

Singapore Management University

## Institutional Knowledge at Singapore Management University

---

Research Collection School Of Computing and Information Systems

School of Computing and Information Systems

---

12-2019

### A mathematical programming model for the green mixed fleet vehicle routing problem with realistic energy consumption and partial recharges

Vincent F. YU

*National Taiwan University of Science and Technology*

Panca JODIWAN

*National Taiwan University of Science and Technology*

Aldy GUNAWAN

*Singapore Management University, aldygunawan@smu.edu.sg*

Audrey Tedja WIDJAJA

*Singapore Management University, audreyw@smu.edu.sg*

Follow this and additional works at: [https://ink.library.smu.edu.sg/sis\\_research](https://ink.library.smu.edu.sg/sis_research)



Part of the [Artificial Intelligence and Robotics Commons](#), [Numerical Analysis and Scientific Computing Commons](#), [Operations Research, Systems Engineering and Industrial Engineering Commons](#), and the [Transportation Commons](#)

---

#### Citation

YU, Vincent F.; JODIWAN, Panca; GUNAWAN, Aldy; and WIDJAJA, Audrey Tedja. A mathematical programming model for the green mixed fleet vehicle routing problem with realistic energy consumption and partial recharges. (2019). *2019 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM): Macau, December 15-18: Proceedings*. 1339-1343.

Available at: [https://ink.library.smu.edu.sg/sis\\_research/7187](https://ink.library.smu.edu.sg/sis_research/7187)

This Conference Proceeding Article is brought to you for free and open access by the School of Computing and Information Systems at Institutional Knowledge at Singapore Management University. It has been accepted for inclusion in Research Collection School Of Computing and Information Systems by an authorized administrator of Institutional Knowledge at Singapore Management University. For more information, please email [cherylids@smu.edu.sg](mailto:cherylids@smu.edu.sg).

# A Mathematical Programming Model for the Green Mixed Fleet Vehicle Routing Problem with Realistic Energy Consumption and Partial Recharges

V. F. Yu<sup>1</sup>, P. Jodiawan<sup>1</sup>, A. Gunawan<sup>2</sup>, A. T. Widjaja<sup>2</sup>

<sup>1</sup>Department of Industrial Management, National Taiwan University of Science and Technology, Taipei, Taiwan

<sup>2</sup>School of Information Systems, Singapore Management University, Singapore, Singapore

([vincent@mail.ntust.edu.tw](mailto:vincent@mail.ntust.edu.tw), [pancajodiawan@gmail.com](mailto:pancajodiawan@gmail.com), [aldygunawan@smu.edu.sg](mailto:aldygunawan@smu.edu.sg), [audreyw@smu.edu.sg](mailto:audreyw@smu.edu.sg))

**Abstract** - A green mixed fleet vehicle routing with realistic energy consumption and partial recharges problem (GMFVRP-REC-PR) is addressed in this paper. This problem involves a fixed number of electric vehicles and internal combustion vehicles to serve a set of customers. The realistic energy consumption which depends on several variables is utilized to calculate the electricity consumption of an electric vehicle and fuel consumption of an internal combustion vehicle. Partial recharging policy is included into the problem to represent the real life scenario. The objective of this problem is to minimize the total travelled distance and the total emission produced by internal combustion vehicles. This is a new variant of problem which is developed from a mixed fleet of electric and internal combustion vehicles, full recharging policy, and operational cost minimization. A mixed integer programming model is then developed to address this problem and commercial software is utilized to solve the model.

**Keywords** – electric vehicle routing problem, mixed fleet, realistic energy consumption, partial recharging

## I. INTRODUCTION

In recent years, the transportation system has played an important factor that contributes to global warming and energy crisis issues. In fact, transportation has become the second largest sector by contributing 23% of global GHG emissions according to International Energy Agency. The European Union (EU) reported that its transportation sector has contributed 24% of total EU-28 GHG emissions in 2016 which shows an increasing trend.

Several governments have attempted to set targets and strategies for the emissions reduction. USA has proposed a target on emissions reduction by 26% - 28% below 2005 levels by 2025. EU has also set a goal to reduce the GHG emissions 60% below 1990 levels by 2050. Particular strategies involving innovations and regulations have also been being proposed and implemented. For example, EU has considered to harness technologies that contribute to lower greenhouse gas emissions, such as electrification of road transport and development of sustainable fuels.

