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NURUL ASYIKEEN BINTE AZHAR

Singapore Management University, [nurula.a.2020@engd.smu.edu.sg](mailto:nurula.a.2020@engd.smu.edu.sg)

Aldy GUNAWAN

Singapore Management University, [aldygunawan@smu.edu.sg](mailto:aldygunawan@smu.edu.sg)

Shih-Fen CHENG

Singapore Management University, [sfcheng@smu.edu.sg](mailto:sfcheng@smu.edu.sg)

Erwin LEONARDI

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# Carbon-Aware Mine Planning with a Novel Multi-Objective Framework<sup>\*</sup>

Nurul Asyikeen Binte Azhar<sup>1,2</sup> ✉, Aldy Gunawan<sup>1</sup>, Shih-Fen Cheng<sup>1</sup>, and Erwin Leonardi<sup>2</sup>

<sup>1</sup> School of Computing and Information Systems, Singapore Management University, 80 Stamford Road 178902, Singapore

{nurula.a.2020, aldygunawan, sfcheng}@smu.edu.sg

<sup>2</sup> Rio Tinto Ltd., Tower 3, 12 Marina Boulevard, Marina Bay Financial Centre, 018982, Singapore {asyikeen.azhar, erwin.leonardi}@riointo.com

**Abstract.** The logistical complication of long-term mine planning involves deciding the sequential extraction of materials from the mine pit and their subsequent processing steps based on geological, geometrical, and resource constraints. The net present value (NPV) of profit over the mine’s lifespan usually forms the sole objective for this problem, which is considered as the NP-hard *precedence-constrained production scheduling problem* (PCPSP) as well. However, increased pressure for more sustainable and carbon-aware industries also calls for environmental indicators to be considered. In this paper, we enhance the generic PCPSP formulation into a multi-objective optimization (MOO) problem whereby carbon cost forms an additional objective. We apply the Non-Dominated Sorting Genetic Algorithm II (NSGA-II) to this formulation and experiment with variants to the solution generation. Our tailored application of the NSGA-II using a set of real-world inspired datasets can form an approximated Pareto front for planners to observe stipulated annual carbon emission targets. It also displays that tailored variants of the NSGA-II can produce diverse solutions that are close to the true Pareto front.

**Keywords:** Precedence-constraint production scheduling · Resource capacity optimization · Multi-objective evolutionary algorithm · Sustainability.

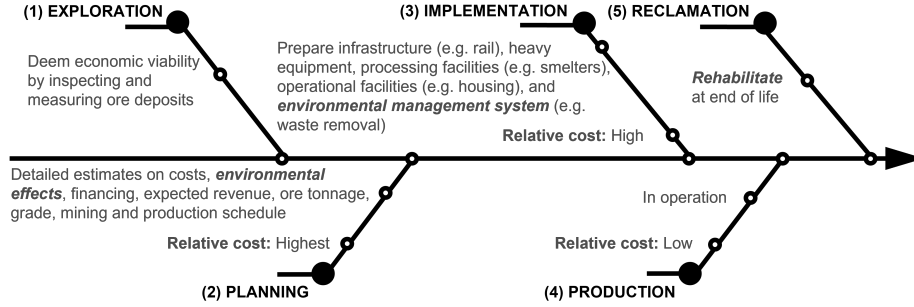
## 1 Introduction

The five phases of mining present a number of logistical challenges (Fig. 1). During the planning, implementation, and production phases, logistics management must forecast, plan, and schedule tasks from strategic, tactical, and operational angles. During the planning phase, environmental concerns are addressed through the environmental impact assessment (EIA) and reclamation plan. These are submitted with the mining plan to the respective government authorities before the implementation phase [7]. As the mining phases mature,

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the mining plans, EIA, and reclamation plans are updated and can become progressively detailed. The reclamation plan is updated and reviewed periodically in certain jurisdictions such as Western Australia. However, regulatory rigor and intra-governmental coordination vary across jurisdictions and may be improved, such as integrating the EIA and reclamation plan processes [12]. These can be classified as the strategic perspective for mine planning.

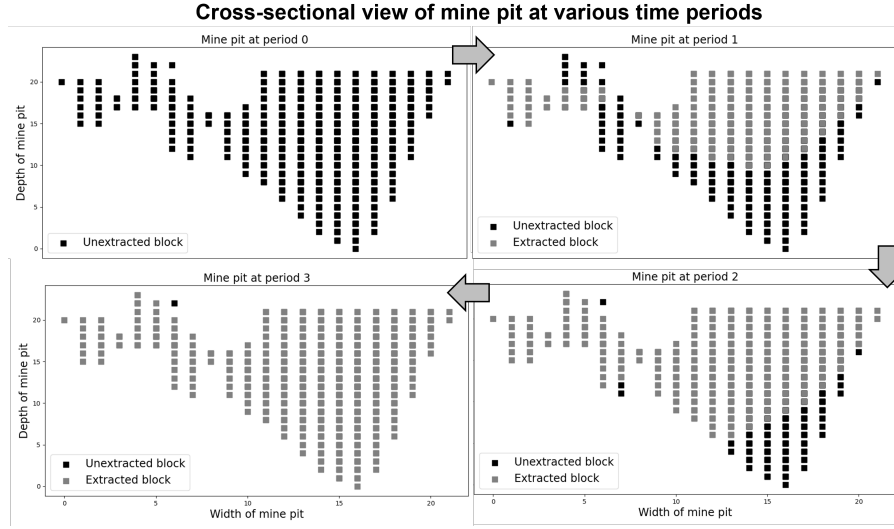


**Fig. 1.** Summary of activities for each mine phase and their influences on costs.

Similarly, the tactical and operational perspectives of mine planning are updated periodically and have the potential to incorporate environmental concerns. Planning from the tactical perspective [13] over the whole mine lifespan (decades) has been tackled as the *precedence-constrained* production scheduling problem (PCPSP) and acknowledged as NP-hard. The logistical challenge in the PCPSP involves the movement of extracted raw materials from the mine pit through the broad sequence of processing and refinement steps to obtain the desired products. To support these activities, there are various operational facilities for processing (e.g., crushing and grinding), refining (e.g., hydrometallurgy), storage (i.e., stockpile), and waste (e.g., tailings pond and dump). These facilities have their own set of resources and corresponding capacity constraints.

