final-exam performance ($r = 0.49$, $\beta = 0.721$, $p = 0.045$), excluding students in the CS Majority, who demonstrate no correlation ($r = -0.025$, $\beta = -0.025$, $p = 0.824$). Refer to Table I: Section A for the detailed results of the correlation analysis.

The number of distinct donors does not show a significant correlation with final-exam performance ($r = 0.031$, $\beta = 0.026$, $p = 0.757$). However, when considering outsider status exclusively, a strong positive correlation emerges between the number of distinct donors and course performance ($r = 0.639$, $\beta = 1.101$, $p = 0.005$), as indicated in the last column of Table I: Section A.

Examining the relationship between final-exam performance and *inter*-cluster peer help, no significant correlations are observed at the cohort level or when analyzing by gender, as presented in Table I: Section B, under Correlation Analysis. However, among female students, a positive correlation is found for *intra*-cluster peer help ($r = 0.476$, $\beta = 0.156$, $p = 0.006$). Interestingly, for female students, higher academic performance is associated with receiving more peer help ($r = 0.361$, $\beta = 0.098$, $p = 0.043$), suggesting that academically inclined female students are more likely to seek and receive assistance from their peers. Refer to Table I: Section C for detailed results of the correlation analysis.

Among the 32 students who reported helping peers from different clusters over 11 weeks, we observe a positive but nonsignificant relationship between the frequency of inter-cluster peer help and final-exam scores ($r = 0.149$, $\beta = 0.103$, $p = 0.141$). This trend persists even after controlling for gender, as depicted in Table I: Section B.

In terms of comparisons between inter-cluster and intra-cluster peer help, students with outsider status generally exhibit lower levels of peer help given or received, as demonstrated in Table I: Sections B & C, under Outsider Status Comparison. Unfortunately, due to the limited number of students, we do not have enough data to split them into CS students and outsiders for correlation analysis. Therefore, the bottom-right portion of Table I is left blank.

Our observations reveal an interesting pattern where high-achieving students are not always the ones who act as donors and assist their peers. This finding suggests that the motivation behind helping a classmate in this context may not necessarily be to enhance the recipient's knowledge or skills. Instead, it is possible that the donor is seeking clarification or reaffirming their own understanding of a specific concept. In this case, offering assistance could be seen as a way to provide emotional support rather than purely academic guidance.

Considering that peer helping can be influenced by homophily, the number of donors that an outsider has can serve as a measure of their assimilation into the CS majority. Building trust within a short timeframe would require the outsider to demonstrate diligence and effort independently, which could directly correlate with their academic ability.

E. ***RQ5****: Do students who receive help typically rely on the same donors?*

For **RQ5**, we analyze the number of distinct donors for each student on a weekly basis throughout the course. This metric corresponds to the student's unweighted in-degree centrality at each particular week. In Fig 5, the blue line represents the weekly average, while the vertical bars indicate the 25^{th} and 75^{th} percentiles, showing the maximum number of distinct donors for 25% and 75% of the class, respectively.

We observe that the blue line initially exhibits an increasing trend, indicating a rise in the number of distinct donors.

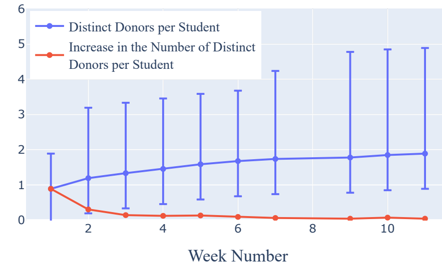

Figure 5: [**RQ5**] Change in number of distinct donors per student over time, with

However, by Week 3 of the course, the increase in the number of distinct donors (presented as the red line) drops to near zero. This suggests that most students start seeking help from a limited set of donors, leading to a decrease in the diversity of donors over time.

On average, each student has approximately 1.89 distinct donors ($\sigma = 1.22$). Furthermore, each donor provides peer help around 4.13 times on average ($\sigma = 2.95$) over the span of 10 weeks. It is interesting to note that 70.87% of the cohort relies on a single donor for more than 50% of their peer help instances. This finding indicates that as the course progresses, students tend to form closer connections with specific peers who consistently provide them with assistance, resulting in a smaller group of individuals on whom they rely.

A possible explanation for this pattern is that students typically select their preferred seats during the first week of the term and tend to maintain those seats in the following weeks. As a result, a seating arrangement is established where students consistently have the same neighboring classmates. In such circumstances, students naturally develop connections with those sitting in close proximity to them. They are unlikely to intentionally relocate within the classroom to seek assistance from unfamiliar individuals. This seating arrangement may contribute to the formation of closer connections as well as promoting reciprocity (discussed in **RQ2**) with specific peers throughout the course.

VII. Limitations & Future Work

There are a few limitations to consider in this study. Firstly, the engagement in peer learning was voluntary, and students were incentivized through class participation marks. This may have influenced their responses and introduced bias. Future studies could explore the impact of mandatory peer learning activities on student engagement and outcomes.

Another limitation is the relatively small sample size of around 100 students over a single term. To enhance the generalizability of the findings, collecting responses from a larger number of students across multiple terms would be beneficial. This would provide more robust statistics and allow for a more comprehensive analysis.

It is important to note that this study focused on a course that implemented the flipped-classroom pedagogy, which inherently promotes collaborative learning and peer helping.

The findings may not be directly applicable to courses with different instructional approaches. Future research could investigate the influence of instructional methods on peer learning dynamics and outcomes.

VIII. Conclusions

In conclusion, this article presented an analysis of student responses regarding their engagement in peer learning activities. The findings reveal a significant level of peer help, with up to 86% of students participating in such activities. Over time, the frequency of peer helping tends to increase and stabilize. Variations in the extent of peer helping and its impact on academic performance were observed, depending on factors such as gender and students' sense of affiliation to the larger cohort. Notably, students tended to seek help and provide assistance within their own community.

The insights gained from this study can inform the design of courses and educational interventions to enhance student interactions, peer helping, and collaborative learning. Further research can build upon these findings to explore the effectiveness of different instructional strategies and the implications for student outcomes in peer learning contexts.

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