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## **Nature of VIX Jumps on Market Timing of Hedge Funds**<sup>∗</sup>

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#### **Abstract**

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The study indicates that Brownian motion, finite and infinite activity jumps are present in the ultra-high frequency VIX data. The total quadratic variation can be split into a continuous component of 29% and a jump component of 71%. Jump activities on ultra-high frequency VIX data are found informative in ex-ante identifying subgroups of hedge funds that deliver significant outperformance. In the months that follow large jumps, strategies exposing to long volatility and extreme risk tend to deliver positive performance in extreme market environments. In the months that follow small jumps, possibly as a result of trading illiquidity, most fund strategies exhibit losses in the jolting market environments. In the months that follow Brownian motion, strategies exposing to short volatility tend to deliver best performance. Hedge funds therefore deliver out-of-sample performance respective of types of jump activities on ultra-high frequency VIX.

#### *JEL classification:* G12; G13; G14

*Keywords:* Ultra-high frequency VIX; Infinite jump activity; Finite jump activity; Brownian motion; Hedge fund strategies

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## **1. Introduction**

The study undertakes a model-free analysis of the ultra-high frequency movements in the Chicago Board Options Exchange (CBOE) Volatility Index (VIX). The CBOE VIX is a key measure of market expectations of near-term volatility conveyed by S&P 500 Index (SPX) option prices. The VIX is widely referred to as a broad signal of investor sentiment and market volatility. Whaley (2000) terms it the "investor fear gauge" or the "market temperature" that can tell us how optimistic or pessimistic investors are. The VIX is also believed to be an indicator of investor appetite for risk (Dash and Moran, 2005).

Volatility in general, and VIX in particular, is widely thought to influence hedge fund returns (Drummond, 2005; Black, 2006; Dash and Moran, 2005; Bondarenko, 2007; Agarwal, Bakshi and Huij, 2009; Avramov, Kosowski, Naik and Teo, 2011). Hedge funds often employ derivatives, short-selling, and leverage (Fung and Hsieh, 2001; Weisman, 2002; Bondarenko, 2007; Diez and Garcia, 2011) to generate returns during extreme states of the equity market, and this can lead to hedge funds being exposed to higher-moment risks of the equity market. There is evidence that VIX can be a proxy for other economic drivers of hedge fund returns (Anson, Ho and Silberstein, 2005). For instance, the change in VIX is correlated with proxies for the credit, liquidity, and correlation risks (Brunnermeier and Pedersen, 2009; Billio,

Getmansky and Pellizon, 2009; Boyson, Stahel and Stulz, 2010; Dudley and Nimalendran, 2011; Akay, Senyuz and Yoldas, 2011). The first differences in VIX have also been used to proxy market volatility in the extant literature (e.g., Ang, Hodrick, Xing and Zhang, 2006; Agarwal, Bakshi and Huij, 2009). Table 1 reports the hedge fund return exposure to change in VIX across quintiles of SPX over October 2003 to July 2010. Within each group, we further split funds into those that have positive change in VIX and those that do not. Diversified hedge funds are negatively correlated to changes in VIX, except in the subgroup of worst SPX accompanied with decreasing VIX, which is consistent with the findings of Liew and French (2005), Schneeweiss, Kazemi and Martin (2002, 2003) and Amenc, El Bied and Martellini (2003). At the individual strategy level, some strategies such as Trend Following and Managed Futures hedge funds outperform at times of market downturns, whereas Macro, Equity Market Neutral and Distressed Restructuring hedge funds benefit from low- to moderate-volatility environments (Anson and Ho, 2003).

#### **[Table 1 about here]**

We report in Figure 1 the average monthly hedge fund returns of each quintile across five states. The SPX, VIX and absolute percentage change in VIX are respectively sorted into five states. Consistent with Fung and Hsieh (2001) and Agarwal and Naik (2004), the results show that a large number of equity-oriented hedge fund strategies such as Equity Hedge, Convertible Arbitrage and Distressed Restructuring exhibit payoffs resembling a short position in an equity index put option and therefore bear significant left-tail risk. Other strategies such as Trend Following and Managed Futures deliver returns resembling those of a portfolio of straddles. Our evidence suggests that the success of these strategies hinges on the behavior of various economic indicators. Therefore, conditioning on jump activities on VIX may allow one to better predict hedge fund returns over the volatility cycle.

#### **[Figure 1 about here]**

The factors that affect movements in the VIX, however, are always in flux. Being able to distinguish between continuity, small jumps, and large jumps in sample path of VIX movements and determining the relative magnitude of those components should be a welcome addition to the toolkit of investors, hedge fund managers, and regulators. It is thereby economically important to develop a statistical understanding of the fine structure of jumps in VIX.

Statistical tests are developed to determine on the basis of the observed log-returns whether a jump component is present (Aït-Sahalia and Jacod, 2009*b*), whether the jumps have finite or infinite activity (Aït-Sahalia and Jacod, 2012), an estimate of a degree of jump activity (Aït-Sahalia and Jacod, 2009*a*), and whether a Brownian motion is needed when infinite activity jumps are included (Aït-Sahalia and Jacod, 2010). Alternative methodologies exist for studying the continuous and jump components from discretely sampled semimartingales, including (i) splitting the quadratic variation into continuous and discontinuous proportions to test for the presence of jumps (Aït-Sahalia, 2002; Carr and Wu, 2003; Barndorff-Nielsen and Shephard, 2004; Huang and Tauchen, 2005; Andersen, Bollerslev and Diebold, 2007; Jiang and Oomen, 2008; Lee and Mykland, 2008; Lee and Hannig, 2010), (ii) using the statistic from Aït-Sahalia and Jacod (2009*b*) to identify the presence of a Brownian component (Tauchn and Todorov, 2010), and (iii) using threshold or truncation-based estimators of the continuous component of the quadratic variation to test for the presence of a continuous component (Mancini, 2001).

The study uses the methodology developed by Aït-Sahalia (2004), Aït-Sahalia and Jacod (2009*a*,*b*) and Aït-Sahalia and Jacod (2010, 2012), which is a unified approach, to detect if there exists Brownian motion, infinite activities (small jumps), or finite activities (big jumps) in the ultra-high frequency VIX data.<sup>1</sup> The basic methodology consists in constructing realized power variations of VIX increments, suitably truncated and/or sampled at different frequencies. For this to work, the VIX data need to have a lot of depth; that is, highly traded, down to the 15 seconds in this study.

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<sup>1</sup> The authors fully acknowledge Aït-Sahalia and Jacod's MATLAB codes to generate various test statistics, which are available on the website at http://www.princeton.edu/~yacine.

This paper begins by undertaking various model-free tests for presence of a continuous component, small jumps and large jumps from discrete observations compiled from movements in ultra-high frequency VIX data. Empirical results indicate that a continuous component, finite and infinite activity jumps are present in the ultra-high frequency VIX data. The degree of jump activity from Q4 2003 to Q2 2010 is in the range from 1.71 to 1.95, indicating a very high degree of jump activity. The total quadratic variation can be split into a continuous component of 29% and a jump component of 71%, which by construction is attributable to small and big jumps. Relative to our findings, some (semi)parametric evidence for presence of jumps in the spot volatility has been proposed by Eraker, Johannes and Polson (2003), Eraker (2004), Broadie, Chernov and Johannes (2007), Todorov (2010), and Bjursell, Wang and Webb (2011).

