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Smart charging of electric vehicle: An innovative business model for utility firms

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Citation

Smart Charging of Electric Vehicles:
An Innovative Business Model for Utility Firms

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Problem Definition: By providing an environmentally friendly alternative to traditional vehicles, electric vehicles will transform urban mobility, particularly in smart cities. In practice, once an electric vehicle is plugged in, the charging station completes charging as soon as possible. Given that the procurement cost of electricity and associated emissions vary significantly during a day, substantial savings can be achieved by smart charging—delaying charging until the cost is lower. In this paper, we study smart charging as an innovative business model for utility firms.

Practical Relevance: Utility firms are already investing in charging stations and they can achieve significant cost savings through smart charging.

Methodology: We consider a mechanism design problem in which a utility firm first announces pairs of charging price and completion time. Then, each customer selects the pair that maximizes their utility. Given the selected completion times, the utility firm solves the optimal control problem of determining the charging schedule that minimizes the cost of charging under endogenous, time-varying electricity procurement cost. We assume that there are ample parking spots with chargers at the charging station.

Results: We devise an intuitive and easy-to-implement policy for scheduling charging of electric vehicles under given completion times. We prove that this policy is optimal if all customers arrive at the station simultaneously. We also characterize the optimal pairs of charging price and completion time. By using real electricity demand and generation data from the largest electricity market in the U.S., we find that cost and emissions savings from smart charging are approximately 20% and 15%, respectively, during a typical summer month.

Managerial Implications: In contrast to the current practice of charging vehicles without delay, we show that it is economically and environmentally beneficial to delay charging some vehicles and to set charging prices based on customers' inconvenience cost due to delays. We also find that most of the savings from implementing smart charging can be achieved during peak-demand days, highlighting the effectiveness of smart charging.

1.   Introduction

Electric vehicles will transform mobility, particularly in smart cities, by providing an environmentally friendly alternative to traditional vehicles (Lombardi et al. 2018). This transformation requires an estimated $50 billion of investment in charging stations across the U.S., Europe, and China by 2030 (Engel et al. 2018), providing an opportunity to create innovative business models for managing vehicle charging. Under the current practice, whenever an electric vehicle is plugged in, its battery is typically charged at the maximum possible speed to complete charging as soon as possible (Myers 2017). Given that the marginal cost of electricity generation and associated emissions vary significantly throughout a day, some experts worry that vehicle charging can result in unnecessarily high cost and emissions (Birnbaum 2015). However, customers do not necessarily require charging as soon as possible and they may agree to later completion times. Therefore, compared to charging vehicles as soon as possible, substantial cost and emissions savings can be achieved by smart charging—delaying charging until the electricity generation cost is lower. In this paper, we study smart charging as an innovative business model for utility firms.

Utility firms expect a significant increase in the number of electric vehicles and some firms, such as Southern California Edison, are already planning to invest in their own charging stations (Wilson et al. 2019, p. 26, Gahran 2019, and Southern California Edison 2016, p. 5). Given that utility firms incur the additional cost of procuring (or generating) electricity for charging a growing number of electric vehicles and they directly observe the resulting emissions, they are in an ideal position to manage charging (Baker et al. 2019).

Motivated by the need for innovative charging solutions, we pose the following research questions. What is the optimal charging policy of a utility firm under given charging completion times? What are the desired charging completion times and how should the utility firm design a pricing scheme that incentivizes customers to choose among them? We consider the two types of utility firms—public and private (or investor-owned, e.g., Southern California Edison). A public utility firm, such as an electric cooperative, is not-for-profit, whereas a private utility firm is for-profit (DOE 2015, p. 27). Within a service region, there is one utility firm that provides traditional electricity service and electric vehicle charging service. For either type of utility firm, a major challenge of managing electric vehicle charging is that customers have heterogeneous levels of sensitivity to delays in charging completion. This heterogeneity affects the utility firm's decisions on when to charge each electric vehicle and, in turn, the total charging cost and resulting emissions. By properly designing a pricing policy, the firm can incentivize customers with different delay-sensitivity

to choose different charging completion times, thus reducing the cost of charging and passing (part of) cost savings to customers.

We consider a mechanism design problem of a utility firm that operates a charging station under an endogenous, time-varying cost of electricity. We assume that the utility firm operates the charging station in a public parking space with ample parking spots and chargers (e.g., ChargePoint 2021) such that there is always an available charger for a customer arriving at the station. The firm designs pairs of charging price and completion time, and each customer chooses the pair that maximizes their utility (including their inconvenience cost) which decreases in the charging price, completion time, and the customer's sensitivity to delay. Each customer's delay-sensitivity is their private information, but the utility firm knows the distribution of the sensitivity across customer population. We consider both a public utility firm, whose objective is to minimize the sum of charging and inconvenience cost, and a private utility firm, whose objective is to maximize the revenue from the charging service minus the charging cost. The utility firm can reduce the charging cost by delaying charging until the cost of electricity is lower, but this leads to a higher inconvenience cost for customers. We also calibrate this analytical model by using real electricity generation and demand data from the PJM Interconnection, the largest electricity market in the U.S., to illustrate our findings.

Our results offer several managerial insights for utility firms that seek innovative solutions to electric vehicle charging. In contrast to the current practice of charging electric vehicles as soon as possible with a uniform pricing, we show that it is economically and environmentally beneficial to delay charging for some vehicles with prices reflecting customers' inconvenience cost due to delays. We do so by first devising an easy-to-implement policy to minimize the cost of charging for given completion times. This policy is an effective solution to a complex optimal control problem that determines the (time-varying) charging speed for each vehicle. The inconvenience cost-based pricing policy incentivizes customers with low delay-sensitivity to choose later completion times, giving the utility firm the opportunity to charge vehicles when the cost of charging is lower. This results in substantial cost savings, compared to charging vehicles as soon as possible. For example, in the PJM Interconnection, average cost savings across the days in a typical summer month are approximately 20%. We also find that this policy avoids the use of less energy-efficient power plants in the PJM Interconnection (because these plants are usually more costly), resulting in an average carbon emissions savings of approximately 15%. That is, from a policy perspective, allowing utility firms to own and operate charging stations can lead to reductions in both cost and emissions. Finally, most of these reductions from smart charging can be achieved during a few days

2

with high electricity demand, making customers potentially more receptive to smart charging and highlighting its practical relevance.

2. Literature Review

Our paper contributes to the emerging literature on smart-city operations by studying the electrification of urban mobility (for reviews, see Mak 2018, Hasija et al. 2020, and Qi and Shen 2019). Recent papers have studied various problems in this context. For example, Qi et al. (2018) analyze how a service provider can jointly use its own fleet and excess capacity from shared mobility providers (e.g., Uber drivers) for last-mile delivery. Moreover, electric vehicles have received particular attention in recent years. Mak et al. (2013), Avci et al. (2014), and Qi et al. (2020b) study battery swapping business model where a service provider leases fully charged batteries to vehicle owners by swapping them with depleted ones. We complement these papers by studying the charging station business model, where customers own their batteries. Chocteau et al. (2011) analyze adoption of electric vehicles in commercial fleets. Lim et al. (2015) show that fast charging reduces range anxiety of customers, whereas leasing batteries can increase adoption as well as profitability under high resale anxiety. Instead of focusing on adoption, we focus on a utility firm's charging and pricing decisions. Several papers consider a fleet of electric vehicles that are shared by customers: He et al. (2017) determine service regions to maximize coverage while keeping costs low, He et al. (2020) jointly optimize infrastructure planning and repositioning, and Zhang et al. (2020) determine service regions and optimize charging/discharging schedules of these shared vehicles. Qi et al. (2020a) investigate how autonomous electric vehicles can provide grid services as well as mobility to customers. We complement this stream of research by considering the interaction between delay-sensitive owners of electric vehicles and charging stations which face time-varying electricity cost. We analyze the key tradeoff between the charging cost and the inconvenience cost to identify economic and environmental benefits of smart charging.

Our paper also contributes to the energy related-operations literature (see Agrawal and Yücel 2020b for a review), which has investigated several topics, including integration of renewable energy with conventional sources (Wu and Kapuscinski 2013 and Zhou et al. 2019), effects of electricity pricing policies on consumption and investments (Ata et al. 2018 and Kök et al. 2018), supply function equilibrium in wholesale electricity markets (Al-Gwaiz et al. 2016 and Sunar and Birge 2019), capacity investment in energy sources (Hu et al. 2015 and Kök et al. 2020), and demand-response programs (Webb et al. 2019 and Agrawal and Yücel 2020a). Our paper is also related to the literature on energy storage operations. Wu et al. (2012) propose a heuristic for managing

seasonal energy storage, Zhou et al. (2016) show the importance of negative electricity prices in operating a storage facility, and Kapuscinski et al. (2020) characterize energy storage operations in an electricity network and optimize storage investment across the network to minimize the total cost of matching supply with demand. Different from the large-scale storage facilities studied in these papers, batteries in electric vehicles are owned by customers and their primary goal is to provide mobility. Therefore, to manage battery charging operations, a utility firm needs to consider customers' preferences. Our paper identifies how the utility firm can incentivize customers with different delay-sensitivity to agree to different charging completion times, thus reducing the cost of charging and passing (part of) cost savings to customers.

Our paper is also related to the extensive literature on price and lead-time quotation (see Keskinocak and Tayur 2004 for a review), which focuses on a manufacturing firm or a service provider that faces customers with varying willingness-to-pay for lead-times. The majority of papers in this stream (e.g., Keskinocak et al. 2001, Charnsirisakskul et al. 2006, Pekgün et al. 2008, and Hafızoğlu et al. 2016) consider a business-to-business setting in which there is no information asymmetry. In our setting, delay-sensitivity is customers' private information, which requires using the mechanism design approach to identify how a principal (utility firm) should design a menu of contracts (charging price and completion time) for agents (customers).

Mechanism design (Myerson 1981 and Lovejoy 2006) has been used in various settings in operations, including price and lead-time quotation (Lutze and Özer 2008, Akan et al. 2012, and Afèche 2013), contract farming in agriculture (Federgruen et al. 2019), and resource allocation (Balseiro et al. 2019). Unlike the majority of the papers in this stream of the literature which either normalize the marginal production cost to zero or consider it as a constant, the cost of charging electric vehicles in our paper is time-varying and endogenously dependent on the charging schedules (i.e., the marginal cost of charging increases in the number of vehicles being charged at that time). Thus, the principal's cost of serving agents is more complex. In fact, the cost is determined by an optimal control problem for which we devise an easy-to-implement policy.

Charging electric vehicles has also received significant attention in the energy and engineering literature (for a review, see Su et al. 2012 and García-Villalobos et al. 2014). Several papers investigate economic and environmental implications of residential charging of electric vehicles (Clement-Nyns et al. 2010 and Muratori 2018). Another stream focuses on engineering challenges, such as grid reliability (Deilami et al. 2011) and ancillary services (Bessa and Matos 2012). An emerging stream of this literature investigates the pricing problem of a private charging station instead of a utility firm. For example, Bitar and Xu (2016) consider a charging station which

procures electricity from a utility firm at a constant marginal cost to charge electric vehicles. Ghosh and Aggarwal (2018) allow for both charging and discharging of an electric vehicle by using a discrete-time formulation and focusing on a single customer without considering optimal charging schedules for customers throughout a day. We complement these papers by studying smart charging of electric vehicles by a utility firm facing time-varying electricity cost. We propose an easy-to-implement policy for solving the complex optimal control problem of determining a charging schedule for each vehicle to minimize the overall charging cost. We also determine how a utility firm should design pairs of charging price and completion time to either minimize total cost or maximize its profit. Our numerical analysis demonstrates that smart charging leads to significant cost and emissions savings, providing both economic and environmental benefits.

3.   Model

We consider a setting where a utility firm owns and operates an electric vehicle charging station which has ample parking spots with chargers. When customers arrive at the charging station, the utility firm offers pairs of charging price and completion time.[1] Each customer selects the pair that maximizes their utility, which decreases in the charging price, completion time, and their sensitivity to delay. We consider the two main types of utility firms—public and investor-owned. A public utility firm minimizes the sum of charging cost and inconvenience cost of customers due to delay; an investor-owned utility firm (referred to as private utility firm hereafter for brevity) maximizes the revenue from customers minus the charging cost. Either type of utility firm must satisfy the total demand for electricity, including the electricity for charging vehicles. We denote the utility firm's planning horizon by T, which is typically 12 to 24 hours. We index customers that arrive during $[0, T]$ by $n = \{1, \ldots, N\}$, where N is the total number of customers. We provide the details of our model for customers and the utility firm below.

3.1   Customers

To focus on the key tradeoff between electric vehicle charging cost and inconvenience cost due to delay, we first consider a model in which all N customers arrive simultaneously at a charging station at time $t = 0$, i.e., no customers arrive during $t \in (0, T]$. We consider non-simultaneous arrivals in Section 6. For simplicity, we assume that each customer needs the same amount of energy, which is normalized to one unit without loss of generality.[2] We let the maximum charging speed be $\bar{a}$. Thus,

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[1] The completion time is not necessarily the customer's departure time but a deadline by which the firm needs to complete charging.

[2] Our results continue to hold if there are multiple customer segments with different energy needs. Details of this analysis are available on request.

the minimum charging time, denoted by $\underline{w}$, is given by $1/\bar{a}$. For example, if each customer needs 20 kWh and the maximum charging speed $\bar{a} = 6.67$ kW, the minimum charging time $\underline{w} = 3$ hours.

For a given charging price p and completion time $\tau \geq \underline{w}$, a customer's utility from the charging service is

$$u(\theta, p, \tau) = u_0 - p - \theta\delta(\tau), \tag{1}$$

where u_0 is a baseline utility (when charging is free and completed without delay), which does not affect optimization and is assumed to be the same for all customers, $\theta \geq 0$ is the customer's sensitivity to delay, and $\delta(\tau) \geq 0$ is the disutility due to delay. We assume $\delta(\tau)$ is continuous and strictly increasing in τ for $\tau \geq \underline{w}$, i.e., a later completion time results in a higher disutility.[3]

Customers are heterogeneous with respect to the delay-sensitivity parameter θ and can be divided into I classes, ordered from low to high delay-sensitivity: $\theta^{(1)} < \theta^{(2)} < \cdots < \theta^{(I)}$. A customer's delay-sensitivity is the customer's private information, which is unobservable to the utility firm. From the utility firm's perspective, customer n's delay-sensitivity θ_n takes value $\theta^{(i)}$ with probability $\beta^{(i)}$ for $i \in \{1, \ldots, I\}$, where $\sum_{i=1}^{I} \beta^{(i)} = 1$ and $\theta_1, \ldots, \theta_N$ are independently and identically distributed.

