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Integrated optimization of farmland cultivation and fertilizer application: Implications for farm management and food security

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Abstract

Motivated by the fresh produce industry, this paper studies a farmer's joint cultivation and fertilizer (a representative farm input) application decisions facing uncertainties in crop's open market price, harvesting cost, and farm yield, where yield is stochastically increasing in the fertilizer application rate. We develop a two-stage stochastic program that captures the trade-offs facing a farmer growing a commodity crop in a single season to maximize the expected profit. We then use the model to evaluate the expected optimal harvest volume (a measure of food security). Our analytical analysis is complemented with numerical experiments calibrated to data. We characterize how the farmer's optimal decisions, profitability as well as the expected optimal harvest volume are affected by fertilizer and cultivation costs and farm yield uncertainty. We find that these effects can be counterintuitive and significantly different from those when only cultivation decision is optimized (as considered in the extant literature); specifically when these effects induce the farmer to change the two decisions in opposite directions. For example, an increase in fertilizer cost may incent the farmer to cultivate more farmland. Another example is that a reduction in cultivation cost or yield variability may decrease the expected optimal harvest volume. This result is useful for policymakers as it demonstrates that commonly used policies in practice, such as distributing discount vouchers for seed procurement (which reduces the cultivation cost) or increasing the availability of disease-resistant seeds (which reduces yield variability) that have been devised for increasing crop production level may backfire.

Keywords: Farm planning, agriculture, integrated optimization, fertilizer, yield uncertainty, price uncertainty, open market, fresh produce, food security, harvesting

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1 Introduction

In this paper, we study the decisions related to farm planning for a farmer growing a commodity crop in a single season. Although applicable to several agricultural industries, our analysis is motivated by the fresh produce (e.g., fruits and vegetables) industry, one of the largest agricultural industries in the world. In the U.S. alone, for instance, this industry is valued at $104.7 billion in 2016, and annual per capita consumption of fruits and vegetables has been increasing at a much faster pace than that of traditional crops (such as wheat), due to, for example, growing awareness of more balanced diets and availability of more disposable income (Ahumada and Villalobos, 2009).

Farm planning in the fresh produce industry involves several decisions that present challenges for the farmer. At the beginning of the growing season, the farmer needs to decide the *size of farmland to cultivate* incurring a cultivation cost—which accounts for plowing and tilling of farmland as well as procurement and sowing of seeds—while facing uncertainty in farm yield due to unfavorable weather conditions and infestation of pest and diseases. At this stage, the farmer also needs to decide the *quantity of farm inputs to apply* on the cultivated farmland, including fertilizer (to increase soil's nutrients), pesticides (to protect the crop from pests), and herbicides (to protect the crop from weeds). One common feature of these farm inputs is that their application, though costly, improves the yield. In this paper, we consider fertilizer as a representative farm input and focus on fertilizer application decision. At the end of the growing season, after the yield is realized, the farmer needs to decide the *harvest volume* based on the crop's profit margin; that is, the difference between crop's open market price and harvesting cost. In practice, significant fluctuations in open market price are commonly observed for a variety of reasons, including its dependence on the farmer's yield (Kazaz and Webster, 2011)—as the yields for other farmers in close proximity share similar characteristics and collectively affect the crop's aggregate supply—as well as changes in macroeconomic conditions and industry regulations (Li et al., 2022). The harvesting cost also shows significant variation specifically when the farmer needs to acquire additional harvesting resources (beyond what is available internally) at the end of the growing season because, crucially, the acquisition cost depends on the demand for these resources. In the fresh produce industry, because most fruits and vegetables are harvested by hand, labor is the most critical resource among the harvesting resources (that also include containers, harvesters, and carriers) and labor cost is the main determinant of harvesting cost.[1] In this industry, the limiting nature of harvesting labor is empirically well documented (see, for example, Gunders and Bloom (2012)) and it is common

[1]For example, in the U.S. fresh produce industry labor costs account for about 42% of the variable production expenses for farms (Calvin and Martin, 2010).

practice to hire seasonal workers for harvesting at the end of the growing season. The labor cost of these seasonal workers shows significant variation (see, for example, Richards (2018)) because it depends on the farmer's yield: the yields for other farmers in close proximity share similar characteristics and collectively affect the demand for these workers. In summary, uncertainties in crop's open market price and harvesting cost should also be taken into account in making the cultivation and fertilizer application decisions.

In recent years farm planning in agricultural industries, including the fresh produce industry have experienced additional challenges. As reported by Hayashi (2022), costs and uncertainties in the farming environment have increased across all agricultural industries due to a variety of factors, including severe weather events, regional conflicts, and the Covid-19 pandemic. It is common knowledge that global warming and climate change have led to an unprecedented number of climate-induced shocks across the globe and, as highlighted by Tigchelaar et al. (2018), these shocks are a major contributor to increasing crop yield variability worldwide. As illustrated in Figure 2 of the U.S. Department of Agriculture (USDA) report, commodity prices for fresh produce have shown significant variability in recent years, more so than other commodities such as grains (USDA Economic Research Service, 2020). Besides the Covid-19 pandemic, recent changes in immigration laws in several countries have led to labour shortages (Gunders and Bloom, 2017) and as a result, increase in wages for harvesting labour. For example, blueberry farmers in the U.K. face significant challenges for finding harvesting labor as a result of Brexit because most of the seasonal workers come from the European Union (Partridge, 2021). The cultivation and fertilizer costs have also experienced significant increases in recent years. In particular, farmers in some areas of the U.S. report more than 300% increase in fertilizer prices (Myers, 2021). The harvest volume for fruits and vegetables for the glasshouse farmers in the U.K. have decreased more than 50% due to surging cost of energy, which is one of the primary inputs used for cultivation in glasshouse farming (Evans, 2022). These observations highlight the need for understanding how the increases in the costs and uncertainties in the farming environment affect the decisions associated with farm planning.

As reviewed by Glen (1987), Lowe and Preckel (2004), and Ahumada and Villalobos (2009), farm planning problem has received considerable attention both in the operations management (OM) and agricultural economics (AE) literatures. Only recently have these literatures started focusing on stochastic models that capture uncertainties facing farmers. In the AE literature, besides the empirical studies that identify the agronomic factors affecting decisions and uncertainties associated with farm planning, a stream of papers use analytical models to establish the effects of different farm inputs on yield (for example, Tembo et al. (2008) propose a model to capture the effect of fertilizer application on the stochastic yield). In this literature, the few papers that develop analytical models for the optimization of joint cultivation and farm-input application decisions under uncertainty either provide numerical solutions (e.g., Babcock et al., 1987) or provide heuristic solutions and evaluate their performance using numerical experiments (e.g., Livingston et al., 2015). In the OM literature, the main focus of related papers is to develop tractable analytical models to optimize a farmer's decisions that incorporate important characteristics of the farm planning problem in a stylized manner based on the findings from the AE literature. The majority of papers in this stream does not consider fertilizer (or any other farm-input) application decision, as their objective is to examine the interplay between cultivation decision and a variety of operational features, including crop rotation benefits across growing seasons (Boyabatlı et al., 2019), equilibrium crop price in the market place (Hu et al., 2019), planting capacity across growing seasons (Zhang and Swaminathan, 2020), rainfall uncertainty (Maatman et al., 2002), yield-dependent crop price (Kazaz, 2004), and yield-dependent open market trading costs (Kazaz and Webster, 2011). Among the few papers that consider the farm-input application decision, Anderson and Monjardino (2019) consider the fertilizer application decision on a single acre and study the fertilizer contract design problem. Federgruen et al. (2019) consider the irrigation decision on a single acre and study the farmer's sales contract choice. Neither of the papers considers the cultivation decision.

The first objective of this paper is to study the implications of farmer's joint cultivation and farm-input (fertilizer) application decisions for farm planning while incorporating the important characteristics of the fresh produce industry. Barring Huh and Lall (2013), there is no work in the OM literature that studies the joint optimization of these decisions under uncertainty. Huh and Lall (2013) model the joint cultivation and irrigation decisions in the presence of uncertainties in rainfall and crop price. They establish the concavity properties of the farmer's optimization problem and provide a computational study. Different from that paper, we characterize the specific strategies that may emerge as part of the optimal decisions. Moreover, motivated by the recent increases in costs and uncertainties in the farming environment, we examine how changes in cultivation and fertilizer costs as well as farm yield uncertainty impact these decisions and farmer's profitability. These analyses are for useful for generating practical insights for farm management. They are also useful for understanding the implications of optimal decisions for food security as discussed next.

Farm planning decisions also have significant implications for food security. According to the USDA, food security is defined as access by all people at all times to sufficient food for an active, healthy life. While achieving food security faces many challenges across different parts of the food supply chains, including reducing the food wasted on the retail end as well as the food lost for spoilage due to poor and limited infrastructure for storage and cooling facilities during transportation (see Akkaş and Gaur (2022) for a detailed discussion), this paper focuses specifically on the challenges associated with increasing the crop production level. As highlighted by Godfray et al. (2010), the world will need at least 70% more food by 2050 and closing the gap between the maximum attainable and the actual crop production levels plays a key role in responding to this need. One of the key reasons for the actual crop production level, or the harvest volume, to be lower than the maximum attainable level is that for any crop volume available for harvesting at the end of the growing season the farmer may optimally choose not to harvest all. In particular, as also highlighted by World Wild Fund (2021), this would happen when the crop's realized profit margin is negative due to low crop price or high harvesting cost owing to limited availability of harvesting resources (including labor and containers for harvested crop). There is no shortage of anecdotal evidence that documents significant amount of unharvested crop left in the field (denoted as production loss in farming) in a variety of agricultural industries including the fresh produce industry. A recent report estimates that 1.2 billion tonnes of food is lost on farms, either left unharvested or disposed immediately after harvest, which corresponds to 15.3% of food produced in farms with an economic value of $370 million per annum (World Wild Fund, 2021). The fresh produce industry is one of the agricultural industries with the highest production losses in farming as reported by Gunders and Bloom (2017): For example, Tesco reports production losses of 17% for salad greens and 15% for berries. According to the USDA, about 2.64% (63,900 acres) of planted vegetable and fruit fields are left unharvested in 2019 where this number can vary widely by crop and can be as high as 10 percent for some crops (USDA National Agricultural Statistics Service, 2022). Another key reason for the harvest volume to be lower than the maximum attainable level is that the crop volume available for harvesting at the end of the growing season crucially depends on the farmer's decisions at the beginning of the growing season. In particular, the farmer may choose not to achieve maximum possible production level (which requires cultivating the whole farmland and applying fertilizer at its agronomic recommendation) because it may not be profitable to do so due to high cultivation and fertilizer costs. Therefore, in closing the gap between the maximum attainable and the actual crop production levels there is a need to understand how farmer's optimal cultivation and fertilizer application decisions as well as the uncertainties in the farming environment affect the crop production level at the end of the growing season.

The second research objective of this paper is to examine the implications of farmer's joint cultivation and farm-input (fertilizer) application decisions for food security while incorporating the important characteristics of the fresh produce industry. To this end, we aim to characterize the crop production level at the profit-maximizing decisions of the farmer. Moreover, motivated by the recent increases in costs and uncertainties in the farming environment, we investigate how changes in cultivation and fertilizer costs as well as farm yield uncertainty impact the farmer's crop production level at the optimal decisions. These analyses are useful for policymakers for understanding the consequences of some commonly used policies in practice that have been devised for increasing the crop production level. Some examples for these policies include distributing discount vouchers for procurement of seeds (which reduces the cultivation cost) or fertilizer (which reduces the fertilizer cost) or increasing availability of disease-resistant seeds (which reduces the farm yield uncertainty). A stream of papers in the OM literature study in a variety of settings how the crop production level at the farmer's profit-maximizing decisions is affected by different policies, including providing crop revenue insurance (Alizamir et al., 2018), reducing the downside risk of crop price uncertainty through minimum support prices (Chintapalli and Tang, 2021), levying taxes on chemical farm-inputs to discourage their usage (Akkaya et al., 2020), and providing low-cost loans to farmers (Kazaz et al., 2016). In all these papers the crop production level is affected by the cultivation decision but there is no consideration of fertilizer application decision. As such, the current work differs from the earlier ones as it provides a more general representation of crop production. The analysis in this paper shows that considering this more general representation critically impacts the effectiveness of various policies. In particular, we will show that for some of the policies considered in our analysis ignoring the farmer's fertilizer decision may lead to the erroneous conclusion that this policy increases the crop production level.

To achieve the two research objectives we propose a two-stage stochastic model that, in a stylized manner, captures the trade-offs facing a farmer growing a single commodity crop to sell in the open market so as to maximize the expected profit. In the first stage (at the beginning of the growing season), the farmer determines the number of acres to cultivate and the quantity of fertilizer to apply per acre on the cultivated farmland facing uncertainties in the farm yield, harvesting cost (labour cost for hiring seasonal workers) and open market price. We assume that farm yield is stochastically increasing in the fertilizer application rate up to a maximum level which we denote as agronomic recommendation. In the second stage (at the end of the growing season), these uncertainties are realized and the farmer decides the crop volume to harvest and sell to the open market as well as the amount of harvesting resource to acquire to support this volume.

We characterize the joint optimal cultivation and fertilizer application decisions and identify five strategies that emerge as a part of the optimal policy: partial farmland cultivation without using any fertilizer, partial farmland cultivation with applying fertilizer at agronomically recommended rate, and full farmland cultivation with three distinct fertilizer application rates; agronomic recommendation, partial (less than agronomic recommendation), and none. We provide specific conditions under which each strategy is optimal based on the cultivation and fertilizer costs.

To examine the implications of farmer's optimal decisions for food security, we contextualize in our setting the gap between maximum attainable and actual crop production levels, a food security measure similar to those commonly used in the literature (see Godfray et al. (2010)), and define the expected gap measure as the difference between the expected maximum harvest volume and the expected optimal harvest volume. The expected optimal harvest volume can be strictly less than the expected maximum harvest volume because the farmer may choose not to cultivate whole farmland or not to apply fertilizer at its agronomic recommendation in the cultivation stage, or it may not be profitable to hire seasonal workers in the harvesting stage due to negative crop margin.

We investigate the effects of cultivation and fertilizer costs as well as farm yield variability on the farmer's optimal decisions, profitability and the expected gap. We also carry out the same analyses using a benchmark model where the farmer only optimizes the cultivation decision in the first stage to highlight how our key results differ from those results based on the knowledge base developed in the extant OM literature. Whenever analytical results are not attainable, we use numerical experiments based on realistic instances. To this end, we calibrate our model to represent a typical fresh tomato farmer in Florida—as tomato is among the most valuable fresh produce, valued at approximately \$1.6 billion in the U.S. in 2019 (USDA, 2020), and Florida is the largest fresh tomato growing region in the U.S. in 2019 (USDA, 2020). The model calibration is based on the publicly available data from USDA and the U.S. Bureau of Labor Statistics as complemented by the data obtained from the extant literature. Our main findings can be summarized as follows:[2]

1) Common intuition may suggest that an increase in cultivation cost has the following effects: (i) it incents the farmer to cultivate fewer acres without changing the fertilizer applied per acre (as this decision is affected by fertilizer cost) and (ii) it decreases the expected optimal harvest volume (hence, increases the expected gap). We prove that this intuition is correct except for the

[2]During this summary, for expositional brevity and practical relevance we focus on more realistic scenarios in which the fertilizer cost is not very high so that the farmer optimally uses some fertilizer; that is, we rule out the following two strategies: full or partial farmland cultivation without using any fertilizer. These strategies are not observed as optimal in our data-calibrated numerical studies in the context of fresh tomato farming.

case when the cultivation and fertilizer costs are in moderate range. In that case, we prove that an increase in cultivation cost also incents the farmer to apply more fertilizer to counteract against the reduction in the number of acres cultivated to increase the crop availability at the harvesting stage. Because the farmer cultivates fewer acres but applies more fertilizer per acre the resulting impact on the expected optimal harvest volume is indeterminate. We numerically observe that the increase in fertilizer application rate may outweigh the reduction in cultivation volume and the expected optimal harvest volume increases (hence, expected gap decreases). In the benchmark model an increase in cultivation cost always decreases the expected optimal harvest volume.

2) While an increase in fertilizer cost intuitively incents the farmer to decrease the fertilizer application rate, the effect on the optimal cultivation volume is more nuanced. Common intuition may suggest that an increase in fertilizer cost incents the farmer to decrease the cultivation volume as the farm input becomes more expensive. We prove that this intuition is correct except for the case when the cultivation and fertilizer costs are in moderate range. Outside of this case, because the farmer cultivates fewer acres and applies less fertilizer per acre it can be proven that the expected optimal harvest volume decreases (hence, expected gap increases). When the cultivation and fertilizer costs are in moderate range, the farmer increases the cultivation volume to counteract against the reduction in crop availability at the harvesting stage due to decreasing fertilizer application rate. Because the farmer cultivates more acres but applies less fertilizer per acre the resulting impact on the expected optimal harvest volume is indeterminate. In our numerical experiments we observe that the increase in cultivation volume does not outweigh the reduction in fertilizer application rate and the expected optimal harvest volume also decreases in this case.

3) An increase in farm yield variability affects the farmer's profitability by increasing the variability of three factors: harvestable crop volume, harvesting cost, and crop's open market price. Our partial analytical characterizations complemented with numerical experiments illustrate that the overall impact is that higher yield variability is detrimental for profitability. Common intuition may suggest that an increase in farm yield variability (which makes farming less predictable) incents the farmer to cultivate fewer acres and apply less fertilizer per acre. We show that this intuition is correct except for the case when the cultivation and fertilizer costs are in moderate range. In that case, an increase in farm yield variability incents the farmer to apply more fertilizer to counteract against the reduction in the number of acres cultivated to increase the crop availability at the harvesting stage. The effect of farm yield variability on the expected gap is more complex than the effects of cultivation and fertilizer costs on the same. This is because, besides the two common

effects—that is, impacting the farmer's optimal cultivation and fertilizer application decisions—there is a third effect as changes in farm yield variability also alters the likelihood of a positive crop margin—that is, the difference between open market price and harvesting cost—in the harvesting stage for any given farmer's decisions. We show that a higher yield variability decreases the likelihood of a positive crop margin for any given farmer's decisions. When the cultivation and fertilizer costs are not in moderate range, as yield variability increases because the farmer cultivates fewer acres and applies less fertilizer per acre, the overall impact is such that expected optimal harvest volume decreases (hence, expected gap increases). When the cultivation and fertilizer costs are in moderate range, the effect of an increase in yield variability on the expected optimal harvest volume is indeterminate because the farmer applies more fertilizer per acre. We numerically observe that the increase in fertilizer application rate may outweigh the other two effects and the expected optimal harvest volume increases (hence, expected gap decreases). In the benchmark model an increase in farm yield variability always decreases the expected optimal harvest volume.

The general insight from our analyses is that the effects of costs and uncertainties on the farmer's optimal decisions and on the expected optimal harvest volume (hence, the expected gap) can be counterintuitive and significantly different from those when only cultivation decision is optimized; specifically when these effects induce the farmer to change the fertilizer application and cultivation decisions in opposite directions. Based on these results, we put forward practical insights for farm management by providing rules of thumb for responding to changes in the farming environment and important policy insights by showcasing the unintended consequences of some commonly adopted policies in practice that have been devised to increase farmer's crop production level and income.

The rest of the paper proceeds as follows. §2 describes our model and assumptions. §3 characterizes the optimal cultivation and fertilizer application decisions. §4 and §5 examine the implications of optimal decisions for farm management and food security, respectively. In particular, we investigate how changes in cultivation and fertilizer costs as well as farm yield uncertainty impact the farmer's optimal decisions, profitability (§4), and the expected gap (§5). §6 provides an application in the context of fresh tomato farming. §7 concludes with a discussion of future research directions.

2 Model Description and Assumptions

We use the following mathematical representation throughout the paper. A realization of a random variable $\tilde{\xi}$ is denoted by ξ. The expectation operator, probability, and indicator function are denoted by $\mathbb{E}$, $\Pr(\cdot)$, and $\mathbb{I}\{\cdot\}$, respectively. We use $(u)^+ = \max(u, 0)$. Monotonic relations are used in the weak sense unless otherwise stated. Subscript c and h denote the parameters and decision variables related to cultivation and harvesting, respectively. The optimal decisions and performance measures evaluated at the optimal solution are denoted by *. We use the following weight units: lb (one pound) and carton (25 pounds). All the proofs are relegated to §B of the online appendix.

We consider a farmer growing a single commodity crop to sell in the open market so as to maximize the expected profit in a single growing season. We model the farmer's decisions in a two-stage stochastic program. In the first stage, the farmer determines the number of acres to cultivate and quantity of fertilizer to apply per acre on the cultivated farmland facing uncertainties in the farm yield, harvesting cost, and open market price. In the second stage, these uncertainties are realized and the farmer decides the crop volume to harvest (and sell to the open market) and the amount of harvesting resource to acquire to support this volume.

We first discuss how we model uncertainties in farm yield, harvesting cost, and open market price. To model the farm yield uncertainty, we use $\tilde{\epsilon} \in [0, \bar{\epsilon}]$ (e.g., carton/acre which is a commonly used unit in fresh tomato farming) to represent the uncertain farm yield per acre in the absence of fertilizer application where $\bar{\epsilon}$ represents the largest realization. Let μ_ϵ and σ_ϵ denote the mean and standard deviation of farm yield, respectively. To model the harvesting cost uncertainty, we make the following two assumptions: First, we assume that the farmer has internal harvesting resource $K_h > 0$ (e.g., in carton) and we normalize the unit harvesting cost when the internal resource is used to zero. Second, we assume that additional resources can be acquired in the harvesting stage at a unit cost $\omega_h(\epsilon) > 0$ (e.g., \$/carton) which is increasing in the farm yield realization ϵ. Because the farm yield is uncertain at the first stage, $\omega_h(\tilde{\epsilon})$ (hereafter, denoted as "external unit harvesting cost") is also uncertain at this stage. In our model, while K_h can represent the capacity of any harvesting equipment/machinery (e.g., containers and harvesters), in the context of fresh-produce industry K_h represents the available labor as most fruits and vegetables are harvested by hand. Our harvesting cost modeling captures the main features of fresh-produce industry practice in a parsimonious way. In particular, the farmer prioritizes using the available labor (e.g., farmer's own family members and existing contracted labor) for harvesting but because this availability is limited the farmer can also hire seasonal workers (as often done in practice) when the realized farm yield is sufficiently high. Our modeling of external unit harvesting cost $\omega_h(\epsilon)$ increasing in ϵ is motivated by the following observations: (i) the cost of hiring seasonal workers increases in the demand for these workers and (ii) when the farm yield is high for our focal farmer, the farm yield will also likely to be high for other farmers in close proximity (because these farmers share similar climatic conditions), increasing the demand for the seasonal workers. While we characterize the farmer's optimal decisions in §3 using a generic $\omega_h(\tilde{\epsilon})$ representation, to study the implications of optimal decisions for farm management and food security, we further assume $\omega_h(\tilde{\epsilon}) = \omega_0 + \beta\tilde{\epsilon}$ where $\omega_0 > 0$ denotes the base labor cost and $\beta \geq 0$ captures the strength of the external unit harvesting cost's dependence on farm yield in §4-6.

To model the open market price uncertainty, we define market uncertainty $\tilde{m} \in [\underline{m}, \overline{m}]$ (e.g., \$/carton)—where $\underline{m} > 0$ and $\overline{m}$ represent the smallest and largest realization, respectively—to capture the uncertainty in open market price associated with factors that are not related to farm yield (e.g., macroeconomic conditions and industry regulations). Let μ_m and σ_m denote the mean and standard deviation of market uncertainty, respectively. We also allow for the open market price to be affected by the farm yield based on the following observations: (i) open market price decreases in crop's aggregate supply and (ii) when the farm yield is high for our focal farmer, the farm yield will also likely to be high for other farmers in close proximity, decreasing the crop's aggregate supply. To this end, we define $p(m, \epsilon) > 0$ as crop's open market price (hereafter, denoted as "crop price" for brevity) which is decreasing in the farm yield realization ϵ and increasing in the market uncertainty realization m. Because $\tilde{\epsilon}$ and $\tilde{m}$ are uncertain at the first stage, $p(\tilde{m}, \tilde{\epsilon})$ is also uncertain at this stage. While we characterize the farmer's optimal decisions in §3 using a generic $p(\tilde{m}, \tilde{\epsilon})$ representation, to study the implications of optimal decisions for farm management and food security in §4-6, we further assume $p(\tilde{m}, \tilde{\epsilon}) = \tilde{m} - \alpha(\tilde{\epsilon} - \mu_\epsilon)$ where $\alpha \geq 0$ captures the strength of crop price's dependence on farm yield. To avoid uninteresting cases in which the farmer optimally does not harvest at all, we assume $\alpha < \underline{m}/(\bar{\epsilon} - \mu_\epsilon)$ so that crop price is always positive for any $\tilde{m}$ and $\tilde{\epsilon}$ realizations.

In summary, we use $\tilde{\epsilon}$ and $\tilde{m}$ to capture all relevant uncertainties for the farmer; that is, uncertainty in farm yield, harvesting cost, and crop price. We characterize the farmer's optimal decisions in §3 without imposing any distributional assumptions on $\tilde{\epsilon}$ and $\tilde{m}$. In the rest of our analysis, we assume $\tilde{\epsilon}$ and $\tilde{m}$ have independent distributions and whenever applicable (which will be specified) we further assume $\tilde{\epsilon}$ and $\tilde{m}$ follow a univariate Normal distribution.

We next discuss how we model the farmer's decisions. Let $Q > 0$ (in acres) denote the available farmland, $r_c > 0$ (in \$/acre) denote the cultivation cost per acre (that accounts for plowing and tilling of farmland as well as procurement and sowing of seeds), and $y_c > 0$ (in \$/lb) denote the unit fertilizer (e.g., nitrogen) cost. In the first (cultivation) stage, the farmer jointly makes the following two decisions. First, the farmer decides the size of the farmland to cultivate, denoted by $x_c \geq 0$ (in acres), within the available farmland Q incurring the cultivation cost $r_c x_c$. Second, the farmer decides the fertilizer application rate per acre, denoted by $s_c \geq 0$ (in lb/acre), on the cultivated farmland x_c incurring the fertilizer cost $y_c s_c x_c$. A key feature of fertilizer application is that it (stochastically) increases the farm yield in the second stage. We model the increase in the yield as a shift in the corresponding distribution; that is, when the farmer applies s_c amount of fertilizer per acre, the farm yield per acre in the harvesting stage is given by $\tilde{\epsilon} + as_c$. Here, $a > 0$ (e.g., in carton/lb) measures the extent to which the farm yield responds to fertilizer, so a larger value of a corresponds to a stronger effect of fertilizer on yield. We assume that the increasing yield response to fertilizer application is relevant for $s_c \in [0, \bar{s}]$, where $\bar{s}$ denotes the agronomically recommended rate beyond which any more fertilizer application does not improve the farm yield. Our model of farm yield response to fertilizer is representative of a linear response plateau model (see, for example, Tembo et al., 2008) as commonly used in the agricultural economics literature to model crop yield response to farm input.

In the second (harvesting) stage, the farmer determines the harvest volume, denoted by $x_h \geq 0$ (e.g., in carton), to sell in the open market at unit price $p(m, \epsilon)$. When the harvest volume x_h is larger than the available harvesting capacity K_h, the farmer also acquires $(K_h - x_h)$ units of additional harvesting resource at external unit harvesting cost $\omega_h(\epsilon)$. To avoid uninteresting cases, we assume $K_h < \bar{\epsilon}Q$; that is, internal harvesting resource is not sufficient for harvesting maximum attainable yield from the whole farmland in the absence of fertilizer application. Otherwise, the farmer does not acquire additional harvesting resource for any farm yield realization ϵ in the absence of fertilizer application and this contradicts with the observation that seasonal workers are often used for harvesting in the fresh produce industry (see, for example, Calvin and Martin, 2010).

We now formulate the farmer's decision problem. In the harvesting stage, farm yield $\tilde{\epsilon}$ and market uncertainty $\tilde{m}$ are realized. Given the decisions from the cultivation stage, namely cultivation volume x_c and fertilizer application rate s_c, these realizations determine the crop volume available for harvesting $x_c(\epsilon + as_c)$, external unit harvesting cost $\omega_h(\epsilon)$, and the crop price $p(m, \epsilon)$. Constrained by $x_c(\epsilon + as_c)$, the farmer chooses the crop volume $x_h \geq 0$ to harvest and sell to the open market while acquiring $(x_h - K_h)^+$ units of additional harvesting resource to maximize the profit $p(m, \epsilon)x_h - \omega_h(\epsilon)(x_h - K_h)^+$. It is easy to establish that the farmer optimally harvests all the available crop (i.e., $x_h^* = x_c(\epsilon + as_c)$) when the crop price is larger than the external unit harvesting cost (i.e., $p(m, \epsilon) \geq \omega_h(\epsilon)$); otherwise (i.e., $p(m, \epsilon) < \omega_h(\epsilon)$), the farmer optimally harvests all the available crop up to the internal harvesting capacity (i.e., $x_h^* = \min(x_c(\epsilon + as_c), K_h)$).

