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First train timetabling and bus service bridging in intermodal bus-and-train transit networks

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First train timetabling and bus service bridging in intermodal bus-and-train transit networks

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A R T I C L E I N F O

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A B S T R A C T

Subway system is the main mode of transportation for city dwellers and is a quite significant backbone to a city's operations. One of the challenges of subway network operation is the scheduling of the first trains each morning and its impact on transfers. To deal with this challenge, some cities (e.g. Beijing) use bus 'bridging' services, temporarily substituting segments of the subway network. The present paper optimally identifies when to start each train and bus bridging service in an intermodal transit network. Starting from a mixed integer nonlinear programming model for the first train timetabling problem, we linearize and reformulate the model using the auxiliary binary variables. Following that, the bus bridging model is developed to cooperate with the first train operation for reducing long transfer waiting times. After realizing the low computational efficiency of solving the integrated model, a tailored algorithm is designed to optimally solve the first train timetabling and bus service bridging problems. The exact models and algorithms are applied to the Beijing subway network to test their effectiveness and computational efficiency. Numerical results show that our approaches decrease the total passenger waiting time by 53.4% by a combined effect of adjusting the first train departure times and operating 27 bridging buses on 7 routes.

1. Introduction

1.1. Study motivation

Since the 1970s, with the increasingly heavy problems of the urbanized energy crisis, resource shortage, traffic congestion and traffic accidents, etc., subway systems have attracted much attention around the world. The level of urban traffic supply has been improved due to the rapid expansion of subway networks; the traffic congestion has been reduced; the urban territories have been expanded. However, subway systems in metropolises have increasingly become complex in recent years. It is, therefore, more difficult to manage the operations of subway systems under concentrated networking.

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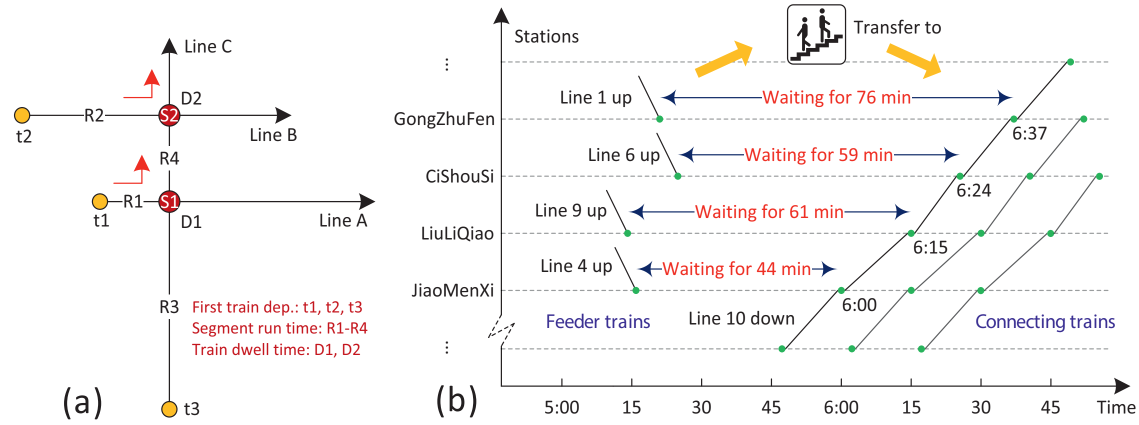

Fig. 1. First train transfer issues.

There is a famous saying that "well begun is half done". The first train transfer problem is crucial to morning passengers. If the transfer connectivity from the first train to other trains in a subway network is well addressed, morning commuters can save plenty of time and it further helps to reduce congestion during morning rush hours. The first train transfer problem is mainly due to the expansion of the subway network. As Fig. 1(a) shows, passengers riding the first trains on line A and line B transfer to the first train on line C. They have to wait (t3 + R3 + D1 − t1 − R1) min and (t3 + R3 + D1 + R4 + D2 − t2 − R2) min, respectively. The transfer waiting times usually increase much more at some stations as the network becomes large (R3 + D1 ≫ R1; R3 + D1 + R4 + D2 ≫ R2), and/or the inappropriate setting of the first trains' departure times (t3 ≫ t1; t3 ≫ t2). Fig. 1(b) shows a sample of the first train diagram from the Beijing subway in 2016, which depicts a serious problem of long waiting time for its first train connection. For instance, when a passenger riding the first train transfers from the up-train direction on line 6 to the down-train direction on line 10 at Ci-Shou-Si station, he/she has to wait for as long as 59 min. Moreover, according to our statistics, the total transfer waiting time of the first train passengers in the full Beijing subway system (more than 240 transfer directions) was over 22,790 min daily in 2016, considering the transfer passenger volumes. It is an unsolved problem of subway network operations that the first train transfers should be well synchronized. Moreover, Mohring et al. (1987) found that passengers often viewed their waiting time twice the actual time. Therefore, this study will address the first train transfer problem to reduce transfer waiting times for the early-morning passengers, aiming to allow for well-timed connection transfers in large subway systems.

Note that the previous study Kang et al. (2016) reduced the total transfer waiting time for passengers from 22,790 min to 16,380 min daily by optimizing the early train departure times for the Beijing subway case (9 bi-directional lines and 31 transfer stations with the first train departure times between 4:50 and 5:10, a range of 20 min adjustment). Although the optimized timetables reduced the total waiting time by 28.1%, a figure of 16,380 min is still a huge waiting period and the longest waiting time of a transfer direction reaches as much as 70 min. An efficient approach to avoid long waiting times for the morning passengers in practice is to coordinate well-designed train operations with bus bridging services, which are provided in the event of inefficient first train transfers. The design of temporary bus services and their routes during the first train periods can improve connectivity in large-size subway networks. This type of temporary network is often referred to as the bus bridging service (Jin et al., 2015), and will be introduced in detail in Section 2.3. When the first train transfer is not efficient in the rail network, bridging buses would ferry affected commuters waiting for the connecting trains (these bridging buses run from one station to the other stations with different locations on the subway lines) and minimize the impact of a single mode of transportation on early morning commuters.

We use the same example in Fig. 1(a) to better illustrate the idea of intermodal bus-and-train cooperation. Assume that the first trains on line A, line B, and line C depart from their depots at time 0. The segment travel time is given on each line of Fig. 2(a). The dwell time at each station is set to 5. The transfer walking time at each transfer station is 0. As shown in Fig. 2(a), the transfer waiting time from line A to line C is $0 + 70 + 5 - 0 - 20 = 55$ min. The transfer waiting time from line B to line C is $0 + 70 + 5 + 30 + 5 - 0 - 50 = 60$ min. If we deploy a bridging bus departing from S1 at time 15 and heading to S2 and line C (bus running time from S1 to S2 is 50), the waiting time from line A to the bus is $15 + 5 - 20 = 0$ min and from line B to the bus is $15 + 5 + 50 + 5 - 50 = 25$ min. As can be observed, transfer waiting times and journey times are reduced effectively by riding the bridging buses. Moreover, one may suggest deploying deadheading trains (serving passengers from S1 on Line C) before the first trains to connect transfer passengers. This strategy is effective in reducing waiting times and journey times. However, it requires subway operators to run two extra deadheading trains from the depot of Line C with an extra running time of 140 min and extra operational costs. Hence, the train-and-bus mode is the best for both passengers and companies. Without losing generality, Fig. 2(b) integrates the feeder train, connecting train and bus bridging timetables into one diagram, which shows the advantage of bridging buses in reducing first train transfer waiting times. As shown, passengers in the feeder train T1 transfer to the connecting train T1' at the exchange station with a

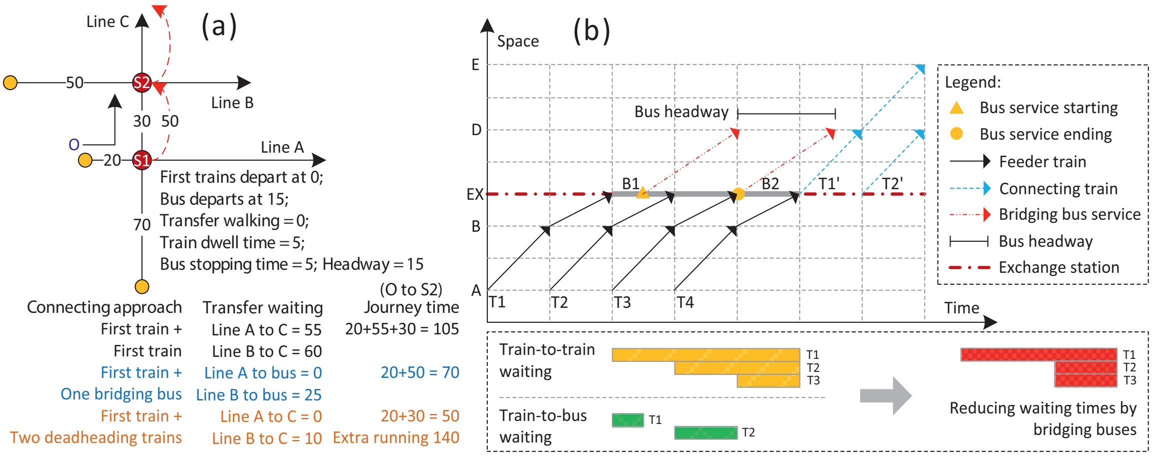

Fig. 2. Advantage of bus bridging services to transfer waiting times.

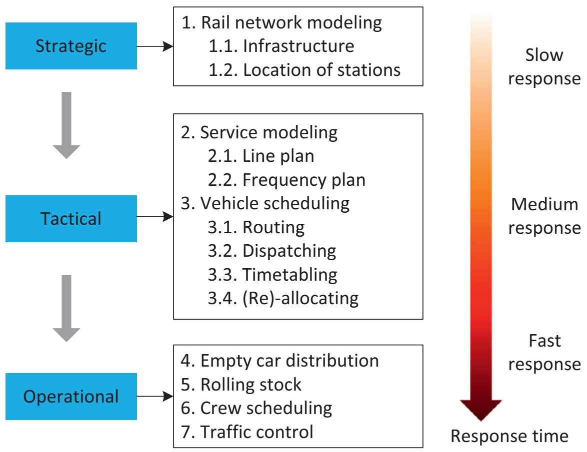

Fig. 3. Problems within different planning horizons.

three-headway waiting time. This scenario occurs because of the differences in first train departure times and network structure. Long transfer waiting time problems will become more prominent and serious with the extending of the subway networks. With bridging buses (denoted by B1 and B2), early morning passengers could enjoy a fast transfer and smooth connection services. The transfer waiting times can be reduced significantly, as shown in Fig. 2(b). A practical case of the cooperation between the first trains and bridging buses is detailed in Appendix A. Indeed, there are additional costs in operating the bridging buses such as fuel costs and labor costs. However, if we do not dispatch bridging buses, passengers may be forced to ride taxis as commuting tools at transfer stations. In the early morning, there are not many taxis cruising in the streets, making it difficult for passengers to take taxis. On the other hand, it is expensive to take a taxi compared to subway trains and bridging buses. In view of the effectiveness of the above approach, this paper will study the problem of coordinating bus bridging services with the first train timetables in intermodal bus-and-train transport networks. In the following, some related literature is reviewed.