Looking to the targets set by governments, the usage of alternative fuel vehicles (AFVs) has recently become an emerging trend. Many organizations, ranging from government agencies to private companies, have started to utilize AFVs as their operational fleets, either to voluntarily reduce their operational negative impacts on

environment or to meet new regulations which have been set by local governments [1]. As electric vehicles (EVs) are considered as a promising type of AFVs, the popularity of EVs has been growing together with the growth of environmental consciousness in society.

EVs have been invented to serve as a substitution of internal combustion vehicles (ICVs) with a main advantage of zero tailpipes emissions. On the other hand, EVs still come with several innate drawbacks, such as, shorter driving range and significant refueling time compared to diesel powered trucks [2]. This brings a new challenge in the Vehicle Routing Problem (VRP). The complexity of the problem is much higher than the conventional VRP.

Instead of fully replacing ICVs with EVs, most companies gradually introduce EVs into their existing ICV fleets [3]. Therefore, the mixed fleet strategy becomes another consideration in the routing planning problem with EVs. In most cases, it is assumed that the energy consumption follows a linear function of travelled distance [4, 5]. In reality, energy consumption actually depends on several other variables [3]. For EVs, there are two recharging policies at each recharging station: full recharge [6, 7] and partial recharge [8, 9, 10]. Recently, Reference [11] studies a combination of the mixed fleet, a realistic energy consumption and the partial recharge policy.

In this paper, we extend the work by adding emission minimization instead of only using the operational cost, which is commonly used by other researchers. Emission minimization was originally proposed as one of the VRP variants, namely Pollution Routing Problem (PRP) [12] and only limits to ICVs. Reference [5] extended the PRP by considering mixed fleet between ICVs and EVs which aimed to study the influence of limiting the emissions without a realistic energy consumption.

Our mathematical model formulates on a mixed fleet routing between ICVs and EVs under a real energy consumption model and the partial recharge policy with the objective of minimizing emission and operational cost. The instances are obtained from [3]. There are 9 datasets where each dataset consists of 20 instances. In this paper, the first two datasets [3] are utilized for our experiments. The first dataset consists of 10 customers and 2 recharging stations and the second one consists of 15 customers and 2 recharging stations. Since each dataset contains 20 different instances, in total there are 40 instances used in this paper.

The paper is organized as follows. Section II summarizes the literature review. In Section III, we introduce the mathematical model. Section IV is devoted to the experimental results and analysis. Finally, Section V concludes and provides some ideas for future works.

## II. LITERATURE REVIEW

The research on routing for EV is originated from [1] who introduced green vehicle routing problem (G-VRP). In G-VRP, a fleet of alternative fuel vehicles (AFVs) is utilized to serve a set of customers. The main difference between G-VRP and VRP is that the refueling process of AFVs should be taken into consideration because of the lack of infrastructures and considerable refueling time for particular types of alternative fuel, i.e. electricity. Later, Reference [7] introduced electric vehicle routing problem with time windows (EVRPTW). This problem aims to minimize the operational cost which is the travelled distance cost of operating EVs. Each EV is able to visit any recharging station to recharge its battery in order to visit all the assigned customers. Every time the EV performs the recharging process, it will recharge into the full state of the battery.

Electric vehicle routing problem (E-VRP) has recently gained more interests. Reference [9] extended the E-VRPTW by allowing partial recharge whenever an EV performs a recharging process. The problem is then mentioned as EVRPTW-PR. Reference [13] developed exact algorithms for both EVRPTW and EVRPTW-PR. In these both researches, partial recharge shows a cost saving benefit compared to full recharge scheme. Reference [8] firstly introduced the multiple technologies concept to G-VRP where these technologies affect the refueling speed performed to an AFV. Later, Reference [10] addressed a similar problem by introducing fast and super-fast recharging stations beside a normal recharging station. Benefits in term of cost savings compared to single technology scenario were shown by these both researches.

