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ABSTRACT

Carbon emission control becomes a challenge in recent years, and carbon emission trading is an effective way to curb carbon emission. This paper investigates the multi-item production planning problem with carbon cap and trade mechanism, in which a firm uses a common capacity and carbon emission quota to produce multiple products for fulfilling independent stochastic demands, and the firm can buy or sell the right to emit carbon on a trading market of carbon emission. A profit-maximization model is proposed to characterize the optimization problem. The optimal policy of production and carbon trading decisions is analyzed, and an efficient solution method with linear computational complexity is presented for solving the optimal solution. The impacts of carbon price, carbon cap on the shadow price of the common capacity, production decisions, carbon emission and the total profit are investigated. The comparisons of the carbon cap and trade policy and the taxation policy are given to show the effectiveness of the policies. Numerical analyses are presented for illustrating our findings and obtaining some managerial insights and policy implication.

1. Introduction

Research has already shown that global warming has a direct relationship with the emission of carbon and other greenhouse gases. Many countries have attempted to enact legislation or design market-based carbon trading mechanism for controlling carbon emission. In comparison with the command-and-control standards, the carbon cap and trade mechanism is more effective in carbon emission reduction (Stavins, 2008; Hua et al., 2011). In past decades, some carbon emission control mechanisms have been launched, such as Kyoto Protocol in 1997 which aims to establish a carbon cap and trade system on international scale. The European Union Emission Trading System (EU-ETS), which is launched by European Union on January 2005, is a cornerstone of European Union climate policy toward its Kyoto commitment and beyond. The EU-ETS has grown to be the world largest carbon trading market, greatly advancing Chicago Climate Exchange (CCX) and Australia Climate Exchange (ACX), etc. As the World Bank report “State and Trends of the Carbon Market Report 2011” shows, the carbon trading volume of EU-ETS carbon allowances reaches 119.8 billion dollars in 2010 and will continue to increase.

Not only applied in the carbon footprint cutting, the cap and trade mechanism is also used to control the emission of other pollutants, e.g., industrial waste and sewage. The most notable case is the SO_2 trading system under Acid Rain Program which is launched by the U.S. Government to curb the emission of sulfur dioxide. This program covers 263 power stations in America at the first stage and expends to all of which the capacity is larger than 25 MW. According to the report from U.S. Environmental Protection Agency (2007),[1] this program curbs 40% of the sulfur emission from 1990 to 2006 with a 37% increase of total electricity generated.

Apart from the emission policy regulation, the customer awareness about climate change has been another factor to drive firms to be greener. According to a U.S. customer survey (Klassen and McLaughlin, 1996), almost 85.7% of the investigated have strong willingness to pay more for products that are environment friendly; shareholders reflect a similar opinion, recommending that the top priority for corporate expenditures be cleaning up the environment. The enterprises in China are feeling significant pressure to introduce green supply chain principles and practices because they keep encountering green barriers when exporting commodity (Zhu et al., 2005). Zhu et al. (2007) investigated 89 automotive enterprises in China, and they found that these enterprises experienced both government regulatory and market pressure to adopt green supply chain practices.

To respond to government regulatory on carbon emission and environment concerns from customers, many firms have tried to improve their products' design or adopt more energy efficient equipments, facilities and carbon-reducing technologies. As a

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[1] The report: The Experience with Emissions Control Policies in the United States, U.S. Environmental Protection Agency, Office of Atmospheric Programs, 2007, p42. http://www.epa.gov/airmarkets/international/china/JES_USexperience.pdf.

result, green supply chain management, which defined as integrating environment concern into supply chain management, has caught attention in academic area. Many research on green supply chain investigated closed-loop supply chain or sustainable usage of material, such as Srivastava (2007), Chaabane et al. (2012) and Shi et al. (2011). Other works have been done to design efficient reverse logistics networks, e.g., Lieckens and Vandaele (2007) and Lee et al. (2010).

Although these actions are very valuable to curb carbon emission, they often need long lead time and costly investments. In fact, some carbon emission can be reduced by incorporating carbon emission concern into operational decision-making, which requires much less or no implementing cost. Up to now, less work has been done to incorporate carbon emission concerns into supply chain management, both in scientific research and in industry practice. Some works have been done for studying the measurement methods of carbon emission in supply chains, for examples see Cholette and Venkat (2009) and Sundarakani et al. (2010). Some works have attempted to study operations decisions in production planning and transportation management with carbon emission regulations. Penkuhn et al. (1997) incorporated carbon emission taxes into a joint production planning problem. Letmathe and Balakrishnan (2005) studied the product portfolio selection and production problems with deterministic demands in the presence of several different types of environmental constraints and production constraints. Kim et al. (2009) studied a tradeoff between carbon emission and transportation costs via multi-objective optimization. Hoen et al. (2010) investigated the effects of emission cost and emission constraint on the transport mode selection decision. Cachon (2011) studied the impact of carbon emission cost on the design of supply chain.

Recently, several papers incorporated carbon emission concern into some classical production and inventory management models. Benjaafar et al. (2012) illustrated how some different carbon emission concerns could be integrated into operational decision-making in single-firm and multi-firm lot-size problems, and they provided a series of insights to highlight the impact of operational decisions on carbon emissions by analyzing numerical examples. Hua et al. (2011) studied the optimal order quantity under the carbon emission trading mechanism by integrating carbon emission concern into the classical economic order quantity model, and they derived some interesting observations. Li and Gu (2012) added the cost of environmental protection to the well-known Arrow–Karlin dynamic production–inventory model, in which the firm could either sell the emission permits in market or deposit for future use. They compared the optimal production–inventory strategies with and without emission permits, and investigated the effect of tradable emission permits with banking on the production–inventory strategy. Song and Leng (2012) studied the single-period production problem under carbon emission policies and obtained the optimum production quantity.

In practice, firms face the challenge of managing multi-product production system in the presence of carbon emission control. For instance, Walkers, the UK's largest snack foods manufacturer, now works with Carbon Trust, an independent organization aims to reducing carbon footprint in business, to manage the product portfolios and to reduce carbon emission. So does Trinity Mirror, the UK's largest newspaper publisher with some 240 local and regional newspapers and five national newspapers (Carbon Trust, 2006). However, the impact of carbon emission control on multi-item production planning is seldom investigated in literature.

In this paper, we study the multi-item production planning problem with carbon cap and trade mechanism, in which a common capacity and carbon emission quota are shared to produce multiple products for fulfilling independent stochastic demands. A certain amount of carbon emission (carbon cap) is allocated to the firm by an external regulatory body, and the firm can buy or sell carbon credit on a trading market of carbon emission, e.g., European Climate Exchange (ECX) and CCX. The carbon price is set by the trading market, and it is an exogenous variable to decisions made by the firm. The firm has to make the decisions on production quantities and the carbon trading quantity for maximizing the expected profit. We present a profit-maximization model to characterize the firm's decisions in the multi-item production planning problem with carbon cap and trade mechanism. We derive the optimal policy of production and carbon trading decisions, and give an efficient solution method solving the optimal solution to the studied problem. We obtain some managerial insights by theoretically and numerically analyzing the impacts of carbon price, carbon cap on the system performance.

In addition, we compare the impacts of the carbon cap and trade policy and the taxation policy on the carbon emission and profit of the regulated firm. Under the taxation policy, the regulated firm pays a carbon tax based on the amount of carbon footprint emitted. The comparison would provide some instructions to the implementation of carbon emission control policy. Although the discussion throughout this paper focuses on carbon footprint, the result can easily be applied to control emission of other pollutants.

The rest of the paper is organized as follows. Section 2 describes the problem. In Section 3, the optimal policy and a solution method are presented. The impacts of carbon cap and trade mechanism on the system performance are investigated in Section 4. Section 5 compares the two different emission control policies. Section 6 provides numerical examples to illustrate our results. Section 7 concludes the paper with a few future research directions. All proofs are presented in Appendix.