When modeling mines, planners discretize the buried materials into three-dimensional blocks to decide the sequence of extraction. Each block would have its own set of preceding blocks that require prior extraction due to the geology and geometry unique to each mine site. This is known as precedence constraints and is illustrated in Fig. 2. There are then diverse resource constraints for each block at the downstream processing and refining facilities. For the efficient and effective movement of extracted materials, these sequence and processing decisions are considered holistically with the approximate net present value (NPV) of profit throughout decades of the mine's lifespan.

Barring profit, environmental considerations and the push for net zero carbon emissions are reverberating within the mining industry due to its extensive processing and operations scale. Hence, the monitoring of carbon emissions throughout the value chain of the PCPSP is increasingly necessary and ties in with schemes such as the emission trading systems (ETS) and emission taxes. In



**Fig. 2.** Block extraction sequence example from year to year following precedence and resource constraints for MineLib’s Newman1 instance [3]

this paper, we boost previous research on the PCPSP by enhancing the generic PCPSP formulation [6] with carbon cost. This allows planners to consider the NPV of carbon cost alongside profit.

Instead of solely maximizing the NPV of profit in the generic PCPSP formulation, we convert it into a multi-objective optimization (MOO) problem with another objective that minimizes the NPV of carbon cost. Next, we apply the NSGA-II [5] whereby components of the algorithm - initial solution generation, crossover and mutation - are tailored to the PCPSP. As far as we know, earlier works of evolutionary algorithms (EA) to the PCPSP did not propose similar frameworks. In fact, details on the initial solution generation, crossover, and mutation are quite lacking. Furthermore, earlier works for the PCPSP employed EA such as genetic algorithm [1] and differential evolution [8] for a single objective formulation problem whereas we use NSGA-II for an MOO problem. We then test variants of our approach with a real-world mine dataset from an operating mine and benchmark instances from Minelib [6]. It turns out that variants to the solution generation can improve results. Furthermore, we demonstrate how our approach can produce approximated Pareto fronts for planners to observe carbon emissions targets when deciding on mining plans.

## 2 Related Work

The angles of strategic, tactical, and operational issues are commonly used for research on the logistics of mining. Lately, qualitative and quantitative research on mine planning that incorporates sustainability elements has increased. However,

quantitative research based on operations research, artificial intelligence, and machine learning trails behind qualitative research [21]. Furthermore, research that incorporates carbon emissions from a tactical angle remains sparse.

From the strategic angle, one of the research covers carbon emissions indirectly by increasing efficiency and thus minimizing haulage costs. The main focus of Rim  l  , Dimitrakopoulos, and Gamache [14] is to minimize land disturbance by optimizing the order of ore extraction. To achieve this, they propose an in-pit waste disposal approach, which involves depositing unprofitable extracted ores in available areas within the pit instead of moving them to temporary dumps. Meanwhile, Xu et al. [20] model carbon emission cost directly as one of the undesirable environmental outputs. In their multi-objective approach, they aim to minimize undesirable environmental outputs while simultaneously maximizing both the NPV of profit and social benefits. It focuses solely on the pit limit, which determines whether or not to extract the ores while ignoring the sequencing and processing decisions thereafter.

From the operational angle, the objective is to explicitly decrease expenses associated with carbon emissions in operational facilities and transportation networks. Valderrama et al. [18] utilize a mixed integer programming (MIP) model to analyze carbon emissions from inter-facility transportation and operating facilities. Similarly, Attari and Torkayesh [2] employ a multi-objective MIP to examine carbon emissions in transportation between facilities and customers. In comparison, Canales-Bustos, Santibañez-González, and Candia-Véjar [4] develop a multi-objective hybrid particle swarm optimization algorithm to minimize investment costs, transportation costs, deviations between product quality and goals, and carbon emissions from facilities and vehicles. These operational angles ignore the tactical decisions of extraction and sequencing of blocks. Lastly, Wang et al. [19] indirectly model carbon emissions by comparing resource efficiency and the NPV of profit with an NSGA-II. Their formulation also pre-defines extraction decisions and focuses only on the processing of extracted ores.

Research from the tactical angle directly models carbon emission cost when examining trade-offs between profit and sustainability. Azhar et al. [3] consider extraction, sequencing, and processing decisions of ores as per the PCPSP. They enhance the generic PCPSP formulation with an additional constraint of carbon emission cost [20] to produce an approximated Pareto front.

Our research is based on the tactical angle of the PCPSP, focusing on decisions of block extraction, extraction sequence, and its processing steps [3]. We also directly model the trade-off between the NPV of profit and the carbon emission cost by adopting the carbon costing framework by Xu et al. [20]. However, we differ from Azhar et al. [3] by including the carbon emission cost as an additional objective function instead of a constraint.

### 3 The PCPSP Definition and Formulation

The PCPSP determines the entire mining process, including extraction and processing decisions for valuable mineral ores, and provides investors with an es-

timate of a mine's value [7]. The mine consists of several components, such as the pit, dump, stockpiles, processing plants, and heavy machinery. In the pit, mineral ore deposits are divided into blocks of the same size for modeling purposes. Each block has a unique value and a set of precedence constraints based on geology, which affects the overall extraction sequence over time and how the ore is processed, as illustrated in Fig. 2. After extraction, the block is transported to processing facilities where it undergoes various treatments, such as crushing, grinding, and screening, to reduce it according to the requirements. Then, the material is refined to improve its quality and obtain various desired end products. Unfortunately, this value chain consumes significant raw materials (e.g., energy, water, gases) and generates harmful by-products (e.g., water and air pollution, chemical waste).