In addition to research field that purely addressed E-VRP, several recent researches were also dedicated to a mixed fleet between EV and ICV routing problem. Reference [3] contributed in this field by introducing a problem named electric vehicle routing problem with time windows and mixed fleet (E-VRPTWMF). A set of EVs and ICVs is ready at a depot to serve a set of customers. Full recharging scheme is utilized in this problem. In addition, this problem is the first problem that introduced realistic energy consumption to calculate electricity consumed by an EV while serving customers. Extensive studies performed in the research, such as, the effect of several different operational costs toward the fleet utilization and the effect of utilizing the realistic energy consumption toward the solution's quality. Reference [4] extended the mixed fleet routing by introducing Plug in Hybrid Electric Vehicle (PHEV) which has two operating modes, fuel-generating energy and electricity-generating energy and each type of vehicle differs in its capacity. In addition, each EV and PHEV differs in its battery

capacity and its energy consumption rate. Most recent, Reference [5] presented a mixed fleet problem where a constraint for pollution emissions emitted by the ICVs was introduced. The problem was studied to show the impact of the constraint to the fleet composition. Later, Reference [11] addressed a mixed fleet problem by simultaneously including realistic energy consumption and partial recharge scheme. The research shows that realistic energy consumption is an important aspect that needs to be included for EV's route planning.

## III. MATHEMATICAL PROGRAMMING MODEL

Consider a directed graph  $G = (V \cup F' \cup 0 \cup N+1, A^+)$  where  $V$  is a set of  $N$  customers:  $V = \{1, \dots, N\}$ ,  $F$  is a set of recharging stations and  $F'$  is the set of visits to nodes in  $F$ , and nodes 0 and  $N+1$  as the starting and ending point respectively. The nodes are connected through a set of arcs  $A^+ = \{(i, j) \mid i, j \in V \cup F' \cup 0 \cup N+1, i \neq j\}$  with a non-negative distance  $d_{ij}$ , travel time  $t_{ij}$ , and a cost of  $c_i$  for each kilometer traveled by the vehicle. For simplifying the mathematical model formulation, we introduce several sets:  $V' = V \cup F'$ ,  $V_0 = V \cup 0$ ,  $V_{N+1} = V \cup N+1$ ,  $V_0' = V \cup F' \cup 0$ , and  $V_{N+1}' = V \cup F' \cup N+1$ .

Each node  $i$  has a non-negative demand  $q_i$  ( $i \in V$ ), service time  $s_i$  ( $i \in V$ ), and hard time window  $[e_i, l_i]$  ( $i \in V \cup F' \cup 0 \cup N+1$ ) where early arrival with waiting is allowed but late arrival is strictly prohibited. A set of EVs  $K_E$  and ICVs  $K_{IC}$  with capacity  $Q$  are available at the depot. EVs having a battery capacity of  $B$  can be charged partially with recharging rate of  $r$ .

The energy consumed by an EV depends on the aerodynamic drag  $c_d$ , air density  $\rho$ , vehicle frontal surface  $A$ , vehicle speed  $v$ , vehicle mass  $m$ , gravitational constant  $g$ , coefficient rolling resistance  $c_r$ , energy efficiency  $\phi_d$ , and discharging efficiency  $\phi_d$ . The energy consumed by an ICV depends on the fuel-to-air mass ratio  $\zeta$ , heating value  $\kappa$ , engine friction factor  $k$ , engine speed  $N$ , engine displacement  $D$ , factor for converting the fuel rate  $\psi$ , efficiency of diesel engines  $\eta$ , and drive train efficiency  $\eta_{tr}$ . The carbon emissions emitted by an ICV depends on the CO<sub>2</sub> emitted per liter of fuel  $FE$  and the emission cost  $c_e$  per gram of CO<sub>2</sub>.

Several decision variables in this model are listed below.

- $x_{ijk_e}^E$  : 1 if an EV  $k_E$  travels from node  $i$  to  $j$  ( $k_E \in K_E$ ;  $i, j \in V \cup F' \cup 0 \cup N+1$ ).
- $x_{ijk_c}^{IC}$  : 1 if an ICV  $k_{IC}$  travels from node  $i$  to  $j$  ( $k_{IC} \in K_{IC}$ ;  $i, j \in V \cup F' \cup 0 \cup N+1$ ).
- $\tau_i$  : arrival time at node  $i$  ( $i \in V_0'$ ).
- $\tau_{N+1}^{k_E}$  : arrival time of an EV  $k_E$  at node  $N+1$  ( $k_E \in K_E$ ).
- $\tau_{N+1}^{k_{IC}}$  : arrival time of an ICV  $k_{IC}$  at node  $N+1$  ( $k_{IC} \in K_{IC}$ ).
- $u_0^{k_E}$  : initial load brought by an EV  $k_E$  ( $k_E \in K_E$ ).
- $u_0^{k_{IC}}$  : initial load brought by an ICV  $k_{IC}$  ( $k_{IC} \in K_{IC}$ ).

