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Shiyang HUANG

Bart Zhou YUESHEN
Singapore Management University, bartyueshen@smu.edu.sg

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Speed Acquisition*

Shiyang Huang† Bart Zhou Yueshen‡

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† School of Economics and Finance, The University of Hong Kong; huangsy@hku.hk; K.K. Leung Building, The University of Hong Kong, Pokfulam Road, Hong Kong.

‡ INSEAD; b@yueshen.me; INSEAD, 1 Ayer Rajah Avenue, Singapore 138676.

Speed Acquisition

Abstract

Speed is a salient feature of modern financial markets. This paper studies investors' speed acquisition, alongside their information acquisition. Speed heterogeneity arises in equilibrium, fragmenting the information aggregation process with a nonmonotone impact on the overall price informativeness. Various competition effects drive speed and information to be either substitutes or complements. The model cautions the possible dysfunction of price discovery: An improving information technology might complement speed acquisition, which shifts the concentration of price discovery over time, possibly hurting price informativeness. Novel predictions are discussed regarding investor composition, their performance, and trading volume.

Keywords: speed, information, technology, price discovery, price informativeness
JEL code: D40, D84, G12, G14

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1 Introduction

Information aggregation is a fundamental function of financial markets. It involves two steps. First, investors acquire information. Second, via trading, such information is incorporated into price. Intrinsically underlying this second step is the notion of speed, because trading takes time: Not all investors with information instantaneously gather together (and neither do their trading orders). If they only slowly arrive in the market, the resulting information aggregation will also be slow.

Indeed, in modern financial markets, investors choose not only how much information to acquire but also how quickly to trade on it. Consider a hedge fund for example. Its information acquisition involves, e.g., visiting firms, buying datasets, or developing valuation models. Such investments determine the amount and the quality of the data (signal precision). Its speed acquisition covers different aspects: First, the fund can invest in equipment or staff to speed up data processing, reaching the same trading strategy sooner. Second, before execution, a trading order needs to journey through the back office (for risk management, due diligence, and compliance; Bouvard and Lee, 2016). The fund can invest in more personnel to streamline such processes. Third, from trading desks and onward, the fund can expedite order execution by investing in computational hardware and connection to exchange servers (e.g., algorithmic and high-frequency trading technology). Importantly, these three speed investments only reduce the time needed to implement trades, not affecting the trading strategies' signal quality.

The canonical frameworks studying information aggregation, like Grossman and Stiglitz (1980) and Verrecchia (1982), do not feature speed, as investors trade at the same time. The recent high-frequency trading literature typically bundles information and speed together for fast traders; e.g., Hoffmann (2014) and Biais, Foucault, and Moinas (2015). Contrary to this view, Dugast and Foucault (2018) and Kendall (2018) argue that processing information is time-consuming and fast traders can only rely on imprecise signals. This paper complements the literature by studying investors' separate acquisition of speed and of information, rather than tying the two together.

A set of questions arises: How much speed technology should different investors acquire? Is investment in speed favored over information? Which securities attract more fast investors? What are the implications for the overall "price discovery" (information aggregation and price informativeness)?

This paper develops a model to address these questions. Consider an economy populated with a continuum of speculators trading a risky asset. Before trading, there are two technologies available. The information technology improves speculators' profitability by increasing signal precision. The speed technology allows one to execute his trade before the information is capitalized into prices—fast speculators have a "first-mover advantage." In equilibrium, speculators optimally choose how much to invest in each technology to maximize his profit, taking into account the investment costs and the competition from others.

A driving feature of the model is the "temporal fragmentation effect" of the speed technology. Though all speculators want to acquire speed to enjoy the "first-mover advantage," in equilibrium, not everyone will be equally fast, for otherwise there is no "first-mover" and some will stay slow to save the speed cost. Speed heterogeneity thus endogenously arises, splitting the trading process temporally into parts, e.g., an early fragment with fast speculators and a late fragment with slow ones. The speed technology, affecting the equilibrium size of fast traders, therefore shifts the concentration of trading over time, affecting various market quality.

There are three main results. First, the speed technology inflicts a nonmonotone impact on the overall price informativeness. With a more advanced (cheaper) speed technology, more speculators become fast, boosting the early price discovery. At the same time, fewer remain slow and the late price discovery decays. The overall price informativeness, therefore, can be either improved or hurt, depending on whether the (early) boost overcomes the (late) decay.

It is worth emphasizing that this nonmonotone impact of speed technology holds even when information acquisition is shut down. This result contributes to the literature with how speed technology *alone* could affect market quality, refining the understanding from existing theory where speed and information are tied together (e.g., Dugast and Foucault, 2018; Kendall, 2018).

Second, speed and information can be either complements or substitutes. Consider, for example, a positive shock in the information technology, upon which all speculators acquire more information. How the demand for speed responds depends on the relative change between fast and slow speculators' rents. As everyone acquires more information, competition intensifies, attenuating the rents within both groups. In addition, the increased early price discovery hurts the slow speculators (if the early price becomes fully-revealing, no rent will be left for the slow).Taken together, if the fast (the slow) are hurt more, some of them are better off staying slow (becoming fast) instead, in which case information substitutes (complements) speed.

Such *endogenous* complementarity or substitution effects between the two technologies are novel to the literature. This insight is made possible precisely because the two technologies are modeled side-by-side, departing from the aforementioned existing models where speed and information are inseparable.

A number of novel empirical predictions in the cross-section of assets follow this second result. For example, in terms of investor composition, the model predicts that fast speculators mostly participate in assets with *moderate* information acquisition cost, e.g., medium size stocks: In very small stocks (high information cost), speculators' information rent is not sufficient to justify the costly speed acquisition—low demand for both speed and information (complementarity). In large stocks (low information cost), all speculators acquire a lot of information and the competition becomes very fierce, making investments in speed unprofitable—high demand for information but low for speed (substitution). Only with moderate information cost, speed acquisition is most profitable; hence a hump-shape relation between demands for speed and information arises. The same intuition predicts that speculators' performance (expected return) also peaks in assets with moderate information costs. Such predictions are consistent with empirical findings[1] and differ

[1] Relating to empirical evidence, this paper argues that hedge funds overall fit the interpretation of fast speculators in the model, while mutual funds and pension funds can be thought of as slow ones. See Remark 1 and 2 in Section 3.1 for details. Consistent with this interpretation, Griffin and Xu (2009) find that hedge funds (fast) are most active in

from existing theories.[2] (More testable implications are discussed in Section 4.4.)

The third result is that, perhaps surprisingly, a more advanced information technology can still worsen price informativeness. The key mechanism at work is the joint force of the temporal fragmentation effect of the speed technology and its endogenous complementarity with information: Due to complementarity, a better information technology stimulates demand for both information and speed. The former improves price informativeness. The latter shifts trading concentration from late to early, thereby improving the initial price discovery but lowering the contribution of subsequent late trading. When the deterioration in the late price discovery dominates, the overall price informativeness is weakened by the information technology.

This third result cautions the dysfunction of information aggregation in financial markets. The "information technology" in the model can be interpreted broadly. For example, recent years have seen strengthened transparency and disclosure requirements by regulators. Policies like Sarbanes-Oxley, Regulation Fair Disclosure, and Rule 10b5-1 have arguably reduced the information acquisition cost. In addition, there is evidence of speed acquisition complementing the accessibility of information. Du (2015) finds that high-frequency traders are constantly crawling the website of U.S. SEC in order to trade on the information in latest company filings. Through such a complementarity channel, this paper argues that transparency and disclosure policies might generate unintended negative impact on price informativeness. Some recent empirical evidence echoes this view. Weller (2018) shows that algorithmic trading has risen at the cost of long-run price discovery. Gider, Schmickler, and Westheide (2016) shows how high-frequency trading hurts the predictability of earnings in the far future.

Different financial assets are exposed to different levels of information and speed technology. Bai, Philippon, and Savov (2016) finds a rising trend of the price informativeness of S&P 500

medium size stocks, relative to mutual funds (slow). Shumway, Szefler, and Yuan (2011) examine funds' holdings and find that the return predictability is highest in medium stocks.

[2] For example, in Dugast and Foucault (2018), the population size of early traders monotonically decreases (increases) in the cost of the early-raw signal reduces. Similarly, in Kendall (2018) investors are always more likely to "rush" (trade early) when the quality of the early signal increases.

nonfinancial firms in a half-century sample period starting from the 1960s. The finding for firms beyond the S&P 500, however, is the opposite. Farboodi, Matray, and Veldkamp (2017) reproduce the patterns and explain these phenomena with a composition effect. This paper adds to the discussion that the distinction in different technologies—speed v.s. information—is important in determining individual stocks' respective price informativeness over the years.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 studies a model of speed acquisition only and Section 4 adds information acquisition alongside to it. Discussions on model assumptions, robustness, and extensions are collated in Section 5. Section 6 then concludes.

2 Related literature

This paper builds on a number of models featuring (two) sequential trading rounds: Grundy and McNichols (1989), Froot, Scharfstein, and Stein (1992), Hirshleifer, Subrahmanyam, and Titman (1994), Holden and Subrahmanyam (1996, 2002), Brunnermeier (2005), Cespa (2008), Malinova and Park (2014), Banerjee, Davis, and Gondhi (2018), Dugast and Foucault (2018), and Kendall (2018). A distinguishing feature of the current model is that investors are allowed to engage in costly speed acquisition, separately from the conventional information acquisition. To compare, in the above, investors either cannot choose their speed at all or have speed investment decision tied with information:

- In Grundy and McNichols (1989), Brunnermeier (2005), and Cespa (2008), all investors trade in both rounds. Hence, there is no speed.
- Investors' speed are exogenously assigned in Froot, Scharfstein, and Stein (1992); Hirshleifer, Subrahmanyam, and Titman (1994); and Banerjee, Davis, and Gondhi (2018).
- In Holden and Subrahmanyam (1996), Dugast and Foucault (2018), and Kendall (2018), speed appears as a by-product of, hence not separable from, different types of information:

short- v.s. long-horizon; raw v.s. processed; early-weak v.s. late-strong signals. As such, these models do not address questions about the interaction between speed and information technologies, like whether they are substitutes or complements.

- Section IV of Holden and Subrahmanyam (2002) directly models speed by letting investors choose at a cost to observe and trade on a signal early. However, investors cannot separately invest in information, as the signal is common across investors and across time (this paper studies such a special case in Section 3). In Malinova and Park (2014), investors with exogenous quality of signals can choose when to trade freely; i.e., no costly speed or information acquisition.

Separating speed from information, this paper makes a novel contribution to the literature by identifying *endogenous* complementarity or substitution between the two technologies. These theoretical results lead to unique predictions on investor composition, holding-performance relation, and trading volume (Section 4.4).

Another key result of the paper is that lowering information cost can hurt price informativeness, even in an environment with *a single information source*.[3] A number of the aforementioned papers share a similar prediction due to some form of "substitution" in different types/sources of information. For example, in Brunnermeier (2005), the existence of an insider who monopolizes the short-run information curbs other analysts' long-run trading aggressiveness; in Banerjee, Davis, and Gondhi (2018), a public disclosure pushes investors to instead learn about others' beliefs, no longer about fundamentals; in Dugast and Foucault (2018), a cheaper raw signal can crowd out investment in the processed signal. Such substitution across various signals does not exist in the current model as there is only one information source.[4] Also worth noting is that in Dugast and

[3] "A single information source" means that there is no public information (Brunnermeier, 2005; Banerjee, Davis, and Gondhi, 2018), no independent fundamentals (Froot, Scharfstein, and Stein, 1992), no short- v.s. long-term information (Holden and Subrahmanyam, 1996), no learning about others' beliefs (Banerjee, Davis, and Gondhi, 2018) or about noise (Froot, Scharfstein, and Stein, 1992). In Dugast and Foucault (2018) and Kendall (2018), there is a single fundamental but there are two different signals with exogenous quality: "raw/weak" for the early signal and "processed/strong" for the late. As such, the two signals can also be thought of as two different information sources.

[4] Hirshleifer, Subrahmanyam, and Titman (1994) and Holden and Subrahmanyam (2002) also have a single

Foucault (2018) and Kendall (2018), the early signal is always less precise than the late one. Hence, when all investors are fast, these models predict a worse price informativeness than when all slow. This is not the case in the current model: When all are fast, price informativeness is the same as when all slow, as everyone learns from and trades on the same, single information source.

The model predicts that better information technology could worsen price informativeness. Two recent studies highlight similar results. It is worth emphasizing that the underlying mechanisms are different, thus leading to contrasting testable implications for other market quality measures.

- Investors in Dugast and Foucault (2018) choose between trading early with a raw (low-quality) signal or late with a processed (high-quality) one. They need to purchase the signals from competitive information vendors, who produce the signals at exogenous fixed costs, as in Veldkamp (2006a,b). When the early signal becomes cheaper to produce, more investors acquire it and trade early, the short-run price discovery increases, and the residual information rent reduces. Under some parameters, information vendors will *raise* the price of the processed signal, hindering investors' incentive to acquire the processed signal, and the overall price informativeness is hurt. The key channel is that through the information market, the cost of producing the raw signal now affects the prices of *both* signals, possibly in opposite directions. Without vendors as in Veldkamp (2006a,b), such a channel does not exist in the current model as the costs of speed and information are the same as their prices and they do not affect each other.
- In Kendall (2018), investors choose whether to "rush" to trade early on an imprecise signal or to "wait" to trade late on a precise one. The benefit of waiting is the better signal quality. The downside is that there might be a public announcement or another investor trading early, reducing the information rent—the "time cost". The key insight is that with such time cost, investors rush more often to trade on the early imprecise signal, forgoing the opportunity to

information source in their model. While not explicitly studied, a higher precision in endowed signals in those settings would always lead to higher price informativeness. This is because they do not study investment in speed (separately from information), hence no endogenous complementarity or substitution between the two.

acquire the late precise signal, thus hurting informational efficiency. Notably, as the focus is on the time cost, Kendall (2018) does not explain how monetary costs of different signals affect market quality. That is, there is no costly technology acquisition and investors obtain signals for free.

The shared feature of these two papers is that trading fast *with* high-quality signal is by construction not possible. Speed and (high-quality) information is mutually exclusive. In contrast, the current model allows and predicts endogenous complementarity (and sometimes substitution) between the two, yielding novel findings. To name a few (see Section 4.3 and 4.4 for details): Improvements in either speed or information can hurt price informativeness in the current model, while in Dugast and Foucault (2018) only the quality of the early (raw) signal could hurt price informativeness, but not the late (precise) signal. A better information technology in the current model initially increases but then reduces investors' demand for speed—a *nonmonotone* effect in terms of investor composition. In Dugast and Foucault (2018), a cheaper raw signal (early) monotonically increases the population of fast investors because the early raw signals, by construction, substitutes the precise but late signals. In Kendall (2018), similarly, a more precise early signals monotonically makes investors more likely to rush (trade early). The current model also shows how different investors' investment performance *nonmonotonically* depends on the technologies. Neither paper studies different types' investors performance and the relation with technologies.

This paper further contributes to three themes of the literature. First, the literature on costly information acquisition largely focuses on the magnitude aspect of price discovery, following the seminal works by Grossman and Stiglitz (1980) and Verrecchia (1982). Recent studies explore other dimensions. To name a few, Peress (2004, 2011) studies the wealth effect on information acquisition. Van Nieuwerburgh and Veldkamp (2009, 2010) analyze information acquisition under limited attention. Goldstein and Yang (2015) explore the implication of information diversity. To compare, the above literature assumes that the market always clears with all investors trading at the same time—they have the same speed. With endogenous speed acquisition, this paper allows to study the process of price discovery with investors arriving and trading asynchronously.

Second, the *temporal* fragmentation (due to speed technology) in this paper differs from the existing literature on *spatial* market fragmentation.[5] Regarding the focus on price discovery, an important feature of temporal fragmentation is that information revealed early naturally carries over to the future—the market never forgets. Such natural accumulation of information over time is critical in determining the complementarity or substitution between the two technologies. In a model of multiple venues (spatial fragmentation), there is no naturally directional "flow" of information from one venue to another (more fundamentally, the notion of speed does not apply to a spatial setting). Speed therefore touches upon a novel angle of market fragmentation.

Third, this paper lends equilibrium support to the literature with *endogenous bundling* of speed and information acquisition. The model predicts that fast investors always acquire more information than the slow. This is because price discovery always accumulates over time and the same piece of information has higher marginal benefit the sooner it is traded. This insight justifies a popular connotation for fast traders that they are also more informed. See, e.g., models by Hoffmann (2014) and Biais, Foucault, and Moinas (2015); evidence by Brogaard, Hendershott, and Riordan (2014) and Shkilko and Sokolov (2016); and surveys by Biais and Foucault (2014), O'Hara (2015), and Menkveld (2016).

In a different line, investors' speed choice has been studied in limit order models with discrete prices. Examples include Yao and Ye (2018) and Wang and Ye (2017). The main driving feature is the binding tick size which limits investors' competition on price and as a result they turn to speed competition. The current paper focuses on the incentive to acquire speed due to the transitory nature of information advantage.

[5] For example, Admati (1985), Pasquariello (2007), Boulatov, Hendershott, and Livdan (2013), Goldstein, Li, and Yang (2014), Cespa and Foucault (2014), among many others, study information and cross-market learning of correlated assets. Pagano (1989), Chowdhry and Nanda (1991), Foucault and Gehrig (2008), and Baruch, Karolyi, and Lemmon (2007) study trading of the same asset on different venues (e.g., dual-listed stocks). More recently, market fragmentation has been theorized in the context of dark v.s. lit trading mechanisms, as in Ye (2011), Zhu (2014), Brolley (2016), and Buti, Rindi, and Werner (2017). Finally, Chao, Yao, and Ye (2017a,b) study the competition among exchanges by zooming in on fee structure and tick size.

3 A model of speed acquisition

This section studies a model of speed acquisition. The framework is akin to Section IV of Holden and Subrahmanyam (2002) (whose focus is on the autocorrelation of price changes, not on price discovery). Section 3.1 details the model setting. Section 3.2 solves the equilibrium. Section 3.3 examines the equilibrium properties.

3.1 Model setup

Assets. There is a risky asset and a risk-free numéraire. At the end of the game, each unit of the risky asset will pay off V units of the numéraire. Unconditionally, V is normally distributed with mean p_0 and variance $1/\tau_0$ (> 0). Without loss of generality, p_0 is normalized to zero.

Speculators. There is a unity continuum of speculators, indexed by $i \in [0, 1]$. They have constant absolute risk-aversion (CARA) preference with the same risk-aversion coefficient γ (> 0).

Speed technology. There is a speed technology that can affect speculators' trading time t_i (see "Timeline" below). All speculators are slow by default, trading at $t_i = 2$. One can instead become fast and trade at $t_i = 1$ by paying $1/g_t$ units of the numéraire. The exogenous parameter g_t measures the level of the speed technology. The larger is g_t, the more advanced (cheaper) is the technology.

Information technology. The information technology is shut down in this section to highlight the effect of the speed technology. Instead, each speculator is born with a noisy signal s_i about the fundamental with fixed precision $h_\circ$: $s_i = V + \varepsilon_i$, where $\{\varepsilon_i\}$ are i.i.d. normal with zero mean and variance $1/h_\circ$ (> 0). Section 4 allows them to endogenously acquire information.

Timeline. There are four dates in the model: $t \in \{0, 1, 2, 3\}$, illustrated Figure 1. At $t = 0$, all speculators independently choose to invest in speed $t_i \in \{1, 2\}$. Time $t \in \{1, 2\}$ are trading rounds. The set of speculators $\{i|\, t_i = t\}$ arrive at $t \in \{1, 2\}$ together and they independently submit demand schedules $\{x_i(p_t; \cdot)\}$ to trade the risky asset, based on his information set—private signal s_i and the

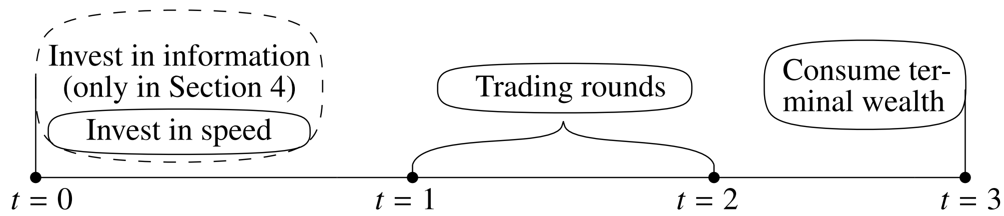


Figure 1: Timeline of the game. The model has four dates: $t \in \{0, 1, 2, 3\}$. At $t = 0$, all speculators invest in technology; at $t \in \{1, 2\}$, speculators arrive in the market at the time according to their speed technology and submit their demand schedules to trade the risky asset; finally, at $t = 3$ the risky asset pays off and all consume their terminal wealth.

public history of past prices. Finally, at $t = 3$, the risky asset pays off at V and all consume their terminal wealth.

Trading. In each trading round $t \in \{1, 2\}$, there is noise demand U_t, which is i.i.d. normal with zero mean and variance $1/\tau_U$ (> 0). The noises capture unmodeled exogenous trading (due to sentiment or hedging) by investors who cannot choose speed. The random variables V, $\{U_t\}$, and $\{\varepsilon_i\}$ are independent. The aggregate demand in round t is

$$L_t(p) = \int_{i \in [0,1]} x_i(p; s_i, p_{t-1}, \ldots) \mathbb{1}_{\{t_i = t\}} \mathrm{d}i + U_t \tag{1}$$

There is a competitive market maker, who sets the price given all existing public information (as in Kyle, 1985). Thus, the trading price in round t is

$$p_t = \mathbb{E}[V | \{L_r(\cdot)\}_{r \le t}]. \tag{2}$$

The setup is consistent with, among many others, Hirshleifer, Subrahmanyam, and Titman (1994), Vives (1995), Holden and Subrahmanyam (1996), and Cespa (2008).

Strategy and equilibrium definition. To sum up, each speculator maximizes his expected utility over the final wealth by optimizing his demand $x_i(\cdot)$ upon trading; and, backwardly, by choosing his speed $t_i \in \{1, 2\}$ at $t = 0$. Denote by $\pi(t_i)$ the speculator i's ex ante certainty equivalent (whose functional form will be derived below). Define $\mathcal{P} := \{t_i\}_{i\in[0,1]}$ as the collection of all speculators' investment policies. A Nash equilibrium is a collection $\mathcal{P}$, such that for any speculator $i \in [0, 1]$, fixing $\mathcal{P}\backslash t_i$, he chooses $t_i = \arg\max_{t_i\in\{1,2\}} \pi(t_i)$.

Remarks regarding the model setup:

Remark 1 (Interpreting speed). The speed technology is fairly stylized in the model. It only generates two relative speed tiers, $t \in \{1, 2\}$. The time lapse (between $t = 1$ and $t = 2$) can be interpreted according to any one of the following three frequencies.

- High-frequency speed (in subseconds to minutes; e.g., Budish, Cramton, and Shim, 2015). An investor (institution) can improve his high-frequency speed by investing in the trading desk—algorithms, colocation to exchange servers, optic-fibre cables, and microwave towers.
- Medium-frequency speed (in hours to days; e.g., Bouvard and Lee, 2016). When implementing a trading idea, managers in an institution are subject to risk management, due diligence, and regulatory compliance, which can take hours if not days, especially for large orders. This process can be expedited by staffing additional personnel in the back office.
- Low-frequency speed (in days to weeks; e.g., Holden and Subrahmanyam, 2002). Processing raw data to form trading ideas takes time. For example, firms' announcements might affect future cash flows projections. It can take analysts days or weeks to update such fundamentals. Recruiting more analysts or investing in faster computers can speed up this process.

Remark 2 (Who is fast). Based on the above three frequencies of speed, this paper suggests that hedge funds fit the description of fast speculators, while the slow speculators can be mutual funds or pension funds. Hedge funds' high-frequency speed advantage arises from their investment in trading technology. The U.S. Securities and Exchanges Commission (SEC) defines high-frequency traders as "proprietary trading firms", "proprietary trading desks of a multi-service broker-dealer", or "hedge funds" (Section IV.B, SEC, 2010). The medium-frequency speed interpretation can be attributed to different regulatory requirement. Mutual funds are registered with the SEC and are subject to extensive regulatory compliance, risk control, and bookkeeping requirements. Hedge funds, on the other hand, are under less regulatory scrutiny, thus able to expeditiously trade on their signals. Under the low-frequency interpretation, the fast investors process information sooner than the slow and should lead subsequent returns. Supporting this view, Swem (2017) documents that hedge funds' trades predict rating changes by sell-side analysts, who are then followed by other buy-side institutions.

Remark 3 (Modeling choices). Several assumptions are worth emphasizing: (1) the speculators' population size is fixed; (2) the amount of noise trading is exogenous; (3) fast investors only trade at $t = 1$; and (4) a competitive market maker sets the price. These assumptions are made purposefully to, and only to, pinpoint the economics of speed acquisition (and, later, its interaction with information acquisition in Section 4). Section 5 studies corresponding extensions to relax these assumptions: (1) to endogenize investor participation; (2) to model the timing of liquidators (noise traders); (3) to allow fast investors trade also more frequently; and (4) to replace the competitive market maker with a fringe of uninformed investors. Both the key mechanisms and the main results of the paper are shown to stand robust to these extensions.

3.2 Equilibrium analysis

The equilibrium can be analyzed in two backward steps. The first is to study speculators' optimal demand schedules, fixing their speed acquisition. Suppose there is a fraction of $\phi_1 \in [0, 1]$ of the speculators who acquire speed and the rest $\phi_2 = 1 - \phi_1$ remain slow; that is

$$\phi_1 := \int_{i\in[0,1]} \mathbb{1}_{\{t_i=1\}} \mathrm{d}i \text{ and } \phi_2 := \int_{i\in[0,1]} \mathbb{1}_{\{t_i=2\}} \mathrm{d}i.$$

Hence, ϕ_1 denotes the aggregate demand for speed. A speculator i demands x_i at $t_i = t$ to solve

$$x_i \in \arg\max_{x_i} \mathbb{E}\Big[-e^{-\gamma\cdot(V-p_t)x_i} \,|\, V + \varepsilon_i = s_i, p_t, p_{t-1}, \ldots\Big]$$

where p_t is given by the market maker's pricing as in Equation (2). Standard conjecture-and-verify analysis as in Vives (1995) yields the following result.

