The heterogeneous vehicle routing problem with multiple time windows for the e-waste collection problem

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Abstract— **Waste from electrical and electronic equipment (WEEE) or e-waste describes end-of-life electronic products that are discarded. Due to their toxic and negative impacts to humans' health, many publications have been proposed to handle, however, studies related to e-waste collection and transportation to waste disposal sites are not widely studied so far. This study proposes a mixed integer linear programming (MILP) model to solve the e-waste collecting problem by formulating it as the heterogeneous vehicle routing problem with multiple time windows (HVRPMTW). The model is validated with newly developed benchmark instances that are solved by commercial software, CPLEX. The model is also adopted for solving a real case study in the context of Singapore. The results show that the proposed mathematical model is a good start for formulating and solving the problem with reasonable problem sizes. From a managerial perspective, this offers significant practical improvements.**

I. INTRODUCTION

The fast development, mass production, and cost advantages of households appliances and personal technologies have changed the lives of many people, but left many health issues and environmental impacts. Waste from electrical and electronic equipment or e-waste describes end-of-life electronic products that are discarded. It covers many electronic devices, ranging from large household appliances and technologies equipment to consumer electronics. In short, it is the remaining waste after the usage life cycle of a technology elapses and is unintentionally reused or recycled [1].

E-waste can be classified into various types including new and functioning electrical and electronic equipment (EEE), used and functioning EEE suitable for direct reuse, used and non-functioning but repairable EEE, used and non-functioning but non-repairable EEE, and waste electrical and electronic equipment (WEEE) with various hazardous exposure of e-waste components to health [2]. The main types of e-waste and their impacts on health by top e-waste generating countries can be found in [3]. For example, some European Union countries classify e-waste as household appliances, lighting equipment, and electrical and electronic tools while some Asian countries, such as China and Japan, consider e-waste as televisions, refrigerators, washing machines, or air conditioners. The negative side effects on the human body can be listed as psychological and neurological abnormal functions, the spreads of cancers, weaker immune systems, and damage at the molecule and cell levels [3]. Compared to e-waste related policy studies, impacts on health, or e-waste recycling operations, the collection and transportation of e-waste, often referred to reverse logistics or e-waste closed-loop supply chain, has caught scant attention from scholars [4][5]. Thus, our work fills the gap in the literature of e-waste collection problems by formulating it as a vehicle routing problem.

In Singapore, e-waste is collected by ALBA, a government-funded corporation with e-waste management and recycling operation [6]. This corporation started in 2021 under a 5-year period contract. ALBA offers 2 choices for collecting e-waste: doorsteps and e-waste collection points (or e-bins). For the former option, customers who want to dispose of the e-waste have to pay ALBA a small compensation, whereas it is cost-free for the latter option. In this work, we provide a schedule that minimizes the total cost for a whole working horizon with e-waste collecting demands or requests that are known in advance. This paper explores the problem by integrating the ordinary vehicle routing problem with multiple time windows (VRPMTW) and the extensively researched heterogeneous vehicle routing problem (HVRP) found in the VRP literature [7] [8] [9]. Thus, we phrase it as the heterogeneous vehicle routing problem with multiple time windows (HVRPMTW). We consider a vehicle routing problem arises when e-waste must be collected by vehicles and sent to the depot.

The vehicle routing problem (VRP) is a well-known combinatorial optimization problem that assigns a set of vehicles to distribute or collect specific goods at a set of designated locations [10]. The practical variants of VRP studies include: capacitated VRP where vehicles with limited carrying capacity need to pick up or deliver items at various locations and VRP with time windows where customers have specific time ranges to be served. Until now, with technology expansions in various aspects, many variations of VRP have been proposed, such as multi-depot VRP (MDVRP), two echelon VRP (2EVRP), green VRP (GVRP), heterogeneous VRP (HVRP), multi-period VRP (MPVRP), and stochastic VRP[11].

