3-2005

On “investment decisions in the theory of finance: Some antinomies and inconsistencies”

Bert DE REYCK
Singapore Management University, bdreyck@smu.edu.sg

Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
European Journal of Operational Research 161 (2005) 499–504
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/dsw

Interfaces with Other Disciplines

On "investment decisions in the theory of finance: Some antinomies and inconsistencies"

Bert De Reyck *

London Business School, Regent's Park, London NW1 4SA, UK

Received 23 May 2003; accepted 6 September 2003
Available online 3 December 2003

Abstract

In the paper "Investment Decisions in the Theory of Finance: Some antinomies and inconsistencies", Magni [Eur. J. Operat. Res. 137 (2002) 206] shows that using the net present value rule for making investment decisions can lead to inconsistencies and antinomies. The author claims that the so-called equivalent-risk tenet of finance, whereby an investor needs to compare an investment opportunity with an asset of equivalent risk, is impossible to implement. In this paper, we show that the main thesis of this paper is incorrect, and that finance theory, when applied correctly, can be used to value investment projects by comparing assets of equivalent risk. We point out the fallacies in the author's reasoning and provide an alternative, and correct, methodology for valuing the projects described in the paper.

Keywords: Finance; Valuation; Net present value; Real options; Equivalent risk

1. Introduction

The net present value rule states that a project should only be undertaken when the discounted value of all the project's cash flows, including the required investments, exceeds zero, whereby a *risk-adjusted* discount rate should be used that is appropriate for the level of risk in the project. This rule is based on the concept of opportunity cost; if there is an alternative investment with higher net present value and equal risk, or with equal net present value and lower risk, the project should not be undertaken. Therefore, financial theory suggests comparing an investment opportunity with other investments or assets with equivalent risk to determine whether or not the investment should be made. The expected rate of return of the equivalent asset(s) then determines the hurdle rate, or risk-adjusted discount rate for the investment opportunity at hand. This leads to what Magni (2002) calls the *equivalent-risk tenet of finance*: "It is not legitimate to compare two different assets with different risks in order to solve the decision problem; we have to render the comparison homogeneous by finding an alternative comparable (in terms of risk) to line of action we are offered." The author then continues by showing that this investment decision rule leads to inconsistencies and antinomies, followed by a conclusion that the equivalent-risk tenet of finance is impossible to implement and that "[we should] reconsider this

* Tel.: +44-20-7706-6884; fax: +44-20-7724-7875.
E-mail address: bdereyck@london.edu (B. De Reyck).

0377-2217/$ - see front matter
doi:10.1016/j.ejor.2003.09.006

index, currently considered a pillar of modern finance theory".

We will show in this paper that the alleged inconsistencies are a result of an incorrect valuation method described in the paper, and develop a method that allows valuing the projects described in the paper correctly. As a result, we refute Magni's claim that it is impossible to implement the equivalent-risk tenet of finance. In Section 2, we show the fallacies contained in Section 2 of Magni (2002), where an antinomy is presented. We also propose a valuation method that can be used for valuing the projects described in the paper. In Section 3, we generalise our findings for a general framework of investment projects. Finally, in Section 4, we provide an intuitive explanation of the erroneous reasoning in Magni (2002).

2. An antinomy revisited

2.1. An example

In Section 2 ("An antinomy") of Magni (2002), the author develops an example to illustrate the inconsistencies embedded in the net present value rule. Three mutually exclusive non-deferrable projects A, B and C are introduced. Project A requires an initial cash outlay of 100 at time zero ($s = 0$) with subsequent cash inflows $\tilde{x}_s$ at times s, $s = 1, 2, \ldots, 8$:

$$\tilde{x}_s = \tilde{x} = \begin{cases} 30, & \text{with probability } 0.6 \\ 10, & \text{with probability } 0.4 \end{cases} \quad \text{for all } s$$

