# Efficient Encrypted Data Search With Expressive Queries and Flexible Update

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Abstract—Outsourcing encrypted data to cloud servers that has become a prevalent trend among Internet users to date. There is a long list of advantages on data outsourcing, such as the reduction cost of local data management. How to securely operate encrypted data (remotely), however, is the top-rank concern over data owner. Liang et al. proposed a novel encrypted cloud-based data share and search system without loss of privacy. The system allows users to flexibly search and share encrypted data as well as updating keyword field. However, the search complexity of the system is of extreme inefficiency,  $O(nd)$ , where d is the total number of system files and n is the size of query formula. This article, for the first time, leverages the "oblivious cross search" technology in public key searchable encryption context to reduce the search complexity to *only*  $O(n_f(w))$ , where  $f(w)$  is the number of files embedded with the "least frequent keyword" w. The new scheme maintains efficient encrypted data share and keyword field update as well. This article further revisits the security models for payload security, keyword privacy and search token privacy (i.e., search pattern privacy) and meanwhile, presents security and efficiency analysis for the new scheme.

 $\bigstar$ 

Index Terms—Secure data search, share, flexible query, update, efficiency

# 1 INTRODUCTION

SEARCHABLE encryption (SE) [30] enables <sup>a</sup> data owner to fulfill search over encrypted outsourced data without loss of data and query secrecy. The data owner needs to build an encrypted search index structure for a cloud server. It next can generate a search token with its secret information, so that the server can locate and return all encrypted data matching the query without knowing the exact "contents" of query and the underlying data. SE is applicable to many real-world cloud applications (e.g., CipherCloud). So far, it is unknown that if there exists a trivial way to combine SE with other secure sharing and computing technologies (e.g., proxy re-encryption and homomorphic encryption), so that a data owner may flexibly operate (e.g., sharing, searching and even computing) its encrypted data stored in cloud [24].

To partially resolve the unknown, an attribute-based searchable proxy re-encryption scheme is proposed in [20]. It is the first of its type to fill the difficulties and technical gaps between SE and proxy re-encryption. Its computation and communication complexity, however, strongly depend on the size of attribute set/policy. Specifically, the size of system public key, re-encryption key and search token are linear in the size of attribute set and furthermore, data share leads to the expansion of ciphertext size. Besides, the keyword update is not flexible enough as the system only allows the update to be done in data sharing stage. The efficiency bottleneck and flexible keyword update are two interesting open problems left by [20].

To tackle the problems, Liang et al. [19] proposed a searchable proxy re-encryption with flexible keyword update in the context of identity-based encryption, in which system users can update keyword at any time and meanwhile, data search and share complexity are much more efficient than that of [20].

Motivation. Although outperforming [20] in the merit of data search, share and keyword update, [19] still cannot achieve "efficient" search. Specifically, the scheme requires the complexity  $O(nd)$  to fulfill a formula query, where n is the size of the formula and  $d$  is the total number of encrypted files. In other words, a cloud server has to check through all encrypted files for a single keyword search, i.e.,  $O(d)$  where  $n = 1$ . While  $n > 1$ , the server needs to repeat the above throughout check  $n$  times. The search complexity then is linear with the size of the formula as well as that of encrypted database. In practice, response time or online client waiting time is significantly related to the search efficiency. How to reduce the complexity that mainly motivates this work. Besides, this work will attempt to consider the privacy of search token.

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Contributions. While deploying more expressive search (e.g., conjunctive), the public key based SE (PKSE) technology leads to expensive computation and communication cost (e.g., [5] with composite order group). It is hard to resolve the efficiency bottleneck beyond the search expressiveness in the context of PKSE.

Inspired by the "oblivious cross search" technique introduced in [7], we propose a feasible solution, to some extent, to fill the gap between efficiency and search expressiveness in this paper. By using the solution, we reduce the search complexity of [19] from  $O(nd)$  to  $O(nf(w))$ . Our technical roadmap is described as follows. We employ TSet and XSet into our scheme, in which *TSet* is an encrypted and scrambled set for database  $DB = (m_i, W_i)_{i=1}^d$ , and *XSet* can be seen as an extra "linked" level between keywords and files seen as an extra "linked" level between keywords and files. We create "links" between keywords and files in TSet, and similarly, generate the "copy" links in  $XSet$ . The files here become the connection between TSet and XSet. Much like the idea presented in [7], our solution leverages the search in the form of  $Q = w_1 \wedge \xi(w_2, \dots, w_n)$ , where  $w_1$  is "least fre-<br>quent keyword" that is used to identify the "smallest" quent keyword" that is used to identify the "smallest" encrypted files set, and  $\xi$  is the unconstrained query formula. The corresponding encrypted files of  $w_1$  can be located in  $TSet$ , while the query  $\xi$  is performed in  $XSet.$ Since the connection between the two sets are the files, if there are existing some files satisfying the search in both TSet and XSet, they are the output of the query Q. The search complexity now is proportional to the number of files embedded with  $w_1$ .

To allow one to search some single keyword in TSet, we reuse the TSet instantiation in [7]. To achieve keyword update, we need to redesign the search token, and to revise TSet instantiation. Note that TSet contains many  $T[w]$ (which will be introduced later), in which a  $T[w]$  corresponds to all encrypted files embedded the same w. Since the encrypted files are stored in  $T[w]$  randomly, the server has no chance to know which one will be going to have keyword update later. We mask the "keyword-file link" into extra ciphertext components, so that the server can first locate  $T[w]$  and next use pairing computation to find the specified item in  $T[w]$  (to be updated). We design an algorithm called  $LotAlg$  to fulfill the above "double" trace function. To update *TSet* accordingly, we add a new algorithm called  $UpTSet$  into  $TSet$  instantiation. We have to require system user to store the location information of XSet as a matrix locally, so that he/she can inform the server that the item located in the position  $(i, j)$  needs to be updated.

This paper is an extension but not trivial incremental progress of the conference version [19]. The contributions of this paper are described as follows.

- In this paper, we mainly follow the system definition given in [19] but with an exception that we revise the data sharing algorithm so as to only allow a data owner to share its encrypted files embedded with a specific keyword with others. Keyword update is necessary no more in the data sharing phase, as user can update the keyword field after being granted the decryption rights of the shared data.
- We define a "weak" search token privacy model matching our system definition in this paper.
- It is the first time to employ SSE's "oblivious cross search" technique into a PKSE scheme. We also effectively integrate the technique with keyword update.
- The new system maintains effective data share, search and keyword update functionalities and meanwhile, it achieves constant size in public key, re-encryption key, search token and ciphertext, and no linearly cost exists in the construction phases of re-encryption key and search token. In other words, we reduce the search complexity without compromising other features and efficiency.
- The security and efficiency analysis show that out system has a relatively better potential in the deployment of large scale database.
- This paper designs algorithms for TSet update and redundancy elimination, which may be of independent interest.

Related Work. Song et al. [30] introduced the first notion of SE. Two main streams of SE have been defined: one is symmetric SE (SSE), and the other is public key based SE (PKSE). Some light-weight cryptographic tools (e.g., pseudorandom function) are usually used in SSE, so that an SEE construction (e.g., [8]) enjoys relatively high search efficiency. In contrast, a PKSE scheme (e.g., [4]) leverages public key technology that yields the loss of efficiency in search. SSE and PKSE have respective pros and cons. SSE cannot easily check data integrity unless data owner downloads all encrypted data from server. Although supporting data integrity check at anytime and by any system user, PKSE suffers from poor search efficiency and less query expressiveness. This paper, for the first time, employs SSE's technique into PKSE so as to tackle the query efficiency and expressiveness issue.

Following [4], Abdalla et al. [1] proposed a generic PKSE construction anonymous identity-based encryption. To date many PKSE variants have been proposed, e.g., authorized keyword search [14], verifiable keyword search [2], fuzzy keyword search [32], conjunctive keyword search [11], range query [5], [29] supporting conjunctive, subset, and range search queries, and attribute-based keyword search [34].

To delegate decryption rights to others, Blaze, Bleumer and Strauss [3] defined the concept of proxy re-encryption (PRE). A PRE scheme enables a semi-trusted proxy to convert a ciphertext of a message intended for a user to another ciphertext of the same message intended for another user without "seeing" the massage. In this paper, we consider a "multiple conversion" case, which we call it multi-hop proxy re-encryption. PRE has been used in various contexts, for example traditional PRE [6], [21], [22], identity-based PRE, e.g., [12], [17], [18], and functional PRE, e.g., [16].

It is unknown that if there is a way to integrate a SSE/ PKSE with a PRE scheme to yield a secure protocol. Some difficulties stand in front of the combination. We need to consider the keyword privacy into two ciphertext levels - an original ciphertext as well as its corresponding re-encrypted ciphertext. Besides, we need to guarantee that an adversary with search ability cannot break the payload security for ciphertexts. To combine PKSE with PRE, Shao et al. [28] introduced a new primitive called PRE with keyword search (PREKS). Hu and Liu propose a "search-but-nodecryption" PREKS scheme. These schemes are built based





on bidirectional technique to achieve data sharing, but they cannot hold against collusion attacks where a proxy colludes with another system user to reveal data owner's secret key. To eliminate this attack, Fang et al. [10] designed a new PREKS system but with the price that the loss of searchability after data sharing.

A new PREKS is recently proposed in [20]. It is the first of its type to explore PREKS in the attribute-based context. It, however, suffers from heavy search complexity due to the leverage of attribute-based technology to relate the size of search token to the size of attribute set. Similarly, Zheng, Xu and Ateniese's attribute-based searchable system [34] does also "overkill" the search efficiency for the purpose of achieving attribute property. Knowing the bottleneck of the previous mentioned schemes, Liang et al. [19] proposed a novel system in the context of identity-based to achieve better search efficiency. However, [19] still suffers from linear search complexity. A search for a formula with size  $n$ , in [19], has to take  $O(nd)$ . Besides, the scheme has not considered the search token (i.e., search pattern) privacy yet. This paper targets to tackle the above open problems of [19]. Note some recent research works have been proposed for real-world applications and attacks on SE, e.g., the searchable chain of PKSE [15], passive attacks over SSE [26] and the PKSE applied to Internet of Things [13]. We notice that there exist some researches on SSE over the topics of forward and backward security [31], [36], [37].

Since this work is the first to combine identity-based encryption, PRE, SE and oblivious cross search techniques, it is briefly compared with the most related works, namely the seminal PKSE [4], an identity-based PRE [12], attributebased PKSE [20], a recent identity-based PKSE [19] and a recent PKSE scheme [13] in Table 1. In the table,  $|S|$  denotes the size of attribute set/policy,  $\perp$  denotes "not applicable", "rk" denotes re-encryption key, "token" denotes search token and "ROM" is short for random oracle model, respectively. By ciphertext expansion we mean that the size of ciphertext will be expanded after the ciphertext is shared. We use "linear" and "constant" to denote if a given size (e.g., the size of ciphertext) grows linearly with keyword/ attribute/re-encryption hops or it is constant (no matter how many numbers of keyword/attribute/hop it has). It can be seen that this work is the first to achieve  $O(n f(w))$  efficiency in search without loss of data share and keyword update functionalities in the context of PKSE. We will present the efficiency analysis and practical comparison with [13] in terms of computation and communication cost in Section 4.

# 2 PROBLEM STATEMENT

# 2.1 System Entities

- A data encryptor forms a database DB as  $(m_i, W_i)_{i=1}^d$ <br>next encrypts DB to be EDB and finally uploads the next encrypts DB to be EDB, and finally uploads the  $EDB$  to a cloud server, where  $d$  is the number of files in DB.
- A data receiver owns the underlying data of EDB (stored in the cloud) intended for it and meanwhile, it (with the valid decryption rights) can construct search token for the EDB queries. It is allowed to share its EDB to a specified system user. It further can update the keyword field of EDB by delivering the server a keyword update token.
- A trusted key issue center is to generate a secret key for each system user. It also takes part in the generation of a special key for decryption rights delegation.
- A cloud server stores system users' EDB. Given a search/share/update token, it can locate and return/ share/update the corresponding EDB matching the token.

Note that a data encryptor can be also a data receiver for itself, i.e., outsourcing its own encrypted data to the cloud.

# 2.2 System Algorithms

The system definition is similar to that of [19].

Definition 1. It consists of the following algorithms:

- $(mpk, msk) \leftarrow Setup(1<sup>k</sup>)$ . On input a security parameter k, the system setup algorithm outputs a master public key mpk and a master secret key msk, where  $k \in \mathbb{N}$ . Hereafter, we implicitly regard mpk as an input for the following algorithms.
- $(pk_{ID}, sk_{ID}) \leftarrow KeyGen(msk, ID)$ . On input msk and an identity ID, the key pair generation algorithm outputs a public key and secret key pair  $(pk_{ID}, sk_{ID})$

for a system user with identity ID  $\in \mathbb{Z}_q^*$ . Hereafter, we<br>assume that why implicitly includes the identity ID assume that  $pk_{ID}$  implicitly includes the identity ID.

