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# A constraint programming approach to capacity planning in container vessels

Byung Kwon Lee<sup>1</sup> · Joyce M. W. Low<sup>2</sup>

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## Abstract

A container vessel carries containers of various characteristics, in terms of size, weight, and contents. The cargo load of a container vessel, being subjected to a set of operational conditions and restrictions regarding ship stability and safety, is a fundamental element in decision-making when a shipping line provides logistics services to clients. This study presents a constraint programming-based model for the capacity planning of a container vessel under various operational conditions. The proposed model generates base solutions and is complemented with a rich scenario-based analysis that utilizes real-life ship data of a container vessel operated by a liner shipping company with a significant market presence. Solutions obtained from the model provide insights on containership capacity planning with differing settings and search strategies. Recommendations to container carriers, regarding improved capacity planning, are the highlights of the study.

**Keywords** Capacity planning · Cargo-mix profiles · Stowage planning · Container vessels · Constraint programming

## 1 Introduction

The container shipping industry has exhibited a rapid growth trend in container volumes for the past 30 years (UNCTAD 2020). To meet the growing demand for cargo transportation services, carriers are deploying larger container vessels. Their use allows carriers to enjoy scale economies, thus lowering their unit (slot) costs. (Merk

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2018; Ge et al. 2019; UNCTAD 2020). As competition in the industry intensifies, it is pertinent for these carriers to seek additional ways to increase revenue while further reducing cost (Glave et al. 2014). Perunovic and Vidic-Perunovic (2011) have previously suggested that carriers ought to re-examine their existing processes to lower operating costs via improved efficiency while providing better services to their clients. In its most basic form, revenue accrued from a shipping service along a route is closely correlated with the number of containers carried on a vessel over its entire voyage (Christensen and Pacino 2017; Helo et al. 2018). Since cargo load planning directly affects the profitability of a voyage, one promising avenue to increasing a carrier's competitiveness is to strive for better load planning, so that more containers could be carried on the vessel (Wu 2014). A vessel capacity planning study facilitates selective deployment of vessels that will meet the capacity demand on a route consisting of specified ports to visit. This study also assists carriers to select the ports that are to be visited on an itinerary sailed by a said vessel. The assessment of vessel capacity provides invaluable information that helps the carrier to better utilize the twenty-foot equivalent unit (TEU) capacity of a vessel and maximize the potential revenue.

In container shipping, vessels are deployed in service networks according to a published schedule. Itineraries are characterized by port rotations where ships call at predetermined ports along fixed routes. Each port presents a unique cargo mix that varies in container size (i.e., 20 ft, 40 ft, high cube, etc.), weight, and type [general, reefer, dangerous goods, and out-of-gauge (OOG)]. For a given capacity, the ship operator has to decide how many of each container type to load at each port in order to maximize capacity utilization of the overall, round service. The placement of the loaded containers will pose another important deliberation as it affects the balance of the ship at sea. The right stowing pattern will also minimize the need for container re-stows, increasing efficiency and reducing cost.

Our study begins with the construction of a capacity planning tool based on constraint programming (CP) for maximizing cargo load capacity of a container vessel. CP models are said to work well with problems that contain complex constraints of special structures, compared with existing optimization methods (Bockmayr and Hooker 2005). A carrier faces a predefined cargo mix specific to the trade routes that it undertakes. The model considers an array of parameters that characterize cargo weight and container size distributions. When dealing with the problem of vessel space utilization, the model takes into account both the soft and the hard constraints, such as physical stowing conditions, logical stacking rules, ship stability and stress, cargo mix<sup>1</sup>, and re-stows. Using the proposed planning tool, the carrier can vary the ratios of cargo type (i.e., 20 ft against 40 ft containers; normal cargo against special cargo such as reefers and OOGs) and configurations of container weight classes that are representative of their respective trade routes to achieve better cargo load performances, such as smaller number of re-stows, higher space utilization (congruent to

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<sup>1</sup> The cargo mix excludes dangerous cargo as demand for the transportation of dangerous cargoes occurs in an ad hoc manner, which cannot be predicted in advance. Such containers are handled on a case-by-case basis.

more load capacity), and lower proportion of cargo placed in slots that are outside of crane reach. Next, the IBM ILOG CP Optimizer is used to solve the CP model and evaluate two search strategies that contribute to the domain reductions and the constraint propagations in different aspects. Real industry data, including vessel and cargo profiles for specific trade routes, are obtained from a carrier with a significant market presence. The data are used to test run the model on the various settings related to container and cargo mix. An empirical analysis is conducted to examine how different combinations of factors may influence the cargo load capacity of a container vessel and minimize the number of empty slots, the number of re-stows, or the number of TEUs located in bays that are outside the predetermined working regions of quay cranes. Sensitivity analyses would allow a carrier to gain strategic insights in the configuration of cargo profiles via selective port visits to maximize the carrying capacity of a vessel. For itineraries that consist of a prespecified set of ports that must be called, the analysis on ship capacity allows the carrier to deploy best-suited vessels that meet the demands.

This study aims to make theoretical and practical contributions in the development of capacity planning models and their applications. Increasing ship sizes, coupled with shrinking profit margins, pinpoints the importance of cargo selection decisions at ports along the sailing itinerary, and the efficient positioning of these cargoes on the vessel will boost the utilization of cargo load capacity in the overall voyage. The study extends the knowledge frontier by examining the potential use of a CP-based model approach to find promising solutions.

From a practical perspective, the construction of an efficient capacity planning model will help to automate the manual and tedious process of capacity planning that many shipping companies are facing. At present, there is no existing software package dedicated to estimating cargo load capacity, to the best of our knowledge. Several commercial packages such as StowMan (developed by NAVIS, <https://www.navis.com/>) and CASP (developed by Total Soft Bank, <http://www.tsb.co.kr/en/>) are available for stowage planning. However, stowage planning for a ship needs to be specific to the ship profile for precise calculations of ship stability. This means that, whenever a container is positioned at a slot, the stowage planning package software should reflect the dynamics that mirror the stability conditions of the ship. Furthermore, stowage planning packages are concerned with how containers are stacked and stowed logically and safely onboard the vessel. While a ship's cargo capacity is indicated on its tonnage certificate, the actual carrying capacity is affected by container slotting. Our study proposes a capacity planning tool that quantifies the amount of cargo that can be carried on a ship. By simplifying ship stability calculations, the model is generalized and made applicable to a range of container ships of different hull structures or size. The objective is to provide directives to ship management on capacity planning, rather than coming up with precise calculation of ship stability for container slot allocations for cargo operations on individual ships.

The rest of the paper is organized in the following fashion: Section “[Literature review](#)” presents a literature review in the related areas of capacity planning, cargo-mix problems, and stowage planning. Section “[Methodology](#)” defines the problem and describes the modeling methodology. Section “[Empirical case study](#)” introduces the CP model. The “[Conclusion](#)” section presents a case study on capacity planning.

Through the case study, we illustrate the process of constructing and running a capacity planning model that seeks to maximize the cargo load capacity for a predefined cargo-mix. The CP solutions and insights gained are then be used to identify a final solution of capacity planning. Section 6 summarizes the findings and concludes the study with suggestions for further research.