Upon customers' arrival, the utility firm offers I pairs of charging price and completion time, $\{(p^{(i)}, \tau^{(i)}) : i = 1, \ldots, I\}$, for each customer to choose from. Note that the utility firm offers exactly I pairs because, by the revelation principle (Myerson 1981), the firm only needs to consider a truth-telling direct mechanism, i.e., a customer in class i finds it optimal to truthfully reveal their delay-sensitivity $\theta^{(i)}$ by choosing the pair $(p^{(i)}, \tau^{(i)})$. Formally, the pairs announced by the utility firm satisfy incentive compatibility constraints:

$$u(\theta^{(i)}, p^{(i)}, \tau^{(i)}) \geq u(\theta^{(i)}, p^{(j)}, \tau^{(j)}), \quad \forall i = 1, \ldots, I, \, j = 1, \ldots, I. \tag{2}$$

Let (p_n, τ_n) denote customer n's optimal choice. Then, (2) implies that

$$(p_n, \tau_n) = (p^{(i)}, \tau^{(i)}) \text{ if } \theta_n = \theta^{(i)}, \quad \forall n = 1, \ldots, N. \tag{3}$$

Note that (2) requires the utility firm to design the pairs of charging price and completion time such that the less delay-sensitive customers are compensated for truthful revelation of their delay-sensitivity classes. This is because a customer's utility decreases in delay sensitivity. Moreover, this compensation is known as the information rent (see detailed discussions in Section 5).

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[3]Our results continue to hold under an alternative formulation for the inconvenience cost where customers face a lower inconvenience while waiting for charging to start than waiting for charging to complete. Details of this analysis are available on request.

The pairs of price and completion time must also satisfy the individual rationality constraints:

$$u(\theta^{(i)}, p^{(i)}, \tau^{(i)}) \geq \underline{u}, \quad \forall i = 1, \ldots, I, \tag{4}$$

where the reservation utility $\underline{u}$ represents the value of a customer's outside option of charging the vehicle at another charging station. For simplicity, we assume that the reservation utility $\underline{u}$ is the same for all customers. Nevertheless, all of our results continue to hold if the reservation utility increases in the customers' delay-sensitivity, i.e., $\underline{u}^{(1)} \leq \underline{u}^{(2)} \leq \cdots \leq \underline{u}^{(I)}$, which corresponds to a setting where customers who have more valuable outside options are also more sensitive to delays (see the proof of Proposition 2).

3.2 Utility Firm

In a given service region, there is typically only one utility firm, either public or private (EIA 2007). Public utility firms are owned by the government or by the customers in their service regions (as electric cooperatives), and their objective is to serve their constituents (i.e., customers) at the minimum cost (EIA 2007 and DOE 2015, p. 27). Private utility firms, on the other hand, are owned by investors, and their objective is to maximize profits, subject to regulations imposed by state and federal governments (DOE 2015, p. 27). These regulations restrict how private utility firms can set retail prices of electricity (Cawley and Kennard 2018, p. 2). However, given the novelty of electric vehicle charging service, private utility firms are not constrained in setting charging prices, which they perceive as a new business opportunity (Baker et al. 2019).

For either type of utility firm, we formulate a two-stage optimization problem. In the first stage, by designing pairs of charging price and completion time that satisfy incentive compatibility and individual rationality constraints in (2) and (4), a public utility firm minimizes the sum of the expected cost of inconvenience to customers (due to delay) and the expected cost of charging, whereas a private utility firm maximizes its expected profit, i.e., the revenue from customers minus the cost of charging. In the second stage, given the charging completion times chosen by customers, the utility firm decides the charging speed for each customer over time to minimize the total cost of charging N vehicles. We formulate the second-stage problem next.

Let $a_n(t) \in [0, \bar{a}]$ denote the charging speed for customer n's vehicle at time $t \in [0, T]$, where $\bar{a}$ is the maximum possible charging speed. The electricity demand due to charging all N electric vehicles at time t is

$$q(t) \doteq \sum_{n=1}^{N} a_n(t). \tag{5}$$

Let $d(t)$ denote the utility firm's electricity demand from other sources at time t, excluding the electricity used for electric vehicle charging. We refer to $d(t)$ as exogenous demand hereafter. Let $\widetilde{c}(q)$ be the cost (per unit of time) of procuring/producing electricity at rate q. Consistent with the practice (see Figure 2 in Section 7) and the literature (e.g., Wu and Kapuscinski 2013, Kök et al. 2018), we assume that $\widetilde{c}(q)$ is differentiable, convex increasing in q because more costly power plants are used to produce electricity as the demand increases. Then, the cost of meeting exogenous demand $d(t)$ is $\widetilde{c}(d(t))$, and the incremental cost of charging vehicles at time t is

$$c(q(t) \mid d(t)) \doteq \widetilde{c}\big(d(t) + q(t)\big) - \widetilde{c}\big(d(t)\big), \tag{6}$$

where $q(t)$ is defined in (5). Because $\widetilde{c}(\cdot)$ is convex, the firm can intuitively minimize the total vehicle charging cost by smoothing the total demand $d(t) + q(t)$ over time $t \in [0, T]$. Formally, the utility firm minimizes the cost of charging by choosing the charging schedule $\{a_n(t) : n = 1, \ldots, N\}$ for given charging completion times $\{\tau_1, \ldots, \tau_N\}$ (selected by the N customers):

$$C(\tau_1, \ldots, \tau_N) \doteq \min_{\{a_1(t),\ldots,a_N(t)\}} \int_0^T c\big(q(t) \mid d(t)\big)\mathrm{d}t \tag{7}$$

$$\text{s.t.} \quad \int_0^{\tau_n} a_n(t)\,\mathrm{d}t = 1, \quad \forall n = 1, \ldots, N, \tag{8}$$

$$q(t) = \sum_{n=1}^N a_n(t), \quad \forall t \in [0, T], \tag{9}$$

$$0 \le a_n(t) \le \overline{a}, \quad \forall t \in [0, \tau_n], \forall n = 1, \ldots, N, \tag{10}$$

$$a_n(t) = 0, \quad \forall t \in (\tau_n, T], \forall n = 1, \ldots, N, \tag{11}$$

where $C(\tau_1, \ldots, \tau_N)$ is the minimum charging cost, (8) ensures that one unit of energy is charged to each vehicle by the customer's chosen completion time, (9) defines the electricity demand due to charging electric vehicles, (10) and (11) ensure that each vehicle is charged within the charging speed limit and before the chosen completion time. Using the minimum charging cost obtained from the second-stage problem (7)-(11), we next formulate the utility firm's first-stage problem.

As discussed in Section 3.1, a utility firm cannot observe customer n's delay-sensitivity θ_n, but the firm knows the distribution of θ across customers: θ_n takes value $\theta^{(i)}$ with probability $\beta^{(i)}$. Accordingly, the public utility firm's problem of minimizing the sum of the expected inconvenience and charging costs can be written as

$$\min_{\left\{\left(p^{(i)}, \tau^{(i)} \in [\underline{w}, T]\right): i = 1, \ldots, I\right\}} \mathbb{E}_{\{\theta_1, \ldots, \theta_N\}} \left[\sum_{n=1}^N \theta_n \delta(\tau_n) + C(\tau_1, \ldots, \tau_N)\right], \tag{12}$$

$$\text{s.t.} \quad (2), (3), \text{ and } (4),$$

8

where $\mathbb{E}_{\{\theta_1,\ldots,\theta_N\}}$ denotes the expectation over all N customers' delay-sensitivities, (2) and (4) are incentive compatibility and individual rationality constraints that the pairs $\{(p^{(i)}, \tau^{(i)}) : i = 1, \ldots, I\}$ must satisfy, respectively, and (3) links customer n's choice to the offered pairs. Note that charging price $p^{(i)}$'s do not appear in the public utility firm's objective in (12) because they are transfer payments between the utility firm and customers. These prices appear in (2) and (4) and they are set to incentivize customers to choose the completion times that minimize the total cost.

In contrast to the public utility firm, the private firm maximizes its expected profit as follows:

$$\max_{\left\{\left(p^{(i)}, \tau^{(i)} \in [\underline{w}, T]\right) : i = 1, \ldots, I\right\}} \mathbb{E}_{\{\theta_1, \ldots, \theta_N\}} \left[\sum_{n=1}^{N} p_n - C(\tau_1, \ldots, \tau_N) \right], \tag{13}$$

$$\text{s.t.} \quad (2), (3), \text{ and } (4).$$

Note that both the problem in (12) and that in (13) involve the minimum charging cost function $C(\tau_1, \ldots, \tau_N)$, which we analyze next.

4. Optimal Charging Schedule Given Completion Times

This section focuses on the second-stage problem, given in (7)-(11), i.e., finding the optimal charging schedule for given completion times. This is a continuous-time optimal control problem, and it can be expressed in the standard form (see the proof of Proposition 1) with state variables $x_n(t) = \int_0^t a_n(s) \, \mathrm{d}s$, representing the amount of electricity charged to vehicle n by time t. Note that constraint (8) translates into N terminal conditions (i.e., $x_n(\tau_n) = 1$, for $n = 1, \ldots, N$ at different τ_n). Furthermore, there are N control variables and N state variables, where N can be large, requiring an efficient method to solve the optimal control problem in order to evaluate and optimize the objectives in (12) and (13) efficiently.

We devise such a method by utilizing the charging problem's special structure (i.e., the convexity of the electricity procurement cost function) and constructing a policy that yields *an* optimal solution to (7)-(11). In this policy, we iteratively determine the charging schedule for one vehicle at a time (in the order of the given completion times), while keeping the resulting total electricity demand as smooth as possible. When the iterations terminate, the total electricity demand (including the demand from charging N vehicles) yields the lowest cost for the utility firm. We present this policy below and show its optimality in Proposition 1, where $L(t)$ denotes the running total electricity demand that is updated as the charging schedules are sequentially determined.

Juice-filling policy

Step 0. Sort the given completion times. Without loss of generality, assume $\tau_N \leq \tau_{N-1} \leq \cdots \leq \tau_1$.

Step 1. Initialize $n = N$ and $L(t) = d(t)$ for $t \in [0, T]$.

Step 2. Find z_n such that

$$\int_0^{\tau_n} \min \left\{ \left(z_n - L(t) \right)^+, \bar{a} \right\} \mathrm{d}t = 1. \tag{14}$$

Step 3. Set $a_n(t) = \min \left\{ \left(z_n - L(t) \right)^+, \bar{a} \right\}$ for $t \in [0, \tau_n]$ and set $a_n(t) = 0$ for $t \in (\tau_n, T]$.

Step 4. Update $L(t)$ to be $L(t) + a_n(t)$ for $t \in [0, T]$. If $n = 1$, then stop. Otherwise, decrease n by 1 and go to step 2.

We refer to the above policy as "juice-filling policy" because it resembles the procedure of filling N containers with juice, where each container represents the battery of an electric vehicle, the juice represents the electricity that is charged to the vehicle's battery, and z_n is the "juice surface level" (or fill-up-to level) after filling a container. Figure 1 shows the first two iterations of the policy for a special case in which the exogenous demand $d(t)$ is a constant D. (The policy is optimal even if $d(t)$ is nonstationary; see Proposition 1.) For illustration purposes, we consider $N = 5$ electric vehicles, with given completion times $\tau_5 \leq \tau_4 \leq \cdots \leq \tau_1$.

Figure 1(a) illustrates the first iteration: We initialize $L(t) = D$ and construct the first container with its base on the demand level $L(t) = D$ from $t = 0$ to $t = \tau_5$, as highlighted in bold lines. Its height is $\bar{a}$, the maximum charging speed. We pour one unit of juice into this container, filling it up to z_5, which satisfies (14). The charging schedule is such that $a_5(t) = z_5 - D$ for $t \in [0, \tau_5]$ and $a_5(t) = 0$ for $t > \tau_5$. Before proceeding to the next electric vehicle, we update $L(t)$ to $L(t) + a_5(t)$.

Figure 1(b) illustrates the second iteration that charges another vehicle by time τ_4: We construct a new container with its base on the updated $L(t)$, which is given by $L(t) = z_5$ for $t \in [0, \tau_5]$ and $L(t) = D$ for $t \in (\tau_5, \tau_4]$, and its top is $\bar{a}$ units above its base. Pouring one unit of juice brings the juice surface level to z_4, which is determined by (14). The charging schedule is $a_4(t) = z_4 - L(t)$ for $t \in [0, \tau_5]$ and $a_4(t) = \bar{a}$ for $t \in (\tau_5, \tau_4]$. We update $L(t)$ to $L(t) + a_4(t)$ and proceed to the next vehicle. After five iterations, the resulting charging schedule for the five vehicles is shown in Figure 1(c).

The next proposition formally states the optimality of the above policy. All proofs are in Online Appendix Section C.

Proposition 1 *Given exogenous demand $d(t)$ and charging completion times $\{\tau_1, \ldots, \tau_N\}$, the charging speed control, $\{a_1(t), \ldots, a_N(t) : t \in [0, T]\}$, found by the juice-filling policy is an optimal solution to the cost minimization problem in (7)-(11).*

Proposition 1 shows that the juice-filling policy is optimal even if the exogenous demand, $d(t)$,

10

Figure 1: Juice-filling policy for charging five vehicles under a constant exogenous demand

(a) First iteration ($n = 5$)

(b) Second iteration ($n = 4$)