In the context of adopting VRP for handling the e-waste problem, [12] proposed a case study in Poland and focused on two main problems: the truck loading problem with various e-waste dimensions with one fixed size container, and the capacitated VRP for transporting e-waste from a set of store

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locations back to the disassembly plant. [13] then developed an artificial intelligence (AI) model for solving HVRPTW in collecting e-waste using 4 metaheuristics, which include simulated annealing (SA), tabu search (TS), greedy algorithm, and bee colony. It is concluded that SA is the best algorithm for handling the problem. [14] introduced a multi-objective model for solving a multi-level green heterogeneous VRP in collecting and delivering e-waste in addition to job shop scheduling by using robotics in disassembling the waste. Moreover, [15] proposed a mathematical model for the general capacitated general routing problem with a time window and prioritized demand on waste pick-up and used Dijkstra's shortest path algorithm with tabu search to handle the problem. Other works can be found in [16].

The remaining sections of this paper are organized as follows. A full problem description and its mathematical model are given in Section II. We introduce a mixed integer linear programming (MILP) model where a set of different configurations of vehicles is adopted (HVRP) to collect e-waste at multiple locations with known quantities. Each location has a set of preferred time windows on different days (VRPMTW) within the planning zone. It is limited in our study that at each day each location only has one preferred time window with known demand. Each location can only be visited at most once during the actual operation, thus split collection is strictly prohibited in our work. The model is then tested and verified by solving some newly generated benchmark instances (or synthetic instances) that represent the e-waste scenario. The results obtained by commercial software are summarized in Section III. We also solve a real problem in the Singapore context. Finally. we discuss and conclude our work in Section IV.

II. PROBLEM DESCRIPTION

We describe the problem in more details in this section. We present and formulate the problem as a mixed integer linear programming (MILP) model. The proposed model can be extended or modified according to some other constraints of requirements.

A. Problem Definition

HVRPMTW is formulated as a complete directed graph $G(N, A)$ with a vertex set $N = \{0, 1, \ldots, n, e\}$ and an arc set $A = \{(i, j) : i, j \in N, i \neq j, i \neq e, j \neq 0\}$. In this graph, each arc (i, j) is bounded with travel time t_{ij}, and both nodes 0 and e represent the depots for vehicles. Note that both can be the same nodes.

Let $N^* = N \setminus \{0, e\} = \{1, \ldots, n\}$ be the set of nodes representing the e-waste collection points, while N^0 and N^e are the set of nodes without nodes 0 and e, respectively. Each node $i \in N^*$ has fixed demand d_i, service time s_i, and a set of time windows W_i. The set of waste collection points N^* is further classified by two categories or subsets: the households (doorstep collections) N^1 and the fixed e-bin collection points N^2.

Let $K = \{1, 2, \ldots, k\}$ be the set of available time periods (e.g., days) in the schedule. The set $W_i = \{w_i^1, w_i^2, \ldots, w_i^k\} = \left\{\left[l_i^1, u_i^1\right], \left[l_i^2, u_i^2\right], \ldots, \left[l_i^k, u_i^k\right]\right\}$ consists of a set of possible time windows for each day $k \in K$, where l_i^k and u_i^k are the earliest and the latest times for the visit at node i during time period k, respectively. Not all households are available every day and their availability in specific time period k is indicated by a binary parameter y_i^k. It means that the available time windows for the household $i \in N^1$ varies. In addition, the time windows of the e-bin collection points $\left[l_i^k, u_i^k\right] : i \in N^2, k \in K$ are fixed based on the operating hours of the depot $\left[l_0^k, u_0^k\right] \equiv \left[l_e^k, u_e^k\right]$.

E-waste is collected by a set of heterogeneous vehicles $V = V^1 \cup V^2$ in which the sets for large and small vehicles are denoted as V^1 and V^2, respectively. Each vehicle $v \in V$ in its respective set has a maximum capacity of Q^v and costs c^v per unit of time for operation. For collecting e-waste from household $i \in N^1$, a nominal fee p^i will be collected. On the other hand, collections at collection points do not incur any fee. We mimic the scenario that takes place in Singapore, as explained in Section I.