1.2. Related studies

The Railway passenger transportation problem is a systematic problem that is usually planned in three levels of hierarchy: strategic, tactical, and operational control, as illustrated in Fig. 3. The tactical and operational problems are at a high dynamic level (Almodóvar and García-Ródenas, 2013). When the planning horizon becomes short, the buffer responding time reduces significantly, which makes the above problems need dynamic approaches (Narayanaswami and Rangaraj, 2011). Table 1 summarizes the motivation and objectives of the recently published literature on scheduling and rescheduling of the railway operations. This table has three columns: Column 1 lists the main railway scheduling and rescheduling operations problems that are discussed in the literature related to this paper. Columns 2 and 3 list scheduling and rescheduling objectives, respectively. In this section, a brief literature review of railway operations will be discussed.

Table 1
Motivations and objectives of public transit scheduling and rescheduling.

Content	Motivation and/or objective: Scheduling	Rescheduling
Line planning	Number of services (Fu et al., 2015)	Energy efficiency (Scheepmaker et al., 2017)
	Line coordination (Dou et al., 2015)	Speed profile optimization (Yang et al., 2020)
	Service response (Jin et al., 2015)	Maximizing capacity (Morlok and Chang, 2004)
	Parallel service (Li et al., 2015)	Reliability (Lamorgese et al., 2016)
Timetabling	Robustness (Fischetti et al., 2009)	Buffer distribution (Smith et al., 2006)
	Delay resistance (Liebchen et al., 2010)	Compensation management (Lao et al., 2015)
	Nominal problem (Cacchiani and Toth, 2012)	Integration of services (Li et al., 2015)
	Operating cost (Ibarra-Rojas et al., 2014)	Disruption handling (Zilko et al., 2016)
	Departure time choices (Kang and Meng, 2017)	Train delays (Kang et al., 2015b)
Rolling stock	Operational cost (Ibarra-Rojas et al., 2014)	Robustness (Andersson et al., 2015)
	Minimizing idle runs (Gafarov et al., 2014)	Punctuality (Šemrov et al., 2016)
	Maximizing utilization (Lamorgese et al., 2016)	
Crew scheduling	Crew utility (Caprara et al., 2002)	Minimizing crews (Deng and Lin, 2011)
	Freight crew (Bach et al., 2016)	Punctuality (Veelenturf et al., 2012)

Scheduling is a time allocation of resources to meet demands in completing a task (Abdolmaleki et al., 2020). The train timetable scheduling problem aims to achieve a conflict-free railway timetable consisting of all the train arrival and departure times at each station. Cacchiani and Toth (2012) performed an intensive study on the train timetabling problems and underlined the differences between methods for dealing with the nominal and robust versions. When designing railway train timetables, objectives developed by the operating companies are profit-oriented, such as reducing operating costs (Ibarra-Rojas et al., 2014), minimizing train travel times (Chevrier et al., 2013), and improving energy efficiency (Wang and Goverde, 2019). From the passengers' perspective, operating companies will always employ measures regularly to minimize waiting times (Niu and Zhou, 2013; Lin and Ku, 2014; Niu et al., 2015; Kang et al., 2015a; Zhou et al., 2019). For example, Dou et al. (2015) optimized network-based bus timetables to coordinate the last train services in a subway network, by using tailor-made coordination principles for a bus network. Other methods to solve the timetable coordination problem can be found in Wong et al. (2008), Shafhi and Khani (2010), and Niu et al. (2015).

Railway disruption management can be divided into two categories: train timetabling and real-time rolling stock rescheduling. For deterministic or stochastic reasons, it sometimes requires the reallocation of resources for task completion, which is referred to as rescheduling. For example, Jin et al. (2015) presented a mixed integer linear programming (MILP) model of deploying bus services for the disruptions of urban railway transit networks. Train timetable rescheduling aims to coordinate the arrival-departure times of trains at stations to improve the level of services. Acuna-Agost et al. (2011b) defined the train rescheduling problem after incidents as a reparation problem, for which a MIP was formulated to minimize the difference between the original timetable and the rescheduling timetable. In such problems, MIP models were usually developed to optimize train delays and reduce the number of canceled trains. There is a wealth of literature on timetable rescheduling, e.g., Carey and Lockwood (1995), Narayanaswami and Rangaraj (2013), Corman et al. (2014), Veelenturf et al. (2015), and many more.

Although there exist plenty of studies on train scheduling or rescheduling problems, few works on the integrated optimization of bus bridging services and first train services in an intermodal public transport network can be found. Two previous studies, Dou et al. (2015) and Kang et al. (2019) investigated how to design bus routes for midnight passengers who failed to catch the last trains. They aimed to improve subway network accessibility. In contrast, this paper focuses on the first train and bus bridging coordination problem to improve the first train connections and synchronizations in large subway networks. The innovative problem setting considered is essentially different from the above-mentioned two studies. The main contributions in this paper are as follows. First, we design a bus bridging approach to cooperate with the first train operations. This is a novel method to cope with the first train transfer problem in a large network. Second, we build an explicitly integrated MILP model for the first train and bus bridging timetabling problems to minimize transfer waiting times for the early-morning passengers. Third, after realizing the low computational efficiency of solving the integrated model, a tailored algorithm is designed for optimally solving the first train timetabling and bus service bridging problems. We apply the model and the approach to the Beijing subway network. The results indicate that the optimal first train and bus bridging timetables decrease the total waiting time from a base of 22,790 min to 10,628.5 min daily by a combined effect of adjusting the first train departure times and operating 27 bridging buses on 7 routes.

2. Modeling the first train timetabling and bus service bridging

2.1. Non-linear first train timetabling model

Fig. 4 shows two cases of first train connections. Line l is a feeder line and line l' is a connecting line. As case A shows, if the first train on line l arrives earlier than the first train on line l', then the first train passengers on line l can transfer to the first connecting train on line l' (this case may lead to long waiting times). Otherwise, these passengers can transfer to

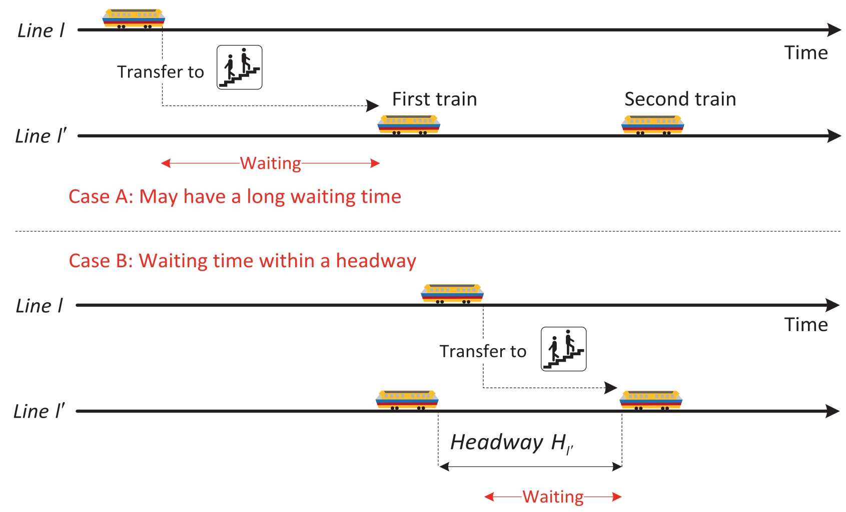

Fig. 4. Illustration of first train connections.

the following connecting trains, e.g., the second train on line l', as illustrated in case B. The transfer waiting time in case B is always within a headway's time. The above transfer processes generate different waiting times for first-train passengers. In practice, subway operators hope to improve service level by reducing such passenger waiting times through optimizing the first train timetables.

To minimize the total first train transfer waiting time (between the first feeder train and the "right" connecting train) in the network, a non-linear first train timetabling model is developed. Eq. (1) is the objective function, in which $p^k_{ll'}$ represents the number of transfer passengers from line l to line l' at station k and $w^k_{ll'}$ represents the corresponding waiting time from line l to line l' at station k. The first train arrival and departure times at and from a station are calculated using Eqs. (2) and (3), respectively, where t^R_l represents the departure time of the first train from the vehicle depot on line l, $t^l_{(s-1)s}$ represents the segment running time between stations $s-1$ and s on line l, and t^l_s represents the dwell time of the train at s on line l. In addition, t^R_l and $t^R_{l'}$ are limited in the upper and lower bounds, as illustrated by Eqs. (4) and (5). The Eq. (6) calculates the transfer waiting times $w^k_{ll'}$ based on the train departure time $Dep^{l'}_k$, the train arrival time Arr^l_k, and the transfer walking time $Tra^{ll'}_k$. According to Fig. 4, $w^k_{ll'}$ is divided into two cases, as shown in Eq. (7). The binary variable, $x^k_{ll'}=1$, represents the first-train-to-first-train connection (case A). On the contrary, $x^k_{ll'}=0$ represents case B. For passengers who arrive earlier than the first connecting train ($x^k_{ll'}=1$), their waiting time is equal to the difference between the first connecting train departure time and the passenger arrival time. Otherwise ($Dep^{l'}_k - Arr^l_k - Tra^{ll'}_k < 0$, $x^k_{ll'}=0$), a certain number of headways of the connecting line (i.e., the value of $n{*}headway$) should be added to the term. This is because waiting time is non-negative. The term $n \bullet H_{l'}$ in $Dep^{l'}_k - Arr^l_k - Tra^{ll'}_k + n \cdot H_{l'}$ ensures $w^k_{ll'}$ to be non-negative. Physically, the value n captured by Eq. (8) represents the number of trains passing the station on connecting line l' when the feeder passengers arrive at the platform. Namely, n is the number of trains missed by the transfer passengers (or the number of trains on line l' that arrive at the platform earlier than the arrival time of transfer passengers from line l). Finally, Eq. (9) shows the non-negative constraint for transfer waiting times.

[MINLP-M1]:

$$\min \sum_{\forall l \in L} \sum_{\forall l' \in L} \sum_{k \in S(l) \cap S(l')} p^k_{ll'} \cdot w^k_{ll'} \tag{1}$$

subject to

$$Arr^l_k = t^R_l + \sum_{s=2}^{k} t^l_{(s-1)s} + \sum_{s=1}^{k-1} t^l_s, \forall k \in S(l), \forall l \in L \tag{2}$$

$$Dep^{l'}_k = t^R_{l'} + \sum_{s=2}^{k} t^{l'}_{(s-1)s} + \sum_{s=1}^{k} t^{l'}_s, \forall k \in S(l), \forall l' \in L \tag{3}$$

$$T^{\min}_l \le t^R_l \le T^{\max}_l, \forall l \in L \tag{4}$$

$$T^{\min}_{l'} \le t^R_{l'} \le T^{\max}_{l'}, \forall l' \in L \tag{5}$$

$$w_{ll'}^{k}=\begin{cases}Dep_{k}^{l'}-Arr_{k}^{l}-Tra_{k}^{ll'},x_{ll'}^{k}=1\\ Dep_{k}^{l'}-Arr_{k}^{l}-Tra_{k}^{ll'}+n\cdot H_{l'},x_{ll'}^{k}=0\end{cases},\forall k\in S(l)\cap^{S}\left(l'\right),\forall l,l'\in L \tag{6}$$

$$x_{ll'}^{k}=\begin{cases}1,Dep_{k}^{l'}-Arr_{k}^{l}-Tra_{k}^{ll'}\geq 1\\ 0,Dep_{k}^{l'}-Arr_{k}^{l}-Tra_{k}^{ll'}<0\end{cases},\forall k\in S(l)\cap^{S}\left(l'\right),\forall l,l'\in L \tag{7}$$

$$n=\mathrm{Int}\left(\frac{\left|Dep_{k}^{l'}-Arr_{k}^{l}-Tra_{k}^{ll'}\right|}{H_{l'}}\right)+1 \tag{8}$$

$$w_{ll'}^{k}\geq 0,\forall k\in S(l)\cap^{S}\left(l'\right),\forall l,l'\in L \tag{9}$$

2.2. Linear first train timetabling model

We linearize the above model as follows. Eqs. (6)–(8) are simplified and replaced by Eqs. (10) and (11). As Eq. (10) shows, variables to capture $w_{ll'}^{k}$ are $Dep_{k}^{l'}$, Arr_{k}^{l} and $h_{l'}$. As defined in Eqs. (2) and (3), $Dep_{k}^{l'}$ and Arr_{k}^{l} are easy to calculate. Variable $h_{l'}$ is an integer and should be non-negative. With Eq. (11) and the minimum objective function, the exact value for variable $h_{l'}$ will be found.

$$w_{ll'}^{k}=Dep_{k}^{l'}-Arr_{k}^{l}-Tra_{k}^{ll'}+h_{l'}\cdot H_{l'},\forall k\in S(l)\cap^{S}\left(l'\right),\forall l,l'\in L \tag{10}$$

$$w_{ll'}^{k}\geq 0,\forall k\in S(l)\cap^{S}\left(l'\right),\forall l,l'\in L \tag{11}$$

Remark. With the minimum objective function of the total first train transfer waiting time, Eqs. (10) and (11) calculate the first train transfer waiting times linearly.

