3-2023

# Asymmetric information of product authenticity on C2C e-commerce platforms: How can inspection services help?

Linqiu LI
*University of Science and Technology of China*

Xin FANG
*Singapore Management University*, xfang@smu.edu.sg

Yun Fong LIM
*Singapore Management University*, yflim@smu.edu.sg

# Asymmetric Information of Product Authenticity on C2C E-Commerce Platforms: How Can Inspection Services Help?

Linqiu LI • Xin FANG • Yun Fong LIM

Lee Kong Chian School of Business, Singapore Management University

Oct 01, 2022

**Abstract**

**Problem definition:** We consider a customer-to-customer (C2C) platform that provides an inspection service. Uncertain about his product's authenticity, a seller sells his product through the platform. Before purchasing, a buyer obtains a signal of the product authenticity from the product's price set by the seller. The platform's inspection service can detect a counterfeit with a probability. If the product passes the inspection, the platform sends it to the buyer and charges the seller a commission fee. Otherwise, the platform returns it to the seller and charges the seller a penalty fee.

**Methodology/results:** We develop a two-stage game-theoretical model. In the first stage, the platform designs a contract specifying the commission and penalty fees. In the second stage, the seller signals his product authenticity by setting a price and the buyer decides whether to purchase it. This results in a contract design problem that governs a signaling game. We find that the effect of inspection is beyond merely detecting counterfeits. The inspection, even an imperfect one, changes the signaling game's structure and incentivizes the seller whose product is likely authentic to sell through the platform. This can only be achieved by carefully choosing the commission and penalty fees. Moreover, a larger platform's expected profit does not imply a larger commission fee or price in equilibrium. Under some mild conditions, the optimal commission increases but the optimal penalty decreases as the platform's inspection capability improves.

**Managerial implications:** The inspection service is not widely available among leading C2C platforms as it is considered imperfect and costly. Our study suggests that its benefit may be underestimated in practice. Moreover, the inspection can eliminate the seller's information rent and generate more revenue for the platform. This paper provides guidance on how to set commission and penalty fees when the inspection service is provided.

**Keywords:** asymmetric information, counterfeit, inspection, platform

## 1 Introduction

Customer-to-Customer (C2C) platforms receive increasing attention among e-commerce business models. Prominent examples of C2C platforms include eBay and Taobao, which act as an intermediary to match individual sellers and buyers. Under this business model, the sellers display their products on a platform and send purchased items to the buyers directly. The platform obtains revenue by charging a commission fee whenever a product is successfully sold.

Most C2C platforms have little control over the authenticity of the products displayed. As a result, complaints about counterfeits are on the rise (Rivera, 2018). According to Suthivarakom (2020), with third-party sellers on Amazon, the keyword "counterfeit" or "fake" appears in 1.725% and 4.275% of all the Amazon's reviews in 2015 and 2019 respectively. Similarly, Taobao was found to have 240,000 vendors selling counterfeits in 2017 (Hawkins and Thorpe, 2019). Recently, sneaker resale is becoming a fiercely contested market as some shoppers collect rare merchandise and sell it at a markup price online (Patel, 2019). With a lucrative profit, there are more and more counterfeits in the sneaker market. The fake Nike footwear in 2019 would be worth more than $472 million if it were real (Rohrlich, 2020). C2C platforms cannot guarantee the authenticity of listed products because they do not physically handle the products. In addition, it is difficult for buyers to judge the authenticity because they cannot physically assess the products before purchases. Thus, buyers are worried about ending up with a counterfeit at a high price (Kim, 2018, Suthivarakom, 2020).

Being aware of the product's origin and able to physically assess the items, sellers generally know more about the authenticity of their products than buyers. Some professional sellers on Amazon and Taobao may know whether their products are authentic when they know the product source. However, many of the sellers on C2C platforms are non-professional and may not be *completely sure* about the authenticity of their products. For example, high-end sneakers are usually offered in a limited quantity and have a long trade tail as the shoes continue to be bought and sold on secondary markets (Dennis, 2019). Although the source is not completely reliable, most collectors purchase such sneakers through the secondary markets instead of official channels. Moreover, some fake items resemble the authentic ones so closely that it becomes very challenging to differentiate them (Koaysomboon, 2018). Even skilled appraisers make mistakes sometimes (Clifford, 2019). It requires professional training and extensive experience to inspect the products. Due to unreliable sources and the lack of professional inspection capability, many sellers on C2C platforms are not completely certain about the authenticity of their products.

To address the issue of product authenticity or quality, the literature (for example, Baiman et al., 2000, Hwang et al., 2006, Babich and Tang, 2012) has investigated *ex ante* and *ex post* inspections. The ex ante inspections certify sellers prior to transactions, whereas the ex post inspections refer to inspections by buyers after receiving the products. Different from the literature, we study *interim* inspections adopted by some C2C e-commerce platforms, where products are inspected after buyers place their orders but before receiving the products. Typical C2C platforms implementing the interim inspections include StockX, Goat, and Poizon (see `stockx.com`, `goat.com`, and `poizon.com` respectively). These platforms improve their inspection capability by investing heavily in advanced technology and experienced inspectors. For example, Goat enhances its platform's inspection technology using machine learning (Kelly, 2018). Although the interim inspections may cause a delay in transactions, buyers value this service, making these platforms popular. For example, in 2020 StockX completed more than 7.5 million transactions and its gross product value reached $1.8 billion (Farrell, 2021).

The transactions on these platforms proceed as follows. A seller first displays his product on a platform and sets a price. If a buyer decides to purchase the product, she pays the price to the platform. The seller then sends the purchased item to the platform for inspection. If the product passes the inspection, it will be shipped to the buyer. In this case, the platform charges a fraction of the selling price as a commission fee and remits the remaining amount to the seller. For example, StockX sets 8% to 14.5% of the price as a commission fee for different product categories, whereas Goat sets 9.5% to 25% of the price as a commission fee. If the product fails the inspection, the platform returns the item to the seller. In that case, the platform refunds the payment to the buyer and may penalize the seller based on the selling price. For example, StockX asks for 15% of the price as a penalty fee, while Goat has zero penalty to the seller. In practice, different platforms adopt different policies for the commission and penalty fees. One of our goals is to identify the optimal policy for the platforms.

We develop a game-theoretical model to capture the interactions among a platform, a seller, and a buyer. Being uncertain about his product's authenticity, the seller sells his product through the platform. Before purchasing the product, the buyer has even less information about the product authenticity and only obtains a signal from the product's price set by the seller. The platform can detect whether the product is a counterfeit with a probability, which measures the platform's inspection capability. If the product passes the inspection, the platform sends the product to the buyer and charges the seller a commission fee. If the product fails the inspection, the platform returns the product to the seller and charges the seller a penalty fee. The game has two stages. In the first stage, the platform designs a contract specifying the commission and penalty fees as fractions of the selling price anticipating the interaction between the seller and the buyer. In the second stage, given the contract, the interaction between the seller and the buyer forms a signaling game. The seller signals his product's authenticity by setting the price. Given the price, the buyer then decides whether to purchase the product.

We analyze the equilibrium decisions of the three parties given that the platform provides the inspection service and the seller is uncertain about his product authenticity. Specifically, we would like to answer the following questions:

1. How does the platform's inspection affect the seller's pricing decision? What are the benefits for the platform to provide the inspection service?

2. How should the platform set the commission and the penalty fractions to maximize its profit? How does the platform's inspection capability affect its commission and penalty?

3. What if the seller knows his product authenticity? How do the equilibrium results and managerial insights differ from the case in which the seller is uncertain about it?

We summarize our findings and contributions as follows. We find that the effect of inspection is beyond merely detecting counterfeit products. The inspection, even an imperfect one, changes the structure of the signaling game completely. It incentivizes only the high-product-authenticity seller to sell through the platform. However, this can only be achieved with carefully designed commission and penalty fees. Since the inspection service is not yet widely available among leading C2C e-commerce platforms, our results suggest that the benefit of product authenticity inspection may be underestimated in practice. In addition, with the inspection service, the platform can eliminate the information rent earned by the seller. This generates more revenue for the platform.

To maximize its expected profit, we identify the optimal contract for the platform. We find that a larger platform's expected profit does not imply a larger commission fraction or a higher price in equilibrium. Under some mild conditions, the optimal commission fraction increases but the optimal penalty fraction decreases as the platform's inspection capability improves. This implies that the platform should adjust the contract parameters accordingly if its inspection capability changes because of the use of new technology or new inspectors. If the inspection is perfect such that the platform can identify all the counterfeits, it is optimal for the platform to remove the penalty fee completely.

We find that the equilibrium results and managerial insights do depend on whether the seller is certain about the product authenticity. Specifically, if the seller knows whether his product is authentic or counterfeit, the platform can design a contract to screen out the seller with a counterfeit. In this case, the product on the platform would be authentic, and the platform's profit and optimal commission would not depend on the inspection capability. In contrast, if the seller is uncertain about his product authenticity, products with different authenticity levels may be sold through the platform, reflecting what happens on C2C platforms selling second-hand collectible goods (such as StockX, Goat, and Poizon).

The rest of this paper is organized as follows. §2 reviews the related literature. §3 describes the two-stage model. §4 solves the signaling game in the second stage. §5 derives the platform's optimal contract in the first stage. §6 conducts sensitivity analysis both analytically and numerically. §7 analyzes the case where the seller is certain about his product authenticity. §8 discusses managerial insights and concludes the paper. Appendix A provides additional results. Appendix B discusses two extensions of our model. Appendix C presents all proofs.

## 2 Related Literature

This paper studies a C2C platform with an inspection service under asymmetric information about product authenticity. It is related to four streams of literature: quality management and inspections in a decentralized supply chain, signaling asymmetric quality information, counterfeit products, and platform business models.

The quality management literature typically focuses on inspection decisions and contract designs in a moral hazard setting in which suppliers' quality decisions cannot be observed. For example, Baiman et al. (2000) analyze a variety of scenarios in which players' actions or product failures may or may not be contractible. Hwang et al. (2006) compare buyers' inspection and vendors' certification. Babich and Tang (2012) consider three mechanisms to deal with product adulteration problems: deferred payment, product inspection, and combined mechanisms. Seung and Taesu (2018) investigate several reward mechanisms for collaborative product quality improvement in a buyer-driven supply chain. Mariya and Edieal (2018) consider buyers' investment efforts to improve supplier quality, monetary incentives to motivate suppliers' quality-improvement efforts, and inspection upon supplier delivery to control outgoing quality. Lee and Li (2018) model the interaction between a buyer and a supplier using relational contracts in which penalties for quality failures are set so that the supplier voluntarily pays them. Different from these papers, our paper falls into the category of adverse selection in games with incomplete information rather than moral hazard. Specifically, we consider that the seller who has better (but not perfect) information about product quality tries to signal that information to the buyer with or without an inspection. This is not analyzed in this stream of literature.

Our work is closely related to signaling quality information. Several different forms of quality signals have been examined in the literature, including advertising (Kihlstrom and Riordan, 1984, Milgrom and Roberts, 1986), pricing (Bagwell and Riordan, 1991, Stiving, 2000), warranties (Lutz, 1989, Boulding and Kirmani, 1993), money-back guarantee (Moorthy and Srinivasan, 1995), umbrella branding (Wernerfelt, 1988), scarcity (Stock and Balachander, 2005), queues (Debo et al., 2012), and brand extension (Moorthy, 2012). Kirmani and Rao (2000) provide a comprehensive review of the literature on signaling quality. Yu et al. (2015) show that capacity rationing in advance selling can be an effective signal of quality and use Pareto dominance to select equilibrium. Different from this stream of literature, we investigate how a platform's inspection affects quality signaling between a seller and a buyer on the platform. To the best of our knowledge, our paper is the first to analyze the impact of inspection in a signaling setting. In addition, different from the literature, we consider a contract-design problem governing a signaling game. The platform's optimal contract depends on the equilibrium of the signaling game between the seller and the buyer. This makes our model technically challenging to solve. Finally, most of the quality signaling papers consider only two quality types: a high type or a low type. In contrast, we assume the seller's type is continuous, representing a probability of his product authenticity. This model element reflects sellers' uncertainty about product authenticity on C2C platforms especially when selling second-hand collectible goods.

Although our model can be applied to general quality problems on C2C platforms, product counterfeiting is a major cause of these quality problems, which relates our paper to the literature on counterfeit products. Wilcox et al. (2009) consider how luxury brands express themselves affects consumers' desire for counterfeits. Zhang et al. (2012) study how non-deceptive counterfeits affect the price, market share, and profitability of brand name products. Hu et al. (2013) consider a distribution channel consisting of a supplier who offers all-unit quantity discounts and a reseller who may divert some goods to the gray markets. Qian (2014) shows that fake products can benefit a manufacturer of high-quality products during the early stage of brand development. Cho et al. (2015) examine the effectiveness of anticounterfeit strategies against both deceptive and non-deceptive counterfeiters. Gao et al. (2017) find that copycats with a high physical resemblance but low product quality are more likely to successfully enter the market. Pun and DeYong (2017) examine the decisions of a manufacturer and a copycat firm who are competing for strategic customers. Yao and Zhu (2018) study the economic impact of anticounterfeit technology and explore measures to help increase the authentic company's profit. Pun et al. (2018) study how blockchain technology can be used to combat counterfeiting. Tang et al. (2020) study online platforms' anticounterfeit efforts to deter the entry of counterfeits under different business models. Guan et al. (2020) examine the impact of supplier copycatting in the presence of information asymmetry on product fit. Yi et al. (2020) investigate how counterfeits influence a global supply chain and how the supply chain should effectively take anticounterfeit actions. Chen and Papanastasiou (2021) examine the interaction between social-learning manipulation and equilibrium market outcomes as well as the impact of anti-manipulation measures. Wang et al. (2020) provide a literature review on counterfeiting and gray market in operations management. None of the above papers analyze how a platform's inspection affects a seller's signaling of product authenticity as in our paper. Furthermore, in these papers, a manufacturer either produces authentic products or counterfeits. In contrast, the seller is not certain about the authenticity of his product in our model.

Finally, our paper is also related to the literature on platform business models. Some papers in this stream focus on the trade-off between platform business models and traditional alternatives: a marketplace versus a reseller in Hagiu and Wright (2015) and Tian et al. (2018), a platform versus a vertically integrated firm in Hagiu and Wright (2018), and agency selling versus reselling in Abhishek et al. (2016). Jiang et al. (2011) consider that an e-retailer can function as a platform, and at the same time, sell directly. They find that functioning as a platform enables the e-retailer to learn about the demand for long-tail products sold by its enrolled sellers, and then to cherry-pick the successful ones for direct selling. Mantin et al. (2014) show that by introducing the platform format, a traditional retailer will create an outside option that improves its bargaining power in a negotiation with a manufacturer. Several papers consider peer-to-peer platforms where each consumer can be a supplier or a buyer. See, for example, Fraiberger and Sundararajan (2015), Jiang and Tian (2016), Benjaafar et al. (2018), Tian and Jiang (2018), and Abhishek et al. (2021). To the best of our knowledge, none of the papers in this stream analyze the contract design of a platform that offers an inspection service as in our paper. The seller in our model pays a penalty if his product fails the inspection. Thus, the contract that we study has its particular structure including both the commission and the penalty, which is different from the settings in the literature.

## 3 Problem Formulation

We consider a C2C e-commerce platform, a seller, and a buyer. The platform (such as StockX) serves as an online marketplace and provides a product inspection service. The platform first designs a contract, and then the seller decides whether to accept it. If the seller accepts the contract, he sells his product through the platform by setting a price. After seeing the price, the buyer decides whether to purchase the product.

The seller's product is either authentic ($\theta = 1$) with probability $\lambda$ or counterfeit ($\theta = 0$) with probability $(1 - \lambda)$, where $\lambda \in [0, 1]$. As discussed in §1, the seller is *not certain* about his product authenticity, but he knows the probability $\lambda$ as his private information. We call $\lambda$ the *type* of the seller. Following the literature of signaling games (see, for example, Kirmani and Rao, 2000), we assume that the platform and the buyer only know the probability distribution of $\lambda$ over $[0, 1]$ with p.d.f. $f(\cdot)$ and c.d.f. $F(\cdot)$. We call this probability distribution the common *prior belief* of the platform and the buyer about $\lambda$. Except the seller's type $\lambda$, all the information is public to all the three parties.

Let $\mu_S$ and $\mu_B$ represent the reservation prices of the seller and the buyer, respectively, for an authentic product. In contrast, for a counterfeit product, we assume that the reservation prices of the seller and the buyer are zero. Thus, the *seller's expected reservation price* is $\lambda\mu_S$, which depends on his type $\lambda$. If the seller chooses not to sell through the platform, then he can earn a revenue that is equal to his expected reservation price through an outside option. Knowing his type $\lambda$, the seller makes his pricing decision $\pi(\lambda) : [0, 1] \to [0, \mu_B] \cup \{\infty\}$, where $\infty$ is a formidable price representing that the seller does not wish to sell his product on the platform. After observing the price $\pi$, the buyer makes her purchase decision $I(\pi) : [0, \mu_B] \to \{0, 1\}$, where $I(\pi)$ equals 1 if she purchases the product and 0 otherwise.

If the buyer decides to purchase the product, she pays the price $\pi$ to the platform. The seller then sends his product to the platform for an inspection. Inspired by the quality inspection literature (Baiman et al., 2000, Hwang et al., 2006, Lee and Li, 2018), we assume that an authentic product always passes the inspection. In contrast, a counterfeit product fails the inspection with probability $P_M$, where $P_M \in [0, 1]$ represents the platform's *inspection capability.* We assume that the inspection is conducted by an independent and professional team and thus cannot be manipulated by the platform.

If the product passes the inspection, then the platform sends the product to the buyer. The platform keeps a fraction $\alpha$ of the payment $\pi$ as a commission and remits the remaining fraction

$(1-\alpha)$ to the seller. If the product fails the inspection, the seller pays a fraction $\beta$ of $\pi$ to the platform as a penalty (see, for example, `stockx.com`). The product is then returned to the seller and the platform refunds the payment $\pi$ to the buyer. To make our model general, we assume $\alpha \in (-\infty, 1]$ and $\beta \in [0, \infty)$. For the inspection, the platform incurs an inspection cost $c$ to handle and inspect the product. To avoid uninteresting equilibria, we assume $0 \leq c < \mu_B - \mu_S$.

After receiving the product, the buyer may inspect it. Likewise, we assume that an authentic product always passes the buyer's inspection, whereas a counterfeit product fails the buyer's inspection with probability $P_B \in [0, 1]$. If the buyer finds that the received product is a counterfeit, an *external failure* occurs and the platform incurs an external failure loss $\ell$, which includes the loss due to customer dissatisfaction (Baiman et al., 2000). The product cannot be returned because it is difficult for the buyer to provide evidences that the product passing the platform's inspection is a counterfeit. For example, StockX does not allow product returns. Figure 1 illustrates the event flow after the buyer purchases the product.

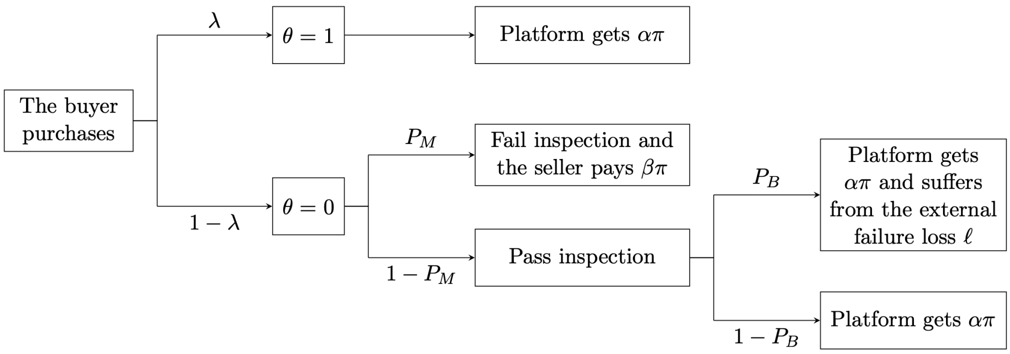



Figure 1: The event flow after the buyer purchases the product

There are two stages in the decision-making process. In the first stage, the platform chooses the contract parameters $\alpha$ and $\beta$. After knowing the contract $(\alpha, \beta)$, the seller decides whether to sell his product through the platform by setting the price $\pi$ based on his type $\lambda$ in the second stage. After observing the price $\pi$, the buyer updates her belief about the seller's type and decides whether to purchase the product. Figure 2 shows the sequence of the decisions.

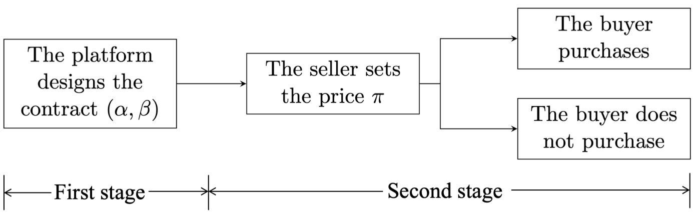



Figure 2: The decision sequence

Given a contract $(\alpha, \beta)$, the interaction between the seller and the buyer forms a signaling game. Following a backward induction, we first analyze the equilibrium of the signaling game in the second stage, and then identify the platform's optimal contract in the first stage taking into account the impact of the contract on the signaling game.

# 4 The Second Stage: Signaling Game

We first analyze the equilibrium of the second-stage signaling game, which is a dynamic game with incomplete information. In this game, the seller uses the price to signal the product's authenticity, and then the buyer decides whether to purchase the product.

We employ the concept of *perfect Bayesian equilibrium* (see its definition in Appendix C) to analyze the game. Following the literature of signaling games (see, for example, Bagwell and Riordan, 1991), we focus on pure-strategy equilibria. Moreover, if multiple equilibria exist, we apply *Pareto dominance* to select an equilibrium (Fudenberg and Tirole, 1991, Yu et al., 2015). Specifically, we choose an equilibrium that Pareto dominates others from the seller's point of view (in this equilibrium, each seller type $\lambda$ obtains a revenue no less than what he earns in any other equilibria). Since the seller makes a decision first in the signaling game, it is reasonable to assume that the seller can choose the equilibrium most appealing to himself. Such an equilibrium is supported by behavioral experiments (Van, 1999). Lastly, to avoid trivialities, we focus on *participating equilibria* (Yu et al., 2015, Janssen et al., 2005). Specifically, we assume that in an equilibrium the seller would not adopt price 0 or a price that is rejected by the buyer. In these cases, it is more profitable for the seller to choose not to sell through the platform. Therefore, we focus on a set of equilibria in which the seller either chooses not to sell through the platform ($\pi = \infty$) or adopts a positive price ($0 < \pi \leq \mu_B$) that is acceptable to the buyer.

Different from the literature on price signaling, we investigate the effect of inspection on the signaling game. We first present the equilibrium results with the platform providing the inspection service in §4.1. After that, we compare these results with a case in which the platform does not provide the inspection service in §4.2.

## 4.1 Equilibrium Results of The Signaling Game

We first analyze the purchase decision of the buyer. According to Figure 1, the probability for the product to pass the platform's inspection is $\lambda + (1 - \lambda)(1 - P_M)$. Thus, the expected cost for the buyer to purchase the product is $(1 - P_M + P_M\lambda)\pi$. Given $\pi \in (0, \mu_B]$ and $\lambda \in [0, 1]$, the buyer's utility is $R_B(I; \pi, \lambda) = [\lambda\mu_B - (1 - P_M + P_M\lambda)\pi]\, I$, where $\lambda\mu_B$ represents the buyer's expected reservation price. After observing the price $\pi$, the buyer generates a *posterior belief* about the seller's type $\lambda$, which is a probability distribution over $[0,1]$ with p.d.f. $m(\cdot|\pi)$. The buyer's expected utility is $R_B(I;\pi) = \int_0^1 R_B(I;\pi,x)m(x|\pi)\mathrm{d}x$. The buyer purchases the product ($I^*(\pi) = 1$) if and only if $R_B(1;\pi) \geq R_B(0;\pi) = 0$, which is equivalent to

$$E[\lambda|\pi] \geq q(\pi) \triangleq \frac{(1-P_M)\pi}{\mu_B - P_M\pi}, \tag{1}$$

where $E[\lambda|\pi] \triangleq \int_0^1 xm(x|\pi)\mathrm{d}x$ is the seller's expected type from the buyer's perspective after she observes the price $\pi$, and $q(\pi)$ is the critical level of the seller's expected type.

**Lemma 1. (The buyer's equilibrium decision)** *After observing the price $\pi$, the buyer purchases the product (that is, $I^*(\pi) = 1$) if and only if the seller's expected type $E[\lambda|\pi]$ is no less than its critical level $q(\pi)$.*

To analyze the seller's decision, we need to determine his expected revenue. Based on Figure 1, the seller obtains the revenue $(1-\alpha)\pi$ if his product passes the inspection with probability $\lambda + (1-\lambda)(1-P_M)$, but pays the penalty $\beta\pi$ if his product fails the inspection with probability $(1-\lambda)P_M$. Thus, if the seller chooses to sell his product with price $\pi \in (0,\mu_B]$, his expected revenue is $R_S(\pi;\lambda,I) = [(1-P_M+P_M\lambda)(1-\alpha)\pi - (1-\lambda)P_M\beta\pi]\,I$. If he chooses not to sell through the platform ($\pi = \infty$), his expected revenue $R_S(\pi;\lambda,I)$ is equal to his expected reservation price $\lambda\mu_S$.

Next, we derive the equilibrium of the second-stage signaling game. We will prove in §5 that the platform's optimal contract does not choose a small penalty fraction ($\beta < \frac{1-P_M}{P_M}(1-\alpha)$). Thus, we present the equilibrium analysis under a small penalty fraction in Appendix A.1 and focus on the equilibrium under a large penalty fraction ($\beta > \frac{1-P_M}{P_M}(1-\alpha)$) below. Specifically, we follow the three steps illustrated in Figure 3 to derive the equilibrium of the signaling game.

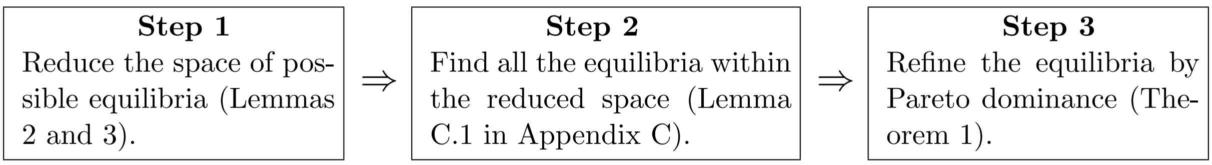



Figure 3: Three steps to derive the equilibrium of the signaling game

#### 4.1.1 Step 1: Reduce The Space of Possible Equilibria

In contrast to signaling games with discrete types, the seller's type is continuous in our model. As a result, the seller's pricing decision is a function $\pi(\lambda) : [0,1] \to (0,\mu_B] \cup \{\infty\}$. There is an infinite number of possible functions mapping from $[0,1]$ to $(0,\mu_B] \cup \{\infty\}$. Thus, we identify some necessary conditions of the seller's equilibrium decision $\pi^*(\lambda)$ in Lemma 2, which will substantially simplify our subsequent analysis. Given $(\alpha, \beta)$, define a constant $\underline{\hat{\lambda}} \triangleq \frac{P_M\beta-(1-P_M)(1-\alpha)}{P_M(1-\alpha+\beta)}$ and a function $A\left(\hat{\lambda}\right) \triangleq \frac{\hat{\lambda}\mu_S}{P_M(1-\alpha+\beta)\hat{\lambda}+(1-P_M)(1-\alpha)-P_M\beta}$.

**Lemma 2. (Structures of the seller's equilibrium decision)**

*If $\beta > \frac{1-P_M}{P_M}(1-\alpha)$, the seller's equilibrium pricing decision has one of the following forms:*
*(i) $\pi^*(\lambda) = \infty$ if $\lambda < \hat{\lambda}$ and $\pi^*(\lambda) = \hat{\pi}$ if $\lambda \geq \hat{\lambda}$, where*

$$\underline{\hat{\lambda}} < \hat{\lambda} \leq 1, \hat{\pi} = A\left(\hat{\lambda}\right). \tag{2}$$

*(ii) $\pi^*(\lambda) = \infty$ for $0 \leq \lambda \leq 1$.*

Lemma 2 implies that the seller, who chooses to sell, adopts the same equilibrium price $\hat{\pi}$ regardless of his type. This is because if the seller chooses to sell, his expected revenue $R_S(\pi; \lambda, I)$ increases with $\pi$ as long as the price is acceptable to the buyer (that is, $I^*(\pi) = 1$). Thus, to maximize his expected revenue, the seller, who chooses to sell, should adopt the highest price acceptable to the buyer regardless of his type. In other words, a strategy profile with different seller types adopting more than one selling price is not an equilibrium in our model, because the seller types adopting the lower prices always have incentives to deviate to the highest price. Thus, only *semi pooling equilibria* (part 1 of Lemma 2) with one selling price or *pooling equilibria* (part 2 of Lemma 2) may arise in this signaling game. This is different from the literature on separating equilibrium (for example, Bagwell and Riordan, 1991, Yu et al., 2015) because the seller's revenue in our problem can be separated in terms of the seller type $\lambda$ and the signal $\pi$ (see the proof of Lemma 2 for details).

