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# Ambiguous optimistic fair exchange: Definition and constructions <sup>☆</sup>



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## ABSTRACT

Optimistic fair exchange (OFE) is a protocol for solving the problem of exchanging items or services in a fair manner between two parties, a signer and a verifier, with the help of an arbitrator which is called in only when a dispute happens between the two parties. In almost all the previous work on OFE, after obtaining a partial signature from the signer, the verifier can present it to others and show that the signer has indeed committed itself to something corresponding to the partial signature *even* prior to the completion of the transaction. In some scenarios, this capability given to the verifier may be harmful to the signer. In this paper, we propose the notion of *ambiguous optimistic fair exchange* (AOFE), which is a variant of OFE and requires additionally that the verifier cannot convince anybody about the authorship of a partial signature generated by the signer. We present a formal security model for AOFE in the multi-user setting and chosen-key model, and propose a generic construction of AOFE that is provably secure under our model. Furthermore, we propose an efficient instantiation of the generic construction, security of which is based on Strong Diffie–Hellman assumption and Decision Linear assumption without random oracles.

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## 1. Introduction

Optimistic Fair Exchange (OFE) allows two parties to fairly exchange information in such a way that at the end of a protocol run, either both parties have obtained the complete information from one another or none of them has obtained anything from the counter party. In an OFE, there is a third party, called Arbitrator, which is only called in when a dispute occurred between the two parties. OFE is a useful tool in practice, for example, it can be used for performing contract signing, fair negotiation and similar applications on the Internet. Since its introduction [1], there have been many OFE schemes proposed [2,16,3,11,33,14,29,32,38,4,34,15,21,35]. For all recently proposed schemes, an OFE protocol for signature

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typically consists of three message flows. The initiator of OFE, Alice, first sends a *partial signature*  $\sigma_P$  to a responder, Bob, where  $\sigma_P$  is considered as Alice's partial commitment to her full signature which will be sent to Bob. But beforehand, Bob should send his full signature back to Alice first in the second message flow. After receiving Bob's full signature, Alice then sends her full signature to Bob in the third message flow. If Bob refuses to send his full signature to Alice in the second message flow,  $\sigma_P$  should have no use to Bob, so that Alice has no concern about giving away  $\sigma_P$ . However, after Bob has sent his full signature to Alice while Alice refuses to send her full signature in the third message flow, then Bob can ask the Arbitrator to retrieve Alice's full signature from  $\sigma_P$  by sending both  $\sigma_P$  and Bob's full signature to the Arbitrator. To the best of our knowledge, among almost all the known OFE schemes, there is one common property about Alice's partial signature  $\sigma_P$  which has neither been captured in any of the security models for OFE nor been considered as a requirement for OFE. The property is that once  $\sigma_P$  is given out, at least one of the following statements is true.

1. Everyone can verify that Alice generates  $\sigma_P$ , because  $\sigma_P$ , similar to a standard digital signature, has the non-repudiation property with respect to Alice's public key;
2. Bob can show to anybody that Alice is the signer of  $\sigma_P$ .

For example, in [15,21], the partial signature of Alice is a standard signature, which can only be generated by Alice. In many other OFE schemes, Alice's signature is encrypted under the arbitrator's public key, and then a non-interactive proof is generated to show that the ciphertext indeed contains a signature of Alice. This is known as *verifiably encrypted signature*. However, regarding the validity and non-repudiation of a signature, as pointed out by Boyd and Foo [10], this raises the question of whether a non-interactive proof that a signature is encrypted is really having any difference from a signature itself, as the proof is already sufficient to convince any third party that the signer has committed to the message.

This property may cause no concern in some applications, for example, in those where only the full signature is deemed to have some actual value to the receiving party. However, it may be undesirable in some other applications. Since  $\sigma_P$  is publicly verifiable and non-repudiative,  $\sigma_P$  has evidently shown Alice's commitment to the corresponding message. This may incur some unfair situation, to the advantage of Bob, if Bob does not send out his full signature. In contract signing applications, this could be undesirable because  $\sigma_P$  can already be considered as Alice's undeniable commitment to a contract in court while there is no evidence showing that Bob has committed to anything. For example, Alice wants to sign with Bob a contract of procuring Bob's company. After sending out her partial signature, Alice has no way to regret and cannot withdraw the procurement if Bob persists. However, Bob can pause the contract signing, and use Alice's partial signature to bargain for better offers with others. He then carries out a new OFE protocol with the one offering the best price to sign the contract. Bob can play the same trick iteratively until that no one can give an even better offer.

For making OFE be applicable to more applications and practical scenarios, in this paper, we propose to enhance the security requirements of OFE and construct a new OFE scheme which does not have the problems mentioned above. One may also think of this as an effort to make OFE more admissible as a viable fair exchange tool for real applications. We will build an OFE scheme which not only satisfies all the existing security requirements of OFE (with respect to the strongest security model available [21]), but in addition to that, will also have  $\sigma_P$  be not self-authenticating and unable for Bob to demonstrate to others that Alice has committed herself to something. We call this enhanced notion of OFE as *Ambiguous Optimistic Fair Exchange* (AOFE). It inherits all the formalized properties of OFE [15,21] and has a new property introduced: *signer ambiguity*. It requires that a partial signature  $\sigma_P$  generated by Alice or Bob should look alike and be indistinguishable even to Alice and Bob.

### 1.1. Related works

There have been many OFE schemes proposed in the past [2,3,11,33,14,29,32,38,4,34,15,21]. In the following, we review some recent ones by starting from 2003 when Park, Chong and Siegel [33] proposed an OFE based on sequential two-party multi-signature. It was later broken and repaired by Dodis and Reyzin [14]. The scheme is *setup-driven* [39,40], which requires all users to register their keys with the arbitrator prior to conducting any transaction. In [32], Micali proposed another scheme based on a CCA2 secure public key encryption with the property of *recoverable randomness* (i.e., both plaintext and randomness used for generating the ciphertext can be retrieved during decryption). Later, Bao et al. [4] showed that the scheme is not fair, where a dishonest party, Bob, can obtain the full commitment of another party, Alice, without letting Alice get his obligation. They also proposed a fix to defend against the attack.

In PKC 2007, Dodis, Lee and Yum [15] considered OFE in a *multi-user* setting. Prior to their work, almost all previous results considered the single-user setting only which consists of a single signer and a single verifier (along with an arbitrator). The more practical multi-user setting considers a system to have multiple signers and verifiers (along with the arbitrator), so that a dishonest party can collude with other parties in an attempt of cheating. Dodis et al. [15] showed that security of OFE in the single-user setting does not necessarily imply the security in the multi-user setting. They also proposed a formal definition of OFE in the multi-user setting, and proposed a generic construction, which is *setup-free* (i.e. no key registration is required between users and the arbitrator) and can be built in the random oracle model [5] if there exist one-way functions, or in the standard model if there exist trapdoor one-way permutations.

In CT-RSA 2008, Huang, Yang, Wong and Susilo [21] considered OFE in the multi-user setting and *chosen-key* model, in which the adversary is allowed to choose public keys arbitrarily without showing its knowledge of the corresponding private keys. Prior to their work, the security of all previous OFE schemes (including the one in [15]) are proven in a

more restricted model, called *certified-key* model, which requires the adversary to prove its knowledge of the corresponding private key before using a public key. In [21], Huang et al. gave a formal security model for OFE in the multi-user setting and chosen-key model, and proposed an efficient OFE scheme based on ring signature. In their scheme, a partial signature is a conventional signature and a full signature is a two-member ring signature in addition to the conventional signature. The security of their scheme was proven without relying on the random oracle assumption.

In [16], Garay, Jakobsson and MacKenzie introduced a similar notion for optimistic contract signing, named *abuse-freeness*. It requires that no party can ever prove to a third party that he is capable of choosing whether to validate or invalidate a contract. They also proposed a construction of abuse-free optimistic contract signing protocol. The security of their scheme is based on DDH assumption under the random oracle model. Besides they did not consider the multi-user setting for their contract signing protocol.

Liskov and Micali [30] proposed an *online-untransferable signature* scheme, which can be considered as an enhanced version of designated confirmer signature. In such a scheme, there is also a party (confirmer) semi-trusted by both the signer and the recipient. A dishonest recipient, who is interacting with a signer, cannot convince a third party that the signature is generated by the signer. But both the signer and the conformer are able to convert a signature so that anyone can identify its owner. The online non-transferability of their scheme is similar to the *signer ambiguity* (see Definition 2) of AOFE. However, the *online attack* considered in [30] would not happen in AOFE, as the signature generation and verification are both non-interactive. Besides, the signing process of their scheme requires several rounds of interaction with the recipient, and the scheme works in the certified-key model and is not setup-free, i.e. there is a setup stage between each signer and the confirmer, and the confirmer needs to store a public/secret key pair for each signer.

*Works after [20]*. There have been multiple works since the introduction of AOFE. To name a few, Chen et al. [13] introduced a new notion of *Verifiable Encryption of Chameleon Signatures*, and used it to construct a three-round abuse-free optimistic protocol for contract signing. A somewhat generic transform from *designated confirmer signature* (DCS) to AOFE was proposed in [22,23]. The underlying DCS scheme is required to enjoy a property named *samplability*, while is satisfied by only a few DCS schemes. The transform was later refined in [25], where the DCS is only required to satisfy standard properties, e.g. unforgeability and anonymity. A new notion called *group-oriented optimistic fair exchange* was proposed in [24], which considers the fair exchange between two groups of users, keeping the anonymity of the signer in its group. Another enhanced version of AOFE, named *Perfect AOFE*, was proposed in [36], in which a partial signature leaks no information about the actual signer or the intended verifier. This is useful for applications where the involved parties of an exchange wish to further protect their privacy on whether they are indeed involved in an exchange or not. This notion was further improved very recently. *Privacy-preserving OFE* was proposed in [26], in which the arbitrator could not learn the full signature even after the resolution process. Very recently, another variant of AOFE called *Attributed-based Optimistic Fair Exchange*, was introduced in [37], which integrates the advantage of both AOFE and (Ciphertext-Policy) Attributed-Based Encryption. Only the verifier who possesses appropriate credentials (issued by a credential center according to its attributes) can convert the signer's partial signature into a full one. It is worthy to notice that the schemes proposed in [22,23,25,26] are secure in the certified-key model, while the schemes proposed in this work and [37] are secure in the chosen-key model.

## 1.2. Our work

In this paper we make the following contributions. First, we propose the notion of *Ambiguous Optimistic Fair Exchange* (Ambiguous OFE, or AOFE in short) which allows a signer Alice to generate a partial signature in such a way that a verifier Bob cannot convince anyone about the authorship of this partial signature, and thus cannot prove to anybody that Alice committed herself to anything prematurely. Realizing the notion needs to make the partial signature ambiguous with respect to Alice and Bob. We will see that this requires us to include both Alice and Bob's public keys into the signing and verification algorithms of AOFE.

Second, for formalizing AOFE, we propose a strong security model in the multi-user setting and chosen-key model. Besides the existing security requirements for OFE, that is, resolution ambiguity (the ambiguity considered in [15,21]), security against signers, security against verifiers and security against the arbitrator, AOFE has an additional requirement: *signer ambiguity*. It requires that the verifier can generate partial signatures which are (computationally) indistinguishable from the partial signatures generated by the signer. We also evaluate the relations among the security requirements and show that if a scheme has security against the arbitrator and (a weaker form of) signer ambiguity, then it has (a weaker form of) security against verifiers.

Third, we revisit two generic methods in constructing OFE [15,21], and show that if we simply extend them to the construction of AOFE, the resulting schemes would be *insecure*. Specifically, they are insecure against the arbitrator under our proposed model.

Fourth, we propose a generic AOFE construction. It is based on the generic OFE construction of [15], but instead of using a CCA secure encryption scheme, an existentially unforgeable signature scheme under chosen message attacks and a simulation-sound NIZK proof system, we employ a selective-tag weakly CCA secure tag-based encryption [28], a weakly unforgeable signature [7], a strong one-time signature and a general NIZK proof system. The security of the generic construction is proven secure under our proposed multi-user setting and chosen-key model.

Last but not least, we propose a concrete and efficient instantiation of our generic construction of AOFE, the security of which is based on the intractability of Strong Diffie–Hellman problem and Decision Linear problem without random oracles.