Since VIX contains ex-ante volatility, it may, in theory, have some predictive power in hedge fund returns that are affected by volatility. By examining out-of-sample hedge fund returns, jump activities on ultra-high frequency VIX data are informative in ex-ante identifying subgroups of hedge funds that deliver significant outperformance. In the months that follow large jumps, strategies exposing to long volatility and extreme market risk such as Trend Following and Managed Futures hedge funds tend to deliver positive performance in extreme market environments. In the months that follow small jumps, possibly as a result of trading illiquidity, most fund strategies exhibit losses in the jumping-around market environments. In the months that follow Brownian motion, strategies exposing to short volatility such as Risk Arbitrage, Merge Arbitrage, Event Driven, Equity Hedge and Relative Value tend to deliver best performance.

To summarize, the study goes nicely with the literature, trying to price VIX futures and options with either diffusion or jumps or both, or market-timing hedge fund strategies. The study points out to which type of models or hedge fund strategies researchers and practitioners should focus on when there exists purely Brownian motion, or jumps, or some combinations in the ultra-high frequency VIX data.

The paper is organized as follows. Section 2 presents the measurement device designed to analyze which components are present, in what relative proportions, and the degree of activity of the jumps. Section 3 describes the data and reports the results of applying the measure devices to the time series of ultra-high frequency VIX. Section 4 uses jump activities on VIX as market-timing signals to exploit predictability in the performance of hedge fund strategies. Section 5 concludes.

## **2. Methodology**

#### **2.1 Measurement Device**

The study examines which component(s) of jumps, finite or infinite activity, and a continuous component, originating in a given unobserved path, need to be included in the observed ultra-high frequency VIX and their relative magnitude. The study uses 15-second VIX data to analyze their finer characteristics such as the degree of activity of jumps. Consider  $[T/\Delta_n]$  observed increments of logarithmic VIX on [0, T], which are collected at a discrete sampling interval  $\Delta_n$ :

$$
\Delta_n^i ln VIX \equiv ln VIX_{i\Delta_n}^* + \varepsilon_i - (ln VIX_{(i-1)\Delta_n}^* + \varepsilon_{i-1})
$$
\n(1)

where  $\left(\ln VIX_{i\Delta_n}^*, \ln VIX_{(i-1)\Delta_n}^*\right)$  denote the true values;  $(\varepsilon_i, \varepsilon_{i-1})$  are *i.i.d.* market microstructure noise, not depending of the observation frequency.

Following Aït-Sahalia and Jacod (2012), the realized power variations of these increments, suitably truncated and/or sampled at different frequencies, are defined as

$$
B(p, u_n, \Delta_n)_T = \sum_{i=1}^{[T/\Delta_n]} \left| \Delta_n^i lnVIX \right|^p \times 1_{\left\{ \left| \Delta_n^i lnVIX \right| \le u_n \right\}} \tag{2}
$$

where  $p \ge 0$  is the power variable to accentuate either the continuous ( $p < 2$ ) or jump ( $p > 2$ ) components or to keep them both present ( $p = 2$ ). A sequence of truncation levels  $u_n > 0$  can eliminate or retain only the increments larger than  $u_n$ . Typically the truncation levels  $u_n$  are usually achieved by taking  $\eta$  units of standard deviations of the continuous part

$$
u_n = \eta \sigma_S \sqrt{\Delta_n} \tag{3}
$$

with  $\eta = (\alpha/\sigma_s)\Delta_n^{\pi^{-1/2}}$  for some constants  $\sigma \in (0,1/2)$  and  $\alpha \in (0,1)$ . The

behavior of the truncated power variations  $B(p, u_n, \Delta_n)_T$  depends on the degree of activity of the jumps when there are infinitely many jumps.

Retaining only the increments larger than  $u_n$  is written as

$$
U(p, u_n, \Delta_n)_T = \sum_{i=1}^{[T/\Delta_n]} \left| \Delta_n^i lnVIX \right|^p \times 1_{\left\{ \left| \Delta_n^i lnVIX \right| > u_n \right\}} \tag{4}
$$

, which can allow one to eliminate all the increments from the continuous part of the model. Then obviously

$$
U(p, u_n, \Delta_n)_T = B(p, \infty, \Delta_n)_T - B(p, u_n, \Delta_n)_T
$$
\n<sup>(5)</sup>

Placing  $u_n \to \infty$ ,  $B(p, \infty, \Delta_n)$ , gives no truncation at all. The number of increments of *lnVIX* from the jump part of the model is therefore counted as taking the power  $p = 0$ :

$$
U(0, u_n, \Delta_n)_T = \sum_{i=1}^{[T/\Delta_n]} 1_{\{|\Delta_n^i ln VIX| > u_n\}} \tag{6}
$$

The study exploits the different asymptotic behavior of the variations  $B(p, u_n, \Delta_n)$  and/or  $U(p, u_n, \Delta_n)$  by varying the power p, the truncation level  $u_n$ and the sampling frequency  $\Delta_n$ . Formally, using observed high-frequency data originating in a given unobserved path, the study adopts the power variations method to address in which set(s) of jumps, finite or infinite activity, and Brownian motion the path of  $lnVIX$  defined pathwise on  $[0, T]$  contains.

## **2.2 Ultra-High Frequency VIX Data**

The study uses ultra-high frequency VIX transactions from October 1, 2003 to

June 30, 2010. The data source is the TickData database. The VIX indicator is only updated by the exchange every 15 seconds since Q4 2003; the VIX series are thus sampled every 15 seconds. The study does not include the overnight changes in VIX. Figure 2 shows the tail distributions of the log differences from VIX over the sample period, respectively sampled at 15-second and daily frequencies. The 15-second VIX log differences centralize around zero with asymmetric long tails in relatively low probabilities. This histogram with large positive skewness and extremely high kurtosis is extraordinarily different from the one in the daily VIX log differences, which has much smaller positive skewness (almost insignificantly) and much less kurtosis (but still greater than 3). Being able to distinguish the distinct features between ultra-high frequency and daily data is therefore important, as it has implications for many high frequency trading strategies that rely on specific components of the model being present or absent.

#### **[Figure 2 about here]**

Using ultra-high frequency log differences as inputs, the study deconstructs the observed series back into its original components, continuous and jumps. Each one of the statistics is computed separately for each quarter of the sample period. The truncation cutoff level  $u_n$  is expressed in terms of a number  $\eta$  of standard deviations of the continuous part of the semimartingale. That initial annualized

standard deviation estimate  $\sigma_s = 0.3467$  that is obtained by using  $B(2, 4 \times$  $\hat{\sigma}_s \sqrt{\Delta_n}$ ,  $\Delta_n$ ) with the median  $\hat{\sigma}_s = 0.25$  of annualized standard deviation of daily changes in logarithmic VIX, serves to identify a reasonable range of values. The study then uses different multiples of it for the truncation level  $u_n$ .