Lemma 3.1 (Equilibrium trading, without information acquisition). *Each speculator i submits a linear demand schedule at $t = t_i \in \{1, 2\}$:*

$$x_i = \frac{h_\circ}{\gamma}\left(s_i - p_{t_i}\right). \tag{3}$$

His ex-ante certainty equivalent at $t = 0$ is

$$\pi_{t_i} = \frac{1}{2\gamma}\ln\left(1 + \frac{h_\circ}{\tau_{t_i}}\right) - \frac{2 - t_i}{g_t}, \tag{4}$$

where the price informativeness $\tau_t := \mathrm{var}[V \mid \{L_r(\cdot)\}_{\forall r \le t}]^{-1}$ satisfies the recursion of

$$\Delta\tau_t = \tau_t - \tau_{t-1} = \frac{\phi_t^2 h_\circ^2 \tau_{\mathrm{U}}}{\gamma^2} \tag{5}$$

with the initial value $\tau_0 = \mathrm{var}[V]^{-1}$. The equilibrium price p_t satisfies the recursion of

$$p_t = \frac{\tau_{t-1}}{\tau_t} p_{t-1} + \frac{\Delta\tau_t}{\tau_t}\left(V + \frac{\gamma U_t}{\phi_t h_\circ}\right)$$

with initial value $p_0 = \mathbb{E}V$ $(= 0)$.

A speculator's demand x_i scales with the difference between his private signal and the trading price $(s_i - p_{t_i})$, where the scaling factor $h_\circ/\gamma$—his trading aggressiveness—increases with the precision of his signal and decreases with his risk-aversion.[6] His certainty equivalent has two components: The first term represents the information rent due to his private information, while the second term corresponds to the cost of speed acquisition.

The second step is to find speculators' optimal speed acquisition decisions at $t = 0$. Equation (4) reveals that one's speed choice t_i affects his ex-ante certainty equivalent in two ways: First, trading early positively affects his information rent, as $\tau_1 \le \tau_2 = \tau_1 + \Delta\tau_2$. Second, however, speed

[6] Note that slow speculators ($t_i = 2$) do *not* directly trade on the fast round price p_1, thanks to the competitive market maker who sets p_2 while recalling the information from $t = 1$ trading. As such, from a slow speculator's perspective, observing only p_2 is as good as observing both p_1 and p_2. This Markov feature inherits from Vives (1995) and goes back to Kyle (1985), where the dynamic equilibrium only uses the contemporaneous price p_t as a state variable, not the entire price history.

acquisition is costly, $1/g_t > 0$. Trading off these two aspects, the marginal speculator must be indifferent between becoming fast and staying slow, in order to sustain an interior equilibrium; i.e., $\pi_1 = \pi_2$. Otherwise, a corner equilibrium arises.

Proposition 3.1 (Equilibrium speed acquisition, without information acquisition). *Fix speculators' information acquisition at $h_\circ$. There exists a unique equilibrium, depending on the speed technology g_t relative to some threshold $\hat{g}_t$ (> 0, see the proof):*

Case 1 (corner). *When $g_t \le \hat{g}_t$, all investors stay slow with $\phi_1 = 0$ and $\phi_2 = 1$.*

Case 2 (interior). *When $\hat{g}_t < g_t$ ($< \infty$), a fraction $\phi_1 \in (0, 1)$ of investors acquire speed and become fast, while the rest $\phi_2 = 1 - \phi_1$ stay slow. The equilibrium population sizes $\{\phi_1, \phi_2\}$ uniquely solve $\pi_1 = \pi_2$ and $\phi_1 + \phi_2 = 1$.*

The equilibrium depends on the level of speed technology: When $g_t \le \hat{g}_t$, investing in speed is too costly for any speculator and nobody acquires speed. Only for sufficiently advanced speed technology ($g_t > \hat{g}_t$) will there be some fast speculators.[7]

Note that the population size pair $\{\phi_1, \phi_2\}$ has an alternative interpretation: They measure speculators' ex-ante probability mix between becoming fast or staying slow. At $t = 0$, each speculator independently chooses to be fast with probability ϕ_1 and to remain slow with probability ϕ_2. Under this interpretation, ϕ_1 also represents the *individual* demand for speed.

3.3 Equilibrium properties

This subsection studies the implications of an advancing speed technology g_t. The first result is the intuitive price effect: The higher is g_t, the lower is the cost to be fast, and the demand for speed increases, as illustrated in Figure 2 (a).

[7] However, there are always nonzero mass of speculators staying slow ($\phi_2 > 0$). To see the reason, suppose there is an equilibrium where all speculators are fast, i.e., $\phi_1 = 1$ and $\phi_2 = 0$. In this case there is no informed trading at $t = 2$; thus, $\tau_1 = \tau_2$. Equation (4) then implies that the marginal fast speculator is strictly better off if he instead does not invest in speed, saving the speed acquisition cost $1/g_t$. Hence, some fast speculators will deviate to staying slow, invalidating such an equilibrium.

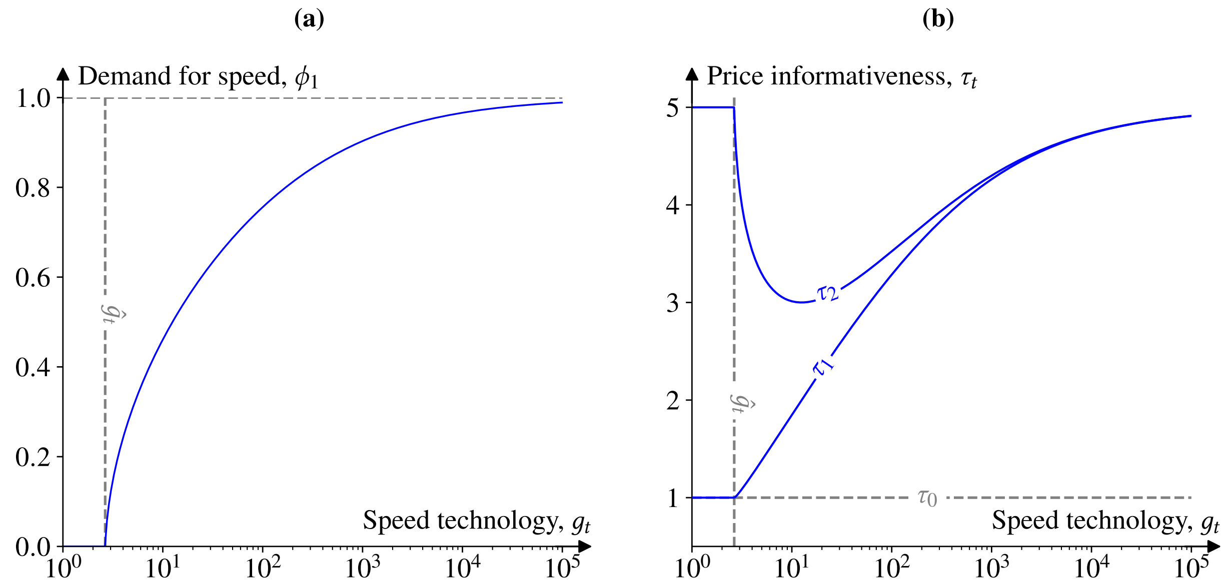


Figure 2: Varying speed technology with fixed information. Panel (a) shows how speed technology g_t affects individual investors' demand for speed ϕ_1 and Panel (b) price informativeness τ_t. To the right of the vertical dashed line, the equilibrium is interior (with both fast and slow investors). The primitive parameters are: $\tau_0 = 1.0$, $\tau_U = 4.0$, and $\gamma = 0.1$. The common signal precision is fixed at $h_\circ = 0.1$.

Proposition 3.2 (Speed technology and speed acquisition). *Fix all speculators' signal precision at $h_i = h_\circ$ (> 0). In the interior equilibrium, as the speed technology g_t advances, more speculators acquire speed: $\partial \phi_1 / \partial g_t > 0$.*

As more speculators acquire speed, the intermediate price informativeness τ_1 increases. However, there is a nonmonotone effect on the overall τ_2, as seen in Figure 2 (b). This is because the speed technology *temporally fragments* price discovery: When speed is affordable ($g_t > \hat{g}_t$), a fraction ϕ_1 of the speculators trade early at $t = 1$, while the rest ϕ_2 $(= 1 - \phi_1)$ trade late at $t = 2$. The price discovery process accordingly fragments into an early $\Delta\tau_1$ and a late $\Delta\tau_2$.[8] Recalling

[8] For clarity, throughout this paper, the price informativeness *increments* $\Delta\tau_1$ and $\Delta\tau_2$ are referred to as the "early" and the "late price discovery", respectively; while the *levels* τ_1 and τ_2 are called the "intermediate" and the "overall price informativeness", respectively. Remark 1 suggests three interpretations of the lengths of "early" v.s. "late" and "intermediate" v.s. "overall".

from Equation (5), the early price discovery increases with g_t:

$$\Delta\tau_1 = \frac{\tau_U}{\gamma^2} h_\circ^2 \phi_1^2,$$

because ϕ_1 increases in g_t (Proposition 3.2). However, the late price discovery drops:

$$\Delta\tau_2 = \frac{\tau_U}{\gamma^2} h_\circ^2 \phi_2^2 = \frac{\tau_U}{\gamma^2} h_\circ^2 \cdot (1-\phi_1)^2.$$

The overall $\tau_2 = \tau_0 + \Delta\tau_1 + \Delta\tau_2$ is subject to the joint force of the two price discovery and, therefore, exhibits a nonmonotone trend in the speed technology g_t.

> **Proposition 3.3 (Speed technology and price informativeness).** *Fix all speculators' signal precision at $h_i = h_\circ$ (> 0). In the interior equilibrium, as the speed technology g_t advances, the intermediate price informativeness τ_1 monotonically increases, while the overall τ_2 initially decreases but eventually increases. Mathematically, $\partial\tau_1/\partial g_t > 0$ for all $g_t > \hat{g}_t$; and $\partial\tau_2/\partial g_t < 0$ (> 0) for sufficiently small (large) g_t.*

Essentially, the speed technology *temporally* fragments speculators' participation in the market, thus shifting the concentration the price discovery process. Jointly, Proposition 3.2 and 3.3 are thus referred to as *the temporal fragmentation effect* of speed technology.

Key to the nonmonotone effect is that each price discovery, $\Delta\tau_t$, is a *convex function* in the population size ϕ_t (Equation 5). Such convexity, inherent from Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982)—just to name a few, is due to that price discovery has *increasing returns to scale*: Each marginal informed investor's trading resolves increasingly more uncertainty (from noise trading). As the speed technology shifts the concentration (ϕ_t) of such a production process ($\sum \Delta\tau_t$), the total productivity is affected nonmonotonically.

Mathematically, the impact of a marginal change in ϕ_t (due to speed technology) on $\Delta\tau_t$ depends on the initial level of ϕ_t. For example, when ϕ_1 is close to zero and ϕ_2 to one (when $g_t \downarrow \hat{g}_t$), a small increase in speed $\mathrm{d}g_t$ prompts a small population $\mathrm{d}\phi_1$ to move from slow to fast. The resulting loss in the late $\Delta\tau_2$ is much larger than the gain in the early $\Delta\tau_1$:

$$\text{(6)} \qquad \mathrm{d}\tau_2 = \frac{\partial\tau_2}{\partial\phi_1}\mathrm{d}\phi_1 = \left(\frac{\partial\Delta\tau_1}{\partial\phi_1} + \frac{\partial\Delta\tau_2}{\partial\phi_1}\right)\mathrm{d}\phi_1 = \frac{\tau_{\mathrm{U}}}{\gamma^2}h_\circ^2 \underbrace{\frac{\partial}{\partial\phi_1}\left(\phi_1^2 + \phi_2^2\right)}_{=2(2\phi_1-1)<0 \text{ for } \phi_1 \text{ close to } 0} \mathrm{d}\phi_1 < 0.$$

The reverse holds true when ϕ_1 is close to one and ϕ_2 close to zero. Indeed, the overall τ_2 is the same at the either extreme of g_t: $\lim_{g_t \downarrow \hat{g}_t} \tau_2 = \lim_{g_t \uparrow \infty} \tau_2 = \tau_0 + \tau_{\mathrm{U}} h_\circ^2/\gamma^2$; see Figure 2 (b). This is because in either extreme the speculators are no longer fragmented.[9] This feature differentiates the mechanism from models like Dugast and Foucault (2018) and Kendall (2018). In such models, the early signal is always less precise than the late one. Hence, when all speculators are fast, these models predict a worse price informativeness than all slow.

While the model is stylized, the temporal fragmentation effect is a robust feature of the speed technology. The modeling choices mentioned in Remark 3 can be relaxed: It does not depend on the assumption that the population size of speculators is fixed (Section 5.1 shows that the effect remains under free-entry). It does not build on exogenously fixed noise sizes (Section 5.2 explicitly models liquidity traders' timing decisions). It does not restrict that fast speculators trade only once at $t = 1$ (see Section 5.3 where speed means not only trading earlier but also more frequently). Neither does it depend on the assumed competitive market maker (Section 5.4).

Proposition 3.3 highlights that speed technology *alone* could negatively affect market quality, even when isolated from information. This insight complements existing models where such adverse effects arise only when information and speed are tied together; see, e.g., Dugast and Foucault (2018) and Kendall (2018). Section 4 below endogenizes speculators' information acquisition to explore the interaction between speed and information and yields additional predictions.

[9] More generally, if there are T tiers of speed (T trading rounds), the overall price informativeness becomes $\tau_T = \tau_0 + \sum_{t=1}^{T} \Delta\tau_t = \tau_0 + \frac{\tau_{\mathrm{U}}}{\gamma^2} h_\circ^2 \sum_{t=1}^{T} \phi_t^2$, where $\sum_{t=1}^{T} \phi_t^2$ is the Herfindahl-Hirschman index measuring speculators' *temporal concentration*. The speed technology g_t nonmonotonically drives such trading concentration.

4 Endogenizing information acquisition

This section studies a model where speculators can endogenously acquire both speed and information. Section 4.1 extends the setup, Section 4.2 derives the equilibrium, Section 4.3 explores the equilibrium properties, and Section 4.4 discusses empirical implications.

4.1 Model setup

Assets, speculators, speed technology, timing, and trading are the same as in Section 3.1.

Information technology. There is an information technology available at $t = 0$ (see Figure 1 where $t = 0$ is shown with the dashed circle). Each speculator i can choose to spend c_i (≥ 0) units of the numéraire to improve his signal precision h_i:

$$h_i = g_h k(c_i),$$

where $k(\cdot)$ is twice-differentiable, concave, and strictly increasing; and g_h (≥ 0) gauges the marginal productivity of this information technology (c.f. g_t). Without investing in this technology, the speculator gets no signal; i.e. $k(0) = 0$.

Due to the monotonicity of $k(\cdot)$, a speculator's information acquisition can be referred to as either h_i (the precision) or c_i (the cost) interchangeably: There exists a convex, strictly increasing cost function $c(\cdot; g_h)$, so that $\forall h_i \geq 0$, $c_i = c(h_i; g_h) := k^{-1}(h_i/g_h)$. To ensure that there is always *some* information in the market, let $\dot{c}(0) = 0$.

Strategy and equilibrium definition. Each speculator maximizes his expected utility over the final wealth by optimizing his demand $x_i(\cdot)$ upon trading; and, backwardly, by choosing his technology pair $(t_i, h_i) \in \{1, 2\} \times [0, \infty)$ at $t = 0$. Denote by $\pi(t_i, h_i)$ speculator i's ex ante certainty equivalent. Define $\mathcal{P} := \{(t_i, h_i)\}_{i \in [0,1]}$ as the collection of all speculators' investment policies. A Nash equilibrium is a collection $\mathcal{P}$, such that for any speculator $i \in [0, 1]$, fixing $\mathcal{P} \backslash (t_i, h_i)$, he chooses $(t_i, h_i) = \arg\max_{(t_i, h_i) \in \{1,2\} \times [0,\infty)} \pi(t_i, h_i)$.

4.2 Equilibrium analysis

The analysis proceeds as in Section 3.2 by first solving speculators' optimal trading and then backwardly their optimal technology investment at $t = 0$.

To begin with, conjecture (and later verify) that all speculators of the same speed $t_i = t \in \{1, 2\}$ acquire the same amount of information $h_i = h_t$. That is, all fast (slow) speculators have signal precision h_1 (h_2). As before, write the population size of the fast (slow) speculators as ϕ_1 ($\phi_2 = 1 - \phi_1$). The trading equilibrium is summarized by the following lemma.

Lemma 4.1 (Equilibrium trading, with information acquisition). *Each speculator i submits a linear demand schedule at $t = t_i \in \{1, 2\}$:*

$$x_i = \frac{h_{t_i}}{\gamma}\left(s_i - p_{t_i}\right). \tag{7}$$

His ex-ante certainty equivalent at $t = 0$ is

$$\pi_{t_i} = \frac{1}{2\gamma}\ln\left(1 + \frac{h_{t_i}}{\tau_{t_i}}\right) - c\left(h_{t_i}; g_h\right) - \frac{2 - t_i}{g_t}, \tag{8}$$

where the price informativeness $\tau_t := \mathrm{var}[V|\, \{L_r(\cdot)\}_{\forall r \le t}\,]^{-1}$ satisfies the recursion of

$$\Delta\tau_t = \tau_t - \tau_{t-1} = \frac{\phi_t^2 h_t^2 \tau_{\mathrm{U}}}{\gamma^2} \tag{9}$$

with the initial value $\tau_0 = \mathrm{var}[V]^{-1}$. The equilibrium price p_t satisfies the recursion of

$$p_t = \frac{\tau_{t-1}}{\tau_t} p_{t-1} + \frac{\Delta\tau_t}{\tau_t}\left(V + \frac{\gamma U_t}{\phi_t h_t}\right)$$

with initial value $p_0 = \mathbb{E}V$ $(= 0)$.

Compared to Lemma 3.1, it can be seen that all results remain in the same structure except that the speed-specific h_{t_i}, which is endogenous and to be determined, takes the place of $h_\circ$.

Turn to the speculators' optimal technology acquisition next. Consider a speculator i's optimal information acquisition h_i, fixing his speed choice $t_i = t$ and all others' choice (t_j, h_{t_j}) for $\forall j \neq i$.

To maximize his ex-ante certainty equivalent (8), speculator i solves the first-order condition

$$\frac{1}{2\gamma}\frac{1}{\tau_t + h_i} - \frac{\partial c(h_i; g_h)}{\partial h_i} = 0 \iff h_i = h^*(\tau_t, g_h), \tag{10}$$

where the solution $h^*(\cdot)$ is unique and nonnegative and satisfies the second-order condition, thanks to the convexity in $c(\cdot; g_h)$. Note that the speculator's individual choice h_i does not affect the price informativeness τ_t, because each i alone is infinitesimally small. By symmetry, therefore, all speculators of the same speed $t_i = t \in \{1, 2\}$ acquire the same amount of information $h_t := h^*(\tau_t, g_h)$. This verifies the conjecture made at the beginning of Section 4.2.

The equilibrium speed acquisition ϕ_1 can be pinned down by $\pi_1 = \pi_2$, i.e. the marginal speculator being indifferent between becoming fast or staying slow. Note that ϕ_t affects price efficiency τ_t (Equation 9), which indirectly also affects information acquisition h_t (first-order condition 10). Together, h_t and τ_t again affect π_t. It is through these channels that the speed acquisition ϕ_1 chains all equilibrium objects together.

Proposition 4.1 (Equilibrium technology acquisition). *There exists a unique equilibrium, depending on the speed technology g_t relative to some threshold $\hat{g}_t$ (> 0, see the proof):*

Case 1 (corner). *When $g_t \le \hat{g}_t$, all speculators invest in $(t_i, h_i) = (2, h_2)$, where h_2 and τ_2 uniquely solve the first-order condition (10) and the recursion (9) with $\phi_1 = 0$ and $\phi_2 = 1$.*

Case 2 (interior). *When $\hat{g}_t < g_t$ ($< \infty$), a fraction $\phi_1 \in (0, 1)$ of speculators invest in $(t_i, h_i) = (1, h_1)$, while the rest ϕ_2 speculators invest in $(t_i, h_i) = (2, h_2)$, such that $\{h_1, h_2, \phi_1, \phi_2\}$ uniquely solve the following equation system:*

Optimal information acquisition:	$h_1 = h^*(\tau_1; g_h)$ *and* $h_2 = h^*(\tau_2; g_h)$;
Indifference in speed:	$\pi_1 = \pi_2$;
Population size identity:	$\phi_1 + \phi_2 = 1$;

where the expressions of $h^*(\cdot)$, π, and τ are given by Equations (10), (8), and (9).

Just like in Proposition 3.1 (no information acquisition), the speculators only acquire speed when it is advanced enough. It turns out the same holds true for the information technology:

Corollary 4.1. *Fixing the speed technology g_t, there exists a threshold $\hat{g}_h$ (> 0) such that the equilibrium is interior if and only if ($\infty >$) $g_h > \hat{g}_h$.*

That is, when the information technology is too poor, the additional information rent of becoming fast is not sufficient to compensate for the cost of speed and as such, all speculators stay slow.

4.3 Equilibrium properties

The equilibrium properties are explored by means of comparative statics regarding the two technology levels, g_t for speed and g_h for information.

4.3.1 The temporal fragmentation effect of speed technology

The first result is that the temporal fragmentation effect of the speed technology remains robust.

Proposition 4.2 (Temporal fragmentation of speed). *The comparative statics stated in Propositions 3.2 and 3.3 hold whether or not the speculators can endogenously acquire information.*

Figure 3 (a) replicates the patterns seen in Figure 2 (b): The intermediate price informativeness τ_1 monotonically increases with the speed technology g_t, while the overall τ_2 initially dips but eventually rises.

It should be emphasized that Proposition 3.3 does not state that τ_2 is strictly U-shape (e.g., quasiconvex) in g_t. It only describes the nonmonotone pattern via the two extremes of $g_t \downarrow \hat{g}_t$ and $g_t \uparrow \infty$. In fact, the pattern of τ_2 for moderate levels of g_t can fluctuate when there is information acquisition, especially when the information technology g_t is high. Figure 3 (b) shows such an example. This novel fluctuating pattern turns out to be driven by the interaction (the complementarity, to be exact) between the two technologies, as explained below.

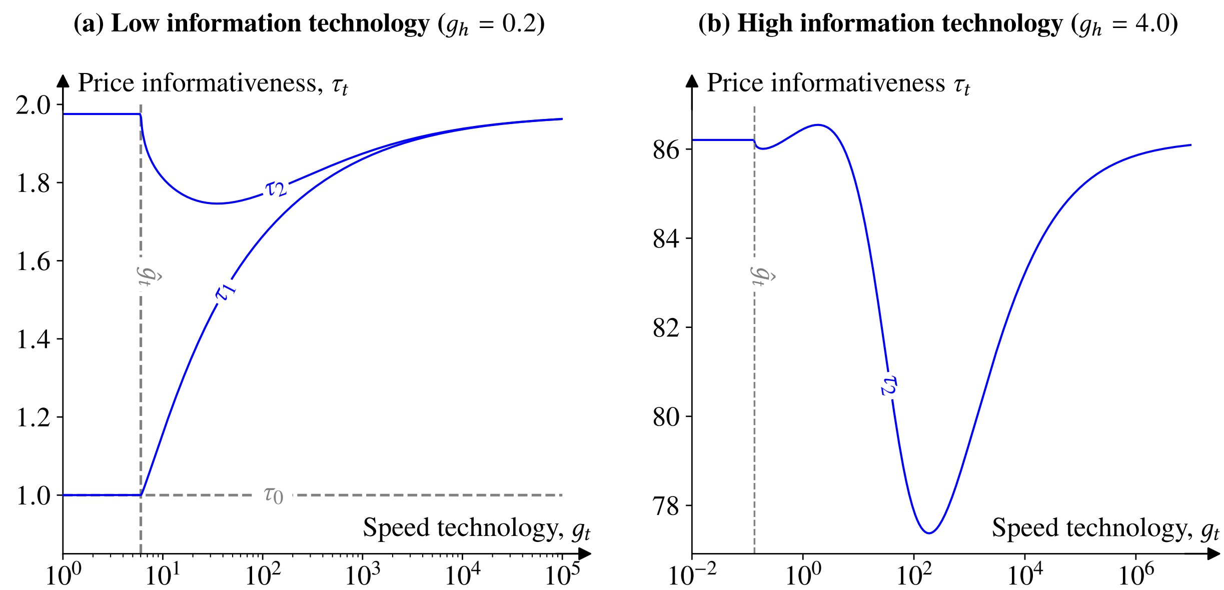


Figure 3: Varying speed technology with information acquisition. This figure shows how the speed technology g_t affects price informativeness τ_t. With a low information technology $g_h = 0.2$, Panel (a) qualitatively replicates the pattern shown in Figure (2.b) where there is no information acquisition. With $g_h = 4.0$, Panel (b) shows a fluctuating overall price informativeness τ_2 as g_t increases. The other primitive parameters are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k(c) = \sqrt{c}$. (There is no endowed signal; i.e. $h_\circ = 0$.)

4.3.2 Cross-technology effects: complementarity and/or substitution

An advancement in g_t or g_h can be equivalently interpreted as a reduction in the respective (marginal) acquisition cost. The impacts on the other technology—the cross-technology effects—are graphed in Figure 4. In both panels, it can be seen that the aggregate demand for one technology is first increasing but eventually decreasing in the other technology level: The technologies can be either complements or substitutes. (Note that in Panel (a), the aggregate demand of information, $\int_{i\in[0,1]} h_i \mathrm{d}i = \phi_1 h_1 + \phi_2 h_2$, is shown in the red-dashed line.)

