11-2021

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Citation

CHEN, Jianqing; GUO, Zhiling; and HUANG, Jian. An economic analysis of rebates conditional on positive reviews. (2021). *Information Systems Research*. 33, (1), 224-243.
Available at: https://ink.library.smu.edu.sg/sis_research/6436

Published in Information Systems Research, 2021, DOI: 10.1287/isre.2021.1048

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Abstract

Strategic sellers on some online selling platforms have been recently using a conditional-rebate strategy to manipulate product reviews under which only purchasing consumers who post positive reviews online are eligible to redeem the rebate. A key concern for the conditional rebate is that it can easily induce fake reviews which might be harmful to society. We develop a micro-behavioral model capturing consumers' review-sharing benefit, review-posting cost, and moral cost of lying to examine the seller's optimal pricing and rebate decisions. We derive three equilibria: the no-rebate, authentic-review equilibrium, the low-rebate, boosted-positive-review equilibrium, and the high-rebate, fake-review equilibrium. We find that the seller's optimal price and rebate decisions critically depend on both review-posting cost and moral cost. The seller adopts the no-rebate strategy when the review-posting cost is low but the moral cost is high, the low-rebate strategy when the review-posting cost is high or when review-posting cost is intermediate and the moral cost is high, and the high-rebate strategy when the review-posting cost is not too high and the moral cost is low. Our results suggest that it is not always profitable for strategic sellers to adopt the conditional-rebate strategy. Even if the conditional-rebate strategy is adopted, it does not always result in fake reviews. Furthermore, we find that when a low (high) rebate is offered, if the review-posting cost is not too high (very low), the conditional-rebate strategy can even lead to higher social welfare than a benchmark with no rebate. Our findings shed new light on the online-platform policy debate about the fake-review phenomenon induced by conditional rebates.

1 Introduction

Online reviews have been well documented as an important information source in consumers' purchase decisions (Zhu and Zhang, 2010; eMarketer, 2017). Firms often offer various incentives to encourage consumers to post reviews online. For example, after a consumer shops at Home Depot, the consumer is often invited to write reviews with the promise that he/she will have a chance to draw a lottery for some prize (e.g., an iPad), and Best Buy offers 25 reward points ($0.50 monetary value) to consumers who write reviews. In recent years, a new approach to encourage reviews has emerged. On Taobao.com, the leading online trading platform in China, many sellers offer consumers mail-in rebates after purchase, but the rebates can be redeemed only if the consumers post positive reviews online. In Google Play, some developers run campaigns offering discounted prices or in-game currency for five-star reviews.[1] This conditional-rebate strategy is different from the typical unconditional-rebate strategy because in some sense sellers "bribe" buyers for positive reviews rather than simply expanding the review pool using monetary incentives.

Despite the prevailing conditional-rebate practice on Taobao.com, a key concern is that it can easily induce fake reviews that might harm society. Recently, leading online platforms in the U.S. have developed various countermeasures and online-review policies to combat fake reviews. For example, Google has deployed a system that combines human intelligence with machine learning to detect fake reviews,[2] and Amazon's review policy clearly disallows "offering compensation or requesting compensation (including free or discounted products) in exchange for creating, modifying, or posting content."[3] However, with mail-in conditional rebates, it is difficult for platforms to distinguish manipulated reviews from legitimate ones because (a) all reviews are written by real consumers who indeed have purchased a focal product and (b) the rebates are provided to consumers offline in an untraceable manner (typically delivered together with the focal product). Motivated by the conditional-rebate offerings and the incentivized-review phenomenon common on today's e-commerce platforms, as well as the technical challenges of detecting online-review manipulation, this study aims to answer the following research questions: Under what conditions do strategic sellers prefer the conditional-rebate strategy? Under what conditions do fake reviews arise as an

[1] https://play.google.com/about/storelisting-promotional/ratings-reviews-installs/
[2] https://www.theinquirer.net/inquirer/news/3068397/google-is-cracking-down-on-fake-play-store-reviews
[3] https://www.amazon.com/gp/help/customer/display.html?nodeId=201929730

equilibrium outcome? How do conditional rebates affect sellers' profits and social welfare?

On the one hand, there is clear benefit associated with positive reviews. For example, prior studies show that one extra star in a Yelp review could increase revenues by 5–9% (Economist, 2015), which explains why restaurants often seek fake acclaim, offering customers discounts in exchange for positive reviews on social networking sites and other online platforms. On the other hand, offering cash rebate incurs a direct cost for sellers. Therefore, it may not always be profitable to pursue such a strategy. Although poor ratings and reviews affect a seller's reputation and sales, favorable product price can mitigate these negative effects. The Better Business Bureau's recent Trust Sentiment Index report shows that roughly one in three consumers says they would still purchase from a business that has poor ratings or reviews if the price is right. Therefore, it is not immediately clear whether a strategic seller should provide monetary incentive to boost its positive reviews, increase consumers' perceived value of its product, and thus charge a price premium, or it should offer a price discount to attract more consumers. This research aims to understand how a strategic seller should trade off the rebate incentive and product pricing to maximize its profit.

