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Jian G. JIN

Qiang MENG

Hai WANG

Singapore Management University, haiwang@smu.edu.sg

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# Column Generation Approach for Feeder Vessel Routing and Synchronization at a Congested Transshipment Port

**Jian Gang Jin**

School of Naval Architecture, Ocean & Civil Engineering,  
Shanghai Jiao Tong University, China

**Qiang Meng**

Department of Civil & Environmental Engineering,  
National University of Singapore, Singapore

**Hai Wang**

School of Information Systems, Singapore Management University, Singapore  
Email: {jiangang.jin@sjtu.edu.cn; ceemq@nus.edu.sg; haiwang@smu.edu.sg}

## 1 Introduction

The global container shipping system is commonly structured as the hub-and-spoke network, in which large container vessels (long-haul services) visit transshipment ports (hubs) while feeder vessels connect the hub with neighboring ports. In the literature on container shipping, most of the works focus on hub-and-spoke network design and long-haul service routing and scheduling [1], while the problem of feeder service design is not well investigated [2]. In this paper, we study the *feeder vessel routing and synchronization problem* (FVRSP) at a congested transshipment port where only fixed time slots are open to feeder vessels. Particular attention is paid to synchronizing the transshipment between long-haul and feeder services as efficient as possible. The key is to adjust the schedules of feeder services with respect to the long-haul service schedules. In doing so, the vessel traffic can be well coordinated and balanced in the temporal dimension, so to avoid schedule conflicts and potential congestions, and improve the transshipment connection between long-haul and feeder services [3]. Another key motivation of this study is to propose a proactive operational strategy for transshipment hub port for the purpose of mitigating the congestion: proactively designating visiting times for feeder services during uncongested periods.

## 2 Set covering model for the FVRSP

In the FVRSP two decisions for designing feeder services are considered: (1) determining the port-call sequence (i.e., routing) and (2) assigning visiting time slot (i.e., synchronization). The former decision concerns the optimal selection of feeder vessels routes (i.e., port-of-call sequence), vessel type (in terms of capacity) and number of vessels to be deployed for maintaining weekly service, and the amount of containers to be picked up and delivered along each route, with the objective of minimizing the operational cost. The latter decision is to determine the hub port calling time for feeder services within the hub port's specified times. Note that this is especially effective for those congested transshipment hub ports to reduce the waiting time of feeder vessels. Besides, in order to ensure efficient transshipment connection (i.e., synchronization) between long-haul and feeder services while respecting the vessel traffic demand at the hub port [4], efficient synchronization should be ensured which helps to keep the transit time for container delivery from origin to destination ports at a minimum level, and thus attract more shipment demand (as is illustrated by Figure 1).

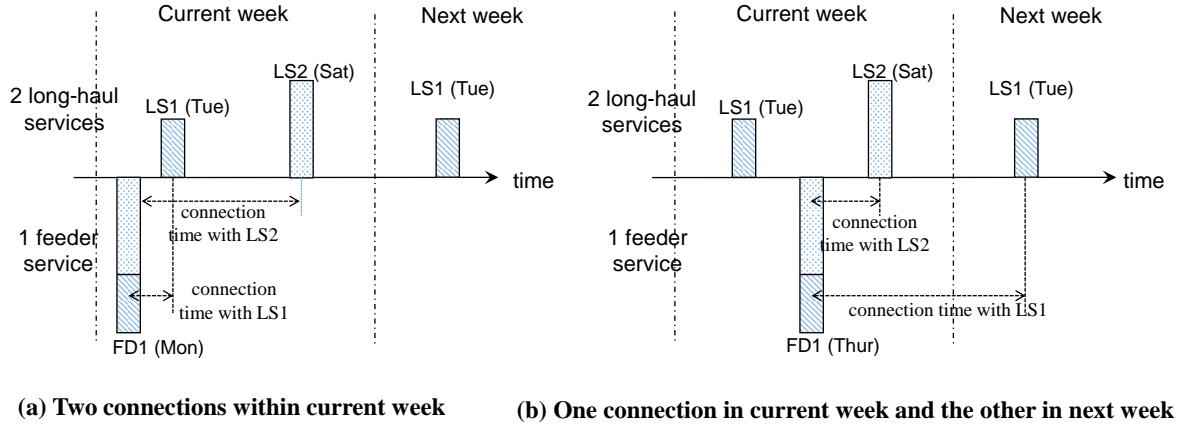


Figure 1: Two cases of transshipment connection between long-haul and feeder services