2. The problem

Product portfolio produced by a firm generally shares some common manufacturing process or resource while incur different carbon footprint for different products. For examples, the three products (Crisps, Quavers and Doritos) from Walkers satisfy similar consumer needs yet the manufacturing processes (e.g., frying and baking processes) are different for each of the three, which incur different amount of carbon emission (Carbon Trust, 2006); Trinity Mirror recognizes that a significant portion of the carbon emissions comes from its manufacturing processes. It uses two types of raw materials (50% recovered fiber and 100% recovered fiber) on paper manufacturing, and the energies consumed in manufacturing newspapers with the two kinds of fibers are 0.6 and 0.44 kW h per paper sold, respectively (Carbon Trust, 2006). In these examples, the snack foods from Walkers and newspapers from Trinity Mirror are typical newsvendor-type products.

We consider the multi-item production system that uses a common capacity and carbon emission quota to produce n different newsvendor-type products, and model the problem as an extended multi-item newsvendor problem with carbon emission control. Let $i=1,\ldots,n$ be the index for all products. The production cost, selling price, and salvage value for one unit of product i is c_i, p_i and s_i, respectively. To avoid the trivial case, we assume $p_i > c_i > s_i$. Random demand for product i is D_i, and $f_i(x)$, $F_i(x)$ and $F_i^{-1}(x)$ are the probability density (positive) function, cumulative distribution and inverse distribution functions, respectively. It is common to assume that all demands are nonnegative, so we assume that $F_i(x)=0$ for all $x \le 0$, $i=1,\ldots,n$. The total common capacity is t, and τ_i unit of common capacity is needed for producing one unit of product i. The carbon cap is a, and the carbon price is c_e.

In this problem, the firm must decide the optimal production quantities $(x_1,\ldots,x_n)$ and the corresponding carbon trading quantity q so as to maximize the total expected profit. Let e_i be the carbon emission for producing one unit of product i, then the total carbon emission will be $\sum_{i=1}^{n} e_i x_i$, and the carbon trading quantity q satisfies $\sum_{i=1}^{n} e_i x_i = a+q$. The carbon trading quantity $q>0$ implies that the firm will buy q unit of carbon credit from the carbon trading market, and $q<0$ means that the firm will sell $-q$ unit of carbon credit on the carbon trading market. The firm will not involve in the carbon trading market if the carbon trading quantity $q=0$.

Now we are ready to present the optimization model for the multi-item production planning problem with carbon cap and trade mechanism. Let $(\cdot)^+ = \max\{\cdot,0\}$ and $x=(x_1,\ldots,x_n)'$, denote by $E(\cdot)$ the expectation operator, then the mathematical model for maximizing the total expected profit is given as follows (problem P):

$$\text{Max } \pi(x,q) = \sum_{i=1}^{n} E[p_i \min(x_i,D_i) + s_i(x_i-D_i)^+ - c_i x_i] - c_e q, \tag{1}$$

subject to

$$\sum_{i=1}^{n} \tau_i x_i \le t, \tag{2}$$

$$\sum_{i=1}^{n} e_i x_i = a+q, \tag{3}$$

$$x_i \ge 0,\ i=1,\ldots,n \tag{4}$$

In problem P, $p_i \min(x_i,D_i)$ is the revenue from selling product $i=1,\ldots,n$, $s_i(x_i-D_i)^+$ is the salvage value of the leftover product $i=1,\ldots,n$, $c_i x_i$ is the production cost of product $i=1,\ldots,n$, $c_e q$ is the cost or revenue from the carbon trading market. Eq. (2) indicates the common capacity constraint, Eq. (3) describes the carbon constraint, and Eq. (4) gives the non-negativity constraints on production quantities. For ease of exposition, we re-index i such that $e_1/\tau_1 \le \cdots \le e_n/\tau_n$

From Eq. (3), we have $q = \sum_{i=1}^{n} e_i x_i - a$. Substituting it into Eq. (1), we have

$$\begin{aligned}\pi(x,q) &= \sum_{i=1}^{n} E[p_i \min(x_i,D_i) + s_i(x_i-D_i)^+ - c_i x_i] - c_e\left(\sum_{i=1}^{n} e_i x_i - a\right)\\ &= \sum_{i=1}^{n} E[p_i \min(x_i,D_i) + s_i(x_i-D_i)^+ - (c_i + c_e e_i)x_i] + c_e a\\ &= \sum_{i=1}^{n} [(p_i - c_i - c_e e_i)x_i - (p_i - s_i)\int_0^{x_i} F_i(u_i)du_i] + c_e a \end{aligned} \tag{5}$$

Since $c_e a$ in Eq. (5) is a constant value, it can be removed from the objective function of problem P. Using the above transformation, problem P becomes a parameter-adjusted multi-item newsvendor model, where $c_i + c_e e_i$ is the adjusted unit production cost by adding the cost of using carbon resource for producing one unit of product i to the original unit production cost of product i.

3. The optimal policy and solution method

In this section, we first investigate the optimal policy for production and carbon trading decisions, then we propose a solution method for solving problem P.

3.1. The optimal policy

Problem P is an extended multi-item newsvendor problem. Multi-product constrained newsvendor problem is a classical inventory management problem, and various versions of multi-item newsvendor problem have been studied in recent years. Some works have focused on analyzing the structural properties of the problems for developing efficient solution methods, such as Vairaktarakis (2000), Abdel-Malek and Montanari (2005) and Zhang et al. (2009). Others investigated various extended multi-item newsvendor problems in complex settings, such as outsourcing (Zhang and Du, 2010), portfolio contracts (Zhang and Hua, 2010), and mixed demands (Zhang, 2011).

Since problem P is a parameter-adjusted multi-item newsvendor model, it is a concave problem (Zhang et al., 2009), and Karush–Kuhn–Tucker (KKT) conditions can characterize its optimality conditions. Let λ be the dual variable corresponding to the constraint in Eq. (2), and let w_i, $i=1,\ldots,n$ be the dual variables corresponding to the constraints $x_i \ge 0$, $i=1,\ldots,n$ in Eq. (4). Then, x_i, $i=1,\ldots,n$, is the optimal solution to problem P if and only if there exists non-negative dual variables λ, w_i, $i=1,\ldots,n$, such that

$$(p_i - c_i - c_e e_i) - (p_i - s_i)F_i(x_i) - \lambda\tau_i + w_i = 0,\ i=1,\ldots,n, \tag{6}$$

$$\sum_{i=1}^{n} w_i x_i = 0, \tag{7}$$

$$\lambda\left(t - \sum_{i=1}^{n} \tau_i x_i\right) = 0. \tag{8}$$

$\lambda \ge 0$ in Eq. (8) represents the shadow price of the common capacity.

We let x^* be the optimal production decision, and λ^* be the optimal shadow price. We denote by $\tilde{x}$ the optimal solution to the unconstrained problem, then $\tilde{x}$ can be solved by setting $\frac{\partial \pi(x,q)}{\partial x_i} = (p_i - c_i - c_e e_i) - (p_i - s_i)F(x_i) = 0$. Thus, we have $\tilde{x}_i = F_i^{-1}\left(\frac{p_i - c_i - c_e e_i}{p_i - s_i}\right)$, $i=1,\ldots,n$. We let $x(\lambda)$ be an optimal solution of Eqs. (6) and (7) for any given $\lambda \ge 0$, then the optimal policy for problem P can be characterized by the following proposition.

Proposition 1.