Proof. For $l\in L$, $l'\in L$, $s\in S(l)\cap^{S}(l')$ and inter-transfer directions (i.e., $l\leftrightarrow l'$), we have $w_{ll'}^{k}=Dep_{k}^{l'}-Arr_{k}^{l}-Tra_{k}^{ll'}+h_{l'}\cdot H_{l'}$. Therefore, the objective function $\min\sum_{\forall l\in L}\sum_{\forall l'\in L}\sum_{k\in S(l)\cap^{S}(l')}p_{ll'}^{k}\cdot w_{ll'}^{k}$ can be reformulated as follows:

$$\min\sum_{\forall l\in L}\sum_{\forall l'\in L}\sum_{k\in S(l)\cap^{S}(l')}\left(Dep_{k}^{l'}-Arr_{k}^{l}-Tra_{k}^{ll'}+h_{l'}\cdot H_{l'}\right)\cdot p_{ll'}^{k},$$

which is further extended by:

$$\min\sum_{\forall l\in L}\sum_{\forall l'\in L}\sum_{k\in S(l)\cap S(l')}\left[\left(t_{l'}^{R}+\sum_{s=2}^{k}t_{(s-1)s}^{l'}+\sum_{s=1}^{k}t_{s}^{l'}\right)-\left(t_{l}^{R}+\sum_{s=2}^{k}t_{(s-1)s}^{l}+\sum_{s=1}^{k-1}t_{s}^{l}\right)-Tra_{k}^{ll'}+h_{l'}\cdot H_{l'}\right]\cdot p_{ll'}^{k}.$$

Finally, we get:

$$\min\sum_{\forall l\in L}\sum_{\forall l'\in L}\sum_{k\in S(l)\cap^{S}(l')}\left[\left(t_{l'}^{R}-t_{l}^{R}+h_{l'}\cdot H_{l'}\right)\cdot p_{ll'}^{k}\right]+\Omega,$$

where Ω is a constant and $\Omega=\sum_{\forall l\in L}\sum_{\forall l'\in L}\sum_{k\in S(l)\cap^{S}(l')}((\sum_{s=2}^{k}t_{(s-1)s}^{l'}+\sum_{s=1}^{k}t_{s}^{l'}-\sum_{s=2}^{k}t_{(s-1)s}^{l}-\sum_{s=1}^{k-1}t_{s}^{l}-Tra_{k}^{ll'})\cdot p_{ll'}^{k})$. It can be observed that with the minimum objective function proposed, Eqs. (10) and (11) calculate the first train transfer waiting times linearly. In other words, the first train transfer waiting times captured by Eqs. (6)–(8) can be replaced by Eqs. (10) and (11). This completes the proof.

Therefore, the first train timetabling problem is reformulated as a MILP model with the same objective function in MINLP-M1:

[MILP-M2]:

$$\min\sum_{\forall l\in L}\sum_{\forall l'\in L}\sum_{k\in S(l)\cap^{S}(l')}p_{ll'}^{k}\cdot w_{ll'}^{k}$$

subject to constraints (2)–(11).

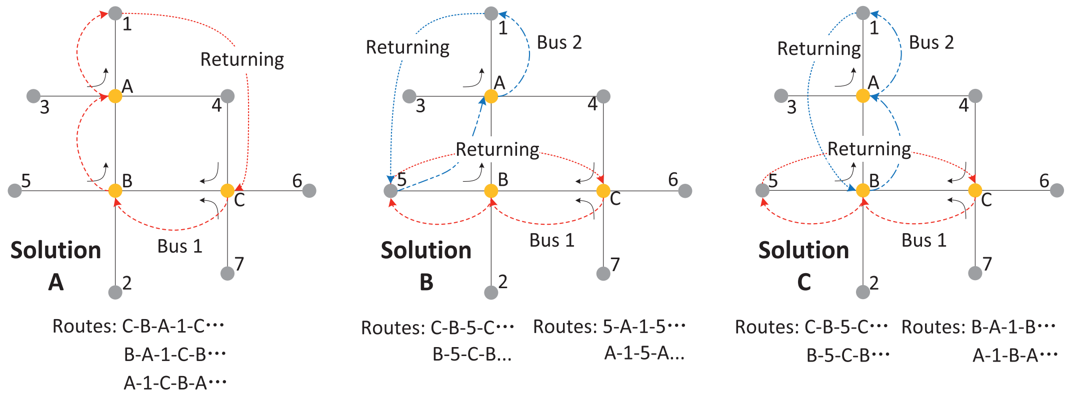

Fig. 5. Different ways of operating bridging buses.

2.3. Connections with bridging buses

Solid arrows in Fig. 5 represent the routes that the bridging buses take to pick up morning transfer passengers. As can be observed in Fig. 5, there are three different types of routing solutions for bridging buses. *Solution A* utilizes one bridging bus to string the key transfer directions together. This method saves the number of required bridging buses but is not effective against large networks (due to a positive correlation between the size of the subway network and the total train-to-bus waiting time). *Solution B* dispatches buses almost according to the connecting lines. Each bridging bus serves the transfer directions that belong to the same connecting line. Compared to *solution B, solution C* adopts the same principle in bus routing, except for the start and/or end stops. In *solution C*, each bridging bus starts at the transfer station, passes through the intermediate stations, and returns to the start station. This method replicates train services by providing bus bridging services in parallel to the subway sections. It is easy to operate and convenient for morning passengers. In this paper, we adopt *solution C* to implement bus bridging services.

Before utilizing bridging buses to reduce long transfer waiting times, we should first define journeys/directions with long transfer waiting times. Here, we select the transfer directions considering two aspects: transfer waiting times and passenger volumes. Understandably, directions with long waiting times for the morning passengers should have bridging buses to improve first train services. Meanwhile, if the number of transfer passengers is small (e.g., $p^k_{ll'} \approx 0$), then the bridging buses would not be allocated to long waiting time directions.

Eq. (12) defines the binary variable $\alpha^k_{ll'}$ that has a value of one if the passenger transfer waiting time from line l to line l' at station k when they ride the first train is larger or equal to π. Otherwise, $\alpha^k_{ll'}$ equals zero. Herein, π is a set parameter that can be adjusted in range, as shown in Eq. (13). To represent and capture $\alpha^k_{ll'}$ linearly, a large enough integer M is used in Eq. (14). When $w^k_{ll'} \ge \pi$, $\alpha^k_{ll'}$ returns to the value of one ($0 \le w^k_{ll'} - \pi < M$). On contrary, when $w^k_{ll'} < \pi$, $\alpha^k_{ll'}$ returns to the value of zero ($-M \le w^k_{ll'} - \pi < 0$).

$$\alpha^k_{ll'} = \begin{cases} 1, w^k_{ll'} \ge \pi \\ 0, w^k_{ll'} < \pi \end{cases}, \forall k \in S(l) \cap^{S} \left(l'\right), \forall l, l' \in L \tag{12}$$

$$H_{l'} < \pi < M, \forall l' \in L \tag{13}$$

$$M \cdot \left(\alpha^k_{ll'} - 1\right) \le w^k_{ll'} - \pi < M \cdot \alpha^k_{ll'}, \forall k \in S(l) \cap^{S} \left(l'\right), \forall l, l' \in L \tag{14}$$

Similarly, Eq. (15) defines the binary variable $\gamma^k_{ll'}$ that is equal to one if the total first train transfer waiting time from line l to line l' at station k is larger or equal to ρ, where the parameter ρ is restricted in Eq. (16). Otherwise, $\gamma^k_{ll'}$ equals zero. The linearization expression of $\gamma^k_{ll'}$ is illustrated in Eq. (17). When $p^k_{ll'} \cdot w^k_{ll'} \ge \rho$, $\alpha^k_{ll'}$ returns to the value of one ($0 \le p^k_{ll'} \cdot w^k_{ll'} - \rho < M$); when $p^k_{ll'} \cdot w^k_{ll'} < \rho$, $\alpha^k_{ll'}$ returns to the value of zero ($-M \le p^k_{ll'} \cdot w^k_{ll'} - \rho < 0$).

$$\gamma^k_{ll'} = \begin{cases} 1, p^k_{ll'} \cdot w^k_{ll'} \ge \rho \\ 0, p^k_{ll'} \cdot w^k_{ll'} < \rho \end{cases}, \forall k \in S(l) \cap^{S} \left(l'\right), \forall l, l' \in L \tag{15}$$

$$W_{\min} < \rho < M \tag{16}$$

$$M \cdot \left(\gamma^k_{ll'} - 1\right) \le p^k_{ll'} \cdot w^k_{ll'} - \rho < M \cdot \gamma^k_{ll'}, \forall k \in S(l) \cap^{S} \left(l'\right), \forall l, l' \in L \tag{17}$$

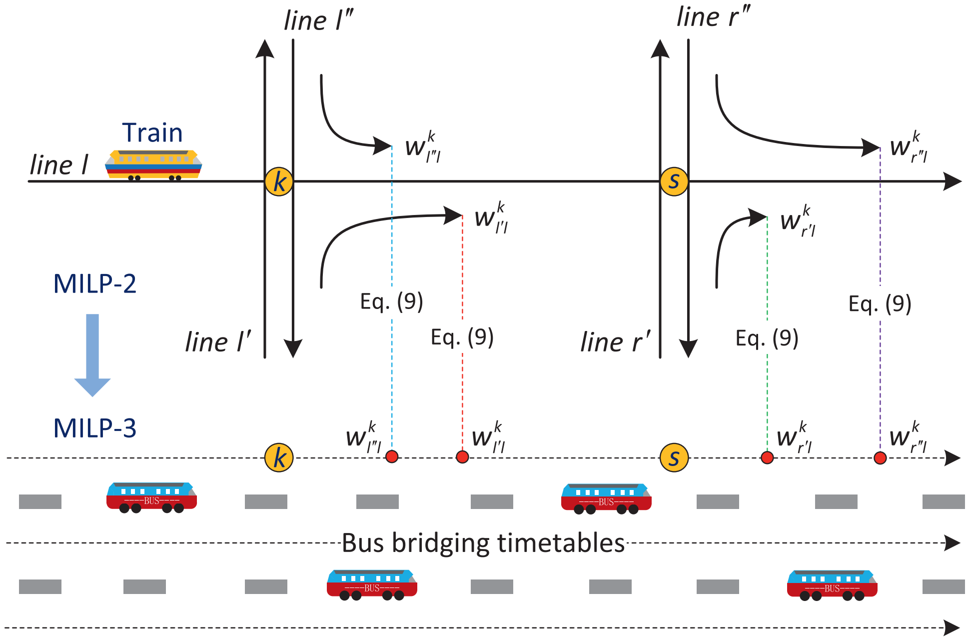

Fig. 6. Determining where and when to implement bus bridging services.