> **Proposition 4.3 (Complementarity and substitution between technologies).** *In the interior equilibrium, as one technology increases, fixing the other, speculators' aggregate speed and information acquisition are initially complements but eventually substitutes. Mathematically,* $\partial\phi_1/\partial g_h > 0$ (< 0) *for small (large)* g_h*; and* $\partial(\phi_1 h_1 + \phi_2 h_2)/\partial g_t > 0$ (< 0) *for small (large)* g_t.

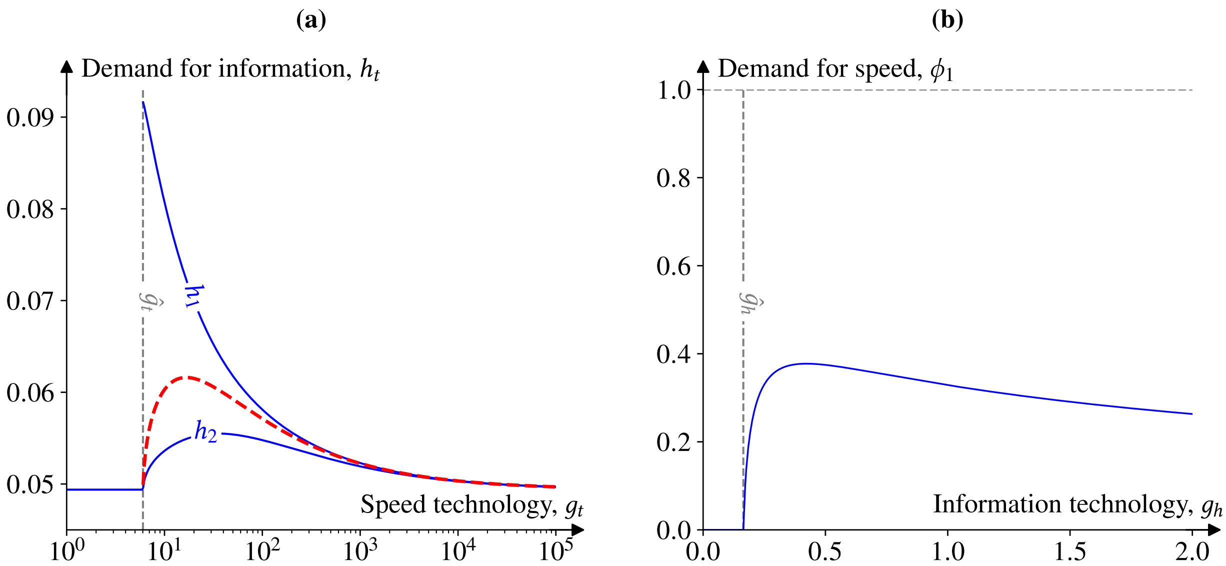


Figure 4: Cross-technology effects. This figure illustrates how an advancement in one technology affects the demand for the other. Panel (a) shows the cross-technology effect of the speed technology g_t and Panel (b) g_h. The vertical dashed lines indicate the thresholds of the corresponding technology, below which all investors stay slow. The red-dashed lines in Panel (a) indicates the aggregate demand for information, $\int_{i\in[0,1]} h_i \mathrm{d}i$. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k(c) = \sqrt{c}$. For Panel (a), $g_h = 0.2$. For Panel (b), $g_t = 10.0$.

▌ *In addition, $\partial h_1/\partial g_t < 0$; but $\partial h_2/\partial g_t > 0$ (< 0) for small (large) g_t.*

Such nonmonotone cross-technology effects are driven by various competition effects, i.e., crowding-out forces. Consider Figure 4 (b) for example. Three crowding-out effects arise after an advancement in g_h:

(1) *intra*temporally, each fast speculator acquires more information h_1 and crowds out each other, as in Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982);

(2) similarly, the slow at $t = 2$ crowd out each other; and

(3) *inter*temporally the fast crowd out the slow, because an increase in the intermediate τ_1 also improves the overall τ_2 (the market never forgets), hurting the certainty equivalent of slow speculators (from Equation 8, π_t decreases in τ_t and hence also in τ_{t-1}).

The first effect hurts fast speculators' rent π_1, making them less willing to acquire speed—reducing demand for speed. The second and the third effects hurt the slow, pushing them to compete with the fast at $t = 1$ instead—raising demand for speed. It is these countervailing crowding-out effects that drive the net demand for the two technologies.[10]

The endogenous complementarity/substitution between the two technologies is a key insight revealed by the model with speed acquisition. Only with endogenous speed heterogeneity, there arise three different *inter*/*intra*temporal crowding-out effects, which interact with investors' demand for information.

Returning to Figure 3 (b), the fluctuating τ_2 can now be understood as a combination of (1) the temporal fragmentation of the speed technology g_t and (2) its complementarity with information acquisition: As g_t starts to increase from $\hat{g}_t$, initially τ_2 is hurt due to the temporal fragmentation effect (Proposition 3.3). Without information acquisition, this would result in a U-shaped τ_2 similar to Figure 2 (b). With information acquisition, however, the initial higher g_t also raises (slow speculators') demand for information (Proposition 4.3) and $\Delta\tau_2$ sees a boost. When such complementarity is strong, it can overturn the temporal fragmentation effect, giving a fluctuating τ_2 as seen in Figure 3 (b). In Figure 3 (a), g_h is very low and the complementarity is not strong enough, thus generating no fluctuation in τ_2.

4.3.3 Information technology might hurt price efficiency

The effects of an advancing information technology are shown in Figure 5. Intuitively, as information becomes cheaper (higher g_h), both the fast and the slow speculators acquire more information, as shown in Panel (a).

[10] When the information technology g_h is low (close to $\hat{g}_h$), there are very few fast speculators (ϕ_1 close to zero). Therefore, Effect (2) dominates, stimulating slow ones to acquire speed and move to $t = 1$. As more speculators have acquired speed, Effect (3) strengthens and the remaining slow ones have growing incentive to become fast. These two forces result in complementarity between speed and information. However, when there are already many fast speculators (ϕ_1 close to one), Effect (1) dominates, as the *intra*temporal crowding-out effect scales with the population size ϕ_1. When it is no longer profitable to acquire speed, information substitutes speed, as shown in Figure 3 (b) for the range of roughly $g_h \geq 0.5$. Figure 3 (a) can be explained with these three crowding-out effects similarly.

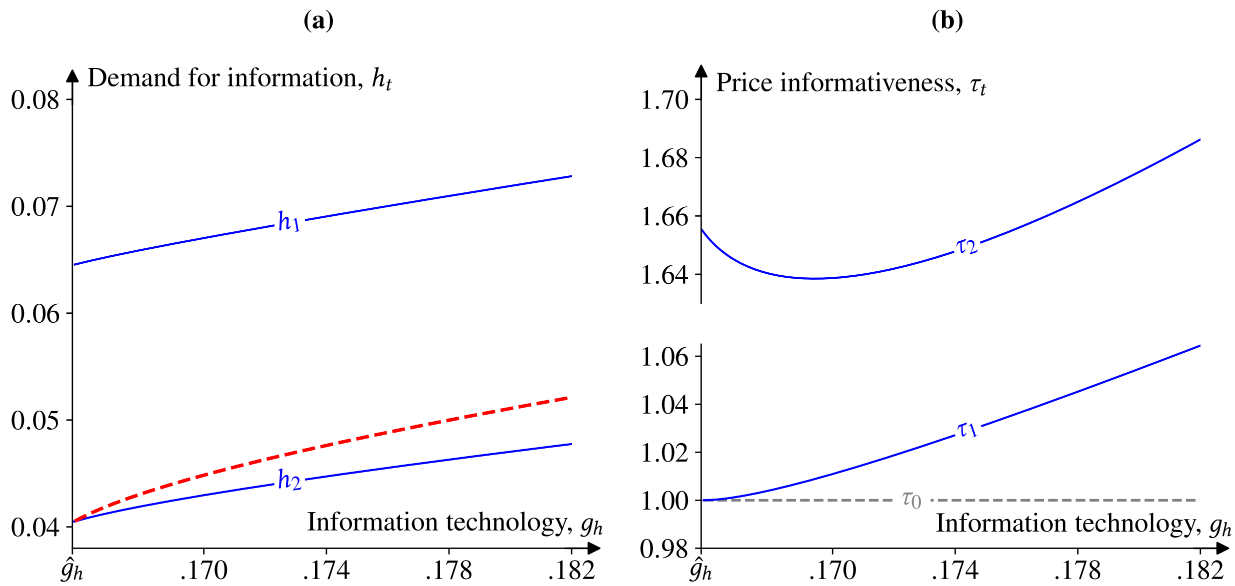


Figure 5: Varying information technology, with speed acquisition. This figure shows how the information technology g_h affects demand for information in Panel (a) and price informativeness τ_t in Panel (b). Only the region of $g_h \geq \hat{g}_h$ where there is interior equilibrium is shown. The red-dashed lines in Panel (a) indicates the aggregate demand for information, $\int_{i\in[0,1]} h_i \mathrm{d}i$. The primitive parameters are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, $g_t = 10.0$, and $k(c) = \sqrt{c}$.

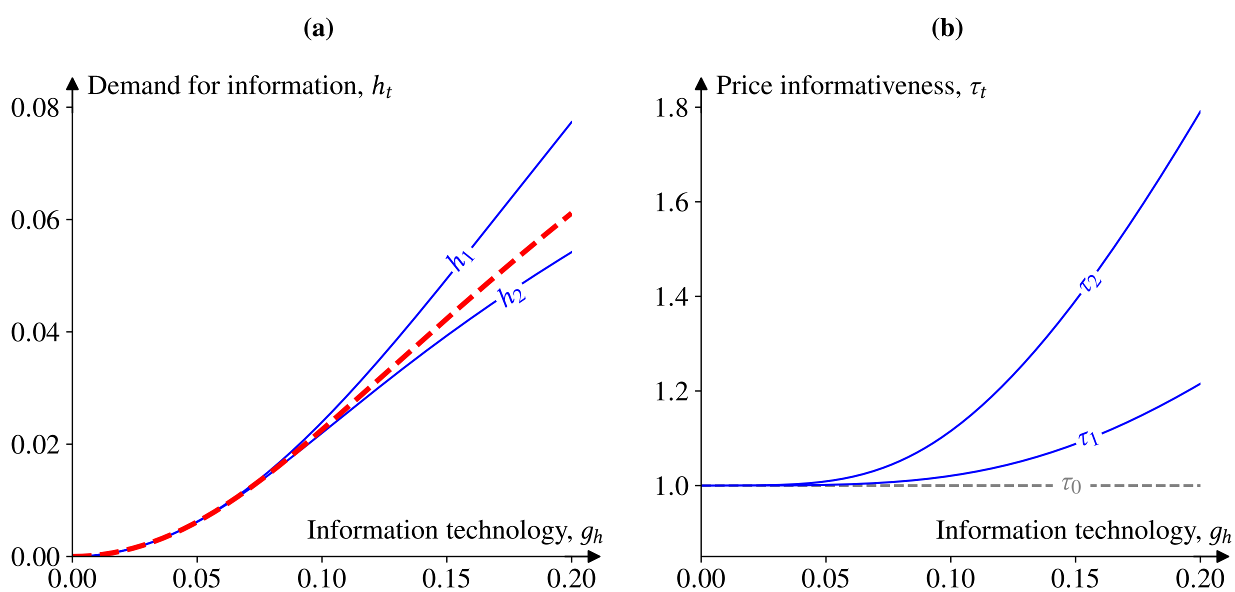


Figure 6: Varying information technology, without speed acquisition. This figure contrasts Figure 5 by shutting down speed acquisition. Specifically, a fixed population of $\phi_1 = 0.3$ are fast speculators and the rest $\phi_2 = 0.7$ are slow. The other primitive parameters are kept the same as in Figure 5: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k(c) = \sqrt{c}$.

Perhaps surprisingly, whereas everyone in the economy acquires more information (higher h_1 and h_2), only the intermediate price informativeness τ_1 is monotone increasing with g_h. The overall τ_2 sees an initial dip, which is only recovered when g_h further improves, as seen in Panel (b).

Proposition 4.4 (Information technology and price informativeness). *In the interior equilibrium, advancement in the information technology always improves the intermediate price informativeness τ_1. However, the overall τ_2 is initially hurt but eventually improved. Mathematically, $\partial \tau_1 / \partial g_h > 0$; and $\partial \tau_2 / \partial g_h < 0$ (> 0) for small (large) g_h.*

How could more intensive information acquisition *by everyone* still worsen the overall price informativeness? The answer lies in the process of price discovery—the speed. Recall (1) that speed technology temporally fragments price discovery; and (2) that the two technologies can exhibit complementarity (when g_h close to the threshold $\hat{g}_h$). As such, when g_h improves from $\hat{g}_h$, due to the complementarity, investors acquire both information and speed, triggering the temporal fragmentation effect. It turns out that when g_h is close to $\hat{g}_h$, the negative temporal fragmentation effect always overturns the positive effect of more intensive information acquisition.

To emphasize, the alarming message that better information technology (e.g., disclosure, announcements, etc.) can hurt price efficiency is a novel insight revealed only through the channel of speed acquisition. For example, without speed acquisition, the model reduces to a version of Verrecchia (1982) (with two sequential trading rounds), who shows that lowering the cost of information always improves price informativeness. Formally, in the current model:

Proposition 4.5 (Information technology and price informativeness, without speed acquisition). *Fixing speculators' speed, an increase in the information technology g_h always monotonically increases the demand for information and improves price informativeness. Mathematically, $\partial h_t / \partial g_h \geq 0$ and $\partial \tau_t / \partial g_h \geq 0$ when $\phi_1 \in [0, 1]$ and $\phi_2 = 1 - \phi_1$ are exogenously given.*

This contrast is illustrated by Figure 6, which shows that the nonmonotone pattern of τ_2 disappears without speed acquisition.

	(1) Information acquisition			**(2)** Speed acquisition			**(3)** Price informativeness	
	h_1	h_2	$\int_0^1 h_i \mathrm{d}i$	ϕ_1	ϕ_2	$\int_0^1 \mathbb{1}_{\{t_i=1\}} \mathrm{d}i$	τ_1	τ_2
(a) Only speed acquisition (Section 3)								
g_t:				$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow\nearrow^{\text{(i)}}$
(b) Both speed and information acquisition (Section 4)								
g_t:	$\searrow$	$\nearrow\searrow$	$\nearrow\searrow^{\text{(ii)}}$	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow\nearrow^{\text{(i)}}$
g_h:	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow\searrow$	$\searrow\nearrow$	$\nearrow\searrow^{\text{(ii)}}$	$\nearrow$	$\searrow\nearrow^{\text{(iii)}}$
(c) Only information acquisition (Proposition 4.5)								
g_h:	$\nearrow$	$\nearrow$	$\nearrow$				$\nearrow$	$\nearrow$

Table 1: Summary of effects of technology shocks. This table summarizes how technology affects the market in terms of (1) investors' information acquisition; (2) speed acquisition; and (3) price informativeness. Three settings are considered: the speculators (a) have exogenous information but can endogenously acquire speed; (b) can endogenously acquire both speed and information; and (c) have exogenous speed but can endogenously acquire information. Each row represents a positive shock in the respective technology, g_h for information and g_t for speed. Monotone increasing and decreasing effects are denoted by $\searrow$ and $\nearrow$, respectively. Nonmonotone effects are denoted by $\searrow\nearrow$ (initially decreasing but eventually increasing) and $\nearrow\searrow$ (initially increasing but eventually decreasing), respectively. Shaded cells highlight the key findings (i)-(iii) summarized in Section 4.3.4.

4.3.4 Summary of results

Table 1 provides a summary of all the comparative statics studied so far. Each column corresponds to a specific equilibrium outcome: information acquisition, speed acquisition, and price informativeness. Each row shows a (positive) shock in a specific technology. Three scenarios are studied: (a) only speed acquisition as in Section 3; (b) both speed acquisition and information acquisition as in this section; and for completeness, (c) only information acquisition.

The shaded cells highlight the three main findings: (i) the temporal fragmentation of speed, Proposition 3.3; (ii) the complementarity and substitution between speed and information, Proposition 4.3; and (iii) the nonmonotone effect of information technology on price informativeness, Proposition 4.4. In particular, comparing Panel (b) and (c), note that effect (iii) only arises when there is endogenous speed acquisition.

4.4 Empirical implications

4.4.1 Endogenous bundling of speed and information technology

It can be seen from the above numerical illustrations (e.g., Figure 4.a, 5.a, and 6.a) that conditional on their speed acquisition, a fast speculator always acquires more information than a slow one:

$$h_1 \geq h_2. \tag{11}$$

This is a robust result, for the price discovery process is always cumulative—rational agents do not forget ($\Delta\tau_t \geq 0$ by Equation 9). As such, the sooner a speculator can trade, the less price discovery has already happened ($\tau_t \leq \tau_{t+1}$), and the more valuable is his private information. To take this advantage, a fast speculator always has stronger incentive to acquire more information.[11]

This insight justifies a common connotation for fast traders that they are also more informed, popular in the high-frequency trading literature like Hoffmann (2014), Biais, Foucault, and Moinas (2015), and Budish, Cramton, and Shim (2015). In contrast, the current model predicts "endogenous bundling" of acquiring both information and speed. A large volume of empirical evidence, surveyed in Biais and Foucault (2014), O'Hara (2015), and Menkveld (2016), also echoes this view.

This result also contrasts against models like Dugast and Foucault (2018) and Kendall (2018) sharply, who assume that early signals are less informative than late ones. In those models, information acquisition and speed acquisition are "negatively" tied together, because information processing is time-consuming. The current model assumes away this friction and allows the speculators to speed up information processing (reaching the same trading strategy sooner).

[11] One should not mistake $h_1 \geq h_2$ for the complementarity between speed and information, as h_1 and h_2 are the information demand *conditional on* a speculator's speed. When acquiring technologies at $t = 0$, the unconditional demand for information is $\phi_1 h_1 + \phi_2 h_2$.

4.4.2 Testable predictions in the cross-section

The discussion below turns to the cross-section of assets for sharper predictions. The premise is that different assets are subject to varying information acquisition cost g_h. In the equity market, the empirical literature has examined changes in g_h using exogenous shocks (Hong and Kacperczyk, 2010; Kelly and Ljungqvist, 2012). The following predictions can be tested in similar settings.

Investor composition. Figure 4 (b) shows that fast speculators' participation in an asset is hump-shaped in its g_h. Consider for example a reduction in analyst coverage (e.g., due to merges of brokerage houses) in a stock, raising its information acquisition cost and lowering g_h. The model then predicts (Proposition 4.3) that fast speculators' participation in the affected stock can either go up or down, depending on the stock's pre-shock coverage. For an extensively covered stock (high g_h), the reduction in g_h will increase the number of fast speculators. For a lesser-known stock (low g_h, close to $\hat{g}_h$), the effect is the opposite.

Two empirical measures can be used to proxy for fast speculators' participation. A stock's quote-to-trade ratio is often used to gauge the prevalence of algorithmic and high-frequency trading (Hendershott, Jones, and Menkveld, 2011). Alternatively, to the extent that hedge funds are subject to less stringent regulatory compliance and reporting requirement (Remark 2), they can be thought of as the faster speculators than, e.g., mutual funds and pension funds. Under this measure of fast speculators, the hump-shape in Figure 4 (b) is consistent with the empirical finding by Griffin and Xu (2009, Figure 3).

Holdings, predictability, and performance. The model also yields cross-sectional predictions on speculators' (e.g., funds') performance. A speed-t speculator's (expected) trading profit is

$$\mathbb{E}[(V - p_t)x_i] = \frac{h_t}{\gamma \tau_t}.$$

Following the literature (e.g., Kacperczyk, van Nieuwerburgh, and Veldkamp, 2016), such performance can be equivalently interpreted as the *return predictability* of funds' holdings: cov[V −

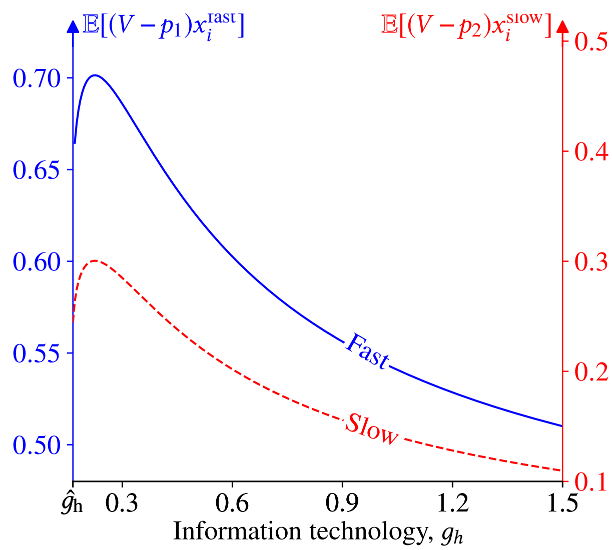


Figure 7: Performance. This figure illustrates fast and slow speculators' performance in the cross-section of assets sorted along information technology g_h. The blue-solid (the red-dashed) line shows the expected trading profit for the fast on the left axis (the slow on the right axis). The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$,$g_t = 10.0$, and $k(c) = \sqrt{c}$.

$p_t, x_i] = \mathbb{E}[(V - p_t)x_i]$. Intuitively, a fund's performance (information rent) is higher if and only if it predicts future return more precisely.

Figure 7 shows that both fast and slow speculators' performance exhibit hump-shapes in information technology g_h. Due to the initial complementarity, there are more speculators acquiring both speed and information. With fewer slow ones, their *intra*temporal competition alleviates, improving their performance. In the meantime, the increasing information technology overcomes the mild competition among the fast, also improving their performance. However, as all traders acquire more and more information, prices become very revealing, eventually crowding out everyone's rent. Both the fast and the slow' performance worsen.

To examine the above patterns, one can again focus on the different return predictability of fast and slow funds' holding changes. Since holding position data is usually at low frequency (monthly or quarterly), the "low-frequency speed" in Remark 1, i.e., the speed of acquiring and processing

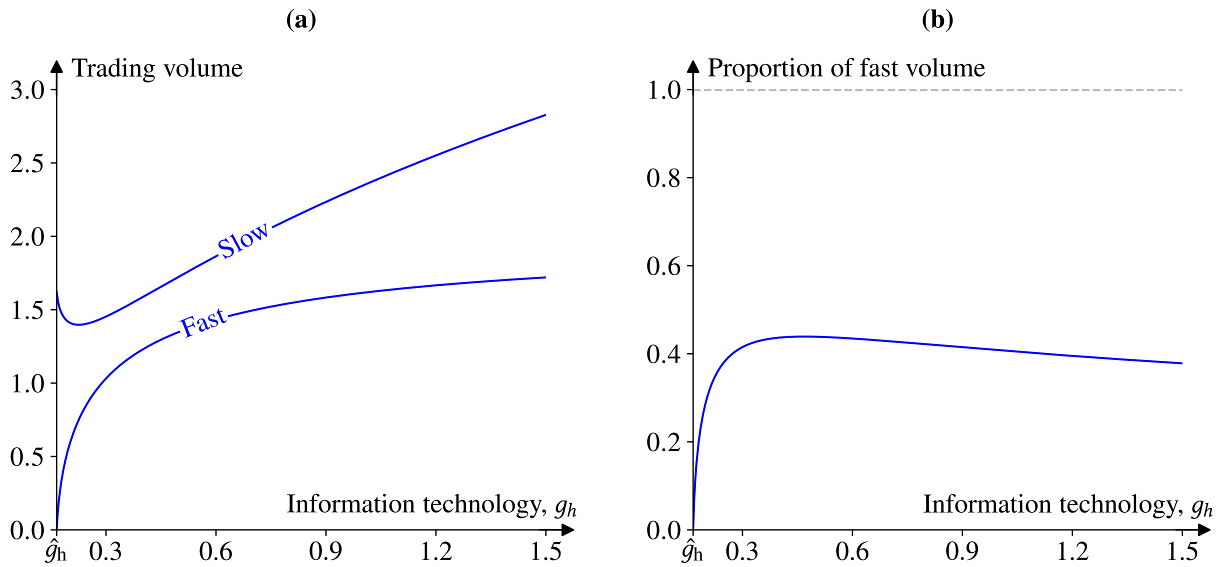


Figure 8: Trading volume and information technology. This figure illustrates how trading volume varies along the information technology g_h in the cross-section. Panel (a) shows the fast and the slow speculators' aggregate volume. Panel (b) plots the proportion of fast volume. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, $g_t = 10.0$, and $k(c) = \sqrt{c}$.

long-term information, is more appropriate. In this context, Swem (2017) provides evidence that (fast) hedge funds' trading predicts future returns better than other (slow) buy-side institutions. Such return predictability in the cross-section of information technology is still subject to further empirical examination. One existing evidence supportive of Figure 7 is by Shumway, Szefler, and Yuan (2011), who find that the return predictability of funds' holdings mainly manifests in medium stocks (assuming g_h is lower for larger stocks).

Trading volume. A more direct measure of fast and slow speculators' participation is to look at their trading volume. The total trading volume across all speed-t speculators is given by

$$\int_{i\in[0,1]} \mathbb{1}_{\{t_i=t\}} |x(s_i, p_t)| \mathrm{d}i = \phi_t \mathbb{E}\left|\frac{h_t}{\gamma}(s_i - p_t)\right| = \frac{\phi_t h_t}{\gamma}\sqrt{\frac{1}{\tau_t} + \frac{1}{h_t}}\sqrt{\frac{2}{\pi}}.$$

Figure 8 plots the volume patterns in the asset cross-section, sorted along exogenous g_h. Panel (a) shows that the fast speculators' volume monotonically increases with g_h. This is intuitive as in assets with easier access to information, speculators' signal precision is generally higher and they trade more aggressively (recall that $x_i = h_i \cdot (s_i - p_t)/\gamma$). This intuition, however, is challenged for the slow speculators, whose trading volume first drops with g_h before increasing again. This is because of the initial complementarity between information and speed: There are more speculators acquire speed, so fewer slow speculators and lower volume by them. Consistently, Panel (b) plots the proportion of fast speculators' volume, relative to the total volume. It is initially increasing but then gradually decreasing along the g_h cross-section of assets.