## **3. Empirical Results**

## **3.1 Jumps: Present or Not**

The test statistic  $S_j$  discriminates between jumps and no jumps based on observed data, but not among different types of jumps. Taking microstructure noise into account, the test statistic is given by

$$
S_j(p, k, \Delta_n) = \frac{B(p, \infty, k\Delta_n)}{B(p, \infty, \Delta_n)}\Big|_{p>2, k\geq 2}
$$
  
\n
$$
\xrightarrow{\mathbb{P}} \begin{cases} 1/k & additive noise dominates\\ 1/\sqrt{k} & rounding error dominates (and jumps have finite activity)\\ 1 & jumps present and no significant noise\\ k^{p/2-1} & no jumps present and no significant noise \end{cases}
$$

$$
(7)
$$

The histogram for empirical values of  $S_j$  is shown in Panel A of Figure 3. The data for the histogram are produced by computing  $S_j$  for the twenty-seven quarters from Q4 2003 to Q2 2010, and for a range of values of  $p$  from 3 to 6,  $\Delta_n$  from 15 seconds to 2 minutes, and  $k = 2, 3$ . As indicated in (7), values around 1 are indicative of jumps presence and the noise is not the major concern. Panel B of Figure 3 displays the mean value of  $S_j$  across values of  $p$  and  $k$  and the twenty-seven quarters as a

function of  $\Delta_n$ . For very small values of  $\Delta_n$ , the noise dominates (limits below 1), then the limit is around 1 as  $\Delta_n$  increases away from the noise-dominated frequencies. In general, the study finds that the average of  $S_j$  stays around 1 for the majority of its sampling frequencies. The conclusion from  $S<sub>j</sub>$  is that the noise is not a major concern at the ultra high frequencies, and the evidence points towards the presence of jumps.

#### **[Figure 3 about here]**

This conclusion confirms simple inspection of the tails of the 15-second log-return distribution in Figure 2. Clearly, a continuous component alone would be very unlikely to generate such returns in such tails.

## **3.2 Jumps: Finite or Infinite Activity**

 $S_J$  tests whether jumps are likely to be present, but it cannot distinguish between finite and infinite activity jumps. The statistic  $S_{FA}$ , which is like  $S_j$  with the addition of truncation, discriminates between finite and infinite activity jumps based on observed data. Taking microstructure noise into account, the test statistic is given by

$$
S_{FA}(p, u_n, k, \Delta_n) = \frac{B(p, u_n, k\Delta_n)}{B(p, u_n, \Delta_n)}\Big|_{p>2, k\geq 2}
$$
  
\n
$$
\longrightarrow \begin{cases}\n1/k & additive noise dominates \\
1 & infinite activity jumps present and no significant noise \\
k^{p/2-1} & finite activity jumps present and no significant noise\n\end{cases}
$$
\n
$$
(8)
$$

The histogram in Panel A of Figure 4 is produced by computing for the twenty-seven quarters from Q4 2003 to Q2 2010 the value of  $S_{FA}$  for a range of values of p from 3 to 6,  $\eta$  from 5 to 10 standard deviations,  $\Delta_n$  from 15 seconds to 2 minutes, and  $k=2$ , 3. The result shows that the empirical values of  $S_{FA}$  are distributed around 1, which is indicative of infinite activity jumps and the noise is not the major concern. The justification is that if only a finite number of jumps had been present, then the statistic should have behaved as if the process were continuous.

Panel B of Figure 4 presents the mean value of  $S_{FA}$  across the twenty-seven quarters and values of p,  $\eta$  and k as a function of  $\Delta_n$ . For very small values of  $\Delta_n$ the limit is below 1 as indicative of noise dominating, then the infinite jump activity dominates (limits around 1) as  $\Delta_n$  increases away from the noise-dominated frequencies. Based on the VIX data, the statistic  $S_{FA}$  identifies the likely presence of infinite activity jumps.

#### **[Figure 4 about here]**

For the robustness check, the study tests the null being infinite activity jumps and the alternative of finite activity jump by choosing  $\gamma > 1$  and  $p' > p > 2$  and the  $S_{IA}$  test statistic as follows:

$$
S_{IA}(p, u_n, \gamma, \Delta_n) = \frac{B(p', \gamma u_n, \Delta_n)B(p, u_n, \Delta_n)}{B(p', u_n, \Delta_n)B(p, \gamma u_n, \Delta_n)}\Big|_{p' > p > 2, \gamma > 1}
$$
  
\n
$$
\longrightarrow \begin{cases}\n1 & \text{finite activity jumps present and no significant noise} \\
\gamma^{p'-p} & \text{infinite activity jumps present and no significant noise}\n\end{cases}
$$
\n(9)

As shown in Figure 5, the statistic  $S_{IA}$  identifies the likely presence of finite jumps.

#### **[Figure 5 about here]**

#### **3.3 Brownian Motion: Present or Not**

When infinitely many jumps are included, there are a number of models in the literature which dispense with the Brownian motion. The price process is then a purely discontinuous Lévy process with infinite activity jumps, or more generally is driven by such a process. In order to decide whether the Brownian motion really exhibits in the data, or if it can be forgone with in favor of a pure jump process with infinite activity, the test statistic  $S_W$  discriminates between the Brownian motion and pure infinite activity jumps based on observed data. The null hypothesis is set to detect the presence of the Brownian motion, whereas the alternative assumes that there is no Brownian motion but exist infinitely active jumps. Taking microstructure noise into account, the test statistic is given by

$$
S_W(p, u_n, k, \Delta_n) = \frac{B(p, u_n, \Delta_n)}{B(p, u_n, k\Delta_n)}\Big|_{1 < p < 2, k \ge 2}
$$
  
\n
$$
\xrightarrow{\mathbb{P}} \begin{cases} 1/k & \text{additive noise dominates} \\ n \text{ of } n \text{
$$

Panel A of Figure 6 displays a histogram of the distribution of  $S_W$  obtained by computing its value for the twenty-seven quarters from Q4 2003 to Q2 2010 for a range of values of p from 1 to 1.75,  $\eta$  from 5 to 10 standard deviations,  $\Delta_n$  from 15 seconds to 2 minutes, and  $k = 2$ , 3. The majority of empirical estimates are on the side of the limit arising in the presence of a continuous component. As the sampling frequency increases, the noise becomes more of a factor, and for very high sampling frequencies, the results are indicative of little noise driving the asymptotics. This is confirmed by Panel B of Figure 6 which displays the mean value of  $S_W$  across the twenty-seven quarters and values of p,  $\eta$  and k as a function of  $\Delta_n$ . As the study downsamples away from the noise-dominated frequencies, the average value of the statistic settles down towards the 1.5 indicating presence of a Brownian motion. The log-price process is not driven by a purely discontinuous Lévy process with infinite activity jumps.