(c) Charging schedule generated from the juice-filling policy
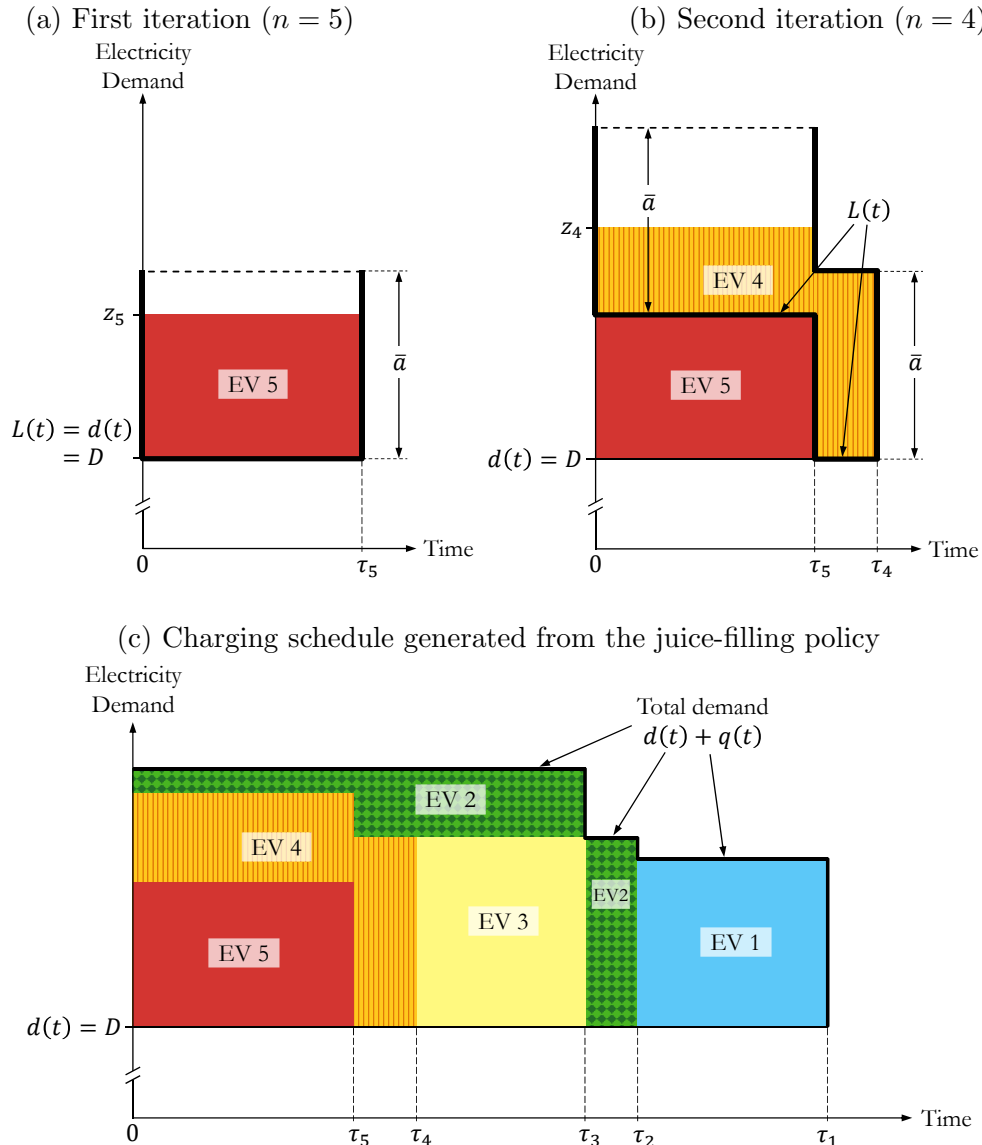
is nonstationary. Intuitively, this policy smooths the running total demand in every iteration in a greedy fashion. This leads to a total demand that is as smooth as possible over time, as illustrated in Figure 1(c). Note that minimizing the charging cost in (7) is equivalent to minimizing the utility firm's cost of serving total demand $\int_0^T \widetilde{c}(d(t) + q(t)) \mathrm{d}t$ minus the cost of meeting only the exogenous demand $\int_0^T \widetilde{c}(d(t)) \mathrm{d}t$, where the latter is a constant that does not affect the optimization. Because $\widetilde{c}(\cdot)$ is a convex function, the cost of meeting the total demand is minimized by smoothing the total demand $d(t) + q(t)$ as much as possible, which is accomplished by the juice-filling policy.

The juice-filling policy identifies only one optimal solution to the charging problem in (7)-(11). Typically, there exist infinitely many optimal charging schedules, e.g., the charging schedule in

11

Figure 1(c) can be modified to achieve the same total demand over time without violating any constraint. Compared to other optimal schedules, the juice-filling policy greatly simplifies the computation due to its sequential nature of determining the charging schedule. Juice-filling is an intuitive and easy-to-implement policy for finding the solution of a complicated problem. This makes the policy particularly suitable for practical use. Note that the optimality of the juice-filling policy for the charging-cost-minimization problem relies on the assumption that all customers arrive simultaneously. This policy can be generalized for non-simultaneous arrivals (see Section 6).

We close this section by investigating structural properties of the minimum charging cost $C(\tau_1, \ldots, \tau_N)$ as a function of the completion times $(\tau_1, \ldots, \tau_N)$.

Lemma 1 *(i) If $\{b_1, \ldots, b_N\}$ is a permutation of $\{1, \ldots, N\}$, then $C(\tau_{b_1}, \ldots, \tau_{b_N}) = C(\tau_1, \ldots, \tau_N)$. (ii) $C(\tau_1, \ldots, \tau_N)$ is continuous and decreasing in $(\tau_1, \ldots, \tau_N)$.*

Lemma 1(i) reveals the symmetry of the optimal charging cost function. This symmetry relies on the assumption that all electric vehicles have the same demand for electricity. Part (ii) of the lemma suggests that earlier completion times limit the utility firm's ability to smooth the total demand, and thus increase the cost of charging electric vehicles. Since earlier completion times clearly reduce customers' inconvenience cost, this result manifests the tradeoff between the charging cost and the inconvenience cost. We investigate this tradeoff in the next section to characterize the optimal charging prices and completion times. Finally, we remark that the charging cost function $C(\tau_1, \tau_2, \ldots, \tau_N)$ is generally not convex in τ_n for any n.

5. Optimal Charging Prices and Completion Times

Given the optimal charging schedule determined by the juice-filling policy, we next consider the utility firm's first-stage problem of designing pairs of charging price and completion time. We consider the public and private utility firms separately and, in each case, we characterize the optimal completion times and charging prices and compare them with a benchmark setting without information asymmetry.

5.1 Public Utility Firm

Let us first consider a benchmark setting where a public utility firm can observe customers' delay-sensitivity upon their arrival. Accordingly, the utility firm is not subject to the incentive compatibility constraints (2) because for customers in class i the firm offers only the pair $\left(p^{(i)}, \tau^{(i)}\right)$ intended for their class. Recall that the public utility firm minimizes the total cost of charging vehicles and customers' inconvenience. Thus, without information asymmetry, the public utility

firm's problem can be written as

$$\min_{\left\{\left(p^{(i)}, \tau^{(i)} \in [\underline{w}, T]\right):\, i=1,\dots,I\right\}} \mathbb{E}_{\{\theta_1,\dots,\theta_N\}} \left[\sum_{n=1}^{N} \theta_n \delta(\tau_n) + C(\tau_1, \dots, \tau_N) \right], \tag{15}$$

$$\text{s.t.} \quad (3) \text{ and } (4),$$

where constraint (3) reflects that the customer with observable class i is offered the i-th pair, which the customer is willing to accept due to the individual rationality constraint (4). Moreover, the expectation in (15) implies that the utility firm designs the menu (i.e., pairs of charging price and completion time) before customers arrive and cannot redesign it based on realizations of $\{\theta_1, \dots, \theta_N\}$. Accordingly, the only difference between the problem in (15) and the problem under information asymmetry given in (12) is the absence of the incentive compatibility constraints (2). This enables us to compare this benchmark against the main model by isolating the effect of information asymmetry.

The identical objective functions in (12) and (15) do not immediately imply that the two problems have identical solutions because, in general, the first-best solution (i.e., the solution to a benchmark setting without information asymmetry) may not be implementable (i.e., incentive-compatible and individually rational) in a setting with information asymmetry. In our context, for the optimal completion times of (15) to be implementable for (12), there must also exist charging prices that, together with the completion times, satisfy the incentive compatibility constraints in (2).

In the analysis below, Lemma 2 shows that the first-best completion times from (15) are decreasing in customers' delay-sensitivity, and Proposition 2 finds the charging prices that can implement these completion times. Consequently, the public utility firm's decision under information asymmetry still achieves the first-best.

Lemma 2 *An optimal solution to* (15) *exists, and the optimal charging completion times satisfy* $\tau^{(1)*} \geq \tau^{(2)*} \geq \cdots \geq \tau^{(I)*}$.

Solving the problem in (15) is challenging because the charging cost $C(\tau_1, \dots, \tau_N)$ characterized in Section 4 is not convex and cannot be expressed in closed form. Nevertheless, Lemma 2 shows that to characterize the optimal pairs of charging price and completion times, it is sufficient to only consider charging completion times that are decreasing in the delay-sensitivity of customers (for a similar result, see Proposition 2 in Lutze and Özer 2008). We next construct the optimal pricing scheme $\{p^{(1)}, \dots, p^{(I)}\}$ for implementing given completion times $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$ (we consider any decreasing sequence of completion times to facilitate comparison with Proposition 3).

Proposition 2 *(i) Given a set of completion times satisfying $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$, for a public utility firm, the following procedure constructs a pricing scheme that satisfies the incentive compatibility and individual rationality constraints in (2) and (4):*

Set $p^{(I)} \leq u_0 - \theta^{(I)}\delta(\tau^{(I)}) - \underline{u}$ and for $i = I - 1, I - 2, \ldots, 1$, choose $p^{(i)}$ such that

$$p^{(i)} \in \left[p^{(i+1)} - \theta^{(i+1)}(\delta(\tau^{(i)}) - \delta(\tau^{(i+1)})), \ p^{(i+1)} - \theta^{(i)}(\delta(\tau^{(i)}) - \delta(\tau^{(i+1)})) \right]. \tag{16}$$

(ii) The problems in (12) and (15) have the same optimal completion times and the same minimum objective values.

Proposition 2(i) reveals an important managerial insight by characterizing how a public utility firm should set prices for charging electric vehicles in accordance with the cost of inconvenience. This inconvenience-cost based pricing policy incentivizes customers to agree to smart charging (i.e., delaying charging when the electricity demand is high) in contrast to the current practice of charging vehicles as soon as possible (Myers 2017). In particular, when a customer in delay-sensitivity class i agrees to completion time τ_i that is later than τ_{i+1} for the next more delay-sensitive class, the customer's price discount should be based on the increased inconvenience cost $\Delta^{(i)} \doteq \delta(\tau^{(i)}) - \delta(\tau^{(i+1)})$:

$$p^{(i+1)} - p^{(i)} \in \left[\theta^{(i)}\Delta^{(i)}, \theta^{(i+1)}\Delta^{(i)} \right]. \tag{17}$$

The above insight continues to hold for the private utility firm, which we analyze in the next section.

Proposition 2(ii) implies that information asymmetry does not distort the public utility firm's choice of completion times. That is, without observing customers' delay-sensitivity but knowing the distribution of delay-sensitivity, the public utility firm can still find the first-best completion times from (15), and construct the corresponding charging prices (based on Proposition 2(i)) to incentivize customers to choose the first-best completion times. These completion times can be determined by solving the problem in (15) subject to $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$.

5.2 Private Utility Firm

Let us revisit the benchmark setting without information asymmetry discussed at the beginning of Section 5.1, but for a private utility firm. Similar to the discussion in Section 5.1, the firm's problem is the same as in the main model (13), but the firm is not subject to incentive compatibility

constraints (2):

$$\max_{\left\{\left(p^{(i)},\, \tau^{(i)} \in [\underline{w}, T]\right):\, i=1,\dots,I\right\}} \mathbb{E}_{\{\theta_1,\dots,\theta_N\}}\left[\sum_{n=1}^{N} p_n - C(\tau_1,\dots,\tau_N)\right], \tag{18}$$

$$\text{s.t.} \quad (3) \text{ and } (4).$$

The firm can maximize its profit in (18) by setting the prices such that the individual rationality constraints (4) are binding, i.e., $p^{(i)} = u_0 - \underline{u} - \theta^{(i)}\delta(\tau^{(i)})$, $\forall i = 1, \cdots, I$ (for a similar result, see Proposition 2 in Lutze and Özer 2008). Substituting $p_n = u_0 - \underline{u} - \theta_n \delta(\tau_n)$ into (18) shows that maximizing the firm's profit is equivalent to minimizing the sum of the customers' inconvenience cost and the charging cost. Therefore, the benchmark problem in (18) for a private utility firm also yields the first-best charging completion times of the benchmark problem in (15) for a public firm. The difference is that the prices in (15) can be any prices that satisfy the individual rationality constraints, whereas the prices in (18) are such that each customer only obtains the reservation utility.

Under information asymmetry, unlike a public utility firm, the private utility firm who solves the problem in (13) has an incentive to depart from the first-best completion times due to the information rent. In the analysis below, we first derive the pricing scheme for given feasible completion times (Proposition 3), and then we explicitly derive the information rent (Proposition 4).

Proposition 3 *(i) An optimal solution to (13) exists and satisfies $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$.*
(ii) Given a set of completion times satisfying $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$, for a private utility firm, the following procedure constructs a pricing scheme that maximizes its profit and satisfies the incentive compatibility and individual rationality constraints in (2) and (4):
Set $p^{(I)} = u_0 - \theta^{(I)}\delta(\tau^{(I)}) - \underline{u}$ and for $i = I-1, I-2, \dots, 1$, set $p^{(i)}$ such that

$$p^{(i)} = p^{(i+1)} - \theta^{(i)}(\delta(\tau^{(i)}) - \delta(\tau^{(i+1)})). \tag{19}$$

Proposition 3(i) shows that, although the optimal completion times for a private utility firm deviate from the first-best completion times of the benchmark setting, they still decrease in customers' delay-sensitivity. This is because, under information asymmetry, the completion times need to be lower for more delay-sensitive customers to ensure incentive compatibility (for a similar result, see Proposition 4 in Lutze and Özer 2008).

Compared to the prices of the public utility firm in Proposition 2, Proposition 3(ii) suggests that the private firm should set the prices at their upper bounds so as to maximize its profit. Moreover, Proposition 3(ii) reveals that the firm should set prices such that each customer would

15

be indifferent between choosing the pair designed for their class and the pair for the next class with a higher delay-sensitivity. To see this, rewrite (19) as

$$\theta^{(i)}\delta(\tau^{(i)}) + p^{(i)} = \theta^{(i)}\delta(\tau^{(i+1)}) + p^{(i+1)}, \tag{20}$$

which implies that a customer with delay-sensitivity $\theta^{(i)}$ is indifferent between choosing $\left(p^{(i)}, \tau^{(i)}\right)$ and $\left(p^{(i+1)}, \tau^{(i+1)}\right)$; the latter is designed for those with delay-sensitivity $\theta^{(i+1)}$.

As discussed in Section 5.1, a public utility firm considers information rent as an internal transfer payment. In contrast, a private utility firm considers information rent as an additional cost. We next explicitly derive the information rent by rewriting the private utility firm's objective function in (13) using the relationship between prices and completion times in Proposition 3.

Proposition 4 *For a private utility firm, the optimal completion times are determined by*

$$\min_{\left\{\tau^{(i)}\in[\underline{w},T],\, i=1,\ldots,I\right\}} \mathbb{E}_{\{\theta_1,\ldots,\theta_N\}}\left[\sum_{n=1}^{N}\theta_n\delta(\tau_n) + C(\tau_1,\ldots,\tau_N)\right] + N\sum_{i=1}^{I-1}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta(\tau^{(j+1)}), \tag{21}$$

where the expected value term is the same as the public utility firm's objective function in (12) and the last term is the expected value of the information rent incurred by the private utility firm.

Proposition 4 implicitly characterizes the optimal completion times for a private utility firm, i.e., these completion times can be determined by solving the problem in (21) subject to $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$ (for a similar result, see Theorem 1 in Lovejoy 2006). Moreover, Proposition 4 shows that the private utility firm accounts for customers' inconvenience cost and the total charging cost as a public firm does, but the private firm differs from the public firm in including the information rent as a cost component. Therefore, the information rent can distort the private firm's decision on charging completion times. That is, compared to the public firm, the private firm may choose earlier completion times at higher charging prices, or later completion times at lower prices. The direction of the distortion depends on the cost function and customers' sensitivity to delay. The industry regulator or policymakers would be concerned about the magnitude of the distortion, which we shall estimate in Section 7.