In the cultivation stage, given unit cultivation cost r_c and unit fertilizer cost y_c the farmer chooses the cultivation volume x_c and fertilizer application rate s_c. Let $\Pi_c^*(r_c, y_c)$ denote the farmer's optimal expected profit in this stage, which is given as follows:

$$\begin{aligned}\Pi_c^*(r_c, y_c) \doteq \max_{x_c, s_c} \quad & \mathbb{E}\left[p(\tilde{m}, \tilde{\epsilon}) \min\left(x_c(\tilde{\epsilon} + as_c), K_h\right) + \left(p(\tilde{m}, \tilde{\epsilon}) - \omega_h(\tilde{\epsilon})\right)^+ \left(x_c(\tilde{\epsilon} + as_c) - K_h\right)^+\right] \\ & - y_c s_c x_c - r_c x_c, \qquad\qquad (1) \\ \text{s.t.} \quad & 0 \le x_c \le Q, \; 0 \le s_c \le \bar{s}.\end{aligned}$$

In (1), the first term in the objective function denotes the expected profit in the harvesting stage. In particular, in the harvesting stage the farmer has a unit crop margin of $p(m, \epsilon)$ for the harvest volume $\min\left(x_c(\epsilon + as_c), K_h\right)$ whereas the unit crop margin for the harvest volume $(x_c(\epsilon + as_c) - K_h)^+$ is given by $(p(m, \epsilon) - \omega_h(\epsilon))^+$ because the farmer optimally chooses to harvest this volume only if this margin is positive. The second and third terms in (1) represent the fertilizer and cultivation cost, respectively. The constraints state that the cultivation volume cannot exceed the available farmland Q and the fertilizer application rate cannot exceed the agronomic recommendation $\bar{s}$.

3 Optimal Cultivation and Fertilizer Application Decisions

In this section, we characterize the farmer's optimal cultivation and fertilizer application decisions, denoted by (x_c^*, s_c^*). For ease of exposition, we present the characterization when the unit fertilizer cost y_c is large, small, and medium.

Proposition 1 (Large unit fertilizer cost) *Let* $y_c^{(0)} \doteq a\mathbb{E}\left[p(\tilde{m}, \tilde{\epsilon})\right]$. *When* $y_c > y_c^{(0)}$, *we have*

$$(x_c^*, s_c^*) = \begin{cases} (0, 0) & \text{if } \Theta(0) \le r_c, \\ (\hat{x}_c^{nf}, 0) & \text{if } \Theta(Q) \le r_c < \Theta(0), \\ (Q, 0) & \text{if } r_c < \Theta(Q), \end{cases}$$

where $\hat{x}_c^{nf} \in (K_h/\bar{\epsilon}, Q]$ *is the unique solution to* $\Theta(\hat{x}_c^{nf}) = r_c$ *with*

$$\Theta(x_c) \doteq \begin{cases} \mathbb{E}\left[\tilde{\epsilon}\, p(\tilde{m}, \tilde{\epsilon})\right] & \text{if } x_c \le \frac{K_h}{\bar{\epsilon}}, \\ \mathbb{E}\left[\tilde{\epsilon}\left(p(\tilde{m}, \tilde{\epsilon}) - \min(p(\tilde{m}, \tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right)\right] & \text{if } x_c > \frac{K_h}{\bar{\epsilon}}. \end{cases}$$

When the fertilizer cost is large, the farmer optimally does not apply any fertilizer (i.e., $s_c^* = 0$). In this case, the marginal cost of cultivating an additional acre is given by the cultivation cost per acre r_c. When r_c is small, the farmer optimally cultivates the whole farmland; when r_c is large, the farmer optimally does not cultivate at all; otherwise, the farmer optimally cultivates $\hat{x}_c^{nf}$ acres. Here, $\hat{x}_c^{nf}$ (where superscript nf represents "no fertilizer") denotes the cultivation volume for which r_c equals the expected marginal revenue of cultivating an additional acre, as given by $\Theta(x_c)$. In the harvesting stage this marginal revenue is characterized by the product of farm yield per acre ϵ and the effective crop margin which takes two different forms based on the availability of harvesting resource K_h. In particular, when K_h is not sufficient for harvesting the yield from additional cultivated acre (i.e., $K_h < \epsilon x_c$), the crop margin is given by $(p(m,\epsilon) - \omega_h(\epsilon))^+$ as the farmer optimally harvests only when the crop price $p(m,\epsilon)$ is larger than the external unit harvesting cost $\omega_h(\epsilon)$. When K_h is sufficient for harvesting the yield from additional cultivated acre, the crop margin is given by the crop price $p(m,\epsilon)$. Using the identity $\min(p(m,\epsilon), \omega_h(\epsilon)) = p(m,\epsilon) - (p(m,\epsilon) - \omega_h(\epsilon))^+$, the effective crop margin at the harvesting stage can be written as $p(m,\epsilon) - \min(p(m,\epsilon), \omega_h(\epsilon))\mathbb{I}\left\{\epsilon > \frac{K_h}{x_c}\right\}$ as given in the characterization of $\Theta(x_c)$ in Proposition 1.

Next we characterize the optimal decisions for a sufficiently small unit fertilizer cost y_c.

Proposition 2 (Small unit fertilizer cost) *Let* $y_c^{(2)} \doteq a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\{\tilde{\epsilon} > K_h/Q - a\bar{s}\}\right]$. *When* $y_c < y_c^{(2)}$, *we have*

$$(x_c^*, s_c^*) = \begin{cases} (0, \bar{s}) & \textit{if } \Gamma(0) \le r_c + \bar{s}y_c, \\ (\hat{x}_c^f, \bar{s}) & \textit{if } \Gamma(Q) \le r_c + \bar{s}y_c < \Gamma(0), \\ (Q, \bar{s}) & \textit{if } r_c + \bar{s}y_c < \Gamma(Q), \end{cases}$$

where $\hat{x}_c^f \in (K_h/(\bar{\epsilon} + a\bar{s}), Q]$ *is the unique solution to* $\Gamma(\hat{x}_c^f) = r_c + \bar{s}y_c$ *with*

$$\Gamma(x_c) \doteq \begin{cases} \mathbb{E}\left[(\tilde{\epsilon} + a\bar{s})\, p(\tilde{m},\tilde{\epsilon})\right] & \textit{if } x_c \le \frac{K_h}{\bar{\epsilon}+a\bar{s}}, \\ \mathbb{E}\left[(\tilde{\epsilon} + a\bar{s})\left(p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\bar{s}\right\}\right)\right] & \textit{if } x_c > \frac{K_h}{\bar{\epsilon}+a\bar{s}}. \end{cases}$$

When the fertilizer cost is small, the farmer optimally applies fertilizer at agronomic recommendation (i.e., $s_c^* = \bar{s}$). Therefore, the marginal cost of cultivating an additional acre is given by the sum of cultivation cost per acre r_c and fertilizer application cost $\bar{s}y_c$. The characterization of optimal cultivation volume x_c^* is structurally similar to that of Proposition 1. In particular, when $r_c + \bar{s}y_c$ is small, the farmer optimally cultivates the whole farmland; when it is large, the farmer optimally does not cultivate at all (and the fertilizer application decision is irrelevant); otherwise, the farmer optimally cultivates $\hat{x}_c^f$ acres. Here, $\hat{x}_c^f$ (where superscript f represents "fertilizer") denotes the cultivation volume for which the expected marginal revenue of cultivating an additional acre (while applying fertilizer at rate $\bar{s}$ per acre), as given by $\Gamma(x_c)$, equals its marginal cost $r_c + \bar{s}y_c$. The expected marginal revenue term $\Gamma(x_c)$ differs from $\Theta(x_c)$ in that the former adds the effect of fertilizer application on the farm yield; that is, the realized farm yield per acre is given by $\epsilon + a\bar{s}$.

So far we have observed that when the unit fertilizer cost y_c is sufficiently small or sufficiently large, the farmer always optimally chooses the same fertilizer application rate regardless of the optimal cultivation volume. When y_c is in the moderate range, the farmer may also optimally change the fertilizer application decision, as illustrated in Proposition 3:

Proposition 3 (Moderate unit fertilizer cost) *Let $\Theta(x_c)$ ($\Gamma(x_c)$) and $y_c^{(0)}$ ($y_c^{(2)}$) be as defined in Proposition 1 (Proposition 2) as well as $y_c^{(1)} \doteq a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\{\tilde{\epsilon} > K_h/Q\}\right]$ where $y_c^{(1)} \in [y_c^{(2)}, y_c^{(0)}]$.*
<u>*Case i:*</u> *When $y_c^{(1)} \leq y_c < y_c^{(0)}$, we have*

$$(x_c^*, s_c^*) = \begin{cases} (0,\bar{s}) & \text{if } \Gamma(0) \leq r_c + \bar{s}y_c, \\ (\hat{x}_c^f, \bar{s}) & \text{if } r_c + \bar{s}y_c < \Gamma(0) \text{ and } r_c \geq \Theta(\overline{x}_c) \\ (\hat{x}_c^{nf}, 0) & \text{if } \Theta(Q) \leq r_c < \Theta(\overline{x}_c) \\ (Q, 0) & \text{if } r_c < \Theta(Q), \end{cases}$$

where $\hat{x}_c^f \in (K_h/(\bar{\epsilon} + a\bar{s}), \underline{x}_c]$ is the unique solution to $\Gamma(\hat{x}_c^f) = r_c + \bar{s}y_c$ and $\hat{x}_c^{nf} \in (\overline{x}_c, Q]$ is the unique solution to $\Theta(\hat{x}_c^{nf}) = r_c$. Here, $\underline{x}_c > K_h/(\bar{\epsilon} + a\bar{s})$ is the unique solution to

$$a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{\underline{x}_c} - a\bar{s}\right\}\right] = y_c;$$

and $\overline{x}_c > K_h/\bar{\epsilon}$ is the unique solution to

$$a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{\overline{x}_c}\right\}\right] = y_c.$$

<u>*Case ii:*</u> *When $y_c^{(2)} \leq y_c < y_c^{(1)}$, we have*

$$(x_c^*, s_c^*) = \begin{cases} (0,\bar{s}) & \text{if } \Gamma(0) \leq r_c + \bar{s}y_c, \\ (\hat{x}_c^f, \bar{s}) & \text{if } r_c + \bar{s}y_c < \Gamma(0) \text{ and } r_c \geq \Theta(\overline{x}_c), \\ (Q, \hat{s}_c) & \text{if } r_c < \Theta(\overline{x}_c), \end{cases}$$

where $\hat{x}_c^f \in (K_h/(\bar{\epsilon}+a\bar{s}), \underline{x}_c]$ *is the unique solution to* $\Gamma(\hat{x}_c^f) = r_c + \bar{s}y_c$ *and* $\hat{s}_c \in (0, \bar{s})$ *is the unique solution to*

$$a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{Q} - a\hat{s}_c\right\}\right] = y_c. \tag{2}$$

We only delineate the intuition behind the second case; the first case can be explained in a similar fashion. Let us consider a given unit fertilizer cost $y_c \in [y_c^{(2)}, y_c^{(1)})$ and examine the optimal solution while changing the cultivation cost per acre r_c. When r_c is sufficiently high, the farmer optimally does not cultivate any farmland (and the fertilizer application decision is irrelevant). As r_c decreases, the farmer increases the cultivation volume, and paralleling the characterization in Proposition 2, optimally cultivates $\hat{x}_c^f$ acres and applies fertilizer at agronomically recommended rate $\bar{s}$. As r_c further decreases, the farmer further increases the cultivation volume and optimally cultivates the whole farmland. In this case, different from the characterization in Proposition 2, the farmer optimally applies fertilizer at a rate $\hat{s}_c$ that is lower than $\bar{s}$ because y_c is higher than the unit fertilizer cost in Proposition 2 and thus, it is not beneficial for the farmer to continue applying $\bar{s}$ amount of fertilizer per acre when the increase in number of acres cultivated is accounted for. Here, $\hat{s}_c$ is the fertilizer rate per acre (applied to the whole farmland Q) for which the marginal cost y_c equals its expected marginal revenue. In the harvesting stage this marginal revenue is characterized by the product of an additional unit of fertilizer's effect on yield per acre, as given by a, and the effective crop margin which follows a similar structure with the effective crop margin that is used to characterize $\Gamma(x_c)$ in Proposition 2 where x_c and s_c are substituted with Q and $\hat{s}_c$, respectively.

Corollary 1 combines the characterizations from Propositions 1, 2, and 3:

Corollary 1 *When* $r_c + \bar{s}y_c \geq \Gamma(0) = \mathbb{E}[(\tilde{\epsilon}+a\bar{s})p(\tilde{m},\tilde{\epsilon})]$ *and* $r_c \geq \Theta(0) = \mathbb{E}[\tilde{\epsilon}p(\tilde{m},\tilde{\epsilon})]$, *we have* $x_c^* = 0$ *and the fertilizer application decision is irrelevant. Otherwise, we have* $x_c^* > 0$ *and the characterization of* (x_c^*, s_c^*) *is as illustrated in Figure 1 for the case* $\Gamma(Q) < \Theta(Q)$ *(the characterization is structurally the same for the case* $\Gamma(Q) \geq \Theta(Q)$*) where*

$$\Xi_1 \doteq \left\{(r_c,y_c) : y_c \leq \hat{y}_c(r_c), y_c^{(2)} \leq y_c \leq y_c^{(0)}\right\} \cup \left\{(r_c,y_c) : \Gamma(Q) \leq r_c + \bar{s}y_c, 0 \leq y_c < y_c^{(2)}\right\},$$

$$\Xi_2 \doteq \{(r_c,y_c) : \Theta(Q) \leq r_c, y_c > \hat{y}_c(r_c)\},$$

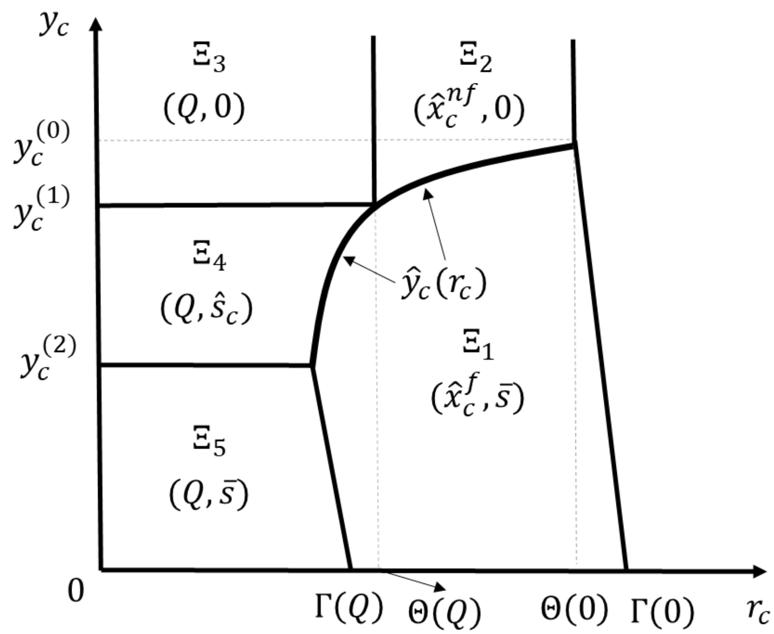
$$\Xi_3 \doteq \left\{(r_c,y_c) : \Theta(Q) > r_c, y_c \geq y_c^{(1)}\right\},$$

$$\Xi_4 \doteq \left\{(r_c,y_c) : y_c > \hat{y}_c(r_c), y_c^{(2)} \leq y_c < y_c^{(1)}\right\},$$

$$\Xi_5 \doteq \left\{(r_c,y_c) : \Gamma(Q) > r_c + \bar{s}y_c, 0 \leq y_c < y_c^{(2)}\right\}.$$

Here, $\hat{y}_c(r_c)$, which can be proven to be concavely increasing in r_c, is the unique solution to $\Theta(\overline{x}_c) = r_c$ where $\overline{x}_c$ is as given by Proposition 3.

Figure 1: Illustration of (x_c^*, s_c^*) within the (r_c, y_c) space for the case $\Gamma(Q) < \Theta(Q)$



Notes. $\Theta(x_c)$ and $\Gamma(x_c)$ are as defined in Propositions 1 and 2, respectively. $y_c^{(0)}$, $y_c^{(1)}$, and $y_c^{(2)}$ are as defined in Propositions 1, 3, and 2, respectively. It follows that $\hat{x}_c^{nf}$ (from Propositions 1 and 3) depends on r_c, but not on y_c; $\hat{x}_c^{f}$ (from Propositions 2 and 3) depends on both r_c and y_c; and $\hat{s}_c$ (from Proposition 3) depends on y_c but not on r_c.

When the farmer optimally cultivates some acres, Corollary 1 identifies five strategies that emerge as optimal: partial farmland cultivation without using any fertilizer (Ξ_2), partial farmland cultivation with applying fertilizer at agronomically recommended rate (Ξ_1), and full farmland cultivation with three distinct fertilizer application rates; agronomic recommendation (Ξ_5), less than agronomic recommendation (Ξ_4), and none (Ξ_3). As we discuss from the next section onward, transitions among these strategies will play a critical role in understanding how the farmer should adjust optimal decisions as a response to a change in the business environment (e.g., an increase in cultivation cost per acre); specifically when this change induces the farmer to switch from one optimal strategy to another in which both cultivation and fertilizer application decisions are different (for instance, a switch from Ξ_4 to Ξ_1 in Figure 1 as the cultivation cost per acre r_c increases.)

As highlighted in the Introduction, one of the objectives of this paper is to examine how joint optimization of the cultivation and fertilizer application decisions affects the key insights associated with farm management and food security in comparison with those insights that are based on optimization of only cultivation decision (as offered by the extant Operations Management literature).

To this end, we consider a benchmark model where the farmer optimizes the cultivation decision without applying any fertilizer.[3] In this case, the optimal cultivation decision is given by the characterization in Proposition 1 without imposing the condition $y_c > y_c^{(0)}$; that is, the characterization is relevant for any $y_c > 0$. Throughout the remainder of our analysis, whenever applicable, we make a comparison with this benchmark model. We also make the following assumptions hereafter:

Assumption 1 *We assume*

(i) $r_c + \bar{s}y_c < \mathbb{E}\left[p(\tilde{m},\tilde{\epsilon})(\tilde{\epsilon}+a\bar{s})\right]$ *and* $r_c < \mathbb{E}\left[p(\tilde{m},\tilde{\epsilon})\tilde{\epsilon}\right]$*;*

(ii) $\tilde{m}$ *and* $\tilde{\epsilon}$ *have independent distributions;*

(iii) $p(\tilde{m},\tilde{\epsilon}) = \tilde{m} - \alpha(\tilde{\epsilon}-\mu_\epsilon)$ *for* $\alpha \in [0, \underline{m}/(\bar{\epsilon}-\mu_\epsilon))$ *and* $\omega_h(\tilde{\epsilon}) = \omega_0 + \beta\tilde{\epsilon}$ *for* $\omega_0 > 0$ *and* $\beta \geq 0$*.*

Assumption 1(i) implies that, as follows from Corollary 1, the farmer optimally cultivates a positive amount of farmland (i.e., $x_c^* > 0$). It also implies that the optimal cultivation volume is given by $\min(\hat{x}_c^{nf}, Q)$ in the benchmark model. Paralleling our discussion in §2, Assumption 1(ii) introduces additional structure on the distributions of $\tilde{m}$ and $\tilde{\epsilon}$ whereas Assumption 1(iii) introduces specific functional forms for the crop price and external unit harvesting cost; these are necessary for the tractability of our analysis in the next section.

4 Implications of Optimal Decisions for Farm Management

In this section, motivated by the recent increasing costs and uncertainties in the farming environment as highlighted in the Introduction, we examine how changes in cultivation and fertilizer costs as well as farm yield variability impact the farmer's optimal decisions and profitability. These analyses are useful for generating important practical insights for farm management (see the end of this section) and for understanding the implications of farmer's optimal decisions for food security (see §5). Our results on the effect of fertilizer cost and how the optimal fertilizer application rate is impacted by cultivation cost and farm yield variability cannot be obtained using the benchmark model. For our remaining results, unless we state any differences it should be understood that they extend those results obtained using the benchmark model to our setting.

We first investigate the effects of cultivation and fertilizer costs. It is easy to establish that an increase in either of these costs decreases the farmer's profitability. We next examine how these costs impact the farmer's optimal decisions using the illustration given by Figure 1.

[3] Another benchmark model is the one that optimizes the cultivation decision for a given fertilizer application rate $s > 0$. In this case, the optimal cultivation decision can be obtained from the characterization in Proposition 2 by substituting $\bar{s}$ with s and removing the condition $y_c < y_c^{(2)}$. Because this model yields identical insights with the model without any fertilizer application, for brevity, we only consider the latter throughout the rest of the analysis.

Proposition 4 (Effect of cultivation cost per acre r_c) *We have* $\frac{\partial \hat{x}_c^{nf}}{\partial r_c} < 0$, $\frac{\partial \hat{x}_c^{f}}{\partial r_c} < 0$, *and* $\frac{\partial \hat{s}_c}{\partial r_c} = 0$. *Moreover,* $\hat{x}_c^{nf} > \hat{x}_c^{f}$ *for* $y_c \in [y_c^{(1)}, y_c^{(0)})$. *When* r_c *increases, (i)* x_c^* *decreases and (ii)* s_c^* *does not change except for the cases when the increase in* r_c *induces a transition from either* Ξ_2 *or* Ξ_4 *to* Ξ_1 *in Figure 1 (in these cases* s_c^* *increases).*

Intuitively, an increase in r_c incents the farmer to decrease the optimal cultivation volume x_c^*. However, the effect on the optimal fertilizer application rate s_c^* is more nuanced. Common intuition may suggest that an increase in r_c does not affect s_c^* because fertilizer cost is the relevant cost for this decision. Proposition 4 shows that this intuition is correct (for example, $\hat{s}_c$ does not change) *unless the increase in* r_c *induces the farmer to switch from one optimal strategy to another in which both* x_c^* *and* s_c^* *are different.* In particular, as illustrated in Figure 1, when an increase in r_c induces the farmer to switch the optimal strategy from either $(Q, \hat{s}_c)$ or $(\hat{x}_c^{nf}, 0)$ to $(\hat{x}_c^{f}, \bar{s})$, s_c^* increases to counteract against the reduction in crop availability at the harvesting stage due to decreasing x_c^*.

Proposition 5 (Effect of unit fertilizer cost y_c) *We have* $\frac{\partial \hat{x}_c^{nf}}{\partial y_c} = 0$, $\frac{\partial \hat{x}_c^{f}}{\partial y_c} < 0$, *and* $\frac{\partial \hat{s}_c}{\partial y_c} < 0$. *When* y_c *increases, (i)* s_c^* *decreases and (ii)* x_c^* *decreases except for the cases when the increase in* y_c *induces a transition from* Ξ_1 *to either* Ξ_2 *or* Ξ_4 *in Figure 1 (in these cases* x_c^* *increases).*

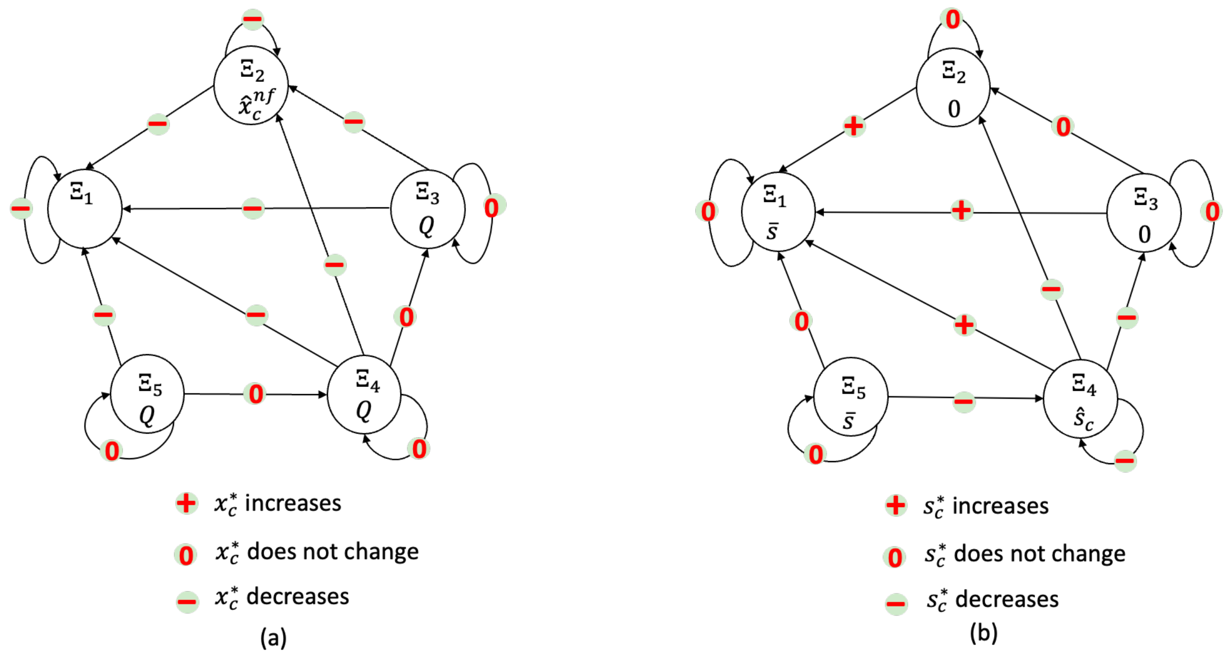
While an increase in unit fertilizer cost y_c intuitively incents the farmer to decrease the optimal fertilizer application rate s_c^*, the effect on the optimal cultivation volume x_c^* is more nuanced. Common intuition may suggest that an increase in y_c incents the farmer to decrease x_c^* because the farm input (fertilizer in this case) becomes more expensive. Proposition 5 shows that this intuition is correct (for example, $\hat{x}_c^{f}$ decreases) *unless the increase in* y_c *induces the farmer to switch from one optimal strategy to another in which both* x_c^* *and* s_c^* *are different.* In particular, as illustrated in Figure 1, when an increase in y_c induces the farmer to switch the optimal strategy from $(\hat{x}_c^{f}, \bar{s})$ to either $(Q, \hat{s}_c)$ or $(\hat{x}_c^{nf}, 0)$, x_c^* increases to counteract against the reduction in crop availability at the harvesting stage due to decreasing s_c^*.

We next examine the effects of farm yield variability σ_ϵ on the farmer's optimal decisions and profitability. To this end, as discussed in §2, we further assume that the farm yield has a Normal distribution. The effects can only be analytically characterized under specific conditions:

Proposition 6 (Effect of farm yield variability σ_ϵ) *Assume* $\tilde{\epsilon} \sim \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$.
(i) When $\alpha = 0$ *and* $\beta = 0$, *we have* $\frac{\partial \Pi_c^*(r_c, y_c)}{\partial \sigma_\epsilon} \leq 0$.
(ii) When $\alpha = 0$ *and* $K_h \geq (\mu_\epsilon + a\bar{s})Q$, *we have* $\frac{\partial \hat{x}_c^{f}}{\partial \sigma_\epsilon} \leq 0$, $\frac{\partial \hat{x}_c^{nf}}{\partial \sigma_\epsilon} \leq 0$, *and* $\frac{\partial \hat{s}_c}{\partial \sigma_\epsilon} \leq 0$. *Figure 2*

characterizes the effect of an increase in σ_m on x_c^* (panel a) and s_c^* (panel b): When σ_ϵ increases, (i) x_c^* decreases and (ii) s_c^* decreases except for cases when it induces a transition from Ξ_2, Ξ_3, or Ξ_4 to Ξ_1 in Figure 2 (in these cases s_c^* increases).

Figure 2: Network representation of the transition of the optimal cultivation volume x_c^* (a) and fertilization application rate s_c^* (b) as σ_ϵ increases.



