11-2023

Tail risk hedging: The search for cheap options

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Citation

NEO, Poh Ling and TEE, Chyng Wen. Tail risk hedging: The search for cheap options. (2023). *Journal of Portfolio Management*. 50, (1), 106-119.

Available at: https://ink.library.smu.edu.sg/lkcsb_research/7326

Tail Risk Hedging: The search for cheap options

Poh Ling Neo Chyng Wen Tee

March 4, 2023

Abstract

The authors discover that a simple heuristic of sorting liquid equity options by dollar price to construct a portfolio of cheap put options leads to a surprisingly robust tail risk hedge – the superior performance holds even when compared against advanced empirical option strategies. Further investigation reveals the asymmetry in market correlation under different market conditions as the mechanism of this robust hedging performance. The put options selected by the heuristic comprises of stocks with diverse firm characteristics. The correlation spike accompanying tail risk events leads to most of these options moving into-the-money (ITM), compensating the losses incurred on a broad-base equity index holding. During normal market conditions, these options benefited from the diversification effect due to a lower market correlation, thus mitigating the portfolio drag effect.

Keywords: *tail risk; portfolio insurance; risk management; portfolio management; hedging; option markets; index options; volatility risk premium*

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Tail risk events are undesirable and difficult to predict – they also inflict significant drawdown to most portfolios. The 1-day move of −12.5% in the S&P 500 index (SPX) on March 16, 2020, due to the COVID-19 pandemic market shock has once again emphasized the importance of tail risk management. Although this 1-day crash recovered swiftly due to the drastic actions taken by the US central bank, fund managers still need to address investors' concerns regarding the pre-emptive action they take to mitigate the impacts of tail risk events to their portfolios.

The heightened volatility in the market in recent years has made tail risk research increasingly vital. Van Oord and Zhou (2016) demonstrate that historical tail betas help predict the future performance of stocks in extreme market downturns. To forecast left-tail risk, Xiong, Idzorek, and Ibbotson (2016) propose looking at forward skewness, which can help the popular low-volatility strategy to reduce tail risk without lowering Sharpe ratio. Schmielewski and Stoyanov (2017) show that low-risk strategies constructed from a large universe of international stocks have better risk performance statistics while exhibiting a less-pronounced sector concentration. Gava, Guevara, and Turc (2021) use extreme risk theory to find the right allocation for defensive systematic strategies, thereby successfully reduce negative asymmetry and sharp losses.

Most academic literature on tail risk focuses on the mathematical exposition of extreme value theory (EVT), with a view of forecasting left-tail risk by modeling the conditional value at risk (CVaR). Bhansali (2008) is among the first to adopt a practitioner's point of view by formulating a comprehensive tail risk management framework, along with its practical implementation in a realistic setting. Bhansali and Davis (2010) subsequently explore the use of option markets to hedge the tail risk of a broad-based equity index, for instance the SPX index. The option monetization strategy of their actively managed hedge portfolio is further refined in Bhansali *et al.* (2020) and demonstrated to perform well in recent market downturn. Bhansali and Holdom (2021) also recommend a portfolio approach through a diversified tail risk hedging strategies by holding gold, bond futures or index options, which is shown to be more robust when combined with a portfolio that has large equity exposure.

The nonlinear option payoff profile is ideally suited for hedging tail risk. The objective of this work is to investigate the feasibility of exploiting the asymmetry in market correlation under different market conditions to construct economical option portfolio for tail risk hedging. We discover that a simple heuristic which sort liquid equity put options by their dollar value to construct a portfolio of cheap options leads to a surprisingly robust hedging performance for tail risk. The superior performance holds even when we compare our simple price-based heuristic against a suite of advanced empirical option strategies based on the volatility risk premium literature. On average, there are 90 options in our hedge portfolio, of which only approximately 40 belongs to the SPX index constituent stocks. Our analyses reveal that the superior performance stems from the empirical behavior of market level correlation – correlation is generally lower during good market conditions when stock returns are more diversified, but spikes up during bad market conditions when stock movements are concentrated. This asymmetric behavior in market correlation has been documented by Longin and Solnik (2001) and Chua, Kritzman, and Page (2009). We leverage on this empirical characteristic to effectively hedge the tail risk exposure of a broad-base SPX index using a portfolio of cheap equity put options in a cost-efficient manner.

Instead of utilizing expensive SPX index options, the portfolio of cheap options constructed by our price-based heuristic provides matching hedging return during tail risk events due to the spike in market level correlation. During normal market conditions, the SPX index options exert significant portfolio drag due to their significant volatility risk premium. On the other hand, the portfolio of cheap options, in addition to their price economy, also benefited from the effect of diversification due to lower market level correlation, leading some of these options to move ITM, further mitigating the detrimental effect portfolio drag.

Tail Risk Hedging Strategies

All tail risk hedging strategies must be implemented economically well before extreme events occur. A natural choice to hedge a broad-based SPX index holding against tail risk is to acquire portfolio insurance through the purchase of the SPX index put options. However, Carr and Wu (2009) have demonstrated the existence of large volatility risk premium for both the SPX and Dow Jones indexes, inflating the option prices due to the implied volatilities being higher than the future realized volatility. This can be attributed to the implications of risk aversion, exposure to tail events, and fatter left tails of the physical index distribution in markets where volatility is traded (see Bakshi and Madan (2006)). For instance, a recent empirical study by Fallon and Park (2016) shows that the 32-year return of a strategy that sells 1-month SPX variance swaps exhibits a decent Sharpe ratio above 1.2, but experiences severe yet infrequent crashes. Welch (2016) points out that this premium can also be accounting for other sudden disasters in addition to crash risks.