We develop an analytical model to study a strategic seller's conditional-rebate and pricing decisions. The focal product has both digital and nondigital attributes. Digital attributes refer to the attributes that can be easily communicated via the Internet, such as size and color, whereas nondigital attributes are those that are hard to evaluate prior to purchase, such as product fit. Consumers have heterogeneous valuation in these two dimensions. They learn the product's digital-attribute value before the purchase, but only form an expectation of the nondigital-attribute value based on online reviews. Consumers make purchase decisions based on their total perceived expected product valuation. Once consumers receive the product, they might receive a conditional rebate which can be redeemed only if they post positive reviews. In general, consumers incur a cost to post online reviews, but enjoy satisfaction by sharing their true opinions. If they post fake reviews, consumers suffer a moral cost. Consumers trade off these benefits and costs to determine whether to post positive reviews, negative reviews, or no reviews.

We derive three equilibrium outcomes: the no-rebate, authentic-review equilibrium, the low-rebate, boosted-positive-review equilibrium, and the high-rebate, fake-review equilibrium. We find that sellers' optimal price and rebate decisions critically depend on review-posting cost, moral cost, and the expected nondigital-attribute value. In the presence of a high expected nondigital-attribute value, when the review-posting cost is low but the moral cost is high, the seller has no incentive to offer the conditional rebate because of a sizable volume of voluntary reviews (due to low review-posting cost) and a high cost to "bribe" unsatisfied consumers (due to the high moral cost). When the review-posting cost is high or it is intermediate and the moral cost is high, the seller offers a low rebate to elicit more positive reviews from satisfied consumers. Only when the review-posting cost is not too high and the moral cost is low, would the seller offer a high rebate. The high rebate not only motivates more satisfied consumers to share their true experiences, but also induces unsatisfied consumers to post fake reviews. In the presence of a low expected nondigital-attribute value, the seller never offers a high rebate. In this case, when the review-posting cost is high, the seller offers a low rebate to boost positive reviews; otherwise, the seller offers no rebate.

Our findings suggest that it is not always profitable for strategic sellers to pursue the conditional-rebate strategy. In addition, the seller's conditional-rebate strategy does not necessarily result in fake reviews. Fake reviews come with a high cost for the firm. Not only does the firm need to provide enough monetary incentive to "bribe" unsatisfied consumers to lie, but the monetary incentive will also be taken by satisfied consumers who would otherwise have posted positive reviews without the rebate. Only when the moral cost is low, the review-posting cost is not too high, and the expected value from nondigital attributes is high would the firm prefer a high-rebate strategy, causing fake reviews to appear in equilibrium. Further, numerically, we find that offering a low rebate is more likely to arise as an equilibrium than a high rebate in the entire feasible parameter space we examine. Under the low-rebate equilibrium, the rebate is used to boost positive reviews from satisfied consumers, not to bribe unsatisfied consumers to lie and post fake reviews. The information shared online still reflects the true opinions from purchasing consumers. These findings shed new light on the criticism and concern about the fake-review phenomenon induced by the conditional-rebate strategy.

The conditional rebate motivates additional consumers to post positive product reviews, and the inflated product reviews increase consumers' perceived nondigital-attribute value of the product, leading to increased overall perceived product valuation. Compared with the benchmark case where no rebate is considered, the use of a conditional rebate not only enables the seller to charge a price premium, but also increases the overall product demand and profit, regardless of low or high rebate amount being optimally offered. However, the profit implications of the review-posting and moral costs are different under the low- and high-rebate equilibria. In the equilibrium when a high rebate is offered, the firm's profit (weakly) decreases in both the review-posting cost and the moral cost. This is intuitive because, to induce additional positive reviews, the seller must both cover consumers' review-posting cost and compensate unsatisfied consumers' moral cost of posting fake reviews. In sharp contrast, surprisingly, in the equilibrium when a low rebate is offered, the firm's profit increases in the review-posting cost because, as the review-posting cost increases, the number of satisfied consumers who would otherwise have posted positive reviews without monetary incentive decreases. Because offering a rebate to this group of consumers incurs a pure cost to the seller, the reduced size in this group significantly benefits the seller and enables the seller to further optimize its price and rebate, resulting in increased rebate offering and profit. In this equilibrium, the firm's profit is independent of the moral cost, because no consumers post fake reviews.

The effect of the conditional rebate on social welfare also depends on the review-posting cost and the moral cost. If the review-posting cost is not too high, social welfare in the low-rebate equilibrium can be higher than that in the benchmark case where no rebate is considered. If the review-posting cost and moral cost are small enough, social welfare in the high-rebate equilibrium can be higher than that in the benchmark case. Therefore, offering a conditional rebate might be socially beneficial. The conditional rebate brings in both social gain and social loss. The social gain comes from the additional transactions due to boosted product reviews. The social loss comes from distorted consumers' review-posting behavior—some consumers posting reviews when their review-sharing benefits are below their review-posting cost and some consumers being induced to post fake reviews, incurring both review-posting cost and moral cost. In the low-rebate equilibrium, the rebate increases in the review-posting cost. When the review-posting cost is high, the rebate is high and the distortion in consumers' review-posting behavior is so significant that the social gain cannot offset the social loss. In the high-rebate equilibrium, the social loss can be pronounced because of additional moral cost, and as a result, social welfare improvement can rarely occur. Only when the review-posting cost and moral cost are very low can the total social gain outweigh the total social loss, leading to increased social welfare. Overall, our findings offer new insights into the fake-review phenomenon induced by sellers' conditional-rebate strategy.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 introduces our baseline model. We analyze consumers' review-posting behavior and derive the firm's equilibrium pricing and rebate decisions in Section 4. Section 5 further examines the effects of the conditional rebate on the firm's profit and social welfare. In Section 6, we extend the baseline model to the case of sophisticated consumers. Section 7 discusses managerial implications and concludes.