4.4.3 Market-wide trends

Speed technology. The advancement of speed technology is one of the most salient phenomenon in securities trading in recent years. Such speed technology speaks not only to the subsecond high-frequency trading technology, but more broadly also to how algorithms automate the whole trading process, from information processing (live streaming of news and data like Bloomberg) to risk management and compliance (Bouvard and Lee, 2016).

With this trend, the model suggests the following narrative: On the one hand, the market has seen a drastic rise of machines, characterized most noticeably by their ability to trade fast. This is consistent with the prediction of increasing fast traders in investor composition (Proposition 3.2). On the other, the implication on price informativeness appears mixed. In particular, short-term (intraday) measures of price efficiency (order flow predictability, serial correlation, etc.) seem to improve (Brogaard, Hendershott, and Riordan, 2014; Hirschey, 2018), while long-term price informativeness seems negatively affected (e.g., Weller, 2018). Such mixed findings are consistent with the patterns shown in Figure 2 (b) (Proposition 3.3).

Information technology (accessibility). Regulators have strengthened transparency and disclosure requirements in recent years. Policies like Sarbanes-Oxley, Regulation Fair Disclosure, and Rule 10b5-1 have arguably reduced the cost of information acquisition. In this context, the model cautions against how the improved information accessibility (technology) might negatively impact price informativeness, through the complementarity with speed acquisition (Proposition 4.4). Such complementarity is evidenced by Du (2015), who finds that high-frequency traders are constantly crawling the website of the SEC in order to trade on the information in latest company filings.

A number of recent studies share qualitatively similar caveats. For example, Dugast and Foucault (2018) shows that the acquisition of raw information can crowd out processed information, thus hurting the overall price informativeness. Banerjee, Davis, and Gondhi (2018) show that a public announcement could worsen price informativeness, because investors would switch to learning about others' beliefs instead of the fundamental. Both mechanisms feature some *substitution* between different sources of information. To compare, the novel mechanism studied here is due to the joint effect of *the endogenous complementarity* between information and speed (Proposition 4.3) and the temporal fragmentation of speed (Proposition 3.3). Note that the effect does not exist if speed acquisition is shutdown; c.f. Panel (c) of Table 1.

5 Discussion and robustness

The model builds on a number of specific assumptions, as highlighted in Remark 3, to facilitate tractability. The purpose of this section is to relax these assumptions. For comparison, the model presented in Section 3 and 4 will be referred to as "the baseline," while the subsections below will be "the extensions." Detailed analyses of the extensions are deferred to the supplementary material. The discussion here mainly focuses on the intuition of the robustness.

5.1 Endogenous population size

In the baseline model, the population size is fixed at $\phi_1 + \phi_2 = 1$. This appears to "mechanically" create the temporal fragmentation effect: As the speed technology prompts speculators to acquire speed, a higher ϕ_1 implies a lower ϕ_2, thus fragmenting price discovery.

An extension in Section S1 in the supplementary material studies the robustness of the results by endogenizing the speculator population. The analysis is briefly summarized here. The only modification of the baseline is the free-entry of speculators: There is a continuum of speculators indexed on $i \in [0, \infty)$ who can choose to trade or not. Following the literature (e.g., Bolton, Santos, and Scheinkman, 2016), they are sorted according to their reservation value $R(i)$ for not trading (e.g., an opportunity cost). Specifically, if speculator i chooses not to trade, he obtains a certainty equivalent of $R(i)$, which is monotone increasing in i. Equivalently, $R(i)$ can be interpreted as speculator i's entry cost and $\forall i < j$, speculator i has higher comparative advantage in trading than j.

As no other model assumptions are changed, conditional on entry, speculators trade just like in the baseline and Lemma 4.1 holds. The (interior) equilibrium is pinned down by conditions similar to those stated in Proposition 4.1. Each speed-t speculator acquires information h_i according to first-order condition $\partial\pi/\partial h_i = 0$; and should be indifferent between fast or slow: $\pi_1 - R(i) = \pi_2 - R(i)$. The condition that endogenously determines the population size is

$$\pi_1 = \pi_2 = R(\phi_1 + \phi_2),$$

which says that the marginal investor is indifferent of trading fast, trading slow, or no trading. Equivalently, if the monotonicity of $R(\cdot)$ is strict, $\phi_1 + \phi_2 = R^{-1}(\pi_1) = R^{-1}(\pi_2)$. To compare, the population size condition under the baseline is $\phi_1 + \phi_2 = 1$.

Consider a speed technology shock in g_t, after which ϕ_1 increases (more fast speculators), implying more early price discovery; i.e., $\Delta\tau_1$ increases. Having resolved more price discovery at $t = 1$, there is less information rent left for the slow. Therefore, there will be fewer speculators who enter to trade slowly; i.e., ϕ_2 decreases. This is the temporal fragmentation effect: an advancement in g_t increases ϕ_1 but reduces ϕ_2. Panel (a) of Figure 9 demonstrates the patterns of the key results under a linear parametrization of $R(i) = i$. They are qualitatively the same as in the baseline. (Extensive numerical analysis suggests that the choice of $R(\cdot)$ does not affect the qualitative patterns.)

The key intuition behind the robustness of temporal fragmentation is that *the fast and the slow* *split the same piece of pie*. When the speed technology benefits the fast, the slow are hurt (relative to the fast) and thus some of them must be crowded out from $t = 2$. Either they become fast as in the baseline (where the total population is fixed), or they stay out of trading as in this extension (where there is free entry). In either case, temporal fragmentation arises: There will be relatively more fast speculators than slow ones after a positive speed shock.

5.2 Liquidity timing

The overall price informativeness can be expressed as

$$\tau_2 = \tau_0 + \Delta\tau_1 + \Delta\tau_2 = \tau_0 + \frac{(\phi_1 h_1/\gamma)^2}{\mathrm{var}[U_1]} + \frac{(\phi_2 h_2/\gamma)^2}{\mathrm{var}[U_2]}$$

where each fragment of price discovery $\Delta\tau_t$ (Equation 9) is essentially the information-to-noise ratio in that trading round. In the baseline model, technology shocks only affect the numerators of $\Delta\tau_t$, via speculators' equilibrium responses.[12] A natural question is whether there are effects on the denominators and if so, how they would affect price informativeness.

To address this question, Section S2 of the supplementary material endogenizes the denominators. The extension is briefly described here. Following the literature (e.g., Admati and Pfleiderer, 1988), two types of liquidity (noise) traders are introduced. First, there are nondiscretionary liquidators who must trade U_1 at $t = 1$ and U_2 at $t = 2$. These correspond to the noises in the baseline. Second, there are discretionary liquidators, who upon paying the same speed acquisition cost $1/g_t$ can trade early at $t = 1$ but otherwise late at $t = 2$. They optimally time their liquidation to minimize expected execution cost. Taken together, the amount of liquidation flow (noise) at t aggregates to $U_t + \psi_t Q_t$, where $\psi_1 \in [0, 1]$ is the discretionary liquidators' endogenous demand for speed and Q_t is their (time-varying) systemic liquidation need. The price discovery $\Delta\tau_t$ can then

[12] Recall from Lemma 4.1 that h_t/γ is a speed-t speculator's trading aggressiveness on his private signal. The numerator in $\Delta\tau_t$ is essentially the square of the aggregate aggressiveness: $(\phi_t h_t/\gamma)^2 = \left(\int_{\{t_i=t\}} h_i/\gamma \mathrm{d}i\right)^2$.

be written as an information-to-noise ratio with numerator and denominator both endogenized:

$$\Delta\tau_t = \frac{(\phi_t h_t/\gamma)^2}{\tau_{\mathrm{U}}^{-1} + \psi_t^2 \tau_{\mathrm{Q}}^{-1}}. \tag{12}$$

Note that this extension nests the baseline as a special case when the discretionary liquidators' trading need vanishes; i.e., when $\mathrm{var}[Q_t] = \tau_{\mathrm{Q}}^{-1} \to 0$. The nondiscretionary liquidators can also be shut down by taking $\mathrm{var}[U_t] = \tau_{\mathrm{U}}^{-1} \to 0$.

The analytic tractability of the extension is very limited, due to the complexity in analyzing the highly nonlinear equilibrium conditions. Fortuitously, extensive numerical analyses seem to suggest that there exists a stable equilibrium. The key results of the baseline remain robust in such an equilibrium, as shown in Panel (b) of Figure 9. In particular, the overall τ_2 is still nonmonotone in the speed technology g_t: The temporal fragmentation effect stays. Note from the left-most panel that ϕ_1 rises but ψ_1 drops with g_t; i.e. more speculators acquire speed but fewer discretionary liquidators do so. This makes $\Delta\tau_1$ increasingly larger, while $\Delta\tau_2$ smaller.

Intuitively, shocks in the speed technology g_t have two opposing forces on discretionary liquidators' demand for speed ψ_1. On the one hand, the speed cost is lower and the demand should rise. On the other, the speculators' demand also rises and their trading at $t = 1$ imposes additional adverse-selection cost for the discretionary liquidators, reducing their demand for speed. In the extensive numerical analyses, the second negative effect always dominates. One potential explanation is that the adverse-selection at $t = 2$ is always reduced with more speculators acquiring speed (more price discovery has already happened at $t = 1$). Thus, the incentive for the discretionary liquidator not to acquire speed becomes stronger, pulling them back to $t = 2$.

The bottom line is that even ϕ_1 and ψ_1 move hand-in-hand, the temporal fragmentation effect of the speed technology is hard to be silenced completely. A necessary condition for the overall τ_2 to be immune from g_t is that the changes in the numerator and the denominator of $\Delta\tau_t$—the information-to-noise ratio—*exactly offset* each other (Equation 12). The only knife-edge case where such offsetting holds seems to be the irrelevance result in Dávila and Parlatore (2017), where all market participants are ex-ante homogeneous, subject to the same hedging shocks. As soon as there is some minimum heterogeneity across agents, $\Delta\tau_t$ will be affected nonmonotonically by the technology shocks and the temporal fragmentation effect will manifest.

5.3 Frequent fast trading

The overall price informativeness τ_2 comprises of two endogenous fragments, $\Delta\tau_1$ and $\Delta\tau_2$. In the baseline model, fast speculators only trade at $t = 1$ and thus only contribute to the early $\Delta\tau_1$. This is a simplification to capture the main intuition of the results. More realistically, speed technology should enable speculators not only to trade early, but also frequently. Would more frequent trading by fast speculators smooth out the price discovery process $\{\Delta\tau_t\}$ and undo the temporal fragmentation of the speed technology?

To answer this question, Section S3 in the supplementary material allows the fast speculators to trade in both $t \in \{1, 2\}$; i.e., "frequent fast trading". All other model structures remain the same as in the baseline. There it is shown that a fast speculator i's *cumulative* demand at time t has the form of $x_{it} = \frac{h_i}{\gamma}(s_i - p_t)$, a special case of Vives (1995) with two trading periods. This means at $t = 2$, he only trades his *net* demand:

$$x_{i2} - x_{i1} = \frac{h_i}{\gamma}(s_i - p_2) - \frac{h_i}{\gamma}(s_i - p_1) = -\frac{h_i}{\gamma}(p_2 - p_1),$$

which does *not* reflect his private signal s_i. In fact, the only reason he trades at $t = 2$ is to rebalance his holding according to the price change $p_2 - p_1$. As such, a fast speculator's trading at $t = 2$ contributes nothing to the price discovery $\Delta\tau_2$. Only the slow speculators' trading adds to $\Delta\tau_2$. The resulting dynamics of price discovery $\Delta\tau_t$ are exactly the same as in the baseline (Lemma 4.1) and all results go through. Panel (c) of Figure 9 replicates the key results.

It is well-known that in such rational expectations equilibrium models, informed investors' aggressiveness on their private signal s_i is always the ratio between the precision of the signal h_i and their risk-aversion γ, even in a dynamic framework, with (Vives, 1995; and Cespa, 2008) or without the competitive market maker (Cespa and Vives, 2012, 2015). This is because the investors are *competitive*. If one does not trade on his private signal as much and as early as possible, his information rent will be eroded by others. Thus, everyone only contributes once to the price discovery process at his earliest opportunity. To this extent, the baseline results remain robust with frequent fast trading, as long as the economy in question can be reasonably approximated by competitive speculators.

Two further extensions along this line are worth exploring in future research: 1) to consider large speculators' price impact and study how that would interact with the technologies differently, e.g., in models like Kyle (1985, 1989); and 2) to allow fast speculators to acquire information multiple times. Such extensions will enrich the current model but its key mechanisms (like the speed technology's temporal fragmentation effect) will remain.

5.4 The market clearing mechanism

The speculators' demand schedules are cleared by a competitive market maker, who sets the price conditional all available information. The purpose of having such a market maker is that he helps ensure the trading price p_t is always (semi-strong) efficient (as in Kyle, 1985; and Vives, 1995) and this suits the focus on price informativeness of this study.

A competitive market maker is not the only way to facilitate trading. An alternative is to determine the price p_t via market clearing, e.g., as in Grossman and Stiglitz (1980). Section S4 of the supplementary material re-examines the model by replacing the competitive market maker with a fringe of uninformed investors of mass n. All other model specifications remain the same as in Section 4. It is shown that a speculator i's *cumulative* demand at round t always takes the well-known form

$$x_{it} = \frac{h_i}{\gamma}(s_i - m_t) - a_{it} \cdot (p_t - m_t)$$

where p_t is the market clearing price, $m_t := E[V \mid p_t, p_{t-1}, \ldots]$ is the (semi-strong) efficient price, and $a_{i,t}$ is some constant up to speculator i and time t. The demand function shows that there are two trading motives for the speculator: First, his demand scales with the valuation difference between his private signal and the efficient price. Second, he trades against the mispricing $p_t - m_t$ (i.e., the price pressure). Compared to the baseline, the second component is new, because when the competitive market maker exists and sets $p_t = m_t$, the above demand reduces to $x_{it} = \frac{h_i}{\gamma}(s_i - p_t)$ as in Lemma 4.1. Similarly, an uninformed investor's demand in round t is shown to be

$$y_{it} = -b_{it} \cdot (p_t - m_t).$$

That is, he only trades against the mispricing, serving as a market maker. (It can be shown that this extension nests the baseline as a special case with the measure of uninformed investors $n \to \infty$.)

It turns out that all baseline results remain robust in this extension. Panel (d) of Figure 9 shows the patterns. Intuitively, as speculators still trade on private signals s_i with the same aggressiveness as in the baseline, the price discovery recursion of $\Delta\tau_t$ remains the same as stated in Equation (9). Their additional trading on mispricing $p_t - m_t$ does not contribute to price discovery. More specifically, consider the aggregate demand $L_t(p_t)$ in round t (similar to Equation 7). In the baseline, the competitive market maker sets price efficiently: $p_t = m_t = \mathbb{E}[V \,|\, L_t(\cdot), L_{t-1}(\cdot), \dots]$. In this extension, while the market clearing price is solved from $L_t(p_t) = 0$, in terms of information and learning, the (semi-strong) efficient price m_t is the same $m_t = \mathbb{E}[V \,|\, L_t(\cdot), L_{t-1}(\cdot), \dots]$. This explains why introducing the competitive market maker in the baseline model is without loss of generality for the purpose of understanding price informativeness and price discovery.[13]

5.5 Dependence between the two technologies

In the baseline model, the two technology levels do not affect each other. Such independence between g_t and g_h need not necessarily be the case. They can complement each other, for example,

[13] It is worth emphasizing that allowing frequent fast trading in this setting still does not affect price discovery. Just as in Section 5.4, fast speculators' *net* demand at $t = 2$, $x_{i2} - x_{i1}$, remains independent of the private signal s_i—they only contribute to the early price discovery $\Delta\tau_1$.

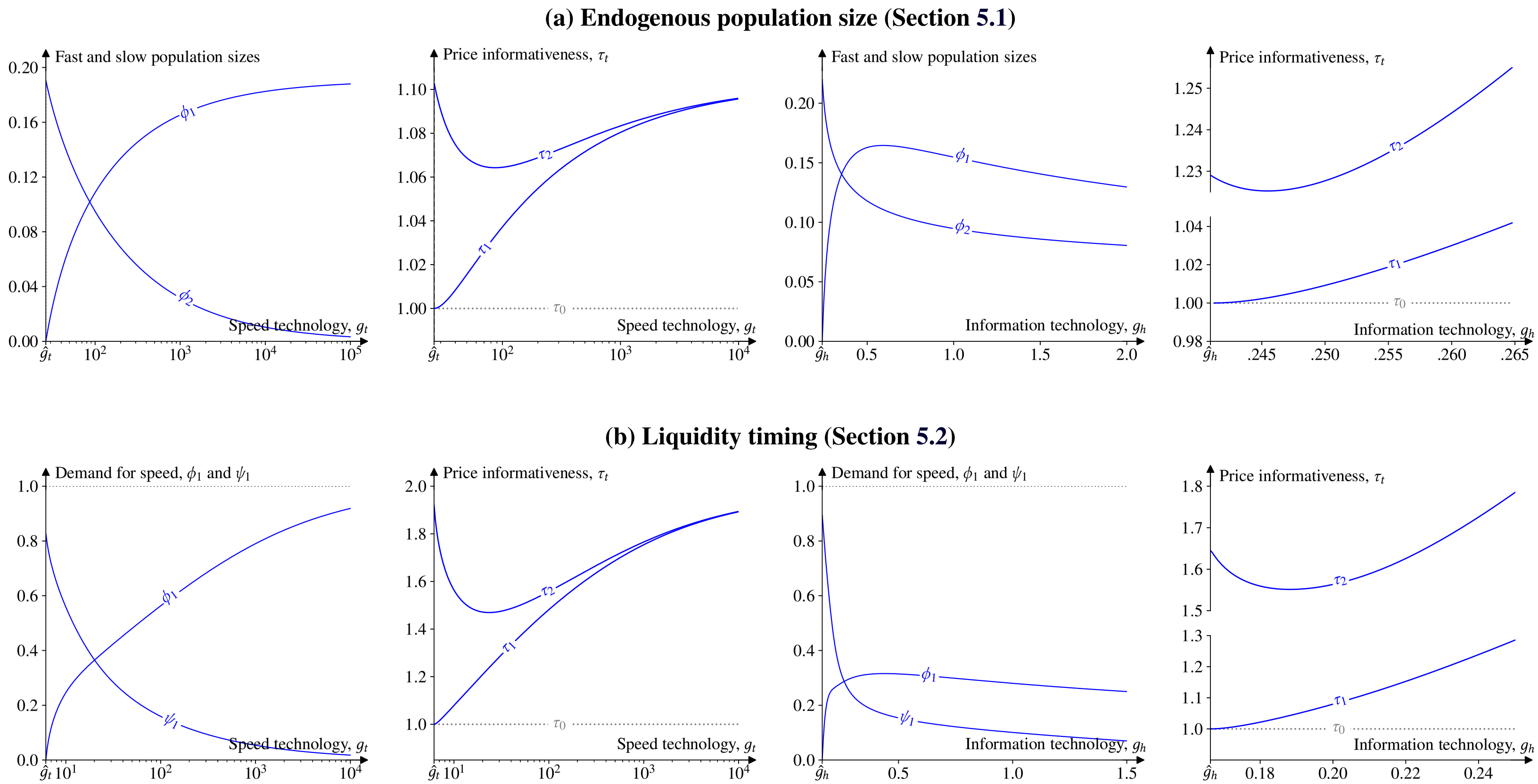
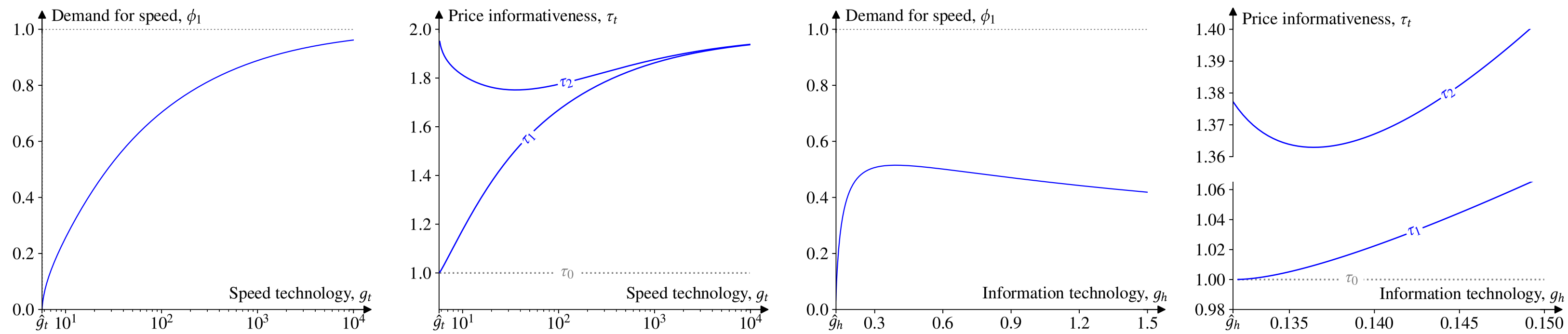


Figure 9: Robustness under various extensions. This figure shows the main results of the baseline model remain robust under various extensions: Panel (a) for endogenous population size; (b) for liquidity timing; (c) for frequent fast trading; and (d) for market clearing. Each panel shows four selected graphs: The first most-left two show the temporal fragmentation effect of the speed technology (Proposition 4.2; c.f. Figure 2); the third shows the initial complementarity and the eventual substitution between speed and information (Proposition 4.3; c.f. Figure 4.b); and the right-most shows how information technology might hurt price informativeness (Proposition 4.4; c.f. Figure 5.b). Only the region with interior equilibrium is plotted. The details of the numerical illustration are deferred to the supplementary material.

(c) Frequent fast trading (Section 5.3)



(d) Market clearing (Section 5.4)

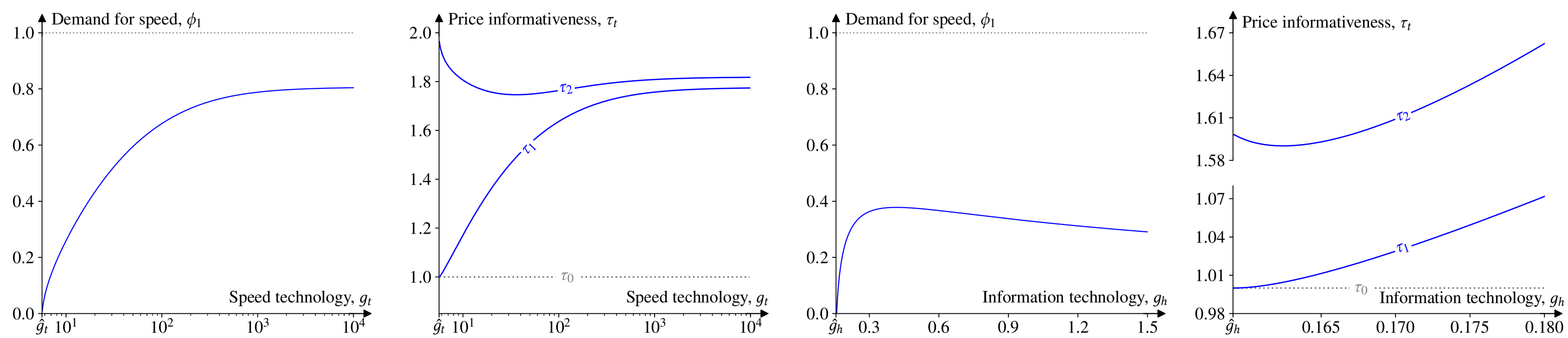


Figure 9 continued.

because of the shared hardware (e.g., CPUs). Having invested in one technology can therefore reduce the cost for the other (e.g., g_h increases in g_t).

Substitution between the two is also possible. Dugast and Foucault (2018) argue that because processing information takes time, investors trading on "processed" information are intrinsically slower than those trading on "raw" information. That is, investing in one technology might increase the (marginal) cost for the other (e.g., g_h decreases in g_t).

Exactly how speed and information technologies interfere with each other is perhaps a question of engineering and computer science. The current model sets a benchmark with independent technologies—an agnostic view. The outcomes of the model, therefore, offer a clean set of predictions on investors'/speculators' *endogenous* demand for the two technologies, as opposed to the exogenous, built-in substitution/complementarity.

One can use the current model as a starting point to study implications of built-in substitution or complementarity between the two technologies. Figure 10 plots price informativeness τ_1 (red-dashed line) and τ_2 (blue-solid line) on a contour of (g_t, g_h).[14] When there is complementarity, the effect of an increase in one technology can be examined by, e.g., the left (green) arrow in the figure (g_h increases from 0.15 to 0.16, while g_t increases from about 10 to 100). If instead the substitution of the technologies dominates, the effect can be seen from, e.g., the right (blue) arrow (g_h mildly increases from 0.125 to 0.135, while g_t drops sharply from about 4,000 to 50). In both examples, the long-run price informativeness τ_2 (blue-solid contour lines) drops. Note that the right (blue) arrow is consistent with Dugast and Foucault (2018) and Kendall (2018), who show that when processing information takes time, better information might hurt price informativeness.

[14] Note the pattern shown is consistent with Propositions 3.3 and 4.4. Moving right on a horizontal cut of Figure 10, the information technology g_h is fixed and as the speed technology g_t improves, the short-run price informativeness τ_1 monotonically increases, while the long-run price informativeness τ_2 first decreases and then increases. Moving upward on a vertical cut, g_t is fixed and as g_h increases, τ_1 monotonically increases but τ_2 first decreases and then increases.