In this paper, we explore the feasibility of exploiting the asymmetry in market level correlation to reduce portfolio drag when hedging tail risk. Instead of acquiring expensive index put option, we use a simple price-based heuristic to construct a portfolio of cheap equity put options instead. Bhansali (2008) has highlighted that tail risk events are systemic in nature and tends to affect all asset classes at the same time, causing a spike in the overall inter- and intra-market correlation, consequently reducing the basis risk when buying insurance on such events. This implies that even if the underlying stocks of our option portfolio is not an exact replica of the SPX index constituents, the spike in market level correlation during crashes will enhance its hedging return to match the protection offered by SPX index options.

Exhibit 1: Cumulative wealth of the SPX index with and without tail risk hedge.

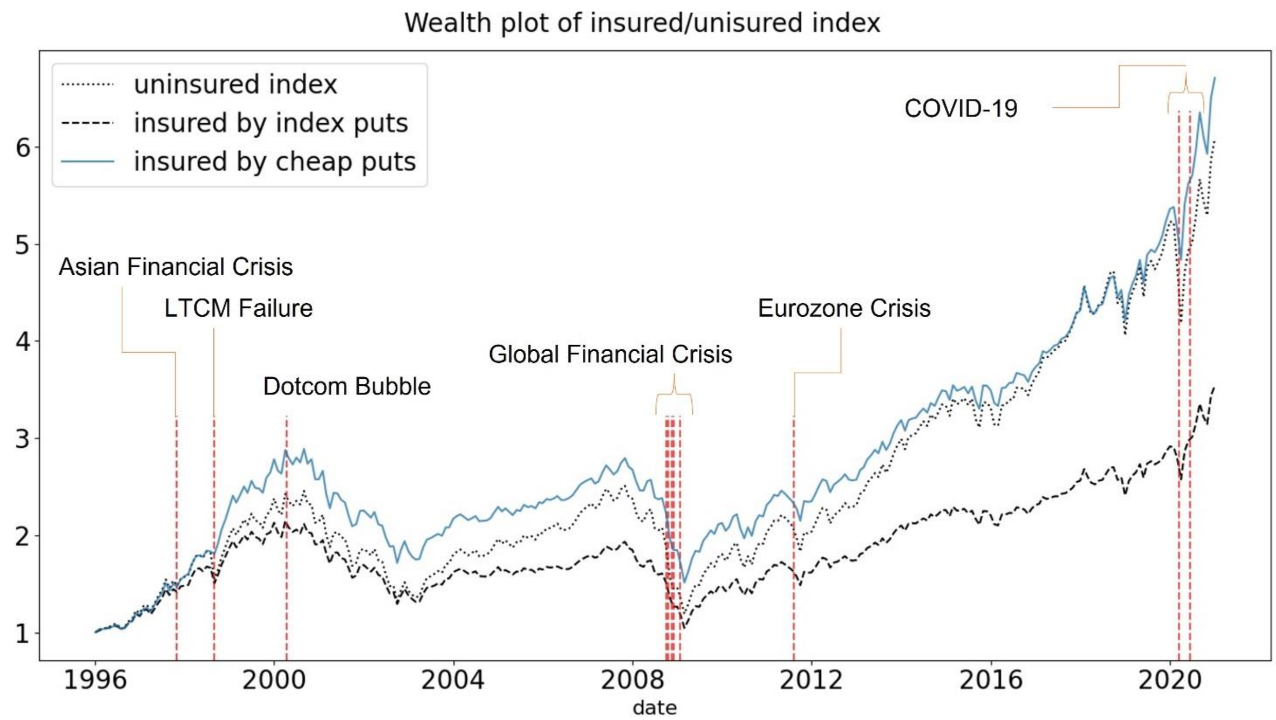


Exhibit 1 presents a graphical summary of the key discovery of this paper. It is well known that hedging tail risk using index put options exerts a drag on portfolio return, exemplified by the wide gap between the unhedged SPX portfolio (dotted line) and the hedged portfolio using index options (dashed line). On the other hand, our price-based heuristic that selects cheap equity options (solid line) successfully exploit the asymmetry in market level correlation under different market conditions to hedge tail risk. The hedged portfolio using either index or cheap options both exhibit reduction in volatility and maximum drawdown (MDD)[1]. However, for our study period of 25 calendar years from 1996 to 2020, if a 2% risk budget is allocated monthly to hedge tail risk by purchasing out-of-the-money (OTM) put options with −10% delta, the difference in annualized return (Sharpe ratio) of the hedged portfolio using our price-based heuristic compared to the benchmark SPX index is +0.4% (+0.131), in stark contrast to the −2.17% (−0.58) obtained using expensive index put options.