We further explain the structure of the equilibrium pricing decision specified in Lemma 2 as follows. If the penalty fraction $\beta$ is large, part 1 of Lemma 2 shows that there may exist a semi pooling equilibrium, in which the seller with type larger than a threshold ($\lambda \geq \hat{\lambda}$) chooses to sell with the same equilibrium price $\hat{\pi}$, while the seller with type smaller than the threshold refuses to sell. In this semi pooling equilibrium, the seller can *partially* reveal his type by the pricing decision. Specifically, the equilibrium price $\hat{\pi}$ signals that the seller's type is no less than $\hat{\lambda}$. Let $\hat{R}_S(\lambda) = R_S\left(\hat{\pi}; \lambda, I^*\left(\hat{\pi}\right)\right) = \left[(1-P_M+P_M\lambda)(1-\alpha)-(1-\lambda)P_M\beta\right]\hat{\pi}$ represent the seller's expected revenue by adopting the equilibrium price $\hat{\pi}$. In Figure 4, the bold solid line represents $\hat{R}_S(\lambda)$ and the thin solid line represents the seller's expected reservation price $\lambda\mu_S$. The two lines intersect at $\hat{\lambda}$, which is implied by the condition $\hat{\pi} = A\left(\hat{\lambda}\right)$ in (2), where type-$\hat{\lambda}$ seller is indifferent between selling with price $\hat{\pi}$ and not selling. The seller with $\lambda < \hat{\lambda}$ refuses to sell through the platform because he will earn less than his expected reservation price if he sells with price $\hat{\pi}$. In contrast, the seller with $\lambda \geq \hat{\lambda}$ earns more than his expected reservation price $\lambda\mu_S$ by selling with price $\hat{\pi}$, and he chooses to sell through the platform.

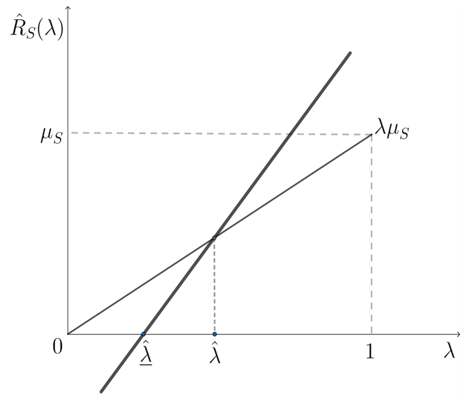



Figure 4: The seller's expected revenue $\hat{R}_S(\lambda)$ by selling with price $\hat{\pi}$

Next, we identify the condition on the equilibrium price $\hat{\pi}$ such that it is acceptable to the buyer. Denote $\bar{F}(x) = 1 - F(x)$. The buyer's *on-equilibrium belief* $m(\cdot|\hat{\pi})$ (the belief associated with the price adopted on the equilibrium path) should satisfy the Bayesian rule: $m(x|\hat{\pi})$ equals $f(x)/\bar{F}\left(\hat{\lambda}\right)$ if $x \geq \hat{\lambda}$, and 0 otherwise. Thus, $E[\lambda|\hat{\pi}] = \int_0^1 xm(x|\hat{\pi})\mathrm{d}x = \int_{\hat{\lambda}}^1 xf(x)\mathrm{d}x/\bar{F}\left(\hat{\lambda}\right)$. According to Lemma 1, $I^*(\hat{\pi}) = 1$ if and only if $E[\lambda|\hat{\pi}] \geq q(\hat{\pi})$, which is equivalent to $\hat{\pi} \leq \frac{\mu_B E[\lambda|\hat{\pi}]}{1-P_M+P_M E[\lambda|\hat{\pi}]}$. Therefore, we obtain

$$\hat{\pi} \leq \hat{\Pi}\left(\hat{\lambda}\right) \triangleq \frac{\mu_B \int_{\hat{\lambda}}^1 xf(x)\mathrm{d}x}{\int_{\hat{\lambda}}^1 (1 - P_M + P_M x)f(x)\mathrm{d}x}, \quad \hat{\lambda} \in [0, 1), \tag{3}$$

where $\hat{\Pi}\left(\hat{\lambda}\right)$ represents the expected value of the product from the buyer's perspective given her posterior belief about the seller type. The buyer is willing to purchase the product with a price no higher than $\hat{\Pi}\left(\hat{\lambda}\right)$ when only the high-type seller with $\lambda \geq \hat{\lambda}$ chooses to sell. Note that the definition of $\hat{\Pi}\left(\hat{\lambda}\right)$ is not applicable to $\hat{\lambda} = 1$ because its denominator is 0 at $\hat{\lambda} = 1$ (see (3)). For consistency, we define $\hat{\Pi}(1) = \lim_{\hat{\lambda} \to 1^-} \hat{\Pi}\left(\hat{\lambda}\right) = \mu_B$. Intuitively, when only the seller with type $\lambda = 1$ (authentic for sure) chooses to sell, the buyer is willing to purchase the product with a price no higher than his reservation price $\mu_B$ for an authentic product. Therefore, we obtain Lemma 3 below.

**Lemma 3.** *In a semi pooling equilibrium where $\pi^*(\lambda) = \infty$ if $\lambda < \hat{\lambda}$ and $\pi^*(\lambda) = \hat{\pi}$ if $\lambda \geq \hat{\lambda}$, the equilibrium price satisfies $\hat{\pi} \leq \hat{\Pi}\left(\hat{\lambda}\right)$.*

#### 4.1.2 Step 2: Find all the equilibria within the reduced space

Lemmas 2 and 3 characterize necessary conditions that an equilibrium needs to satisfy. This significantly reduces the space of possible equilibria. Next, we identify all the equilibria within this reduced space. Specifically, we prove that any pair of $\hat{\lambda}$ and $\hat{\pi}$ satisfying conditions (2) and (3) can lead to a semi pooling equilibrium with appropriately specified off-equilibrium beliefs, and a pooling equilibrium also exists. Due to space limitation, we summarize all the equilibria for different contract parameter values in Lemma C.1 in Appendix C.

#### 4.1.3 Step 3: Refine the equilibria by Pareto dominance

Within the reduced space, still there are infinitely many equilibria (see part 1 of Lemma C.1 in Appendix C). Among these equilibria, we choose an equilibrium that Pareto dominates the others such that in this equilibrium, each seller type $\lambda$ obtains a revenue no less than what he earns in any other equilibria. We prove in Theorem 1 that after refining the equilibria by Pareto dominance, there exists a unique semi pooling equilibrium determined by $\hat{\lambda}^*$, which is the unique root of $A\left(\hat{\lambda}\right) = \hat{\Pi}\left(\hat{\lambda}\right)$. Although the equilibrium threshold $\hat{\lambda}^*$ cannot be expressed as a closed-form function of $\alpha$ and $\beta$, we are able to derive a set of $(\alpha, \beta)$ that induces the equilibrium with $\hat{\lambda}^*$. Define $\Omega\left(\hat{\lambda}^*\right) = \left\{(\alpha,\beta)\middle|\alpha < 1 - \frac{\mu_S}{\hat{\Pi}(\hat{\lambda}^*)},\ \beta = \frac{\left(1-P_M+P_M\hat{\lambda}^*\right)(1-\alpha)\hat{\Pi}\left(\hat{\lambda}^*\right)-\hat{\lambda}^*\mu_S}{\left(1-\hat{\lambda}^*\right)P_M\hat{\Pi}\left(\hat{\lambda}^*\right)}\right\}$ for $0 < \hat{\lambda}^* < 1$, and $\Omega\left(\hat{\lambda}^*\right) = \left\{(\alpha,\beta)\middle|\alpha = 1 - \frac{\mu_S}{\hat{\Pi}(\hat{\lambda}^*)}, \beta > \frac{1-P_M}{P_M}(1-\alpha)\right\}$ for $\hat{\lambda}^* = 1$. Theorem 1 summarizes the complete equilibrium result of the signaling game after the refinement.

**Theorem 1. (Refined equilibrium of the signaling game)** *If the platform provides the inspection service ($P_M > 0$), under the contract $(\alpha,\beta) \in \Omega\left(\hat{\lambda}^*\right)$ and $\hat{\lambda}^* \in (0,1]$, there exists a unique pure-strategy perfect Bayesian equilibrium that Pareto dominates other equilibria from the seller's point of view. We have the following results in this equilibrium.*

*The seller's pricing decision is*

$$\pi^*(\lambda) = \begin{cases} \infty, & if \quad \lambda < \hat{\lambda}^*; \\ \hat{\pi}^* \triangleq \hat{\Pi}\left(\hat{\lambda}^*\right), & if \quad \lambda \geq \hat{\lambda}^*. \end{cases}$$

*The buyer's purchase decision is*

$$I^*(\pi) = \begin{cases} I(E[\lambda|\pi] \geq q(\pi)), & if \quad 0 < \pi < \hat{\pi}^*; \\ 1, & if \quad \pi = \hat{\pi}^*; \\ 0, & if \quad \hat{\pi}^* < \pi \leq \mu_B. \end{cases} \tag{4}$$

*The buyer's beliefs $m^*(\cdot|\pi)$ are determined as follows:*

*(i) If $0 < \pi < \hat{\pi}^*$, then $m^*(x|\pi)$ is arbitrary.*
*(ii) If $\pi = \hat{\pi}^*$, then*

$$m^*(x|\pi) = \begin{cases} 0, & if \quad x < \hat{\lambda}^*; \\ \frac{f(x)}{\bar{F}\left(\hat{\lambda}^*\right)}, & if \quad x \geq \hat{\lambda}^*. \end{cases}$$

*(iii) If $\hat{\pi}^* < \pi \leq \mu_B$, then $m^*(x|\pi)$ satisfies $E[\lambda|\pi] = \int_0^1 x m^*(x|\pi)\mathrm{d}x < q(\pi)$.*

There exists a unique semi pooling equilibrium for the signaling game: The seller chooses to sell his product with the same equilibrium price $\hat{\pi}^*$ if $\lambda \geq \hat{\lambda}^*$, but he chooses not to sell through the platform if $\lambda < \hat{\lambda}^*$. Note that the equilibrium price $\hat{\pi}^*$ attains its upper bound $\hat{\Pi}\left(\hat{\lambda}^*\right)$ in Lemma 3. The equilibrium price $\hat{\pi}^*$ reveals that the seller's type is no less than $\hat{\lambda}^*$ and the on-equilibrium belief $m^*(\cdot|\pi)$ for $\pi = \hat{\pi}^*$ satisfies the Bayesian rule. The off-equilibrium belief is specified for the price $\pi > \hat{\pi}^*$ so that the expected type of the seller conditioned on the price $\pi$ cannot be too high. Although the off-equilibrium belief can be arbitrary for the price $\pi < \hat{\pi}^*$, the buyer's decision in (4) should be consistent with her belief. We provide additional details of the beliefs in Appendix C.

Theorem 1 shows that the inspection, even an imperfect one, incentivizes the high-type seller with $\lambda \geq \hat{\lambda}^*$ to sell through the platform. However, this can only be achieved by carefully choosing the commission fraction $\alpha$ and penalty fraction $\beta$. Specifically, the equilibrium in which the seller with $\lambda \geq \hat{\lambda}^*$ chooses to sell is sustained by contract parameters $(\alpha, \beta) \in \Omega\left(\hat{\lambda}^*\right)$. Figure 5 illustrates the set $\Omega\left(\hat{\lambda}^*\right)$ on the $\alpha$-$\beta$ plane. Given $\hat{\lambda}^* \in [0, 1]$, $\Omega\left(\hat{\lambda}^*\right)$ is a ray (without the end point) in the dotted region of Figure 5, where $\alpha$ is small and $\beta$ is large. All $(\alpha, \beta)$ points on the ray $\Omega\left(\hat{\lambda}^*\right)$ can induce the equilibrium in Theorem 1.

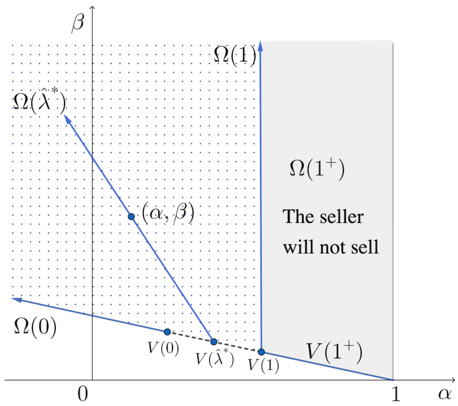



Figure 5: Equilibria under different contract parameter values

Define $\Omega\left(1^+\right) = \left\{(\alpha, \beta) \middle| \alpha > 1 - \frac{\mu_S}{\mu_B}, \beta > \frac{1-P_M}{P_M}(1-\alpha)\right\}$, which corresponds to the shaded region in Figure 5. Corollary 1 summarizes the equilibrium in $\Omega\left(1^+\right)$.

**Corollary 1.** *If the platform provides the inspection service ($P_M > 0$), under the contract $(\alpha, \beta) \in \Omega(1^+)$, the seller will not sell through the platform with $\pi^*(\lambda) = \infty$ for $0 \leq \lambda \leq 1$.*

Intuitively, a larger commission fraction $\alpha$ or penalty fraction $\beta$ leads to a smaller set of seller types that want to sell through the platform. For large $\alpha$ and $\beta$ as given in the shaded region $\Omega\left(1^+\right)$ in Figure 5, the seller earns less than his expected reservation price by adopting any price $\pi \in (0, \mu_B]$. Thus, the seller chooses not to sell through the platform regardless of his type.

Theorem 1 and Corollary 1 analyze the equilibria of the second-stage signaling game under a large penalty fraction ($\beta > \frac{1-P_M}{P_M}(1-\alpha)$), and next we analyze the equilibria of boundary cases ($\beta = \frac{1-P_M}{P_M}(1-\alpha)$). The boundary cases consist of the following three parts: (i) $\Omega(0) = \{(\alpha,\beta)|\alpha < 1 - \frac{\mu_S}{\hat{\Pi}(0)}, \beta = \frac{1-P_M}{P_M}(1-\alpha)\}$, which is a ray (without its end point) shown in Figure 5; (ii) $V\left(\hat{\lambda}^*\right) = \left\{(\alpha,\beta)\middle|\alpha = 1 - \frac{\mu_S}{\hat{\Pi}(\hat{\lambda}^*)}, \beta = \frac{1-P_M}{P_M}(1-\alpha)\right\}$, which is the end point of the ray $\Omega\left(\hat{\lambda}^*\right)$, and the dashed line in Figure 5 represents all such end points for $0 \leq \hat{\lambda}^* \leq 1$; (iii) $V(1^+) = \left\{(\alpha,\beta)\middle|\alpha > 1 - \frac{\mu_S}{\hat{\Pi}(1)}, \beta = \frac{1-P_M}{P_M}(1-\alpha)\right\}$, which is the bottom boundary of the shaded region in Figure 5. Corollary 2 summarizes the equilibria of the boundary cases.

**Corollary 2.** *Suppose the platform provides the inspection service ($P_M > 0$).*

*(i) If $(\alpha,\beta) \in \Omega(0)$, the seller chooses to sell with price $\pi^*(\lambda) = \hat{\Pi}(0)$ for $\lambda \in [0,1]$.*

*(ii) If $(\alpha,\beta) = V\left(\hat{\lambda}^*\right)$, there exist multiple equilibria. Among these equilibria, the seller earns the same expected revenue for $\lambda \in [0,1]$.*

*(iii) If $(\alpha,\beta) \in V(1^+)$, the seller will not sell through the platform with $\pi^*(\lambda) = \infty$ for $\lambda \in [0,1]$.*

For $(\alpha,\beta) = V\left(\hat{\lambda}^*\right)$, the unique equilibrium identified by Theorem 1, in which $\pi^*(\lambda) = \infty$ for $\lambda < \hat{\lambda}^*$ and $\pi^*(\lambda) = \hat{\pi}^* = \hat{\Pi}\left(\hat{\lambda}^*\right)$ for $\lambda \geq \hat{\lambda}^*$, is in the set of equilibria in part 2 of Corollary 2. We select this equilibrium as the refined equilibrium for $(\alpha,\beta) = V\left(\hat{\lambda}^*\right)$ to facilitate the description of the optimal contract in §5.

## 4.2 Effect of The Inspection Service on The Seller's Decision

What is the impact of the platform's inspection on the seller's decision? To answer this research question in §1, we analyze the case without the inspection service, and compare it with our base model with the inspection. After the buyer purchases the product, without the inspection, the seller directly sends the product to the buyer (for example, see `ebay.com` and `taobao.com`). This means $P_M = 0$ and $c = 0$ in our model. Note that the commission fraction $\alpha$ is the only contract parameter in this case.

Proposition 1 summarizes the equilibrium result for the case without the platform's inspection. Define $\check{\Pi}\left(\check{\lambda}\right) = \frac{\mu_B \int_0^{\check{\lambda}} x f(x)\mathrm{d}x}{F\left(\check{\lambda}\right)}$ for $0 < \check{\lambda} \leq 1$ and $\check{\Pi}(0) = \lim_{\check{\lambda}\to 0^+} \check{\Pi}\left(\check{\lambda}\right) = 0$. Define $\check{\lambda}^* = \max\left\{\check{\lambda}\middle|(1-\alpha)\check{\Pi}\left(\check{\lambda}\right) \geq \check{\lambda}\mu_S, 0 \leq \check{\lambda} \leq 1\right\}$.

**Proposition 1.** *If the platform does not provide the inspection service ($P_M = 0$), there exists a unique pure-strategy perfect Bayesian equilibrium that Pareto dominates other equilibria from the seller's point of view. We have the following results in this equilibrium.*

*The seller's pricing decision is*

$$\pi^*(\lambda) = \begin{cases} \breve{\pi}^* \triangleq \breve{\Pi}\left(\check{\lambda}^*\right), & if \quad \lambda \leq \check{\lambda}^*; \\ \infty, & if \quad \lambda > \check{\lambda}^*. \end{cases}$$

*The buyer's purchase decision is*

$$I^*(\pi) = \begin{cases} I\left(E[\lambda|\pi] \geq \frac{\pi}{\mu_B}\right), & if \quad 0 < \pi < \breve{\pi}^*; \\ 1, & if \quad \pi = \breve{\pi}^*; \\ 0, & if \quad \breve{\pi}^* < \pi \leq \mu_B. \end{cases}$$

*The buyer's beliefs $m^*(\cdot|\pi)$ are determined as follows:*

*(i) If $0 < \pi < \breve{\pi}^*$, then $m^*(x|\pi)$ is arbitrary.*
*(ii) If $\pi = \breve{\pi}^*$, then*

$$m^*(x|\pi) = \begin{cases} \frac{f(x)}{F(\check{\lambda}^*)}, & if \quad x \leq \check{\lambda}^*; \\ 0, & if \quad x > \check{\lambda}^*. \end{cases}$$

*(iii) If $\breve{\pi}^* < \pi \leq \mu_B$, then $m^*(x|\pi)$ satisfies $E[\lambda|\pi] = \int_0^1 xm^*(x|\pi)\mathrm{d}x < \frac{\pi}{\mu_B}$.*

If the platform does not inspect the product, then the low-type seller with $\lambda \in \left[0, \check{\lambda}^*\right]$, who likely holds a counterfeit product, chooses to sell the product through the platform in the equilibrium. This is because for the low-type seller with a small $\lambda$, it is easy to earn a revenue that matches his expected reservation price $\lambda\mu_S$. This phenomenon is similar to the classical "adverse selection". That is, the quality of goods traded in a market can decay in the presence of information asymmetry (Akerlof, 1970). In contrast to Akerlof (1970), who only considers discrete quality levels, the seller's product authenticity level $\lambda$ is continuous in our model. It is worth noting that only the low-type seller sells his product in Proposition 1, implying that without the inspection service, there is a high chance for the buyer to receive a counterfeit product. This is *not equivalent* to only counterfeits are sold on the platform. Our result is consistent with the observations in §1.

Recall from Theorem 1 that if the platform provides the inspection service and uses an appropriate contract $(\alpha, \beta)$, only the high-type seller sells his product through the platform, which is opposite to the result in Proposition 1. This is because without the inspection service, the platform screens the seller only through the commission fraction $\alpha$. In contrast, with the inspection service, the platform can screen the seller with an additional contract parameter (penalty fraction $\beta$). If the platform uses a contract $(\alpha, \beta)$ in the dotted region of Figure 5, Theorem 1 shows that the seller with a low type cannot earn a revenue matching his expected reservation price and chooses not to sell. As the product authenticity level on the platform improves, the buyer is willing to pay a higher price, which attracts a higher-type seller whose expected reservation price is higher. This increases the authenticity level and selling price even further, thus attracting the seller with an even higher type. Comparing Theorem 1 and Proposition 1 shows that the effect of the inspection service is beyond merely detecting counterfeit products. It also improves product authenticity by altering the seller's equilibrium decision.

The above comparison implies that the effect of the inspection service may be larger than one would expect. Specifically, Babich and Tang (2012) show that even with a sufficiently-high inspection capability, the seller cannot be completely deterred from product adulteration in a moral hazard setting. In contrast, in a signaling setting, we find that the inspection can completely reverse the seller's decision even with a low inspection capability. Given that the inspection service is largely missing from leading C2C e-commerce platforms, our results suggest that the benefit of inspection in improving product authenticity may be underestimated in practice.

Furthermore, if no inspection service is provided, the low-type seller with $\lambda \in \left[0, \check{\lambda}^*\right]$ earns more than his expected reservation price, which leads to the following corollary.

**Corollary 3.** *If the platform does not provide the inspection service ($P_M = 0$), it has to pay information rent to the seller.*

Note that the information rent determines how the revenue is distributed between the platform and the seller. §5.1 investigates the effect of the inspection service on the platform in terms of the information rent and compares it with Corollary 3.

# 5 The First Stage: Optimal Contract

We first analyze how the platform's contract parameters with the inspection service affect the seller's expected revenue in §5.1, and then identify the platform's optimal contract in §5.2.

## 5.1 Effect of The Contract Parameters on The Seller's Revenue

Theorem 1 and Corollary 2 show that in the second-stage equilibrium generated by any contract $(\alpha, \beta) \in \Omega\left(\hat{\lambda}^*\right) \cup V\left(\hat{\lambda}^*\right)$, the seller with $\lambda \geq \hat{\lambda}^*$ chooses to sell his product through the platform. However, different contracts lead to different expected revenues of the seller. The following proposition analyzes how the platform's contract parameters $(\alpha, \beta) \in \Omega\left(\hat{\lambda}^*\right) \cup V\left(\hat{\lambda}^*\right)$ affect the seller's expected revenue $R_S^*(\lambda) = R_S\left(\pi^*(\lambda); \lambda, I^*\right)$ in the second-stage equilibrium.

**Proposition 2.** *If the platform provides the inspection service ($P_M > 0$), then we have the following results.*

1. *Given $\hat{\lambda}^* \in [0,1)$, as $\alpha$ decreases and $\beta$ increases along the ray $\Omega\left(\hat{\lambda}^*\right)$, the seller's expected revenue $R_S^*(\lambda)$ in the second-stage equilibrium increases for $\lambda \in \left[\hat{\lambda}^*, 1\right]$.*
2. *Given $\hat{\lambda}^* \in [0,1]$ and the contract $(\alpha, \beta) = V\left(\hat{\lambda}^*\right)$, the seller's expected revenue $R_S^*(\lambda)$ in the second-stage equilibrium equals his expected reservation price $\lambda\mu_S$ for $\lambda \in \left[\hat{\lambda}^*, 1\right]$.*

Proposition 2 shows that the platform with the inspection service can alter the seller's expected revenue $R_S^*(\lambda)$ in the second-stage equilibrium by adjusting the contract parameters $\alpha$ and $\beta$. Specifically, as $\alpha$ decreases and $\beta$ increases along the ray $\Omega\left(\hat{\lambda}^*\right)$ in Figure 5, the high-type seller with $\lambda \in \left[\hat{\lambda}^*, 1\right]$ earns more revenue in the equilibrium. Moreover, if the platform adopts the contract $(\alpha, \beta) = V\left(\hat{\lambda}^*\right)$, which corresponds to the end point of the ray $\Omega\left(\hat{\lambda}^*\right)$ in Figure 5, the seller's expected revenue equals his expected reservation price regardless of $\lambda$.

**Corollary 4.** *If the platform provides the inspection service ($P_M > 0$), the seller's information rent is eliminated under the contract $(\alpha, \beta) = V\left(\hat{\lambda}^*\right)$.*

In contrast to Corollary 3, Corollary 4 shows that the inspection service is a useful tool for the platform to eliminate the seller's information rent. Moreover, we can easily prove that the inspection service can increase the total revenue of the platform and the seller. Therefore, by providing the inspection service, the platform can generate more revenue for itself through increasing the total "pie" and decreasing the share of the seller. This may explain why some platforms begin to offer the inspection service in practice. For example, eBay recently starts to inspect sneakers, watches, and handbags from its sellers (Holt, 2020).

With multiple seller types, the literature of contract design considers a menu of contracts to extract the system profit and reduce the information rent (Baron and Myerson, 1982). However, we consider a single contract $(\alpha, \beta)$ for all the seller types because it is impractical for the platform to choose continuous $\alpha$ and $\beta$ for different seller types in practice (see StockX and Goat). In contrast to the literature, by offering only a single contract for all the seller types, our platform can eliminate the seller's information rent if $(\alpha, \beta) = V\left(\hat{\lambda}^*\right)$.

## 5.2 Platform's Optimal Contract

Anticipating the equilibrium of the second-stage signaling game given each contract, we design the optimal contract of the platform. Recall from Theorem 1 and Corollary 2 that the contracts $(\alpha, \beta) \in \Omega\left(\hat{\lambda}^*\right) \cup V\left(\hat{\lambda}^*\right)$ lead to the equilibrium in which the seller with $\lambda \in \left[\hat{\lambda}^*, 1\right]$

chooses to sell. For a given threshold $\hat{\lambda}^*$ of the seller type, we first find the optimal contract $\left(\alpha^*\left(\hat{\lambda}^*\right), \beta^*\left(\hat{\lambda}^*\right)\right)$ in $\Omega\left(\hat{\lambda}^*\right) \cup V\left(\hat{\lambda}^*\right)$ in Figure 5. Then, we identify the optimal threshold $\hat{\lambda}^{**}$ for the platform.

We first find the optimal contract $\left(\alpha^*\left(\hat{\lambda}^*\right), \beta^*\left(\hat{\lambda}^*\right)\right)$ for a given threshold $\hat{\lambda}^*$ such that the platform's expected profit is maximized. To derive the platform's expected profit, we first consider the total expected profit of both the platform and the seller. In the equilibrium given $\hat{\lambda}^*$, the platform and the seller earn a total revenue of $\hat{\Pi}\left(\hat{\lambda}^*\right)$ (see Theorem 1) if the seller's product passes the inspection, which happens with probability $1 - P_M + P_M\lambda$. On the other hand, the platform incurs the external failure loss $\ell$ if the buyer finds that the product that passes the platform's inspection is a counterfeit, which happens with probability $(1-\lambda)(1-P_M)P_B$ (see Figure 1). Moreover, the platform incurs the inspection cost $c$ for each product inspection. Thus, in the equilibrium with threshold $\hat{\lambda}^*$, the total expected profit of both the platform and the seller is $\int_{\hat{\lambda}^*}^{1}\left[(1-P_M+P_Mx)\hat{\Pi}\left(\hat{\lambda}^*\right)-(1-x)(1-P_M)P_B\ell-c\right]f(x)\mathrm{d}x$. Note that $(\alpha,\beta)$ do not affect the total expected profit, but determine how the total profit is distributed between the platform and the seller.