2 Related Literature

Our work examines a new rebate mechanism to encourage online product reviews. Two streams of research are particularly relevant to our study—the research on rebates and on product reviews.

Rebates as promotional tools have been widely studied in economics and marketing for decades (e.g., Gerstner and Hess, 1991; Chen et al., 2005). One common explanation for rebates is a pricing device to achieve market segmentation (e.g., Chen et al., 2005; Lu and Moorthy, 2007)—compared with uniform pricing, the firm raises the price for consumers who do not redeem the rebate but decreases it for the others who do redeem it, reaping benefits from price discrimination. However, in the presence of the online cash-back mechanism, Ho et al. (2017) show that offering rebates may not always benefit consumers. Different from this stream of the literature, we focus on a new conditional-rebate mechanism under which only purchasing consumers posting positive reviews are eligible to redeem the rebate.

Our work is primarily related to product reviews. Consumers today are increasingly influenced by online product reviews in a variety of purchase decisions (Lu et al., 2013; Kwark et al., 2014). Prior research finds that online product reviews are an important source of information to reduce consumers' uncertainty about products, especially nondigital attributes such as product fit where consumers have ex ante incomplete knowledge (Godes and Mayzlin, 2004; Chen and Xie, 2008). Several analytical papers thus focus on firms offering reviews to facilitate consumer learning of product fit (Sun, 2012; Kwark et al., 2014). Similar to Chen and Xie (2008), who model consumer reviews as an imperfect signal of whether the product is a match or mismatch, we focus on the impact of online reviews on consumers' nondigital-attribute evaluation.

A growing body of literature studies the information role of online reviews and their impact on firms' pricing and marketing strategies (e.g., Chen and Xie, 2008; Li and Hitt, 2010; Liu et al., 2017; Feng et al., 2018; Zimmermann et al., 2018). Chen and Xie (2008) investigate how consumer reviews influence a monopolistic firm's incentive to provide fit-revealing information. Liu et al. (2017) study how online reviews and past sales-volume information jointly affect consumers' purchasing decisions and firms' pricing strategies in a framework with herding consumers being uncertain about products. While interesting, this stream of research takes the online-review-generation mechanisms as given and focuses on firms' marketing responses to indirectly influence online reviews. We add to this line of inquiry by considering how firms can strategically influence the online-review-generation mechanisms and directly manipulate the online review provisions.

Like many voluntarily provided public goods (Gallus, 2017), online reviews may be underprovisioned (Anderson, 1998; Avery et al., 1999), which limits their helpfulness to other consumers (Mudambi and Schuff, 2010). Prior studies find that financial incentives effectively motivate individuals to write reviews on Airbnb.com (Fradkin et al., 2015) and Best Buy (Khern-am-nuai et al., 2018). Li and Xiao (2014) consider a rebate mechanism in which sellers provide a rebate to cover the buyer's feedback-reporting cost, regardless of whether the feedback is positive or negative. They find that, compared with the no-rebate market, the seller's rebate decision has a significant impact on the buyer's purchasing decision and result in more efficient trades. In contrast, Cabral and Li (2015) conduct controlled field experiments on eBay and find evidence that buyers reciprocate with sellers favorably if the sellers provide a feedback rebate. As such, sellers can "buy" feedback, but such feedback is likely to be biased. A distinct feature of these studies is that the rebate is used to compensate buyers' review-posting cost and is independent of whether the feedback is positive or negative. In contrast to these studies and taking a step further, we consider a conditional-rebate strategy in which sellers only reward buyers who post positive online reviews.

Because higher ratings positively impact sales and revenue, strategic sellers may engage in online review manipulation. Dellarocas (2006) examines firms' incentive to manipulate reviews in public forums and the implications to consumer welfare when firms introduce a product to a new market. Different from that work, we consider a conditional-rebate strategy in which sellers only reward buyers who post positive online reviews. We study firms' product pricing and rebate strategies to manipulate consumer reviews in online markets. This focus is also different from prior research studying firms' other strategic behaviors, such as deceptive advertising to fool consumers about the product's true quality (Piccolo et al., 2017) and false advertising to overstate the product's value (Rhodes and Wilson, 2018). We complement this stream of literature and enrich our understanding of firms' and consumers' strategic interaction in online markets.

Finally, our research contributes to ongoing research in online platforms' information policies (Hao and Tan, 2019; Gutt et al., 2019). Jiang and Guo (2015) consider product valuation and consumer misfit cost, and study their impact on a firm's pricing and review-system design decisions. By focusing on the sellers' strategic decisions in offering a conditional rebate to encourage reviews, we examine how the sellers' conditional-rebate strategy affects the review outcomes, product sales, and social welfare. We uncover the different conditions under which unbiased (authentic) reviews or biased (fake) reviews may emerge as equilibrium outcomes. Our findings provide important implications for platform owners in the design and implementation of online-review guidelines and policies.