Note. Inside each node is the Ξ_i region and its corresponding optimal decisions: x_c^* (a) and s_c^* (b). When σ_ϵ increases, (x_c^*, s_c^*) starting from any region Ξ_i $(i \in \{1, \cdots, 5\})$ can transition to another region as indicated by the arrows in each panel. Two panels illustrate how an increase in σ_ϵ affects the optimal decisions locally (within each Ξ region) and globally (across these Ξ regions). For example, for $(Q, \hat{s}_c)$ in Ξ_4 while a small increase in σ_ϵ does not impact x_c^* and decreases s_c^* (as depicted by the sign on the loop-arrow on Ξ_4 in panels (a) and (b), respectively), a large increase in σ_ϵ may induce the farmer to change the optimal strategy to $(\hat{x}_c^f, \bar{s})$ in Ξ_1 and thus, decreases x_c^* and increases s_c^* (as depicted by the sign on the arrow from Ξ_4 to Ξ_1 in panels (a) and (b), respectively).

When $\alpha = 0$, the uncertain crop price is given by $\tilde{m}$ (see Assumption 1(iii)) and it is not affected by the farm yield uncertainty. In this case, an increase in farm yield variability σ_ϵ decreases profitability because while low yield realizations are detrimental (owing to low crop availability for harvesting), high yield realizations are not as beneficial: the farmer is exposed to external unit harvesting cost $\omega_0 + \beta\epsilon$. Proposition 6 proves this result for the special case of $\beta = 0$ but we find in our numerical studies that this result continues to hold for the $\beta > 0$ case. Based on the same argument (about how profitability is affected), common intuition may suggest that an increase in σ_ϵ incents the farmer to cultivate fewer acres and apply less fertilizer per acre. Proposition 6 demonstrates that this intuition is correct for the effect on optimal cultivation volume x_c^*. However, the intuition is correct for the effect on optimal fertilizer application rate s_c^* (for example, $\hat{s}_c$ decreases) *unless the increase in σ_ϵ induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different.* In particular, when an increase in σ_ϵ induces the farmer to switch the optimal strategy from $(Q, 0)$, $(Q, \hat{s}_c)$, or $(\hat{x}_c^{nf}, 0)$ to $(\hat{x}_c^f, \bar{s})$, s_c^* increases to counteract against the reduction in crop availability at the harvesting stage due to decreasing x_c^*. Proposition 6 proves the results associated with optimal decisions for the case with sufficiently high harvesting capacity K_h (i.e., $K_h \geq (\mu_\epsilon + a\bar{s})Q$). As we discuss in the next section, this K_h range corresponds to a realistic representation of farming environment in practice.

When $\alpha > 0$, characterizing the effect of farm yield variability is not analytically tractable. This is because in comparison with the $\alpha = 0$ case there is an additional impact that works in the opposite direction. To illustrate this, let us focus on the effect of σ_ϵ on farmer's profitability. Different from the $\alpha = 0$ case, an increase in σ_ϵ also increases the variability of crop price $\tilde{m} - \alpha(\tilde{\epsilon} - \mu_\epsilon)$ which, in turn, increases the farmer's profitability. This is because while the farmer benefits from high crop price realizations, low crop price realizations are not as detrimental: the farmer optimally chooses not to acquire additional resource to increase the harvest volume beyond the available capacity when the crop price is less than the external unit harvesting cost. Nevertheless, we find in our data-calibrated numerical studies, where the estimated parameters satisfy $\alpha > 0$ and $\beta > 0$, the farmer's profitability always decreases in σ_ϵ. Similarly, we find in our numerical studies that the results associated with optimal decisions for the $\alpha = 0$ case, as presented in Proposition 6, continues to hold for the $\alpha > 0$ case. We refer the reader to §6.2 for the details of these analyses.

Our results in this section have important practical insights for farm management. In discussing these insights, for expositional brevity and practical relevance we focus on more realistic scenarios in which the fertilizer cost is not very high so that the farmer optimally uses some fertilizer; that is, the optimal strategies are those given in regions Ξ_1, Ξ_4, and Ξ_5 in Figure 1. These are the optimal strategies that we observe in our data-calibrated numerical studies in the context of fresh tomato farming. We summarize our insights below:

(1) Based on the knowledge base developed in the extant OM literature (that focuses only on cultivation optimization), the farmer's best response to increasing cultivation cost is to reduce cultivation volume. This response is also commonly observed in practice. For example, as highlighted by Evans (2022) glasshouse farmers in the U.K. reduce their cultivation volume of cucumbers and sweet peppers as a response to increasing cost of energy which is one of the primary inputs used for cultivation in glasshouse farming. Our results suggest that the farmer should also apply more

fertilizer as a response to increasing cultivation cost; specifically when the cultivation and fertilizer costs are moderate (where we observe a transition from region Ξ_4 to Ξ_1 in Figure 1).

(2) In practice there is no shortage of anecdotal evidence that documents farmers reducing their fertilizer application rate as a response to increasing fertilizer cost. For example, Thomas and Maltais (2021) reports that escalating fertilizer costs lead some farmers in the U.S. to cut back on their overall fertilizer use. Our results highlight that reducing the fertilizer application rate without changing the cultivation volume is the best response to increasing fertilizer cost only when the cultivation cost per acre r_c is low (specifically, lower than the r_c level that solves $y_c^{(2)} = \hat{y}_c(r_c)$ in Figure 1). Otherwise, our results demonstrate that the best response to increasing fertilizer cost is one of the following: (i) cultivate more acres and use less fertilizer and (ii) cultivate fewer acres without changing the fertilizer application rate. In particular, the former response should be employed when the cultivation and fertilizer costs are moderate (where we observe a transition from region Ξ_1 to Ξ_4 in Figure 1) whereas the latter response should be employed otherwise.

(3) As discussed in the Introduction, there is widespread empirical evidence that showcases climate-induced shocks increasing the farm yield variability in a variety of agricultural industries including the fresh produce industry. These climate-induced shocks affect the farmer's profitability by increasing the uncertainties in harvestable crop volume, external harvesting cost, and crop price as the latter two factors also depend on farm yield variability. Our results in the context of fresh tomato farming illustrate that the overall impact of an increase in yield variability is detrimental for profitability. In terms of farmer's best response to increasing yield variability, the knowledge base developed in the extant OM literature (that focuses only on cultivation optimization) suggests reducing the cultivation volume. Our results identify that the farmer should also apply less fertilizer while cultivating fewer acres as a response to increasing yield variability unless the cultivation and fertilizer costs are moderate (where we observe a transition from region Ξ_4 to Ξ_1 in Figure 1). In that case, the farmer should apply more fertilizer while cultivating fewer acres.

The general insight from our results is that in designing an effective response to changes in the farming environment it is important to have a holistic approach that jointly considers the two levers of increasing the crop volume at the harvesting stage: cultivating more acres or applying more fertilizer per acre. As we discuss in the next section, this joint consideration will also play a key role in understanding the implications of farmer's optimal decisions for food security.

5 Implications of Optimal Decisions for Food Security

In this section, we contextualize in our setting the gap between maximum attainable and actual crop production levels, a food security measure similar to those commonly used in the literature (see, for example, Godfray et al. (2010)). In particular, we define the expected gap as the difference between the expected maximum attainable yield $(\mu_\epsilon + a\bar{s})Q$ and the expected optimal harvest volume $\mathbb{E}[x_h^*(\tilde{m}, \tilde{\epsilon})]$ at the farmer's optimal cultivation and fertilizer application decisions. Paralleling §4, motivated by the recent increasing costs and uncertainties in the farming environment, we investigate how changes in cultivation and fertilizer costs as well as farm yield variability impact the expected gap. We say that a change is beneficial for food security when the expected gap decreases and harmful otherwise. As we discuss at the end of this section, our analyses are useful for understanding the consequences of some commonly adopted policies in practice that have been devised to increase farmer's crop production level and income. Throughout our analysis, whenever applicable, we will again highlight how considering the fertilizer application decision affects the insights offered based on the benchmark model.

Let $J^*(r_c, y_c) = (\mu_\epsilon + a\bar{s})Q - \mathbb{E}[x_h^*(\tilde{m}, \tilde{\epsilon})]$ denote the expected gap for a given cultivation cost per acre r_c and unit fertilizer cost y_c. Here, the expected optimal harvest volume is given by

$$\mathbb{E}[x_h^*(\tilde{m}, \tilde{\epsilon})] = \mathbb{E}\left[x_c^*(\tilde{\epsilon} + a s_c^*)\mathbb{I}\{p(\tilde{m}, \tilde{\epsilon}) \geq \omega_h(\tilde{\epsilon})\} + \min\left(x_c^*(\tilde{\epsilon} + a s_c^*), K_h\right)\mathbb{I}\{p(\tilde{m}, \tilde{\epsilon}) < \omega_h(\tilde{\epsilon})\}\right], \quad (3)$$

where in the harvesting stage the farmer optimally harvests all the available crop $x_c^*(\epsilon + a s_c^*)$ when the crop price is larger than the external unit harvesting cost (i.e., $p(m, \epsilon) \geq \omega_h(\epsilon)$); otherwise, the farmer optimally harvests all the available crop up to the internal harvesting capacity K_h. The expected optimal harvest volume in (3) can be strictly less than the expected maximum attainable yield $(\mu_\epsilon + a\bar{s})Q$ because the farmer may choose not to cultivate whole farmland (i.e., $x_c^* \neq Q$) or not to apply fertilizer at its agronomic recommendation (i.e., $s_c^* \neq \bar{s}$) in the cultivation stage, or it may not be profitable to acquire additional resource in the harvesting stage (i.e., $p(m, \epsilon) < \omega_h(\epsilon)$) when the available crop is larger than the internal harvesting capacity K_h.

In this section, to obtain sharper insights we make the following additional assumption:

Assumption 2 *$K_h \geq (\mu_\epsilon + a\bar{s})Q$.*

This assumption states that the farmer has sufficient internal capacity to harvest the expected maximum attainable yield. Using Assumption 2, it is easy to establish that in the absence of uncertainty; that is, when the farm yield and market uncertainty realizations always equal their respective means, the farmer always optimally cultivates the whole farmland (i.e., $x_c^* = Q$) and applies fertilizer at agronomically recommended rate (i.e., $s_c^* = \bar{s}$) for any r_c and y_c in the cultivation stage and the farmer does not need to acquire additional resource to harvest the available crop in harvesting stage. Therefore, $J^*(r_c, y_c) = 0$ in the absence of uncertainty (which is a realistic representation of farming environment in practice). In other words, when Assumption 2 holds, farm yield and market uncertainties are the key drivers of a positive expected gap.

We first examine how changes in cultivation cost per acre r_c and unit fertilizer cost y_c impact the expected gap $J^*(r_c, y_c)$. As follows from (3), a change in each cost affects the expected gap only by altering the optimal decisions (x_c^*, s_c^*) in the cultivation stage.

Proposition 7 (Effect of cultivation cost per acre r_c) *When r_c increases, $J^*(r_c, y_c)$ increases except for the cases when it induces a transition from either Ξ_2 or Ξ_4 to Ξ_1 in Figure 1.*

Common intuition may suggest that an increase in r_c (which makes farming more expensive) decreases the expected optimal harvest volume, and thus, increases the expected gap. Proposition 7 proves that this intuition is correct (that is, an increase in r_c is harmful for food security) unless it induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different. In particular, when r_c increases, as follows from Proposition 4, the farmer optimally cultivates fewer acres and does not alter optimal fertilizer application rate except for the cases when the increase in r_c induces the farmer to switch the optimal strategy from either $(Q, \hat{s}_c)$ or $(\hat{x}_c^{nf}, 0)$ to $(\hat{x}_c^f, \bar{s})$ in Figure 1. Outside of these cases, because x_c^* decreases and s_c^* does not change, the expected optimal harvest volume in (3) decreases, and thus, the expected gap increases as shown in Proposition 7. When these cases happen, the farmer optimally increases s_c^* to counteract against the reduction in crop availability at the harvesting stage due to decreasing x_c^*. Because x_c^* decreases and s_c^* increases the resulting impact on the expected optimal harvest volume is indeterminate. We find in our data-calibrated numerical studies that the increase in fertilizer application rate may outweigh the decrease in cultivation volume and the expected optimal harvest volume increases (see §6.3 for details). In other words, an increase in r_c can be *beneficial* for food security. This behavior cannot be observed in the benchmark model where it can be proven that an increase in r_c always increases the expected gap and thus, it is always harmful for food security.

We next examine the effect of unit fertilizer cost y_c on the expected gap:

Proposition 8 (Effect of unit fertilizer cost y_c) *When y_c increases, $J^*(r_c, y_c)$ increases except for the cases when it induces a transition from Ξ_1 to either Ξ_2 or Ξ_4 in Figure 1.*

Proposition 8 proves that an increase in y_c is harmful for food security unless it induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different. In particular, when y_c increases, as follows from Proposition 5, the farmer optimally applies less fertilizer per acre and cultivates fewer acres except for the cases when the increase in y_c induces the farmer to switch the optimal strategy from $(\hat{x}_c^f, \bar{s})$ to either $(Q, \hat{s}_c)$ or $(\hat{x}_c^{nf}, 0)$ in Figure 1. Outside of these cases, because x_c^* and s_c^* decrease, the expected optimal harvest volume decreases, and thus, the expected gap increases as shown in Proposition 8. When these cases happen, the farmer optimally increases x_c^* to counteract against the reduction in crop availability at the harvesting stage due to decreasing s_c^*. Because x_c^* increases and s_c^* decreases the resulting impact on the expected optimal harvest volume is indeterminate. Nevertheless, we find in our numerical studies that the increase in cultivation volume does not outweigh the decrease in fertilizer application rate and thus, the result in Proposition 8 continues to hold in general (see §6.3 for details).

We next examine the effect of farm yield variability σ_ϵ. As follows from (3), a change in σ_ϵ affects the expected gap by altering the expected optimal harvest volume for any given farmer's decisions (x_c, s_c) as well as the farmer's optimal decisions (x_c^*, s_c^*) in the cultivation stage.

Proposition 9 (Effect of farm yield variability σ_ϵ) *Assume $\tilde{\epsilon} \sim \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$ and $\alpha = 0$. When σ_ϵ increases, $J^*(r_c, y_c)$ increases except for the cases when it induces a transition from Ξ_2, Ξ_3, or Ξ_4 to Ξ_1 in Figure 2.*

Recall that when $\alpha = 0$, the uncertain crop price is given by $\tilde{m}$ and it is not affected by the farm yield uncertainty. In this case, Proposition 9 proves that an increase in farm yield variability σ_ϵ decreases the expected gap–that is, it is harmful for food security—unless it induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different. As follows from (3), how an increase in σ_ϵ affects the expected optimal harvest volume for a given farmer's decisions (x_c, s_c) crucially depends on how it impacts the (stochastic) ordering between the crop price $\tilde{m}$ and the external unit harvesting cost $\omega_0 + \beta\tilde{\epsilon}$. This is because the farmer optimally harvests all the available crop $x_c(\epsilon + a s_c)$ only when the crop price is larger than this cost in the harvesting stage. It can be proven that an increase in σ_ϵ increases the likelihood that the external unit harvesting cost will be larger than crop price in the harvesting stage which, in turn, decreases the expected optimal harvest volume for a given (x_c, s_c). Proposition 6 has already established that an increase in σ_ϵ incents the farmer to cultivate fewer acres and apply less fertilizer per acre except for the cases when it induces the farmer to switch the optimal strategy from $(Q, 0)$, $(Q, \hat{s}_c)$, or $(\hat{x}_c^{nf}, 0)$ to $(\hat{x}_c^f, \bar{s})$ in Figure 2. Therefore, outside of these cases because x_c^* and s_c^* decrease, these changes further decrease the expected optimal harvest volume in (3), and thus, increase the expected gap as shown in Proposition 9. When these cases happen, because x_c^* decreases and s_c^* increases the resulting impact on the expected gap is indeterminate. This behavior is different from the benchmark model where it can be proven that an increase in σ_ϵ always increases the expected gap.

When $\alpha > 0$, the uncertain crop price $\tilde{m} - \alpha(\tilde{\epsilon} - \mu_\epsilon)$ is affected by the farm yield variability σ_ϵ and characterizing the effect of σ_ϵ on the expected gap is not analytically tractable. Nevertheless, we find in our data-calibrated numerical studies, where the estimated α has a positive value, the results associated with the $\alpha = 0$ case, as presented in Proposition 9, continues to hold for the $\alpha > 0$ case as well. We also find that when an increase in σ_ϵ induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different (specifically, from $(Q, \hat{s}_c)$ to $(\hat{x}_c^f, \bar{s})$), because s_c^* decreases the expected gap may also decrease; that is, an increase in σ_ϵ can be *beneficial* for food security. We refer the reader to §6.3 for the details of these analyses.

Our results in this section are useful for policymakers to understand the consequences of some commonly adopted policies in practice that have been devised to increase the farmer's crop production level and income. An example is distributing discount vouchers for procurement of seeds (which reduces the cultivation cost per acre) or fertilizer (which reduces the unit fertilizer cost); we refer the reader to Giné et al. (2022) for an application in the context of Tanzania's farming environment. Another example is increasing the availability of disease-resistant seeds that reduces the farm yield variability as highlighted by Kazaz et al. (2016); we refer the reader to Arndt et al. (2016) for an application in the context of Malawi's farming environment. The general insights from our analysis are that while a policy that reduces the cultivation or fertilizer cost or yield variability always increases the farmer's income (as measured by the optimal expected profit), its impact on the farmer's crop production level (as measured by the expected optimal harvest volume) is more nuanced. Consider a policy that reduces the cultivation cost or yield variability. Based on the knowledge base developed in the extant OM literature (that focuses only on cultivation optimization), this policy always increases the crop production level. However, our results demonstrate that this policy is proven to increase the crop production level only when it does not induce the farmer to switch from one optimal strategy to another in which the optimal cultivation volume is higher and the optimal fertilizer application rate is lower. In other cases (where our results in the previous section identify specific conditions under which they appear), this policy may backfire because the reduction in optimal fertilizer usage may decrease the farmer's crop production level as observed in the context of fresh tomato farming. Similar insights are also relevant for a policy that reduces the fertilizer cost except for one difference: we do not observe the reduction in optimal fertilizer usage decreasing the farmer's crop production level in the context of fresh tomato farming. In summary, it is important for the policymakers to consider the farmer's joint cultivation and farm-input application decisions as otherwise the devised policies may have unintended consequences.

6 Numerical Analysis: Application to Fresh Tomato Farming

In this section, we discuss an application of our model in the context of (fresh) tomato farming which is among the most valuable fresh produce, valued at approximately \$1.6 billion in the U.S. in 2019 (USDA, 2020). We calibrate our model parameters to represent a tomato farmer in Florida which is the largest fresh tomato growing region in the U.S. in 2019 (USDA, 2020). We provide a brief description of the data and calibration used for our numerical experiments (§6.1) and relegate its detailed discussion to §A of the online appendix. Using these experiments, we complement our analytical analyses in the previous two sections that examine the implications of optimal decisions for farm management and food security, respectively. To this end, we explore how changes in cultivation and fertilizer costs as well as farm yield variability impact the farmer's optimal decisions and profitability (§6.2) as well as the expected gap (§6.3). Throughout this section, we report our results in a selective fashion to complement the analytical results proven in §4 and §5 under specific conditions by numerically investigating the effects without imposing these conditions.

6.1 Data, Model Calibration, and Computation for Numerical Experiments

We obtain the historical fresh tomato selling price and farm yield in Florida from USDA (USDA, 2010, 2018) and obtain the historical harvesting labor wage from Bureau of Labor Statistics (United States Department of Labor, 2021). We denote any calibrated parameter z by $\acute{z}$ and display these parameters in Table 1. To represent the baseline scenario, besides using these calibrated parameter values we set $K_h = (\acute{\mu}_\epsilon + \acute{a}\acute{\bar{s}})\acute{Q}$ for the internal harvesting capacity and normalize the farmland to a single acre (i.e., $\acute{Q} = 1$). For this baseline scenario, we obtain the optimal expected profit as 6,770.83 (\$/acre) which is in the range of profits that could be obtained by substituting the selling price in Florida in 2014 from USDA (2018) to VanSickle and McAvoy (2015).

Numerical computation. In examining the implications of optimal decisions for farm management (§6.2) and food security (§6.3), we extend our numerical instances around the baseline scenario by varying several key parameters around their calibrated values. In particular, we consider the four key parameters, cultivation cost per acre r_c, unit fertilizer cost y_c, market variability σ_m, and farm yield variability σ_ϵ, to change by -45% to 45% from their calibrated values with a 15%

Table 1: Calibrated parameters for fresh tomato farming in Florida

$\acute{\mu}_\epsilon$	Mean of farm yield (carton/acre)	787.89
$\acute{\sigma}_\epsilon$	Standard deviation of farm yield (carton/acre)	117.45
$\acute{\mu}_m$	Normalized mean of crop price ($/carton)	11.54
$\acute{\sigma}_m$	Standard deviation of market uncertainty ($/carton)	2.80
$\acute{\alpha}$	Dependence of price on farm yield ($/(carton/acre))	0.01
$\acute{\beta}$	Dependence of external harvesting (labor) cost on farm yield ($/(carton/acre))	0.0067
$\acute{\omega}_0$	Base external harvesting (labor) cost ($)	2.28
$\acute{r}_c$	Cultivation cost per acre ($/acre)	4379.59
$\acute{\bar{s}}$	Maximum fertilizer application rate (agronomic recommendation) (lb/acre)	823.74
$\acute{y}_c$	Unit fertilizer cost ($/lb)	3.46
$\acute{a}$	Yield response to fertilizer application (carton/lb)	0.57

Note. We assume that market uncertainty $\tilde{m}$ and farm yield uncertainty $\tilde{\epsilon}$ follow a univariate Normal distribution. The weight units are the following: lb (one pound) and carton (25 pounds).

increment. We also consider K_h to be $0\%, 15\%$, and 30% away from its baseline values.[4] In total, we evaluate $7 \times 7 \times 7 \times 7 \times 3$ =7,203 numerical instances. In illustrating how a measure of interest (i.e., optimal cultivation volume, optimal fertilizer application rate, optimal expected profit, and the expected gap) at a given numerical instance (e.g., baseline scenario) changes with respect to r_c, y_c, or σ_ϵ, we plot our figures using a finer increment than 15% (specifically, 0.1% increment) within the range of [−45%,45%] of the calibrated value.

In all the instances considered, the optimal strategies that emerge are those when the fertilizer cost is either low or moderate; that is, the farmer either cultivates the whole farmland Q while applying fertilizer at agronomically recommended rate $\bar{s}$ or partial rate $\hat{s}_c$, or cultivates $\hat{x}_c^f$ acres while applying fertilizer at $\bar{s}$ (i.e., Ξ_5, or Ξ_4, or Ξ_1, respectively, in Figure 1). We note here that we observe a transition from one optimal strategy to another in which both cultivation volume and fertilizer application rate are different, specifically when there is a transition between regions Ξ_4 and Ξ_1 where the optimal decisions are $(Q, \hat{s}_c)$ and $(\hat{x}_c^f, \bar{s})$, respectively. As discussed in the previous two sections, this transition will have critical implications for our results.

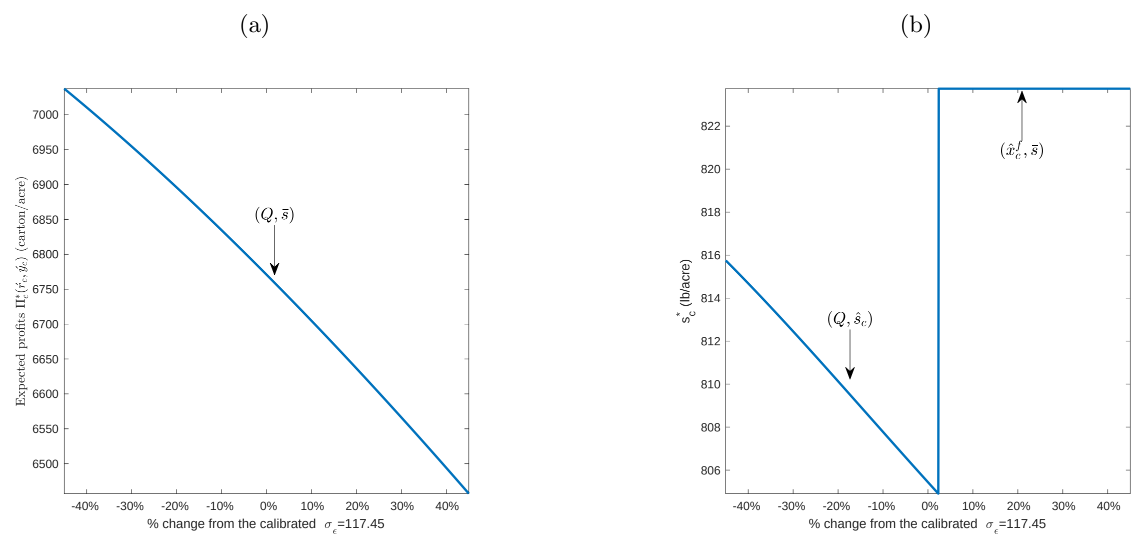
6.2 Implications of optimal decisions for farm management

Propositions 4 and 5 fully characterize the effects of cultivation cost per acre r_c and unit fertilizer cost y_c on the farmer's optimal decisions, respectively. Therefore, we focus on the effects of farm yield variability σ_ϵ on the farmer's optimal decisions and profitability. Proposition 6 proves under the special case of $\alpha = 0$ and $\beta = 0$ that the optimal expected profit decreases in σ_ϵ. In our

[4] Throughout our numerical experiments, we only consider K_h values that are no smaller than $(\mu_\epsilon + a\bar{s})Q$ to be consistent with Assumption 2 in §5.

numerical experiments, we have $\acute{\alpha} > 0$ and $\acute{\beta} > 0$ as shown in Table 1. As discussed above, we consider 1,029 numerical instances (generated by changing r_c, y_c, σ_m, and K_h around the baseline scenario) and examine in each instance how σ_ϵ affects the farmer's profitability. In all these instances we consistently observe that, paralleling our result in Proposition 6, the optimal expected profit decreases in σ_ϵ; see Figure 3(a) for an illustration. We also consistently observe that when σ_ϵ increases, (i) the optimal cultivation volume x_c^* decreases and (ii) optimal fertilizer application rate s_c^* decreases except for the cases when it induces a transition from Ξ_4 to Ξ_1 (in these cases s_c^* increases). These results are the same as those proven in Proposition 6 for the special case of $\alpha = 0$. Figure 3(b) provides an illustration for the optimal fertilizer application rate s_c^*. In this example as σ_ϵ increases, s_c^* first decreases—where $\hat{s}_c$ decreases as proven in Proposition 6 for $\alpha = 0$—and then increases to $\bar{s}$ as the optimal strategy changes from $(Q, \hat{s}_c)$ (Ξ_4) to $(\hat{x}_c^f, \bar{s})$ (Ξ_1). In the latter case, s_c^* is increased from $\hat{s}_c$ to $\bar{s}$ to counteract against the reduction in crop availability at the harvesting stage due to a decrease in x_c^* from Q to $\hat{x}_c^f$.

Figure 3: Effects of Farm Yield Variability σ_ϵ on the Optimal Expected Profit Π_c^* (Panel a) and the Optimal Fertilizer Application Rate s_c^* (Panel b)



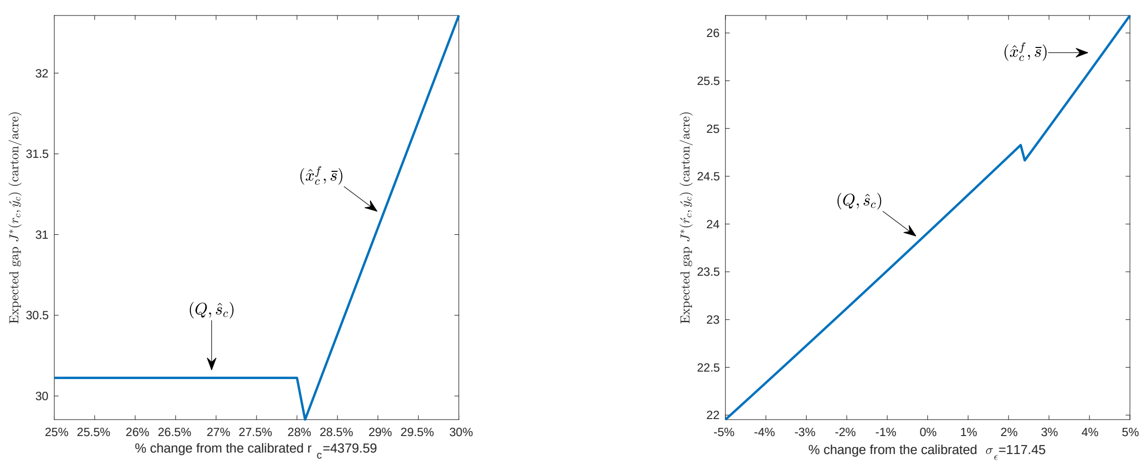
Notes. In each panel, $\sigma_\epsilon \in [-45\%, 45\%]$ changes from the baseline value $\acute{\sigma}_\epsilon = 117.45$ with 0.1% increments. In panel b, $r_c = 1.3\acute{r}_c$, $y_c = 1.3\acute{y}_c$, and the rest of the parameters in both panels are at their calibrated (baseline) levels.