Based on Eqs. (14) and (17), we use $\beta^k_{ll'}$ to represent the transfer directions that need bridging buses to reduce transfer waiting times. As shown in Eq. (18), $\beta^k_{ll'}$ is an intersection of $\alpha^k_{ll'}$ and $\gamma^k_{ll'}$. If $\beta^k_{ll'} = 1$, transfer direction from line l to line l' at station k should be equipped with bridging buses. As a result, the set $B_{l'}$ in Eq. (19) shows a sequence of binary variables, explicitly indicating whether we should provide bus bridging connections to line l'.

$$\beta_{ll'k} = \alpha_{ll'k} \cap^{\gamma}_{ll'k}, \forall k \in S(l) \cap^{S} \left(l'\right), \forall l, l' \in L \tag{18}$$

$$B_{l'} = \left\{\beta_{ll'k} | \forall l' \in L, \forall k \in S(l) \cap^{S} \left(l'\right)\right\} \tag{19}$$

2.4. Bus bridging service model

After selecting the journeys/directions with long transfer waiting time, this section models the bus bridging services to cooperate with the first train operations. Fig. 6 shows the bus bridging timetabling problem, which determines the place of deployment for each bridging bus and the departure time of each bridging bus. With a given bus headway for each bus route, the bus bridging is assigned a route at each headway time interval. Although the bus speed is smaller than the train speed, the journey time can significantly be saved by reducing the long first train transfer waiting time. Note that we also implicitly consider the bus operation cost by limiting the number of bridging buses deployed. The model is formulated in MILP-M3 to minimize the total train-to-bus waiting time under a certain number of deployed bridging buses.

[MILP-M3]:

$$\min \sum_{k \in S(l')} \sum_{l \in L} \sum_{l' \in L} b_{ll'k} \cdot Bw_{ll'k} \tag{20}$$

subject to

$$b_{ll'k} = \beta_{ll'k} \cdot p_{ll'k}, \forall k \in S(l) \cap^{S} \left(l'\right), \forall l, l' \in L \tag{21}$$

$$Bw_{ll'k} = tb^{l'}_k - Arr^l_k - \theta^{l'}_k + Bh_{l'} \cdot H^B_{l'}, \forall k \in S(l) \cap^{S} \left(l'\right), \forall l, l' \in L \tag{22}$$

$$Bw_{ll'k} \geq 0, \forall k \in S(l) \cap^{S} \left(l'\right), \forall l, l' \in L \tag{23}$$

$$tb^{l'}_k = Bt^{l'}_0 + \sum_k bt^{l'}_{(k-1)k} + \sum_k bt^{l'}_k, \forall k \in S(l) \cap^{S} \left(l'\right), \forall l' \in L \tag{24}$$

$$Bt^{l'\min}_0 \leq Bt^{l'}_0 \leq Bt^{l'\max}_0, \forall l' \in L \tag{25}$$

$$tb_k^{l'} + Bh_{l'} \cdot H_{l'}^B \le \mathrm{Int}\left(\frac{w_{ll'k}}{H_{l'}^B}\right) \times H_{l'}^B, \forall k \in S(l) \cap^S (l'), \forall l, l' \in L \tag{26}$$

$$\frac{w_{ll'k} - H_{l'}^B}{H_{l'}^B} < \mathrm{Int}\left(\frac{w_{ll'k}}{H_{l'}^B}\right) < \frac{w_{ll'k} + H_{l'}^B}{H_{l'}^B}, \forall k \in S(l) \cap^S (l'), \forall l, l' \in L \tag{27}$$

Eq. (20) aims to minimize the total transfer waiting time for train-to-bus passengers. The $b_{ll'k}$ in Eq. (21) represents the number of passengers exchanging from train to bus and its value indirectly shows the place of deployment for each bridging bus (i.e., at station k of rail line l and bus line l') when $b_{ll'k} \neq 0$ and $Bt_0^{l'} \neq 0$ (which will be introduced later). $Bw_{ll'k}$, captured by Eq. (22), represents the bus waiting time from subway line l to bus line l' at station k. With $\beta_{ll'k}$ utilized in Eq. (21), the number of passengers of each long waiting time direction is captured. $Bw_{ll'k}$ is positive in Eq. (23) and is determined using the bridging bus departure time $tb_k^{l'}$, the first train arrival time Arr_k^l, the passenger walking time $\theta_k^{l'}$, and the headway of bridging buses $H_{l'}^B$. $tb_k^{l'}$ is obtained by Eq. (24), where $Bt_0^{l'}$ limited in bounds in Eq. (25) represents the first bridging bus departing from line l' at station k, $bt_{(k-1)k}^{l'}$ represents the segmental bus running time, and $bt_k^{l'}$ represents the bus dwell time. Eq. (26) limits the number of bridging buses $Bh_{l'}$ for each connecting line l' in time horizon. In this way, the total bus operation cost can be considered and indirectly under control by different values of $Bh_{l'}$. Note that function Int(•) in Eq. (27) is easy to linearize by constraining the term within the range $(\frac{w_{ll'k}-H_{l'}^B}{H_{l'}^B}, \frac{w_{ll'k}+H_{l'}^B}{H_{l'}^B})$.

3. Approach

3.1. Solution algorithm

A direct way to solve the above two MILP models is to build an integrated model for the first train timetabling and bus bridging timetabling problem. The constraints are similar to those in the aforementioned sub-problems. The objective in Eq. (28) is to minimize the total transfer waiting time for the first train transfers and the bridging bus passengers.

[MILP-M4]:

$$\min \sum_{\forall l \in L} \sum_{\forall l' \in L} \sum_{k \in S(l) \cap^S (l')} (p_{ll'k} - b_{ll'k}) \cdot w_{ll'k} + \sum_{k \in S(l')} \sum_{l \in L} \sum_{l' \in L} b_{ll'k} \cdot Bw_{ll'k} \tag{28}$$

subject to constraints (2)–(11), (13), (14), (16)–(19) and (21)–(27).

We tried to use CPLEX to solve the MILP-M4 on a 2.5 GHz CPU and 16 GB of RAM laptop for the Beijing subway case. Unfortunately, the CPLEX returned a gap of 10.2% after running over 24 h. As a result, it is computationally intractable to solve MILP-M4 efficiently by only utilizing the commercial solver CPLEX. Thus, we come up with the idea of sequentially solving the integrated model. Before showing the sequential approach, let us see the following example.

Fig. 7 contains 3 subway lines intersecting at stations A and B, where each dot represents the first station on each line. Train segmental running times are also given on the side of each segment. For a better understanding and illustration, $\forall s \in S(l) \cap^S (l'), \forall l, l' \in L$, we set train dwell time and transfer walking time to be zero, i.e., $t_s^l = 0$ and $Tra_s^{ll'} = 0$. The passenger flow $p_{ll'}^s = 1$ and train headway $H_{l'} = 1$. Moreover, we set the decision variables t_l^R as binary variables (Ibarra-Rojas and Rios-Solis, 2012). In this way, all the eight possible scenarios of the first train synchronizations can be listed using the enumeration method (the number of solutions enumerated by this simplified problem is $(t_l^R)^{|L|}$, where $|L|$ represents the number of subway lines in the network). As shown, the optimal solution for the MILP-2 is "0–1–1″, indicating that the first trains on line 1, line 2 and line 3 depart at time 0, time 1 and time 1, respectively, i.e., $t_1^R = 0$, $t_2^R = 1$, and $t_3^R = 1$. The total transfer waiting time is 3, which will be eliminated by deploying bridging buses at station B after solving MILP-3, i.e., $Bt_0^1 = 3$. As a result, the optimal solutions for solving MILP-4 are equivalent to sequentially solving MILP-2 and MILP-3 by utilizing problem-specific knowledge.

The integrated first train timetabling and bus service bridging problem is a systematic and combined optimization problem, which consists of two sequential timetabling problems (the first train and bus bridging timetabling) as shown in Fig. 8. Based on an intuitive model for the first train timetabling problem, we further solve the model by utilizing problem-specific knowledge. Transfer waiting times for all directions can be obtained after solving the MILP-M2. The key directions (routes) that should be served by bridging buses are selected considering the transfer waiting time and the passenger volumes. Then, the sub-problem (i.e., the bus bridging timetabling problem) is solved based on the results from the first train timetabling. The overall procedures of Algorithm 1 for the first train timetabling and bus service bridging problem is summarized as follows. Note that Algorithm 1 can solve the MILP-4 model to global optimality, and the proof can be found in Appendix B.

3.2. Calibration

Here, we use a sample network to illustrate the effectiveness of the developed model and algorithm. Three subway lines in Fig. 9 cross the three transfer stations (A, B, and C). Each line consists of several stations, and the segment running times

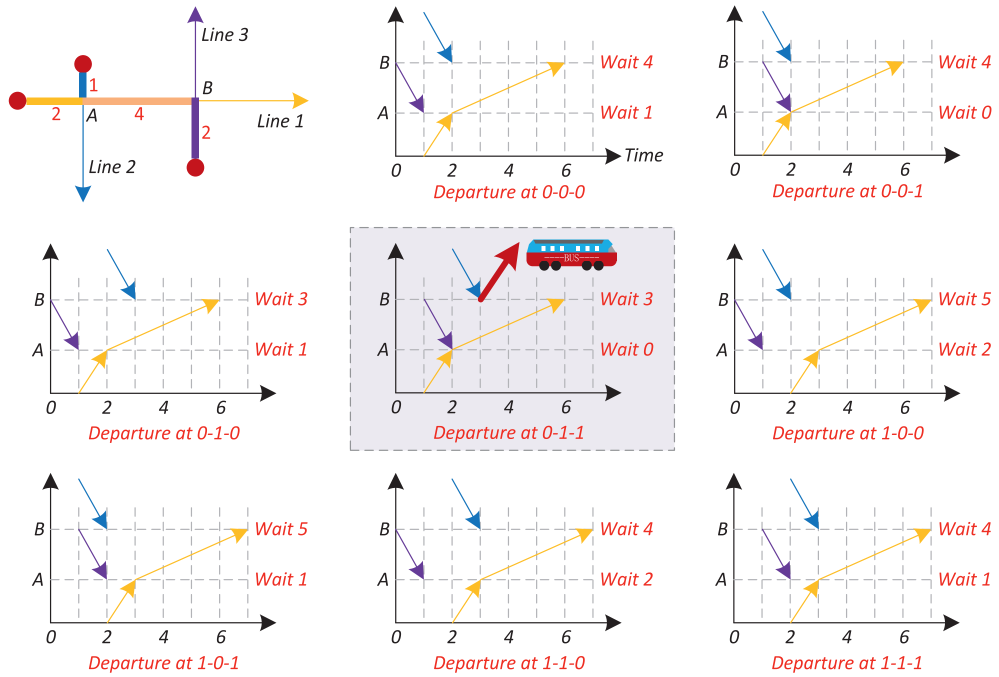

Fig. 7. Different scenarios of the first train synchronization.