3 Model

We consider an online seller selling a product to a continuum of potential consumers. As in Lal and Sarvary (1999), we distinguish two types of product properties—digital attributes and nondigital attributes. Digital attributes (also known as search attributes) refer to the attributes that can be easily communicated to and assessed by consumers via the Internet before purchase. Nondigital attributes (also called experience attributes) refer to those that are difficult to evaluate online, which can be determined only by trying, inspecting, or even consuming the product. For instance, size and color of a product are examples of digital attributes, and how well the product fits a consumer's specific setting can be an example of a nondigital attribute (e.g., whether a jacket fits a consumer's figure or whether a piece of furniture fits the consumer's room design/style). We denote X as the part of the valuation associated with digital attributes and Y as the part of valuation associated with nondigital attributes. A consumer's valuation of the product is $X + Y$, determined by both the digital and nondigital attributes. Among all consumers, we denote $\bar{X}$ and $\bar{Y}$ as the maximum value derived from digital and nondigital attributes, respectively. We can then express a consumer's overall product valuation as $X + Y = \left(\frac{\bar{X}}{\bar{X}+\bar{Y}}\frac{X}{\bar{X}} + \frac{\bar{Y}}{\bar{X}+\bar{Y}}\frac{Y}{\bar{Y}}\right)\left(\bar{X} + \bar{Y}\right)$, where $\frac{\bar{X}}{\bar{X}+\bar{Y}}$ and $\frac{\bar{Y}}{\bar{X}+\bar{Y}}$ are the weights of digital and nondigital attributes, and $\frac{X}{\bar{X}}$ and $\frac{Y}{\bar{Y}}$ are normalization. By letting $x \equiv \frac{X}{\bar{X}}$, $y \equiv \frac{Y}{\bar{Y}}$, $q \equiv \bar{X} + \bar{Y}$, and $\theta = \frac{\bar{Y}}{\bar{X}+\bar{Y}}$, we can rewrite the product valuation as

$$[(1 - \theta)x + \theta y]q. \tag{1}$$

In the rest of the paper, we use the notations in Equation (1). We call x and y a consumer's (normalized) digital-attribute and nondigital-attribute values. Notice $\theta \in [0,1]$ represents the consumer's weight on the nondigital attributes between the two types of attributes, and q stands for the product's maximum valuation among all consumers. We consider that the product has a certain proportion of digital attributes because, otherwise, the product would not be suitable for online shopping in the first place. For ease of exposition, technically, we let $\theta \leq \frac{1}{2}$. The other cases can be similarly analyzed, and we can show that our main results remain the same.

Consumers are generally heterogeneous in their valuation. We assume that before purchase each consumer learns her digital-attribute value x, based on information provided by the seller, such as product description. We assume x follows a uniform distribution over $[0,1]$. In contrast, before purchase, consumers cannot exactly know their nondigital-attribute value y or its distribution, although they may have some expectation based on the available information such as online reviews. For ease of exposition, we assume that ex post y can be either high or low: Consumers derive a high value if the product fits their needs well; otherwise, they derive a low value. Without loss of generality, we normalize the high value to 1 and the low value to 0; that is, $y = 1$ and $y = 0$, respectively. As a result, consumers are either satisfied or unsatisfied after their purchase. We assume that consumers are equally likely to be satisfied or unsatisfied. In other words, they can derive high or low value from the nondigital attributes with an equal probability.

Satisfied consumers may post positive reviews about the product, and unsatisfied consumers may post negative reviews, upon weighing the review-posting benefits and costs. One important reason for consumers to post reviews is that consumers have the desire for sharing.[4] We thus assume that, on the one hand, consumers derive value v from sharing their true opinions, which follows a uniform distribution over $[0,1]$. On the other hand, consumers incur a cost c ($c \geq 0$) for posting reviews due to the time and effort required. Without additional incentive, whether a consumer posts reviews is determined by her review-sharing benefit and the review-posting cost. We assume $c < 1$ to ensure that even in the absence of additional incentive, some consumers still post reviews.

To motivate more consumers to post positive reviews, the seller may offer a monetary incentive. Following the common practice on Taobao.com, we consider that the seller gives a rebate s ($s \geq 0$) to each purchasing consumer who posts positive reviews online. When the rebate is zero, this setting

[4] https://www.bostonwebdesigners.net/wp-content/uploads/POS_PUBLIC0819-1.pdf

reduces to the classical pricing problem. Note that this rebate is conditional on posting positive reviews. While the monetary incentive naturally motivates more satisfied consumers to share their true opinion (i.e., provide positive reviews), the effect of this incentive on unsatisfied consumers is more nuanced. When unsatisfied consumers post positive reviews, although they can enjoy the rebate, they not only fail to derive the value v resulting from sharing their true opinions, but also suffer from lying. Thus, those unsatisfied consumers who post fake reviews would incur a moral cost m ($m \geq 0$). To focus on more general cases, we assume that the maximum value that consumers may derive from nondigital attributes is not too high (e.g., $\theta q \leq \frac{3}{2}$); otherwise, the firm would always have incentive to offer a rebate.