6.3 Implications of optimal decisions for food security

Propositions 7 and 8 prove that when either cultivation cost per acre r_c or unit fertilizer cost y_c increases, the expected gap $J^*(r_c, y_c)$ also increases except for cases when it induces the farmer to switch from one optimal strategy to another in which both optimal cultivation volume and fertilizer application rate are different (in these cases the effects on $J^*(r_c, y_c)$ are indeterminate). In our numerical experiments such a transition only exists between regions Ξ_4 and Ξ_1. As unit fertilizer cost y_c increases, in all numerical experiments that includes a transition from Ξ_1 to Ξ_4 we observe that expected gap continues to increase. As cultivation cost per acre r_c increases, in some of the numerical experiments that includes a transition from Ξ_4 to Ξ_1 we observe that the expected gap decreases; see Figure 6(a) for an example. In this example as r_c increases, the expected gap is non-decreasing except when the farmer's optimal strategy switches from $(Q, \hat{s}_c)$ (Ξ_4) to $(\hat{x}_c^f, \bar{s})$ (Ξ_1). In that case, a higher r_c increases the expected gap because the increase in the fertilizer application rate (from $\hat{s}_c$ to $\bar{s}$) outweighs the decrease in the cultivation volume (from Q to $\hat{x}_c^f$) which, in turn, increases the optimal expected harvest volume in (3).

Figure 4: Effects of Cultivation Cost Per Acre r_c (Panel a) and Farm Yield Variability σ_ϵ (Panel b) on the Expected Gap $J^*(r_c, y_c)$

(a) (b)



Notes. In panel a, $r_c \in [25\%, 30\%]$ away from the baseline value $\acute{r}_c = 4379.59$ with 0.1% increments, $y_c = 1.3\acute{y}_c$, and $\sigma_\epsilon = 1.15\acute{\sigma}_\epsilon$. In panel b, $\sigma_\epsilon \in [-5\%, 5\%]$ away from the baseline value $\acute{\sigma}_\epsilon = 117.45$ with 0.1% increments, $r_c = 1.3\acute{r}_c$, and $y_c = 1.3\acute{y}_c$. In both panels, the rest of the parameters are at their calibrated (baseline) levels.

We next examine the effect of farm yield variability σ_ϵ. Proposition 9 proves under the $\alpha = 0$ assumption that when σ_ϵ increases, the expected gap $J^*(r_c, y_c)$ also increases except for cases when it induces the farmer to switch from one optimal strategy to another in which both optimal cultivation volume and fertilizer application rate are different (in these cases the effect on $J^*(r_c, y_c)$ is indeterminate). In our numerical experiments, we verify that this result continues to hold when $\alpha > 0$. We also find that when an increase in σ_ϵ induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different, the expected gap may also decrease; see

Figure 6(b) for an example. In this example as σ_ϵ increases, the expected gap is non-increasing except when the farmer's optimal strategy switches from $(Q, \hat{s}_c)$ (Ξ_4) to $(\hat{x}_c^f, \bar{s})$ (Ξ_1). In that case, a higher σ_ϵ increases the expected gap because the increase in the fertilizer application rate (from $\hat{s}_c$ to $\bar{s}$) outweighs the decrease in the cultivation volume (from Q to $\hat{x}_c^f$) which, in turn, increases the optimal expected harvest volume in (3).

7 Conclusions

Motivated by the fresh produce industry, this paper studies a farmer's joint cultivation and fertilizer (a representative farm input) application decisions in the presence of uncertainties in crop's open market price, harvesting cost (labor cost for hiring seasonal workers) and farm yield where yield is stochastically increasing in the fertilizer application rate. Motivated by the recent changes in the farming environment as summarized in the Introduction, we provide insights on how the farmer's optimal decisions and profitability as well as the resulting expected optimal harvest volume (a measure of food security) are affected by increasing cultivation and fertilizer costs as well as farm yield uncertainty. We show that these effects can be significantly different from those when only cultivation decision is optimized (as is the case in the extant OM literature); specifically when these effects induce the farmer to change the fertilizer application and cultivation decisions in opposite directions. Based on our results, we put forward practical insights for farm management. We also provide policy insights by shedding light on unintended consequences of some commonly adopted policies in practice that have been devised to increase farmer's crop production level and income.

We conduct additional analyses to examine other research questions relevant to our setting; the details of these analyses are relegated to §C of the online appendix. First, in §C.1, we examine how an increase in market variability σ_m (which increases the crop price variability) impact the farmer's optimal decisions and profitability as well as the expected gap. This analysis is motivated by the observation that crop price variability is one of the key reasons that drives the farmers to leave their crop unharvested on their farmland (World Wild Fund, 2021), and thus, it is important for the farmers in practice to understand how to respond to changes in crop price variability. We find that the effects of an *increase* in σ_m on the farmer's optimal decisions and profitability are structurally the same as the effects of a *decrease* in yield variability σ_ϵ on these two measures (as given by Proposition 6). Similarly, we also find that the effect of an *increase* in σ_m on the expected gap is structurally the same as the effect of a *decrease* in yield variability on the same measure (as given by Proposition 9) when the expected crop price is sufficiently low. In our data-calibrated baseline scenario this condition is not satisfied and an increase in σ_m increases the expected gap in the context of fresh tomato farming. As an increase in σ_m increases the farmer's profitability, but also increases the expected gap, this implies that a policymaker has to carefully balance the two potential policy objectives. Second, in §C.2, we examine whether a policy that reduces the labour cost for hiring seasonal workers always increases the crop production level. To this end, we investigate how changes in base labor cost ω_0 impact the crop production level (as measured by the expected optimal harvest volume). We find that while a reduction in ω_0 always increases the crop production level in the benchmark model, it increases the crop production level in our setting except for the cases when the reduction in ω_0 induces the farmer to switch the optimal strategy from $(\hat{x}_c^f, \bar{s})$ to $(\hat{x}_c^{nf}, 0)$, $(Q, 0)$, or $(Q, \hat{s}_c)$. Outside of these cases, a reduction in ω_0 induces the farmer to cultivate more acres and apply more fertilizer per acre, increasing the crop production level. When these cases happen, a reduction in ω_0 induces the farmer to apply less fertilizer per acre while cultivating more acres. We find in our data-calibrated numerical studies that in these cases a reduction in ω_0 may decrease the crop production level. Finally, in §C.3, we extend our model to consider contract farming, a commonly observed practice in agricultural industries (Huh and Lall, 2013), where the farmer sells the harvested crop to a buyer at a fixed unit price r up to a maximum volume D and sells the remaining crop to the open market. We show under realistic assumptions for contract parameters (r, D) that all of our results continue to hold in this setting.

Our work has several limitations due to our specific modeling assumptions and further research is needed to validate the relevance of our insights when those assumptions are relaxed. First, we assume that the farmer's cultivation, fertilizer application, and subsequent harvesting decisions have no effect on the crop's open market price. This is a reasonable assumption for commodity crops sold in open markets (including fresh produce) as considered in this paper where production volume of an individual farmer is insignificant in comparison to the aggregate production volume traded in the open market.[5] However, for a non-commodity crop or a commodity crop sold in the local market where the crop price is not benchmarked to the open market price, the farmer's decisions may affect the crop price by altering its availability in the market. Examining our research questions in this setting requires a quantity-dependent crop price modeling. While we expect that our results that showcase differences from the knowledge base developed in the OM literature (that focuses only on cultivation decision) would continue to be relevant in this setting, future research is still needed to verify this conjecture. Second, we model the farmer's objective as expected profit maximization which assumes that the farmer does not have an aversion to profit variability. While

[5]An equivalent interpretation of our setting is a farmer growing a commodity crop to sell through a bilateral contract where the contract price is benchmarked on the crop's open market price.

this is a reasonable assumption for a non-smallholder farmer in a developed country (e.g., U.S.) as considered in our model calibration, it may not be reasonable for a smallholder farmer in a developing country (e.g., India) whose minimum income needed for subsistence heavily depends on farm profit. A smallholder farmer may have an aversion to farm profit variability, specifically to the scenarios in which the realized profit is lower than the subsistence income. One potential approach to factor in the aversion to low profit scenarios is to consider contract farming as discussed above. As we demonstrate in §C.3 of the online appendix, it can be shown that contract farming creates value for the farmer by substituting the profit from open market sale at low profit realizations with profit from contract sale; that is, engaging in contract farming enables the farmer to reduce the exposure to low profit scenarios. We have already discussed above that our main results continue to hold in the presence of contract farming. Another potential approach is to consider a risk-averse utility function for the farmer. Examining the robustness of our insights in this setting should prove to be an interesting avenue for future research.

Our work can be extended to examine other interesting research questions in the context of food security challenges in farming. Based on our analysis in §5 and §6 we provide insights on how a specific subsidy policy—for example, distributing a voucher for seed procurement or distributing a voucher for fertilizer procurement—affects the farmer's crop production level and income. It would be interesting to make comparisons across these subsidy policies for helping policymakers in choosing the right subsidy to implement. To this end, our results underline one important characteristic in the context of fresh tomato farming: distributing a voucher for fertilizer procurement (which decreases the unit fertilizer cost) always increases the crop production level whereas distributing a voucher for seed procurement (which decreases the cultivation cost per acre) may not. Using our model, future research can be conducted to make further comparisons across subsidy policies based on how each policy affects the farmer's crop production level and income to (i) determine conditions under which one policy outperforms the others and (ii) examine how consideration of fertilizer application decision affects these conditions. Moreover, it would be interesting to investigate how a government that maximizes food security (as measured by crop production level) should choose which subsidy policy to implement in a farming ecosystem. This analysis would require an equilibrium model that captures the interaction between the government and a population of farmers (that represent the farming ecosystem) and it is beyond the scope of this paper. Our paper's insights will be useful in understanding this interaction because there is a need to capture how each subsidy policy affects an individual farmer's optimal decisions in that setting.

References

Ahumada, O., J. R. Villalobos. 2009. Application of planning models in the agri-food supply chain: A review. *European Journal of Operational Research* **196**(1) 1–20.

Akkaş, A., V. Gaur. 2022. OM Forum—Reducing food waste: An operations management research agenda. *Manufacturing & Service Operations Management* **24**(3) 1261–1275.

Akkaya, D., K. Bimpikis, H. Lee. 2020. Government interventions to promote agricultural innovation. *Manufacturing & Service Operations Management* **23**(2) 437–452.

Alizamir, S., F. Iravani, H. Mamani. 2018. An analysis of price vs. revenue protection: Government subsidies in the agriculture industry. *Management Science* **65**(1) 32–49.

Anderson, E., M. Monjardino. 2019. Contract design in agriculture supply chains with random yield. *European Journal of Operational Research* **277**(3) 1072–1082.

Arndt, C., K. Pauw, J. Thurlow. 2016. The economy-wide impacts and risks of Malawi's farm input subsidy program. *American Journal of Agricultural Economics* **98**(3) 962–980.

Babcock, B. A., J. A. Chalfant, R. N. Collender. 1987. Simultaneous input demands and land allocation in agricultural production under uncertainty. *Western Journal of Agricultural Economics* **12**(2) 207–215.

Boyabatlı, O., J. Nasiry, Y. Zhou. 2019. Crop planning in sustainable agriculture: Dynamic farmland allocation in the presence of crop rotation benefits. *Management Science* **65**(5) 2060–2076.

Calvin, L., P. Martin. 2010. The U.S. produce industry and labor: Facing the future in a global economy. `https://www.ers.usda.gov/webdocs/publications/44764/err-106.pdf?v=0`, last accessed in December, 2018.

Chintapalli, P., C. S. Tang. 2021. The value and cost of crop minimum support price: Farmer and consumer welfare and implementation cost. *Management Science* **67**(11) 6839–6861.

Evans, J. 2022. UK cucumber and pepper crops face energy and labour crunch. `https://www.ft.com/content/f2d7494d-429e-49e1-aa37-97dc9eb1ecbc`, Last Accessed in April 2023.

Federgruen, A., U. Lall, A. S. Şimşek. 2019. Supply chain analysis of contract farming. *Manufacturing & Service Operations Management* **21**(2) 361–378.

Giné, X., S. Patel, B. Ribeiro, I. Valley. 2022. Efficiency and equity of input subsidies: Experimental evidence from Tanzania. *American Journal of Agricultural Economics* **104**(5) 1625–1655.

Glen, J. J. 1987. Mathematical models in farm planning: A survey. *Operations Research* **35**(5) 641–666.

Godfray, H. C. J., J. R. Beddington, I. R. Crute, L. Haddad, D. Lawrence, J. F. Muir, J. Pretty, S. Robinson, S. M. Thomas, C. Toulmin. 2010. Food security: The challenge of feeding 9 billion people. *Science* **327**(5967) 812–818.

Gunders, D., J. Bloom. 2012. Wasted: How America is losing up to 40 percent of its food from farm to fork to landfill `https://www.nrdc.org/sites/default/files/wasted-food-IP.pdf`, Last Accessed in April 2023.

Gunders, D., J. Bloom. 2017. Wasted: How America is losing up to 40 percent of its food from farm to fork to landfill. https://www.nrdc.org/sites/default/files/wasted-2017-report.pdf, Last Accessed in April 2023.

Hayashi, Y. 2022. Ukraine War Creates Worst Global Food Crisis Since 2008, IMF Says. https://www.wsj.com/articles/ukraine-war-creates-worst-global-food-crisis-since-2008-imf-says-11664553601, Last Accessed in April 2023.

Hochmuth, G., E. Hanlon. 2020. A Summary Of N, P, And K Research With Tomato In Florida. https://edis.ifas.ufl.edu/publication/CV236, last accessed in May 2021.

Hu, M., Y. Liu, W. Wang. 2019. Socially beneficial rationality: The value of strategic farmers, social entrepreneurs, and for-profit firms in crop planting decisions. *Management Science* **65**(8) 3654–3672.

Huh, W. T., U. Lall. 2013. Optimal crop choice, irrigation allocation, and the impact of contract farming. *Production and Operations Management* **22**(5) 1126–1143.

Kazaz, B. 2004. Production planning under yield and demand uncertainty with yield-dependent cost and price. *Manufacturing & Service Operations Management* **6**(3) 209–224.

Kazaz, B., S. Webster. 2011. The impact of yield-dependent trading costs on pricing and production planning under supply uncertainty. *Manufacturing & Service Operations Management* **13**(3) 404–417.

Kazaz, B., S. Webster, P. Yadav. 2016. Interventions for an artemisinin-based malaria medicine supply chain. *Production and Operations Management* **25**(9) 1576–1600.

Li, B., O. Boyabatlı, B. Avcı. 2022. Economic and environmental implications of biomass commercialization in agricultural processing. *Management Science* Forthcoming.

Livingston, M., M. J. Roberts, Y. Zhang. 2015. Optimal sequential plantings of corn and soybeans under price uncertainty. *American Journal of Agricultural Economics* **97**(3) 855–878.

Lowe, T. J., P. V. Preckel. 2004. Decision technologies for agribusiness problems: A brief review of selected literature and a call for research. *Manufacturing & Service Operations Management* **6**(3) 201–208.

Maatman, A., C. Schweigman, A. Ruijs, M. H. van Der Vlerk. 2002. Modeling farmers' response to uncertain rainfall in Burkina Faso: A stochastic programming approach. *Operations Research* **50**(3) 399–414.

Miyao, G., B. Aegerter, D. Sumner, D. Stewart. 2017. Sample costs to produce processing tomatoes. https://ucanr.edu/sites/colusa/files/277960.pdf, last accessed in April, 2023.

Myers, S. 2021. Too Many to Count: Factors Driving Fertilizer Prices Higher and Higher. https://www.fb.org/market-intel/too-many-to-count-factors-driving-fertilizer-prices-higher-and-higher, Last Accessed in April 2023.

Partridge, R., J. Partington. 2021. The anxiety is off the scale': UK farm sector worried by labour shortages. https://www.theguardian.com/business/2021/aug/25/the-anxiety-is-off-the-scale-uk-farm-sector-worried-by-labour-shortages, Last Accessed in April 2023.

Richards, T. J. 2018. Immigration reform and farm labor markets. *American Journal of Agricultural Economics* **100**(4) 1050–1071.

Tembo, G., B. W. Brorsen, F. M. Epplin, E. Tostão. 2008. Crop input response functions with stochastic plateaus. *American Journal of Agricultural Economics* **90**(2) 424–434.

Thomas, P., K. Maltais. 2021. Surging Fertilizer Costs Push Farmers to Shift Planting Plans, Raise Prices. https://www.wsj.com/articles/surging-fertilizer-costs-push-farmers-to-shift-planting-plans-raise-prices-11639580768, Last Accessed in April 2023.

Tigchelaar, M., D. S. Battisti, R. L. Naylor, D. K. Ray. 2018. Future warming increases probability of globally synchronized maize production shocks. *Proceedings of the National Academy of Sciences* **115**(26) 6644–6649.

United States Department of Labor. 2021. Bureau of Labor Statistics quarterly census of employment and wages, NAICS-Based Data Files (1975 - most recent). https://www.bls.gov/cew/downloadable-data-files.htm, last accessed in June, 2021.

USDA. 2010. U.S. Tomato statistics historical data. https://www.ers.usda.gov/data-products/vegetables-and-pulses-data/vegetables-and-pulses-historical-data/, last accessed in June, 2021.

USDA. 2018. Quick Stats, National agriculture statistics service. https://quickstats.nass.usda.gov/, last accessed in May, 2018.

USDA. 2020. Vegetables 2020 Summary, USDA National Agricultural Statistics Service (NASS). https://downloads.usda.library.cornell.edu/usda-esmis/files/02870v86p/j6731x86f/9306tr664/vegean21.pdf, last accessed in June, 2021.

USDA Economic Research Service. 2020. Economic Drivers of Food Loss at the Farm and Pre-Retail Sectors: A Look at the Produce Supply Chain in the United States. https://www.ers.usda.gov/webdocs/publications/95779/eib-216.pdf?v=2683.3, last accessed in April, 2023.

USDA National Agricultural Statistics Service. 2022. National Agricultural Statistics Service. Vegetables 2021 Summary. https://downloads.usda.library.cornell.edu/usda-esmis/files/02870v86p/zs25zc490/9593vz15q/vegean22.pdf, Last Accessed in April 2023.

VanSickle, J., E. McAvoy. 2015. Production budget for tomatoes grown in southwest Florida and in the Palmetto-ruskin area of Florida. https://view.officeapps.live.com/op/view.aspx?src=http%3A%2F%2Ffred.ifas.ufl.edu%2Fpdf%2Fiatpc%2Ffiles%2FSWFLTomBudget2015.docx, last accessed in June, 2021.

World Wild Fund. 2021. Driven to Waste: The Global Impact of Food Loss and Waste on Farms. https://files.worldwildlife.org/wwfcmsprod/files/Publication/file/6yoepbekgh_wwf_uk__driven_to_waste___the_global_impact_of_food_loss_and_waste_on_farms.pdf?_ga=2.36480992.1328403993.1681743677-314194109.1681743677, Last Accessed in April 2023.

Zhang, Y., J. M. Swaminathan. 2020. Improved crop productivity through optimized planting schedules. *Manufacturing & Service Operations Management* **22**(6) 1165–1180.

Online Appendix for *Integrated Optimization of Cultivation and Fertilizer Application: Implications for Farm Management and Food Security*

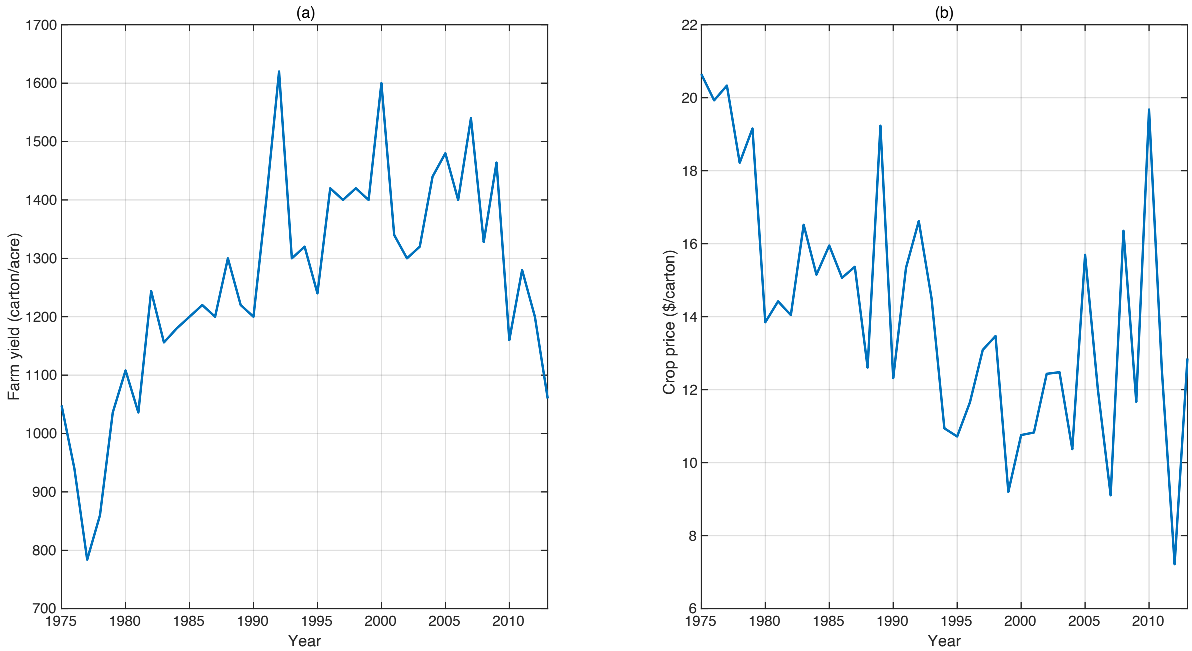
Appendix A Data and Calibration

Data. We obtain the historical fresh tomato annual yield (in carton/acre) and annual selling price (in \$/carton) in Florida for 1975-1997 from USDA Economic Resource Service (USDA, 2010) and for 1998-2013 from USDA National Agricultural Statistics Service QuickStats (USDA, 2018). Figure 5 plots the farm yield in panel a and consumer price index (CPI)-adjusted crop price in panel b, respectively. We obtain the fresh tomato harvesting labor wage (in \$/week) as the historical labor wage for the vegetable and melon farming industry (where fresh tomato is categorized under) from Bureau of Labor Statistics (United States Department of Labor, 2021). In particular, we use the data under the industry classification NAICS with code 111219 for 1990-2013 and the data under the industry classification SIC with code 016 for 1975-1989. As the wage data under these two classifications differ slightly for the overlapping yeas 1990-2000, we first compute $\sum_{i=1}^{11} \frac{h_i}{h_i'}/11 = 0.98$, where h_i (h_i'), $i \in \{1, \cdots, 11\}$, represents harvesting wage under NAICS (SIC) classification for year 1990 to 2000. Then, we multiply the harvesting wage under SIC in each year from 1975-1989 by 0.98 to obtain those under NAICS for each year in 1975-1989. We next convert these harvesting labor wage data from \$/week to \$/carton: We divide the harvesting labor wage each year in 1975-2013 by (436\$/week)/ 2.05\$/carton, where 436 is the average weekly labor wage for the farming industry with NAICS code 111219 in Florida in 2014 and 2.05 is the harvesting labor cost from VanSickle and McAvoy (2015)—both numbers are obtained for 2014, the year of sample cost used to calibrate other parameters. We finally adjust both the crop price and harvesting wage using the U.S. CPI with the base year 2014.

Model and its calibration. Recall that $\tilde{p}(m, \epsilon) = \tilde{m} - \alpha(\tilde{\epsilon} - \mu_\epsilon)$ and we assume that $\tilde{m}$ and $\tilde{\epsilon}$ are independently normally distributed with mean μ_m and μ_ϵ and standard deviation σ_μ and σ_ϵ, respectively. We can show that $\tilde{p}$ is normally distributed with $\mu_p = \mu_m$ and standard deviation $\sigma_p = \sqrt{\sigma_m^2 + \alpha^2\sigma_\epsilon^2}$. We can also show that $\tilde{p}$ and $\tilde{\epsilon}$ follow a bivariate distribution with the covariance $\text{cov}(\tilde{p}, \tilde{\epsilon}) = -\alpha\sigma_\epsilon^2$ and correlation coefficient $\rho = \frac{-\alpha\sigma_\epsilon}{\sigma_p}$. We use Henze-Zirkler test to verify whether the price and yield data follow a bivariate normal distribution and find that one cannot reject the null hypothesis that $\tilde{p}$ and $\tilde{\epsilon}$ are bivariate normal random variables, where the p value of the Henze-Zirkler test is 0.38. Thus, in order to calibrate μ_m, μ_ϵ, α, σ_μ and σ_ϵ, we first use the price data series to obtain $\mu_p = 14.16$ and $\sigma_p = 3.37$, use the yield data series to obtain $\mu_\epsilon = 1260.62$ and $\sigma_\epsilon = 187.92$, and then obtain the correlation coefficient between price and yield as $\rho = -0.58$. We next obtain the other parameters as follows: $\mu_m = \mu_p = 14.16$, $\acute{\alpha} = -\rho\sigma_p/\sigma_\epsilon \doteq 0.01$, and $\acute{\sigma}_m = \sqrt{\sigma_p^2 - \alpha^2\sigma_\epsilon^2} \doteq 2.80$. (Note that μ_m will be further adjusted below.)

Recall also $\omega_h(\tilde{\epsilon}) = \omega_0 + \beta\tilde{\epsilon}$ and ω_h follows a normal distribution with mean μ_ω and standard deviation σ_ω, we can easily show that $\mu_\omega = \omega_0 + \beta\mu_\epsilon$ and $\sigma_\omega^2 = \beta^2\sigma_\epsilon^2$. To calibrate ω_0 and β, we first use the wage data series to obtain $\mu_\omega = 1.63$ and $\sigma_\omega = 0.17$ and use yield data to obtain

Figure 5: Florida Fresh Tomato Farm Yield (Panel a) and CPI-adjusted Crop Price (Panel b)



$\mu_\epsilon = 1288.67$ and $\sigma_\epsilon = 163.74$. Note that the values for μ_ϵ and σ_ϵ differ slightly from those obtained previously as we use wage and yield data from 1978 to 2013 due to the fact that the wage data for 1975 to 1977 are abnormally high. Then we can obtain $\beta = \frac{\sigma_\omega}{\sigma_\epsilon} \doteq 0.001$ and $\omega_0 = \mu_\omega - \beta\mu_\epsilon \doteq 0.34$. Then, we multiply these two values by 1.4 to account for the 40% labor cost overhead (Miyao et al., 2017) to obtain $\beta = 0.0014$ and $\omega_0 = 0.34 \times 1.4 = 0.476$.

As our model implicitly assumes that when yield is low, we use contracted labor and implicitly normalize their harvesting wage to be zero, we thus subtract the mean of the harvesting wage from the mean of the crop price to obtain the mean of crop price as $\mu_m = 14.16 - 1.63 = 12.53$.

Calibration of other parameters. We calibrate other parameters using a sample cost for fresh tomato growers in Southwest Florida (VanSickle and McAvoy, 2015). We calculate the cultivation cost as all cultivation-related costs minus the cost of yield-enhancing resources, including fertilizer, fumigant, herbicide, and insecticide. We calibrate $\acute{r}_c$ as the total cultivation cost (7,231.84 \$/acre) minus the cost of yield-enhancing resources (2,852.25 \$/acre), i.e., $\acute{r}_c = 7,231.84 - 2,852.25 = 4,379.59$ (\$/acre). We set $\acute{\bar{s}} = 823.74$ (lb/acre) as the sum of the weight of all yield-enhancing resources; we then compute $\acute{y}_c$ as the total cost of all these yield-related resources divided by $\acute{\bar{s}}$ to obtain $2,852.25/823.74 \doteq 3.46$ (\$/lb). We also obtain harvesting cost excluding harvesting labor cost from VanSickle and McAvoy (2015) to be \$0.99/ carton. And since our model explicitly normalizes the unit harvesting cost to be zero, we thus subtract this unit harvesting cost from the mean of the harvesting price and obtain the final mean of the crop price to be $\acute{\mu}_m = 12.53 - 0.99 = 11.54$.

