10-2021

Improving the performance of transportation networks: A semi-centralized pricing approach

Zhiguang CAO
Singapore Management University, zgcao@smu.edu.sg

Hongliang GUO

Wen SONG

Kaizhou GAO

Liujiang KANG

See next page for additional authors

Citation

CAO, Zhiguang; GUO, Hongliang; SONG, Wen; GAO, Kaizhou; KANG, Liujiang; ZHANG, Xuexi; and WU, Qilun. Improving the performance of transportation networks: A semi-centralized pricing approach. (2021). *IEEE Transactions on Intelligent Transportation Systems*. 22, (10), 6353-6364.
Available at: https://ink.library.smu.edu.sg/sis_research/8124

Improving the Performance of Transportation Networks: A Semi-Centralized Pricing Approach

Article *in* IEEE Transactions on Intelligent Transportation Systems · October 2021

DOI: 10.1109/TITS.2020.2991759

7 authors, including:

Zhiguang Cao
Singapore Management University

Hongliang Guo
Agency for Science, Technology and Research (A*STAR)

Wen Song
Shandong University

Kaizhou Gao
Macau University of Science and Technology

Improving the Performance of Transportation Networks: A Semi-Centralized Pricing Approach

Zhiguang Cao, Hongliang Guo, Wen Song, Kaizhou Gao, Liujiang Kang, Xuexi Zhang, and Qilun Wu

***Abstract*— Improving the performance of transportation network is a crucial task in traffic management. In this paper, we start with a cooperative routing problem, which aims to minimize the chance of road network breakdown. To address this problem, we propose a subgradient method, which can be naturally implemented as a semi-centralized pricing approach. Particularly, each road link adopts the pricing scheme to calculate and adjust the local toll regularly, while the vehicles update their routes to minimize the toll costs by exploiting the global toll information. To prevent the potential oscillation brought by the subgradient method, we introduce a heavy-ball method to further improve the performance of the pricing approach. We then test both the basic and improved pricing approaches in a real road network, and simultaneously compare them with several baselines. The experimental results demonstrate that, our approaches significantly outperform others, by comprehensively evaluating them in terms of various metrics including average travel time and travel distance, winners and losers, potential congestion occurrence, last arrival time, toll costs and average traffic flows, with two different O-D profiles.**

***Index Terms*— Traffic control, public transportation, vehicle routing, path planning.**

Manuscript received October 26, 2018; revised May 14, 2019, October 14, 2019, March 25, 2020, and April 22, 2020; accepted April 28, 2020. Date of publication May 21, 2020; date of current version October 4, 2021. This work was supported in part by the National Natural Science Foundation of China under Grant 61803104, in part by the Fundamental Research Funds for the Central Universities under Grant 63192616, in part by the Fundamental Research Funds of Shandong University under Grant 62420079614084, in part by the Science and Technology Project of Shenzhen Power Supply Bureau under Grant 090000KK52180114, in part by the Science and Technology Planning Project of Guangzhou City, China, under Grant 201802020028, in part by the MOE Academic Research Fund of Singapore under Grant R266000096133, Grant R266000096731, and Grant MOE2017-T2-2-153, in part by the Singapore National Research Foundation under Grant NRF-RSS2016004, and in part by the Zhejiang Lab's International Talent Fund for Young Professionals. The Associate Editor for this article was F. Viti. *(Corresponding author: Hongliang Guo.)*

Zhiguang Cao is with the Department of Industrial Systems Engineering and Management, National University of Singapore, Singapore 119077 (e-mail: isecaoz@nus.edu.sg).

Hongliang Guo is with the School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu 610054, China (e-mail: guohl1983@uestc.edu.cn).

Wen Song is with the Institute of Marine Science and Technology, Shandong University, Qingdao 266237, China (e-mail: wensong@email.sdu.edu.cn).

Kaizhou Gao is with the Macau Institute of Systems Engineering, Macau University of Science and Technology, Macau 999078, China (e-mail: kzgao@must.edu.mo).

Liujiang Kang is with the Key Laboratory of Transport Industry of Big Data Application Technologies for Comprehensive Transport, Ministry of Transport, Beijing Jiaotong University, Beijing 100044, China (e-mail: ljkang@bjtu.edu.cn).

Xuexi Zhang and Qilun Wu are with the School of Automation, Guangdong University of Technology, Guangzhou 510006, China (e-mail: zxxnet@gdut.edu.cn; wu_qilun@hotmail.com).

Digital Object Identifier 10.1109/TITS.2020.2991759

I. Introduction

ADDRESSING the issue of congestion has been recognized as a primary task for traffic management, as well as for the sustainable development of many cities [1]. It draws extensive attentions from industry, research community and city authority, since it directly affects peoples' daily life in various ways [2]. Traffic congestion always results in undesired traffic delays, which not only displease the commuters with respects to traveling experience, but also engender an increase in fuel consumption and economic loss. Particularly, the fuel consumption from the transportation systems plays a large role in urban greenhouse gases (GHG) emissions, which causes inestimable impacts to the environments [3], [4]. To alleviate the traffic congestion impacts, a lot of strategies have been proposed, which mainly focus on providing efficient route guidance for single independent vehicles [5]–[8], and multiple cooperative vehicles [9]–[12], respectively. In view of the potential benefits to the whole transportation network and social welfare, the latter strategies are widely adopted and applied.

Various cooperative routing solutions have been studied. Papageorgiou [13] proposed a general framework for traffic dynamic modelling and control, which can integrate with traffic assignment. Within this framework, dynamic system optimum and user optimum can be achieved by designing corresponding control strategies, respectively. Adler and Blue [14] developed a cooperative multiagent transportation management and route guidance system to reduce the average travel time of the whole transportation network. In the system, they fostered the scheme of interaction and cooperation between network operator agents and vehicle agents. The results demonstrated that the system had desirable capability in terms of scalability, and the performance for the transportation network was significantly improved by an information provider agent. Yamashita *et al.* [15] proposed a centralized multiagent routing approach, which aimed to cooperatively minimize the expected travel time for all vehicle agents. In this approach, a vehicle agent was assumed to be willing to share its real time information (i.e., current location, destination, and the expected remaining route to destination) with a central server. Then the central server assigned a higher weight to a block of road link if more vehicles were likely to pass by exploiting the route information it collected. Thus, the route guidance for each vehicle could be derived through minimizing the corresponding weights. Chow *et al.* [16] proposed a cooperative and adaptive traffic control method implemented in a decentralized way. They adopted a linear quadratic formulation based upon a cyclic store-and-forward model, given the need of computational effectiveness for real time application. Moreover, the decentralized control also incorporated an user-optimal re-routing algorithm with purpose to improve the performance through better utilizing network capacity. Knoop *et al.* [17] investigated four cooperative routing strategies based on macroscopic fundamental diagram (MFD). The results based on simulation demonstrate that the situation without control performs worst, and the situation with control in conjunction with full speed information performs the best. Moreover, all situations with control based on variables aggregated over a sub-network generate performance in between. He *et al.* [18] proposed a cooperative routing approach to avoid potential congestion by exploring large-scale social signals obtained from two kinds of devices, namely, mobile phone and subway card. In this approach, two types of routing strategies were considered, i.e., shortest path and minimum cost, the latter of which included travel time and cost related to congestion. Kordonis *et al.* [19] proposed a mechanisms for cooperative freight routing, which adopted monetary incentives to balance the traffic load and alleviate the time delays experienced by both truck and passenger vehicle drivers. With the assumption of voluntary participation, the presented scheme was budget-balanced, and it did not penalize the truck drivers compared to the user equilibrium. Desai *et al.* [20] proposed a practical multiagent-based approach to cooperatively achieve desired route allocation, which was supposed to minimize the expected travel time of all the routes. More specifically, in this approach, vehicle agents (VAs) propagated important traffic information by using inter-vehicular communication and then undertook its distributed processing. VAs exchanged their route preference information to achieve an initial allocation of routes. The allocation was then further improved via successive virtual negotiation "deals". Yildirimoglu *et al.* [21] developed a network-level traffic management scheme based on MFD (Macroscopic Fundamental Diagram), which aims to mitigate congestion in urban areas by considering the effect of route choice at an aggregated level. Particularly, this scheme includes a route guidance system that advises drivers a sequence of subregions to assist them in reaching their destination. Wang *et al.* [22] presented a multiagent system to avoid the potential traffic congestion en-route. In this system, algorithm of NRR (Next Road Rerouting) was conceived to distribute the vehicle agents affected by an en-route event, to the sub-optimal paths. The rerouting strategy took into account four crucial real time factors, which were supposed to help in reducing the average travel time in comparison with the scenario without using NRR. Menelaou *et al.* [23] presented a low-complexity route reservation scheme to control the road congest, which decomposes the road infrastructure into slots in the spatial and temporal domains for each vehicle. With this scheme, vehicles could either be delayed at their origin or are routed through longer but congestion-free routes so that their traveling time would be minimized. Menelaou *et al.* [24] improved the route reservation approach by solving two problems under MFD, which are the earliest destination arrival time problem and the traffic load balancing problem, respectively. Particularly, the former aims to minimize the travel time for all vehicles while the road links they travelled through should not exceed the critical density, and the latter aims to minimize the variance of densities across the road network, both of which are solving using dynamic programming solution. Sirmatel and Geroliminis [25] proposed a network-level economic MPC scheme to improve mobility in urban road networks, which integrates perimeter control and regional route guidance. In particular, this scheme formulates the problem of finding the perimeter control and route guidance inputs to a multi-region urban network to minimize total time spent (TTS) as an economic MPC problem.