Finally, we discuss how optimal completion times offered by the private utility firm are distorted by the information rent in (21) by examining a special case where delay-sensitivity follows a discrete uniform distribution with $\beta^{(i)} = \frac{1}{I}$ for $i = 1, \ldots, I$. In this case, the information rent term simplifies to $\frac{N}{I}\sum_{i=2}^{I}(i-1)\left(\theta^{(i)} - \theta^{(i-1)}\right)\delta(\tau^{(i)})$ (see the proof of Proposition 4). To understand the intuition behind this term, consider a customer in class i and let the customer's disutility of

16

inconvenience $\delta(\tau^{(i)})$ increase by one unit and let the price $p^{(i)}$ decrease by $\theta^{(i)}$ units so that the customer's utility remains the same. Under these changes, if a customer in class $i-1$ chooses the pair $(p^{(i)}, \tau^{(i)})$, then based on (20), the customer's inconvenience cost increases by $\theta^{(i-1)}$ while enjoying a price decrease of $\theta^{(i)}$, resulting in a net gain of $\theta^{(i)} - \theta^{(i-1)}$. To maintain incentive compatibility, i.e., to incentivize customers in class $i-1$ to choose the pair $(p^{(i-1)}, \tau^{(i-1)})$, $p^{(i-1)}$ must decrease by $\theta^{(i)} - \theta^{(i-1)}$. This change, in turn, results in decreases in each of the prices $\{p^{(j)}, j = 1, 2 \ldots, i-1\}$ by $\theta^{(i)} - \theta^{(i-1)}$. That is, one unit increase in $\delta(\tau^{(i)})$ should be traded off against $(i-1)(\theta^{(i)} - \theta^{(i-1)})$ units of revenue loss, which is exactly the expression for the information rent.

6. Non-simultaneous Arrivals of Customers

In this section we extend our model to a more general setting, where customers arrive at the charging station at different times. We show that the results in Section 5 on optimal pairs of charging price and completion time continue to hold, but finding the optimal charging schedule is more complicated under non-simultaneous arrivals. We find that the optimal charging policy has a threshold structure. We finally generalize the juice-filling policy for non-simultaneous arrivals.

6.1 Utility Firm's Problem when Customers Choose Completion Times Upfront

We first consider a setting where customers' arrival times are exogenous and at time $t = 0$ they choose charging prices and completion times from the menu offered by the utility firm. Specifically, prior to $t = 0$, the utility firm designs a menu that is time-varying but piecewise constant over time. At $t = 0$, each customer informs the firm of their arrival time and is presented with the menu that applies to that arrival time. Then, the customer chooses the pair that maximizes their utility. Given the arrival and completion times, the utility firm then determines the charging schedule. This setting corresponds to a reservation system, where reservations for charging service are made at time 0. This is an immediate extension of the main model. In Online Appendix Section A, we present a more complex setting where customers choose their completion times upon arrival.

We first reformulate the utility firm's second-stage problem of minimizing the charging cost for given arrival and completion times. Let s_n be the exogenous arrival time of customer n. We redefine τ_n as the service duration for customer n. Then $s_n + \tau_n$ is the charging completion time. Given the service duration $\{\tau_1, \ldots, \tau_N\}$ selected by customers, the utility firm's second-stage

problem is

$$C(s_1, \ldots, s_N, \tau_1, \ldots, \tau_N) \doteq \min_{\{a_1(t), \ldots, a_N(t)\}} \int_0^T c\big(q(t) \mid d(t)\big) \mathrm{d}t \tag{22}$$

$$\text{s.t.} \quad \int_{s_n}^{s_n + \tau_n} a_n(t)\, \mathrm{d}t = 1, \quad \forall n = 1, \ldots, N, \tag{23}$$

$$q(t) = \sum_{n=1}^N a_n(t), \ \forall t \in [0, T], \tag{24}$$

$$0 \le a_n(t) \le \overline{a}, \quad \forall t \in [s_n, s_n + \tau_n], \forall n = 1, \ldots, N, \tag{25}$$

$$a_n(t) = 0, \qquad \forall t \notin [s_n, s_n + \tau_n], \forall n = 1, \ldots, N. \tag{26}$$

If $s_n = 0$ for $n = 1, \ldots, N$, the above formulation is equivalent to the second-stage problem of the main model, given in (7)-(11).

We next formulate the utility firm's first-stage problem of designing a time-varying menu. The firm divides the time interval $[0, T]$ into L periods: $[t_1, t_2), [t_2, t_3), \ldots, [t_L, t_{L+1})$, where $t_1 = 0$ and $t_{L+1} = T$. During $[t_\ell, t_{\ell+1})$ or period ℓ, the firm uses a period-specific menu $\big\{\big(p_\ell^{(i)}, \tau_\ell^{(i)}\big) : i = 1, \ldots, I\big\}$ for the customers arriving in this period. These L menus are designed by the utility firm at time $t = 0$. A public utility firm designs $\big\{\big(p_\ell^{(i)}, \tau_\ell^{(i)}\big) : i = 1, \ldots, I, \ell = 1, \ldots, L\big\}$ to minimize the total expected cost $\mathbb{E}_{\{\theta_1, \ldots, \theta_N\}} \Big[\sum_{n=1}^N \theta_n \delta(\tau_n) + C(s_1, \ldots, s_N, \tau_1, \ldots, \tau_N)\Big]$, where $(p_n, \tau_n) = (p_\ell^{(i)}, \tau_\ell^{(i)})$ if $\theta_n = \theta^{(i)}$ and $s_n \in [t_\ell, t_{\ell+1})$ because the menu $\big\{\big(p_\ell^{(i)}, \tau_\ell^{(i)}\big) : i = 1, \ldots, I\big\}$ for each period ℓ must satisfy the incentive compatibility and individual rationality constraints in (2) and (4), respectively. That is, the firm is subject to $L \times I$ incentive compatibility and individual rationality constraints. On the other hand, a private utility firm maximizes its profit $\mathbb{E}_{\{\theta_1, \ldots, \theta_N\}} \Big[\sum_{n=1}^N p_n - C(s_1, \ldots, s_N, \tau_1, \ldots, \tau_N)\Big]$ subject to the same constraints.

Because the analysis in Section 5 does not rely on the expected charging cost function, it can be readily verified that the results obtained for simultaneous arrivals in Section 5 continue to apply for each period $\ell \in \{1, \cdots, L\}$. That is, for given service durations, the public and private utility firms should set charging prices as in Propositions 2 and 3, respectively, for each period. However, the second-stage problem is more involved as we analyze below.

Note that we assume that arrival times are exogenous, i.e., customers do not strategically determine their arrival times based on their expectation of how the menu may change over time. This ensures tractability and isolates the tradeoff between charging cost and inconvenience cost due to delayed completion.

6.2 Charging Schedule Under Given Arrival and Completion Times

We next solve the second-stage cost-minimization problem in (22)-(26), which is common for public and private utility firms. Note that for this second-stage problem, both arrival times $\{s_1, \ldots, s_N\}$ and completion times $\{s_1 + \tau_1, \ldots, s_N + \tau_N\}$ are given. For ease of exposition within this section, we assume $s_1 = 0$ and no two arrival or completion times are equal. To facilitate the analysis, we sort the arrival and completion times in ascending order and label them as $\{t_1, \ldots, t_{2N}\}$ such that $0 = s_1 = t_1 < t_2 < \cdots < t_{2N} < T$. Each time interval $[t_j, t_{j+1}]$ begins and ends with either an arrival or a charging completion. Let $\mathcal{I}(t)$ be the set of customers that are present at the charging station at time t, i.e., $\mathcal{I}(t) \doteq \{k : t \in [s_k, \tau_k]\}$.

Unlike the simultaneous-arrival case in Section 4, where there exists an optimal policy that can be implemented by a juice-filling procedure with only one juice surface level z_n for each vehicle n, in the case of non-simultaneous arrivals, we shall show that there exists an optimal policy that can be characterized by one juice surface level $z_{n,j}$ for each time interval $(t_j, t_{j+1}]$ for every vehicle n. These juice surface levels for the same vehicle may differ in different time intervals and they can be determined via the optimization problem given in (A.15)-(A.17) in the Online Appendix. Because these thresholds are scalars, optimizing them is simpler than the original optimal control problem (22)-(26). For given $z_{n,j}$'s, the charging policy is as follows.

Charging policy for non-simultaneous arrivals (given thresholds $z_{n,j}$'s)

Step 1. Initialize $j = 1$ and $L(t) = d(t)$ for $t \in [0, T]$.

Step 2. Set $n = \arg\min_k \{\tau_k : k \in \mathcal{I}(t_j)\}$, which is the vehicle with the earliest completion time in the feasible set $\mathcal{I}(t_j)$.

Step 3. Set $a_n(t) = \min\left\{ \left(z_{n,j} - L(t)\right)^+, \bar{a} \right\}$ for $t \in [t_j, t_{j+1}]$.

Step 4. Update $L(t)$ to be $L(t) + a_n(t)$ for $t \in [t_j, t_{j+1}]$. If $n = \arg\max_k \{\tau_k : k \in \mathcal{I}(t_j)\}$ and $j = 2N - 1$, then stop; if $n = \arg\max_k \{\tau_k : k \in \mathcal{I}(t_j)\}$ but $j < 2N - 1$, then increase j by 1 and go to step 2; otherwise, increase n by 1 and go to step 3.

The above charging policy is similar to the juice-filling policy within each time interval because by construction, neither an arrival nor a charging completion occurs during (t_j, t_{j+1}). Accordingly, the charging speed in Step 3 above is similar to that in Step 3 of the juice-filling policy. We next prove that there exits $z_{n,j}$'s such that this policy is optimal.

Proposition 5 *Given exogenous demand $d(t)$, there exists $\{z_{n,j} : n = 1, \ldots, N, j = 1, \ldots, 2N - 1\}$ such that the control policy $\{a_1(t), \ldots, a_N(t)\}$ found by the charging policy for non-simultaneous*

arrivals is an optimal solution to the cost minimization problem in (22)-(26).

Although the optimal charging policy can be characterized as described above, thresholds $z_{n,j}$'s remain to be optimized by solving (A.15)-(A.17) in the Online Appendix. As the number of electric vehicles increases, such an optimization problem becomes challenging to solve, making it practically intractable to solve the first-stage problem. We next design an intuitive and computationally efficient charging policy for the utility firm by generalizing the juice-filling policy introduced in Section 4 to accommodate non-simultaneous arrivals. We use $\{a_1^G(t), \ldots, a_N^G(t)\}$ to denote the charging schedule under this policy.

Generalized juice-filling policy

Step 0. Sort the given completion times. Without loss of generality, assume $s_1 + \tau_1 \leq \cdots \leq s_N + \tau_N$.

Step 1. Initialize $n = 1$ and $L(t) = d(t)$ for $t \in [0, T]$.

Step 2. Find z_n such that

$$\int_{s_n}^{s_n + \tau_n} \min\left\{\left(z_n - L(t)\right)^+, \bar{a}\right\} \mathrm{d}t = 1. \tag{27}$$

Step 3. Set $a_n^G(t) = \min\left\{\left(z_n - L(t)\right)^+, \bar{a}\right\}$ for $t \in [s_n, s_n + \tau_n]$ and set $a_n^G(t) = 0$ for $t \notin [s_n, s_n + \tau_n]$.

Step 4. Update $L(t)$ to be $L(t) + a_n^G(t)$ for $t \in [0, T]$. If $n = N$, then stop. Otherwise, increase n by 1 and go to step 2.

The above policy is equivalent to the juice-filling policy if $s_n = 0$ for each customer n. As with the juice-filling policy, for each vehicle n, there is one juice surface level z_n, which can be computed sequentially in closed-form (see Step 2). Therefore, the generalized juice-filling policy is much more efficient to compute than the optimal policy parameterized by threshold $z_{n,j}$'s, which require solving an optimization problem.

The generalized juice filling policy is likely to perform well when the number of vehicles is large. This is because, similar to the juice-filling policy, this generalized policy smooths the demand in a greedy fashion in each iteration. Intuitively, as the number of vehicles increases, the policy involves more iterations, thus further smoothing the demand. Therefore, we use this generalized policy in the next section, where we show that it enables a utility firm to achieve significant cost savings through smart charging, compared to the current practice of charging vehicles as soon as possible.

7. Numerical Analysis

In this section, we illustrate our findings by using real electricity demand and supply data from the PJM Interconnection, the largest wholesale electricity market in the U.S., which serves 65

million customers across thirteen states (PJM 2018). Our main goal is to quantify the economic and environmental benefits of smart charging under a public or a private utility firm.

7.1 Parameter Estimation

Electricity Procurement Cost and Emissions. We first estimate the cost of procuring electricity, i.e., $\widetilde{c}(\cdot)$ in (6), by constructing the daily supply curve for PJM Interconnection (see Figure 2, also see Online Appendix Section B for details). To do so, we calculate the generation cost for each of the 448 conventional power plants in the PJM by multiplying the heat rate of a plant with the cost of the fuel source used by the plant. We then estimate the cumulative capacity in PJM by calculating each plant's effective capacity (accounting for planned outages) and sorting the plants in the increasing order of their generation costs. Finally, to account for unplanned outages, we calibrate the supply curve so that the observed electricity price is consistent with the estimated electricity price from the supply curve and the observed electricity demand. Figure 2 illustrates the calibrated supply curve for August 25, 2016. We follow these steps to also estimate carbon emissions of electricity generation.

Figure 2: Supply curve for August 25, 2016
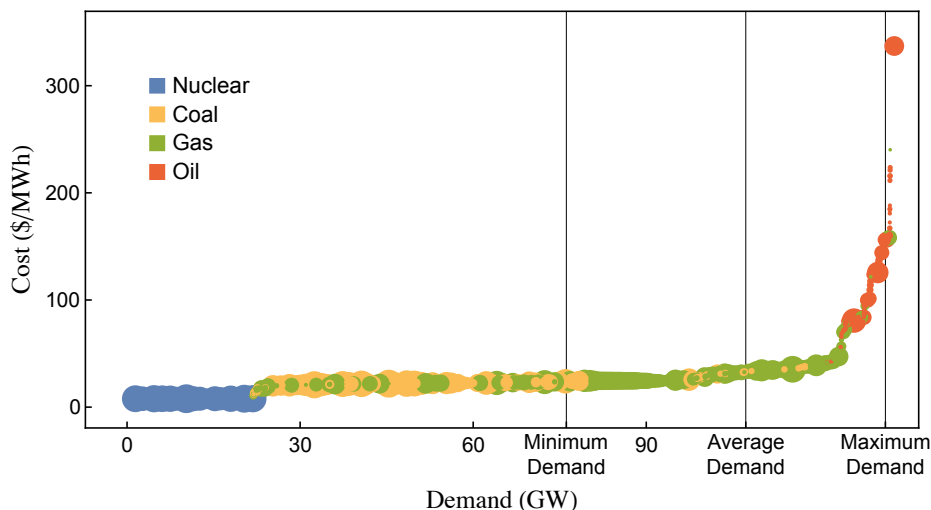
Notes. This figure is based on data for PJM Interconnection. Each circle represents a power plant with its generation cost on the vertical axis and the cumulative capacity up to that plant on the horizontal axis. The size of a circle is proportional to the plant's capacity.