- $u_{ij}^{k_E}$  : amount of load brought by an EV  $k_E$  when travels from node  $i$  to  $j$  ( $k_E \in K_E; i, j \in V \cup F' \cup 0 \cup N+1$ ).
- $u_{ij}^{k_{IC}}$  : amount of load brought by an ICV  $k_{IC}$  when travels from node  $i$  to  $j$  ( $k_{IC} \in K_{IC}; i, j \in V \cup F' \cup 0 \cup N+1$ ).
- $y_i^{k_E}$  : the remaining electric energy of an EV  $k_E$  upon arrival at node  $i$  ( $k_E \in K_E; i, j \in V \cup F' \cup 0 \cup N+1$ ).
- $Y_i^{k_E}$  : amount of electric energy obtained by an EV  $k_E$  after recharging process at recharging station  $i$  ( $k_E \in K_E; i \in F$ ).
- $p_{ij}^{k_E}(u_{ij}^{k_E})$  : amount of mechanical power spent by an EV  $k_E$  when travels from node  $i$  to  $j$  and carries a load of  $u_{ij}^{k_E}$  ( $k_E \in K_E; i, j \in V \cup F' \cup 0 \cup N+1$ ).
- $p_{ij}^{k_{IC}}(u_{ij}^{k_{IC}})$  : amount of mechanical power spent by an ICV  $k_{IC}$  when travels from node  $i$  to  $j$  and carries a load of  $u_{ij}^{k_{IC}}$  ( $k_{IC} \in K_{IC}; i, j \in V \cup F' \cup 0 \cup N+1$ ).
- $b_{ij}^{k_E}(u_{ij}^{k_E})$  : amount of electric energy consumed by an EV  $k_E$  when travels from node  $i$  to  $j$  and carries a load of  $u_{ij}^{k_E}$  ( $k_E \in K_E; i, j \in V \cup F' \cup 0 \cup N+1$ ).
- $f_{ij}^{k_{IC}}(u_{ij}^{k_{IC}})$  : amount of fuel consumed by an ICV  $k_{IC}$  when travels from node  $i$  to  $j$  and carries a load of  $u_{ij}^{k_{IC}}$  ( $k_{IC} \in K_{IC}; i, j \in V \cup F' \cup 0 \cup N+1$ ).

$$\min \sum_{k_E \in K_E} \sum_{i \in V_0} \sum_{j \in V_{N+1}} c_t d_{ij} x_{ijk_E}^E + \sum_{k_{IC} \in K_{IC}} \sum_{i \in V_0} \sum_{j \in V_{N+1}} c_t d_{ij} x_{ijk_{IC}}^{IC} + \sum_{k_{IC} \in K_{IC}} \sum_{i \in V_0} \sum_{j \in V_{N+1}} c_e FE f_{ij}^{k_{IC}}(u_{ij}^{k_{IC}}) \quad (1)$$

### Flow constraints

$$\sum_{k_E \in K_E} \sum_{j \in V_{N+1}} x_{ijk_E}^E + \sum_{k_{IC} \in K_{IC}} \sum_{j \in V_{N+1}} x_{ijk_{IC}}^{IC} = 1 \quad \forall i \in V \quad (2)$$

$$\sum_{j \in V_{N+1}} x_{ijk_E}^E \leq 1 \quad \forall i \in V', \forall k_E \in K_E \quad (3)$$

$$\sum_{j \in V_{N+1}} x_{ijk_{IC}}^{IC} \leq 1 \quad \forall i \in V_0, \forall k_{IC} \in K_{IC} \quad (4)$$

$$\sum_{i \in V_0} x_{ijk_E}^E = \sum_{i \in V_{N+1}} x_{jik_E}^E \quad \forall j \in V', \forall k_E \in K_E \quad (5)$$

$$\sum_{i \in V_0} x_{ijk_{IC}}^{IC} = \sum_{i \in V_{N+1}} x_{jik_{IC}}^{IC} \quad \forall j \in V, \forall k_{IC} \in K_{IC} \quad (6)$$

$$\sum_{j \in V} x_{0jk_E}^E \leq K_E \quad \forall k_E \in K_E \quad (7)$$

$$\sum_{j \in V} x_{0jk_{IC}}^{IC} \leq K_{IC} \quad \forall k_{IC} \in K_{IC} \quad (8)$$

The objective function is formulated in Equation (1) to minimize the vehicles travelling cost and the ICVs' emissions cost. Equation (2) ensures that every customer is visited exactly once by one vehicle, either EV or ICV. Equations (3) and (4) ensure that every vehicle can only leave the depot maximum once (multi-trip is not allowed). Equation (5) and (6) make sure that once a vehicle arrives at a certain node, it will depart from that node. Equations (7) and (8) limit the number of EV and ICV used.