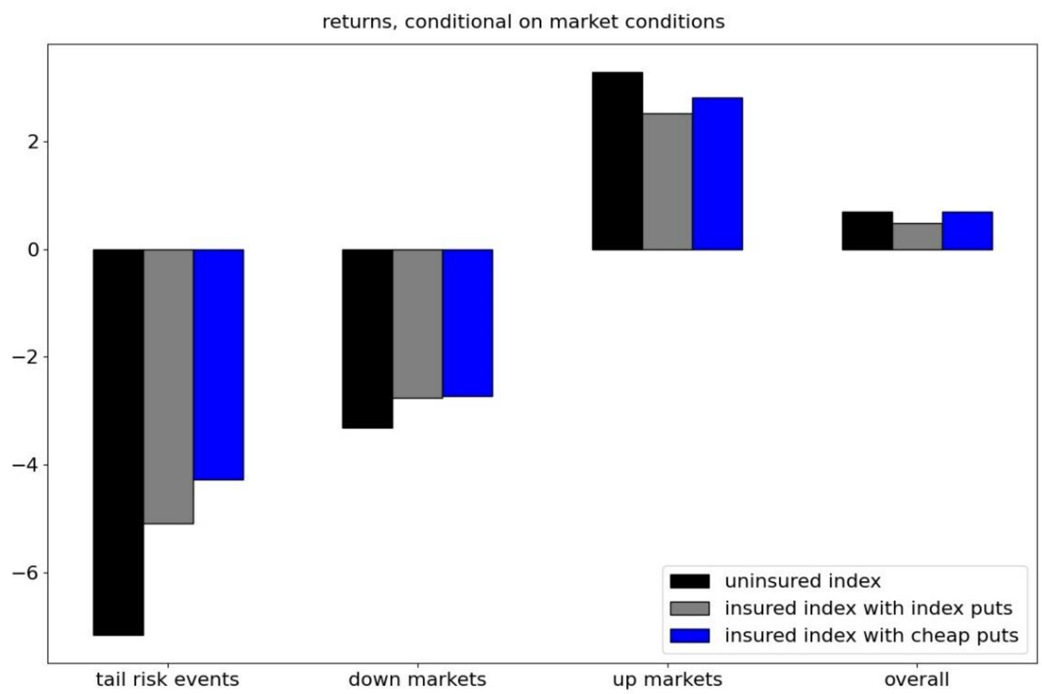
We quantify performance on a monthly basis and adopt the following terminology in this paper: a "*tail risk event*" refers to a month containing at least one day with a daily return lower than −5%. A "*down market*" refers to a month where the overall monthly return is negative, with daily losses not exceeding -5%. An "*up market*" refers to a month with positive monthly return, with daily losses not exceeding -5%. Exhibit 2 illustrates the SPX index and the hedged portfolio returns under different market conditions. The exhibit shows that that the drawdowns experienced by the hedged portfolio during both "tail risk events" and "down markets" are noticeably smaller. During "up markets", although there is also a reduction in the gains experienced by the hedged portfolio due to portfolio drag, hedging tail risk using our price-based heuristic clearly provides a superior hedging performance under all market conditions.

Indeed, our analysis shows that the overall performance of the option portfolio constructed by our price-based heuristic exhibits a portfolio drag of −0.40%, the negative sign here implies that the hedged portfolio outperformed the unhedged benchmark SPX index, despite the risk budget expenditure. This is because during up markets, the portfolio of cheap options has an average raw return of −20.85%, compared to the index option portfolio return of −34.90%, a marked reduction

[1] See Exhibit 4 for full details of the performance metric comparisons reported here.

in portfolio drag. During down markets, the average raw return of the cheap options (index options) is +50.24% (+24.78%), thereby giving significantly better downside protection for the same risk budget. Crucially, when tail risk events occur, the average monthly raw returns provided by the cheap options (index options) is +137.03% (+96.23%). The combined benefits of reduced portfolio drag during good market conditions and enhanced downside risk protection during bad market conditions using our price-based heuristic resulted in an overall improvement in Shape ratio, Sortino ratio, and Calmar ratio, over the SPX index and the hedged portfolio using index options.

Exhibit 2: Monthly returns of unhedged (raw SPX) and hedged (SPX + options) portfolios, using different insurance portfolios in the period 1996 – 2020.



Note that the portfolio drag experienced when hedging tail risk using index options is significant due to the scarcity of major market crashes and the inevitable theta decay of option time value during normal market conditions. In our analysis period, there were only 11 tail risk instances when the market experienced a daily movement lower than −5%. Moreover, the index option markets embody critical information about the market risk premia and its dynamics which is essentially unidentifiable from the stock market data alone (see Andersen, Fusari, and Todorov (2015)). Bollerslev, Todorov, and Xu (2015) also suggest that this variance risk premia is associated with the special compensation demanded by investors for bearing tail risk. These insights are consistent with our observation that using a portfolio of cheap equity put options to hedge tail risk results in better overall returns due to a reduction in portfolio drag during up markets.

When market condition is bad, we observe that the portfolio of cheap options selected using our price-based heuristic can provide an effective hedge during tail risk events, even though it is not an exact replica of the SPX index constituents. As mentioned earlier, this superior performance is due to the empirical characteristic of market correlation. During up markets, the average correlation of the underlying stocks of our option portfolio and the SPX index is 40.95% which resulted in an average of 18.93% of the options to move into-the-money (ITM). The effect of diversification stemming from a lower correlation which reduces portfolio drag as some of the options expires ITM, while index options predominantly expire OTM. During down markets, the average correlation of the stocks and index increases to 48.58%, which resulted in an average of 50.24% of the options to move ITM. Crucially, during tail risk events, the average correlation spikes to 64.95%, with 65.36% of the options moving ITM, thus providing the necessary hedging return to compensate for the losses experienced by SPX index. In other words, the heightened correlation causes the portfolio of cheap options to provide in the same downside risk protection as index options. Our strategy is therefore able to take advantage of the correlation asymmetry observed under different market conditions to generate robust hedging performance.