## 2 Literature review

The cargo load capacity planning problem has become more complex following the emergence of bigger vessels with higher TEU capacities. As highlighted in Allianz Global Corporate & Specialty (2019), the growing vessel size gives rise to a growing need for operational research in this area of vessel usage optimization. While the revenue of shipping operations is positively associated with the amount of TEUs carried on each service, the efficiency of a cargo load capacity plan varies, with solutions pursued through different approaches and search algorithms. Currently, such approaches fall under three broad categories: combinatorial optimization, mathematical programming, and constraint programming. A pioneering work is found in Wilson and Roach (1999) where the authors introduced combinatorial optimization as an approach to break down the optimization of the entire vessel into blocks in each bay. In their model, the branch-and-bound search technique was first used to assign slots to a block. A tabu search algorithm was subsequently run to assign containers to slots within individual blocks, which is a blocked cargo space. A block (or a blocked cargo space) refers to a set of slots belonging to a section of a single hatch lid. This blocked cargo space has the effect of reducing crane movements and hatch-lid movements predicted during planning. This research approach has inspired researchers to adopt a model decomposition approach. Ting and Tzeng (2004) and Kos and Zenzerovic (2004) both approached the cargo load capacity optimization problem through mathematical programming. The models were similar in terms of selecting of parameters and constraints, but differed in their objective functions. Specifically, Ting and Tzeng (2004) focused on maximizing the number of TEUs allocated to an outbound ship, whereas Kos and Zenzerović (2004) maximized revenue while considering the different profit margins of the different cargo types. While the two studies presented mathematical models offering effective solutions to optimize vessel utilization, both had unfortunately failed to incorporate stability and dangerous cargo class constraints into their models. Later, Delgado et al. (2012) proposed a CP approach that took most of vessel physical constraints into consideration. Their study adopted high-fidelity modeling, where the analysis is dedicated to ship profiles of interest. However, the authors neglected 45 ft containers and other cargo type constraints. Delgado et al., therefore, opened up many avenues of extension of the CP approach to take into account the aforementioned areas.

Closely related to the capacity planning problem is the cargo-mix problem that aims to find the cargo composition needed to maximize a vessel's revenue or space utilization on a given service. In solving a cargo-mix problem, researchers (for examples, see Delgado 2013; Parreño 2016; Christensen et al. 2019) have bifurcated into deterministic optimization and stochastic optimization. As the term implies,

**Table 1** Cargo-mix problem

	Deterministic optimization	Stochastic optimization
Block stowage: positioning a set of cargo into a region of a vessel	Christensen and Pacino (2017) and Delgado (2013)	Christensen et al. (2019)
Cargo-slot assignment	Parreño et al. (2016) and Delgado (2013)	Christensen et al. (2019)

deterministic optimization is used when there is no uncertainty in the amount of cargo flows at each port pair. On the other hand, stochastic optimization is used when cargo flows are uncertain. Table 1 lists some of the most significant works in this area.

The stowage planning problem has a similar thrust to the cargo load capacity planning problem. The stowage planning problem aims to find the load configurations that best match the cargo to be loaded at the current port *and* the specifications of the particular vessel, while also making sure that vessel capacity could be fully utilized in subsequent port calls of the itinerary. The purpose of stowage planning is to facilitate efficient discharging and loading operations at the ports while meeting the demand for cargo carrying capacity from one port to another and ensuring the navigational safety requirement of the ship (i.e., ship stability). Under the umbrella of stowage planning problems, researchers have examined the topic of *block stowage* involving the positioning of a set of cargoes into an area of the vessel and of the terminal (i.e., load sequencing from yard to vessel, focusing on stowing cargo on the vessel). Methodologies can be broadly classified as mathematical programming, search-based heuristics, and rule-based heuristics (Table 2). Ambrosino et al. (2006, 2009) proposed a three-phase heuristic approach to construct a stowage plan. The first phase finds an optimal solution of subsets of bays related to independent portions of the ship; the second phase applies a binary linear programming model to optimally solve multiple (i.e., the number of destinations) single-destination stowage plans with limited ship stability conditions (i.e., a trial stowage plan), and the third phase applies a tabu-search-based heuristic to efficiently find an improved stowage plan based on the union of solutions of the second phase. The authors also developed a prototype system visualizing stowage plans. Prior to this, Delgado (2013) demonstrated the practicality of the proposed model for block stowage and cargo slotting by implementing it in a stowage planning package. Ambrosino and Sciomachen (2018) studied a stowage planning problem that focuses on hazardous containers. The authors categorize containers into nine classes according to international regulations and provide a rule-based heuristic to assign hazardous containers together with dry containers. Chou and Fang (2021) incorporated knowledge of domain experts to construct practical stowage plans from general stowage plans of a ship to meet the requirement of various ship stability measures (e.g., metacentric height, trim, heel, bending moment, etc.). Chao and Lin (2021) introduced class-based master bay plans in which the proposed mathematical model has a structure of multicommodity network, with the objective of minimizing re-stows, allowing the allocation of ship slots to containers of different lengths. Recently, Larsen and

**Table 2** Literature on stowage planning problems

	Mathematical programming	Search-based heuristics	Rule-based heuristics
Cargo-slot assignment	Delgado (2013), Delgado et al. (2012), Pacino (2012)	Ding and Chou (2015), Pacino (2012), Delgado et al. (2012), Ambrosino et al. (2009), Wilson and Roach (2000), Avriel et al. (1998)	Low et al. (2011)
Block stowage: positioning a set of cargo into a region of a vessel	Chao and Lin (2021), Ambrosino et al. (2017), Ambrosino et al. (2015), Delgado (2013), Pacino (2012), Ambrosino et al. (2009), and Kang and Kim (2002)	Azevedoa et al. (2018), Ambrosino et al. (2017), Dubrovsky et al. (2002), Kang and Kim (2002), and Wilson and Roach (2000)	Chou and Fang (2021), Ambrosino et al. (2015), and Ambrosino et al. (2006)
Terminal-side stowage planning (load sequencing from yard to vessel, focusing on stacking cargo on the vessel)	Iris et al. (2018), Monaco et al. (2014), and Imai et al. (2006)	Iris et al. (2018), Araújo et al. (2016), Monaco et al. (2014), and Imai et al. (2006)	

Pacino (2021) published benchmark instances for stowage planning with practical data on vessels and ship stability measures to further promote the stowage planning research.

In summary, the stowage planning problem deals with the loading of cargo for the given load lists of the ports, whereas the cargo-mix problem concerns itself with the cargo load capacity for various cargo mixes. Nevertheless, the two problems have the same constraints for stowing cargo on a vessel. Helo et al. (2018) demonstrated the importance of fundamental components, such as cargo profiles (cargo mix) and cargo systems (vessel physical structure), in determining the cargo load capacity of a vessel and the necessity of considering such components in stowage planning.