Remove the effect of yield-enhancing resources on the calibration. As farm yield data are related to those applied with yield-enhancing cultivation resources (such as fertilizer), we remove this effect from the calibration to obtain the calibration without these resources. We experiment with different values for the percentage increase from the set {60%, 70%, 80%, 90%}, which are among the most frequent values in Hochmuth and Hanlon (2020). Therefore, we obtain the new calibration of the yield-related parameters as the values of these parameters (i.e., $\mu_\epsilon = 1260.62$ and $\sigma_\epsilon = 187.92$) divided by the value from the set $\{1.6,\ 1.7,\ 1.8,\ 1.9\}$. For instance, given a percentage increase of 60%, we obtain the mean farm yield with the yield-enhancing effect removed through dividing μ_ϵ by 1.6 to obtain $\acute{\mu}_\epsilon = 1260.62/1.6 \doteq 787.89$. Therefore, we compute $\acute{a}$ as the value such that applying the maximum rate of yield-enhancing resources (i.e., $\acute{\bar{s}}$) results in 60% increase in the mean yield, i.e., $\acute{a}\acute{\bar{s}} = 787.89 \times 0.6$, so we obtain $\acute{a} = 787.89 \times 0.6/823.74 \doteq 0.57$ (carton/lb). To maintain the same magnitude of harvesting labor wage after the yield-enhancing effect is removed, we multiply the wage parameters β and ω_0 by 1.6 , that is, $\acute{\beta} = 0.0014 \times 1.6 \doteq 0.0024$ and $\acute{\omega}_0 = 0.476 \times 1.6 \doteq 0.76$. This way of removing the effect in the calibration is equivalent to removing this effect from the yield data before calibration.

Note that the harvesting labor wage parameters obtained previously are for the cases when labor wage can be low or high. As there is no shortage of evidence that farmers leave the fields unharvested when labor wage is high (USDA Economic Research Service, 2020), we focus on such scenarios in the numerical experiments in §6. To obtain the parameters related to the harvesting wage in such scenarios, we experimented with different multipliers of such parameters obtained previously, so that the probability of observing that the crop price is less than the harvesting wage is not too low. In particular, we multiple β and ω_0 by three, that is, $\acute{\beta} = 0.0024 \times 3 \doteq 0.0067$ and $\acute{\omega}_0 = 0.76 \times 3 \doteq 2.28$. In this case, the probability that the crop price is less than the harvesting labor wage is 12.3%. We also experimented even larger values of the multipliers and find the qualitative insights are similar.

Appendix B Proofs of Main Results

Throughout the Appendix, we denote stage-1 objective function for a given cultivation volume x_c and fertilizer application rate s_c as $\pi_c(x_c, s_c)$ where, as follows from (1),

$$\pi_c(x_c, s_c) \doteq \mathbb{E}\left[p(\tilde{m}, \tilde{\epsilon}) \min\left(x_c(\tilde{\epsilon} + as_c), K_h\right) + \left(p(\tilde{m}, \tilde{\epsilon}) - \omega_h(\tilde{\epsilon})\right)^+ \left(x_c(\tilde{\epsilon} + as_c) - K_h\right)^+\right] - y_c s_c x_c - r_c x_c,$$

Using the identity $\min(p(m,\epsilon), \omega_h(\epsilon)) = p(m,\epsilon) - (p(m,\epsilon) - \omega_h(\epsilon))^+$ we can rewrite $\pi_c(x_c, s_c)$ as $\mathbb{E}\left[p(\tilde{m}, \tilde{\epsilon}) x_c(\tilde{\epsilon} + as_c) - \min(p(\tilde{m}, \tilde{\epsilon}), \omega_h(\tilde{\epsilon}))(x_c(\tilde{\epsilon} + as_c) - K_h)^+\right] - y_c s_c x_c - r_c x_c$. We will use this expression throughout this appendix.

We prove Propositions 1-3 using the same two-step approach: we first optimize the fertilizer application rate s_c for a given cultivation volume $x_c > 0$ (in Lemma 1), and then obtain the optimal x_c^*.

Lemma 1 *The optimal fertilizer application rate for a given $x_c > 0$ is*

$$s_c^*(x_c) = \begin{cases} 0, & \text{if } y_c \geq a\mathbb{E}\left[p(\tilde{m}, \tilde{\epsilon}) - \min(p(\tilde{m}, \tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right], \\ \hat{s}_c(x_c), & \text{if } a\mathbb{E}\left[p(\tilde{m}, \tilde{\epsilon}) - \min(p(\tilde{m}, \tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\bar{s}\right\}\right] \leq y_c \text{ and} \\ & \quad y_c < a\mathbb{E}\left[p(\tilde{m}, \tilde{\epsilon}) - \min(p(\tilde{m}, \tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right], \\ \bar{s}, & \text{if } y_c < a\mathbb{E}\left[p(\tilde{m}, \tilde{\epsilon}) - \min(p(\tilde{m}, \tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\bar{s}\right\}\right], \end{cases}$$

where $\hat{s}_c(x_c) > (K_h/x_c - \bar{\epsilon})^+/a$ *is the unique solution to*

$$y_c = a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\hat{s}_c(x_c)\right\}\right]. \tag{A-1}$$

When $x_c = 0$*, any fertilizer application rate within* $[0, \bar{s}]$ *is optimal.*

Proof of Lemma 1 $\pi_c(x_c, s_c)$ is concave in s_c for a given $x_c > 0$, as we have

$$\frac{\partial \pi_c(x_c,s_c)}{\partial s_c} = x_c a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - as_c\right\}\right] - y_c x_c,$$

which decreases in $s_c \in [0, \bar{s}]$. We consider the following three cases:

(i) if $\frac{\partial \pi_c(x_c,s_c)}{\partial s_c}|_{s_c=0} \leq 0$, i.e., $y_c \geq a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right]$, $s_c^*(x_c) = 0$;

(ii) if $\frac{\partial \pi_c(x_c,s_c)}{\partial s_c}|_{s_c=0} > 0$ and $\frac{\partial \pi_c(x_c,s_c)}{\partial s_c}|_{s_c=\bar{s}} \leq 0$, i.e., $a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\bar{s}\right\}\right] \leq y_c < a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right]$, $s_c^*(x_c) = \hat{s}_c(x_c)$, where $\hat{s}_c(x_c)$ is the unique solution to the first order condition (A-1).

(iii) if $\frac{\partial \pi_c(x_c,s_c)}{\partial s_c}|_{s_c=\bar{s}} > 0$, i.e., $y_c < a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\bar{s}\right\}\right]$, $s_c^*(x_c) = \bar{s}$.

Combining the above three cases gives $s_c^*(x_c)$ as shown in the lemma. ■

Proof of Propositions 1-3 Using Lemma 1, we solve for the optimal x_c. Noting the bounds of x_c (i.e., $x_c \in [0, Q]$) as well as the conditions in Lemma 1, we consider three cases depending on the value of y_c:

Large y_c*:* $y_c \geq a\mathbb{E}[p(\tilde{m},\tilde{\epsilon})]$;

Small y_c*:* $y_c < a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{Q} - a\bar{s}\right\}\right]$;

Moderate y_c*:* (i) $a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{Q}\right\}\right] \leq y_c < a\mathbb{E}[p(\tilde{m},\tilde{\epsilon})]$, and (ii) $a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{Q} - a\bar{s}\right\}\right] \leq y_c < a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{Q}\right\}\right]$.

The above three cases correspond to Propositions 1-3, respectively.

Proof of Proposition 1.

When $y_c \geq a\mathbb{E}[p(\tilde{m},\tilde{\epsilon})]$, we obtain from Lemma 1 that $s_c^*(x_c) = 0$ for all $x_c \in (0, Q]$. Substituting $s_c^*(x_c) = 0$ into the objective function $\pi_c(x_c, s_c)$, we obtain

$$\pi_c(x_c, 0) = \mathbb{E}\left[p(\tilde{m},\tilde{\epsilon})x_c\tilde{\epsilon} - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))(x_c\tilde{\epsilon} - K_h)^+\right] - r_c x_c.$$

For $x_c \leq K_h/\bar{\epsilon}$, $\pi_c(x_c, 0) = \mathbb{E}[p(\tilde{m},\tilde{\epsilon})\tilde{\epsilon}]x_c - r_c x_c$, which is a linear function of x_c. For $x_c > K_h/\bar{\epsilon}$, we take the derivative of $\pi_c(x_c, 0)$ with respect to x_c and obtain

$$\frac{d\pi_c(x_c,0)}{dx_c} = \mathbb{E}\left[\tilde{\epsilon}\left(p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right)\right] - r_c,$$

which decreases in x_c. Thus $\pi_c(x_c, 0)$ is linear in x_c for $x_c \leq K_h/\bar{\epsilon}$ and is concave in x_c for $x_c > K_h/\bar{\epsilon}$. Then using the definition of $\Theta(x_c)$ in Proposition 1, we obtain the optimal solution as shown in Proposition 1.

Proof of Proposition 2.

When $y_c < a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{Q} - a\bar{s}\right\}\right]$, we obtain from Lemma 1 that $s_c^*(x_c) = \bar{s}$ for all $x_c \in (0, Q]$. The proof follows the same approach as that of Proposition 1 and is omitted here.

Proof of Proposition 3.

We only prove case (i) $a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{Q}\right\}\right] \le y_c < a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon})\right]$, and omit that for case (ii), which can be done analogously. In case (i), we obtain from Lemma 1 that $s_c^*(x_c)$ may take one of the three forms depending on the value of x_c. Noting that both $a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\bar{s}\right\}\right]$ and $a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right]$ decrease in x_c, we can rewrite the conditions in Lemma 1 with respect to the value of x_c (instead of y_c). Let $\underline{x}_c$ be the unique solution to

$$a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\bar{s}\right\}\right] = y_c,$$

and $\bar{x}_c$ be the unique solution to

$$a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right] = y_c.$$

From these definitions we obtain $K_h/\underline{x}_c - a\bar{s} = K_h/\bar{x}_c$, and thus $\underline{x}_c < \bar{x}_c$. Substituting $s_c^*(x_c)$ into $\pi_c(x_c, s_c)$, we obtain $\pi_c(x_c, s_c^*(x_c))$ as follows:

$$\begin{cases} \mathbb{E}\left[p(\tilde{m},\tilde{\epsilon})x_c(\tilde{\epsilon}+a\bar{s}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))(x_c(\tilde{\epsilon}+a\bar{s}) - K_h)^+\right] - y_c\bar{s}x_c - r_c x_c, \\ \hfill \text{if } 0 < x_c \le \underline{x}_c, \\ \mathbb{E}\left[p(\tilde{m},\tilde{\epsilon})x_c(\tilde{\epsilon}+a\hat{s}_c(x_c)) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))(x_c(\tilde{\epsilon}+a\hat{s}_c(x_c)) - K_h)^+\right] - y_c\hat{s}_c(x_c)x_c - r_c x_c, \\ \hfill \text{if } \underline{x}_c < x_c \le \bar{x}_c, \\ \mathbb{E}\left[p(\tilde{m},\tilde{\epsilon})x_c\tilde{\epsilon} - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))(x_c\tilde{\epsilon} - K_h)^+\right] - r_c x_c, \\ \hfill \text{if } \bar{x}_c < x_c \le Q. \end{cases}$$

The proofs of Propositions 1 and 2 have shown the properties of the above piecewise function when $0 < x_c \le \underline{x}_c$ and $\bar{x}_c < x_c \le Q$. Now taking the derivative of the second expression (for $\underline{x}_c < x_c \le \bar{x}_c$) with respect to x_c, we have

$$\begin{aligned}
&\frac{d\pi_c(x_c,\hat{s}_c(x_c))}{dx_c} \\
=&\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon})(\tilde{\epsilon}+a\hat{s}_c(x_c)) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))(\tilde{\epsilon}+a\hat{s}_c(x_c))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\hat{s}_c(x_c)\right\}\right] \\
&+ a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon})x_c - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))x_c\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\hat{s}_c(x_c)\right\}\right]\frac{\partial\hat{s}_c(x_c)}{\partial x_c} \\
&- y_c x_c\frac{\partial\hat{s}_c(x_c)}{\partial x_c} - y_c\hat{s}_c(x_c) - r_c \\
=&\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon})\tilde{\epsilon} - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\tilde{\epsilon}\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\hat{s}_c(x_c)\right\}\right] - r_c \\
&+ \left(x_c\frac{\partial\hat{s}_c(x_c)}{\partial x_c} + \hat{s}_c(x_c)\right)\left(a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\hat{s}_c(x_c)\right\}\right] - y_c\right) \\
=&\mathbb{E}\left[\tilde{\epsilon}\left(p(\tilde{m},\tilde{\epsilon}) - \min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\hat{s}_c(x_c)\right\}\right)\right] - r_c,
\end{aligned}$$

where the last equality follows from the optimality condition for $\hat{s}_c(x_c)$ as given by equation (A-1).

Therefore, the derivative of $\pi_c(x_c, s_c^*(x_c))$ with respect to x_c is as follows:

$$\frac{d\pi_c(x_c, s_c^*(x_c))}{dx_c}$$
$$=\begin{cases} \mathbb{E}\left[(\tilde{\epsilon}+a\bar{s})\left(p(\tilde{m},\tilde{\epsilon})-\min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon}>\frac{K_h}{x_c}-a\bar{s}\right\}\right)\right]-y_c\bar{s}-r_c & \text{if } 0<x_c\le \underline{x}_c,\\ \mathbb{E}\left[\tilde{\epsilon}\left(p(\tilde{m},\tilde{\epsilon})-\min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon}>\frac{K_h}{x_c}-a\hat{s}_c(x_c)\right\}\right)\right]-r_c & \text{if } \underline{x}_c<x_c\le \bar{x}_c,\\ \mathbb{E}\left[\tilde{\epsilon}\left(p(\tilde{m},\tilde{\epsilon})-\min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon}>\frac{K_h}{x_c}\right\}\right)\right]-r_c, & \text{if } \bar{x}_c<x_c\le Q.\end{cases}$$
$$=\begin{cases} \Gamma(x_c)-y_c\bar{s}-r_c, & \text{if } 0<x_c\le \underline{x}_c,\\ \Theta\left(\frac{K_h}{\frac{K_h}{x_c}-a\hat{s}_c(x_c)}\right)-r_c & \text{if } \underline{x}_c<x_c\le \bar{x}_c,\\ \Theta(x_c)-r_c, & \text{if } \bar{x}_c<x_c\le Q.\end{cases}$$

Recall that the left hand side of equation (A-1) is independent of x_c and the right hand side of the equation is the expectation of a function of $\tilde{m}$ and $\tilde{\epsilon}$ over the intervals $m \in [\underline{m}, \bar{m}]$ and $\epsilon \in (K_h/x_c - a\hat{s}_c(x_c), \bar{\epsilon}]$. Thus, $K_h/x_c - a\hat{s}_c(x_c)$ must be a constant. This shows that $\Theta(K_h/(K_h/x_c - a\hat{s}_c(x_c))) - r_c$ does not depend on x_c, implying that the objective function is linear in x_c for $x_c \in [\underline{x}_c, \bar{x}_c]$. Following the same approach as in the proof of Proposition 1, we can show that $\pi_c(x_c, s_c^*(x_c))$ is linear in $x_c \in [0, K_h/(\bar{\epsilon}+a\bar{s})]$, strictly concave in $x_c \in [K_h/(\bar{\epsilon}+a\bar{s}), \underline{x}_c]$, linear in $x_c \in [\underline{x}_c, \bar{x}_c]$ and strictly concave in $x_c \in [\bar{x}_c, Q]$, and is also globally concave. Note that $\Gamma(\underline{x}_c) - y_c\bar{s} = \Theta(\bar{x}_c)$, so $\Gamma(\underline{x}_c) \le r_c + \bar{s}y_c$ is equivalent to $r_c \ge \Theta(\bar{x}_c)$. Then we obtain the optimal solution as shown in Proposition 3. Since $\underline{x}_c < \bar{x}_c$, it follows that $\hat{x}_c^f < \hat{x}_c^{nf}$. ■

Proof of Corollary 1 We only prove that $\hat{y}_c(r_c)$ is increasing and concave in r_c as the other results are straightforward. Recall that $\hat{y}_c(r_c)$ solves the equation $r_c = \Theta(\bar{x}_c)$, or more explicitly,

$$r_c = \mathbb{E}\left[\tilde{\epsilon}\left(p(\tilde{m},\tilde{\epsilon})-\min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon}>\frac{K_h}{\bar{x}_c}\right\}\right)\right], \tag{A-2}$$

where $\bar{x}_c$ is defined in Proposition 3 with y_c replaced by $\hat{y}_c$. That is, $\bar{x}_c$ satisfies the following equation

$$a\mathbb{E}\left[p(\tilde{m},\tilde{\epsilon})-\min(p(\tilde{m},\tilde{\epsilon}),\omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon}>\frac{K_h}{\bar{x}_c}\right\}\right]=\hat{y}_c. \tag{A-3}$$

Differentiating both sides of equation (A-2) with respect to r_c yields

$$1 = -\frac{K_h^2}{\bar{x}_c^3}\frac{\partial \bar{x}_c}{\partial r_c}g_\epsilon\left(\frac{K_h}{\bar{x}_c}\right)\mathbb{E}\left[\min\left(p\left(\tilde{m},\frac{K_h}{\bar{x}_c}\right),\omega_h\left(\frac{K_h}{\bar{x}_c}\right)\right)\right], \tag{A-4}$$

from which we obtain $\frac{\partial \bar{x}_c}{\partial r_c} < 0$. Now differentiating both sides of equation (A-3) with respect to r_c yields

$$\begin{aligned}\frac{d\hat{y}_c}{dr_c} &= -\frac{K_h}{\bar{x}_c^2}\frac{\partial \bar{x}_c}{\partial r_c}g_\epsilon\left(\frac{K_h}{\bar{x}_c}\right)a\mathbb{E}\left[\min\left(p\left(\tilde{m},\frac{K_h}{\bar{x}_c}\right),\omega_h\left(\frac{K_h}{\bar{x}_c}\right)\right)\right]\\ &= a\frac{\bar{x}_c}{K_h} > 0,\end{aligned}$$

where we have used equation (A-4) to derive the second equality. The concavity of $\hat{y}_c$ follows because $\frac{d^2\hat{y}_c}{dr_c^2} = \frac{a}{K_h}\frac{\partial \bar{x}_c}{\partial r_c} < 0$. ■

Proof of Proposition 4 From the definition of $\hat{x}_c^{nf}$, we know $\Theta(\hat{x}_c^{nf}) = r_c$. Differentiating both sides of this equation with respect to r_c gives $\frac{\partial \Theta(\hat{x}_c^{nf})}{\partial \hat{x}_c^{nf}}\frac{\partial \hat{x}_c^{nf}}{\partial r_c} = 1$. Since $\Theta(x_c)$ decreases in x_c from its definition in Proposition 1, we know that $\frac{\partial \hat{x}_c^{nf}}{\partial r_c} < 0$. The same argument can be used to show $\frac{\partial \hat{x}_c^{f}}{\partial r_c} < 0$. Using the definition of $\hat{s}_c$ in equation (2), we know that $\hat{s}_c$ does not depend on r_c which implies that $\frac{\partial \hat{s}_c}{\partial r_c} = 0$. The relationship $\hat{x}_c^{nf} > \hat{x}_c^{f}$ has been shown in the proof of Proposition 3.

As r_c increases to a larger extent, the optimal solution may take a different form, which corresponds to region transitions in Figure 1. Note that the boundaries do not move as r_c changes. There are four possible transitions after an increase in r_c: (a) $\Xi_5 \to \Xi_1$: x_c^* decreases from Q to $\hat{x}_c^f$, and s_c^* remains at $\bar{s}$. (b) $\Xi_3 \to \Xi_2$: x_c^* decreases from Q to $\hat{x}_c^{nf}$, and s_c^* remains at 0. (c) $\Xi_4 \to \Xi_1$: x_c^* decreases from Q to $\hat{x}_c^f$, and s_c^* increases from $\hat{s}_c$ to $\bar{s}$. (d) $\Xi_2 \to \Xi_1$: x_c^* decreases from $\hat{x}_c^{nf}$ to $\hat{x}_c^f$, and s_c^* increases from 0 to $\bar{s}$. Therefore, x_c^* decreases whenever an increase in r_c results in a shift across regions, while s_c^* does not change except for the cases when the increase in r_c induces a transition from either Ξ_2 or Ξ_4 to Ξ_1. This together with the first statement established earlier shows that when r_c increases, (i) x_c^* decreases and (ii) s_c^* does not change except for the cases when it induces a transition from either Ξ_2 or Ξ_4 to Ξ_1 in Figure 1 (in these cases s_c^* increases). ■

Proof of Proposition 5 The proof is similar to that of Proposition 4. The first statement follows directly from the definitions of $\hat{x}_c^{nf}$, $\hat{x}_c^f$, and $\hat{s}_c$ in Propositions 1-3, respectively.

As y_c increases to a larger extent, the optimal solution may take a different form, which corresponds to region transitions in Figure 1. Note that the boundaries do not move as y_c changes. There are five possible transitions after an increase in y_c: (a) $\Xi_5 \to \Xi_1$: x_c^* decreases from Q to $\hat{x}_c^f$, and s_c^* remains at $\bar{s}$. (b) $\Xi_5 \to \Xi_4$: x_c^* remains at Q, and s_c^* decreases from $\bar{s}$ to $\hat{s}_c$. (c) $\Xi_4 \to \Xi_3$: x_c^* remains at Q, and s_c^* decreases from $\hat{s}_c$ to 0. (d) $\Xi_1 \to \Xi_4$: x_c^* increases from $\hat{x}_c^f$ to Q, and s_c^* decreases from $\bar{s}$ to $\hat{s}_c$. (e) $\Xi_1 \to \Xi_2$: x_c^* increases from $\hat{x}_c^f$ to $\hat{x}_c^{nf}$, and s_c^* decreases from $\bar{s}$ to 0. Therefore, s_c^* always decreases whenever an increase in y_c results in a shift across regions, and x_c^* decreases in y_c except for the cases where an increase in y_c results in an increase in x_c^* from $\hat{x}_c^f$ in Ξ_1 to either Q in Ξ_4 or $\hat{x}_c^f$ in Ξ_2. This together with the first statement established earlier shows that when y_c increases, (i) s_c^* decreases and (ii) x_c^* decreases except for the cases when the increase in y_c induces a transition from Ξ_1 to either Ξ_2 or Ξ_4 in Figure 1 (in these cases x_c^* increases). ■

We now present Lemma 2 which will be used in the proof of Proposition 6.

Lemma 2 *Assume $\tilde{\epsilon} \sim \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$. When $\alpha = 0$ and $K_h \geq (\mu_\epsilon + a\bar{s})Q$, we obtain that for a given r_c, $\hat{y}_c(r_c)$ increases in σ_ϵ.*

Proof of Lemma 2 Let $u_l = (K_h/\overline{x}_c - \mu_\epsilon)/\sigma_\epsilon$ and $u_h = ((m-\omega_0)/\beta - \mu_\epsilon)/\sigma_\epsilon$. With $\tilde{\epsilon} \sim \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$, $\hat{y}_c$ is the unique solution to

$$\hat{y}_c = \int_0^\infty \left[am - a\int_{u_l}^{u_h} (\omega_0 + \beta(\mu_\epsilon + z\sigma_\epsilon))\phi(z)dz - a\int_{u_h}^\infty m\phi(z)dz \right] g_m(m)dm, \tag{A-5}$$

where $\overline{x}_c$ is the unique solution to

$$r_c = \int_0^\infty \left[m\mu_\epsilon - a\int_{u_l}^{u_h} (\omega_0 + \beta(\mu_\epsilon + z\sigma_\epsilon))(\mu_\epsilon + z\sigma_\epsilon)\phi(z)dz - a\int_{u_h}^\infty m(\mu_\epsilon + z\sigma_\epsilon)\phi(z)dz \right] g_m(m)dm. \tag{A-6}$$

Differentiating both sides of equation (A-5) with respect to σ_ϵ yields

$$\begin{aligned}\frac{\partial \hat{y}_c}{\partial \sigma_\epsilon} =& \int_0^\infty \left[-a \int_{u_l}^{u_h} \beta z \phi(z) dz - a(\omega_0 + \beta(\mu_\epsilon + u_h \sigma_\epsilon))\phi(u_h) \frac{\partial u_h}{\partial \sigma_\epsilon} \right. \\ & \left. + a(\omega_0 + \beta(\mu_\epsilon + u_l \sigma_\epsilon))\phi(u_l) \frac{\partial u_l}{\partial \sigma_\epsilon} + am\phi(u_h)\frac{\partial u_h}{\partial \sigma_\epsilon} \right] g_m(m) dm \\ =& \int_0^\infty \left[-a \int_{u_l}^{u_h} \beta z \phi(z) dz + a(\omega_0 + \beta(\mu_\epsilon + u_l \sigma_\epsilon))\phi(u_l) \frac{\partial u_l}{\partial \sigma_\epsilon} \right] g_m(m) dm.\end{aligned}$$

Differentiating both sides of equation (A-6) with respect to σ_σ yields

$$\begin{aligned}0 =& \int_0^\infty \left[- \int_{u_l}^{u_h} \left[\beta z(\mu_\epsilon + z\sigma_\epsilon) + z(\omega_0 + \beta(\mu_\epsilon + z\sigma_\epsilon)) \right] \phi(z) dz - (\omega_0 + \beta(\mu_\epsilon + u_h \sigma_\epsilon)(\mu_\epsilon + u_h \sigma_\epsilon)\phi(u_h) \frac{\partial u_h}{\partial \sigma_\epsilon} \right. \\ & \left. + (\omega_0 + \beta(\mu_\epsilon + u_l \sigma_\epsilon)(\mu_\epsilon + u_l \sigma_\epsilon)\phi(u_l) \frac{\partial u_l}{\partial \sigma_\epsilon} + m(\mu_\epsilon + u_h\sigma_\epsilon)\phi(u_h) \frac{\partial u_h}{\partial \sigma_\epsilon} \right] g_m(m) dm \\ =& \int_0^\infty \left[- \int_{u_l}^{u_h} \left(\beta z(\mu_\epsilon + z\sigma_\epsilon) + z(\omega_0 + \beta(\mu_\epsilon + z\sigma_\epsilon)) \right) \phi(z) dz \right. \\ & \left. + (\omega_0 + \beta(\mu_\epsilon + u_l \sigma_\epsilon)(\mu_\epsilon + u_l \sigma_\epsilon)\phi(u_l) \frac{\partial u_l}{\partial \sigma_\epsilon} \right] g_m(m) dm \qquad \text{(A-7)}\end{aligned}$$

Substituting equation (A-7) into the expression of $\frac{\partial \hat{y}_c}{\partial \sigma_\sigma}$ and simplifying gives

$$\frac{\partial \hat{y}_c}{\partial \sigma_\epsilon} = \int_0^\infty \left[\frac{a}{\mu_\epsilon + u_l \sigma_\epsilon} \int_{u_l}^{u_h} \left[\beta z \sigma_\epsilon (z - u_l) + z(\omega_0 + \beta(\mu_\epsilon + z\sigma_\epsilon)) \right] \phi(z) dz \right] g_m(m) dm > 0.$$

Therefore $\hat{y}_c$ increases in σ_ϵ. ■

Proof of Proposition 6 (i) For $\alpha = 0$ and $\beta = 0$, the farmer's profit for a given x_c and s_c can be written as follows:

$$\begin{aligned}\pi_c(x_c, s_c) =& x_c \mathbb{E} \left[\tilde{m}(\tilde{\epsilon} + as_c) - \min(\tilde{m}, \omega_0) \left(\tilde{\epsilon} + as_c - \frac{K_h}{x_c} \right) \mathbb{I} \left\{ \tilde{\epsilon} > \frac{K_h}{x_c} - as_c \right\} \right] - r_c x_c - y_c s_c x_c \\ =& x_c \mu_m (\mu_\epsilon + as_c) - x_c \mathbb{E} \left[\min(\tilde{m}, \omega_0) \right] \int_{u_l}^\infty \left(\mu_\epsilon + z\sigma_\epsilon + as_c - \frac{K_h}{x_c} \right) \phi(z) dz - r_c x_c - y_c s_c x_c,\end{aligned}$$

where $u_l = \frac{K_h/x_c - (\mu_\epsilon + as_c)}{\sigma_\epsilon}$. Taking the first derivative of $\pi_c(x_c, s_c)$ with respect to σ_ϵ yields

$$\frac{\partial \pi_c(x_c, s_c)}{\partial \sigma_\epsilon} = -x_c \mathbb{E} \left[\min(\tilde{m}, \omega_0) \right] \int_{u_l}^\infty z\phi(z) dz = -x_c \mathbb{E} \left[\min(\tilde{m}, \omega_0) \right] \phi(u_l),$$

where we have used the result that $\phi'(z) = -z\phi(z)$ for the standard normal distribution. Therefore, $\frac{\partial \pi_c(x_c, s_c)}{\partial \sigma_\epsilon} \leq 0$, and from the envelope theorem we obtain $\frac{\partial \Pi_c^*(r_c, y_c)}{\partial \sigma_\epsilon} \leq 0$.