(a) *For any given* $\lambda \ge 0$, $x(\lambda)$ *satisfies Eqs. (6) and (7) if and only if*

$$x_i(\lambda) = F_i^{-1}\left(\left(\frac{p_i - c_i - c_e e_i - \lambda\tau_i}{p_i - s_i}\right)^+\right),\ i=1,\ldots,n,, \tag{9}$$

(b) *If* $(x(\lambda),\lambda)$ *satisfies* $\lambda = 0$ *or* $\sum_{i=1}^{n} \tau_i x_i(\lambda) = t$, *then we have* $x^* = x(\lambda)$.

3.2. The solution method

Since problem P has similar structure as the classical multi-item newsvendor problem, the idea for solving the classical multi-item newsvendor problem developed by Zhang et al. (2009) and Zhang (2012) can be extended for solving problem P. $\sum_{i=1}^{n} \tau_i x_i(\lambda)$ is decreasing in λ, a binary search procedure can be used for solving λ^*, and then the optimal solution can be determined by using Eq. (9). Before giving the solution method, we first find an upper bound of λ^*, which will be applied in the binary search procedure. If $\lambda^* \ge (\max_{i=1,\ldots,n}\{(p_i - c_i - c_e e_i)/\tau_i\})^+$, according to Eq. (9), we know $x_i(\lambda^*) = 0$, $i=1,\ldots,n$, and hence

Step 1: Let $\lambda^L = 0, \lambda^U = (\max_{i=1,\ldots,n}\{(p_i - c_i - c_e e_i)/\tau_i\})^+$;

Step 2: Let $\lambda = (\lambda^L + \lambda^U)/2$;

Step 3: If $\lambda = 0$, then let $x_i^* = F_i^{-1}\left(\left(\frac{p_i - c_i - c_e e_i}{p_i - s_i}\right)^+\right)$, $i = 1,\ldots,n$, stop;

Step 4: Let $x_i(\lambda) = F_i^{-1}\left(\left(\frac{p_i - c_i - c_e e_i - \lambda\tau_i}{p_i - s_i}\right)^+\right)$, $i = 1,\ldots,n$, ;

Step 5: If $\sum_{i=1}^{n} \tau_i x_i(\lambda) > t$, then let $\lambda^L = \lambda$, go to *Step* 2;

If $\sum_{i=1}^{n} \tau_i x_i(\lambda) < t$, then let $\lambda^U = \lambda$, go to *Step* 2;

Step 7: Let $x_i^* = x_i(\lambda)$, stop.

Fig. 1. The solution method for solving problem *P*.

$\sum_{i=1}^{n} \tau_i x_i(\lambda^*) = 0$, which violates the optimality condition $\sum_{i=1}^{n} \tau_i x_i(\lambda^*) = t$ proved in Proposition 1(b). Thus, we have $\lambda^* \in [0, (\max_{i=1,\ldots,n}\{(p_i - c_i - c_e e_i)/\tau_i\})^+)$. Main steps of the solution procedure are summarized in Fig. 1.

The solution method in Fig. 1 terminates if either $\lambda = 0$ or $\sum_{i=1}^{n} \tau_i x_i(\lambda) = t$. If the constraint $\sum_{i=1}^{n} \tau_i x_i \le t$ is inactive, then the iterating process will lead to $\lambda^L = \lambda^U = 0$, and the solution procedure will go to *Step* 3, and we have $x_i^* = F_i^{-1}\left(\left(\frac{p_i - c_i - c_e e_i}{p_i - s_i}\right)^+\right)$. *Step* 4 solves $x_i(\lambda)$ from Eq. (9) for any given $\lambda > 0$. Based on the fact that $\sum_{i=1}^{n} \tau_i x_i(\lambda)$ is decreasing in λ, *Step* 5 chooses the half-interval for λ in the binary search procedure by comparing $\sum_{i=1}^{n} \tau_i x_i(\lambda)$ and t with an implicit stopping condition $\sum_{i=1}^{n} \tau_i x_i(\lambda) = t$ in *Step* 6.

Since the outer loop of the binary search procedure has constant complexity $O(\log_2(1/\varepsilon))$, where ε is the error target for the binary search, and the computational complexity of Steps 3–5 is $O(n)$, the computational complexity of the proposed solution method is $O((\log_2(1/\varepsilon))n)$. Thus proposed solution method has linear computational complexity, and it is very efficient for solving large-scale instances of problem *P*.

Notice that the problem without carbon consideration is a special case of problem *P* with $c_e = 0$, which can be solved by directly applying the proposed solution method.

4. The impact of carbon cap and trade mechanism

In this section, we analyze the impacts of carbon price and carbon cap on the shadow price of the common capacity, production decisions, carbon emission and total profit.

We define $\rho_i = \frac{1}{f_i(x_i(\lambda^*))(p_i - s_i)} > 0$, $i = 1,\ldots,n$, and we denote by $I(\lambda) = \{i | \lambda < (p_i - c_i - c_e e_i)/\tau_i, i = 1,\ldots,n\}$ for any given $\lambda \ge 0$. According to Eq. (9), we know that $I(\lambda)$ is the set of products with non-negative production quantities for the given λ. By analyzing the relationship between the shadow price and the carbon price, we have the following proposition.

Proposition 2. *The optimal shadow price λ^* is decreasing in the carbon price c_e.*

Proposition 2 implies that the value of the common capacity decreases as the carbon price increases. This is true because that the increasing of the carbon price will incur higher actual production cost and hence the firm tends to produce less. As a result, less production will lead to low value of common capacity.

From Proposition 2, we can further obtain some clear results for two special cases: (1) the ratios of carbon emission to common capacity consumption are equal, i.e., $e_1/\tau_1 = \cdots = e_n/\tau_n$, and (2) demands for all products are uniformly distributed. Let $\Theta(c_e)$ be the set of c_e such that $c_e = (p_i - c_i - \lambda^*\tau_i)/e_i$ for $\lambda^* > 0$, $c_e = (p_i - c_i\tau_i)/e_i$, $i = 1,\ldots,n$, and $\lambda^* = 0$. Then we have the following corollaries.

Corollary 1. *The optimal shadow price λ^* is linear decreasing in the carbon price c_e if the ratios of carbon emission to common capacity consumption are equal for all products.*

Corollary 2. *If demands for all products are uniformly distributed, then (a) the optimal shadow price λ^* is piecewise linear decreasing in the carbon price c_e , and (b) there are at most $2n+1$ breakpoints on the piecewise linear curve, and the breakpoints must be in the set $\Theta(c_e)$.*

From Proposition 2, we know that the linear relationship between the value of the common capacity and the carbon price does not hold for general case. This is because the optimal shadow price depends on the carbon price and production arrangement, and production arrangement in the general case destroys the linear relationship. In the case of $e_1/\tau_1 = \cdots = e_n/\tau_n$, there is no difference on carbon emission to use the common capacity for producing one unit of different products, and Corollary 1 shows that production arrangement will not affect the relationship between the value of the common capacity and the carbon price, and then the linear relationship holds if $e_1/\tau_1 = \cdots = e_n/\tau_n$. Corollary 2 says that the optimal shadow price curve over the carbon price is a piecewise linear curve, and the breakpoints on this curve can be found in the set $\Theta(c_e)$.

By investigating how the carbon price affects the optimal production decisions, we have the following result.