*Differences from [20].* In this paper we make the following changes, compared with [20]. First of all, the model of signer ambiguity is revised to correct some minor issue in [20]. Second, the proof of the theorem on the relation between signer ambiguity and security against verifiers (Theorem 1) is revised to improve the reduction factor. Third, a new section (Section 3) is added to discuss about the insecurity of simple extensions of two generic constructions of OFE to AOFE. Fourth, a generic construction of AOFE is proposed (in Section 4) with detailed security proofs, which covers the efficient construction proposed in [20].

### 1.3. Paper organization

In the next section, we define AOFE and propose a security model. We then show some relation among the formalized security requirements. A popular generic construction used in building OFE schemes is revisited in Section 4, and a new generic construction of AOFE is proposed and proved. In Section 5 we give an efficient instantiation of the construction, security of which does not rely on the random oracle heuristic. The paper is concluded in Section 6.

## 2. Ambiguous optimistic fair exchange

In AOFE, we require that after receiving a partial signature  $\sigma_P$  from Alice (the signer), Bob (the verifier) cannot convince others but himself that Alice has committed to  $\sigma_P$ . This property is analogous to the non-transferability of designated verifier signature [27] and the ambiguity of concurrent signature [12]. Also, the AOFE verification algorithm should take the public keys of both signer and (designated) verifier as inputs, in contrast to that in the traditional definition of OFE [2,15,21].

**Definition 1** (*Ambiguous Optimistic Fair Exchange*). A (non-interactive) *Ambiguous Optimistic Fair Exchange* (AOFE) scheme involves two users, i.e. a signer and a verifier, and an arbitrator, and consists of the following probabilistic polynomial-time (PPT) algorithms:

- PMGen: On input  $1^k$  where  $k$  is a security parameter, it outputs the system parameter PM.
- Setup<sup>TTP</sup>: On input PM, the algorithm generates a public arbitration key APK and a secret arbitration key ASK.
- Setup<sup>User</sup>: On input PM and (optionally) APK, the algorithm outputs a public/secret key pair  $(PK, SK)$ . For user  $U_i$ , we use  $(PK_i, SK_i)$  to denote its key pair.
- Sig and Ver: Sig( $M, SK_i, PK_i, PK_j, APK$ ) outputs a (full) signature  $\sigma_F$  on  $M$  of user  $U_i$  with the designated verifier  $U_j$ , where message  $M$  is chosen by user  $U_i$  from the message space  $\mathcal{M}$  defined under  $PK_i$ , while Ver( $M, \sigma_F, PK_i, PK_j, APK$ ) outputs 1 or 0, indicating  $\sigma_F$  is  $U_i$ 's valid full signature on  $M$  with designated verifier  $U_j$  or not.
- PSig and PVer: They are partial signing and verification algorithms respectively. PSig( $M, SK_i, PK_i, PK_j, APK$ ) outputs a partial signature  $\sigma_P$ , while PVer( $M, \sigma_P, \mathbf{PK}, APK$ ) outputs 1 or 0, where  $\mathbf{PK} = \{PK_i, PK_j\}$ .
- Res: This is the resolution algorithm. Res( $M, \sigma_P, ASK, \mathbf{PK}$ ), where  $\mathbf{PK} = \{PK_i, PK_j\}$ , outputs a full signature  $\sigma_F$ , or  $\perp$  indicating the failure of resolving a partial signature.

It is required that there is an efficient algorithm which given a pair  $(PK, SK)$ , verifies if  $SK$  matches  $PK$ , i.e.  $(PK, SK)$  is an output of algorithm Setup<sup>User</sup>. As in [15], PSig together with Res should be functionally equivalent to Sig.

**Correctness:** for any  $k \in \mathbb{N}$ ,  $PM \leftarrow \text{PMGen}(1^k)$ ,  $(APK, ASK) \leftarrow \text{Setup}^{\text{TTP}}(PM)$ ,  $(PK_i, SK_i) \leftarrow \text{Setup}^{\text{User}}(PM, APK)$ ,  $(PK_j, SK_j) \leftarrow \text{Setup}^{\text{User}}(PM, APK)$ , and  $M \in \{0, 1\}^*$ , let  $\mathbf{PK} = \{PK_i, PK_j\}$ ,  $\sigma_P \leftarrow \text{PSig}(M, SK_i, PK_i, PK_j, APK)$ ,  $\sigma_F \leftarrow \text{Sig}(M, SK_i, PK_i, PK_j, APK)$ , we have:

$$\text{PVer}(M, \sigma_P, \mathbf{PK}, APK) = 1,$$

$$\text{Ver}(M, \sigma_F, PK_i, PK_j, APK) = 1,$$

$$\text{and } \text{Ver}(M, \text{Res}(M, \sigma_P, ASK, \mathbf{PK}), PK_i, PK_j, APK) = 1.$$

### 2.1. Security properties

**Chosen-key model.** We consider the security of AOFE in the multi-user setting [15] and the chosen-key model, which was introduced by Lysyanskaya et al. [31] in the context of sequential aggregate signature. In the chosen-key model, the adversary can choose any public key, except that the challenge/target public key(s) should be honestly generated. We also require that there exists an efficient algorithm for checking the validity of the challenge key pair(s) output by the adversary, i.e. if  $(PK, SK)$  is a possible output of Setup<sup>User</sup>.

**Resolution ambiguity.** During an exchange, the verifier may be unable to receive the signer's full signature due to the poor internet connection. In this case the verifier could ask the arbitrator to resolve the signer's partial signature that it has already received. If the full signature output by algorithm Res has a different structure from that output by algorithm Sig,

it may bring bad effect to the credit of the signer, as others may think the signer was cheating in the exchange. To protect the interest of the signer, we need *resolution ambiguity*<sup>1</sup> in this case.

Resolution ambiguity requires that the ‘resolved signatures’, e.g.  $\text{Res}(M, \text{PSig}(M, SK_i, PK_i, PK_j, APK), \text{ASK}, \{PK_i, PK_j\})$ , output by the arbitrator should be (computationally) *indistinguishable* from the ‘actual signatures’ generated by the signer, e.g.  $\text{Sig}(M, SK_i, PK_i, PK_j, APK)$ . We stress that there are some cases in which a signer who is unable or refuses to return its full commitment, should be accounted, then the resolution ambiguity is not required any more, i.e. the OFE scheme should be accountable.

**Signer ambiguity.** Informally, given a partial signature  $\sigma_P$  from a signer  $A$ , a verifier  $B$  should not be able to convince others that  $\sigma_P$  was generated by  $A$ . To capture this, we borrow the idea of defining the *ambiguity* in concurrent signatures [12], and require that  $B$  should be able to simulate partial signatures that look *indistinguishable* from those generated by  $A$ . We need the existence of a simulation algorithm  $\text{FPSig}$ , that takes as input  $(M, SK_B, PK_A, PK_B, APK)$  and outputs a partial signature  $\sigma_P$  that is valid under  $PK_A, PK_B$ . This is also the reason why a verifier should be equipped with a public/secret key pair, and its public key should be included in the inputs of  $\text{PSig}$  and  $\text{Sig}$ . Formally, we define an experiment in which  $D$  is a probabilistic polynomial-time distinguisher.

$$\begin{aligned} & \text{PM} \leftarrow \text{PMGen}(1^k) \\ & (APK, \text{ASK}) \leftarrow \text{Setup}^{\text{TTP}}(\text{PM}) \\ & (M, \{(PK_i, SK_i)\}_{i \in \{A, B\}}, st) \leftarrow D^{O_{\text{Res}}}(APK) \\ & b \leftarrow \{0, 1\} \\ & \sigma_P \leftarrow \begin{cases} \text{PSig}(M, SK_A, PK_A, PK_B, APK) & \text{if } b = 0 \\ \text{FPSig}(M, SK_B, PK_A, PK_B, APK) & \text{if } b = 1 \end{cases} \\ & b' \leftarrow D^{O_{\text{Res}}}(st, \sigma_P) \\ & \text{Succ. of } D := [b' = b \wedge (M, \sigma_P, \{PK_A, PK_B\}) \notin \text{Query}(D, O_{\text{Res}})] \end{aligned}$$

where  $st$  is the state information of  $D$ ; oracle  $O_{\text{Res}}$  takes as input a valid partial signature  $\sigma_P$  of user  $U_i$  on message  $M$  with respect to verifier  $U_j$ , i.e.  $(M, \sigma_P, \{PK_i, PK_j\})$ , and outputs a full signature  $\sigma_F$  on  $M$  under  $PK_i, PK_j$ ; and  $\text{Query}(D, O_{\text{Res}})$  is the set of valid queries that  $D$  issued to oracle  $O_{\text{Res}}$ . The advantage of  $D$ , denoted by  $\text{Adv}_D^{\text{SA}}(k)$ , is defined as the gap between its success probability in the experiment above and  $1/2$ , i.e.  $\text{Adv}_D^{\text{SA}}(k) = |\Pr[b' = b] - 1/2|$ .

**Definition 2** (*Signer ambiguity*). An AOFE scheme is *signer ambiguous* if for any PPT algorithm  $D$ ,  $\text{Adv}_D^{\text{SA}}(k)$  is negligible in  $k$ .

**Remark 1.** A similar notion introduced in [16,30] requires that the signer’s partial signature can be simulated in an indistinguishable way. However, their ‘*indistinguishability*’ [16,30] is defined in the CPA fashion, i.e. the adversary has no oracle access for resolving partial signatures; while our definition of signer ambiguity is of CCA type, i.e. the adversary has access to  $O_{\text{Res}}$ .

We also remark that this level of signer ambiguity may be the best that we can get. In Definition 2,  $D$  is required to output well-formed key pairs for both  $A$  and  $B$ . If, for example,  $PK_B$  is maliciously chosen by  $D$  so that it is the hash of  $PK_A$ , then no one probably knows the corresponding secret key  $SK_B$ , and there is no way to ensure that the verifier can simulate the signer’s partial signatures. As far as we know, the definition of anonymity of ring signatures also imposes a similar requirement. In the definition of *anonymity w.r.t. adversarially-chosen keys* and *anonymity against full key exposure* [6], the two target key pairs,  $(PK_{i_0}, SK_{i_0})$  and  $(PK_{i_1}, SK_{i_1})$ , are required to be well-formed/honestly generated. The difference is that the two target key pairs in [6] are prepared by the challenger, while in Definition 2 they are chosen by the distinguisher. It is readily seen that our definition of signer ambiguity is stronger (or at least not weaker).

**Security against signers.** It requires that no PPT adversary  $A$  should be able to produce a partial signature with non-negligible probability, which looks good to a verifier but cannot be resolved to a full signature by the arbitrator. This ensures the fairness for verifiers, that is, if the signer has committed to a message w.r.t an (honest) verifier, the verifier should always be able to get the signer’s full commitment. Formally, we consider the following experiment:

$$\begin{aligned} & \text{PM} \leftarrow \text{PMGen}(1^k) \\ & (APK, \text{ASK}) \leftarrow \text{Setup}^{\text{TTP}}(\text{PM}) \\ & (PK_B, SK_B) \leftarrow \text{Setup}^{\text{User}}(\text{PM}, APK) \end{aligned}$$

<sup>1</sup> Resolution ambiguity is the same as the *ambiguity* defined in [15,21]. Here we rename it in order to avoid any confusion, as we will define another kind of ambiguity, e.g. *signer ambiguity*.

$$\begin{aligned}
(M, \sigma_P, PK_A) &\leftarrow A^{O_{\text{PSig}}^B, O_{\text{Res}}}(\text{APK}, PK_B) \\
\sigma_F &\leftarrow \text{Res}(M, \sigma_P, \text{ASK}, \{PK_A, PK_B\}) \\
\text{Succ. of } A &:= [\text{PVer}(M, \sigma_P, \{PK_A, PK_B\}, \text{APK}) = 1 \wedge \text{Ver}(M, \sigma_F, PK_A, PK_B, \text{APK}) \\
&= 0 \wedge (M, PK_A) \notin \text{Query}(A, O_{\text{PSig}}^B)]
\end{aligned}$$

where oracle  $O_{\text{Res}}$  is described in the previous experiment;  $O_{\text{PSig}}^B$  takes as input  $(M, PK_i)$  and outputs a partial signature on  $M$  valid under  $PK_i, PK_B$  generated using  $SK_B$ ; and  $\text{Query}(A, O_{\text{PSig}}^B)$  is the set of queries made by  $A$  to oracle  $O_{\text{PSig}}^B$ . Note that the adversary is not allowed to corrupt  $PK_B$ ; otherwise it can easily succeed in the experiment by simply using  $SK_B$  to produce a partial signature under public keys  $PK_A, PK_B$ . The advantage of  $A$  in the experiment, denoted by  $\text{Adv}_A^{\text{SAS}}(k)$ , is defined to be  $A$ 's success probability.