### 3.1 Carbon Costing Formulation

Our main aim is to augment the PCPSP formulation with carbon costing. To measure carbon emissions, we use the metric by Xu et al. [20], which defines the cost of carbon emissions  $\mathcal{C}_{i,e}$  from energy consumption. This cost reflects the amount of carbon dioxide that is absorbed during ore excavation, processing, and refining. The formula includes two key quantities: the amount of ore extracted from the pit and sent for further processing  $\mathcal{Q}_{i,o}$ , and the amount of waste material extracted and treated  $\mathcal{Q}_{i,w}$ . These quantities are multiplied by the energy consumed using coal to extract one unit tonne of material (either ore or waste) from the pit  $e_m$ , and the energy consumed to process one unit tonne of ore  $e_p$ . These values are then multiplied by the carbon factor of coal  $f_c$ , the conversion coefficient of carbon dioxide from carbon  $f_a$ , and the absorption cost of carbon dioxide  $\mathcal{C}_c$ .

$$\mathcal{C} = \frac{(\mathcal{Q}_{i,o} + \mathcal{Q}_{i,w})e_m + \mathcal{Q}_{i,o}e_p}{1000} f_c f_a \mathcal{C}_c \quad (1)$$

### 3.2 Enhanced Multi-Objective PCPSP Formulation

The PCPSP is a problem in scheduling mining activities that aims to maximize the NPV of profit while satisfying several requirements related to mineral ore grade, equipment availability, and processing plant capacity [6]. This requires expertise in multiple domains such as geology, chemistry, engineering, economics, and customer relations. Firstly, geologists provide information on the ore components and grades based on multiple drill samples and the structure of the surrounding materials. Secondly, mining engineers use this information to assess the structure, methods, and equipment needed to access the ore. Next, geologists and chemists determine the type of processing and refining required for different ores, which can result in varying products. Then, economists estimate the economic value of each block based on demand and supply worldwide. Lastly, customer relations evaluate the demand of current and potential customers. All these considerations make mine scheduling a complex task.

Mine scheduling research usually relies on real-world case studies because mining operations are unique and shaped by geo-metallurgical factors [13]. However, this approach leads to solution techniques that cannot be directly compared with others. The MineLib library [6] provides a set of generalized mathematical formulations and instances for three problem variants, including the PCPSP, which is the most complex problem. By adopting the PCPSP formulation, our work enables other researchers to build upon it.

The generic formulation for the PCPSP [6] defines  $\mathcal{B}$  as the set of blocks,  $\mathcal{B}_b$  as the subset of predecessors for block  $b \in \mathcal{B}$ , and  $\mathcal{D}$  as the set of destinations. The profit  $\tilde{p}_{bdt}$  is obtained by extracting a block  $b$  and processing it at a destination  $d$  during a specific period, using  $q_{bdr}$  units of operational resource  $r \in \mathcal{R}$ . A binary decision variable  $x_{bt}$  is used to indicate whether block  $b$  is extracted during period  $t$ . A continuous decision variable  $y_{bdt}$  represents portions of block  $b$  delegated to destination  $d$  during period  $t$ . The augmented multi-objective PCPSP has two objective functions. The first objective maximizes the NPV of profit for periods  $\mathcal{T}$ , while the second objective minimizes the carbon emission cost pegged to the extraction and processing of mineral ores.

The first objective function derives the profit  $\tilde{p}_{bdt}$  for a given period  $t \in \mathcal{T}$  with  $\frac{p_{bd}}{(1+\alpha)^t}$ , where  $\alpha$  denotes the discount factor. The estimated value of a block is determined by geologists using its ore composition and grade. The summation of the NPV of profit throughout the mine's lifespan emphasizes the significance of extracting blocks with higher value earlier.

$$\textbf{(Objective function 1)} \quad Z_1 = \max \sum_{b \in \mathcal{B}} \sum_{d \in \mathcal{D}} \sum_{t \in \mathcal{T}} \tilde{p}_{bdt} y_{bdt} \quad (2)$$

The second objective function calculates the discounted carbon cost  $\tilde{c}_{bdr}$  from using operational resource  $r \in \mathcal{R}$  to extract or process block  $b$  at destination  $d \in \mathcal{D}$  during period  $t \in \mathcal{T}$ . It is derived using  $\frac{c_{bdr}}{(1+\alpha)^t}$ . Practically, this function can be crucial within the framework of carbon credit trading, a market-based instrument aimed at reducing carbon dioxide emissions. Under this system, economies that exceed their allocated emissions can purchase credits from those that have reduced their emissions below their carbon emission permits.

$$\textbf{(Objective function 2)} \quad Z_2 = \min \sum_{b \in \mathcal{B}} \sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} \tilde{c}_{bdr} y_{bdt} \quad (3)$$

Constraint (4) sets conditions for the order in which blocks can be extracted, and it applies to all blocks and periods. It states that block  $b'$  must be extracted in the same or an earlier period than block  $b$ , as  $b'$  is a predecessor of  $b$ . This constraint is decided by mining engineers based on the materials surrounding the ore, including sand, silt, and clay, as well as the ore's type and composition. The latter is determined by geologists.

$$\sum_{\tau \leq t} x_{b\tau} \leq \sum_{\tau \leq t} x_{b'\tau} \quad \forall b \in \mathcal{B}, b' \in \mathcal{B}_b, t \in \mathcal{T} \quad (4)$$

Constraint (5) specifies that a block should be completely dispatched to one or multiple destinations if it is mined. Otherwise, it should not be sent to any destination. The selection of destination is influenced by the ore's grade and composition, as well as the demands of customers.

$$x_{bt} = \sum_{d \in \mathcal{D}} y_{bdt} \quad \forall b \in \mathcal{B}, t \in \mathcal{T} \quad (5)$$

Constraint (6) limits block extraction to only once during the mine's lifespan.

$$\sum_{t \in \mathcal{T}} x_{bt} \leq 1 \quad \forall b \in \mathcal{B} \quad (6)$$

Constraint (7) guarantees that for each time period  $t$ , the use of every operational resource  $r$  is within the limits of minimum  $\underline{\mathcal{R}}_{rt}$  and maximum  $\bar{\mathcal{R}}_{rt}$ . These resources are managed by mining engineers and technicians, and they include diggers, haulage trucks, grinders, and processing plants.

$$\underline{\mathcal{R}}_{rt} \leq \sum_{b \in \mathcal{B}} \sum_{d \in \mathcal{D}} q_{bdr} y_{bdt} \leq \bar{\mathcal{R}}_{rt} \quad \forall r \in \mathcal{R}, t \in \mathcal{T} \quad (7)$$