### 3 Methodology

This section introduces a set of practical conditions and requirements, exerting an impact on the cargo load capacity of a container vessel. Subsequently, a CP-based model is developed to determine the cargo load capacity taking into consideration the stacking restrictions of 20 ft and 40 ft containers, weight and height limits, reefer cargo positions, weight balance of the ship, etc. These requirements will be elaborated on in the subsequent subsections.

#### 3.1 Problem definition

This study aims to address the inefficient use of load capacity in cargo vessels due to inferior cargo mix presented at selected loading ports.<sup>2</sup> To do so, we will need to emulate the way an actual planner goes about drawing up a cargo loading plan that fulfills an array of objectives, such as maximizing cargo load, minimizing re-stows, etc., while taking into account all the conditions affecting cargo load capacity. These conditions can range from trade/route, physical stacking, logical stacking, vessel balance, and cargo type to re-stowing issues.

As part of the load planning process, the type of containers that should be placed in each of the individual slots of a ship must be determined. Clearly, this will depend on the cargo-mix profile of the specific trade route. As depicted in Fig. 1, the container weight should be balanced for safety reasons. According to the Verified Gross Mass (VGM) guidelines<sup>3</sup> of the International Maritime Organization (IMO), a container will no longer be allowed to be loaded on vessels unless its VGM has been provided by the shipper to the ocean carrier, and/or port terminal representatives, prior to the load list cut-off date. The new guideline was adopted by the IMO to increase maritime safety and reduce the risks to cargo, containers, and all those

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<sup>2</sup> By inferior cargo mix, we refer to cargo size and weight that do not optimize the capacity of the vessel.

<sup>3</sup> VGM guidelines require the mandatory verification of the gross mass of packed containers to ensure the safety of ships, seafarers, and shore-side workers from any discrepancy between the declared gross mass and the actual gross mass of a packed container. We refer readers to <https://www.imo.org/en/OurWork/Safety/Pages/Verification-of-the-gross-mass.aspx>.



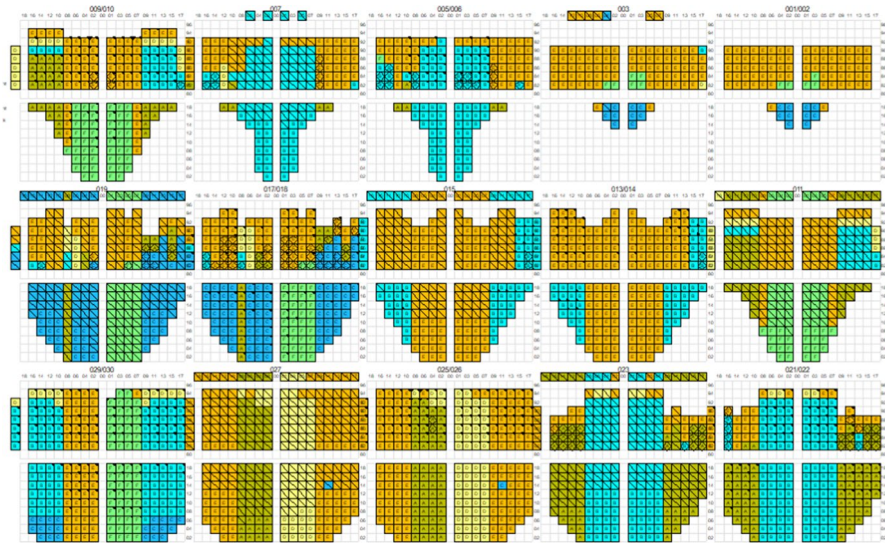


Fig. 1 Part of bay views of a container vessel

involved in container transport throughout the supply chain. In addition, containers that need to be offloaded in an earlier port should be placed on top to minimize unnecessary and costly handling in the process known as re-stows. As a vessel has limited capacity, the carrier will have to decide on the number and type of containers to be loaded at each port of call, to maximize the carrying capacity and revenue of the voyage, subject to the structural design and safety characteristics of the ship.

### 3.2 Modeling

A capacity planning CP model is formulated with the objective to maximize vessel space utilization. Owing to the complexity of our constraints, the CP is chosen to address the issue of capacity utilization maximization, subject to the constraints of container stowing onboard. Constraints were divided into two main categories, namely hard constraints and soft constraints. Hard constraints relate to weight and height stacking restrictions, allocations to reefer slots, and lashing regulations that must be adhered to. Soft constraints, instead, relate to ship stability, with cargo weight balance included in the model. The implementation of the soft constraints is linked to the objective function, and they are taken into consideration during the solution search process of the problem (Pacino 2012; Delgado 2013; Delgado et al. 2012; Parreño et al. 2016; Christensen and Pacino 2017; Christensen et al. 2019). Penalties for not meeting the soft constraints are laid out in the objectives and soft constraints, i.e., maximizing the use of slot space, minimizing the re-stows, minimizing container stacking at predetermined “out of crane working range,” etc.; (subsection 3.3 describes the objectives and constraints in depth). In practice, there are other constraints such as shear force and torsion that should also be taken into account, in addition to the hard and soft constraints laid out in VGM guidelines.

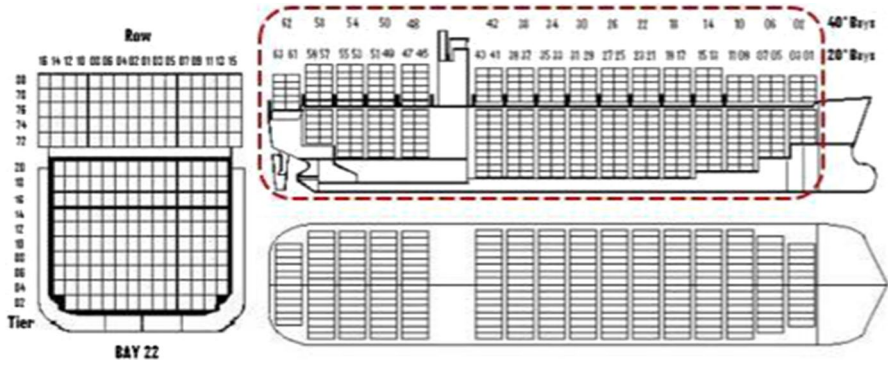


Fig. 2 Detailed view on bay, row, and tier numbering system (Sciomachen and Tanfani 2007)

The model applies the operational constraints instead of calculating the precise mechanical dynamics.

### 3.3 CP model

This subsection describes the proposed CP model for the capacity planning problem.