Utility Function. To estimate customers' utility function, we first focus on the waiting cost. Incentivizing customers to wait longer to reduce the cost of charging electric vehicles is a novel business idea and, therefore, no prior study has estimated the waiting cost in such a context. It is reasonable to assume that the waiting cost varies significantly across customers. For example,

21

Akşin et al. (2013) estimate that waiting cost in a call center varies from a negligible level to approximately \$1/minute and they consider three classes of customers. We also consider a wide range of waiting cost, reflected by the heterogeneity in customers' sensitivity to delays, and we assume that there are five classes of delay-sensitivity: $\theta^{(1)} = 0.1$, $\theta^{(2)} = 2$, $\theta^{(3)} = 4$, $\theta^{(4)} = 6$, and $\theta^{(5)} = 8$ with equal probability. These values also represent the inconvenience cost (in dollars) for the first hour of delay.[4]

We assume that each customer needs to charge 20 kWh for their electric vehicle and and the maximum charging speed is 6.67 kW. Thus, the minimum time required for charging is $\underline{w} = 3$ hours. If the charging completion time τ is longer than 3 hours, an inconvenience cost is incurred. We assume the utility function (1) is $u(\theta, \tau, p) = 50 - \theta(\tau - 3)^2 - p$, for $\tau \geq 3$. Here, any charging service longer than the 3-hour minimum time leads to an inconvenience cost that quadratically increases in the delay. That is, a longer wait leads to a higher marginal waiting cost, which is supported by the psychology literature (Osuna 1985). We set the reservation utility at $\underline{u} = \$40$ so that the customers' willingness to pay for no-delay charging is $u_0 - \underline{u} = \$10$. This translates to 50 cents per kWh, a typical price at charging stations (Blink Charging Co. 2019).

Customer Demand. We consider a total of $N = 2,500$ customers, arriving evenly at five different times over the course of a day: 8:00 a.m., 10:30 a.m., 1:00 p.m, 3:30 p.m., and 6:00 p.m. This approximates the realistic situation where customers arrive at the charging station during different times, depending on their daily schedules. The differences in customers' daily schedules are also approximated by their heterogeneous delay-sensitivity. For example, a customer who does not need their vehicle for an extended period of time has a low delay-sensitivity. Note that two adjacent arrival times are 2.5 hours apart, shorter than the minimum charging time of 3 hours, which reflects the reality that demand for charging vehicles can significantly increase if all vehicles are charged as soon as possible as in the current practice (Myers 2017).

At each arrival time, 500 customers arrive at the charging station, with 100 customers of each type $\theta^{(i)}$, $i = 1, \ldots, 5$. That is, instead of independently drawing each customer's delay-sensitivity from the delay-sensitivity distribution, we assume that the same number of customers from each class request charging. This assumption simplifies the computation and it is reasonable given that the number of customers N is much greater than the number of customer classes I. Because the objective function of a utility firm in the first-stage problem is generally neither convex nor concave, we employ a global search algorithm with the following constraints: (a) The charging completion

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[4]We tried expanding the range of θ to include higher values of θ, but those highly delay-sensitive customers are served almost without delay in the optimal solution. As a result, their electricity demand can be regarded as non-schedulable and included in the exogenous demand $d(t)$.

times decrease in the customers' sensitivity to delay (Lemma 2 and Proposition 3), and (b) For customers with the same level of delay-sensitivity, later arrivals also have later completion. For the solution of the second-stage problem of the utility firm, we use the generalized juice-filling policy. Finally, to better illustrate our results, we scale down both the supply curve and the exogenous demand by a factor of 1,000, so that the demand from electric vehicle charging is approximately 2% of the total electricity demand.

7.2 Optimal Charging Schedule

We first discuss charging decisions of a public utility firm whose objective is to minimize the total cost. Figure 3 illustrates the optimal charging schedule from 8:00 a.m. to 11:00 p.m. on a day with high demand for electricity. This time interval represents typical hours of usage for a public charging station. On that day, the marginal cost of electricity generation during the peak hours is more than six times as high as that during the off-peak hours ($157/MWh versus $25/MWh, indicated by the numbers below the exogenous demand curve). The color-coding scheme illustrates different levels of delay-sensitivity and arrival times, with lighter (darker) colors for customers arriving earlier (later) in the day. Each color represents 100 customers.
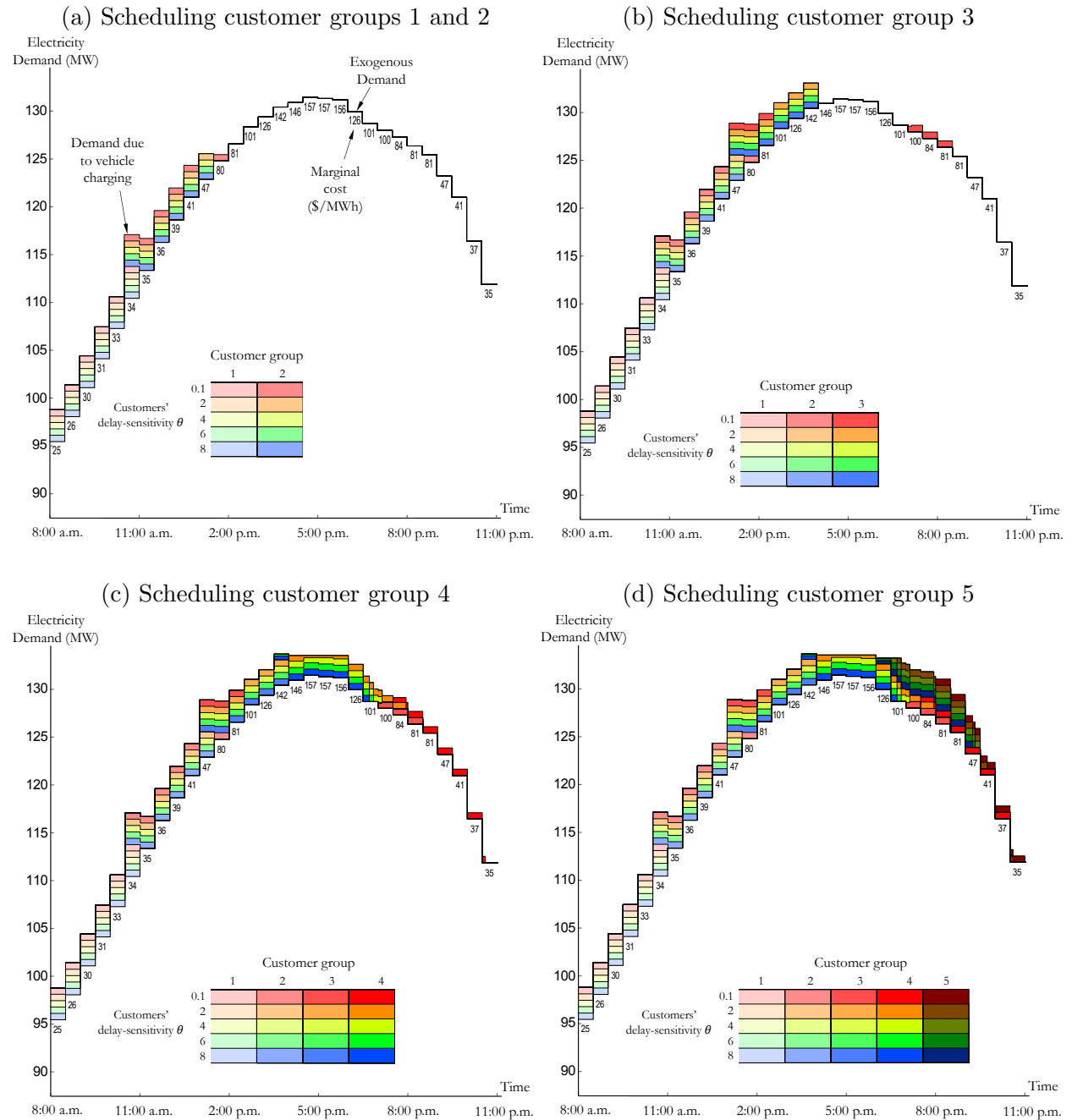
Figure 3 (a) shows the charging schedules for the first 1,000 arriving customers when the exogenous demand is rapidly increasing. The charging schedule is close to as soon as possible (ASAP) policy because delaying charging for these customers will only incur higher inconvenience cost, but a slight delay is planned for the least delay-sensitive customers in the second group. Such a delay smooths the total demand from 1:00 to 2:00 p.m., which is reflected in Figure 3 (b). Smoothing the demand is particularly important when the marginal cost rapidly increases near the peak hours.

Figure 3 (b) reveals an important feature of the optimal schedule: The least delay-sensitive customers in group 3 start charging as soon as they arrive at 1:00 p.m, but the charging pauses at 2:30 p.m. and resumes at 7:00 p.m. until it finishes shortly after 8:30 p.m. This 4.5-hour delay allows the utility firm to schedule more delay-sensitive customers in the next two groups.

In Figure 3 (c), the least delay-sensitive customers in group 4 have their vehicles charged from 7:30 p.m. to shortly after 10:30 p.m. In addition, customers who are more sensitive to delays also have to experience some delays. These delays flatten the total demand during the peak hours, leading to a lower charging cost. Observe that the schedule actually completes charging for group 4 customers with $\theta^{(i)}$, $i \geq 2$, earlier than the aforementioned group 3 customers with $\theta^{(1)}$, who are least delay-sensitive and the utility firm finishes charging their vehicles at approximately 8:30 p.m.

Figure 3 (d) shows that as the exogenous demand decreases in the evening hours, more vehicles

Figure 3: Electric vehicle charging schedule on a typical peak day (August 25, 2016) under total cost minimization policy of a public utility firm

(a) Scheduling customer groups 1 and 2

(b) Scheduling customer group 3

(c) Scheduling customer group 4

(d) Scheduling customer group 5

are being charged. From 7:00 to 8:30 p.m., some vehicles from groups 3, 4, and 5 are charged at the same time. Overall, under smart charging, the demand due to vehicle charging during the peak time (when marginal cost exceeds \$150/MWh) is reduced to about half of that from 6:00 to 9:00 p.m.

We omit the illustration of a private utility firm's optimal charging schedule because it is qualitatively similar to the public firm's schedule shown in Figure 3.

24

7.3 Economic and Environmental Benefits of Smart Charging

Finally, we quantify economic and environmental benefits of smart charging by considering three cases—current practice of charging ASAP, a public utility firm's smart charging for total cost minimization, and a private utility firm's smart charging for profit maximization (see Table 1).

Table 1 indicates that although charging ASAP without delays leads to zero inconvenience cost, it leads to a higher total cost and emissions, compared to smart charging. On a typical peak-demand day, the charging cost per kWh is $0.285 under ASAP charging (total cost divided by 50,000 kWh for charging 2,500 vehicles), which is more than twice that under smart charging by a public or a private utility firm ($0.114/kWh or $0.125/kWh, respectively). Moreover, charging ASAP also results in higher emissions because the additional electricity used during peak-demand hours is mostly generated by the least efficient peaking power plants with the highest emissions intensity.

We next compare smart charging of a private and public utility firm by considering the last two columns of Table 1. The inconvenience cost is lower under a private utility firm because it completes charging service slightly earlier for most customers than a public utility firm. Accordingly, a private utility firm charges customers higher prices (3.6% higher on average), yielding a total revenue that is also 3.6% or $802.5 higher. The charging cost is also 9.2% or $523.4 higher. Therefore, the

Table 1: Electric vehicle charging on a typical peak day (August 25, 2016)

	As soon as possible	Public utility firm	Private utility firm
Inconvenience cost ($)	0	1420.5	1010.1
Charging cost ($)	14268.6	5707.5	6230.9
Charging cost per kWh ($ per kWh)	0.285	0.114	0.125
Total cost ($)	14268.6	7128.0	7241.0
Prices ($) $[\,p_t^{(i)}\,]$ ($p_t^{(i)}$ in row i column t)	10	$\begin{bmatrix} 10 & 9.98 & 7.96 & 3.25 & 7.72 \\ 10 & 10 & 10 & 4.73 & 8.06 \\ 10 & 10 & 10 & 8.06 & 8.62 \\ 10 & 10 & 10 & 9.20 & 9.25 \\ 10 & 10 & 10 & 9.66 & 9.53 \end{bmatrix}$	$\begin{bmatrix} 10 & 9.98 & 7.96 & 5.53 & 8.26 \\ 10 & 10 & 9.98 & 7.05 & 8.62 \\ 10 & 10 & 10 & 8.88 & 9.18 \\ 10 & 10 & 10 & 9.52 & 9.51 \\ 10 & 10 & 10 & 9.88 & 9.70 \end{bmatrix}$
Total payment ($)	25000	22600.4	23402.9
Profit (= payment − charging cost)	10731.4	16892.9	17172.0
Total information rent ($)	0	979.1	586.9
CO_2 emissions (kilograms per kWh of EV charging)	3.48	1.65	1.88
Total CO_2 emissions from charging (metric tons)	174.0	82.7	94.0

private utility firm achieves a profit that is \$279.1 or 1.7% higher than the public utility firm. However, the CO_2 emissions increase from 82.7 to 94.0 metric tons, which is a 13.7% increase. This emissions difference may be concerning, but the emissions differences are generally small for most of the days (see Table 2). For the entire August, the total CO_2 emissions under the private utility firm is only 1.8% higher than that under a public utility firm.

Table 1 also shows how a utility firm can pass part of the cost savings from smart charging through pricing. For the first customer group that arrives at 8:00 a.m., both public and private utility firms charge \$10 (first column of the price matrix), the same price as in ASAP charging, because charging is performed without delay in the early morning when the exogenous demand is low. Customers that arrive later in the day may choose delayed completion times and receive a price discount. In particular, least delay-sensitive customers choose to have the longest delay and pay as low as \$3.25 and \$5.53 (for the 4th customer group) to the public and private utility firm, respectively, compared to \$10 flat fee under the current practice of ASAP charging. Customers who are more sensitive to delay pay more (or save less) in exchange for less delay in charging.

Finally, for each day in August 2016, we present the cost and emissions savings due to smart charging by a private or public utility firm compared to the current practice of ASAP charging in Table 2. For the entire month, smart charging reduces the total cost of charging electric vehicles by \$22,249 for a public utility firm and by \$20,926 for a private utility firm, which is 22.4% and 21.1% cost reduction, respectively. Smart charging leads to the highest cost reduction on August 11th and 25th when peak demand was particularly high and the wholesale electricity prices reached \$175.65 and \$198.40 per MWh, respectively. Consequently, the ASAP charging cost exceeds \$14,000 (or \$0.28 per kWh), and smart charging reduces the charging cost by over 50%. Smart charging also achieves a cost reduction of 7 to 35% during five days (August 10th, 12th, 13th, 18th, 26th) for which the ASAP charging cost ranges from \$3,000 to approximately \$6,000. Cost reduction by smart charging is between 2 to 4% for 14 days, and for the remaining 10 days, the cost reduction is no more than 0.25%. Note that, if smart charging by a public or a private utility firm is implemented on only 4 peak days (August 10th, 11th, 12th, 25th), the charging cost can be reduced by 20.5% or 19.3%, respectively. That is, implementing smart charging on peak days is particularly beneficial for a utility firm because 92% of the cost savings for the entire month is achieved on 4 peak days.