Proposition 3. *The optimal production quantity x_i^* increases (decreases) as the carbon price c_e increases if $-d\lambda^*/dc_e > e_i/\tau_i$ ($e_i/\tau_i > -d\lambda^*/dc_e$), and it does not change if $e_i/\tau_i = -d\lambda^*/dc_e$.*

Proposition 3 shows that production quantities of low-emission products will be favored over high-emission products in the presence of cap-and-trade. The impact of the carbon price on the optimal production quantity depends on the comparison of e_i/τ_i and $-d\lambda^*/dc_e$. The ratio e_i/τ_i can be viewed as the carbon emission of unit common capacity consumption for producing product *i*. The larger e_i/τ_i is, the higher relative carbon utilization cost of product *i* is. Since λ^* is the value of the common capacity, $-d\lambda^*/dc_e$ is the ratio of the system production value to the carbon trading value from unit carbon price increase, which can be viewed as the relative system carbon utilization value. Proposition 3 implies that the firm will produce more product *i* if the relative system carbon utilization value is larger than the relative carbon utilization cost of product *i* (i.e., $-d\lambda^*/dc_e > e_i/\tau_i$), and that the firm will produce less product *i* if the relative system carbon utilization value is smaller than the relative carbon utilization cost of product *i* (i.e., $e_i/\tau_i > -d\lambda^*/dc_e$). The firm will not change the production of product *i* if the relative system carbon utilization value equals to the relative carbon utilization cost of product *i* (i.e., $e_i/\tau_i = -d\lambda^*/dc_e$). Notice that in the case of $e_1/\tau_1 = \cdots = e_n/\tau_n$, according to Eq. (A.3), we know $-d\lambda^*/dc_e = e_1/\tau_1$. In this situation, Proposition 3 implies that the change of the carbon price will not affect any production arrangement, and the optimal production decisions are the same as the optimal production decisions in the production planning without carbon consideration.

Since the production planning without carbon consideration is equivalent to the case that the carbon price is zero, according to Proposition 3, we can compare the production decisions in the production planning with and without carbon consideration. Let x^0 be the optimal production decision in the production planning without carbon consideration, and λ^0 be the corresponding optimal shadow price. Then based on Proposition 3, we have the following corollary.

Corollary 3. *For* $i=1,...,n$ *, we have (a)* $x_i^* \geq x_i^0$ *if* $(\lambda^0-\lambda^*)/c_e > e_i/\tau_i$ *, (b)* $x_i^* \leq x_i^0$ *if* $(\lambda^0-\lambda^*)/c_e < e_i/\tau_i$ *, and (c)* $x_i^* = x_i^0$ *if* $(\lambda^0-\lambda^*)/c_e = e_i/\tau_i$ *.*

By investigating how the carbon price affects the total carbon emission, we have the following result.

Proposition 4. *The total carbon emission decreases as the carbon price* c_e *increases, and there are no carbon emission if* $c_e \geq \max_{i=1,...,n}\{(p_i-c_i)/e_i\}$ *.*

Proposition 4 illustrates that the carbon price is a good lever for controlling the carbon emission in the studied system. When the carbon price increases, the actual production costs of products will change. According to Propositions 3 and 4, we know that the firm will adjust its production arrangement by producing more carbon efficient products (with small e_i/τ_i) and producing less carbon inefficient products (with large e_i/τ_i) in order to utilize the carbon resource more efficiently. As a result, the total carbon emission can be reduced by increasing the carbon price. When $c_e \geq \max_{i=1,...,n}\{(p_i-c_i)/e_i\}$, the actual production cost of the most profitable product outweighs the selling price, the firm would rather sell all its carbon quota on the market.

Let q^* be the optimal carbon trading quantity, by analyzing the expected profit function, we have the following proposition.

Proposition 5. *The expected profit* $\pi(x^*,q^*)$ *is increasing (decreasing) in the carbon price* c_e *if the carbon trading quantity* $q^*<0$ ($q^*>0$).

Proposition 5 implies that the increase of the carbon price will generate more profit in the case that the firm has carbon quota surplus for sale, and that the increase of the carbon price will reduce the firm's profit when the firm needs to buy carbon credit from the market.

From Proposition 4, we know that the total carbon emission increases its the carbon price decrease, then the total carbon emission reaches its extreme value $\sum_{i=1}^n e_i x_i^0$ when $c_e=0$. According to Propositions 4 and 5, we have the following properties for the expected profit by analyzing the different intervals for the carbon cap: (a) $a=0$, (b) $a \geq \sum_{i=1}^n e_i x_i^0$ and (c) $0<a<\sum_{i=1}^n e_i x_i^0$.

Corollary 4.

(a) *The expected profit* $\pi(x^*,q^*)$ *decreases with the carbon price* c_e *if* $a=0$ *.*
(b) *The expected profit* $\pi(x^*,q^*)$ *increases with the carbon price* c_e *if* $a \geq \sum_{i=1}^n e_i x_i^0$ *.*
(c) *The expected profit* $\pi(x^*,q^*)$ *initially decreases and then increases with the carbon price* c_e *if* $0<a<\sum_{i=1}^n e_i x_i^0$ *, and the turning point is* $q^*=0$ *.*

In the case of $a=0$, any production in the system needs buying some carbon credit from the market. It is obvious that the increase of the carbon price will reduce the firm's profit. In the case of $a \geq \sum_{i=1}^n e_i x_i^0$, the firm will have carbon quota surplus for any $c_e>0$. As the carbon price increases, the total carbon emission will decrease, and the firm can sell more carbon quota surplus at the higher carbon price. As a result, the firm can obtain more profit. In the case of $0<a<\sum_{i=1}^n e_i x_i^0$, the expected profit decreases with carbon price first when $\sum_{i=1}^n e_i x_i^* > a$ and increases with carbon price after $\sum_{i=1}^n e_i x_i^* < a$, the expected profit reaches the bottom at $\sum_{i=1}^n e_i x_i^* = a$.

In the following, we investigate how the carbon cap affects the system performance. It is obvious that the carbon cap has no influence on the optimal shadow price of the common capacity, the optimal production quantities, and the total carbon emission. The results in Proposition 5 and Corollary 4 inspire an idea to find the combinations of the carbon price and carbon cap for maintaining the expected profit in the system without carbon emission consideration. Let $\pi_{a=0}(x^*,q^*)$ be the expected profit for the case of $a=0$. By investigating the effect of the combination of the carbon cap and the carbon price on the expected profit, we have the following proposition.

Proposition 6. *For any given carbon price* $c_e>0$ *, there exists a threshold* $\hat{a}=(\pi(x^0)-\pi_{a=0}(x^*,q^*))/c_e$ *for the carbon cap such that* $\pi(x^*,q^*)=\pi(x^0)$ *, and* $\pi(x^*,q^*)>\pi(x^0)$ *for* $a>\hat{a}$ *,* $\pi(x^*,q^*)<\pi(x^0)$ *for* $a<\hat{a}$ *.*

Proposition 6 implies that there is an indifference curve of combinations of the carbon cap and carbon price with the expected profit $\pi(x^0)$, which is defined by $\hat{a}c_e=\pi(x^0)-\pi_{a=0}(x^*,q^*)$. The points on the indifference curve indicate that the carbon emission consideration does not affect the expected profit. The points below the indifference curve imply that the allocated carbon cap is not enough to maintain the expected profit in the system without carbon consideration. The points above the indifference curve mean that the allocated carbon cap can generate more profit than the expected profit in the system without carbon consideration.

This indifference curve has an important implication for policy makers or government regulators to design some suitable carbon trading mechanisms to curb carbon emission and simultaneously make firms be profitable. If the carbon trading policy makers want to curb carbon emission without hurting the firm's profit, the combinations of the carbon price and carbon cap should be chose from the area above or on the indifference curve, i.e., the carbon cap allocated to the firm should not be less than the threshold carbon cap value for any carbon price.

5. Comparison with the taxation policy

The multi-item production planning under the taxation policy is a special case of the multi-item production planning with carbon cap and trade mechanism. Denote by P^t the multi-item production problem under the taxation policy, and let c_e^t indicate the unit emission tax rate under the taxation policy, then problem P^t can modeled as problem P given in Eqs. (1)–(4) with $a=0$ and $c_e^t = c_e$, where $c_e^t q$ in problem P^t is the carbon tax for the total carbon emission. Since problem P^t is equivalent to problem P with $a=0$, the results for cap and trade system described in Section 4 also hold for the taxation system. We can compare the cap and trade policy with the taxation policy in the multi-item production planning in the following propositions.