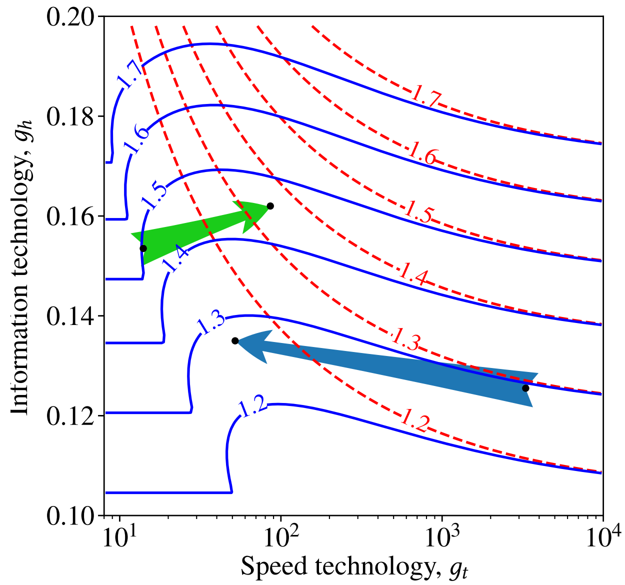


Figure 10: Price informativeness plotted against both technologies. This contour graph plots how the long-run price informativeness τ_2, in blue-solid line, and the short-run price informativeness τ_1, in red-dashed line, vary with the two technologies, g_t and g_h. The two arrows illustrates the different effects of an information technology advancement. The left arrow (green) shows complementarity between the two, while the right arrow (blue) shows substitution. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k(c) = \sqrt{c}$.

6 Conclusion

This paper studies a model with endogenous speed acquisition, alongside the conventional information acquisition. In the interior equilibrium, some speculators acquire speed and become fast, while the rest stay slow—trading is endogenously fragmented into parts, and so is the price discovery process. Such "temporal fragmentation effect" of the speed technology drives the fraction of fast speculators in the economy, affecting the concentration of trading and price discovery. In addition, speculators' demand for information and speed can be either complements or substitutes, depending on the relative strengths of various competition effects. Based on the interaction of these two novel mechanisms, the model generates testable implications for how technologies could affect various market quality. Most notably, when either the speed or the information technology improves, price informativeness can be hurt. This provides a cautionary tale of the disruptive effects of how technological advancement, as seen in recent years, might hinder the price discovery function of financial markets.

Appendix

Proofs

Lemma 3.1 and 4.1

Proof. This proof encompasses both lemmas as special cases by allowing each speculator's signal precision h_i to differ, as analyzed more generally in Vives (1995). Conjecture that a fast speculator i's demand schedule is $x_i = a_{i,1}s_i - b_{i,1}p_1$ and that a slow speculator i's demand schedule is $x_i = a_{i,2}s_i - b_{i,2}p_1 - c_{i,2}p_2$. At $t = 1$, there are only fast speculators and the aggregate demand is

$$L_1(p_1) = \int_{i\in[0,1]} x_i(p_1, s_i)\mathbb{1}_{\{t_i=1\}}\mathrm{d}i + U_1 = \left(\int_{t_i=1} a_{i,1}\mathrm{d}i\right)V - \left(\int_{t_i=1} b_{i,1}\mathrm{d}i\right)p_1 + U_1,$$

where the convention $\int \varepsilon_i \mathrm{d}i = 0$ is used. From the market maker's perspective, the sufficient summary statistic, therefore, is the intercept of the above linear demand, which can be transformed into $z_1 := V + U_1/\left(\int_{t_i=1} a_{i,1}\mathrm{d}i\right)$. Therefore, using standard property of normal distribution,

$$\tau_1 = \mathrm{var}[V \,|\, L_1(\cdot)]^{-1} = \tau_0 + \left(\int_{t_i=1} a_{i,1}\mathrm{d}i\right)^2 \tau_{\mathrm{U}}. \tag{13}$$

The incremental price discovery is $\Delta\tau_1 = \left(\int_{t_i=1} a_{i,1}\mathrm{d}i\right)^2 \tau_{\mathrm{U}}$. The maker maker sets the efficient price

$$p_1 = \mathbb{E}[V \,|\, L_1(\cdot)] = \mathbb{E}[V \,|\, z_1] = \frac{\tau_0}{\tau_1}p_0 + \frac{\Delta\tau_1}{\tau_1}z_1. \tag{14}$$

As such, the trading price p_1 is an equivalent statistic of z_1. From a fast speculator's perspective, $\mathrm{var}[V \,|\, s_i, p_1]^{-1} = \mathrm{var}[V \,|\, s_i, z_1]^{-1} = h_i + \tau_1$ and $\mathbb{E}[V \,|\, s_i, p_1] = \mathbb{E}[V \,|\, s_i, z_1] = (\tau_0 p_0 + h_i s_i + \Delta\tau_1 z_1)/(\tau_1 + h_i)$. Using the above, a CARA fast speculator i's optimal demand is

$$x_i = \frac{\mathbb{E}[V \,|\, s_i, p_1] - p_1}{\gamma \mathrm{var}[V \,|\, s_1, p_1]} = \frac{1}{\gamma}(h_i s_i + \Delta\tau_1 z_1 - (\tau_0 + h_i + \Delta\tau_1)p_1) = \frac{h_i}{\gamma}(s_i - p_1).$$

(Recall the normalization $p_0 = 0$.) The conjectured linear demand $x_i = a_{i,1}s_i - b_{i,1}p_1$ for fast speculators has thus been verified with coefficients $a_{i,1} = b_{i,1} = h_i/\gamma$.

At $t = 2$, the slow speculators' aggregate demand is

$$L_2(p_2; p_1) = \int_{i\in[0,1]} x_i(p_2, s_i; p_1)\mathbb{1}_{\{t_i=2\}}\mathrm{d}i + U_2$$
$$= \left(\int_{t_i=2} a_{i,2}\mathrm{d}i\right)V - \left(\int_{t_i=2} b_{i,2}\mathrm{d}i\right)p_1 - \left(\int_{t_i=2} c_{i,2}\mathrm{d}i\right)p_2 + U_2,$$

Recalling p_1, the market maker updates his information set to $\{p_1, z_2\}$, where $z_2 := V + U_2/\left(\int_{t_i=2} a_{i,2}\mathrm{d}i\right)$ summarizes the new information in $L_2(\cdot)$. Then,

$$\tau_2 = \mathrm{var}[V|\, p_1, L_2(\cdot)]^{-1} = \mathrm{var}[V|\, z_1, z_2\,]^{-1} = \tau_1 + \left(\int_{t_i=2} a_{i,2}\mathrm{d}i\right)^2 \tau_{\mathrm{U}}, \tag{15}$$

where the incremental price discovery $\Delta\tau_2 = \left(\int_{t_i=2} a_{i,2}\mathrm{d}i\right)^2 \tau_{\mathrm{U}}$. The market maker then sets the efficient price

$$p_2 = \mathbb{E}[V|\, p_1, L_2(\cdot)] = \mathbb{E}[V|\, z_1, z_2\,] = \frac{\tau_0}{\tau_2}p_0 + \frac{\Delta\tau_1}{\tau_2}z_1 + \frac{\Delta\tau_2}{\tau_2}z_2. \tag{16}$$

A slow speculator updates $\mathrm{var}[V|\, s_i, p_1, p_2\,]^{-1} = \mathrm{var}[V|\, s_i, z_1, z_2\,]^{-1} = h_1 + \tau_2$ and $\mathbb{E}[V|\, s_i, p_1, p_2\,] = \mathbb{E}[V|\, s_i, z_1, z_2\,] = (\tau_0 p_0 + \Delta\tau_1 z_1 + \Delta\tau_2 z_2 + h_i s_i)/(\tau_2 + h_i)$. Solving a quadratic optimization problem, a CARA slow speculator's optimal demand is

$$x_i = \frac{\mathbb{E}[V|\, s_i, p_1, p_2\,] - p_2}{\gamma \mathrm{var}[s_1, p_1, p_2]} = \frac{1}{\gamma}(h_i s_i + \Delta\tau_1 z_1 + \Delta\tau_2 z_2 - (\tau_0 + \Delta\tau_1 + \Delta\tau_2 + h_i)p_2) = \frac{h_i}{\gamma}(s_i - p_2).$$

Thus the conjectured linear demand for slow speculators is also verified with coefficients $a_{i,2} = c_{i,2} = h_i/\gamma$ and $b_{i,2} = 0$. That is, the slow speculator's demand is independent of p_1.

The analysis so far has proved the speculators' optimal demand as stated in the lemmas. In particular, for Lemma 3.1, $h_i = h_\circ$ for all $i \in [0, 1]$; and for Lemma 4.1, $h_i = h_{t_i}$. In the meantime, Equations (13) through (16) verify the recursion systems of p_t and $\Delta\tau_t$. It remains to compute the speculators' ex ante certainty equivalent. Consider a fast speculator. Before accounting for the technology acquisition cost, his expected utility at $t = 0$ is $-\mathbb{E}\Big[\exp\Big\{-\frac{[\mathbb{E}[V|\, s_i, p_1\,] - p_1]^2}{2\mathrm{var}[V|\, s_i, p_1\,]}\Big\}\Big]$. The expressions derived earlier yield the following: $\mathbb{E}[V|\, s_i, p_1\,] - p_1 = \frac{h_i}{\tau_1 + h_i}\left(\frac{\tau_0}{\tau_1}V + \varepsilon_i - \frac{\Delta\tau_1}{\tau_1}\frac{U_1}{\int_{t_j=1}(h_j/\gamma)\mathrm{d}j}\right)$ and $\mathrm{var}[V|\, s_i, p_1\,]^{-1} = \tau_1 + h_i$. Plug the above into the $t = 0$ expected utility for a fast speculator, simplify, and the resulting ex ante certainty equivalent *before technology acquisition costs* is $\frac{1}{2\gamma}\ln\left(1 + \frac{h_i}{\tau_1}\right)$. Subtracting the information acquisition cost and the speed acquisition cost gives the expression stated in the lemmas, with $h_i = h_\circ$ for Lemma 3.1; and $h_i = h_{t_i}$ for Lemma 4.1. The calculation for slow speculators repeats the above and is omitted. □

Proposition 3.1 and 4.1

Proof. This proof encompasses both propositions. Write the speculators' certainty equivalent π_1 and π_2 as functions of the fast population size $\phi_1 \in [0, 1]$. To see this, note from Equation (8) that π_t is a function of h_t and τ_t. Then, from the first-order condition (10), speculators' endogenous choice of h_i can be written as a function of τ_{t_i}. (For Proposition 3.1, there is no information acquisition and $h_i = h_\circ$ is a degenerate function of τ_{t_i}.) Finally, $\tau_1 = \tau_0 + \Delta\tau_1$ and $\tau_2 = \tau_1 + \Delta\tau_2$, where $\Delta\tau_t = \tau_U \phi_t^2 h_t^2/\gamma^2$. Hence, τ_1 is effectively a function of ϕ_1, while τ_2 of both ϕ_1 and $\phi_2 = 1 - \phi_1$. As such, speculators' certainty equivalent π_t are functions of ϕ_1. Then, depending on ϕ_1, there are three cases.

Case 1: Suppose $\phi_1 = 1$ and $\phi_2 = 0$; i.e. all speculators pay the speed technology cost $1/g_t$ and become fast. If this is the case, then in equilibrium $\pi_1 \geq \pi_2$ must hold. Consider a speculator i's unilateral deviation to not investing in the speed technology, saving the cost of $1/g_t$ and becomes slow. By Equation (9), the price informativeness remains the same, $\tau_1 = \tau_2$, because a single speculator's deviation has zero population measure. Then i's optimal technology investment h_i, by the first-order condition (10), remains the same as if he were fast: $h_i = h^*(\tau_2; g_h) = h^*(\tau_1; g_h) = h_1$. (For Proposition 3.1, without information acquisition, $h_i = h_\circ$ always holds.) As a result, his certainty equivalent $\pi_2 = \pi_1 + 1/g_t > \pi_1$ and he indeed will deviate. Such a case of $\phi_1 = 1$ and $\phi_2 = 0$, therefore, can never be an equilibrium.

Case 2: Consider next the case of $\phi_1 = 0$ and $\phi_2 = 1$. (This will correspond to the corner equilibrium stated in the propositions.) If this is an equilibrium, it has to be the case that $\pi_1 \leq \pi_2$, i.e., all stay slow. The argument below shows that fixing all other primitive parameters, $\pi_1 \leq \pi_2$ holds if and only if $g_t < \hat{g}_t$, for some threshold $\hat{g}_t$. At $\phi_1 = 0$, $\tau_1 = \tau_0 < \tau_2$ and thus a slow speculator's unilateral deviation to fast yields $\pi_1|_{\phi_1=0} = \frac{1}{2\gamma}\ln\left(1 + \frac{h_1}{\tau_0}\right) - \dot{c}(h_1) - \frac{1}{g_t}$, where h_1 is the unique solution implied by the first-order condition (10) with $\tau_1 = \tau_0$. By envelope theorem, $\partial\pi_1/\partial g_t = 1/g_t^2 > 0$. (For Proposition 3.1, $h_1 = h_\circ$ is constant and the above inequality also holds.) Therefore, $\pi_1|_{\phi_1=0}$ is monotone increasing in g_t with limits $\lim_{g_t \downarrow 0} \pi_1 = -\infty < 0 < \pi_2 < \lim_{g_t \uparrow \infty} \pi_1$. (Note that $\pi_2|_{\phi_1=0}$ is a finite number unaffected by g_t in this case.) By continuity, therefore, there exists a unique $\hat{g}_t$ such that $\pi_1 = \pi_2$ when $\phi_1 = 0$. As such, $\pi_1 \leq \pi_2$, supporting $\phi_1 = 0$ and $\phi_2 = 1$, if and only if $g_t \leq \hat{g}_t$. When instead $g_t > \hat{g}_t$, this corner equilibrium does not hold.

Case 3: Now consider the interior case of $\phi_1 \in (0, 1)$, requiring $\pi_1 = \pi_2$. The key is to show that the difference $\pi_1 - \pi_2$ strictly decreases in ϕ_1. Evaluate the partial derivative of $\pi_1 - \pi_2$ with respect to ϕ_1 and after some simplification,

$$\frac{\partial(\pi_1 - \pi_2)}{\partial \phi_1} \cdot 2\gamma = \left[\frac{h_2/\tau_2}{\tau_2 + h_2} - \frac{h_1/\tau_1}{\tau_1 + h_1}\right]\frac{\partial \tau_1}{\partial \phi_1} + \frac{h_2/\tau_2}{\tau_2 + h_2}\frac{\partial \Delta\tau_2}{\partial \phi_1}.$$

Note that the term in the square-brackets is nonpositive, because $\tau_2 \geq \tau_1$ by construction and because $h_t = h^*(\tau_t; g_h)$ decreases in τ_t as implied by the first-order condition (10). (For Proposition 3.1, $h_t = h_\circ$ is constant and the term in the square-brackets is still negative.)

Both $\partial\tau_1/\partial\phi_1$ and $\partial\Delta\tau_2/\partial\phi_1$ still need to be signed. For Proposition 3.1, this is straightforward as one can immediately see from Equation (9) with constant $h_t = h_\circ$ that $\partial\tau_1/\partial\phi_1 > 0$ and $\partial\Delta\tau_2/\partial\phi_1 < 0$. For Proposition 4.1, rearrange the first-order condition (10) for $t = 1$ as $(\tau_0 + \Delta\tau_1 + g_h k(c_1))/\dot{k}(c_1) = g_h/(2\gamma)$ with $\Delta\tau_1 = \phi_1^2 g_h^2 k(c_1)^2 \tau_U/\gamma^2$ following Equation (9). It can then immediately be concluded that the information expense c_1 must decrease in ϕ_1, as otherwise the left-hand side of the above equation is always increasing in ϕ_1, unable to maintain the equality. (Recall that $k(\cdot)$ is concavely increasing.) Using the same argument, it is also known that τ_1 $(= \tau_0 + \Delta\tau_1)$ must decrease in c_1. Hence, τ_1 (and $\Delta\tau_1$) increases in ϕ_1. For $t = 2$, $(\tau_0 + \Delta\tau_1 + \Delta\tau_2 + h_2)/\dot{k}(c_2) = g_h/(2\gamma)$ with $\Delta\tau_2 = (1 - \phi_1)^2 g_h^2 k(c_2)^2 \tau_U/\gamma^2$. Note that $\frac{\partial\Delta\tau_2}{\partial\phi_1} = \left(-2(1-\phi_1)h_2^2 + 2(2-\phi_1)^2 h_2 \frac{\partial h_2}{\partial \phi_1}\right)\frac{\tau_U}{\gamma^2}$. As such, if $\Delta\tau_2$ increases in ϕ_1, then it has to be the case that $\partial h_2/\partial\phi_1 > 0$. Because $h_2 = g_h k(c_2)$, c_2 is also increasing in ϕ_1. It then follows that the left-hand side of the above equation strictly increases in ϕ_1—$\Delta\tau_1$, $\Delta\tau_2$, and c_2 all increase with ϕ_1, invalidating the equality. Therefore, it must be $\Delta\tau_2$ decreases in ϕ_1. As τ_1 increases in ϕ_1 but $\Delta\tau_2$ decreases in ϕ_1, one can conclude from the above partial derivative that the difference $\pi_1 - \pi_2$ indeed strictly decreases in ϕ_1.

To sum up from the above three cases, recall from the first cases that at $\phi_1 = 1$, $\pi_1 < \pi_2$. From the second case, at $\phi_1 = 0$, $\pi_1 > \pi_2$ if and only if $g_t > \hat{g}_t$. Hence, when $g_t \leq \hat{g}_t$, the equilibrium with interior ϕ_1 does not exist due to the above monotonicity of $\pi_1 - \pi_2$ in ϕ_1. When $g_t > \hat{g}_t$, there exists a unique $\phi_1 \in (0, 1)$ such that $\pi_1 = \pi_2$, sustaining this equilibrium. This completes the proof of both propositions. □

Proposition 3.2 (also part of Proposition 4.2)

Proof. In the interior equilibrium, $\pi_1 - \pi_2 = 0$. From the proof of Proposition 3.1 and 4.1 above, it is known that $\pi_1 - \pi_2$ can be written as a function of the endogenous ϕ_1 and hence in the form of $F(\phi_1(g_t); g_t)$. Take derivative of $F(\cdot) = 0$ with respect to g_t on both sides gives $(\partial(\pi_1 - \pi_2)/\partial\phi_1)(\partial\phi_1/\partial g_t) + 1/g_t^2 = 0$. Case 3 in the proof of Proposition 3.1 and 4.1 shows that $\partial(\pi_1 - \pi_2)/\partial\phi_1 < 0$. Therefore, $\partial\phi_1/\partial g_t > 0$ in the interior equilibrium. (The above holds whether

h_i is endogenous or not, thus proving both propositions.) □

Proposition 3.3 (also part of Proposition 4.2)

Proof. For Proposition 3.3 (no information acquisition), as shown in the proof of Proposition 3.2, ϕ_1 is increasing with g_t, which directly implies that τ_1 is increasing with g_t. For the overall τ_2, by the implicit function theorem, $\partial\tau_2/\partial\phi_1 = 2\tau_U h_o^2\phi_1/\gamma^2 - 2\tau_U h_o^2\phi_2/\gamma^2$, or $\partial\tau_2/\partial g_t = 2(\tau_U h_o^2\phi_1/\gamma^2 - 2\tau_U h_o^2\phi_2/\gamma^2)(\partial g_t/\partial\phi_1)$. It is clear that $\partial\tau_2/\partial g_t < 0$ when ϕ_1 is close to zero and ϕ_2 close to one (i.e., g_t is small), and $\partial\tau_2/\partial g_t > 0$ when ϕ_1 is close to one and ϕ_2 close to zero (i.e., g_t is large).

For Proposition 4.2 (with information acquisition), two steps are involved. The first step is to prove that $\partial\tau_1/\partial g_t > 0$. In the interior equilibrium, the first-order condition (10) for $t = 1$, together with $\tau_1 = \tau_0 + \tau_U h_1^2\phi_1^2/\gamma^2$, implies an implicit function of $h_1 = g_h k(c_1)$ and ϕ_1, from which $\partial h_1/\partial\phi_1 = -\frac{2\tau_U\phi_1 h_1^2/\gamma^2}{2\tau_U\phi_1^2 h_1/\gamma^2+1-\ddot{k}(c_1)/\dot{k}(c_1)} < 0$, where the inequality follows because $k(\cdot)$ is concavely increasing. From the effect of speed technology and population of sizes, $\partial\phi_1/\partial g_t > 0$. Therefore, by chain rule, $\partial h_1/\partial g_t < 0$. The first-order condition (10) also implies that τ_1 decreases with c_1 and, hence, also with h_1, yielding $\partial\tau_1/\partial g_t > 0$.

The second step is to prove that τ_2 first decreases and then increases with g_t. Recall $\tau_2 = \tau_0 + \tau_U\tau_1^2\phi_1^2/\gamma^2 + \tau_U\tau_2^2\phi_2^2/\gamma^2$. By implicit function theorem on the first-order condition (10),

$$\frac{\partial h_2}{\partial\phi_2} = -\frac{4\tau_U}{\gamma}\frac{\phi_2 h_2^2 - \phi_1 h_1^2 - \phi_1^2 h_1\partial h_1/\partial\phi_1}{-\ddot{k}(c_2)/\dot{k}(c_2) + 2\gamma + 4\tau_U\phi_2^2\tau_2/\gamma}.$$

As done in the proof of step 1, the idea is to first sign the above partial derivative and then sign $\partial h_2/\partial g_t$ using chain rule: $\frac{\partial h_2}{\partial g_t} = \frac{\partial h_2}{\partial\phi_2}\frac{\partial\phi_2}{\partial\phi_1}\frac{\partial\phi_1}{\partial g_t}$, where $\partial\phi_2/\partial\phi_1 = -1$ following the identity $\phi_1 + \phi_2 = 1$ and $\partial\phi_1/\partial g_t > 0$. In particular, consider the limits of $\partial h_2/\partial\phi_2$ as $g_t \uparrow \infty$ and $g_t \downarrow \hat{g}_t$, respectively.

To evaluate these limits, one needs to show that h_1, h_2, and $\partial h_1/\partial\phi_1$ are have finite bounds. The finite bounds for h_t can be easily established by noting from the first-order condition (10) that τ_t in equilibrium is monotone decreasing in τ_t. From the model setting, it is known that τ_t has strictly positive lower bound τ_0. Therefore, both h_1 and h_2 have finite upper bounds. (They also have lower bounds of zero by construction.) Finally, from the expression of $\partial h_1/\partial\phi_1$ derived in the proof of the previous step, it can be seen that $\phi_1 \cdot (\partial h_1/\partial\phi_1) = -\frac{2\tau_U\phi_1^2 h_1/\gamma^2}{2\tau_U\phi_1^2\tau_1/\gamma^2+1-\ddot{k}(c_1)/\dot{k}(c_1)}h_1 > -h_1$ is also bounded.

Now the limits can be evaluated. When speed technology $g_t \uparrow \infty$, almost all speculators become fast and $\phi_2 \downarrow 0$ and $\lim_{\phi_2\downarrow 0}(\frac{\partial h_2}{\partial\phi_2}) = -\frac{4\tau_U}{\gamma}\frac{-\phi_1 h_1^2-\phi_1^2 h_1\partial h_1/\partial\phi_1}{-\ddot{k}(c_2)/\dot{k}(c_2)+2\gamma} > 0$. Similarly, when speed technology $g_t \downarrow \hat{g}_t$, almost all speculators stay slow, $\phi_1 \downarrow 0$, and $\lim_{\phi_1\downarrow 0}(\frac{\partial h_2}{\partial\phi_2}) = -\frac{4\tau_U}{\gamma}\frac{\phi_2 h_2^2}{-\ddot{k}(c_2)/\dot{k}(c_2)+2\gamma+4\tau_U\phi_2^2 h_2/\gamma} < 0$. As the above shows, for sufficiently large (low) g_t, h_2 increases (decreases) in ϕ_2 and hence decreases (increases) in g_t by the chain rule expression above. The first-order condition (10) implies that τ_2 decreases with τ_2 and the stated results are proved. □

Proposition 4.3

Proof. **Fixing g_t, g_h increases from $\hat{g}_h$ to ∞.** The aggregate demand for speed in the economy is $\int_{[0,1]} \mathbb{1}_{\{t_i=1\}} \mathrm{d}i = \phi_1$. From $\Delta\tau_1 = \tau_U h_1^2 \phi_1^2/\gamma^2$, by implicit function theorem,

$$\frac{\partial \phi_1}{\partial g_h} = \frac{\gamma^2}{2\tau_U \phi_1 h_1^2}\left(\frac{\partial \Delta\tau_1}{\partial g_h} - \frac{2\tau_U \phi_1^2 h_1}{\gamma^2}\frac{\partial h_1}{\partial g_h}\right). \tag{17}$$

Hence, the sign of $\partial\phi_1/\partial g_h$ depends on the difference between the two terms in the brackets. Consider first the case of a very small g_h. Corollary 4.1 establishes the existence of a lower bound $\hat{g}_h$ for g_h, such that the equilibrium is interior if and only if $g_h \geq \hat{g}_h$. In particular, when $g_h \downarrow \hat{g}_h$, the marginal speculator is just indifferent between becoming fast or not, implying $\phi_1 \downarrow 0$. The first-order condition (10) at this limit gives $1/(2(\tau_0 + h_1)\gamma) - \dot{c}(h_1) = 0$, which has interior solution of $0 < h_1 < \infty$, thanks to the assumption of $\dot{c}(0) = 0$. By differentiability, therefore, $\partial h_1/\partial g_h$ is finite in this limit as well. Taken together, the second term in the above brackets has limit zero as $\phi_1 \downarrow 0$, when $g_h \downarrow \hat{g}_h$. The remaining term is $\partial\Delta\tau_1/\partial g_h$, which is shown by Proposition 4.4 to be strictly positive. Thus, $\partial\phi_1/\partial g_h$ is positive in the case of a very small g_h, close to the lower bound of $\hat{g}_h$.