### Time constraints

$$\tau_0 = 0 \quad (9)$$

$$\tau_i + (s_i + t_{ij}) \left( \sum_{k_E \in K_E} x_{ijk_E}^E + \sum_{k_{IC} \in K_{IC}} x_{ijk_{IC}}^{IC} \right) - M \left( 1 - \left( \sum_{k_E \in K_E} x_{ijk_E}^E + \sum_{k_{IC} \in K_{IC}} x_{ijk_{IC}}^{IC} \right) \right) \leq \tau_j \quad \forall i \in V_0, \forall j \in V \quad (10)$$

$$\tau_i + (s_i + t_{ij}) \sum_{k_E \in K_E} x_{ijk_E}^E - M \left( 1 - \sum_{k_E \in K_E} x_{ijk_E}^E \right) \leq \tau_j \quad \forall i \in V_0, \forall j \in F' \quad (11)$$

$$\tau_i + \frac{1}{r} (Y_i^{k_E} - y_i^{k_E}) + t_{ij} x_{ijk_E}^E - M(1 - x_{ijk_E}^E) \leq \tau_j \quad \forall i \in F', \forall j \in V', \forall k_E \in K_E \quad (12)$$

$$\tau_i + \frac{1}{r} (Y_i^{k_E} - y_i^{k_E}) + t_{i,N+1} x_{i,N+1,k_E}^E - M(1 - x_{i,N+1,k_E}^E) \leq \tau_{N+1} \quad \forall i \in F', \forall k_E \in K_E \quad (13)$$

$$\tau_i + (s_i + t_{i,N+1}) x_{i,N+1,k_E}^E - M(1 - x_{i,N+1,k_E}^E) \leq \tau_{N+1} \quad \forall i \in V_0, \forall k_E \in K_E \quad (14)$$

$$\tau_i + (s_i + t_{i,N+1}) x_{i,N+1,k_{IC}}^{IC} - M(1 - x_{i,N+1,k_{IC}}^{IC}) \leq \tau_{N+1} \quad \forall i \in V_0, \forall k_{IC} \in K_{IC} \quad (15)$$

$$e_i \leq \tau_i \leq l_i \quad \forall i \in V' \quad (16)$$

$$e_{N+1} \leq \tau_{N+1} \leq l_{N+1} \quad \forall k_{IC} \in K_{IC} \quad (17)$$

$$e_{N+1} \leq \tau_{N+1} \leq l_{N+1} \quad \forall k_E \in K_E \quad (18)$$

Equation (9) guarantees every vehicle leaves depot at time 0. Equations (10) - (15) calculate the arrival time of each vehicle at the visited nodes. Equations (16) - (18) are the time windows constraints.

### Load constraints

$$\sum_{i \in V_0} u_{ij}^{k_E} - \sum_{i \in V_{N+1}} u_{ji}^{k_E} = q_j \sum_{i \in V_0} x_{ijk_E}^E \quad \forall j \in V', \forall k_E \in K_E \quad (19)$$

$$\sum_{i \in V_0} u_{ij}^{k_{IC}} - \sum_{i \in V_{N+1}} u_{ji}^{k_{IC}} = q_j \sum_{i \in V_0} x_{ijk_{IC}}^{IC} \quad \forall j \in V, \forall k_{IC} \in K_{IC} \quad (20)$$

$$\sum_{i \in V} u_{iN+1}^{k_E} = 0 \quad \forall k_E \in K_E \quad (21)$$

$$\sum_{i \in V} u_{iN+1}^{k_{IC}} = 0 \quad \forall k_{IC} \in K_{IC} \quad (22)$$

$$0 \leq u_{ij}^{k_E} \leq Q \cdot x_{ijk_E}^E \quad \forall i \in V_0, \forall j \in V_{N+1}, \forall k_E \in K_E \quad (23)$$

$$0 \leq u_{ij}^{k_{IC}} \leq Q \cdot x_{ijk_{IC}}^{IC} \quad \forall i \in V_0, \forall j \in V_{N+1}, \forall k_{IC} \in K_{IC} \quad (24)$$

Equations (19) and (20) ensure the amount of load in a vehicle before and after visiting a customer equals to the demand of the visited customer. Equations (21) and (22) guarantee a vehicle carries nothing when returns to the depot. Equations (23) and (24) are the vehicle capacity constraints.