- $EDB \leftarrow Enc(pk_{ID}, DB)$ . On input a  $pk_{ID}$ , and a DB (i.e.,  $(m_i, W_i)_{i=1}^d$ , including the files' identities  $m_d$ , and<br>a keyword description set  $W \in \{0, 1\}^*$   $d = |DR|$ ) the a keyword description set  $W\in\{0,1\}^*$ ,  $d=|DB|$ ), the<br>data encruntion algorithm outnuts an encrunted EDB data encryption algorithm outputs an encrypted EDB.
- $TK \leftarrow TKGen(sk_{ID}, w)$ . On input a sk<sub>ID</sub>, and a keyword description w, the search token generation algorithm outputs a search token TK, which is used to search user ID's EDB with keyword description w.
- $rk_{ID_i\rightarrow ID_j|w_i} \leftarrow ReKeyGen(msk, ID_i, ID_j, w_i)$ . On input msk, an  $ID_i$ , an  $ID_j$ , and a  $w_i$ , the re-encryption key generation algorithm outputs a re-encryption key  $rk_{ID_i\rightarrow ID_j|w_i}$  which can be used to share the encrypted files tagged with  $w_i$  in  $ID_i$ 's EDB to  $ID_i$ .
- $uptk_{w_i\rightarrow w_j} \leftarrow UpTKGen(sk_{ID}, m_d, w_i, w_j)$ . On input a sk<sub>ID</sub>, a file identity  $m_d$ , an old keyword description  $w_i$  tagged with  $m_d$  and a new one  $w_i$ , the keyword update token generation algorithm outputs a update token  $uptk_{w_i \to w_j}$  which can be used to update the encrypted file  $m_d$  in EDB (intended for ID) with the old description  $w_i$  to the one with the new one  $w_i$ .
- $EDB \leftarrow ReEnc(r_{kID_i \rightarrow ID_j|w_i}, EDB)$ . On input a<br>share token  $rk_{ID}$ , and an EDB the re-encryption share token  $rk_{ID_i \to ID_j|w_i}$  and an EDB, the re-encryption algorithm converts all encrypted files tagged with  $w_i$  in  $ID_i$ 's EDB into those encrypted files of the same message under  $ID_j$  and  $w_i$ . We note that this conversion maintains keyword update ability for  $ID_i$ . In addition, we state that the user  $ID_i$  cannot obtain the decryption and search rights of the new (re-encrypted) ciphertext after the conversion.
- $EDB \leftarrow Update(uptk_{w_i \rightarrow w_i}, EDB)$ . On input a keyword update token  $uptk_{w_i \rightarrow w_j}$  and an EDB, the keyword update algorithm updates all ciphertexts with an old keyword  $w_i$  to those with the new keyword  $w_i$ within the given EDB.
- $1/0 \leftarrow Search(TK, EDB)$ . On input a search token TK generated by the user with decryption rights on EDB, and an EDB, the search algorithm outputs 1 if they match, and 0 otherwise. We further note that a cloud server will choose to return either the matching ciphertext(s) or nothing to the user based on the outputs of the algorithm.
- $Dec(sk_{ID}, EDB)$ . On input a  $sk_{ID}$  and an EDB, the ciphertext decryption algorithm outputs all files  $m_d$ .

The main system work flow is described as follows.

- The setup phase. A trusted authority first runs the algorithm *Setup* to generate the  $mpk$  for all system users, trusted key issue center and a cloud server, to initialize the system and to keep the  $msk$  only for the key issue center.
- The key pair generation phase. The trusted key issue center generates a key pair for a system user via running the algorithm KeyGen. A user ID publishes the public key  $pk_{ID}$  and keeps  $sk_{ID}$  secret.
- The encryption phase. A data encryptor, Bob, runs the algorithm *Enc* to generate an *EDB* for Alice, with the corresponding keyword description  $K1$ , and further uploads the *EDB* to the cloud server.

Note here for simplicity we set  $|W| = 1$  and K1 is the only keyword in W.

- The data search phase.
	- 1. Alice (with the decryption rights of the EDB) runs the algorithm TKGen and delivers the search token corresponding to  $K1$  to the server.
	- 2. The server intakes the token and the EDB to run the algorithm Search. If finding a match, the server outputs 1 and returns the corresponding ciphertexts, and outputs 0 and returns nothing otherwise.
	- 3. If receiving a successful return from the server, Alice runs the algorithm Dec with its secret key to recover the underlying message.
- The keyword description update phase.
	- 1. To update keyword description from  $K1$  to  $K2$ , Alice (with the decryption rights of the ciphertext) runs the algorithm  $UpTKGen$  to construct a keyword update token  $K1 \rightarrow K2$ , and next delivers it along with a search token for  $K1$  to the server.
	- 2. If there are encrypted files tagged with  $K1$ , the server runs the algorithm Update intaking the token and the EDB to update the keyword description to K2.
- The ciphertext share phase.
	- 1. To share its encrypted data tagged with  $K1$  to Carol, Alice can generate a re-encryption key from Alice  $\rightarrow$  Carol with the help of the trusted key issue center by running the algorithm ReKeyGen.
	- 2. If there are encrypted files tagged with  $K1$ , the server runs the algorithm  $ReEnc$  to convert the ciphertext's decryption rights to Carol.
	- 3. Carol can retrieve the shared files by searching.

# 2.3 Threat Models

We define the data confidentiality model, the keyword privacy model and search token privacy model below. Generally speaking, the first model is used to guarantee that a Probabilistic Polynomial Time (PPT) adversary cannot compromise the information of a message by given an encryption of the message; the second model is to ensure a PPT adversary cannot reveal the keyword embedded in a given ciphertext; the last model is for preventing a PPT adversary from extracting the keyword embedded into a given search token. Note we will consider the privacy of keyword update token as well. This can be captured in the keyword privacy model.

We assume that cloud server, data encryptor and data receiver are honest-but-curious, while the authority for system initialization and the key issue center are fully trusted. We further assume either data encryptor or data receiver will not collude with the server to reveal the underlying keyword description and database. Note that we here leave the collusion attacks for our future work. By honest-butcurious (i.e., semi-honest) we mean that one will honestly run a protocol by following the specification of a protocol but curiously collecting some information (in which it is interested) during the protocol execution.

Definition 2. Our system achieves chosen plaintext (CPA) security if the advantage Adv<sup>CPA</sup> is negligible for any PPT adver-<br>sary A in the following experiment: sary <sup>A</sup> in the following experiment:

$$
|Pr[b = b' : (mpk, msk) \leftarrow Setup(1^k); (DB_0, DB_1, ID^*,state) \leftarrow \mathcal{A}^O(mpk); b \in_R \{0, 1\}; EDB^* \leftarrow Enc(pk_{ID^*},DB_b^*); b' \leftarrow \mathcal{A}^O(EDB^*, state)] - \frac{1}{2}|,
$$

where *state* is the state information,  $DB_0$ ,  $DB_1$  are two equal-size databases with the form  $(m_i, W_i)_{i=1}^d$ ,  $W^* = \sum W_i$ <br>is the challenge keyword set  $ID^*$  is the challenge identity is the challenge keyword set,  $ID^*$  is the challenge identity,  $\mathcal{O} = \{ \mathcal{O}_{pk}, \mathcal{O}_{sk}, \mathcal{O}_{rk}, \mathcal{O}_{uptk}, \mathcal{O}_{TK} \}$ . By querying the public key oracle  $\mathcal{O}_{pk}$ , A is given the corresponding public key of the system user (it issues). For the secret key oracle  $\mathcal{O}_{sk}$ , intaking ID, the oracle outputs  $sk_{ID}$  for A, where  $ID \neq ID^*$  indicating the challenge identity cannot be corrupted by A. For the re-encryption key oracle  $\mathcal{O}_{rk}$ , intaking a tuple  $(ID_i, ID_j,$  $w_i$ ), the oracle outputs  $rk_{ID_i\rightarrow ID_j}|w_i$ . If  $ID_i$  (resp.  $ID_j$ ) is in an an analyzing position of  $ID^*$  (reds  $ID$  mean be honest re-encryption path including  $ID^*$  (note  $ID_i$  may be equal to  $ID^*$ ). and meanwhile,  $ID_j$  (resp.  $ID_i$ ) is in a corrupted re-encryption path,  $\mathcal{O}_{rk}$  outputs  $\perp$ . By a re-encryption path (of a given ciphertext) we mean a path that is used to record the re-encryption history of the ciphertext among different system users (in which the nodes of the path stands for users), for example, a re-encryption path (of a ciphertext) between user A and user C could be  $A - B - C$ . If one of the users is corrupted, then the path is defined as a corrupted one; otherwise, it is an honest path. For the keyword update token oracle  $\mathcal{O}_{uptk}$ , intaking a tuple (*ID*,  $w_i$ ,  $w_j$ ), the oracle outputs a token  $uptk_{w_i \to w_j}$  for keyword description update. For the search token oracle  $\mathcal{O}_{TK}$ , intaking a tuple (ID, w), the oracle outputs a search token TK. We here do not offer reencryption, update and search oracles to A. A, however, can run the corresponding re-encryption, update and search algorithms with the re-encryption keys, keyword update tokens and search tokens given by the above defined oracles. We further note that the given two databases are shared with the same keyword set  $W^*$ . It won't affect the security level of the game, since we focus on the database secrecy other than that of keyword.

Definition 3. Our system achieves keyword privacy if the advantage Adv $_A^{\chi_P}$  is negligible for any PPT adversary A in the<br>following experiment: following experiment:

$$
|Pr[b = b' : (mpk, msk) \leftarrow Setup(1^k); (DB, W_0^*, W_1^*,ID^*, state) \leftarrow A^{\mathcal{O}}(mpk); b \in_R \{0, 1\}; EDB^* \leftarrow Enc(\npk_{ID^*}, W_b^*, DB); b' \leftarrow A^{\mathcal{O}}(EDB^*, state)] - \frac{1}{2}|,
$$

where *state* is the state information,  $m$  is the challenge message,  $W_0^*$ ,  $W_1^*$  are two challenge distinct keyword sets (for clarity, we highlight them as an "individual" input for  $Enc$ ),  $ID^*$  is the challenge identity, and  $\mathcal{O} = \{ \mathcal{O}_{pk}, \mathcal{O}_{sk}, \mathcal{O}_{rk}, \mathcal{O}_{uptk}, \mathcal{O}_{uptk} \}$  $\mathcal{O}_{TK}$ . The oracle  $\mathcal{O}_{pk}$  returns public keys for A. For the secret key oracle  $\mathcal{O}_{sk}$ , intaking ID, the oracle outputs  $sk_{ID}$ , where  $ID \neq ID^*$ . For the re-encryption key oracle  $\mathcal{O}_{rk}$ , intaking a tuple  $(ID_i, ID_j, w_i)$ , the oracle outputs  $rk_{ID_i\rightarrow ID_j|w_i}$ . If  $ID_i$  (resp.  $ID_j$ ) is in an honest re-encryption

path including  $ID^*$  and meanwhile,  $ID_i$  (resp.  $ID_i$ ) is in a corrupted re-encryption path,  $\mathcal{O}_{rk}$  outputs  $\perp$ . For the keyword update token oracle  $\mathcal{O}_{uptk}$ , intaking a tuple (ID,  $w_i$ ,  $w_i$ ), the oracle outputs a token  $uptk_{w_i\rightarrow w_i}$  for keyword description update. For the search token oracle  $\mathcal{O}_{TK}$ , intaking a tuple  $(ID, w)$ , the oracle outputs a search token TK. If  $ID = ID^*$  and w is in a keyword update path including at least one of the challenge keywords (note  $w$  may be equal to one of the challenge keywords),  $\mathcal{O}_{TK}$  outputs  $\bot$ . If  $ID \neq ID^*$ is in a re-encryption path including  $ID^*$  and meanwhile,  $w$ is in a keyword update path including at least one of the challenge keywords,  $\mathcal{O}_{TK}$  outputs  $\perp$  as well. By a keyword update path we mean a path records all the keywords (of a given ciphertext) which have been updated so far, e.g.,  $w_1 - w_2 - w_3$ .

Definition 4. Our system achieves weak search token privacy if the advantage Adv<sup>STP</sup> is negligible for any PPT adversary A in<br>the following experiment: the following experiment:

$$
|Pr[b = b' : (mpk, msk) \leftarrow Setup(1^k); (w_0^*, w_1^*, ID^*) \leftarrow
$$
  

$$
\mathcal{A}(mpk); b \in_R \{0, 1\}; TK_0 \leftarrow TKGen(sk_{ID^*}, w_R^*),
$$
  

$$
TK_1 \leftarrow TKGen(sk_{ID^*}, w_b^*); b' \leftarrow \mathcal{A}^{\mathcal{O}}(TK_0, TK_1)] - \frac{1}{2}|,
$$

where  $w_0^*$ ,  $w_1^*$  are two challenge distinct keyword set, each of the set includes a pair of distinct keywords,  $w_R^*$  is a random keyword set with two distinct keywords in the keyword space,  $ID^*$  is the challenge identity, and  $\mathcal{O} = \{ \mathcal{O}_{TK} \}.$ For the search token oracle  $\mathcal{O}_{TK}$ , intaking a tuple (ID, w), the oracle outputs a search token *TK*. If  $ID = ID^*$ ,  $w = w_b^*$ ,  $\mathcal{O}_{\text{max}}$  outputs  $\perp$  $\mathcal{O}_{TK}$  outputs  $\perp$ .