Each vehicle $v \in V$ departs from the depot on day $k \in K$ at time $t_0^{kv} : l_0^k \leq t_0^{kv} \leq u_{n+1}^k$ and reaches location $i \in N^*$ at time t_i^{kv} to collect e-waste. It then departs from the node at $t_i^{kv} + s_i$ by carrying the waste with a load of $L_i^{kv} \leq Q^v$. If a vehicle arrives at node $i \in N^*$ at time t_i^{kv} where $t_i^{kv} < l_i^k$ or $t_i^{kv} > u_i^k$, then a penalty cost of ε^v (\$/unit time) will be imposed for the penalty time of Δ_i^{kv}. In addition, the vehicles may arrive earlier; e. g., by idling an amount of time δ_i^v before approaching node $i \in N$ for collecting the e-waste, which costs them π^v (\$/unit time).

Each vehicle $v \in V$ departs from the depot $\{0\}$ as well as returns to the depot $\{e\}$ once only. Each location in the vertex set $i \in N^*$ can only be served by at most one vehicle. The entire travel time for vehicle $v \in V$ cannot exceed the maximum time per time period, which is $R = u_e^k - l_0^k$.

Figure 1 illustrates the HVRPTMTW problem. In this example, there are 15 nodes with e-waste to be collected in which 8 of them are the e-bin collection points and the rest are households. A set of time windows for each node is shown by 3 indices: the available day in the planning horizon, and the lower and upper bound values of the time window. Given the time periods in this example are $\{1, 2, \ldots, 6\}$, five vehicles are selected for collecting e-waste. Two of them are small ones, and the other three are large ones.

The route for each vehicle is as follows: $\{0 \rightarrow 13 \rightarrow 1 \rightarrow 16\}$: Route 1, $\{0 \rightarrow 8 \rightarrow 15 \rightarrow 14 \rightarrow 16\}$: Route 2, $\{0 \rightarrow 9 \rightarrow 2 \rightarrow 10 \rightarrow 16\}$: Route 3, $\{0 \rightarrow 7 \rightarrow 6 \rightarrow 12 \rightarrow 16\}$: Route 4, and $\{0 \rightarrow 5 \rightarrow 11 \rightarrow 4 \rightarrow 3 \rightarrow 16\}$: Route 5, respectively. Except for Routes 1 and 3, the other routes violate the households' available time windows in which the vehicles arrive earlier than expected in Route 2 (Node 15) and Route 5 (Node 11), while the vehicle in Route 4 arrives later at Node 12 than it should. Thus, in these routes, a penalty will be imposed. It is assumed that each household can only be visited once.