Consider next the case of a very large g_h, i.e. $g_h \uparrow \infty$. First, there exists an upper bound for speculators' expense on information acquisition, c_t. To see this, note from the first-order condition (10):

$$\frac{1}{2\gamma}\dot{k}(c_t) > \frac{1}{2\gamma}\dot{k}(c_t) - \frac{1}{g_h}\tau_t = k(c_t) \geq k(0) + c_t\dot{k}(c_t) = c_t\dot{k}(c_t) \tag{18}$$

where the first inequality holds because $\tau_t \geq \tau_0 > 0$ and the last inequality holds by concavity of $k(\cdot)$ and by $k(0) = 0$. Therefore, for $t \in \{1, 2\}$, there exists an upper bound for $c_t \leq 1/(2\gamma)$, an upper bound for $k(c_t) \leq k(1/(2\gamma))$, and a lower bound for $\dot{k}(c_t) \geq \dot{k}(1/(2\gamma)) > 0$. Second, in the limit of $g_h \uparrow \infty$, the equilibrium is always interior (following Corollary 4.1). Hence, the limit of the fast speculator's ex ante certainty equivalent $\lim_{g_h\uparrow\infty} \pi_1 = \frac{1}{2\gamma}\lim_{g_h\uparrow\infty} \ln\left(1 + \frac{h_1}{\tau_1}\right) - \lim_{g_h\uparrow\infty} c_1 - \frac{1}{g_t}$ exists and must be nonnegative to sustain the interior equilibrium. Since c_1 is bounded from above, it follows that $\lim_{g_h\uparrow\infty}(h_1/\tau_1)$ also exists and is strictly positive. That is, there exists some $a \in (0, \infty)$, such that $\lim_{g_h\uparrow\infty}(\tau_1/h_1) = a$. Equivalently, as τ_0 is a finite constant, $\lim_{g_h\uparrow\infty}(\Delta\tau_1/h_1) = a$. Further, a fast speculator's first-order condition (10) can be rewritten as $\frac{1}{2\gamma}\frac{g_h}{\tau_1+h_1} - \dot{c}(h_1/g_h) = 0$. Since the above holds under $g_h \uparrow \infty$, it follows that $h_1 \sim g_h$; or $\lim_{g_h\uparrow\infty}(h_1/g_h) = b \in (0, \infty)$. (If h_1 is of higher magnitude than g_h, the limit of the first term above falls to zero, while the limit of the second term is strictly positive as $c(\cdot)$ is strictly convex. If instead h_1 is of lower magnitude than g_h, the limit of the first term approaches infinity, while the second term falls to zero.) Now consider the limit of the difference in the brackets of Equation (17):

$$\lim_{g_h\uparrow\infty}\left(\frac{\partial\Delta\tau_1}{\partial g_h}-2\frac{\tau_U\phi_1^2h_1}{\gamma^2}\frac{\partial h_1}{\partial g_h}\right)=\lim_{g_h\uparrow\infty}\left(\frac{\partial\Delta\tau_1}{\partial g_h}-2\frac{\Delta\tau_1}{h_1}\frac{\partial h_1}{\partial g_h}\right)=(ab-2ab)<0$$

where the last equality follows L'Hôpital's rule. Therefore, in the limit of $g_h\uparrow\infty$, $\partial\phi_1/\partial g_h<0$. Finally, consider the value of ϕ_1 in this limit. Note that $\Delta\tau_1=\tau_0+\tau_U\phi_1^2h_1^2/\gamma^2$. Therefore, in order for $\lim_{g_h\uparrow\infty}(\Delta\tau_1/h_1)=a\in(0,\infty)$ to hold, it must be such that $\lim_{g_h\uparrow\infty}(\phi_1^2h_1)=c\in(0,\infty)$, i.e., ϕ_1 in this limit is of magnitude $h_1^{-1/2}$. As $h_1\uparrow\infty$, this also implies that $\phi_1\downarrow 0$ in this limit.

Fixing g_h, g_t increases from $\hat{g}_t$ to ∞. The aggregate demand for information is $\bar{h}:=\phi_1h_1+\phi_2h_2$. Since ϕ_1 is monotone in g_t (Proposition 3.2), it is sufficient to examine the partial derivative of the above aggregate demand with respect to ϕ_1: $\partial\bar{h}/\partial\phi_1=h_1-h_2+\phi_1\cdot(\partial h_1/\partial\phi_1)-\phi_2\cdot(\partial h_2/\partial\phi_2)$. At the initial extreme of $g_t\downarrow\hat{g}_t$, the proof of Proposition 3.3 has shown that 1) $\phi_1\downarrow 0$, 2) $\phi_1\cdot\partial h_1/\partial\phi_1$ is bounded, and 3) $\partial h_2/\partial\phi_2<0$. Taking these into the above partial derivative yields $\partial\bar{h}/\partial\phi_1\to h_1-h_2-\phi_2\cdot(\partial h_2/\partial\phi_2)>0$, recalling that $h_1\geq h_2$ from Equation (11). At the eventual extreme of $g_t\uparrow\infty$, the proof of Proposition 3.3 has shown that 1) $\phi_2\downarrow 0$, 2) $\partial h_1/\partial\phi_1<0$, and 3) $\partial h_2/\partial\phi_2>0$. In addition, since $\phi_2\downarrow 0$, $\Delta\tau_2=\phi_2^2h_2^2\tau_U/\gamma^2\downarrow 0$ (h_2 is bounded), implying $\tau_2\downarrow\tau_1$ and 4) $h_2\uparrow h_1$. Taking the above into $\bar{h}$ yields $\partial\bar{h}/\partial\phi_1\to\phi_1\cdot(\partial h_1/\partial\phi_1)-\phi_2\cdot(\partial h_2/\partial\phi_2)<0$. □

Proposition 4.4

Proof. Two inequalities (20) and (22) below will be useful. The first-order condition (10) can be written as $\dot{k}(c_t)/(2\gamma)-k(c_t)=\tau_t/g_h$, which uniquely solves c_t. Fixing g_h,

$$\frac{\partial c_t}{\partial\tau_t}=\frac{1}{g_h}\left(\frac{\ddot{k}(c_t)}{2\gamma}-\dot{k}(c_t)\right)^{-1}\leq 0 \tag{19}$$

where the inequality follows the concavity of $k(c)$. In addition,

$$\frac{\partial c_t}{\partial g_h}=-\frac{\tau_t}{g_h^2}\left(\frac{\ddot{k}(c_t)}{2\gamma}-\dot{k}(c_t)\right)^{-1}=-\frac{\tau_t}{g_h}\frac{\partial c_t}{\partial\tau_t}\geq 0. \tag{20}$$

Note that the above hold with or without speed acquisition.

The proof then proceeds as follows. By construction, $\tau_1=\tau_0+\Delta\tau_1$ and $\tau_2=\tau_0+\Delta\tau_1+\Delta\tau_2$. The first-order condition implicitly has c_1 and c_2 as functions of $c_1(\Delta\tau_1)$ and $c_2(\Delta\tau_1,\Delta\tau_2)$. Further, $\Delta\tau_t=\tau_U g_h^2k(c_t)^2\phi_t^2/\gamma^2$, or $\phi_t=\frac{\gamma}{\sqrt{\tau_U}}\frac{\sqrt{\Delta\tau_t}}{g_hk(c_t)}$. Therefore, the unconstrained equilibrium (with endogenous acquisition of both speed and information) is pinned down by a two-equation, two-unknown system: $\pi_1 - \pi_2 = 0$ and $\phi_1 + \phi_2 - 1 = 0$; or, equivalently, with a vector function $F(\Delta\tau_1, \Delta\tau_2; g_h)$,

$$F = \begin{bmatrix} \left(\frac{1}{2\gamma}\ln\left(1+\frac{g_h k(c_1)}{\tau_1}\right) - c_1 - \frac{1}{g_t}\right) - \left(\frac{1}{2\gamma}\ln\left(1+\frac{g_h k(c_2)}{\tau_2}\right) - c_2\right) \\ \frac{\sqrt{\Delta\tau_1}}{k(c_1)} + \frac{\sqrt{\Delta\tau_2}}{k(c_2)} - \frac{\sqrt{\tau_U}}{\gamma} g_h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{21}$$

where $\{c_t\}_{t\in\{1,2\}}$ are functions of $\Delta\tau_1$ and $\Delta\tau_2$ following the first-order condition (10), which can be rewritten as $\dot{k}(c_t)/(2\gamma) - k(c_t) = \tau_t/g_h$.

Take total derivatives with respect to g_h on the equilibrium condition $F = 0$ to get $\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}\begin{bmatrix} \mathrm{d}\Delta\tau_1 \\ \mathrm{d}\Delta\tau_2 \end{bmatrix} + \begin{bmatrix} F_{1g} \\ F_{2g} \end{bmatrix}\mathrm{d}g_h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. One can easily evaluate, using envelope theorem,

$$F_{1g} = \frac{1}{2\gamma}\frac{k(c_1)}{\tau_1 + g_h k(c_1)} - \frac{1}{2\gamma}\frac{k(c_2)}{\tau_2 + g_h k(c_2)} = \frac{1}{g_h}\left(\frac{k(c_1)}{\dot{k}(c_1)} - \frac{k(c_2)}{\dot{k}(c_2)}\right) > 0,$$

where the second equality follows the first-order condition (10), while the last inequality follows the concavity of $k(c)$, knowing that $c_1 > c_2$. Also,

$$\begin{aligned} F_{2g} &= -\frac{\sqrt{\Delta\tau_1}}{k(c_1)^2}\dot{k}(c_1)\frac{\partial c_1}{\partial g_h} - \frac{\sqrt{\Delta\tau_2}}{k(c_2)^2}\dot{k}(c_2)\frac{\partial c_2}{\partial g_h} - \frac{\sqrt{\tau_U}}{\gamma} \\ &= -\frac{\sqrt{\tau_U}}{\gamma}\phi_1 g_h \frac{\dot{k}(c_1)}{k(c_1)}\frac{\partial c_1}{\partial g_h} - \frac{\sqrt{\tau_U}}{\gamma}\phi_2 g_h \frac{\dot{k}(c_2)}{k(c_2)}\frac{\partial c_2}{\partial g_h} - \frac{\sqrt{\tau_U}}{\gamma} < -\frac{\sqrt{\tau_U}}{\gamma} < 0, \end{aligned}$$

where the equality uses the expression of ϕ_t and the inequality holds because $\partial c_t/\partial g_h$ is derived earlier to be positive (inequality 20).

The elements in the Jacobian matrix can also be evaluated. Using envelope theorem,

$$F_{11} = -\frac{k(c_1)}{\dot{k}(c_1)\tau_1} + \frac{k(c_2)}{\dot{k}(c_2)\tau_2} \leq 0$$

where the inequality holds because $k(c_1)/\dot{k}(c_1) \geq k(c_2)/\dot{k}(c_2)$ (concavity) and $\tau_1 \leq \tau_2$. Similarly,

$$F_{12} = \frac{k(c_2)}{\dot{k}(c_2)\tau_2} > 0.$$

Now consider the partial derivatives with respect to F_2:

$$\begin{aligned} F_{21} &= \frac{1}{2\sqrt{\Delta\tau_1}k(c_1)} - \frac{\sqrt{\Delta\tau_1}}{k(c_1)^2}\dot{k}(c_1)\frac{\partial c_1}{\partial \tau_1}\cancel{\frac{\partial \tau_1}{\partial \Delta\tau_1}} - \frac{\sqrt{\Delta\tau_2}}{k(c_2)^2}\dot{k}(c_2)\frac{\partial c_2}{\partial \tau_2}\cancel{\frac{\tau_2}{\Delta\tau_1}} \\ &= \frac{1}{2\sqrt{\Delta\tau_1}k(c_1)} - \frac{\sqrt{\tau_U}}{\gamma}\phi_1 g_h \frac{\dot{k}(c_1)}{k(c_1)}\frac{\partial c_1}{\partial \tau_1} - \frac{\sqrt{\tau_U}}{\gamma}\phi_2 g_h \frac{\dot{k}(c_2)}{k(c_2)}\frac{\partial c_2}{\partial \tau_2} > 0 \end{aligned}$$

where the equality follows the expression of ϕ_t and the inequality holds because $\partial c_t/\partial \tau_t \leq 0$ as shown before (inequality 22).

$$\frac{\partial m_t}{\partial \tau_t} = \frac{1}{g_h}\left(\frac{\ddot{k}(m_t)}{2\gamma} - \dot{k}(m_t)\right)^{-1} \leq 0 \tag{22}$$

Similarly,

$$F_{22} = \frac{1}{2\sqrt{\Delta\tau_2}k(c_2)} - \frac{\sqrt{\Delta\tau_2}}{k(c_2)^2}\dot{k}(c_2)\frac{\partial c_2}{\partial \tau_2}\cancel{\frac{\partial \tau_2}{\partial \Delta\tau_2}} > 0.$$

By Cramer's rule,

$$\frac{\partial \Delta\tau_1}{\partial g_h} = \frac{\begin{vmatrix} -F_{1g} & F_{12} \\ -F_{2g} & F_{22} \end{vmatrix}}{\begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix}} \quad \text{and} \quad \frac{\partial \Delta\tau_2}{\partial g_h} = \frac{\begin{vmatrix} F_{11} & -F_{1g} \\ F_{21} & -F_{2g} \end{vmatrix}}{\begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix}}.$$

The the denominator is easy to sign: $F_{11}F_{22} - F_{12}F_{21} < 0$. It remains to examine the numerators. For τ_1, it can be seen that $-F_{1g}F_{22} + F_{12}F_{2g} < 0$; hence $\partial\tau_1/\partial g_h = \partial\Delta\tau_1/\partial g_h > 0$.

To sign $\partial\tau_2/\partial g_h$ is equivalent to signing the sum of the numerators of $\partial\Delta\tau_1/\partial g_h$ and $\partial\Delta\tau_2/\partial g_h$:

$$(-F_{1g}F_{22} + F_{12}F_{2g}) + (-F_{11}F_{2g} + F_{1g}F_{21}) = (F_{21} - F_{22})F_{1g} + (F_{12} - F_{11})F_{2g}.$$

To prove the statement made in the proposition, the objective is to show that under the limits of $g_h \uparrow \infty$ and of $g_h \downarrow \hat{g}_h$, the sign of the above term is negative and positive, respectively (recall that the determinant for the denominator is negative). The proof of Proposition 4.3 shows that in the upper limit, $\phi_1 \downarrow 0$ and $\phi_2 \uparrow 1$. The proof of Corollary 4.1 shows that in the lower limit, speculators are just indifferent between acquiring the speed or not, implying again $\phi_1 \downarrow 0$ and $\phi_2 \uparrow 1$. Using these limiting values of ϕ_1 and ϕ_2, the above simplifies to

$$\left(\frac{1}{2\sqrt{\Delta\tau_1}k_1} - \frac{1}{2\sqrt{\Delta\tau_2}k_2}\right)F_{1g} + \frac{k_1}{\dot{k}_1\tau_1}F_{2g}, \tag{23}$$

where, simplifying the notation, $k(\cdot)$ and $\dot{k}(\cdot)$ are replaced by subscripts of $t \in \{1, 2\}$.

Consider the limit of $g_h \uparrow \infty$ first. Equation (23) satisfies the following inequality:

$$\left(\frac{1}{2\sqrt{\Delta\tau_1}k_1} - \frac{1}{2\sqrt{\Delta\tau_2}k_2}\right)F_{1g} + \frac{k_1}{\dot{k}_1\tau_1}F_{2g} < \frac{F_{1g}}{2\sqrt{\Delta\tau_1}k_1}$$

because $F_{1g} > 0$ and $F_{2g} < -\sqrt{\tau_{\mathrm{U}}}/\gamma < 0$. The proof of Proposition 4.3 establishes that $\Delta\tau_1 \to \infty$. In addition, the inequality (18) establishes that in equilibrium, both c_1 and c_2 have finite upper and lower bounds, implying that both k_1 and F_{1g} is also finite (since $k(\cdot)$ is twice-differentiable).

Therefore, $\lim_{g_h \uparrow \infty}(F_{1g}/(2\sqrt{\Delta\tau_1}k_1) = 0$ and

$$\lim_{g_h \uparrow \infty}\left[\left(\frac{1}{2\sqrt{\Delta\tau_1}k_1} - \frac{1}{2\sqrt{\Delta\tau_2}k_2}\right)F_{1g} + \frac{k_1}{\dot{k}_1\tau_1}F_{2g}\right] < \lim_{g_h \uparrow \infty}\frac{F_{1g}}{2\sqrt{\Delta\tau_1}k_1} = 0.$$

This proves that in this upper limit, τ_2 is increasing with g_h.

Finally, consider the limit of $g_h \downarrow \hat{g}_h$. As $g_h \downarrow \hat{g}_h$, clearly F_{1g} and F_{2g} are finite. However, $\phi_1 \downarrow 0$, $\Delta\tau_1 \downarrow 0$, and the first term of Equation (23) approaches $+\infty$. The sum of numerators above therefore has a positive sign. Given the negative sign of the denominator, it can be concluded that $\partial\tau_2/\partial g_h < 0$ in the limit of $g_h \downarrow \hat{g}_h$. □

Proposition 4.5

Proof. Recall from inequality (22) that c_t is always increasing in g_h, which holds true irrespective of the speed technology. Therefore, with or without speed acquisition, the equilibrium $h_t = k(c_t)$ is increasing in the information technology g_h.

At $t = 1$, as g_h increases, h_1 also increases as shown above. It then follows that $\partial\tau_1/\partial g_h > 0$ because $\tau_1 = \tau_0 + \tau_U h_1^2\phi_1^2/\gamma^2$ with ϕ_1 exogenous. For $t = 2$, suppose the opposite, $\partial\tau_2/\partial g_h < 0$, is true. Then h_2 should be decreasing with g_h because $\tau_2 = \tau_1 + \tau_U h_2^2\phi_2^2/\gamma^2$ with τ_1 is increasing in g_h. However, the transformation of first-order condition (10), $g_h/(2\gamma) = (\tau_2 + h_2)k^{-1}(h_2/g_h)$, shows that it is impossible for both τ_2 and h_2 to be decreasing with g_h at the same time. Thus, the assumed inequality is wrong and τ_2 increases with g_h. □

Corollary 4.1

Proof. Consider the threshold $\hat{g}_t$, at which the benefit of investing in speed to trade at $t = 1$ is small enough, so that the marginal speculator is just willing to stay slow. Therefore, at this threshold $\phi_1 = 0$ and $\phi_2 = 1$, implying $\pi_1 = \frac{1}{2\gamma}\ln\left(1 + \frac{g_h k(c_1)}{\tau_0}\right) - c_1 - \frac{1}{\hat{g}_t}$ and $\pi_2 = \frac{1}{2\gamma}\ln\left(1 + \frac{g_h k(c_2)}{\tau_2}\right) - c_2$, where $\tau_2 = \tau_0 + \tau_U g_h^2 k(c_2)^2/\gamma^2$. In equilibrium, it has to be such that $\pi_1 = \pi_2 = \pi^*$, which implies $\tau_2/(\tau_2 + g_h k(c_2)) > \tau_0/(\tau_0 + g_h k(c_1))$ because $c_1 + 1/\hat{g}_t > c_1 > c_2$. Subtract by 1 on both sides and rearrange to get $k(c_2)/(\tau_2 + g_h k(c_2)) < k(c_1)/(\tau_0 + g_h k(c_1))$.

Next, from the expression of π_1, by envelope theorem, $\frac{\partial\pi^*}{\partial g_h} = \frac{1}{2\gamma}\frac{k(c_1)}{\tau_0 + g_h k(c_1)} + \frac{1}{\hat{g}_t^2}\frac{\partial\hat{g}_t}{\partial g_h}$. Similarly, from the expression of π_2, $\frac{\partial\pi^*}{\partial g_h} = \frac{1}{2\gamma}\frac{1}{\tau_2 + g_h k(c_2)}\left(1 - \frac{2hg_h^2 k(c_2)^2}{\gamma^2\tau_2}\right)k(c_2) < \frac{1}{2\gamma}\frac{k(c_2)}{\tau_2 + g_h k(c_2)} < \frac{1}{2\gamma}\frac{k(c_1)}{\tau_0 + g_h k(c_1)} = \frac{\partial\pi^*}{\partial g_h} - \frac{1}{\hat{g}_t^2}\frac{\partial\hat{g}_t}{\partial g_h}$. Therefore, $\partial\hat{g}_t/\partial g_h < 0$.

Further, consider the extremes of $g_h \downarrow 0$ and $g_h \uparrow \infty$. Toward the lower bound 0, from the expression of π_1 it can be seen that the first term in π_1 drops down to zero. Since a speculator always has the option not to trade, π_1 is bounded below by zero. This leads to $c_1 \downarrow 0$ and $1/\hat{g}_t \downarrow 0$, implying $\lim_{g_h \downarrow 0} \hat{g}_t = \infty$. On the other hand, the first-order condition (10) applied to π_1 implies $0 = \frac{\dot{k}(c_1)}{2\gamma} - k(c_1) - \frac{\tau_0}{g_h} < \left(\frac{1}{2\gamma} - c_1\right)\dot{k}(c_1)$, where the inequality follows because $\tau_0/g_h > 0$ and because $k(m) \geq \dot{k}(m)m$ by concavity. Hence, c_1 is always bounded from above by $1/(2\gamma)$. From the first-order condition, with τ_1 fixed at τ_0, it follows the concavity of $k(\cdot)$ that c_1 monotone increases in g_h, and so does $k(c_1)$. Taken together, $\lim_{g_h \uparrow \infty} \pi_1 > \frac{1}{2\gamma} \lim_{g_h \uparrow \infty} \ln\left(1 + \frac{g_h k(c_1)}{\tau_0}\right) - \frac{1}{2\gamma} - \lim_{g_h \uparrow \infty} \frac{1}{\hat{g}_t}$. If $\lim_{g_h \uparrow \infty} \hat{g}_t > 0$, then the above limit of π_1 shoots to infinity. In that case, the assumed equilibrium will not hold, however, because all slow speculators will have incentive to acquire speed by paying $1/\hat{g}_t$ to earn infinite profit. Therefore, it has to be the case that $\lim_{g_h \uparrow \infty} \hat{g}_t = 0$.

Finally, the above concludes that $\hat{g}_t$ is a strictly decreasing function in g_h, with $\hat{g}_t(0) \to \infty$ and $\hat{g}_t(\infty) \to 0$. As the strict monotonicity implies invertibility, there exists $\hat{g}_h(g_t)$ for all $g_t \in (0, \infty)$ such that the equilibrium is interior if and only if $g_h \geq \hat{g}_h(g_t)$. □

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Supplementary material for

"Speed Acquisition"

This note adds to the paper by studying four extensions and verifying the robustness of the main results: endogenous population size in Section S1; liquidity timing in Section S2; repeated fast trading in Section S3; and and market clearing in Section S4 . Numerical illustrations are contained in Section S5.

S1 Endogenous population size

The baseline model fixes the speculator population size at 1. This seems to drive the temporal fragmentation effect (Proposition 3.2): When the speed technology g_t improves, more speculators becoming fast (higher ϕ_1) "mechanically" implies fewer remaining slow (lower ϕ_2), thus temporally fragmenting trading and price discovery.

To clarify that the temporal fragmentation does not build on such mechanical structure, this extension studies speculators' free entry. Consider be an infinite-measure continuum of speculators, indexed on $i \in [0, \infty)$. Following the literature (see, e.g., Bolton, Santos, and Scheinkman, 2016), sort them according to their reservation value $R(i)$ for not trading. If speculator i chooses not to trade, he obtains a certainty equivalent of $R(i)$, which is monotone increasing in i. To ensure at least some participation, normalize $R(0) = 0$.

As no other model assumptions are changed, speculators trade just like in the baseline and Lemma 4.1 holds. Speculators' certainty equivalents are also in the same form as in Equations (8) in the paper. The conditions characterizing technology acquisition in an interior equilibrium are:

Optimal information acquisition:	$h_1 = h^*(\tau_1; g_h)$ and $h_2 = h^*(\tau_2; g_h)$;
Indifference in speed:	$\pi_1 = \pi_2$;
Indifference in entry:	$R(\phi_1 + \phi_2) = \pi_1 = \pi_2$.

Compared with Proposition 4.1, the only different condition is the last one which determines the population size $\phi_1 + \phi_2$. In the baseline, the population is fixed, hence $\phi_1 + \phi_2 = 1$. Here, due to free entry, the marginal (i.e., the $(\phi_1 + \phi_2)$-th) speculator, must be indifferent from trading or not.

It turns out that this only change in the equilibrium conditions does not affect the main results from the baseline. In particular, the temporal fragmentation effect of the speed technology remains robust. Figure S1 numerically demonstrates the pattern. It can be seen from the top-left panel that as the speed technology increases, the population size of fast speculators ϕ_1 increases, while the slow ϕ_2 reduces. The same intuition as in the baseline applies here: The fast and the slow compete for the same piece of pie. With better speed technology, it is more advantageous (upon entry) to become fast rather than slow. In the baseline, the worse-off slow speculators are "crowded in" to acquire speed and trade fast. Here, they might get "crowded out" of trading. In either case, ϕ_1 rises and ϕ_2 reduces.[1] The other panels replicate the other key results: complementarity and substitution; and the nonmonotone effects on cumulative price informativeness.