Although a lot of success has been achieved for cooperative routing, most of them directly optimize the metric of expected travel time or travel distance. Since traffic is always random and dynamic, using simple mean values of travel time or travel distance as the optimization objective may not handle well the stochastic nature of the traffic. For instance, the routes of least expected travel time with a large variance (refers to risk in this context) may not make sense in practice. On the other hand, probability is usually deemed as a robust metric to measure a dynamic or stochastic process, as it always takes into account both mean value and variance (or standard deviation) [26], [27]. In view of this, we focus on minimizing the probability of breakdown to improve the performance of the whole transportation network in this paper.

To arrive at this goal, we first leverage a static and centralized cooperative vehicle routing model [28], which considers the breakdown probability of each road link, and the decision variable is the assigned traffic load to a road link. To make the solution much more practical, we further equivalently reformulate the problem so that the decision variable is converted into the assigned routes for vehicle, while the objective becomes minimizing all the possible violations for each road link, which may lead to potential breakdown. Afterwards, rather than solving the optimization problem in one shot, we propose a subgradient method to iteratively solve the problem with expectation to better tackle the traffic dynamics. More importantly, by doing so, the subgradient method can be naturally implemented as a semi-centralized pricing approach: one of the decision variables in the subgradient method can be endorsed with a physical meaning, i.e., the toll for a each road. Thus, each road link may update the local toll regularly according to the rules of subgradient method, while the vehicles adjust their routes to minimize the toll costs by exploiting the latest information of global tolls. We would like to note that, various pricing or tolling solutions have been studied [29]–[33]. Our approach differs from them in that, our original goal has nothing to do with tolls directly. To summarize, our contributions are stated as follows:

1) We aim to improve the performance of road networks by extending a centralized and static optimization model in [28] that considers reducing the chance of road network breakdown. In specific, we reformulate the problem and solve it by proposing a sub-gradient method,

so that the model turns to be semi-centralized and capable of coping with traffic dynamics.

2) We theoretically proved that the proposed sub-gradient method could guarantee convergence.
3) To apply the sub-gradient method into practice in an iterative way, we endorse the parameters in sub-gradient method with physical meanings, which further turns to be a dynamic pricing approach by complying certain assumptions. And experimental results demonstrated its superiority to the baselines.

The remainder of the paper is organized as follows. Section II elaborates the problem definition and transformation. Section III develops an adaptive semi-centralized pricing solution, and an improvement based on the heave-ball method is proposed as well. Section IV demonstrates the comprehensive experimental results and analysis, and the best setting for the solution is concluded and recommended as well. Section VI concludes the paper and states the future works.

II. Problem Definition and Transformation

In this section, we first formulate a cooperative routing problem aiming at minimizing the probability of breakdown for the whole road network, by optimizing the traffic load for each road link. Then we reformulate the problem as a cardinality minimization so that the decision variable is converted from traffic load into the routes for each vehicle.

A. Formulation as Minimizing Breakdown Probability

Given a transportation network represented as a directed graph G, i.e., $G = (E, V)$, E denotes the set of edges (i.e., road links), and V denotes the set of nodes (i.e., intersections). In [34], *breakdown* refers to a local first-order phase transition from free flow to synchronized flow. According to the traffic flow theory in [34], the optimum of a transportation network is achieved when dynamic traffic control is performed in such a way that the probability for the breakdown occurrence in any road links reaches the minimum possible value. This is equivalent to maximizing the probability that none traffic breakdown occurred in any road link [28]. And we would like to note that, readers may refer to [28], [34] for more details about the definition of breakdown. Consequently, the cooperative routing problem can be formulated as:

$$\max_{\hat{\boldsymbol{x}}} \ln\left(\prod_{i=1}^{|E|}\left(1 - Prob(r_i + \hat{x}_i)\right)\right) \quad \middle| \quad \begin{array}{l} \boldsymbol{A}\hat{\boldsymbol{x}} = \boldsymbol{b}; \\ \hat{x}_i \in \hat{\boldsymbol{x}}, \quad \hat{\boldsymbol{x}} \succeq \boldsymbol{0}, \end{array} \tag{1}$$

where $\prod_{i=1}^{|E|}(1 - Prob(r_i + x_i))$ represents the probability that no breakdown occurred on any of the road links; $\ln(\cdot)$ refers to the log operation, which helps to simplify the actual computation process, but does not change the optimal point of the problem; $Prob(\hat{x}_i) = \frac{e^{w_i \hat{x}_i + \tau_i}}{1 + e^{w \hat{x}_i + \tau_i}}$ is the road breakdown probability for road link e_i; $\hat{x}_i$ is the decision variable denoting the controllable load of the road link e_i; r_i is the default road network load at road link e_i; w_i represents the unit impact of road traffic load to road link breakdown; τ_i represents other miscellaneous factors (e.g., weather condition and road width), which also influence the road breakdown; $|E|$ refers to the total number of road links in the road network; $\boldsymbol{A}\hat{\boldsymbol{x}} = \boldsymbol{b}$ is the network flow conservation constraint; $\boldsymbol{A}$ is a $|V| \times |E|$ matrix, which describes the road network topology; $\boldsymbol{b}$ is a column vector specifying the origin and destination of the vehicles; the controllable load should be non-negative, defined as $\hat{\boldsymbol{x}} \succeq \boldsymbol{0}$ [28]. Solving the problem in Eq. (1) is equivalent to minimizing the following equation:

$$\min_{\hat{\boldsymbol{x}}} \sum_{i=1}^{|E|} \ln\left(1 + e^{w_i(\hat{x}_i + r_i) + \tau_i}\right) \quad \middle| \quad \begin{array}{l} \boldsymbol{A}\hat{\boldsymbol{x}} = \boldsymbol{b}; \\ x_i \in \hat{\boldsymbol{x}}, \quad \hat{\boldsymbol{x}} \succeq \boldsymbol{0}. \end{array} \tag{2}$$

B. Reformulation as a Practical Routing Engine

It is straightforward to prove that Eq. (2) satisfies Slater's condition of convex optimization [35], therefore, the solution procedure is tractable. However, the decision variable $\hat{\boldsymbol{x}}$ in Eq. (2) is the expected traffic load for each road link, which is impractical as it does not explicitly instruct a vehicle which way to go. Therefore, we further reformulate the cooperative routing problem in Eq. (2) as a routing engine, which directly engenders a route for each vehicle as follows:

$$\min_{\substack{\boldsymbol{x}_1, \ldots, \\ \boldsymbol{x}_N, \boldsymbol{\xi}}} Card(\boldsymbol{\xi}) \quad \middle| \quad \begin{array}{l} \boldsymbol{A}\boldsymbol{x}_j = \boldsymbol{b}_j, \quad j = \{1, 2, \ldots N\}; \\ \sum_{j=1}^{N} \boldsymbol{x}_j \preceq \boldsymbol{c} + \boldsymbol{\xi}; \\ \boldsymbol{x}_j = \{0, 1\}^{|E|}, \quad \boldsymbol{\xi} \succeq \boldsymbol{0}, \end{array} \tag{3}$$

where $\boldsymbol{x}_j$ is the decision variable, which explicitly represents the route for vehicle j; $Card(\cdot)$ is the cardinality function, which returns the quantity of non-zero elements for the input parameter; N is the quantity of the vehicles; $\boldsymbol{c}$ and $\boldsymbol{\xi}$ are both vectors with size of $1 \times |E|$; $\boldsymbol{c}$ represents the capacity for all road links; $\boldsymbol{\xi}$ represents the potential violations with respect to the road link capacities. Thus, the cooperative routing problem becomes minimizing the quantity of the road links on which a breakdown may happen. And in this paper, we aim to solve the problem described in Eq. (3), as it is much more practical. We would like to note that, the problem in Eq. (3) is similar to the problems in [13] and [24], especially the Earliest Destination Arrival Time (EDAT) problem in [24], as they both consider traffic balance, either explicitly or implicitly. In particular, the EDAT problem is to minimize the traveling time while keeping the traffic density not exceeding a critical value, and our problem is to minimize the chance that the quantity of vehicles for a road link will exceed the corresponding capacity.

III. Solution to Reduce the Chance of Breakdown

In this section, we introduce an approximation to the cardinality minimization problem. To derive an iterative solution to the approximation, we first look into its dual problem, and then prove the concavity, so that a subgradient method can be adopted to solve the problem. More importantly, to make the subgradient method meaningful and practical, we conceive a semi-centralized pricing scheme, in which each road link updates its local toll, while the vehicles will minimize their toll costs by updating their routes according to the global tolls.