$E(\tilde{x}) = 22$. Project A is financed by a loan equal to 40, repaid by three instalments of 20 at times 1, 2 and 3. The risk-adjusted discount rate for project A, reflecting the rates of return of equivalent-risk assets in capital markets, is denoted by i and the discount rate for the loan, reflecting the market rates for interest of similar loans, by δ. The net present value of the levered project equals:

$$G_A = -100 + \sum_{s=1}^{8} \frac{22}{(1+i)^s} + 40 - \sum_{s=1}^{3} \frac{20}{(1+\delta)^s}. \tag{2.1}$$

Project B contains two projects, $B1$ and $B2$, where $B1$ is riskless with cash flows -40, -10 and 10 at time 0, 1 and 2, respectively, followed by five cash inflows equal to 10 from time 4 to time 8. $B2$ has an initial outlay of 20 with cash inflows at time $s = 1, 2, \ldots, 9$ equal to $\tilde{y}_s = \tilde{x} - 10$, $s = 1, 4, 5, 6, 7, 8$, $\tilde{y}_2 = \tilde{x} - 30$, and $\tilde{y}_3 = \tilde{x} - 20$. The author then states that the net present value of project B equals:

$$G_B = -40 - \frac{10}{1+r_f} + \frac{10}{(1+r_f)^2} + \sum_{s=4}^{8} \frac{10}{(1+r_f)^s} - 20 + \frac{12}{1+i} - \frac{8}{(1+i)^2} + \frac{2}{(1+i)^3} + \sum_{s=4}^{3} \frac{12}{(1+i)^s}. \tag{2.2}$$

The rationale behind this formula is that the risk-free rate is appropriate for the risk-free cash flows of the project, whereas the risky cash flows bear the same risk as the ones in project A (cash flow series $\tilde{x}$), so that the same discount rate can be used.

Project C requires an initial cash outlay of 60, followed by cash flows $\tilde{z}_s = \tilde{x} - 20$, $s = 1, 2, 3$, and $\tilde{z}_s = \tilde{x}$, $s = 4, 5, 6, 7, 8$. The author states again that the risk of the cash flows z is identical to the risk in cash flows x, so that the net present value of project C equals:

$$G_C = -60 + \sum_{s=1}^{3} \frac{2}{(1+i)^s} + \sum_{s=4}^{8} \frac{22}{(1+i)^s}. \tag{2.3}$$

The cash flows of the different projects are depicted in Table 1.

Since in general the discount rates i, δ and r_f are different, so are the net present values of the three projects according to formulas (2.1)–(2.3). However, the net cash flows for the three projects in time periods 1 through 8 are identical, which should result in identical net present values. Hence, an antinomy is shown.

2.2. Correct analysis

The flaw in the analysis above is the fact that actually, the cash flows of projects A, B and C do not exhibit the same level of risk, so that the use of

Table 1
Cash flows of projects A, B and C

	$s=0$	$s=1$	$s=2$	$s=3$	$s=4$	$s=5$	$s=6$	$s=7$	$s=8$
A	−100	$\tilde{x}$	$\tilde{x}$	$\tilde{x}$	$\tilde{x}$	$\tilde{x}$	$\tilde{x}$	$\tilde{x}$	$\tilde{x}$
Loan A	40	−20	−20	−0					
$B1$	−40	−10	10		10	10	10	10	10
$B2$	−20	$\tilde{x}-10$	$\tilde{x}-30$	$\tilde{x}-20$	$\tilde{x}-10$	$\tilde{x}-10$	$\tilde{x}-10$	$\tilde{x}-10$	$\tilde{x}-10$
C	−60	$\tilde{x}-20$	$\tilde{x}-20$	$\tilde{x}-20$	$\tilde{x}$	$\tilde{x}$	$\tilde{x}$	$\tilde{x}$	$\tilde{x}$

the same discount rate is not warranted. True, the variance in the cash flows is identical, but the variance in the returns offered by the three projects is not. Finance theory and the Capital Asset Pricing Model (Sharpe, 1965; Lintner, 1965) define risk as a function of returns, not cash flows.