It is acknowledged that the nonmonotone effects on cumulative price informativeness τ_2 (the lower two panels) relies on the heterogeneity across the pool of speculators; that is, $R(i)$ is strictly increasing. In the special case of $R(i) = \bar{R}\ \forall i$, both the fast and the slow speculators' ex-ante certainty equivalents $\pi_1 = \pi_2 = \bar{R}$ are exogenously fixed. In particular, from Equation (8),

$$\bar{R} = \pi_2 = \frac{1}{2\gamma} \ln\left(1 + \frac{h^*(\tau_2; g_h)}{\tau_2}\right) - c(h^*(\tau_2; g_h); g_h),$$

where it is easy to see that the right-hand side is monotone in τ_2. The exogenous constant $\bar{R}$ hence uniquely pins down the cumulative price informativeness τ_2—neither the speed or information technology would affect price discovery! This case, however, appears as rather special, for it requires all agents to be homogeneous. Arguably, it is more reasonable to assume different agents

[1] More formally, the result can be proved by contradiction. Suppose ϕ_2 also (weakly) increases with g_t. Then the total speculator population $\phi_1 + \phi_2$ increases and the marginal speculator's certainty equivalent $\pi_1 = \pi_2 = R(\phi_1 + \phi_2)$ must increase to support his entry. However, a higher π_2 can only be achieved with a lower τ_2 (less price discovery, hence more information rent left; Equation 8). This leads to a contradiction as the increasing ϕ_1 and ϕ_2 imply a higher τ_2: more informed trading, more price discovery.

in the economy face heterogeneous outside options (e.g., aptness in financial securities' trading). The case of monotone increasing $R(i)$ is therefore possibly the more suitable assumption. The numerical illustration in Figure S1 uses a linear $R(i) = i$ but the patterns remain robust for a variety versions of convex or concave $R(i)$.

S2 Liquidity timing

In the baseline model, the amount of liquidity trading (noise) is exogenous and fixed. This extension investigates into endogenous liquidity timing. Following the literature (e.g., Admati and Pfleiderer, 1988), two types of liquidity traders are introduced. First, there are nondiscretionary liquidity traders, who in aggregate trade U_t units of the asset at each t. These correspond to the noise trading in the baseline. Second, there is a continuum of discretionary liquidators, indexed by $j \in [0, 1]$. At both dates $t \in \{1, 2\}$, each of them suffers random liquidation needs (specified below), which if not executed results in a holding cost of δ (Han, Tang, and Yang, 2016). They are discretionary in the sense that by paying a speed acquisition cost of $1/g_t$, one can liquidate early at $t_j = 1$; otherwise he stays slow and liquidates at $t_j = 2$. A discretionary liquidator j's liquidation need at t is $\eta_{jt} + Q_t$ units of the asset, where $\{\eta_{jt}\}$ is i.i.d. with zero mean, representing the idiosyncratic liquidation shocks; and $\{Q_t\}$ the systemic component, i.i.d. normal with zero mean and variance τ_Q^{-1}. To be consistent with the baseline, a fast liquidator can only trade once at $t = 1$, thus incurring the holding cost δ at $t = 2$; vice versa.

The setup above deserves some comments. First, it nests the baseline. By setting $\tau_Q \to \infty$, i.e., $\text{var}[Q_t] \to 0$, the discretionary liquidation needs shrink to zero (the idiosyncratic components η_{jt} always average out). In this case, only the nondiscretionary shocks U_t remain, returning to the baseline setting (Section 4 in the paper). Second, the independence between the systemic components Q_1 and Q_2 is assumed for simplicity. This can be read as a special case of Cespa and Vives (2012, 2015), where the amount of noise trading follows an autoregressive structure. Here the autoregressive coefficient is set to zero. When there is non-zero autocorrelation, the analysis will become more complicated as an additional learning channel arises (from price p_1, one can imperfectly infer the amount of noise Q_2). Third, the structure is in fact a special case of Section 4 of Admati and Pfleiderer (1988), with the key simplification of replacing strategic large investors with the continuum of competitive speculators and liquidators. The structure has also been used to study geographical (not temporal) fragmentation by Chowdhry and Nanda (1991), Baruch, Karolyi, and Lemmon (2007), and Foucault and Gehrig (2008). In this line of the literature, the discretionary liquidator optimally times his trading to minimize the total execution cost. In doing so, they jointly determine the amount of "noise" in each trading round (venue), thus affecting market quality.

The analysis proceeds as follows. First, Section S2.1 derives the equilibrium trading for the speculators. Second, backwardly, Section S2.2 studies the equilibrium condition for the speculators and the discretionary liquidators. Finally, the numerical results are discussed in Section S2.3.

S2.1 Trading

Suppose there are $\phi_1 \in [0, 1]$ fast speculators and each fast (slow) speculator has signal precision h_1 (h_2). In addition, a proportion of $\psi_1 \in [0, 1]$ of the liquidators choose to acquire speed and trade early, while the rest $\psi_2 = 1 - \psi_1$ remain slow. Conjecture (and verify later) that a speculator i with speed t submits demand schedule $x_{it} = a_t s_i - b_t p_t$. Then the aggregate demand function at $t \in \{1, 2\}$ is

$$L_t(p_t) = \int_0^1 \mathbb{1}_{\{t_i=t\}}(a_t s_i - b_t p_t)\mathrm{d}i + \int_0^1 \mathbb{1}_{\{t_j=t\}}(\eta_{jt} + Q_t)\mathrm{d}i + U_t$$

$$= \phi_t \cdot (a_t V - b_t p_t) + \psi_t Q_t + U_t = \phi_t a_t \cdot \left(V + \frac{W_t}{\phi_t a_t} - \frac{b_t}{a_t} p_t\right)$$

where $W_t := \psi_t Q_t + U_t$. Hence, the new information in the aggregate demand $L(p_t)$ can be summarized as $z_t := V + W_t/(\phi_t a_t)$. Standard filtering gives the market maker's competitive prices:

$$\text{(S1)} \qquad \begin{aligned} p_1 &= \mathbb{E}[V \mid L_1(\cdot)] = \frac{\tau_0}{\tau_1} p_0 + \frac{\Delta\tau_1}{\tau_1} z_1 \\ p_2 &= \mathbb{E}[V \mid L_1(\cdot), L_2(\cdot)] = \frac{\tau_1}{\tau_2} p_1 + \frac{\Delta\tau_2}{\tau_2} z_2 \end{aligned}$$

where the price informativeness τ_t satisfies $\tau_t = \tau_{t-1} + \Delta\tau_t$ with

$$\text{(S2)} \qquad \Delta\tau_t = \frac{\phi_t^2 a_t^2}{\mathrm{var}[W_t]} = \phi_t^2 a_t^2 \frac{\tau_Q \tau_U}{\psi_t^2 \tau_U + \tau_Q}.$$

It remains to verify the conjecture of the speculators' demand schedules. The analysis is standard. From the first-order condition of his utility maximization, a speed-t speculator has demand schedule

$$x_{it} = \frac{\mathbb{E}[V \mid s_i, p_t, p_{t-1}, \ldots] - p_t}{\gamma \mathrm{var}[V \mid s_i, p_t, p_{t-1}, \ldots]} = \frac{1}{\gamma}(h_t s_i + \tau_{t-1} p_{t-1} + \Delta\tau_t z_t - (\tau_t + h_t) p_t) = \frac{h_t}{\gamma}(s_i - p_t).$$

This verifies the conjecture with $a_t = b_t = h_t/\gamma$. It is easy to show that the speculators' ex-ante certainty equivalents remain in the same form of π_t shown in Equation (8). Summing up, the trading equilibrium in this extension has the same structure as in Lemma 4.1, except that $\psi_t Q_t + U_t$ replaces U_t.

S2.2 Technology acquisition

The speculators' information acquisition follows the same first-order condition (10) as in the baseline. Similarly, their speed acquisition must be such that $\pi_1 = \pi_2$ (for an interior equilibrium).

Turn to the discretionary liquidators next. A liquidator j with speed $t_j = t$ expects a total cost of

$$\mathbb{E}\big[p_t \cdot (\eta_{jt} + Q_t)\big] + \frac{2-t}{g_t} + \delta = \lambda_t \frac{\psi_t}{\tau_Q} + \frac{2-t}{g_t} + \delta$$

where

$$\lambda_t := \frac{\Delta\tau_t}{\phi_t a_t \tau_t}$$

is the liquidator's price impact at time t.[2] To sustain an interior equilibrium, the total costs for a fast and a slow liquidator must be the same: $\frac{1}{g_t} + \lambda_1 \frac{\psi_1}{\tau_Q} + \delta = \lambda_2 \frac{\psi_2}{\tau_Q} + \delta$; i.e.,

$$\frac{1}{g_t} + \lambda_1 \frac{\psi_1}{\tau_Q} - \lambda_2 \frac{\psi_2}{\tau_Q} = 0.$$

This condition is reminiscent of those seen in Chowdhry and Nanda (1991), Baruch, Karolyi, and Lemmon (2007), and Foucault and Gehrig (2008), where discretionary liquidators split their orders across venues. The differences are two-fold. First, here the fast liquidators incur an additional speed acquisition cost of $1/g_t$. Second, implicit in the expressions of $\{\lambda_t\}$ lies how the *temporal* split of noise affects the expected execution costs *sequentially*. That is, fast liquidators' total flow $\psi_1 Q_1$ affects the early price discovery $\Delta\tau_1$, which affects both λ_1 and λ_2. Such a feature does not exist in the above models, where $\Delta\tau_1$ does not accumulate across markets.

To sum up, to characterize the equilibrium, for the speculators there are two parameters for speed acquisition ϕ_1 and ϕ_2 and two for information acquisition h_1 and h_2. There are two new parameters ψ_1 and ψ_2 for the discretionary liquidators' speed acquisition. In an (interior) equilibrium, the six parameters must jointly solve the following equation system:

Speculators' information acquisition	$h_1 = h^*(\tau_1; g_h)$ and $h_2 = h^*(\tau_2; g_h)$;
Speculators' indifference in speed:	$\pi_1 = \pi_2$;
Liquidators' indifference in speed:	$\frac{1}{g_t} + \lambda_1 \frac{\psi_1}{\tau_Q} - \lambda_2 \frac{\psi_2}{\tau_Q} = 0$;
Population size identity:	$\phi_1 + \phi_2 = 1$; and $\psi_1 + \psi_2 = 1$.

S2.3 Results

The six-equation system is highly nonlinear and complex to analyze. The properties of equilibrium existence, uniqueness, or corners are not known. Fortunately, extensive numerical analysis seems to suggest there exists a stable equilibrium. Figure S2 shows a specific example, demonstrating the

[2] While each liquidator is infinitesimally small, he knows that his liquidation position $\eta_{jt} + Q_t$ contains the systemic component Q_t. This creates expected price impact for his liquidation.

robustness of the key results highlighted in the baseline. In particular, the temporal fragmentation effect of the speed technology remains, as seen in the top-left panel. As the speed technology g_t increases, it is the discretionary liquidators who first acquire speed ($\psi_1 > 0$ while $\phi_1 = 0$). This is intuitive, because without sufficient noise at $t = 1$, the speculators' information rent would not be enough to justify the speed acquisition cost $1/gt$. As g_t continues to rise, speculators also start to acquire speed ($\phi_1 > 0$). This hurts the fast discretionary liquidators as they are adversely selected when trading with the informed: The fraction ψ_1 therefore starts to drop as the fraction ϕ_1 increases.

The lower-left panel shows that in the interior equilibrium ($\phi_1 > 0$ and $\psi_1 > 0$), the cumulative price informativeness τ_2 is again nonmonotone in g_t (Proposition 4.2). To see the intuition of the robustness, recall from the above analysis that

$$\Delta\tau_t = \frac{h_t^2}{\gamma^2}\phi_t^2 \frac{\tau_{\mathrm{Q}}\tau_{\mathrm{U}}}{\psi_t^2\tau_{\mathrm{U}} + \tau_{\mathrm{Q}}} = \frac{h_t^2}{\gamma^2}\frac{\phi_t^2\tau_{\mathrm{U}}}{\psi_t^2\tau_{\mathrm{U}}\tau_{\mathrm{Q}}^{-1} + 1}.$$

(When there is no common liquidity shock Q_t, i.e., when $\tau_{\mathrm{Q}} \uparrow \infty$, the above converges to the baseline Equation 9.) For the sake of the argument, consider exogenous information; i.e., $h_t = h_\circ$ as in Section 3 in the baseline. A necessary condition to overturn the nonmonotonicity of τ_2 is to have the effects on ϕ_t and ψ_t exactly offset. In particular, this will require ϕ_t (in the numerator) and ψ_t (in the denominator) to move in the same direction. However, this can hardly be the case: If more speculators trade at t, the discretionary liquidators will want to avoid the escalated adverse-selection and avoid trading at t. Therefore, whenever a shock in g_t stimulates higher ϕ_1, ψ_1 will go down (see the top-left panel), thus making the signal-to-noise ratio at $t = 1$ even higher than the baseline, where the denominator does not change. That is, the temporal fragmentation effect is amplified, rather than neutralized, by the endogenous liquidity timing. Indeed, comparing the lower-left panels in Figure S0 and S2, it can be seen that the initial drop in τ_2 after g_t rises beyond the threshold $\hat{g}_t$ is more salient with liquidity timing.

The complementarity and substitution between the two technologies are shown in the top-right and the middle-left panels. The patterns predicted by Proposition 4.3 remain robust. This is unsurprising as the various crowding-out effects among speculators persist in the interior equilibrium. Note that the complementarity still drives the nonmonotone price informativeness τ_2 when the information technology g_h rises (Proposition 4.4), as shown in the lower-right panel.

S3 Frequent fast trading

There are two aspects of speed: being able to trade early and frequently. The baseline model only looks at the "trading early" aspect. This extension allows fast speculators to trade also "frequently". Specifically, the setting is largely the same as in Section 4 in the baseline, except that now the fast speculators can trade at both $t = 1$ and $t = 2$.

The main concern is whether fast speculators would "hoard" their private information and evenly trade it across the two rounds, thus affecting the intuition behind the temporal fragmentation effect of the speed technology. This turns out to be not the case. While they do trade in both rounds, the fast speculators do not reveal additional information in $t = 2$. Just like in the baseline, they only contribute to price discovery once at $t = 1$. As such, frequent fast trading does not affect any of the baseline results. The detailed analysis below proceeds backwardly as usual, by first solving the trading equilibrium and speculators' certainty equivalents, and then solving the optimal technology acquisition.

Consider speculators' demand at $t = 2$ first. All speculators, $i \in [0, 1]$, submit their demand schedule $x_{i2}(\cdot)$. Specifically, an arbitrary speculator i solves

$$\max_{x_{i2}} E\big[-\exp\{-\gamma \cdot [(p_2 - p_1)x_{i1} + (V - p_2)x_{i2}]\}\big|\, x_{i1}, s_i, p_1, p_2\,\big],$$

where x_{i1} is his inventory after $t = 1$ (for a slow speculator, $x_{i1} = 0$). Equivalently,

$$\exp\{-\gamma \cdot [(p_2 - p_1)x_{i1}]\} \max_{x_{i2}} E\big[-\exp\{-\gamma \cdot [(V - p_2)x_{i2}]\}|s_i, p_1, p_2\big].$$

Hence, the optimization problem reduces to the same one as the one faced by the slow speculators in the main model. The same conjecture-and-verify analysis as in Lemma 4.1 applies and gives the optimal linear *cumulative* demand,

$$x_{i2} = \frac{h_i}{\gamma}(s_i - p_2),$$

for both the fast and the slow speculators.

It remains to solve for the fast speculators' trading at $t = 1$. A fast speculator i's terminal wealth is $(p_2 - p_1)x_{i1} + (V - p_2)x_{i2}$, where x_{i1} is to be solved and x_{i2} follows the above. Recall that $p_2 = \mathbb{E}[V|\, p_1, p_2\,]$ (competitive market making) and $\mathrm{var}[V|\, s_i, p_1, p_2\,]^{-1} = \tau_2 + h_i$. Therefore, a fast speculator's $t = 1$ optimization becomes

$$\max_{x_{i1}} \left[-\exp\left\{ -\gamma \cdot (p_2 - p_1)x_{i1} - \frac{h_i^2}{2(\tau_2 + h_i)}(s_i - p_2)^2 \right\} \Big|\, s_i,p_1 \right],$$

or, equivalently,

$$\max_{x_{i1}} \left[-\exp\left\{ \gamma \cdot (s_i - p_2)x_{i1} - \gamma(s_i - p_1)x_{i1} - \frac{h_i^2}{2(\tau_2 + h_i)}(s_i - p_2)^2 \right\} \Big| s_i,p_1 \right].$$

To simplify notations, let $z_i = s_i - p_2$ and then it follows:

$$\begin{aligned} \mathbb{E}[z_i|\, s_i, p_1\,] &= s_i - \frac{\tau_1}{\tau_2}p_1 - \frac{\Delta\tau_2}{\tau_2}\left(\frac{h_i}{\tau_1 + h_i}s_i + \frac{\tau_1}{\tau_1 + h_i}p_1\right); \\ \mathrm{var}[z_i\big|\, s_i,p_1\,] &= \left(\frac{\Delta\tau_2}{\tau_2}\right)^2\left(\frac{1}{\tau_1 + h_i} + \frac{1}{\Delta\tau_2}\right). \end{aligned}$$

Denote also by

$$\begin{aligned} \mu &:= \mathbb{E}[z_i\big|\, s_i,p_1\,] = \left(1 - \frac{\Delta\tau_2}{\tau_2}\frac{h_i}{\tau_1 + h_i}\right)(s_i - p_1); \\ \beta &:= \mathrm{var}[z_i\big|\, s_i,p_1\,] = \left(\frac{\Delta\tau_2}{\tau_2}\right)^2\left(\frac{1}{\tau_1 + h_i} + \frac{1}{\Delta\tau_2}\right). \end{aligned}$$

The above $t = 1$ optimization problem reduces to:

$$\max_{x_{i1}} \frac{-1}{\sqrt{1 + \frac{h_i^2}{(\tau_2 + h_i)}\mathrm{var}(z_i|s_i,p_1)}} \cdot \exp\left\{ \gamma \cdot \mu x_{i1} - \gamma(s_i - p_1)x_{i1} - \frac{h_i^2}{2(\tau_2 + h_i)}\mu^2 + \frac{1}{2}\frac{(\gamma x_{i1} - \frac{h_i^2}{(\tau_2 + h_i)}\mu)^2\beta}{1 + \frac{h_i^2}{(\tau_2 + h_i)}\beta} \right\}$$

or after some simplification,

$$\max_{x_{i1}} \frac{-1}{\sqrt{1+\frac{h_i^2}{(\tau_2+h_i)}\mathrm{var}(z_i|s_i,p_1)}} \cdot \exp\left\{-\gamma(s_i-p_1)x_{i,1} + \frac{1}{2}\frac{(\gamma x_{i1})^2\beta + 2\gamma\cdot\mu x_{i1} - \frac{h_i^2}{(\tau_2+h_i)}\mu^2}{1+\frac{h_i^2}{(\tau_2+h_i)}\beta}\right\}$$

The first-order condition with respect to $x_{i,1}$ is

$$-(s_i-p_1) + \frac{x_{i1}\beta\gamma+\mu}{1+\frac{h_i^2}{(\tau_2+h_i)}\beta} = 0,$$

which uniquely pins down x_{i1}. Substituting in μ and β and simplifying yield

$$x_{i1} = \frac{h_i}{\gamma}(s_i-p_1),$$

which is exactly the same form as in the main model.

Next consider the recursions of τ_t and p_t. They can be found using the above optimal demand functions. At $t=1$, since the fast speculators' optimal demand is the same as shown in Lemma 4.1, the same results hold (see the proof of Lemma 4.1): $\Delta\tau_1 = \tau_1 - \tau_0 = \left(\int_{\{t_j=1\}}\frac{h_j}{\gamma}\mathrm{d}j\right)^2\tau_{\mathrm{U}}$ and $p_1 = p_0 + \frac{\Delta\tau_1}{\tau_1}\left(V + \frac{\gamma U_1}{\int_{\{t_j=1\}}h_j\mathrm{d}j}\right)$. At $t=2$, the market maker observes the aggregate demand

$$\begin{aligned}L_2(p_2) &= \int_{\{t_j=1\}}\big(x_{j2}(s_j,p_2) - x_{j1}(s_j,p_1)\big)\mathrm{d}j + \int_{\{t_j=2\}}x_{j2}(s_j,p_2)\mathrm{d}j + U_2\\ &= p_1\int_{\{t_j=1\}}\frac{h_j}{\gamma}\mathrm{d}j - p_2\int_{\forall j}\frac{h_j}{\gamma}\mathrm{d}j + V\int_{\{t_j=2\}}\frac{h_j}{\gamma}\mathrm{d}j + U_2,\end{aligned}$$

where the second equality follows the optimal demand schedules derived earlier. Observe how the fast speculators' signals are exactly offset, not contributing to the price discovery in the second fragment ($t=2$).This is an interesting insight: The "trading frequently" aspect of speed does not contribute to price discovery. Only fast speculators' first trading at $t=1$ contributes their private signals to the market. This result is robust also in similar settings like Cespa and Vives (2012, 2015). The resulting recursions are just like in Lemma 4.1: $\Delta\tau_2 = \tau_2 - \tau_1 = \left(\int_{\{t_j=2\}}\frac{h_j}{\gamma}\mathrm{d}j\right)^2\tau_{\mathrm{U}}$ and $p_2 = p_1 + \frac{\Delta\tau_2}{\tau_2}\left(V + \frac{\gamma U_2}{\int_{\{t_j=2\}}h_j\mathrm{d}j} - p_1\right)$.

Finally, consider speculators' ex ante certainty equivalents. Since slow speculators only trade once at $t = 2$, they expect the same certainty equivalent as given in Equation (8):

$$\pi_2 = \frac{1}{2\gamma}\ln\left(1 + \frac{h_i}{\tau_2}\right) - c(h_i; g_h).$$

The first-order condition leads to the same amount of information acquisition by all slow speculators: $h_i = h_2 = h^*(\tau_2; g_h)$. A fast speculator i's unconditional expected utility, before paying the technology cost, is

$$\begin{aligned}&\mathbb{E}[-\exp\{-\gamma \cdot (p_2 - p_1)x_{i1} - \gamma \cdot (V - p_2)x_{i2}\}]\\ =&\mathbb{E}\left[-\exp\left\{-h_i \cdot (s_i - p_1)^2 + h_i \cdot (s_i - p_2)(s_i - p_1) - \frac{h_i^2}{2(\tau_2 + h_i)}(s_i - p_2)^2\right\}\right]\end{aligned}$$

where the equality follows the optimal demand $x_{i1}(\cdot)$ and $x_{i2}(\cdot)$ derived above. Define $Y :=$ $[s_i - p_1; s_i - p_2]$ as a bivariate normal (column) random vector, with

$$\mathbb{E}Y = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } \operatorname{var}Y = \begin{bmatrix} \tau_1^{-1} + h_i^{-1} & \tau_2^{-1} + h_i^{-1} \\ \tau_2^{-1} + h_i^{-1} & \tau_2^{-1} + h_i^{-1} \end{bmatrix}.$$

Then the above expected utility can be rewritten as $E[-e^{Y^T AY}]$ where the coefficient matrix A is given by $A = [-h_i, h_i/2; h_i/2, -h_i^2/(2(\tau_2 + h_i))]$. Evaluating the expectation with the density of the bivariate normal Y yields the expected utility of $-\tau_1\tau_2/\sqrt{\tau_1 \cdot (h_i + \tau_2)(-h_i\tau_1 + (h_i + \tau_1)\tau_2)}$. Solving the certainty equivalent yields

$$\pi_1 = \frac{1}{2\gamma}\ln\left(1 + \frac{h_1}{\tau_1} + \frac{h_1}{\tau_2^2}\frac{\Delta\tau_2}{\tau_1}\right) - c(h_1; g_h) - \frac{1}{g_t},$$

where the symmetric $h_i = h_1$ is solved by the first-order condition. Compared to the π_1 in the baseline, it can be seen that there arises an extra term in the $\ln(\cdot)$. This extra term represents the additional benefit of "trading frequently" and it is the only difference with the baseline model.

The equilibrium technology acquisition are then characterized by the following conditions:

$$\begin{aligned} \text{Optimal information acquisition:} \quad & \partial\pi_1/\partial h_1 = \partial\pi_2/\partial h_2 = 0; \\ \text{Indifference in speed:} \quad & \pi_1 = \pi_2; \\ \text{Population size identity:} \quad & \phi_1 + \phi_2 = 1. \end{aligned}$$

(It is easy to verify the second-order conditions for optimal information acquisition hold.)

It turns out that due to the additional term in fast speculators' certainty equivalent, the model is no longer analytically tractable. Extensive numerical searches however do suggest the existence of a stable equilibrium. Very much like the baseline, depending on the relative level of the technologies, the equilibrium can either be cornered (no fast speculator) or interior. The key patterns are illustrated in Figure S3, where all patterns are qualitatively the same as in the baseline in Figure S0. This is unsurprising given that frequent fast trading does not affect how the acquired information is aggregated. The price and the trade informativeness dynamics remain exactly the same as in the baseline.

S4 Market clearing

In the baseline, there is always a competitive market maker who clears the market at the efficient price (à la Kyle, 1985). This extension studies an alternative setting, where the market marker is replaced by a continuum of uninformed investors of measure n, as in, e.g., Grossman and Stiglitz (1980). All agents, speculators and market makers, have the same constant absolute risk-aversion utility with a risk-aversion coefficient γ. All other assumptions are the same as in the baseline. In particular, all agents only trade once (buy-and-hold investors). The benefit of such a structure is that it nests the baseline as a special case of $n \to \infty$, i.e., the uninformed market makers in aggregate have infinite risk-bearing capacity.