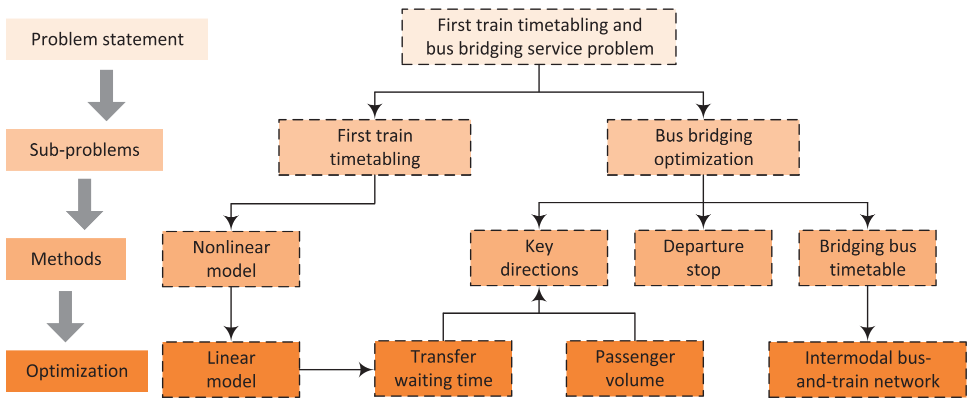

Fig. 8. Algorithm flowchart for the problem.

for trains are given along the segments. Train dwell times are set to 1 min; train headways are given to 30 min; and the first train departure times are restricted in [0, 30] min. Besides, the bridging bus dwell times are set to 1 min; bus headways are given to 15 min; and the first bus departure times are limited in [0, 20] min. Transfer walking times between lines at stations A, B, and C are 2, 3, and 4 min, respectively.

The model and the algorithm were applied to the sample network. As a result, Table 2 shows the optimal timetables for the first trains operating in the sample network. As observed, 24 directions were analyzed based on 3 transfer stations. Each direction contains information on the transfer passenger volume, first connecting train departure time, first feeder train arrival time, transfer waiting time, and the total transfer waiting time. There are 10 directions with $w_{ll'k} \geq 20$. Nine key directions with $w_{ll'k} \geq 20$ and $p_{ll'k} \cdot w_{ll'k} \geq 100$ are selected by the algorithm, as shown in italic in the table.

The 9 key directions (curved arrows) are illustrated clearly in Fig. 10. To reduce passenger waiting times, 5 bridging buses (dotted arrow lines) are adopted in the figure. The result is a significant reduction in waiting times. For example, two long waiting times (28 min of each) at station A are reduced to 15 min and 3 min by bridging buses respectively. For the entire sample network, without considering the passenger volumes, the total waiting time reduces from 313 min to 141 min, which equates to 54.95% of the time saved. With passenger volumes considered, the total waiting time reduces from 3061 min to 1331 min, which saves 56.5% of the time. The bus bridging strategy shows good efficiency in the morning passenger transport. With the optimal bridging timetables, the total waiting time and passenger waiting times are greatly

Algorithm 1
Procedure of the first train timetabling and bus service bridging.

Input: subway network and all model parameters;
Output: first train timetables and bridging bus timetables;
Initialize $t_l^R = 0,\ t_k^{l'} = 0,\ \forall k \in S(l) \cap S(l'),\ \forall l, l' \in L$;
For $l = 1: n$
    For $k = 1: m$
        Calculate train arrival and departure times;
    End for
End for
Calculate $w_{ll'}^k$ according to Eq. (10);
Solve MILP-M2 to develop the first train timetable;
Return $t_l^R,\ \forall l \in L$ and $w_{ll'}^k,\ \forall k \in S(l) \cap S(l'),\ \forall l, l' \in L$;
If $w_{ll'}^k - \pi \geq 0$
    If $p_{ll'}^k \times w_{ll'}^k - \rho \geq 0$
        Select $\beta_{ll'}^k$ as bus bridging directions into $B_{l'}$;
    End if
End if
For $l' = 1: n$
Abstract $w_{ll'}^k$ with $\beta_{ll'}^k = 1$;
    Calculate passengers volumes by Eq. (21);
    Calculate $Bw_{ll'}^k$ according to Eq. (22);
End for
With the above key directions, solve MILP-M3;
Return $tb_k^{l'},\ \forall k \in S(l) \cap S(l'),\ \forall l, l' \in L$.

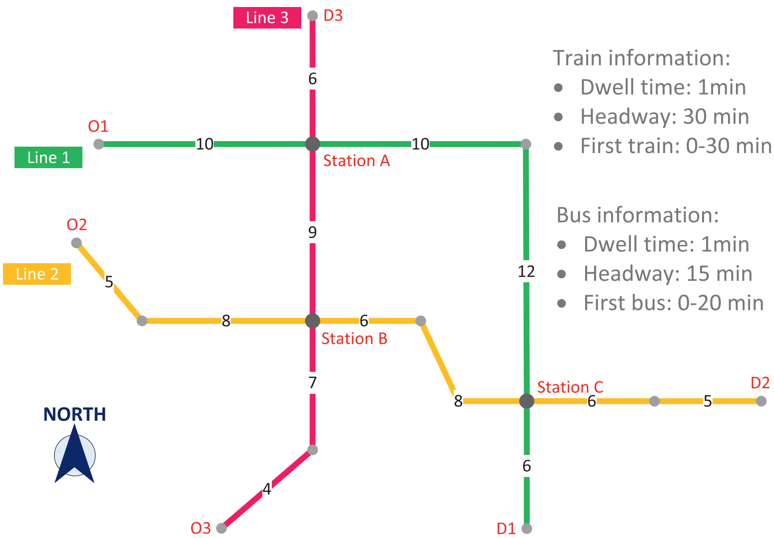

Fig. 9. Map of the sample network.

reduced. Table 3 shows the optimal solutions (routes and timetables) for the bus bridging services. As illustrated, there are 5 bus routes designed. The origins, destinations, departure times and headways are also clearly shown in Table 3.

4. Case study

Fig. 11 shows the map of the Beijing subway network, consisting of 9 bi-directional lines and 31 transfer stations. Note that 303 non-transfer stations are removed from the map. The up-train directions are from the south to the north, from the west to the east, and they are operating in a counter-clockwise direction. For instance, the up-train direction on Line 2 (L2U) starts from the vehicle depot and passes by XiZhiMen (XZM in short), CheGongZhuang, and so forth. A complete first train trip is defined as: the train starts from the vehicle depot of a line and terminates at another depot on the other end of the line (except circle lines). As for Line 13, the XiZhiMen/DongZhiMen station plays the role of vehicle depot. The headway of each line (both up-train and down-train directions) is constant. Except for line 2 and line 10 (5-min headway), all the other lines have a 10-min headway.

Table 2
Optimal solutions for the first train timetabling (min).

No.	Direction	$p^k_{ll'}$	$Dep^{l'}_k$	Arr^l_k	$w^k_{ll'}$	$p^k_{ll'} \cdot w^k_{ll'}$	$Bw^k_{ll'}$	$b^k_{ll'} \cdot Bw^k_{ll'}$
1	L1U-L3U	5	5:34	5:15	17	85	17	85
2	L1U-L3D	10	5:17	5:15	0	0	0	0
3	L1D-L3U	12	5:34	5:34	**28**	**336**	**3**	**36**
4	L1D-L3D	8	5:17	5:34	11	88	11	88
5	L3U-L1U	9	5:16	5:33	11	99	11	99
6	L3U-L1D	20	5:35	5:33	0	0	0	0
7	L3D-L1U	14	5:16	5:16	**28**	**392**	**15**	**210**
8	L3D-L1D	11	5:35	5:16	17	187	17	187
9	L1U-L2U	3	5:37	5:39	25	75	25	75
10	L1U-L2D	7	5:13	5:39	1	7	1	7
11	L1D-L2U	8	5:37	5:10	**24**	**192**	**0**	**0**
12	L1D-L2D	19	5:13	5:10	0	0	0	0
13	L2U-L1U	10	5:40	5:36	1	10	1	10
14	L2U-L1D	21	5:11	5:36	2	42	2	42
15	L2D-L1U	9	5:40	5:12	**25**	**225**	**5**	**45**
16	L2D-L1D	18	5:11	5:12	**26**	**468**	**8**	**144**
17	L2U-L3U	23	5:24	5:20	0	0	0	0
18	L2U-L3D	17	5:27	5:20	3	51	3	51
19	L2D-L3U	7	5:24	5:28	**22**	**154**	**8**	**56**
20	L2D-L3D	4	5:27	5:28	**25**	**100**	**3**	**12**
21	L3U-L2U	16	5:21	5:23	**24**	**384**	**9**	**144**
22	L3U-L2D	20	5:29	5:23	2	40	2	40
23	L3D-L2U	6	5:21	5:26	**21**	**126**	**0**	**0**
24	L3D-L2D	15	5:29	5:26	0	0	0	0
Total		/	/	/	313	3061	141	1331

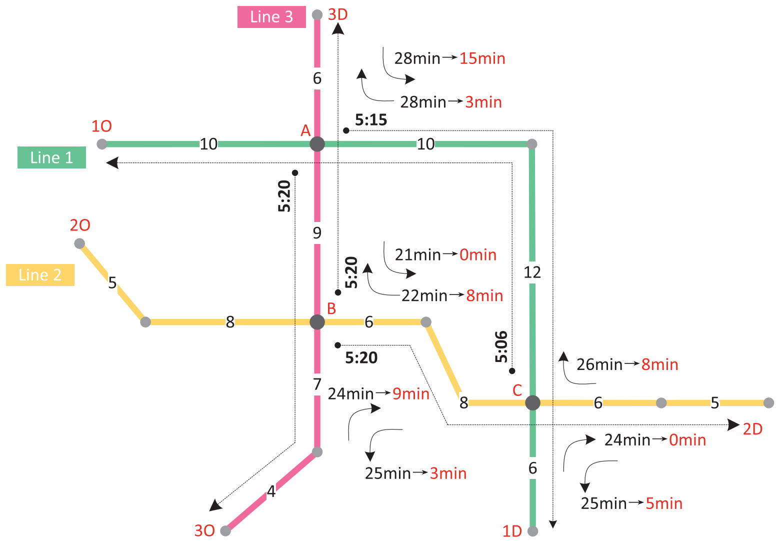

Fig. 10. Map of the sample network.

4.1. Optimal first train and bridging bus timetables

There are a total number of 240 transfer directions in the Beijing subway system. In this case study, the total number of transfer passengers in the network is 2136. Their first train transfer waiting times, according to the original and optimal timetables, are shown in Fig. 12. As illustrated, the first train model is effective in reducing waiting times, which significantly increases the number of directions with smaller $w_{ll'k}$, e.g., $w_{ll'k}$ in [0,5] min from 113 to 134. Concerning larger $w_{ll'k}$ ($w_{ll'k}$ in (10, 20] min and (20, 30] min), the optimal timetables reduce the number of such directions. However, there are 58 directions with larger waiting times, which accounts for 24.2% of the total number of transfer directions, even under the optimal timetables. Besides, we also find that the first train optimization model is not as effective in reducing the long waiting times ($w_{ll'k}$ in (30, 76] min). After experiments, we find that even though we remove constraints (4) and (5), the waiting time is only reduced by 2830 min. Therefore, bridging buses are utilized here.