**Definition 3** (Security against signers). An AOFE scheme is *secure against signers* if there is no PPT adversary  $A$  such that  $\text{Adv}_A^{\text{SAS}}(k)$  is non-negligible in  $k$ .

**Security against verifiers.** This security notion requires that any PPT verifier  $B$  should not be able to transform a partial signature into a full signature with non-negligible probability if no help has been obtained from the signer or the arbitrator. This requirement has some similarity to the notion of *opacity* for verifiably encrypted signature [8]. Formally, we consider the following experiment:

$$\begin{aligned}
\text{PM} &\leftarrow \text{PMGen}(1^k) \\
(\text{APK}, \text{ASK}) &\leftarrow \text{Setup}^{\text{TTP}}(\text{PM}) \\
(PK_A, SK_A) &\leftarrow \text{Setup}^{\text{User}}(\text{PM}, \text{APK}) \\
(M, PK_B, \sigma_F) &\leftarrow B^{O_{\text{PSig}}, O_{\text{Res}}}(PK_A, \text{APK}) \\
\text{Succ. of } B &:= [\text{Ver}(M, \sigma_F, PK_A, PK_B, \text{APK}) = 1 \wedge (M, \cdot, \{PK_A, PK_B\}) \notin \text{Query}(B, O_{\text{Res}})]
\end{aligned}$$

where oracle  $O_{\text{Res}}$  is described in the experiment of signer ambiguity,  $\text{Query}(B, O_{\text{Res}})$  is the set of valid queries  $B$  issued to the resolution oracle  $O_{\text{Res}}$ , and oracle  $O_{\text{PSig}}$  takes as input a message  $M$  and a public key  $PK_j$  and returns a valid partial signature  $\sigma_P$  on  $M$  under  $PK_A, PK_j$  generated using  $SK_A$ . In the experiment,  $B$  can ask the arbitrator for resolving any partial signature with respect to any pair of public keys adaptively chosen by  $B$ , with the limitation described in the experiment. The advantage of  $B$  in the experiment, denoted by  $\text{Adv}_B^{\text{SAV}}(k)$ , is defined to be  $B$ 's success probability in the experiment above.

**Definition 4** (Security against verifiers). An AOFE scheme is *secure against verifiers* if there is no PPT adversary  $B$  such that  $\text{Adv}_B^{\text{SAV}}(k)$  is non-negligible in  $k$ .

**Security against the arbitrator.** Intuitively, security against the arbitrator requires that no PPT adversary  $C$  including the arbitrator, should be able to generate with non-negligible probability a valid full signature without explicitly asking the signer to do so. It ensures the fairness for signers, that is, no one can frame the actual signer on a message with a forgery. Formally, we consider the following experiment:

$$\begin{aligned}
\text{PM} &\leftarrow \text{PMGen}(1^k) \\
(\text{APK}, \text{ASK}^*) &\leftarrow C(\text{PM}) \\
(PK_A, SK_A) &\leftarrow \text{Setup}^{\text{User}}(\text{PM}, \text{APK}) \\
(M, PK_B, \sigma_F) &\leftarrow C^{O_{\text{PSig}}}(\text{ASK}^*, \text{APK}, PK_A) \\
\text{Succ. of } C &:= [\text{Ver}(M, \sigma_F, PK_A, PK_B, \text{APK}) = 1 \wedge (M, PK_B) \notin \text{Query}(C, O_{\text{PSig}})]
\end{aligned}$$

where the oracle  $O_{\text{PSig}}$  is described in the previous experiment,  $\text{ASK}^*$  is  $C$ 's state information, which might not be the corresponding private key of  $\text{APK}$ , and  $\text{Query}(C, O_{\text{PSig}})$  is the set of queries  $C$  issued to the oracle  $O_{\text{PSig}}$ . The advantage of  $C$  in this experiment, denoted by  $\text{Adv}_C^{\text{SAA}}(k)$ , is defined to be  $C$ 's success probability.

**Definition 5** (Security against the arbitrator). An AOFE scheme is said to be *secure against the arbitrator* if there is no PPT adversary  $C$  such that  $\text{Adv}_C^{\text{SAA}}(k)$  is non-negligible in  $k$ .

**Remark 2.** In AOFE, both signer  $A$  and verifier  $B$  are equipped with public/secret key pairs (of the same structure), and both of them can generate indistinguishable partial signatures on the same message. If the security against the arbitrator holds for  $A$ , it holds for  $B$  as well. That is, even when colluding with  $A$  (and other signers), the arbitrator should not be able to frame  $B$  for a full signature on a message, if it has not obtained a partial signature on the message generated by  $B$ .

**Definition 6 (Secure AOFE).** An AOFE scheme is *secure* in the multi-user setting and chosen-key model if it is resolution ambiguous, signer ambiguous, secure against signers, secure against verifiers and secure against the arbitrator.

## 2.2. Weak variants of security models

Intuitively, if an AOFE scheme is not secure against verifiers, the scheme cannot be signer ambiguous, because a malicious verifier can convert a signer's partial signature to a full one, which allows the verifier to win the signer ambiguity game. For technical reasons, we first describe some weakened models below. In the definition of signer ambiguity (Definition 2), the two public/secret key pairs are selected by  $D$ . In a slightly weaker variant, the two key pairs are selected by the challenger, and then given to  $D$ . This is comparable to the ambiguity definition of concurrent signature [12], or the strongest definition of anonymity of ring signature considered in [6], namely *anonymity against full key exposure*. We can also define an even weaker version of signer ambiguity, in which  $D$  is given  $(PK_A, PK_B, SK_B)$  and oracle access to  $O_{\text{PSig}}$ . We call this variant *the weak signer ambiguity*.

In the definition of security against verifiers (Definition 4), the verifier's public key  $PK_B$  is adaptively selected by the adversary  $B$ . In a weaker variant, the challenger selects  $(PK_B, SK_B)$  and gives the pair to  $B$ . The rest of the model remains unchanged. We call this variant *weak security against verifiers*. Below we show that if an AOFE scheme is weakly signer ambiguous and secure against the arbitrator, then it is weakly secure against verifiers.

**Theorem 1.** In AOFE, weak signer ambiguity and security against the arbitrator (Definition 5) together imply weak security against verifiers.

**Proof.** Suppose that an AOFE scheme is not weakly secure against verifiers. Let  $B$  be the PPT adversary that has non-negligible advantage  $\epsilon$  in the experiment of weak security against verifiers after making at most  $q$  queries of the form  $(\cdot, PK_B)$  to oracle  $O_{\text{PSig}}$ . By the security against the arbitrator, with overwhelming probability  $B$  has queried  $O_{\text{PSig}}$  in the form  $(\cdot, PK_B)$ . Hence the value of  $q$  is at least one.

Denote the experiment of weak security against verifiers by  $\text{Ex}^{(0)}$ . Note that in  $\text{Ex}^{(0)}$  all queries to  $O_{\text{PSig}}$  are answered with partial signatures generated using  $SK_A$ . We now define a series of experiments,  $\text{Ex}^{(1)}, \dots, \text{Ex}^{(q)}$ , so that  $\text{Ex}^{(i)}$  ( $i \geq 1$ ) is the same as  $\text{Ex}^{(i-1)}$  except that the  $(q+1-i)$ -th,  $\dots$ ,  $q$ -th queries of the form  $(\cdot, PK_B)$  submitted to  $O_{\text{PSig}}$  are answered with partial signatures generated using  $SK_B$ . Let  $B$ 's success probability in  $\text{Ex}^{(i)}$  be  $\epsilon_i$ . Note that  $\epsilon_0 = \epsilon$ , and in  $\text{Ex}^{(q)}$  all queries  $(\cdot, PK_B)$  to  $O_{\text{PSig}}$  are answered with partial signatures generated using  $SK_B$ . Since  $B$  also knows  $SK_B$  (via corruption), it can use  $SK_B$  to generate partial signatures using  $SK_B$  on any message. Therefore, making queries  $(\cdot, PK_B)$  to  $O_{\text{PSig}}$  does not help  $B$  on winning the experiment if the answers are generated using  $SK_B$ . It is equivalent to the case that  $B$  does not issue any query  $(\cdot, PK_B)$  to  $O_{\text{PSig}}$ . Hence guaranteed by the security against the arbitrator, we have that  $B$ 's advantage in  $\text{Ex}^{(q)}$  is negligible as  $B$  has to output a full signature without getting any corresponding partial signature.

The non-negligible gap,  $\epsilon_0 - \epsilon_q$ , between  $B$ 's advantage in  $\text{Ex}^{(0)}$  and that in  $\text{Ex}^{(q)}$  translates into a non-negligible gap between  $B$ 's advantage in a pair of neighboring hybrid experiments. We construct a PPT algorithm  $D$  that uses  $B$  as a subroutine to break the weak signer ambiguity.

Given  $(APK, PK_A, PK_B, SK_B)$ , algorithm  $D$  selects  $i$  uniformly from  $\{1, \dots, q\}$ , invokes  $B$  on input  $(APK, PK_A, PK_B, SK_B)$ , and then simulates the oracles for  $B$ . The oracle  $O_{\text{Res}}$  is simulated by  $D$  using its own resolution oracle. If  $B$  makes a query  $(M, PK_j)$  to  $O_{\text{PSig}}$  where  $PK_j \neq PK_B$ ,  $D$  forwards this query to its own partial signing oracle, and returns the obtained answer back to  $B$ . Now consider the  $\ell$ -th query of the form  $(M, PK_B)$  made by  $B$  to  $O_{\text{PSig}}$ . If  $\ell < q+1-i$ ,  $D$  forwards it to its own oracle, and returns the obtained answer. If  $\ell = q+1-i$ ,  $D$  requests its challenger for the challenge partial signature  $\sigma_p^*$  on  $M$  and returns the obtained signature to  $B$ . If  $\ell > q+1-i$ ,  $D$  simply uses  $SK_B$  to produce a partial signature on  $M$ . At the end of the simulation, when  $B$  outputs  $(M^*, \sigma_f^*)$ . If  $B$  succeeds in the experiment,  $D$  outputs  $b' = 0$ ; otherwise,  $D$  outputs a random bit  $b'$ .

If  $D$ 's challenger uses  $SK_A$  and follows PSig algorithm to produce  $\sigma_p^*$ , i.e.  $b = 0$ , the view of  $B$  is identical to that in  $\text{Ex}^{(i-1)}$  and the probability that  $D$  outputs  $b' = 0$  is

$$\bar{\epsilon}_i \stackrel{\text{def}}{=} \epsilon_{i-1} + \frac{1}{2}(1 - \epsilon_{i-1}) = \frac{1}{2} + \frac{1}{2}\epsilon_{i-1}.$$

If the challenger uses  $SK_B$  and follows the simulation algorithm FPSig to produce  $\sigma_p^*$ , i.e.  $b = 1$ , the view of  $B$  is identical to that in  $\text{Ex}^{(i)}$ , and  $D$  outputs  $b' = 0$  with probability

$$\epsilon_i \stackrel{\text{def}}{=} \epsilon_i + \frac{1}{2}(1 - \epsilon_i) = \frac{1}{2} + \frac{1}{2}\epsilon_i.$$



Denote by  $\Pr[b' = 0|b = 0]$  (resp.  $\Pr[b' = 0|b = 1]$ ) the probability that  $D$  outputs  $b' = 0$  when  $\sigma_p^*$  is output by algorithm Sig (resp. FPSig). Since the index  $i$  is uniformly distributed in  $\{1, \dots, q\}$ , we have that

$$\begin{aligned} \Pr[b' = b] &= \Pr[b' = 0 \wedge b = 0] + \Pr[b' = 1 \wedge b = 1] \\ &= \frac{1}{2} + \frac{1}{2}(\Pr[b' = 0|b = 0] - \Pr[b' = 0|b = 1]) \\ &= \frac{1}{2} + \frac{1}{2}\left(\frac{1}{q}\sum_{j=1}^q \bar{\epsilon}_j - \frac{1}{q}\sum_{j=1}^q \epsilon_j\right) \\ &= \frac{1}{2} + \frac{1}{2q}\left(\sum_{j=1}^q \left(\frac{1}{2} + \frac{1}{2}\epsilon_{i-1}\right) - \sum_{j=1}^q \left(\frac{1}{2} + \frac{1}{2}\epsilon_i\right)\right) \\ &= \frac{1}{2} + \frac{1}{4q}(\epsilon_0 - \epsilon_q) \end{aligned}$$

which is non-negligibly larger than  $\frac{1}{2}$  since  $q$  is polynomial in the security parameter. This contradicts with the weak signer ambiguity assumption.  $\square$

**Corollary 1.** In AOFE, signer ambiguity (Definition 2) and security against the arbitrator (Definition 5) together imply weak security against verifiers.