In the example, it is assumed that the appropriate risk-adjusted discount factor for cash flows $\tilde{x}_s$, $s=1,2,\ldots,8$, is equal to i. This does not mean, however, that this is also the appropriate discount rate for cash flows $\tilde{y}_s=\tilde{x}-10$, $s=1$, 4, 5, 6, 7, 8, $\tilde{y}_2=\tilde{x}-30$, or $\tilde{y}_3=\tilde{x}-20$. We will now derive a correct valuation of cash flows $\tilde{y}_s$, $s=1,2,\ldots,8$.

Proposition 1. *Let project X be a one-period project resulting in cash flows $\tilde{x}=(x_1,x_2)$, with probabilities p and $1-p$, respectively, and let i be project X's cost of capital, obtained through the market valuation of a security or project with exactly the same payoff pattern. Project Y with cash flows $\tilde{y}=(y_1,y_2)$ and probabilities p and $1-p$ can then be valued as follows:*

$$G_Y = G_{\tilde{y}} = \frac{E(\tilde{y}) - \rho(\tilde{y},\tilde{x})(i-rf)G_{\tilde{x}}\frac{\sigma(\tilde{y})}{\sigma(\tilde{x})}}{1+rf},$$
$$\rho(\tilde{y},\tilde{x}) \in \{-1,1\}, \tag{2.4}$$

where $E(\tilde{y})$ denotes the expected value of cash flows $\tilde{y}$ and rf is the risk-free rate.

Proof. The Capital Asset Pricing model (Sharpe, 1965; Lintner, 1965) allows to value the cash flows of a project using Certainty Equivalents and the following formula (Brealey and Myers, 2000, p. 248):

$$G_{\tilde{y}} = \frac{E(\tilde{y}) - \lambda \operatorname{cov}(\tilde{y}, r\tilde{m})}{1+rf} \quad \text{with } \lambda = \frac{E(r\tilde{m})-rf}{\sigma^2(r\tilde{m})}, \tag{2.5}$$

where $r\tilde{m}$ denotes the market return and λ is the so-called market price of risk. This formula can be derived from the present value of the project:

$$G_{\tilde{y}} = \frac{E(\tilde{y})}{1+E(r\tilde{y})} \tag{2.6}$$

with $r\tilde{y}=(ry_1,ry_2)$ the returns of the project in each project state, and $E(r\tilde{y})$ the expected return:

$$\begin{aligned} E(r\tilde{y}) &= rf + \beta(\tilde{y})(E(r\tilde{m})-rf) \\ &= rf + \frac{\operatorname{cov}(r\tilde{y}, r\tilde{m})}{\sigma^2(r\tilde{m})}(E(r\tilde{m})-rf), \end{aligned} \tag{2.7}$$

where $\beta(\tilde{y})$ denotes the beta of project Y. Substituting (2.7) in (2.6) and using $ry_s=\frac{y_s}{G_{\tilde{y}}}-1$, $s=1,2$, such that $\operatorname{cov}(r\tilde{y},r\tilde{m})=\frac{\operatorname{cov}(\tilde{y},r\tilde{m})}{G_{\tilde{y}}}$, gives:

$$G_{\tilde{y}} = \frac{E(\tilde{y})}{1+\left(rf+\frac{\operatorname{cov}(\tilde{y},r\tilde{m})}{G_{\tilde{y}}\sigma^2(r\tilde{m})}(E(r\tilde{m})-rf)\right)},$$

which results into formula (2.5).