We develop a set covering model for the FVRSP, in which each column (i.e. variable) is characterized by compact information related with a feeder vessel, including a sequence of port-of-call, vessel type, number of vessels, cycle time and hub visiting time slot assignment. Let  $R$  denote the set of feeder service plans. Define binary decision variable  $\delta_r$  to be 1 if service route  $r$  is employed, and 0 otherwise. Then, the problem can be formulated

as follows:

$$\min \quad \sum_{r \in R} (c_{fixed}^r + c_{variable}^r + c_{connection}^r) \delta_r \quad (1)$$

$$\text{s.t.} \quad \sum_{r \in R} \lambda_i^r \delta_r = 1 \quad \forall i \in N \quad (2)$$

$$\sum_{r \in R} e_s^r \delta_r = 1 \quad \forall s \in S \quad (3)$$

$$\sum_{r \in R} \beta_k^r \delta_r = 1 \quad \forall k \in \Phi \quad (4)$$

$$\delta_r \in \{0, 1\} \quad \forall r \in R \quad (5)$$

The objective function (1) minimizes the sum of the fixed and variable costs of employed feeder services as well as the transshipment connection cost. Constraint (2) ensures that each feeder port is served by one feeder route exactly. Constraint (3) guarantees the total number of each type of vessels respect the fleet size. Constraint (4) restricts that each hub calling time slot can only be assigned to at most one feeder service.

### 3 Column generation based approach

Considering of the exponential number of columns in the set covering model, we employ a column generation algorithm to solve the linear relaxation of the model. The pricing sub-problem needs to be activated for identifying feeder service plans (i.e., columns) with negative reduced cost. Let  $\pi_i^1, \pi_s^2, \pi_k^3$  be the dual variables associated with Constraints (2)-(4), respectively. Define decision variable  $x_{ij}$  be 1 if leg  $(i, j)$  is served;  $e$  as the required number of vessels; and  $y_k$  be 1 if time slot  $k$  is assigned to the route. Then, the reduced cost corresponding to the feeder service plan is:

$$\bar{c} = \sum_{(i,j) \in A} (c_{ij} - \pi_i^1) x_{ij} + (c_s - \pi_s^2) e + \sum_{k \in \Phi} (c_k - \pi_k^3) y_k$$

where  $c_{ij}, c_s$  and  $c_k$  are coefficients associated with link(i.e., voyage) cost, vessel fixed cost and connection cost, respectively. We find the pricing sub-problem is a variant of shortest path problem with negative weights, path dependent cost and time slot assignment related connection cost (corresponding to the above three items). We develop a divide-and-conquer method by enumerating the time slot assignment decision and solving the reduced problem via efficient heuristic method. In order to accelerate the computational efficiency, we further develop an enhanced column generation based heuristic method.

**Column generation scheme (COL0).** The standard column generation procedure is firstly conducted. The master and pricing sub-problems are solved in an interactive

manner until no feeder service plan with negative reduced cost can be found. At this stage, we obtain an initial column set  $R_1$ .

**Extended column generation scheme (COL1).** At the end of the first step, we can find those saturated (i.e., completely utilized) vessel fleet and time slots by identifying those constraints with negative dual variables. For those saturated fleet and time slots, we reduce the right-hand-side of the corresponding constraints by  $m_s/\rho$  and  $1/\rho$ , respectively. With the updated constraints, the column generation procedure is further activated to expand the column set. Such an extended column generation procedure can be activated for  $t$  times and finally obtain the column set  $R_2$ .

**Obtaining integer solutions.** With the column set  $R_2$  generated from the above two steps, we impose the binary integer restriction for the decision variables  $\delta_r$  and employ a post branch-and-bound procedure to find integer solutions.

## 4 Computational tests and remarks

A case study based on the Southeast Asia container shipping network is conducted indicating that the solution method is applicable for solving real-world size feeder network design problems. The developed method outperforms CPLEX both in solution quality and computational efficiency, while CPLEX fails to generate feasible solutions for instances with 15 ports. Results also demonstrate that container transshipment synchronization at the hub port can be significantly enhanced at the cost of introducing marginal changes to the feeder network configuration.

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