Letting an adversary select the two challenge public keys gives the adversary more power in attacking signer ambiguity. Therefore, signer ambiguity defined in Section 2.1 is at least as strong as the weak signer ambiguity. Hence this corollary follows the theorem above directly.

### 3. Previous constructions revisited

In this section we first analyze the extension of two generic constructions to AOFE, and show that the resulting schemes are actually insecure under our proposed security model.

#### 3.1. The first try

In [21] Huang et al. provided a simple and straightforward method in constructing efficient optimistic fair exchange schemes. In their proposal, each user has two key pairs, one for public key signature, and the other for ring signature. The partial signature of a user consists of only a (standard) signature on the message, while the full signature includes additionally a ring signature under the ring consisting of the signer and the arbitrator.

The main difference between OFE and AOFE is that in addition to all the security properties of OFE, AOFE also enjoys signer ambiguity. A natural way to extend their method in the construction of AOFE is to use two ring signatures in the scheme, i.e. the partial signature consists of a ring signature on  $M$  under the ring of the signer and the verifier, and the full signature includes additionally a ring signature on  $M$  under the ring of the signer and the arbitrator.

**(Insecurity):** It seems that we achieve signer ambiguity via this method, due to the anonymity of the underlying ring signature scheme. However, this construction is insecure against the arbitrator. That is, the arbitrator can frame any signer on any message without asking the signer to sign any message. To do this, the arbitrator  $C$  colludes with a verifier  $U_j$ , and selects a target user  $U_i$  and a target message  $M$ .  $U_j$  generates a partial signature  $\sigma_P$  on  $M$  with regard to the group  $\{U_i, U_j\}$ , and claims that  $\sigma_P$  was generated by  $U_i$ . Then  $C$  resolves it to  $\sigma_F$  by producing a ring signature under the ring  $\{C, U_i\}$ . The two rings intersect at  $U_i$ , and thus  $\sigma_F$  is binding to  $U_i$ .

#### 3.2. The second try

In [15] Dodis et al. revisited a generic construction of optimistic fair exchange. Roughly, the partial signature of a user includes an encryption  $c$  of his signature  $\sigma$  on the message  $M$  generated under the arbitrator's public key, and an NIZK proof  $\pi$  showing that  $c$  contains the user's signature on  $M$ . The user's full signature on  $M$  is  $\sigma$ . They showed that this generic construction is a secure OFE scheme in the multi-user setting (under the certified-key model).

One may trivially extend this scheme to AOFE. Namely, the NIZK proof shows that  $c$  contains either the signer's signature on  $M$  or the verifier's signature on  $M$ , using the signature and the randomness for the encryption as the witness. This may seem to be a secure AOFE scheme. Namely, the NIZK proof shows the membership of the following language:

$$L = \left\{ \left( (\mathcal{P}k_{TA}, PK_i, PK_j, c, M), (\sigma, r) \right) : c = \mathcal{E}.\text{Enc}(\mathcal{P}k_{TA}, \sigma; r) \wedge (S.\text{Ver}(PK_i, \sigma, M) = 1 \vee S.\text{Ver}(PK_j, \sigma, M) = 1) \right\}$$

where  $\text{pk}_{\text{TA}}$  is the public key of the arbitrator,  $\mathcal{E}$  is the encryption scheme and  $\mathcal{S}$  is the signature scheme. However, the following attack demonstrates that it is actually *insecure* under the security model proposed in Section 2.1.

**(The attack):** We consider the security against the arbitrator (see Definition 5). Let  $C$  be an adversary. Given the challenge public key  $PK_A$ ,  $C$  randomly selects a message  $M$ , and generates two public keys, say,  $PK_B, PK_D$ . Then it asks the oracle  $O_{\text{PSig}}$  to sign  $M$  w.r.t.  $PK_D$ , and obtains a ciphertext  $c$ , and an NIZK proof  $\pi_{AD}$  showing that  $c$  contains a signature generated by either user  $A$  or user  $D$ .  $C$  then uses the arbitrator's secret key to recover user  $A$ 's signature  $\sigma$ , and re-encrypts  $\sigma$  under the arbitrator's public key using a fresh randomness  $r'$ . Let the new ciphertext be  $c'$ . Finally, the adversary produces a new NIZK proof  $\pi_{AB}$  showing that  $c'$  is an encryption of a signature on  $M$  generated by either  $A$  or  $B$ , using  $\sigma, r'$  as the witness. By the validity of  $(c, \pi_{AD})$ , we can easily get that  $(c', \pi_{AB})$  is also a valid output. Note that in the whole attack  $C$  didn't submit  $(M, PK_B)$  to the oracle  $O_{\text{PSig}}$ . Thus, the security against the arbitrator breaks. However, it's not hard to prove that this generic construction is secure against the arbitrator under a weaker model which differs from the one described in Section 2.1 only in that it's required  $(M, \cdot) \notin \text{Query}(C, O_{\text{PSig}})$  instead of  $(M, PK_B) \notin \text{Query}(C, O_{\text{PSig}})$ . This weak security against the arbitrator is guaranteed by the unforgeability (EUFCMA) of the underlying signature scheme.

From the attacks above, we learn that constructing a secure AOFE scheme is not a trivial task. The introduction of signer ambiguity to OFE makes the security against the arbitrator more subtle. Besides, the security against the arbitrator of AOFE seems to be stronger than that of OFE defined in [15,21]. However, they in fact are not comparable. In the model considered in this paper, the public key of the verifier is involved in the generation of a partial signature, thus every query the adversary submits to oracles includes an additional public key; while this is not the case in the model of OFE. In the security against the arbitrator (Definition 5), the adversary is allowed to obtain the signer's signature on  $M$  (but with respect to any public key rather than  $PK_B$ ). This is similar with the strong unforgeability of digital signatures [19].

#### 4. Our generic construction

Now we propose a generic construction of AOFE in the standard model, which is similar with the one widely used in building OFE schemes. Namely, the signer's signature is encrypted under the arbitrator's public key, and then a non-interactive proof is given to show that the ciphertext contains the signer's signature on the message. As pointed by Boyd et al. [10], the non-interactive proof is not much different from the signer's signature, as it's also sufficient to prove to others that the signer is bound to the message. Since AOFE requires that a verifier cannot prove to others that the signer is bound to a message, in the generic construction the signer has the verifier involved in the proof. That is, the signer provides a non-interactive proof showing that the ciphertext contains either the signer's signature or the verifier's signature on the message.

##### 4.1. The proposal

Let  $\mathcal{S} = (\text{Kg}, \text{Sig}, \text{Ver})$  be a public key signature scheme that is *weakly existentially unforgeable under chosen message attacks* [7,19], and  $\text{OTS} = (\text{Kg}, \text{Sig}, \text{Ver})$  be a *strong one-time* signature scheme. Let  $\mathcal{E} = (\text{Kg}, \text{Enc}, \text{Dec})$  be a tag-based public key encryption scheme that is *selective-tag weakly CCA secure*, and  $\Pi = (\text{Kg}, \text{Prv}, \text{Ver}, (\text{Sim}_1, \text{Sim}_2))$  be an NIZK proof system for the following language:

$$L = \left\{ ((\text{pk}_{\text{TA}}, PK_i, PK_j, c, \text{tag}, M), (\sigma, r)) : \right. \\ \left. c = \mathcal{E}.\text{Enc}(\text{pk}_{\text{TA}}, \text{tag}, \sigma; r) \wedge (S.\text{Ver}(PK_i, \sigma, M) = 1 \vee S.\text{Ver}(PK_j, \sigma, M) = 1) \right\}.$$

Algorithm  $\Pi.\text{Kg}$  takes as input  $1^k$  and outputs a common reference string  $\text{crs}$ .  $\text{Prv}$  and  $\text{Ver}$  are the prover strategy and verification algorithm respectively.  $(\text{Sim}_1, \text{Sim}_2)$  is the simulator, where  $\text{Sim}_1$  takes as input  $1^k$  and outputs a simulated common reference string  $\text{crs}$  that's indistinguishable from a real one, along with the corresponding trapdoor  $\tau_S$ , and  $\text{Sim}_2$  takes as input  $(\text{crs}, \tau_S, x)$  and outputs a non-interactive proof whose distribution is indistinguishable from that of proofs output by the prover. Note that the membership of the language above is efficiently checkable, so we have that  $L \in \mathcal{NP}$ . Fig. 1 (on page 186) describes our generic construction of AOFE, named GAOFE.

Note that in Fig. 1 (page 186) we omit the description of the parameter generation algorithm just for simplicity. This algorithm simply calls the corresponding parameter generation algorithms of the encryption scheme and the signature scheme to generate their parameters, which will be used in the other algorithms of GAOFE.

**Remark 3.** Note that in the construction, the public keys of both the signer and the verifier, i.e.  $PK_i$  and  $PK_j$ , are included in the message to be signed of the one-time signature  $\delta$ . This is important, as otherwise the scheme would be vulnerable to an attack which compromises the security against signers. Specifically, the signer Alice runs as the verifier an execution of the protocol with Bob. After obtaining Bob's partial signature  $\sigma_{P,B}$ , Alice aborts this execution, and then restarts a new execution as the signer with Bob. She sends  $\sigma_{P,B}$  as her partial signature to Bob. As  $\sigma_{P,B}$  is a valid signature with regard to the group consisting of Alice and Bob, and the partial signature doesn't specify who the signer is and who the receiver is, Bob would view it as a valid one, and then returns his full signature. At this time, Alice aborts this execution again. Bob then resorts to the arbitrator for resolving  $\sigma_{P,B}$  to a full one. However, since it was originally generated by Bob himself, the arbitrator can only resolve it to Bob's full signature. So in this case, Bob cannot get Alice's full signature, and thus the security against

- **Setup<sup>TPP</sup>**: Given PM, the arbitrator runs  $\mathcal{E}.Kg(1^k)$  to generate a key pair,  $(pk_{TA}, sk_{TA})$ , and invokes  $\Pi.Kg(1^k)$  to produce a common reference string. It publishes  $APK = (pk_{TA}, crs)$  and stores  $ASK = sk_{TA}$  secretly.
- **Setup<sup>User</sup>**: Each user  $U_i$  runs  $\mathcal{S}.Kg(1^k)$  to generate a key pair for the signature scheme,  $(pk_i, sk_i)$ , and publishes  $PK_i = pk_i$  and stores  $SK_i = sk_i$ .
- **PSig**: To partially sign a message  $M$  with verifier  $U_j$ , the user  $U_i$  does the following.
  - Generate a new pair of one-time key for OTS, i.e.  $(otvk, otsk) \leftarrow \text{OTS}.Kg(1^k)$ .
  - Compute a signature  $\sigma$  on  $otvk$ , i.e.  $\sigma \leftarrow \mathcal{S}.Sig(SK_i, otvk)$ .
  - Encrypt the signature under the arbitrator's public key using randomness  $r$  with respect to tag  $otvk$ , i.e.  $c \leftarrow \mathcal{E}.Enc(pk_{TA}, otvk, \sigma; r)$ .
  - Produce an NIZK proof  $\pi$  using witness  $(\sigma, r)$ , i.e.

$$\pi \leftarrow \Pi.Priv(crs, (pk_{TA}, PK_i, PK_j, c, otvk, otvk), (\sigma, r)).$$

- Sign the ciphertext, the proof and the message using  $otsk$ , i.e.

$$\delta \leftarrow \text{OTS}.Sig(otsk, c \parallel \pi \parallel M \parallel PK_i \parallel PK_j).$$

The partial signature  $\sigma_p$  is composed of  $(c, \pi, \delta, otvk)$ .