The rest of the paper is organized as follows. In the next section, we discuss the data and methodology used to select the put option universe, followed by a discussion of the main results.

For robustness studies, we examine the performance of different hedging portfolios constructed using advanced empirical volatility strategies. We then proceed to investigate the impact of trading frictions due to transaction costs. We also explore the benefit of an active option monetization strategy introduced by Bhansali *et al*. (2020), before moving on to our conclusions.

Data and Methodology

Our research requires data of both equity and index options, along with the stock market's daily return. The option data is obtained from the IvyDB database provided by OptionMetrics, and the stock daily return is obtained from the CRSP database. Our study period covers 25 calendar years from January-1996 to December-2020, an extensive period encompassing numerous notable tail risk events.

We use the following procedure to construct an equity put option universe for our hedging strategy. On each month end, we select an out-of-the-money (OTM) put option for each optionable stock. The put is chosen to have a delta closest to -10%, and an expiry between 6 months to 1 year. The choice of delta is to maintain a balance between downside risk protection and risk budget[2], while the option expiry is selected to minimize the number of transactions required, subject to liquidity constraint, due to the need to roll the hedging positions as the option expires. We exclude stocks that have ex-dividend dates prior to option expiry and apply the data filters described by Goyal and Saretto (2009) to minimize the impact of data recording errors in OptionMetrics[3]. We also

[2] As it is not always possible to select puts with a delta of -10%, we include puts whose delta is between −15% and −5%. If there are multiple puts that satisfy our selection criteria, we will select the put whose delta is closest to −10%.
[3] We remove all instances where the ask price is lower than the bid price or the quoted spread is lower than the minimum tick size.

verify that the selected options exhibit non-zero open interests to ensure sufficient liquidity during actual implementation.

For practical consideration, we further impose additional liquidity constraints. From the OTM put universe, we only select options with bid-ask spread less than 50% of the mid-price. Applying this trading constraint resulted in an average of 3,583 optionable stocks and 453 OTM puts every month. The average delta and time-to-expiry of the OTM put options are −10.62% and 0.57 years, respectively[4].

The market capitalization of the sample of stocks that are included in our OTM put option universe is $21.20 billion, which is larger than the market cap of an average optionable stock of $4.17 billion. This is not surprising, as small cap stocks are not expected to have sufficiently liquid options to meet our data selection criteria. It is also desirable to construct hedge portfolio with options on large and liquid stocks to ensure the robustness of our results.

We use SPX 500 index put options with similar delta and expiry as a benchmark to evaluate the performance of our hedge portfolios. Exhibit 3 tabulates the descriptive statistics of the put options. Note that the index put options have an average delta and time to expiry of −10% and 0.68 years respectively, matching closely our equity put options universe.

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[4] In the entire analysis period, without our additional data filter, the OTM put universe consists of a total of 5,576 unique stocks and 196,032 OTM puts.

Exhibit 3: Comparison of the OTM equity and index put option universe.

	Liquid OTM puts	Index puts
Mean market cap of optionable stocks ($ ‘M)	4169.93	-
Mean market cap of selected puts ($ ‘M)	21200.96	-
Mean number of optionable stocks	3583	-
Mean number of available puts	453	-
Mean Delta (%)	-10.93	-9.97
Mean Expiry (Years)	0.57	0.68
Mean IV (%)	48.33	27.50
Mean HV (%)	43.90	15.30
Mean IV-HV (%)	4.43	12.2
Mean price ($)	1.10	17.91

We also report the stocks’ average historical volatility (HV) and the options’ implied volatility (IV) from our OTM put universe in Exhibit 3. The historical volatility of a stock is calculated as the standard deviation of daily stock returns for the most recent 12 months, while the implied volatility is based on the quoted option price reported in OptionMetrics. The relationship between implied and realized volatilities has been studied extensively in the literature (see, for instance, Christensen and Prabhala (1998)). Note that the average IV is higher than the HV for both the index option and our OTM put option universe. Moreover, the IV and HV of our equity options are consistently higher than the index option. Given that our OTM put option universe generally comprises large market cap stocks, this is consistent with Bakshi and Kapadia (2003)’s observation that the IV of large market cap stocks are greater than its HV, while the volatility risk premia observed in individual stock options are lower than index options.

Although equity put options are generally more volatile than index put options, we find that IV – HV is greater for index put options. This observation is in line with Bollen and Whaley (2004)[5],

[5] They have observed that the largest 20 stocks exhibit this IV & HV profiles over 1996-2000. We have shown that the trend continues to 2020 and across all large market cap stocks.

as the high IV – HV is caused by a strong buying demand for portfolio insurance. The elevated IV – HV will also lead to a significant portfolio drag if index put options are used to hedge tail risk. This assertion is supported by empirical evidence in the academic literature – Baltussen *et al.* (2018) have shown that stocks with high uncertainty about risk, as measured by the volatility of expected volatility, robustly underperform stocks with low uncertainty about risk by 8% per year. On the other hand, Chabi-Yo, Ruenzi, and Weigert (2018) examines whether investors receive compensation for holding crash-sensitive stocks and found that stocks with strong lower-tail dependence have higher average future returns than stocks with weak lower-tail dependence, consistent with the notion that investors are crash-averse, and will pay a premium for portfolio insurance.