Because of the binomial nature of the uncertainty in the cash flows generated by projects X and Y, we have that $\rho(\tilde{y},\tilde{x})\in\{-1,1\}$. Consequently: $\rho(\tilde{y},r\tilde{m})=\rho(\tilde{y},\tilde{x})\rho(\tilde{x},r\tilde{m})$ or:

$$\operatorname{cov}(\tilde{y}, r\tilde{m}) = \rho(\tilde{y},\tilde{x})\operatorname{cov}(\tilde{x}, r\tilde{m})\frac{\sigma(\tilde{y})}{\sigma(\tilde{x})}. \tag{2.8}$$

Moreover:

$$\begin{aligned} i &= E(r\tilde{x}) \\ &= rf + \beta(\tilde{x})(E(r\tilde{m})-rf) \quad \text{or} \quad \beta(\tilde{x}) = \frac{i-rf}{E(r\tilde{m})-rf} \end{aligned} \tag{2.9}$$

and also:

$$\beta(\tilde{x}) = \frac{\operatorname{cov}(r\tilde{x}, r\tilde{m})}{\sigma^2(r\tilde{m})} = \frac{\operatorname{cov}(\tilde{x}, r\tilde{m})}{P(\tilde{x})\sigma^2(r\tilde{m})}, \tag{2.10}$$

where $r\tilde{x} = (rx_1, rx_2)$ denote the project returns in the different scenarios.

Using (2.9) and (2.10), we obtain:

$$\mathrm{cov}(\tilde{x}, r\tilde{m}) = \frac{k - rf}{E(r\tilde{m}) - rf} P(\tilde{x})\sigma^2(r\tilde{m}). \tag{2.11}$$

Substituting (2.11) into (2.8) and the result into (2.5), gives, after some algebra, the result (2.4). □

De Reyck et al. (2003) use formula (2.4) to value real options in projects represented as binomial trees. If we apply formula (2.4) to cash flows $\tilde{x}_1$ and $\tilde{y}_1$ in the example, with $\rho(\tilde{y}_1, \tilde{x}_1) = 1$ and $\sigma(\tilde{x}_1) = \sigma(\tilde{y}_1)$, we obtain:

$$G_{\tilde{y}_1} = \frac{E(\tilde{y}_1) - (i - rf)G_{\tilde{x}_1}}{1 + rf}$$
$$= \frac{E(\tilde{x}_1) - 10 - (i - rf)\frac{E(\tilde{x}_1)}{1+i}}{1 + rf} = \frac{E(\tilde{x}_1)}{1 + i} - \frac{10}{1 + r_f},$$

and not

$$G_{\tilde{y}_1} = \frac{E(\tilde{y}_1)}{1 + i} = \frac{E(\tilde{x}_1) - 10}{1 + i}$$

as proposed by Magni (2002). As a result, the net present values for all three projects are identical.

3. Generalising

3.1. Generic example

In Section 3 (“Generalising”) of Magni (2002), the author provides the following generic three-project situation. Consider an investment schema A consisting of certain cash flow a_0 at time 0 and cash flows $\tilde{a}_s$ at time s, $s = 1, 2, \ldots, n$. Project A is financed by a loan contract with debt cash flows $\tilde{f}_s$ at time s, $s = 0, 1, \ldots, n$. The risk-adjusted discount rates for the project and the loan are i and δ, respectively. The net present value of the levered project is:

$$G_A = \sum_{s=0}^{n} \frac{a_s}{(1 + i)^s} + \sum_{s=0}^{n} \frac{f_s}{(1 + \delta)^s}, \tag{3.1}$$

where $a_s = E(\tilde{a}_s)$ and $f_s = E(\tilde{f}_s)$, $s = 1, 2, \ldots, n$.