- **PVer**: On receiving  $U_i$ 's partial signature  $\sigma_p = (c, \pi, \delta, otvk)$  on message  $M$ , user  $U_j$  checks if  $\text{OTS}.Ver(otvk, \delta, c \parallel \pi \parallel M \parallel PK_i \parallel PK_j) = 1$  and  $\Pi.Ver(crs, (pk_{TA}, PK_i, PK_j, c, otvk, otvk), \pi) = 1$ . If either fails, it rejects; otherwise, it accepts.
- **Sig**: To sign a message  $M$  with verifier  $U_j$ , user  $U_i$  first computes a partial signature  $\sigma_p = (c, \pi, \delta, otvk)$  as above, and then sets its full signature to be  $\sigma_F = (\sigma, \sigma_p)$ .
- **Ver**: On receiving  $U_i$ 's full signature  $\sigma_F = (\sigma, (c, \pi, \delta, otvk))$  on message  $M$ , the verifier first checks if  $\text{PVer}(M, (c, \pi, \delta, otvk), \{PK_i, PK_j\}, APK) = 1$  and  $\mathcal{S}.Ver(pk_i, \sigma, otvk) = 1$ . If either fails, it outputs 0 (reject); otherwise, it outputs 1 (accept).
- **Res**: On receiving from  $U_j$  a partial signature  $\sigma_p = (c, \pi, \delta, otvk)$  claimed to be generated by  $U_i$ , the arbitrator checks the validity of  $\sigma_p$  by calling algorithm PVer. If it's invalid, it returns  $\perp$  to  $U_j$ . Otherwise, it recovers  $\sigma$  from  $c$  by computing  $\sigma \leftarrow \mathcal{E}.Dec(sk_{TA}, otvk, c)$ . If  $\mathcal{S}.Ver(pk_i, \sigma, otvk) = 1$ , the arbitrator returns  $\sigma$  to  $U_j$ ; otherwise, it returns  $\perp$ .

**Fig. 1.** Our generic construction of AOFE, GAOFE.

signers is broken. Therefore, it seems necessary to specify in the message to be signed the identities of the signer and the verifier. Besides the validity check of the partial signature, the verifier should also check if the public key of the receiver specified in the signature, i.e.  $PK_j$ , is his. If not, it should reject. However, the inclusion of public keys doesn't contradict the signer ambiguity. The embedding of  $PK_i$  and  $PK_j$  to the message to be signed doesn't require any private information, and anyone can do it. The verifier is still able to use his secret key to produce indistinguishable partial signatures. We also note that the attack above does not happen in ordinary OFE at all. Thus, this attack shows a big difference between AOFE and OFE.

#### 4.2. Security analysis

**Theorem 2.** GAOFE is a secure ambiguous optimistic fair exchange scheme.

The theorem follows the following lemmas directly:

**Lemma 1.** GAOFE is resolution ambiguous.

**Proof.** Guaranteed by the security against signers (as shown in Lemma 3), if a partial signature  $\sigma_p$  is valid, then with overwhelming probability that the arbitrator can extract the signer's signature  $\sigma$  on the message. Conditioned on that the resolution succeeds, the signature output by the arbitrator is the same as that output by the signer. Therefore, the distribution of the output by the arbitrator is indistinguishable from that of the signer's signatures. So the construction above is resolution ambiguous.  $\square$

**Lemma 2.** GAOFE is signer ambiguous.

Before presenting the proof, we describe how the simulation algorithm FPSig works. To simulate a partial signature on  $M$ , the verifier  $U_j$  generates a fresh one-time key pair  $(otvk, otsk)$  and uses its own secret key  $SK_j$  to generate its signature  $\sigma$  on  $otvk$ . The rest remains the same as algorithm PSig.

**Proof.** Let  $D$  be a PPT adversary against the security against verifiers. We modify the experiment so that the challenger runs  $\Pi.\text{Sim}_1$  algorithm to generate the common reference string  $crs$  along with a simulation trapdoor  $\tau_S$ . By the common

reference string indistinguishability of  $\Pi$ ,  $D$ 's advantage in new experiment is negligibly close to that in the original experiment.

Second, we modify the experiment so that to answer each query the adversary submits to  $O_{\text{PSig}}$  and to prepare the challenge partial signature, the challenger calls  $\Pi.\text{Sim}_2$  on input  $\tau_5$  to produce the proof  $\pi$ , instead of calling the prover strategy with the witness  $(\sigma, r)$ . Guaranteed by the zero-knowledge property of  $\Pi$ , this modification brings only a negligible difference to  $D$ 's advantage.

Third, the experiment is modified again so that for each valid query submitted by the adversary to oracle  $O_{\text{Res}}$ , i.e.  $(M, \sigma_P, \{PK_i, PK_j\})$  where  $\sigma_P = (c, \pi, \delta, \text{otvk})$ , if  $\text{otvk}$  was ever used by  $O_{\text{PSig}}$  in answering  $B$ 's partial signing query, i.e.  $(M', PK')$  is the query and  $\sigma' = (c', \pi', \delta', \text{otvk})$  is the answer, but  $(c, \pi, M, \{PK_i, PK_j\}, \delta) \neq (c', \pi', M', \{PK_A, PK'\}, \delta')$ , the experiment is aborted. As will be discussed in the proof of Lemma 5, this case happens with negligible probability, guaranteed by the strong one-time unforgeability of OTS. Hence,  $D$ 's advantage does not change noticeably.

Assume that  $D$  can win this experiment with non-negligible advantage  $\epsilon_D$ , we then use it to build another PPT algorithm  $D'$  to break the selective-tag CCA security of  $\mathcal{E}$ , as below.

$D'$  first calls  $\text{OTS.Kg}(1^k)$  to generate a one-time key pair  $(\text{otvk}^*, \text{otsk}^*)$ , and submits  $\text{otvk}^*$  to its challenger as the challenge tag, which then returns a public key  $pk$  of  $\mathcal{E}$ .  $D'$  runs  $\Pi.\text{Sim}_1(1^k)$  to generate  $(c_{\text{RS}}, \tau_5)$ , and calls  $\mathcal{S}.\text{Kg}(1^k)$  to generate a key pair for the honest user  $U_A$ , say,  $(PK_A, SK_A)$ . It invokes  $D$  on input  $(pk, PK_A) = ((pk, c_{\text{RS}}), PK_A)$ , and then begins to simulate oracle  $O_{\text{Res}}$  for  $D$ .

Given a query  $(M, \sigma_P, \{PK_i, PK_j\})$  where  $\sigma_P = (c, \pi, \delta, \text{otvk})$ ,  $D'$  first checks if the query passes the PVer algorithm. If not, it returns  $\perp$  to  $D$ ; otherwise, the soundness of  $\Pi$  implies that  $c$  contains a valid signature  $\sigma$  on  $\text{otvk}$  with respect to either  $PK_i$  or  $PK_j$ .  $D'$  forwards the ciphertext  $c$  and the tag  $\text{otvk}$  to its own decryption oracle, and obtains  $\sigma$ . If either  $\mathcal{S}.\text{Ver}(PK_i, \sigma, \text{otvk}) = 1$  or  $\mathcal{S}.\text{Ver}(PK_j, \sigma, \text{otvk}) = 1$  holds,  $D'$  returns  $\sigma$  to  $D$ .

At some time,  $D$  submits  $(M^*, (PK_A, SK_A), (PK_B, SK_B))$ .  $D'$  first checks if both the key pairs are valid. If not,  $D$  aborts and outputs a random bit. Otherwise, it does the following:

1. Run  $\mathcal{S}.\text{Sig}$  twice to generate signatures  $\sigma_0, \sigma_1$  on  $\text{otvk}^*$  using  $SK_A, SK_B$  respectively.
2. Submit  $\sigma_0, \sigma_1$  to its own challenger, which returns a ciphertext  $c^*$  of  $\sigma_b$  with respect to tag  $\text{otvk}^*$  for some random bit  $b \in \{0, 1\}$ .
3. Call the simulator  $\Pi.\text{Sim}_2$  on input the trapdoor  $\tau_5$  to generate a proof  $\pi^*$  for  $(pk, PK_A, PK_B, c^*, \text{otvk}^*)$ .
4. Use  $\text{otsk}^*$  to compute a one-time signature  $\delta^*$  on  $c^* \parallel \pi^* \parallel M^* \parallel PK_A \parallel PK_B$ .

The challenge signature prepared by  $D'$  is  $\sigma_p^* = (c^*, \pi^*, \delta^*, \text{otvk}^*)$ . If  $b = 0$ ,  $\sigma_p^*$  is a valid partial signature output by algorithm PSig; otherwise,  $\sigma_p^*$  is a simulated partial signature output by algorithm FPSig.

$D'$  returns  $\sigma_p^*$  to the adversary, and then continues to simulate the oracle  $O_{\text{Res}}$ . Let  $(M, \sigma_P, \{PK_i, PK_j\})$  be any of its valid queries, where  $\sigma_P = (c, \pi, \delta, \text{otvk})$ . We distinguish the following two cases:

1.  $\text{otvk} \neq \text{otvk}^*$ . In this case,  $D'$  simulates  $O_{\text{Res}}$  in the same way as above.
2.  $\text{otvk} = \text{otvk}^*$ . We say, this case will not happen in the experiment. First of all, we can exclude the subcase  $(c^* \parallel \pi^* \parallel M^* \parallel PK_A \parallel PK_B, \delta^*) = (c \parallel \pi \parallel M \parallel PK_i \parallel PK_j, \delta)$ , because the adversary is prohibited from asking  $O_{\text{Res}}$  for resolving  $(M^*, \sigma_p^*, \{PK_A, PK_B\})$ . On the other hand, if  $(c^* \parallel \pi^* \parallel M^* \parallel PK_A \parallel PK_B, \delta^*) \neq (c \parallel \pi \parallel M \parallel PK_i \parallel PK_j, \delta)$ , according to the experiment's specification, the experiment is aborted.

Finally,  $A$  outputs a bit  $d$ .  $D$  then outputs a bit  $b' = d$  and halts.

It's readily seen that the oracle  $O_{\text{Res}}$  is simulated indistinguishably by  $D'$ . If  $D$  succeeds in the experiment,  $D'$  also succeeds in outputting the bit  $b'$ . So  $D$ 's advantage is

$$\text{Adv}_{D'}^{\text{T-PKE}}(k) \geq \text{Adv}_D^{\text{SA}}(k) = \epsilon_D$$

which is non-negligible by hypothesis. Therefore, the selective-tag CCA security of  $\mathcal{E}$  is broken.  $\square$

**Lemma 3.** GAOFE is secure against signers.

**Proof.** If a partial signature  $(c, \pi, \delta, \text{otvk})$  of  $U_A$  is valid on message  $M$  with respect to verifier  $U_B$ , by the soundness of  $\Pi$  and the perfect completeness of  $\mathcal{E}$ , with overwhelming probability the arbitrator can recover from the ciphertext  $c$  a valid signature  $\sigma$  on  $M$  generated by either  $U_A$  or  $U_B$ . On the other hand, due to the security against the arbitrator (as shown in Lemma 5), we know that only with negligible probability can the adversary  $A$  forge a signature on behalf of an honest user  $U_B$ . Therefore, the valid signature must be generated by the adversary itself. Hence, the adversary can only break the security against signers with negligible probability.  $\square$

**Lemma 4.** GAOFE is secure against verifiers.

**Proof.** Let  $B$  be an efficient adversary against the security against verifiers. Let  $\mathcal{P}$  be the set of partial signatures returned by the oracle  $O_{\text{PSig}}$ , and let  $(M^*, PK_B, \sigma_f^*)$  be  $B$ 's final output, where  $\sigma_f^* = (\sigma_p^*, \sigma^*)$ . First of all, we modify the experiment

so that if  $\sigma_p^* \notin \mathcal{P}$ , we abort it. Guaranteed by the security against the arbitrator (shown in Lemma 5), the modification leads to only a negligible difference in  $B$ 's success probability.

Second, we modify the experiment so that the challenger runs  $\Pi.\text{Sim}_1$  algorithm to generate the common reference string  $\text{crs}$  along with a simulation trapdoor  $\tau_S$ , and to answer each query the adversary submits to  $O_{\text{PSig}}$  the challenger calls  $\Pi.\text{Sim}_2$  with  $\tau_S$  to produce the proof  $\pi$ , instead of calling the prover strategy with the witness  $(\sigma, r)$ . By the zero knowledge property of  $\Pi$ , we know that the modification brings a negligible difference to  $B$ 's success probability.