This alternative setting has been studied in, e.g, Section 1 of Brennan and Cao (1996). For completeness, the key steps are reproduced below. Conjecture that at t, a speculator has linear demand $x_{it} = a_t s_i - b_t p_t + c_t$, while an uninformed investor has $y_{it} = -d_t p_t + e_t$. The aggregate demand function then becomes

$$\begin{aligned} L_t(p_t) &= \int_{\{t_i=1\}} x_{it}(s_i, p_t)\mathrm{d}i + n y_{it} + U_t = \phi_t \cdot (a_t V - b_t p_t + c_t) + n \cdot (-d_t p_t + e_t) + U_t \\ &= -(\phi_t b_t + n d_t) p_t + (\phi_t c_t + n e_t) + \phi_t a_t \cdot \left(V + \frac{U_t}{\phi_t a_t} \right) \end{aligned}$$

Market clearing means to solve p_t from $L_t(p_t) = 0$, while in the baseline $p_t = \mathbb{E}[V \mid L_t(\cdot)]$ is set by competitive market makers. Note that in either case, the informationally sufficient statistic for $L_t(\cdot)$ is the same

$$z_t := V + \frac{U_t}{\phi_t a_t};$$

see, e.g., the proof of Lemma 3.1 and 4.1. Further, as will be shown very soon, in equilibrium, speculators' aggressiveness on their private information a_t are also exactly the same in either case. Therefore, under either market structure, the price informativeness dynamics $\Delta\tau_t$ will be exactly the same. This is the key intuition why, for the purpose of understanding price informativeness τ_t, it is equivalent to set the price by competitive market makers or by market clearing.

Standard optimization for CARA speculators and uninformed investors yield the following demand functions:

$$x_{it} = \frac{\mathbb{E}[V \mid s_i, p_t, p_{t-1}, \dots] - p_t}{\gamma \mathrm{var}[V \mid s_i, p_t, p_{t-1}, \dots]} \text{ and } y_{it} = \frac{\mathbb{E}[V \mid p_t, p_{t-1}, \dots] - p_t}{\gamma \mathrm{var}[V \mid p_t, p_{t-1}, \dots]}.$$

The joint normal distribution of the random variables ensures that the above demand functions have linear structures:

$$\text{fast speculator:} \qquad x_{i1} = \frac{h_i}{\gamma}(s_i - m_1) - \frac{h_i + \tau_1}{\gamma}(p_1 - m_1),$$
$$\text{slow speculator:} \qquad x_{i2} = \frac{h_i}{\gamma}(s_i - m_2) - \frac{h_i + \tau_2}{\gamma}(p_2 - m_2),$$
$$\text{fast uninformed:} \qquad y_{i1} = -\frac{\tau_1}{\gamma}(p_1 - m_1),$$
$$\text{slow uninformed:} \qquad y_{i2} = -\frac{\tau_2}{\gamma}(p_2 - m_2).$$

where

$$m_1 = \mathbb{E}[V \,|\, p_1\,] = \mathbb{E}[V \,|\, z_1\,] = \frac{1}{\tau_1}(\tau_0 p_t + \Delta\tau_1 z_1)$$
$$m_2 = \mathbb{E}[V \,|\, p_2, p_1\,] = \mathbb{E}[V \,|\, z_2, z_2\,] = \frac{1}{\tau_2}(\tau_1 m_1 + \Delta\tau_2 z_2)$$

and under the market clearing condition $L_t(p_t) = 0$, $z_t = \left(\frac{\phi_t b_t + n d_t}{\phi_t a_t}\right)p_t - \frac{\phi_t c_t + n e_t}{\phi_t a_t}$. The above verifies that speculators' aggressiveness, a_{it}, on their private signals remains the same as in the baseline. The market clearing prices can be then solved as (with $p_0 = 0$ for notation simplicity):

$$p_1 = \frac{\phi_1 h_1 + (\phi_1 + n)\gamma\Delta\tau_1}{\phi_1 h_1 + (\phi_1 + n)\gamma\tau_1} z_1;$$
$$p_2 = \frac{(\phi_2 + n)\gamma\Delta\tau_1}{\phi_2 h_2 + (\phi_2 + n)\gamma n} z_1 + \frac{\phi_2 h_2 + (\phi_2 + n)\gamma\Delta\tau_2}{\phi_2 h_2 + (\phi_2 + n)\gamma\tau_2} z_2.$$

Brennan and Cao (1996) show that the speculators' ex-ante certainty equivalents are

$$\pi_t = \frac{1}{2\gamma}\ln\left(1 + \frac{h_t}{\tau_t}\right) + \frac{1}{2\gamma}\ln(\tau_t \mathrm{var}[V - p_t]) - c(h_t; g_h) - \frac{2-t}{g_t}.$$

Compared to the baseline, the difference lies in the second $\ln(\cdot)$ term. When the asset price is set by the competitive market maker, $p_t = \mathbb{E}[V \,|\, p_t, p_{t-1}, \ldots]$ and $\mathrm{var}[V - p_t] = 1/\tau_t$, under which the above certainty equivalents reduce to those stated in the paper (Equation 8). When such market maker is replaced by uninformed liquidity providers, $\mathrm{var}[V - p_t]$ no longer takes such simple form and the certainty equivalents change accordingly. (Intuitively, as the price is no longer efficient, speculators acquire additional trading gains from providing liquidity to noise trading.)[3]

[3] Recall that the current setup nests the baseline. To see this, consider the special case of $n \to \infty$. In this limit, the

The conditions for an interior equilibrium of technology acquisition remain the same as before:

Optimal information acquisition:	$\partial\pi_1/\partial h_1 = \partial\pi_2/\partial h_2 = 0$;
Indifference in speed:	$\pi_1 = \pi_2$;
Population size identity:	$\phi_1 + \phi_2 = 1$.

In particular, note that the speculators' first-order conditions for information acquisition remains exactly the same as in Equation (10). The only affected part in the analysis is the $\ln(\tau_t \text{var}[V - p_t])$ in the certainty equivalent π_t. As alluded earlier, this component is related to the market making capacity n. With finite market makers, the noise trading U_t is able to push the market clearing price away from the efficient price $\mathbb{E}[V \mid p_t, p_{t-1}, \dots]$ and speculators can trade against such price pressure and extract rent from the noise traders. Such rent, however, is not related to information or price discovery. Therefore, it does not affect the intuition of the results in the baseline model, as confirmed by extensive numerical analysis. Figure S4 illustrates the patterns, which are qualitatively the same as seen in the baseline (Figure S0).

S5 Numerical illustrations

This section collects the numerical illustrations studied in this note, from Figures S1 to S4. To compare, the patterns from the baseline model is shown in Figure S0. In each figure, there are two columns. The left columns show patterns after shocks in the speed technology g_t and the right columns the information technology g_h. Each column has three panels, in the sequence of demand for speed, demand for information, and price informativeness, from top to bottom. The common primitive parameters used in these numerical exercises are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k(c) = \sqrt{c}$; in the left column, $g_h = 0.2$; and in the right column, $g_t = 10.0$.

uninformed investors approximate the competitive market maker in the baseline. Indeed, it is easy to see in this case that $p_1 = \mathbb{E}[V \mid z_1] = \mathbb{E}[V \mid L_1(\cdot)]$ and $p_2 = \mathbb{E}[V \mid z_1, z_2] = \mathbb{E}[V \mid L_1(\cdot), L_2(\cdot)]$.

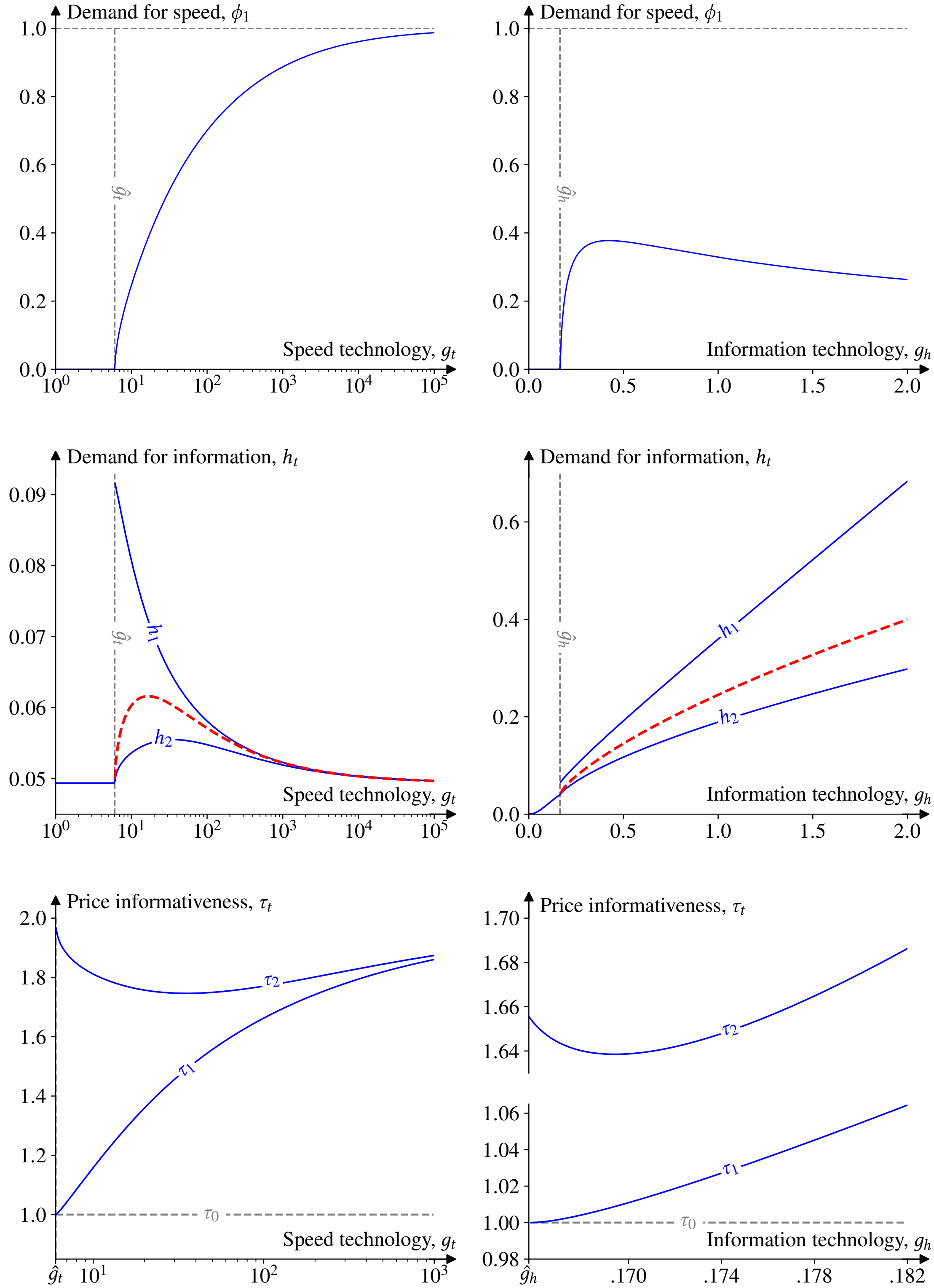

Figure S0: Baseline as in the paper.

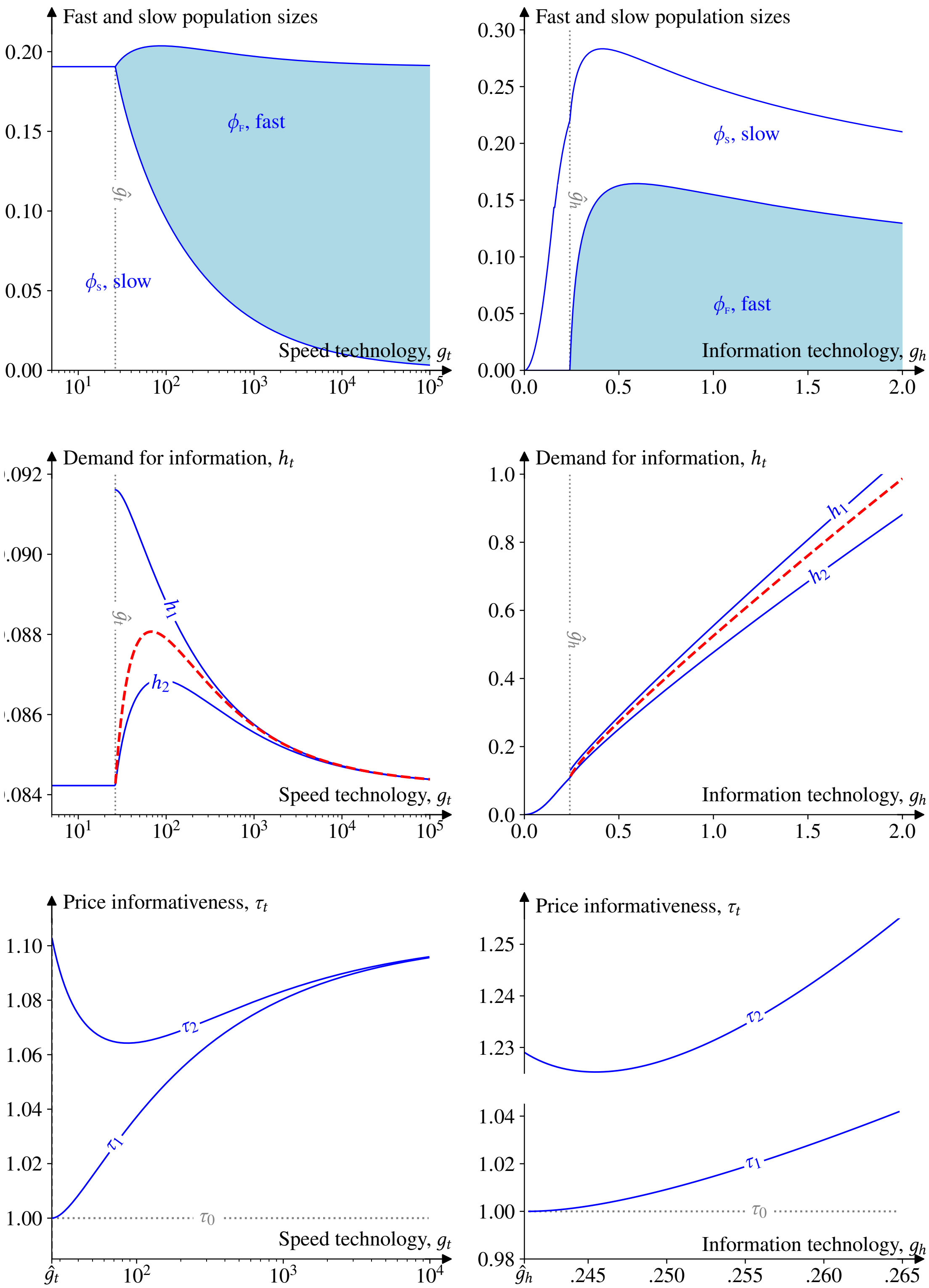

Figure S1: Endogenous population size. The setting specific primitive parametrization used in this numerical illustration is $R(i) = i$, agent i's reservation value for not trading.

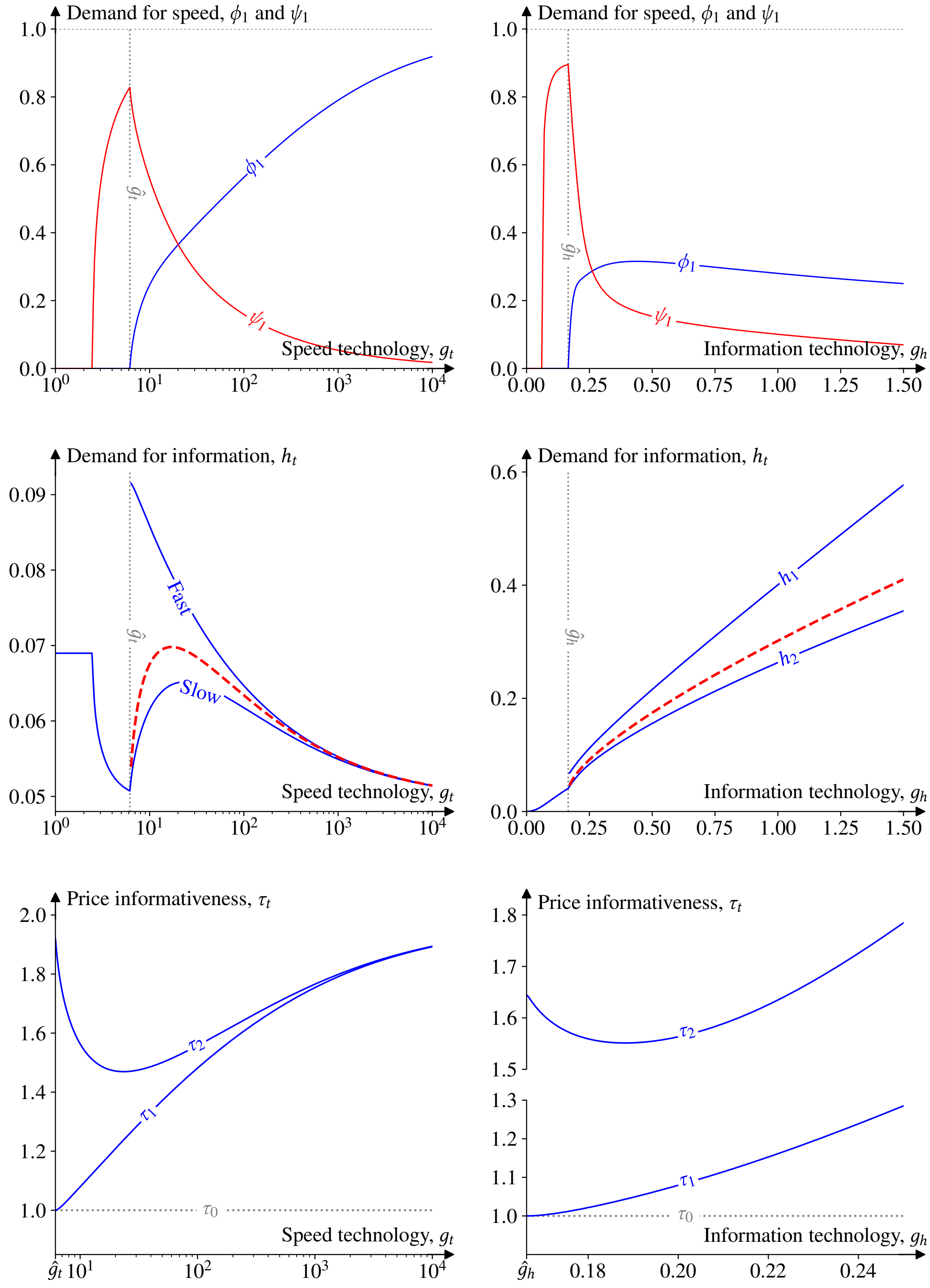

Figure S2: Liquidity timing. The extension specific primitive parameter used in this numerical illustration is $\tau_Q = 1.0$, the precision of the systemic component in discretionary liquidators' shocks.

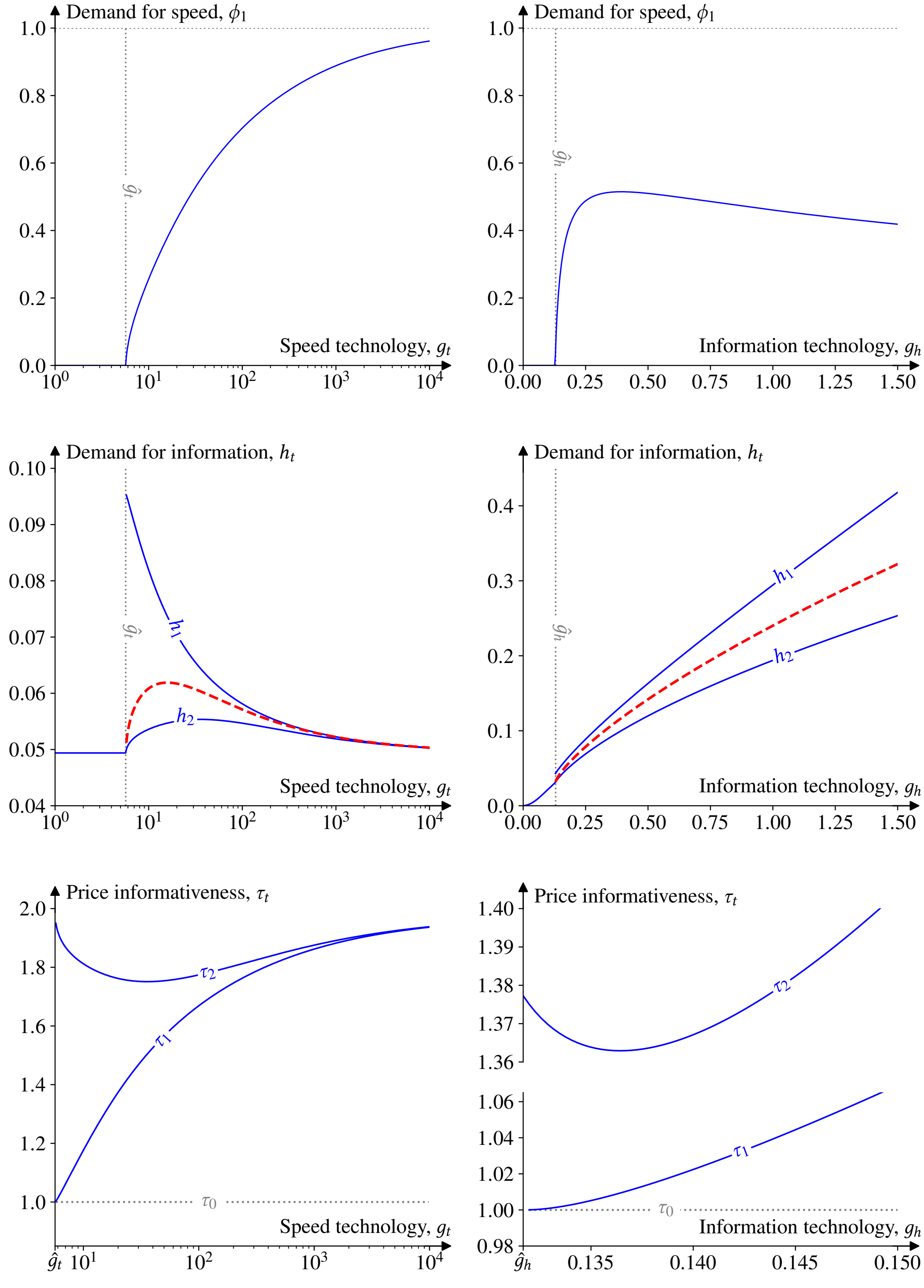

Figure S3: Frequent fast trading. There are no primitive parametrization specific to this extension.

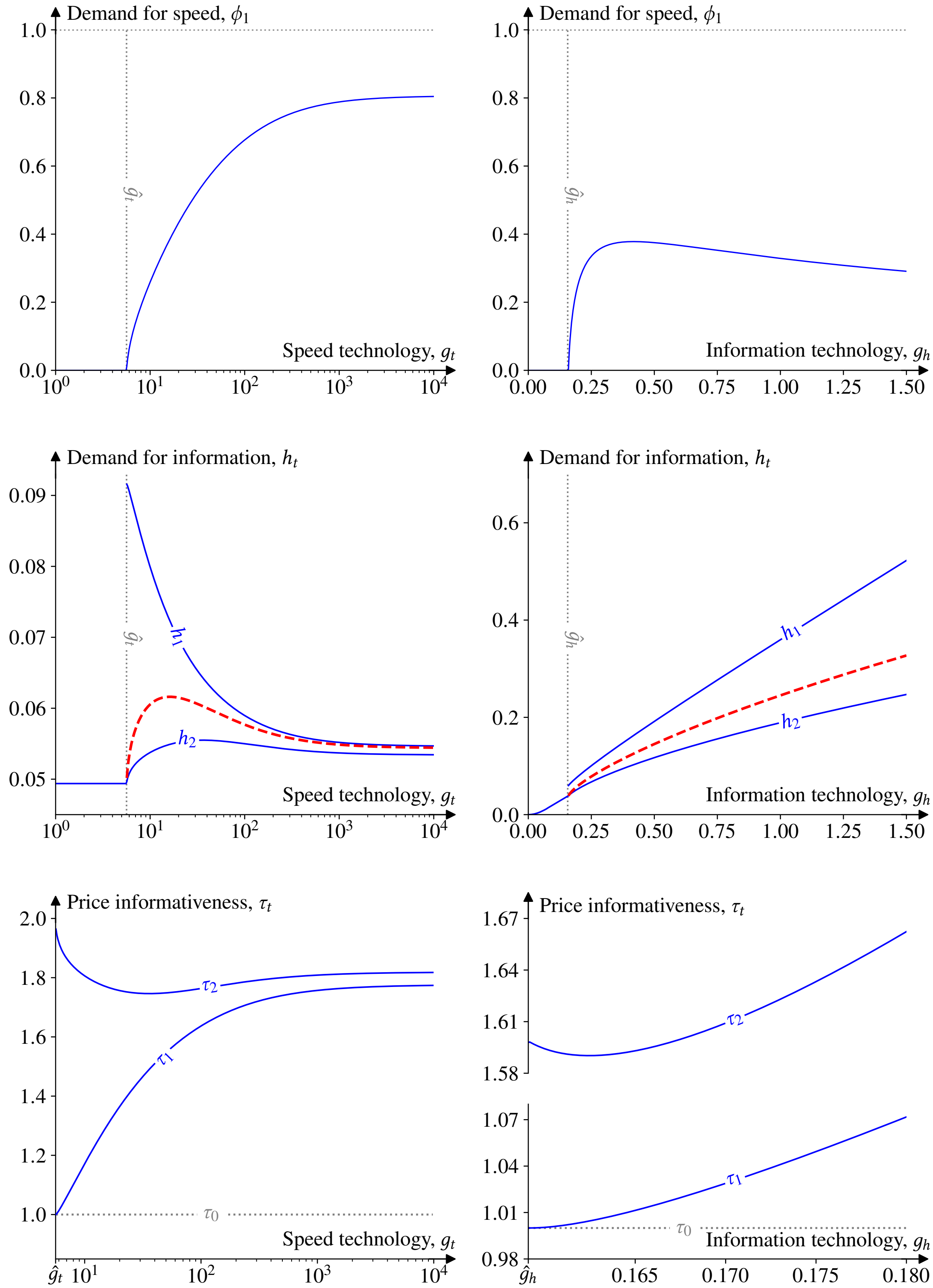

Figure S4: Market clearing. The extension-specific parameter used in this numerical illustration is $n = 1.0$, the size of uninformed investors.

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