Constraint (8) represents side constraints with lower bound  $\underline{a}$  and upper bound  $\bar{a}$ . It can represent various mining situations, including grade constraints.

$$\underline{a} \leq \mathcal{A}y \leq \bar{a} \quad (8)$$

Finally, constraints (9) and (10) reflect the range of values.

$$x_{bt} \in \{0, 1\} \quad \forall b \in \mathcal{B}, t \in \mathcal{T}, \quad (9)$$

$$y_{bdt} \in [0, 1] \quad \forall b \in \mathcal{B}, d \in \mathcal{D}, t \in \mathcal{T}. \quad (10)$$

## 4 NSGA-II for the Augmented Multi-Objective PCPSP

Diverse techniques are available for MOO problems with their own advantages and disadvantages [11]. Due to the NP-hard nature of the PCPSP, exact methods such as the MIP and constraint programming [15], are found to be efficient for smaller instances but become intractable as the instance size increases. Hence, alternatives have to be explored. One of them is the extension of the genetic algorithm framework. Genetic algorithms use a population of randomly generated solutions that are evaluated for improvement at each iteration, making it possible to converge on the entire Pareto set in one run. Multi-objective evolutionary algorithms (MOEAs) belong to the class of genetic algorithms used for MOO problems. They utilize additional advanced methods to maintain a varied population of Pareto optimal solutions throughout the iterations. The MOEAs are differentiated by their fitness assignment, diversity mechanism, elitism, and the use of external population [9].



We utilize NSGA-II [5] as it remains a popular choice due to its effective mechanisms of non-domination sorting, crowding distance sorting, and elitism, which contribute to a diverse Pareto optimal set and accelerated convergence. Its wide application across other industries, including medicine [10] and manufacturing [17], further demonstrates its strength. Our implementation of the NSGA-II for the augmented multi-objective PCPSP is summarized in Fig 3.

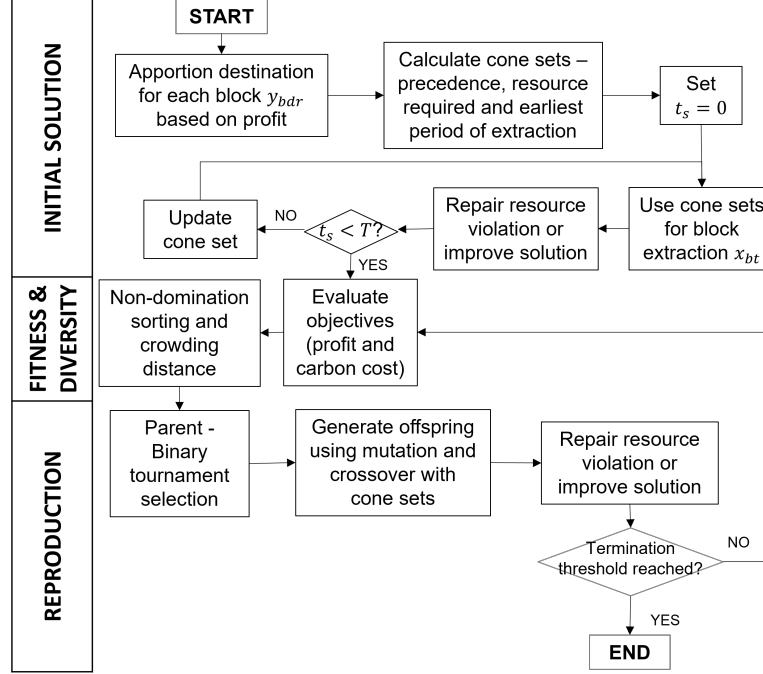
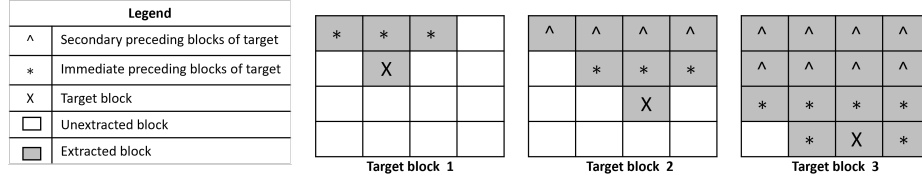


Fig. 3. Overview of NSGA-II implementation.

#### 4.1 Initial Population of Solutions with Cone Sets

The repair of precedence constraint violation is computationally intensive and hence, it should be prevented as much as possible. To do so, we compute the complete set of preceding blocks that need to be extracted before the current block of interest with a directed acyclic graph (DAG). The shape of this set, together with the block of interest, can be described as a cone set, illustrated in Fig. 4. For each cone set, we compute the resources required, the profit, and the earliest period the cone set can be extracted based on the cumulative resource capacity for each period  $t$ . The apportionment of blocks to destinations at each time period,  $y_{bdt}$  is determined by the profit associated with each destination; we utilize **argmax** and **softmax** (details in Section 5). These components form the heuristic for the initial solution, shown in Algorithm 1.



**Fig. 4.** Cross-sectional view of mine pit when excavating different target blocks.

Algorithm 1 produces a population of solutions  $\Omega_j$  where each solution  $j$  is the set of blocks extracted at each time period  $x_{bt}$ . Each solution is processed progressively through each time period  $t \in \mathcal{T}$  (line 6). At each time period, blocks are pre-selected based on the earliest period of extraction (line 9) with allowance for resource violation (line 15) using an upper bound resource multiplier  $\rho$ . The resource multiplier is randomly generated where  $\rho \in [0, 1]$  (line 5). This allows variants amongst solutions in the population set  $\Omega_j$ . Then, the resources required for the solution are checked against the upper bound of resources for the time period  $t$ . If there are no violations, a local improvement heuristic is run (line 20). Otherwise, a repair operator is invoked (line 22).

Both the improvement heuristic and the repair operator rely on finding fringe blocks. These are blocks situated at the periphery of the solution that may prevent major disruptions to the current solution when added or removed. If there is no resource violation, the priority is to add more profitable blocks. Otherwise, blocks that consume the least resources are added. Conversely when removing blocks, blocks that are least profitable are prioritized. Finally, the solutions are ranked based on the two objective functions with non-domination sorting and crowding distance [5].