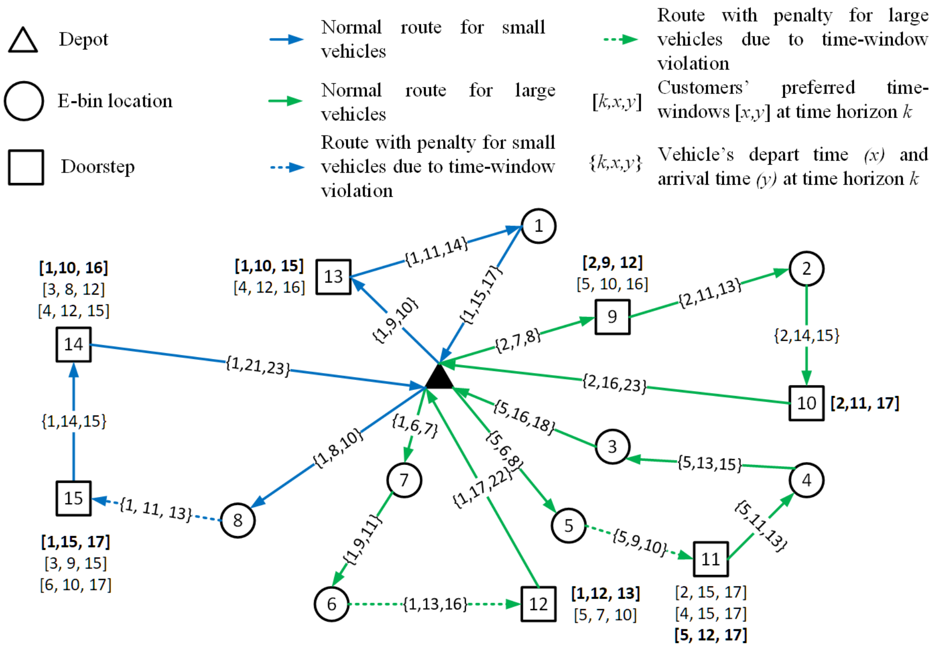

Fig. 1: An example illustrating HVRPMTW

Let binary variable X_{ij}^{kv} be 1 if and only if vehicle $v \in V$ visits node $j \in N \setminus \{0\}$ after node $i \in N \setminus \{e\}$ on day $k \in K$. B is a very large arbitrary constant value. $IJKV$ is the set of tuples $\langle i, j, k, v \rangle$ that has all feasible combinations of $i \in N^e, j \in N^0, k \in K$, and $v \in V$, while VIK is the set of tuples $\langle v, i, k \rangle$ that consists of all available combinations of $v \in V, i \in N, k \in K$. The following mathematical model is proposed in Section II-B.

B. Mathematical Model

$$min\Bigg(\sum_{\langle i,j,v,k\rangle \in IJKV}\Big(\big(t_{ij}+s_j\big)\times c^v + \delta_j^{kv}\times \pi^v + \Delta_i^{kv}\times \varepsilon^v\Big)\times X_{ij}^{kv}\Bigg) - \sum_{i\in N^1} p^i \tag{1}$$

The objective function 1 aims to minimize the total cost during the waste collecting operation, which includes the total operation cost based on the travel time between nodes and the service time at the associated nodes. It also includes the idle cost and penalty cost due to collecting e-waste that is outside the node's time window minus the total gains from the e-waste collection from households. Subject to

$$\sum_{k\in K}\sum_{v\in V}\sum_{i\in N^e} X_{ij}^{kv} = 1 \qquad \forall j \in N^* \tag{2}$$

$$\sum_{v\in V}\sum_{i\in N^e} X_{ij}^{kv} \le y_j^k \qquad \forall j \in N^*, \forall k \in K \tag{3}$$

$$\sum_{k\in K}\sum_{j\in N^0} X_{0j}^{kv} = 1 \qquad \forall v \in V \tag{4}$$

$$\sum_{k\in K}\sum_{i\in N^e} X_{ie}^{kv} = 1 \qquad \forall v \in V \tag{5}$$

$$X_{0j}^{kv} \le 1 \qquad \forall \langle 0, j, k, v \rangle \in IJKV : v \in V, k \in K \tag{6}$$

$$X_{ie}^{kv} \le 1 \qquad \forall \langle i, e, k, v \rangle \in IJKV : v \in V, \forall k \in K \tag{7}$$

$$\sum_{v\in V}\sum_{j\in N^0} X_{0j}^{kv} - \sum_{v\in V}\sum_{i\in N^e} X_{ie}^{kv} = 0 \qquad \forall k \in K \tag{8}$$

$$\sum_{k\in K}\sum_{i\in N^e} X_{ij}^{kv} - \sum_{k\in K}\sum_{h\in N^0} X_{jh}^{kv} = 0 \qquad \forall j \in N^*, \forall v \in V \tag{9}$$

$$\sum_{i\in N^e} X_{ij}^{kv} - \sum_{h\in N^0} X_{jh}^{kv} = 0 \qquad \forall \langle v, j, k \rangle \in VIK \tag{10}$$