The seller charges price p. In addition to product price and the learned digital-attribute value, consumers make their purchase decisions based on their expected nondigital-attribute value, which could be influenced by online reviews. In our baseline model, we assume that consumers are naive: They do not factor in the effect of the rebate on the reviews in their expectation, because, for example, they are unaware of the possible review manipulation at the time of purchase or they are simply not sophisticated in their decision making. In the extension, we relax this assumption by incorporating sophisticated consumers and show that our major results remain qualitatively the same. We denote $\hat{\lambda}$ as the perceived proportion of satisfied consumers among those who purchased the product, and consumers think that with probability $\hat{\lambda}$ they will be satisfied as well (i.e., derive high value from the nondigital attributes). We denote n_g, n_b, and n_o as the numbers of purchasing consumers who post positive, negative, and no reviews, respectively. Those who post no reviews are believed to be either satisfied or unsatisfied with an equal probability. Based on these notations, we have $\hat{\lambda} = \frac{n_g + 0.5 n_o}{n_g + n_b + n_o}$. Accordingly, the consumer's perceived expected utility at the time of purchase can be formulated as

$$U(p, s) = [(1 - \theta)x + \theta\hat{\lambda}]q - p. \tag{2}$$

The seller's expected profit is

$$\Pi(p, s) = (p - s)\ n_g(p, s) + p\ [n_b(p, s) +\ n_o(p, s)], \tag{3}$$

where the first term on the right-hand side is the revenue from the purchasing consumers who post positive reviews and receive the rebate, and the second and third terms represent the revenue from the purchasing consumers who post negative and no reviews, respectively. The marginal production cost is assumed to be zero.

Because the rebate affects the reviews and the reviews in turn influence consumers' purchase decisions, we focus on the stationary equilibrium where the outcome of the consumers' review-posting decisions is consistent with that when formulating consumers' expected valuation. The timing is as follows. First, the seller chooses its product price and rebate to maximize its profit in Equation (3), and announces the price (but not the rebate, since it is unobservable at the time of purchase). Then, consumers make purchase decisions based on the perceived expected utility in Equation (2); If $U(p, s) \geq 0$, they purchase. Finally, consumers make their review-posting decisions, based on the realized nondigital-attribute value, review-sharing benefit, review-posting cost, rebate, and moral cost.

4 Equilibrium Analysis

In this section, using the approach of backward induction, we first analyze consumers' review-posting and purchase decisions, and then examine the firm's pricing and rebate decisions.

4.1 Review-Posting and Purchase Decisions

Once consumers receive the product, they observe their nondigital-attribute value, which is either high or low. As a result, consumers are either satisfied or unsatisfied. Together with the product, consumers might receive the rebate. Based on rebate s, possibly moral cost m, the review-sharing benefit v, and review-posting cost c, both satisfied and unsatisfied consumers make their review-posting decisions.

1. Satisfied consumers: A satisfied consumer chooses to post either positive or no reviews. If she posts a positive review, the consumer derives net value $v + s - c$ by sharing her true opinion and redeeming the rebate. The consumer derives zero value if not posting. A satisfied consumer does not have incentive to post a negative review as she derives negative value in this case.

2. Unsatisfied consumers: An unsatisfied consumer chooses to post negative, positive, or no reviews. The consumer derives net value $v - c$ if she shares her true opinion and posts a negative review, and zero value from not posting. Incentivized by the rebate, the consumer might consider posting a positive (fake) review, but doing so incurs a moral cost m from lying, leading to net value $s - m - c$.

In light of the trade-off in these options, we can derive consumers' review-posing decisions as follows.

Lemma 1. *In equilibrium, satisfied consumers with v such that $v + s \geq c$ post positive reviews, and the other satisfied consumers post no reviews. Unsatisfied consumers with v such that $v \geq max\{s - m, c\}$ post negative reviews, those with v such that $s - m \geq max\{v, c\}$ post positive (fake) reviews, and the other unsatisfied consumers post no reviews.*

Proof. All proofs are in the appendix unless indicated otherwise. □

Satisfied consumers post positive reviews if the benefit from review sharing and the rebate is greater than the review-posting cost (i.e., $v + s \geq c$). When the rebate is large enough such that $s \geq c$, all satisfied consumers are motivated to post positive reviews; Otherwise, some satisfied consumers (i.e., $v < c-s$) do not post, as illustrated in Figures 1(a) and 1(b). Unsatisfied consumers may consider posting positive reviews only if the rebate is high enough to compensate for the moral and review-posting costs (i.e., $s \geq m + c$); Otherwise, no unsatisfied consumers post (fake) positive reviews. In the former case, as shown in Figure 1(c), the unsatisfied consumers who derive high value from sharing their true opinion (i.e., $v \geq s - m$) post negative reviews and the others post (fake) positive reviews. In the latter case, as illustrated in Figure 1(d), the unsatisfied consumers post negative reviews if the review-sharing benefit is greater than the cost (i.e., $v \geq c$) and post no reviews otherwise.