Proposition 7. *The cap and trade policy and taxation policy are indifferent in curbing carbon emission if the carbon price and the carbon tax rate are same.*

Proposition 7 implies that the same effect of carbon emission control can be achieved via the cap and trade policy and the taxation policy by setting the same carbon price and carbon tax rate.

Denote by π the profit under the cap and trade policy, and π^t the profit under the taxation policy, we have the following results for the firm's profit under the two policies.

Proposition 8.

(a) $\pi > \pi^t$ *if* $c_e^t = c_e$;
(b) $\pi(x^0) > \pi^t$;
(c) $\pi > \pi^t$ *if* $a \geq \sum_{i=1}^{n} e_i x_i^0$;
(d) In the case of $a < \sum_{i=1}^{n} e_i x_i^0$, if $\pi > \pi(x^0)$ *then* $\pi > \pi^t$, *if* $\pi < \pi(x^0)$, *there exists a threshold* $\hat{c}_e^t < c_e$ *such that* $\pi^t = \pi$, *and* $\pi^t < \pi$ *for* $c_e^t > \hat{c}_e^t$, $\pi^t > \pi$ *for* $c_e^t < \hat{c}_e^t$.

Proposition 8(a) indicates that the firm will get more profit under the cap and trade policy if the carbon price and the carbon tax rate are same. Proposition 8(b) means the carbon tax will reduce the profit of the regulated firm. Proposition 8(c) and Proposition 8(d) imply that the profit under the taxation policy can never be larger than profit under the cap and trade if $\pi > \pi(x^0)$. Proposition 8(d) also indicates that the firm can achieve the same profit under the two different policies by adjusting the carbon tax rate or the carbon price if the profit with carbon consideration is smaller than that without carbon consideration.

6. Numerical study

In this section, numerical results are presented to illustrate our theoretical results and obtain some managerial insights. Since it is difficult to obtain the parameters set from the practical system, we design the parameters sets for illustrating our theoretical results.

We set $n=3$ in the numerical study, and use normal demand distribution examples to show our main results. In the base case, we set the carbon quota $a=800$, the common capacity $t=500$, and $c_e=1$. Other problem parameters and information are listed in Table 1, in which μ_i, σ_i, $i=1,\ldots,n$, are parameters of mean and standard deviation of normal demands. We also test other parameters set in the numerical study, we found that any meaningful parameters set gives similar results. Although we set different e_i for different products in the example, the analysis results are similar if all e_i are equal. That is to say, we do not exclude the case with the same carbon emissions for producing one unit of all products.

Table 1
Parameters and solution for the normal demand example.

Product	p_i	c_i	s_i	μ_i	σ_i	τ_i	e_i	x_i^0	x_i^*
1	60	40	10	70	20	2	2	56.51	61.20
2	80	55	15	120	40	3	7	87.96	92.05
3	50	35	10	150	30	4	12	30.78	25.37

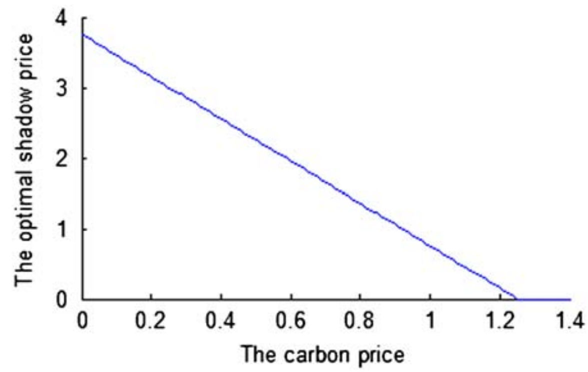

Fig. 2. The curve of the optimal shadow price.

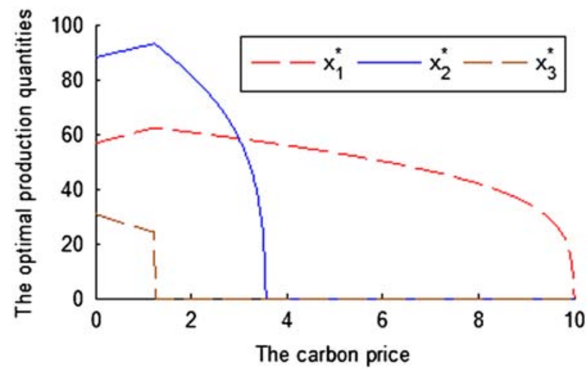

Fig. 3. The curves of the optimal production quantities.

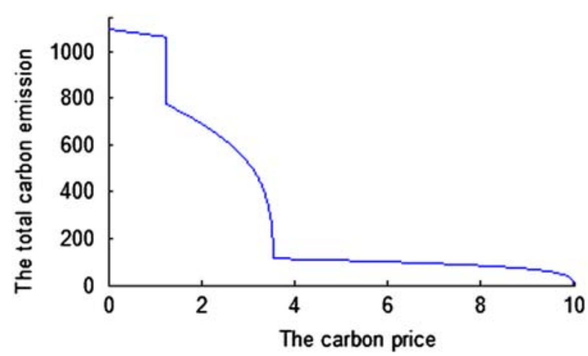

Fig. 4. The curve of the total carbon emission.

To show the impact of the carbon price on the system performance, we investigate more cases by varying the value of the carbon price in the base case, and keep other parameters unchanged. The curves of optimal shadow price, the optimal production quantities and the total carbon emission with different carbon price are plotted in Figs. 2–4, respectively.

Fig. 2 verifies the conclusion that the optimal shadow price decreases as the carbon price increases, which is proved in Proposition 2. From Fig. 2, we also know the optimal shadow price becomes zero at $c_e=1.25$. According to Fig. 3, we observe

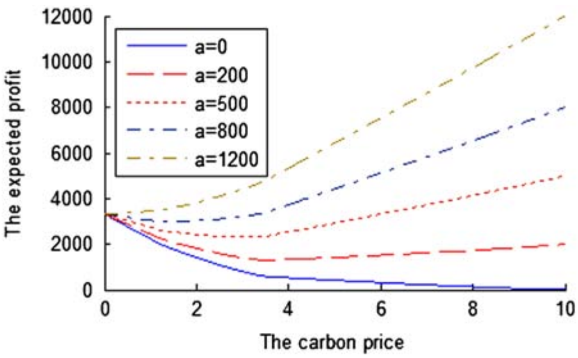

Fig. 5. The curves of the expected profit for different carbon caps.

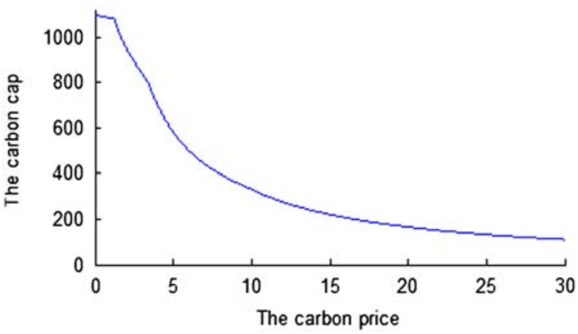

Fig. 6. The indifference curve of combinations of the carbon cap and carbon price.

that x_3^* decreases as the carbon price increases since $-d\lambda^*/dc_e < e_3/\tau_3$ always holds, and that x_1^* and x_2^* increase first and then decrease as the carbon price increases since $-d\lambda^*/dc_e > e_1/\tau_1$, $-d\lambda^*/dc_e > e_2/\tau_2$ over $c_e \in [0,1.25)$, and $-d\lambda^*/dc_e < e_1/\tau_1$, $-d\lambda^*/dc_e < e_2/\tau_2$ over $c_e \in (1.25,+\infty)$. These observations coincide with the result given in Proposition 3. Fig. 4 verifies that results in Proposition 4 that the total carbon emission decreases with the carbon price. Since the carbon emission from each product is linear with its production quantity, the total carbon emission is linearly weighted sum for the production quantities of all products. This result can be observed from the shapes of the curves reported in Figs. 3 and 4.