Constraint 2 indicates that all customers have to be served. Constraint 3 ensures that each customer can only be served on the day when he or she is available. Constraints 4 to 7 make sure that each vehicle only departs and returns at respective depots at most once during the available time periods. Constraint 8 ensures that all departed vehicles return to the depot on the same day. Flow conservation constraints are given by Constraints 9 and 10.

$$L_j^{kv} - L_i^{kv} - Q^v \times \Big(1 - X_{ij}^{kv}\Big) \le d_j \qquad \forall \langle i, j, k, v \rangle \in IJKV \tag{11}$$

$$L_j^{kv} - L_i^{kv} + Q^v \times \Big(1 - X_{ij}^{kv}\Big) \ge d_j \qquad \forall \langle i, j, k, v \rangle \in IJKV \tag{12}$$

$$L_j^{kv} \le Q^v \times \sum_{i\in N^e} X_{ij}^{kv} \qquad \forall \langle v, j, k \rangle \in VIK \tag{13}$$

$$L_0^{kv} = 0 \qquad \forall v \in V, \forall k \in K \tag{14}$$

$$L_e^{kv} \le Q^v \times \sum_{j\in N^0} X_{0j}^{kv} \qquad \forall v \in V, \forall k \in K \tag{15}$$

Constraints 11 and 12 are the sub-tour elimination constraints for the vehicles' loads, which make sure that they pick-up the exact amount of waste from node $i \in N^*$. Constraint 13 shows that the loads of each vehicle cannot exceed its maximum capacity at any node i in the vertex set N. Constraint 14 initializes the load for each vehicle at Depot 0. Constraint 15 shows that the load of vehicle $v \in V$

at Depot e must not exceed its capacity.

$$t_j^{kv} - t_i^{kv} - s_i - \delta_j^{kv} - M \times \left(1 - X_{ij}^{kv}\right) \leq t_{ij} \\ \forall j \in N^*, \forall k \in K, \langle i,j,k,v \rangle \in IJKV \tag{16}$$

$$t_j^{kv} - t_i^{kv} - s_i - \delta_j^{kv} + M \times \left(1 - X_{ij}^{kv}\right) \geq t_{ij} \\ \forall j \in N^*, \forall k \in K, \langle i,j,k,v \rangle \in IJKV \tag{17}$$

$$\delta_i^{kv} \leq \left(u_e^k - l_0^k\right) \times X_{ij}^{kv} \qquad \forall v \in V, \forall k \in K \tag{18}$$

$$t_i^{kv} + t_{ie} + \left(u_e^k - l_0^k\right) \times \left(1 - X_{ie}^{kv}\right) \geq l_e^k \\ \forall v \in V, \forall k \in K, i \neq e \tag{19}$$

$$t_i^{kv} + t_{ie} \leq u_e^k + \left(u_e^k - l_0^k\right) \times \left(1 - X_{ie}^{kv}\right) \\ \forall v \in V, \forall k \in K, i \neq e \tag{20}$$

Constraints 16 and 17 are the sub-tour elimination constraints of the vehicles' $v \in V$ travel time in the respective time period $k \in K$. Constraint 18 bounds the waiting time at any location $i \in N^*$ to the maximum allowed operation time per time period. Constraints 19 and 20 ensure all vehicles return to the depot within the operation hour of Depot e.

$$\Delta_i^{kv} = \begin{cases} l_i^k - t_i^{kv}, & \text{if } l_i^k - t_i^{kv} > 0 \\ & \forall i \in N, \forall k \in K, \forall v \in V \\ t_i^{kv} - u_i^k, & \text{if } t_i^{kv} - u_i^k > 0 \\ & \forall i \in N, \forall k \in K, \forall v \in V \\ 0 & \text{otherwise} \\ & \forall i \in N, \forall k \in K, \forall v \in V \end{cases} \tag{21}$$

$$X_{ij}^{kv} \qquad \text{binary} \qquad \forall \langle i,j,k,v \rangle \in IJKV \tag{22}$$

$$L_i^{kv}, t_i^{kv}, \Delta_i^{kv} \qquad \text{non-negative} \qquad \forall v \in V, \forall i \in N \tag{23}$$