#### 3.3.1 Notations

Parameters for container slots

$i$	The index for ship bay, $i \in I$ (refer to Fig. 2). $I = I_{bow} \cup I_{stern}$ where $I_{bow} \cap I_{stern} = \emptyset$ . $I_{bow}$ is the set of ship bays belonging to the bow side of the ship and $I_{stern}$ is the set of ship bays belonging to the stern side of the ship
$j$	The index for tier, $j \in J$ (refer to Fig. 2)
$k$	The index for row, $k \in K$ (refer to Fig. 2)
$p$	The port of visiting, $p \in P$
$slot_{ijk}$	The container slot identified by ship bay $i$ , tier $j$ and row $k$
$slot_{ijk}^{(R)}$	The index for each container slot, which is reefer compatible, of ship bay $i$ , tier $j$ and row $k$
$h_{ijk}^p$	The container height located at $slot_{ijk}$ in port $p$
$w_{ijk}^p$	The weight class of the container located at $slot_{ijk}$ in port $p$ . It is represented by an integer number such as 1, 2, ..., 6. where smaller number indicates a lighter container class
$cnt$	The unique container identification (ID), $cnt \in CNT$
$cnt^{(R)}$	The unique container ID for reefers, $cnt^{(R)} \in CNT^{(R)}$
$stack_{ik}$	The stack identified by the two indexes of ship bay $i$ and row $k$
$stack_r^{(CI)}$	The stack that is out of crane intensity region, $r \in R$

## Parameters for container stacking

$re^p(stack_{ik})$	The number of containers that need to be re-stowed at $stack_{ik}$ in port $p$ as determined by the count of the number of positive integers ( $x_{ijk}^{od}$ ) with the port $d > p$ for every pair of slots at the stack (refer to Fig. 7)
$c^p(stack_r^{(CI)})$	The number of containers that need to be discharged at $stack_r^{(CI)}$ in port $p$ as determined by the count of the number of positive integers ( $x_{ijk}^{od}$ ) with the port $d = p$ at the stack (refer to Fig. 7)
$lim^w(stack_{ik})$	The weight limit of $stack_{ik}$
$lim^h(stack_{ik})$	The height limit of $stack_{ik}$
$slt(cnt)$	The slot position ( $slot_{ijk}$ ) where $cnt$ is located. It can be $slot_{ijk}^{(R)}$ for $cnt^{(R)}$

As can be seen in Fig. 2, the indexing convention for ship bays is dependent on the ship profiles. Although the numbering for *above deck* and *below deck* is separated, the numbering for tiers is only even numbers. The numbering for rows starts from inner of the ship bay to outer of the ship bay and progresses in a zig-zag fashion. The modeling in this study applies the same numbering styles.

### 3.3.2 Decision variables

To devise a complete and optimized container loading plan, the decision variables are defined to represent each and every slot on the vessel, as shown in Fig. 2. Each decision variable in the solution is assigned a unique container ID, which is the container to be allocated to the specific container slot.

$x_{ijk}^{od}$	The container ID assigned at ship bay $i$ , tier $j$ , and row $k$ , where its port of origin and port of destination are $o$ and $d$ , respectively. It is represented by a positive integer
$c_{ijk}^p$	0 if the slot of ship bay $i$ , tier $j$ , and row $k$ is empty in port $p$ , and otherwise 1

Therefore, the mathematical formulation for the load capacity planning model is as follows:

*Minimize*

$$-\sum_{p \in P} \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} c_{ijk}^p + \sum_{p \in P} \sum_{i \in I} \sum_{k \in K} re^p(stack_{ik}) + \sum_{p \in P} \sum_{i \in I} \sum_{k \in K} c^p(stack_r^{(CI)}) + \sum_m |y_m| \quad (1)$$

*subject to*

$$c_{ijk}^p - c_{i(j-2)k}^p \leq 0, \forall i, j, k, p \quad (2)$$

$$\sum_{j \in J} w_{ijk}^p \leq lim^w(stack_{ik}), \forall i, k, p \quad (3)$$

$$\sum_{j \in J} h_{ijk}^p \leq lim^h(stack_{ik}), \forall i, k, p \quad (4)$$

$$slt(x_{ijk}^{od}) \neq slot_{ijk} if x_{ijk}^{od} \in CNT^{(R)}, \forall i, j, k, o, d \quad (5)$$

$$w_{ijk}^p - w_{i(j-2)k}^p \leq 0, \forall i, j, k, p \quad (6)$$

$$\sum_{i \in I} \sum_{j \in J} \sum_{k \in \{K|k=odd\}} w_{ijk}^p - \sum_{i \in I} \sum_{j \in J} \sum_{k \in \{K|k=even\}} w_{ijk}^p = y_1, \forall p \quad (7)$$

$$\sum_{i \in I_{bow}} \sum_{j \in J} \sum_{k \in K} w_{ijk}^p - \sum_{i \in I_{stern}} \sum_{j \in J} \sum_{k \in K} w_{ijk}^p = y_2, \forall p \quad (8)$$

$$\sum_{j \in J} \sum_{k \in K} w_{ijk}^p - \sum_{j \in J} \sum_{k \in K} w_{(i+1)jk}^p = y_3, \forall i, p \quad (9)$$

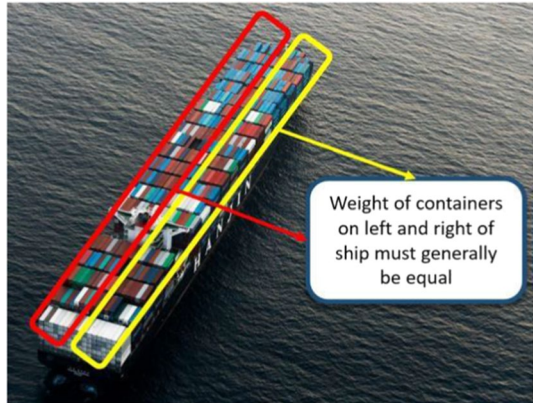
$$\sum_{i \in I_{bow}} \sum_{j \in J} \sum_{k \in \{K|k=odd\}} w_{ijk}^p - \sum_{i \in I_{stern}} \sum_{j \in J} \sum_{k \in \{K|k=even\}} w_{ijk}^p = y_4, \forall p \quad (10)$$

The objective function (1) is made up of three terms. The first term represents the number of empty TEU slots on the ship. Since the revenue derived from a shipping service is directly related to the number of TEUs carried on the vessel, minimization of the number of empty TEU slots will maximize profits from the shipping service. The second term relates to the number of re-stows. Re-stows occur when containers with planned discharge at the current port are located or stored below containers that are only scheduled to be discharged at a later port in the vessel's itinerary. Such a situation leads to containers not scheduled for discharge at the current port being discharged as well in the process, and then reloaded back onto the vessel as a re-stow. By minimizing the number of re-stows, the time needed for container vessels to remain berthed for cargo handling operations can be reduced. This helps to cut cost and increase profits of a shipping service. The third term in the objective function relates to the number of TEUs located in bays that are not covered by cranes, which depends on the crane intensity (CI) at the specific cargo handling operation.<sup>4</sup> The amount of time a ship remains at berth will be shorter with higher quay crane intensity, *ceteris paribus*. Since there is a minimum required distance between quay cranes (Low et al. 2011), there will be regions of bays that each quay crane generally covers and regions that are uncovered. Containers that are positioned at the uncovered bays (i.e., out of the predetermined quay crane working region) give rise to inefficiencies when quay cranes are required to be redeployed or shifted to continue their work. As an illustration, the crane (or cranes) that are adjacent will need to be shifted to give space to the crane that is handling the containers. In addition, an

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<sup>4</sup> The quay crane intensity is an estimation of the number of cranes used to handle a vessel. It is calculated by dividing the total number of container moves by the number of moves the longest crane will perform (Pacino 2018).