### Energy constraints

$$p_{ij}^{k_{IC}}(u_{ij}^{k_{IC}}) = \left( \frac{1}{2} \cdot c_d \cdot \rho \cdot A \cdot v^2 + m_{truck} \cdot g \cdot c_r \right) x_{ijk_{IC}}^{IC} + g \cdot c_r \cdot u_{ij}^{k_{IC}} \quad \forall i \in V_0, \forall j \in V_{N+1}, \forall k_{IC} \in K_{IC} \quad (25)$$

$$f_{ij}^{k_{IC}}(u_{ij}^{k_{IC}}) = \left( \frac{\xi}{\kappa \cdot \psi} \left( kND + \frac{P_{ij}^{k_{IC}}(u_{ij}^{k_{IC}})}{\eta \cdot \eta_{ij}} \right) \right) x_{ijk_{IC}}^{IC} \cdot t_{ij} \quad \forall i \in V_0, \forall j \in V_{N+1}, \forall k_{IC} \in K_{IC} \quad (26)$$

$$p_{ij}^{k_E}(u_{ij}^{k_E}) = \left( \frac{1}{2} \cdot c_d \cdot \rho \cdot A \cdot v^2 + m_{truck} \cdot g \cdot c_r \right) x_{ijk_E}^E + g \cdot c_r \cdot u_{ij}^{k_E} \quad \forall i \in V_0, \forall j \in V_{N+1}, \forall k_E \in K_E \quad (27)$$

$$b_{ij}^{k_E}(u_{ij}^{k_E}) \geq \Phi \cdot \varphi_d \cdot p_{ij}^{k_E}(u_{ij}^{k_E}) \cdot t_{ij} \quad \forall i \in V_0, \forall j \in V_{N+1}, \forall k_E \in K_E \quad (28)$$

$$b_{ij}^{k_E}(u_{ij}^{k_E}) + M(1 - x_{ijk_E}^E) \leq y_i^{k_E} - y_j^{k_E} \leq b_{ij}^{k_E}(u_{ij}^{k_E}) - M(1 - x_{ijk_E}^E) \quad \forall i \in V_0, \forall j \in V_{N+1}, \forall k_E \in K_E \quad (29)$$

$$b_{ij}^{k_E}(u_{ij}^{k_E}) + M(1 - x_{ijk_E}^E) \leq Y_i^{k_E} - y_i^{k_E} \leq b_{ij}^{k_E}(u_{ij}^{k_E}) - M(1 - x_{ijk_E}^E) \quad \forall i \in F', \forall j \in V_{N+1}, \forall k_E \in K_E \quad (30)$$

$$y_0^{k_E} = B \quad \forall k_E \in K_E \quad (31)$$

$$y_i^{k_E} \leq Y_i^{k_E} \quad \forall i \in F', \forall k_E \in K_E \quad (32)$$

$$y_i^{k_E} \leq B \quad \forall i \in V_0, \forall k_E \in K_E \quad (33)$$

$$Y_i^{k_E} \leq B \quad \forall i \in F', \forall k_E \in K_E \quad (34)$$

The fuel consumption of an ICV is calculated by Equations (25) and (26), while the electricity consumption of an EV is calculated by Equations (27) and (28). Equation (29) calculates the remaining energy level of an EV upon travelling through nodes  $i$  to  $j$ . Equation (30) calculates the remaining energy level of an EV upon performing recharging process. Equations (31) – (34) limit the energy level of an EV.

## IV. COMPUTATIONAL RESULTS

This section is devoted for the computational results of the proposed mathematical model. First, we describe how the experiments are set up. We also present the benchmark instances. The results obtained are also summarized.