According to Proposition 2, as $\alpha$ decreases and $\beta$ increases along the ray $\Omega\left(\hat{\lambda}^*\right)$ in Figure 5, the revenue of the seller with $\lambda \in \left[\hat{\lambda}^*, 1\right]$ increases. Since the total expected profit is fixed given $(\alpha,\beta) \in \Omega\left(\hat{\lambda}^*\right)$, the platform's expected profit decreases. So, the platform's expected profit is the largest at the end point $(\alpha,\beta) = V\left(\hat{\lambda}^*\right)$ in Figure 5. That is, given $\hat{\lambda}^* \in [0,1]$, the optimal contract is $\left(\alpha^*\left(\hat{\lambda}^*\right), \beta^*\left(\hat{\lambda}^*\right)\right)$, where

$$\alpha^*\left(\hat{\lambda}^*\right) = 1 - \frac{\mu_S}{\hat{\Pi}\left(\hat{\lambda}^*\right)}, \tag{5}$$

$$\beta^*\left(\hat{\lambda}^*\right) = \frac{(1-P_M)\mu_S}{P_M\hat{\Pi}\left(\hat{\lambda}^*\right)}. \tag{6}$$

Next, we identify the optimal threshold $\hat{\lambda}^{**}$ for the platform. According to Proposition 2, the seller's expected revenue is equal to his expected reservation price under the contract $(\alpha,\beta) = V\left(\hat{\lambda}^*\right)$. Thus, the platform's expected profit $R_M\left(\hat{\lambda}^*\right)$ equals the two parties' total expected profit minus the seller's expected reservation price. That is, $R_M\left(\hat{\lambda}^*\right) = \int_{\hat{\lambda}^*}^{1}\Big[(1-P_M+P_Mx)\hat{\Pi}\left(\hat{\lambda}^*\right)$ $-(1-x)(1-P_M)P_B\ell - c - x\mu_S\Big] f(x)\mathrm{d}x$. Substituting $\hat{\Pi}\left(\hat{\lambda}^*\right)$ in (3) into the expression of $R_M\left(\hat{\lambda}^*\right)$, we obtain Lemma 4.

**Lemma 4.** *Under the contract $(\alpha,\beta) = V\left(\hat{\lambda}^*\right)$, the platform's expected profit is*

$$R_M\left(\hat{\lambda}^*\right) = \int_{\hat{\lambda}^*}^{1} [x(\mu_B - \mu_S) - (1-x)(1-P_M)P_B\ell - c] f(x)\mathrm{d}x. \tag{7}$$

Lemma 4 shows that the platform's expected profit equals the gap of the expected reservation prices between the buyer and the seller minus the expected external failure loss and the inspection cost. Since a higher seller type results in a larger gap of the expected reservation prices and a smaller expected external failure loss, the platform prefers the seller with a higher type. To maximize its expected profit $R_M\left(\hat{\lambda}^*\right)$, the platform should choose a proper $\hat{\lambda}^*$ in (7). Theorem 2 determines the optimal threshold $\hat{\lambda}^{**}$ for the platform.

**Theorem 2. (Optimal contract of the platform)** *If the platform provides the inspection service ($P_M > 0$), the platform's optimal contract is $\left(\alpha^*\left(\hat{\lambda}^{**}\right), \beta^*\left(\hat{\lambda}^{**}\right)\right)$, where $\hat{\lambda}^{**} = \frac{(1-P_M)P_B\ell+c}{(1-P_M)P_B\ell+\mu_B-\mu_S}$. Furthermore, the platform's optimal expected profit is $R_M^* = R_M\left(\hat{\lambda}^{**}\right)$.*

Interestingly, the optimal threshold $\hat{\lambda}^{**}$ is independent of the distribution $F(\cdot)$ of the seller type $\lambda$. This is appealing because the distribution $F(\cdot)$ may not be accurately estimated in practice. Regardless of the distribution $F(\cdot)$, the platform should adopt a contract to attract the seller with $\lambda$ in the same set $\left[\hat{\lambda}^{**}, 1\right]$.

Since the contract in Theorem 2 is optimal for the platform, we have the following corollary.

**Corollary 5.** *Any contract with $\beta < \frac{1-P_M}{P_M}(1-\alpha)$ (the bottom blank triangle of Figure 5) is strictly dominated by the optimal contract $(\alpha^*, \beta^*) = V\left(\hat{\lambda}^{**}\right)$ for the platform.*

Some platforms, such as StockX and Goat, claim that their customers will receive 100% authentic products. To ensure that, the platform needs to have a perfect inspection service with $P_M = 1$. In this case, the platform's optimal contract is given in the following corollary.

**Corollary 6. (No penalty for perfect inspection)** *If the platform provides a perfect inspection service ($P_M = 1$), then $\hat{\lambda}^{**} = \frac{c}{\mu_B-\mu_S}$, $\alpha^* = 1 - \frac{\mu_S}{\mu_B}$, $\beta^* = 0$, and $\hat{\Pi}\left(\hat{\lambda}^{**}\right) = \mu_B$.*

Corollary 6 shows that the platform should remove the penalty if it has the perfect inspection service. This result is interesting and can be explained as follows. If the inspection service is perfect, an external failure will not happen. In this case, it is optimal for the platform to remove the penalty completely by setting $\beta^* = 0$ such that it can attract as many seller types as possible. This benefits the platform because even the seller with low $\lambda$ can potentially contribute to the platform's profit if his product turns out to be authentic. Nevertheless, the seller with $\lambda < \hat{\lambda}^{**} = \frac{c}{\mu_B-\mu_S}$ is still rejected by the platform. This is because the expected revenue generated from such a low seller type cannot cover the platform's inspection cost.

# 6 Sensitivity Analysis

We now analyze how market conditions influence the equilibrium results under the platform's optimal contract, and answer our second research question in §1. Specifically, we perform sensitivity analysis on the distribution of $\lambda$ and the platform's inspection capability $P_M$ in §6.1 and §6.2 respectively. We conduct sensitivity analysis on other parameters in Appendix A.2.

## 6.1 Sensitivity to The Seller-Type Distribution

Although Theorem 2 shows that the optimal threshold $\hat{\lambda}^{**}$ is independent of the seller-type distribution $F(\cdot)$, the other equilibrium results under the platform's optimal contract depend on the distribution. Proposition 3 describes how the characteristics of $F(\cdot)$ affect these equilibrium results. Consider markets 1 and 2 with seller-type distributions $F_1(\cdot)$ and $F_2(\cdot)$ respectively.

**Proposition 3. (Sensitivity to the seller-type distribution)**

1. *The platform's optimal expected profit can be written as* $R_M^* = (\mu_B - \mu_S + (1-P_M)P_B\ell)\int_{\hat{\lambda}^{**}}^1 \bar{F}(x)\mathrm{d}x$. *Thus,* $R_M^*$ *is larger in market 1 than in market 2 if and only if*

$$\int_{\hat{\lambda}^{**}}^1 \bar{F}_1(x)\mathrm{d}x \geq \int_{\hat{\lambda}^{**}}^1 \bar{F}_2(x)\mathrm{d}x. \tag{8}$$

2. *The optimal commission fraction* $\alpha^*$ *is larger, the optimal penalty fraction* $\beta^*$ *is smaller, and the equilibrium price* $\hat{\Pi}\left(\hat{\lambda}^{**}\right)$ *is larger in market 1 than in market 2 if and only if*

$$\frac{\int_{\hat{\lambda}^{**}}^1 \bar{F}_1(x)\mathrm{d}x}{\bar{F}_1\left(\hat{\lambda}^{**}\right)} \geq \frac{\int_{\hat{\lambda}^{**}}^1 \bar{F}_2(x)\mathrm{d}x}{\bar{F}_2\left(\hat{\lambda}^{**}\right)}. \tag{9}$$

To satisfy (8), it is sufficient to have $\bar{F}_1(x) \geq \bar{F}_2(x)$ for all $x \in [0,1]$ (that is, $F_1(\cdot)$ is stochastically larger than $F_2(\cdot)$). According to the discussion after Lemma 4, the platform earns more profit from the seller with a larger $\lambda$. Therefore, it is intuitive that the platform generates more profit in the market with a stochastically larger distribution.

According to part 1 of Proposition 3, the platform's optimal expected profit is proportional to $\int_{\hat{\lambda}^{**}}^1 \bar{F}(x)\mathrm{d}x$. Since the seller chooses to sell his product with probability $\bar{F}\left(\hat{\lambda}^{**}\right)$, the platform generates an expected profit proportional to $\int_{\hat{\lambda}^{**}}^1 \bar{F}(x)\mathrm{d}x / \bar{F}\left(\hat{\lambda}^{**}\right)$ conditioned on the seller chooses to sell. Note that it is possible for two distributions $F_1(\cdot)$ and $F_2(\cdot)$ to satisfy (8) but not (9) (see the discussion after the proof of Proposition 3 in Appendix C). Therefore, a larger expected profit does not imply a larger commission fraction or a higher price in equilibrium.

### 6.2 Sensitivity to Inspection Capability $P_M$

According to Proposition 2, under the platform's optimal contract, the seller's expected revenue equals his expected reservation price $\lambda\mu_S$, which does not depend on the inspection capability $P_M$. Theorem 3 shows how the other equilibrium results change with $P_M$. Define $\lambda^\dagger = \frac{(1-P_M)P_B\ell}{(1-P_M)P_B\ell+\mu_B-\mu_S}$, $\xi = \bar{F}\left(\hat{\lambda}^{**}\right) = \int_{\hat{\lambda}^{**}}^1 f(x)\mathrm{d}x$, and $\eta = \int_{\hat{\lambda}^{**}}^1 xf(x)\mathrm{d}x$.

**Theorem 3. (Sensitivity to $P_M$)**

*1. The optimal threshold $\hat{\lambda}^{**}$ decreases with $P_M$. In contrast, the equilibrium price $\hat{\Pi}\left(\hat{\lambda}^{**}\right)$ increases with $P_M$ if and only if*

$$\lambda^\dagger \left(1-\hat{\lambda}^{**}\right) f\left(\hat{\lambda}^{**}\right) \leq \frac{(\xi-\eta)\eta}{\eta-\hat{\lambda}^{**}\xi}. \tag{10}$$

*2. The platform's optimal expected profit $R_M^*$ increases with $P_M$. In contrast, the optimal commission fraction $\alpha^*$ increases with $P_M$ if and only if (10) holds. The optimal penalty fraction $\beta^*$ decreases with $P_M$ if and only if*

$$\lambda^\dagger \left(1-\hat{\lambda}^{**}\right) f\left(\hat{\lambda}^{**}\right) \leq \frac{P_M^2\eta^2+(1-P_M^2)\xi\eta}{(1-P_M)P_M\left(\eta-\hat{\lambda}^{**}\xi\right)}. \tag{11}$$

As the platform's inspection capability $P_M$ increases, the seller faces a higher risk of paying the penalty. Intuitively, one may think that the seller is less likely to sell his product through the platform. However, part 1 of Theorem 3 shows the opposite result: $\hat{\lambda}^{**}$ is decreasing in $P_M$, implying that the seller is more likely to sell through the platform. Since it is more likely to screen out a counterfeit product as $P_M$ increases, the platform optimizes the contract to induce a lower threshold $\hat{\lambda}^{**}$ of the seller type to increase the probability of a successful transaction.

Part 1 of Theorem 3 shows that the equilibrium price $\hat{\Pi}\left(\hat{\lambda}^{**}\right)$ increases with $P_M$ if and only if (10) holds. To understand this, define *input quality* $q^I$ and *output quality* $q^O$ as follows:

$$q^I = \frac{\int_{\hat{\lambda}^{**}}^1 xf(x)\mathrm{d}x}{\bar{F}\left(\hat{\lambda}^{**}\right)}, \tag{12}$$

$$q^O = \frac{\int_{\hat{\lambda}^{**}}^1 xf(x)\mathrm{d}x}{\int_{\hat{\lambda}^{**}}^1 (1-P_M+P_Mx)f(x)\mathrm{d}x} = \frac{q^I}{1-P_M+P_Mq^I}. \tag{13}$$

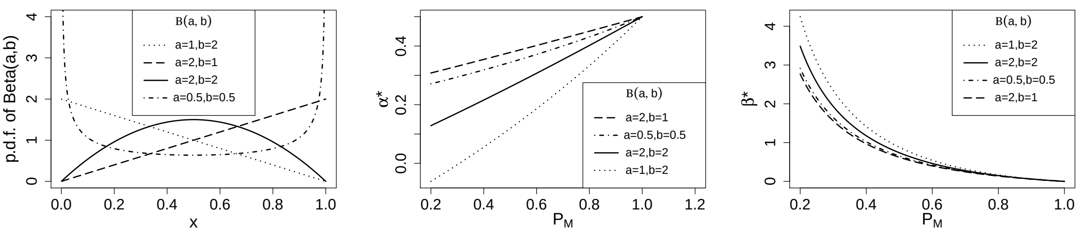
The input quality $q^I$ represents the probability of receiving an authentic product from the seller by the platform before its inspection. The output quality $q^O$ represents the probability of sending an authentic product to the buyer by the platform after its inspection. Note that $q^I \leq q^O$. If $P_M = 0$, then $q^O = q^I$; if $P_M = 1$, then $q^O = 1$.

On the one hand, since the optimal threshold $\hat{\lambda}^{**}$ decreases with $P_M$, we know from (12) that $q^I$ becomes lower as $P_M$ increases. On the other hand, a counterfeit product is more likely to be screened out as $P_M$ increases. If (10) holds, the positive effect of the improved inspection capability dominates the negative effect of the lower input quality. In that case, the output quality $q^O$ improves as the inspection capability $P_M$ increases. According to (3), the equilibrium price $\hat{\Pi}\left(\hat{\lambda}^{**}\right) = \mu_B q^O$. Thus, $\hat{\Pi}\left(\hat{\lambda}^{**}\right)$ increases with $P_M$ under condition (10).

Part 2 of Theorem 3 shows that the platform generates more profit if its inspection capability $P_M$ increases. This may sound intuitive, but in fact there are several drivers for this result. First, both the equilibrium price and the commission fraction increase with $P_M$ under condition (10), thus the platform earns more commission. Second, since the optimal threshold $\hat{\lambda}^{**}$ decreases with $P_M$, the seller is more likely to sell through the platform. Third, it becomes less likely to have an external failure if $P_M$ increases, and thus the platform is less likely to suffer from the external failure loss. In summary, the combination of these three drivers implies that the platform's expected profit increases with $P_M$.

Part 2 of Theorem 3 also shows that the optimal commission fraction $\alpha^*$ and penalty fraction $\beta^*$ are not always monotonic in $P_M$ and the monotonicity is guaranteed by conditions (10) and (11) respectively. Under these conditions, the platform should adjust the contract parameters accordingly if its inspection capability increases. Specifically, as $P_M$ increases to 100%, $\alpha^*$ increases to its upper bound $1 - \frac{\mu_S}{\mu_B}$ and $\beta^*$ reduces to 0 (see Corollary 6). We provide the following numerical examples to illustrate how conditions (10) and (11) can be satisfied. Example 1 gives a wide range of distributions that satisfy the conditions.

**Example 1.** *Consider the seller type $\lambda$ follows a Beta distribution $B(1,2)$, $B(2,1)$, $B(2,2)$ or $B(0.5,0.5)$, with $P_B = 0.2$, $\ell = 3$, $c = 0.2$, $\mu_S = 5$, and $\mu_B = 10$. Figure 6(a) shows the p.d.f. of each distribution above. Figures 6(b) and (c) show that the platform's optimal contract parameters $\alpha^*$ and $\beta^*$, respectively, are monotonic in the inspection capability $P_M$.*

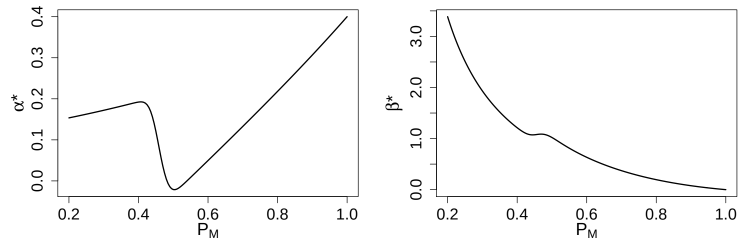



(a) (b) (c)

Figure 6: The impact of $P_M$ on the platform's optimal contract parameters

In contrast, if conditions (10) and (11) are violated, the optimal contract parameters are no longer monotonic in $P_M$. We present such an example below.

**Example 2.** *Consider the p.d.f. of the seller type* $f(x) = \frac{\varphi(x,0.25,0.01^2)+1}{2}$, *where* $\varphi\left(x, 0.25, 0.01^2\right)$ *is the p.d.f. of a truncated normal distribution on* $[0, 1]$ *with mean* $0.25$ *and standard deviation* $0.01$*. We set* $P_B = 0.2$, $\ell = 10$, $c = 0.2$, $\mu_S = 6$, *and* $\mu_B = 10$*. The density is* $20.45$ *at* $x = 0.25$, *which violates conditions (10) and (11). Figures 7(a) and (b) show that the platform's optimal contract parameters* $\alpha^*$ *and* $\beta^*$, *respectively, are not monotonic in the inspection capability* $P_M$*.*




(a) (b)

Figure 7: The impact of $P_M$ on the platform's optimal contract parameters when conditions (10) and (11) are violated

In summary, $\alpha^*$ increases but $\beta^*$ decreases as $P_M$ increases under some mild conditions (see Figure 6). However, if the density of a certain type is high as in Example 2, then $\alpha^*$ and $\beta^*$ may not be monotonic in $P_M$ (see Figure 7). Therefore, the platform should pay close attention to the seller-type distribution when choosing the contract parameters.

## 7 A Model with Two Seller Types

Sellers may know whether their products are authentic when product sources are clear. To investigate such a setting, we consider a case in which $\lambda$ is either 1 or 0 such that the seller is certain about his product authenticity in this section. Comparing this case with our base model, where $\lambda$ is continuous, allows us to investigate the impact of the seller's uncertainty about his product authenticity on equilibrium results. This will answer our third research question in §1.

In this section, we assume that $\lambda$ is either 1 or 0, which means the seller knows that his product is authentic (type 1) or counterfeit (type 0) respectively. Again, $\lambda$ is the seller's private information. The platform and the buyer have a common prior belief about $\lambda$, which is a discrete probability distribution: $P(\lambda = 1) = w$ and $P(\lambda = 0) = 1-w$. The rest of the setting is the same as the base model. Note that in our base model the buyer has two "layers" of uncertainty about the product authenticity as she has a belief about $\lambda$, which itself is a probability of the product authenticity. In contrast, in this section, the buyer has only one layer of uncertainty about the product authenticity because $\lambda = 1$ or 0 completely reveals the product authenticity. Theorem

4 summarizes the equilibrium results if the seller is certain about his product authenticity. The definitions of $\Omega(0)$, $\Omega(1)$, $\Omega(1^+)$, $\widetilde{\Omega}$, and $V(1)$ are provided in Appendix C.

**Theorem 4.** *Suppose the seller knows the product authenticity ($\lambda = 1$ or $0$) and the platform provides the inspection service ($P_M > 0$).*

1. *Under a contract $(\alpha, \beta) \in \Omega(1)$, only the type-1 seller chooses to sell through the platform with price $\mu_B$. Under a contract $(\alpha, \beta) \in \Omega(0)$, the seller chooses to sell through the platform with price $\mu_B w/(1 - P_M + P_M w)$ regardless of his type. Under a contract $(\alpha, \beta) \in \Omega(1^+) \cup \widetilde{\Omega}$, the seller does not sell through the platform.*
2. *The set of optimal contracts is $V(1)$, in which only the type-1 seller chooses to sell through the platform with price $\mu_B$. The platform's expected profit is $w(\mu_B - \mu_S - c)$.*

Figure 8 illustrates the seller's decision in the second-stage signaling game for different contracts. In the upper-left region $\Omega(1)$ where $\alpha$ is small and $\beta$ is large, only the type-1 seller chooses to sell. In the lower-left region $\Omega(0)$ where both $\alpha$ and $\beta$ are small, both seller types choose to sell. In the upper-right region $\Omega(1^+)$ and lower-right region $\widetilde{\Omega}$, the seller does not sell. Part 2 of Theorem 4 shows that it is optimal for the platform to adopt a contract in $V(1)$ where the penalty fraction $\beta$ is large. This screens out the type-0 seller with a counterfeit and only the type-1 seller chooses to sell (see Figure 8).

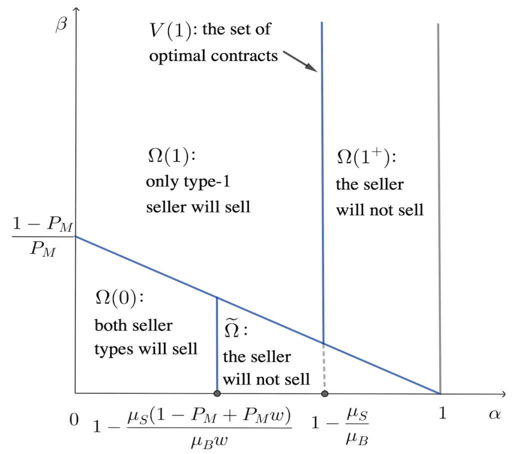



Figure 8: The seller's decision in the signaling game when he knows the product authenticity

Theorem 4 shows that the inspection service still incentivizes the high-type (type-1) seller to sell as long as the platform chooses the commission and penalty fractions carefully. However, other results are different from our base model. For example, if the seller knows the product authenticity, an external failure would not happen because only the type-1 seller chooses to sell under the platform's optimal contracts. Interestingly, this result holds for any positive platform's inspection capability. So, the platform can consider adopting the inspection service because even a low inspection capability would eliminate external failures with a carefully designed contract. In contrast, in our base model, the platform may receive a counterfeit from the seller who is not certain about the product authenticity, and thus an external failure may happen, which reflects the reality of the second-hand market of collectible goods mentioned in §1.

Theorem 4 also generates different insights from our base model. Firstly, in our base model (see Theorem 3), the platform has an incentive to improve its inspection capability (Snow, 2020) as it can increase the platform's expected profit. In contrast, if the seller knows the product authenticity (see Theorem 4), the platform would not have an incentive to enhance its inspection capability because it does not impact the platform's expected profit. Secondly, in our base model (see Theorem 3), the optimal commission and penalty fractions can be either monotonic or non-monotonic in the inspection capability. The platform should adjust the commission and penalty fractions accordingly if its inspection capability changes. In contrast, if the seller knows the product authenticity (see Figure 8), the optimal commission fraction is independent of its inspection capability ($\alpha^* = 1 - \mu_S/\mu_B$). Therefore, the seller-type distribution (continuous or discrete) plays an important role in generating different insights.

# 8 Conclusion

We study a C2C platform functioning as a marketplace that provides an inspection service. Selling a product through the platform, a seller is uncertain about the product authenticity. The seller is characterized by his type, representing the probability of his product being authentic. Compared to the seller, a buyer has even less information about the product authenticity. Before the buyer purchases the product, she obtains a signal of the product authenticity from its selling price. The platform's inspection service helps to detect whether the product is counterfeit with a probability. We develop a two-stage game-theoretical model capturing the interactions among the platform, the seller, and the buyer. In the first stage of the game, the platform designs a contract determining a commission fraction and a penalty fraction. In the second stage, the seller first decides whether to sell the product through the platform by setting its price. Given the price, the buyer then decides whether to purchase the product. Our model provides guidance and insights into the platform's contract design and provision of the inspection service.

We have identified the following effects of providing the inspection service. Firstly, even with a very low inspection capability, the inspection service completely alters the seller's decision and improves the product authenticity (see Theorem 1 and Proposition 1). Our results suggest that the benefit of inspection service may be underestimated in practice as the service is not widely available among leading C2C e-commerce platforms. Secondly, with the inspection service, the platform can influence the seller's revenue and generate more revenue for itself by adjusting the commission and penalty fractions, and even eliminate the seller's information rent despite the asymmetric information of product authenticity (see Corollaries 3 and 4). Finally, the platform's profit increases with its inspection capability (see Theorem 3). This partially explains why some C2C platforms invest heavily to improve their inspection capability (Snow, 2020). This inherent incentive for the platform to improve its inspection capability helps combat counterfeits.

To maximize its expected profit, the platform should carefully choose the commission and penalty fractions (see Theorem 2). Interestingly, a larger platform's expected profit does not imply a larger commission fraction or a higher price in equilibrium (see Proposition 3). Under some mild conditions, the optimal commission fraction increases but the optimal penalty fraction decreases as the platform's inspection capability increases (see Theorem 3). This partially explains why platforms without the inspection service (such as Taobao) usually charge a lower commission fee than platforms with the inspection service (such as StockX and Goat), where the commission fraction can be as high as 25%. However, the optimal contract parameters may not be monotonic in the inspection capability if the density of a certain seller type is high (see Figure 7). Therefore, the platform should pay attention to the seller-type distribution when designing the contract. With a perfect inspection capability, the platform can remove the penalty completely, attracting more sellers who are less confident about their product authenticity (see Corollary 6). For example, Goat sets a zero penalty fee (see `goat.com`).

We also analyze a model with two seller types ($\lambda = 1$ or $0$), which generates different insights from our base model. Specifically, when the seller knows the product authenticity, the platform can adopt a contract to screen out the seller with a counterfeit given asymmetric information of product authenticity (see Theorem 4). Thus, the platform would not receive a counterfeit and an external failure would not happen. Moreover, the platform's expected profit and the optimal commission fraction are independent of the inspection capability. In contrast, in our base model, the seller who is uncertain about the product authenticity may sell through the platform, reflecting what happens on platforms selling second-hand collectible goods (such as StockX, Goat, and Poizon). Moreover, the platform's expected profit and the optimal contract parameters in our base model depend on the inspection capability.

Appendix B analyzes two extensions of our model. The first extension assumes that the buyer has a positive reservation price for a counterfeit. The second extension considers that the buyer has a private reservation price for an authentic product. Our main results continue to hold under the two extensions. Since our paper is the first to analytically investigate C2C platforms providing the inspection service, more future research is possible. Firstly, it may be interesting to consider a rating system on the seller's product authenticity, where the belief about the seller type is updated dynamically over time. Secondly, one may endogenize the platform's inspection capability to investigate its incentive to manipulate the inspection result. Thirdly, in the second stage, considering the bargaining between the seller and the buyer, or the buyer making a price offer can be interesting.

## Acknowledgement

The authors thank the Department Editor, Associate Editor, and two anonymous referees for their valuable comments that have substantially improved the paper. Portions of this paper were presented in the INFORMS Annual Meeting, Anaheim, 2021 and the 12th POMS-HK International Conference, Hong Kong, 2022. The authors thank the audiences for many insightful comments and stimulating questions. The first author was supported by the National Natural Science Foundation of China [Grant 72071198]. The second and third authors are grateful for the support from the Lee Kong Chian School of Business, Singapore Management University under the Maritime and Port Authority (MPA) Research Fellowship and Retail Centre of Excellence Research Grant. The third author was supported by the ASEAN Business Research Initiative Grant (ABRI) [Grant #G17C20421], the Research Grants Council of Hong Kong [Grants 15501920 and 15501221], and the Key Program of National Natural Science Foundation of China [No. 71931009].

## References

Abhishek V, Guajardo JA, Zhang Z (2021) Business models in the sharing economy: Manufacturing durable goods in the presence of peer-to-peer rental markets. *Inform. Systems Res.* **32**(4):1450–1469.

Abhishek V, Jerath K, Zhang ZJ (2016) Agency selling or reselling? Channel structures in electronic retailing. *Management Sci.* **62**(8):2259–2280.

Akerlof G (1970) The market for "lemons": Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics* **84**(3):488–500.

Babich V, Tang C (2012) Managing opportunistic supplier product adulteration: Deferred payments, inspection, and combined mechanisms. *Manufacturing Service Oper. Management* **14**(2):301–314.

Baiman S, Fischer PE, Rajan MV (2000) Information, contracting, and quality costs. *Management Sci.* **46**(6):776–789.

Bagwell K, Riordan M (1991) High and declining prices signal product quality. *Amer. Econom. Rev.* **81**(1):224–239.

Banks JS, Sobel J (1987) Equilibrium selection in signaling games. *Econometrica* **55**(3):647–661.

Baron DP, Myerson RB (1982) Regulating a monopolist with unknown costs. *Econometrica*, **50**(4):911–930.

Benjaafar S, Kong G, Li X, Courcoubetis C (2018) Peer-to-peer product sharing: Implications for ownership, usage and social welfare in the sharing economy. *Management Sci.* **65**(2):477–493.

Boulding W, Kirmani A (1993) A consumer-side experimental examination of signaling theory: do consumers perceive warranties as signals of quality? *Journal of Consumer Research* **20**(1):111–123.

Chen L, Papanastasiou Y (2021) Seeding the herd: pricing and welfare effects of social learning manipulation. *Management Sci.* **67**(11):6734–6750.

Cho I-K, Kreps DM (1987) Signaling games and stable equilibria. *The Quarterly Journal of Economics* **102**(2):179–221.