Because consumers' expected utilities increase in their observed digital-attribute value x when making purchase decisions, there exists a threshold x^* such that the consumers with $x \geq x^*$ purchase and those with $x < x^*$ do not purchase, as illustrated in Figure 1. By solving $U(p, s) = 0$ in Equation (2), we obtain the marginal consumer

$$x^* = \frac{p - \theta\hat{\lambda}q}{(1-\theta)q} \tag{4}$$

and the demand for the product is $(1 - x^*)$. Furthermore, because product price p affects the

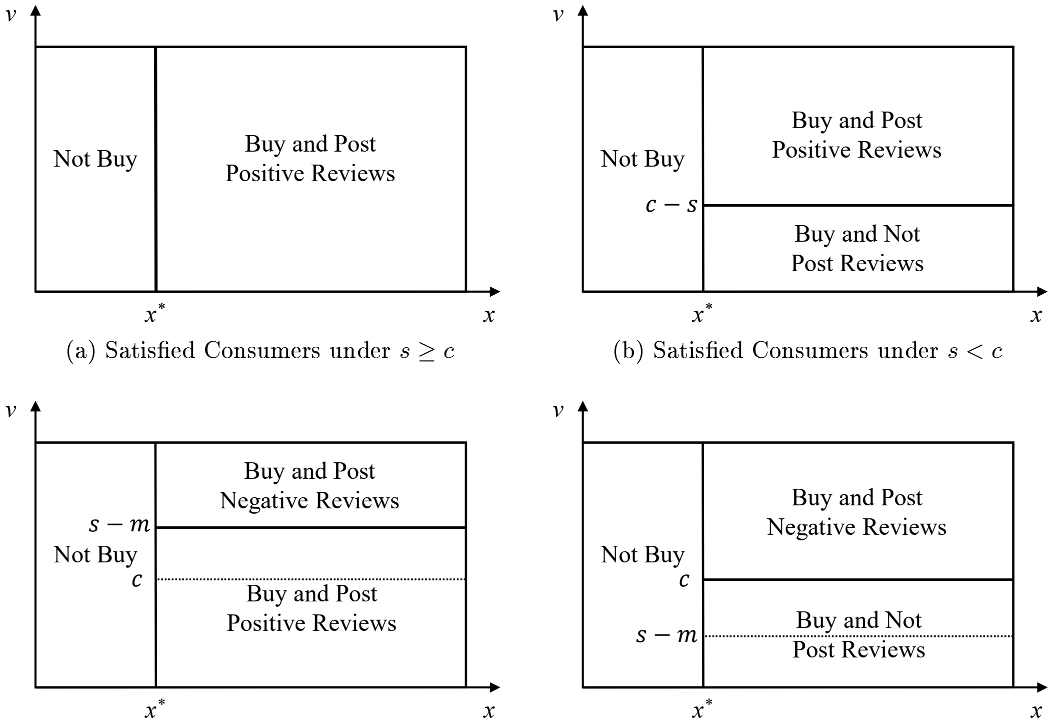

Figure 1: Consumers' Review-Posting Decisions

total number of purchasing consumers and the rebate s affects their review-posting decisions, the numbers of purchasing consumers who post positive, negative, and no reviews (i.e., $n_g(p, s)$, $n_b(p, s)$, and $n_o(p, s)$) are functions of p and s. We next distinguish cases by the rebate level and derive the segmentation of the three consumer groups accordingly.

- **High-Rebate Case ($s \geq c + m$).** In this case, the rebate is so high that all satisfied consumers post positive reviews. Meanwhile, the unsatisfied consumers with v less than or equal to $s - m$ post (fake) positive reviews, and those with v greater than $s - m$ post negative reviews. Therefore, we have

$$
\begin{aligned}
n_g(p, s) &= 0.5(1 - x^*) + 0.5(1 - x^*)(s - m) \\
n_b(p, s) &= 0.5(1 - x^*)[1 - (s - m)] \\
n_o(p, s) &= 0.
\end{aligned}
\tag{5}
$$

- **Intermediate-Rebate Case ($c \leq s < c + m$).** In this case, the rebate is high enough to motivate all satisfied consumers to post positive reviews, but not high enough to induce any unsatisfied consumers to post positive reviews. The unsatisfied consumers with v greater than or equal to c post negative reviews, and those with lower v do not post. Therefore, we have

$$\begin{aligned} n_g(p,s) &= 0.5(1-x^*) \\ n_b(p,s) &= 0.5(1-x^*)(1-c) \\ n_o(p,s) &= 0.5(1-x^*)c. \end{aligned} \tag{6}$$

- **Low-Rebate Case ($0 \leq s < c$).** In this case, the rebate is low such that not all satisfied consumers are incentivized to post positive reviews. The satisfied consumers with v greater than or equal to $c-s$ post positive reviews, and those with lower v do not post. The unsatisfied consumers with v greater than or equal to c post negative reviews, and those with lower v do not post. Therefore, we have

$$\begin{aligned} n_g(p,s) &= 0.5(1-x^*)[1-(c-s)] \\ n_b(p,s) &= 0.5(1-x^*)(1-c) \\ n_o(p,s) &= 0.5(1-x^*)(c-s) + 0.5(1-x^*)c. \end{aligned} \tag{7}$$

As expected, the numbers of purchasing consumers who post positive, negative, and no reviews change with the rebate level. Moreover, the composition of each consumer segment also differs under different rebate levels, leading to different functional forms for the same consumer segment in the three cases. For example, with a high rebate, both satisfied and unsatisfied consumers post positive reviews, whereas with a low rebate only satisfied consumers post positive reviews.

4.2 Price and Rebate Decisions

Anticipating the consumer segmentation, the firm maximizes its profit in Equation (3) by optimally choosing the price and rebate. The following proposition summarizes the optimal decisions in equilibrium.