(ii) For $\alpha = 0$, equation (A-1) which characterizes the optimal fertilizer application rate $\hat{s}_c$ reduces to

$$y_c = a\mu_m - a\mathbb{E} \left[\min(\tilde{m}, \omega_0 + \beta\tilde{\epsilon}) \mathbb{I} \left\{ \tilde{\epsilon} > \frac{K_h}{Q} - a\hat{s}_c \right\} \right].$$

Define $\Omega(s_c|m) \doteq \mathbb{E} \left[\min(m, \omega_0 + \beta\tilde{\epsilon}) \mathbb{I} \left\{ \tilde{\epsilon} > \frac{K_h}{Q} - as_c \right\} \right]$ for a given m. We now examine how Ω changes with σ_ϵ. There are two cases depending on the value of m.

When m is small, $\Omega(s_c|m) = \mathbb{E} \left[m\mathbb{I} \left\{ \tilde{\epsilon} > \frac{K_h}{Q} - as_c \right\} \right] = m \left(1 - \Phi \left(\frac{K_h/Q - as_c - \mu_\epsilon}{\sigma_\epsilon} \right) \right)$. Taking the derivative with respect to σ_ϵ yields

$$\frac{\partial \Omega(s_c|m)}{\partial \sigma_\epsilon} = -m\phi \left(\frac{K_h/Q - as_c - \mu_\epsilon}{\sigma_\epsilon} \right) \left(-\frac{K_h/Q - as_c - \mu_\epsilon}{\sigma_\epsilon^2} \right) \geq 0,$$

where the inequality follows from the assumption $K_h \geq (\mu_\epsilon + a\bar{s})Q$.

When m is large, we have

$$\Omega(s_c|m) = \int_{K_h/Q-as_c}^{(m-\omega_0)/\beta} m g_\epsilon(\epsilon)d\epsilon + \int_{(m-\omega_0)/\beta}^{\infty} (\omega_0+\beta\epsilon)g_\epsilon(\epsilon)d\epsilon$$
$$=(\omega_0+\beta\mu_\epsilon)\left(\Phi(u_h)-\Phi(u_l)\right)+\beta\sigma_\epsilon(\phi(u_l)-\phi(u_h))+m(1-\Phi(u_h)),$$

where $u_l = (K_h/Q - as_c - \mu_\epsilon)/\sigma_\epsilon$ and $u_h = ((m-\omega_0)/\beta - \mu_\epsilon)/\sigma_\epsilon$. Taking the first derivative with respect to σ_ϵ yields

$$\frac{\partial\Omega(s_c|m)}{\partial\sigma_\epsilon} =(\omega_0+\beta\mu_\epsilon)\left(\phi(u_l)\frac{u_l}{\sigma_\epsilon}-\phi(u_h)\frac{u_h}{\sigma_\epsilon}\right)+\beta(\phi(u_l)-\phi(u_h))$$
$$+\beta\sigma_\epsilon\left(u_l\phi(u_l)\frac{u_l}{\sigma_\epsilon}-u_h\phi(u_h)\frac{u_h}{\sigma_\epsilon}\right)+m\phi(u_h)\frac{u_h}{\sigma_\epsilon}$$
$$=\frac{\omega_0+\beta\mu_\epsilon}{\sigma_\epsilon}u_l\phi(u_l)+\beta(\phi(u_l)-\phi(u_h))+\beta\mu_l^2\phi(u_l).$$

Since $K_h > (\mu_\epsilon + a\bar{s})Q$, we obtain $u_h > u_l > 0$ and thus $\phi(u_l)-\phi(u_h) > 0$. This shows that $\frac{\partial\Omega(s_c|m)}{\partial\sigma_\epsilon} \geq 0$.

The above two cases combined, we have shown that $\frac{\partial\Omega(s_c|m)}{\partial\sigma_\epsilon} \geq 0$ regardless of the realization of $\tilde{m}$, and so $\frac{\partial\mathbb{E}[\Omega(s_c|\tilde{m})]}{\partial\sigma_\epsilon} \geq 0$. Moreover, from the implicit function theorem, sgn $\left(\frac{\partial\hat{s}_c}{\partial\sigma_\epsilon}\right)$ is opposite to sgn $\left(\frac{\partial\mathbb{E}[\Omega(s_c|\tilde{m})]}{\partial\sigma_\epsilon}\right)$ as $\mathbb{E}[\Omega(s_c|\tilde{m})]$ increases in s_c. Thus, we obtain $\frac{\partial\hat{s}_c}{\partial\sigma_\epsilon} \leq 0$.

We now examine how $\hat{x}^{nf}$ changes with σ_ϵ. For $\alpha = 0$ we obtain

$$\Theta(x_c) = \mathbb{E}\left[\tilde{m}\tilde{\epsilon} - \min(\tilde{m}, \omega_0+\beta\tilde{\epsilon})\tilde{\epsilon}\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right].$$

Define $\Theta(\sigma_\epsilon|m) \doteq \mathbb{E}\left[m\tilde{\epsilon} - \min(m,\omega_0+\beta\tilde{\epsilon})\tilde{\epsilon}\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right]$ for a given m. There are two cases depending on the value of m.

When m is small, we have $\Theta(\sigma_\epsilon|m) = \mathbb{E}\left[m\tilde{\epsilon}\mathbb{I}\left\{\tilde{\epsilon} < \frac{K_h}{x_c}\right\}\right] = m\mu_\epsilon\Phi(u_l) - m\sigma_\epsilon\phi(u_l)$ where $u_l = (K_h/x_c - \mu_\epsilon)/\sigma_\epsilon$. Taking the derivative of $\Theta(\sigma_\epsilon|m)$ with respect to σ_ϵ yields

$$\frac{\partial\Theta(\sigma_\epsilon|m)}{\partial\sigma_\epsilon} =m\mu_\epsilon\phi(u_l)\frac{\partial u_l}{\partial\sigma_\epsilon} - m\phi(u_l) - m\sigma_\epsilon\phi'(u_l)\frac{\partial u_l}{\partial\sigma_\epsilon}$$
$$=m\mu_\epsilon\phi(u_l)\left(-\frac{u_l}{\sigma_\epsilon}\right) - m\phi(u_l) + m\sigma_\epsilon u_l\phi(u_l)\left(-\frac{u_l}{\sigma_\epsilon}\right)$$
$$=-m\phi(u_l)\left(1+\frac{u_l}{\sigma_\epsilon}\frac{K_h}{x_c}\right).$$

Since $K_h > (\mu_\epsilon + a\bar{s})Q$, we obtain $K_h/x_c > \mu_\epsilon$ and thus $u_l > 0$. Therefore, we have $\frac{\partial\Theta(\sigma_\epsilon|m)}{\partial\sigma_\epsilon} \leq 0$.

When m is large, we have

$$\Theta(\sigma_\epsilon|m) =\mathbb{E}\left[m\tilde{\epsilon} - \min(m,\omega_0+\beta\tilde{\epsilon})\tilde{\epsilon}\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right]$$
$$=\int_{-\infty}^{u_l} m(\mu_\epsilon + z\sigma_\epsilon)\phi(z)dz + \int_{u_l}^{u_h}(m-(\omega_0+\beta\mu_\epsilon)-\beta\sigma_\epsilon z)(\mu_\epsilon+z\sigma_\epsilon)\phi(z)dz$$
$$=m\mu_\epsilon\Phi(u_l) - m\sigma_\epsilon\phi(u_l) + (m-(\omega_0+\beta\mu_\epsilon))\mu_\epsilon(\Phi(u_h)-\Phi(u_l))$$
$$+\sigma_\epsilon(m-(\omega_0+\beta\mu_\epsilon)-\beta\mu_\epsilon)(\phi(u_l)-\phi(u_h)) - \beta\sigma_\epsilon^2(\Phi(u_h)-\Phi(u_l)+u_l\phi(u_l)-u_h\phi(u_h)),$$

where $u_l = (K_h/x_c - \mu_\epsilon)/\sigma_\epsilon$ and $u_h = ((m-\omega_0)/\beta - \mu_\epsilon)/\sigma_\epsilon$. Taking the derivative of $\Theta(\sigma_\epsilon|m)$

with respect to σ_ϵ and after some simplifications we obtain

$$\frac{\partial\Theta(\sigma_\epsilon|m)}{\partial\sigma_\epsilon} = -\frac{\mu_\epsilon}{\sigma_\epsilon}(\omega_0+\beta\mu_\epsilon)u_l\phi(u_l) - (\omega_0+\beta\mu_\epsilon)\phi(u_l)(1+u_l^2) - \beta\mu_\epsilon u_l^2\phi(u_l) - \beta\sigma_\epsilon u_l^3\phi(u_l)$$
$$-\beta\mu_\epsilon(\phi(u_l)-\phi(u_h)) - \beta\sigma_\epsilon u_h\phi(u_h) - 2\beta\sigma_\epsilon(\Phi(u_h)-u_h\phi(u_h)-(\Phi(u_l)-u_l\phi(u_l))).$$

Since $K_h > (\mu_\epsilon + a\bar{s})Q$, we obtain $K_h/x_c > \mu_\epsilon$ and thus $u_h > u_l > 0$ and $\phi(u_l) - \phi(u_h) > 0$. In addition, we can show that $\Phi(u_h) - u_h\phi(u_h) > \Phi(u_l) - u_l\phi(u_l)$. To see this, define $f(x) \doteq \Phi(x) - x\phi(x)$ for $x > 0$. Taking the derivative with respect to x yields $f'(x) = \phi(x) - x\phi'(x) - \phi(x) = x^2\phi(x) > 0$, and thus $f(u_h) > f(u_l)$. This proves that $\frac{\partial\Theta(\sigma_\epsilon|m)}{\partial\sigma_\epsilon} \leq 0$.

The above two cases combined, we have shown that $\frac{\partial\Theta(\sigma_\epsilon|m)}{\partial\sigma_\epsilon} \leq 0$ regardless of the realization of $\tilde{m}$ and so $\frac{\partial\Theta(x_c)}{\partial\sigma_\epsilon} \leq 0$. Moreover, from the implicit function theorem, $\text{sgn}\left(\frac{\partial\hat{x}_c^{nf}}{\partial\sigma_\epsilon}\right) = \text{sgn}\left(\left.\frac{\partial\Theta(x_c)}{\partial\sigma_\epsilon}\right|_{x_c=\hat{x}_c^{nf}}\right)$ as $\Theta(x_c)$ decreases in x_c. Thus we obtain $\frac{\partial\hat{x}_c^{nf}}{\partial\sigma_\epsilon} \leq 0$. The same approach can be applied to show that $\frac{\partial\hat{x}_c^{f}}{\partial\sigma_\epsilon} \leq 0$.

Finally, we show the possible transitions across regions after an increase in σ_ϵ. This requires us to first show how the boundaries in Figure 1 change with an increase in σ_ϵ. It is straightforward that $\Gamma(0)$ and $\Theta(0)$ are independent of σ_ϵ. $\Gamma(Q)$ and $\Theta(Q)$ increase in σ_ϵ, since we have shown earlier that $\frac{\partial\Theta(x_c)}{\partial\sigma_\epsilon} \leq 0$ and $\frac{\partial\Gamma(x_c)}{\partial\sigma_\epsilon} \leq 0$ for any given $x_c > 0$. In addition, Lemma 2 shows how $\hat{y}_c(r_c)$ changes with σ_ϵ. Also, $y_c^{(0)} = a\mu_m$ is independent of σ_ϵ, and following the same method as in the proof of $\frac{\partial\hat{s}_c}{\partial\sigma_\epsilon} \leq 0$, it can be readily shown that $\frac{\partial y_c^{(1)}}{\partial\sigma_\epsilon} \leq 0$ and $\frac{\partial y_c^{(2)}}{\partial\sigma_\epsilon} \leq 0$. With these intermediary results, we can check the changes in the optimal solution due to a change in σ_ϵ. We only present the corresponding changes in x_c^* due to the region shifts caused by a decrease in σ_ϵ, as the changes in s_c^* can be established in a similar fashion. We obtain: (i) $\Xi_1 \to \Xi_2$: x_c^* increases from $\hat{x}_c^f$ to $\hat{x}_c^{nf}$. (ii) $\Xi_1 \to \Xi_3$ and $\Xi_1 \to \Xi_4$: x_c^* increases from $\hat{x}_c^f$ to Q. (iii) $\Xi_1 \to \Xi_5$ ($\Xi_2 \to \Xi_3$): x_c^* increases from $\hat{x}_c^f$ to Q (from $\hat{x}_c^{nf}$ to Q). (iv) $\Xi_4 \to \Xi_5$ and $\Xi_3 \to \Xi_4$: x_c^* remains at Q. ■

Proof of Propositions 7 and 8 $J^*(r_c, y_c)$ depends on r_c and y_c only through their impact on the optimal decisions, and thus for $\tau \in \{r_c, y_c\}$ we have

$$\frac{\partial J^*(r_c,y_c)}{\partial\tau} = \frac{\partial x_c^*}{\partial\tau}\frac{\partial J^*(r_c,y_c)}{\partial x_c^*} + \frac{\partial s_c^*}{\partial\tau}\frac{\partial J^*(r_c,y_c)}{\partial s_c^*}. \tag{A-8}$$

Proof of Proposition 7: When $(r_c, y_c) \in \Xi_i$ for $i = 3, 4, 5$, x_c^* and s_c^* do not change with r_c, and thus $\frac{\partial J^*(r_c,y_c)}{\partial r_c} = 0$. When $(r_c, y_c) \in \Xi_1$, $x_c^* = \hat{x}_c^f$ and $s_c^* = \bar{s}$. We know from Proposition 4 that $\frac{\partial\hat{x}_c^f}{\partial r_c} < 0$. This together with the result $\frac{\partial J^*(r_c,y_c)}{\partial x_c^*} \leq 0$ (by the definition of $J^*(r_c, y_c)$) shows that $\frac{\partial J^*(r_c,y_c)}{\partial r_c} \geq 0$. Similarly, we can show that when $(r_c, y_c) \in \Xi_2$, $\frac{\partial J^*(r_c,y_c)}{\partial r_c} \geq 0$.

As r_c increases to a larger extent, the optimal solution may take a different form, which corresponds to region transitions in Figure 1. Note that the boundaries do not move as r_c changes. As shown in Proposition 4, when r_c increases, (i) x_c^* decreases and (ii) s_c^* does not change except for the cases when it induces a transition from either Ξ_2 or Ξ_4 to Ξ_1 in Figure 1 (in these cases s_c^* increases). Moreover, $\frac{\partial J^*(r_c,y_c)}{\partial x_c^*} \leq 0$. Then using (A-8) we conclude that when r_c increases, $J^*(r_c, y_c)$ increases except for the cases when it induces a transition from either Ξ_2 or Ξ_4 to Ξ_1 in Figure 1.

Proof of Proposition 8: When $(r_c, y_c) \in \Xi_i$ for $i = 3, 5$, x_c^* and s_c^* do not change with y_c and thus

$\frac{\partial J^*(r_c,y_c)}{\partial y_c} = 0$. When $(r_c, y_c) \in \Xi_1$, $x_c^* = \hat{x}_c^f$ and $s_c^* = \bar{s}$. We know from Proposition 5 that $\frac{\partial \hat{x}_c^f}{\partial y_c} < 0$. This together with the result $\frac{\partial J^*(r_c,y_c)}{\partial x_c^*} \le 0$ (by the definition of $J^*(r_c, y_c)$) shows that $\frac{\partial J^*(r_c,y_c)}{\partial y_c} \ge 0$. Similarly, we can show that when $(r_c, y_c) \in \Xi_2$, $\frac{\partial J^*(r_c,y_c)}{\partial y_c} \ge 0$. When $(r_c, y_c) \in \Xi_4$, $x_c^* = Q$ and $s_c^* = \hat{s}_c$. From Proposition 5, we know $\frac{\partial \hat{s}_c}{\partial y_c} < 0$. This together with $\frac{\partial J^*(r_c,y_c)}{\partial s_c^*} \le 0$ (by the definition of $J^*(r_c, y_c)$) shows that $\frac{\partial J^*(r_c,y_c)}{\partial y_c} \ge 0$.

As y_c increases to a larger extent, the optimal solution may take different forms, which corresponds to region transitions in Figure 1. Note that the boundaries do not move as y_c changes. From Proposition 5, when y_c increases, (i) s_c^* decreases and (ii) x_c^* decreases except for the cases when the increase in y_c induces a transition from Ξ_1 to either Ξ_2 or Ξ_4 in Figure 1 (in these cases x_c^* increases). Moreover, $\frac{\partial J^*(r_c,y_c)}{\partial x_c^*} \le 0$ and $\frac{\partial J^*(r_c,y_c)}{\partial s_c^*} \le 0$. Then using (A-8) we conclude that when y_c increases $J^*(r_c, y_c)$ increases except for cases when it induces a transition from Ξ_1 to either Ξ_2 or Ξ_4 in Figure 1. ■

Lemma 3 *For a given x_c and s_c, define*

$$J(x_c, s_c) \doteq (\mu_\epsilon + a\bar{s})Q - \mathbb{E}\left[x_c(\tilde{\epsilon} + as_c)\mathbb{I}\{p(\tilde{m}, \tilde{\epsilon}) > \omega_h(\tilde{\epsilon})\} + \min\left(x_c(\tilde{\epsilon} + as_c), K_h\right)\mathbb{I}\{p(\tilde{m}, \tilde{\epsilon}) \le \omega_h(\tilde{\epsilon})\}\right].$$

(i) Assume $\tilde{m} \sim \mathcal{N}(\mu_m, \sigma_m^2)$ and $\mathbb{E}[p(\tilde{m}, K_h/Q - a\bar{s}) \le \omega_h(K_h/Q - a\bar{s})$. Then $\frac{\partial J(x_c,s_c)}{\partial \sigma_m} \le 0$.
(ii) Assume $\tilde{\epsilon} \sim \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$ and $\alpha = 0$. Then $\frac{\partial J(x_c,s_c)}{\partial \sigma_\epsilon} \ge 0$.

Proof of Lemma 3 (i) $J(x_c, s_c)$ depends on σ_m only through the second term which we denote as Ω. We rewrite Ω as follows:

$$\begin{aligned}\Omega =& -\mathbb{E}\left[x_c(\tilde{\epsilon} + as_c) - (x_c(\tilde{\epsilon} + as_c) - K_h)^+\mathbb{I}\{p(\tilde{m}, \tilde{\epsilon}) \le \omega_h(\tilde{\epsilon})\}\right] \\ =& -x_c(\mu_\epsilon + as_c) + \int_{K_h/x_c - as_c}^{\bar{\epsilon}} (x_c(\tilde{\epsilon} + as_c) - K_h)\mathbb{E}[\mathbb{I}\{p(\tilde{m}, \epsilon) \le \omega_h(\epsilon)\}]g_\epsilon(\epsilon)d\epsilon \\ =& -x_c(\mu_\epsilon + as_c) + \int_{K_h/x_c - as_c}^{\bar{\epsilon}} (x_c(\tilde{\epsilon} + as_c) - K_h)\Phi\left(\frac{k(\epsilon) - \mu_m}{\sigma_m}\right) g_\epsilon(\epsilon)d\epsilon,\end{aligned}$$

where $k(\epsilon) = \omega_h(\epsilon) + \alpha(\epsilon - \mu_\epsilon)$.

Taking the derivative of Ω with respect to σ_m yields

$$\frac{\partial \Omega}{\partial \sigma_m} = \int_{K_h/x_c - as_c}^{\bar{\epsilon}} (x_c(\tilde{\epsilon} + as_c) - K_h)\phi\left(\frac{k(\epsilon) - \mu_m}{\sigma_m}\right)\left(-\frac{k(\epsilon) - \mu_m}{\sigma_m^2}\right) g_\epsilon(\epsilon)d\epsilon \le 0,$$

where the inequality follows from the assumption $\mu_m \le \omega_0 - \alpha\mu_\epsilon + (\alpha + \beta)(K_h/Q - a\bar{s})$ as well as $K_h/x_c - as_c \ge K_h/Q - a\bar{s}$ for $x_c \in [0, Q]$ and $s_c \in [0, \bar{s}]$.

(ii) For $\alpha = 0$, we define $T(\sigma|m) \doteq \mathbb{E}\left[x_c(\tilde{\epsilon} + as_c)\mathbb{I}\{m > \omega_h(\tilde{\epsilon})\} + \min\left(x_c(\tilde{\epsilon} + as_c), K_h\right)\mathbb{I}\{m \le \omega_h(\tilde{\epsilon})\}\right]$ for a given x_c, s_c and realization of $\tilde{m}$. We rewrite $T(\sigma|m)$ as follows:

$$T(\sigma|m) = \mathbb{E}\left[x_c(\tilde{\epsilon} + as_c) - (x_c(\tilde{\epsilon} + as_c) - K_h)^+\mathbb{I}\{m \le \omega_0 + \beta\tilde{\epsilon}\}\right].$$

Let $u_l = (K_h/x_c - as_c - \mu_\epsilon)/\sigma_\epsilon$ and $u_h = ((m - \omega_0)/\beta - \mu_\epsilon)/\sigma_\epsilon$. We have two cases to consider depending on the value of m.

When m is small, $T(\sigma|m)$ reduces to

$$T(\sigma|m) = \int_{-\infty}^{u_l} x_c(\mu_\epsilon + as_c + z\sigma_\epsilon)\phi(z)dz + \int_{u_l}^{\infty} K_h\phi(z)dz$$
$$= x_c(\mu_\epsilon + as_c)\Phi(u_l) - \sigma_\epsilon x_c\phi(u_l) + K_h(1 - \Phi(u_l)).$$

Taking the derivative of $T(\sigma|m)$ with respect to σ_ϵ yields

$$\frac{\partial T(\sigma|m)}{\partial \sigma_\epsilon} = - x_c(\mu_\epsilon + as_c)\phi(u_l)\frac{u_l}{\sigma_\epsilon} - x_c\phi(u_l) + \sigma_\epsilon x_c\phi'(u_l)\frac{u_l}{\sigma_\epsilon} + K_h\phi(u_l)\frac{u_l}{\sigma_\epsilon}$$
$$= - x_c\phi(u_l) \leq 0.$$

When m is large, $T(\sigma|m)$ can be rewritten as

$$T(\sigma|m) = \int_{-\infty}^{u_h} x_c(\mu_\epsilon + as_c + z\sigma_\epsilon)\phi(z)dz + \int_{u_h}^{\infty} K_h\phi(z)dz$$
$$= x_c(\mu_\epsilon + as_c)\Phi(u_h) - \sigma_\epsilon x_c\phi(u_2) + K_h(1 - \Phi(u_h)).$$

Taking the derivative of $T(\sigma|m)$ with respect to σ_ϵ yields

$$\frac{\partial T(\sigma|m)}{\partial \sigma_\epsilon} = - x_c(\mu_\epsilon + as_c)\phi(u_h)\frac{u_h}{\sigma_\epsilon} - x_c\phi(u_h) + \sigma_\epsilon x_c\phi'(u_h)\frac{u_h}{\sigma_\epsilon} + K_h\phi(u_h)\frac{u_h}{\sigma_\epsilon}$$
$$= - x_c\phi(u_h) - x_c u_h\phi(u_h)(u_h - u_l) \leq 0,$$

where the inequality follows from the assumption $K_h > (\mu_\epsilon + a\bar{s})Q$, and $u_h > u_l > 0$.

The above two cases combined, we have shown that $\frac{\partial T(\sigma|m)}{\partial \sigma_\epsilon} \leq 0$ regardless of the value of m. Thus, $J(x_c, s_c) = (\mu_\epsilon + a\bar{s})Q - \mathbb{E}[T(\sigma|\tilde{m})]$ increases in σ_ϵ. ■

Proof of Proposition 9 We obtain

$$\frac{\partial J^*(r_c, y_c)}{\partial \sigma_\epsilon} = \underbrace{\frac{\partial J(x_c, s_c)}{\partial \sigma_\epsilon}\bigg|_{(x_c^*, s_c^*)}}_{\text{direct effect}} + \underbrace{\frac{\partial x_c^*}{\partial \sigma_\epsilon}\frac{\partial J(x_c^*, s_c^*)}{\partial x_c^*} + \frac{\partial s_c^*}{\partial \sigma_\epsilon}\frac{\partial J(x_c^*, s_c^*)}{\partial s_c^*}}_{\text{indirect effect}}. \tag{A-9}$$

When $(r_c, y_c) \in \Xi_i$ for $i = 3, 5$, x_c^* and s_c^* do not change with σ_ϵ. Thus, only the direct effect of equation (A-9) exists. Thus from Lemma 3 we obtain $\frac{\partial J^*(r_c,y_c)}{\partial \sigma_\epsilon} \geq 0$. When $(r_c, y_c) \in \Xi_1$, $x_c^* = \hat{x}_c^f$, which decreases in σ_ϵ from Proposition 6, and $s_c^* = \bar{s}$ which is independent of σ_ϵ. We also know that $J(x_c, s_c)$ decreases in x_c. Thus, both the direct and indirect effects in equation (A-9) are positive; that is, $\frac{\partial J^*(r_c,y_c)}{\partial \sigma_\epsilon} \geq 0$ when $(r_c, y_c) \in \Xi_1$. Similarly, we can show that $\frac{\partial J^*(r_c,y_c)}{\partial \sigma_\epsilon} \leq 0$ when $(r_c, y_c) \in \Xi_2$ and $(r_c, y_c) \in \Xi_4$.

As σ_ϵ changes to a larger extent, the optimal solution may take a different form corresponding to the region shift in Figure 1. From Proposition 6 (ii), we know that x_c^* always increases, while s_c^* increases except when the decrease in σ_ϵ induces a transition from Ξ_1 to either Ξ_2, Ξ_3 or Ξ_4 in Figure 2. It can be readily checked that both the direct and indirect effects are positive except when there is a transition from Ξ_1 to either Ξ_2, Ξ_3 or Ξ_4. This completes the proof as required. ■

Appendix C Additional Analysis

In this section, we provide the detailed analyses of the extensions mentioned in the Conclusion section of our paper. The proofs for our technical statements in this section are omitted for brevity and they are available upon request.

C.1 Analysis for the Effects of Market Variability

In this section, we investigate the effects of market uncertainty on the implications of optimal decisions for farm management and food security. Recall that we define market uncertainty $\tilde{m}$ to capture the uncertainty in open market price associated with factors that are not related to farm yield (e.g., macroeconomic conditions and regulations). As highlighted by USDA Economic Research Service (2020), it is well-documented that open market prices for fresh produce have significant variability and this variability is one of the key reasons driving the farmers to leave their crop unharvested on their farmland as the crop price may not be sufficiently large to economically justify harvesting. Therefore, it is important for the farmers in practice to understand how changes in crop price variability affect their farm operations and crop production. To this end, we investigate how changes in market variability σ_m affect the farmer's optimal decisions and profitability as well as the expected gap. Paralleling our analysis in the main paper, we will rely on Assumption 1 throughout this analysis. In characterizing the effects of market variability σ_m, as discussed in §2, we further assume that the market uncertainty $\tilde{m}$ has a Normal distribution.

We first examine the effects of σ_m on the farmer's optimal decisions and profitability.