## **[Figure 6 about here]**

## **3.4 Relative Magnitude of the Components**

The previous empirical results indicate the presence of a jump and a continuous component. This section examines what fraction of the quadratic variation  $(QV)$  is attributable to the continuous and jump components. The relative magnitude of the two jump and continuous components in the total  $QV$  after taking microstructure noise into account is determined as

\n $\begin{cases}\n 0 & \text{additive noise dominates} \\  0 & \text{rounding error dominates} \\  \frac{B(2, u_n, \Delta_n)}{B(2, \infty, \Delta_n)} & \text{for } QV \text{ due to the continuous component} \\  1 - \frac{B(2, u_n, \Delta_n)}{B(2, \infty, \Delta_n)} & \text{for } QV \text{ due to the jump component} \\  & \text{and no significant noise}\n \end{cases}$ \n	(11)
---	------

Since the conclusion from  $S_J$ ,  $S_{FA}$  and  $S_W$  statistics is that the noise is not the major concern, it is feasible to calculate the relative magnitude of the two jump and continuous components in the total  $QV$ . Panel A of Figure 7 displays empirical distribution of the proportion of  $QV$  attributable to the Brownian component using the twenty-seven quarters, values of  $\eta$  ranging from 2 to 5 standard deviations, and  $\Delta_n$  from 15 seconds to 2 minutes. The study finds values attributable to the continuous component around 29%. In Panel B of Figure 7, the average fraction of  $QV$  attributable to the continuous component is fairly stable as the sampling frequency varies.

#### **[Figure 7 about here]**

The percentage of  $QV$  attributable to jumps can be further decomposed into a small jump and a big jump component depending on the cutoff level  $\varepsilon$ :

$$
\%QV\ due\ to\ big\ jumps = \frac{U(2,\varepsilon,\Delta_n)}{B(2,\infty,\Delta_n)}\tag{12}
$$

$$
\%QV\ due\ to\ small\ jumps\ =\ 1 - \frac{B(2,u_n,\Delta_n)}{B(2,\infty,\Delta_n)} - \frac{U(2,\varepsilon,\Delta_n)}{B(2,\infty,\Delta_n)}\tag{13}
$$

## **3.5 Estimating the Degree of Jump Activity**

The previous statistics above indicate the presence of Brownian motion, finite

activity jumps and infinite activity jumps in the data. Following Aït-Sahalia and Jacod (2012), the study indexes the activity of  $lnVIX$  as the Blumenthal-Getoor index  $\beta$ that characterizes the local behavior of the Lévy measure  $\nu$  near 0.<sup>2</sup> Aït-Sahalia and Jacod (2012) propose an estimator of  $\beta$  in the presence of a continuous component of the model; that is, the test statistic allows one to eliminate the increments due to the continuous component. The test statistic is based on varying the actual cutoff level: fix  $0 < \eta < \eta'$  and consider two cutoffs  $u_n = \eta \sigma_S \sqrt{\Delta_n}$  and  $u'_n = \eta' \sigma_S \sqrt{\Delta_n}$  with  $\gamma = \eta'/\eta$ :

$$
\beta_n(\eta, \gamma, \Delta_n)|_{\gamma > 1} = \ln \left( \frac{U(0, u_n, \Delta_n)}{U(0, \gamma u_n, \Delta_n)} \right) / \ln(\gamma) \tag{14}
$$

Estimating  $\beta$  requires the largest sample size due to its reliance on truncating from the right in the power variations U. The degree of infinite jump activity  $\beta$  is thus estimated using a number of observations to the right of a cutoff  $u_n$  given by 2 standard deviations of the continuous part, values of  $\gamma$  ranging from 1.50 to 1.75, and various sampling frequencies from 15 seconds to 2 minutes on a quarterly basis.

Panel A of Figure 8 displays the empirical distribution of the index of jump activity  $\beta$  computed for the twenty-seven quarters from Q4 2003 to Q2 2010. Panel B of Figure 8 presents the average value of estimated  $\beta$  as a function of the sampling interval  $\Delta_n$  employed. In the presence of Brownian motion and infinite activity

 $\overline{a}$ 

<sup>&</sup>lt;sup>2</sup> The activity index is 2 asymptotically in a continuous martingale and 0 in a purely finite activity jumps model.

jumps, the estimated  $\beta$  is in the range from 1.71 to 1.95, indicating a very high degree of jump activity. A slightly dissenting result is Tauchen and Todorov (2011) who suggest that the latent spot volatility, extracted from high frequency VIX data, is a pure jump process with jumps of infinite variation and activity close to that of a continuous martingale.

### **[Figure 8 about here]**

## **4. Market Timing Hedge Funds**

This section classifies ultra-high VIX activities as Brownian motion, small jumps and large jumps, and uses these categories augmented with the sign of change in VIX as predictive variables to see whether jumping, moving around randomly, or Brownian motion could predict hedge fund performance.

The study analyzes the ex-post out-of-sample performance of hedge funds using monthly returns reported in the Dow Jones Credit Suisse, Hedge Fund Research, and Institutional Advisory Services Group datasets over November 2003 to July 2010 ― a time period that covers both market upturns and downturns, as well as relatively calm and turbulent periods. Following Agarwal, Daniel and Naik (2009) and Avramov, Kosowski, Naik and Teo (2011) to classify hedge funds into different strategy classes, the multitude of hedge funds we use includes five investment categories: (i) directional traders consisting of Managed Futures, Macro/CTA and Trend Following,

(ii) relative value consisting of Relative Value Arbitrage and Convertible Arbitrage, (iii) security selection consisting of Equity Hedge and Equity Market Neutral, (iv) multi-process consisting of Event Driven, Merger Arbitrage, Distressed Restructuring and Risk Arbitrage, and (v) fund of funds consisting of Equal Weighted and Global. $3$ 

To understand the impact that jump activities on VIX have on fund performance, the study sorts funds into three groups based on whether their previous months are dominated by large jumps, small jump, or Brownian motion. Next, within each group, the study further splits funds into those that have positive monthly change in VIX in their previous months and those that do not. The jump activities are detected based on intraday VIX activities within one month of data and are reformed every month. Given the sample period October 2003 to June 2010, there are in total 60 months dominated by large jumps including the Q4 2008 financial crisis, 3 by small jumps, and 18 by Brownian motion.<sup>4</sup> Table 2 illustrates in-sample and out-of-sample market exposure to Brownian motion, small jumps and big jumps, respectively. The big jumps are usually accompanied with high VIX, high and, in particular, low SPX, as well as large variation in absolute percentage monthly changes in VIX. The Brownian

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<sup>&</sup>lt;sup>3</sup> We acknowledge that there are many issues with hedge fund databases, such as selection, survivorship and instant history biases, which provide an upward bias to hedge fund returns. The purpose of our research is not to determine the absolute size of hedge fund returns, but rather to observe how hedge fund returns react to VIX activities.

<sup>4</sup> The months dominated by small jumps consist of June 2006, July 2006 and July 2007, while those mainly attributable to Brownian motion include October-November 2003, February-March 2004, May-August 2004, October 2004, January 2005, June 2005, April 2006, August 2006, October 2006, January 2007, April 2007, June 2007 and March 2010. The rest of the sample is dominated by large jumps.

motion usually exists with low VIX, medium SPX and low absolute percentage monthly change in VIX. Noticeably, in-sample market exposure of absolute percentage monthly change in VIX to small jumps significantly drops into a relatively low out-of-sample level.