**Fig. 3** Illustrated descriptions of horizontal balance for constraint (7)

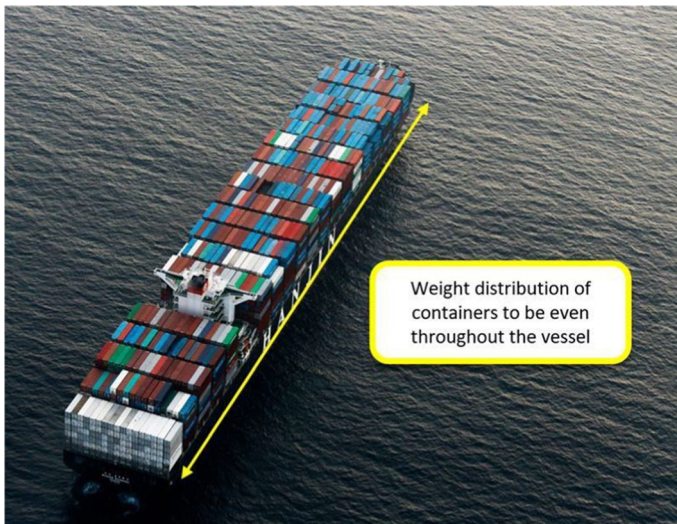


imbalanced distribution of container re-stows will also result in some quay cranes finishing their work ahead of other cranes. The under-optimized work distribution leads to time wastage and increased cost for shipping companies.

Constraints that are identified to be crucial are hard constraints (2)–(6), and they define boundaries that the model solution must meet. Soft constraints (7)–(10), on the other hand, are helpful in enhancing the effectiveness of the solution, but they are not mandatory. More specifically, constraint (2) shows the stacking requirements that each container must meet to be placed on top of another container. Since the index numbering for tiers is only represented in even numbers (Fig. 2), the consecutive tiers are  $j - 2$  and  $j$  instead of  $j - 1$  and  $j$ . Constraint (3) is based on the stack restrictions that each stack of containers below and above deck must not exceed the defined limit (based on ship characteristics). Constraint (4) gives the stack height limits such that each and every column of the containers below and above deck must not be over the defined limit (based on ship structure). Constraint (5) imposes the condition that reefer cargo type must be placed at reefer points (mostly above deck). A reefer container cannot be located at a slot without an electric plug, but a general container could be possibly placed at a reefer slot if available. Constraint (6) is the lashing constraints that require containers stacked on top to be lighter than those under them. Horizontal balance constraint (7) ensures that the weights on the left and right of the ship are generally balanced so that the ship does not tilt to either side (Fig. 3). Indexes for rows ( $k$ ) on the left of the ship are even, while rows ( $k$ ) on the right of the ship are odd. The index number for rows increases from inner to outer of the ship bay in a zig-zag pattern. Front and back balance constraint (8) ensures that the weights in the front and back of the ship are generally balanced such that the weight will not affect the trim of the ship (Fig. 4). Bays on a ship are numbered sequentially from bow to stern. The other ship stability constraints are reflected in (9)–(10). Constraint (9) is imposed to ensure that the weight of all the cargo is spread out equally throughout the vessel and does not concentrate at a certain bay (Fig. 5). Constraint (10) ensures that the front and right (left) of the ship is generally balanced with the back and left (right) of the ship (Fig. 6). Constraints (7)–(10), corresponding to Figs. 3–6 respectively, conceptualize the ship stability



**Fig. 4** Illustrated descriptions of the front and back balance for constraint (8)



**Fig. 5** Illustrated description of overall balance for constraint (9)

measures such as metacentric height, trim, list, bending moment, shear force, and torsion (Pacino 2012; Delgado 2013; Delgado et al. 2012; Parreño et al. 2016).

## 4 Empirical case study

With the world becoming progressively fast-paced, there is a need for loading plans to be generated more quickly to keep the shipping business profitable (Lee et al. 2014). Coupled with the increasingly competitive business environment, there is an urgent necessity for carriers to develop the capabilities to furnish load plans that are backed by mathematical computations to guarantee that profits are maximized

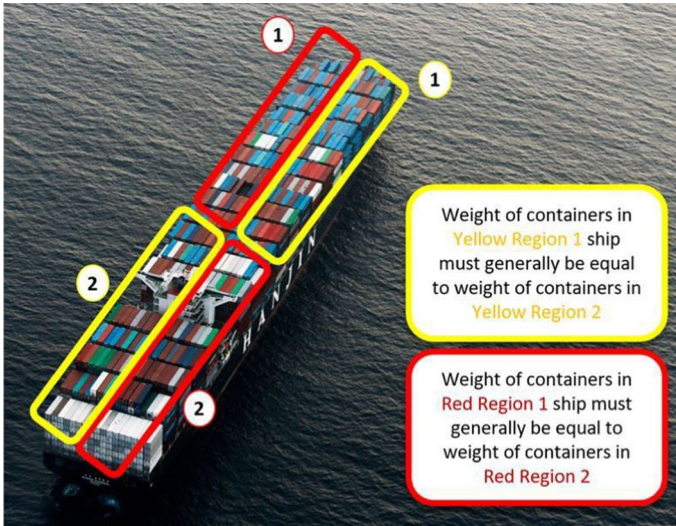


Fig. 6 Illustrated description of overall balance for constraints (10)

(Glave et al. 2014). At present, the capacity planning is typically done using a manual container positioning process with a computerized planning package, which generally takes hours to be completed. The resulting loading plan is highly subjective as it differs substantially among planners according to their judgment and experience. Considering that load planning is an extremely tedious process to be completed manually, it will be useful to devise a support tool to speed up the process of load planning and to ensure that vessel capacity is maximized.

The subsequent empirical case study looks into the contemporary issue of capacity planning at a liner shipping company. The purpose is to achieve maximum vessel utilization via an optimized cargo intake while ensuring that navigational constraints are not breached in all of its shipping services. The data on vessel and cargo profiles for a specific trade route, obtained from a carrier with a significant market presence, help with understanding bay structures, checking stacking restrictions on ships, and generating cargo demand at ports.

#### 4.1 Design of experiment

The proposed model exemplifies a variety of scenarios by experimenting over a range of parameters characterizing the cargo profiles and examining how cargo load capacity will be affected when these parameter values are changed. The sample ship profile used in the experiment is a container ship registered and sailing under the flag of Singapore. The ship possesses a gross tonnage of 151,015 and deadweight of 150,166. The length overall (LOA) is 368.5 m, and the beam is 51 m, along with 86 bays and a container capacity of 13,892 TEUs. In the series of experiments run, a set of 12 ship bays, excluding (standard) middle portion ones but including bays with reefer points, was extracted from a ship profile of 86 bays.