## 4.2 Reproduction

The reproduction of offspring from the initial solution or parent solution consists of the binary tournament operator, mutation, and crossover. Once offspring solutions are produced, they are again ranked based on the two objective functions with non-domination sorting, and crowding distance. This reproduction step is run till the maximum number of generations (i.e., the termination threshold).

The binary tournament operator randomly selects two individuals (or solutions) from a population and chooses the best (based on the rank function of NSGA-II) for the next generation. This procedure is repeated until the desired number of individuals for the next generation is obtained. The set of parent solutions from this step then leads to the crossover operator.

Crossover facilitates the creation of new offspring solutions by combining genetic material from two parent solutions. The process involves selecting a crossover point in the parents' genetic code and exchanging genetic information beyond that point to generate two new offspring solutions. The crossover rate parameter  $\eta$  determines whether this step occurs for the parent pair. A high

**Algorithm 1** Generation of the population of initial solutions.

**Input:** Block model  $\mathcal{B}$ , destinations  $\mathcal{D}$ , predecessor edges  $\mathcal{E}$ , time periods  $\mathcal{T}$ , resource types  $\mathcal{R}$ , resource capacity required per block at each destination and resource type  $q_{bdr}$ , resource bounds for each resource type  $\bar{\mathcal{R}}_{rt}$ , profit of block when sent to destination  $\tilde{p}_{bd}$ , apportionment of the block to each destination  $y_{bdt}$  upper bound resource multiplier  $\rho$ , population size  $\Omega$