Cho SH, Fang X, Tayur S (2015) Combating strategic counterfeiters in licit and illicit supply chains. *Manufacturing Service Oper. Management* **17**(3):273–289.

Clifford T (2019) The RealReal CEO says: 'We keep changing' to fend off fakes. https://www.cnbc.com/2019/11/13/the-realreal-ceo-says-we-keep-changing-to-fend-off-fakes.html?qsearchterm=luxury

Debo L, Parlour C, Rajan U (2012) Signaling quality via queues. *Management Sci.* **58**(5):876–891.

Dennis G (2019) These are the most expensive sneakers to be resold in the last year as the sneaker-resale market explodes. https://www.businessinsider.com/most-expensive-sneakers-sold-last-year-resale-2019-4

Farrell M (2021) StockX valued at $3.8 billion and lets employees sell shares. *The Wall Street Journal*. Retrieved April 7, 2021. https://www.wsj.com/articles/stockx-valued-at-3-8-billion-and-lets-employees-sell-shares-11617830099

Fraiberger S, Sundararajan A (2015) Peer-to-peer rental markets in the sharing economy. Working paper, New York University.

Fudenberg D, Tirole J (1991) Game theory. MIT Press, Cambridge, MA.

Gao SY, Lim WS, Tang C (2017) Entry of copycats of luxury brands. *Marketing Sci.* **36**(2):272–289.

Gibbons R (1992) Game theory for applied economists. Princeton University Press, Princeton, NJ.

Guan X, Pun H, Mao Y, Chen YJ (2020) Strategic communication in a supply chain with copycat threat. Working paper.

Hagiu A, Wright J (2015) Marketplace or reseller? *Management Sci.* **61**(1):184–203.

Hagiu A, Wright J (2018) Controlling versus enabling. *Management Sci.* **65**(2):577–595.

Hawkins A, Thorpe J (2019) Buyer beware: Chinese shoppers' weapon to dodge cheap knock-offs. https://www.ft.com/content/aaf2b2ae-3108-11e9-80d2-7b637a9e1ba1

Holt K (2020) EBay will authenticate sneakers from resellers. https://sg.finance.yahoo.com/news/ebay-collectible-sneakers-authenticity-guarantee-164108922.html

Hu M, Pavlin JM, Shi M (2013) When gray markets have silver linings: all-unit discounts, gray markets, and channel management. *Manufacturing Services Oper. Management* **15**(2):250–262.

Hwang I, Radhakrishnan S, Su L (2006) Vendor certification and appraisal: Implications for supplier quality. *Management Sci.* **52**(10):1472–1482.

Janssen M, Moraga-Gonzalez J, Wildenbeest M (2005) Truly costly sequential seach and oligopolistic pricing. *International Journal of Industrial Organization* **23**:451–466.

Jiang B, Jerath K, Srinivasan K (2011) Firm strategies in the “mid tail” of platform-based retailing. *Marketing Sci.* **30**(5):757–775.

Jiang B, Tian L (2016) Collaborative consumption: Strategic and economic implications of product sharing. *Management Sci.* **64**(3):1171–1188.

Kelly H (2018) This sneaker-selling startup is trying to stomp out fakes. https://money.cnn.com/2018/01/05/technology/goat-sneaker-app/index.html

Kihlstrom RE, Riordan MH (1984) Advertising as a signal. *J. Political Econom.* **92**(3):427–450.

Kim D (2018) How to avoid counterfeits when looking for deals? *The New York Times* Retrieved 16 August 2018.

Kirmani A, Rao AR (2000) No pain, no gain: A critical review of the literature on signaling unobservable product quality. *Journal of Marketing* **64**(2):66–79.

Koaysomboon T (2018) Things you only know if you are... a luxury product inspector. https://www.timeout.com/bangkok/news/things-you-only-know-if-you-are-a-luxury-product-inspector-060118

Lariviere MA (2006) A note on probability distributions with increasing generalized failure rates. *Oper. Res.* **54**(3):602-604.

Lee H, Li C (2018) Supplier quality management: investment, inspection, and incentives. *Production Oper. Management* **27**:304–322.

Lutz NA (1989) Warranties as signals under consumer moral hazard. *RAND J. Econom.* **20**(2):239–255.

Mariya B, Edieal P (2018) Dynamic relational contracts for quality enforcement in supply chains. *Management Sci.* **65**(3):1305–1321.

Mantin B, Krishnan H, Dhar T (2014) The strategic role of third-party marketplaces in retailing. *Production Oper. Management* **23**(11):1937–1949.

Milgrom P, Roberts J (1986) Price and advertising signals of product quality. *J. Political Econom.* **94**(4):796–821.

Moorthy S (2012) Can brand extension signal product quality? *Marketing Sci.* **31**(5):756–770.

Moorthy S, Srinivasan K (1995) Signaling quality with a moneyback guarantee: The role of transaction costs. *Marketing Sci.* **14**(4):442–466.

Patel S (2019) Sneaker marketplace stockx seeks new customers with bleacher report deal. *The Wall Street Journal*. Retrieved September 17, 2019.

Pun H, DeYong GD (2017) Competing with copycats when customers are strategic. *Manufacturing Service Oper. Management* **19**(3):403–418.

Pun H, Swaminathan JM, Hou P (2021) Blockchain adoption for combating deceptive counterfeits. *Production Oper. Management* **30**:864–882.

Qian Y (2014) Counterfeiters: Foes or friends? How counterfeits affect sales by product quality tier. *Management Sci.* **60**(10):2381–2400.

Rivera A (2018) What is C2C? *Business News Daily* Retrieved July 21, 2018. https://www.businessnewsdaily.com/5084-what-is-c2c.html

Rohrlich J (2020) How $470 million worth of fake Nikes get into the US. *New American Economy* Retrieved January 2, 2020.

Seung HY, Taesu C (2018) Quality improvement incentive strategies in a supply chain. *Transportation Research Part E: Logistics and Transportation Review* **114**:331–342.

Stiving M (2000) Price-endings when prices signal quality. *Management Sci.* **46**(12):1617–1629.

Stock A, Balachander S (2005) The making of a “hot product”: A signaling explanation of marketers' scarcity strategy. *Management Sci.* **51**(8):1181–1192.

Snow J (2020) AI is a new weapon in the battle against counterfeits. *The Wall Street Journal* Retrieved August 10, 2020.

Suthivarakom G (2020) Welcome to the era of fake products. *The New York Times* Retrieved 11 February 2020.

Tang Q, Chu J, Lim WS, Zhang X (2020) Online platform's optimal efforts to combat counterfeits of luxury brands. Working paper.

Tian L, Jiang B (2018) Effects of consumer-to-consumer product sharing on distribution channel. *Production Oper. Management* **27**(2):350–367.

Tian L, Vakharia AJ, Tan Y, Xu Y (2018) Marketplace, reseller, or hybrid: strategic analysis of an emerging e-commerce model. *Production Oper. Management* **27**(8):1595–1610.

Wang Y, Lin J, Choi TM (2020) Gray market and counterfeiting in supply chains: A review of the operations literature and implications to luxury industries. *Transportation Research Part E: Logistics and Transportation Review* **133**.

Wernerfelt B (1988) Umbrella branding as a signal of new product quality: An example of signaling by posting a bond. *RAND J. Econom.* **19**(3):458–466.

Wilcox K, Kim HM, Sen S (2009) Why do consumers buy counterfeit luxury brands? *J. Marketing Res.* **46**(2):247–259.

Van WF (1999) "Experimental studies of signaling games" in L. Luini, Ed. *Uncertain Decisions*, Springer, Boston.

Yao S, Zhu K (2018) The role of anticounterfeit technology in combating counterfeit products. Working Paper.

Yi Z, Yu M, Cheung KL (2020) Impacts of counterfeiting on a global supply chain. *Manufacturing Service Oper. Management.*

Yu M, Ahn H-S, Kapuscinski R (2015) Rationing capacity in advance selling to signal quality. *Management Sci.* **61**(3):560–577.

Zhang J, Hong LJ, Zhang RQ (2012) Fighting strategies in a market with counterfeits. *Ann. Oper. Res.* **192**:49–66.

# Appendix

# A Supplementary Results

## A.1 Equilibrium Under Contracts with A Small Penalty Fraction $\beta$

Proposition 4 summarizes the complete equilibrium result of the signaling game for $\beta < \frac{1-P_M}{P_M}(1-\alpha)$. Denote $\check{\Pi}\left(\check{\lambda}\right) = \frac{\mu_B \int_0^{\check{\lambda}} x f(x)\mathrm{d}x}{\int_0^{\check{\lambda}}(1-P_M+P_M x)f(x)\mathrm{d}x}$ for $0 < \check{\lambda} \leq 1$ and $\check{\Pi}(0) = \lim_{\check{\lambda}\to 0^+} \check{\Pi}\left(\check{\lambda}\right) = 0$. Given $(\alpha, \beta)$, define $\check{\lambda}^* = \max\left\{\check{\lambda}\middle| A\left(\check{\lambda}\right) \leq \check{\Pi}\left(\check{\lambda}\right), 0 \leq \check{\lambda} \leq 1\right\}$.

**Proposition 4.** *If the platform provides the inspection service ($P_M > 0$), for $\beta < \frac{1-P_M}{P_M}(1-\alpha)$, there exists a unique pure-strategy perfect Bayesian equilibrium that Pareto dominates other equilibria from the seller's point of view. We have the following results in this equilibrium.*

*The seller's pricing decision and the buyer's purchase decision are*

$$\pi^*(\lambda) = \begin{cases} \check{\pi}^* \triangleq \check{\Pi}\left(\check{\lambda}^*\right), & if \quad \lambda \leq \check{\lambda}^*; \\ \infty, & if \quad \lambda > \check{\lambda}^*; \end{cases} \quad and \quad I^*(\pi) = \begin{cases} I\left(E[\lambda|\pi] \geq q(\pi)\right), & if \quad 0 < \pi < \check{\pi}^*; \\ 1, & if \quad \pi = \check{\pi}^*; \\ 0, & if \quad \check{\pi}^* < \pi \leq \mu_B. \end{cases}$$

*The buyer's beliefs $m^*(\cdot|\pi)$ are determined as follows:*

*(i) If $0 < \pi < \check{\pi}^*$, then $m^*(x|\pi)$ is arbitrary.*
*(ii) If $\pi = \check{\pi}^*$, then*

$$m^*(x|\pi) = \begin{cases} \frac{f(x)}{F\left(\check{\lambda}^*\right)}, & if \quad x \leq \check{\lambda}^*; \\ 0, & if \quad x > \check{\lambda}^*. \end{cases}$$

*(iii) If $\check{\pi}^* < \pi \leq \mu_B$, then $m^*(x|\pi)$ satisfies $E[\lambda|\pi] = \int_0^1 x m^*(x|\pi)\mathrm{d}x < q(\pi)$.*

If the penalty fraction is small (that is, $\beta < \frac{1-P_M}{P_M}(1-\alpha)$), the equilibrium of the signaling game with the platform's inspection service is similar to that without inspection (see Proposition 1). Specifically, the seller chooses to sell his product with the same equilibrium price $\check{\pi}^*$ if $\lambda \leq \check{\lambda}^*$, but he chooses not to sell through the platform if $\lambda > \check{\lambda}^*$. Therefore, only the low-type seller sells his product through the platform if the penalty fraction is small.

## A.2 Sensitivity to Other Parameters

Proposition 5 provides the sensitivity analysis on other parameters.

**Proposition 5. (Sensitivity to $P_B$, $\ell$, $c$, $\mu_S$, and $\mu_B$)**

1. *The optimal threshold $\hat{\lambda}^{**}$ increases with $P_B$, $\ell$, $c$, and $\mu_S$, and decreases with $\mu_B$. The equilibrium price $\hat{\Pi}\left(\hat{\lambda}^{**}\right)$ increases with $P_B$, $\ell$, $c$, and $\mu_S$, and may increase or decrease with $\mu_B$.*
2. *The optimal commission fraction $\alpha^*$ increases with $P_B$, $\ell$, and $c$, and may increase or decrease with $\mu_S$ and $\mu_B$. The optimal penalty fraction $\beta^*$ always has the opposite sensitivity results to that of $\alpha^*$ with respect to $P_B$, $\ell$, $c$, $\mu_S$, and $\mu_B$. The platform's optimal expected profit $R_M^*$ decreases with $P_B$, $\ell$, $c$, and $\mu_S$, and increases with $\mu_B$.*

As the buyer's inspection capability $P_B$ increases, an external failure is more likely to happen. The platform also incurs a larger loss from the external failure as $\ell$ increases. Moreover, the platform incurs a larger inspection cost as $c$ increases. Thus, part 1 of Proposition 5 shows that the platform prefers a higher seller type ($\hat{\lambda}^{**}$ increases) as $P_B$, $l$, or $c$ increases. If $\mu_S$ increases, it is more difficult for the seller to match his expected reservation price. The seller becomes less likely to sell through the platform, implying that $\hat{\lambda}^{**}$ increases. In contrast, if $\mu_B$ increases, then the seller can charge a higher price and becomes more likely to sell through the platform, implying that $\hat{\lambda}^{**}$ decreases. Note that $\hat{\lambda}^{**}$ decreases as $\mu_B$ increases, causing a lower average authenticity level of the seller. This may lower the equilibrium price $\hat{\Pi}\left(\hat{\lambda}^{**}\right)$.

Part 2 of Proposition 5 implies that the platform should adjust the commission fraction and the penalty fraction in the opposite directions as $P_B$, $\ell$, $c$, $\mu_S$, and $\mu_B$ vary. For example, as social media becomes more accessible, the buyer's inspection capability $P_B$ or the external failure loss $\ell$ may increase. The platform should increase the commission fraction and reduce the penalty fraction accordingly. It is intuitive that the platform's profit decreases with $P_B$, $\ell$, and $c$. If $\mu_B$ increases or $\mu_S$ decreases, the platform, which benefits from the gap between the two reservation prices, earns more as an intermediary.

# B Extensions

## B.1 Positive Reservation Price for a Counterfeit

In our base model, the buyer's reservation price of a counterfeit is 0. In this section, we assume that the buyer is willing to purchase the counterfeit at a positive price. Specifically, the buyer's reservation prices of the authentic product and the counterfeit are $\mu_B^H$ and $\mu_B^L$, respectively. We assume $\mu_B^H > \mu_B^L > 0$ and $\mu_B^H - \mu_S > c$. The set of the seller's feasible prices is $\left[\mu_B^L, \mu_B^H\right] \cup \{\infty\}$. The other settings are the same as that in the base model.

Given price $\pi$, the buyer generates a posterior belief $m(\cdot|\pi)$. The buyer's expected utility is given as $R_B(I;\pi) = I \cdot \int_0^1 \left[x\left(\mu_B^H - \pi\right) + (1-x)(1-P_M)\left(\mu_B^L - \pi\right)\right] m(x|\pi)\mathrm{d}x$. The buyer purchases the product with price $\pi$ (that is $I^*(\pi) = 1$) if and only if $R_B(1;\pi) \geq R_B(0;\pi) = 0$, which is equivalent to

$$E[\lambda|\pi] = \int_0^1 x m(x|\pi)\mathrm{d}x \geq q(\pi) \triangleq \frac{(1-P_M)\left(\pi - \mu_B^L\right)}{\mu_B^H - P_M\pi - (1-P_M)\mu_B^L},$$

where $q(\pi)$ can be interpreted as a critical level of the seller's expected type. It is the lowest expected type that makes price $\pi$ acceptable to the buyer. The expected revenue of the seller is given as $R_S(\pi;\lambda,I) = [(1-P_M+P_M\lambda)(1-\alpha)\pi - (1-\lambda)P_M\beta\pi]\, I$, which is the same as that in the base model. Therefore, Lemma 2 still holds in this extension. We focus on the equilibrium in which $\pi^*(\lambda) = \infty$ for $\lambda \in \left[0, \hat{\lambda}\right)$ and $\pi^*(\lambda) = \hat{\pi}$ for $\lambda \in \left[\hat{\lambda}, 1\right]$. Similar to (3), the equilibrium price $\hat{\pi}$ has the following upper bound:

$$\hat{\pi} \leq \hat{\Pi}\left(\hat{\lambda}\right) = \frac{\mu_B^H \int_{\hat{\lambda}}^1 x f(x)\mathrm{d}x + \mu_B^L(1-P_M)\int_{\hat{\lambda}}^1 (1-x)f(x)\mathrm{d}x}{\int_{\hat{\lambda}}^1 (1-P_M+P_M x)f(x)\mathrm{d}x}.$$

We summarize the equilibrium results in the following theorem. The definitions of $\Omega\left(\hat{\lambda}^*\right)$ and $V\left(\hat{\lambda}^*\right)$ are placed in Appendix C.

**Theorem 5.** *Suppose the platform provides the inspection service ($P_M > 0$).*

1. *For $(\alpha,\beta) \in \Omega\left(\hat{\lambda}^*\right)$, $\hat{\lambda}^* \in (0,1]$, there exists a unique pure-strategy perfect Bayesian equilibrium that Pareto dominates other equilibria from the seller's point of view. We have the following results in this equilibrium.*

   *The seller's pricing decision and the buyer's purchase decision are*

$$\pi^*(\lambda) = \begin{cases} \infty, & if \quad \lambda < \hat{\lambda}^*; \\ \hat{\pi}^* \triangleq \hat{\Pi}\left(\hat{\lambda}^*\right), & if \quad \lambda \geq \hat{\lambda}^*; \end{cases} \quad and \quad I^*(\pi) = \begin{cases} I\left(E[\lambda|\pi] \geq q(\pi)\right), & if \quad \mu_B^L \leq \pi < \hat{\pi}^*; \\ 1, & if \quad \pi = \hat{\pi}^*; \\ 0, & if \quad \hat{\pi}^* < \pi \leq \mu_B^H. \end{cases}$$

   *The buyer's beliefs $m^*(\cdot|\pi)$ are determined as follows:*
   *(i) If $\mu_B^L \leq \pi < \hat{\pi}^*$, then $m^*(x|\pi)$ is arbitrary.*
   *(ii) If $\pi = \hat{\pi}^*$, then*

$$m^*(x|\pi) = \begin{cases} 0, & if \quad x < \hat{\lambda}^*; \\ \frac{f(x)}{\bar{F}\left(\hat{\lambda}^*\right)}, & if \quad x \geq \hat{\lambda}^*. \end{cases}$$

   *(iii) If $\hat{\pi}^* < \pi \leq \mu_B^H$, then $m^*(x|\pi)$ satisfies $E[\lambda|\pi] = \int_0^1 x m^*(x|\pi)\mathrm{d}x < q(\pi) = \frac{(1-P_M)\left(\pi-\mu_B^L\right)}{\mu_B^H - P_M\pi - (1-P_M)\mu_B^L}$.*

2. *If $(1-P_M)\left(P_B\ell - \mu_B^L\right) + \mu_B^H - \mu_S > 0$ , then the platform's optimal contract $(\alpha^*,\beta^*)$ is $V\left(\hat{\lambda}^{**}\right)$, where $\hat{\lambda}^{**} = \max\{\frac{(1-P_M)\left(P_B\ell-\mu_B^L\right)+c}{(1-P_M)\left(P_B\ell-\mu_B^L\right)+\mu_B^H-\mu_S}, 0\}$. The platform's optimal expected profit is $R_M^* = \int_{\hat{\lambda}^{**}}^1 \left[x\mu_B^H + (1-x)(1-P_M)\mu_B^L - x\mu_S - (1-x)(1-P_M)P_B\ell - c\right] f(x)\mathrm{d}x$.*

Note that all the results in Theorem 5 can degenerate to that of the base model if $\mu_B^L = 0$. However, if the buyer has a positive reservation price of the counterfeit, the equilibrium price $\hat{\Pi}\left(\hat{\lambda}^*\right)$ becomes larger than that in the base model. Since the buyer is willing to purchase the counterfeit at a positive price, the seller with low types becomes more valuable from the viewpoint of the platform. It is straightforward to find that the optimal threshold $\hat{\lambda}^{**}$ is lower than that in the base model if $\mu_B = \mu_B^H$. In other words, the seller is more likely to sell his product through the platform if the buyer has a positive reservation price of the counterfeit. If the buyer's reservation price of the counterfeit $\mu_B^L$ is larger than $P_B\ell + \frac{c}{1-P_M}$, then all the seller types in $[0,1]$ are preferred by the platform. Even the type-0 seller who holds a counterfeit chooses to sell through the platform in equilibrium.

## B.2 Buyer's Private Reservation Price

In our base model, the seller's expected reservation price $\lambda\mu_S$ is his private information because the platform and the buyer do not know the seller's type $\lambda$. In this section, we consider that the buyer has a private reservation price $\mu_B$ for the authentic product. The platform and the seller hold a common belief on $\mu_B$ with p.d.f $g(\cdot)$, c.d.f. $G(\cdot)$, and support $\left[\underline{\mu}_B, \overline{\mu}_B\right]$. To avoid uninteresting equilibria, we assume $\underline{\mu}_B - \mu_S > c$. The buyer's reservation price of the counterfeit is still 0. We make the following mild assumption on the distribution of $\mu_B$: $G(\cdot)$ has an increasing general failure rate, that is, $\frac{yg(y)}{\bar{G}(y)}$ is an increasing function of $y$. Increasing general failure rate is a common assumption in supply chain contracting problems. For more information on this assumption, please refer to Lariviere (2006).

The seller's pricing decision is a function $\pi(\lambda) : [0,1] \to [0, \overline{\mu}_B] \cup \{\infty\}$. Let $I(\pi, \mu_B) : [0, \overline{\mu}_B] \times \left[\underline{\mu}_B, \overline{\mu}_B\right] \to \{0,1\}$ be the buyer's purchase decision, where $I(\pi,\mu_B) = 1$ (respectively, $I(\pi,\mu_B) = 0$) indicates that the buyer with reservation price $\mu_B$ chooses (respectively, refuses) to purchase with price $\pi$. With a little ambiguity on notations, we let $I(\pi)$ denote the probability of the buyer purchasing the product with price $\pi$. Other assumptions are the same as that in the base model.

Given price $\pi$, the buyer generates a posterior belief $m(\cdot|\pi)$. The expected utility of the buyer with reservation price $\mu_B$ is given as $R_B(I;\pi,\mu_B) = I \cdot \int_0^1 \left[x\mu_B - (1 - P_M + P_M x)\pi\right] m(x|\pi)\mathrm{d}x$. The buyer purchases the product with price $\pi$ if and only if $R_B(1;\pi,\mu_B) \geq R_B(0;\pi,\mu_B) = 0$, which is equivalent to $\mu_B \geq \left[\frac{1-P_M}{E[\lambda|\pi]} + P_M\right]\pi$. Thus, given price $\pi$, the buyer purchases the product with probability $I^*(\pi) = \bar{G}\left(\left[\frac{1-P_M}{E[\lambda|\pi]} + P_M\right]\pi\right)$. The expected revenue of the type-$\lambda$ seller is $R_S(\pi;\lambda, I^*(\pi)) = \left[(1 - P_M + P_M\lambda)(1-\alpha) - (1-\lambda)P_M\beta\right]\pi I^*(\pi)$.

We summarize the equilibrium results in the following theorem. The definitions of $\Omega\left(\hat{\lambda}^*\right)$ and $V\left(\hat{\lambda}^*\right)$ are given in Appendix C. Denote

$$\mu_B^0 = \inf\left\{y \in \left[\underline{\mu}_B, \overline{\mu}_B\right] \middle| \frac{yg(y)}{\bar{G}(y)} \geq 1\right\}. \tag{14}$$

**Theorem 6.** *Suppose the platform provides the inspection service ($P_M > 0$).*

1. *For $(\alpha,\beta) \in \Omega\left(\hat{\lambda}^*\right)$, $\hat{\lambda}^* \in (0,1]$, there exists a unique pure-strategy perfect Bayesian equilibrium that Pareto dominates other equilibria from the seller's point of view. We have the following results in this equilibrium.*

   *The seller's pricing decision is*

$$\pi^*(\lambda) = \begin{cases} \infty, & if \quad \lambda < \hat{\lambda}^*; \\ \hat{\pi}^* \triangleq \hat{\Pi}\left(\hat{\lambda}^*\right) \triangleq \frac{\mu_B^0 \int_{\hat{\lambda}^*}^1 x f(x)\mathrm{d}x}{\int_{\hat{\lambda}^*}^1 (1-P_M+P_M x) f(x)\mathrm{d}x}, & if \quad \lambda \geq \hat{\lambda}^*. \end{cases}$$

   *The buyer's purchase decision is $I^*(\pi) = \bar{G}\left(\left[\frac{1-P_M}{E[\lambda|\pi]} + P_M\right]\pi\right)$. In particular, $I^*(\hat{\pi}^*) = \bar{G}\left(\mu_B^0\right)$.*
   *The buyer's beliefs $m^*(\cdot|\pi)$ are determined as follows:*
   *(i) If $\pi = \hat{\pi}^*$, then*

$$m^*(x|\pi) = \begin{cases} 0, & if \quad x < \hat{\lambda}^*; \\ \frac{f(x)}{\bar{F}(\hat{\lambda}^*)}, & if \quad x \geq \hat{\lambda}^*. \end{cases}$$

   *(ii) If $\pi \neq \hat{\pi}^*$ and $\pi\bar{G}(\pi) \leq \hat{\pi}^*\bar{G}\left(\mu_B^0\right)$, then $m^*(x|\pi)$ is arbitrary.*
   *(iii) If $\pi \neq \hat{\pi}^*$ and $\pi\bar{G}(\pi) > \hat{\pi}^*\bar{G}\left(\mu_B^0\right)$, then $m^*(x|\pi)$ satisfies $\int_0^1 xm^*(x|\pi)\mathrm{d}x \leq \frac{(1-P_M)\pi}{\bar{G}^{-1}\left(\hat{\pi}^*\bar{G}\left(\mu_B^0\right)/\pi\right) - P_M\pi}$.*

2. *The platform's optimal contract $(\alpha^*,\beta^*)$ is $V\left(\hat{\lambda}^{**}\right)$, where $\hat{\lambda}^{**} = \frac{(1-P_M)P_B\ell + c}{(1-P_M)P_B\ell + \mu_B^0 - \mu_S}$. The optimal expected profit of the platform is $R_M^* = \bar{G}\left(\mu_B^0\right)\int_{\hat{\lambda}^{**}}^1 \left[\left(\mu_B^0 - \mu_S\right)x - (1-x)(1-P_M)P_B\ell - c\right] f(x)\mathrm{d}x$.*

The main difference between this extension and the base model is that the seller's expected revenue is no longer linear in the price. The seller optimizes his revenue by balancing the price $\pi$ and the buyer's purchase probability $I^*(\pi)$. The buyer accepts the equilibrium price with probability $\bar{G}(\mu_B^0)$, which indicates that the buyer purchases the product if and only if her reservation price $\mu_B$ is no smaller than $\mu_B^0$. According to (14), the threshold $\mu_B^0$ only depends on the distribution of the buyer's reservation price. So $\mu_B^0$ is independent of the distribution of the seller's type and the inspection capability of the platform. Moreover, no matter what contract the platform chooses, the seller will always set a price to attract the buyer with reservation price $\mu_B$ in the same set (that is, $\mu_B \in [\mu_B^0, \overline{\mu}_B]$). If $\underline{\mu}_B g\left(\underline{\mu}_B\right) \geq 1$, then $\mu_B^0 = \underline{\mu}_B$ and the buyer accepts the equilibrium price with probability 1.