#### **[Table 2 about here]**

Table 3 and Figure 9 evaluate the out-of-sample performance of hedge funds in each sub-group over the sample period. Traditional risk/return measures such as Sharpe ratios and standard deviations are inadequate to measure risk for hedge funds with highly non-normal distributions and large tails. These are the three measures to gauge hedge performance when applied to a single hedge fund strategy: (i) using maximum drawdown as a downside risk measure; (ii) using adjusted conditional Value-at-Risk as a measure of extreme tail risk; and (iii) using extended Sharpe ratio as a measure of the excess return relative to risk with highly non-normal distributions and large tails.

First measure is the magnitude of maximum drawdown for monthly returns on the hedge fund:

$$
MaxDD(T;R) = \max_{0 \le t \le T} \left[ R_{0 \le t \le t}^{peak} - R(t) \right] \tag{15}
$$

where  $R(t)$  is the out-of-sample monthly return respective of VIX jump activities, and  $R_{0 \leq \tau \leq t}^{peak} = \max_{0 \leq \tau \leq t} [R(\tau)]$  is the maximum monthly return in the [0,t] period.

 $MaxDD(T)$  is defined as the maximum sustained decline (peak to trough) for period  $[0, T]$ , which provides an intuitive and well-understood empirical measure of the loss arising from potential extreme events (Magdon-Ismail et al., 2004; Magdon-Ismail and Atiya, 2004).

Second measure is the magnitude of the expected shortfall or conditional Value-at-Risk  $(CVaR)$  (Rockafellar and Uryasev, 2002) for monthly returns on the hedged fund at the confidence level  $1 - \varphi$ :

$$
CVaR_{1-\varphi}(R_{CF}) = \mu(R) + \sigma(R) \cdot E(z_{cf,1-\kappa} | \kappa > 1 - \varphi)
$$
  
=  $\mu(R) + \sigma(R) \times E\left[\begin{array}{l} z_{1-\kappa} + \frac{1}{6}(z_{1-\kappa}^2 - 1)S(R) \\ + \frac{1}{24}(z_{1-\kappa}^3 - 3z_{1-\kappa})K(R) \\ - \frac{1}{36}(2z_{1-\kappa}^3 - 5z_{1-\kappa})S(R)^2 \end{array}\right] \kappa > 1 - \varphi$  (16)

where  $R_{CF}$  is the monthly returns on the hedge fund that uses the Cornish-Fisher expansion to incorporate skewness and kurtosis into the return distribution (Cornish and Fisher, 1938; Baillie and Bollerslev, 1992; Liang and Park, 2010).  $z_{1-\kappa}$  is the critical value for probability  $1 - \kappa$  with standard normal distribution (e.g.  $z_{1-\kappa} = -1.64$  at  $\kappa = 95\%$ ), while  $\mu$ ,  $\sigma$ , S and K follow the standard definitions of mean, volatility, skewness and excess kurtosis, respectively, as computed from the monthly returns on the hedge fund.

Third measure is the magnitude of the extended Sharpe ratio (denoted  $ESR$ ) for monthly returns on the hedge fund:

$$
ESR = \frac{1}{z_{\pi} \sigma_{\pi}} \left( e_{\pi} + \frac{1}{2} (z_{\pi}^{+} \sigma_{\pi})^{2} - \frac{1}{2} (z_{\pi}^{-} \sigma_{\pi})^{2} \right)
$$
(17)

where 
$$
e_{\pi}
$$
 = excess monthly return rate of the hedge fund  $\pi$ ;  $\sigma_{\pi}$  = volatility of  $\pi$ ;  
\n $z_{\pi}^{+} = \frac{\max(z_{cf,\kappa}(\pi).0)}{z_{\kappa}}$ ;  $z_{\pi}^{-} = \frac{\min(z_{cf,\kappa}(\pi).0)}{z_{1-\kappa}}$ ; for example,  $z_{\kappa} = 2.33$  at  $\kappa = 1\%$ ,  
\n $z_{1-\kappa} = -2.33$  at  $1 - \kappa = 99\%$ . Note that *ESR* is an omega-function-like measure.  
\nThe numerator is a measure of upside cumulants while the standard deviation of  
\nreturns in the denominator is replaced by a measure of downside cumulants (Karatzas  
\nand Shreve, 1998; Fernholz, 2002; Keating and Shadwick, 2002). This is a more  
\nbalanced measure from the perspective of not only minimizing risk (which also tends  
\nto minimize returns) but also achieving a balance between upside and downside  
\nmoments, and is generally consistent with the real-world practice in that hedge fund  
\nmanagers tend to take risk to preserve upside.

The hedge funds are found to deliver performance respective of market conditions. There is always some volatility-based risk in any hedge fund, but these sharp changes in value can be positive as well as negative. In one or more specific market conditions, market volatility can be exploited to the benefit of investors. Big jumps on VIX offer potentially sizable directions of markets to those who long or short markets attempting to capture their rise and fall, while Brownian motion on VIX attract trading styles of investment that is expected to be long and short comparable securities to capture value while eliminating the systematic risk of the markets.

Finally, small infinite activity jumps on VIX are more likely to reflect immediate jolting market environment such as trading illiquidity.

Trend Following, Managed Futures and Macro funds employ the market timing approach that bets on the directions of markets dynamically to achieve absolute return targets. Therefore, they are found to outperform in the months following big jumps relative to the months following small jumps and Brownian motion. In contrast, Relative Value, Equity Hedge, Event Driven, Convertible Arbitrage and Distressed Restructuring funds are the non-directional style that attempt to extract value from a set of diversified arbitrage opportunities targeted at exploiting structural anomalies of markets. As a non-directional approach, it is a low volatility approach, and the returns resemble that of a high yielding bond-like instrument without the equivalent interest rate or credit risk. They are found to outperform in the months following Brownian motion. Further, many hedge funds outperform by buying illiquid securities and short-selling liquid securities. Naturally such funds are also susceptible to liquidity shocks like the 2008 Lehman Brothers collapse. When markets are illiquid, hedge fund performance is highly sensitive to changes in funding liquidity as well as asset liquidity and leads to higher volatility. Perhaps arbitrage funds in general are short liquidity as well as short volatility. They perform best in calm markets, and worse in the jumping-around markets that are driven by small jumps on VIX. It is likely that a

common exposure to preceding small jumps on VIX drives negative performance in most of the funds. The returns of Risk Arbitrage, Convertible Arbitrage and Merger Arbitrage funds are the only investment styles that achieve positive average returns in the months following small jumps. However, only Merger Arbitrage funds escape the downturn in performance during the period following small jumps accompanied with increasing VIX. As a result, the ESR of Merger Arbitrage is found positive.

#### **[Table 3 about here]**

#### **[Figure 9 about here]**

Hedge funds are considered riskier than other types of investment vehicles because they employ strategies which can result in sharp losses if managed poorly. The study uses the following regression specification to evaluate the economic significance of manager skill conditional on jump activities on VIX. To adjust for risk, the study evaluates the performance of hedge funds relative to the Fung and Hsieh  $(2004)$  seven-factor model:<sup>5</sup>

$$
r_{i,t} = \sum_{k=1}^{7} \beta_{i,k} F_{k,t} + \alpha_{i,con} D_{con,t-1} + \alpha_{i,sj} D_{sj,t-1} + \alpha_{i,bj} D_{bj,t-1} + \varepsilon_{i,t}
$$
(18)

where  $r_{i,t}$  is the return on fund i in excess of the one-month Treasury bill return (the risk-free rate) in month t.  $F_{k,t}$  is the kth risk factor of the benchmark factor model, including (i) the S&P 500 return minus the risk-free rate,  $SNPMRF$ ; (ii) the Russell

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<sup>5</sup> Our results are not sensitive to augmenting the Fung and Hsieh (2004) model with the MSCI emerging markets index excess return, the Fama and French (1993) high-minus-low book-to-market factor, the Jegadeesh and Titman (1993) momentum factor, or the Pástor and Stambaugh (2003) liquidity factor.