Proposition 10 (Effect of market variability σ_m) *Assume $\tilde{m} \sim \mathcal{N}(\mu_m, \sigma_m^2)$. We have $\frac{\partial \Pi_c^*(r_c, y_c)}{\partial \sigma_m} \geq 0$, $\frac{\partial \hat{x}_c^f}{\partial \sigma_m} \geq 0$, $\frac{\partial \hat{x}_c^{nf}}{\partial \sigma_m} \geq 0$, and $\frac{\partial \hat{s}_c}{\partial \sigma_m} \geq 0$. Moreover, the effect of a decrease in σ_m on x_c^* and s_c^* is identical to the characterizations given in panel a and panel b of Figure 2, respectively.*

An increase in market variability σ_m increases the variability of crop price $\tilde{m} - \alpha(\tilde{\epsilon} - \mu_\epsilon)$ which, in turn, increases the farmer's profitability. This is because while the farmer benefits from high crop price realizations, low crop price realizations are not as detrimental: the farmer optimally chooses not to acquire additional resource to increase the harvest volume beyond the available capacity when the crop price is less than the external unit harvesting cost. Based on the same argument, common intuition may suggest that an increase in σ_m incents the farmer to cultivate more acres and apply more fertilizer per acre. Proposition 10 demonstrates that this intuition is correct for the effect on optimal cultivation volume x_c^*. However, the intuition is correct for the effect on optimal fertilizer application rate s_c^* (for example, $\hat{s}_c$ increases) *unless the increase in σ_m induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different.* In particular, as illustrated in Figure 2, when an increase in σ_m induces the farmer to switch the optimal strategy from $(\hat{x}_c^f, \bar{s})$ to $(Q, 0)$, $(Q, \hat{s}_c)$, or $(\hat{x}_c^{nf}, 0)$, s_c^* decreases because of the increase in x_c^*. Because the characterization of the effects of a decrease in σ_m on the farmer's optimal decisions and profitability is identical to the characterization of the effects of an increase in yield variability σ_ϵ on those, we have the opposite managerial insights for farm management associated with yield variability as discussed at the end of §4 of the main paper.

We next examine how changes in market variability σ_m impact the expected gap $J^*(r_c, y_c)$. To this end, paralleling our analysis in §5 of the main paper we rely on Assumption 2; that is, we assume $K_h \geq (\mu_\epsilon + a\bar{s})Q$. We complement our analytical analysis with data-calibrated numerical experiments as discussed in §6 of the main paper. Recall that we allow for market variability σ_m to change by -45% to 45% from their calibrated values with a 15% increment. In illustrating how a measure of interest (i.e., optimal cultivation volume, optimal fertilizer application rate, optimal expected profit, and the expected gap) at a given numerical instance (e.g., baseline scenario) changes with respect to σ_m, we plot our figures using a finer increment than 15% (specifically, 0.1% increment) within the range of [−45%,45%] of the calibrated value.

As follows from (3), a change in σ_m affects the expected gap by altering the expected optimal harvest volume for any given farmer's decisions (x_c, s_c) as well as the farmer's optimal decisions (x_c^*, s_c^*) in the cultivation stage. For expositional brevity, we consider the effect of a decrease in σ_m:

Proposition 11 (Effect of market variability σ_m) *Assume $\tilde{m} \sim \mathcal{N}(\mu_m, \sigma_m^2)$ and $\mu_m \leq \omega_0 - \alpha\mu_\epsilon + (\alpha + \beta)(K_h/Q - a\bar{s})$. When σ_m decreases, $J^*(r_c, y_c)$ increases except for the cases when it induces a transition from Ξ_2, Ξ_3, or Ξ_4 to Ξ_1 in Figure 2.*

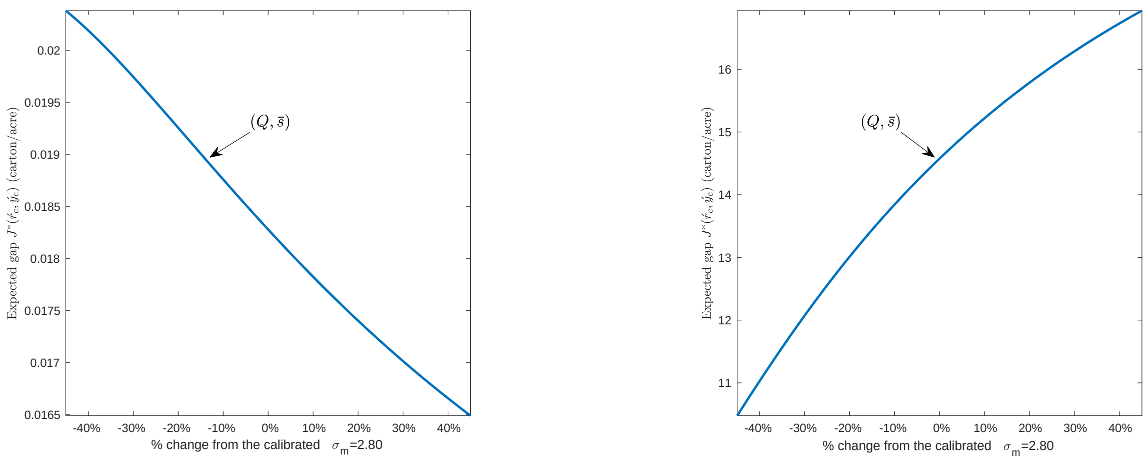
Recall that an increase in market variability σ_m increases the variability of crop price $p(\tilde{m}, \tilde{\epsilon}) = \tilde{m} - \alpha(\tilde{\epsilon} - \mu_\epsilon)$. Proposition 11 proves under a specific condition that an increase in crop price variability is beneficial for food security unless it induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different. As follows from (3), how an increase in crop price variability affects the expected optimal harvest volume for a given farmer's decisions (x_c, s_c) crucially depends on how it impacts the (stochastic) ordering between the crop price and the external unit harvesting cost $\omega_h(\tilde{\epsilon}) = \omega_0 + \beta\tilde{\epsilon}$. This is because the farmer optimally harvests all the available crop $x_c(\epsilon + as_c)$ only when the crop price is larger than this cost in the harvesting stage. When the condition in Proposition 11 holds (equivalently, $\mathbb{E}[p(\tilde{m}, K_h/Q - a\bar{s})] \leq \omega_h(K_h/Q - a\bar{s})$), using Assumption 2 it can be proven that an increase in crop price variability increases the likelihood that crop price will be larger than the external unit harvest cost which, in turn, increases the expected optimal harvest volume for a given (x_c, s_c). We have already established in Proposition 10 that an increase in σ_m incents the farmer to cultivate more acres and apply more fertilizer per acre except for the cases when it induces the farmer to switch the optimal strategy from $(\hat{x}_c^f, \bar{s})$ to $(Q, 0)$, $(Q, \hat{s}_c)$, or $(\hat{x}_c^{nf}, 0)$ in Figure 2. Therefore, outside of these cases because x_c^* and s_c^* increase, these changes further increase the expected optimal harvest volume in (3), and thus, decrease the expected gap as shown in Proposition 11. When these cases happen, the farmer optimally decreases s_c^* because of the increase in x_c^*. Because x_c^* increases and s_c^* decreases the resulting impact on the expected optimal harvest volume is indeterminate. In our numerical studies that satisfy the condition in Proposition 11, we do not observe a transition in which x_c^* increases and s_c^* decreases; see Figure 6(b) for an example. In this example, K_h is sufficiently large so that the condition in Proposition 11 is satisfied when the rest of the parameters are at their calibrated values. In this instance, as σ_m increases the farmer's optimal decisions $(Q, \bar{s})$ do not change and the expected gap increases. Based on our analytical and numerical analyses, we conclude that when the condition in Proposition 11 is satisfied, an increase in market variability σ_m (which increases the crop price variability) is beneficial for food security. This behavior is consistent with the benchmark model where it can be proven under the same condition that an increase in σ_m decreases the expected gap.

We note here that the condition $\mathbb{E}[p(\tilde{m}, K_h/Q - a\bar{s})] \leq \omega_h(K_h/Q - a\bar{s})$ is not satisfied in our data-calibrated baseline scenario in §6.[6]When this condition is not satisfied, the effect of an increase

[6]This condition states that when the farmer chooses $(Q, \bar{s})$ in the cultivation stage, in the harvesting stage for

Figure 6: Effect of Market Variability σ_m on the Expected Gap $J^*(r_c, y_c)$

(a) (b)



Notes. In panel a (b), $K_h = (\acute{\mu}_\epsilon + \acute{\alpha}\acute{\bar{s}})\acute{Q}$ ($K_h = 1.3(\acute{\mu}_\epsilon + \acute{\alpha}\acute{\bar{s}})\acute{Q}$). In both panels, $\sigma_m \in [-45\%, 45\%]$ away from the baseline value $\acute{\sigma}_m = 2.8$ with 0.1% increments and the rest of the parameters are at their calibrated (baseline) levels.

in σ_m on the expected gap is indeterminate because it may decrease the expected optimal harvest volume for a given (x_c, s_c) in (3). In our data-calibrated baseline scenario, we observe that an increase in σ_m decreases the expected optimal harvest volume for a given (x_c, s_c) which in turn, increases the expected gap; see Figure 6(a) for the illustration. In the baseline scenario, as σ_m increases the farmer's optimal decisions $(Q, \bar{s})$ do not change and the expected gap decrease. In that case an increase in market variability σ_m (which increases the crop price variability) is harmful for food security.

C.2 Analysis for the Effects of Harvesting Cost

In this section, we investigate how changes in harvesting cost affect the expected gap. This analysis is important for understanding the consequences of a policy that reduces the labour cost for hiring seasonal workers on the crop production level. To this end, we consider the external harvesting cost function in Assumption 1(ii); that is, $\omega_h(\epsilon) = \omega_0 + \beta\epsilon$ where $\omega_0 > 0$ and $\beta \geq 0$ and examine how changes in the base labor cost ω_0 affects the expected gap. Because the expected gap depends on the farmer's optimal decisions, we first examine how these decisions are impacted by ω_0:

Proposition 12 (Effect of harvesting cost ω_0 on optimal decisions) *We have $\frac{\partial \hat{x}_c^{nf}}{\partial \omega_0} < 0$, $\frac{\partial \hat{x}_c^{f}}{\partial \omega_0} < 0$, and $\frac{\partial \hat{s}_c}{\partial \omega_0} < 0$. Moreover, when ω_0 increases, (i) x_c^* decreases and (ii) s_c^* decreases except for the cases when it induces a transition from Ξ_2, Ξ_3 or Ξ_4 to Ξ_1 in Figure 1 (in these cases s_c^* increases).*

sufficiently high farm yield realizations that the farmer considers acquiring additional harvesting resource, market uncertainty realization m should be larger than its mean μ_m for the crop price $m - \alpha(\epsilon - \mu_\epsilon)$ to be larger than the external unit harvesting cost $\omega_0 + \beta\epsilon$. In other words, the expected crop price is not sufficient for economically justifying acquiring of additional harvesting resource.

Intuitively, an increase in harvesting cost ω_0 incents the farmer to decrease the optimal cultivation volume x_c^*. However, the effect on the optimal fertilizer application rate s_c^* is more nuanced. Common intuition may suggest that an increase in ω_0 (which makes farming more expensive) also incents the farmer to decrease s_c^*. Proposition 12 shows that this intuition is correct (for example, $\hat{s}_c$ decreases) unless the increase in ω_0 induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different. In particular, when an increase in ω_0 induces the farmer to switch the optimal strategy from $(\hat{x}_c^{nf}, 0)$ (in Ξ_2) $(Q, 0)$ (in Ξ_3), or $(Q, \hat{s}_c)$ (in Ξ_4) to $(\hat{x}_c^f, \bar{s})$ (in Ξ_1), s_c^* is increased to counteract against the reduction in crop availability at the harvesting stage due to decreasing x_c^*.

We next examine how changes in the harvesting cost ω_0 impact the expected gap $J^*(r_c, y_c)$. As follows from (3), a change in ω_0 affects the expected gap only by altering the optimal decisions (x_c^*, s_c^*) in the cultivation stage.

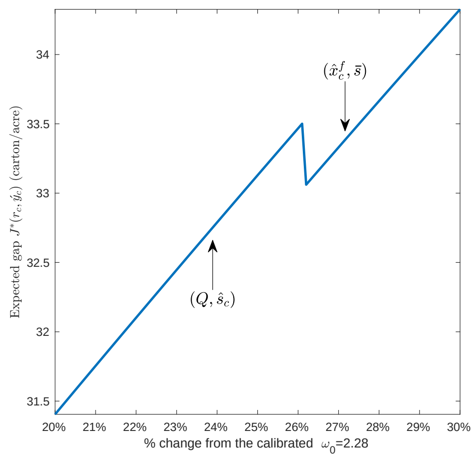
Proposition 13 (Effects of harvesting cost ω_0 on expected gap) *When ω_0 increases, $J^*(r_c, y_c)$ increases except for the cases where it induces a transition from Ξ_2, Ξ_3 or Ξ_4 to Ξ_1 in Figure 1.*

Common intuition may suggest that an increase in ω_0 (which makes farming more expensive) decreases the expected optimal harvest volume, and thus, increases the expected gap. Proposition 13 proves that this intuition is correct; that is, an increase in ω_0 is harmful for food security unless it induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different. In particular, when ω_0 increases, as follows from Proposition 12, the farmer optimally cultivates fewer acres and applies less fertilizer except for the cases when the increase in ω_0 induces the farmer to switch the optimal strategy from $(\hat{x}_c^{nf}, 0)$ (in Ξ_2), $(Q, 0)$ (in Ξ_3), or $(Q, \hat{s}_c)$ (in Ξ_4) to $(\hat{x}_c^f, \bar{s})$ (in Ξ_1). Outside of these cases, because x_c^* and s_c^* decrease, the expected optimal harvest volume in (3) decreases, and thus, the expected gap increases as shown in Proposition 13. When these cases happen, the farmer optimally increases s_c^* to counteract against the reduction in crop availability at the harvesting stage due to decreasing x_c^*. Because x_c^* decreases and s_c^* increases the resulting impact on the expected optimal harvest volume is indeterminate. We find in our data-calibrated numerical studies that the increase in fertilizer application rate may outweigh the decrease in cultivation volume and the expected optimal harvest volume increases (see Figure 7 for an illustration). In other words, an increase in ω_0 can be *beneficial* for food security. This behavior cannot be observed in the benchmark model where it can be proven that an increase in ω_0 always increases the expected gap and thus, it is always harmful for food security.

C.3 Analysis for Contract Farming

In this section, we study an extension of our model in which the farmer, besides selling the crop to the open market, also engages in contract farming with a buyer. In particular, we assume an exogenously given contract with unit price r and maximum delivery volume D. In the harvesting stage the farmer first sells to the buyer up to the maximum delivery volume D and the remaining harvested crop (if any) is sold to the open market. We replicate the entire analysis of our paper in this extended model. As we discuss below we show that our analytical results are structurally the same in this extended model. Moreover, we verify that our main numerical results continue to

Figure 7: Effect of the External Harvesting Cost ω_0 on the Expected Gap $J^*(r_c, y_c)$



Notes. $\omega_0 \in [20\%, 30\%]$ changes from the baseline value $\acute{\omega}_0 = 2.28$ with 0.1% increments. $y_c = 1.3\acute{y}_c$, $r_c = 1.3\acute{r}_c$, and all the rest of the parameters are at their calibrated (baseline) levels. In this example as ω_0 increases, the expected gap is non-decreasing except when the farmer's optimal strategy switches from $(Q, \hat{s}_c)$ (in Ξ_4) to $(\hat{x}_c^f, \bar{s})$ (in Ξ_1).

hold in this extended model. In summary, our main insights of the paper continue to hold in the presence of contract farming.

C.3.1 Model Discussion

Throughout this section, we focus on the case with $D \leq K_h$ so that the farmer has sufficient internal resources to harvest the quantity to satisfy the maximum volume D. While we do not impose any assumptions on the contract price r at this point, arguably the most realistic case is to assume $r = \mu_m$; that is, the contract price is given by the expected open market price. This is because at the time of contracting (which is not modeled in our paper and which happens before the harvesting stage) the buyer knows that the crop can be sourced from the open market in the harvesting stage which has an expected price μ_m; therefore, a buyer would not be interested in paying more than μ_m. Similarly, at the time of contracting the farmer also knows that the crop can be sold to the open market at the harvesting stage which has an expected price μ_m; therefore, the farmer would not be interested in accepting less than μ_m. We keep contract price as r for our structural analysis and we assume $r = \mu_m$ for our numerical experiments in this section.

We now formulate the farmer's decision problem. In the harvesting stage, farm yield $\tilde{\epsilon}$ and market uncertainty $\tilde{m}$ are realized. Given the decisions in the cultivation stage, namely cultivation volume x_c and fertilizer application rate s_c, the farmer's optimization problem in the harvesting stage is formulated as follows:

$$\Pi_h(x_c, s_c, \epsilon, m) \doteq \max_{x_h \geq 0} \quad r \min(D, x_h) + p(m, \epsilon)(x_h - D)^+ - \omega_h(\epsilon)(x_h - K_h)^+ \qquad \text{(A-10)}$$
$$\text{s.t.} \quad x_h \leq x_c(\epsilon + a s_c).$$

The farmer maximizes the profit by choosing an optimal crop volume to harvest, subject to the crop availability constraint as captured by the realized yield $x_c(\epsilon + as_c)$. Here the first term of the objective function is the farmer's revenue from contract which is given by the product of unit crop revenue r and the delivered volume; that is, the minimum of the harvesting volume x_h and the maximum contract delivery volume D. The second term is the revenue from open market sales which is given by the product of crop price $p(m, \epsilon)$ and the remaining harvest volume after the contract is satisfied $(x_h - D)^+$. The third term is the cost for additional harvesting resources. Using the assumption $D \leq K_h$, it can be shown that the optimal harvesting volume for a given (ϵ, m) is the same as in our main model:

$$x_h^*(\epsilon, m) = \begin{cases} x_c(\epsilon + as_c) & \text{if } p(m, \epsilon) \geq \omega_h(\epsilon), \\ \min\left(x_c(\epsilon + as_c), K_h\right) & \text{if } p(m, \epsilon) < \omega_h(\epsilon). \end{cases} \tag{A-11}$$

In the cultivation stage, given unit cultivation cost r_c and unit fertilizer cost y_c the farmer chooses the cultivation volume x_c and fertilizer application rate s_c. Let $\Pi_c^*(r_c, y_c)$ denote the farmer's optimal expected profit in this stage, which is given as follows:

$$\begin{aligned} \Pi_c^*(r_c, y_c) \doteq \max_{x_c, s_c} \quad & \mathbb{E}\left[r \min(D, x_c(\tilde{\epsilon} + as_c)) + p(\tilde{m}, \tilde{\epsilon})(x_c(\tilde{\epsilon} + as_c) - D)^+\right. \\ & \left. - \min(p(\tilde{m}, \tilde{\epsilon}), \omega_h(\tilde{\epsilon}))(x_c(\tilde{\epsilon} + as_c) - K_h)^+\right] - y_c s_c x_c - r_c x_c, \\ \text{s.t.} \quad & 0 \leq x_c \leq Q, \ 0 \leq s_c \leq \bar{s}. \end{aligned} \tag{A-12}$$

In (A-12), the first term in the objective function is the expected profit in the harvesting stage. In particular, the first two terms within the expectation represent the farmer's revenue if all the available crops are harvested. However, additional harvesting resources are needed beyond the internal resources, i.e., for the harvesting amount $(x_c(\epsilon + as_c) - K_h)^+$, and their cost is given in the third term within the expectation. The second and third terms in the objective function represent the fertilizer and cultivation cost, respectively. The constraints state that the cultivation volume cannot exceed the available farmland Q and the fertilizer application rate cannot exceed the agronomic recommendation $\bar{s}$. To make a comparison with the expected stage-1 profit in the main model as given by the objective function in (1), using the identity $\min(p(m, \epsilon), \omega_h(\epsilon)) = p(m, \epsilon) - (p(m, \epsilon) - \omega_h(\epsilon))^+$, we can rewrite the expected stage-1 profit in the main model as

$$\mathbb{E}\left[p(\tilde{m}, \tilde{\epsilon})x_c(\tilde{\epsilon} + as_c) - \min(p(\tilde{m}, \tilde{\epsilon}), \omega_h(\tilde{\epsilon}))(x_c(\tilde{\epsilon} + as_c) - K_h)^+\right] - y_c s_c x_c - r_c x_c.$$

We observe that the only difference in (A-12) from (1) is the farmer's expected revenue (i.e., the first two terms within the expectation) and the two expressions become identical when we set $D = 0$.

Before solving the farmer's decision problem, it is useful to examine under what conditions the farmer benefits from contract farming. We can check the derivative of the objective function in (A-12) with respect to D and show that when

$$\mathbb{E}\left[(r - p(\tilde{m}, \tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{D}{Q} - a\bar{s}\right\}\right] \geq 0, \tag{A-13}$$

engaging in contract farming will be beneficial to the farmer. Specifically, the above condition ensures that the marginal value of contracting is nonnegative for any given farmers decisions (x_c, s_c) at the cultivation stage (the condition is obtained using $x_c = Q$ and $s_c = \bar{s}$). We note that when the open market price p is independent of farm yield, because $\mathbb{E}[p(\tilde{m})] = \mu_m$ the condition in (A-13) reduces to $r \geq \mu_m$; that is, the contract price must be greater than the expected open market price for the farmer to strictly benefit from contracting. In other words, in the realistic case of $r = \mu_m$ contract farming does not have value for the farmer. However, in our focal case where the open market price decreases in the farm yield (i.e., $p(m, \epsilon)$ decreases in ϵ), the farmer may benefit from contract farming even when $r = \mu_m$. This is because when the maximum delivery volume is D, an additional unit of contract (maximum delivery volume) substitutes the open market sales revenue $p(m, \epsilon)$ with the contract revenue r when the maximum harvest volume $Q(\epsilon + a\bar{s})$ is larger than D; that is, when the yield realization is sufficiently high (i.e., $\epsilon > D/Q - a\bar{s}$). At these high yield realizations, the open market price $p(m, \epsilon)$ is low because $p(m, \epsilon)$ decreases in ϵ. In other words, contract farming creates value for the farmer by substituting the open market sales revenue at low revenue realizations with the fixed unit revenue r.

To further illustrate the value of engaging in contract farming, let us focus on the case $p(\tilde{m}, \tilde{\epsilon}) = \tilde{m} - \alpha(\tilde{\epsilon} - \mu_\epsilon)$ where $\alpha > 0$ as assumed in the main paper. In this case when $r = \mu_m$, (A-13) reduces to

$$\mathbb{E}\left[\alpha(\tilde{\epsilon} - \mu_\epsilon)\mathbb{I}\left\{\tilde{\epsilon} > \frac{D}{Q} - a\bar{s}\right\}\right] \geq 0. \tag{A-14}$$

It is easy to see that (A-14) is strictly positive when $\frac{D}{Q} - a\bar{s} > 0$ where the minimum yield realization is assumed to be zero.

C.3.2 Optimal cultivation and fertilizer application decisions

We now characterize the farmer's optimal cultivation and fertilizer application decisions, denoted by (x_c^*, s_c^*). For tractability we make the following assumptions hereafter for our contracting model.

Assumption 3 *We assume (i)* $r - \mathbb{E}\left[p\left(\tilde{m}, \frac{D}{Q} - a\bar{s}\right)\right] \geq 0$*; and (ii)* $D = K_h$*.*

These conditions are needed to ensure that the objective function of the farmer's decision problem is well-behaved. Part (i) of this assumption states that the contract price must be no lower than the expected open market price even when the yield realization is low in which case the farmer can still meet the contract demand by cultivating the whole farmland and applying fertilizer at the agronomically recommended rate. When the open market price is independent of farm yield, this condition is equivalent to (A-13) since both reduce to $r \geq \mu_m$. However, in our focal case where the open market price decreases in the farm yield, this condition is stronger and it implies the condition in (A-13), thereby ensuring that engaging in contract farming is beneficial to the farmer. Part (ii) of the assumption is only needed for the case with moderate y_c values when we prove Proposition 16. This is a reasonable assumption considering that with part (i) and $D \leq K_h$, it is profitable for the farmer to increase the maximum delivery volume D to K_h.

For ease of exposition, we present the characterization in three cases based on the range of unit fertilizer cost y_c starting with the large y_c case.

Proposition 14 (Large unit fertilizer cost) *When $y_c > y_c^{(0)} \doteq ar$, we have*

$$(x_c^*, s_c^*) = \begin{cases} (0,0) & \text{if } \Theta(0) \le r_c, \\ (\hat{x}_c^{nf}, 0) & \text{if } \Theta(Q) \le r_c < \Theta(0), \\ (Q,0) & \text{if } r_c < \Theta(Q), \end{cases}$$

where $\hat{x}_c^{nf} \in (D/\bar{\epsilon}, Q]$ is the unique solution to $\Theta(\hat{x}_c^{nf}) = r_c$ with

$$\Theta(x_c) \doteq \begin{cases} \mathbb{E}\left[r\tilde{\epsilon}\right] & \text{if } x_c \le \frac{D}{\bar{\epsilon}}, \\ \mathbb{E}\left[\tilde{\epsilon}\left(r\mathbb{I}\left\{\tilde{\epsilon} < \frac{D}{x_c}\right\} + p(\tilde{m},\tilde{\epsilon})\mathbb{I}\left\{\tilde{\epsilon} > \frac{D}{x_c}\right\} - \min(p(\tilde{m},\tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right)\right] & \text{if } x_c > \frac{D}{\bar{\epsilon}}. \end{cases}$$

When the unit fertilizer cost is large, the farmer optimally does not apply any fertilizer. In this case, the marginal cultivation cost is given by r_c, the value of which determines the optimal cultivation volume. When r_c is small, the farmer optimally cultivates the whole farmland; when r_c is large, the farmer optimally does not cultivate at all; otherwise, the farmer optimally cultivates $\hat{x}_c^{nf}$ acres. As can be seen, the optimal solution is structurally the same as that for our main model in Proposition 1. Nevertheless, there are some differences in the detailed expressions of the cutoff value $y_c^{(0)}$ and $\Theta(x_c)$. In particular, for a given yield realization ϵ we observe from $\Theta(x_c)$ that the marginal revenue for cultivating an additional acre equals ϵr for small cultivation volumes (i.e., $x_c \le D/\bar{\epsilon}$), but equals $\epsilon p(m,\epsilon)$ for large cultivation volumes (i.e., $x_c > D/\bar{\epsilon}$). In contrast, the marginal revenue always equals $\epsilon p(m,\epsilon)$ in our main model without forward contracting. This is due to the fact the farmer has two selling channels: first through contract and then through the open market.

Next we characterize the optimal decisions for a sufficiently small unit fertilizer cost y_c.

Proposition 15 (Small unit fertilizer cost)
Let $y_c^{(2)} \doteq a\mathbb{E}\left[r\mathbb{I}\left\{\tilde{\epsilon} < \frac{D}{Q} - a\bar{s}\right\} + p(\tilde{m},\tilde{\epsilon})\mathbb{I}\left\{\tilde{\epsilon} > \frac{D}{Q} - a\bar{s}\right\} - \min(p(\tilde{m},\tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\{\tilde{\epsilon} > K_h/Q - a\bar{s}\}\right]$. When $y_c < y_c^{(2)}$, we have

$$(x_c^*, s_c^*) = \begin{cases} (0,\bar{s}) & \text{if } \Gamma(0) \le r_c + \bar{s}y_c, \\ (\hat{x}_c^{f}, \bar{s}) & \text{if } \Gamma(Q) \le r_c + \bar{s}y_c < \Gamma(0), \\ (Q,\bar{s}) & \text{if } r_c + \bar{s}y_c < \Gamma(Q), \end{cases}$$

where $\hat{x}_c^f \in (K_h/(\bar{\epsilon} + a\bar{s}), Q]$ is the unique solution to $\Gamma(\hat{x}_c^f) = r_c + \bar{s}y_c$ with

$$\Gamma(x_c) \doteq \begin{cases} \mathbb{E}\left[r(\tilde{\epsilon} + a\bar{s})\right] & \text{if } x_c \le \frac{K_h}{\bar{\epsilon}+a\bar{s}}, \\ \mathbb{E}\Big[(\tilde{\epsilon} + a\bar{s})\Big(r\mathbb{I}\left\{\tilde{\epsilon} < \frac{D}{x_c} - a\bar{s}\right\} + p(\tilde{m},\tilde{\epsilon})\mathbb{I}\left\{\tilde{\epsilon} > \frac{D}{x_c} - a\bar{s}\right\} & \\ \qquad - \min(p(\tilde{m},\tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c} - a\bar{s}\right\}\Big)\Big] & \text{if } x_c > \frac{K_h}{\bar{\epsilon}+a\bar{s}}. \end{cases}$$

When the fertilizer cost is small, the farmer optimally applies fertilizer at agronomic recommendation (i.e., $s_c^* = \bar{s}$). Therefore, the marginal cost of cultivating an additional acre is given by the sum of cultivation cost per acre r_c and fertilizer application cost $\bar{s}y_c$. The characterization of optimal cultivation volume x_c^* is structurally similar to that of Proposition 14. In particular, when $r_c + \bar{s}y_c$ is small, the farmer optimally cultivates the whole farmland; when it is large, the farmer optimally does not cultivate at all (and the fertilizer application decision is irrelevant); otherwise, the farmer optimally cultivates $\hat{x}_c^f$ acres.