**Output:** Population of solutions  $\Omega_j$  where each solution is block extracted at each time period

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```

 $x_{bt}$ 
1:  $G \leftarrow \text{ConstructDAG}(\mathcal{B}, \mathcal{E})$ 
2:  $\theta \leftarrow \text{ConeSetComputations}(G, \mathcal{D}, \mathcal{T}, \mathcal{R}, q_{bdr}, \bar{\mathcal{R}}_{rt}, \tilde{p}_{bd})$ 
3:  $\Omega_j \leftarrow \emptyset$ 
4: for  $j$  in  $\Omega$  do
5:    $\rho \leftarrow \text{RANDOM}(0, 1)$ 
6:   for  $t$  in  $\mathcal{T}$  do
7:      $\tilde{\mathcal{B}} \leftarrow \emptyset$ 
8:     for  $b$  in  $\theta$  do
9:       if  $b$  earliest extraction period =  $t$  then
10:         $\tilde{\mathcal{B}} \leftarrow \tilde{\mathcal{B}} \cup b$ 
11:      end if
12:    end for
13:    for  $b$  in  $\tilde{\mathcal{B}}$  do
14:       $\tilde{\mathcal{I}} \leftarrow \emptyset$ 
15:      if  $\sum_{b \in \tilde{\mathcal{B}}} \sum_{d \in \mathcal{D}} q_{bdr} y_{bdt} \leq \bar{\mathcal{R}}_{rt} * \rho$  then
16:         $\tilde{\mathcal{I}} \leftarrow \tilde{\mathcal{I}} \cup b$ 
17:      end if
18:    end for
19:    if  $\sum_{b \in \tilde{\mathcal{I}}} \sum_{d \in \mathcal{D}} q_{bdr} y_{bdt} < \bar{\mathcal{R}}_{rt}$  then
20:       $x_{btj} \leftarrow \text{ImproveSolution}(G, q_{bdr}, \tilde{p}_{bd}, \tilde{\mathcal{I}})$ 
21:    else if  $\sum_{b \in \tilde{\mathcal{I}}} \sum_{d \in \mathcal{D}} q_{bdr} y_{bdt} > \bar{\mathcal{R}}_{rt}$  then
22:       $x_{btj} \leftarrow \text{RepairResourceViolation}(G, q_{bdr}, \tilde{p}_{bd}, \tilde{\mathcal{I}})$ 
23:    end if
24:  end for
25:   $\Omega_j \leftarrow \Omega_j \cup x_{bt\omega}$ 
26: end for
27: return  $\Omega_j$ 

```

---

crossover rate reflects a higher chance that crossover occurs. Due to the multi-period structure of the PCPSP, we design an interdependent-period single-point crossover that accounts for the entire mine lifespan. It combines two parent solutions by exchanging portions of their periodic components. We use a crossover cut  $\zeta \in [0, 1]$ , drawn randomly, to determine the crossover index using  $\zeta * \sum_{t \in \mathcal{T}} x_{bt}$ . This index can fall in any time period. Fig. 5 illustrates a crossover between two feasible solutions. The last step in this illustration is an immediate fix to prevent block extraction across multiple periods.

Subsequently, mutation introduces a small random alteration to the genetic code of an individual. In the mutation step, we choose blocks for extraction (i.e.,  $x_{bt} = 1$ ) with no precedence blocks. The current time period  $t'$  is mutated randomly to another time period  $t \in \{\mathcal{T} \setminus t'\}$ . The mutation step occurs for all parent pairs, but we used the mutation rate  $\gamma$  to select the percentage of blocks with no precedence  $|\{\hat{b}\}|$  to mutate. The subset of blocks with no precedence is selected uniformly using  $\text{SAMPLE}(\text{RANDOM}(0, \gamma) * |\{\hat{b}\}|, \{\hat{b}\})$ .

Solutions may become infeasible after crossover and mutation. The crossover step may result in precedence violation whereas the mutation step may result in resource violation. Hence, the repair operator is invoked. It uses the pre-computed cone sets that are akin to the initial solution generation.

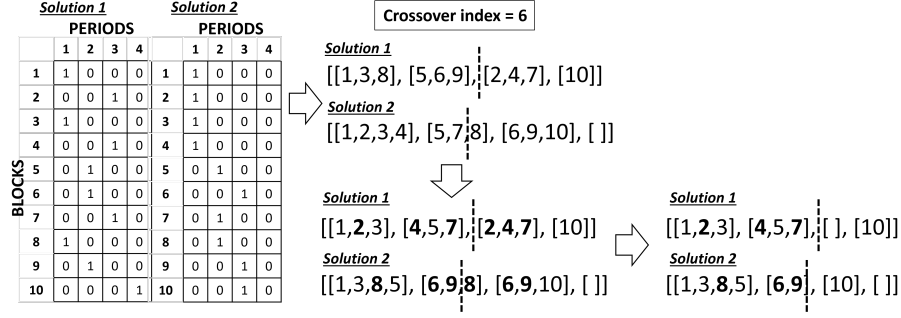


Fig. 5. Crossover between two feasible solutions.

## 5 Experiments

We utilize a small-sized real-world inspired dataset from an operating copper and gold mine, Wilma1. It was adapted for the generic formulation and mocked up. We complemented it with benchmark instances from MineLib - Newman1 and Kd. Newman1 is small-sized whilst Kd is medium-sized (Kd is a copper mine in North America). Table 1 informs the number of blocks  $|\mathcal{B}|$ , precedence  $|\mathcal{B}_b|$ , time periods  $|\mathcal{T}|$ , destinations  $|\mathcal{D}|$  and operational resource constraints  $|\mathcal{R}|$ .

Table 1. Key characteristics of the dataset.

Name	Block	Precedence	Periods	Destinations	Resources
Newman1	1,060	3,922	6	2	2
Wilma1	1,960	7,112	4	3	3
Kd	14,153	219,778	12	2	2

### 5.1 Experimental Setup

There are two NSGA-II variants - based on the **argmax** and **softmax** functions – for the portions of block  $b$  delegated to destination  $d$  during period  $t$ ,  $y_{bdt} \in [0, 1]$ . This directly affects Objective Function 1. The first variant, using the **argmax** function, apportions a block fully to the destination with the most profit. Meanwhile, the **softmax** function variant, apportions a block across all destinations with a multiplier  $\lambda$  on the profit of that block when sent to each destination,  $p_{bd}$ :

$$\text{softmax} = \frac{e^{\lambda p_{bd}}}{\sum_{d=1}^{\mathcal{D}} e^{\lambda p_{bd}}} \quad (11)$$

We run the two variants three times, each with 100 solutions in a population and generations ranging from eight to ten. From the six populations of solution

sets, we derive the true Pareto front of non-dominated solutions. Then, the solution sets from each population are compared using the ratio of non-dominated individuals (RNI) [16], distance, and diversity metrics [5]. Firstly, the RNI metric yields the proportion of best-known solutions  $\phi$  (i.e., solutions that form part of the Pareto front) that exist in a population  $\Omega_j$  with  $\mathcal{N}$  solutions:

$$\text{Ratio of non-dominated individuals (RNI)} = \frac{|\phi_j|}{\mathcal{N}} \quad (12)$$