To investigate how the carbon price and carbon cap affect the expected profit, we plot Fig. 5 to show the curves of the expected profit with different carbon price for the carbon cap $a=0$, 200, 500, 800 and 1200, respectively. The bottom curve in Fig. 5 verifies that the expected profit decreases as the carbon price increases in the case of $a=0$. From the solution x_i^0 given in Table 1, we have $\sum_{i=1}^{n} e_i x_i^0 = 1098.04$. The top curve verifies that the expected profit is increasing in the carbon price in the case of $a=2000>1098.04$. The other curves in Fig. 5 verify that the expected profit decreases first and then increases with carbon price in the case of $0<a<1098.04$. These results are theoretically proved in Proposition 6.

To show the results in Proposition 6, we plot the indifference curve of combinations of the carbon cap and the carbon price with the expected profit $\pi(x^0)$ in Fig. 6. Notice that we do not plot the curve after $c_e>30$ because the threshold carbon cap value on this curve will get closer to 0 for large carbon price.

7. Conclusions

As the regulation and legislation on carbon emission come into effect, it is helpful for firms to incorporate carbon emission concern into their operation decisions. In this paper, we establish a profit-maximization model to combine the carbon cap and trade mechanism into a single-period capacitated multi-item production planning with stochastic demands. By analyzing the structural properties of the problem, we derive the optimal policy and propose a simple solution method with linear computational complexity. The impacts of the taxation policy on the firm's profit and carbon emission are compared with that of the cap and trade policy.

By analyzing impacts of carbon price, carbon cap on the shadow price of the common capacity, production decisions, carbon emission and total profit, we obtain some interesting managerial insights: (1) the higher the carbon price is, the lower the marginal value of the common capacity is, thus any capacity investment should be evaluated with the carbon price; (2) the firm tends to produce more carbon efficient products and to produce less carbon inefficient products under the cap-and-trade mechanism; (3) the cap-and-trade mechanism induces the firm to reduce carbon emission; (4) the higher the carbon price is, the more profit the firm makes if he has carbon quota surplus for sale; on the contrary, the higher the carbon price is, the less profit the firm gains if he need to buy carbon emission credit from the market; (5) there is an indifference curve of combinations of the carbon cap and carbon price for maintaining the firm's profit of the system without the carbon cap and trade mechanism, which gives a good reference for policy makers to design efficient carbon cap and trade mechanism. Policy comparison means that the taxation policy and the cap and trade policy can achieve the same carbon emission or profit of the regulated firm, but they cannot provide the same carbon emission and the same profit simultaneously. In addition, the cap and trade policy can provide a wide range for the firm's profit.

The work in this paper can be extended in some directions. It is interesting to consider some multi-period production planning with carbon cap and trade mechanism, in which the firm can utilize the carbon credit surplus from the previous periods, or hold carbon credit for production in future periods, and a carbon emission trading market exists and the carbon price may vary over periods. Another extension of our work is to analyze some multi-firm production optimization and competition problems under carbon cap and trade mechanism, in which different firms can trade their carbon quota, and their products may compete for the same customer base.

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Appendix A

Proof of proposition 1

(a) From Eq. (6), we know $x_i = F_i^{-1}(\frac{p_i-c_i-c_e e_i-\lambda\tau_i+w_i}{p_i-s_i})$. If $p_i-c_i-c_e e_i-\lambda\tau_i>0$, we have $x_i>0$, then the slackness condition $w_i x_i=0$ implies $w_i=0$ and $x_i = F_i^{-1}(\frac{p_i-c_i-c_e e_i-\lambda\tau_i}{p_i-s_i})$. If $p_i-c_i-c_e e_i-\lambda\tau_i \le 0$, according to $w_i x_i=0$, we have $x_i=0$ and $w_i>0$. These results can be summarized as Eq. (9).

(b) $\lambda=0$ or $\sum_{i=1}^{n}\tau_i x_i(\lambda)=t$ implies $\lambda(t-\sum_{i=1}^{n}\tau_i x_i)=0$. If $\lambda=0$ or $\sum_{i=1}^{n}\tau_i x_i(\lambda)=t$, since $x(\lambda)$ satisfies Eqs. (6) and (7), $x(\lambda)$ will satisfy all KKT conditions. Thus, we have $x^*=x(\lambda)$ if $(x(\lambda),\lambda)$ satisfies $\lambda=0$ or $\sum_{i=1}^{n}\tau_i x_i(\lambda)=t$.

Proof of proposition 2

In the case that the common capacity constraint is inactive, we have $\lambda^*=0$. For any $\lambda^*>0$, we know the common capacity constraint is active, according to Proposition 1, we have $\sum_{i\in I(\lambda^*)}\tau_i x_i(\lambda^*)-t=0$. Denote by

$$G(\lambda^*,c_e)=\sum_{i\in I(\lambda^*)}\tau_i F_i^{-1}\left(\frac{p_i-c_i-c_e e_i-\lambda^*\tau_i}{p_i-s_i}\right)-t=0 \tag{A.1}$$

Using the derivative of implicit function, we have

$$\frac{d\lambda^*}{dc_e}=-\frac{\partial G(\lambda^*,c_e)/\partial c_e}{\partial G(\lambda^*,c_e)/\partial\lambda^*}=-\frac{\sum_{i\in I(\lambda^*)}\rho_i\tau_i e_i}{\sum_{i\in I(\lambda^*)}\rho_i\tau_i^2}<0 \tag{A.2}$$

Thus, the optimal shadow price λ^* is decreasing in carbon price c_e.

Proof of corollary 1

Since $e_1/\tau_1=\cdots e_n/\tau_n$, we let $r=e_i/\tau_i$, $i=1,\ldots,n$. Substitute $e_i=r\tau_i$, $i=1,\ldots,n$ into Eq. (A.2), we have

$$\frac{d\lambda^*}{dc_e}=-\frac{\sum_{i\in I(\lambda^*)}r\rho_i\tau_i^2}{\sum_{i\in I(\lambda^*)}\rho_i\tau_i^2}=-r<0 \tag{A.3}$$

Thus, λ^* is linear decreasing in the carbon price c_e if $e_1/\tau_1=\cdots e_n/\tau_n$.

Proof of corollary 2

(a) If D_i is uniformly distributed on $[\alpha_i,\beta_i]$, then the probability density function $f_i(x_i(\lambda^*))=\frac{1}{\beta_i-\alpha_i}$. Substitute it into Eq. (A.2), we have

$$\frac{d\lambda^*}{dc_e}=-\frac{\sum_{i\in I(\lambda^*)}((\beta_i-\alpha_i)\tau_i e_i/(p_i-s_i))}{\sum_{i\in I(\lambda^*)}((\beta_i-\alpha_i)\tau_i^2/(p_i-s_i))} \tag{A.4}$$

From Eq. (A.4), we observe that $\frac{d\lambda^*}{dc_e}$ is a negative constant value for any given $I(\lambda^*)$. If the change of c_e does not affect $I(\lambda^*)$, then λ^* is linear decreasing in c_e. Thus, λ^* is piecewise linear decreasing in c_e.