2000 minus S&P 500 monthly total return,  $SCMLC$ ; (iii) the monthly change in the 10-year Treasury constant maturity yield, *BD10RET*; (iv) monthly changes in the credit spread defined as Moody's Baa bond yield minus the 10-year Treasury bond yield, BAAMTSY; and (v) excess returns on portfolios of lookback straddles on bonds (PTFSBD), currencies (PTFSFX) and commodities (PTFSCOM).<sup>6</sup> The data on SNPMRF, SCMLC, BD10RET, and BAAMTSY are available from Datastream. Finally,  $\varepsilon_{i,t}$  is fund *i*'s residual return in month *t*. The three dummy variables are a convenient means of building discrete shifts of the excess fund returns in accordance with jump activities on VIX in month  $t-1$ . We use the dummy variables so that we can pool different jump activities in the same regression. Specifically,  $D_{b j, t-1}$  equals 1 if month t-1 is dominated by large jumps, and 0 otherwise. Similarly,  $D_{cont-1}$ equals 1 if month  $t-1$  is dominated by Brownian motion, and 0 otherwise. Finally,  $D_{s,j,t-1}$  equals 1 if month t-1 is dominated by small jumps, and 0 otherwise.

The intercept, or alpha, in Table 4, Figures 10 and 11 shows the risk-adjusted return to the level of expertise of the manager. The thirteen hedge funds, on average, earn a monthly alpha of 0.4243% and 0.6553% in the months following Brownian motion and big jumps, respectively, which lead to a monthly average return of 0.3487% and 0.2054%. In contrast, the return following the months dominated by

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 $6$  The returns on PTFSBD, PTFSFX and PTFSCOM are obtained from Fung and Hsieh's data library http://faculty.fuqua.duke.edu/~dah7/HFRFData.htm/.

small jumps averages -0.8823%, because the funds have moved to a negative alpha of -1.0651% per month. This indicates that most of hedge fund managers fail to time occasional and small illiquidity by adjusting their portfolios' market exposure.

The results are consistent across the most individual hedge fund styles. In almost every hedge style, the big-jump and Brownian regimes have the positive impact on subsequent hedge fund alphas with the small-jump regime having the negative impact on subsequent hedge fund return alphas. There are three exceptions. The first is Risk Arbitrage and Merger Arbitrage hedge funds where big-jump VIX regime has a negative impact on subsequent alphas. Second, Risk Arbitrage, Merger Arbitrage and Trend Following hedge funds show negative impact on subsequent alphas from VIX in Brownian motion regime. Third, with Merger Arbitrage and Distressed Restructuring hedge funds, the alphas following small-jump regime are positive. It is evidence that hedge fund alphas are enhanced more by preceding extreme volatility levels, both high and low, than by preceding mid-volatility levels.

#### **[Table 4 about here]**

#### **[Figure 10 about here]**

#### **[Figure 11 about here]**

To summarize, our results are consistent with the view that allowing for predictability based on jump activities on VIX is important in ex-ante identifying subgroups of hedge funds that deliver significant outperformance. Conditioning on large VIX jumps, funds that long volatility (such as Trend Following and Managed Futures) deliver significantly higher out-of-sample returns relative to funds that short volatility (such as Relative Value, Equity Hedge, Event Driven and Distressed Restructuring), which coincides with extreme bear markets. Many shorting volatility strategies, following the spike in volatility in Q4 2008, have been susceptible to sudden large losses and were exposed to the high (positive) downside market beta. Long volatility strategies have gained popularity since 2008, primarily as a hedge against catastrophic scenarios, often referred to as "tail risk."

In the months that follow Brownian motion as a result of excluding the period with the 2008 financial crisis, hedge funds with negative volatility exposure tend to outperform those with positive volatility exposure. Using volatility as an asset class prior to the Q4 2008 financial crisis, therefore, tends to capture historical excess returns by selling volatility as well as various strategies involving combinations of option positions. This is consistent with the extant literature including Hafner and Wallmeier (2008) who analyze the implications of optimal investments in sizable short positions on variance swaps. Egloff, Leippold and Wu (2010) have an extensive analysis of how variance swaps fit into optimal portfolios in dynamic context that improve the ability of the investor to hedge time-variations in investment

opportunities. Finally, in the months that follow small jumps on VIX, possibly as a result of trading illiquidity, the majority of hedge funds deliver negative returns, in particular when augmented with positive change in VIX. Therefore, detecting jump activities on ultra-high frequency VIX data can help forecast cross-sectional differences in hedge fund performance through their exposure to long or short volatility risk evaluated conditional on different VIX jump quintiles.

#### **5. Conclusions**

The empirical results of the spectrogram methodology appear to indicate the presence of a continuous component, occasional large jumps, and infinite activity jumps with a fairly high degree of jump activity in the ultra-high frequency VIX data. Jump components represent approximately 71% of the total quadratic variation with a degree of infinite activity jumps in the range from 1.71 to 1.95.

There is evidence that the change in VIX is correlated with proxies for the credit, liquidity and correlation risks. This study evaluates out-of-sample monthly performance of hedge funds conditioning on the jump activities on ultra-high frequency VIX data. The results suggest the economic value of predictability obtains for short-volatility strategies in the months that follow Brownian motion, while long-volatility strategies outperform in the months that follow large VIX jumps. Our evidence suggests that the success of hedge fund strategies hinges on the behavior of VIX jump activities.

The contribution of this study is twofold. First, the study distinguishes between continuity, small jumps and large jumps on ultra-high frequency VIX data, and determines their relative magnitudes. Our results are informative as to the relevant directions such volatility traders may take. Second, allowing for predictability based on jump activities on VIX is important in ex-ante identifying subgroups of hedge funds that deliver significant outperformance.

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#### **Table 1 Correlation of Hedge Fund Returns with Monthly Change in VIX across Quintiles of the S&P 500 Index**

The monthly hedge fund returns are sorted into five groups based on the S&P 500 index (SPX). Quintile 1 (Q1) consists of the worst months, and Quintile 5 (Q5) the best months. Next, within each group, we further split funds into those that have positive change in VIX and those that have negative change in VIX. We report the correlation between monthly returns and monthly change in VIX ( $\Delta VIX$ ) of each sub-group. The sample is over the period October 2003 to July 2010.

**Table 2 Summary Statistics for Market Status across Jump Activities on VIX**  The table illustrates in-sample and out-of-sample market exposure of VIX, SPX and absolute percentage monthly change in VIX (denoted " $|\Delta VIX/VIX|\%$ ") to Brownian motion, small jumps and big jumps, respectively. The in-sample (out-of-sample) data period is from October (November) 2003 to June (July) 2010.