Investment schema B consists of two projects, $B1$ and $B2$. $B1$ yields a certain sequence b_s at time s, $s = 0, 1, \ldots, n$. $B2$ generates a certain cash flow c_0 at time 0 and subsequent random cash flows $\tilde{c}_s$ at time s, $s = 1, 2, \ldots, n$. We then have:

$$G_B = \sum_{s=0}^{n} \frac{b_s}{(1 + r_f)^s} + \sum_{s=0}^{n} \frac{c_s}{(1 + j)^s}, \tag{3.2}$$

where $c_s = E(\tilde{c}_s)$, $s = 1, 2, \ldots, n$, and j the appropriate risk-adjusted discount rate for the cash flow series $\tilde{c}_s$. A third project C consists of an initial flow r_0 at time 0 and a stream of cash flows $\tilde{r}_s$ at time $s = 1, 2, \ldots, n$. We then have:

$$G_C = \sum_{s=0}^{n} \frac{r_s}{(1 + y)^s} \tag{3.3}$$

with $r_s = E(\tilde{r}_s)$, $s = 1, 2, \ldots, n$, and y the risk-adjusted discount rate for the cash flow series $\tilde{r}_s$.

If we assume that the following conditions hold:

$$b_0 = a_0 + f_0 - c_0$$

$$r_0 = a_0 + f_0$$

$$b_s = \tilde{f}_s + k_s \quad \forall s,\ 1 \leqslant s \leqslant n,$$

$$\tilde{c}_s = \tilde{a}_s - k_s \quad \forall s,\ 1 \leqslant s \leqslant n,$$

$$\tilde{r}_s = \tilde{a}_s + \tilde{f}_s \quad \forall s,\ 1 \leqslant s \leqslant n$$

with $k_s \in \Re$, $s = 1, 2, \ldots, n$, then the cash flows generated by the three projects are identical, and should have identical net present values. However, the author argues that because $\tilde{a}_s$, $\tilde{c}_s$ and $\tilde{r}_s$ have the same risk, $y = j = i$ holds. Hence, (3.1)–(3.3) result in different net present values.

3.2. Correct analysis

The reasoning that the cash flow series $\tilde{a}_s$, $\tilde{c}_s$ and $\tilde{r}_s$ have the same risk and therefore warrant the same risk-adjusted discount rate is not correct. Using (2.4), a correct valuation of cash flow $\tilde{c}_1$, yields:

$$G_{\tilde{c}_1} = \frac{E(\tilde{c}_1) - \rho(\tilde{c}_1, \tilde{a}_1)(i - rf)G_{\tilde{a}_1}\frac{\sigma(\tilde{c}_1)}{\sigma(\tilde{a}_1)}}{1 + rf},$$

with $\rho(\tilde{c}_1, \tilde{a}_1) = 1$ and $\sigma(\tilde{c}_1) = \sigma(\tilde{a}_1)$, so that:

$$G_{\tilde{c}_1} = \frac{E(\tilde{c}_1) - (i - rf)G_{\tilde{a}1}}{1 + rf} = \frac{a_1 - k_1 - (i - rf)\frac{a_1}{1+i}}{1 + rf} = \frac{a_1}{1 + i} - \frac{k_1}{1 + rf}. \quad (3.4)$$

From (3.2), we derive:

$$G_{\tilde{c}_1} = \frac{c_1}{1 + j} = \frac{a_1 - k_1}{1 + j}. \quad (3.5)$$

Combining (3.4) and (3.5), we obtain that $i \neq j$ (if $i \neq r_f$). The same applies to $i \neq y$ and $j \neq y$.

4. The framing

In Section 4, the author provides an intuitive explanation for the cause of the inconsistencies, where the alleged biases of the net present value methodology are uncovered. The author claims that a different investment valuation results if the problem is framed differently, which would be inconsistent with financial theory.

4.1. Framework

The author uses the following framework. Suppose we expect to receive a sum $a = E(\tilde{a})$ at time s. Its present value is $\frac{a}{(1+i)^s}$, with i the appropriate risk-adjusted discount rate. Suppose we also receive the certain sum k at time s. Its present value is $\frac{k}{(1+r_f)^s}$. Summing the two values yields the net present value of $\tilde{a} + k$. The author then states that a project where you receive a sum $\tilde{b}$ equal to $\tilde{a} + k$ at time s is equivalent in risk to $\tilde{a}$, and computes the net present value of this future sum as $\frac{b}{(1+i)^s}$ with $b = E(\tilde{b})$. Because $\frac{b}{(1+i)^s} = \frac{a+k}{(1+i)^s} \neq \frac{a}{(1+i)^s} + \frac{k}{(1+r_f)^s}$ (except when $i = r_f$), an antinomy is shown.