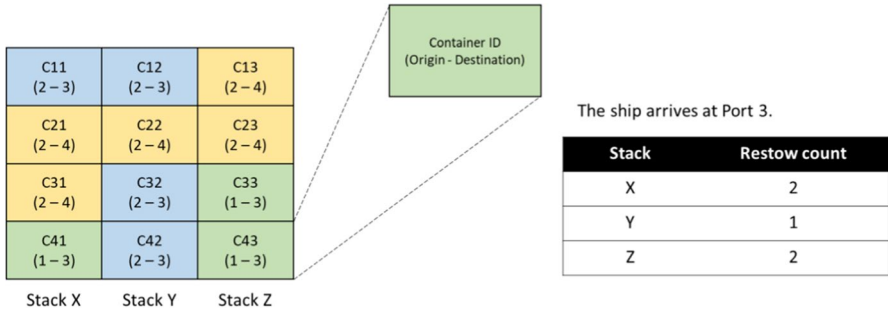


Fig. 7 Counting of re-stows

The basic setting of the experiment is as follows: The ship visits three ports to discharge and load containers where one to two quay cranes are randomly generated from a uniform distribution at each port. The ratio of the number of 20 ft containers to 40 ft containers is set to one to one, and the percentage of reefer containers among them is 10% with a uniform distribution. Each size of container falls into six weight classes (empty, light, medium, heavy, extra heavy, and double extra heavy). We followed industry practices in defining weight classes. For the 20 ft containers, the weights of the empty, light, medium, heavy, extra heavy, and double extra heavy are less than 2.5 tons, between 2.5 and 8 tons, between 8 and 16 tons, between 16 and 24 tons, between 24 and 31 tons, and between 31 and 48 tons, respectively. For 40 ft containers, the corresponding weights of the empty, light, medium, heavy, extra heavy, and double extra heavy will be less than 4.5 tons, between 4.5 and 12 tons, between 12 and 18 tons, between 18 and 24 tons, between 24 and 32 tons, and between 32 and 54 tons, respectively. Uniformly equal probabilities are applied at each weight class for generating containers. Since this experiment only uses a limited number of ship bays that can be covered by one to two quay cranes, it will not be meaningful to measure crane intensity.

In this series of experiments, factors that shipping companies can control based on their shipping plan are identified and adjusted to improve cargo load capacity for their specific trade routes. The performance outcomes are evaluated on two dimensions, namely the number of TEU capacity utilized and the number of TEU re-stowed. As explained earlier, the number of TEU capacity utilized is clearly an important criterion as it directly determines the revenue that can be earned on a voyage. The number of TEU re-stowed indicates the level of inefficiency arising from unnecessary container movements. Figure 7 shows the way that the number of TEU re-stows are tabulated in this model. Suppose that the ship arrives at port 3 and the containers are stacked as in Fig. 7. The containers destined for port 3 should be discharged. In stack X, containers C21 and C31 need to be re-stowed to discharge container C41. After discharging container C41, the two containers are likely to be returned to the bay. In stack Y, the container C22 is located in between the containers that are to be discharged at the current port 3. The container C22 is singled out for re-stow. The same applies to the stack Z. As



the destination of containers C13 and C23 is port 4, these containers also need to be re-stowed to allow the discharge of containers C33 and C43.

The last component of the objective function, i.e., the number of TEUs located in bays that are not covered by cranes, is excluded from the set of performance measures because we are considering a diminished ship profile for this set of experiments. Hence, it would not be feasible to consider quay crane intensity in the forming of regions covered by quay cranes.

Two different search strategies were applied in the proposed model: the depth-first search strategy and the multipoint strategy<sup>5</sup> (Russell and Norvig 2003). The proposed model is implemented in Java API of IBM ILOG CP Optimizer 12.8, and the experiments are run on a computer of 64-bit operating systems with central processing unit (CPU) 1.6 GHz and 1.6 GB memory. Five runs are conducted for each experimental setting, and the average outcomes (i.e., performance measure) for randomness of probabilistic parameters are recorded.

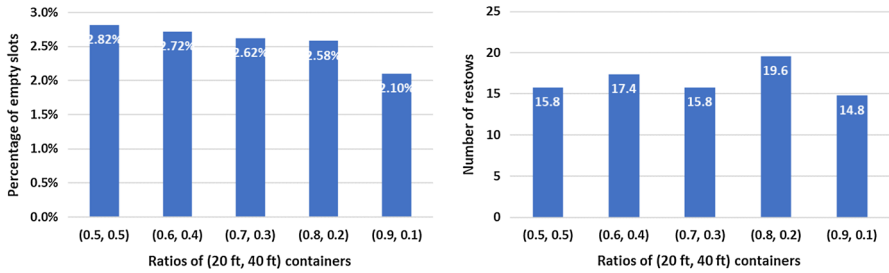
## 4.2 Constraint programming solutions and discussion

### 4.2.1 Configurations of container sizes

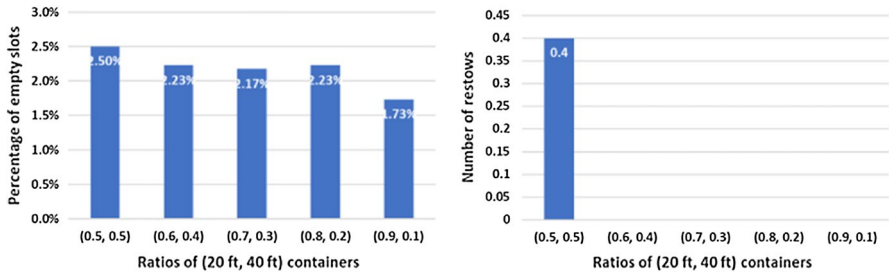
Different ship services and routes encounter different cargo profiles that vary in their distributions of container sizes. For instance, a shipping service from Asia to Europe generally has a higher percentage of 40 ft and OOG cargo compared with a shipping service from Asia to America. In this subsection, the effect of various ratios of 20 ft and 40 ft containers on the performance measures, namely the percentage of empty slots and the number of re-stows, is discussed.

As the percentage of 20 ft containers increases from 50% to 90% with a corresponding decrease in the percentage of 40 ft containers from 50% to 10%, the percentage of empty slots decreases under the two search strategies. The multipoint search strategy reported a reduction in empty slots by an average of 0.18% for every 10% reduction of 40 ft containers, while with the depth-first search strategy empty slots decreased by an average of 0.19% under the same condition (Figs. 8 and 9). Comparing the search strategies, the multipoint search strategy

<sup>5</sup> The depth-first search strategy is a tree search algorithm such that each instantiation of a decision variable can be thought of as a branch in a search tree. The optimizer works on the subtree of one branch until it has found a solution or has proven that there is no solution in that subtree. The optimizer will not move to work on another section of the tree until the current one has been fully explored. For computational efficiency, the termination criterion for the solution search process will be set to the maximum number of branches. The number of branches generating is limited to 100,000 for the two search strategies. On the other hand, the multipoint search strategy creates a set of solutions using the search points and combines the solutions in the set to produce better solutions. The multipoint search strategy is typically known to be more diversified than depth-first, but it does not necessarily prove the optimality or the inexistence of a solution. This experiment utilizes 50 random search points. The rest of the settings for the two search strategies follow the default configurations of the IBM ILOG CP Optimizer (IBM Documentation 2016). It is typically known that the multipoint search strategy is more efficient, but the depth-first search strategy provides better solutions. The experiment adopts the two search strategies as they could represent the effectiveness and efficiency of searching solutions (Russell and Norvig 2003).