Proposition 1. *The firm's optimal price and rebate are*

$$(p^*, s^*) = \begin{cases} \left(\frac{(2-\theta)q}{4}, 0\right) & \text{if } m \geq \hat{m}(c) \text{ and } c \leq \hat{c} \\ \left(\frac{4q(8-5\theta+c\theta)+3\theta^2q^2-4(1-c)^2}{64}, \frac{\theta q-2(1-c)}{4}\right) & \text{if } m \geq \hat{m}(c) \text{ and } c > \hat{c} \\ \left(\frac{2q(4-3\theta-m\theta)+3\theta^2q^2-(1-m)^2}{16}, \frac{\theta q-(1-m)}{2}\right) & \text{if } m < \hat{m}(c) \text{ and } m \leq \bar{m}(c) \\ \left(\frac{q(2-\theta+c\theta)+(1+c)(c+m)}{4}, c+m\right) & \text{otherwise,} \end{cases} \tag{8}$$

where $\hat{c} = \frac{2-\theta q}{2}$, $\bar{m}(c) = \theta q - 1 - 2c$, *and*

$$\hat{m}(c) = \begin{cases} (\sqrt{\theta q} - 1)^2 & \text{if } c \leq \sqrt{\theta q} - 1 \\ \frac{(\theta q - c - 1)c}{1+c} & \text{if } \sqrt{\theta q} - 1 < c \leq \hat{c} \\ \frac{4c(3\theta q-2)-20c^2-(2-\theta q)^2}{16(c+1)} & \text{if } \hat{c} < c. \end{cases} \tag{9}$$

The proposition shows that the firm's optimal price and rebate decisions critically depend on both moral and review-posting costs. As illustrated in Figure 2, there are three equilibrium rebate strategies (no rebate, low rebate, and high rebate), corresponding to three review outcomes (authentic reviews, boosted positive reviews, and fake reviews).[5] Only when the moral cost is low and the review-posting cost is not too high would the firm offer a high rebate to both motivate more satisfied consumers to post positive reviews and induce unsatisfied consumers to post fake positive reviews (the high-rebate, fake-review equilibrium). When the review-posting cost is high or when it is intermediate and the moral cost is high, the firm prefers to offer a low rebate to motivate more satisfied consumers to post positive reviews (the low-rebate, boosted-positive-review equilibrium). When the review-posting cost is low but the moral cost is high, the firm prefers not to offer a rebate, resulting in authentic reviews in the absence of any monetary incentive (the no-rebate, authentic-review equilibrium).

The intuition is as follows. A conditional rebate can benefit the firm by boosting consumers' perceived expected utilities, but it comes with a cost. When the moral cost is high, it is very costly to induce unsatisfied consumers to lie and post fake positive reviews. As a result, the firm has no

[5] The thin dotted line in the "High Rebate (fake reviews)" area is the curve $\bar{m}(c)$, which segments the third case in Proposition 1 (with an interior solution for the optimal rebate amount) from the fourth case (with a corner solution).

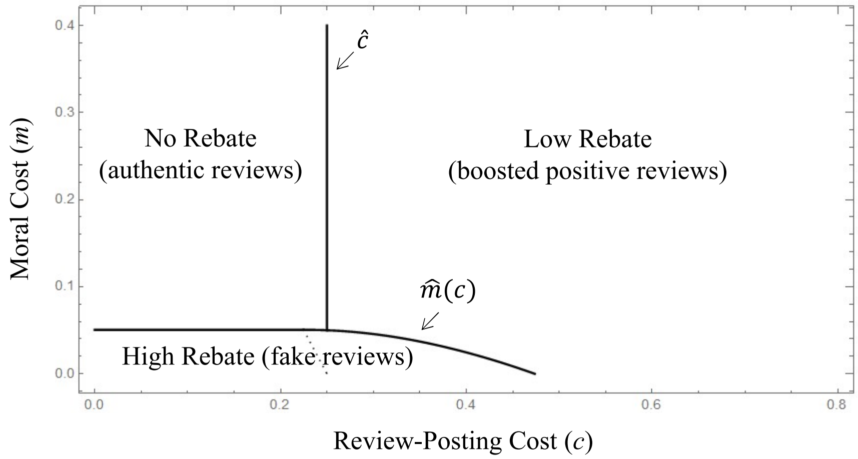

Figure 2: Equilibrium Rebate Decisions and Review Outcomes ($\theta = \frac{1}{8}$ and $q = 12$)

incentive to "bribe" unsatisfied consumers but only considers motivating satisfied consumers. There are two distinct cases: low review-posting cost and high review-posting cost. If the review-posting cost is low, a sizable proportion of satisfied consumers voluntarily share their true opinions and post positive reviews in the absence of monetary incentive. If the firm offers the rebate, a large number of satisfied consumers who would otherwise have posted positive reviews without monetary incentive also redeem the rebate, representing a high cost to the firm. As a result, the firm chooses not to offer any rebate, yielding the no-rebate, authentic-review equilibrium. On the other hand, if the review-posting cost is high, the number of consumers who voluntarily share their true opinion and post reviews is relatively small. Therefore, the benefit of inducing additional positive reviews using monetary incentives can outweigh the cost of offering the rebate. Consequently, the firm prefers to offer a low rebate to elicit more positive reviews from satisfied consumers, thus the low-rebate, boosted-positive-review equilibrium.