Main Results and Analyses

All tail risk hedges are constrained by a limited risk budget. Bhansali and Davis (2010) have emphasized the considerations to implement tail risk hedges economically. Following our price-based heuristic, we sort the OTM put options in our universe according to their dollar price. On each month end, we allocate a risk budget of 2% to acquire the cheapest 20% of the put options in our universe with equal dollar-weighting – this corresponds to 90 options each period on average, of which approximately 40 options' underlying are SPX index constituent stocks.

In addition to our price-based heuristic, we also construct tail risk hedging option portfolios using other advanced empirical strategies based on option volatilities to provide further basis for performance comparison. Option market frequently employs implied volatility (IV) and historical volatility (HV) as measures of option prices, and there exist an extended stream of academic literature on this topic. We construct option portfolios from our option universe by sorting them according to (1) IV, (2) HV, and (3) IV − HV, and then allocate the same 2% risk budget to acquire the lowest 20% of the options in each case.

Exhibit 4 documents the performance of the hedged portfolios aggregated over the entire analysis period. We also provide the performance of the raw SPX for direct comparison. The monthly return at time t is calculated as

$$r(t) = (1 - b)r_{idx}(t) + br_{pf}(t) \quad \text{Eq (1)}$$

where $b=2\%$ is the risk budget, while $r_{idx}(t)$ and $r_{pf}(t)$ are the returns of the SPX index and hedge portfolio at time t, respectively.

Exhibit 4: Performance of SPX index along with different hedging portfolios from 1996 – 2020. Bracketed numbers are *t*-stats (*vs.* SPX + index options) and *F*-stats (*vs.* raw SPX) for return and volatility, respectively, with *** denoting statistical significance at the 0.01% level.

	Return (%)	Volatility (%)	Sharpe Ratio	Sortino Ratio	CVaR (%)	MDD (%)	Calmar Ratio
SPX	7.24	15.29	0.474	0.904	11.10	52.56	0.138
SPX + index puts	5.07	12.17 (1.58***)	0.416	0.831	8.56	51.31	0.098
SPX + cheap puts	7.64 (5.42***)	12.62 (1.47***)	0.605	1.196	8.82	47.63	0.158
SPX + low HV puts	6.26 (3.29***)	12.55 (1.48***)	0.498	0.992	8.99	52.37	0.12
SPX + low IV puts	7.01 (4.98***)	12.54 (1.49***)	0.559	1.107	8.97	49.30	0.142
SPX + low IV−HV puts	7.98 (7.51***)	12.63 (1.47***)	0.632	1.246	8.91	45.12	0.176

It is evident from Exhibit 4 that the hedging performance of our price-based heuristic (SPX + cheap puts) has superior risk-return profile when compared to the raw index (SPX) and the hedged portfolio using index options (SPX + index puts). A main driver of this improvement is the smaller portfolio drag – the hedged portfolio using cheap options exhibits a return of 7.64%, which is higher than the raw SPX index return of 7.24% despite the risk budget expenditure. This performance is in stark contrast to the mere 5.07% return from the hedged portfolio using index options[6].

In Exhibit 4, our results indicate that the hedged portfolios of SPX + low HV puts and SPX + low IV puts can also improve the risk-return profile, with the former having a better overall performance. This agrees with Cao and Han (2013)'s observation that the correlation of IV with option returns is higher, when compared to HV with option returns. Sorting options by IV – HV is insightful, as it provides information on the variance risk premia investors are paying for options. Goyal and Saretto (2009) have demonstrated that both HV and IV provide valuable information about future volatility. In their cross-sectional analysis of option returns, they show that low IV-HV options provide good returns due to investors' overreactions to current events. Indeed, we find that the tail risk hedging performance of low IV − HV options is comparable to our simple price-based heuristic.

We also provide *t*- and *F*-statistics in Exhibit 4 to assess statistical significance in our results. For returns, we use the hedged portfolio using index put options as the base case (SPX + index puts), and compute *t*-statistics of the difference in returns of the hedge portfolio using other option strategies compared to the base case. Note that the tail risk hedge portfolios created using our

[6] Looking solely at the option portfolio, the average annualized return of index put options across the entire analysis period is negative, while the cheap put options generate a positive return. This translates to the negative portfolio drag of −0.4% reported in the preceding discussion.

price-based heuristic as well as other empirical strategies based on volatility all outperform the base case with economic and statistical significance at the 0.01% level.

For volatility, we use the raw SPX index as the base case, and compute *F*-statistics of all hedge portfolios for comparison. We observe a volatility reduction from approximately 15% to 12% across all hedge portfolios. The improvement in volatility performance is statistically significant at the 0.01% level. The combination of the *t*-tests and *F*-tests in Exhibit 4 shed light on the performance and cost efficiency of all tail risk hedging portfolios studied in this paper. Notably, our simple price-based heuristic as well as the other volatility-based option strategies can all provide matching the protection offered by index options during tail risk events, but they achieve this without incurring the same portfolio drag during normal market condition.