Constraint 21 calculates the penalty time if the vehicle arrives earlier than the nodes' time window lower bound or later than its upper bound, which is a non-linear constraint. Constraints 22 and 23 are the feasible region of the decision variables. To linearize constraint 21, we introduce constraints 24 - 30 and a set of decision variables that are λ_i^{kv}, μ_i^{kv}, θ_i^{kv}, and ϕ_i^{kv}. λ_i^{kv} and μ_i^{kv} are two binary decision variables indicating whether vehicle $v \in V$ arrives at node $i \in N^0$ earlier/later than the given time windows at day $k \in K$ or not. θ_i^{kv} and ϕ_i^{kv} are two non-negative decision variables for computing the earlier/later arrival time than the preferred time window at node $i \in N^0$ at specific time period $k \in K$.

$$\theta_i^{kv} \leq M \times \lambda_i^{kv} \qquad \forall v \in V, \forall i \in N, \forall k \in K \tag{24}$$

$$l_i^k - t_i^{kv} \leq \theta_i^v \leq l_i^k - t_i^{vk} + M \times (1 - \lambda_i^{kv}) \\ \forall i \in N, \forall v \in V, \forall k \in k \tag{25}$$

$$\phi_i^{kv} \leq M \times \mu_i^{kv} \qquad \forall v \in V, \forall i \in N, \forall k \in k \tag{26}$$

$$(t_i^{kv} - u_i^k) \leq \phi_i^{kv} \leq (t_i^{vk} - u_i^k) + M \times (1 - \mu_i^{kv}) \\ \forall i \in N, \forall v \in V, \forall k \in k \tag{27}$$

$$\Delta_i^{kv} = \phi_i^{kv} + \theta_i^{kv} \qquad \forall v \in V, \forall i \in N \tag{28}$$

$$\lambda_i^{kv}, \mu_i^{kv} \text{ binary} \qquad \forall v \in V, \forall (i,j) \in A, \forall k \in K \tag{29}$$

$$\delta_i^{kv}, \theta_i^{kv}, \phi_i^{kv} \text{ non-negative} \qquad \forall v \in V, \forall i \in N \tag{30}$$

TABLE I: HVRPMTW Parameter Values

Instance	20 instances generated from Gehring & Homberger's instances		
N^1	4-15	π^1	\$3 / time unit
N^2	4-15	V^2	4-15
K	4	Q^2	100
V^1	4-15	c^2	\$1 / time unit
Q^1	500	ε^2	\$2 / time unit
c^1	\$2 / time unit	π^2	\$5 / time unit
ε^1	\$7 / time unit	$p^i : i \in N^1$	\$300 / node

Constraints 24 and 25 compute the penalty time for the early arrival of vehicle $v \in V$ at node $i \in N$, while constraints 26 and 27 compute the penalty time for the late arrival of vehicle $v \in V$ at node $i \in N$ as $\max(\text{Early/Late penalty time}, 0)$. Constraint 28 identifies the amount of penalty time that vehicle $v \in V$ will incur when collecting waste at node $i \in N$. Constraints 29 and 30 are the feasible regions of these mentioned variables.

III. COMPUTATIONAL EXPERIMENTS

In this section, we first present the newly generated benchmark instances and the experimental set-up, which includes details about the environment used. We then summarize the results together with the limitations of commercial software in solving the instances. Some further parameter analyses are also presented. Finally, we discuss the case study result.

A. *Benchmark Instances and Experimental Set-up*

We developed new benchmark instances, which are adopted and modified from the Gehring & Homberger instances. We selected four instances for our study: C_1_6_1, C_2_6_1, R_1_6_1, R_2_6_1.Compared with the original data, the following parameters are provided and some of them are randomly generated: the number and capacity of the second type of vehicles, the related costs of each type of vehicles, the number of households, the number of e-bin locations, the number of time periods (days) in the planning horizon, and availability of households including the payments involved.