**Fig. 8** Percentage of empty slots (left) and the number of re-stows (right) under different configurations of container sizes—the multipoint search strategy

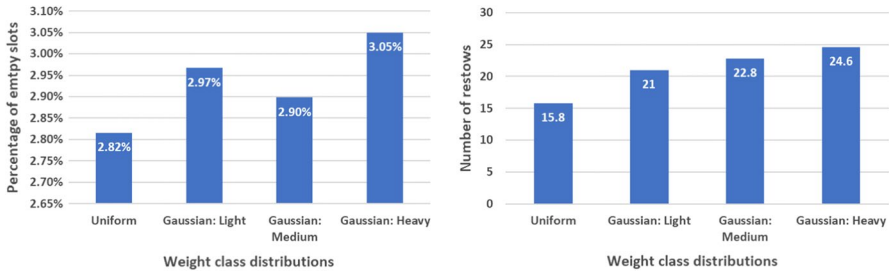


**Fig. 9** Percentage of empty slots (left) and the number of re-stows (right) under different configurations of container sizes—the depth-first search strategy

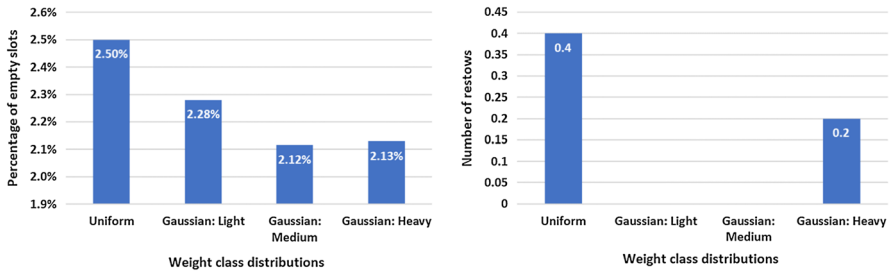
gives a percentage of empty slots that is 18.6% higher than that of the depth-first search strategy across the ratios of 40 ft containers considered in the experiment runs.

The findings can be explained by the fact that the opportunity of filling up the empty slots is expected to decrease when the percentage of 40 ft containers increases. Given that every slot capacity can either accommodate two 20 ft containers or one 40 ft container, the chances that a single slot is going to be occupied by only one 20 ft container instead of two 20 ft containers are higher when the percentage of 40 ft containers increases (or the percentage of 20 ft containers decreases). Once a single 20 ft container occupies a slot, a 40 ft container cannot be located above the 20 ft container. Vice versa, a higher percentage of 20 ft containers relative to that of the 40 ft containers would contribute to higher cargo load capacity.

On the contrary, there is no noticeable trend on the number of re-stows vis-à-vis the percentage breakdown between the two types of containers. This is because re-stows are more likely to occur as a result of the visiting ports sequence instead of the configurations of container sizes. Nevertheless, the depth-first search strategy is found to provide solutions that are significantly better (i.e., 99.5% lower) compared with the multipoint search strategy if the performance is measured by the number of re-stows.



**Fig. 10** Percentage of empty slots (left) and the number of re-stows (right) under configurations of container weight classes—the multipoint search strategy



**Fig. 11** Percentage of empty slots (left) and the number of re-stows (right) under different configurations of container weight classes—the depth-first search strategy

#### 4.2.2 Configurations of weight classes

Other than container sizes, weight distribution of cargo profile also differs among trade routes. This experiment proceeds to consider six weight classes for each size of container and examines the effect of the various distributions of weight classes on number of empty slots onboard and the number of re-stows required. To this end, a uniform distribution is employed. This assigns equal probabilities across all weight classes when containers are generated in the simulation experiments. Other generated weight classes are discretized and bounded between 1 and 6, with the smaller number indicating the lighter class. More specifically, “Gaussian (Light)” applies the normally distributed random probabilities with mean of 2.33, while “Gaussian (Medium)” and “Gaussian (Heavy)” are normally distributed with means of 3.5 and 4.67, respectively. The standard deviations are taken to be 3 in all these “Gaussian” distributions. Note that the generated numbers are discretized by rounding them up to exactly correspond to the weight classes.

Having Gaussian weight distributions is helpful in reducing the percentage of empty slots under the depth-first search strategy. However, the same observation does not apply to the multipoint search strategy. Figure 11 shows a possible reduction in empty slots that is nearly 13% when taking the biased generation of containers in place of the equal probability-based generation on weight classes (even though the marginal reduction is rather insignificant at an average of 0.33%).

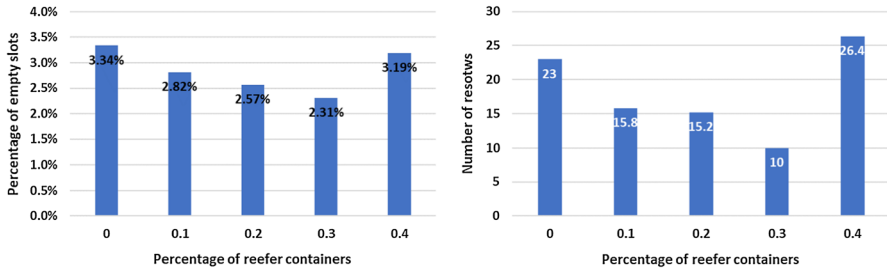
On the other hand, the number of re-stows increases when the weight class is biased towards heavier containers under the multipoint search strategy, or towards a certain weight class under the depth-first search strategy (Fig. 10). Compared with the equal-probability-based generation, the biased generation of weight classes approximated by a Gaussian distribution produces a 44.3% increase in number of re-stows on average under the multipoint search strategy. Particularly, when a higher proportion of heavier weight class containers is generated, the number of re-stows increases by 8.2%. There will be higher chances of a larger number of re-stows when heavier containers are planned to be loaded onto the vessel as the proposed model stacks containers by considering not only the sequence of visiting ports but also weight classes. In relation to the latter, the lashing regulations require heavier containers to be placed below lighter ones. Meanwhile, the depth-first search strategy is able to provide solutions that have minimal or no re-stows (Fig. 11).

### 4.2.3 Configurations of reefer containers

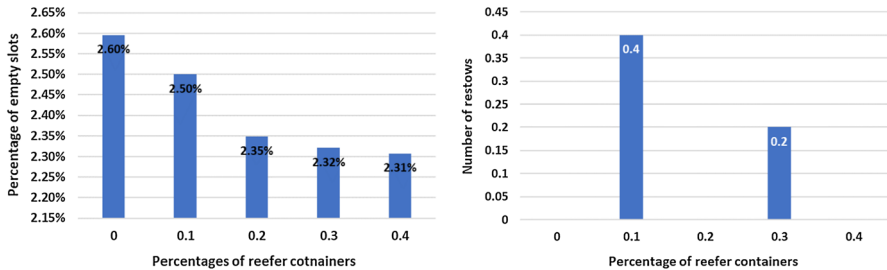
While reefer containers must be loaded in the reefer slots with electricity plugs that control the temperatures, the general containers can also be loaded in reefer slots if they are available. In the experiments that follow, the percentage of reefer containers among the generated containers will be varied between 0% and 40% to examine the effect of these proportions on the percentage of empty slots and the number of re-stows.