## 3 SYSTEM CONSTRUCTION

## 3.1 The Intuition

Let a bilinear map tuple be  $(q, g, \hat{g}, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e)$ , where  $\mathbb{G}_1$ ,  $\mathbb{G}_2$  ( $\mathbb{G}_1$  and  $\mathbb{G}_2$  are not the same group) and  $\mathbb{G}_T$  are multiplicative cyclic groups of prime order  $q$ ,  $|q| = k$ , and g is a random generator of  $\mathbb{G}_1$ ,  $\hat{g}$  is a random generator of  $\mathbb{G}_2$ . The mapping  $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$  has three properties: (1) Bilinearity: for all  $a, b \in_R \mathbb{Z}_q^*$ ,  $e(g^a, \hat{g}^b) = e(g, \hat{g})^{ab}$ ; (2) Non-degeneracy:<br> $e(g, \hat{g}) \neq 1$  where  $1g$  is the unit of  $\mathbb{C}_{\infty}$ ; (3) Computability e  $e(g, \hat{g}) \neq 1_{\mathbb{G}_T}$ , where  $1_{\mathbb{G}_T}$  is the unit of  $\mathbb{G}_T$ ; (3) Computability: e can be efficiently computed.

A concrete encrypted cloud-based data share and search construction is proposed in [19]. The construction supports single keyword search, conjunctive keyword and even formula queries. For example, a system user can launch a query  $(w_1 \wedge w_2 \wedge w_3) \vee w_4$  to a "curious-but-honest" cloud server. The query indicates that the server needs to return all the ciphertexts containing the keywords  $w_1$ ,  $w_2$  and  $w_3$  at the same time, and those with the keyword  $w_4$ . The direct way for the server to fulfill the search is to first obtain the set  $A_1$  of the ciphertexts  $\mathbb{C}_{w_1}$  embedded with  $w_1$ , the set  $A_2$ of  $\mathbb{C}_{w_2}$  with  $w_2$ , the set  $A_3$  of  $\mathbb{C}_{w_3}$  with  $w_3$ , and the set  $A_4$  of  $\mathbb{C}_{w_4}$  with  $w_4$ , and finally return the intersection of the three sets,  $A_1 \wedge A_2 \wedge A_3$ , and the set  $A_4$  to the user.

However, the search efficiency is extremely low. This is because the server has to exhaustedly search the whole encrypted database for each keyword. Namely, the complexity of each keyword search is linear to the size of the

database. Accordingly, the complexity of the formula query search is at least  $O(mn)$ , where m is the size of the formula, and  $n$  is the size of the database (i.e., how many encrypted data are stored in the database).

To tackle the efficiency problem, we choose to leverage the latest technique introduced in [7], which we named it as "oblivious cross search". Below, we show that how to apply oblivious cross search into our construction to relieve the complexity to  $O(f(w)m)$ , where  $f(w)$  is the complexity of locating the "least frequent keyword"  $w$  in the database. Note that we refer readers to the Section 3.1.1 in [7] about the way of choosing least frequent keyword in a database. The premise of the design is to build up a second level of search index for the cross searching. Meanwhile, we also need a new and black-box building block, called T-Set protocol [7]. The primitive includes three main algorithms, namely  $(TSet, K_T) \leftarrow TSetSetup(T)$ ,  $stag \leftarrow TSetGetTag(K_T, w)$  and  $T[w] \leftarrow TSetRetrieve(TSet, stag)$ , where T is an array of lists of equal-length bit strings indexed by the elements of keywords, so that for any function  $n(k)$  of the security parameter k, for each keyword  $w T[w]$  is a list  $t = (s_1, \ldots, s_{T_w})$  of strings (note  $T_w = |T[w]|$ ,  $n(k) = |s_i|$  and  $i \in [1, T_w]$ ). The function of the primitive are the followings: (1) Put the tuples of  $w$  and the corresponding related file identity into  $T[w]$ ; all  $T[w]$ form a  $T$ ; (2) Intake  $T$ , the algorithm  $TSetSetup$  outputs an encrypted "scrambled" set TSet and a secret information  $K_T$ ; (3) Intake a keyword w and  $K_T$ , the algorithm  $TSetGetTag$ outputs a trapdoor  $stag$  under  $w$ ; the  $stag$  is used to locate the corresponding  $T[w]$  later; (4) Intake stag and TSet, the algorithm TSetRetrieve identifies the location and next returns  $T[w]$ . We will introduce the usage of TSet later. Both  $T[w]$ and TSet are designed for a level of search index and further will be efficiently used in cross searching. We state that the above techniques allow server to fasten the search complexity (i.e., shortening search response time/client online waiting time) but they also require data owner and cloud server to increase some level of computation and storage. We further note that this "increased burden" for data owner and server are acceptable in practice (please see Section 4 for efficiency analysis).

In addition to the above techniques, we make use of IBE and PRE technologies. We use IBE to embed the keyword for ciphertext which is a typical public key searchable encryption mode; while the PRE is used for encrypted data sharing for the case where the encrypted files can be further shared to others in cloud server.

We assume the notation m to be a file identity in  $\{0,1\}^k$ ,<br>3 to be a dataset filling with files  $ID \subseteq \mathbb{Z}^*$  is the identity *DB* to be a dataset filling with files,  $ID \in \mathbb{Z}_q^*$  is the identity of system user  $w \in \mathbb{Z}^*$ . In practice, we may put  $w \in \{0, 1\}^*$ of system user,  $w \in \mathbb{Z}_q^*$ . In practice, we may put  $w \in \{0,1\}^*$ <br>in a Target Collision Resistant (TCR) hash function [9] to in a Target Collision Resistant (TCR) hash function [9] to yield an element in  $\mathbb{Z}^*_q$  before using it. Notations used in our construction are summarized in Table 2.

#### 3.2 The Construction

Below we show how to design an efficient search construction by using oblivious cross search technique.

Setup. A fully trusted authority will initialize the system, and set up msk and mpk below.

1). Choose an asymmetric pairing group  $(q, g, \hat{g}, \mathbb{G}_1, \mathbb{G}_2,$  $\mathbb{G}_T$ , e).

TABLE 2 Frequently Used Notations

| $\boldsymbol{k}$            | security parameter   |
|-----------------------------|--|
| ID                          | user identity  |
| msk                         | master secret key  |
| mpk                         | master public key  |
| <b>PKG</b>                  | private key generator  |
| $sk_{ID}/pk_{ID}$           | the user <i>ID's</i> secret/public key   |
| DB/EDB                      | database/encrypted database  |
| DB(w)<br>d.                 | a set of files' identities where the files are with $w$<br>the file number of DB, i.e., $ DB  = d$ |
| m<br>$W/w$                  | file identity<br>keyword set/keyword   |
| $List_{sk}$                 | a list for storing users' secret keys  |
| $List_{up}$                 | a list for storing keyword update details  |
| $List_{rk}$                 | a list for storing re-encryption details   |
| T                           | an array for storing tuples of $w$ and $m$   |
| T[w]                        | a list for storing all $m$ embedded with $w$   |
| <i>XSet</i>                 | a set for storing xtag   |
| <b>TSet</b>                 | a scramble and secure set for storing all $T[w]$   |
| XMat                        | a set for storing $(w, m, c_1, c_2)$   |
| L                           | a list for storing $(e, y)$  |
| c <sub>1</sub>              | a counter: the repetition of $w$   |
| c <sub>2</sub>              | a counter: the update number of $m$ for its<br>keyword field                                       |
| $uptk_{w_i\rightarrow w_j}$ | a keyword update token, from $w_i$ to $w_j$  |
| $\delta_i$                  | a fresh random factor for $w_i$  |
| $\sigma_{i\rightarrow j}$   | a fresh random factor for re-encryption from<br>$ID_i$ to $ID_i$                                   |
| ТK                          | a query/search token   |
| Pos                         | a list for storing position value  |
|                             |  |

- 2). Choose  $\theta$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\alpha_4$ ,  $\alpha_5$ ,  $\beta_1$ ,  $\beta_2 \in_R \mathbb{Z}_q^*$ , and set  $b_1 = \theta_1^{\beta_1}$ ,  $b_2 = \theta_2^{\beta_2}$ ,  $a_3 = \theta_1^{\alpha_3}$ ,  $a_4 = \theta_2^{\alpha_4}$ ,  $a_5 = \theta_3^{\alpha_5}$  $h_1 = g^{\beta_1}$ ,  $h_2 = g^{\beta_2}$ ,  $g_1 = g^{\alpha_1}$ ,  $g_2 = g^{\alpha_2}$ ,  $g_3 = g^{\alpha_3}$ ,  $g_4 = g^{\alpha_4}$ ,  $g_5 = g^{\alpha_5}$ ,  $K = g^{\theta}$ ,  $\hat{h}_1 = \hat{g}^{\beta_1}$ ,  $\hat{h}_2 = \hat{g}^{\beta_2}$ ,  $\hat{g}_1 = \hat{g}^{\alpha_1}$ ,  $\hat{g}_2 = \hat{g}^{\alpha_2}$ ,  $\hat{g}_3 = \hat{g}^{\alpha_3}$ ,  $\hat{g}_4 = \hat{g}^{\alpha_4}$ ,  $\hat{g}_5 = \hat{g}^{\alpha_5}$ , and  $\hat{K} = \hat{g}^{\theta}$ .
- 3). Choose TCR hash functions  $H_1 : \mathbb{G}_T \to \mathbb{G}_1$ ,  $H_2 : \mathbb{G}_T \to$  $\mathbb{Z}_q^*$ , and  $H_3: \{0,1\}^k \to \mathbb{Z}_q^*.$
- 4). Output  $msk = (\hat{g}_1^{\alpha_1}, \alpha_5, \hat{h}_1, \hat{K})$ , and  $mpk = (q, h_1, h_2,$ <br> $\hat{h}_2, q, \hat{q}_1, q_2, \hat{q}_2, q_3, \hat{q}_3, q_4, \hat{q}_5, q_6, K, F, H, H_2)$  $h_2$ , g,  $\hat{g}$ ,  $g_1$ ,  $\hat{g}_1$ ,  $g_2$ ,  $\hat{g}_2$ ,  $g_3$ ,  $\hat{g}_3$ ,  $g_4$ ,  $\hat{g}_4$ ,  $g_5$ ,  $K$ ,  $F$ ,  $H_1$ ,  $H_2$ ). KeyGen. This algorithm is run by a trusted PKG.
- 1). Choose an  $r \in_R \mathbb{Z}_{q'}^*$  and set  $(\hat{g}_2^{\alpha_1}(\hat{h}_1^{ID}\hat{g}_3)^r, \hat{g}^r)$  for the user ID user ID.
- 2). Set  $g_6 = g^{\alpha_6}$ ,  $g_7 = g^{\alpha_7}$ ,  $g_8 = g^{\alpha_8}$ ,  $g_9 = g^{\alpha_9}$ ,  $\hat{g}_6 = \hat{g}^{\alpha_6}$ ,  $\hat{g}_7 = \hat{g}^{\alpha_7}$ ,  $\hat{g}_8 = \hat{g}^{\alpha_8}$ ,  $\hat{g}_9 = \hat{g}^{\alpha_9}$ , where  $\alpha_6$ ,  $\alpha_7$ ,  $\alpha_8$ ,  $\alpha_9 \in_R \mathbb{Z}_q^*$ .<br>Make use of a list List, to store the tuple (LD)
- 3). Make use of a list  $List_{sk}$  to store the tuple  $(I\hat{D}, I\hat{D})$  $\hat{g}_2^{\alpha_1}(\hat{h}_1^{ID}\hat{g}_3)^r$ ,  $\hat{g}^r$ ,  $r$ ,  $\alpha_6$ ,  $\alpha_7$ ,  $\alpha_8$ ,  $\alpha_9$ ).
- 4). Output  $sk_{ID}$  as  $(\hat{g}_2^{\alpha_1}(\hat{h}_i^{ID}\hat{g}_3)^r, \hat{g}^r, \alpha_6, \alpha_7, \alpha_8, \alpha_9)$ , and  $g_{N,D}$  as  $(a_2, a_3, a_3, a_9)$  $pk_{ID}$  as  $(g_6, g_7, \hat{g}_7, g_8, \hat{g}_8, g_9).$

Enc. The algorithm is run by a data encryptor to encrypt  $m$  with a description  $w$  for a user ID. (1). Parse a DB to be  $(m_i, W_i)_{i=1}^d$ , where  $W_i$  is the keyword set. Initialize an empty set array  $T$  (indexed by keywords from  $W$ ) and an empty set XSet.