Considering waiting times and transfer passengers, 21 key directions that $w_{ll'k} \geq 20$ and $p_{ll'k} \cdot w_{ll'k} \geq 100$ are selected from the Beijing subway system, as shown in the first and second columns of Table 4. The third column shows the number

Table 3
Optimal solutions for the bus bridging services.

Bus line	From	To	Departure time	Headway
Bus 1	Station C	Station 1O	5:06	15 min
Bus 2	Station A	Station 1D	5:15	15 min
Bus 3	Station B	Station 2D	5:20	15 min
Bus 4	Station B	Station 3D	5:20	15 min
Bus 5	Station A	Station 3O	5:20	15 min

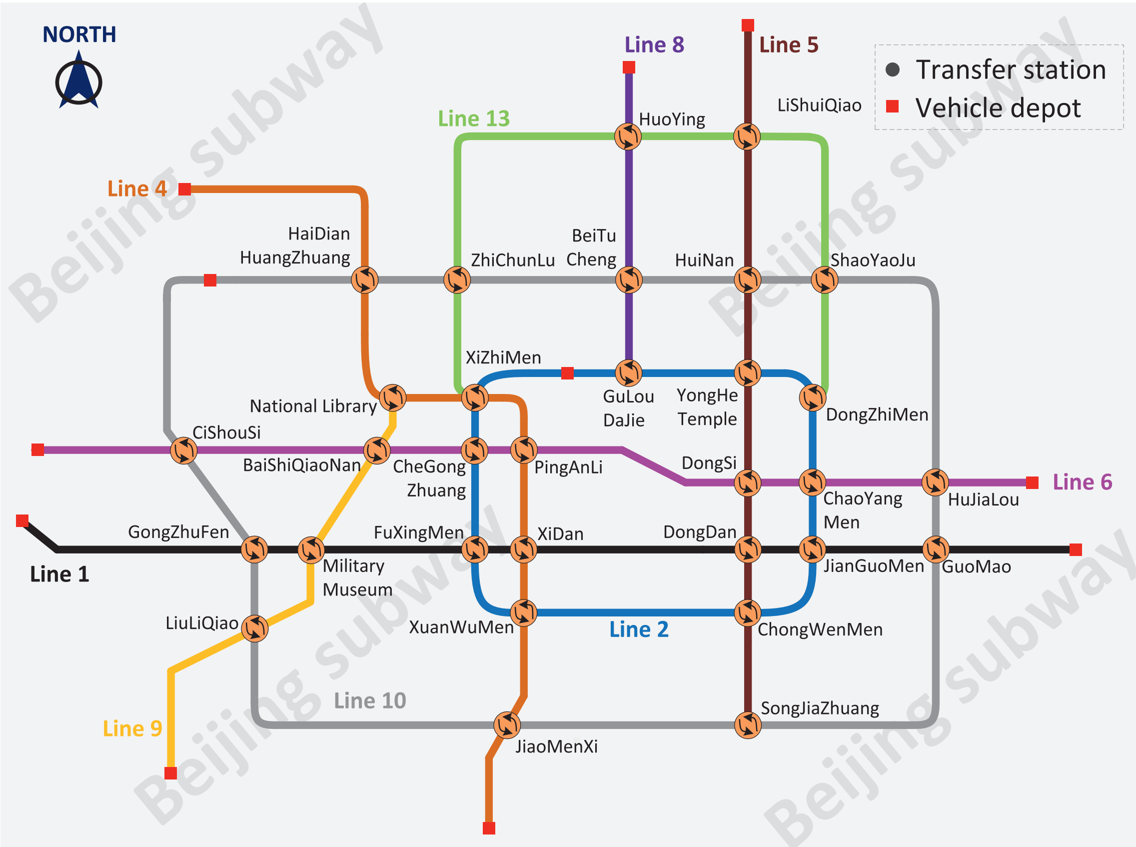

Fig. 11. Map of the Beijing subway system.

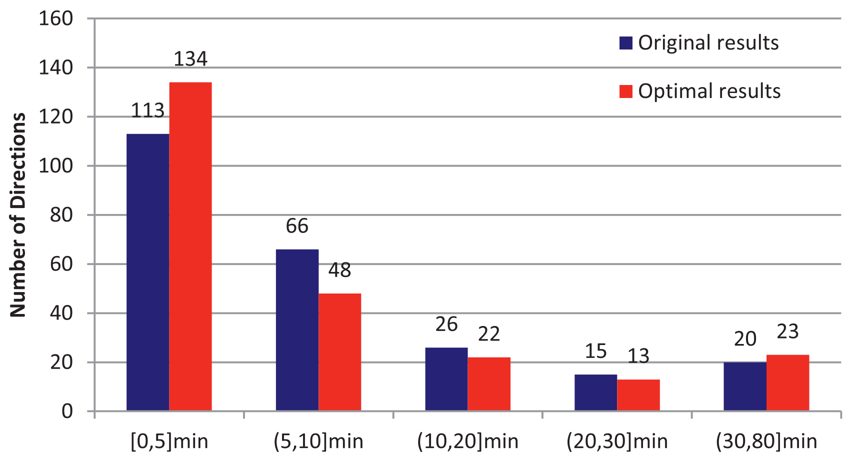

Fig. 12. Distributions of the first train transfer waiting times.

Table 4
Optimal solutions for the first train timetabling and bus bridging timetabling.

Station	Direction	$p^k_{ll'}$	$Dep^{l'}_k$	Arr^l_k	$w^k_{ll'}$	$p^k_{ll'} \cdot w^k_{ll'}$	$tb^{l'}_k$	$Bw^k_{ll'}$	$b^k_{ll'} \cdot Bw^k_{ll'}$
GZF	1-L1U-L10D	15	6:37	5:18	76	1140	5:20	0	0
GZF	2-L10U-L1D	5	5:55	5:20	32	160	5:25:30	13	65
GZF	3-L1D-L10D	7	6:37	5:54	40	280	5:28:30	5	35
MM	4-L9U-L1D	18	5:52	5:09	40	720	5:14	0	0
FXM	5-L2U-L1D	14	5:45	5:19	24.5	343	5:18	7	98
GM	6-L1D-L10U	20	6:13	5:23	45.5	910	5:25	0	0
GM	7-L1D-L10D	15	5:55	5:23	27.5	412.5	6:12	5	75
XZM	8-L2U-L4U	20	5:37	5:12	23	460	5:37	9	180
YHG	9-L2D-L5U	19	5:44	5:08	33	627	5:40	10	190
JMX	10-L4U-L10D	10	6:00	5:13	44	440	5:36:30	15	150
HDHZ	11-L4D-L10U	3	6:21	5:14	63	189	5:40	10	30
HN	12-L10D-L5U	12	5:51	5:23	26.5	318	5:22	4	48
HN	13-L5D-L10U	17	6:08	5:20	46.5	790.5	5:07	3	51
LSQ	14-L5D-L13U	6	5:37	5:06	28	168	5:18	2	12
LSQ	15-L5D-L13D	11	5:48	5:06	39	429	5:16	2	22
CSS	16-L6U-L10D	3	6:24	5:22	59	177	5:46	10	30
HJL	17-L6D-L10U	17	5:55	5:27	25	425	5:35:30	15	255
BTC	18-L8D-L10U	4	6:11	5:26	41	164	5:40	15	60
HY	19-L8D-L13D	7	5:43	5:06	33.5	234.5	5:09	0	0
LLQ	20-L9U-L10D	20	6:15	5:11	61	1220	5:10	0	0
LLQ	21-L9D-L10D	12	6:15	5:33	39	468	5:18:30	9	108
Total		255	/	/	847	10,075.5	/	134	1409

Table 5
Optimal bus bridging routes.

Routes	Serving line	Time	No. of buses	Headway	From	Passing by and to stations
#1	L1D	5:20	2	15 min	FXM	MM and GZF
#2	L4U	5:14	2	15 min	XZM	Vehicle depot
#3	L5U	5:18	2	15 min	YHG	HN
#4	L10U	5:02	4	15 min	GZF	GM, HJL, HN, BTC, and HDHZ
#5	L10D	5:00	4	15 min	GM	JMX, LLQ, GZF, and CSS
#6	L13U	5:09	2	15 min	LSQ	Vehicle depot
#7	L13D	5:10	3	15 min	HY	LSQ

of transfer passengers. The arrival and departure times of the feeder and connecting trains are given in the fourth and fifth columns. As a result, the first train waiting times without passenger volumes are illustrated in the sixth column. The next column represents the waiting times with passenger volumes considered. The last three columns report the results of the bus bridging model.

We report the optimal bus bridging routes and timetables in Table 5. As observed, there are seven routes returned. For each route, we give the service time, the number of dispatched buses, headway, starting station, intermediate stations, and final station. Waiting times saved by the bridging buses are illustrated in Fig. 13, where "$x \to y$" represents the comparison results of waiting times. Herein, "x" indicates a waiting time with the optimal first train timetable, and "y" indicates a waiting time with the optimal bus bridging timetable. All long waiting times are shortened effectively.

In addition, we give the waiting times using different timetables for two cases in Fig. 14. Considering passenger volumes, the optimal first train timetables reduce the total waiting time from 22,790 min to 19,295 min, and the optimal bus bridging timetables decrease the total waiting time by 53.4% (with 27 bridging buses on 7 routes), from a base of 22,790 min to 10,628.5 min. Without considering passenger volumes, the total waiting time is decreased from 2562 min to 2273 min, and further to 1560 min. In this way, we can directly find the effectiveness of the model without the impacts of daily fluctuation of passenger data.

4.2. Analysis of constraints of bus departure times

In this subsection, we turn our attention to the constraints of bus bridging departure times – Eq. (25), which could be relaxed to release the limits on bridging buses. Without Eq. (25), the bridging buses can operate at any time of the day (i.e., from 00:00 to 24:00). In this way, we can test how much transfer waiting time can be reduced only by optimizing the first train timetables.