When the proportion of reefer containers increases from 0% to 30%, the percentage of empty slots and the number of re-stows decrease under both search strategies. The increased proportion of reefer containers is seen to exert a positive reduction of the solution space as the reefer slots are limited. Once the reefer slots are fully occupied, a search strategy will be left with the task of finding the positions for the remaining dry containers. Such segregation helps the search algorithms to find better solutions, thereby lowering the percentage of empty slots and the number of re-stows required at the same time. However, when the proportion of reefer containers becomes very high (i.e., 0.4 and above), reefer containers need to be randomly abandoned due to slot limitations. The multipoint search algorithm produces container position allocations that result in the number of empty slots increasing by 38.1% and the number of re-stows increasing by 1.64 times on average (Fig. 12).

On the other hand, the depth-first search strategy is able to reduce the percentages of empty slots over the entire range of the percentage of reefer containers. However, the marginal reductions progressively decrease from 10% to 1% as the percentages of reefer containers increase (Fig. 13). Especially in the case of the number of re-stows, the depth-first search strategy provides noticeably better solutions, compared with the multipoint search strategy. Congruent with the existing literature on algorithms, it is found that the depth-first search strategy is able to find better solutions than the multipoint search strategy even though it requires longer response time.



**Fig. 12** Percentage of empty slots (left) and the number of re-stows (right) under different configurations of reefer container quantity—the multipoint search strategy



**Fig. 13** Percentage of empty slots (left) and the number of re-stows (right) under different configurations of reefer container quantity—the depth-first search strategy

## 5 Conclusions

This study presents an initial stage of research on cargo load capacity planning with the ultimate goal of developing a load planning software. In the foreseeable future, the carrier involved in this study will further develop this cargo load capacity planning to devise a capacity sharing plan in a ship among its alliance members. The plan would allocate a certain percentage of the ship space to alliance members based on the carrier's prior knowledge of the shipping network. For the route that (partially) overlaps, the carrier can use some of the ship space to carry cargo on the overlapped part of the sailing itinerary. Even though the space allocation is predetermined, the space occupancy levels can still be flexibly adjusted during the operations. Apart from achieving a better distributed space allocation to alliance members, a well-prepared cargo load capacity plan will potentially improve the efficiency of the shipping logistics business.

In its current form, the model proposed here provides a low-fidelity, but generalized, view of cargo load capacity without being tied down by the exact specifications of the ship profile. For management purposes, these solutions are often sufficient for decision-makers who do not desire the technical details. Although the complete model was not used in this experiment, our cargo load capacity model offers realistic applications, and their solutions provide a good-enough

feel on the loading capacity helpful for fleet planning. It should be noted that, by excluding only the middle portion of the vessel, which is a standard portion, the main shape of the vessel is preserved in the experiment, despite the vessel profile being scaled down. Other main characteristics of the vessel profile, such as the number of slots in a tier, height and weight of a stack, stacks above (and below) deck, and slot types (general or reefers), are also duly considered in the experiment. While the optimal solution search was limited to some extent, the study presents a local optimal solution with a significant number of possible solutions that can be evaluated to achieve the local optimal solution.

The base model, together with the experiments conducted, illustrates the importance of capacity planning and sets the groundwork for further research in this area. Prior to this study, a systematic and consistent comparison of different capacity planning decisions has not been possible, due to the strong element of subjectivity when loading plans are drawn up by different planners that elude a readily available common base of comparison.

In summary, results from the study show that higher percentage of 20 ft containers would contribute to higher cargo load capacity and fewer re-stows simultaneously. On the other hand, a weight-class biased distribution affects the two performance measures differently. While a Gaussian weight-class distribution is helpful in reducing the percentage of empty slots especially under the depth-first search strategy, the number of re-stows also increases when the weight class is biased toward a certain weight class, or the heavier containers. We note that, while the industry trend is towards 40 ft containers, given that the effort to handle a 40 ft container appears to be quite similar to that of a 20 ft container, the actual cost of handling a 40 ft container is higher considering the larger numbers of empty slots and re-stows. This should be reflected in the pricing of the container handling charges. With regard to the container types, an increase in the proportion of reefer containers is beneficial in lowering both the percentage of empty slots and the number of re-stows under the two search strategies if the proportion of reefers is below 30%. However, beyond that threshold, a higher percentage of reefer containers could induce random abandonment of these containers due to the space limitation, giving rise to a higher percentage of empty slots.

Being exploratory in nature, this study is admittedly not without its shortcomings. The current model is implemented as a simplistic three-port problem that considers empty-slot rate, re-stows, and crane intensity violations. Only one ship profile was considered in the construction of the base model and the running of the experiments. Due to its complexity, the model presents solutions under a limited computational budget. The other shortcomings can be more easily addressed. Firstly, several constraints related to ship stability and stress, such as trim, bending moment, and shearing force, etc., influenced by a ship's hull structures require detailed consideration of the entire vessel profile (Pacino 2012; Delgado 2013; Delgado et al. 2012; Parreño et al. 2016). As these are too complex to model by the CP Optimizer, the model in this study has formulated these constraints using a generalized formula. While such formulations may not be entirely reflective of the real-life restrictions computed by commercial stowage planning packages, these constraints could be overcome with integration with commercial packages

to allow for complex constraints to be accurately reflected in the solution search process.

Going forward, more work should be conducted to further improve the completeness of the model and decrease the computational effort for industry practicality. One avenue in which the realism of the current model could be improved is by considering other industry practices such as the use of different types of vessels, lashing requirements for specific vessel types, and the carrying of dangerous cargoes (in particular, to include constraints that account for dangerous cargo that must be stored away from other cargo, highly flammable goods that must be stored away from light, under deck, and away from sources of heat, etc.). Another avenue is to run more experiments using cargo profiles from other trade routes and other vessels of different hull structures. This would allow for a broader understanding of cargo load capacity planning in the global service network of an international carrier. To reduce the computational time and required runtime memory usage of the model, Gent et al. (2006) have recommended the implementation of symmetry breaking and implied constraints in combinatorial CP models similar to those in this study. The design of constraints and the ordering of constraints together with the implementation of high-quality filtering algorithms could also be explored as possible approaches to improve the efficiency of the solution search process of the CP model (Régim 2004). Effectively, this should facilitate a reduction of the required search space during the run of the model and directly improve the efficiency of the model itself in finding the optimal solution.

The development of the capacity planning model will also allow running of experiments with different parameters that may have been impractical (if not, impossible) in the past owing to a time-consuming and labor-intensive but subjective planning process. In the future, there is a possibility that emerging technologies (including artificial intelligence and machine learning) may be applied to automatically generate or identify the ship profiles that are similar to a new ship or a new shipping network. Instead of optimizing cargo load capacity, an alternative approach could be to leverage on the planners' experiences or identify stacking rules to predict the cargo load capacity along a new network.

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