A. ℓ_1-Norm Approximation to Cardinality Minimization

Generally, the most efficient approach to solve the cardinality optimization is the ℓ_1-norm approximation [35], [36]. Consequently, the cooperative routing problem in Eq. (3) can be further expressed as follows:

$$\min_{\substack{x_1,\ldots,\\ x_N,\xi}} \sum_{j=1}^{|E|} \xi_j \quad \left| \begin{array}{l} Ax_j = b_j, \quad j = \{1, \ldots N\}; \\ \sum_{j=1}^{N} x_j \preceq c + \xi; \\ x_j = \{0,1\}^{|E|}, \quad \xi_j \geq 0, \end{array} \right. \tag{4}$$

where the ℓ_1-norm operator is explicitly removed due to the constraint $\xi_j \geq 0$. The problem in Eq. (4) is a mixed integer linear programming (MILP) problem and hence is NP-hard. Normally, it can be solved as a typical one-short optimization using a popular solver, e.g., branch and price algorithm in Cplex. However, on one hand, the computation for the one-short optimization might be prohibitively time-consuming, given the fact that both the scale of the road network and the quantity of the vehicles are usually huge. On the other hand, traffic is always random, and pre-computed routes by one-short optimization might not handle well the traffic dynamics [37]. Therefore, we will develop an adaptive approach to iteratively solve the problem in Eq. (4), which not only improves the computation efficiency, but also has the potential to tackle the traffic dynamics.

B. Reformulation for an Adaptive Solution

The optimization problem described in the preceding subsection assumes fixed model parameters, which have already been pre-gauged. However, in reality, traffic condition may change dramatically in a short time due to various factors (e.g., weather, road work and traffic jam occurrence), and drivers may also need to accordingly adjust their routes on the fly. Therefore, an adaptive solution, which leverages both the objective of reducing network breakdown and traffic dynamics, is much more desired. In view of this, we first define the problem in Eq. (4) as Problem **I**, and then look at a slightly transformed problem (i.e., Problem **II**) as follows:

$$\min_{\substack{x_1,\ldots,\\ x_N,\xi}} \max_{\lambda} (\boldsymbol{\xi} + \boldsymbol{\lambda}^\top ((\sum_{j=1}^{N} x_j) - c - \boldsymbol{\xi})) \quad \left| \begin{array}{l} Ax_j = b_j, \\ x_j = \{0,1\}^{|E|}, \\ \boldsymbol{\lambda} \succeq \mathbf{0}, \quad \boldsymbol{\xi} \succeq \mathbf{0}, \\ j = \{1, \ldots N\}, \end{array} \right. \tag{5}$$

where $\boldsymbol{\lambda}$ is the Lagrangian multiplier. To verify that the problem **II** has the same solution with Problem **I**, we define the solution to Problem **I** as p^*, and that to Problem **II** as d^*. The detailed logic is stated in two steps as follows:

1) $d^* \leq p^*$: Since p^* is the optimal solution to Problem **I**, it satisfies $(\sum_j^N x_j - c - \boldsymbol{\xi}) \preceq \mathbf{0}$. In this case, the optimum for Problem **II** will set $\boldsymbol{\lambda} = \mathbf{0}$. Consequently, p^* is reachable in Problem **II**. Thus, we have $d^* \leq p^*$.

2) $d^* \geq p^*$: On one hand, whenever one of the elements in the vector $(\sum_j^N x_j - c - \boldsymbol{\xi})$ is greater than zero, we can set the corresponding element of $\boldsymbol{\lambda}$ to be $+\infty$, since the 'inner' optimization in problem **II** is to maximize the objective function. In this case, no matter what we do with the 'outer' optimization parameters in problem **II**, the objective value will be $+\infty$. Therefore, none single element of the vector $(\sum_i^N x_i - c - \boldsymbol{\xi})$ will be greater than zero. On the other hand, no matter whether all the elements of the vector $(\sum_j^N x_j - c - \boldsymbol{\xi})$ are less than or equal to zero, the best value for the corresponding λ must be zero, since any other value will make the objective decrease, which contradicts the 'inner' optimization. In this case, it is already the same as Problem **I**, which means $d^* = p^*$. In view of the two aspects together, we can conclude that d^* is either $+\infty$ or equal to p^*, which leads to $d^* \geq p^*$.

By combining the two seminal conclusions, we may draw that $d^* = p^*$. Therefore, we will concentrate on Problem **II** instead, as it is much easier to solve, and the adaptive solution to which may have potentials to handle traffic dynamics well.

C. Concavity Proof

Normally, subgradient method is a desirable solution to iteratively solve large-scale problems [38], such as Eq. (5). However, the precondition is that the target optimization should satisfy the concavity condition. To prove it, we first perform a slight transformation to Problem **II**, which can be expressed as follows:

$$\max_{\lambda} \min_{\substack{x_1,\ldots,\\ x_N,\xi}} (\boldsymbol{\xi} + \boldsymbol{\lambda}^\top ((\sum_{j=1}^{N} x_j) - c - \boldsymbol{\xi})) \quad \left| \begin{array}{l} Ax_j = b_j, \\ x_j = \{0,1\}^{|E|}, \\ \boldsymbol{\lambda} \succeq \mathbf{0}, \quad \boldsymbol{\xi} \succeq \mathbf{0}, \\ j = \{1, \ldots N\}, \end{array} \right. \tag{6}$$

where the order of maximization and minimization in the objective function has changed. Then, we take $\boldsymbol{\lambda}$ as the parameter out of Eq. (6), and define the objective value as $g(\boldsymbol{\lambda})$, which is expressed as follows:

$$g(\boldsymbol{\lambda}) = \min_{\substack{x_1,x_2,\ldots,\\ x_N,\xi}} \left(\boldsymbol{\xi} + \boldsymbol{\lambda}^\top (\sum_{j}^{N} x_j - c - \boldsymbol{\xi}) \right). \tag{7}$$

The key step for concave satisfaction is to prove that $g(\boldsymbol{\lambda})$ is a concave function as the constraint $Ax_j = b_j$ in Eq. (6) is affine. To this end, we need to show that $\forall \boldsymbol{\lambda_1}$, $\forall \boldsymbol{\lambda_2}$ and $\forall \theta \in$[0,1], the below inequality holds:

$$\theta g(\boldsymbol{\lambda_1}) + (1-\theta) g(\boldsymbol{\lambda_2}) \leq g(\theta \boldsymbol{\lambda_1} + (1-\theta) g(\boldsymbol{\lambda_2})). \tag{8}$$

Then, we begin the proof from the left hand side of Eq. (8), and we have:

$$\begin{aligned} &\theta g(\boldsymbol{\lambda_1}) + (1-\theta) g(\boldsymbol{\lambda_2}) \\ &= \min_{\substack{x_1,\ldots,\\ x_N,\xi}} \theta \left(\boldsymbol{\xi} + \boldsymbol{\lambda_1}^\top (\sum_{i}^{N} x_i - c - \boldsymbol{\xi}) \right) \\ &\quad + \min_{\substack{x_1,\ldots,\\ x_N,\xi}} (1-\theta) \left(\boldsymbol{\xi} + \boldsymbol{\lambda_2}^\top (\sum_{i}^{N} x_i - c - \boldsymbol{\xi}) \right) \\ &\leq \min_{\substack{x_1,\ldots,\\ x_N,\xi}} \theta \left(\boldsymbol{\xi} + \boldsymbol{\lambda_1}^\top (\sum_{i}^{N} x_i - c - \boldsymbol{\xi}) \right) \end{aligned}$$

$$
\begin{aligned}
&+ (1-\theta)\Bigg(\boldsymbol{\xi} + \boldsymbol{\lambda_2}^\top(\sum_i^N \boldsymbol{x_i} - \boldsymbol{c} - \boldsymbol{\xi})\Bigg)\\
&= \min_{\substack{\boldsymbol{x_1},\dots,\\ \boldsymbol{x_N},\boldsymbol{\xi}}} \Bigg(\boldsymbol{\xi} + \theta\boldsymbol{\lambda_1}^\top(\sum_i^N \boldsymbol{x_i} - \boldsymbol{c} - \boldsymbol{\xi})\\
&\quad + (1-\theta)\boldsymbol{\lambda_2}^\top(\sum_i^N \boldsymbol{x_i} - \boldsymbol{c} - \boldsymbol{\xi})\Bigg)\\
&= \min_{\substack{\boldsymbol{x_1},\dots,\\ \boldsymbol{x_N},\boldsymbol{\xi}}} \Bigg(\boldsymbol{\xi} + \Big(\theta\boldsymbol{\lambda_1}^\top + (1-\theta)\boldsymbol{\lambda_2}^\top\Big)(\sum_i^N \boldsymbol{x_i} - \boldsymbol{c} - \boldsymbol{\xi})\Bigg)\\
&= g(\Big(\theta\boldsymbol{\lambda_1}^\top + (1-\theta)\boldsymbol{\lambda_2}^\top\Big)) \qquad (9)
\end{aligned}
$$

Thus, $g(\boldsymbol{\lambda})$ is indeed a concave function. Consequently, the subgradient method can be exploited to solve the Eq. (5).