(2). For each  $w \in W$ , build the tuple list  $T[w]$  and  $XSet$  as

- Initialize an empty list L.
- -For all  $m \in DB(w)$ , initialize a counter  $c_1 = 0$ , set  $C_1 = m \cdot e(g_2, \hat{g}_1)^t$ ,  $C_2 = g^t$ ,  $C_3 = (h_1^{ID}g_3)^t$ ,  $C_4 =$



Fig. 1. The structures used to store  $T$  and  $XSet$  belonging to user  $ID$ .

 $(g_5g_4^{-H_2(m)})^t$ ,  $C_5 = e(g_4, \hat{g}_4)^t$ ,  $C_6 = H_1(e(h_2, \hat{g}_4)^t)$  $C_7 = K^t$ ,  $y = (g_7^{H_3(w||c_1)}g_6)^t$ ,  $xtag = (g_8^{H_3(w)}g_9)^t$ , where  $t\in_R \mathbb{Z}_q^*.$ Sot $e-$ 

- Set  $e = (C_1, C_2, C_3, C_4, C_5, C_6, C_7)$ , and append  $(e, y)$ to  $L$ , add  $xtag$  to  $XSet$ . The structure of  $XSet$  is shown in Fig. 1. We assume that  $XSet$  is formed as an  $n \times t$  matrix, and meanwhile, the user will locally save an  $n \times t$  matrix *XMat*, where the position  $(i, j)$ of *XMat* stores a tuple ( $w_i \rightarrow m_j$ ,  $c_1$ ), the position  $(i, j)$  of *XSet* stores a *xtag* indicating a relationship  $(w_i, m_j)$ ,  $|W| = n$  and  $t \leq kd$ . For instance,  $(i, j)$  in *XMat* could be (1,0), meaning  $w_i$  is tagged with  $m_i$ with no repetition. We further assume that  $XMat$ will be automatically synchronized by user if there is any update, and moreover,  $(i, j)$  in *XSet* is "linked" to the  $(i, j)$  in  $XMat$ .
- Put L into  $T[w]$ . Note that we show the structure of T in Fig. 1.

(3). Run  $(TSet, K_T) \leftarrow TSetSetup(T)$ , where TSet is a "scrambled and encrypted" version of  $T$ , and  $ID$  is implicitly included in TSet so that the server knows the TSet belongs to the user ID. Output  $K_T$  and  $EDB = (TSet, XSet)$ . We note that the  $EDB$  receiver needs to know  $K_T$  and  $XMat$ . The data encryptor may choose a key  $k_0$  to mask  $K_T$  and  $XMat$ , encrypt  $k_0$  for the receiver, and further publish all the encryption on a bulletin board, so that the receiver can download and save the necessary data locally.

UpTKGen. To update the description from  $w_i$  to  $w_j$ , the user *ID* uses a list  $List_{up}$  to store tuples  $(z, * \rightarrow ID, ...)$ <br> $w \rightarrow w \rightarrow \delta \rightarrow \delta \rightarrow \delta \rightarrow \gamma \rightarrow \delta^{c} \rightarrow \delta^{c+1}$ wi ! wiþ<sup>1</sup>, <sup>d</sup><sup>i</sup> ! <sup>d</sup><sup>i</sup>þ<sup>1</sup>, ci ! ciþ<sup>1</sup>, <sup>d</sup> ci <sup>i</sup> ! <sup>d</sup> ciþ1 <sup>i</sup>þ<sup>1</sup> , <sup>s</sup>), where <sup>z</sup> <sup>2</sup>  $[1, |List_{up}|]$ ,  $*$  is wildcard,  $\delta_i \in_R \mathbb{Z}_q^*$  can randomize the *i*th legacy is the underly the random keyword update token (chosen by the user), the random seed  $\delta_i^{c_i}$  is used to randomize the counter  $c_i$  (of the keyword  $w_i$ ), and  $\sigma$  will be introduced later. Each user maintains his own  $List_{up}$  in the system. The  $List_{up}$  records a keyword update path; each keyword description  $w_i$  is tagged with "fresh" random factors  $\delta_i$  and  $\delta_i^c$ ; a given tuple in the list indicates a re-encryption path from an identity  $*$  to  $ID$  - if  $\perp$  - ID, ID does not have any delegator and  $\sigma$  is equal to 1. Accordingly, if a ciphertext for ID is tagged with a keyword that has not been updated yet by a keyword update token, the tuple stored in List<sub>up</sub> is  $(z, * \rightarrow ID, w_i \rightarrow \bot \text{ or } * \rightarrow w_i,$  $\delta_i \rightarrow \perp$ ,  $\delta_i^c \rightarrow \perp$ ,  $\ast$ ), in which we say  $w_i$  (with its random factor  $\delta_i$ ) is the starting node of the current keyword undate nath  $\delta_i$ ) is the starting node of the current keyword update path (held by *ID*), and we hereafter may use  $\delta^*$  and  $\delta^{c*}$  to specify such  $\delta_i$  and  $\delta_i^c$ . To generate the token  $\mathit{uptk}_{w_i \to w_j}$ , the user first

recovers  $\delta_i$  and  $\delta_i^c$ (corresponding to the current keyword description  $w_i$ ) from  $List_{up}$  and next chooses a new  $\delta_j, \delta_j^c \in_R \mathbb{Z}_q^*$  for  $w_j$ . The user *ID* works as follows.

- 1). Search  $List_{up}$  to recover the starting random factor  $\delta^*$ of the keyword update path including  $w_i$ , the current keyword  $w_i$  and its random factors  $\delta_i$ ,  $\delta_i^c$ , and recovers  $\sigma_{x\rightarrow y}$  from the re-encryption relationship remark  $ID_x \rightarrow ID_y = ID$  (suppose *ID* has a delegator only; the multi-delegator case has been discussed in [19]). Note that if there is no keyword update (via ID's keyword update token) yet,  $\delta_i = \delta^*$ ,  $\delta_i^c = \delta^{c*}$ ; if  $\perp \rightarrow ID$ , the value of  $\sigma$  is equal to 1. Run *locToken* =  $(\omega_0, \omega_1)$ . the value of  $\sigma$  is equal to 1. Run *locToken* =  $(\omega_{p0}, \omega_{p1})$  $\omega_{p2}) \leftarrow LotAlg. locTokenGen(TSet, w_i, m, K_T, skID).$
- 2). Set  $\omega_{r1} = (\alpha_7 H_3(w_j||c'_1) + \alpha_6)\delta_j \delta_j^{c'_1}/(\alpha_7 H_3(w_i||c_1) + \alpha_6)$  $\delta_i \delta_i^c$ ,  $\omega_{r2} = (\alpha_8 H_3(w_j) + \alpha_9)\delta_j/(\alpha_8 H_3(w_i) + \alpha_9)\delta_i$ where  $c_1$  and  $c'_1$  can be retrieved from  $XMat$ .
- 3). Run  $stag \leftarrow TSetGetTag(K_T, w_j)$  to achieve a stag which will be used by the server to locate the  $T[w_j]$ from TSet.
- 4). Retrieve the row and column numbers  $i$  and  $j$  from *XMat* by using  $(w_j \rightarrow m, c_1)$  and  $(w_i \rightarrow m, c_1)$ . Put  $(i, j)^{(w_i)}$  and  $(i, j)^{(w_j)}$  to a list *Pos*. Note the user will<br>update *XMat* locally for example by setting the old update XMat locally, for example, by setting the old  $(i, j)$  tuple to be (0,0) and the new tuple to be (1,0).
- 5). Finally, output  $uptk_{w_i \to w_j} = (Pos, stag, \omega_{r1}, \omega_{r2}, locToken)$ .

ReKeyGen. When a user  $ID_i$  decides to share encrypted data under keyword  $w_i$  with another  $ID_i$ , a re-encryption key is generated and delivered to the server as follows.

- 1). The PKG sets  $rk_1 = (\hat{h}_1^{ID_i}\hat{g}_3)^{r_i}(\hat{h}_1^{ID_j}\hat{g}_3)^{-r_j}\hat{K}^{\xi}$ ,  $rk_2 = \hat{g}^{r_i-r_j}$ and  $rk_3 = \hat{g}^\xi$ , where  $\xi \in_R \mathbb{Z}_q^*$ ,  $r_i$  and  $r_j$  are stored in the <br>List corresponding to LD and LD, respectively.  $List_{sk}$  corresponding to  $ID_i$  and  $ID_j$ , respectively.
- 2). By sharing the data with  $ID_j$ ,  $ID_i$  delegates the keyword update and search abilities to  $ID_i$ . To get rid of the re-encryption key construction cost,  $ID_i$  can choose to share  $List_{up}$  with the PKG. The PKG constructs a list  $List_{rk}$  to store tuples  $(z, ID_i \rightarrow ID_j, w_i, ...)$  $\sigma_{i\to j}$ , where z is the index for a tuple,  $\sigma_{i\to j} \in_R \mathbb{Z}_q^*$  is chosen by the PKG and will be set to 1 for the case chosen by the PKG and will be set to 1 for the case where the user has no delegator. The PKG here maintains re-encryption path in  $List_{rk}$ .
- 3). The PKG verifies if  $ID_i$  has a single delegator, say  $ID<sub>o</sub>$ , in all the re-encryption paths. Note we have considered the case where  $ID_i$  has multiple delegators in [19]. If yes, set  $rk_4 = (\sigma_{i\to j}/\sigma_{o\to i})(\alpha_7^{(ID_j)}H_3(w_i||c_1) +$ <br>(*IDs*), sec (*IDs*) and set in a (*IDs*) contained in a set of the s  $\alpha_6^{(ID_j)})/((\alpha_7^{(ID_i)}H_3(w_i||c_1)+\alpha_6^{(ID_i)})(\delta_i\delta_i^{c_1})$  $\int_{i}^{c_1} / \delta^* \delta^{c*}$ )), and  $\begin{split} r k_5 &= (\sigma_{i\to j}/\sigma_{o\to i}) (\alpha_8^{(ID_j)} H_3(w_i) + \alpha_9^{(ID_j)})/((\alpha_8^{(ID_i)} H_3(w_i) + \alpha_{o\to i}^{(ID_i)})^2) \end{split}$  $\alpha_{9}^{(ID_i)})(\delta_i/\delta^*))$ ; if no, construct  $rk_4, rk_5$  as above except for setting  $\sigma_{o\rightarrow i} = 1$ , where  $\delta_i$  is related to the current keyword  $w_i$  embedded in the ciphertext (of  $ID_i$ ), and  $\delta^*$  can be traced back in  $List_{up}$  with knowledge of  $ID_i$ and  $w_i$ . The PKG then may encrypt  $\sigma_{i\rightarrow j}$  for the corresponding delegatee  $ID_j$  via a simple IBE encryption<sup>1</sup> and next to publish the encryption to a bulletin board,

1. The encryption may be as  $C_0 = \sigma_{i \to j} \cdot H_0(Y)$ ,  $C_1 = Y \cdot e(g_2, \hat{g}_1)^t$ ,<br>  $= \sigma^t C_2 = (h_1^{D_j} g_2)^t$  where  $Y \in \mathbb{C} \pi$ ,  $t \in p \mathbb{Z}^*$   $H_0 \cdot \mathbb{C} \pi \to \mathbb{Z}^*$  $C_2 = g^t$ ,  $C_3 = (h_1^{F_1} g_3)^t$ , where  $Y \in \mathbb{G}_T$ ,  $t \in_R \mathbb{Z}_{q^t}^*$ ,  $H_0 : \mathbb{G}_T \to \mathbb{Z}_q^*$ .



Fig. 2. Keyword description update for user ID only.

so that the delegatee can download the ciphertext, recover the  $\sigma_{i\rightarrow i}$  and store it into List<sub>up</sub>. Note the encryption does not need to be a part of re-encryption key.