**Table 3 Out-of-Sample Hedge Fund Return and Risk across VIX Jump Activities**  The table reports summary statistics for monthly hedge fund returns (%) and risk over the out-of-sample period, November 2003 to July 2010. Thirteen funds are sorted into three groups based on whether their previous months are dominated by Brownian motion, small jumps or big jumps. These are the three measures to gauge hedge fund performance when applied to a single hedge fund strategy: (i) using maximum drawdown (MaxDD) as a downside risk measure; (ii) using adjusted conditional Value-at-Risk (CVaR) at the confidence level 95% as a measure of extreme tail risk; and (iii) using extended Sharpe ratio (ESR) as a measure of the excess return relative to risk with highly non-normal distributions and large tails.

		<b>Brownian Motion</b>	Small Jumps	<b>Big Jumps</b>
	${\bf N}$	18	3	60
<b>Equal Weighted</b>	M	0.4055	$-0.6544$	0.0163
	Mdn	0.3284	$-0.2566$	0.4141
	Maximum	1.7548	0.5550	2.2767
	Minimum	$-1.0853$	$-2.2615$	$-9.9303$
	Stdev	0.8161	1.4498	1.9300
	MaxDD	0.0258	0.0282	0.1218
	CVaR(95%)	$-0.0113$	$-0.0364$	$-0.0613$
	<b>ESR</b>	0.2454	$-0.6738$	$-0.0754$
Global	M	0.4305	$-0.7847$	0.0417
	Mdn	0.3727	$-0.5700$	0.3545
	Maximum	2.5479	0.7630	3.1499
	Minimum	$-1.3140$	$-2.5472$	$-9.3470$
	Stdev	1.1532	1.6655	2.1287
	MaxDD	0.0348	0.0331	0.1216
	CVaR(95%)	$-0.0170$	$-0.0407$	$-0.0630$
	<b>ESR</b>	0.1962	$-0.6812$	$-0.0603$
Relative Value Arbitrage	M	0.5937	$-0.4338$	0.0121
	Mdn	0.5285	0.5107	0.3362
	Maximum	1.6514	0.8080	6.8138
	Minimum	$-1.2280$	$-2.6202$	$-14.1105$
	Stdev	0.7688	1.8993	3.1299
	<b>MaxDD</b>	0.0286	0.0343	0.1713
	CVaR(95%)	$-0.0117$	$-0.0447$	$-0.0972$
	<b>ESR</b>	0.4553	$-0.3981$	$-0.0575$
<b>Event Driven</b>	M	0.7250	$-0.7334$	0.1373
	Mdn	0.7581	$-0.7298$	0.8613
	Maximum	3.1533	0.5120	2.8059
	Minimum	$-1.5361$	$-1.9824$	$-7.5259$
	Stdev	1.2762	1.2472	2.1158
	MaxDD	0.0389	0.0249	0.1033
	CVaR(95%)	$-0.0163$	$-0.0305$	$-0.0570$
	<b>ESR</b>	0.4172	$-0.9020$	$-0.0228$
<b>Equity Hedge</b>	M	0.4331	$-0.4720$	$-0.0386$
	Mdn	0.4799	$-1.0010$	0.3760
	Maximum	3.0191	1.1620	4.4771
	Minimum	$-2.5518$	$-1.5769$	-9.9866
	Stdev	1.5589	1.4441	2.5866
	MaxDD	0.0511	0.0274	0.1289
	CVaR(95%)	$-0.0280$	$-0.0258$	$-0.0729$
	<b>ESR</b>	0.1369	$-0.6676$	$-0.0789$
<b>Risk Arbitrage</b>	M	0.4611	0.0533	0.4593
	Mdn	0.4250	0.3500	0.4950
	Maximum	3.0100	0.4600	3.2200
	Minimum	$-1.5200$	$-0.6500$	$-3.4900$





#### **Table 4 Out-of-Sample Risk-Adjusted Alphas Respective of VIX Jump Activities**

This table shows the result that hedge fund excess returns regress against seven risk factors and three dummy variables:

 $r_{i,t} = \alpha_{i,con}D_{con,t-1} + \alpha_{i,sj}D_{sj,t-1} + \alpha_{i,bj}D_{bj,t-1} + \beta_{i,1}SNPMRF_t + \beta_{i,2}SCMLC_t + \beta_{i,3}BD10RET_t + \beta_{i,4}BAAMTSY_t + \beta_{i,5}PTFSBD_t + \beta_{i,6}PTFSFX_t + \beta_{i,7}PTFSCOM_t + \varepsilon_{i,6}DTTSSFA_t + \varepsilon_{i,7}DTTSCOM_t + \varepsilon_{i,8}DTTSSFA_t + \varepsilon_{i,9}DTTSSCA_t + \varepsilon_{i,9}DTTSSCA_t + \varepsilon_{i$ where  $r_{i,t}$  is the return on fund *i* in excess of the one-month Treasury bill return (the risk-free rate) in month t.  $D_{b,t-1}$  equals 1 if month t-1 is dominated by large jumps, and 0 otherwise.  $D_{cont-1}$  equals 1 if month t-1 is dominated by Brownian motion, and 0 otherwise.  $D_{s,i,t-1}$  equals 1 if month t-1 is dominated by small jumps, and 0 otherwise. The Fung and Hsieh (2004) model proposes seven risk factors to evaluate hedge fund performance: the S&P 500 return minus the risk-free rate, SNPMRF; the Russell 2000 minus S&P 500 monthly total return, *SCMLC*; the monthly change in the 10-year Treasury constant maturity yield, *BD*10RET; monthly changes in the credit spread defined as Moody's Baa bond yield minus the 10-year Treasury bond yield, BAAMTSY; and excess returns on portfolios of lookback straddles on bonds (PTFSBD), currencies (PTFSFX) and commodities (PTFSCOM).<sup>\*\*\*</sup> (\*\*, \*) indicates that the t statistics after the Newey-West correction of standard errors for heteroscedasticity and autocorrelation are significance in the 99% (95%, 90%) confidence interval.  $R(\%)$  under the headings  $D_{con,t-1}$ ,  $D_{S,i,t-1}$  and  $D_{b,i,t-1}$  represents the out-of-sample average monthly fund return (%) following the months dominated by Brownian motion, small jumps and big jumps, respectively. The groups are also further split into the scenarios  $\Delta VIX < 0$  and  $\Delta VIX > 0$ . The out-of-sample period is from November 2003 to July 2010.













**Panel B. Quintiles of absolute percentage change in VIX** 



**Figure 1. Hedge fund monthly returns across five quintiles of VIX, absolute percentage change in VIX and SPX.** Quintile 1 (Q1) consists of the months with the smallest VIX in Panel A, absolute percentage change in VIX ([Percentage Change in VIX]) in Panel B, and S&P 500 Index (SPX) in Panel C. Quintile 5 (Q5) indicates the months with the largest VIX, absolute percentage change in VIX, and SPX, respectively. The sample period is October 2003 to July 2010.