There are two scenarios for households' time windows: strict and relaxed time windows. For the former, the time windows are kept similar with the original instances, while for the latter the time windows are extended by random generated numbers. Moreover, customers' different time windows are taken from the original G&H instances, but their availability is randomly generated for the whole planning horizon, which is four days for all of the simulated instances. The range of specified parameters appears in Table I, and the complete datasets are available upon request.

The mathematical model is solved by commercial software, IBM ILOG CPLEX Optimization Studio - Academic vesion 12.10.0.0 on a personal computer with the following specifications: Intel Core i7-12700 2.10GHz, 16GB RAM, 1000GB HDD + 500GB SSD, Windows 10 Education version 22H2 64-bit. The experimental designs are set up with the maximum computational times of 3 hours or 13GB of memory occupation running due to the limitation of computer specifications.

B. *Computational Results*

The results of solving 10 benchmark instances by commercial software, CPLEX, with two different options of time windows are presented in Table II. In this experiment, our largest instance consists of a maximum of 30 nodes, including 15 households and 15 e-bins. Initially, the experimental results demonstrate that CPLEX can obtain feasible solutions for all benchmark instances within a three-hour running time. However, only four of them are solved optimally, namely instances 4-4 with strict and relaxed time windows, 6-6 with relaxed time windows, and 4-8 with strict time windows. The percentage gap between the best bound and the best integer obtained during the computation process indicates the quality of the solutions achieved by CPLEX within the global time limit. Notably, the gap percentage tends to increase as the instance size grows.

Another observation is that handling a larger number of households in the test instances requires more effort to find solutions compared to increasing the number of e-bins. This complexity arises due to the time windows associated with household e-waste collection requests. It is shown that CPLEX is not efficient in solving the HVRP-MTW instances. Our experiment aims to validate the proposed mathematical formulation of HVRP-MTW. For future work, we plan to develop an algorithm based on a metaheuristic approach to efficiently solve larger HVRP-MTW instances within a reasonable computational time.

C. *Parameter Analysis*

In this work, the difference between time window settings exponentially boosts the difficulty in finding the optimal solution. In our experiments, there is a significant gap between the best found integer solution and its bound for the instance of 6-6 with strict time windows, which is nearly 16000%. The reason for this is because the best found solution for this case is -\$5 while the boundary for the case is -\$791.18.Moreover, starting from Instance 12-12 for both scenarios, there is a significant difference between the bound and best found integer solution so far with considerable gap values. The reason for this results is because of the difference in results' sign. For example, with Instance 12-12 relaxed time windows, the best found integer cost is \$506, while the boundary for the problem at that time is -\$3618.03.

To be more specific, the instances are limited to be solved within three hours only, which may not be enough time for handling the problem, especially for larger instances. However, we do increase the time limitation to four hours for solving some larger instances, but the solver consumes more memory than the current specs of our machine and returns out-of-memory status after 3 hours and 20 minutes. The complexity of the problems mainly comes from how strict the provided time windows are, the availability status of the existing households, and the number of days in the planning periods.

To verify this, we tested with small instances of 4-4 instances with 3 days instead of the current experimental design. The problem can be solved within less than 1 minute compared against the same instance with larger time windows and availability day (e. g. , taking over six minutes to solve). This is one limitation of using commercial software that can be extended in our future work by proposing heuristics to solve larger instances.

D. *Case study*

After validating our work with a set of generated instances, real data are adopted for testing the validity of the model. In this real case scenario, a dataset on partial waste collecting locations provided by the National Environment Agency (NEA) website is used for our case study. We only focus on a particular region since in practice, e-waste collection are done based on regions.

Vehicle capacities are converted from volumes to load units with 10% of the capacity lost due to inappropriate arrangement. Vehicles' original volumes are retrieved from the light and heavy rigid truck dimensions given by [17]. The e-waste dimensions are randomly collected from a reseller website and then converted into load units. The operation costs are computed based on the average income of waste truck drivers in Singapore, which is provided by [18]. The penalty and idle costs are randomly generated. The coordinates of locations are collected from `http:\maps.google.com`. We then compute their distance matrix using the Euclidean method. The details are summarized in Table III.