4.2. Correct analysis

The flaw in this reasoning lies where the author states that b and a are equivalent in risk. In financial investment analysis, risk is defined as the variance of a project's or asset's *return*. Although the variance in the cash flows of b and a are identical, this is not true for their returns. In fact, the returns of $\tilde{b}$ in the two scenarios, $r\tilde{b} = (rb_1, rb_2)$, equal $rb_1 = \frac{b_1}{G_{\tilde{b}}} = \frac{a_1+k}{G_{\tilde{b}}}$ and $rb_2 = \frac{b_2}{G_{\tilde{b}}} = \frac{a_2+k}{G_{\tilde{b}}}$, which are clearly different from the returns of $\tilde{a}$: $r\tilde{a} = (ra_1, ra_2)$, $ra_1 = \frac{a_1}{G_{\tilde{a}}}$ and $ra_2 = \frac{a_2}{G_{\tilde{a}}}$.

Application of formula (2.4) for $s = 1$ gives:

$$G_{\tilde{b}} = \frac{b - \rho(\tilde{b}, \tilde{a})(i - rf)G_{\tilde{a}}\frac{\sigma(\tilde{b})}{\sigma(\tilde{a})}}{1 + rf} = \frac{b - (i - rf)G_{\tilde{a}}}{1 + rf} = \frac{a + k - (i - rf)\frac{a}{1+i}}{1 + rf} = \frac{a}{1 + i} + \frac{k}{1 + rf}.$$

And for arbitrary s: $G_{\tilde{b}} = \frac{a}{(1+i)^s} + \frac{k}{(1+rf)^s}$, the correct result.

Consequently, no matter how ones perceives a project, i.e. comprised of a risk-free and a risky component, the valuation remains the same. Therefore, different frames for the same investment project do not result in inconsistent valuations, as proposed by the author. Also the principle of additivity is still valid.

4.3. Clarifying example

An example will clarify this. Suppose we have a project that delivers a cash flow $\tilde{x}$ one period later equal to 20 or 40 with equal probabilities, and that a discount rate of $i = 10\%$ is the correct risk-adjusted discount rate for this project. This results in a present value of $G_{\tilde{x}} = \frac{30}{1.1} = 27.273$. This project can also be seen as yielding a certain cash flow y of 20 and an additional risky cash flow $\tilde{z}$ equal to 0 or 20 with equal probability. If we assume that $\tilde{z}$ is equivalent in risk compared to $\tilde{x}$, we can use the same discount rate and obtain: $G_{\tilde{z}} = \frac{10}{1.1} = 9.091$. If we assume the risk-free rate is 5%, this would mean that the value of y and $\tilde{z}$ combined would be $G_{y+\tilde{z}} = \frac{20}{1.05} + \frac{10}{1.1} = 28.139$, which is inconsistent with the value of $G_{\tilde{x}}$. Using formula (2.4) instead, we obtain:

$$G_{\tilde{z}} = \frac{E(\tilde{z}) - \rho(\tilde{z}, \tilde{x})(i - rf)G_{\tilde{x}}\frac{\sigma(\tilde{z})}{\sigma(\tilde{x})}}{1 + rf} = \frac{10 - (0.05)27.273\frac{10}{10}}{1.05} = 8.225.$$

Consequently, $G_{\tilde{x}} = 27.273 = G_y + G_{\tilde{z}} = 19.048 + 8.225$.