**Panel A. VIX log difference ("Log-Return") densities sampled at the 15 second frequency** 



**Panel B. VIX log difference ("Log-Return") densities sampled at the daily frequency**



**Figure 2. VIX log difference densities over the period from October 1, 2003 to June 30, 2010.** Overnight log-returns are excluded from the sample. Panel A displays the VIX log difference density and its tails sampled at the 15 second frequency. Panel B presents the VIX log difference density and its tails sampled at the daily frequency. Tails of log-returns R are given by  $|R| \ge 0.05$ . Simple visual inspection of the tails of both log difference distributions suggests the presence of jumps and right-skewed. Excess skewness and kurtosis for 15-second log differences are, however, significantly larger than those for daily log differences, as indicative of more small jumps activity in ultra-high frequency VIX data.

**Panel A. Empirical distribution of**  $S_j$  **for VIX** 



**Panel B.** Average value of  $S_j$  as a function of the sampling interval for VIX



**Figure 3. Test statistic of**  $S_j$  **for VIX from Q4 2003 to Q2 2010. Test statistic**  $S_j$  **is to test for the** presence of jumps. Panel A displays empirical distribution of  $S_j$  for VIX, whereas Panel B shows average value of  $S_j$  as a function of the sampling interval for VIX. The value of  $S_j$  is calculated for a range of values of p from 3 to 6,  $\Delta_n$  from 15 seconds to 2 minutes, and  $k = 2$ , 3. The values of  $S_j$ between  $min(1/k) = 0.3333$  and  $max(1/\sqrt{k}) = 0.7071$  are indicative of noise dominating, the values around 1 indicate the presence of jumps, and the values approach  $k^{p/2-1} \in [1.4142, 9]$  if no jumps exist.

**Panel A. Empirical distribution of**  $S_{FA}$  **for VIX** 



**Panel B.** Average value of  $S_{FA}$  as a function of the sampling interval for VIX



**Figure 4. Test statistic of**  $S_{FA}$  **for VIX from Q4 2003 to Q2 2010. Test statistic**  $S_{FA}$  **is to test** whether jumps have finite or infinite activity. Panel A displays empirical distribution of  $S_{FA}$  for VIX, whereas Panel B shows average value of  $S_{FA}$  as a function of the sampling interval for VIX. The value of  $S_{FA}$  is calculated for a range of values of p from 3 to 6,  $\eta$  from 5 to 10 standard deviations,  $\Delta_n$ from 15 seconds to 2 minutes, and  $k = 2$ , 3. The values of  $S_{FA}$  between  $min(1/k) = 0.3333$  and  $max(1/k) = 0.5$  are indicative of noise dominating, the values around 1 indicate infinite activity, and the values approach  $k^{p/2-1} \in [1.4142, 9]$  if finite activity exists.

**Panel A. Empirical distribution of**  $S<sub>IA</sub>$  **for VIX** 



**Figure 5. Test statistic of**  $S_{IA}$  **for VIX from Q4 2003 to Q2 2010. Test statistic**  $S_{IA}$  **is to test the null** being infinite activity jumps present and no significant noise, and the alternative of finite activity jumps present and no significant noise, by choosing  $\gamma > 1$  and  $p' > p > 2$ . Panel A displays empirical distribution of  $S_{IA}$  for VIX, whereas Panel B shows average value of  $S_{IA}$  as a function of the sampling interval for VIX. The value of  $S_{IA}$  around  $\gamma^{p'-p}$  is indicative of infinite jump activity, and the value around 1 indicates jumps have finite activity.





**Panel B.** Average value of  $S_W$  as a function of the sampling interval for VIX



**Figure 6. Test statistic of**  $S_W$  **for VIX from Q4 2003 to Q2 2010. Test statistic**  $S_W$  **is to test whether** Brownian motion is present. Panel A displays empirical distribution of  $S_W$  for VIX, whereas Panel B shows average value of  $S_W$  as a function of the sampling interval for VIX. The value of  $S_W$  is calculated for a range of values of  $p$  from 1 to 1.75,  $\eta$  from 5 to 10 standard deviations,  $\Delta_n$  from 15 seconds to 2 minutes, and  $k = 2$ , 3. The values of  $S_W$  between  $min(k^{1-p/2}) = 1.0905$  and  $max(k^{1-p/2}) = 1.7321$  are indicative of the presence of Brownian motion, the values around 1 indicate no Brownian motion, and the values between  $min(1/k) = 0.3333$  and  $max(1/k) = 0.5$  indicate noise dominating.



Panel A. Empirical distribution of fraction of  $QV$  attributable to Brownian component for VIX

Panel B. Average fraction of QV attributable to Brownian component as a function of the **sampling interval**



Figure 7. Fraction of *QV* attributable to Brownian component for VIX from Q4 2003 to Q2 2010.  $QV$  relative magnitude is examined in the presence of a jump and a continuous component. Panel A displays empirical distribution of fraction of  $QV$  attributable to Brownian component for VIX, whereas Panel B shows average proportion of  $QV$  attributable to the continuous component as a function of the sampling interval for VIX. The fraction of  $QV$  from the Brownian component using the twenty-seven quarters, values of  $\eta$  from 2 to 5 standard deviations, and  $\Delta_n$  from 15 seconds to 2 minutes.



**Panel A. Empirical distribution of the index of jump activity**  $\beta$  **for VIX** 

**Panel B.** Average value of the index of jump activity  $\beta$  as a function of the sampling interval



**Figure 8. The index of jump activity**  $\beta$  **for VIX from Q4 2003 to Q2 2010.** Panel A displays Empirical distribution of the index of jump activity  $\beta$  for VIX, whereas Panel B shows Average value of the index of jump activity  $\beta$  as a function of the sampling interval for VIX. The degree of infinite jump activity  $\beta$  is estimated using the twenty-seven quarters, values of  $\gamma$  ranging from 1.5 to 1.75, the value of  $\eta$  at 2, and  $\Delta_n$  from 15 seconds to 2 minutes.



**Figure 9. Out-of-sample monthly fund returns conditional on jump activities on ultra-high frequency VIX data.** The graph reports out-of-sample average monthly returns of each sub-group over the sample period November 2003 to July 2010. Thirteen funds are sorted into three groups based on whether their previous months are dominated by Brownian motion, small jumps or big jumps. Within each group, we further split funds into those that have negative change in VIX in the previous month and those that do not.



**Figure 10. Out-of-sample monthly average risk-adjusted alpha.** Thirteen hedge funds are sorted into three groups based on whether their previous months are dominated by Brownian motion, small jumps or big jumps. Within each group, we further split funds into those that have negative change in VIX in the previous month and those that do not. To adjusted risk, the study evaluates the performance of hedge funds relative to the 7-factor model. The graph reports out-of-sample risk-adjusted alpha of each sub-group over the sample period November 2003 to July 2010.



**Figure 11. Out-of-sample risk-adjusted alpha.** Thirteen hedge funds are sorted into three groups based on whether their previous months are dominated by Brownian motion, small jumps or big jumps. Within each group, we further split funds into those that have negative change in VIX in the previous month and those that do not. To adjusted risk, the study evaluates the performance of hedge funds relative to the 7-factor model. The graph reports out-of-sample risk-adjusted alpha of each sub-group over the sample period November 2003 to July 2010.