(b) Since the breakpoints on the optimal shadow price curve over the carbon price appear at the change points of $I(\lambda^*)$. From the definition of $I(\lambda)$, we know that $I(\lambda^*)$ changes if the change of c_e affects the sign of $p_i-c_i-(e_i c_e+\lambda^*\tau_i)$. That is to say, a breakpoint will appear if $p_i-c_i-(e_i c_e+\lambda^*\tau_i)=0$ is met. From Proposition 2, we know that λ^* is decreasing in c_e, and hence λ^* will become zero for large enough c_e. Thus there are two possible breakpoints for each $i=1,\ldots,n$, i.e., $c_e=(p_i-c_i-\lambda^*\tau_i)/e_i$ for $\lambda^*>0$, and $c_e=(p_i-c_i\tau_i)/e_i$. In addition, the value of c_e such that $\lambda^*=0$ is an intuitive breakpoint. Thus, there are at most $2n+1$ breakpoints on the piecewise linear curve.

Proof of proposition 3

From Eq. (9), we have $x_i^*=F_i^{-1}(\frac{p_i-c_i-c_e e_i-\lambda^*\tau_i}{p_i-s_i})$ for $i\in I(\lambda^*)$. Taking the derivative of x_i^* with respect to c_e, we have

$$\begin{aligned}\frac{dx_i^*}{dc_e}&=-\frac{1}{f_i(x_i(\lambda^*))(p_i-s_i)}\frac{d(c_e e_i+\lambda^*\tau_i)}{dc_e}\\&=-\frac{\tau_i}{f_i(x_i(\lambda^*))(p_i-s_i)}\left(\frac{e_i}{\tau_i}+\frac{d\lambda^*}{dc_e}\right)\end{aligned} \tag{A.5}$$

Thus we have $\frac{dx_i^*}{dc_e}>0$ if $-\frac{d\lambda^*}{dc_e}>\frac{e_i}{\tau_i}$, $\frac{dx_i^*}{dc_e}<0$ if $-\frac{d\lambda^*}{dc_e}<\frac{e_i}{\tau_i}$, and $\frac{dx_i^*}{dc_e}=0$ if $-\frac{d\lambda^*}{dc_e}=\frac{e_i}{\tau_i}$.

Proof of corollary 3

From Eq. (9), we have $x_i^*=F_i^{-1}\left(\left(\frac{p_i-c_i-c_e e_i-\lambda^*\tau_i}{p_i-s_i}\right)^+\right)$ and $x_i^0=F_i^{-1}\left(\left(\frac{p_i-c_i-\lambda^0\tau_i}{p_i-s_i}\right)^+\right)$. According to Eq. (8), we know $\lambda^0(t-\sum_{i=1}^{n}\tau_i x_i^0)=0$ and $\lambda^*(t-\sum_{i=1}^{n}\tau_i x_i^*)=0$. Since x_i^* and x_i^0 are both deceasing in λ, we have $\lambda^0\geq\lambda^*$.

In the case of $\lambda^0=\lambda^*$, we have $(\lambda^0-\lambda^*)/c_e=0<e_i/\tau_i$, and $x_i^*<x_i^0$.

In the case of $\lambda^0>\lambda^*$, if $(\lambda^0-\lambda^*)/c_e>e_i/\tau_i$, then we have $\frac{p_i-c_i-c_e e_i-\lambda^*\tau_i}{p_i-s_i}>\frac{p_i-c_i-\lambda^0\tau_i}{p_i-s_i}$, and hence $x_i^*\geq x_i^0$; if $(\lambda^0-\lambda^*)/c_e>e_i/\tau_i$, then $\frac{p_i-c_i-c_e e_i-\lambda^*\tau_i}{p_i-s_i}<\frac{p_i-c_i-\lambda^0\tau_i}{p_i-s_i}$, and hence $x_i^*\leq x_i^0$; if $(\lambda^0-\lambda^*)/c_e=e_i/\tau_i$, then we have $x_i^*=x_i^0$.

Proof of proposition 4

The total carbon emission is $\sum_{i=1}^{n}e_i x_i^*$. Taking the derivative of $\sum_{i=1}^{n}e_i x_i^*$ with respect to c_e, we have

$$\frac{d\sum_{i=1}^{n}e_i x_i^*}{dc_e}=-\sum_{i\in I(\lambda^*)}\rho_i e_i\left(e_i+\tau_i\frac{d\lambda^*}{dc_e}\right) \tag{A.6}$$

Substituting $\frac{d\lambda^*}{dc_e}$ in Eq. (A.2) into Eq. (A.6), we have

$$\begin{aligned}\frac{d\sum_{i=1}^{n}e_i x_i^*}{dc_e}&=-\sum_{i\in I(\lambda^*)}\rho_i e_i\left(e_i-\tau_i\frac{\sum_{j\in I(\lambda^*)}\rho_j\tau_j e_j}{\sum_{j\in I(\lambda^*)}\rho_j\tau_j^2}\right)\\&=-\sum_{i\in I(\lambda^*)}\rho_i e_i^2+\frac{(\sum_{i\in I(\lambda^*)}\rho_i\tau_i e_i)^2}{\sum_{i\in I(\lambda^*)}\rho_i\tau_i^2}\\&=\frac{-1}{\sum_{i\in I(\lambda^*)}\rho_i\tau_i^2}\left[\left(\sum_{i\in I(\lambda^*)}\rho_i e_i^2\right)\left(\sum_{i\in I(\lambda^*)}\rho_i\tau_i^2\right)-\left(\sum_{i\in I(\lambda^*)}\rho_i\tau_i e_i\right)^2\right]\end{aligned} \tag{A.7}$$

Let $m=|I(\lambda^*)|$. If $m=1$, then we have

$$\frac{d\sum_{i=1}^{n}e_i x_i^*}{dc_e}=\frac{-1}{\rho_i\tau_i^2}\left[\rho_i e_i^2\rho_i\tau_i^2-(\rho_i\tau_i e_i)^2\right]=0 \tag{A.8}$$

If $m>1$, we re-index $i\in I(\lambda^*)$ as $i=1,\ldots,m$, then we have

$$\begin{aligned}\frac{d\sum_{i=1}^{n}e_i x_i^*}{dc_e}&=\frac{-1}{\sum_{i\in I(\lambda^*)}\rho_i\tau_i^2}\left[\left(\sum_{i\in I(\lambda^*)}\rho_i e_i^2\right)\left(\sum_{i\in I(\lambda^*)}\rho_i\tau_i^2\right)-\left(\sum_{i\in I(\lambda^*)}\rho_i\tau_i e_i\right)^2\right]\\&=-\frac{\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}\rho_i\rho_j(\tau_i e_j-\tau_j e_i)^2}{\sum_{i=1}^{m}\rho_i\tau_i^2}\leq 0\end{aligned} \tag{A.9}$$

From Eqs. (A.8) and (A.9), we know that $\sum_{i=1}^{n}e_i x_i^*$ decreases as carbon price increases.

From Eq. (9), we have $x_i^*=F_i^{-1}\left(\left(\frac{p_i-c_i-c_e e_i-\lambda^*\tau_i}{p_i-s_i}\right)^+\right)$. If $c_e\geq\max_{i=1,\ldots,n}\{(p_i-c_i)/e_i\}$, we have $p_i-c_i-c_e e_i\leq 0$, then $p_i-c_i-c_e e_i-\lambda^*\tau_i\leq 0$ for all $i=1,\ldots,n$, and there must be $x_i^*=0$ for $i=1,\ldots,n$, and hence $\sum_{i=1}^{n}e_i x_i^*=0$.