Fig. 15 shows the test results. The biggest bar chart provides the bus bridging departure times with and without Eq. (25), where the x-axis shows 7 bus routes and the y-axis shows the departure times of bridging buses. As shown, most lines have similar results with or without limits, except route #4. The bridging bus for route #4 originally departs at 5:02:00. After relaxing Eq. (25), it now departs at 4:43:00, which equates to a 19-min gap. The waiting time results are also provided in Fig. 15. First, we remove the passenger information. The results indicate that the total waiting time is 134 min with

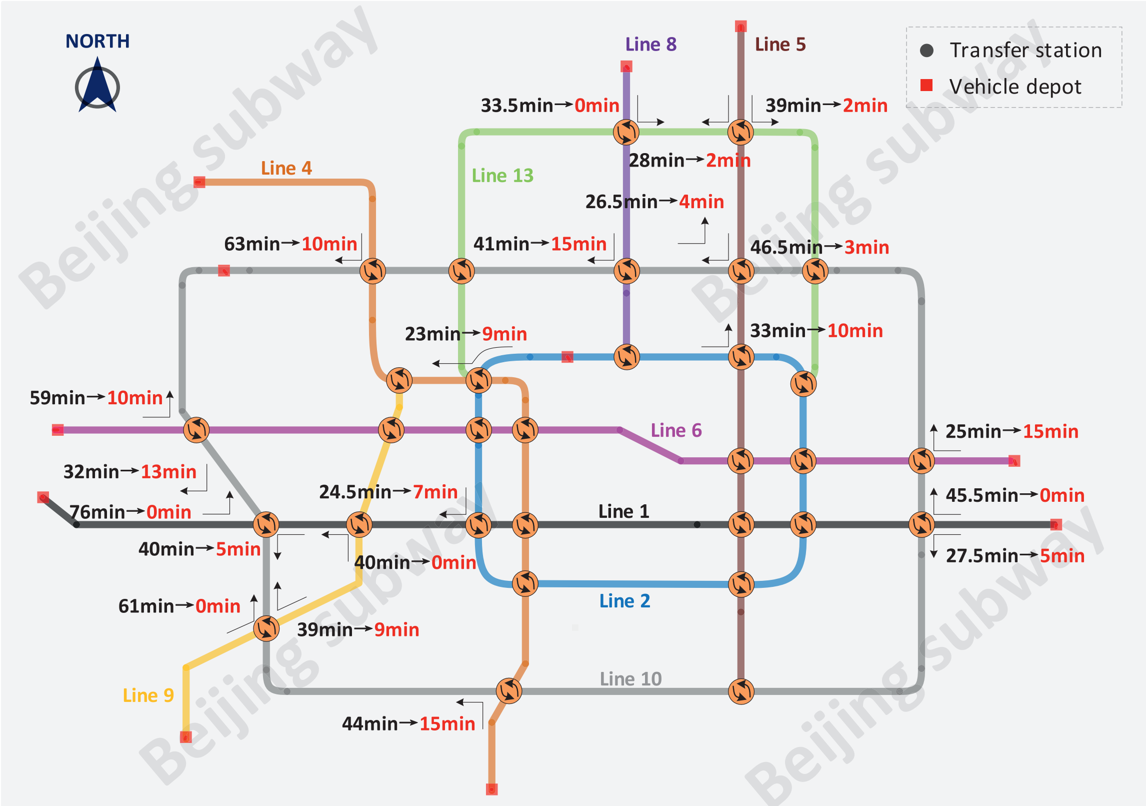

Fig. 13. Reduction of transfer waiting times by bridging buses.

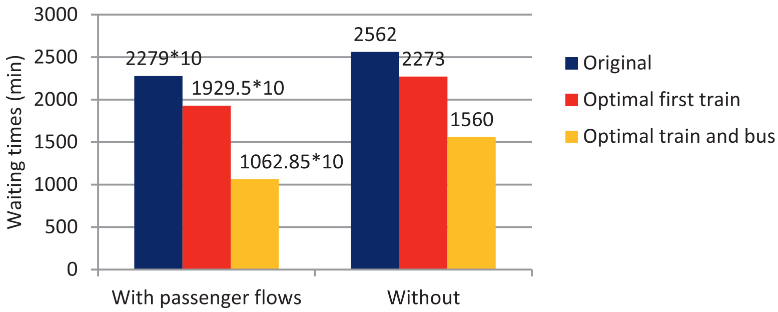

Fig. 14. Comparison between the original and optimal timetables.

Eq. (25) and is 129 min without Eq. (25). Then, we add passenger volumes in the program, and the total waiting time is 1409 min with start time limits and is 1320 min without start time limits. Therefore, we can conclude that the bus bridging departure time constraints can also be relaxed when designing timetables for real operations.

4.3. Impacts of the number of routes and bridging buses on transfer waiting times

On a typical day, the Beijing Bus Group runs more than 160,000 buses in the entire public transport network. Adding bridging buses will increase the operational costs. Therefore, there is a need to consider the operator's point of view as well as passenger service aspects. Table 6 shows the numerical analysis of routes, buses, fuel costs, operating times, and waiting times. We decreased the number of bus bridging routes from 7 to 1. As a result, the number of bridging buses was reduced from 27 to 7 correspondingly. When we run 7 bus bridging routes, the total fuel cost is 3863.7 Yuan for 53.7 h of operation. If we consider the hourly bus operating cost to be 650 Yuan (empirical data), the total operating cost is 34,905 Yuan, which is acceptable to the public transport operators in large cities like Beijing and Hong Kong. With each bus route and its bridging buses, the total saved waiting time is shown in the sixth column of Table 6. The waiting time improvement, the average time saving per bus, and the marginal saving on each additional route are illustrated in the last three columns.

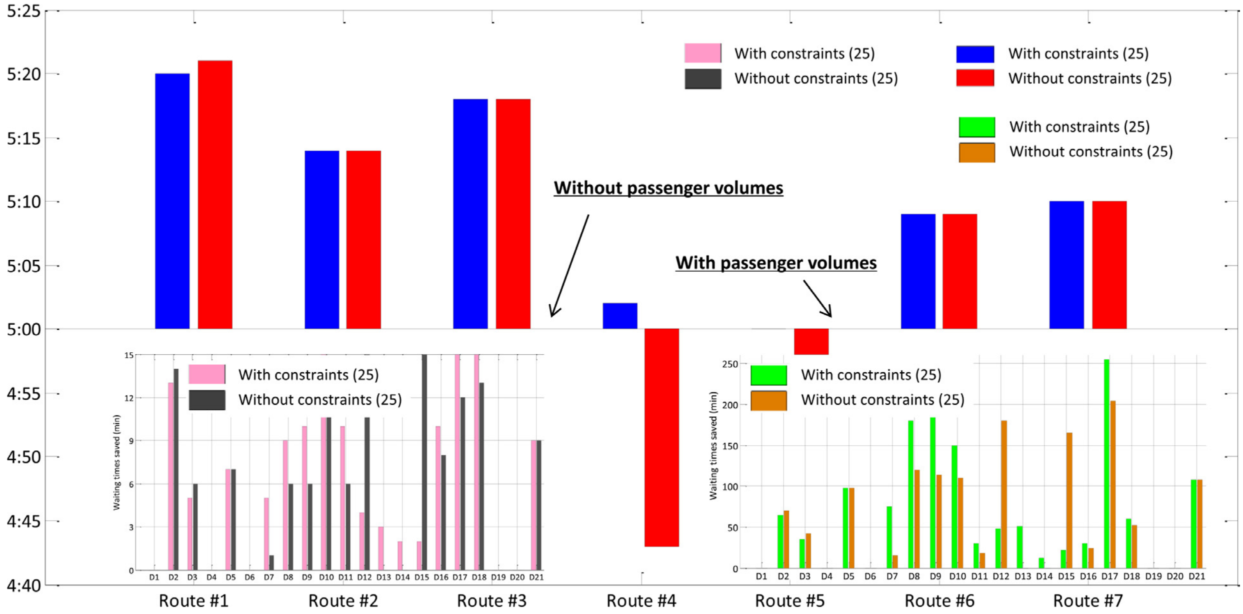

Fig. 15. Results and impacts of constraints (25) on passenger waiting times.

Table 6
Analysis of bus fuel costs and transfer waiting time reductions.

Number of bus routes	Route mileage (km)	Number of bridging buses	Fuel cost (RMB)*	Bus operating time (h)#	Waiting time savings (min)	Improving based on 22,790 min of waiting time	Average savings per bus (min)	Marginal savings on each additional route (RMB)%
7	79.5	27	3863.7	53.7	8666.5	38.0%	321.0	195.0
6	54.2	25	2439.0	33.9	8510.5	37.3%	340.4	350.0
5	40.0	23	1656.0	23.0	8230.5	36.1%	357.8	801.9
4	35.2	20	1267.2	17.6	7589.0	33.3%	379.5	883.8
3	31.1	17	951.7	13.2	6882.0	30.2%	404.8	2909.4
2	20.5	10	369.0	5.1	4554.5	20.0%	455.5	1325.0
1	16.4	7	206.6	2.9	3494.5	15.3%	499.2	4368.1

* Based on https://www.chinabuses.com/buses/2015/1201/article_67433.html, the fuel cost of each bus is 1.8 RMB/km approximately.

The fastest bus speed is 40 km/h based on traffic regulations of China cities (in the morning, bridging buses can reach the fastest speed). Based on different hourly bus operating costs (for example, the bus operating costs in metropolises likes Beijing, Shanghai, and some US cities range from 650 RMB to 1300 RMB empirically), the total operating cost can be estimated.

% Based on Tirachini et al. (2010), the value of waiting time savings is equivalent to 75 RMB per hour. Thus, the waiting time can be converted to a monetary unit.

Fig. 16 shows the relationship between the number of additional routes, bus operating time, and waiting time improvement. Adding three bus bridging routes (with 13.2 h of operating time in total) can reduce the total waiting time by 33.3%. With more bus routes, the effect on reducing waiting times is not as prominent (reducing by 4.7% with another four routes). Therefore, operators can make a reasonable decision based on the results discussed above. Compared to increasing subway frequency (from 10-min headway to 5-min headway), the increased number of trains running in the network is 14 (serving on L1U, L1D, L4U, L4D, L5U, L5D, L6U, L6D, L8U, L8D, L9U, L9D, L13U, and L13D). This case reduces the transfer waiting time to 18,019 min (19,295 min for the 10-min headway, saving waiting time by 1276 min). Hence, it is cost-effective to dispatch the bridging buses, compared to increasing train frequency.

4.4. Analysis of key directions

As observed from the model, bridging buses are utilized and dispatched for the key directions in the network. That is, the number and the distribution of key directions partly determine the utilization of bridging buses. Therefore, we discuss the definition of key directions in this subsection. First, we divide passenger waiting times into 7 groups based on their values, as shown in the x-axis of Fig. 17. After this, the values of waiting times, without considering passenger volumes, are treated as benchmarks. The number of key directions for each scenario selected by the program is shown in Fig. 17.

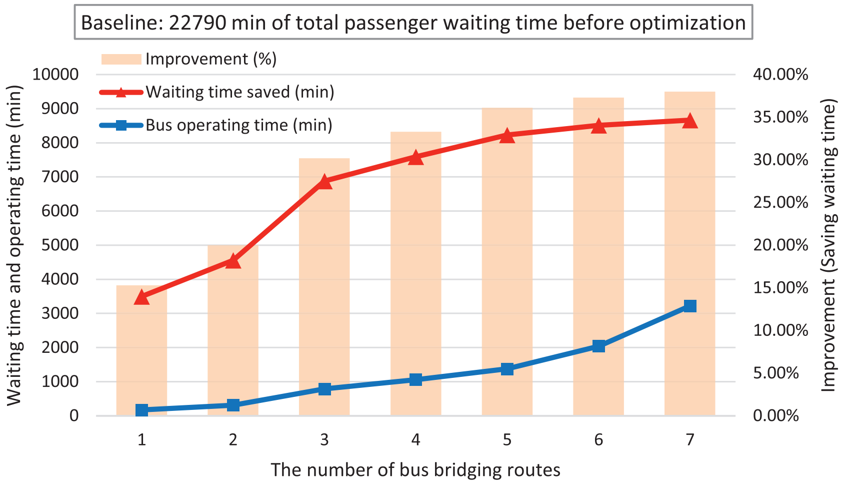

Fig. 16. Waiting time reduction with different numbers of bus bridging routes and bus operating times.