- 4). The user  $ID_i$  runs  $stag_{w_i} \leftarrow TSetGetTag(K_I^{(ID_i)}, w_i)$ . It further locates  $(i, j)$  from  $XMat$  by  $w_i$ ,  $m_i$  c, and puts further locates  $(i, j)$  from  $XMat$  by  $w_i$ ,  $m$ ,  $c_1$ , and puts  $(i, j)$  to Pos.
- 5). The user  $ID_j$  runs  $\hat{stag} \leftarrow TSetGetTag(K_j^{(ID_j)}, w_i)$  to achieve a rkc  $-$  stag which will be used by the server achieve a  $rk_6 = st\hat{a}q$  which will be used by the server to locate the  $T[w_i]$  from *TSet* belonging to  $ID_j$ .
- 6). Finally, the re-encryption key  $rk_{ID_i\rightarrow ID_i|w_i}$  is set to be  $(Pos, rk_1, rk_2, rk_3, rk_4, rk_5, rk_6, stag_{w_i}).$
- TKGen. To generate a search token TK for query  $\bar{w} = (w_1, \dots, w_n)$ , the user *ID* works as follows.
- 1). Run  $stag \leftarrow TSetGetTag(K_T, w_1)$ . Recall that  $w_1$  is the least frequent keyword.
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- 2). For  $c_1 = 1, 2, \ldots$  till server sends "stop",<br>
 For each  $i = 2, \ldots, n$ , set  $xtoken[c_1, i] =$ For each  $i = 2, \ldots, n$ , set  $xtoken[c_1, i] =$ <br> $((\alpha(a, H_3(w_i)+\alpha_9)/((\alpha_7H_3(w_1||c_1)+\alpha_6)(\delta_1^{c_1}/\delta^{c_4})))^T \circ x)$  where  $\left( (\hat{g}^{(\alpha_8H_3(w_i)+\alpha_9)/((\alpha_7H_3(w_1||c_1)+\alpha_6)(\delta_i^{c_1}/\delta^{c_*}))} \right)^r, \hat{g}^r),$  where<br>r is a fresh random seed in  $\mathbb{Z}^*$   $\delta^{c_*}$  and  $\delta^{c_1}$  are the *r* is a fresh random seed in  $\mathbb{Z}_q^*$ ,  $\delta^{c*}$  and  $\delta_i^{c_1}$  are the current and the original random seeds for the counter which can be retrieved from  $List_{uv}$ .
	- Set  $xtoken[c_1] = (xtoken[c_1, 2], \ldots,xtoken[c_1, n]).$
- 3). Put all  $xtoken$  and  $stag$  to TK and send TK to the server.

Update. The algorithm fulfills the "update" function, namely updating the keyword description of encrypted files and sharing encrypted files among system users. The update functionality will not expand the size of encrypted files. No matter how many times an encrypted file is updated, its size remains constant.

- 1). Update keyword description (see Fig. 2) the server runs the algorithm  $Update(uptk_{w_i \rightarrow w_i}, EDB)$  as
	- To locate the items, run  $(e_i, y_i) \leftarrow LotAlg. locateItem$  $(locToken, TSet).$
	- For each tuple  $(e_i, y_i)$ , set  $y_i = y_i^{\omega_{r1}}$ , and next updates the resulting  $y_i$  into  $(e_i, y_i)$ .
	- Run  $TSet \leftarrow UpTSet(TSet, stag, UpE)$ , where  $UpE \leftarrow (e_i, y_i)$ . Note that we will introduce the algorithm  $UpTSet$  and the update of old tuples  $(e_i, y_i)$  in *TSet* later.
	- Exercise  $(i, j)^{w_i}$  from Pos, locates all *xtag* in *XSet*,<br>and undates *xtag xtag*<sup>(*v*<sub>2</sub>)</sub> It further sets the</sup> and updates  $xtag = xtag^{\omega_{r2}}$ . It further sets the *xtags* of the positions  $(i, j)^{w_i}$  to "null" and puts<br>the new *xtags* to the positions  $(i, j)^{w_j}$ the new *xtag*s to the positions  $(i, j)^{w_j}$ .

Update TSet[w]



Fig. 3. Share encrypted data - from user  $ID_i$  to user  $ID_i$ .

- 2). Share encrypted data (see Fig. 3) the server runs the algorithm  $ReEnc(rk_{ID_i \rightarrow ID_i|w}, EDB)$  as:
	- To locate the items, the server runs  $(e_i, y_i) \leftarrow$  $T[w] = L \leftarrow TSetRetrieve(TSet^{ID_i}, stag_w).$
	- For each  $(e_i, y_i)$ , the server sets

$$
C_1 = C_1 \cdot e(C_2, rk_1)^{-1} \cdot e(C_3, rk_2) \cdot e(C_7, rk_3)
$$
  
= 
$$
\frac{m \cdot e(g_2, \hat{g}_1)^t \cdot e((h_1^{ID_i}g_3)^t, \hat{g}^{r_i-r_j}) \cdot e(K^t, \hat{g}^{\xi})}{e(g^t, (\hat{h}_1^{ID_i}\hat{g}_3)^{r_i}(\hat{h}_1^{ID_j}\hat{g}_3)^{-r_j}\hat{K}^{\xi})}
$$
  
= 
$$
\frac{m \cdot e(g_2, \hat{g}_1)^t \cdot e(g^t, (\hat{h}_1^{ID_j}\hat{g}_3)^{r_j})}{e((h_1^{ID_i}g_3)^t, \hat{g}^{r_j})}, y_i = y_i^{rk_4}.
$$

- The server runs  $TSet^{ID_j} \leftarrow UpTSet(TSet^{ID_j}, rk_6,$  $UpE$ ), where  $UpE \leftarrow (e_i, y_i)$ .
- The server retrieves  $(i, j)$  from Pos, locates all xtags in  $XSet^{ID_i}$ , and further appends  $xtag^{rk_5}$  to the  $(i, j)$  of  $XSet^{ID_j}$ .

Search. With the search token  $TK$ ,  $TSet$  and  $XSet$ , the server runs the search process as follows.

- 1). The server extracts stag from TK, and set  $L \leftarrow TSetRetrieve(TSet, stag).$
- 2). For  $c = 1, ..., |L|$ , the server works as follows:<br>• Retrieve each  $(e_c, y_c)$  from L.
	- Retrieve each  $(e_c, y_c)$  from L.
	- -If  $\forall i = 2,...,n$ , check if  $e(y_c, xtoken[c, i]_1) =$ <br> $e(xtag, xtoken[c, i]_1)$  If yes send e to the user  $e(xtag, xtoken[c, i]_2)$ . If yes, send  $e_c$  to the user.<br>en the last tuple in L is reached, the server set
- 3). When the last tuple in  $L$  is reached, the server sends "stop" to the user and halt.

After receiving all  $e$  from the server, the user can easily run  $m = C_1 \cdot e(C_3, sk_2)/e(C_2, sk_1)$  to recover all file identities, where  $C_3$ ,  $C_2$  are the components of each  $e$ .

Dec. The user *ID* recovers *m* as follows.

(1). Recover  $sk_1 = \hat{g}_2^{\alpha_1}(\hat{h}_1^{ID}\hat{g}_3)^T$ ,  $sk_2 = \hat{g}^T$  from  $sk_{ID}$ .<br>(2) Because the masses of  $(G, \mathcal{A})$ .

(2). Recover the message as  $m = C_1 \cdot e(C_3, sk_2)/e(C_2, sk_1)$ . For the original ciphertext, one can compute as

$$
C_1 \cdot e(C_3, sk_2)/e(C_2, sk_1)
$$
  
=  $m \cdot e(g_2, \hat{g}_1)^t e((h_1^{ID} g_3)^t, \hat{g}^r)/e(g^t, \hat{g}_2^{a_1}(\hat{h}_1^{ID}\hat{g}_3)^r)$   
=  $m \cdot e(g_2, \hat{g}_1)^t/e(g^t, \hat{g}_2^{a_1}) = m.$ 

For the re-encrypted ciphertext, one can decrypt as

$$
C_1 \cdot e(C_3, sk_2)/e(C_2, sk_1)
$$
  
= 
$$
\frac{m \cdot e(g_2, \hat{g}_1)^t e(g^t, (\hat{h}_1^{ID_j}\hat{g}_3)^{r_j}) e((h_1^{ID_i}g_3)^t, \hat{g}^{r_j})}{e((h_1^{ID_i}g_3)^t, \hat{g}^{r_j}) e(g^t, \hat{g}_2^{a_1}(\hat{h}_1^{ID_j}\hat{g}_3)^{r_j})}
$$
  
= 
$$
m \cdot e(g_2, \hat{g}_1)^t / e(g^t, \hat{g}_2^{a_1}) = m,
$$

where  $ID_i$  is the delegator of the ciphertext, and  $ID_i$  is the delegatee, i.e., the current ciphertext holder, who can decrypt the message by using  $sk_{ID_i}$ .

(3). The user can also reveal the pseudorandom keys  $k_1$ ,  $k_2$  and  $k_3$  by  $C_0 \oplus H_3(e(g_2, \hat{g}_1)^t)$ . With the keys, the user obtains search ability obtains search ability.

## 3.3 The Construction of  $LotAlg$

The algorithm  $LotAlg$  (which is executed by both data owner and server) includes two sub-algorithms, namely locTokenGen and locateItem. It can locate the items (which are stored in *TSet*) needed to be updated. The sub-algorithm locTokenGen (run by data owner) intakes TSet, w, m,  $K_T$ ,  $sk_{ID}$ , and outputs a locate token  $locToken$  that helps server locate the items; while locTokenGen (run by server) takes locToken, TSet as input, and outputs all matching tuples  $(e_i, e_j)$  $y_i$ ). In the algorithms,  $\omega_{p0}$  is used to locate the set  $T[w]$ , and  $\omega_{p1}$ ,  $\omega_{p2}$  are to identify the files  $m_d$  within  $T[w]$ . Since  $\alpha_5$  is a part of  $msk$  belonging to PKG, the PKG will involve in Step 1. (2) to help generate  $\omega_{p1}$ ,  $\omega_{p2}$ .

# Algorithm 1.  $LotAlg$  - Locate Item in  $TSet$

- 1.  $locToken \leftarrow locTokenGen(TSet, w, m, K_T, skID)$
- (1). Run  $stag_w \leftarrow TSetGetTag(K_T, w)$ .<br>
(2). Generate  $(r_{tk}, (\hat{h}_2\hat{q}_4^{-r_{tk}})^{(\overline{a}_5 H_2(m))\delta_t \sigma_{xx}}$
- (2). Generate  $(r_{tk}, (\hat{h}_2\hat{g}_4^{-r_{tk}})^{\overline{(\alpha_5-H_2(m))\delta_t\alpha_{x-y}}})$ , where  $r_{tk} \in_R \mathbb{Z}_q^*$ .<br>
(3) Set the locate telem as legislated  $\overline{(\alpha_5+\alpha_6)}$ .
- (3). Set the locate token as  $locToken = (\omega_{p0}, \omega_{p1}, \omega_{p2}) =$  $(\textit{stag}_w, r_{tk}, (\hat{h}_2\hat{g}_4^{-rk})^{\overline{(\alpha_5-H_2(m))\delta_t\sigma_{x\rightarrow y}}}).$
- 2.  $(e_i, y_i) \leftarrow locateItem(locToken, TSet)$
- (1). Retrieve  $T[w] = L \leftarrow TSetRetrieve(TSet, stag_w)$ .<br>(2). For each  $e_i$  within  $T[w]$  ( $i \in [1, t]$ ), check  $H_1(e(C_4))$
- For each  $e_i$  within  $T[w]$  ( $i \in [1, t]$ ), check  $H_1(e(C_4, \omega_{p2})$  $C_{5}^{\varphi_{p1}}$  =  $C_{6}$ , where  $C_{4}$ ,  $C_{5}$ ,  $C_{6}$  are elements in each  $e_{i}$ .<br>Output, all the tuples  $(e, w)$  that make the abo
- (3). Output all the tuples  $(e_i, y_i)$  that make the above equation hold.

## **3.4 The Constructions of**  $TSet$ ,  $XSet$ , and  $XMat$

We state that the construction of TSet can follow the one introduced in [7]. We refer the reader to [7] for more construction details of *TSet*. The *TSet* is instantiated as a hash table with  $B$  buckets of size  $S$  each. Here, we define the size of B to be  $|W|$ , i.e., total number of keywords in a DB, and the size of S to be  $|T[w]|$ , i.e., the size of  $|DB(w)|$ . Accordingly, the size of the hash table is  $\sum_{w \in W} |DB(w)|$ . We use  $TSet[i][i] \in [1, S]$  to denote  $T[w]$ TSet[i]  $(i \in [1, B])$  and TSet[i, j]  $(j \in [1, S])$  to denote  $T[w_i]$ and the *j*th item of  $T[w_i]$ , respectively. The *j*th item includes a record tuple (label, value), in which label is used to indicate if files tagged with the same  $w$ , and  $value$  is used to store  $(e, y)$ . We show the structure of *TSet* in Fig. 4.

To adapt the design of [7] to our construction, we need to fix the size of B,  $|T[w]|$  and S, and meanwhile fix the



Fig. 4. The structure of  $TSet$ .

universal keyword set  $W$  in advance, where  $S$  has to be sufficient large. For example, we can set  $B = |W|$ , the number of keywords, to be 1,000,000 throughout the whole system, and meanwhile, for each keyword, we allow it to be repeatedly tagged with at most 10,000 files. If  $T[w]$  is not fully taken, we then automatically set the corresponding tuple  $(e, y)$  to be "null". For instance, a tuple  $T[w]$  can be  $((e_1, y_1))$ ,  $(e_2, y_2), \ldots, (e_t, y_t)$ , where  $(e_t, y_t)$ ="null". Accordingly, we need to set  $TSet[b, j].value \leftarrow "null"$  in the algorithm  $TSetSetup(T)\text{, but still setting }TSet[b,j].label \leftarrow L.$ 

The structure of *XSet* (storing *xtags*),  $n \times t$  matrix (in which the rows are for keywords and the columns are for files), is reflected onto  $XMat$  (storing  $1/0$  tuples). The position  $(i, j)$  indicates the relationship between a keyword  $w_i$ and a file  $m_j$ . Both of the structures also need to fix the size in advance, where the  $n$  and  $t$  must be identical to the size of universal keyword set and file set defined in TSet. We note that the above operations and set design are required to be done by data owner rather than server.