Both Xiong and Idzorek (2018) and Chang, Holdom, and Bhansali (2022) have pointed out the importance of using a comprehensive measure to assess tail risk hedging performance. In Exhibit 4 we also report conditional value at risk (CVaR), which is a risk measure that quantifies the improvement in the return distribution's left tail through tail risk hedging. Hedged portfolio using cheap (index) put options successfully reduces the CVaR from 11.10% down to 8.82% (8.56%). Another important metric is the maximum drawdown (MDD), a path-dependent risk measure. As shown in Exhibit 4, the portfolio of cheap (index) put options reduces the MDD from 52.56% to 47.63% (51.31%), leading to an improvement of the Calmar ratio from 0.098 to 0.158.

We compare the return and volatility of the tail risk hedging strategies using equity options in Exhibit 5. Note that the hedging performance of our price-based heuristic performs statistically better than the performance of low HV and low IV options. The *t*-statistic of the hedged portfolio return differences between cheap options and HV (IV) options is 4.46 (2.03), which is significant at the 0.01% (5%) level. We also find that a hedging portfolio using low IV – HV options provides the highest return of 7.98% (see Exhibit 4), although the difference is not statistically significant when compared to our simple price-based heuristics (Exhibit 5).

Exhibit 5: Statistical tests for the hedged portfolios' return and volatility differences using different option strategies. The figures reported are the *t*- and *F*- test statistics, where * and *** denote statistical significance at the 5% and 0.01% level, respectively.

	Return Differences			
	Cheap puts	HV puts	IV puts	IV-HV puts
Cheap puts		4.46***	2.03*	-0.97
HV puts			5.40***	4.32***
IV puts				2.49*
IV-HV				

	Volatility Differences			
	Cheap puts	HV puts	IV puts	IV-HV
Cheap puts		1.01	1.01	1.00
HV puts			1.00	1.00
IV puts				1.01
IV-HV				

Having established the improvement in the overall risk-return performance in the hedge portfolios using different option strategies, Exhibit 6 furnishes further empirical evidence that these hedge portfolio provides sufficient protection during critical periods by tabulating the strategies's return under different market conditions. The three panels compare performance of the hedge portfolios under task risk events (top panel), down markets (middle panel) and up markets (bottom panel). In each panel, we show the return of the raw index and the hedging option portfolio separately, followed by the overall hedged portfolio *r(t)* with a risk budget of 2%. The elevated market level correlation accompanying tail risk events and down markets resulted in significantly higher return in the cheap put option portfolio of +137.03% and +50.24%, respectively, outperforming not just the index options, but also all the other empirical option strategies. During up markets, the lower market correlation leads to a lower portfolio drag of −20.85%, which is second only to the low IV−HV strategy at −18.78%.

Exhibit 6: Comparison of option insurance portfolios' return under different market conditions.

Market Conditions	Tail Risk Events				
Occurrences (Months)	11				
Raw index return (%)	-7.16				
Option portfolios	Index	Cheap	HV	IV	IV-HV
Raw option return (%)	96.23	137.03	120.84	114.47	108.78
Overall *r(t)*, *b* = 2%	-5.1	-4.28	-4.6	-4.73	-4.84
% ITM	72.72	65.36	67.70	65.99	68.71
Correlation (%)		64.95	70.63	69.36	67.15

Market Conditions	Down				
Occurrences (Months)	100				
Raw index return (%)	-3.31				
Option portfolios	Index	Cheap	HV	IV	IV-HV
Raw option return (%)	24.78	50.24	21.84	27.10	29.66
Overall *r(t)*, *b* = 2%	-2.75	-2.24	-2.81	-2.7	-2.65
% ITM	65.00	48.58	49.51	50.77	54.22
Correlation (%)		44.62	56.43	55.34	50.06

Market Conditions	Up				
Occurrences (Months)	189				
Raw index return (%)	3.28				
Option portfolios	Index	Cheap	HV	IV	IV-HV
Raw option return (%)	-34.92	-20.85	-26.90	-24.33	-18.78
Overall *r(t)*, *b* = 2%	2.52	2.8	2.68	2.73	2.84
% ITM	3.17	18.93	15.01	17.01	21.57
Correlation (%)		40.95	47.29	46.46	41.11

Although only approximately 40 out of the 90 options in our cheap option portfolio are constituents of the SPX index, our results have clearly demonstrated that they provide a hedging return higher than index put options during tail risk events by riding on the market correlation spike. In our analyses, we observe that the average stock-index *ex post* correlation increases from 40.95% during up markets to 64.95% during tail risk events[7]. Because of this, 65% of the cheap put options move into-the-money when tail risk events occur.

We also observe that during up markets, diversification leads to 18.93% of the cheap options to expire in ITM, which ameliorates the portfolio drag, allowing the portfolio of cheap options to exhibit a raw return of −20.85% during up markets, as compared to the −34.92% for the index put options. Given that 189 months out of 300 months in the entire analysis period constitute up market condition, the smaller losses incurred by the portfolio of cheap put options during these periods significantly reduce the effect of portfolio drag. The combined effects of improved portfolio drag during up markets, and superior downside protection during down markets and tail risk events in the entire analysis period result in an improvement of the overall portfolio metrics stated in Exhibit 4.