## C Proofs

A pure-strategy **perfect Bayesian equilibrium** in a signaling game is a pair of strategies $\pi^*$ and $I^*$ and a posterior belief $m^*$ such that the following three conditions are satisfied (see Gibbons, 1992, Fudenberg and Tirole, 1991):

**Condition 1.** *Given posterior belief $m^*(\cdot|\pi)$, the buyer's decision $I^*(\pi)$ maximizes her expected utility:*

$$\text{For all } \pi \in [0, \mu_B], I^*(\pi) \in \underset{I\in\{0,1\}}{\arg\max} \int_0^1 R_B(I; \pi, x) m^*(x|\pi) \mathrm{d}x.$$

**Condition 2.** *Given the buyer's purchase decision $I^*(\pi)$, the seller pricing decision $\pi^*(\lambda)$ maximizes his expected revenue:*

$$\text{For all } \lambda \in [0, 1], \pi^*(\lambda) \in \underset{\pi\in(0,\mu_B]\cup\{\infty\}}{\arg\max} R_S(\pi; \lambda, I^*(\pi)).$$

**Condition 3.** *On-equilibrium belief follows the Bayesian rule:*
*If $\pi^*(\lambda_0) = \pi_0$, then*

$$m^*(\lambda_0|\pi_0) = \frac{f(\lambda_0)}{\int_{\Lambda(\pi_0)} f(x)\mathrm{d}x},$$

*where $\Lambda(\pi_0) = \{\lambda \in [0, 1]|\pi^*(\lambda) = \pi_0\}$.*

***Proof of Lemma 1.*** The result immediately follows the discussion before Lemma 1. □

***Proof of Lemma 2.*** Given the buyer's purchase decision $I^*(\pi)$, if the seller chooses to sell with price $\pi \in (0, \mu_B]$, his expected revenue is

$$\begin{aligned} R_S(\pi; \lambda, I^*(\pi)) &= [(1 - P_M + P_M\lambda)(1 - \alpha)\pi - (1 - \lambda)P_M\beta\pi] I^*(\pi) \\ &= [(1 - P_M + P_M\lambda)(1 - \alpha) - (1 - \lambda)P_M\beta] \pi I^*(\pi) \\ &= [P_M(1 - \alpha + \beta)\lambda + (1 - P_M)(1 - \alpha) - P_M\beta] \pi I^*(\pi). \end{aligned}$$

If the seller chooses not to sell, his expected revenue is $R_S(\infty; \lambda, I^*(\infty)) = \lambda\mu_S$. Note that $P_M(1 - \alpha + \beta)\lambda + (1 - P_M)(1 - \alpha) - P_M\beta > 0$ if and only if $\lambda > \hat{\underline{\lambda}} \triangleq \frac{P_M\beta-(1-P_M)(1-\alpha)}{P_M(1-\alpha+\beta)}$. Since $\beta > \frac{1-P_M}{P_M}(1-\alpha)$, then $P_M\beta-(1-P_M)(1-\alpha) > 0$ and $P_M(1-\alpha+\beta) = P_M(1-\alpha)+P_M\beta > P_M(1-\alpha)+(1-P_M)(1-\alpha) = 1-\alpha \geq 0$. Therefore, $\hat{\underline{\lambda}} > 0$.

We analyze the seller's equilibrium decision by considering the following three cases of different $\lambda$.
**Case 1**: If $\lambda = 0$, then $P_M(1 - \alpha + \beta)\lambda + (1 - P_M)(1 - \alpha) - P_M\beta = (1 - P_M)(1 - \alpha) - P_M\beta < 0$. Since we focus on participating equilibria, the seller will adopt a price $\pi$ that is acceptable to the buyer in equilibrium ($I^*(\pi) = 1$). Moreover, for $\pi \in (0, \mu_B]$ and $I^*(\pi) = 1$, we have $R_S(\pi; \lambda, I^*(\pi))|_{\lambda=0} = [(1 - P_M)(1 - \alpha) - P_M\beta] \pi I^*(\pi) = [(1 - P_M)(1 - \alpha) - P_M\beta] \pi < 0$, which implies that choosing not to sell is a profitable deviation. Thus, any price $\pi \in (0, \mu_B]$ satisfying $I^*(\pi) = 1$ cannot be an equilibrium price for the type-0 seller because it violates Condition 2 of the perfect Bayesian equilibrium above. Therefore, we obtain $\pi^*(\lambda) = \infty$ if $\lambda = 0$.
**Case 2**: If $0 < \lambda \leq \hat{\underline{\lambda}}$, then $P_M(1-\alpha+\beta)\lambda+(1-P_M)(1-\alpha)-P_M\beta \leq 0$. If the seller chooses price $\pi \in (0, \mu_B]$, then his expected revenue is $R_S(\pi; \lambda, I^*(\pi)) = [P_M(1 - \alpha + \beta)\lambda + (1 - P_M)(1 - \alpha) - P_M\beta] \pi I^*(\pi) \leq 0 < \lambda\mu_S = R_S(\infty; \lambda, I^*(\infty))$, which implies that choosing not to sell is a profitable deviation. Thus, any price $\pi \in (0, \mu_B]$ cannot be an equilibrium price for $0 < \lambda \leq \hat{\underline{\lambda}}$ because it violates Condition 2. Therefore, we obtain $\pi^*(\lambda) = \infty$ if $0 < \lambda \leq \hat{\underline{\lambda}}$.
**Case 3**: If $\hat{\underline{\lambda}} < \lambda \leq 1$, then $P_M(1 - \alpha + \beta)\lambda + (1 - P_M)(1 - \alpha) - P_M\beta > 0$. Note that if $I^*(\pi) = 0$ for all $\pi \in (0, \mu_B]$, then the seller chooses not to sell by setting $\pi^*(\lambda) = \infty$ for $\hat{\underline{\lambda}} < \lambda \leq 1$. In the following, we focus on the case where there exists $\pi \in (0, \mu_B]$ such that $I^*(\pi) = 1$. In this case, we define $\hat{\pi} = \max\{\pi \in (0, \mu_B]|I^*(\pi) = 1\}$, which is the highest price that can be accepted by the buyer. Recall that the seller's expected revenue is $R_S(\pi; \lambda, I^*(\pi)) = [P_M(1 - \alpha + \beta)\lambda + (1 - P_M)(1 - \alpha) - P_M\beta] \pi I^*(\pi)$ for $\pi \in (0, \mu_B]$. Since $P_M(1 - \alpha + \beta)\lambda + (1 - P_M)(1 - \alpha) - P_M\beta > 0$ for $\hat{\underline{\lambda}} < \lambda \leq 1$, the seller's expected revenue is increasing in $\pi$ if $I^*(\pi) = 1$. Thus, the optimal price in $(0, \mu_B]$ is $\hat{\pi}$ and

$$R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) = [P_M(1 - \alpha + \beta)\lambda + (1 - P_M)(1 - \alpha) - P_M\beta] \hat{\pi}.$$

According to Condition 2, if $R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) > \lambda\mu_S$, then $\pi^*(\lambda) = \hat{\pi}$. If $R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) < \lambda\mu_S$, then $\pi^*(\lambda) = \infty$. If $R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) = \lambda\mu_S$, then the seller is indifferent between $\pi^*(\lambda) = \hat{\pi}$ and $\pi^*(\lambda) = \infty$. For simplicity of exposition of equilibria, we focus on equilibria in which $\pi^*(\lambda) = \hat{\pi}$ if $R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) = \lambda\mu_S$. (If we consider equilibria in which $\pi^*(\lambda) = \infty$ for $\lambda$ satisfying $R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) = \lambda\mu_S$, then part 1 of Lemma 2 becomes $\pi^*(\lambda) = \infty$ for $\lambda \leq \hat{\lambda}$ and $\pi^*(\lambda) = \hat{\pi}$ for $\lambda > \hat{\lambda}$. In this case, the equilibrium expected revenue of each seller type and expected profit of the platform would not change.) Next, we characterize the set of $\lambda$ such that $\pi^*(\lambda) = \hat{\pi}$. That is the set of $\lambda$ such that $R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) \geq \lambda\mu_S$, which is equivalent to $\Sigma(\lambda) \geq 0$, where

$$\begin{aligned} \Sigma(\lambda) &\triangleq R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) - \lambda\mu_S \\ &= [P_M(1-\alpha+\beta)\hat{\pi} - \mu_S]\,\lambda + [(1-P_M)(1-\alpha) - P_M\beta]\,\hat{\pi}. \end{aligned} \tag{15}$$

Note that $\Sigma(\lambda)$ is linear and monotonic in $\lambda$. To find the set of $\lambda \in (\underline{\hat{\lambda}}, 1]$ such that $\Sigma(\lambda) \geq 0$, we evaluate $\Sigma(\lambda)$ at the following two end points: $\lambda = \underline{\hat{\lambda}}$ and $\lambda = 1$. For $\lambda = \underline{\hat{\lambda}}$, we have $\Sigma(\lambda) = R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) - \lambda\mu_S < 0$, because $R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) = 0$ at $\lambda = \underline{\hat{\lambda}}$. For $\lambda = 1$, we have $\Sigma(1) = P_M(1-\alpha+\beta)\hat{\pi} - \mu_S + [(1-P_M)(1-\alpha) - P_M\beta]\,\hat{\pi} = (1-\alpha)\hat{\pi} - \mu_S$.

(i) If $\Sigma(1) \geq 0$, then by (15), we obtain $P_M(1-\alpha+\beta)\hat{\pi} - \mu_S \geq [P_M\beta - (1-P_M)(1-\alpha)]\,\hat{\pi} > 0$. Therefore, $\Sigma(\lambda)$ is strictly increasing in $\lambda$, and we obtain

$$R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) \geq \lambda\mu_S \iff \Sigma(\lambda) \geq 0 \iff \lambda \geq \hat{\lambda}, \tag{16}$$

where $\hat{\lambda} = \frac{[P_M\beta-(1-P_M)(1-\alpha)]\hat{\pi}}{P_M(1-\alpha+\beta)\hat{\pi}-\mu_S}$ and $\underline{\hat{\lambda}} < \hat{\lambda} \leq 1$. Moreover, $\hat{\lambda} = \frac{[P_M\beta-(1-P_M)(1-\alpha)]\hat{\pi}}{P_M(1-\alpha+\beta)\hat{\pi}-\mu_S}$ is equivalent to $\hat{\pi} = A\left(\hat{\lambda}\right) \triangleq \frac{\hat{\lambda}\mu_S}{P_M(1-\alpha+\beta)\hat{\lambda}+(1-P_M)(1-\alpha)-P_M\beta}$.

(ii) If $\Sigma(1) < 0$, then $\Sigma(\lambda)$ is negative at the two end points $\lambda = \underline{\hat{\lambda}}$ and $\lambda = 1$. Thus, $\Sigma(\lambda) < 0$ for all $\underline{\hat{\lambda}} < \lambda \leq 1$ due to its monotonicity.

Cases 1-3 analyze the seller's equilibrium pricing decision for different $\lambda \in [0,1]$. Combining the above three cases, we obtain the following result on the seller's equilibrium decision. Define $\hat{\pi} = \max\{\pi \in (0, \mu_B] | I^*(\pi) = 1\}$ if there exists $\pi \in (0, \mu_B]$ such that $I^*(\pi) = 1$; otherwise define $\hat{\pi} = 0$.

(i) If $\Sigma(1) = (1-\alpha)\hat{\pi} - \mu_S \geq 0$, then $\pi^*(\lambda) = \infty$ for $\lambda \in \left[0, \hat{\lambda}\right)$ and $\pi^*(\lambda) = \hat{\pi}$ for $\lambda \in \left[\hat{\lambda}, 1\right]$, where $\underline{\hat{\lambda}} < \hat{\lambda} \leq 1$ and $\hat{\pi} = A\left(\hat{\lambda}\right)$.

(ii) If $\Sigma(1) = (1-\alpha)\hat{\pi} - \mu_S < 0$, then $\pi^*(\lambda) = \infty$ for $\lambda \in [0,1]$.

Then Lemma 2 can be obtained immediately. □

***Proof of Lemma 3.*** The result immediately follows the discussion before Lemma 3. □

**Lemma C.1. (All equilibria of the signaling game)**

*1. For $\beta > \frac{1-P_M}{P_M}(1-\alpha)$ and $\alpha \leq 1 - \frac{\mu_S}{\mu_B}$, there exist multiple equilibria:*

*(a) For any $\hat{\lambda}$ that satisfies $\underline{\hat{\lambda}} < \hat{\lambda} \leq 1$ and $A\left(\hat{\lambda}\right) \leq \hat{\Pi}\left(\hat{\lambda}\right)$, there exists a semi pooling equilibrium in which $\pi^*(\lambda) = \infty$ for $\lambda < \hat{\lambda}$ and $\pi^*(\lambda) = \hat{\pi} = A\left(\hat{\lambda}\right)$ for $\lambda \geq \hat{\lambda}$.*

*(b) There also exists a pooling equilibrium in which $\pi^*(\lambda) = \infty$ for $0 \leq \lambda \leq 1$.*

*2. For $\beta > \frac{1-P_M}{P_M}(1-\alpha)$ and $\alpha > 1 - \frac{\mu_S}{\mu_B}$, there exists a unique equilibrium in which $\pi^*(\lambda) = \infty$ for $0 \leq \lambda \leq 1$.*

Lemma C.1 summarizes all the equilibria for different contract parameter values. Under a large penalty fraction $\beta$, Lemma C.1 shows that for a small commission fraction ($\alpha \leq 1 - \frac{\mu_S}{\mu_B}$), there exist a continuum of semi pooling equilibria parameterized by $\hat{\lambda}$ and a pooling equilibrium. In contrast, for a large commission fraction ($\alpha > 1 - \frac{\mu_S}{\mu_B}$), there exists a unique equilibrium in which all the seller types pool on choosing not to sell.

***Proof of Lemma C.1.***
1. We first focus on the semi pooling equilibria in which $\pi^*(\lambda) = \infty$ for $\lambda < \hat{\lambda}$ and $\pi^*(\lambda) = \hat{\pi}$ for $\lambda \geq \hat{\lambda}$. Lemmas 2 and 3 characterize necessary conditions (2) and (3) for a semi pooling equilibrium to be sustained. In the following, we first characterize the set of $\hat{\lambda}$ and $\hat{\pi}$ satisfying conditions (2) and (3) and show this set is non-empty. Then we prove that any pair of $\hat{\lambda}$ and $\hat{\pi}$ in this set can be supported as a semi pooling equilibrium by specifying off-equilibrium beliefs appropriately.

Note that $\hat{\lambda}$ and $\hat{\pi}$ satisfy conditions $\underline{\hat{\lambda}} < \hat{\lambda} \leq 1$, $\hat{\pi} = A\left(\hat{\lambda}\right)$, and $\hat{\pi} \leq \hat{\Pi}\left(\hat{\lambda}\right)$ according to Lemmas 2 and 3. Equivalently, $\hat{\lambda}$ satisfies $\underline{\hat{\lambda}} < \hat{\lambda} \leq 1$ and $A\left(\hat{\lambda}\right) \leq \hat{\Pi}\left(\hat{\lambda}\right)$, and $\hat{\pi}$ satisfies $\hat{\pi} = A\left(\hat{\lambda}\right)$. If $\beta > \frac{1-P_M}{P_M}(1-\alpha)$ and $\alpha \leq 1 - \frac{\mu_S}{\mu_B}$, then $\underline{\hat{\lambda}} = \frac{P_M\beta-(1-P_M)(1-\alpha)}{P_M(1-\alpha+\beta)} \in (0,1)$. Recall that

$$A\left(\hat{\lambda}\right) = \frac{\hat{\lambda}\mu_S}{P_M(1-\alpha+\beta)\hat{\lambda} + (1-P_M)(1-\alpha) - P_M\beta}. \tag{17}$$

Then for $\hat{\lambda} = 1$, we obtain $A\left(\hat{\lambda}\right) = \frac{\mu_S}{1-\alpha} \leq \mu_B = \hat{\Pi}\left(\hat{\lambda}\right)$ because $\alpha \leq 1 - \frac{\mu_S}{\mu_B}$. Therefore, at least $\hat{\lambda} = 1$ satisfies $\underline{\hat{\lambda}} < \hat{\lambda} \leq 1$ and $A\left(\hat{\lambda}\right) \leq \hat{\Pi}\left(\hat{\lambda}\right)$. Then, the set $\left\{\hat{\lambda} | \underline{\hat{\lambda}} < \hat{\lambda} \leq 1, A\left(\hat{\lambda}\right) \leq \hat{\Pi}\left(\hat{\lambda}\right)\right\}$ is non-empty.

We next prove that any pair of $\hat{\lambda} \in \left\{\hat{\lambda}|A\left(\hat{\lambda}\right) \leq \hat{\Pi}\left(\hat{\lambda}\right), \underline{\hat{\lambda}} < \hat{\lambda} \leq 1\right\}$ and $\hat{\pi} = A\left(\hat{\lambda}\right)$ can be supported as an equilibrium by specifying off-equilibrium beliefs appropriately. Specifically, we prove that the seller's pricing decision

$$\pi^*(\lambda) = \begin{cases} \infty, & if \quad \lambda < \hat{\lambda}; \\ \hat{\pi} = A\left(\hat{\lambda}\right), & if \quad \lambda \geq \hat{\lambda}. \end{cases}$$

can be supported as an equilibrium by specifying the following buyer's decision and beliefs.
The buyer's purchase decision is

$$I^*(\pi) = \begin{cases} I(E[\lambda|\pi] \geq q(\pi)), & if \quad 0 < \pi < \hat{\pi}; \\ 1, & if \quad \pi = \hat{\pi}; \\ 0, & if \quad \hat{\pi} < \pi \leq \mu_B. \end{cases} \tag{18}$$

The buyer's beliefs $m^*(\cdot|\pi)$ are determined as follows:
(i) If $0 < \pi < \hat{\pi}$, then $m^*(x|\pi)$ is arbitrary.
(ii) If $\pi = \hat{\pi}$, then

$$m^*(x|\pi) = \begin{cases} 0, & if \quad x < \hat{\lambda}; \\ \frac{f(x)}{\bar{F}(\hat{\lambda})}, & if \quad x \geq \hat{\lambda}. \end{cases}$$

(iii) If $\hat{\pi} < \pi \leq \mu_B$, then $m^*(x|\pi)$ satisfies $E[\lambda|\pi] = \int_0^1 xm^*(x|\pi)\mathrm{d}x < q(\pi)$.

Now we prove that the above $(\pi^*, I^*, m^*)$ satisfies Conditions 1–3. First, for $\pi \neq \hat{\pi}$, it is obvious that the buyer's purchase decision $I^*(\pi)$ is optimal given the off-equilibrium beliefs $m^*(\cdot|\pi)$ according to Lemma 1. Note that $\hat{\pi} = A\left(\hat{\lambda}\right) \leq \hat{\Pi}\left(\hat{\lambda}\right)$. Then for $\pi = \hat{\pi}$, the buyer's purchase decision $I^*(\pi) = 1$ is optimal given the on-equilibrium belief because of (3). Moreover, the on-equilibrium belief satisfies Bayesian rule. Thus, $(\pi^*, I^*, m^*)$ satisfies Conditions 1 and 3. Finally, we prove that the seller's decision maximizes his expected revenue $R_S(\pi; \lambda, I^*(\pi))$. According to (18), the highest price that can be accepted by the buyer is $\hat{\pi}$. To maximize his expected revenue, the seller would compare $R_S(\hat{\pi}; \lambda, I^*(\hat{\pi}))$ and his expected reservation price $R_S(\infty; \lambda, I^*) = \lambda\mu_S$. According to (16), $R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) \geq \lambda\mu_S$ if and only if $\lambda \geq \hat{\lambda}$. Thus, the seller with $\lambda \geq \hat{\lambda}$ chooses to sell with price $\pi^*(\lambda) = \hat{\pi}$ while the seller with $\lambda < \hat{\lambda}$ refuses to sell. Therefore, $(\pi^*, I^*, m^*)$ also satisfies Condition 2, which completes the proof.

Lastly, we consider the pooling equilibrium in which $\pi^*(\lambda) = \infty$ for $\lambda \in [0, 1]$. We can similarly prove that it can be supported as an equilibrium by specifying the following buyer's decision and beliefs:
The buyer's purchase decision is $I^*(\pi) = 0$ for $0 < \pi \leq \mu_B$.
The buyer's belief $m^*(x|\pi)$ satisfies $E[\lambda|\pi] = \int_0^1 xm^*(x|\pi)\mathrm{d}x < q(\pi)$ for $0 < \pi \leq \mu_B$.

2. For $\beta > \frac{1-P_M}{P_M}(1-\alpha)$ and $\alpha > 1 - \frac{\mu_S}{\mu_B}$, we can prove that there does not exist $\hat{\lambda}$ satisfying $\underline{\hat{\lambda}} < \hat{\lambda} \leq 1$ and $A\left(\hat{\lambda}\right) \leq \hat{\Pi}\left(\hat{\lambda}\right)$. Recall (17) and (3), and one can easily obtain

$$\frac{\mathrm{d}A\left(\hat{\lambda}\right)}{\mathrm{d}\hat{\lambda}} = \frac{\mu_S\left[(1-P_M)(1-\alpha) - P_M\beta\right]}{\left[P_M(1-\alpha+\beta)\hat{\lambda} + (1-P_M)(1-\alpha) - P_M\beta\right]^2} < 0, \tag{19}$$

and

$$\frac{\mathrm{d}\hat{\Pi}\left(\hat{\lambda}\right)}{\mathrm{d}\hat{\lambda}} = \frac{\mu_B(1-P_M)f\left(\hat{\lambda}\right)\int_{\hat{\lambda}}^1(x-\hat{\lambda})f(x)\mathrm{d}x}{[(1-P_M)\bar{F}\left(\hat{\lambda}\right) + P_M\int_{\hat{\lambda}}^1 xf(x)\mathrm{d}x]^2} > 0. \tag{20}$$

Therefore, $A\left(\hat{\lambda}\right) - \hat{\Pi}\left(\hat{\lambda}\right)$ is strictly decreasing in $\hat{\lambda}$. Since $\alpha > 1 - \frac{\mu_S}{\mu_B}$, we obtain $A\left(\hat{\lambda}\right) - \hat{\Pi}\left(\hat{\lambda}\right) = \frac{\mu_S}{1-\alpha} - \mu_B > 0$ for $\hat{\lambda} = 1$. Then, $A\left(\hat{\lambda}\right) - \hat{\Pi}\left(\hat{\lambda}\right) > 0$ for all $\underline{\hat{\lambda}} < \hat{\lambda} \leq 1$. As a result, there does not exist $\hat{\lambda}$ satisfying $\underline{\hat{\lambda}} < \hat{\lambda} \leq 1$ and $A\left(\hat{\lambda}\right) \leq \hat{\Pi}\left(\hat{\lambda}\right)$. Therefore, there does not exist a semi pooling equilibrium if $\beta > \frac{1-P_M}{P_M}(1-\alpha)$ and $\alpha > 1 - \frac{\mu_S}{\mu_B}$. However, the pooling equilibrium in which $\pi^*(\lambda) = \infty$ for $\lambda \in [0, 1]$ can still be supported as an equilibrium by specifying the buyer's decision and beliefs given in the proof of part 1. Therefore, if $\beta > \frac{1-P_M}{P_M}(1-\alpha)$ and $\alpha > 1 - \frac{\mu_S}{\mu_B}$, there exists a unique pooling equilibrium in which $\pi^*(\lambda) = \infty$ for $\lambda \in [0, 1]$. □

***Proof of Theorem 1***. Lemma C.1 characterizes all the equilibria. For $\beta > \frac{1-P_M}{P_M}(1-\alpha)$ and $\alpha \leq 1 - \frac{\mu_S}{\mu_B}$, there exist a continuum of semi pooling equilibria and a pooling equilibrium. To select the most plausible equilibrium, we apply Pareto dominance. In the following, we first show that any semi pooling equilibrium, in which the seller with $\lambda \geq \hat{\lambda}$ chooses to sell, Pareto dominates the pooing equilibrium. We then show that the semi pooling equilibrium with the equilibrium price $\hat{\Pi}\left(\hat{\lambda}\right)$ Pareto dominates the other semi pooling equilibria.

We first show that any semi pooling equilibrium Pareto dominates the pooing equilibrium. Note that in the pooling equilibrium where $\pi^*(\lambda) = \infty$ for $\lambda \in [0, 1]$, each seller type obtains his expected reservation price $\lambda\mu_S$. We next calculate the seller's expected revenue in a semi pooling equilibrium, in which the seller with $\lambda < \hat{\lambda}$ chooses not to sell by setting $\pi^*(\lambda) = \infty$ and the seller with $\lambda \geq \hat{\lambda}$ chooses to sell with the same price $\pi^*(\lambda) = \hat{\pi}$. For $\lambda < \hat{\lambda}$, the seller's equilibrium expected revenue is $\lambda\mu_S$. For $\lambda \geq \hat{\lambda}$, the seller's equilibrium expected revenue is

$$R_S\left(\hat{\pi}; \lambda, I^*\left(\hat{\pi}\right)\right) = \left[P_M(1-\alpha+\beta)\lambda + (1-P_M)(1-\alpha) - P_M\beta\right]\hat{\pi}. \tag{21}$$

According to (16), $R_S(\hat{\pi}; \lambda, I^*(\hat{\pi})) \geq \lambda\mu_S$ for $\lambda \geq \hat{\lambda}$. Thus, a semi pooling equilibrium Pareto dominates the pooing equilibrium in which $\pi^*(\lambda) = \infty$ for $\lambda \in [0, 1]$.

Next, we select an equilibrium among the semi pooling equilibria by Pareto dominance. For $\lambda \geq \hat{\lambda}$, we have $P_M(1-\alpha+\beta)\lambda + (1-P_M)(1-\alpha) - P_M\beta \geq P_M(1-\alpha+\beta)\hat{\lambda} + (1-P_M)(1-\alpha) - P_M\beta > P_M(1-\alpha+\beta)\underline{\hat{\lambda}} + (1-P_M)(1-\alpha) - P_M\beta = 0$. That is, the coefficient of $\hat{\pi}$ in (21) satisfies $P_M(1-\alpha+\beta)\lambda + (1-P_M)(1-\alpha) - P_M\beta > 0$ for $\lambda \geq \hat{\lambda}$. Thus, the seller's equilibrium revenue $R_S\left(\hat{\pi}; \lambda, I^*\left(\hat{\pi}\right)\right)$ is increasing in $\hat{\pi}$. Applying Pareto dominance, we should select the equilibrium with the largest $\hat{\pi}$. Note that $\hat{\pi}$ and $\hat{\lambda}$ have a one-to-one correspondence $\hat{\pi} = A\left(\hat{\lambda}\right)$, where $A\left(\hat{\lambda}\right)$ is strictly decreasing in $\hat{\lambda}$ according to (19). Thus, to find the equilibrium with the largest $\hat{\pi}$, it is equivalent to find the equilibrium with the smallest $\hat{\lambda}$. Recall from part 1(a) of Lemma C.1 that $\hat{\lambda}$ satisfies $\underline{\hat{\lambda}} < \hat{\lambda} \leq 1$ and $A\left(\hat{\lambda}\right) \leq \hat{\Pi}\left(\hat{\lambda}\right)$, so we need to find the smallest $\hat{\lambda}$ from the set $\left\{\hat{\lambda} \middle| \underline{\hat{\lambda}} < \hat{\lambda} \leq 1, A\left(\hat{\lambda}\right) - \hat{\Pi}\left(\hat{\lambda}\right) \leq 0\right\}$. By (19) and (20), $A\left(\hat{\lambda}\right) - \hat{\Pi}\left(\hat{\lambda}\right)$ is strictly decreasing in $\hat{\lambda}$. As $\hat{\lambda} \to \underline{\hat{\lambda}}$, we obtain $\lim_{\hat{\lambda} \to \underline{\hat{\lambda}}^+}\left[A\left(\hat{\lambda}\right) - \hat{\Pi}\left(\hat{\lambda}\right)\right] = +\infty$, because $\lim_{\hat{\lambda} \to \underline{\hat{\lambda}}^+} A\left(\hat{\lambda}\right) = +\infty$ and $\lim_{\hat{\lambda} \to \underline{\hat{\lambda}}^+} \hat{\Pi}\left(\hat{\lambda}\right) \leq \hat{\Pi}(1) = \mu_B$. For $\hat{\lambda} = 1$, we obtain $A\left(\hat{\lambda}\right) - \hat{\Pi}\left(\hat{\lambda}\right) = \frac{\mu_S}{1-\alpha} - \mu_B \leq 0$ because $\alpha \leq 1 - \frac{\mu_S}{\mu_B}$. Denote $\hat{\lambda}^*$ as the unique root of $A\left(\hat{\lambda}\right) - \hat{\Pi}\left(\hat{\lambda}\right) = 0$. Then, the set of $\hat{\lambda}$ is $\left\{\hat{\lambda} \middle| \underline{\hat{\lambda}} < \hat{\lambda} \leq 1, A\left(\hat{\lambda}\right) - \hat{\Pi}\left(\hat{\lambda}\right) \leq 0\right\} = \left\{\hat{\lambda} \middle| \hat{\lambda}^* \leq \hat{\lambda} \leq 1\right\}$ and the smallest $\hat{\lambda}$ in this set is $\hat{\lambda}^*$. Therefore, we should select the equilibrium with $\hat{\lambda} = \hat{\lambda}^*$ and $\hat{\pi}^* = A\left(\hat{\lambda}^*\right) = \hat{\Pi}\left(\hat{\lambda}^*\right)$. Note that $\hat{\pi}^*$ attains the upper bound $\hat{\Pi}\left(\hat{\lambda}^*\right)$ of the equilibrium price in an equilibrium with $\hat{\lambda} = \hat{\lambda}^*$ (see Lemma 3).