Third, the experiment is modified again so that for each valid query submitted by the adversary to oracle  $O_{\text{Res}}$ , i.e.  $(M, \sigma_p, \{PK_i, PK_j\})$  where  $\sigma_p = (c, \pi, \delta, \text{otvk})$ , if  $\text{otvk}$  was ever used by  $O_{\text{PSig}}$  in answering  $B$ 's partial signing query, i.e.  $(M', PK')$  is the query and  $\sigma' = (c', \pi', \delta', \text{otvk})$  is the answer, but  $(c, \pi, M, \{PK_i, PK_j\}, \delta) \neq (c', \pi', M', \{PK_A, PK'\}, \delta')$ , the experiment is aborted. If the case happens, obviously, it indicates that the adversary produces a forgery for the signature scheme OTS. Guaranteed by the strong one-time unforgeability of OTS, we have that the modification affects  $B$ 's success probability negligibly as well.

Next we show that  $B$ 's success probability in this experiment, say,  $\epsilon_B$ , is negligible in  $k$ . Assume that  $\epsilon_B$  is non-negligible, we then use  $B$  to build another PPT algorithm  $D$  to break the selective-tag CCA security of  $\mathcal{E}$ .

$D$  first invokes the algorithm  $\mathcal{S}.\text{Kg}(1^k)$  to generate a pair of one-time key,  $(\text{otvk}^*, \text{otvk}^*)$ , submits  $\text{otvk}^*$  to its challenger as the target tag, and receives a challenge public key  $pk$  of  $\mathcal{E}$  from the challenger. It runs  $\Pi.\text{Sim}_1(1^k)$  to generate  $(\text{crs}, \tau_S)$ , and calls  $\mathcal{S}.\text{Kg}$  to generate  $(PK_A, SK_A)$ . Suppose that  $B$  will submit at most  $q$  distinct query to oracle  $O_{\text{PSig}}$ , which is polynomial in the security parameter.  $D$  picks  $i$  at random from the set  $\{1, \dots, q\}$ , and sets  $\text{pk}_{\text{TA}} = pk$ ,  $APK = (\text{crs}, \text{pk}_{\text{TA}})$ . It invokes  $B$  on input  $(APK, PK_A)$ , and then begins to simulate oracles for  $B$  as follows.

- $O_{\text{Res}}$ : On input a resolution query,  $(M, \sigma_p, PK_i, PK_j)$ ,  $D$  first checks the validity of the query, and returns  $\perp$  if it doesn't pass the PVer algorithm. It asks its decryption oracle to decrypt  $c$  with respect to tag  $\text{otvk}^*$ , and receives  $\sigma$  from it.  $D$  returns  $\sigma$  if  $\mathcal{S}.\text{Ver}(PK_i, \sigma, \text{otvk}^*) = 1$  or  $\mathcal{S}.\text{Ver}(PK_j, \sigma, \text{otvk}^*) = 1$ .
- $O_{\text{PSig}}$ : Let  $(M, PK')$  be the  $j$ -th distinct query  $B$  submits to the oracle. If  $j \neq i$ , this query is dealt with by  $D$  like a real user. If  $j = i$ ,  $D$  computes  $\sigma^* \leftarrow \mathcal{S}.\text{Sig}(SK_A, \text{otvk}^*)$ . It sets  $\sigma_0 = \sigma^*$  and randomly selects  $\sigma_1$  from the range of  $\mathcal{S}.\text{Sig}(\cdot, \cdot)$ , and forwards  $\sigma_0, \sigma_1$  to its challenger, which randomly chooses one of them say,  $\sigma_b$  for some bit  $b \in \{0, 1\}$ , to encrypt. Let the ciphertext be  $c^*$ .  $D$  then calls algorithm  $\Pi.\text{Sim}_2$  with the simulation trapdoor  $\tau_S$  to produce a proof  $\pi^*$  on  $(\text{pk}_{\text{TA}}, PK_A, PK', c^*, \text{otvk}^*, \text{otvk}^*)$ . It then completes the generation of the partial signature by computing  $\delta^* = \text{OTS}.\text{Sig}(\text{otsk}^*, c^* \parallel \pi^* \parallel M \parallel PK_A \parallel PK')$ , and returns  $\sigma_p^* = (c^*, \pi^*, \delta^*, \text{otvk}^*)$  back to the adversary  $B$ .

Finally  $B$  outputs  $(M^*, PK_B, \sigma_F^*)$ , where  $\sigma_F^* = (\sigma_p^*, \sigma^*)$ . If the output doesn't satisfy the winning condition,  $D$  aborts and outputs a random bit. By the specification of the experiment, we know that  $\sigma_p^* \in \mathcal{P}$ . If  $(M^*, PK_B)$  is not the  $i$ -th distinct query to oracle  $O_{\text{PSig}}$ , alternatively,  $D$ 's guess of  $i$  is incorrect,  $D$  aborts and outputs a random bit. If  $\text{Ver}(M^*, \sigma_F^*, PK_A, PK_B, APK) = 1$ ,  $D$  outputs 0; otherwise, it outputs 1.

We now consider the simulation of oracles  $O_{\text{Res}}$  and  $O_{\text{PSig}}$ . Let  $(M, \sigma_p = (c, \pi, \delta, \text{otvk}), \{PK_i, PK_j\})$  be a valid query to  $O_{\text{Res}}$ . If  $c \neq c^*$ ,  $D$  can handle this query as above. We then focus on the other case,  $c = c^*$ .

- If  $\text{otvk} \neq \text{otvk}^*$ ,  $D$  can simply forward  $(\text{otvk}, c^*)$  to its decryption oracle as the tag  $\text{otvk}$  is different from the challenge one,  $\text{otvk}^*$ , and obtain the signature  $\sigma$ .
- If  $\text{otvk} = \text{otvk}^*$ ,  $D$  is not allowed to ask its oracle to decrypt  $c^*$  with respect to  $\text{otvk}^*$ . However, as discussed in the proof of Lemma 2, this case will not happen.

Regarding the oracle  $O_{\text{PSig}}$ , it's perfectly simulated when  $j \neq i$ . For  $j = i$ , if  $c^*$  is an encryption of  $\sigma_0$ , the view of  $B$  is identical to that in a real attack, and  $B$  will succeed in the experiment with probability  $\epsilon_B$ , which is non-negligible. If  $c^*$  is an encryption of  $\sigma_1$ , the view of the adversary is indistinguishable from that in a real attack, due to the zero knowledge property of  $\Pi$ . Since  $c^*$  is independent of  $PK_A$ , and thus provides no help to  $B$  in generating  $\sigma_F^*$ . In this case, if  $B$  successfully produces a valid full signature  $\sigma_F^*$ , it should be that  $B$  forges a full signature on behalf of the honest user  $U_A$ , thus breaking the security against arbitrator. As shown in Lemma 5,  $B$ 's success probability in this case is negligible.

Let  $b'$  be the bit that  $D$  outputs. We want to show that  $|\Pr[b' = 1 \wedge b = 1] - \Pr[b' = 0 \wedge b = 1]|$  is non-negligible in  $k$ . No matter  $b = 0$  or  $b = 1$ , the probability that  $D$  aborts due to an incorrect guess of  $i$  is the same, and when it aborts, it outputs 0 with probability exactly one-half. Therefore, we only need to focus on the case in which  $D$  guesses  $i$  correctly, which happens with probability  $1/q$ . Denote this event by  $\text{Corr}$ .

If  $b = 0$ , as we discussed above, the probability that  $B$  outputs a valid  $\sigma_F^*$  is  $\epsilon_B$ . Thus,  $D$  outputs 0 with probability at least  $\epsilon_B$ . On the other side, i.e.  $b = 1$ , as discussed above, the probability that  $D$  outputs 0 is  $\epsilon$ , which is the maximum probability that an efficient can break the security against the arbitrator, and is negligible. So we get the following:

$$\begin{aligned} \text{Adv}_D^{\text{T-PKE}}(k) &= \left| \Pr[b' = b] - \frac{1}{2} \right| \\ &= \left| \Pr[b' = b \wedge \text{Corr}] + \Pr[b' = b \wedge \neg \text{Corr}] - \frac{1}{2} \right| \end{aligned}$$

$$\begin{aligned}
&= \left| \Pr[\text{Corr}]\Pr[b' = b|\text{Corr}] + \Pr[\neg\text{Corr}]\Pr[b' = b|\neg\text{Corr}] - \frac{1}{2} \right| \\
&= \left| \frac{1}{q} \left( \frac{1}{2} \Pr[b' = 0|b = 0 \wedge \text{Corr}] + \frac{1}{2} (1 - \Pr[b' = 0|b = 1 \wedge \text{Corr}]) \right) + \left( 1 - \frac{1}{q} \right) \frac{1}{2} - \frac{1}{2} \right| \\
&= \frac{1}{2q} |\Pr[b' = 0|b = 0 \wedge \text{Corr}] - \Pr[b' = 0|b = 1 \wedge \text{Corr}]| \\
&\geq \frac{1}{2q} |\epsilon_B - \epsilon|
\end{aligned}$$

which is also non-negligible. Therefore, the selective-tag CCA security of  $\mathcal{E}$  is broken.  $\square$

**Lemma 5.** GAOFE is secure against the arbitrator.

**Proof.** Let  $C$  be a PPT adversary  $C$  which breaks the security against the arbitrator, and  $(M^*, \sigma_F^*, PK_B)$  be its final output, where  $\sigma_F^* = (\sigma_p^*, \sigma^*)$  and  $\sigma_p^* = (c^*, \pi^*, \delta^*, \text{otvk}^*)$ . We distinguish the following two cases.

1.  $\text{otvk}^*$  never appeared in oracle  $O_{\text{PSig}}$ 's answers. In this case, we can use  $C$ 's capability to build a PPT adversary  $F_0$  to break the weak unforgeability of  $\mathcal{S}$ .

Suppose that  $C$  issues at most  $q$  distinct queries to oracle  $O_{\text{PSig}}$ .  $F_0$  runs  $\mathcal{S}.\text{Kg}(1^k)q$  times to generate  $q$  one-time key pairs, say,  $(\text{otvk}_1, \text{otvk}_1), \dots, (\text{otvk}_q, \text{otvk}_q)$ . It submits  $\{\text{otvk}_1, \dots, \text{otvk}_q\}$  to its challenger, which then returns a public key  $pk$  and the corresponding signatures,  $\{\sigma_1, \dots, \sigma_q\}$ .  $F_0$  then invokes  $C$  on input  $1^k$ , which returns a public key of the arbitrator,  $APK = (\text{pk}_{\text{TA}}, \text{crs})$ .  $F_0$  then returns  $PK_A = pk$  to  $C$ , and begins to simulate the partial signing oracle  $O_{\text{PSig}}$  for  $C$ . On input the  $i$ -th distinct query  $(M, PK_{j_i})$ ,  $F_0$  chooses a random string  $r$  from the randomness space of  $\mathcal{E}$ , uses  $r$  to encrypt  $\sigma_i$  under  $\text{pk}_{\text{TA}}$  with respect to the tag  $\text{otvk}_i$ , and uses  $(\sigma_i, r)$  as the witness to compute an NIZK proof  $\pi$ .  $F_0$  runs  $\text{OTS}.\text{Sig}$  with secret key  $\text{otvk}_i$  to generate a one-time signature  $\delta$  on  $c \parallel \pi \parallel M \parallel PK_A \parallel PK_{j_i}$ . It returns  $(c, \pi, \delta, \text{otvk}_i)$  to  $C$ . It's readily seen that the simulation of  $O_{\text{PSig}}$  is perfect.

Finally,  $C$  outputs  $(M^*, PK_B, \sigma_F^*)$  where  $\sigma_F^* = ((c^*, \pi^*, \delta^*, \text{otvk}^*), \sigma^*)$ , and wins the experiment with non-negligible probability  $\epsilon_C$ .  $F_0$  outputs  $(\text{otvk}^*, \sigma^*)$ . By the validity of  $C$ 's output, we know that  $\mathcal{S}.\text{Ver}(pk, \sigma^*, \text{otvk}^*) = 1$ . Since  $\text{otvk}^*$  is fresh, i.e.,  $\text{otvk}^* \notin \{\text{otvk}_1, \dots, \text{otvk}_q\}$ ,  $F_0$  didn't ask its signing oracle to return a signature on  $\text{otvk}^*$ . Thus,  $(\text{otvk}^*, \sigma^*)$  is a valid forgery for  $\mathcal{S}$ . Therefore,  $F_0$  breaks the security of  $\mathcal{S}$  with probability at least the same as  $C$ .