#### 3.5 Feasible Update Function

Below we define and design the algorithm  $UpTSet$ . The algorithm intakes TSet, stag  $\leftarrow$  F(K<sub>T</sub>, w) and a set UpE, and outputs a new TSet, where  $UpE$  is a set for  $((e_1,$  $y_1), \ldots, (e_{|UpE|}, y_{|UpE|})).$ 

# Algorithm 2. *UpTSet* - Update the TSet

- 1. Set a bit  $\beta = 1$ , and a counter  $z = 1$ ,<br>2. For each (*e*, *y*), repeat the following
- For each (*e*, *y*), repeat the following loop while  $\beta = 1$ :
- (1). Set  $(b, L, K) \leftarrow H(F(stag, z))$ , retrieve an array  $B \leftarrow TSet[b],$
- (2). Search for index  $c \in \{1, ..., S\}$  s.t.  $B[j].label = L$ ,<br>(3). Randomly choose a  $B[j].value = "null",$  and 1
- Randomly choose a  $B[j].value = "null",$  and reset  $B[j].value \leftarrow (\beta|s) \oplus K$ , where  $s \leftarrow (e, y)$ , if the item is the last one in  $B[j]$ , set  $\beta = 0$ , and  $\beta = 1$  otherwise.
- (4). If either the above reset cannot be done or all  $(e, y)$ are not reset yet, increment z.

The  $UpTSet$  can update the new tuples, say  $(e_{w_j}, y_{w_j})$  to the new "null" positions in TSet. With similar technique and knowledge of  $stag_{w_i}$ , the server can also locate the old tuples  $(e_{w_i}, y_{w_i})$  in *TSet* and further set the *B*.[*j*].*value* to "null".

#### 3.6 Get Rid of Redundancy

The update of TSet will definitely incur the redundancy of (e, y). For example, updating a tuple  $(e_{w_i}, y_{w_i})$  to become  $(e_{w_i}, y_{w_i})$  by using the algorithm  $UpTSet$ , the old tuple  $(e_{w_i},$ 

 $y_{w_i}$ ) is still there, stored in TSet. After being shared with many  $(e_w, y_w)$  from  $ID_i$ ,  $ID_j$  may find out that some  $(e, y)$ may point to the same keyword and file identity. We state that these "overlap" cannot be noticed by the server, as they are in the "well-formed" encryption. The overlap phenomenon will be told while  $ID_i$  proceeds to the decryption of  $e$ . To reduce the redundancy, we design the algorithm  $ReMov$ for the server. The algorithm intakes TSet, locToken, and outputs a new *TSet*, where  $locToken = (stag, \omega_{p1}, \omega_{p2})$ (please refer to Algorithm 1).

Algorithm 3.  $ReMov$  - Remove the Redundancy of TSet

- 1. Set a bit  $\beta = 1$ , and two counters  $z = 1$ ,  $q = 0$ ,<br>2. Repeat the following loop while  $\beta = 1$ :
- Repeat the following loop while  $\beta = 1$ :
- (1). Set  $(b, L, K) \leftarrow H(F(stag, z))$  and retrieve an array  $B \leftarrow TSet[b],$
- (2). Search for index  $c \in \{1, ..., S\}$  s.t.  $B[j].label = L$ ,<br>(3). Let  $v \leftarrow B[j].value \oplus K$ , where  $\beta$  is the first bit
- Let  $v \leftarrow B[j].value \oplus K$ , where  $\beta$  is the first bit of v, and the rest of  $v$  is  $s$ .
- (4). Extract e from s, check  $H_1(e(C_4, \omega_{p2})C_5^{\omega_{p1}}) = C_6$ , where  $C_4$ ,  $C_5$ ,  $C_6$  are elements in e where  $C_4$ ,  $C_5$ ,  $C_6$  are elements in  $e$ .
- (5). If the above equation holds, set  $q + +$ ; otherwise, proceed.
- (6). If  $q > 1$ , reset  $B[j].value = "null"$ ; otherwise, proceed. (7). Increment z.
- Increment z.

A data owner can generate locToken for the server to run ReMov at any time. Recall that locToken includes stag and  $\omega_{p1}, \omega_{p2}$  stag is used to locate the  $T[w]$ , while  $\omega_{p1}, \omega_{p2}$  are for tracking a specific pair of  $(e, y)$  within  $T[w]$ . locToken is an individual "pointer" for the file(s) embedded with a keyword  $w$ . Whilst the equation of the step 2. (4) has been repeated, it indicates the current  $(e, y)$  is a redundant tuple.

# 4 SYSTEM ANALYSIS

# 4.1 Security Analysis

We make use of the generic bilinear group model and the random oracle model to prove that no PPT adversary can break the chosen plaintext security and keyword privacy. We consider three random encodings  $\delta_1$ ,  $\delta_2$ ,  $\delta_T$  of the additive group  $\mathbb{F}_q$  with injective maps  $\delta_1, \delta_2, \delta_T : \mathbb{F}_q \to \{0,1\}^k$ ,<br>where  $k > 3log(a)$ . For  $i = 1, 2, T$  set  $\mathbb{G}_i = \{8, (x) : x \in \mathbb{F}_i\}$ . where  $k > 3log(q)$ . For  $i = 1, 2, T$ , set  $\mathbb{G}_i = {\delta_i(x) : x \in \mathbb{F}_q}$ . The game simulator  $B$  is given oracles to compute the induced group action on  $\mathbb{G}_1$ ,  $\mathbb{G}_2$ ,  $\mathbb{G}_T$  and an oracle to compute a non-degenerate bilinear map  $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ . It is also given random oracles for representing hash functions.

- **Theorem 1.** Let  $Q_1$  be a bound on the total number of group elements an adversary A receives from queries of hash functions, groups  $\mathbb{G}_1$ ,  $\mathbb{G}_2$ ,  $\mathbb{G}_T$  and the bilinear map e, and from interactions with the chosen plaintext security game. We have that the advantage of  $\mathcal A$  in winning the game is  $O(Q_1^2/q).$
- Proof 1. In the normal chosen plaintext security game, a challenge ciphertext includes  $C_1 = m_b \cdot e(g, \hat{g})^{\alpha_1 \alpha_2 t}$ . We can revise the game so that  $C_1$  is either  $e(g, \hat{g})^{\xi}$  or  $e(g, \hat{g})^{\alpha_1 \alpha_2 t}$  instead, where  $\xi \in_{\mathbb{R}} \mathbb{R}$ . We state that any  $e(g, \hat{g})^{\alpha_1 \alpha_2 t}$  instead, where  $\xi \in_R \mathbb{F}_q$ . We state that any<br>adversary with advantage  $\epsilon$  in the normal game can be adversary with advantage  $\epsilon$  in the normal game can be converted into an adversary with advantage  $\epsilon/2$  in the revised game. The adversary is required to distinguish

 $m_0e(g, \hat{g})^{\alpha_1\alpha_2t}$  from  $e(g, \hat{g})^\xi$ , and  $e(g, \hat{g})^\xi$  from  $m_1e(g, \hat{g})^{\alpha_1\alpha_2t}$ .<br>Below we let  $g^x \hat{g}^y$  and  $e(g, \hat{g})^\xi$  denote  $\delta_1(x) \delta_2(y)$  and Below we let  $g^x$ ,  $\hat{g}^y$ , and  $e(g, \hat{g})^z$  denote  $\delta_1(x)$ ,  $\delta_2(y)$  and  $\delta_x(z)$  respectively Note B will maintain  $List_{x}$ ,  $List_{x}$  $\delta_T(z)$ , respectively. Note B will maintain List<sub>sk</sub>, List<sub>rk</sub> and  $List_{uv}$  as in the real scheme.

- Setup Phase. B chooses  $\theta$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\alpha_4$ ,  $\alpha_5$ ,  $\beta_1$ ,  $\beta_2 \in_R \mathbb{F}_q$ , and further sets  $h_1$ ,  $\hat{h}_1$ ,  $h_2$ ,  $\hat{h}_2$ ,  $g_1$ ,  $\hat{g}_1$ ,  $g_2$ ,  $\hat{g}_2$ ,  $g_3$ ,  $\hat{g}_3$ ,  $g_4$ ,  $\hat{g}_4$ ,  $g_5$ ,  $\hat{g}_5$ ,  $K$  and  $\hat{K}$  as in the real scheme. *B* sends the master public key  $mpk$  to  $A$ .
- Random Oracle Queries. When  $A$  queries  $H_1$  on a  $\mathbb{G}_T$  element, B chooses a random  $s \in \mathbb{F}_q$  and outputs  $g^s$ . Similarly,  $\beta$  responds the corresponding values (in  $\mathbb{F}_q$ ) to other random oracles ( $H_2$  and  $H_3$ ) queries.
- Phase 1.
	- (1) Public key and Secret Key Queries. For an identity ID, B chooses  $r, \alpha_6, \alpha_7, \alpha_8, \alpha_9 \in_R \mathbb{F}_q$ , and next computes  $\hat{g}_2^{\alpha_1}(\hat{h}_1^{ID}\hat{g}_3)^r$ ,  $\hat{g}_r$ ,  $g_6 = g^{\alpha_6}$ ,<br> $g_2 = g^{\alpha_7}$ ,  $g_9 = g^{\alpha_8}$ ,  $g_9 = g^{\alpha_9}$ ,  $\hat{g}_6 = \hat{g}^{\alpha_6}$ ,  $\hat{g}_7 = \hat{g}^{\alpha_7}$  $g_7 = g^{\alpha_7}$ ,  $g_8 = g^{\alpha_8}$ ,  $g_9 = g^{\alpha_9}$ ,  $\hat{g}_6 = \hat{g}^{\alpha_6}$ ,  $\hat{g}_7 = \hat{g}^{\alpha_7}$ ,  $\hat{g}_8 = \hat{g}^{\alpha_8}$  ,  $\hat{g}_9 = \hat{g}^{\alpha_9}$  .  $\mathcal B$  sends the secret key tuple  $\left(\hat{g}_2^{\alpha_1}(\hat{h}_1^{ID}\hat{g}_3)^r, \hat{g}^r, \alpha_6, \alpha_7, \alpha_8, \alpha_9\right)$  and the corre-<br>sponding public key to 4 Finally B stores sponding public key to  $A$ . Finally,  $B$  stores  $(ID, \hat{g}^{\alpha_1}_2(\hat{h}^{ID}_1\hat{g}_3)^r, \hat{g}^r, r, \alpha_6, \alpha_7, \alpha_8, \alpha_9) \text{ into } List_{sk}.$ <br>Re-Encryption Key Queries 4 issues the tuple
	- (2) Re-Encryption Key Queries. <sup>A</sup> issues the tuple  $(ID_i, ID_j, w_i)$  to B. B recovers the tuples  $(ID_i,$  $\hat{g}^{\alpha_1}_2(\hat{h}^{ID_i}_1\hat{g}_3)^{r_{ID_i}}, \hat{g}^{r_{ID_i}}, \,r_{ID_i}, \,a^{ID_i}_i,\,a^{ID_i}_7,\,a^{ID_i}_8,\,a^{ID_i}_9) \ \hat{g}^{\alpha_1}_1(\hat{h}^{ID_i}_1\hat{g}_3)^{r_{ID_i}}, \, \hat{g}^{\alpha_1}_2(\hat{h}^{ID_i}_2\hat{g}_3)^{r_{ID_i}}, \,r_{ID_i}, \,r_{ID_i}, \,r_{ID_i}, \,r_{ID_i}, \,r_{ID_i}, \,r_{ID_i}, \,r_{ID_i}, \,r_{ID_i}, \,r_{ID$ and  $(ID_j, \hat{g}^{\alpha_1}_2 (\hat{h}^{ID_j}_1 \hat{g}_3)^{r_{ID_j}}, \hat{g}^{r_{ID_j}}, r_{ID_j}, \alpha^{ID_j}_6, \alpha^{ID_j}_7,$  $\alpha_8^{ID_j}$ ,  $\alpha_9^{ID_j}$ ) from  $List_{sk}$ , and next computes

$$
\begin{aligned} rk_2 &= \hat{g}^{r_{D_i} - r_{D_j}}, \ rk_1 = (\hat{h}_1^{ID_i} \hat{g}_3)^{r_{D_i}} (\hat{h}_1^{ID_j} \hat{g}_3)^{-r_{D_j}} \hat{K}^{\nu}, \\ rk_3 &= \hat{g}^{\nu}, \ rk_4 = (\sigma_{i \to j} / \sigma_{o \to i}) \frac{(\alpha_j^{(D_j)} H_3(w_i^{(ID_i)} || c_1) + \alpha_6^{(D_j)})}{((\alpha_j^{(D_i)} H_3(w_i^{(ID_i)} || c_1) + \alpha_6^{(D_i)})} \\ (\delta_i \delta_i^{c_1} / \delta_i^* \delta_i^{c*})), \ rk_5 &= (\sigma_{i \to j} / \sigma_{o \to i}) \frac{(\alpha_s^{(D_j)} H_3(w_i) + \alpha_9^{(D_j)})}{((\alpha_s^{(ID_i)} H_3(w_i) + \alpha_9^{(ID_i)}) (\delta_i / \delta^*))} \end{aligned}
$$

as in the real scheme, where  $v \in_R \mathbb{F}_q$  and the values of  $\sigma$  and  $\delta$  are from  $List_{rk}$  and  $List_{up}$ .  $\beta$  further generates  $Pos, rk_6, locToken, stag_{w_i}$  as in the real game. Finally,  $\beta$  sends the re-encryption key to  $A$ . Besides,  $B$  will publish an encryption of  $\sigma_{i\rightarrow j}$ , and add  $\sigma_{i\rightarrow j}$  to the corresponding tuple stored in  $List_{up}$ .