D. Velocity Based Pricing Approach

In real transportation scenario, it is difficult to gauge the dynamic road link capacity (i.e., $\boldsymbol{c}$) and the O-D information for all the drivers. However, it is relatively easy to acquire the real time travel speed information of all road links. Coherently, we assume that the speed is inversely proportional to the density of vehicles in the road link, thus we have $\boldsymbol{\pi} = f(\sum_{j=1}^N \boldsymbol{x_j})$, where $\boldsymbol{\pi}$ represents the speed of all road links, which decreases as the amount of vehicles for each road link becomes larger. On the other hand, if we only take $\boldsymbol{\lambda}$ as the parameter out of Problem **II** in Eq. (5), then we can follow the subgradient method to gradually update $\boldsymbol{\lambda}$ to reach the final optimal solution by exploiting the optimal $\boldsymbol{x^*}$ in each iteration.

Thus, it is natural to conceive a dynamic pricing approach, in which $\boldsymbol{\lambda}$ is considered as the tolls for each road link, and updated in each iteration, while each vehicle tries to minimize its own toll costs by choosing the optimal route $\boldsymbol{x_j^*}$. Since the travel speed $\boldsymbol{\pi}$ of road links is assumed to be a function of $\boldsymbol{x_j}$, we can automatically adjust the tolls to encourage or discourage vehicles to take the link. In other words, we are trying to minimize the number of road links which have a lower-than-tolerance velocity by adjusting the road link tolls. Combining those concerns with the subgradient method, we have:

$$\lambda_i^{k+1} = \Big(\lambda_i^k - \alpha(\pi_i^k - \pi_i^{tol})\Big)_+, \qquad (10)$$

where λ_i^{k+1} is the toll of road link e_i in the $k+1_{th}$ iteration; α is the step size; π_i^k is the average velocity of e_i in the k_{th} iteration; $\pi_i^{tol} = \rho\pi_i^f$ is the road-breakdown velocity, and π_i^f is the maximum allowed velocity of e_i. In view of those explanations, each road link will adjust the local toll according to Eq. (10), and the vehicles will update their routes by minimizing the toll costs in each iteration. Since we assume that the vehicles know the real time global tolls, our pricing approach is semi-centralized in nature. Moreover, with appropriate definition of α, we can guarantee that the solution will become better and better as iteration goes on.

TABLE I
PROPERTIES OF TESTING ROAD NETWORK

City	Network Area	# Road Links	# Road Intersections
Guangzhou	34,000,000m^2	503	212

In addition, to make the solution much concise, we add one more step to normalize the toll for each road link as follows:

$$\lambda_i^{k+1} = \frac{\lambda_i^{k+1}}{\boldsymbol{1}^\top\boldsymbol{\lambda^{k+1}}}. \qquad (11)$$

E. Further Improvement to the Proposed Solution

In Eq. (10), the road-breakdown velocity may vary with different road links. However, to a specific road link, π_i^{tol} is fixed. Since π_i^k is supposed to dynamically change with each iteration, a potential oscillation might occur for the toll λ_i^k, which may cause undesirable influences to the routes chosen by the vehicles. Therefore, to address this potential issue, we introduce the 'heavy-ball' method [39] into our pricing approach as follows:

$$\lambda_i^{k+1} = \Big(\lambda_i^k - \alpha(\pi_i^k - \pi_i^{tol}) - \beta(\lambda_i^{k-1} - \lambda_i^k)\Big)_+, \qquad (12)$$

where the term $\beta(\lambda_i^{k-1} - \lambda_i^k)$ functions as a stabilizer to eliminate the possible oscillation phenomenon. Similarly, it is also followed by a normalization step as Eq. (11).

IV. EXPERIMENTATION AND EVALUATION

In this section, we conduct extensive experimentation in various settings to compare the proposed pricing approach and the improvement with existing methods, and demonstrate their advantages over others. Particularly, we first introduce the experimental settings, then we focus on evaluating our approach in the aspects of travel time, travel distance, winners and losers, potential congestion, arrival time of last vehicle and toll costs. Finally, we summarize all the solutions and recommend the most desirable and practical one, by taking all metrics into account.

A. Experimentation Setup

The experimentation is conducted on the simulator, SUMO (i.e., simulation of urban mobility platform). SUMO is an open source, highly portable, microscopic and continuous traffic simulation for handling both artificial and real-world road networks. In this experimentation, we take an island, i.e., the Guangzhou Higher Education Mega Center, China, as the testing bed. The road network is displayed in Fig. 1 (a), which is extracted from OpenStreetMap.[1] The network properties are summarized in Table I.

Particularly, all experimentation in this section is conducted on an ordinary PC with Intel Core i7-7700 processor and 32.00 GB RAM. Pertaining to the vehicles in the simulator, the configurations are set as follows:

1) The length of vehicle is 5m, the minimal gap is 2.5m, and the vehicle following model is Krauss [40].

[1] http://www.openstreetmap.org

(a) real road network (b) example of low traffic density (c) example of high traffic density

Fig. 1. The real road network and examples of different traffic densities. [**Best Viewed in Color**].

2) We use a binomial distribution to randomly generate 9000 O-D pairs (a pair for a vehicle) over the whole road network within the first 1000 simulation steps.[2]
3) The default routes are initialized by "duarouter" in SUMO, which seeks for the shortest distance path.
4) When a vehicle arrives its destination, it will park and not occupy any road resource.
5) The traffic light use the default settings.
6) We use the built-in functions *getLastStepMeanSpeed* and *getMaxSpeed* to return the average travel speed and maximum allowed speed of each road link for Eq. (10).
7) Each road link updates its local toll according to Eq. (10) or Eq. (12), with certain frequency.
8) The parameters in Eq. (12) are set as: $\alpha = 0.9$ and $\beta = 0.5$; The parameter for π_i^{tol} in Eq. (10) is set as: $\rho = 0.5$.

To comprehensively verify our approach, we compare with two benchmarks: (1) *Default* method: it uses the default router in SUMO, which aims to minimize the travel distance; (2) *RIS* method [15]: a server collects information from all vehicles and reroutes them in each step using a Route Information Sharing (RIS) scheme, which is centralized. In this scheme, a weight is added to a road block (normally, it is a small part of a road link) by each vehicle who is likely to pass it, then the central server minimizes the total weights for each vehicle. Particularly, in the experimentation, we divide each road link into segments of 10 meters length, and each of the segments is defined as a road block. Then, all blocks along the intended route of a vehicle will be given weights in a descending order by that vehicle, with the furthest one being 0. Finally, the weight of each block will be calculated as the summation of the weights imposed by all vehicles, which will be used to adjust the routes of all vehicles in the next step. With respects to our own approach, we assume that all the vehicles are sensitive to the dynamic toll, and will minimize the toll costs along the route. In this regard, we also develop some variants to our basic pricing and improved pricing approach: (3) *Pricing: $\mathcal{N}$ steps*: each road link employs Eq. (10) to update the local toll every $\mathcal{N}$ simulation steps; (4) *Improved Pricing: $\mathcal{N}$ steps*: each road link employs Eq. (12) to update the local toll every $\mathcal{N}$ simulation steps. At the same time, the vehicles will update their routes accordingly. In our experimentation, $\mathcal{N}$ is set as 10, 20 and 30 simulation steps, respectively. Then, we run the simulation for 10 times on SUMO according to the above settings, the results of which are recorded and analyzed in Section IV-B, IV-C, IV-D. Before elaborating the results, we first record the vehicle population for each simulation in Table. II. From Table. II we can see that, the minimum population of the vehicles is above 7,000, which means that traffic density is comparatively high in view of the properties in Table. I. Moreover, we also present some examples of different traffic densities in the midst of the simulation, which are shown in Fig. 1 (b) and (c), respectively.

[2]If there already exists a vehicle when that location is chose as the origin of a new vehicle, then that new vehicle would not be generated. Thus, the vehicle population might be less than 9000.

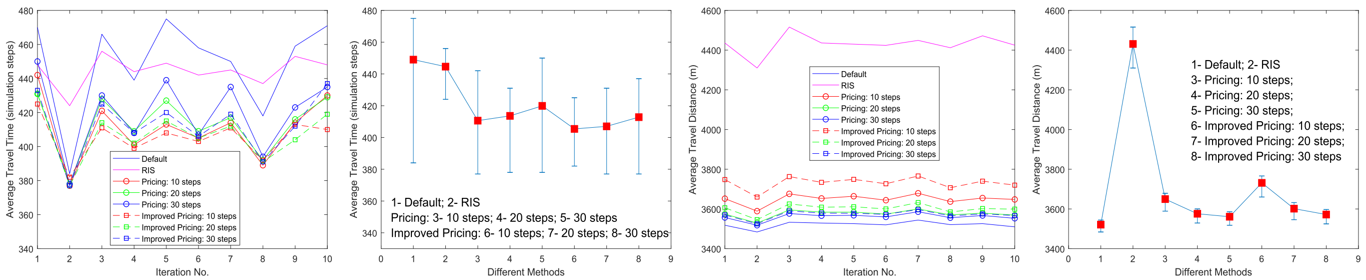
TABLE II
VEHICLE POPULATION GENERATED

1	2	3	4	5	6	7	8	9	10
8,682	7,060	8,959	8,402	8,936	8,199	8176	7,320	8,701	8,816

B. Analysis of Travel Time and Distance

For each of the 10 simulations, we implement our two pricing approaches (with variants) and the benchmark methods using the same settings, and each simulation terminates only when the last vehicle arrived at its destination. Then we record the results with respects to average travel time and average travel distance in Fig. 2 (a)-(d). More specifically, the average travel time in each simulation is calculated as follows: we first add the travel time for all vehicles together, then we use the summation to divide the vehicle population. Likewise, the average travel distance is obtained in a similar way.