The computation is performed on a computer with Intel(R) Xeon(R) CPU E5 -1620 v2 @ 3.70GHz processor, 40.0 GB RAM. The mathematical model is solved by a commercial software, AMPL with CPLEX 12.8.0.0 solver. The computational time for all experiments is restricted to 7200 seconds.

The instances obtained from [3] are based on PRP instances proposed in [14]. The customer locations in these instances represent real cities in the UK, while customer demands and time windows are generated randomly. On the contrary to [14], the vehicle's speed is set to a constant value, 90 km/hour. The number of recharging stations is calculated based on the number of customers. More detail, Reference [3] located  $\lfloor 0.1|N| \rfloor$  recharging stations at randomly selected customer locations.

All aforementioned parameters to calculate energy consumption are also obtained from [14]. Battery capacity, energy efficiency, and discharging efficiency are adopted from [3]. In addition, a constant value to convert fuel consumption into equivalent amount of CO<sub>2</sub> is necessary. This value is provided by [15], 2.6 gram of CO<sub>2</sub> per liter of fuel.

Table I shows the results of GMVRP-REC-PR solved by CPLEX. The column *Instances* lists the name of selected instances. Number of utilized fleet of ICV and EV are represented by column *Number of ICV* and *Number of EV*, respectively. Column *Total Cost* represents the minimum objective value that could be obtained by CPLEX within the time restriction. The last column, *Time*, shows the computational time which is measured in second.

Each 10-customer instance needed less than five minutes (300 seconds) to be solved to optimality. The time is still reasonable since the solver provides optimal solutions for all instances. However, when the instance becomes larger, in this case when the number of customers increases to 15 customers, the computational time to solve the problem increases significantly. This can be seen that most of the 15-customer instances needed more than a half of an hour to be solved to optimality.

Several 15-customer instances were solved only to the feasible solution in which no guarantee for optimality because the CPLEX had reached the time restriction. These instances are marked with asterisk sign (\*) in Table I.

TABLE I  
COMPUTATIONAL RESULT OF GMFVRP-REC-PR WITH CPLEX

Instances	Number of ICV	Number of EV	Total Cost	Time (sec)
E-UK10_01	1	1	544.319	25.031
E-UK10_02	1	1	692.87	10
E-UK10_03	1	1	664.81	18.859
E-UK10_04	1	1	689.8	22.547
E-UK10_05	1	1	575.95	11.937
E-UK10_06	1	1	805.95	69.576
E-UK10_07	1	1	729.888	226.295
E-UK10_08	1	1	779.61	23.946
E-UK10_09	1	1	644.87	131.134
E-UK10_10	1	1	755.47	297.275
E-UK10_11	1	1	987.842	10.483
E-UK10_12	1	1	578.846	47.377
E-UK10_13	1	1	756.9	85.66
E-UK10_14	1	1	578.12	38.797
E-UK10_15	1	1	343.64	45.661
E-UK10_16	1	1	548.095	28.923
E-UK10_17	1	1	569.836	26.083
E-UK10_18	1	1	535.95	200.321
E-UK10_19	1	1	575.138	22.604
E-UK10_20	1	1	495.153	65.505

  

Instances	Number of ICV	Number of EV	Total Cost	Time
E-UK15_01	2	0	1071.18	35.474
E-UK15_02	1	1	654.05	306.2
E-UK15_03	2	1	997.13	3294.88
E-UK15_04*	2	1	1014.06	7361.45
E-UK15_05	2	0	1094.87	3.026
E-UK15_06*	2	1	806.71	7629.62
E-UK15_07	2	1	865.24	2649.22
E-UK15_08*	1	1	538.34	7208.15
E-UK15_09	2	1	866.07	2019.9
E-UK15_10	1	1	769.57	3210.42
E-UK15_11	2	0	972.35	4337
E-UK15_12	2	1	1061.66	1174.41
E-UK15_13*	2	1	882.072	7300.38
E-UK15_14*	2	1	1221.93	7346.17
E-UK15_15*	1	1	731.7	7228.24
E-UK15_16*	1	1	734.15	7209.13
E-UK15_17*	2	1	996.58	7249.99
E-UK15_18	2	1	1101.34	4315.18
E-UK15_19	1	1	498.81	119.59
E-UK15_20	2	1	687.24	4046.73

The computational time increases in a faster rate compared to the increase of number of customers. Consequently, the CPLEX could only solve small instances since it will take a huge amount of time to solve larger instances.