- (3) Keyword Update Token Queries. <sup>A</sup> issues <sup>a</sup> tuple  $(ID, w_i, w_j)$  to  $B. B$  generates  $Pos$ , stag, locToken,  $\omega_{r1} = (\alpha_7 H_3(w_j||c'_1) + \alpha_6)\delta_j \delta_j^{c'_1}/(\alpha_7 H_3(w_i||c_1) +$  $\alpha_6\delta_i\delta_i^{c_1}$ ,  $\omega_{r2} = (\alpha_8H_3(w_j) + \alpha_9)\delta_j/(\alpha_8H_3(w_i) + \alpha_9)\delta_i$  $\alpha_9\delta_i$  as in the real scheme, and returns  $uptk_{w_i\rightarrow w_j}$ to A.
- (4) Search Token Queries.  $\beta$  computes the search token  $xtoken[c_1] = (xtoken[c_1, 2], \ldots,xtoken[c_1, n])$  as in the real scheme, in which  $xtoken[c_1, i] =$  $((\hat{g}^{(\alpha_{8}H_{3}(w_{i})+\alpha_{9})/((\alpha_{7}H_{3}(w_{1}||c_{1})+\alpha_{6})(\delta_{i}^{c_{1}}/\delta^{c_{*}}))})^{r}, \hat{g}^{r})$  and<br>  $\hat{g} \in [9, n]$  $i \in [2, n]$ .
- Challenge Phase. A commits to  $m_0$ ,  $m_1$ ,  $w^*$  and ID<sup>\*</sup>. B chooses  $\xi$ ,  $t \in_R \mathbb{F}_q$ , and computes the cinhertox as  $C_t = e(a, \hat{a})^{\xi}$ ,  $C_s = a^t$ ,  $C_s = (h^{ID^*}a)^t$ ciphertext as  $C_1 = e(g, \hat{g})^{\xi}$ ,  $C_2 = g^t$ ,  $C_3 = (h_1^{ID^*} g_3)^t$ ,  $C_4 = (g_5 g_4^{-H_2(m_b)})^t$ ,  $C_5 = e(g_4, \hat{g}_4)^t$ ,  $C_6 = H_1(e(h_2, \hat{g}_4)^t)$ ,

 $C_7 = K^t$ ,  $y = (g_7^{*H_3(w^*||c_1)}g_6^*)^t$ ,  $xtag = (g_8^{*H_3(w^*)}g_9^*)^t$ ,<br>where  $\frac{*}{s} \frac{*}{s} \frac{*}{s}$  are the griding level algorithm  $\begin{pmatrix} 6 & 9 & 96 \end{pmatrix}$ ,  $\begin{pmatrix} 6 & 96 \end{pmatrix}$ ,  $\begin{pmatrix} 6 & 96 \end{pmatrix}$ , where  $g_6^*$ ,  $g_7^*$ ,  $g_8^*$ ,  $g_9^*$  are the public key elements of  $ID^*$  generated by  $B$  as in the real scheme.

- Phase 2. Same as Phase 1 but with the restrictions.
- - $\bullet$  Guess. *A* outputs a guess bit  $b'$ .<br>We assume 4 can query the group

We assume  $A$  can query the group oracles by using its popes from the simulations and some intermediate responses from the simulations and some intermediate values obtains from the oracles; there are  $q$  distinct values in the ranges of  $\delta_{1,2,T}$  with probability  $1-O(1/q)$ . We seen an oracle query as a rational function  $f = x/y$  in the variables  $\xi$ ,  $\beta_z$ ,  $\alpha_l$ ,  $\theta$ ,  $t$ ,  $\nu$ , the random factors  $\delta$ ,  $\sigma$  and  $r$ , where  $z \in \{1, 2\}$  and  $l \in [1, 9]$ . We here consider a collision event where two queries for two distinct rational functions  $f = x/y$  and  $f' = x'/y'$  with two sets of random<br>choices of variables that yields the same output For choices of variables that yields the same output. For any query pair (in  $\mathbb{G}_1$ ,  $\mathbb{G}_2$ , or  $\mathbb{G}_T$ ) corresponding to two distinct  $f$  and  $f'$ , the collision will happen only if the non-zero polynomial  $xy - x'y$  leads to zero, where the<br>total degree of the equation is at most 7. By the Schwartztotal degree of the equation is at most 7. By the Schwartz-Zippel lemma [27], [35], we have that the probability of the collision is at most  $O(1/q)$ . By a union bound, we have  $O(Q_1^2/q)$ . The simulations do not have collision event with probability  $1 - O(Q_1^2/q)$ event with probability  $1 - O(Q_1^2/q)$ .<br>We here consider the view of

We here consider the view of  $A$  in the case where  $\xi = \alpha_1 \alpha_2 t$ . Since there is no any collision for queries to oracles (with overwhelming probability) and each group oracles (with overwhelming probability) and each group element (responded by  $\beta$ ) is uniformly chosen, the view of A should be identically distributed. However, one remaining possibility that  $A$ 's view is distinct in the above case is that there are two distinct queries  $f$  and  $f'$ to  $\mathbb{G}_T$  but yielding the same output. Since the  $\xi$  is an exponent of the element in  $\mathbb{G}_T$ , we can have some additive computation to output an exponent  $\gamma \xi$  with a nonzero  $\gamma$ . Similarly, we have  $\gamma'\alpha_1\alpha_2t$  as well. Accordingly, we have  $f - f' = \gamma \xi - \gamma' \alpha_1 \alpha_2 t$ , and then  $f - f' - \gamma \xi$ <br> $\gamma' \alpha_1 \alpha_2 t$ . To hold the equation 4 has to obtain the elements  $\gamma' \alpha_1 \alpha_2 t$ . To hold the equation, A has to obtain the element<br>with exponent  $\gamma' \alpha_1 \alpha_2 t$  from queries to  $\mathbb{G}_T$ . Namely, if A with exponent  $\gamma' \alpha_1 \alpha_2 t$  from queries to  $\mathbb{G}_T$ . Namely, if A can achieve the element it can tell the difference to win can achieve the element, it can tell the difference to win the game.

But A cannot construct a query for  $e(g, \hat{g})^{\gamma' a_1 a_2 t}$  for me constant  $\gamma'$  Since our system is built on top of asymsome constant  $\gamma'$ . Since our system is built on top of asymmetric pairing groups, it is much easier to make observation on oracle queries. By observation, only group  $\mathbb{G}_1$ provides elements with exponent t, namely,  $g^t$ ,  $g^{(\beta_1ID + \alpha_3)t}$ ,  $g^{(\alpha_5-\alpha_4H_2(m_b))t}$ ,  $g^{\theta t}$ ,  $g^{(\alpha_7H_3(w||c_1)+\alpha_6)t}$  and  $g^{(\alpha_8H_3(w)+\alpha_9)t}$ . Since there are no factors  $1/\theta$ ,  $1/\beta_1$  and  $1/\alpha_3$  existing in group  $\mathbb{G}_2$  for the elimination of the corresponding exponents,  $\mathcal A$ may consider the rest of the elements. Recall that  $\alpha_6, \alpha_7, \alpha_8, \alpha_9 \in \mathbb{F}_q$  are designed for keyword field that is unrelated to the target component. Moreover,  $\alpha_4$  and  $\alpha_5$ have no direct computation relationship with  $\alpha_1$  and  $\alpha_2$ . A can only focus on  $g^t$ . A is given an element  $e(g^{\alpha_4}, \hat{g}^{\alpha_4})^t$  in  $\mathbb{G}_T$ . However, it cannot help A break the game as it is only used in additive operations.

Given  $g^t$ , A needs the elements with exponent  $k\alpha_1\alpha_2$  in group  $\mathbb{G}_2$ . From the simulations, we can see that there is only the response of secret key query, a  $\hat{g}^{\alpha_1\alpha_2+(ID\beta_1+\alpha_3)r}$ , satisfying the requirement. A accordingly has a  $K_1 =$  $t(\alpha_1\alpha_2 + (ID_i\beta_1 + \alpha_3)r_{ID_i}) = \alpha_1\alpha_2t + ID_i\beta_1r_{ID_i}t + \alpha_3r_{ID_i}t,$  where  $i$  is the index for  $i$ th query. To cancel out the part  $ID_i\beta_1r_{ID_i}t + \alpha_3r_{ID_i}t$ , A needs to find a  $g^t$  and the elements with exponents  $\beta_1 r_{ID_i}$  and  $\alpha_3 r_{ID_i}$  in  $\mathbb{G}_2$ . By observation, A has  $K_2 = (\beta_1 ID_i + \alpha_3)r_{ID_i} - (\beta_1 ID_i + \alpha_3)r_{ID_i} +$  $\theta$ <sup>v</sup> (via re-encryption key queries), such that it can create a query  $\alpha_1\alpha_2t + t\beta_1ID_jr_{ID_i} + t\alpha_3r_{ID_i} - \theta vt$  by subtracting  $K_1$  with  $tK_2$ . We can see that the computation indicates that the given ciphertext under  $ID_i$  is re-encrypted to *ID<sub>i</sub>*. We set  $K_3 = \alpha_1 \alpha_2 t + \Delta - \theta \delta t$ . Since A is given  $g^{\theta t}$  and  $\hat{g}^{\nu}$ , it can cancel out the last part of  $K_3$  to have  $K_4 = \alpha_1 \alpha_2 t + \Delta.$ 

If A finds a way to eliminate  $\Delta$ , it can recover  $\alpha_1 \alpha_2 t$ from  $K_4$ . A only needs to require a query between  $g^{(\beta_1 ID_j + \alpha_3)t}$  and a "special"  $\hat{g}^{rID_j}$ . But a pair of identities  $ID_i$ ,  $ID_j$  in a re-encryption key cannot be corrupted in the security game if one of them has a re-encryption path with  $ID^*$ . Besdies,  $K_4$  is computed from the elements of the challenge ciphertext. Therefore, the special element  $\hat{g}^{r_{Dj}}$ , a part of the secret key  $sk_{ID_j}$ , will not be given to A.<br>There is no other term A gains access to that can cancel There is no other term  $A$  gains access to that can cancel out the part  $\Delta$  of  $K_4$ . A cannot construct a query for  $e(a, \hat{a})^{\gamma'\alpha_1\alpha_2t}$  with some constant  $\nu'$ .  $e(g,\hat{g})^{\gamma'\alpha_1\alpha_2t}$  with some constant  $\gamma'$ . The contract of  $\Box$ 