From Fig. 2 (a) we can see that, regarding each method, the average travel time varies with different simulations. Even though, for most of the cases, the *Pricing* and *Improved Pricing* dominantly achieve lower values than the *Default* method and the *RIS* method, which indicates that our pricing approaches work competitively well against others, considering that average travel time is one of the most important metrics for network breakdown. Comparing the two pricing approaches with the *Default* method, the former is superior at large because pricing based approaches update the tolls by taking into account the travel speed in Eq. (10), which directly relates to the travel time. While the latter only adopts the metric of distance as the objective, which is also the default setting in most of the navigation systems. This superiority



(a) average travel time per simulation (b) mean, max and min of average time (c) average travel distance per simulation (d) mean, max and min of average distance

Fig. 2. Results of travel time and distance. [**Best Viewed in Color**].

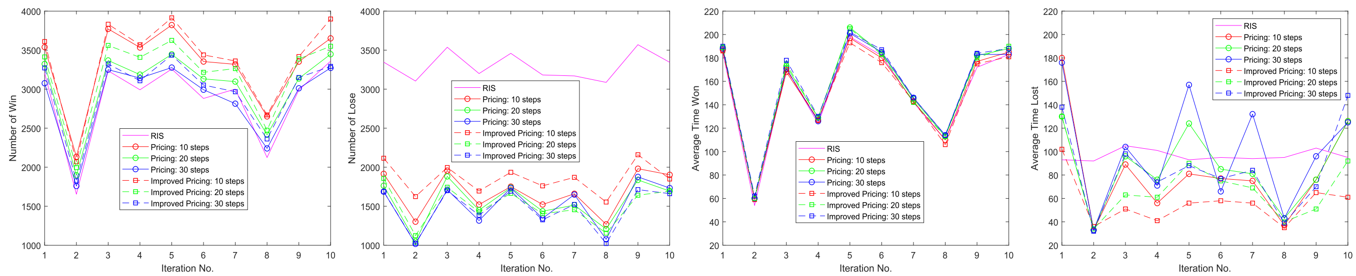
is not obvious for the 2nd simulation. However, the vehicle population in that simulation is the smallest (i.e., 7,060 as shown in Table. II), which implies a lower traffic density in comparison with the other 9 simulations. Comparing the two pricing approaches with the *RIS* method, the latter performs surprisingly worse although it adopts a centralized cooperative routing scheme. The rationale behind this observation for the *RIS* method might come down to two aspects: (1) it is too greedy in that the central server updates the traffic condition at each simulation step, and reroutes all the vehicles whenever they arrive at an intersection, which might be unnecessary; (2) although the central server collects the route information from each vehicle and assign a weight to all the road blocks that they are likely to pass, it might not 100% accurately consider the time when those assigned weights take effect. For example, if two vehicles will pass a same road block, but at significantly different times, then the influence to each other can be neglected. On the other hand, the two pricing approaches are semi-centralized, which are deployed on each road link. And each road link only manages the local toll updates according to the detected traffic condition (i.e., average travel speed) in a real-time manner.

We record the mean of the average travel time over the 10 simulations in Fig. (2) (b), as well as the longest and shortest average travel time for each method. From Fig. (2) (b) we can observe again that, the two pricing approaches are better than the *RIS* method, and the *RIS* method is better than the *Default* method, in terms of the mean of average travel time. More importantly, comparing the two pricing approaches, the mean of average travel time for the *Improved Pricing* approach is shorter than the *Pricing* approach for each corresponding $\mathcal{N}$, which indicates that the proposed improvement takes effect. Because the heavy ball method can help to alleviate the potential oscillation of the dynamic price, which might cause a vehicle to constantly switch among two or more candidate routes. As a consequence, the improvement not only helps to reduce the "hesitation" time, but also improve the quality of the chosen route. Looking into the two pricing approaches, respectively, we can observe that, the average travel time slightly increases as $\mathcal{N}$ becomes larger, e.g., *Improved Pricing: 10 steps* is smaller than *Improved Pricing: 30 steps*, and *Pricing: 10 steps* is smaller than *Pricing: 30 steps*. It is straightforward since smaller $\mathcal{N}$ indicates more fresh traffic condition information, and the up-to-date information is more useful to plan a high-quality route. However, it is neither desired or practical to make $\mathcal{N}$ too small. On one hand, the time might be wasted in swinging forward and backward among several candidate solutions. On the other hand, the drivers dislike frequently changing the route.

Similarly, we also record the average travel distance and mean of the average travel distance in Fig. 2 (c) and (d). From Fig. 2 (c), we can obviously see that the *RIS* method traveled the longest distance for all the cases. This happened because the *RIS* method reroutes the vehicles too frequently, and the way to calculate the weight to a road block may not accurately reflect the real traffic condition. As a consequence, the vehicles may travel on unexpected or unnecessary longer routes. The *Default* method achieved the shortest travel distance as expected, because it directly takes the shortest distance as the optimal criterion, the whole route of which is fixed once the O-D is finalized. The travel distances for the *pricing* and the *Improved Pricing* approaches are slightly longer than the *Default* method in all the 10 simulations, which demonstrate competitive performance. This can be further justified by Fig. 2 (d), in which the means of the travel distance for the two pricing approaches are close to that of the *Default* method. It also needs to be noted that, unlike the pattern of the average travel time in Fig. 2 (b), the average travel distance for the *Improved Pricing* approach is slightly longer than that of the *Pricing* approach. This phenomenon further explains the nature of our approach that they achieve shorter travel time at the price of longer travel distance. It is reasonably practical and desirable as travel time is much more important than distance in real life. On the other hand, if all vehicles constantly seek for the shortest distance, they may get stuck into congestion in the same route. At the same time, we observe that the average travel distance becomes longer as $\mathcal{N}$ decreases. This occurred because more rerouting is likely to be conducted if $\mathcal{N}$ is small, and vehicles may traveled on additionally longer paths, but traffic conditions of which are more favourable.

C. Analysis of Winner and Loser for Travel Time and Distance

Since the travel time is a more important metric, we record the results of winners and losers by comparing all methods with the *Default* method, in the aspect of travel time. More specifically, only a vehicle saved (lost) more than 10 simulation steps, we consider it as a winner (loser). In particular, the average quantities of winners and loser, and the average time saved and lost are shown in Fig. 3 (a)-(d).



(a) average counts of winners (b) average counts of losers (c) average time saved (d) average time lost

Fig. 3. Win and lose for travel time and distance. [**Best Viewed in Color**].

From Fig. 3 (a) and (b) we can observe that, the quantities of winners for the two pricing approaches are significantly higher than that of losers. The proportion of winners is almost half of the vehicle population (see Table II), while the quantities of winners are almost twice as the losers. Looking into the two pricing approaches, the *Improved Pricing* approach achieves slightly higher amount of winners than that of the *Pricing* approach. We also notice that, although the quantity of winners for the *RIS* method is close to the two pricing approaches, the amount of losers for the former is also significantly large. Simultaneously, we observe that the quantity of winners for the pricing approaches will slightly go up as $\mathcal{N}$ decreases.

We record the average travel time saved and lost in Fig. 3 (c) and (d), respectively. From Fig. 3 (c) we observe that, the *Pricing* and *Improved Pricing* approaches saved similar average travel time for all the winners in comparison with the *Default* method. However, the two pricing approaches lost less average travel time than the *RIS* method in most of the cases, as shown in Fig. 3 (d). It happened might due to that some of the vehicles in the *RIS* method waste too much time in frequent rerouting and on the routes which do not reflect the 100% accurate traffic conditions. At the same time, the average travel time lost becomes shorter as $\mathcal{N}$ becomes smaller for the two pricing approaches. On the other hand, the lost time for the *Improved Pricing* approach is slightly lower than the *Pricing* approach, which again verifies the effectiveness of the improvement.

Combining the four sub-figures, we can obviously see that the two pricing approaches result in higher amounts of winners than losers, and the average travel time saved is also much longer than the time lost. Comparing the *Improved Pricing* approach with the *Pricing* approach, the former has both more winners and losers, but shorter average time lost, while both approaches have longer average time saved. Moreover, smaller $\mathcal{N}$ usually leads to more winners and comparatively shorter travel time lost. In addition, the *RIS* method has many winners as well as losers, but the average time lost is shorter than the average time saved, which explained the reason that the *RIS* method has slightly better overall performance than the *Default* method in terms of average travel time.