Transaction Cost Analysis

Our analyses thus far have clearly demonstrated that the different hedging strategies discussed all offer adequate tail risk protection, but our price-based heuristic stands out in terms of its simplicity and cost efficiency. Since option prices are market observables, practical implementation of our strategy is straightforward without having to estimate volatility parameters empirically.

In this section, we perform transaction cost analysis on the tail risk hedging strategies explored in this paper. Although the strategies discussed improve the overall risk-return profile when

[7] The correlation between a stock and the SPX index in a month is calculated using the daily stock return and the daily SPX index returns. The average stock-index correlation is calculated by averaging the pairwise correlations for all the underlying stocks in the option portfolio.

compared to index put options, it is important to assess whether the outperformance persists after accounting for the effect of transaction costs. Empirical research carried out by Goyal and Saretto (2009) and Cao and Han (2013) have shown that transaction costs exert a significant impact on the profitability of their option trading strategies. Exhibit 7 presents the impact of transaction costs on the performance of the option tail-risk hedging strategies. Our preceding analyses have assumed the execution of the option transactions at their mid-prices, i.e., by assuming that the effective spread (ESPR)[8] between the buy and sell prices is zero. Here we repeat our analysis by varying ESPR from 0% to 50% of the quoted spread (QSPR)[9] – a larger ESPR/QSPR ratio implies greater transaction costs.

Exhibit 7: Hedged portfolios' return with transaction costs.

	ESPR/QSPR					
	0%	10%	20%	30%	40%	50%
Index puts	5.07	4.91	4.76	4.60	4.45	4.30
Cheap puts	7.64	6.75	5.90	5.07	4.28	3.51
HV put	6.26	5.66	5.07	4.51	3.96	3.43
IV puts	7.01	6.38	5.78	5.19	4.62	4.07
IV-HV puts	7.98	7.33	6.71	6.10	5.51	4.93

Consistent with the existing literature on option strategies with TCA, our results show that increasing transaction costs will gradually erode the returns of tail risk hedging strategies. However, when compared to using index put options, the outperformance using our price-based heuristic continues to hold for ESPR/QSPR ratio up till 30%. This is remarkable, given that SPX

[8] Effective spread is defined as the difference between the traded price and the quoted mid-price.
[9] Quoted spread is defined as the difference between the quoted best bid and best ask prices.

index options generally have tighter bid/offer spread compared to equity options. Our results provide a strong support on the feasibility of our price-based option portfolio for tail risk hedging.

Option Monetization through Active Management

The bulk of our analysis in this paper is based on aggregated monthly performance of the tail risk hedging portfolio. However, if we track the portfolios' daily performance, we observe that there are multiple occasions when the option portfolios spike in value on days when market experienced severe crash, only for their value to fall towards month end when market partially recovered.

The implication of this observation will affect managers who carry out active tail risk management to monetize their option portfolio, as suggested by Bhansali *et al.* (2020). Monetization refers to the decision to fully or partially sell (i.e., monetize) a hedging position that has appreciated in value. There are two parameters involved in this decision process, namely: 1) the threshold amount of option price appreciation before monetization is triggered, and 2) how much position to monetize. Following Bhansali *et al.* (2020), we use 5 × 50%, signifying that when the options' value increased by a factor of 5, we will monetize 50% of the option positions by liquidating the holding, and redeploy the proceed into the underlying SPX index. To calculate the impacts of monetarization, we replace r_{pf} in Equation (1) by r_{pf}^{m} to include the effects of monetarization. Equation (2) provides the definition of return from the hedge portfolio with monetarization:

$$r_{pf}^{m} = m\left(r_{pf}^{-} + r_{idx}^{+} + r_{pf}^{-} r_{idx}^{+}\right) + (1 - m) r_{pf} \quad \text{Eq (2)}$$

where m is the proportion of insurance portfolio that was liquidated upon meeting the monetarization target. The return of the insurance portfolio is determined by r_{pf}^{-}, the return of the insurance portfolio during monetarization, and r_{idx}^{+}, the return of the index, post monetization.

In our analysis, we observe an average additional return of 66.29% for the portfolio of cheap options and 170.94% for the index options[10]. For our analysis period, we note both the index options and the cheap options monetarized at the same time period. This therefore suggests that during tail risk events, the portfolio of cheap options behave similarly to the index options, thereby removing the basis risk between the two. Considering the effects of monetarization, the overall return of the hedged portfolio reported in Exhibit 4, using cheap (index) options increases from 7.64% (5.07%) to 7.75% (5.36%). This confirms Bhansali *et al.* (2020)'s observation that suitable use of monetarization can improve the efficacy of tail risk insurance through incremental return.

Conclusions

Efficient tail risk hedging has important implication on a portfolio's ability to weather through extreme market conditions. Given the challenge in predicting tail risk events, it is common for portfolio managers to adjust their portfolio holding to account for this contingency. This is achieved either through a diversified allocation or the acquisition of insurance securities – the latter requires the allocation of a risk budget, which is prone to exert a performance drag on the portfolio.