In summary, for $\beta > \frac{1-P_M}{P_M}(1-\alpha)$ and $\alpha \leq 1 - \frac{\mu_S}{\mu_B}$, there exists a unique equilibrium that Pareto dominates other equilibria from the seller's point of view. In this equilibrium, $\pi^*(\lambda) = \infty$ for $\lambda \in \left[0, \hat{\lambda}^*\right)$ and $\pi^*(\lambda) = \hat{\pi}^* = \hat{\Pi}\left(\hat{\lambda}^*\right)$ for $\lambda \in \left[\hat{\lambda}^*, 1\right]$, where $\hat{\lambda}^*$ is uniquely determined by

$$A\left(\hat{\lambda}^*\right) = \hat{\Pi}\left(\hat{\lambda}^*\right) \iff \frac{\hat{\lambda}^*\mu_S}{P_M(1-\alpha+\beta)\hat{\lambda}^* + (1-P_M)(1-\alpha) - P_M\beta} = \hat{\Pi}\left(\hat{\lambda}^*\right).$$

Moreover, $\hat{\lambda}^*$ satisfies $\hat{\lambda}^* > \underline{\hat{\lambda}} = \frac{P_M\beta - (1-P_M)(1-\alpha)}{P_M(1-\alpha+\beta)} > 0$ because $\beta > \frac{1-P_M}{P_M}(1-\alpha)$. Thus, $0 < \hat{\lambda}^* \leq 1$. The buyer's decision and beliefs for this equilibrium can be found in the proof of Lemma C.1.

Note that $\hat{\lambda}^*$ cannot be written as a closed-form function of $\alpha$ and $\beta$, which leads difficulty to derive the optimal contract $(\alpha, \beta)$ in the first stage of our model. To solve this problem, we identify the set of $(\alpha, \beta)$ that induces the equilibrium with $\hat{\lambda}^* \in (0, 1]$. That is, $\Omega\left(\hat{\lambda}^*\right) = \left\{(\alpha, \beta) \middle| \frac{\hat{\lambda}^*\mu_S}{P_M(1-\alpha+\beta)\hat{\lambda}^* + (1-P_M)(1-\alpha) - P_M\beta} = \hat{\Pi}\left(\hat{\lambda}^*\right), \beta > \frac{1-P_M}{P_M}(1-\alpha), \alpha \leq 1 - \frac{\mu_S}{\mu_B}\right\}$. In the following, we simplify the formulation of $\Omega\left(\hat{\lambda}^*\right)$. For $\hat{\lambda}^* \in (0, 1)$, $\frac{\hat{\lambda}^*\mu_S}{P_M(1-\alpha+\beta)\hat{\lambda}^* + (1-P_M)(1-\alpha) - P_M\beta} = \hat{\Pi}\left(\hat{\lambda}^*\right) \iff \beta = \frac{(1-P_M+P_M\hat{\lambda}^*)(1-\alpha)\hat{\Pi}\left(\hat{\lambda}^*\right) - \hat{\lambda}^*\mu_S}{(1-\hat{\lambda}^*)P_M\hat{\Pi}\left(\hat{\lambda}^*\right)}$. Then, $\frac{\beta P_M}{1-P_M} = \frac{(1-P_M+P_M\hat{\lambda}^*)(1-\alpha)\hat{\Pi}\left(\hat{\lambda}^*\right) - \hat{\lambda}^*\mu_S}{(1-P_M)(1-\hat{\lambda}^*)\hat{\Pi}\left(\hat{\lambda}^*\right)}$. Moreover,

$$\begin{aligned}
& \beta > \frac{1-P_M}{P_M}(1-\alpha) \\
\iff \quad & \frac{\beta P_M}{1-P_M} > 1-\alpha \\
\iff \quad & (1-P_M+P_M\hat{\lambda}^*)(1-\alpha)\hat{\Pi}\left(\hat{\lambda}^*\right) - \hat{\lambda}^*\mu_S > (1-P_M)(1-\hat{\lambda}^*)\hat{\Pi}\left(\hat{\lambda}^*\right)(1-\alpha) \\
\iff \quad & \alpha < 1 - \frac{\mu_S}{\hat{\Pi}\left(\hat{\lambda}^*\right)},
\end{aligned}$$

which also guarantees $\alpha \le 1-\frac{\mu_S}{\mu_B}$ because $\hat{\Pi}\left(\hat{\lambda}^*\right) \le \hat{\Pi}(1) = \mu_B$. Therefore, the set of $(\alpha,\beta)$ that induces the equilibrium with $\hat{\lambda}^* \in (0,1)$ is

$$\Omega\left(\hat{\lambda}^*\right) = \left\{(\alpha,\beta)\middle|\alpha < 1-\frac{\mu_S}{\hat{\Pi}\left(\hat{\lambda}^*\right)}, \beta = \frac{\left(1-P_M+P_M\hat{\lambda}^*\right)(1-\alpha)\hat{\Pi}\left(\hat{\lambda}^*\right)-\hat{\lambda}^*\mu_S}{\left(1-\hat{\lambda}^*\right)P_M\hat{\Pi}\left(\hat{\lambda}^*\right)}\right\},$$

which is a ray (without its end point) on $\alpha$-$\beta$ plane because $\beta$ is linear in $\alpha$.
For $\hat{\lambda}^* = 1$, $\frac{\hat{\lambda}^*\mu_S}{P_M(1-\alpha+\beta)\hat{\lambda}^*+(1-P_M)(1-\alpha)-P_M\beta} = \hat{\Pi}\left(\hat{\lambda}^*\right)$ for $\hat{\lambda}^* = 1 \iff \alpha = 1-\frac{\mu_S}{\mu_B}$. Thus, the set of $(\alpha,\beta)$ that induces the equilibrium with $\hat{\lambda}^* = 1$ is

$$\Omega\left(\hat{\lambda}^*\right) = \left\{(\alpha,\beta)|\alpha = 1-\frac{\mu_S}{\mu_B}, \beta > \frac{1-P_M}{P_M}(1-\alpha)\right\}.$$

□

***Comment on the belief.*** The buyer purchases the product with price $\pi$ if and only if $E[\lambda|\pi] \ge q(\pi)$ according to (1), where both $E[\lambda|\pi]$ and $q(\pi)$ are functions of $\pi$. Recall that $q(\pi)$ is the critical level of the seller's expected type, which is the lowest expected type such that the price $\pi$ is acceptable to the buyer. The blue curve in Figure 9 represents $q(\pi)$, which is increasing in $\pi$. This implies that the buyer has a higher requirement for the product authenticity as the price increases. The buyer accepts the price $\pi$ if and only if the conditional expectation $E[\lambda|\pi]$ is above or on the blue curve $q(\pi)$.

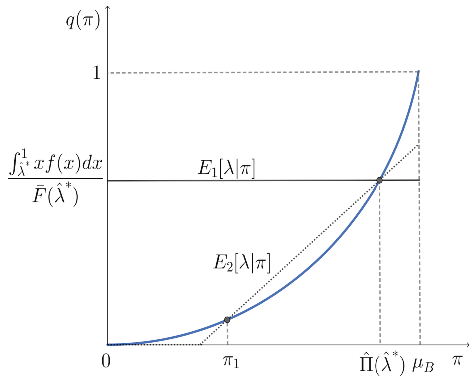



Figure 9: Critical level $q(\pi)$ and conditional expectation $E[\lambda|\pi]$

In Figure 9, $E_1[\lambda|\pi]$ and $E_2[\lambda|\pi]$ are generated by two feasible beliefs. For $E_1[\lambda|\pi]$ (the solid horizontal line in Figure 9), the buyer accepts price $\pi$ if and only if $\pi \le \hat{\Pi}\left(\hat{\lambda}^*\right)$. For $E_2[\lambda|\pi]$ (the dotted polyline in Figure 9), the buyer accepts price $\pi$ if and only if $\pi_1 \le \pi \le \hat{\Pi}\left(\hat{\lambda}^*\right)$. In both cases, $\hat{\Pi}\left(\hat{\lambda}^*\right)$ is the highest price acceptable to the buyer. This guarantees that $\hat{\Pi}\left(\hat{\lambda}^*\right)$ is the equilibrium price. □

***Proof of Corollary 1.*** The result can be immediately obtained from part 2 of Lemma C.1. □

***Proof of Corollary 2.*** If $\beta = \frac{1-P_M}{P_M}(1-\alpha)$, then the seller's expected revenue is $R_S(\pi;\lambda,I^*(\pi)) = [(1-P_M+P_M\lambda)(1-\alpha)\pi-(1-\lambda)P_M\beta\pi]\,I^*(\pi) = (1-\alpha)\lambda\pi I^*(\pi)$ for $\pi \in (0,\mu_B]$. Note that if $I^*(\pi) = 0$ for all $\pi \in (0,\mu_B]$, then the seller chooses not to sell by setting $\pi^*(\lambda) = \infty$ for $\lambda \in [0,1]$. So, we next focus on the case where there exists $\pi \in (0,\mu_B]$ such that $I^*(\pi) = 1$. In this case, we also define $\hat{\pi} = \max\{\pi \in (0,\mu_B]|I^*(\pi) = 1\}$, which is the highest price that can be accepted by the buyer.

(i) If $(\alpha,\beta) \in \Omega(0)$, then $\alpha < 1-\frac{\mu_S}{\hat{\Pi}(0)}$. We consider the following three cases.
**Case 1**: If $\hat{\pi} < \frac{\mu_S}{1-\alpha}$, then $R_S(\hat{\pi};\lambda,I^*(\hat{\pi})) = (1-\alpha)\lambda\hat{\pi} < \lambda\mu_S = R_S(\infty;\lambda,I^*(\infty))$ for $0 < \lambda \le 1$ and $R_S(\hat{\pi};\lambda,I^*(\hat{\pi})) = R_S(\infty;\lambda,I^*(\infty)) = 0$ for $\lambda = 0$. Thus, the seller chooses not to sell by setting $\pi^*(\lambda) = \infty$ for $\lambda \in [0,1]$. (If only the type-0 seller chooses to sell, then the equilibrium price is 0. This is not a participating equilibrium so we do not consider it.) In this equilibrium, each seller type earns his expected reservation price $\lambda\mu_S$.
**Case 2**: If $\hat{\pi} = \frac{\mu_S}{1-\alpha}$, then $R_S(\hat{\pi};\lambda,I^*(\hat{\pi})) = (1-\alpha)\lambda\hat{\pi} = \lambda\mu_S = R_S(\infty;\lambda,I^*(\infty))$ for $\lambda \in [0,1]$. In this case, each seller type is indifferent between selling with price $\hat{\pi}$ and not selling with price $\infty$. Thus, each seller type also earns his expected reservation price $\lambda\mu_S$ in equilibrium.
**Case 3**: If $\hat{\pi} > \frac{\mu_S}{1-\alpha}$, then $R_S(\hat{\pi};\lambda,I^*(\hat{\pi})) = (1-\alpha)\lambda\hat{\pi} > \lambda\mu_S = R_S(\infty;\lambda,I^*(\infty))$ for $0 < \lambda \le 1$ and $R_S(\hat{\pi};\lambda,I^*(\hat{\pi})) = R_S(\infty;\lambda,I^*(\infty)) = 0$ for $\lambda = 0$. For simplicity of exposition of equilibria, we focus on equilibria in which $\pi^*(\lambda) = \hat{\pi}$ for $\lambda \in [0,1]$, where each seller type earns no less than his expected reservation price $\lambda\mu_S$.

According to Pareto dominance, we select Case 3: $\hat{\pi} > \frac{\mu_S}{1-\alpha}$ and $\pi^*(\lambda) = \hat{\pi}$ for $\lambda \in [0,1]$. Similar to (3), we obtain $\hat{\pi} \le \hat{\Pi}(0)$. We can prove that any price $\hat{\pi}$ satisfying $\frac{\mu_S}{1-\alpha} < \hat{\pi} \le \hat{\Pi}(0)$ can be sustained as an equilibrium price by specifying proper off-equilibrium beliefs. Applying Pareto dominance again, we choose the equilibrium in which $\pi^*(\lambda) = \hat{\Pi}(0)$ for $\lambda \in [0,1]$.

(ii) If $(\alpha, \beta) = V\left(\hat{\lambda}^*\right)$ for $0 \le \hat{\lambda}^* \le 1$, then $\alpha = 1 - \frac{\mu_S}{\hat{\Pi}\left(\hat{\lambda}^*\right)}$. Similar to the proof of part 1, we consider the following three cases.
**Case 1**: If $\hat{\pi} < \hat{\Pi}\left(\hat{\lambda}^*\right)$, then $R_S\left(\hat{\pi}; \lambda, I^*\left(\hat{\pi}\right)\right) = (1-\alpha)\lambda\hat{\pi} = \frac{\mu_S}{\hat{\Pi}\left(\hat{\lambda}^*\right)}\lambda\hat{\pi} < \lambda\mu_S = R_S(\infty; \lambda, I^*(\infty))$ for $0 < \lambda \le 1$ and $R_S\left(\hat{\pi}; \lambda, I^*(\hat{\pi})\right) = R_S(\infty; \lambda, I^*(\infty)) = 0$ for $\lambda = 0$. Thus, the seller chooses not to sell by setting $\pi^*(\lambda) = \infty$ for $\lambda \in [0,1]$, where each seller type earns his expected reservation price $\lambda\mu_S$.
**Case 2**: If $\hat{\pi} = \hat{\Pi}\left(\hat{\lambda}^*\right)$, then $R_S\left(\hat{\pi}; \lambda, I^*(\hat{\pi})\right) = (1-\alpha)\lambda\hat{\pi} = \frac{\mu_S}{\hat{\Pi}\left(\hat{\lambda}^*\right)}\lambda\hat{\pi} = \lambda\mu_S = R_S(\infty; \lambda, I^*(\infty))$ for $\lambda \in [0,1]$. In this case, each seller type is indifferent between selling with price $\hat{\pi} = \hat{\Pi}\left(\hat{\lambda}^*\right)$ and not selling with price $\infty$. Then, there exist a continuum of equilibria, and the seller earns his expected reservation price $\lambda\mu_S$ in each equilibrium. Note that the unique equilibrium identified by Theorem 1, in which $\pi^*(\lambda) = \infty$ for $\lambda < \hat{\lambda}^*$ and $\pi^*(\lambda) = \hat{\pi}^* = \hat{\Pi}\left(\hat{\lambda}^*\right)$ for $\lambda \ge \hat{\lambda}^*$, is in the set of equilibria.
**Case 3**: If $\hat{\pi} > \hat{\Pi}\left(\hat{\lambda}^*\right)$, then $R_S\left(\hat{\pi}; \lambda, I^*(\hat{\pi})\right) = (1-\alpha)\lambda\hat{\pi} = \frac{\mu_S}{\hat{\Pi}\left(\hat{\lambda}^*\right)}\lambda\hat{\pi} > \lambda\mu_S = R_S(\infty; \lambda, I^*(\infty))$ for $0 < \lambda \le 1$ and $R_S\left(\hat{\pi}; \lambda, I^*(\hat{\pi})\right) = R_S(\infty; \lambda, I^*(\infty)) = 0$ for $\lambda = 0$. Without loss of generality, we focus on equilibria in which $\pi^*(\lambda) = \hat{\pi}$ for $\lambda \in [0,1]$. Similar to (3), we obtain $\hat{\pi} \le \hat{\Pi}(0)$. This contradicts with $\hat{\pi} > \hat{\Pi}\left(\hat{\lambda}^*\right)$ because $\hat{\Pi}\left(\hat{\lambda}^*\right) \ge \hat{\Pi}(0)$ for $0 \le \hat{\lambda}^* \le 1$ according to (20). Thus, Case 3 is impossible.

Combining the above three cases, there exist multiple equilibria. In each equilibrium, the seller earns his expected reservation price $\lambda\mu_S$.

(iii) If $(\alpha, \beta) \in V(1^+)$, then $\alpha > 1 - \frac{\mu_S}{\hat{\Pi}(1)} = 1 - \frac{\mu_S}{\mu_B}$. Since $\hat{\pi} \le \mu_B$, we obtain $R_S\left(\hat{\pi}; \lambda, I^*(\hat{\pi})\right) = (1-\alpha)\lambda\hat{\pi} < \frac{\mu_S}{\mu_B}\lambda\hat{\pi} \le \lambda\mu_S$ for $0 < \lambda \le 1$. Thus, the seller chooses not to sell by setting $\pi^*(\lambda) = \infty$ for $\lambda \in [0,1]$. □

***Proof of Proposition 1***. We first analyze the buyer's decision. Given $\pi \in (0, \mu_B]$ and $\lambda \in [0,1]$, the buyer's utility is $R_B(I; \pi, \lambda) = (\lambda\mu_B - \pi)\, I$. After observing the price $\pi \in (0, \mu_B]$, the buyer updates her belief $m(\cdot|\pi)$ about the seller's type $\lambda$. Thus, the buyer's expected utility is $R_B(I; \pi) = \int_0^1 R_B(I; \pi, x)m(x|\pi)\mathrm{d}x$. The buyer purchases the product ($I^*(\pi) = 1$) if and only if $R_B(1; \pi) \ge R_B(0; \pi) = 0$, which is equivalent to $E[\lambda|\pi] = \int_0^1 xm(x|\pi)\mathrm{d}x \ge \frac{\pi}{\mu_B}$.

Next, similar to the case with the platform's inspection service, we derive the equilibrium of the signaling game through the following three steps.
**Step 1: Reduce the space of possible equilibira.**

For $\pi \in (0, \mu_B]$, the seller's expected revenue is $R_S(\pi; \lambda, I^*(\pi)) = (1-\alpha)\pi I^*(\pi)$. For $\pi = \infty$, the seller's expected revenue is $R_S(\pi; \lambda, I^*(\pi)) = \lambda\mu_S$. Note that if $I^*(\pi) = 0$ for all $\pi \in (0, \mu_B]$, then the seller chooses not to sell by setting $\pi^*(\lambda) = \infty$ for $\lambda \in [0,1]$. In this equilibrium, each seller type earns his expected reservation price. We next consider there exists $\pi \in (0, \mu_B]$ such that $I^*(\pi) = 1$ and define $\check{\pi} = \max\{\pi \in (0, \mu_B] | I^*(\pi) = 1\}$, which is the highest price that can be accepted by the buyer. We consider the following two cases.
**Case 1**: If $(1-\alpha)\check{\pi} \ge \mu_S$, then $R_S\left(\check{\pi}; \lambda, I^*(\check{\pi})\right) = (1-\alpha)\check{\pi} \ge \lambda\mu_S = R_S(\infty; \lambda, I^*(\infty))$ for $\lambda \in [0,1]$. Thus, the seller chooses to sell by setting $\pi^*(\lambda) = \check{\pi}$ for $\lambda \in [0,1]$. Similar to (3), we obtain $\check{\pi} \le \check{\Pi}(1) = \mu_B \int_0^1 xf(x)\mathrm{d}x$. We can prove that any price $\check{\pi}$ satisfying $\frac{\mu_S}{1-\alpha} \le \check{\pi} \le \check{\Pi}(1)$ can induce an equilibrium with $\pi^*(\lambda) = \check{\pi}$ for $\lambda \in [0,1]$ by specifying proper off-equilibrium beliefs.
**Case 2**: If $(1-\alpha)\check{\pi} < \mu_S$, denote $\check{\lambda} = \frac{(1-\alpha)\check{\pi}}{\mu_S} < 1$. Then, $R_S\left(\check{\pi}; \lambda, I^*(\check{\pi})\right) = (1-\alpha)\check{\pi} \ge \lambda\mu_S = R_S(\infty; \lambda, I^*(\infty))$ for $\lambda \in [0, \check{\lambda}]$ and $R_S\left(\check{\pi}; \lambda, I^*(\check{\pi})\right) = (1-\alpha)\check{\pi} < \lambda\mu_S = R_S(\infty; \lambda, I^*(\infty))$ for $\lambda \in (\check{\lambda}, 1]$. Thus, the seller chooses to sell by setting $\pi^*(\lambda) = \check{\pi}$ for $\lambda \in \left[0, \check{\lambda}\right]$ and the seller chooses not to sell by setting $\pi^*(\lambda) = \infty$ for $\lambda \in \left(\check{\lambda}, 1\right]$. Similar to (3), we obtain $\check{\pi} \le \check{\Pi}\left(\check{\lambda}\right) = \frac{\mu_B \int_0^{\check{\lambda}} xf(x)\mathrm{d}x}{F\left(\check{\lambda}\right)}$. Therefore, in an equilibrium in which $\pi^*(\lambda) = \check{\pi}$ for $\lambda \in \left[0, \check{\lambda}\right]$ and $\pi^*(\lambda) = \infty$ for $\lambda \in \left(\check{\lambda}, 1\right]$, $\check{\lambda}$ and $\check{\pi}$ satisfy $0 \le \check{\lambda} = \frac{(1-\alpha)\check{\pi}}{\mu_S} < 1$ and $\check{\pi} \le \check{\Pi}\left(\check{\lambda}\right)$. Equivalently, $\check{\lambda}$ satisfies $0 \le \check{\lambda} < 1$ and $(1-\alpha)\check{\Pi}\left(\check{\lambda}\right) \ge \check{\lambda}\mu_S$, and $\check{\pi}$ satisfies $\check{\pi} = \frac{\check{\lambda}\mu_S}{1-\alpha}$. We can prove that any $\check{\lambda}$ satisfying $0 \le \check{\lambda} < 1$ and $(1-\alpha)\check{\Pi}\left(\check{\lambda}\right) \ge \check{\lambda}\mu_S$ can lead to an equilibrium in which $\pi^*(\lambda) = \check{\pi} = \frac{\check{\lambda}\mu_S}{1-\alpha}$ for $\lambda \in \left[0, \check{\lambda}\right]$ and $\pi^*(\lambda) = \infty$ for $\lambda \in \left(\check{\lambda}, 1\right]$ by specifying proper off-equilibrium beliefs (similar to the proof of Lemma C.1).
**Step 2: Find all the equilibria within the reduced space.**

Through the analysis in Step 1, we can summarize all the equilibria as follows.

1. If $(1-\alpha)\check{\Pi}(1) \ge \mu_S$, there exist multiple equilibria:

(a) For any $\check{\pi}$ satisfying $\frac{\mu_S}{1-\alpha} \leq \check{\pi} \leq \check{\Pi}(1)$, there exists a pooling equilibrium in which $\pi^*(\lambda) = \check{\pi}$ for $\lambda \in [0,1]$.

(b) For any $\check{\lambda}$ satisfying $0 \leq \check{\lambda} < 1$ and $(1-\alpha)\check{\Pi}\left(\check{\lambda}\right) \geq \check{\lambda}\mu_S$, there also exists a semi pooling equilibrium in which $\pi^*(\lambda) = \check{\pi} = \frac{\check{\lambda}\mu_S}{1-\alpha}$ for $\lambda \in \left[0, \check{\lambda}\right]$ and $\pi^*(\lambda) = \infty$ for $\lambda \in \left(\check{\lambda}, 1\right]$.

(c) There also exists a pooling equilibrium in which $\pi^*(\lambda) = \infty$ for $\lambda \in [0,1]$.

2. If $(1-\alpha)\check{\Pi}(1) < \mu_S$, there exist multiple equilibria:

(a) For any $\check{\lambda}$ satisfying $0 \leq \check{\lambda} < 1$ and $(1-\alpha)\check{\Pi}\left(\check{\lambda}\right) \geq \check{\lambda}\mu_S$, there exists a semi pooling equilibrium in which $\pi^*(\lambda) = \check{\pi} = \frac{\check{\lambda}\mu_S}{1-\alpha}$ for $\lambda \in \left[0, \check{\lambda}\right]$ and $\pi^*(\lambda) = \infty$ for $\lambda \in \left(\check{\lambda}, 1\right]$.

(b) There also exists a pooling equilibrium in which $\pi^*(\lambda) = \infty$ for $\lambda \in [0,1]$.

**Step 3: Refine the equilibria by Pareto dominance.**

Note that the seller's expected revenue is increasing in the equilibrium price $\check{\pi}$. According to Pareto dominance, we select the equilibrium with the largest $\check{\pi}$.

1. If $(1-\alpha)\check{\Pi}(1) \geq \mu_S$, we select the equilibrium in which $\pi^*(\lambda) = \check{\pi}^* = \check{\Pi}(1)$ for $\lambda \in [0,1]$. (Note that $\check{\Pi}\left(\check{\lambda}\right)$ is strictly increasing in $\check{\lambda}$ because $\frac{\mathrm{d}\check{\Pi}(\check{\lambda})}{\mathrm{d}\check{\lambda}} = \mu_B \frac{f(\check{\lambda})\left[\check{\lambda}F(\check{\lambda}) - \int_0^{\check{\lambda}} xf(x)\mathrm{d}x\right]}{\left[F(\check{\lambda})\right]^2} > 0$.)

2. If $(1-\alpha)\check{\Pi}(1) < \mu_S$, we select the equilibrium with the largest $\check{\pi}$. Since $\check{\pi} = \frac{\check{\lambda}\mu_S}{1-\alpha}$, the equilibrium price $\check{\pi}$ is strictly increasing in $\check{\lambda}$. Thus, it is equivalent to select the equilibrium with the largest $\check{\lambda}$, that is $\check{\lambda}^* = \max\{\check{\lambda} | (1-\alpha)\check{\Pi}\left(\check{\lambda}\right) \geq \check{\lambda}\mu_S, 0 \leq \check{\lambda} < 1\}$. Since $(1-\alpha)\check{\Pi}\left(\check{\lambda}\right) - \check{\lambda}\mu_S$ is a continuous function of $\check{\lambda}$ and $(1-\alpha)\check{\Pi}\left(\check{\lambda}\right) - \check{\lambda}\mu_S < 0$ for $\check{\lambda} = 1$, we obtain that $\check{\lambda}^*$ satisfies $(1-\alpha)\check{\Pi}\left(\check{\lambda}^*\right) = \check{\lambda}^*\mu_S$. Therefore, the equilibrium price $\check{\pi}^*$ satisfies $\check{\pi}^* = \frac{\check{\lambda}^*\mu_S}{1-\alpha} = \check{\Pi}\left(\check{\lambda}^*\right)$.

Combining the above two cases, for any $\alpha \leq 1$, we have $\check{\lambda}^* = \max\left\{\check{\lambda} | (1-\alpha)\check{\Pi}\left(\check{\lambda}\right) \geq \check{\lambda}\mu_S, 0 \leq \check{\lambda} \leq 1\right\}$ and $\check{\pi}^* = \check{\Pi}\left(\check{\lambda}^*\right)$, which completes the proof of Proposition 1. □

***Proof of Corollary 3.*** The result is immediately obtained from the proof of Proposition 1. □

***Proof of Proposition 2.***
1. For $(\alpha, \beta) \in \Omega\left(\hat{\lambda}^*\right)$ and $\hat{\lambda}^* \in [0,1)$, if $\lambda \in [\hat{\lambda}^*, 1]$, then

$$\begin{aligned} R_S^*(\lambda) &= (1-P_M+P_M\lambda)(1-\alpha)\hat{\Pi}\left(\hat{\lambda}^*\right) - (1-\lambda)P_M\beta\hat{\Pi}\left(\hat{\lambda}^*\right) \\ &= (1-P_M+P_M\lambda)(1-\alpha)\hat{\Pi}\left(\hat{\lambda}^*\right) - (1-\lambda)\frac{(1-P_M+P_M\hat{\lambda}^*)(1-\alpha)\hat{\Pi}\left(\hat{\lambda}^*\right) - \hat{\lambda}^*\mu_S}{1-\hat{\lambda}^*} \\ &= (1-\alpha)\hat{\Pi}\left(\hat{\lambda}^*\right)\left[1-P_M+P_M\lambda - (1-\lambda)\frac{1-P_M+P_M\hat{\lambda}^*}{1-\hat{\lambda}^*}\right] + (1-\lambda)\frac{\hat{\lambda}^*\mu_S}{1-\hat{\lambda}^*}. \end{aligned} \tag{22}$$

The second equality is obtained by the definition of $\Omega\left(\hat{\lambda}^*\right)$. If $\lambda \geq \hat{\lambda}^*$, the coefficient of $\alpha$ in (22) satisfies $-\hat{\Pi}\left(\hat{\lambda}^*\right)\left[1-P_M+P_M\lambda-(1-\lambda)\frac{1-P_M+P_M\hat{\lambda}^*}{1-\hat{\lambda}^*}\right] \leq 0$. So the seller's expected revenue is decreasing in $\alpha$, which yields the result.