Our model is used to solve the real data with the gaps between best integer and best bound less than 0.6%. The result shows that only 3 large vehicles are used for handling the e-waste collections. This concludes that our proposed model can handle a realistic scenario although it takes long computational times.

IV. CONCLUSION

This study presents a mathematical model of the Heterogeneous Vehicle Routing Problem with Multiple Time Windows (HVRP-MTW) in the context of e-waste collection. The problem is formulated as a Mixed Integer Linear Program (MILP) with the objective of minimizing the overall cost of the e-waste collection process. The proposed model takes into account various constraints that arise when collecting e-waste in the real world, including the capacity constraints of a heterogeneous vehicle fleet, customer requests availability within specific collection periods, and time windows for household on-demand requests on different collection days.

To validate the proposed mathematical formulation, a computational experiment is conducted using ten benchmark instances with two options of time windows for HVRP-MTW, along with a real-world instance. The results of the experiment reveal the limitations of commercial software such as CPLEX, in solving HVRP-MTW instances. CPLEX only manages to achieve optimal solutions for four out of the twenty scenarios within a three-hour time frame based on the ten benchmark instances. This underscores the need to develop an efficient approach to address the

TABLE II: Experiment Results

Home - E-bin Nodes Pairs	4-4	4-8	4-12	8-4	12-4	6-6	8-8	10-10	12-12	15-15
Time Windows Extension*	1	1	1	1	1	1	1	1	1	1
CPLEX Best Integer	136	586	1636	-803	-1166	104	-504	-480	506	2456
CPLEX Best bound	136	455.08	574.86	-1603.75	-2496.24	103.99	-1293.73	-1732.18	-3618.03	-2806.51
Gap (%)	0%	22.34%	64.86%	99.72%	114.09%	0.01%	156.69%	260.87%	815.03%	214.27%
Time (s)	351	14400	14400	14400	14400	2069	14400	14400	14400	14400
Time Windows Extension*	0	0	0	0	0	0	0	0	0	0
CPLEX Best Integer	444	649	1373	-632	-1189	-5	-574	-422	2456	108498
CPLEX Best bound	444	649	682.28	-1344.13	-2396.03	-791.18	-1375.64	-1983.94	-2806.51	-2364.7
Gap (%)	0%	0%	50.31%	112.68%	101.52%	15723.6%	139.66%	370.13%	214.27%	100.2%
Time (s)	30	1110	14400	14400	14400	14400	14400	14400	14400	14400

Time Windows Extension*: 0 - The instances' time windows are kept similar to the original ones; 1 - The instances' original time windows are modified with extended bounds for either upper-bound or lower-bound values.

TABLE III: Real-World Instance

Case study	Retrieved from the NEA website		
N^1	5	π^1	$\$4 \times 10^{-6}$ / time unit
N^2	15	V^2	10
K	4	Q^2	$19\,m^3 \approx 190$ load unit [17]
V^1	5	c^2	$\$7 \times 10^{-6}$ / time unit [18]
Q^1	$59\,m^3 \approx 590$ load unit [17]	ε^2	$\$2 \times 10^{-6}$ / time unit
c^1	$\$12 \times 10^{-6}$ / time unit [18]	π^2	$\$10^{-6}$ / time unit
ε^1	$\$8 \times 10^{-6}$ / time unit	$p^i : i \in N^1$	Demand based: \$10 / load unit

Collections of e-waste: refrigerators - 6 load unit, TV sets - 1 load unit, washing machines - 1 load unit, dryers - 3 load unit, and air conditioners - 2 load unit (considered as same size for similar category). Working hours per day: 16 hours.

HVRP-MTW problem, particularly through the utilization of approximation methods like metaheuristics. This future research direction aims to provide effective solution methods for larger instances of the e-waste collection problem.

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