A comparison between Proposition 2 and Proposition 15 shows that the optimal solution for the extended model has the same structure as that for our main model. Similar to the case with a sufficiently large unit fertilizer cost, because of the contract sales channel, the farmer's marginal revenue of cultivating an additional acre for a given yield realization ϵ is different for a different value of x_c. Specifically, it is equal to $(\epsilon + a\bar{s})r$ for small cultivation volumes (i.e., $\epsilon < D/x_c - a\bar{s}$) but is equal to $(\epsilon + a\bar{s})p(m, \epsilon)$ for large cultivation volumes (i.e., $\epsilon > D/x_c - a\bar{s}$).

So far we have observed that when the unit fertilizer cost y_c is sufficiently small or sufficiently large, the farmer always optimally chooses the same fertilizer application rate regardless of the optimal cultivation volume. When y_c is in the moderate range, the farmer may also optimally change the fertilizer application decision, as illustrated in Proposition 16:

Proposition 16 (Moderate unit fertilizer cost) *Let* $\Theta(x_c)$ $(\Gamma(x_c))$ *and* $y_c^{(0)}$ $(y_c^{(2)})$ *be as defined in Proposition 14 (Proposition 15) and*
$y_c^{(1)} \doteq a\mathbb{E}\left[r\mathbb{I}\left\{\tilde{\epsilon} < \frac{D}{Q}\right\} + p(\tilde{m}, \tilde{\epsilon})\mathbb{I}\left\{\tilde{\epsilon} > \frac{D}{Q}\right\} - \min(p(\tilde{m}, \tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{Q}\right\}\right]$ *where* $y_c^{(1)} \in [y_c^{(2)}, y_c^{(0)}]$.
Case i: When $y_c^{(1)} \leq y_c < y_c^{(0)}$, *we have*

$$(x_c^*, s_c^*) = \begin{cases} (0, \bar{s}) & \text{if } \Gamma(0) \leq r_c + \bar{s}y_c, \\ (\hat{x}_c^f, \bar{s}) & \text{if } r_c + \bar{s}y_c < \Gamma(0) \text{ and } r_c \geq \Theta(\overline{x}_c), \\ (\hat{x}_c^{nf}, 0) & \text{if } \Theta(Q) \leq r_c < \Theta(\overline{x}_c), \\ (Q, 0) & \text{if } r_c < \Theta(Q), \end{cases}$$

where $\hat{x}_c^f \in (D/(\bar{\epsilon} + a\bar{s}), \underline{x}_c]$ *is the unique solution to* $\Gamma(\hat{x}_c^f) = r_c + \bar{s}y_c$ *and* $\hat{x}_c^{nf} \in (\bar{x}_c, Q]$ *is the unique solution to* $\Theta(\hat{x}_c^{nf}) = r_c$. *Here* $\underline{x}_c > D/(\bar{\epsilon} + a\bar{s})$ *is the unique solution to*

$$a\mathbb{E}\left[r\mathbb{I}\left\{\tilde{\epsilon} < \frac{D}{\underline{x}_c} - a\bar{s}\right\} + p(\tilde{m}, \tilde{\epsilon})\mathbb{I}\left\{\tilde{\epsilon} > \frac{D}{\underline{x}_c} - a\bar{s}\right\} - \min(p(\tilde{m}, \tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{\underline{x}_c} - a\bar{s}\right\}\right] = y_c,$$

and $\bar{x}_c > D/\bar{\epsilon}$ *is the unique solution to*

$$a\mathbb{E}\left[r\mathbb{I}\left\{\tilde{\epsilon} < \frac{D}{\bar{x}_c}\right\} + p(\tilde{m}, \tilde{\epsilon})\mathbb{I}\left\{\tilde{\epsilon} > \frac{D}{\bar{x}_c}\right\} - \min(p(\tilde{m}, \tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{\bar{x}_c}\right\}\right] = y_c.$$

Case ii: When $y_c^{(2)} \leq y_c < y_c^{(1)}$, *we have*

$$(x_c^*, s_c^*) = \begin{cases} (0, \bar{s}) & \text{if } \Gamma(0) \leq r_c + \bar{s}y_c, \\ (\hat{x}_c^f, \bar{s}) & \text{if } r_c + \bar{s}y_c < \Gamma(0) \text{ and } r_c \geq \Theta(\overline{x}_c), \\ (Q, \hat{s}_c) & \text{if } r_c < \Theta(\overline{x}_c), \end{cases}$$

where $\hat{x}_c^f \in (K_h/(\bar{\epsilon} + a\bar{s}), Q]$ *is the unique solution to* $\Gamma(\hat{x}_c^f) = r_c + \bar{s}y_c$ *and* $\hat{s}_c \in (0, \bar{s})$ *is the unique solution to*

$$y_c = a\mathbb{E}\left[r\mathbb{I}\left\{\tilde{\epsilon} < \frac{D}{Q} - a\hat{s}_c\right\} + p(\tilde{m}, \tilde{\epsilon})\mathbb{I}\left\{\tilde{\epsilon} > \frac{D}{Q} - a\hat{s}_c\right\} - \min(p(\tilde{m}, \tilde{\epsilon}), \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{Q} - a\hat{s}_c\right\}\right].$$

Recall that the above proposition requires both conditions in Assumption 3, and in particular $D = K_h$. We only delineate the intuition behind the second case; the first case can be explained in a similar fashion. Let us consider a given unit fertilizer cost $y_c \in [y_c^{(2)}, y_c^{(1)})$ and examine the optimal solution while changing the cultivation cost per acre r_c. When r_c is sufficiently high, the farmer optimally does not cultivate any farmland (and the fertilizer application decision is irrelevant). As

r_c decreases, the farmer increases the cultivation volume, and paralleling the characterization in Proposition 15, optimally cultivates $\hat{x}_c^f$ acres and applies fertilizer at agronomically recommended rate $\bar{s}$. As r_c further decreases, the farmer further increases the cultivation volume and optimally cultivates the whole farmland. In this case, different from the characterization in Proposition 15, the farmer optimally applies fertilizer at a rate $\hat{s}_c$ that is lower than $\bar{s}$ because y_c is higher than the unit fertilizer cost in Proposition 15 and thus, it is not beneficial for the farmer to continue applying $\bar{s}$ amount of fertilizer per acre when the increase in number of acres cultivated is accounted for. Here, $\hat{s}_c$ is the fertilizer rate per acre (applied to the whole farmland Q) for which the marginal cost y_c equals its expected marginal revenue. In the harvesting stage this marginal revenue is characterized by the product of an additional unit of fertilizer's effect on yield per acre, as given by a, and the effective crop margin which follows a similar structure with the effective crop margin that is used to characterize $\Gamma(x_c)$ in Proposition 15, where x_c and s_c are substituted with Q and $\hat{s}_c$, respectively.

Similar to the cases with a sufficiently large or small unit fertilizer cost y_c, a comparison between Proposition 3 and Proposition 16 reveals that the optimal solution has the same structure as that for the main model, although the detailed expressions may be different. This is again due to the fact that in the extended model the farmer first sells the crop through contract and then sells the remaining crops (if any) to the open market.

Based on the characterization results for the three cases of y_c we can summarize the optimal decisions in the same way as Corollary 1.

Corollary 2 *When $r_c + \bar{s}y_c \geq \Gamma(0) = \mathbb{E}\left[(\tilde{\epsilon} + a\bar{s})r\right]$ and $r_c \geq \Theta(0) = \mathbb{E}\left[\tilde{\epsilon}r\right]$, we have $x_c^* = 0$ and the fertilizer application decision is irrelevant. Otherwise, we have $x_c^* > 0$ and the characterization of (x_c^*, s_c^*) can be illustrated using the same figure, Figure 1, for the case $\Gamma(Q) < \Theta(Q)$ (the characterization is structurally the same for the case $\Gamma(Q) \geq \Theta(Q)$) where*

$$\Xi_1 \doteq \left\{(r_c, y_c) : y_c \leq \hat{y}_c(r_c), y_c^{(2)} \leq y_c \leq y_c^{(0)}\right\} \cup \left\{(r_c, y_c) : \Gamma(Q) \leq r_c + \bar{s}y_c, 0 \leq y_c < y_c^{(2)}\right\},$$

$$\Xi_2 \doteq \left\{(r_c, y_c) : \Theta(Q) \leq r_c, y_c > \hat{y}_c(r_c)\right\},$$

$$\Xi_3 \doteq \left\{(r_c, y_c) : \Theta(Q) > r_c, y_c \geq y_c^{(1)}\right\},$$

$$\Xi_4 \doteq \left\{(r_c, y_c) : y_c > \hat{y}_c(r_c), y_c^{(2)} \leq y_c < y_c^{(1)}\right\},$$

$$\Xi_5 \doteq \left\{(r_c, y_c) : \Gamma(Q) > r_c + \bar{s}y_c, 0 \leq y_c < y_c^{(2)}\right\}.$$

Here, $\hat{y}_c(r_c)$, which can be proven to be concavely increasing in r_c, is the unique solution to $\Theta(\overline{x}_c) = r_c$ where $\overline{x}_c$ is as given by Proposition 16.

When the farmer optimally cultivates some acres, Corollary 2 identifies the same five strategies with the main model that emerge as optimal: partial farmland cultivation without using any fertilizer (Ξ_2), partial farmland cultivation with applying fertilizer at agronomically recommended rate (Ξ_1), and full farmland cultivation with three distinct fertilizer application rates; agronomic recommendation (Ξ_5), less than agronomic recommendation (Ξ_4), and none (Ξ_3).

Similar to the main model, we make the following assumptions hereafter:

Assumption 4 *We assume*
(i) $r_c + \bar{s}y_c < \mathbb{E}[r(\tilde{\epsilon} + a\bar{s})]$ *and* $r_c < \mathbb{E}[r\tilde{\epsilon}]$*;*
(ii) $\tilde{m}$ *and* $\tilde{\epsilon}$ *have independent distributions;*
(iii) $p(\tilde{m}, \tilde{\epsilon}) = \tilde{m} - \alpha(\tilde{\epsilon} - \mu_\epsilon)$ *for* $\alpha \in [0, \underline{m}/(\bar{\epsilon} - \mu_\epsilon))$ *and* $\omega_h(\tilde{\epsilon}) = \omega_0 + \beta\tilde{\epsilon}$ *for* $\omega_0 > 0$ *and* $\beta \geq 0$*.*

Assumption (i) 4 implies that, as follows from Corollary 2, the farmer optimally cultivates a positive amount of farmland (i.e., $x_c^* > 0$). Assumption 1(ii) introduces additional structure on the distributions of $\tilde{m}$ and $\tilde{\epsilon}$ whereas Assumption 1(iii) introduces specific functional forms for the crop price and external unit harvesting cost; these are necessary for the tractability of our sensitivity analysis in the subsequent sections. When $r = \mu_m$, using part (ii) and (iii) of Assumption 4, it can be shown that Assumption 3 reduces to $K_h \geq Q(\mu_\epsilon + a\bar{s})$, which is the same as Assumption 2 in the main paper.

C.3.3 Analysis of optimal decisions for farm management

We now examine how changes in cultivation and fertilizer costs as well as farm yield variability impact the farmer's optimal decisions and profitability. This analysis follows the same approach as in our main model. After repeating the proofs of Propositions 4, 5, and 6 for our extended contract farming model, we can replicate all the sensitivity analysis results about the effects of costs and uncertainties on the optimal decisions and profitability. That is, the results in these propositions continue to be relevant in our extended model. For brevity we omit the details here but it is worthwhile pointing out the similarities and distinctions in the analyses. First, it is straightforward to establish the effects of cultivation and fertilizer costs on the optimal decisions since $\Theta(x_c)$ and $\Gamma(x_c)$ continue to decrease in x_c, which underpins the proofs of Propositions 4 and 5. To show the effects of yield variability on the optimal decisions, we examine how a change in σ_ϵ affects $\Theta(x_c)$, $\Gamma(x_c)$, and the boundaries of Figure 1. Again, these effects remain the same as those for the main model. Take the effect of σ_ϵ on $\Theta(x_c)$ for example. With $\alpha = 0$ as assumed in Proposition 6, we rewrite $\Theta(x_c)$ as follows:

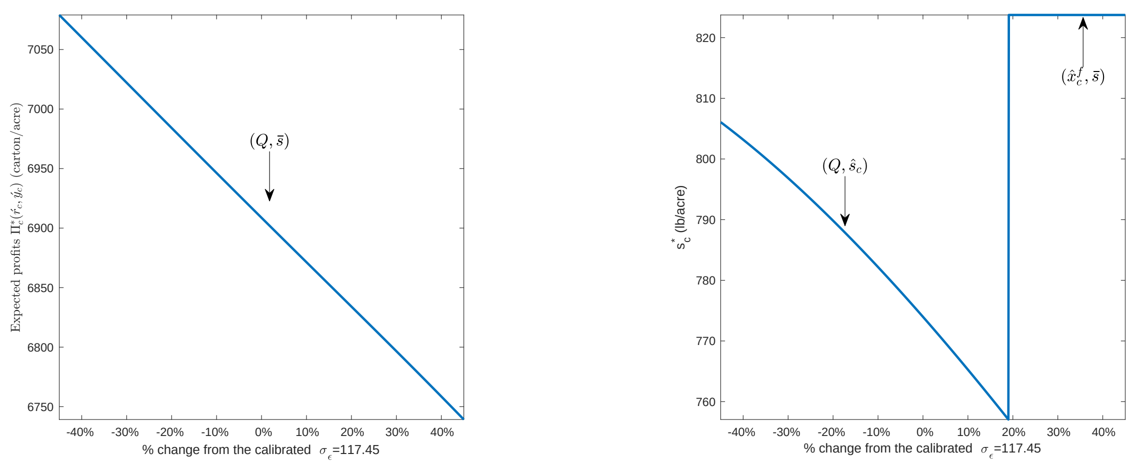
$$\begin{aligned}\Theta(x_c) =& \mathbb{E}\left[\tilde{\epsilon}\left(r - (r - \tilde{m})\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\} - \min(\tilde{m}, \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right)\right] \\ =& r\mu_\epsilon - \mathbb{E}\left[\tilde{\epsilon}\left((r - \mu_m)\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\} + \min(\tilde{m}, \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right)\right].\end{aligned}$$

Further, Assumption 3 reduces to $r \geq \mu_m$ when $\alpha = 0$. It can be shown that $\mathbb{E}\left[\tilde{\epsilon}\left((r - \mu_m)\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right)\right]$ increases in yield variability σ_ϵ based on the condition $K_h \geq Q(\mu_\epsilon + a\bar{s})$ again assumed in Proposition 6. Therefore, the effect of σ_ϵ on the term $\mathbb{E}\left[\tilde{\epsilon}\left((r - \mu_m)\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right)\right]$ is consistent with that on the term $\mathbb{E}\left[\tilde{\epsilon}\left(\min(\tilde{m}, \omega_h(\tilde{\epsilon}))\mathbb{I}\left\{\tilde{\epsilon} > \frac{K_h}{x_c}\right\}\right)\right]$. This explains why the effects of σ_ϵ on the optimal decisions in the extended model remain the same as those for the main model. Overall, we find that our analytical sensitivity results about the effects of cost and uncertainty on optimal decisions are robust and they continue to be relevant for our model with contract farming.

We also replicate our main paper's numerical experiments in this extended model by assuming $r = \mu_m$ and $D = K_h$; otherwise, the numerical setup (including the calibrated values and numerical instances considered) remains the same. We verify that our numerical results for the effect of yield variability continue to hold in this extended model. In particular, paralleling the main paper, in all our numerical instances (where we use the same calibrated $\acute{\alpha} > 0$ and $\acute{\beta} > 0$) as yield variability σ_ϵ increases we consistently observe that (i) the optimal expected profit decreases and (ii) optimal cultivation volume x_c^* decreases whereas the optimal fertilizer application rate s_c^* decreases except for the cases when it induces a transition from Ξ_4 to Ξ_1 (in these cases s_c^* increases). We refer the reader to Figure 8(a) for illustration of (i) and Figure 8(b) for illustration of (ii) for the behavior of s_c^*. These illustrations parallel those in Figure 3 of the main paper.

Figure 8: Effects of Farm Yield Variability σ_ϵ on the Optimal Expected Profit Π_c^* (Panel a) and the Optimal Fertilizer Application Rate s_c^* (Panel b) with Contract Farming

(a) (b)



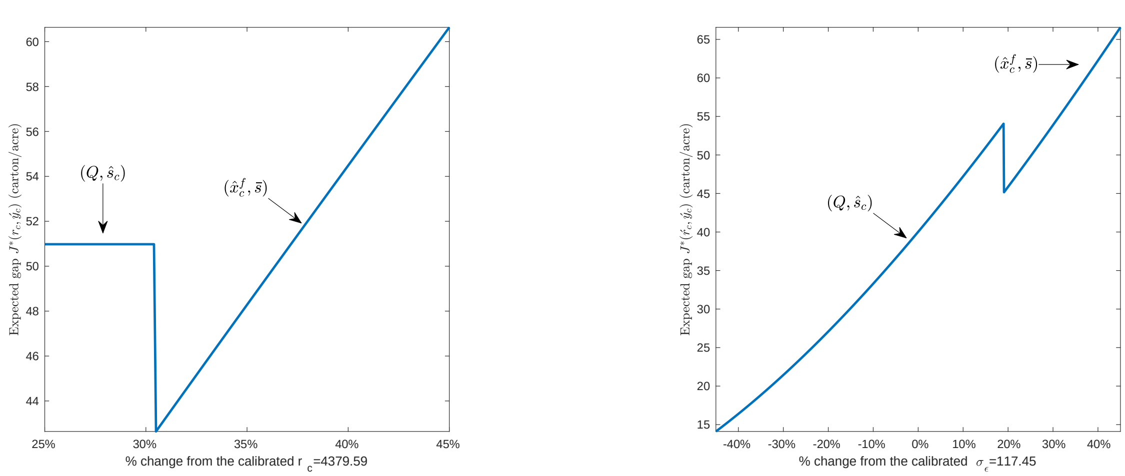
Notes. In each panel, $\sigma_\epsilon \in [-45\%, 45\%]$ changes from the baseline value $\acute{\sigma}_\epsilon = 117.45$ with 0.1% increments. In panel b, $r_c = 1.3\acute{r}_c$, $y_c = 1.3\acute{y}_c$. In both panels, the rest of the parameters are at their calibrated (baseline) levels.

C.3.4 Analysis of optimal decisions for food security

We now examine the implications of the farmer's optimal decisions for food security in this extension. In addition to Assumption 3 and Assumption 4, this analysis also uses Assumption 2 as is the case for our main model. We use the same measure of food security as the expected gap for a given unit cultivation cost r_c and unit fertilizer cost y_c; that is, $J^*(r_c, y_c) = (\mu_\epsilon + a\bar{s})Q - \mathbb{E}[x_h^*(\tilde{m}, \tilde{\epsilon})]$, where x_h^* is given by (A-11) with (x_c, s_c) replaced by the optimal decisions (x_c^*, s_c^*) as summarized in Corollary 2. It is noted that $J^*(r_c, y_c)$ in the extended model is the same as that for the main model except for the fact that the optimal decisions take a different form but remain structurally the same (see Propositions 14-16 and Corollary 2). Recall that we have already shown in the previous section that the effects of unit cultivation cost r_c, unit fertilizer cost y_c, and yield variability σ_ϵ on the optimal decisions are structurally the same with the main model. As a result, we can replicate all the analytical results about the effects of unit cultivation cost r_c, unit fertilizer cost y_c, and yield variability σ_ϵ on the expected gap $J^*(r_c, y_c)$. That is, the results in Propositions 7, 8, and 9 continue to be relevant in our extended model.

Figure 9: Effects of Cultivation Cost Per Acre r_c (Panel a) and Farm Yield Variability σ_ϵ (Panel b) on the Expected Gap $J^*(r_c, y_c)$ with Contract Farming

(a) (b)



Notes. In panel a, $r_c \in [25\%, 45\%]$ away from the baseline value $\acute{r}_c = 4379.59$ with 0.1% increments, $y_c = 1.3\acute{y}_c$, and $\sigma_\epsilon = 1.15\acute{\sigma}_\epsilon$. In panel b, $\sigma_\epsilon \in [-5\%, 5\%]$ away from the baseline value $\acute{\sigma}_\epsilon = 117.45$ with 0.1% increments, $r_c = 1.3\acute{r}_c$, and $y_c = 1.3\acute{y}_c$. In both panels, the rest of the parameters are at their calibrated (baseline) levels.

We also replicate our main paper's numerical experiments in this extended model by assuming $r = \mu_m$ and $D = K_h$. We make the same observations with the main model. In particular, as unit fertilizer cost y_c increases, in all numerical experiments that include a transition from Ξ_1 to Ξ_4 we observe that expected gap continues to increase. As cultivation cost per acre r_c increases, in some of the numerical experiments that include a transition from Ξ_4 to Ξ_1 we observe that the expected gap decreases; see Figure 9(a) for an example. We next examine the effect of farm yield variability σ_ϵ. In this extended model, similar to Proposition 9 of the main paper, we prove under the $\alpha = 0$ assumption that when σ_ϵ increases, the expected gap $J^*(r_c, y_c)$ also increases except for cases when it induces the farmer to switch from one optimal strategy to another in which both optimal cultivation volume and fertilizer application rate are different (in these cases the effect on $J^*(r_c, y_c)$ is indeterminate). In our numerical experiments, we verify that this result continues to hold without the $\alpha = 0$ assumption. We also find that when an increase in σ_ϵ induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different, the expected gap may also decrease; see Figure 9(b) for an example. The illustrations in Figure 9 parallel those in Figure 6 of the main paper.

C.4 Implications of Optimal Decisions for Food Waste

In this section, we consider the implications of farmer's optimal decisions on an alternative food security measure: food waste, which is the amount of crops left unharvested due to a high harvesting labor cost or a low open market price. The objective of this section is two fold: (i) to contextualize in our setting the food waste measure and (ii) to determine whether there is a discrepancy between this measure and the expected gap (as used in the main paper) in terms of how changes in costs and uncertainties in the farming environment affect each measure. Throughout our analysis, paralleling §5 of the main paper, we use Assumption 2 (i.e., $K_h \geq Q(\mu_\epsilon + a\bar{s})$).

In our model, food waste can be defined as the difference between the expected amount of crops available for harvesting given the farmer's optimal decisions and the expected optimal harvesting amount; that is, for a given unit cultivation cost r_c and unit fertilizer cost y_c, food waste is given by $W^*(r_c, y_c) = (\mu_\epsilon + as_c^*)x_c^* - \mathbb{E}[x_h^*(\tilde{m}, \tilde{\epsilon})]$. Substituting $\mathbb{E}[x_h^*(\tilde{m}, \tilde{\epsilon})]$ (as given in (3)) into this definition, we rewrite the food waste measure as follows:

$$W^*(r_c, y_c) = \mathbb{E}\left[(x_c^*(\tilde{\epsilon} + as_c^*) - K_h)^+ \mathbb{I}\{p(\tilde{m}, \tilde{\epsilon}) \leq \omega_h(\tilde{\epsilon})\}\right]. \tag{A-15}$$

Unlike the expected gap defined in (3), food waste increases as the optimal cultivation volume or the optimal fertilizer application rate increases.

We first examine how changes in cultivation cost per acre r_c and unit fertilizer cost y_c impact the food waste measure $W^*(r_c, y_c)$. As can be seen from (A-15), a change in each cost affects the food waste only by altering the optimal decisions (x_c^*, s_c^*) in the cultivation stage.

Proposition 17 (Effect of cultivation cost per acre on food waste) *When r_c increases, $W^*(r_c, y_c)$ decreases except for the cases when it induces a transition from either Ξ_2 or Ξ_4 to Ξ_1 in Figure 1.*

Proposition 17 shows that an increase in r_c reduces food waste unless it induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different. In particular, as r_c increases, as follows from Proposition 4, the farmer optimally cultivates fewer acres and does not alter optimal fertilizer application rate except for the cases when it induces the farmer to switch the optimal strategy from either $(Q, \hat{s}_c)$ or $(\hat{x}_c^{nf}, 0)$ to $(\hat{x}_c^f, \bar{s})$ in Figure 1. Outside of these cases, x_c^* decreases and s_c^* does not change. Since the food waste increases in both x_c^* and s_c^*, the food waste decreases. The effect of cultivation cost on food waste is opposite to that on the expected gap as shown in Proposition 7. This is because the maximum expected harvesting volume in the definition of $J^*(r_c, y_c)$; that is, $(\mu_\epsilon + a\bar{s})$, does not change in r_c, while the expected amount of crops available for harvesting given the optimal decisions in the definition of $W^*(r_c, y_c)$, $(\mu_\epsilon + as_c^*)x_c^*$, may decrease in r_c.

A similar result is relevant for the effect of unit fertilizer cost y_c on the food waste:

Proposition 18 (Effect of unit fertilizer cost on food waste) *As y_c increases, $W^*(r_c, y_c)$ decreases except for the cases when it induces a transition from Ξ_1 to either Ξ_2 or Ξ_4 in Figure 1.*

Proposition 18 proves that an increase in y_c reduces food waste unless it induces the farmer to switch from one optimal strategy to another in which both x_c^* and s_c^* are different. In particular, when y_c increases, as follows from Proposition 5, the farmer optimally applies less fertilizer per acre and cultivates fewer acres except for the cases when the increase in y_c induces the farmer to switch the optimal strategy from $(\hat{x}_c^f, \bar{s})$ to either $(Q, \hat{s}_c)$ or $(\hat{x}_c^{nf}, 0)$ in Figure 1. Outside of these cases, x_c^* and s_c^* decrease, and thus food waste decreases as shown in Proposition 18. Again, the effect of fertilizer cost on food waste is opposite to that on the expected gap as shown in Proposition

8, because food waste increases in both the optimal cultivation volume and the optimal fertilizer application.

We next examine how changes in farm yield variability σ_ϵ impact the food waste. As follows from (A-15), a change in σ_ϵ affects the expected gap by altering the expected food waste for any given farmer's decisions (x_c, s_c) as well as the farmer's optimal decisions (x_c^*, s_c^*) in the cultivation stage.

Proposition 19 (Effect of yield variability on food waste) *Assume* $\tilde{\epsilon} \sim \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$ *and* $\alpha = 0$. *When* $(r_c, y_c) \in \Xi_i$ *for* $i \in \{3, 5\}$, *we have* $\frac{\partial W^*(r_c, y_c)}{\partial \sigma_\epsilon} \geq 0$.

For any given decisions (x_c, s_c) food waste can be proven to increase in yield variability σ_ϵ. This is the direct effect of yield variability on food waste. When $(r_c, y_c) \in \Xi_i$ for $i \in \{3, 5\}$, the optimal decisions are constant, and the indirect effect of yield variability on the food waste through its effect on the optimal decisions is null. Therefore, the aggregate effect is positive, which explains the result that the food waste increases in yield variability in these two regions. Again, this result is consistent with that for the expected gap as shown in Proposition 9. However, when the optimal decisions fall in other regions or there is a transition between regions, as follows from Proposition 6, when σ_ϵ increases, x_c^* decreases and s_c^* decreases except for cases when it induces a transition from either Ξ_2 or Ξ_4 to Ξ_1 (in these cases s_c^* increases). This together with the result that food waste increases in x_c^* and s_c^* implies that the indirect effect of yield variability on the food waste through its effect on the optimal decisions is negative, thereby contradicting the positive direct effect. As a result, it remains unclear which effect dominates.

Overall, our analyses in this section reveal that the effects of cultivation and fertilizer costs on the expected gap and food waste are opposite, while the effects of farm yield variability on these two measures are the same. These results have some implications for food security and waste. While decreasing cultivation or fertilizer cost (e.g., through government subsidies) helps to increase the expected crop production and reduce the expected gap, it may lead to an increase in food waste. This unintended consequence is undesirable since the unharvested crop could have been harvested to further reduce the expected gap. Another implication is that decreasing yield variability (e.g., through provision of pest-resistant seed) may benefit the society as it helps to reduce both the expected gap and the food waste.

References

United States Department of Labor. 2021. Bureau of Labor Statistics quarterly census of employment and wages, NAICS-Based Data Files (1975 - most recent). `https://www.bls.gov/cew/downloadable-data-files.htm`, last accessed in June, 2021.

USDA. 2010. U.S. Tomato statistics historical data. `https://www.ers.usda.gov/data-products/vegetables-and-pulses-data/vegetables-and-pulses-historical-data/`, last accessed in June, 2021.

USDA. 2018. Quick Stats, National agriculture statistics service. `https://quickstats.nass.usda.gov/`, last accessed in May, 2018.