2. For $\alpha = 1 - \frac{\mu_S}{\hat{\Pi}(\hat{\lambda}^*)}$ and $\beta = \frac{(1-P_M)(1-\alpha)}{P_M} = \frac{(1-P_M)\mu_S}{P_M\hat{\Pi}(\hat{\lambda}^*)}$, if $\lambda \in [\hat{\lambda}^*, 1]$, then $R_S^*(\lambda) = (1-P_M+P_M\lambda)(1-\alpha)\hat{\Pi}\left(\hat{\lambda}^*\right) - (1-\lambda)P_M\beta\hat{\Pi}\left(\hat{\lambda}^*\right) = (1-P_M+P_M\lambda)\mu_S - (1-\lambda)(1-P_M)\mu_S = \lambda\mu_S$. □

***Proof of Corollary 4.*** The result immediately follows the discussion before Corollary 4. □

***Proof of Lemma 4.*** The result is obtained by substituting $\hat{\Pi}\left(\hat{\lambda}^*\right)$ in (3) into $R_M\left(\hat{\lambda}^*\right)$. □

***Proof of Theorem 2.*** To find the platform's optimal contract, we first show that the contract $V\left(\hat{\lambda}^{**}\right)$ is optimal within the region $\beta \geq \frac{1-P_M}{P_M}(1-\alpha)$, and then prove that all the contracts with $\beta < \frac{1-P_M}{P_M}(1-\alpha)$ are strictly dominated by the optimal contract $V\left(\hat{\lambda}^{**}\right)$.

Recall that for a given $\hat{\lambda}^* \in [0,1]$, the platform's optimal contract is $V\left(\hat{\lambda}^{**}\right) = \left(\alpha^*\left(\hat{\lambda}^*\right), \beta^*\left(\hat{\lambda}^*\right)\right)$, which is given by (5) and (6). Now we determine the optimal $\hat{\lambda}^*$. By taking the derivative with respect to $\hat{\lambda}^*$, we obtain $\frac{\mathrm{d}R_M(\hat{\lambda}^*)}{\mathrm{d}\hat{\lambda}^*} = -\left[\left[\mu_B - \mu_S + (1-P_M)P_B\ell\right]\hat{\lambda}^* - (1-P_M)P_B\ell - c\right]f\left(\hat{\lambda}^*\right)$. Note that $\frac{\mathrm{d}R_M(\hat{\lambda}^*)}{\mathrm{d}\hat{\lambda}^*}$ is positive for $\hat{\lambda}^* < \frac{(1-P_M)P_B\ell+c}{(1-P_M)P_B\ell+\mu_B-\mu_S}$ and negative for $\hat{\lambda}^* > \frac{(1-P_M)P_B\ell+c}{(1-P_M)P_B\ell+\mu_B-\mu_S}$. Thus,

$R_M\left(\hat{\lambda}^*\right)$ is a unimodal function and attains its maximum at $\hat{\lambda}^{**} = \frac{(1-P_M)P_B\ell+c}{(1-P_M)P_B\ell+\mu_B-\mu_S}$. Therefore, within the contracts with $\beta \geq \frac{1-P_M}{P_M}(1-\alpha)$, the platform's optimal contract is $V\left(\hat{\lambda}^{**}\right)$.

Next, we show that all the contracts with $\beta < \frac{1-P_M}{P_M}(1-\alpha)$ are strictly dominated by the contract $V\left(\hat{\lambda}^{**}\right)$. Specifically, we derive an upper bound of the platform's expected profit for $\beta < \frac{1-P_M}{P_M}(1-\alpha)$, then we show this upper bound is smaller than $R_M\left(\hat{\lambda}^{**}\right)$. Proposition 4 summarizes the equilibrium for $\beta < \frac{1-P_M}{P_M}(1-\alpha)$. In the equilibrium, $\pi^*(\lambda) = \check{\pi}^*$ for $\lambda \in \left[0, \check{\lambda}^*\right]$ and $\pi^*(\lambda) = \infty$ for $\lambda \in \left(\check{\lambda}^*, 1\right]$, where the equilibrium price $\check{\pi}^* = \check{\Pi}\left(\check{\lambda}^*\right) = \frac{\mu_B \int_0^{\check{\lambda}^*} x f(x)\mathrm{d}x}{\int_0^{\check{\lambda}^*}(1-P_M+P_M x)f(x)\mathrm{d}x}$. Similar to §5.2, the total expected profit of the platform and the seller is $\int_0^{\check{\lambda}^*}\left[(1-P_M+P_M x)\check{\Pi}\left(\check{\lambda}^*\right) - (1-x)(1-P_M)P_B\ell - c\right] f(x)\mathrm{d}x$. The expected revenue of the seller with type $\lambda \in \left[0, \check{\lambda}^*\right]$ in equilibrium is no less than his expected reservation price $\lambda\mu_S$. Therefore, given threshold $\check{\lambda}^*$, the platform's expected profit $\check{R}_M\left(\check{\lambda}^*\right)$ satisfies $\check{R}_M\left(\check{\lambda}^*\right) \leq \bar{R}_M\left(\check{\lambda}^*\right) \triangleq \int_0^{\check{\lambda}^*}\left[(1-P_M+P_M x)\check{\Pi}\left(\check{\lambda}^*\right) - (1-x)(1-P_M)P_B\ell - c - x\mu_S\right] f(x)\mathrm{d}x$. Substituting the definition of $\check{\Pi}\left(\check{\lambda}^*\right)$, we obtain $\bar{R}_M\left(\check{\lambda}^*\right) = \int_0^{\check{\lambda}^*}[x(\mu_B-\mu_S) - (1-x)(1-P_M)P_B\ell - c]f(x)\mathrm{d}x$. Note that $\frac{\mathrm{d}\bar{R}_M\left(\check{\lambda}^*\right)}{\mathrm{d}\check{\lambda}^*} = \left[\check{\lambda}^*(\mu_B-\mu_S) - (1-\check{\lambda}^*)(1-P_M)P_B\ell - c\right] f\left(\check{\lambda}^*\right)$, which is negative for $\check{\lambda}^* < \hat{\lambda}^{**} = \frac{(1-P_M)P_B\ell+c}{(1-P_M)P_B\ell+\mu_B-\mu_S}$ and positive for $\check{\lambda}^* > \hat{\lambda}^{**}$. Therefore, $\bar{R}_M\left(\check{\lambda}^*\right)$ first decreases and then increases with $\check{\lambda}^*$, which yields that $\bar{R}_M\left(\check{\lambda}^*\right)$ reaches its maximum at $\check{\lambda}^* = 0$ or $\check{\lambda}^* = 1$. For $\check{\lambda}^* = 0$, $\bar{R}_M\left(\check{\lambda}^*\right) = 0 < R_M\left(\hat{\lambda}^{**}\right)$; for $\check{\lambda}^* = 1$, $\bar{R}_M\left(\check{\lambda}^*\right) = \bar{R}_M(1) = R_M(0) < R_M\left(\hat{\lambda}^{**}\right)$. Therefore, all the contracts with $\beta < \frac{1-P_M}{P_M}(1-\alpha)$ are strictly dominated by the optimal contract $V\left(\hat{\lambda}^{**}\right)$. □

***Proof of Corollaries 5 and 6.*** The results are immediately obtained from the proof of Theorem 2. □

***Proof of Proposition 3.***
1. The expected profit of the platform equals

$$
\begin{aligned}
R_M^* &= \int_{\hat{\lambda}^{**}}^1 [x(\mu_B-\mu_S) - (1-x)(1-P_M)P_B\ell - c]\, f(x)\mathrm{d}x \\
&= -\int_{\hat{\lambda}^{**}}^1 \Big[x\,[\mu_B-\mu_S+(1-P_M)P_B\ell] - (1-P_M)P_B\ell - c\Big]\mathrm{d}\bar{F}(x) \\
&= -\Big[x\,[\mu_B-\mu_S+(1-P_M)P_B\ell] - (1-P_M)P_B\ell - c\Big]\bar{F}(x)\Big|_{\hat{\lambda}^{**}}^1 \\
&\quad + [\mu_B-\mu_S+(1-P_M)P_B\ell]\int_{\hat{\lambda}^{**}}^1 \bar{F}(x)\mathrm{d}x \\
&= [\mu_B-\mu_S+(1-P_M)P_B\ell]\int_{\hat{\lambda}^{**}}^1 \bar{F}(x)\mathrm{d}x,
\end{aligned}
$$

where the third equality is obtained by partial integration formula, and the last equality is obtained by the definition of $\hat{\lambda}^{**}$ in Theorem 2. Then, part 1 of Proposition 3 can be obtained immediately.

2. If we denote $\gamma\left(\hat{\lambda}^{**}\right) = \bar{F}\left(\hat{\lambda}^{**}\right)/\int_{\hat{\lambda}^{**}}^1 x f(x)\mathrm{d}x$, then $\hat{\Pi}\left(\hat{\lambda}^{**}\right) = \frac{\mu_B}{(1-P_M)\gamma\left(\hat{\lambda}^{**}\right)+P_M}$. Note that $\frac{1}{\gamma\left(\hat{\lambda}^{**}\right)} = \int_{\hat{\lambda}^{**}}^1 x f(x)\mathrm{d}x/\bar{F}\left(\hat{\lambda}^{**}\right) = -\int_{\hat{\lambda}^{**}}^1 x\mathrm{d}\bar{F}(x)/\bar{F}\left(\hat{\lambda}^{**}\right) = -\left[x\bar{F}(x)\Big|_{\hat{\lambda}^{**}}^1 - \int_{\hat{\lambda}^{**}}^1 \bar{F}(x)\mathrm{d}x\right]/\bar{F}\left(\hat{\lambda}^{**}\right) = \int_{\hat{\lambda}^{**}}^1 \bar{F}(x)\mathrm{d}x/\bar{F}\left(\hat{\lambda}^{**}\right) + \hat{\lambda}^{**}$, where $\hat{\lambda}^{**}$ does not depend on the distribution $F(\cdot)$. So the equilibrium price is larger under market $F_1(\cdot)$ if and only if $\int_{\hat{\lambda}^{**}}^1 \bar{F}_1(x)\mathrm{d}x/\bar{F}_1\left(\hat{\lambda}^{**}\right) \geq \int_{\hat{\lambda}^{**}}^1 \bar{F}_2(x)\mathrm{d}x/\bar{F}_2\left(\hat{\lambda}^{**}\right)$. Then, it is straightforward to prove the counterparts for the commission fraction and the penalty fraction. □

***Comment on conditions (8) and (9).*** Here, we present two distributions $F_1(\cdot)$ and $F_2(\cdot)$, which satisfy (8) but violate (9). Let $\bar{F}_1(x)$ equal $-\frac{1}{2}x+1$ if $0 \leq x \leq \frac{1}{2}$, and $x^2-3x+2$ if $\frac{1}{2} < x \leq 1$. Let $\bar{F}_2(x) = 1-x$ and $\hat{\lambda}^{**} = \frac{1}{2}$. It is easy to check that $F_1$ is stochastically larger than $F_2$: $\bar{F}_1(x) \geq \bar{F}_2(x)$ for $0 \leq x \leq 1$. However, we have $\int_{\hat{\lambda}^{**}}^1 \bar{F}_1(x)\mathrm{d}x/\bar{F}_1\left(\hat{\lambda}^{**}\right) < \int_{\hat{\lambda}^{**}}^1(-\frac{3}{2}x+\frac{3}{2})\mathrm{d}x/\bar{F}_1\left(\hat{\lambda}^{**}\right) = \frac{1}{4} = \int_{\hat{\lambda}^{**}}^1 \bar{F}_2(x)\mathrm{d}x/\bar{F}_2\left(\hat{\lambda}^{**}\right)$, because $\bar{F}_1(x)$ is convex on $\left[\hat{\lambda}^{**}, 1\right]$ while $\bar{F}_2(x)$ is linear on $\left[\hat{\lambda}^{**}, 1\right]$. □

***Proof of Theorem 3.*** Denote $\xi = \bar{F}\left(\hat{\lambda}^{**}\right)$, $\eta = \int_{\hat{\lambda}^{**}}^1 x f(x)\mathrm{d}x$, and $\lambda^\dagger = \frac{(1-P_M)P_B\ell}{(1-P_M)P_B\ell+\mu_B-\mu_S}$, where $\xi \geq \eta \geq \hat{\lambda}^{**}\xi$.

1. Take the first-order derivative of $\hat{\lambda}^{**}$ with respect to $P_M$: $\frac{\partial \hat{\lambda}^{**}}{\partial P_M} = -\frac{P_B \ell(\mu_B - \mu_S - c)}{[(1-P_M)P_B\ell + \mu_B - \mu_S]^2} < 0$. So the optimal threshold $\hat{\lambda}^{**}$ decreases with $P_M$. Since $\alpha^* = 1 - \frac{\mu_S}{\hat{\Pi}(\hat{\lambda}^{**})}$, the optimal commission fraction $\alpha^*$ and equilibrium price $\hat{\Pi}\left(\hat{\lambda}^{**}\right)$ have the same result of sensitivity analysis with respect to $P_M$. Therefore, we omit the sensitivity analysis of $\hat{\Pi}\left(\hat{\lambda}^{**}\right)$ and only provide the analysis of $\alpha^*$ in the following.

2. First, we provide the sensitivity analysis of the platform's optimal profit $R_M^*$. The optimal expected profit of the platform equals $R_M^* = \int_{\hat{\lambda}^{**}}^1 [x(\mu_B - \mu_S) - (1-x)(1-P_M)P_B\ell - c] f(x)\mathrm{d}x$. Its first-order derivative is given as:

$$\frac{\partial R_M^*}{\partial P_M} = -\left[\hat{\lambda}^{**}(\mu_B - \mu_S) - \left(1 - \hat{\lambda}^{**}\right)(1-P_M)P_B\ell - c\right] f\left(\hat{\lambda}^{**}\right) \frac{\partial \hat{\lambda}^{**}}{\partial P_M} + \int_{\hat{\lambda}^{**}}^1 (1-x)P_B\ell f(x)\mathrm{d}x.$$

The first term is 0 according to the definition of $\hat{\lambda}^{**}$ in Theorem 2. Note that the second term is non-negative. So the platform's optimal expected profit $R_M^*$ increases with $P_M$.

Next, we provide the sensitivity analysis of the optimal commission fraction $\alpha^*$. Denote $\gamma\left(\hat{\lambda}^{**}\right) = \frac{\bar{F}(\hat{\lambda}^{**})}{\int_{\hat{\lambda}^{**}}^1 x f(x)\mathrm{d}x}$, then $\hat{\Pi}\left(\hat{\lambda}^{**}\right) = \frac{\mu_B}{(1-P_M)\gamma(\hat{\lambda}^{**}) + P_M}$. Since $\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}} = -\frac{f(\hat{\lambda}^{**})\int_{\hat{\lambda}^{**}}^1 (x - \hat{\lambda}^{**}) f(x)\mathrm{d}x}{[\int_{\hat{\lambda}^{**}}^1 x f(x)\mathrm{d}x]^2} < 0$ for $\hat{\lambda}^{**} \in (0,1)$, $\gamma\left(\hat{\lambda}^{**}\right)$ is decreasing in $\hat{\lambda}^{**}$. Note that $\alpha^* = 1 - \frac{\mu_S}{\hat{\Pi}(\hat{\lambda}^{**})} = 1 - \frac{\mu_S}{\mu_B}\left[(1-P_M)\gamma\left(\hat{\lambda}^{**}\right) + P_M\right]$. Its first-order derivative is given as: $\frac{\partial \alpha^*}{\partial P_M} = -\frac{\mu_S}{\mu_B}\left[-\gamma\left(\hat{\lambda}^{**}\right) + (1-P_M)\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial \hat{\lambda}^{**}}{\partial P_M} + 1\right]$, where $\gamma\left(\hat{\lambda}^{**}\right) = \frac{\xi}{\eta}$, $\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}} = -f\left(\hat{\lambda}^{**}\right)\frac{\eta - \hat{\lambda}^{**}\xi}{\eta^2}$, and $\frac{\partial \hat{\lambda}^{**}}{\partial P_M} = -\frac{P_B\ell(\mu_B - \mu_S - c)}{[(1-P_M)P_B\ell + \mu_B - \mu_S]^2}$. It is easy to check that $(1-P_M)\frac{\partial \hat{\lambda}^{**}}{\partial P_M} = -\lambda^\dagger\left(1 - \hat{\lambda}^{**}\right)$. So $\frac{\partial \alpha^*}{\partial P_M} = -\frac{\mu_S}{\mu_B}\left[-\frac{\xi}{\eta} + \lambda^\dagger\left(1 - \hat{\lambda}^{**}\right) f\left(\hat{\lambda}^{**}\right)\frac{\eta - \hat{\lambda}^{**}\xi}{\eta^2} + 1\right]$. Hence, the optimal commission fraction $\alpha^*$ increases with $P_M$ if and only if $\lambda^\dagger\left(1 - \hat{\lambda}^{**}\right) f\left(\hat{\lambda}^{**}\right)\left(\eta - \hat{\lambda}^{**}\xi\right) \leq (\xi - \eta)\eta$.

Lastly, we provide the sensitivity analysis of the optimal penalty fraction $\beta^*$. Note that $\beta^* = \frac{(1-P_M)\mu_S}{P_M \hat{\Pi}(\hat{\lambda}^{**})}$ and $\hat{\Pi}\left(\hat{\lambda}^{**}\right) = \frac{\mu_B}{(1-P_M)\gamma(\hat{\lambda}^{**}) + P_M}$. Then, $\beta^* = \frac{(1-P_M)\mu_S}{P_M \mu_B}\left[(1-P_M)\gamma\left(\hat{\lambda}^{**}\right) + P_M\right]$. Its first-order derivative is $\frac{\partial \beta^*}{\partial P_M} = \frac{\mu_S}{\mu_B}\left[-\frac{1-P_M^2}{P_M^2}\gamma\left(\hat{\lambda}^{**}\right) + \frac{(1-P_M)^2}{P_M}\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial \hat{\lambda}^{**}}{\partial P_M} - 1\right]$, where $\gamma\left(\hat{\lambda}^{**}\right) = \frac{\xi}{\eta}$, $\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}} = -f\left(\hat{\lambda}^{**}\right)\frac{\eta - \hat{\lambda}^{**}\xi}{\eta^2}$, and $\frac{\partial \hat{\lambda}^{**}}{\partial P_M} = -\frac{P_B\ell(\mu_B - \mu_S - c)}{[(1-P_M)P_B\ell + \mu_B - \mu_S]^2}$. It is easy to check that $(1-P_M)\frac{\partial \hat{\lambda}^{**}}{\partial P_M} = -\lambda^\dagger\left(1 - \hat{\lambda}^{**}\right)$. Then $\frac{\partial \beta^*}{\partial P_M} \leq 0$ if and only if $\frac{(1-P_M)}{P_M}\frac{\eta - \hat{\lambda}^{**}\xi}{\eta^2}\lambda^\dagger\left(1 - \hat{\lambda}^{**}\right) f\left(\hat{\lambda}^{**}\right) \leq \frac{1-P_M^2}{P_M^2}\frac{\xi}{\eta} + 1$, which is equivalent to $\lambda^\dagger\left(1 - \hat{\lambda}^{**}\right) f\left(\hat{\lambda}^{**}\right) \leq \frac{P_M^2\eta^2 + (1-P_M^2)\xi\eta}{(1-P_M)P_M\left(\eta - \hat{\lambda}^{**}\xi\right)}$. □

***Comment on condition (10).*** In Example 1, we show that condition (10) can be satisfied in many distributions by numerical study. Here, we give an example that violates condition (10).

**Example 3.** *If* $f(\lambda) = \frac{1}{4\epsilon}I\{1/4 - \epsilon < \lambda < 1/4 + \epsilon\} + 1/2$, $\mu_B = 10$, $\mu_S = 6$, $P_B = 0.2$ *and* $l = 10$, $c = \frac{1}{2}$, *then we have* $\lambda^\dagger\left(1 - \hat{\lambda}^{**}\right) f\left(\hat{\lambda}^{**}\right)\left(\eta - \hat{\lambda}^{**}\xi\right) > (\xi - \eta)\eta$ *for* $P_M = \frac{2}{3}$.

For $P_M = \frac{2}{3}$, we obtain $\lambda^\dagger = \frac{1}{7}$, $\hat{\lambda}^{**} = \frac{1}{4}$, $\xi = \bar{F}\left(\hat{\lambda}^{**}\right) = \int_{1/4}^{1/4+\epsilon}(\frac{1}{4\epsilon} + \frac{1}{2})\mathrm{d}x + \int_{1/4+\epsilon}^1 \frac{1}{2}\mathrm{d}x = \frac{5}{8}$ and $\eta = \int_{\hat{\lambda}^{**}}^1 x f(x)\mathrm{d}x = \int_{1/4}^{1/4+\epsilon} x\left(\frac{1}{4\epsilon} + \frac{1}{2}\right)\mathrm{d}x + \int_{1/4+\epsilon}^1 \frac{1}{2}x\mathrm{d}x = \frac{19}{64} + \frac{\epsilon}{8}$. Then $\lambda^\dagger\left(1 - \hat{\lambda}^{**}\right) f\left(\hat{\lambda}^{**}\right)\left(\eta - \hat{\lambda}^{**}\xi\right) = \frac{3}{28}\left(\frac{1}{4\epsilon} + \frac{1}{2}\right)\left(\frac{9}{64} + \frac{\epsilon}{8}\right) = O\left(\frac{1}{\epsilon}\right)$, and $(\xi - \eta)\eta = \left(\frac{21}{64} - \frac{\epsilon}{8}\right)\left(\frac{\epsilon}{8} + \frac{19}{64}\right) = O(1)$. Thus, when $\epsilon$ is sufficiently small, we obtain $\lambda^\dagger\left(1 - \hat{\lambda}^{**}\right) f\left(\hat{\lambda}^{**}\right)\left(\eta - \hat{\lambda}^{**}\xi\right) > (\xi - \eta)\eta$. □

***Comment on condition (11).*** We prove that $\frac{f(\hat{\lambda}^{**})}{\bar{F}(\hat{\lambda}^{**})} \leq 8$ is a sufficient condition to guarantee $\frac{\partial \beta^*}{\partial P_M} \leq 0$. This sufficient condition is easier to identify than (11). Since $\lambda^\dagger\left(1 - \hat{\lambda}^{**}\right) \leq \hat{\lambda}^{**}\left(1 - \hat{\lambda}^{**}\right) \leq \frac{1}{4}$, we have $\frac{\partial \beta^*}{\partial P_M} = \frac{\mu_S}{\mu_B}\left[-\frac{1-P_M^2}{P_M^2}\gamma\left(\hat{\lambda}^{**}\right) + \frac{(1-P_M)^2}{P_M}\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial \hat{\lambda}^{**}}{\partial P_M} - 1\right] = \frac{\mu_S}{\mu_B}\left[-\frac{1-P_M^2}{P_M^2}\frac{\xi}{\eta} + \frac{(1-P_M)}{P_M}\frac{\eta - \hat{\lambda}^{**}\xi}{\eta^2}\lambda^\dagger\left(1 - \hat{\lambda}^{**}\right)\right.$ $\left. f\left(\hat{\lambda}^{**}\right) - 1\right] \leq \frac{\mu_S}{\mu_B}\left[-\frac{1-P_M^2}{P_M^2}\frac{\xi}{\eta} + \frac{1}{4}\frac{1-P_M}{P_M}\frac{\eta - \hat{\lambda}^{**}\xi}{\eta^2} f\left(\hat{\lambda}^{**}\right) - 1\right] = \frac{\mu_S}{\mu_B P_M^2 \eta^2}\left[-(1-P_M^2)\xi\eta + \frac{1}{4}(1-P_M)P_M\eta\right.$ $\left. f\left(\hat{\lambda}^{**}\right) - \frac{1}{4}(1-P_M)P_M\hat{\lambda}^{**}\xi f\left(\hat{\lambda}^{**}\right) - P_M^2\eta^2\right]$. If $\frac{f(\hat{\lambda}^{**})}{\bar{F}(\hat{\lambda}^{**})} = \frac{f(\hat{\lambda}^{**})}{\xi} \leq 8$, then $-(1-P_M^2)\xi\eta + \frac{1}{4}(1-P_M)P_M\eta f\left(\hat{\lambda}^{**}\right) = -\frac{1}{4}(1-P_M)P_M\xi\eta\left[4\frac{1+P_M}{P_M} - \frac{f(\hat{\lambda}^{**})}{\xi}\right] \leq -\frac{1}{4}(1-P_M)P_M\xi\eta\left[8 - \frac{f(\hat{\lambda}^{**})}{\xi}\right] \leq 0$. Thus, we have $\frac{\partial \beta^*}{\partial P_M} \leq 0$ if $\frac{f(\hat{\lambda}^{**})}{\bar{F}(\hat{\lambda}^{**})} \leq 8$. □

***Proof of Theorem 4.*** Denote $\Omega(1)=\left\{(\alpha,\beta)\middle|\alpha\le 1-\frac{\mu_S}{\mu_B},\beta>\frac{1-P_M}{P_M}(1-\alpha)\right\}$, $\Omega(0)=\left\{(\alpha,\beta)\middle|\alpha\le 1-\frac{\mu_S(1-P_M+P_Mw)}{\mu_Bw},\beta<\frac{1-P_M}{P_M}(1-\alpha)\right\}$, $\Omega(1^+)=\left\{(\alpha,\beta)\middle|\alpha>1-\frac{\mu_S}{\mu_B},\beta>\frac{1-P_M}{P_M}(1-\alpha)\right\}$, $\widetilde{\Omega}=\left\{(\alpha,\beta)\middle|\alpha>1-\frac{\mu_S(1-P_M+P_Mw)}{\mu_Bw},\beta<\frac{1-P_M}{P_M}(1-\alpha)\right\}$, and $V(1)=\left\{(\alpha,\beta)\middle|\alpha=1-\frac{\mu_S}{\mu_B},\beta>\frac{1-P_M}{P_M}(1-\alpha)\right\}$.

1. Given $\pi\in(0,\mu_B]$, the buyer generates a posterior belief $m(\pi)$ about the seller's type, that is $P(\lambda=1|\pi)=m(\pi)$. The buyer's expected utility is $R_B(I;\pi)=[m(\pi)(\mu_B-\pi)-(1-m(\pi))(1-P_M)\pi]I$. The buyer purchases the product with price $\pi$ ($I^*(\pi)=1$) if and only if $R_B(1;\pi)\ge R_B(0;\pi)=0$, that is

$$m(\pi)\ge\frac{(1-P_M)\pi}{\mu_B-P_M\pi}\iff\pi\le\frac{m(\pi)\mu_B}{1-P_M+m(\pi)P_M}. \tag{23}$$

If $\pi\in(0,\mu_B]$, the type-1 seller's expected revenue is $R_S(\pi;1,I)=(1-\alpha)\pi I$ and the type-0 seller's expected revenue is $R_S(\pi;0,I)=[(1-\alpha)(1-P_M)-\beta P_M]\pi I$; if $\pi=\infty$, the type-1 seller's expected revenue is $\mu_S$ and the type-0 seller's expected revenue is 0. Next, we analyze the equilibrium of the signaling game for the following sets of contract parameters: $\Omega(1)$, $\Omega(0)$, and $\Omega(1^+)\cup\widetilde{\Omega}$.

(a) For $(\alpha,\beta)\in\Omega(1)$, we analyze the seller's equilibrium decision by considering the following 4 cases.

**Case 1**: Both seller types choose to sell.

This case is impossible because the type-0 seller's expected revenue $R_S(\pi;0,I)$ is negative for any acceptable price $\pi\in(0,\mu_B]$ if $\beta>(1-\alpha)\frac{1-P_M}{P_M}$.

**Case 2**: Both seller types refuse to sell.

In this case, $\pi^*(1)=\pi^*(0)=\infty$. The type-1 seller earns $\mu_S$ and the type-0 seller earns 0.

**Case 3**: Only type-1 seller chooses to sell.

In this case, $\pi^*(1)=\pi_1$ and $\pi^*(0)=\infty$, where $0<\pi_1\le\mu_B$.

**Case 4**: Only type-0 seller chooses to sell.

This case is impossible because the type-0 seller's expected revenue $R_S(\pi;0,I)$ is negative for any acceptable price $\pi\in(0,\mu_B]$ if $\beta>(1-\alpha)\frac{1-P_M}{P_M}$.

Among the above 4 cases, only Cases 2 and 3 are possible in equilibrium. By Pareto dominance, we choose the equilibrium in which $\pi^*(1)=\mu_B$ and $\pi^*(0)=\infty$.

(b) For $(\alpha,\beta)\in\Omega(0)$, we similarly consider the above 4 cases.