2.  $\text{otvk}^*$  appeared in one of  $O_{\text{PSig}}$ 's answers to  $C$ 's partial signing queries. Again, we assume that  $C$  issues at most  $q$  queries to  $O_{\text{PSig}}$ , which is polynomial in  $k$ . We use  $C$  to build an algorithm  $F_1$  to break the security of  $\text{OTS}$ .

Given a public key  $\text{otvk}^*$  of  $\text{OTS}$  and a one-time signing oracle,  $F_1$  invokes  $C$  on input  $1^k$  and obtains  $APK = (\text{pk}_{\text{TA}}, \text{crs})$  from it. It then calls  $\mathcal{S}.\text{Kg}(1^k)$  to generate a key pair for user  $A$ , say,  $(PK_A, SK_A)$ , and randomly selects  $i$  from  $\{1, \dots, q\}$ .  $F_1$  gives  $PK_A$  to  $C$ , and then begins to simulate oracle  $O_{\text{PSig}}$  for it.

If  $j \neq i$ ,  $F_1$  simulates the oracle like an honest user  $U_A$  does. If  $j = i$ , it uses  $SK_A$  to generate a signature  $\sigma$  on  $\text{otvk}^*$ , selects a random string  $r$ , computes  $c \leftarrow \mathcal{E}.\text{Enc}(\text{pk}_{\text{TA}}, \text{otvk}^*, \sigma; r)$  and uses  $(\sigma, r)$  as the witness to produce an NIZK proof  $\pi$ .  $F_1$  then submits  $c \parallel \pi \parallel M \parallel PK_A \parallel PK_i$  to its signing oracle, and obtains a signature  $\delta^*$ . It then returns  $(c, \pi, \delta, \text{otvk}^*)$  back to  $C$ .

Finally  $C$  outputs  $(M^*, \sigma_F^*, PK_B)$  where  $\sigma_F^* = (\sigma_p^*, \sigma^*)$  and  $\sigma_p^* = (c^*, \pi^*, \delta^*, \text{otvk}^*)$ .  $F_1$  outputs  $(c^* \parallel \pi^* \parallel M^* \parallel PK_A \parallel PK_B, \delta^*)$ . Assume that  $C$  wins the experiment. So we have that  $\text{OTS}.\text{Ver}(\text{otvk}^*, \delta^*, c^* \parallel \pi^* \parallel M^* \parallel PK_A \parallel PK_B) = 1$  and  $C$  didn't issue a query on input  $(M^*, PK_B)$ . Since the pair  $(M^*, PK_B)$  is fresh, we have that

$$c^* \parallel \pi^* \parallel M^* \parallel PK_A \parallel PK_B \neq c \parallel \pi \parallel M \parallel PK_A \parallel PK_i.$$

So  $\delta^*$  is a valid signature on a new message. Therefore, the security of  $\text{OTS}$  is broken.

Since we do not know which of the two cases above will happen, we simply toss a coin  $b$ , and run the algorithm  $F_b$ . Still, we have non-negligible probability to break the security of either  $\mathcal{S}$  or  $\text{OTS}$ , if  $C$  wins its experiment with non-negligible probability.  $\square$

**Remark 4.** In our construction, the signer uses its secret key to generate a signature on a fresh one-time verification key, while the message is signed using the corresponding one-time signing key. As shown by Huang et al. in [19], this combination leads to a strongly unforgeable signature scheme. It's not hard to see that our proposed AOFE scheme actually achieves a stronger version of security against the arbitrator. That is, even if the adversary sees the signer  $U_A$ 's full signature  $\sigma_F$  on a message  $M$  with verifier  $U_B$ , it cannot produce another full signature on  $M$ , say,  $\sigma'_F$ , such that  $\text{Ver}(M, \sigma'_F, PK_A, PK_B, APK) = 1$ . The claim can be shown using the proof given above without much modification.

## 5. A concrete scheme without random oracles

In this section, we propose an AOFE scheme, which is based on Groth and Sahai's idea of constructing a fully anonymous group signature scheme [17,18]. Before describing the scheme, we first introduce the assumptions and building tools used in our construction.

### 5.1. Assumptions

**(Admissible pairing):** Let  $\mathbb{G}_1$  and  $\mathbb{G}_T$  be two cyclic groups of large prime order  $p$ .  $\hat{e}$  is an *admissible pairing* if  $\hat{e}: \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_T$  is a map with the following properties: (1) *Bilinearity*:  $\forall R, S \in \mathbb{G}_1$  and  $\forall a, b \in \mathbb{Z}$ ,  $\hat{e}(R^a, S^b) = \hat{e}(R, S)^{ab}$ ; (2) *Non-degeneracy*:  $\exists R, S \in \mathbb{G}_1$  such that  $\hat{e}(R, S) \neq 1$ ; and (3) *Computability*: there exists an efficient algorithm for computing  $\hat{e}(R, S)$  for any  $R, S \in \mathbb{G}_1$ .

**(Decision Linear Assumption (DLIN) [9]):** Let  $\mathbb{G}_1$  be a cyclic group of large prime order  $p$ . The Decision Linear Assumption for  $\mathbb{G}_1$  holds if for any PPT adversary  $\mathcal{A}$ , the following probability is negligibly close to  $1/2$ :

$$\Pr[F, H, W \leftarrow \mathbb{G}_1; r, s \leftarrow \mathbb{Z}_p; Z_0 \leftarrow W^{r+s}; Z_1 \leftarrow \mathbb{G}_1; d \leftarrow \{0, 1\}; \mathcal{A}(F, H, W, F^r, H^s, Z_d) = d].$$

**(Strong Diffie–Hellman Assumption (SDH) [7]):** The  $q$ -SDH problem in  $\mathbb{G}_1$  is defined as follows: given a  $(q+1)$ -tuple  $(g, g^x, g^{x^2}, \dots, g^{x^q})$ , output a pair  $(g^{1/(x+c)}, c)$  where  $c \in \mathbb{Z}_p^*$ . The  $q$ -SDH assumption holds if for any PPT adversary  $\mathcal{A}$ , the following probability is negligible:

$$\Pr[x \leftarrow \mathbb{Z}_p^*; \mathcal{A}(g, g^x, \dots, g^{x^q}) = (g^{\frac{1}{x+c}}, c)].$$

### 5.2. Building tools

**(Tag-based encryption scheme):** We use a tag-based public key encryption scheme  $\mathcal{E}$  with security based on the DLIN assumption reviewed above. Below is a brief review of a suitable scheme due to Kiltz [28].

**Key generation:**  $(pk, sk) = ((F, H, K, L), (\kappa, \lambda)) \leftarrow \mathcal{E}.K(p, \mathbb{G}_1, \mathbb{G}_T, \hat{e}, g)$ , where  $F = g^\kappa$  and  $H = g^\lambda$ ;

**Encryption:** Let  $M \in \mathbb{G}_1$  be a message,  $\text{tag} \in \mathbb{Z}_p$  a tag, and  $r, s \in \mathbb{Z}_p$  the randomness. The ciphertext  $y$  is computed as  $y = (y_1, y_2, y_3, y_4, y_5) = (F^r, H^s, M \cdot g^{r+s}, (g^{\text{tag}K})^r, (g^{\text{tag}L})^s)$ ;

**Decryption:** The validity of a ciphertext can be checked without knowing the secret key. Anyone can check if  $\hat{e}(F, y_4) = \hat{e}(y_1, g^{\text{tag}K})$  and  $\hat{e}(H, y_5) = \hat{e}(y_2, g^{\text{tag}L})$ . If any one fails,  $\perp$  is returned. Otherwise,  $M$  is recovered by computing  $M = y_3 \cdot y_1^{-1/\kappa} \cdot y_2^{-1/\lambda}$ .

Note that the plaintext can also be recovered if the discrete logarithms of  $K$  and  $L$  with respect to  $g$  are known. Assume that  $K = g^{\kappa'}$  and  $L = g^{\lambda'}$ . After checking the validity of  $y = (y_1, y_2, y_3, y_4, y_5)$ ,  $M$  can be computed as  $M = y_3 \cdot y_4^{-1/(\text{tag}+\kappa')} \cdot y_5^{-1/(\text{tag}+\lambda')}$ . Kiltz proved [28] that under the DLIN assumption, the tag-based encryption scheme is selective-tag weakly chosen-ciphertext secure, that is, an adversary  $\mathcal{A}$  cannot tell which message was encrypted under  $\text{tag}^*$  (selected by  $\mathcal{A}$  though) before seeing the public key, even when it has access to a decryption oracle that decrypts ciphertexts under any tag other than  $\text{tag}^*$ . Readers may refer to [28] for the definition of selective-tag weakly CCA security.

**(Non-interactive proofs):** Recently, Groth and Sahai [18] proposed a general methodology for constructing simple and efficient non-interactive witness indistinguishable (NIWI) proofs and non-interactive zero-knowledge (NIZK) proofs that work for bilinear groups, without requiring complex NP-reductions. They proposed efficient non-interactive (NI) proofs for a set of equations in a bilinear group  $(p, \mathbb{G}_1, \mathbb{G}_T, \hat{e}, g)$  over variables in  $\mathbb{G}_1$  and  $\mathbb{Z}_p$ , such as pairing products, i.e.  $\hat{e}(x_1, y_1) \cdot \hat{e}(x_2, y_2) = T$ , or multi-exponentiations, i.e.  $x_1^{\delta_1} x_2^{\delta_2} = 1$ , having solutions  $x_i \in \mathbb{G}_1$ ,  $\delta_j \in \mathbb{Z}_p$ , so that all equations are simultaneously satisfied. Their NI proofs can be based on subgroup decision assumption, (symmetric) external Diffie–Hellman ((S)XDH) assumption, and decision linear (DLIN) assumption. In our construction, we will use their DLIN-based technique.

**(Commitment scheme):** Groth–Sahai's DLIN-based proofs consist of two commitment schemes, one for committing to variables of  $\mathbb{G}_1$ , and the other one to variables in  $\mathbb{Z}_p$ . The common reference string for either of the two schemes, can be generated in either of two indistinguishable ways.

For the first commitment scheme, a real common reference string is set up to  $U = F^R$ ,  $V = H^S$ , and  $W = g^{R+S}$ , a commitment to a variable  $x \in \mathbb{G}_1$  can be respectively expressed as  $c = (c_1, c_2, c_3) = (F^r U^t, H^s V^t, g^{r+s} W^t x)$  for randomness  $r, s \in \mathbb{Z}_p$ . The key for extracting  $x$  from  $c$  is  $xk = (\kappa, \lambda) = (\log_g F, \log_g H)$ , so the scheme is perfectly binding. A simulated common reference string consists of  $F, H, U = F^R, V = H^S$  and  $W = g^T$  where  $T \neq R + S$ , and thus the scheme is also perfectly hiding.

For the commitment to a variable  $\delta \in \mathbb{Z}_p$ , a real common reference string consists of  $F, H, U' = F^{R'}$ ,  $V' = H^{S'}$  and  $W' = g^{T'}$  where  $T' \neq R' + S'$ , and a commitment to  $\delta$  is expressed as  $c' = (c'_1, c'_2, c'_3) = (F^{r'} (U')^\delta, H^{s'} (V')^\delta, g^{r'+s'} (W')^\delta)$

for randomness  $r', s' \in \mathbb{Z}_p$ . The commitment scheme is perfectly binding. A simulated common reference string consists of  $F, H, U = F^{R'}, V = H^{S'}, W = g^{R'+S'}$ . The commitment scheme then becomes perfectly hiding. The simulation trapdoor is  $tk = (R', S')$ , and we can use it to reveal a commitment to 0 to any other value  $\delta \in \mathbb{Z}_p$ . Due to the DLIN assumption, we have that any PPT adversary cannot tell a real common reference string apart from a simulated one.

(Groth–Sahai Proofs): Groth–Sahai NI proof system for bilinear groups consists of four PPT algorithms,  $(K_{NI}, P, V, X)$ , where the key generator  $K_{NI}$  takes as input a system parameter  $(p, \mathbb{G}_1, \mathbb{G}_T, \hat{e}, g)$  and outputs a common reference string  $crs = (F, H, U, V, W, U', V', W')$  and an extraction key  $xk$ ; the algorithm  $P$  takes as input  $crs$ , a problem instance and a witness  $(\dots, x_i, \dots, \delta_j, \dots)$  and outputs a proof  $\pi$ ; the verification algorithm  $V$  takes as input  $crs$ , a problem instance, and a proof  $\pi$ , and outputs 1 indicating that  $\pi$  is valid or 0 indicating that  $\pi$  is invalid; and the extraction algorithm  $X$  takes as input  $crs$ , a problem instance, and a proof  $\pi$ , and outputs  $(\dots, x_i, \dots)$ .