These cost savings are sufficient to justify the investment cost in additional workplace or public chargers which are capable of smart charging by remotely communicating with the utility firm and customers (e.g., ChargePoint 2021 and SmartCar 2021). To estimate the number of additional

Table 2: Costs and emissions of electric vehicle charging for a typical summer month

Notes. Based on PJM demand and power plants data in August 2016
ASAP = Charge as soon as possible by the current practice
TCM = Total cost minimization by a public utility firm
PM = Profit maximization by a private utility firm

day	Cost of EV charging ($)					CO_2 emissions due to EV charging (metric tons)				
	ASAP	TCM	$\frac{ASAP-TCM}{ASAP}$	PM	$\frac{ASAP-PM}{ASAP}$	ASAP	TCM	$\frac{ASAP-TCM}{ASAP}$	PM	$\frac{ASAP-PM}{ASAP}$
1	2291.5	2222.6	3.01%	2225.4	2.89%	38.7	38.7	0.10%	38.7	0.12%
2	2218.6	2159.5	2.67%	2162.2	2.55%	37.7	37.6	0.37%	37.6	0.27%
3	2366.6	2304.2	2.64%	2306.8	2.53%	38.8	38.5	0.73%	38.5	0.75%
4	2011.7	1967.6	2.19%	1970.8	2.03%	35.7	35.7	-0.06%	35.7	-0.01%
5	1557.7	1557.3	0.03%	1557.3	0.03%	29.2	29.3	-0.03%	29.3	-0.03%
6	1450.9	1449.2	0.12%	1449.3	0.12%	29.1	29.1	0.07%	29.1	0.08%
7	1552.5	1551.8	0.04%	1551.8	0.04%	30.7	30.7	0.03%	30.7	0.03%
8	2891.8	2792.0	3.45%	2801.9	3.11%	40.2	40.0	0.56%	40.1	0.48%
9	2601.6	2507.6	3.61%	2513.3	3.39%	38.6	38.3	0.94%	38.3	0.86%
10	6342.7	4079.4	35.68%	4174.7	34.18%	77.8	47.1	39.45%	47.9	38.44%
11	14587.8	6454.8	55.75%	6986.5	52.11%	185.9	89.0	52.12%	98.1	47.22%
12	5257.0	3836.6	27.02%	3910.4	25.61%	63.7	45.8	28.07%	46.4	27.10%
13	3953.4	3611.4	8.65%	3622.9	8.36%	48.2	43.7	9.30%	43.8	9.10%
14	2186.1	2139.4	2.14%	2142.4	2.00%	36.9	36.6	0.89%	36.6	0.78%
15	1625.6	1625.0	0.03%	1625.1	0.03%	31.4	31.4	0.07%	31.4	0.07%
16	2557.1	2477.5	3.11%	2480.8	2.98%	38.6	38.5	0.34%	38.5	0.29%
17	2900.2	2788.8	3.84%	2798.2	3.52%	39.5	39.2	0.90%	39.2	0.85%
18	3439.8	3144.0	8.60%	3159.7	8.14%	43.8	41.0	6.34%	41.1	6.07%
19	1745.4	1741.2	0.25%	1741.3	0.24%	33.9	33.8	0.09%	33.8	0.09%
20	1619.8	1619.1	0.04%	1619.2	0.04%	31.1	31.1	0.05%	31.1	0.05%
21	1209.6	1209.6	0.00%	1209.6	0.00%	26.5	26.5	-0.01%	26.5	-0.01%
22	1440.4	1438.6	0.13%	1438.7	0.12%	29.2	29.2	-0.04%	29.2	-0.03%
23	1437.2	1435.9	0.09%	1435.9	0.09%	27.9	27.9	0.00%	27.9	0.00%
24	1962.3	1922.5	2.03%	1924.9	1.91%	34.8	34.7	0.47%	34.6	0.49%
25	14268.6	5707.5	60.00%	6230.9	56.33%	174	82.7	52.46%	94.0	45.95%
26	3082.0	2860.2	7.20%	2872.2	6.81%	40.6	39.4	3.08%	39.4	2.96%
27	1440.6	1438.9	0.12%	1439.0	0.12%	29.5	29.5	0.05%	29.5	0.06%
28	2245.3	2182.9	2.78%	2185.8	2.65%	36.2	36.1	0.29%	36.1	0.33%
29	2433.0	2344.5	3.64%	2347.8	3.50%	37.6	37.0	1.41%	37.1	1.27%
30	2239.0	2175.2	2.85%	2178.0	2.72%	36.4	36.1	0.76%	36.2	0.66%
31	2380.8	2302.9	3.27%	2308.4	3.04%	38.6	38.7	-0.08%	38.7	-0.12%
monthly total	99296.8	77047.8	22.4%	78371.2	21.1%	1460.9	1212.7	17.0%	1235.0	15.5%

chargers needed to allow extended parking time, we focus on the last three groups of customers who experience most of the delays under smart charging. These customers require a minimum of 4,500 hours (1,500 customers, 3 hours per customer) of charging service time. Under smart charging, these customers experience 1,507 hours of delay, which is approximately 33% of 4,500 hours. Accordingly, under smart charging, 33% more chargers are needed. Under ASAP policy, with an average of 200 customers per hour and 3 hours of service time, 600 vehicles are being charged on average. Thus, we assume that 750 chargers are currently installed to practically eliminate waiting time.[5] Therefore, under smart charging, increasing capacity by 33% requires installing 250 additional chargers. We next estimate total cost savings under smart charging, assuming that there are 20 peak days over a year.[6] Then, the annual cost savings from smart charging are approximately 5 times as much as that in August (recall that 92% of the savings in August are from the 4 peak days), which suggests that the total cost savings are approximately $1.6 million over a 15-year economic life of a charger. Thus, the average cost savings for one of the 250 additional chargers is estimated to be $6,400, which is significantly higher than the cost of a workplace or public charger. Including emissions reduction enhances the benefits of the investment and further justifies it.

Table 2 also shows that smart charging by a public or private utility firm reduces the monthly CO_2 emissions of vehicle charging by 17% or 15.5%, respectively, compared to the current practice of ASAP charging. These emissions reductions generally coincide with the cost reductions, but there are two caveats. First, the emissions reduction percentages are typically lower than the cost reduction percentages because a utility firm's objective does not include emissions. In fact, for five days, smart charging increases emissions slightly (less than 0.12%). Second, the emissions reduction percentages can exceed the cost reduction percentages (notably on August 10th and 12th), primarily because of the increased use of natural gas-fired generators with low emissions. Similar to cost savings, if smart charging by a public or a private utility firm is implemented only on 4 peak days (August 10th, 11th, 12th, 25th), emissions can be reduced by 16.2% or 14.7%, respectively. That is, implementing smart charging on peak days is particularly beneficial for the environment because 95% of the emissions savings for the entire month is achieved on 4 peak days.

These results have important practical implications: First, to achieve most of the benefits of smart charging, the incentives for customers to delay electric vehicle charging need to be used only during the peak days. Second, although the profit-maximization objective of a private utility firm distorts the electric vehicle charging schedules that would otherwise minimize the overall cost, the

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[5]In particular, using Pollaczek-Khinchine formula and assuming an M/D/c queue with an arrival rate of 200 customers per hour and service time of 3 hours, the average wait time is 0.0001 minutes if the number of servers, c, is 750.

[6]This is a conservative estimate because we observe four peak days in a month (i.e., August) in 2016.

distortions are concentrated on peak days and the overall differences in cost and emissions of smart charging between public and private utility firms appear to be small. Therefore, regulators should encourage both private and public utility firms to operate smart charging stations.

8.   Conclusion

Electric vehicles will transform urban mobility in smart cities, which requires innovative business models for charging stations. We propose smart charging of electric vehicles—delaying charging until electricity generation cost is lower during a day—as an alternative to the current practice of completing charging as soon as possible. By considering the cost minimization and profit maximization objectives of public and private utility firms, respectively, we characterize pricing schemes that motivate customers to allow smart charging.

Our results have significant managerial and policy implications. We devise the juice-filing policy which can be easily implemented to determine the smart charging schedule of electric vehicles. By analyzing one of the largest electricity markets in the U.S., we demonstrate that compared to the current practice, smart charging leads to approximately 20% cost savings and 15% CO_2 emissions reductions in a typical summer month. Moreover, most of these benefits can be achieved by implementing smart charging on only a few peak-demand days. From a policy perspective, our results show that allowing either public or private utility firms to own and operate charging stations can lead to significant cost and emissions savings.

In this paper, we assume that charging stations have ample parking spots and we demonstrate that the cost savings from smart charging can justify investment in additional chargers. If spots are limited, the benefits of smart charging would be less pronounced, but the fundamental tradeoff between the charging cost and the inconvenience cost that we focused on remains valid. Finally, note that we consider only charging of electric vehicles, whereas vehicles can also be discharged (if customers agree) to further smooth the net electricity demand. This remains as an open question for future research.

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Online Appendix for *Smart Charging of Electric Vehicles:*
An Innovative Business Model for Utility Firms

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A. Charging Schedule for Non-simultaneous Arrivals when Customers Choose Completion Times upon Arrival

In this section, we consider non-simultaneous arrivals where arrival times $\{s_1, \ldots, s_N\}$ are exogenously given, but each customer chooses their completion time upon their arrival. As in Section 6.1, the utility firm divides time interval $[0, T]$ into L periods and at time $t = 0$ sets a period-specific menu $\big\{\big(p_\ell^{(i)}, \tau_\ell^{(i)}\big) : i = 1, \ldots, I\big\}$ for the customers arriving in period ℓ. Because the customers' delay-sensitivity is uncertain from the utility firm's perspective and only revealed through their choices upon arrival, optimizing the charging schedule requires solving a stochastic dynamic control problem, which is beyond the scope of this paper. Instead, we provide a heuristic method that generates a feasible charging schedule under a given time-varying menu, $\big\{\big(p_\ell^{(i)}, \tau_\ell^{(i)}\big) : i = 1, \ldots, I, \ell = 1, \ldots, L\big\}$. Using the same logic as in Section 6.1, we can see that the menu satisfies the properties in Propositions 2 and 3 in each period. Below, we describe our heuristic, which is based on the generalized juice-filling policy in Section 6.2.

First, we solve an auxiliary scheduling problem as follows. At each arrival time s_n, we replace the n-th customer by I customers all arriving at s_n, indexed as n_i, $i = 1, \ldots, I$. Customer n_i belongs to class i and demands $\beta^{(i)}$ units of energy. Customer n_i's vehicle can be charged at a maximum speed of $\overline{a}\beta^{(i)}$. Because the given menu is incentive compatible, customer n_i will choose $\big(p_\ell^{(i)}, \tau_\ell^{(i)}\big)$, where ℓ is the period index such that $s_n \in [t_\ell, t_{\ell+1})$. Therefore, the utility firm faces a total of NI customers with known arrival and completion times, which allows us to apply the generalized juice-filling policy in Section 6.2 to find a charging schedule for these NI customers. Let $a_{n_i}(t)$ denote the charging schedule for customer n_i.

Next, we construct a charging schedule for each customer upon their arrival. If customer n arriving at $s_n \in [t_\ell, t_{\ell+1})$ is in class i, we set the charging schedule $a_n(t)$ to be $a_{n_i}(t)/\beta^{(i)}$. This charging schedule is feasible because, by construction, $a_n(t)$ charges one unit of energy during $[s_n, s_n + \tau_\ell^{(i)}]$ at maximum speed of $\overline{a}$.

The above heuristic prepares charging schedules for customers with different levels of delay-sensitivity at each arrival time. Note that if there is a large number of customers arriving together, the charging schedules of customers in the same class can be combined, resulting in block charging schedules that are easier to demonstrate, as shown in Figure 3.

B. Details of the Parameter Estimation for Numerical Analysis

We first estimate the generation cost and emissions intensity of the conventional sources in the PJM Interconnection by using the Emissions and Generation Resource Integrated Database (eGRID) of the Environmental Protection Agency (EPA 2019). eGRID provides detailed information for each power plant, including the heat rate (the amount of thermal energy needed to generate one unit of electrical energy, measured in MMBTU/kWh), nameplate capacity (the theoretical maximum

power output of a generator in MW), and emissions rate (the amount of CO_2-equivalent emissions for each unit of electricity generated in lb/MWh). Multiplying the heat rate with the cost of the fuel source of a power plant (given by EIA 2017 in \$/MMBTU) gives the plant's cost of generating electricity (in \$/kWh).

We convert the nameplate capacity to effective capacity for each plant by multiplying the plant's nameplate capacity with the equivalent availability factor (provided by NERC 2019), which accounts for the unavailability due to planned outages for maintenance. We then sort the power plants in the increasing order of their generation costs, which gives a supply curve but does not take into account unplanned outages. To account for them, we calibrate the supply curve by multiplying the capacity with a daily scaling factor $\alpha_m \leq 1$ for each day m. We find the factor α_m by minimizing the sum of squared differences between the actual electricity prices and the estimated electricity prices (based on the adjusted supply curve and the net demand). To find the net electricity demand that should be satisfied by conventional sources, we subtract the hourly generation of renewable sources from the electricity demand. Figure 2 illustrates the resulting calibrated supply curve.

In the numerical setup, we also experimented using a finer division of periods (e.g., 10 or 15 arrival times) for arrival times, and the results are found to be similar. We choose to present the results of the simpler model for the ease of illustration (see Figure 3) as well as for computational efficiency. The computation is still demanding: For each of the 31 days in August 2016, we optimize the charging schedule under the objectives of a public and a private utility firm and compare the results with the current practice of as soon as possible (ASAP) charging policy. The computational procedure takes approximately two hours to converge to a set of optimal completion times (with optimal charging schedule) for each day.

C. Proofs

Proof of Proposition 1: The utility firm's second-stage problem (7)-(11) can be expressed in the standard form of a continuous-time optimal control problem (Bertsekas 2017, Section 7.1) as

$$\min_{\{a_n(t),\, n=1,\ldots,N\}} h\big(x_1(\tau_1),\ldots,x_N(\tau_N)\big) + \int_0^T \widetilde{c}\Big(\sum_{n=1}^N a_n(t) + d(t)\Big)\mathrm{d}t - \int_0^T \widetilde{c}(d(t))\mathrm{d}t, \qquad (A.1)$$

$$\text{s.t.} \quad \frac{\mathrm{d}x_n(t)}{\mathrm{d}t} = a_n(t) \qquad \forall n = 1,\ldots,N, \qquad (A.2)$$

$$0 \leq a_n(t) \leq \overline{a} \qquad \forall n = 1,\ldots,N, \qquad (A.3)$$

where $x_n(t) = \int_0^t a_n(s)\,\mathrm{d}s$ is the state variable, representing the amount of electricity charged to vehicle n by time t, $a_n(t)$ is the control variable, and

$$h\big(x_1(\tau_1),\ldots,x_N(\tau_N)\big) = \begin{cases} 0, & \text{if } x_n(\tau_n) = 1, \quad \forall n \in \{1,\ldots,N\}, \\ \infty, & \text{otherwise,} \end{cases}$$

represents the terminal conditions in (8). The cost function $c(q(t)|(t))$ in (7) is substituted by (5)-(6), as reflected in (A.1). The last term in (A.1), $\int_0^T \widetilde{c}(d(t))\mathrm{d}t$, is independent of the control variables and can be omitted from the optimization below. A unique feature of this problem is that the terminal condition imposed on $x_n(t)$ through $h(x_1(\tau_1),\ldots,x_N(\tau_N))$ applies at different times τ_n. Accordingly, the number of control variables decreases over time.