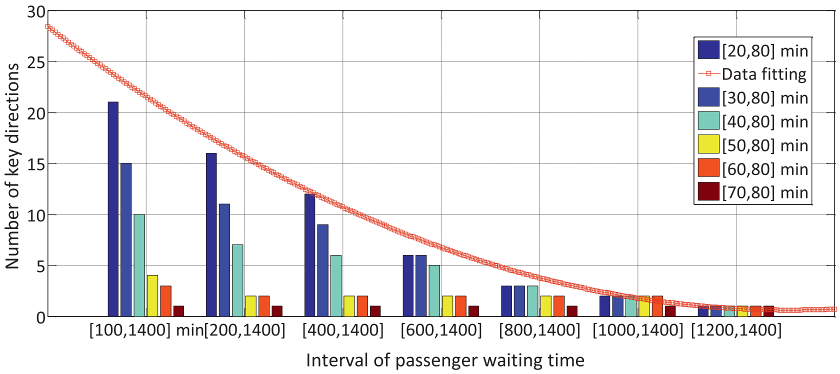

Fig. 17. Components of key directions.

Table 7
Total waiting times defined by different key directions.

Total waiting time (min)	Without passenger volumes					
With passenger volumes	[20, 80]	[30, 80]	[40, 80]	[50, 80]	[60, 80]	[70, 80]
[100, 1400]	10,075.5	7949	6030.5	2726	2549	1220
[200, 1400]	9217.5	7259	5500.5	2360	2360	1220
[400, 1400]	8042	6744.5	5220.5	2360	2360	1220
[600, 1400]	5407.5	5407.5	4780.5	2360	2360	1220
[800, 1400]	3270	3270	3270	2360	2360	1220
[1000, 1400]	2360	2360	2360	2360	2360	1220
[1200, 1400]	1220	1220	1220	1220	1220	1220

As observed, the trend of the number of key directions is decreasing with the increasingly “strict” standards. Moreover, the corresponding first train waiting times of all scenarios are given in Table 7. These waiting times represent the time intervals bridging buses should have to further reduce. Thus, the stricter the standards, the less the necessity of deploying bridging buses.

5. Conclusions

In a large subway network, one urgent problem is the first train transfer problem, which results in a long waiting time for early morning passengers when they transfer. This paper developed a bus bridging approach to cooperate with the first train services. First, we established two first train and bus bridging timetabling models to minimize transfer waiting times for early-morning passengers. This allows well-timed connection transfers in large subway systems. Then, an exact algorithm for the MILP models was designed. Finally, the models and the algorithm were applied to the Beijing subway network, which demonstrated the computational efficiency of the exact models and the approach.

The approach proposed above can be improved in several ways. For instance, on different days (workdays, weekends, and holidays) and for different periods in a day (peak hours and non-peak hours), passenger demands can be defined with weights, which would be assigned to different periods. This changes the problem to a dynamic weighted optimization problem. In this paper, we got the data of passenger volumes from the Beijing subway company to make the study more practical. Moreover, the bus bridging model could be improved to a more comprehensive demand-based model from the first bus time to peak periods (or even an entire day). In this case, bus headways of each route can be decreased gradually by inserting additional buses based on various traffic demands. Other approaches such as real-time synchronization of trains and buses can also be considered for further studies. Moreover, the discussion is open on whether to consider the bus travel time in the objective function. For large cases such as Beijing subway and Shanghai metro, first train transfer waiting times are much longer than bridging bus travel times. In this case, the waiting times dominate the total journey time. Hence, bus travel times can be ignored in the objective function. However, for small networks, it is worth optimizing bus travel times together with the transfer waiting times. Finally, approaches such as Benders decomposition and branch-and-price algorithms may be efficient to solve the large-scale first train timetabling and bus service bridging problems. With an increase in problem sizes, heuristics can also be applied.

Authorship statement

Liujiang Kang: Conceptualization, Methodology, Software, Investigation, Writing - original draft, Writing - review & editing, Funding acquisition. Hao Li: Conceptualization, Methodology, Writing- Original draft preparation, Writing - review & editing. Huijun Sun: Conceptualization, Methodology, Investigation, Writing- Original draft preparation, Writing - review & editing, Funding acquisition. Jianjun Wu: Writing- Original draft preparation, Writing- Reviewing and Editing. Zhiguang Cao: Conceptualization, Methodology, Writing- Original draft preparation, Writing- Reviewing and Editing. Nsabimana Buhigiro: Writing- Reviewing and Editing.

Declaration of Competing Interest

No potential competing interest was reported by the authors.

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Appendix A

As shown in Fig. 18, the origin is the WSL station and the destination is the CSS station. For the first train – first train mode, the total travel time is 61 min (without taking into account the transfer walking time at each station). Passengers take the first train on Line 1 U at the WSL station and head to the GZF station, which takes 2 min by train (arriving at 5:14). Then, they have to wait at the GZF station for nearly 54 min as the first train on Line 10D arrives at the GZF station at 6:08. Finally, it takes passengers 5 min to ride the first train on Line 10D from the GZF station to the CSS station. For

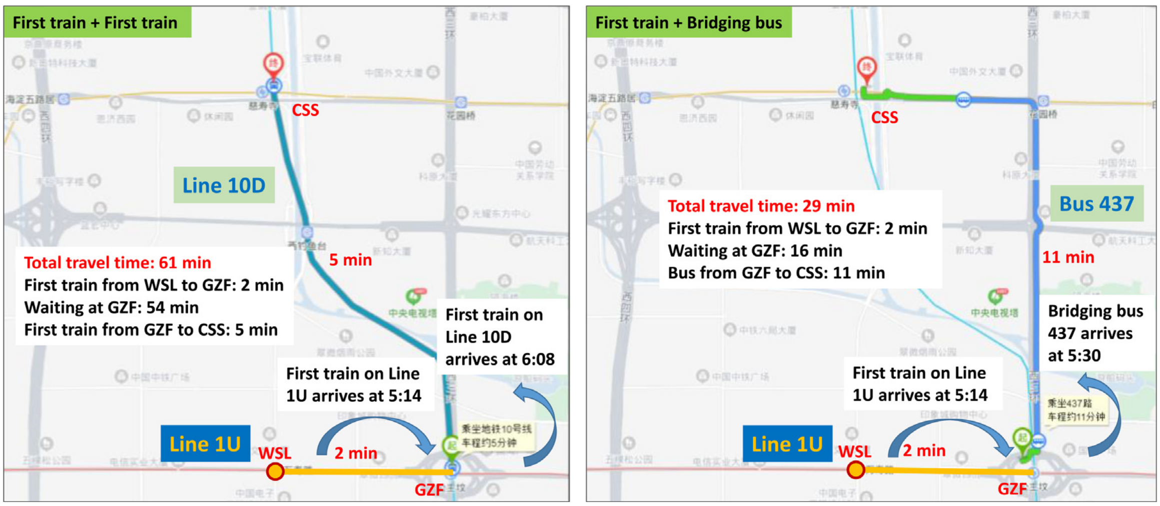

Fig. 18. A comparison between the first train – first train travel and first train – bridging bus travel (data from the Beijing subway in 2021).

the first train – bridging bus mode, the total travel time is 29 min (without taking into account the transferring passengers walking time at each station). They take the first train on Line 1 U at the WSL station and head to the GZF station, which takes 2 min by train (arriving at 5:14). Then, they wait at the GZF station for 16 min as the bridging bus starts service at the GZF station at 5:30. Finally, it takes passengers 11 min to ride the bridging bus from the GZF station to the CSS station.

Appendix B

Proposition 1. *Algorithm 1 solves the MILP-4 model to global optimality*.

Proof. Consider an optimal solution of MILP-4 as $w_{ll'}^{k*}$ and $Bw_{ll'}^{k*}$. Let T_{opt} be the minimum transfer waiting time in the intermodal bus-and-train transit network. The objective function in Eq. (28) can be transformed into Eq. (29):

$$T_{opt} = \min \sum_{\forall l \in L} \sum_{\forall l' \in L} \left[\sum_{k \in S(l) \cap^S (l')} (p_{ll'k} - b_{ll'k}) \cdot w_{ll'}^{k*} + \sum_{k \in S(l')} b_{ll'k} \cdot Bw_{ll'}^{k*} \right] \tag{29}$$

T_{opt} is less than or equal to T'_{opt}, which is defined by Eq. (30) because $\forall l,\ l' \in L$, we have $S(l') \supset \{S(l) \cap^S (l')\}$. Therefore, the relationship $T_{opt} = T'_{opt}$ holds under the condition that $S(l') = \{S(l) \cap^S (l')\}$ in Eq. (29). Therefore, to prove $T_{opt} = T'_{opt}$, it is necessary to show $S(l') = \{S(l) \cap^S (l')\}$.

$$T'_{opt} = \min \sum_{\forall l \in L} \sum_{\forall l' \in L} \sum_{k \in S(l) \cap^S (l')} [(p_{ll'k} - b_{ll'k}) \cdot w_{ll'k} + b_{ll'k} \cdot Bw_{ll'k}] \tag{30}$$

Note that $\forall k \in S(l) \cap^S (l'), \forall l, l' \in L,\ b_{ll'}^k = p_{ll'}^k - (1 - \beta_{ll'}^k) \cdot p_{ll'}^k$. Thus, we have

$$T'_{opt} = \min \sum_{\forall l \in L} \sum_{\forall l' \in L} \sum_{k \in S(l) \cap^S (l')} [(p_{ll'k} - b_{ll'k}) \cdot w_{ll'k} + \beta_{ll'k} \cdot p_{ll'k} \cdot Bw_{ll'k}] \tag{31}$$

To solve Eq. (31), it is equivalent to solving Eq. (32):

$$T'_{opt} = \min \sum_{\forall l \in L} \sum_{\forall l' \in L} \sum_{k \in S(l) \cap S(l')} [(p_{ll'k} - b_{ll'k}) \cdot w_{ll'k}] + \sum_{\forall l \in L} \sum_{\forall l' \in L} \sum_{k \in S(l) \cap S(l')} [\beta_{ll'k} \cdot (p_{ll'k} - b_{ll'k}) \cdot Bw_{ll'k}] \tag{32}$$

When $\beta_{ll'}^k \in B_{l'}$ holds, we can get $b_{ll'}^k = \beta_{ll'}^k \cdot p_{ll'}^k$, indicating that the passengers from line l to line l' at station k will take the bridging buses. In this case, for any $k \in S(l')$, k represents a transfer station on line l', namely $k \in \{S(l) \cap^S (l')\}$. Thus, when $\beta_{ll'}^k \in B_{l'}$, $S(l') = \{S(l) \cap^S (l')\}$ holds.

In this case, the following Eq. (33) can be decomposed into Eq. (34).

$$T'_{opt} = \min \sum_{\forall l \in L} \sum_{\forall l' \in L} \sum_{k \in S(l) \cap^S (l')} (p_{ll'k} - b_{ll'k}) \cdot w_{ll'k} + \sum_{\forall l \in L} \sum_{\forall l' \in L} \sum_{k \in S(l) \cap^S (l')} b_{ll'}^k \cdot Bw_{ll'k} \tag{33}$$

$$T'_{opt} = \min \sum_{\forall l \in L} \sum_{\forall l' \in L} \sum_{k \in S(l) \cap^S (l')} (p_{ll'k} - b_{ll'k}) \cdot w_{ll'k} + \min \sum_{\forall l \in L} \sum_{\forall l' \in L} \sum_{k \in S(l) \cap^S (l')} b_{ll'}^k \cdot Bw_{ll'k} \tag{34}$$

Based on the special condition $S(l') = \{S(l) \cap^S (l')\}$, we finally obtain $T_{opt} = T'_{opt}$. Thus, the developed Algorithm 1 solves the MILP-4 model to global optimality.

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