Clearly, despite the fact that the variance of $\tilde{x}$ and $\tilde{z}$ are the same, their risk is different. If we calculate the returns generated by $\tilde{x}$, we obtain $\frac{20}{G_x} - 1 = -26.66\%$ and $\frac{40}{G_x} - 1 = 46.66\%$, whereas the returns generated by $\tilde{z}$ are $\frac{0}{G_z} - 1 = -100\%$ and $\frac{20}{G_{\tilde{z}}} - 1 = 143.16\%$. Clearly, z entails much higher risk, warranting a higher risk-adjusted discount rate, namely 21.58% instead of 10%.

Alternatively, we can use the Capital Asset Pricing Model to derive the same result: $E(r\tilde{x}) = rf + \beta(\tilde{x})(E(r\tilde{m}) - rf)$, or $0.1 = 0.05 + \beta(\tilde{x})\,(E(r\tilde{m}) - rf)$, or $\beta(\tilde{x})(E(r\tilde{m}) - rf) = 0.05$. We have that $\beta(\tilde{x}) = \frac{\mathrm{cov}(r\tilde{x},r\tilde{m})}{\sigma^2(r\tilde{m})} = \frac{\rho(r\tilde{x},r\tilde{m})\sigma(r\tilde{x})\sigma(r\tilde{m})}{\sigma^2(r\tilde{m})}$ and similarly $\beta(\tilde{z}) = \frac{\rho(r\tilde{z},r\tilde{m})\sigma(r\tilde{z})\sigma(r\tilde{m})}{\sigma^2(r\tilde{m})}$. We have $\rho(r\tilde{x}, r\tilde{m}) = \rho(r\tilde{z}, r\tilde{m})$, $\sigma(r\tilde{x}) = 0.3666$ and $\sigma(r\tilde{z}) = 1.2158$, so that $\beta(\tilde{z}) = 3.316\beta(\tilde{x})$, and $E(r\tilde{z}) = rf + \beta(\tilde{z})(E(r\tilde{m}) - rf) = rf + 3.316\beta(\tilde{x})(E(r\tilde{m}) - rf) = 0.05 + 3.316 \times 0.05 = 21.58\%$ Clearly, the high volatility in the returns associated with the cash flows $\tilde{z}$ result in a higher risk-adjusted discount rate. The value y and $\tilde{z}$ combined is now $G_{y+\tilde{z}} = \frac{20}{1.05} + \frac{10}{1.2158} = 27.27$, which is consistent with the value of $G_{\tilde{x}}$.

5. Conclusions

We have shown that the claim by Magni (2002), that applying the net present value rule for making investment decisions leads to inconsistencies and antinomies, is not valid. Magni (2002) presents several examples and generic frameworks to prove and illustrate these inconsistencies, but the analysis presented in the paper is flawed. In fact, the net present value rule, when applied correctly, can be used to value investment projects by comparing assets of equivalent risk. The flaw in the reasoning in the abovementioned paper lies in the fact that projects that yield identical cash flows except for a constant, deterministic, factor are not equivalent in risk. Risk, as defined in the Capital Asset Pricing Model, is defined in terms of returns generated by the project. These returns are affected by adding or subtracting a constant cash flow. Therefore, the claim made by Magni (2002) that the equivalent-risk tenet of finance is impossible to implement, is incorrect. We have shown that the analysis presented in Sections 2, 3 and 4 of Magni (2002) is incorrect, and we have provided an alternative, and correct, valuation method for the projects and investment schemes presented in the paper.

References

Brealey, R.A., Myers, S.C., 2000. Principles of Corporate Finance, sixth ed. McGraw-Hill, London.

De Reyck, B., Degraeve, Z., Vandenborre, R., 2003. Project Options Valuation with Net Present Value and Decision Tree Analysis, Working Paper, London Business School.

Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. Review of Economics and Statistics 47, 13–37.

Magni, C.A., 2002. Investment decisions in the theory of finance: Some antinomies and inconsistencies. European Journal of Operational Research 137, 206–217.

Sharpe, W.F., 1965. Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance 19, 425–442.