The objective of this work is to explore the possibility to economize risk budget expenditure through the selection of less conventional insurance securities to hedge tail risk. We use the SPX

[10] For instance, we noted that the index put monetarized on 18-Mar-2020, during the COVID-19 pandemic market crash. With and without monetarization, the index put provide a raw return of 412.24% and 217.25% respectively.

index as the benchmark portfolio. Instead of using the SPX index option or its full constituent equity options as the insurance securities, we instead construct an option universe comprising large- and mid-cap stocks and sort them by their dollar price. We further impose liquidity constraints on open interest and bid/offer spread in our methodology. We show that a 2% monthly risk budget expenditure on the cheapest 20% of the options in our universe exhibit a robust tail risk hedging performance, and is superior to the hedging performance of the SPX index option. This performance is robust even when compared to other advance empirical option portfolio construction strategies based on implied and historical volatilities.

Our analysis reveals that the equity options selected by our price-based heuristic have diverse firm characteristics, with less than half of them being SPX constituent stocks. During normal market conditions, their diversified return behavior leads to almost 20% of the OTM put options moving into-the-money, thus compensating for the risk expenditure and mitigating portfolio drag. When market is under distress, the heightened correlation in the market results in more than 65% of these OTM put options moving deep into-the-money, thus providing the necessary tail risk hedge for the SPX holding. Our heuristic is therefore able to exploit the asymmetric behavior of market level correlation under different market conditions to provide an economic and effective tail risk hedge.

References

Andersen, T. G., Fusari, N. and Todorov, V. 2015. "The risk premia embedded in index options." *Journal of Financial Economics* 117 (3): 558-584.

Baltussen, G., van Bekkum, S. and van der Grient, B. 2018. "Unknown unknowns: uncertainty about risk and stock returns." *Journal of Financial and Quantitative Analysis* 53 (4): 1615-1651.

Bakshi, G and Kapadia, N. 2003. "Volatility risk premiums embedded in individual equity options: Some new insights" *Journal of Derivatives* 11 (1): 45-54.

Bakshi, G. and Madan, D. 2006. "A theory of volatility spreads." *Management Science* 52 (12): 1945-1956.

Bhansali, V. 2008. "Tail Risk Management." *Journal of Portfolio Management* 34 (4): 68-75.

Bhansali, V. and Davis, J.M. 2010. "Offensive Tail Risk Management II: The Case for Active Tail Hedging." *Journal of Portfolio Management* 37 (1): 78-91.

Bhansali, V. and Holdom, J. 2021. "Diversifying diversification: downside risk management with portfolios of insurance securities." *Journal of Portfolio Management* 47 (6): 101-113.

Bhansali, V., Chang, L., Holdom, J. and Rappaport, M. 2020. "Monetization Matters: Active Tail Risk Management and the Great Virus Crisis." *Journal of Portfolio Management* 47 (1): 16-28.

Bollen, N. P. B. and Whaley, R. E. 2004. "Does net buying pressure affect the shape of implied volatility functions?" *Journal of Finance* 59 (2): 711-753.

Bollerslev, T., Todorov, V. and Xu, L. 2015. "Tail risk premia and return predictability." *Journal of Financial Economics* 118 (1): 113-134.

Cao, J. and Han, B. 2013. "Cross section of option returns and idiosyncratic stock volatility." *Journal of Financial Economics* 108 (2013): 231-249.

Carr, P. and Wu, L. 2009. "Variance risk premiums." *Review of Financial Studies* 22 (3): 1311-1341.

Chabi-Yo, F., Ruenzi, S. and Weigert, F. 2018. "Crash sensitivity and the cross section of expected stock returns." *Journal of Financial and Quantitative Analysis* 53 (3): 1059-1100.

Chang, L., Holdom, J. and Bhansali, V. 2022. Tail risk hedging performance: measuring what counts." *Journal of Portfolio Management* 48 (5): 25-39.

Christensen, B. J. and Prabhala, N. R. 1998. "The relation between implied and realized volatility." *Journal of Financial Economics* 50 (2): 125-150.

Chua, D. B., Kritzman, M. and Page, S. 2009. "The Myth of Diversification." *Journal of Portfolio Management* 40 (4):89-105.

Fallon, W. and Park, J. 2016. "An asset class characterization of the US equity index volatility risk premium." *Journal of Portfolio Management* 43 (1): 72-84.

Gava, J., Guevara, F. and Turc, J. 2021. "Turning tail risks into tailwinds." *Journal of Portfolio Management* 47 (4): 41-70.

Goyal, A. and Saretto, A. 2009. "Cross-section of option returns and volatility." *Journal of Financial Economics* 94 (2): 310-326.

Longin, F. and Solnik, B. 2001. "Extreme correlation of international equity markets." Journal of Finance 56 (2): 649-676.

van Oord, M. R. C. and Zhou, C. 2016. "Systematic Tail Risk." *Journal of Financial and Quantitative Analysis* 51 (2): 685-705.

Schmielewski, F. and Stoyanov, S. 2017. "Defensive portfolio construction based on extreme value at risk." *Journal of Portfolio Management* 43 (3): 42-50.

Welch, I. 2016. "The (time-varying) importance of disaster risk." *Financial Analysts Journal* 72 (5): 14-30.

Xiong, J.X. and Idzorek, T.M. 2018. "A Case for Tail-Risk-Based Sharpe Ratios." Journal of Portfolio Management 44 (3): 114-125.

Xiong, J. X., Idzorek, T. M. and Ibbotson, R. G. 2016. "The economic value of forecasting left-tail risk." Journal of Portfolio Management 42 (3): 114-123.