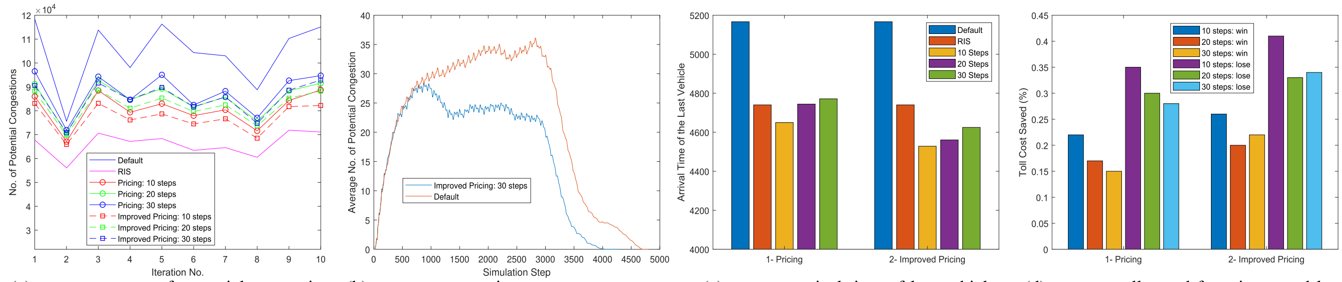
D. Analysis of Congestion, Last Arrival Time and Toll Costs

We evaluate our approach in the aspect of potential congestion occurrence. To test a worse case on purpose, we suppose that, the congestion would occur in a road link as long as the average travel speed is lower than 50% of the maximum speed.[3] Then we count all the occurrence of potential congestion over the whole simulation. It should be noted that multiple congestion might happen on the same road link at different time. The results are recorded in Fig. 4 (a).

From Fig. 4 (a), we can observe that, the two pricing approaches and the *RIS* method achieve lower congestion occurrence than that of the *Default* method. It is straightforward, as the latter never reroutes the vehicle no matter how crowded the traffic is. On the other hand, the occurrence of congestion for the two pricing approaches is slightly higher than the *RIS* method. This phenomenon happened because the *RIS* method updates the traffic condition information at each step, and it reroutes the vehicles even the sign of congestion is not obvious at all. This scheme is helpful to prevent congestion in short term, but will engender unnecessarily additional travel time or travel distance in the long term. Fig. 4 (a) can also tell that, between the two pricing approaches, *Improved Pricing: 30 steps* is comparatively inferior in terms of potential congestion occurrence. Therefore, we look into this approach by comparing it with the *Default* method, and record the average occurrence for each road link over 10 simulations, which are shown in Fig. 4 (b). From Fig. 4 (b), we can observe that, *Improved Pricing: 30 steps* has lower occurrences of congestion than the *Default* method in most of the cases, especially after the first 1000 simulation steps, when the *Improved Pricing: 30 steps* approach takes effect for smoothing the traffic. It is reasonable as we cannot expect the approach to work well at the very beginning when the traffic density is comparatively low. Meanwhile, we observe slight oscillations for both our approach and the *Default* method (without dynamic pricing), and they might be caused by the regular traffic signal phases, which is normal and acceptable. Particularly, we only display the detailed results of *Improved Pricing: 30 steps* to save space, but the remaining pricing based approaches are expected to perform better than it in terms of congestion occurrence, considering the curves in Fig. 4 (a).

Besides, the time of last arrival is also a key metric to evaluate the proposed solution, as it may partially reflect the severe sacrifice made to some individuals. Therefore, we calculate the average time of last arrival over the 10 simulations, which are displayed in Fig. 4 (c). From Fig. 4 (c), we can observe that, the *RIS* method and the two pricing approaches need

[3] In real life, this situation might be still far away from congestion.



(a) average counts of potential congestion (b) average congestion occurrence per step (c) average arrival time of last vehicle (d) average toll saved for winner and loser

Fig. 4. Performance regarding congestion, last arrival time and tolls. [**Best Viewed in Color**].

shorter average arrival time for the last vehicle, compared with the *Default* method. The average last arrival time for the *Pricing* approach is close to the *RIS* method, while the *Improved Pricing* approach is obviously lower than the latter. Simultaneously, we observe a slight increase of last arrival time for both pricing approaches, as $\mathcal{N}$ goes up to 30. However, considering that *Improved Pricing: 30 steps* is already able to significantly alleviate the congestion at an early stage, as demonstrated in Fig. 4 (c), all the proposed pricing approaches are expected to be favorably competitive in speeding up the arrival of the last vehicle.

Additionally, we also analyze the toll costs for vehicles who followed the two pricing approaches. However, we only focus on the winners and losers mentioned in Fig. 3 (a) and (b). To this end, we first compute the tolls for the *Default* method according to the corresponding pricing approaches, although the vehicles are unaware of it. Then we compute the difference of toll costs between the *Default* method and the relevant pricing approaches, which is followed by a standard normalization. Afterwards, we display the ratios of the saved toll costs in Fig. 4 (d), in comparison with the *Default* method. From Fig. 4 (d), we can see that, both the winners and losers significantly saved toll costs, as it is one of the objectives for the two pricing approaches. At the same time, the *Improved Pricing* approach reduces more toll costs than the *Pricing* approach. More importantly, in both approaches, the losers save more toll costs, which can be considered as certain compensation for their lost in travel time or travel distance.

E. MFD Analysis

We further evaluate our approach by analyzing the macroscopic fundamental diagram (MFD). To this end, we implement a *Dynamic Tolling* strategy, i.e., Feedback-Control Approach [29] as the baseline. For both methods, we vary the number of vehicles from 1000 to 10000, with an interval of 1000, and update the toll or price every 30 steps. Then we record the average traffic flows, the results of which are plotted in Fig. 5. From Fig. 5 we can observe that, the average traffic flows increase as the vehicle density becomes larger, until the vehicle amount reaches 7000 and 8000 for our approach and the dynamic tolling method, respectively. Although the traffic with our approach becomes saturated earlier, its overall performance is better than that of the *Dynamic Tolling* method, especially when the traffic is saturated, i.e., vehicle number

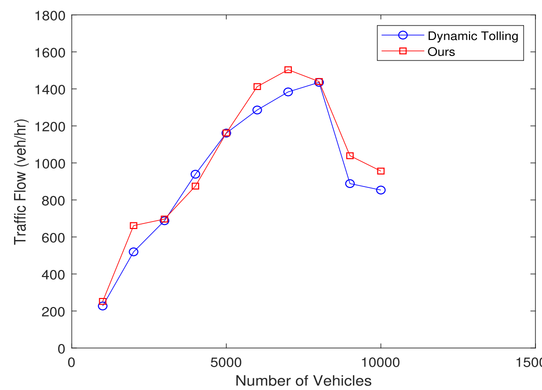
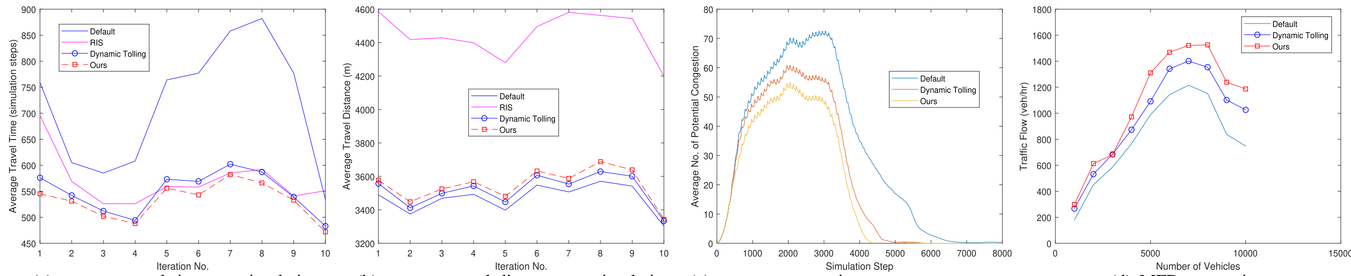


Fig. 5. MFD comparison.

of 9000 and 10000. The superiority might come from the stabilizer in Eq. (12), which leads to more stable pricing.

F. Overall Performance With a More Realistic O-D Profile

Previously, we randomly generated about 9000 O-D pairs and one vehicle for each pair. To make the O-D profile more realistic, in each simulation, we randomly select 1000 O-D pairs and generate 6 to 12 vehicles per pair. Then we implement the methods of *Default*, *RIS*, *Dynamic Tolling* and *Improved Pricing: 30 steps* (Ours), and run the simulation for 10 times, the results of which are recorded in Fig. 6. From Fig. 6 (a) we can see that our method always consumes the shortest average travel time, and the *Dynamic Tolling* method consumes slightly longer time than ours. Although the *RIS* method seeks the path of least expected travel time at each step, its average travel time in turn is much longer than ours in many cases. The *Default* method consumes the longest travel time as it only takes into account the static distance. Therefore, in Fig. 6 (b), the *Default* method achieves the shortest travel distance in all cases. Our method and the *Dynamic Tolling* method achieve only slightly longer travel distance. Meanwhile, the *RIS* method achieves the significantly longest travel distance due to the greedy rerouting. Thereby, we focus on comparing our method with *Dynamic Tolling* method. From Fig. 6 (c) we can observe that both our method and the *Dynamic Tolling* method incur much less links of congestion than that of the *Default* method, and our method leads to less congested links than that of *Dynamic Tolling* method, while this superiority is much obvious around the peak. Then



(a) average travel time per simulation (b) average travel distance per simulation (c) average congestion occurrence per step (d) MFD comparison

Fig. 6. Overall performance for a more realistic O-D profile. [**Best Viewed in Color**].

we stick to the same rule to select O-D pairs, but vary the number of vehicles from 1000 to 10000, with an interval of 1000 to conduct the MFD analysis. From Fig. 6 (d) we can see that, the traffic flows for all three methods tend to decrease as the vehicle number is not less than 8000. However, our method and the *Dynamic Tolling* method always outperform the *Default* method. And in most of cases, the *Dynamic Tolling* method is inferior to our method, especially for larger number of vehicles. To sum up, our method achieves the best overall performance with this more realistic O-D profile.