We first show that the control policy found through the juice-filling policy is an optimal solution to the problem (A.1)-(A.3) for the special case with $N = 1$ vehicle, using Pontryagin's minimum principle (Bertsekas 2017, Section 7.3). In particular, the Hamiltonian is

$$H(x, a, p) = \widetilde{c}(a + d) + pa,$$

where the adjoint equation $p(t)$ (not to be confused with charging price $p^{(i)}$) is given by

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = -\frac{\partial H\left(x^*(t), a^*(t), p(t)\right)}{\partial x} = 0,$$

where $x^*(t)$ is the state trajectory corresponding to the optimal control $a^*(t)$. This implies that $p(t)$ is a constant for $t \in [0, T]$, i.e., $p(t) = p$ for some p. We next minimize the Hamiltonian, subject to the terminal condition:

$$\min_{0 \leq a \leq \bar{a}} \{H(x, a, p) = \widetilde{c}(a + d) + pa\}$$

$$s.t. \int_0^T a(t)\mathrm{d}t = 1. \tag{A.4}$$

Because $\widetilde{c}(\cdot)$ is convex, $H(x, a, p)$ is also convex in a for a given x and p, and the first-order condition is given as $\frac{\mathrm{d}H(x,a,p)}{\mathrm{d}a} = \widetilde{c}'(a + d) + p = 0$, where $\widetilde{c}'(\cdot)$ is the derivative of the cost function. Taking the constraint that $0 \leq a(t) \leq \bar{a}$ into account, the optimal action is to set $a^*(t) = \min\left((z - d(t))^+, \bar{a}\right)$, where $\widetilde{c}'(z) = -p$. Note that z is constant because p is a constant. To find the value of z, we substitute $a^*(t)$ into the terminal condition (A.4) so that z is implicitly given by

$$\int_0^T \min\left((z - d(t))^+, \bar{a}\right)\mathrm{d}t = 1.$$

This control policy is the same as the one found by the juice-filling policy for the special case with one electric vehicle (i.e., when $N = 1$), where z is given in Step 2 and $L(t) = d(t)$. Therefore, the juice-filling policy finds an optimal solution to the problem (A.1)-(A.3).

We next prove that the juice-filling policy is optimal for $N > 1$. We index electric vehicles with n such that $\tau_N \leq \cdots \leq \tau_1$. Let $a_n^*(t)$ for $n \in \{1, \ldots, N\}$ be an optimal solution to the charging problem (A.1)-(A.3). Using the juice-filling policy, we construct another solution $\hat{a}_n(t)$ for $n \in \{1, \ldots, N\}$, which we show is also optimal. We next describe the construction of this solution in the interval $t \in [0, \tau_N]$, where τ_N is the earliest completion time. In essence, we find $\hat{a}_n(t)$ for $n \in \{1, \ldots, N\}$ for N vehicles with simultaneous arrival (i.e., $t = 0$) and completion times (i.e., $t = \tau_N$) such that each vehicle is charged by time τ_N to the same level as that under the optimal policy, i.e., $\hat{x}_n(\tau_N) = x_n^*(\tau_N) = \int_0^{\tau_N} a_n^*(t)\mathrm{d}t$ for $n \in \{1, \ldots, N\}$. Then, we show that the charging policy $\hat{a}_n(t)$ achieves the same cost as the optimal policy $a_n^*(t)$.

The charging policy $\hat{a}_n(t)$ for $n \in \{1, \ldots, N\}$ in the interval $t \in [0, \tau_N]$ can be constructed by applying the juice-filling policy with two modifications: Let $T = \tau_N$ and in Step 2, find z_n such that $\int_0^{\tau_N} \min\left\{(z_n - L(t))^+, \bar{a}\right\}\mathrm{d}t = x_n^*(\tau_N)$. We refer to this policy as the modified juice-filling policy. Note that, for vehicle $n = N$, the modified juice-filling policy is equivalent to the juice-filling policy described in Section 4 because $x_N^*(\tau_N) = 1$. That is, the charging schedule of vehicle N, i.e., $\hat{a}_N(t)$, is the same as that found through the juice-filling policy.

We next verify that the resulting charging cost in $[0, \tau_N]$ under the modified juice-filling policy is the same as that under the juice-filling policy for charging one electric vehicle. Specifically, consider the juice-filling policy for one vehicle, where the vehicle's completion time is τ_N, its maximum

A3

charging speed is $\bar{a}N$, and the vehicle needs to be charged with $\sum_{n=1}^{N} x_n^*(\tau_N)$ units of energy. Following similar steps as above, we can show that there exists z such that

$$\int_0^{\tau_N} \min\left((z - d(t))^+, \bar{a}N\right) \mathrm{d}t = \sum_{n=1}^{N} x_n^*(\tau_N). \tag{A.5}$$

Moreover, it is straightforward to verify that z in (A.5) is the same as z_N of the modified juice-filling policy such that the two policies have the same cost. Given the optimality of the juice-filling policy for one vehicle, charging policy $\hat{a}_n(t)$ for $n \in \{1, \ldots, I\}$ is an optimal policy for $t \in [0, \tau_N]$. That is, $\int_0^{\tau_N} \widetilde{c}\left(\sum_{n=1}^{N} \hat{a}_n(t) + d(t)\right) \mathrm{d}t = \int_0^{\tau_N} \widetilde{c}\left(\sum_{n=1}^{N} a_n^*(t) + d(t)\right) \mathrm{d}t$.

Notice that in constructing $\hat{a}_N(t)$ for $t \in [0, \tau_N]$, we have used the juice-filling policy, where $\int_0^{\tau_N} \min\left\{(z_N - d(t))^+, \bar{a}\right\} \mathrm{d}t = 1$ so that $\hat{a}_N(t) = \min\left\{(z_N - d(t))^+, \bar{a}\right\}$ for $t \in [0, \tau_N]$.

We then update $L(t)$ with $L(t) + \hat{a}_N(t)$ and proceed to the time interval $t \in [0, \tau_{N-1}]$, where τ_{N-1} is the completion time of vehicle $N - 1$. By the same arguments as above, we can construct $\hat{a}_n(t)$ for all $n \in \{1, \ldots, N-1\}$ in $t \in [0, \tau_{N-1}]$ and show that it achieves the same cost as the optimal policy. This procedure can be repeated for all vehicles and time intervals. Therefore, the control policy $\hat{a}_n(t)$ for $n \in \{1, \ldots, N\}$ found through the juice-filling policy identifies an optimal solution to the charging problem (A.1)-(A.3). ∎

Proof of Lemma 1: (i) For given $(\tau_1, \ldots, \tau_N)$, let $\{a_1^*(t), \ldots, a_N^*(t)\}$ be an optimal control. Given that $\{b_1, \ldots, b_N\}$ is a permutation of $\{1, \ldots, N\}$, the control $\{a_{b_1}^*(t), \ldots, a_{b_N}^*(t)\}$ must be feasible under completion time $(\tau_{b_1}, \ldots, \tau_{b_N})$ because $\int_0^{\tau_{b_n}} a_{b_n}^*(t) \mathrm{d}t = 1, \forall n = 1, \ldots, N$. Furthermore, this feasible control yields the same objective value in (7) as the control $\{a_1^*(t), \ldots, a_N^*(t)\}$ because $\sum_{n=1}^{N} a_{b_n}^*(t) = \sum_{n=1}^{N} a_n^*(t)$. If, however, there exists another control $\{\widetilde{a}_{b_1}^*(t), \ldots, \widetilde{a}_{b_N}^*(t)\}$ that yields a strictly lower objective value in (7) than $\{a_{b_1}^*(t), \ldots, a_{b_N}^*(t)\}$, then $\{\widetilde{a}_1^*(t), \ldots, \widetilde{a}_N^*(t)\}$ must be feasible under $(\tau_1, \ldots, \tau_N)$ and yield a strictly lower objective value in (7) than $\{a_1^*(t), \ldots, a_N^*(t)\}$, leading to a contradiction. Hence, we have $C(\tau_{b_1}, \ldots, \tau_{b_N}) = C(\tau_1, \ldots, \tau_N)$.

(ii) The optimal control for (7)-(11) under any given $(\tau_1, \ldots, \tau_N)$ remains a feasible control under $(\hat{\tau}_1, \ldots, \hat{\tau}_N) \geq (\tau_1, \ldots, \tau_N)$. It follows that $C(\tau_1, \ldots, \tau_N)$ decreases in τ_n for any n.

To show the continuity, first note that, by construction of the juice-filling policy, the fill-up-to level z_n is continuous in $(\tau_1, \ldots, \tau_N)$. Consider the cost $\widetilde{c}(d(t) + q^*(t))$ as function of time. The area under this cost function is completely determined by $(\tau_1, \ldots, \tau_N)$ and $(z_1, \ldots, z_N)$. Because $(z_1, \ldots, z_N)$ is continuous in $(\tau_1, \ldots, \tau_N)$, and $\widetilde{c}(\cdot)$ is convex and hence continuous, the area under the curve, $\int_0^T \widetilde{c}(d(t) + q^*(t)) \mathrm{d}t$, is also continuous in $(\tau_1, \ldots, \tau_N)$. Therefore, $C(\tau_1, \cdots, \tau_N) = \int_0^T \widetilde{c}(d(t) + q^*(t)) \mathrm{d}t - \int_0^T \widetilde{c}(d(t)) \mathrm{d}t$ is also continuous in $(\tau_1, \cdots, \tau_N)$. ∎

Proof of Lemma 2: The objective function in (15) can be written as

$$\mathbb{E}_{\{\theta_1, \ldots, \theta_N\}} \left[\sum_{n=1}^{N} \theta_n \delta(\tau_n) + C(\tau_1, \ldots, \tau_N) \right]$$

$$= \sum_{i_1=1}^{I} \sum_{i_2=1}^{I} \cdots \sum_{i_N=1}^{I} \left\{ \prod_{n=1}^{N} \beta^{(i_n)} \left[\sum_{n=1}^{N} \theta^{(i_n)} \delta\left(\tau^{(i_n)}\right) + C\left(\tau^{(i_1)}, \tau^{(i_2)}, \cdots, \tau^{(i_N)}\right) \right] \right\}. \tag{A.6}$$

The objective function in (A.6) is continuous in $(\tau^{(1)}, \ldots, \tau^{(N)})$ because of the continuity of $\delta(\tau)$ and the continuity of $C(\tau_1, \ldots, \tau_N)$ from Lemma 1. Minimizing the continuous function in (A.6) over the compact set $\{\tau^{(i)} \in [\underline{w}, T] : i = 1, \ldots, I\}$ ensures the existence of an optimal solution by Weierstrass Theorem (Sundaram 1996, p. 90). To complete the existence proof, we note that there always exists prices low enough to ensure that the individual rationality constraints are satisfied.

For any solution with $\tau^{(j)} < \tau^{(k)}$ for $j < k$, we can strictly reduce the objective value by swapping $\tau^{(j)}$ and $\tau^{(k)}$. This swap does not affect the charging cost $C(\tau^{(i_1)}, \tau^{(i_2)}, \cdots, \tau^{(i_N)})$ due to Lemma 1(i), but it reduces the inconvenience cost $\sum_{n=1}^{N} \theta^{(i_n)} \delta(\tau^{(i_n)})$ because

$$\theta^{(j)} \delta(\tau^{(k)}) + \theta^{(k)} \delta(\tau^{(j)}) < \theta^{(j)} \delta(\tau^{(j)}) + \theta^{(k)} \delta(\tau^{(k)}),$$

or equivalently, $\theta^{(j)} \left(\delta(\tau^{(k)}) - \delta(\tau^{(j)}) \right) < \theta^{(k)} \left(\delta(\tau^{(k)}) - \delta(\tau^{(j)}) \right)$. This inequality follows from $\theta^{(j)} < \theta^{(k)}$ and $\delta(\tau)$ strictly increases in τ. Therefore, it is not possible to have $\tau^{(j)} < \tau^{(k)}$ for $j < k$ in any optimal solution. Hence, the optimal completion times are such that $\tau^{(1)*} \geq \tau^{(2)*} \geq \cdots \geq \tau^{(I)*}$. ∎

Proof of Proposition 2: (i) For simplicity, let $\delta^{(i)} \doteq \delta(\tau^{(i)})$ for $i = 1, \ldots, I$. Because the disutility $\delta(\tau)$ increases in τ and $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$, we have $\delta^{(1)} \geq \delta^{(2)} \geq \cdots \geq \delta^{(I)}$.

First, we set $p^{(I)} \leq u_0 - \theta^{(I)} \delta^{(I)} - \underline{u}$, so that a customer with $\theta^{(I)}$ receives at least the reservation utility if the customer chooses completion time $\tau^{(I)}$. Then, for $i = I-1, I-2, \ldots, 1$, we sequentially set $p^{(i)}$, such that the following $2(I-i)$ incentive compatibility constraints are satisfied:

$$\theta^{(j)} \delta^{(j)} + p^{(j)} \leq \theta^{(j)} \delta^{(i)} + p^{(i)}, \qquad \theta^{(i)} \delta^{(i)} + p^{(i)} \leq \theta^{(i)} \delta^{(j)} + p^{(j)}, \qquad \forall j = i+1, \ldots, I. \quad (A.7)$$

At the end of the above procedure, all of $2 + 4 + \cdots + 2(I-1) = I(I-1)$ incentive compatibility constraints for all I customer classes are satisfied.

The constraints in (A.7) can be equivalently written as

$$p^{(i)} \in \left[\max_{j>i}\{p^{(j)} - \theta^{(j)}(\delta^{(i)} - \delta^{(j)})\}, \ \min_{j>i}\{p^{(j)} - \theta^{(i)}(\delta^{(i)} - \delta^{(j)})\} \right]. \quad (A.8)$$

The lower (upper) bound ensures that a customer from class i strictly prefers not to choose any lower (higher) class than their class.