Secondly, the distance metric assesses how closely a solution  $j$  in population  $\Omega_j$  converges to the true Pareto front. Initially, the metric calculates the minimum Euclidean distance between a solution from population  $\Omega_j$  and all solutions  $k$  from the true front  $\Omega_k$ . This is then averaged across all  $\mathcal{N}$  solutions. A value close to zero is desired.

$$\text{Distance metric} = \frac{\sum_{j=1}^{\mathcal{N}} \min d_{jk}}{\mathcal{N}} \quad (13)$$

Finally, the diversity metric evaluates the even spread of solutions over the true Pareto front using the Euclidean distance. It considers the distance between successive solutions  $d_i$ , the average distance  $\bar{d}$  of all  $d_i$ , distances between extreme solutions of the true Pareto-optimal front  $d_k$ , and distances between extreme solutions in a population set  $d_j$ . A value close to one indicates better diversity.

$$\text{Diversity metric} = \frac{d_k + d_j + \sum_{i=1}^{\mathcal{N}-1} |d_i - \bar{d}|}{d_k + d_j + (\mathcal{N} - 1)\bar{d}} \quad (14)$$

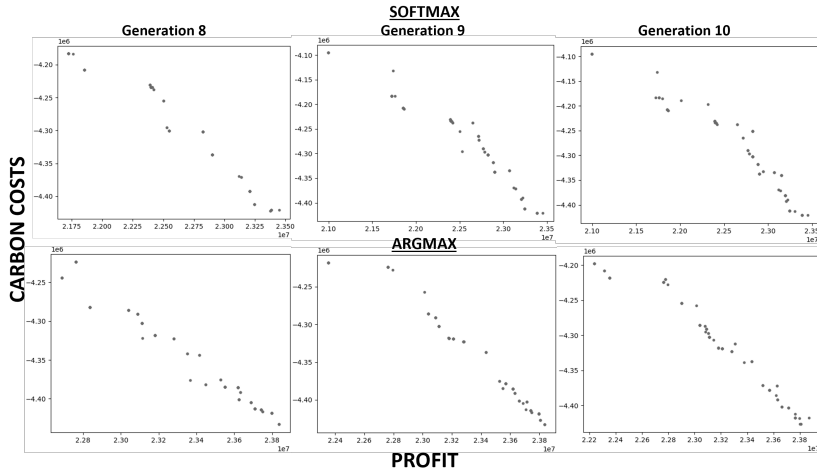
The NSGA-II model for the augmented multi-objective PCPSP was built using Python. The algorithm was executed on a Linux operating system with 3.5 GHz 3rd generation Intel Xeon Scalable processor, 128 vCPUs and 128 Gb memory. The initial solution was produced with the upper bound resource multiplier  $\rho$  of 1.2. In the reproduction, the mutation  $\gamma$  and crossover  $\eta$  rates were set to 0.2 and 0.6, respectively. Lastly, the softmax multiplier  $\lambda$  was 4.

## 5.2 Results

The performance metrics of RNI, distance, and diversity for the argmax and softmax variants are summarized in Table 2. Overall for Newman1 and Wilma1 instances, the **argmax** variants, compared to the **softmax** variants, generate *sets of solutions* that are closer to the true Pareto front (distance metric), but with less assortment (diversity metric) and lesser *individual solutions* that form part of the true Pareto front (RNI). For Newman1, the **argmax** and **softmax** average distance metrics are 0.290 and 0.304, respectively, while for Wilma1, their values are 0.056 (argmax) and 0.074 (softmax). Next, the diversity metric averages 0.753 (argmax) and 0.830 (softmax) for Newman1, and 0.863 (argmax) and 1.017 (softmax) for Wilma1. Finally, the RNI averages 0.003 (argmax) and 0.007 (softmax) for Newman1, and 0.007 (argmax) and 0.037 (softmax) for Wilma1.

**Table 2.** Performance metrics of experiment variants across datasets.

Experiment	Softmax			Argmax		
Generation	RNI	Distance	Diversity	RNI	Distance	Diversity
<b>Newman1</b>						
8	0	0.264	<b>0.862</b>	0	<b>0.254</b>	0.780
9	0	0.332	0.820	0	0.295	0.767
10	<b>0.02</b>	0.317	0.807	0.01	0.321	0.713
Average	0.007	0.304	0.830	0.003	0.290	0.753
<b>Wilma1</b>						
8	<b>0.09</b>	0.082	1.037	0.01	<b>0.039</b>	0.803
9	0.01	0.078	0.977	0	0.070	0.875
10	0.01	0.063	<b>1.038</b>	0.01	0.058	0.911
Average	0.037	0.074	1.017	0.007	0.056	0.863
<b>Kd</b>						
8	0	0.332	0.749	<b>0.03</b>	0.322	<b>0.946</b>
9	0	0.251	0.684	0	0.324	0.905
10	0	<b>0.236</b>	0.685	0.01	0.333	0.925
Average	0	0.273	0.706	0.01	0.326	0.925

**Fig. 6.** Newman1 Pareto front across generations for two NSGA-II variants.

Meanwhile for the medium-sized instance of Kd, these observations between the **argmax** and **softmax** variants are reversed. The **softmax** variants, compared to the **argmax** variants, generate *sets of solutions* that are closer to the true Pareto front (distance metric), but with less assortment (diversity metric) and lesser *individual solutions* that form part of the true Pareto front (RNI). The average values are found in Table 2.

A Pareto optimal front is also produced for each population variant. This visual aid allows mine planners to appreciate the trade-off between the NPV of profit and carbon cost for decision making. Fig. 6, Fig. 7 and Fig. 8 display

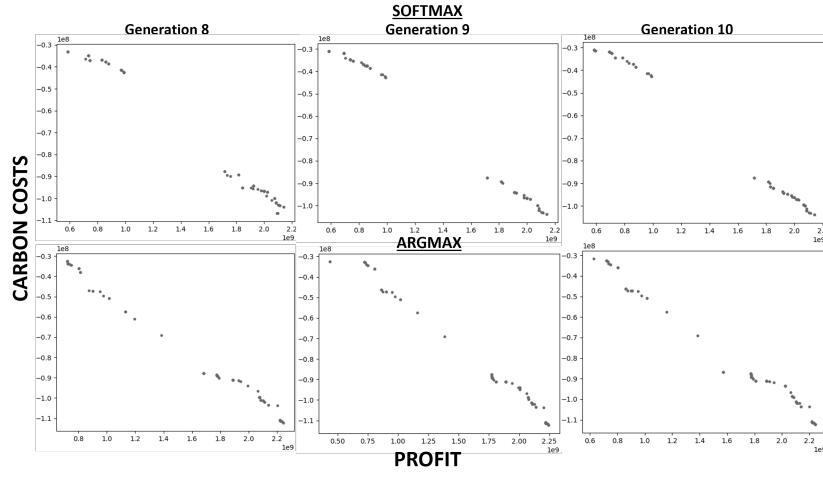


Fig. 7. Wilma1 Pareto front across generations for two NSGA-II variants.

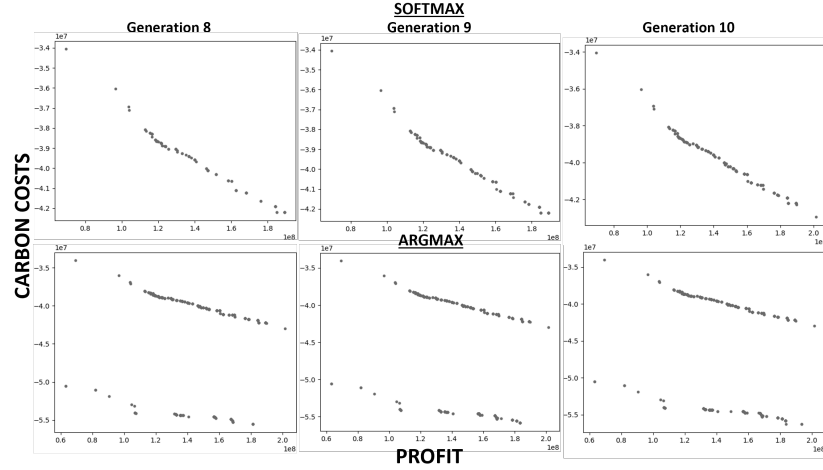


Fig. 8. Kd Pareto front across generations for two NSGA-II variants.

the Pareto front of the population variants across generations for Newman1, Wilma1, and Kd respectively. The computation time for each population variant was less than an hour for Newman1. However, when the model is applied to larger datasets, the computation time increases. Hence, the traversal of the search space may be improved for this framework to be applied to much larger instances.

## 6 Conclusion

In this paper, we implement the NSGA-II model to balance the NPV of profit with carbon emissions when managing the movement of ores from the mine

pit to the production facilities. Our framework for the open pit mine addresses sustainability concerns by augmenting the generic PCPSP into a multi-objective problem that can be catered to diverse environmental concerns. It is applied to real-world instances of an operating mine and MineLib that have been extended for carbon emission considerations. Our approach demonstrates the effective formation of Pareto fronts with profit and carbon cost axes so that mine planners can consider both aspects concurrently.

Previous work has used the generic PCPSP for carbon emissions trade-off but through an additional constraint. To the best of our knowledge, this is the first reformulation into a MOO problem. Future work can focus on improving the computation time to extend this proof of concept to larger datasets. Additionally, other MOEAs can be evaluated against the NSGA-II model and the objective functions can be further expanded for other environmental concerns.

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## References