G. Summary

In view of the metrics of travel time, travel distance, winners and losers, congestion occurrence, last arrival time, toll costs saved and average traffic flows together, as well as the semi-centralized nature, it is safe to conclude that our pricing approach outperform the *Default*, the *RIS* and the dynamic tolling methods under two different O-D profiles. Between the two pricing approaches, *Improved Pricing: 30 steps* is more desirable for two reasons: (1) the overarching performance of the *Improved Pricing* approach is better than the *Pricing* approach; (2) $\mathcal{N} = 30$ is more favorable than $\mathcal{N} = 10$ as it is not practical for vehicles to update their routes too frequently.

V. Discussion

Out goal is to improve the overall performance of the network. There might exist various means to achieve this, e.g., directly minimizing the expected total travel time or delays; balancing the traffic over the whole network; and achieving certain equilibrium. In this paper, we choose the metric of breakdown probability as major means to improve the overall performance. We did not particularly study the distribution relationships between breakdown and other relevant factors, as we directly adopt the analytical model of breakdown probability defined in Eq. (1) [28]. Our target is to develop a practical solution to Eq. (1) so that it could handle traffic dynamics and improve the overall performance of road network, rather than statistically validating the relationships between inputs and outputs of the model. We approximately reformulate Eq. (1) and Eq. (2) as a routing engine in Eq. (3) while considering the objectives and constraints in Eq. (1) and Eq. (2). Thus, the decision variable in Eq. (3) is the route for a vehicle. Considering the approximate equivalence between Eq. (3) and Eq. (1), we could adopt the number of breakdown links in Eq. (3) to verify the chance of road network breakdown, which is further approximately evaluated by the number of congested road links in experimentation. However, it is worth to statistically evaluate the relationships between breakdown and other relevant factors in future, as did in [41].

To make the pricing scheme much easier to apply into practice, we adopted a speed based measurement, as shown in Eq. (10), to regulate the price. However, according to the original formulation in Eq. (5), occupancy or density based measurement would be more desirable. Nevertheless, speed is much easier to be acquired than that of occupancy or density. Therefore, we made an assumption of linear relationship to convert the quantity of vehicles to the average speed, which might not be 100% accurate although practical. Due to those assumptions and approximations, we may not completely guarantee global convergence in practice, but hope that the adapted sub-gradient method is still able to improve the overall performance, which is empirically justified by experimentation. We will investigate more rational ways to further improve the performance in future, and may also explore reinforcement learning based method [42] to update the price.

VI. Conclusion and Future Work

In this paper, we developed a subgradient method to solve a cooperative routing problem, which is originally supposed to reduce the chance of network breakdown. This subgradient method can be naturally implemented as a semi-centralized pricing approach, in which each road link calculates and updates its toll, while the vehicles try to minimize the total toll costs by dynamically adjusting their routes. The approach is semi-centralized, in the sense that each road link only uses the local information to update the toll, but the vehicles are assumed to know the latest global tolls. To mitigate the potential negative effects of oscillation brought by the subgradient method, we propose the heavy-ball method as an improvement. The experimental results of a real road network on SUMO, demonstrated the advantages of our approaches over others, in the aspects of travel time, travel distance, winners and losers, potential congestion occurrence, last arrival time, toll costs saved, as well as the average traffic flows.

We do acknowledge the limitations of our current approach and would like to point out several issues that need to be considered and addressed in future: (1) The parameters α and β are important to the performance, and some smart ways need to be investigated to calculate the optimal values for them.

(2) In the current method, we assume that we know the speed information for all road links, and all drivers are sensitive to the toll, which may not hold in real application. Therefore, the toll updates should consider the case where the speed information is only available for some of the road links, and only a portion of the drivers comply with it. (3) All road links update their tolls synchronously in the current method, while asynchronous updating might be more practical, which can be achieved by individually optimizing the updating frequency for each road link. (4) The scheme to collect the tolls is not the concern of this paper, however, it is a premise to implement the proposed method into practice. Some practice schemes should be investigated, such as the ERP2 in Singapore, which will use the satellite based techniques to charge tolls nationwide.

REFERENCES

[1] K. Gao, Y. Zhang, Y. Zhang, R. Su, and P. N. Suganthan, "Meta-heuristics for bi-objective urban traffic light scheduling problems," *IEEE Trans. Intell. Transp. Syst.*, vol. 20, no. 7, pp. 2618–2629, Jul. 2019.

[2] Z. Cao, H. Guo, J. Zhang, and U. Fastenrath, "Multiagent-based route guidance for increasing the chance of arrival on time," in *Proc. 30th AAAI Conf. Artif. Intell. (AAAI)*, 2016, pp. 1180–1187.

[3] S. Mehar, S. Zeadally, G. Remy, and S. M. Senouci, "Sustainable transportation management system for a fleet of electric vehicles," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 3, pp. 1401–1414, Jun. 2015.

[4] L. Deng, M. H. Hajiesmaili, M. Chen, and H. Zeng, "Energy-efficient timely transportation of long-haul heavy-duty trucks," *IEEE Trans. Intell. Transp. Syst.*, vol. 19, no. 7, pp. 2099–2113, Jul. 2018.

[5] C. Tilk, A.-K. Rothenbächer, T. Gschwind, and S. Irnich, "Asymmetry matters: Dynamic half-way points in bidirectional labeling for solving shortest path problems with resource constraints faster," *Eur. J. Oper. Res.*, vol. 261, no. 2, pp. 530–539, Sep. 2017.

[6] Y. Sun, X. Yu, R. Bie, and H. Song, "Discovering time-dependent shortest path on traffic graph for drivers towards green driving," *J. Netw. Comput. Appl.*, vol. 83, pp. 204–212, Apr. 2017.

[7] Z. Cao, H. Guo, F. Oliehoek, J. Zhang, and U. Fastenrath, "Maximizing the probability of arriving on time: A practical q-learning method," in *Proc. 31th AAAI Conf. Artif. Intell. (AAAI)*, 2017, pp. 4481–4487.

[8] Z. Cao *et al.*, "An accurate solution to the cardinality-based punctuality problem," *IEEE Intell. Transp. Syst. Mag.*, early access, Nov. 22, 2018, doi: 10.1109/MITS.2018.2880260.

[9] F. Köster, M. W. Ulmer, and D. C. Mattfeld, "Cooperative traffic control management for city logistic routing," *Transp. Res. Proc.*, vol. 10, pp. 673–682, Jan. 2015.

[10] M. Zimmermann *et al.*, "Carrot and stick: A game-theoretic approach to motivate cooperative driving through social interaction," *Transp. Res. Part C, Emerg. Technol.*, vol. 88, pp. 159–175, Mar. 2018.

[11] Z. Cao, S. Jiang, J. Zhang, and H. Guo, "A unified framework for vehicle rerouting and traffic light control to reduce traffic congestion," *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 7, pp. 1958–1973, Jul. 2017.

[12] Z. Cao, H. Guo, and J. Zhang, "A multiagent-based approach for vehicle routing by considering both arriving on time and total travel time," *ACM Trans. Intell. Syst. Technol.*, vol. 9, no. 3, pp. 1–21, Feb. 2018.

[13] M. Papageorgiou, "Dynamic modeling, assignment, and route guidance in traffic networks," *Transp. Res. B, Methodol.*, vol. 24, no. 6, pp. 471–495, Dec. 1990.

[14] J. L. Adler and V. J. Blue, "A cooperative multi-agent transportation management and route guidance system," *Transp. Res. C, Emerg. Technol.*, vol. 10, nos. 5–6, pp. 433–454, Oct. 2002.

[15] T. Yamashita, K. Izumi, K. Kurumatani, and H. Nakashima, "Smooth traffic flow with a cooperative car navigation system," in *Proc. 4th Int. Joint Conf. Auton. Agents Multiagent Syst. (AAMAS)*, 2005, pp. 478–485.

[16] A. H. F. Chow, R. Sha, and Y. Li, "Adaptive control strategies for urban network traffic via a decentralized approach with user-optimal routing," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 4, pp. 1697–1704, Apr. 2020.

[17] V. L. Knoop, S. P. Hoogendoorn, and J. W. C. Van Lint, "Routing strategies based on macroscopic fundamental diagram," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2315, no. 1, pp. 1–10, Jan. 2012.

[18] K. He, Z. Xu, P. Wang, L. Deng, and L. Tu, "Congestion avoidance routing based on large-scale social signals," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 9, pp. 2613–2626, Sep. 2016.

[19] I. Kordonis, M. M. Dessouky, and P. A. Ioannou, "Mechanisms for cooperative freight routing: Incentivizing individual participation," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 5, pp. 2155–2166, May 2020.

[20] P. Desai, S. W. Loke, A. Desai, and J. Singh, "CARAVAN: Congestion avoidance and route allocation using virtual agent negotiation," *IEEE Trans. Intell. Transp. Syst.*, vol. 14, no. 3, pp. 1197–1207, Sep. 2013.