We next simplify the bounds for $p^{(i)}$ in (A.8) to obtain the bounds given in Proposition 2. The incentive compatibility for any two $\theta^{(j)}$ and $\theta^{(k)}$ customers requires $\theta^{(k)} \delta^{(k)} + p^{(k)} \leq \theta^{(k)} \delta^{(j)} + p^{(j)}$ and $\theta^{(j)} \delta^{(j)} + p^{(j)} \leq \theta^{(j)} \delta^{(k)} + p^{(k)}$, which are equivalent to

$$\theta^{(k)}(\delta^{(k)} - \delta^{(j)}) \leq p^{(j)} - p^{(k)} \leq \theta^{(j)}(\delta^{(k)} - \delta^{(j)}). \quad (A.9)$$

Consider the ordering of the terms, $\{p^{(j)} - \theta^{(j)}(\delta^{(i)} - \delta^{(j)}), j = i+1, \ldots, I\}$. Let $i < j < k$. Then,

$$p^{(j)} - p^{(k)} - \theta^{(j)}(\delta^{(i)} - \delta^{(j)}) + \theta^{(k)}(\delta^{(i)} - \delta^{(k)}) \geq \theta^{(k)}(\delta^{(k)} - \delta^{(j)}) - \theta^{(j)}(\delta^{(i)} - \delta^{(j)}) + \theta^{(k)}(\delta^{(i)} - \delta^{(k)})$$
$$= (\theta^{(k)} - \theta^{(j)})(\delta^{(i)} - \delta^{(j)}) \geq 0,$$

where the first inequality is due to the first inequality in (A.9), and the last inequality is due to $\theta^{(j)} \leq \theta^{(k)}$ and $\delta^{(i)} \geq \delta^{(j)}$. Therefore, $j = i+1$ maximizes the lower bound in (A.8), i.e., $\max_{j>i}\{p^{(j)} - \theta^{(j)}(\delta^{(i)} - \delta^{(j)})\} = p^{(i+1)} - \theta^{(i+1)}(\delta^{(i)} - \delta^{(i+1)})$.

Consider the ordering of the terms $\{p^{(j)} - \theta^{(i)}(\delta^{(i)} - \delta^{(j)}), j = i+1, \ldots, I\}$. Let $i < j < k$. Then,

$$p^{(j)} - p^{(k)} - \theta^{(i)}(\delta^{(i)} - \delta^{(j)}) + \theta^{(i)}(\delta^{(i)} - \delta^{(k)}) \leq \theta^{(j)}(\delta^{(k)} - \delta^{(j)}) - \theta^{(i)}(\delta^{(i)} - \delta^{(j)}) + \theta^{(i)}(\delta^{(i)} - \delta^{(k)})$$
$$= (\theta^{(j)} - \theta^{(i)})(\delta^{(k)} - \delta^{(j)}) \leq 0,$$

where the first inequality is due to the second inequality in (A.9), and the last inequality follows from $\theta^{(i)} \leq \theta^{(j)}$ and $\delta^{(j)} \geq \delta^{(k)}$. Therefore, $j = i + 1$ minimizes the upper bound in (A.8), i.e., $\min\limits_{j>i}\{p^{(j)} - \theta^{(i)}(\delta^{(i)} - \delta^{(j)})\} = p^{(i+1)} - \theta^{(i)}(\delta^{(i)} - \delta^{(i+1)})$.

Therefore, the feasible interval for p_i in (A.8) becomes:

$$p^{(i)} \in \left[p^{(i+1)} - \theta^{(i+1)}(\delta^{(i)} - \delta^{(i+1)}), \ p^{(i+1)} - \theta^{(i)}(\delta^{(i)} - \delta^{(i+1)}) \right], \tag{A.10}$$

which is clearly nonempty, because $\delta^{(i)} \geq \delta^{(i+1)}$ and $\theta^{(i)} \leq \theta^{(i+1)}$.

To see that the pricing scheme is individually rational (i.e., reservation utility is met), note that

$$(u_0 - \theta^{(i)}\delta^{(i)} - p^{(i)} - \underline{u}) - (u_0 - \theta^{(i+1)}\delta^{(i+1)} - p^{(i+1)} - \underline{u}) \geq \theta^{(i)}(\delta^{(i+1)} - \delta^{(i)}) - p^{(i)} + p^{(i+1)} \geq 0, \tag{A.11}$$

where the first inequality is due to $\theta^{(i+1)} > \theta^{(i)}$ and the second inequality follows from (A.10). That is, the net utility of a customer in class i is higher than that of a customer in class $i + 1$. Given that the pricing scheme is individually rational for a customer in class I by construction, then it is also individually rational for all classes. Moreover, this result also holds if the reservation utility of customers increase in their delay-sensitivity, i.e., if $\underline{u}^{(1)} \leq \underline{u}^{(2)} \leq \cdots \leq \underline{u}^{(I)}$.[7] In this case, the inequalities in (A.11) continue to hold because

$$(u_0 - \theta^{(i)}\delta^{(i)} - p^{(i)} - \underline{u}^{(i)}) - (u_0 - \theta^{(i+1)}\delta^{(i+1)} - p^{(i+1)} - \underline{u}^{(i+1)})$$
$$\geq \theta^{(i)}(\delta^{(i+1)} - \delta^{(i)}) - p^{(i)} + p^{(i+1)} + \underline{u}^{(i+1)} - \underline{u}^{(i)} \geq 0,$$

where $\underline{u}^{(i+1)} - \underline{u}^{(i)} \geq 0$. Accordingly, the pricing scheme is individually rational even if $\underline{u}^{(1)} \leq \underline{u}^{(2)} \leq \cdots \leq \underline{u}^{(I)}$.

(ii) Using the result from part (i), substituting the first-best solution $\{\tau^{(i)*} : i = 1, \ldots, I\}$ from (15) into (12), there exists prices that satisfy incentive compatibility and individual rationality constraints. This implies that the optimal completion times to (12) is the first-best solution. ∎

Proof of Proposition 3: (i) Suppose $\tau^{(j)} < \tau^{(k)}$ for some $j < k$. Incentive compatibility requires

$$p^{(j)} + \theta^{(j)}\delta(\tau^{(j)}) \leq p^{(k)} + \theta^{(j)}\delta(\tau^{(k)}), \tag{A.12}$$
$$p^{(j)} + \theta^{(k)}\delta(\tau^{(j)}) \geq p^{(k)} + \theta^{(k)}\delta(\tau^{(k)}). \tag{A.13}$$

Subtracting (A.13) from (A.12) and simplifying, we have $(\theta^{(j)} - \theta^{(k)})(\delta(\tau^{(j)}) - \delta(\tau^{(k)})) \leq 0$, which contradicts $\theta^{(j)} < \theta^{(k)}$ and $\tau^{(j)} < \tau^{(k)}$ because $\delta(\tau)$ strictly increases in τ. Therefore, any feasible solution must have declining completion times. To prove the existence of an optimal solution, note that the objective function (13) is continuous in $(p^{(i)}, \tau^{(i)})$ because of the continuity of $C(\tau_1, \ldots, \tau_N)$ from Lemma 1. Moreover, charging prices and completion times are bounded by $0 \leq p^{(i)} \leq u_0$ and $\underline{w} \leq \tau^{(i)} \leq T$. Maximizing the continuous function in (13) over this compact set ensures the existence of an optimal solution (with sufficiently low prices that ensure that the individual rationality constraints are satisfied). Finally, because each feasible solution must have declining completion times, the optimal completion times must also be declining.

(ii) Note that the given prices in the proposition correspond to the upper bounds of the prices

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[7] We index customer class I such that it has the highest delay-sensitivity $\theta^{(I)}$ and the highest reservation utility $\underline{u}^{(I)}$, but this reservation utility can still be met by the utility firm. Formally, $u_0 - \theta^{(I)}\delta(\underline{w}) > \underline{u}^{(I)}$, where $\underline{w}$ is the minimum charging completion time. This ensures that it is feasible for the utility firm to serve all customer classes i with delay-sensitivity $\theta^{(i)} \leq \theta^{(I)}$ and reservation utility $\underline{u}^{(i)} \leq \underline{u}^{(I)}$.

in Proposition 2. Therefore, these prices continue to satisfy individual rationality constraints (4) and incentive compatibility constraints (2). Moreover, given that they are the upper bounds, they maximize the utility firm's profit. ∎

Proof of Proposition 4: The private utility firm's objective function in (13) is given as

$$\mathbb{E}_{\{\theta_1,\ldots,\theta_N\}}\left[\sum_{n=1}^N p_n - C(\tau_1,\ldots,\tau_N)\right] = N\mathbb{E}[p_n] - \mathbb{E}[C(\tau_1,\ldots,\tau_N)]. \tag{A.14}$$

Consider the expected revenue $N\mathbb{E}[p_n]$. Based on the iterative procedure given in (19), $p^{(I)} = u_0 - \theta^{(I)}\delta(\tau^{(I)}) - \underline{u}$, and letting $\delta^{(i)} \doteq \delta(\tau^{(i)})$, we can write

$$p^{(i)} = u_0 - \theta^{(I)}\delta^{(I)} - \theta^{(I-1)}(\delta^{(I-1)} - \delta^{(I)}) - \cdots - \theta^{(i)}(\delta^{(i)} - \delta^{(i+1)}) - \underline{u}$$
$$= u_0 - (\theta^{(I)} - \theta^{(I-1)})\delta^{(I)} - \cdots - (\theta^{(i+1)} - \theta^{(i)})\delta^{(i+1)} - \theta^{(i)}\delta^{(i)} - \underline{u}$$
$$= u_0 - \theta^{(i)}\delta^{(i)} - \underline{u} - \sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)},$$

for $i = 1,\ldots,I-1$. Therefore, the expected revenue is

$$N\mathbb{E}[p_n] = N\sum_{i=1}^I \beta^{(i)}p^{(i)} = N\sum_{i=1}^I \beta^{(i)}\left(u_0 - \theta^{(i)}\delta^{(i)} - \underline{u}\right) - N\sum_{i=1}^{I-1}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)}$$
$$= N(u_0 - \underline{u}) - N\sum_{i=1}^I \beta^{(i)}\theta^{(i)}\delta^{(i)} - N\sum_{i=1}^{I-1}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)}$$
$$= N(u_0 - \underline{u}) - \mathbb{E}\left[\sum_{n=1}^N \theta_n\delta_n\right] - N\sum_{i=1}^{I-1}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)},$$

where $\delta_n \doteq \delta(\tau_n)$. By substituting the above expression of the expected revenue into (A.14), the private utility firm's objective in (13) can be rewritten as

$$\max_{\{\tau^{(i)}\in[\underline{w},T],\,i=1,\ldots,I\}} N(u_0 - \underline{u}) - \mathbb{E}\left[\sum_{n=1}^N \theta_n\delta_n\right] - \mathbb{E}[C(\tau_1,\ldots,\tau_I)] - N\sum_{i=1}^{I-1}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)}.$$

This is equivalent to the private utility firm's objective in Proposition 4 because $N(u_0 - \underline{u})$ is constant in $\tau^{(i)}$'s. Finally, note that for $\beta^{(i)} = \frac{1}{I}$, the information rent term $N\sum_{i=1}^{I-1}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} -$

$\theta^{(j)})\delta^{(j+1)} = \frac{N}{I}\sum_{i=1}^{I-1}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)} = \frac{N}{I}\sum_{i=2}^I (i-1)\left(\theta^{(i)} - \theta^{(i-1)}\right)\delta^{(i)}.$ ∎

Proof of Proposition 5: Thresholds $z_{n,j}$'s can be computed via the following problem:

$$\min_{\{z_{n,j},\,n=1,\ldots,N,\,j=1,\ldots,2N-1\}} \int_0^T c\big(q(t) \mid d(t)\big)\mathrm{d}t \tag{A.15}$$

s.t. (23), (24), (26), $\quad z_{n,j} = 0 \quad \forall j = 1,\ldots,2N-1, \forall n \notin \mathcal{I}(t_j)$ and $\tag{A.16}$

$$a_n(t) = \min\left\{\left(z_{n,j} - d(t) - \sum_{k<n}a_k(t)\right)^+, \bar{a}\right\}, \forall t \in [t_j, t_j+1], \forall j = 1,\ldots,2N-1, \forall n \in \mathcal{I}(t_j). \tag{A.17}$$

Note that the decision variables of the original objective function (22), i.e., $\{a_n(t),\,n = 1,\ldots,N\}$, are replaced with the thresholds $z_{n,j}$'s in (A.15). In this formulation, (A.17) implicitly defines the

charging schedule as a function of the thresholds.

Similar to the simultaneous arrivals case, the charging problem (22)-(26) can be expressed in the standard form (A.1)-(A.3), with the additional constraint that vehicle n cannot be charged if it is not at the station at time t, i.e., $a_n(t) = 0, \forall t \notin [s_n, \tau_n]$.

We prove the optimality of the charging policy for non-simultaneous arrivals (given $z_{n,j}$'s) by induction. First consider the time interval $[t_{2N-1}, t_{2N}]$ in which there is only one electric vehicle at the station. Let its index be k. Consistent with the standard form (A.1)-(A.3), denote the level of electricity charged by time t_{2N-1} to vehicle k by $x_k(t_{2N-1})$. Given that vehicle k is the only vehicle in the station in this time interval, we can use Pontryagin's minimum principle to identify the optimal charging schedule as in the proof of Proposition 1: There exists a threshold $z_{k,2N-1}$, where $\int_{t_{2N-1}}^{t_{2N}} \min\left\{ (z_{k,2N-1} - d(t))^+, \bar{a} \right\} \mathrm{d}t = 1 - x_k(t_{2N-1})$ such that it is optimal to set $a_k^*(t) = \min\left\{ (z_{k,2N-1} - d(t))^+, \bar{a} \right\}$ for $t \in [t_{2N-1}, t_{2N}]$. It is straightforward to verify that the charging policy for non-simultaneous arrivals essentially leads to the same charging decision. Therefore, the policy is optimal for $t \in [t_{2N-1}, t_{2N}]$.

Suppose the policy is optimal for intervals $[t_{j+1}, t_{j+2}], \ldots, [t_{2N-2}, t_{2N-1}]$ for some $j \in \{1, \ldots, 2N-3\}$. We next prove the policy is also optimal in $[t_j, t_{j+1}]$. For this time interval, let $a_n^*(t)$ be an optimal solution for $n \in \mathcal{I}(t_j)$. The charging policy for non-simultaneous arrivals (given $z_{n,j}$'s) produces a charging schedule for vehicles $k \in \mathcal{I}(t_j)$ such that each vehicle is charged $\int_{t_j}^{t_{j+1}} a_k^*(t)\mathrm{d}t$ units of electricity by time t_{j+1}. As discussed in the proof of Proposition 1, this schedule is optimal for $t \in [t_j, t_{j+1}]$ because it is equivalent to charging one electric vehicle with $\sum_{k \in \mathcal{I}(t)} \int_{t_j}^{t_{j+1}} a_k^*(t)\mathrm{d}t$ units of electricity by time t_{j+1}. Given that the policy for non-simultaneous arrivals is also optimal for $[t_j, t_{j+1}]$, by induction, it is optimal for all intervals.

Finally, in the above analysis, without loss of generality, we assume that when a customer arrives at the charging station there is at least another customer whose vehicle is being charged. Otherwise, the utility firm's problem can be decoupled into multiple problems, which can be solved separately. ∎

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