Groth–Sahai NI proofs have perfect completeness, and perfect soundness on a real common reference string. Besides, they have perfect partial knowledge: the extraction algorithm will extract  $(\dots, x_i, \dots)$  from the proof, such that there is a solution for the equations using these  $x_i$ 's. Groth–Sahai proofs also have perfect witness-indistinguishability on a simulated common reference string: if there are many possible witnesses for the equations being satisfiable, the proof  $\pi$  does not reveal anything about which witness was used by the prover in generating  $\pi$ .

In our construction (Section 5.4), we use two NI proof systems: NIWI and NIZK. The NIWI proof system is used for showing that a BB-signature  $\bar{\sigma}$  is valid with respect to either  $PK_i$  or  $PK_j$ , i.e.

$$\text{NIWI}\{\alpha : \hat{e}(\alpha, g^{\mathbb{H}(\text{otvk})} PK_i) = \hat{e}(g, g) \vee \hat{e}(\alpha, g^{\mathbb{H}(\text{otvk})} PK_j) = \hat{e}(g, g)\}.$$

The NIZK proof system is used for showing that the commitment  $c = (c_1, c_2, c_3) = (F^{r_c} U^t, H^{s_c} V^t, g^{r_c+s_c} W^t \bar{\sigma})$  in the NIWI proof  $\pi_1$  and the ciphertext  $y = (y_1, y_2, y_3, y_4, y_5) = (F^{r_y}, H^{s_y}, g^{r_y+s_y} \bar{\sigma}, (g^{\text{tag}} K)^{r_y}, (g^{\text{tag}} L)^{s_y})$ , where  $\text{tag} = \mathbb{H}(\text{otvk})$ , contain the same  $\bar{\sigma}$ . This is equivalent to showing the knowledge of a solution to the equation  $(c_1^{-1} y_1) F^r U^t = 1 \wedge (c_2^{-1} y_2) H^s V^t = 1 \wedge (c_3^{-1} y_3) g^{r+s} W^t = 1$ . If  $c$  and  $y$  contain different messages, then there will be no  $r, s, t$  satisfying all the equations above. Groth and Sahai [18] showed a way to turn the set of equations above to a tractable set, which has zero-knowledge proofs. The set of equations above is equivalent to the following one, which is tractable:

$$\phi = 1 \wedge (c_1^{-1} y_1)^\phi F^r U^t = 1 \wedge (c_2^{-1} y_2)^\phi H^s V^t = 1 \wedge (c_3^{-1} y_3)^\phi g^{r+s} W^t = 1.$$

This NIZK proof system was also used by Groth in constructing a fully anonymous group signature scheme [17]. Readers may refer to [17,18] for more details.

### 5.3. High level description of our construction

As mentioned in the introduction, many OFE schemes in the literature follows a generic framework: Alice encrypts her signature under the arbitrator's public key, and provides a proof showing that the ciphertext contains her signature. To extend this framework to AOFE, we may let Alice encrypt her signature under the arbitrator's public key and provide a proof showing that the ciphertext contains either her signature or Bob's signature.

Our concrete construction below follows the framework, which is based on the idea of Groth in constructing a fully anonymous group signature [17]. In detail, Alice's signature consists of a weakly secure BB-signature [7] and a strong one-time signature. Since only the BB-signature is related to Alice's identity, we encrypt it under the arbitrator's public key using Kiltz' tag-based encryption scheme [28], with the one-time verification key as the tag. The non-interactive proof is based on a technique due to Groth and Sahai [18]. It is efficient and does not require any complex NP-reduction. The proof consists of two parts. The first part includes a commitment to Alice's BB-signature along with a non-interactive witness indistinguishable (NIWI) proof showing that either Alice's BB-signature or Bob's BB-signature on the one-time verification key is in the commitment. The second part is non-interactive zero-knowledge (NIZK) proof (of knowledge) showing that the commitment and the ciphertext contains the same thing. These two parts together imply that the ciphertext contains a BB-signature on the message generated by either Alice or Bob. Both the ciphertext and the proof are authenticated using the one-time signing key. Guaranteed by the strong unforgeability of the one-time signature, no efficient adversary can modify the ciphertext or the proof.

### 5.4. The concrete scheme and security analysis

Our proposed concrete scheme, AOFE, is shown in Fig. 2.

**Theorem 3.** *The proposed AOFE is secure in the multi-user setting and chosen-key model without random oracles, provided that DLIN assumption and SDH assumption hold.*

AOFE follows the framework of the generic construction proposed in Section 4. Intuitively, the resolution ambiguity is guaranteed by the extractability and soundness of the NIWI proof of knowledge system. The signer ambiguity and security against verifiers are due to the CCA security of the encryption scheme. Security against signers and security against the



- PMGen takes  $1^k$  and outputs  $\text{PM} = (1^k, p, \mathbb{G}_1, \mathbb{G}_T, \hat{e}, g)$  so that  $\mathbb{G}_1$  and  $\mathbb{G}_T$  are cyclic groups of prime order  $p$ ;  $g$  is a random generator of  $\mathbb{G}_1$ ;  $\hat{e}: \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_T$  is an admissible bilinear pairing; and group operations on  $\mathbb{G}_1$  and  $\mathbb{G}_T$  can be efficiently performed.
- Setup<sup>TTP</sup>: The arbitrator runs the key generation algorithm of the non-interactive proof system to generate a common reference string  $\text{crs}$  and an extraction key  $xk$ , i.e.  $(\text{crs}, xk) \leftarrow K_{NI}(1^k)$ , where  $\text{crs} = (F, H, U, V, W, U', V', W')$ . It also randomly selects  $K, L \leftarrow \mathbb{G}_1$ , and sets  $(\text{APK}, \text{ASK}) = ((\text{crs}, K, L), xk)$ , where  $F, H, K, L$  together form the public key of the tag-based encryption scheme [28], and  $xk$  is the extraction key of the NIWI proof system [17,18], which is also the decryption key of the tag-based encryption scheme.
- Setup<sup>User</sup>: Each user  $U_i$  randomly selects  $x_i \leftarrow \mathbb{Z}_p$ , and sets  $(PK_i, SK_i) = (g^{x_i}, x_i)$ .
- PSig: To partially sign a message  $m$  with verifier  $U_j$ , user  $U_i$  does the following.
  1. Call the key generation algorithm of  $\mathcal{S}$  to generate a one-time key pair  $(otvk, otsk)$ .
  2. Use  $SK_i$  to compute a BB-signature  $\bar{\sigma}$  on  $\mathbb{H}(otvk)$ , i.e.  $\bar{\sigma} \leftarrow g^{1/(x_i + \mathbb{H}(otvk))}$ .
  3. Compute a tag-based encryption [28]  $y$  of  $\bar{\sigma}$ , i.e.  $y = (y_1, y_2, y_3, y_4, y_5) \leftarrow \mathcal{E}_{pk}(\bar{\sigma}, \mathbb{H}(otvk))$ , where  $pk = (F, H, K, L)$ .
  4. Compute an NIWI proof  $\pi_1$  showing that  $\bar{\sigma}$  is a valid signature under either  $PK_i$  or  $PK_j$ , i.e.  $\pi_1 \leftarrow P_{WI}(\text{crs}, (\hat{e}(g, g), PK_i, PK_j, \mathbb{H}(otvk)), (\bar{\sigma}))$ , which shows that the following holds:
 
$$\hat{e}(\bar{\sigma}, PK_i \cdot g^{\mathbb{H}(otvk)}) = \hat{e}(g, g) \vee \hat{e}(\bar{\sigma}, PK_j \cdot g^{\mathbb{H}(otvk)}) = \hat{e}(g, g).$$

5. Compute an NIZK proof  $\pi_2$  showing that  $y$  and the commitment  $C$  to  $\bar{\sigma}$  in  $\pi_1$  contain the same  $\bar{\sigma}$ , i.e.  $\pi_2 \leftarrow P_{ZK}(\text{crs}, (y, \pi_1), (r, s, t))$ .
6. Use  $otsk$  to sign the whole transcript and the message  $M$ , i.e.  $\sigma_{ot} \leftarrow \mathcal{S}_{otsk}(M \| \pi_1 \| y \| \pi_2 \| PK_i \| PK_j)$ . The partial signature  $\sigma_p$  of  $U_i$  on message  $M$  then consists of  $(otvk, \sigma_{ot}, \pi_1, y, \pi_2)$ .
- PVer: After obtaining  $U_i$ 's partial signature  $\sigma_p = (otvk, \sigma_{ot}, \pi_1, y, \pi_2)$ , the verifier  $U_j$  checks the following. If any one fails,  $U_j$  rejects; otherwise, it accepts.
  1. If  $\sigma_{ot}$  is a valid one-time signature on  $M \| \pi_1 \| y \| \pi_2 \| PK_i \| PK_j$  under  $otvk$ .
  2. If  $\pi_1$  is a valid NIWI proof, i.e.  $V_{WI}(\text{crs}, (\hat{e}(g, g), PK_i, PK_j, \mathbb{H}(otvk)), \pi_1) \stackrel{?}{=} 1$ .
  3. If  $\pi_2$  is a valid NIZK proof, i.e.  $V_{ZK}(\text{crs}, (y, \pi_1), \pi_2) \stackrel{?}{=} 1$ .
- Sig: To sign a message  $M$  with verifier  $U_j$ , user  $U_i$  generates a partial signature  $\sigma_p$  as in PSig, and set the full signature  $\sigma_F$  as  $\sigma_F = (\sigma_p, \bar{\sigma})$ .
- Ver: After receiving  $\sigma_F$  on  $M$  from  $U_i$ , user  $U_j$  checks if  $PVer(M, \sigma_p, \{PK_i, PK_j\}, \text{APK}) \stackrel{?}{=} 1$ , and if  $\hat{e}(\bar{\sigma}, PK_i \cdot g^{\mathbb{H}(otvk)}) \stackrel{?}{=} \hat{e}(g, g)$ . If any of the checks fails,  $U_j$  rejects; otherwise, it accepts.
- Res: After receiving  $U_i$ 's partial signature  $\sigma_p$  on message  $M$  from user  $U_j$ , the arbitrator firstly checks the validity of  $\sigma_p$ . If invalid, it returns  $\perp$  to  $U_j$ . Otherwise, it extracts  $\bar{\sigma}$  from  $\pi_1$  by calling  $\bar{\sigma} \leftarrow X_{xk}(\text{crs}, \pi_1)$ . The arbitrator returns  $\bar{\sigma}$  to  $U_j$ .

Fig. 2. Our proposed concrete scheme of AOFE, AOFE.

arbitrator are guaranteed by the weak unforgeability of BB-signature scheme. Proof of the security of the scheme almost follows that of our generic construction of AOFE.

One may notice that an NIWI proof  $\pi_1$  and an NIZK proof  $\pi_2$  are used in the generation of partial signatures in AOFE, while only an NIZK proof is required in the generic construction GAOFE. This is not to say the instantiation deviates from the generic construction. In fact, proofs  $\pi_1$  and  $\pi_2$  functionally serve as the NIZK proof in GAOFE. To see it, notice that the simulator of the NIZK proof (in GAOFE) is mainly used in the proof of signer ambiguity. A simulated partial signature is generated by first using the intended verifier  $U_j$ 's secret key to generate a (conventional) signature and then running the NIZK simulator to produce a simulated proof to show that the ciphertext contains either the signer  $U_i$ 's signature or  $U_j$ 's signature. While in the instantiation, the NIWI proof  $\pi_1$  is simulated by using  $U_j$ 's signature to show that there is a signature of either  $U_i$  or  $U_j$ , and the NIZK proof  $\pi_2$  is simulated by calling the corresponding simulator to show that the (tag-based) ciphertext contains a signature same as the witness used in  $\pi_1$ .

## 6. Conclusion

In this paper, we proposed the notion of ambiguous optimistic fair exchange (AOFE), and gave a formal security model for it. We discussed the relationship among some variants of the model, and showed that signer ambiguity and security against the arbitrator together imply security against verifiers (in a weaker sense). We revisited two generic constructions of OFE, and showed that they cannot be simply extended to AOFE. We then proposed a generic construction of AOFE, and proved its security under the proposed multi-user setting and chosen-key model. We also proposed a concrete and efficient construction of AOFE in bilinear groups, security of which is based on Decision Linear assumption and Strong Diffie–Hellman assumption without random oracles.

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