1. Alipour, A., khodaiari, A.A., Jafari, A., Tavakkoli-Moghaddam, R.: A genetic algorithm approach for open-pit mine production scheduling. *Int. Journal of Mining & Geo-Engineering* **51**(1) (Jun 2017), <https://doi.org/10.22059/ijmge.2017.62152>
2. Attari, M.Y.N., Torkayesh, A.E.: Developing benders decomposition algorithm for a green supply chain network of mine industry: Case of Iranian mine industry. *Operations Research Perspectives* **5**, 371–382 (2018), <https://doi.org/10.1016/j.orp.2018.11.002>
3. Azhar, N.A.B., Gunawan, A., Cheng, S.F., Leonardi, E.: A carbon-aware planning framework for production scheduling in mining. In: *Computational Logistics: 13th International Conference, ICCL 2022, Barcelona, Spain, September 21–23, 2022, Proceedings*. pp. 441–456. Springer (2022), [https://doi.org/10.1007/978-3-031-16579-5\\_30](https://doi.org/10.1007/978-3-031-16579-5_30)
4. Canales-Bustos, L., Santibañez-González, E., Candia-Véjar, A.: A multi-objective optimization model for the design of an effective decarbonized supply chain in mining. *International Journal of Production Economics* **193**, 449–464 (2017), <https://doi.org/10.1016/j.ijpe.2017.08.012>
5. Deb, K., Agrawal, S., Pratap, A., Meyarivan, T.: A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. In: *International conference on parallel problem solving from nature*. pp. 849–858. Springer (2000), [https://doi.org/10.1007/3-540-45356-3\\_83](https://doi.org/10.1007/3-540-45356-3_83)
6. Espinoza, D., Goycoolea, M., Moreno, E., Newman, A.: MineLib: a library of open pit mining problems. *Annals of Operations Research* **206**(1), 93–114 (Dec 2012), <https://doi.org/10.1007/s10479-012-1258-3>
7. Hustrulid, W.A., Kuchta, M., Martin, R.K.: *Open pit mine planning and design, two volume set & CD-ROM pack*. CRC Press (2013)
8. Khan, A., Niemann-Delius, C.: A differential evolution based approach for the production scheduling of open pit mines with or without the condition of grade uncertainty. *Applied Soft Computing* **66**, 428–437 (May 2018), <https://doi.org/10.1016/j.asoc.2018.02.010>



9. Konak, A., Coit, D.W., Smith, A.E.: Multi-objective optimization using genetic algorithms: A tutorial. *Reliability engineering & system safety* **91**(9), 992–1007 (2006), <https://doi.org/10.1016/j.res.2005.11.018>
10. Luong, N.H., Alderliesten, T., Bel, A., Niatsetski, Y., Bosman, P.A.: Application and benchmarking of multi-objective evolutionary algorithms on high-dose-rate brachytherapy planning for prostate cancer treatment. *Swarm and Evolutionary Computation* **40**, 37–52 (2018), <https://doi.org/10.1016/j.swevo.2017.12.003>
11. Marler, R.T., Arora, J.S.: Survey of multi-objective optimization methods for engineering. *Structural and multidisciplinary optimization* **26**(6), 369–395 (2004), <https://doi.org/10.1007/s00158-003-0368-6>
12. Morrison-Saunders, A., McHenry, M.P., Sequeira, A.R., Gorey, P., Mtegha, H., Doepel, D.: Integrating mine closure planning with environmental impact assessment: challenges and opportunities drawn from african and australian practice. *Impact Assessment and Project Appraisal* **34**(2), 117–128 (Apr 2016), <https://doi.org/10.1080/14615517.2016.1176407>
13. Newman, A.M., Rubio, E., Caro, R., Weintraub, A., Eurek, K.: A review of operations research in mine planning. *Interfaces* **40**(3), 222–245 (2010), <https://doi.org/10.1287/inte.1090.0492>
14. Rim  l  , M.A., Dimitrakopoulos, R., Gamache, M.: A stochastic optimization method with in-pit waste and tailings disposal for open pit life-of-mine production planning. *Resources Policy* **57**, 112–121 (Aug 2018), <https://doi.org/10.1016/j.resourpol.2018.02.006>
15. Soto, R., Crawford, B., Almonacid, B., Johnson, F., Olgu  n, E.: Solving open-pit long-term production planning problems with constraint programming a performance evaluation. In: 2014 9th International Conference on Software Engineering and Applications (ICSOFT-EA). pp. 70–77. IEEE (2014), <https://doi.org/10.5220/0005093900700077>
16. Tan, K.C., Lee, T.H., Khor, E.F.: Evolutionary algorithms for multi-objective optimization: Performance assessments and comparisons. *Artificial intelligence review* **17**(4), 251–290 (2002), <https://doi.org/10.1023/a:1015516501242>
17. Touzout, F.A., Benyoucef, L.: Multi-objective multi-unit process plan generation in a reconfigurable manufacturing environment: a comparative study of three hybrid metaheuristics. *International Journal of Production Research* **57**(24), 7520–7535 (2019), <https://doi.org/10.1080/00207543.2019.1635277>
18. Valderrama, C.V., Santib  n  ez-Gonz  lez, E., Pimentel, B., Candia-Vejar, A., Canales-Bustos, L.: Designing an environmental supply chain network in the mining industry to reduce carbon emissions. *Journal of Cleaner Production* **254**, 119688 (2020), <https://doi.org/10.1016/j.jclepro.2019.119688>
19. Wang, X., Gu, X., Liu, Z., Wang, Q., Xu, X., Zheng, M.: Production process optimization of metal mines considering economic benefit and resource efficiency using an NSGA-II model. *Processes* **6**(11), 228 (2018), <https://doi.org/10.3390/pr6110228>
20. Xu, X.c., Gu, X.w., Qing, W., Zhao, Y.q., Wang, Z.k.: Open pit limit optimization considering economic profit, ecological costs and social benefits. *Transactions of Nonferrous Metals Society of China* **31**(12), 3847–3861 (2021), [https://doi.org/10.1016/s1003-6326\(21\)65769-2](https://doi.org/10.1016/s1003-6326(21)65769-2)
21. Zeng, L., Liu, S.Q., Kozan, E., Corry, P., Masoud, M.: A comprehensive interdisciplinary review of mine supply chain management. *Resources Policy* **74**, 102274 (2021), <https://doi.org/10.1016/j.resourpol.2021.102274>