**Case 1**: Both seller types choose to sell.

We first consider the two seller types adopt different prices. Without loss of generality, we assume that $\pi^*(0)=\pi_0$ and $\pi^*(1)=\pi_1$, where $0<\pi_0<\pi_1\le\mu_B$. Since we only consider participating equilibria, then $I^*(\pi_0)=I^*(\pi_1)=1$. Since $\beta<\frac{1-P_M}{P_M}(1-\alpha)$, we obtain $R_S(\pi_0;0,I^*(\pi_0))=[(1-\alpha)(1-P_M)-\beta P_M]\pi_0<[(1-\alpha)(1-P_M)-\beta P_M]\pi_1=R_S(\pi_1;0,I^*(\pi_1))$. This implies that the price $\pi_1$ is a profitable deviation for the type-0 seller, and thus it violates Condition 2.

Therefore, we only need to consider that the two seller types adopt the same price. That is, $\pi^*(0)=\pi^*(1)=\hat{\pi}$, where $0<\hat{\pi}\le\mu_B$. According to Condition 3, the on-equilibrium belief should satisfy the Bayesian rule: $m^*(\hat{\pi})=w$. By (23), we obtain $\hat{\pi}\le\frac{w\mu_B}{1-P_M+wP_M}$.

**Case 2**: Both seller types refuse to sell.

In this case, $\pi^*(1)=\pi^*(0)=\infty$. The type-1 seller earns $\mu_S$ and the type-0 seller earns 0.

**Case 3**: Only type-1 seller chooses to sell.

In this case, $\pi^*(1)=\pi_1$ and $\pi^*(0)=\infty$, where $0<\pi_1\le\mu_B$ and $I^*(\pi_1)=1$. Since $\beta<\frac{1-P_M}{P_M}(1-\alpha)$, we obtain $R_S(\infty;0,I^*)=0<[(1-\alpha)(1-P_M)-\beta P_M]\pi_1=R_S(\pi_1;0,I^*(\pi_1))$. This implies that the price $\pi_1$ is a profitable deviation for the type-0 seller and thus it violates Condition 2. Therefore, Case 3 cannot be an equilibrium.

**Case 4**: Only type-0 seller chooses to sell.

In this case, $\pi^*(1)=\infty$ and $\pi^*(0)=\pi_0$, where $0<\pi_0\le\mu_B$ and $I^*(\pi_0)=1$. According to Condition 3, the on-equilibrium belief satisfies the Bayesian rule: $m(\pi_0)=0$. By (23), we obtain $\pi_0\le 0$, which contradicts with $0<\pi_0\le\mu_B$. Therefore, Case 4 cannot be an equilibrium.

Among the above 4 cases, only Cases 1 and 2 are possible in equilibrium. By Pareto dominance, we select the equilibrium in which $\pi^*(0)=\pi^*(1)=\frac{w\mu_B}{1-P_M+wP_M}$.

(c) For $(\alpha,\beta)\in\Omega(1^+)\cup\widetilde{\Omega}$, we can similarly consider the above 4 cases. It is easy to prove that only Case 2 can be an equilibrium. Therefore, the seller chooses not to sell for $(\alpha,\beta)\in\Omega(1^+)\cup\widetilde{\Omega}$.

2. To derive the platform's optimal contract, we consider the sets $\Omega(1)$, $\Omega(0)$, and $\Omega(1^+)\cup\widetilde{\Omega}$ as follows.

(a) If $(\alpha,\beta)\in\Omega(1)$, only the type-1 seller chooses to sell through the platform. It is optimal for the platform to choose the largest commission fraction that can be accepted by the type-1 seller. Thus, in $\Omega(1)$ the set of optimal contracts is $V(1)$ and the platform's expected profit is $w(\mu_B-\mu_S-c)$.

(b) If $(\alpha,\beta)\in\Omega(0)$, the seller chooses to sell with price $\frac{w\mu_B}{1-P_M+P_M w}$ regardless of his type. We next prove that for any $(\alpha,\beta)\in\Omega(0)$ the platform's expected profit is smaller than $w(\mu_B-\mu_S-c)$. The total expected profit of the platform and the seller is $[w+(1-w)(1-P_M)]\frac{w\mu_B}{1-P_M+P_M w}-(1-w)(1-P_M)P_B\ell-c=w\mu_B-(1-w)(1-P_M)P_B\ell-c$. The seller's expected revenue is larger than or equal to his expected reservation price. Thus, the platform's expected profit is smaller than or equal to $w(\mu_B-\mu_S)-(1-w)(1-P_M)P_B\ell-c$, which is smaller than $w(\mu_B-\mu_S-c)$.

(c) If $(\alpha,\beta)\in\Omega(1^+)\cup\widetilde{\Omega}$, then the seller does not sell. In this case, the platform's expected profit is 0.

Overall, the set of the platform's optimal contracts is $V(1)$ and the platform's optimal expected profit is $w(\mu_B-\mu_S-c)$. $\square$

***Proof of Proposition 4.*** The proof of Proposition 4 is similar to that of Proposition 1, and hence omitted. $\square$

***Proof of Proposition 5.***

1. The sensitivity analysis is obtained directly from the first-order derivative: $\frac{\partial\hat{\lambda}^{**}}{\partial P_B}=\frac{(1-P_M)(\mu_B-\mu_S-c)\ell}{[(1-P_M)P_B\ell+\mu_B-\mu_S]^2}>0$, $\frac{\partial\hat{\lambda}^{**}}{\partial \ell}=\frac{(1-P_M)(\mu_B-\mu_S-c)P_B}{[(1-P_M)P_B\ell+\mu_B-\mu_S]^2}>0$, $\frac{\partial\hat{\lambda}^{**}}{\partial c}=\frac{1}{(1-P_M)P_B\ell+\mu_B-\mu_S}>0$, $\frac{\partial\hat{\lambda}^{**}}{\partial \mu_S}=\frac{(1-P_M)P_B\ell+c}{[(1-P_M)P_B\ell+\mu_B-\mu_S]^2}>0$, and $\frac{\partial\hat{\lambda}^{**}}{\partial \mu_B}=-\frac{(1-P_M)P_B\ell+c}{[(1-P_M)P_B\ell+\mu_B-\mu_S]^2}<0$.

Denote $\xi=\bar{F}\left(\hat{\lambda}^{**}\right)=\int_{\hat{\lambda}^{**}}^1 f(x)\mathrm{d}x$, $\eta=\int_{\hat{\lambda}^{**}}^1 xf(x)\mathrm{d}x$, and $\gamma\left(\hat{\lambda}^{**}\right)=\frac{\bar{F}(\hat{\lambda}^{**})}{\int_{\hat{\lambda}^{**}}^1 xf(x)\mathrm{d}x}=\frac{\xi}{\eta}$. Then, $\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}=-\frac{f(\hat{\lambda}^{**})\int_{\hat{\lambda}^{**}}^1(x-\hat{\lambda}^{**})f(x)\mathrm{d}x}{\left[\int_{\hat{\lambda}^{**}}^1 xf(x)\mathrm{d}x\right]^2}=-f\left(\hat{\lambda}^{**}\right)\frac{\eta-\hat{\lambda}^{**}\xi}{\eta^2}<0$. The equilibrium price is $\hat{\Pi}\left(\hat{\lambda}^{**}\right)=\frac{\mu_B}{(1-P_M)\gamma(\hat{\lambda}^{**})+P_M}$. Take the first-order derivative: $\frac{\partial\hat{\Pi}(\hat{\lambda}^{**})}{\partial P_B}=-\frac{(1-P_M)\mu_B}{\left[(1-P_M)\gamma(\hat{\lambda}^{**})+P_M\right]^2}\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial\hat{\lambda}^{**}}{\partial P_B}>0$, $\frac{\partial\hat{\Pi}(\hat{\lambda}^{**})}{\partial \ell}=-\frac{(1-P_M)\mu_B}{\left[(1-P_M)\gamma(\hat{\lambda}^{**})+P_M\right]^2}\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial\hat{\lambda}^{**}}{\partial \ell}>0$, $\frac{\partial\hat{\Pi}(\hat{\lambda}^{**})}{\partial c}=-\frac{(1-P_M)\mu_B}{\left[(1-P_M)\gamma(\hat{\lambda}^{**})+P_M\right]^2}\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial\hat{\lambda}^{**}}{\partial c}>0$, and $\frac{\partial\hat{\Pi}(\hat{\lambda}^{**})}{\partial \mu_S}=-\frac{(1-P_M)\mu_B}{\left[(1-P_M)\gamma(\hat{\lambda}^{**})+P_M\right]^2}\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial\hat{\lambda}^{**}}{\partial \mu_S}>0$. The derivative to $\mu_B$ is $\frac{\partial\hat{\Pi}(\hat{\lambda}^{**})}{\partial \mu_B}=\frac{1}{\left[(1-P_M)\gamma(\hat{\lambda}^{**})+P_M\right]^2}\left[(1-P_M)\gamma\left(\hat{\lambda}^{**}\right)+P_M-\mu_B(1-P_M)\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial\hat{\lambda}^{**}}{\partial \mu_B}\right]=\frac{1}{\left[(1-P_M)\gamma(\hat{\lambda}^{**})+P_M\right]^2}\left[(1-P_M)\frac{\xi}{\eta}+P_M-\mu_B(1-P_M)\right.$ $\left.f\left(\hat{\lambda}^{**}\right)\frac{\eta-\hat{\lambda}^{**}\xi}{\eta^2}\frac{(1-P_M)P_B\ell+c}{[(1-P_M)P_B\ell+\mu_B-\mu_S]^2}\right]$, which can be either positive or negative.

2. Note that $\alpha^*=1-\frac{\mu_S}{\mu_B}\left[(1-P_M)\gamma\left(\hat{\lambda}^{**}\right)+P_M\right]$ and $\beta^*=\frac{(1-P_M)\mu_S}{P_M\mu_B}\left[(1-P_M)\gamma\left(\hat{\lambda}^{**}\right)+P_M\right]$, so the optimal commission fraction and the optimal penalty fraction always have opposite results of the sensitivity analysis with respect to $P_B$, $\ell$, $c$, $\mu_S$ and $\mu_B$. Therefore, we only provide the proof of the sensitivity analysis of the optimal commission $\alpha^*$. Take the first-order derivative: $\frac{\partial\alpha^*}{\partial P_B}=-(1-P_M)\frac{\mu_S}{\mu_B}\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial\hat{\lambda}^{**}}{\partial P_B}>0$, $\frac{\partial\alpha^*}{\partial \ell}=-(1-P_M)\frac{\mu_S}{\mu_B}\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial\hat{\lambda}^{**}}{\partial \ell}>0$, $\frac{\partial\alpha^*}{\partial c}=-(1-P_M)\frac{\mu_S}{\mu_B}\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial\hat{\lambda}^{**}}{\partial c}>0$, $\frac{\partial\alpha^*}{\partial \mu_S}=-\frac{1}{\mu_B}\left[(1-P_M)\gamma\left(\hat{\lambda}^{**}\right)+P_M+\mu_S(1-P_M)\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial\hat{\lambda}^{**}}{\partial \mu_S}\right]=-\frac{1}{\mu_B}\left[(1-P_M)\gamma\left(\hat{\lambda}^{**}\right)+P_M-\mu_S\right.$ $\left.(1-P_M)\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial\hat{\lambda}^{**}}{\partial \mu_B}\right]$, and $\frac{\partial\alpha^*}{\partial \mu_B}=\frac{\mu_S}{(\mu_B)^2}\left[(1-P_M)\gamma\left(\hat{\lambda}^{**}\right)+P_M-\mu_B(1-P_M)\frac{\mathrm{d}\gamma(\hat{\lambda}^{**})}{\mathrm{d}\hat{\lambda}^{**}}\frac{\partial\hat{\lambda}^{**}}{\partial \mu_B}\right]$, where the last two derivatives can be either positive or negative. Therefore, the optimal commission fraction and the optimal penalty fraction may increase or decrease with $\mu_S$ and $\mu_B$.

3. Take the first-order derivative with respect to $P_B$: $\frac{\partial R_M^*}{\partial P_B}=-\left[\hat{\lambda}^{**}(\mu_B-\mu_S)-\left(1-\hat{\lambda}^{**}\right)(1-P_M)P_B\ell-c\right]$ $f\left(\hat{\lambda}^{**}\right)\frac{\partial\hat{\lambda}^{**}}{\partial P_B}-\int_{\hat{\lambda}^{**}}^1(1-x)(1-P_M)\ell f(x)\mathrm{d}x$, where the term in the square bracket is zero according to the definition of $\hat{\lambda}^{**}$ in Theorem 2. Therefore, $\frac{\partial R_M^*}{\partial P_B}\le 0$. The proof of other parameters is similar. $\square$

***Proof of Theorem 5.*** For $\hat{\lambda}^*\in(0,1)$, $\Omega\left(\hat{\lambda}^*\right)=\left\{(\alpha,\beta)\middle|\alpha<1-\frac{\mu_S}{\hat{\Pi}(\hat{\lambda}^*)},\beta=\frac{(1-P_M+P_M\hat{\lambda}^*)(1-\alpha)\hat{\Pi}(\hat{\lambda}^*)-\hat{\lambda}^*\mu_S}{(1-\hat{\lambda}^*)P_M\hat{\Pi}(\hat{\lambda}^*)}\right\}$.

For $\hat{\lambda}^*=1$, $\Omega\left(\hat{\lambda}^*\right)=\left\{(\alpha,\beta)\middle|\alpha=1-\frac{\mu_S}{\mu_B},\beta>\frac{1-P_M}{P_M}(1-\alpha)\right\}$.

For $\hat{\lambda}^*\in[0,1]$, let $V\left(\hat{\lambda}^*\right)=\left\{(\alpha,\beta)\middle|\alpha=1-\frac{\mu_S}{\hat{\Pi}(\hat{\lambda}^*)},\beta=\frac{1-P_M}{P_M}(1-\alpha)\right\}$.

The proof of Theorem 5 is similar to that of Theorems 1 and 2, and hence omitted. $\square$

***Proof of Theorem 6.***
1. Similar to Theorem 1, we only focus on $\beta > \frac{1-P_M}{P_M}(1-\alpha)$. To derive the equilibrium of the signaling game, we follow the three steps in Figure 3.
**Step 1: Reduce the space of possible equilibria.**

The seller's expected revenue is $R_S(\pi;\lambda,I^*(\pi)) = [(1-P_M+P_M\lambda)(1-\alpha)-(1-\lambda)P_M\beta]\,\pi I^*(\pi) = [P_M(1-\alpha+\beta)\lambda+(1-P_M)(1-\alpha)-P_M\beta]\,\pi I^*(\pi)$. Note that the seller's revenue function is the product of two separate functions: one only in terms of $\lambda$ and the other only in terms of $\pi$. Consequently, the seller will use the same equilibrium price if he chooses to sell. Specifically, for $\lambda \le \underline{\hat{\lambda}} \triangleq \frac{P_M\beta-(1-P_M)(1-\alpha)}{P_M(1-\alpha+\beta)}$, we obtain $R_S(\pi;\lambda,I^*(\pi)) \le 0$. Thus, the seller with $\lambda \in \left[0,\underline{\hat{\lambda}}\right]$ chooses not to sell by setting $\pi^*(\lambda)=\infty$. For $\lambda > \underline{\hat{\lambda}}$, define $\hat{\pi} \in \arg\max_{\pi\in(0,\overline{\mu}_B]} \pi I^*(\pi)$. Then, $R_S(\pi;\lambda,I^*(\pi)) \ge \lambda\mu_S$ if and only if $\lambda \ge \hat{\lambda}$, where $\hat{\lambda} = \frac{[P_M\beta-(1-P_M)(1-\alpha)]\hat{\pi}I^*(\hat{\pi})}{P_M(1-\alpha+\beta)\hat{\pi}I^*(\hat{\pi})-\mu_S}$, which is equivalent to $\hat{\pi}I^*(\hat{\pi}) = A\left(\hat{\lambda}\right) \triangleq \frac{\hat{\lambda}\mu_S}{P_M(1-\alpha+\beta)\hat{\lambda}+(1-P_M)(1-\alpha)-P_M\beta}$. Similar to the proof of Lemma 2, the seller's equilibrium decision has the following structure.

(a) $\pi^*(\lambda)=\infty$ for $\lambda\in\left[0,\hat{\lambda}\right)$ and $\pi^*(\lambda)=\hat{\pi}$ for $\lambda\in\left[\hat{\lambda},1\right]$, where $\underline{\hat{\lambda}}<\hat{\lambda}\le 1$ and $\hat{\pi}I^*(\hat{\pi}) = A\left(\hat{\lambda}\right)$.

(b) $\pi^*(\lambda)=\infty$ for $\lambda\in[0,1]$.

Now we focus on the semi pooling equilibrium (that is, part (a) above) in which $\pi^*(\lambda)=\infty$ for $\lambda\in\left[0,\hat{\lambda}\right)$ and $\pi^*(\lambda)=\hat{\pi}$ for $\lambda\in\left[\hat{\lambda},1\right]$, and identify the condition on the equilibrium price $\hat{\pi}$ (similar to Lemma 3). The on-equilibrium belief $m(\cdot|\hat{\pi})$ should satisfy the Bayesian rule: $m(x|\hat{\pi})$ equals $\frac{f(x)}{\bar{F}(\hat{\lambda})}$ if $x\ge\hat{\lambda}$, and 0 otherwise. Thus, we obtain $E[\lambda|\hat{\pi}] = \frac{\int_{\hat{\lambda}}^1 xf(x)\mathrm{d}x}{\bar{F}(\hat{\lambda})}$ and

$$I^*(\hat{\pi}) = \bar{G}\left(\left[\frac{1-P_M}{E[\lambda|\hat{\pi}]}+P_M\right]\hat{\pi}\right) = \bar{G}\left(h\left(\hat{\lambda}\right)\hat{\pi}\right), \tag{24}$$

where $h\left(\hat{\lambda}\right) = \frac{(1-P_M)\bar{F}(\hat{\lambda})+P_M\int_{\hat{\lambda}}^1 xf(x)\mathrm{d}x}{\int_{\hat{\lambda}}^1 xf(x)\mathrm{d}x} \ge 1$. To identify the condition on the equilibrium price $\hat{\pi}$, we next derive the upper bound of $\hat{\pi}I^*(\hat{\pi})$. For simplicity, we make a linear variable substitution: let $\mathring{\pi} = h\left(\hat{\lambda}\right)\hat{\pi}$. Then, $\hat{\pi}I^*(\hat{\pi}) = \frac{1}{h(\hat{\lambda})}\chi(\mathring{\pi})$, where $\chi(\mathring{\pi}) \triangleq \mathring{\pi}\bar{G}(\mathring{\pi})$ and $\mathring{\pi}\in\left(0,h\left(\hat{\lambda}\right)\overline{\mu}_B\right]$.

(a) For $\mathring{\pi}<\underline{\mu}_B$, $\chi(\mathring{\pi})=\mathring{\pi}$ is increasing in $\mathring{\pi}$.

(b) For $\mathring{\pi}>\overline{\mu}_B$, $\chi(\mathring{\pi})=0$.

(c) For $\underline{\mu}_B\le\mathring{\pi}\le\overline{\mu}_B$, the first-order derivative of $\chi(\mathring{\pi})$ is $\frac{\mathrm{d}\chi(\mathring{\pi})}{\mathrm{d}\mathring{\pi}} = \bar{G}(\mathring{\pi})\left[1-\mathring{\pi}\frac{g(\mathring{\pi})}{\bar{G}(\mathring{\pi})}\right]$, which changes its sign at most once on the interval $\left[\underline{\mu}_B,\overline{\mu}_B\right]$ because $\mathring{\pi}\frac{g(\mathring{\pi})}{\bar{G}(\mathring{\pi})}$ increases with $\mathring{\pi}$. If $1-\mathring{\pi}\frac{g(\mathring{\pi})}{\bar{G}(\mathring{\pi})}<0$ for all $\mathring{\pi}\in\left[\underline{\mu}_B,\overline{\mu}_B\right]$, then $\chi(\mathring{\pi})$ decreases with $\mathring{\pi}$ in $\left[\underline{\mu}_B,\overline{\mu}_B\right]$ and $\chi(\mathring{\pi})$ attains its maximum at $\underline{\mu}_B$. Otherwise, $\chi(\mathring{\pi})$ is a unimodal function of $\mathring{\pi}$ in $\left[\underline{\mu}_B,\overline{\mu}_B\right]$ and the optimal $\mathring{\pi}$ satisfies the first-order condition: $\mathring{\pi}\frac{g(\mathring{\pi})}{\bar{G}(\mathring{\pi})}=1$.

Combining the above analysis, $\chi(\mathring{\pi})$ attains its maximum at $\mathring{\pi}=\mu_B^0 \triangleq \inf\left\{y\in[\underline{\mu}_B,\overline{\mu}_B] \,\middle|\, \frac{yg(y)}{\bar{G}(y)}\ge 1\right\}$. Changing back into the original variable $\hat{\pi}$, then $\hat{\pi}I^*(\hat{\pi})$ attains its maximum at $\hat{\pi}=\hat{\Pi}\left(\hat{\lambda}\right)\triangleq\mu_B^0/h\left(\hat{\lambda}\right)$. Note that $I^*\left(\hat{\Pi}\left(\hat{\lambda}\right)\right) = I^*\left(\mu_B^0/h\left(\hat{\lambda}\right)\right) = \bar{G}\left(\mu_B^0\right)$ according to (24). Thus, in the semi pooling equilibrium with $\hat{\lambda}$, the equilibrium price $\hat{\pi}$ satisfies $\hat{\pi}I^*(\hat{\pi})\le\hat{\Pi}\left(\hat{\lambda}\right)\bar{G}\left(\mu_B^0\right)$, where $I^*(\hat{\pi}) = \bar{G}\left(h\left(\hat{\lambda}\right)\hat{\pi}\right)$.

In summary, for a semi pooling equilibrium in which $\pi^*(\lambda)=\infty$ for $\lambda\in\left[0,\hat{\lambda}\right)$ and $\pi^*(\lambda)=\hat{\pi}$ for $\lambda\in\left[\hat{\lambda},1\right]$, we obtain the following *necessary condition*: $\hat{\pi}$ and $\hat{\lambda}$ satisfy $\underline{\hat{\lambda}}<\hat{\lambda}\le 1$, $\hat{\pi}I^*(\hat{\pi}) = A\left(\hat{\lambda}\right)$, and $\hat{\pi}I^*(\hat{\pi})\le\hat{\Pi}\left(\hat{\lambda}\right)\bar{G}\left(\mu_B^0\right)$, where $I^*(\hat{\pi}) = \bar{G}\left(h\left(\hat{\lambda}\right)\hat{\pi}\right)$.
**Step 2: Find all the equilibria within the reduced space.**

We can prove that any pair of $\hat{\pi}$ and $\hat{\lambda}$ satisfying the above condition can be supported as a semi pooling equilibrium by specifying off-equilibrium beliefs appropriately (similar to the proof of Lemma C.1). Specifically, to guarantee that $\hat{\pi}$ is the seller's equilibrium price, we need $\pi I^*(\pi)\le\hat{\pi}I^*(\hat{\pi})$ for any $\pi\in(0,\overline{\mu}_B]$, where $I^*(\pi) = \bar{G}\left(\left[\frac{1-P_M}{E[\lambda|\pi]}+P_M\right]\pi\right)$ and $I^*(\hat{\pi}) = \bar{G}\left(h\left(\hat{\lambda}\right)\hat{\pi}\right)$. Thus, for the semi pooling equilibrium in which $\pi^*(\lambda)=\infty$ for $\lambda\in\left[0,\hat{\lambda}\right)$ and $\pi^*(\lambda)=\hat{\pi}$ for $\lambda\in\left[\hat{\lambda},1\right]$, we specify the following off-equilibrium beliefs. For $\pi\ne\hat{\pi}$ and $\pi\bar{G}(\pi)\le\hat{\pi}I^*(\hat{\pi})$, the off-equilibrium belief $m^*(x|\pi)$ can be arbitrary. However, for $\pi \neq \hat{\pi}$ and $\pi\bar{G}(\pi) > \hat{\pi}I^*(\hat{\pi})$, the off-equilibrium belief $m^*(x|\pi)$ should satisfy the following condition: $\pi I^*(\pi) \leq \hat{\pi}I^*(\hat{\pi})$, which is equivalent to $E(\lambda|\pi) \leq \frac{(1-P_M)\pi}{\bar{G}^{-1}\left(\hat{\pi}\bar{G}\left(h\left(\hat{\lambda}\right)\hat{\pi}\right)/\pi\right)-P_M\pi}$.

**Step 3: Refine the equilibria by Pareto dominance.**

Similar to the proof of Theorem 1, we can prove that for $\beta > \frac{1-P_M}{P_M}(1-\alpha)$ and $\alpha \leq 1 - \frac{\mu_S}{\mu_B^0\bar{G}(\mu_B^0)}$, there exists a unique equilibrium that Pareto dominates other equilibria from the seller's point of view. In this equilibrium, $\pi^*(\lambda) = \infty$ for $\lambda < \hat{\lambda}^*$ and $\pi^*(\lambda) = \hat{\pi}^* = \hat{\Pi}\left(\hat{\lambda}^*\right)$ for $\lambda \geq \hat{\lambda}^*$, where $\hat{\lambda}^*$ is uniquely determined by $A\left(\hat{\lambda}^*\right) = \hat{\Pi}\left(\hat{\lambda}^*\right)\bar{G}(\mu_B^0) \iff \frac{\hat{\lambda}^*\mu_S}{P_M(1-\alpha+\beta)\hat{\lambda}^*+(1-P_M)(1-\alpha)-P_M\beta} = \hat{\Pi}\left(\hat{\lambda}^*\right)\bar{G}\left(\mu_B^0\right)$. The off-equilibrium beliefs for this equilibrium are specified in Step 2. Moreover, similar to the proof of Theorem 1, we can derive the set of $(\alpha, \beta)$ that induces the equilibrium with $\hat{\lambda}^*$. For $\hat{\lambda}^* \in (0,1)$, $\Omega\left(\hat{\lambda}^*\right) = \left\{(\alpha,\beta)\middle|\alpha < 1 - \frac{\mu_S}{\hat{\Pi}\left(\hat{\lambda}^*\right)\bar{G}\left(\mu_B^0\right)}, \beta = \frac{\left(1-P_M+P_M\hat{\lambda}^*\right)(1-\alpha)\hat{\Pi}\left(\hat{\lambda}^*\right)\bar{G}\left(\mu_B^0\right)-\hat{\lambda}^*\mu_S}{\left(1-\hat{\lambda}^*\right)P_M\hat{\Pi}\left(\hat{\lambda}^*\right)\bar{G}\left(\mu_B^0\right)}\right\}$, which is a ray on $\alpha$-$\beta$ plane because $\beta$ is linear in $\alpha$. For $\hat{\lambda}^* = 1$, $\Omega\left(\hat{\lambda}^*\right) = \left\{(\alpha,\beta)\middle|\alpha = 1 - \frac{\mu_S}{\mu_B^0\bar{G}(\mu_B^0)}, \beta > \frac{1-P_M}{P_M}(1-\alpha)\right\}$.

2. Denote $V\left(\hat{\lambda}^*\right) = \left\{(\alpha,\beta)\middle|\alpha = 1 - \frac{\mu_S}{\hat{\Pi}\left(\hat{\lambda}^*\right)\bar{G}\left(\mu_B^0\right)}, \beta = \frac{(1-P_M)\mu_S}{P_M\hat{\Pi}\left(\hat{\lambda}^*\right)\bar{G}\left(\mu_B^0\right)}\right\}$, which is the end point of the ray $\Omega\left(\hat{\lambda}^*\right)$. Similar to the base model, we can prove that for a given threshold $\hat{\lambda}^*$, the contract $(\alpha,\beta) = V\left(\hat{\lambda}^*\right)$ is optimal for the platform. Under the contract $V\left(\hat{\lambda}^*\right)$, the seller's expected revenue is equal to his expected reservation price $\lambda\mu_S$, so the platform's expected profit can be written as $R_M\left(\hat{\lambda}^*\right) = \bar{G}\left(\mu_B^0\right)\int_{\hat{\lambda}^*}^1\left[(1-P_M+P_Mx)\hat{\Pi}\left(\hat{\lambda}^*\right) - x\mu_S - (1-x)(1-P_M)P_B\ell - c\right]f(x)\mathrm{d}x = \bar{G}\left(\mu_B^0\right)\int_{\hat{\lambda}^*}^1\left[x(\mu_B^0-\mu_S)\right.$ $\left.-(1-x)(1-P_M)P_B\ell - c\right]f(x)\mathrm{d}x$, which is a unimodal function of $\hat{\lambda}^*$. By the first-order condition, the optimal threshold is $\hat{\lambda}^{**} = \frac{(1-P_M)P_B\ell+c}{(1-P_M)P_B\ell+\mu_B^0-\mu_S}$. □