- **Theorem 2.** Let  $Q_2$  be a bound on the total number of group elements an adversary <sup>A</sup> receives from queries of hash function, groups  $\mathbb{G}_1$ ,  $\mathbb{G}_2$ ,  $\mathbb{G}_T$  and the bilinear map e, and from interactions with the keyword privacy game. We have that the advantage of  $\mathcal A$  in winning the game is  $O(Q_2^2/q)$ .
- Proof 2. Due to the similarities in the proofs between Theorems 4.1 and 4.1, we here only present a proof stretch, and state that the proof can be easily completed by following the roadmap of the proof of Theorem 4.1. The game challenger here sets up system and responds the queries of random oracles, public and secret key, re-encryption key, keyword update token and search token as in the proof of Theorem 4.1 but being limited to the restrictions listed in the Definition 3. In the challenge phase, the challenger constructs the challenge ciphertext to A by intaking  $(m, w_0^*, w_1^*,$ <br> $ID^*)$ . Following the restrictions given in the Definition 3, A  $ID^*$ ). Following the restrictions given in the Definition 3,  $A$ attempts to find either  $\hat{g}^{\alpha_7H_3(\bar{w}_b||c_1)+\alpha_6}$  or  $\hat{g}^{\alpha_8H_3(w_b)+\alpha_9}$  to match pairings  $e(g^{(\alpha_7H_3(w_b||c_1)+\alpha_6)t},\hat{g})$  or  $e(g^{(\alpha_8H_3(w_b)+\alpha_9)t},\hat{g})$ .<br>Since  $\hat{g}_s$  and  $\hat{g}_s$  are not given as the public key of  $IP^*$  and Since  $\hat{g}_6$  and  $\hat{g}_9$  are not given as the public key of  $ID^*$  and meanwhile, no  $\hat{g}^{K\alpha_6}$  or  $\hat{g}^{K\alpha_9}$  can be achieved via oracle queries (where  $K$  is a constant), the only way  $A$  can reach the goal is to corrupt the secret key of  $ID^*$  or to obtain some "sensitive" search tokens. But those queries are forbidden (Definition 3).  $A$  fails to win the game.
- **Theorem 3.** Assume the decisional Diffie-Hellman assumption [7] holds in  $\mathbb{G}_2$ , all the pseudorandom functions and hash functions are secure and target collision resistant, our scheme achieves the weak search token privacy.
- **Proof 3.** Given two distinct search tokens  $TK_1$  and  $TK_2$ , the PPT adversary  $A$  is target to tell the difference. Assume  $TK_1 = ((\hat{g}^{\alpha_8H_3(w_2^{(R)})/\alpha_7H_3(w_1^{(R)}||c_1)\beta_1})^{r_1}, \hat{g}^{r_1})$  and  $TK_2 =$  $((\hat{g}^{\alpha_{8}H_{3}(w_{2}^{(b)})/\alpha_{7}H_{3}(w_{1}^{(b)}||c_{1})\beta_{2})^{r_{2}}, \hat{g}^{r_{2}}),$  where  $r_{1}$  and  $r_{2}$  are distinct random factors in  $\mathbb{Z}$ ,  $\beta$  and  $\beta$  are the respective  $\delta$ tinct random factors in  $\mathbb{Z}_q$ ,  $\beta_1$  and  $\beta_2$  are the respective  $\delta$ computation,  $(R)$  and  $(b)$  represent the sign of random and bit b, respectively. Since our bilinear pairings are





designed as  $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$  with no easy homomorphism between  $\mathbb{G}_1$  and  $\mathbb{G}_2$ , A cannot use the parings computation directly. Furthermore,  $r_1$  and  $r_2$  are two fresh and distinct random factors, A cannot find the corresponding elements in  $\mathbb{G}_1$  to compute correct pairings to verify the equality of  $TK_1$  and  $TK_2$ . Similarly, A cannot generate correct elements in  $\mathbb{G}_1$  without knowing  $\alpha_6$ ,  $\alpha_7$ ,  $\alpha_8$ ,  $\alpha_9$ , the parts of secret key of the target ID. But if A can output  $\hat{g}^{r_1r_2\Delta}$  and  $\hat{g}^{r_1r_2\Omega}$  by using  $TK_1$  and  $TK_2$ , it can definitely tell the difference between the search tokens. If so, it can break the DDH problem. That contradicts to our assumption. This weak

## 4.2 Efficiency Analysis

To present a fair computational cost analysis, we denote the exponent cost in  $\mathbb{G}_1$ ,  $\mathbb{G}_2$  and  $\mathbb{G}_T$  as  $exp_1$ ,  $exp_2$ ,  $exp_3$ , respectively. For the pairing cost in  $\mathbb{G}_T$ , we denote it as p. We will consider the cost w.r.t. a trusted system setup party, system user, a PKG and a server. We note that the computational cost of the algorithm Enc shown in Table 3 is for the case where a file is only tagged with a single keyword. From the Table 3, it can be seen that a system user is only required to take less than 50 percent of the system total cost,  $13exp<sub>1</sub> + 3exp<sub>2</sub> + 5p$ , to achieve flexible data query, keyword update and data share, while the trusted party, PKG and server are responsible for the rest of the cost, which is more than 50 percent of the total cost.

In the following communication analysis, we use  $mpk$ , msk, pk, sk,  $CT$ , uptk, rk, TK to denote master secret key, master public key, public key, secret key, ciphertext, keyword update token and search token, respectively. We note that the communication cost of  $TK$  and  $CT$  normally have to respectively multiply factors  $|SF|$  and  $|\Delta|$ , where  $|SF|$  is the size of the search formula,  $|\Delta| = \sum_{w \in W} |DB(w)|$ . Below<br>we only set  $|SE|$  and  $|\Delta|$  to be 1, namely, the following comwe only set  $|SF|$  and  $|\Delta|$  to be 1, namely, the following communication cost is for the case where a file is tagged with a single keyword.

In Table 4, we can see that a system user only needs to spend  $3\mathbb{Z}_q + 7\mathbb{G}_1 + 3\mathbb{G}_2 + 2\mathbb{G}_T$  in communication cost with a server during the execution of the system. The  $\mathbb{G}_T$  part is due to the pairings  $(C_1, C_5)$  used in the ciphertext. It is not difficult to see that the cost of the user is approximately 45 percent of the total system cost.

#### 4.3 Practical Simulation

We further implement our scheme using PBC library [23] which is one of the most widely used library for pairing computation. We choose the asymmetric pairing which is constructed on ordinary curves with embedding degree 6, and its orders are prime or a prime multiplied by a small constant. It is first discovered by Miyaji, Nakabayashi and Takano [25], and it is usually more efficient compared with other curves. The simulation is performed on a mac pro with 2.2 GHz Intel Core i7 and 16 GB 1,600 MHz DDR3 memory. Similar to the theoretical analysis, we demonstrate our practical simulation results in Tables 5 and 7.

We verify our theoretical analysis through the simulation that the workload of a system user indeed is lightened significantly. From Table 5 (computational cost), we can see that the computational time for Trust Party, Server and PKG are 0.0738, 0.0409 and 0.1510 seconds, while the user side takes only 0.0682 seconds. As a result, the user side takes only 20 percent of the total computational cost.

We further compare our design with the recent and lightweight PKES scheme [13]. Since [13] do not consider

TABLE 4 Theoretical Communication Cost

|                           | Communication Cost   |          |              |                                 |                 |                |          |               |
|---------------------------|--|----------|--------------|---------------------------------|-----------------|----------------|----------|---------------|
| Groups                    | mpk  |          |              |                                 | msk pk sk TK rk |                | uptk     | CT            |
| $\mathbb{Z}_q$            | 0  | 1        | $\theta$     | $4 \quad$                       | $\theta$        | $\overline{2}$ | 3        | $\Omega$      |
| $\mathbb{G}_1$            | 8  | $\Omega$ | 4            | $\begin{matrix} 0 \end{matrix}$ | $\theta$        | $\Omega$       | $\Omega$ | 7             |
| $\mathbb{G}_2$            | 6  | 3        | $\mathbf{2}$ |                                 | $2 \quad 2$     | 3              | 1        | $\theta$      |
| $\mathbb{G}_T$            | 0  | $\Omega$ | $\Omega$     | $\Omega$                        | $\Omega$        | $\Omega$       | 0        | $\mathcal{P}$ |
| $TP \rightarrow PKG$      | $\mathbb{Z}_q + 3\mathbb{G}_2$   |          |              |                                 |                 |                |          |               |
| $PKG \rightarrow User$    | $5\mathbb{Z}_a + 4\mathbb{G}_1 + 5\mathbb{G}_2$                            |          |              |                                 |                 |                |          |               |
| $PKG \rightarrow Server$  | $2\mathbb{Z}_q + 3\mathbb{G}_2$  |          |              |                                 |                 |                |          |               |
| $User \rightarrow Server$ | $3\mathbb{Z}_q + 7\mathbb{G}_1 + 3\mathbb{G}_2 + 2\mathbb{G}_T$            |          |              |                                 |                 |                |          |               |
| Total                     | $10\mathbb{Z}_{q} + 19\mathbb{G}_{1} + 19\mathbb{G}_{2} + 2\mathbb{G}_{T}$ |          |              |                                 |                 |                |          |               |
|                           |  |          |              |                                 |                 |                |          |               |



| <b>Trust Party</b> | 0.0738 |
|--------------------|--------|
| Server             | 0.0409 |
| <b>PKG</b>         | 0.1510 |
| <b>User Side</b>   | 0.0682 |
| Total              | 0.3267 |
|                    |        |

TABLE 6 Computational Cost Comparison (Second)



keyword update (which is mainly on server side) and decryption functionalities, we only compare the computational cost in terms of KeyGen, Enc, TKGen and Search to achieve comparison fairness. Based on the test bar set in [13], we can have our KeyGen, Enc, TKGen and Search require 0.02598, 0.046789, 0.006495 and 0.010854 s, respectively. From Table 6, it can be seen that our scheme outperforms [13] in trapdoor generation and a single keyword search, while the rest of our functions require more run time than [13]. But, in general, our run time cost in the four functions is acceptable in practice.

As for the communication cost, we need to find out the length of elements in groups  $\mathbb{G}_1$ ,  $\mathbb{G}_2$ ,  $\mathbb{G}_T$  and  $\mathbb{Z}_n$ . According to the implementation of the MNT curve, the group elements have 40; 120; 120 and 20 bytes in length accordingly. As a result, from the trusted party to PKG, 380 bytes data are sent. From PKG, 860 and 400 bytes data are sent to User and Server respectively. The data communication cost from user to server comes from the ciphertext and token delivery as well as the search query costing about 940 bytes. In total the practical communication cost is about 2,580 bytes.

We also compare our scheme with [13] w.r.t. communication cost in Table 8. But we only consider the related cost over the Keys, Ciphertext and Trapdoor. We note that for Keys we consider the size of both public key and secret key. As for the size of *Keys*, our scheme requires 4 more  $\mathbb{G}_2$  than [13]; and our ciphertext size is larger than that of [13] due to

TABLE 7 Practical Communication Cost (Byte)

| Trusted Party $\rightarrow$ PKG | 380  |
|---------------------------------|------|
| $PKG \rightarrow User$          | 860  |
| $PKG \rightarrow Server$        | 400  |
| User $\rightarrow$ Server       | 940  |
| Total                           | 2580 |

TABLE 8 Communication Cost Comparison (Byte)

| Item/Scheme | Ours | [13] |
|-------------|------|------|
| Keys        | 720  | 240  |
| Ciphertext  | 520  | 80   |
| Trapdoor    | 240  | 120  |



Fig. 5. Time complexity for large scale searching.

the support of our decryption function (which needs extra  $5\mathbb{G}_1 + 2\mathbb{G}_T$  as compared to [13]). But we state that the above cost is acceptable in the viewpoint of practical user since the cost is < 1 MB which is bearable for common network/ communication device, e.g., smartphone.

The "least frequent keyword" technique proposed in this paper can help to improve the keyword searching speed dramatically. Now we would like to set up an experiment to simulate this event. Assume that there are in total  $r$  files, and the keyword set  $A$  belongs to  $m$  files, and keyword set *B* belongs to *n* files, where  $m = 10 \times n$ ,  $m = r \times 0.9$ , and  $m, n \leq r$ , such that the number of files contain A dominants the whole set.  $B$  is defined to be the "least frequent keyword" according to the previous definition. To search the files that contain both  $A$  and  $B$ , traditionally, we would first search  $r$  files to locate files contain either  $A$  or  $B$ . And then search  $m$  or  $n$  times depending on which keyword is searched first. So the time complexity is  $T_{search} \times r +$  $T_{search} \times m$  or  $T_{search} \times r + T_{search} \times n$ . However, if we assume that  $B$  is the "least frequent keyword" that has been prepared, then the searching strategy becomes that we search the files contain  $B$  first, and then search the  $A$ . In this case, the time complexity becomes  $T_{search} \times r + T_{search} \times n$ , which is obviously better than the previous searching strategy if we are sure  $n < m$ . In the first case, we do not know which keyword will be searched first, thus each time the user will have 50 percent chance to pick  $A$  or  $B$ . In detail, the average time follows the binomial distribution with  $p = 1/2$ , but for the simplicity we omit the probability discussion here. We further state that the bitmap index (via BigInteger) is used to denote file identifier, and our experiment tests the search and update time for one keyword in which each keyword has 20 entries.

As shown in Fig. 5, the number of total files  $(r)$  is from 100 to 2000, and  $m, n$  can be easily computed accordingly. By "traditional searching time" we mean that is the nonoblivious-cross-search approach with encryption. It is clear that our proposed keyword searching strategy can greatly improve over the previous designed searching strategy as the number of files grows large.

## 5 CONCLUSION

We have revised encrypted cloud-based data share and search (with keyword update) framework as well as its security notion. We have further proposed an enhanced system satisfying the notion by leveraging identity-based encryption, asymmetric pairing group conversion, identitybased proxy re-encryption and "least frequent keyword" searchable technique. We have proved the security of the system in the generic bilinear group model. Our system is cost-effective as compared to its conference version, and has great potential in being deployed in large scale database.

This paper also leaves some interesting open problems. In the context of PKSE, it may be desirable to consider the forward and backward security, if the system provides file add and deletion. One may consider to use simulationbased or universal composability model define the security for forward/backward PKSE. Another research direction may be to consider the use of the technique proposed in [33] to hold against active online attacks.

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