## V. CONCLUSION

This research introduces the GMFVRP-REC-PR as an operational logistics planning which deals with a mixed fleet of EVs and ICVs where partial recharging policy is employed in order to minimize the total cost which consists of not only travelled distance cost but also emission cost.

In order to solve GMFVRP-REC-PR, a mathematical programming model was proposed. The model was solved using commercial software AMPL. However, the experiments show that it takes a huge computational time to solve the problem when the problem size increase. Therefore, future work includes how to design a heuristic that provides good solutions and solve the problem more efficiently. Real-world applications will also be considered in order to provide evidences about applicability of the problem.

## ACKNOWLEDGMENT

This research was partially supported by the Ministry of Science and Technology of the Republic of China (Taiwan) under grant MOST 106-2410-H-011-002-MY3. This support is gratefully acknowledged.

## REFERENCES

- [1] S. Erdoğan and G. Laporte, "A Green Vehicle Routing Problem," *Transportation Research Part E: Logistics and Transportation Review*, vol. 48, no. 1, pp. 100-114, 2012.
- [2] N. Touati-Moungla and V. Jost, "Combinatorial optimization for electric vehicles management," *Journal of Energy and Power Engineering*, vol. 6, no. 5, pp. 738-743, 2012.
- [3] D. Goeke and M. Schneider, "Routing a mixed fleet of electric and conventional vehicles," *European Journal of Operational Research*, vol. 245, no. 1, pp. 81-99, 2015.
- [4] G. Hiermann, R. F. Hartl, J. Puchinger, and T. Vidal, "Routing a mix of conventional, plug-in hybrid, and electric vehicles," *European Journal of Operational Research*, vol. 272, no. 1, pp. 235-248, 2019.
- [5] G. Macrina, L. D. P. Pugliese, F. Guerriero, and G. Laporte, "The green mixed fleet vehicle routing problem with partial battery recharging and time windows," *Computers & Operations Research*, vol. 101, pp. 183-199, 2019.
- [6] G. Hiermann, J. Puchinger, S. Ropke, and R. F. Hartl, "The electric fleet size and mix vehicle routing problem with time windows and recharging stations," *European Journal of Operational Research*, vol. 252, no. 3, pp. 995-1018, 2016.
- [7] M. Schneider, A. Stenger, and D. Goeke, "The electric vehicle-routing problem with time windows and recharging stations," *Transportation Science*, vol. 48, no. 4, pp. 500-520, 2014.
- [8] Á. Felipe, M. T. Ortuño, G. Righini, and G. Tirado, "A heuristic approach for the green vehicle routing problem with multiple technologies and partial recharges," *Transportation Research Part E: Logistics and Transportation Review*, vol. 71, pp. 111-128, 2014.
- [9] M. Keskin and B. Çatay, "Partial recharge strategies for the electric vehicle routing problem with time windows," *Transportation Research Part C: Emerging Technologies*, vol. 65, pp. 111-127, 2016.
- [10] M. Keskin and B. Çatay, "A matheuristic method for the electric vehicle routing problem with time windows and fast chargers," *Computers & Operations Research*, vol. 100, pp. 172-188, 2018.
- [11] G. Macrina, G. Laporte, F. Guerriero, and L. D. P. Pugliese, "An energy-efficient green-vehicle routing problem with mixed vehicle fleet, partial battery recharging and time windows," *European Journal of Operational Research*, vol. 276, no. 3, pp. 971-982, 2019.
- [12] T. Bektaş and G. Laporte, "The pollution-routing problem," *Transportation Research Part B: Methodological*, vol. 45, no. 8, pp. 1232-1250, 2011.
- [13] G. Desaulniers, F. Errico, S. Irnich, and M. Schneider, "Exact algorithms for electric vehicle-routing problems with time windows," *Operations Research*, vol. 64, no. 6, pp. 1388-1405, 2016.
- [14] E. Demir, T. Bektaş, and G. Laporte, "An adaptive large neighborhood search heuristic for the pollution-routing problem," *European Journal of Operational Research*, vol. 223, no. 2, pp. 346-359, 2012.
- [15] J. Li, D. Wang, and J. Zhang, "Heterogeneous fixed fleet vehicle routing problem based on fuel and carbon emissions," *Journal of Cleaner Production*, vol. 201, pp. 896-908, 2018.