[21] M. Yildirimoglu, M. Ramezani, and N. Geroliminis, "Equilibrium analysis and route guidance in large-scale networks with MFD dynamics," *Transp. Res. Proc.*, vol. 59, pp. 404–420, Oct. 2015.

[22] S. Wang, S. Djahel, Z. Zhang, and J. Mcmanis, "Next road rerouting: A multiagent system for mitigating unexpected urban traffic congestion," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 10, pp. 2888–2899, Oct. 2016.

[23] C. Menelaou, P. Kolios, S. Timotheou, C. G. Panayiotou, and M. P. Polycarpou, "Controlling road congestion via a low-complexity route reservation approach," *Transp. Res. C, Emerg. Technol.*, vol. 81, pp. 118–136, Aug. 2017.

[24] C. Menelaou, S. Timotheou, P. Kolios, and C. G. Panayiotou, "Improved road usage through congestion-free route reservations," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2621, no. 1, pp. 71–80, Jan. 2017.

[25] I. I. Sirmatel and N. Geroliminis, "Economic model predictive control of large-scale urban road networks via perimeter control and regional route guidance," *IEEE Trans. Intell. Transp. Syst.*, vol. 19, no. 4, pp. 1112–1121, Apr. 2018.

[26] A. Christman and J. Cassamano, "Maximizing the probability of arriving on time," in *Proc. Int. Conf. Anal. Stochastic Modeling Techn. Appl.*, 2013, pp. 142–157.

[27] Z. Cao, H. Guo, J. Zhang, D. Niyato, and U. Fastenrath, "Finding the shortest path in stochastic vehicle routing: A cardinality minimization approach," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 6, pp. 1688–1702, Jun. 2016.

[28] H. Guo, Z. Cao, M. Seshadri, J. Zhang, D. Niyato, and U. Fastenrath, "Routing multiple vehicles cooperatively: Minimizing road network breakdown probability," *IEEE Trans. Emerg. Topics Comput. Intell.*, vol. 1, no. 2, pp. 112–124, Apr. 2017.

[29] Y. Yin and Y. Lou, "Dynamic tolling strategies for managed lanes," *J. Transp. Eng.*, vol. 135, no. 2, pp. 45–52, Feb. 2009.

[30] P. Kachroo, S. Gupta, S. Agarwal, and K. Ozbay, "Optimal control for congestion pricing: Theory, simulation, and evaluation," *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 5, pp. 1234–1240, May 2017.

[31] N. Zheng, G. Rérat, and N. Geroliminis, "Time-dependent area-based pricing for multimodal systems with heterogeneous users in an agent-based environment," *Transp. Res. C, Emerg. Technol.*, vol. 62, pp. 133–148, Jan. 2016.

[32] Y. Liu and Y. Li, "Pricing scheme design of ridesharing program in morning commute problem," *Transp. Res. C, Emerg. Technol.*, vol. 79, pp. 156–177, Jun. 2017.

[33] Z. Liu, S. Wang, B. Zhou, and Q. Cheng, "Robust optimization of distance-based tolls in a network considering stochastic day to day dynamics," *Transp. Res. C, Emerg. Technol.*, vol. 79, pp. 58–72, Jun. 2017.

[34] B. S. Kerner, "Optimum principle for a vehicular traffic network: Minimum probability of congestion," in *Proc. IEEE Forum Integr. Sustain. Transp. Syst.*, Jun./Jul. 2011, pp. 195–200.

[35] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[36] Z. Cao, H. Guo, J. Zhang, D. Niyato, and U. Fastenrath, "Improving the efficiency of stochastic vehicle routing: A partial Lagrange multiplier method," *IEEE Trans. Veh. Technol.*, vol. 65, no. 6, pp. 3993–4005, Jun. 2016.

[37] A. A. Agafonov and V. V. Myasnikov, "Method for the reliable shortest path search in time-dependent stochastic networks and its application to GIS-based traffic control," *Comput. Opt.*, vol. 40, no. 2, pp. 275–283, 2016.

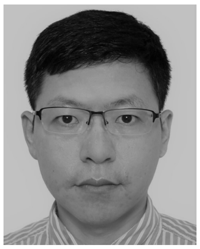
[38] Y. Zhang, O. Yang, and H. Liu, "A Lagrangean relaxation and subgradient framework for the routing and wavelength assignment problem in WDM networks," *IEEE J. Sel. Areas Commun.*, vol. 22, no. 9, pp. 1752–1765, Nov. 2004.

[39] E. Ghadimi, H. R. Feyzmahdavian, and M. Johansson, "Global convergence of the heavy-ball method for convex optimization," in *Proc. Eur. Control Conf. (ECC)*, Jul. 2015, pp. 310–315.

[40] M. Behrisch, L. Bieker, J. Erdmann, and D. Krajzewicz, “SUMO—Simulation of urban mobility: An overview,” in *Proc. 3rd Int. Conf. Adv. Syst. Simul. (SIMUL)*, 2011, pp. 55–60.

[41] W. Brilon, J. Geistefeldt, and M. Regler, “Reliability of freeway traffic flow: A stochastic concept of capacity,” in *Proc. 16th Int. Symp. Transp. Traffic Theory*, vol. 125143, 2005, pp. 1–20.

[42] Z. Cao *et al.*, “Using reinforcement learning to minimize the probability of delay occurrence in transportation,” *IEEE Trans. Veh. Technol.*, vol. 69, no. 3, pp. 2424–2436, Mar. 2020.



Zhiguang Cao received the B.Eng. degree in automation from the Guangdong University of Technology, Guangzhou, China, in 2009, the M.Sc. degree in signal processing from Nanyang Technological University, Singapore, in 2012, respectively, and the Ph.D. degree from the Interdisciplinary Graduate School, Nanyang Technological University, in 2017.

He has been a Research Fellow with the Future Mobility Research Lab, and the Energy Research Institute @ NTU (ERI@N), since 2016. He is currently a Research Assistant Professor with the Department of Industrial Systems Engineering and Management, National University of Singapore, Singapore. His research interests include the applications of AI and optimization for intelligent systems.

Hongliang Guo received the Bachelor of Engineering degree in dynamic engineering and the Master of Engineering degree in dynamic control from the Beijing Institute of Technology, China, and the Ph.D. degree in electrical and computer engineering from the Stevens Institute of Technology, USA.

He later joined Almende, Rotterdam, the Netherlands, as a Postdoctoral Research in 2011. In 2013, he joined NTU as a Research Fellow. In September 2016, he become an Associate Professor with the University of Electronics Science and Technology of China. His research interests include self-organizing systems and agent-based technologies.

Wen Song received the B.S. degree in automation and the M.S. degree in control science and engineering from Shandong University, China, in 2011 and 2014, respectively, and the Ph.D. degree in computer science from Nanyang Technological University, Singapore, in 2018.

He was a Research Fellow with the Singtel Cognitive and Artificial Intelligence Lab for Enterprises (SCALE@NTU). He is currently an Associate Professor with the Institute of Marine Science and Technology, Shandong University, China. His current research interests include artificial intelligence, planning and scheduling, multiagent systems, and operations research.

Kaizhou Gao received the B.Sc. degree from Liaocheng University, China, in 2005, and the master’s degree from Yangzhou University, China, in 2008, and the Ph.D. degree from Nanyang Technological University (NTU), Singapore, in 2016.

From 2008 to 2012, he was with the School of Computer, Liaocheng University. From 2012 to 2013, he was a Research Associate with the School of Electronic and Electrical Engineering, NTU, where he was a Research Fellow from 2015 to 2018. He is currently an Assistant Professor with the Macau Institute of Systems Engineering, Macau University of Science and Technology. He has published over 60 refereed articles. His research interests include intelligent computation, optimization, scheduling, and intelligent transportation.

Liujiang Kang received the Ph.D. degree in traffic and transportation planning and management from Beijing Jiaotong University, China, in 2016.

He was a Research Fellow with the Department of Civil and Environmental Engineering, National University of Singapore. He is currently a Professor with Beijing Jiaotong University. He is a reviewer of more than ten international journals, such as *Transportation Research Part A: Policy and Practice*, *Transportation Research, Part B: Methodological*, *Transportation Research, and Part C: Emerging Technologies*, the *Journal of Transportation Engineering*, the *Journal of Advanced Transportation*, and the *Journal of Rail and Rapid Transit*. His research interests include operations research, optimization theory, and transportation and logistics.

Xuexi Zhang received the B.Eng. degree in electric and automation from the Zhengzhou Engineering Institute, China, in 2000, the M.Eng. degree in control theory and engineering from the Guangdong University of Technology, China, in 2003, and the Ph.D. degree from the School of Automation, Guangdong University of Technology, China, in 2009.

He is currently an Associate Professor with the School of Automation, Guangdong University of Technology. His current research interests include machine learning and operations research.

Qilun Wu received the B.Eng. degree in automation from the School of Automation, Guangdong University of Technology, Guangzhou, China, in 2016, where he is currently pursuing the master’s degree in control theory and control engineering.

He received the second prize in RoboCup China Open in 2016 and 2017, and second prize in China Family Service Robot Special Competition-Home Robot Simulation Project in 2016. His research interests include robotics, route planning, and machine learning.