Proof of proposition 5

The first derivative of $\pi(x^*,q^*)$ with respect to c_e is

$$\frac{d\pi(x^*,q^*)}{dc_e}=\sum_{i\in I(\lambda^*)}\frac{d[(p_i-c_i-c_ee_i)x_i^*-(p_i-s_i)\int_0^{x_i^*}F_i(u_i)du_i]}{dc_e}+a$$
$$=\sum_{i\in I(\lambda^*)}\left(\frac{d[(p_i-c_i-c_ee_i)x_i^*]}{dc_e}-(p_i-s_i)\frac{d[\int_0^{x_i^*}F_i(u_i)du_i]}{dc_e}\right)+a$$
$$=\sum_{i\in I(\lambda^*)}\left(-e_ix_i^*+[(p_i-c_i-c_ee_i)-(p_i-s_i)F_i(x_i^*)]\frac{dx_i^*}{dc_e}\right)+a \quad (A.10)$$

From Eq. (9), we have $F_i(x_i^*)=\frac{p_i-c_i-c_ee_i-\lambda^*\tau_i}{p_i-s_i}$ for $i\in I(\lambda^*)$. Substitute it into Eq. (A.10), we have

$$\frac{d\pi(x^*,q^*)}{dc_e}=\sum_{i\in I(\lambda^*)}\left(-e_ix_i+\lambda^*\tau_i\frac{dx_i^*}{dc_e}\right)+a$$
$$=\lambda^*\sum_{i\in I(\lambda^*)}\left(\tau_i\frac{dx_i^*}{dc_e}\right)+(a-\sum_{i\in I(\lambda^*)}e_ix_i^*) \quad (A.11)$$

Using the similar mathematical manipulation as that in the proof of proposition 3, we have

$$\sum_{i\in I(\lambda^*)}\left(\tau_i\frac{dx_i^*}{dc_e}\right)=\frac{-1}{\sum_{i\in I(\lambda^*)}\rho_i\tau_i^2}\left[\left(\sum_{i\in I(\lambda^*)}\rho_i\tau_i^2\right)^2-\left(\sum_{i\in I(\lambda^*)}\rho_i\tau_i^2\right)^2\right]=0 \quad (A.12)$$

Substituting Eq. (A.12) it into Eq. (A.11), we have

$$\frac{d\pi(x^*,q^*)}{dc_e}=a-\sum_{i\in I(\lambda^*)}e_ix_i^*=-q^* \quad (A.13)$$

Proof of corollary 4:

(a) In the case of $a=0$, any production in the system needs buying some carbon credit from the market, then we have $q^*=\sum_{i=1}^n e_ix_i^*-a=\sum_{i=1}^n e_ix_i^*-0>0$. From Proposition 5, we know that $\pi(x^*,q^*)$ decreases with c_e for the case of $q^*>0$. Thus, $\pi(x^*,q^*)$ decreases with c_e if $a=0$.

(b) Since $\sum_{i=1}^n e_ix_i^0$ is the total carbon emission for $c_e=0$, and Proposition 4 indicates that the total carbon emission decreases as c_e increases, thus we $\sum_{i=1}^n e_ix_i^*<\sum_{i=1}^n e_ix_i^0$ for any $c_e>0$. In the case of $a\geq\sum_{i=1}^n e_ix_i^0$, we have $q^*=\sum_{i=1}^n e_ix_i^*-a<\sum_{i=1}^n e_ix_i^0-a\leq 0$. From Proposition 5, we know that $\pi(x^*,q^*)$ increases with c_e for the case of $q^*<0$. Thus, $\pi(x^*,q^*)$ increases with c_e if $a\geq\sum_{i=1}^n e_ix_i^0$.

(c) In the case of $0<a<\sum_{i=1}^n e_ix_i^0$, we know $q^*=\sum_{i=1}^n e_ix_i^*-a=\sum_{i=1}^n e_ix_i^0-a>0$ for $c_e=0$, and $q^*=\sum_{i=1}^n e_ix_i^*-a=\sum_{i=1}^n e_ix_i^0-a>0$. From Proposition 4, we know $q^*=\sum_{i=1}^n e_ix_i^*-a=0-a<0$ for $c_e\geq\max_{i=1,\ldots,n}\{(p_i-c_i)/e_i\}$. Since $\sum_{i=1}^n e_ix_i^*$ is a decreasing function of c_e over the interval $[0,\max_{i=1,\ldots,n}\{(p_i-c_i)/e_i\}]$, we know q^* is also a decreasing function of c_e over this interval. Thus, there must exist a threshold value $\hat{c}_e$ on the interval $[0,\max_{i=1,\ldots,n}\{(p_i-c_i)/e_i\}]$ such that $q^*=0$ at $c_e=\hat{c}_e$, $q^*>0$ for $c_e<\hat{c}_e$, and $q^*<0$ for $c_e>\hat{c}_e$. According to Proposition 5, we know that $\pi(x^*,q^*)$ initially decreases and then increases with the carbon price c_e if $0<a<\sum_{i=1}^n e_ix_i^0$. The turning point is $c_e=\hat{c}_e$, i.e., $q^*=0$.

Proof of proposition 6

Since $\pi_{a=0}(x^*,q^*)$ is the expected profit for the case of $a=0$, then we have

$$\pi(x^*,q^*)=\pi_{a=0}(x^*,q^*)+c_ea \quad (A.14)$$

That is to say, the expected profit for any $a>0$ can be expressed as the sum of the expected profit without carbon cap ($a=0$) and the market value of the carbon cap (c_ea).

If the expected profit in the system with carbon emission consideration equals to that in the system without carbon emission consideration, we need

$$\pi(x^*,q^*)=\pi(x^0) \quad (A.15)$$

Combining Eqs. (A.14) and (A.15), we have

$$\pi(x^0)-\pi_{a=0}(x^*,q^*)=ac_e \quad (A.16)$$

Then we can solve the threshold carbon cap value as $\hat{a}=(\pi(x^0)-\pi_{a=0}(x^*,q^*))/c_e$.

Since the expected profit for any $a>0$ can be expressed as the sum of the expected profit with the threshold carbon cap ($a=\hat{a}$) and the market value of the remaining carbon quota ($c_e(a-\hat{a})$), then we have

$$\pi(x^*,q^*)=\pi_{a=\hat{a}}(x^*,q^*)+c_e(a-\hat{a})=\pi(x^0)+c_e(a-\hat{a}) \quad (A.17)$$

From Eq. (A.17), we know $\pi(x^*,q^*)>\pi(x^0)$ for $a>\hat{a}$, $\pi(x^*,q^*)<\pi(x^0)$ for $a<\hat{a}$.

Proof of proposition 7

From the analysis in Section 4, we know that the carbon cap a has no influence on the optimal production quantities. Since problem P^t is equivalent to problem P with $a=0$, if the carbon tax rate in problem P^t equals to the carbon price in problem P, then the optimal production quantities in these two problem are also equal, and hence the total carbon emissions in two problems are also equal.

Proof of proposition 8

(a) Since the firm can generate additional profit c_ea from the allocated carbon quota under the cap and trade policy, if the carbon price and the carbon tax rate are same, the firm will get more profit under the cap and trade policy.

(b) $\pi(x^0)=\pi_{a=0,c_e=0}=\pi^t_{c_e^t=0}>\pi^t_{c_e^t>0}=\pi^t$.

(c) From Corollary 4(b), we have $\pi=\pi_{a>0,c_e>0}>\ \pi_{a>0,c_e=0}>\pi_{a=0,c_e=0}=\pi(x^0)$ for the case of $a\geq\sum_{i=1}^n e_ix_i^0$.

(d) Combining it with Proposition 8(b) gives $\pi>\pi^t$

In the case of $a<\sum_{i=1}^n e_ix_i^0$, if $\pi>\pi(x^0)$, from Proposition 8(b), we have $\pi>\pi^t$. If $\pi<\pi(x^0)$, we have $\pi^t_{c_e^t=0}=\pi(x^0)>\pi$, and

$\pi^t_{c^t_e = c_e} < \pi$, there must *exist a threshold* $\hat{c}^t_e < c_e$ *such that* $\pi^t = \pi$ *, and* $\pi > \pi^t$ *for* $c^t_e > \hat{c}^t_e$, $\pi < \pi^t$ *for* $c^t_e < \hat{c}^t_e$.

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