In contrast, when the moral cost is very low, unsatisfied consumers are easily induced to post fake positive reviews with a monetary incentive. Therefore, offering a high rebate to engage both satisfied and unsatisfied consumers might become a valuable option to the firm. In general, fake positive reviews would positively bias consumers' perceived valuation of the nondigital attributes of the product and lead to increased sales, which is beneficial to the firm, especially when the valuation of the nondigital attributes is high. Meanwhile, satisfied consumers who would otherwise have posted positive reviews without monetary incentive also redeem the rebate, a high cost for the firm. In the presence of high valuation of the nondigital attributes, when the moral cost is very low, the total rebate cost can be compensated by the total benefit from the inflated positive reviews and the boost in sales resulting from consumers' increased perceived product utility. Thus, offering a high rebate to induce fake reviews is more profitable than offering no rebate. Notice that an alternative strategy is to offer a low rebate, and its value critically depends on the review-posting cost. As discussed, if the review-posting cost is low, a sizable proportion of satisfied consumers who would otherwise voluntarily post positive reviews in the absence of any monetary incentive redeem the rebate, a high cost to the firm. The total benefit of inducing more positive reviews can be outweighed by the total rebate cost. As a result, offering a low rebate is not profitable. Therefore, if the moral cost is low and review-posting cost is not too high, offering a high rebate to induce fake reviews is optimal, resulting in the high-rebate, fake-review equilibrium.

Nevertheless, when the review-posting cost is high, offering a low rebate becomes profitable because the total number of authentic reviews is not large and thus the total benefit of eliciting more reviews can outweigh the total rebate cost. The firm must trade off the high-rebate strategy against the low-rebate strategy. As the review-posting cost increases, the firm needs to increase the rebate to entice consumers to post reviews. The increase in total rebate cost under the high-rebate strategy is more significant than that under the low-rebate strategy because of a larger volume of rebate redemption in the former. As a result, when the review-posting cost increases, the firm becomes less likely to offer a high rebate (i.e., $\hat{m}(c)$ decreases in c). Further, when the review-posting cost is high enough, the firm gives up bribing unsatisfied consumers and only offers a low rebate to compensate satisfied consumers to post reviews, resulting in the low-rebate, boosted-positive-review equilibrium. Corollary 1 summarizes the conditions under which offering a high rebate cannot be an equilibrium.

Corollary 1. *(a) When the review-posting cost increases, the firm becomes less likely to offer a high rebate; that is, $\hat{m}(c)$ decreases in c.*

(b) When the review-posting cost $c \geq \bar{c}$, the firm would never offer a high rebate to induce unsatisfied consumers to post fake positive reviews, where

$$\bar{c} = \begin{cases} \theta q - 1 & \text{if } \theta q \leq \frac{4}{3} \\ \frac{1}{10}(3\theta q - 2 + 2\sqrt{(2+\theta q)\theta q - 4}) & \text{if } \theta q > \frac{4}{3}. \end{cases}$$

This corollary and Proposition 1 imply that fake positive reviews can be induced by a high rebate in equilibrium only if the moral cost is low and the review-posting cost is not too high. To induce unsatisfied consumers to post fake positive reviews, the firm needs at least to offer $m + c$ to compensate their moral and review-posting costs. Because this offer increases in the review-posting cost, when the review-posting cost is high (even if the moral cost is low), the rebate offer can be high. Further, this high rebate will also be redeemed by satisfied consumers who post reviews. Therefore, when the review-posting cost is high enough (i.e., $c \geq \bar{c}$), the cost to induce fake positive reviews can be too high to be justified by its benefit, and the firm has no incentive to offer a high rebate. Instead, a low rebate to motivate satisfied consumers to post positive reviews can be desirable—the high review-posting cost limits the number of unsatisfied consumers who post negative reviews, and thus the overall effect of additional positive reviews can be significant to offset the monetary incentive offered to the satisfied consumers.

Moreover, whether the firm has incentive to offer a rebate also depends on consumers' maximum valuation of the product (q) and their weight on the nondigital attributes (θ). Analysis of q and θ leads to the following results.

Corollary 2. *(a) The segmentation curves $\hat{c}$ decreases and $\hat{m}(c)$ increases in θq for $c \in [0, 1]$.*

(b) When θq decreases, the firm is less likely to offer a rebate (either low or high).

(c) When $\theta q \leq 1$, the firm never offers a high rebate regardless of the moral cost or the review-posting cost.

The firm benefits from offering a conditional rebate to boost consumers' perceived expected utility, particularly their perceived value from the nondigital attributes. Because θq measures the maximum value consumers may derive from the nondigital attributes, it plays a crucial role in determining the firm's rebate strategy. As illustrated in Figure 3, when θq becomes smaller, the value from the nondigital attributes decreases, and the firm's incentive to offer either high or low rebate decreases (the $\hat{m}(c)$ line moves downwards and the $\hat{c}$ line moves to the right). In particular, when θq is small enough (i.e., $\theta q \leq 1$), the firm no longer has incentive to offer a high rebate (when $\theta q = 1$, $\hat{m}(c)$ shrinks to the origin point at the low-bottom corner in Figure 3 and $\bar{c} = 0$ in Corollary 1). In contrast, the firm continues to have incentive to offer a low rebate in the case where the review-posting cost is high (i.e., $c > \hat{c}$).

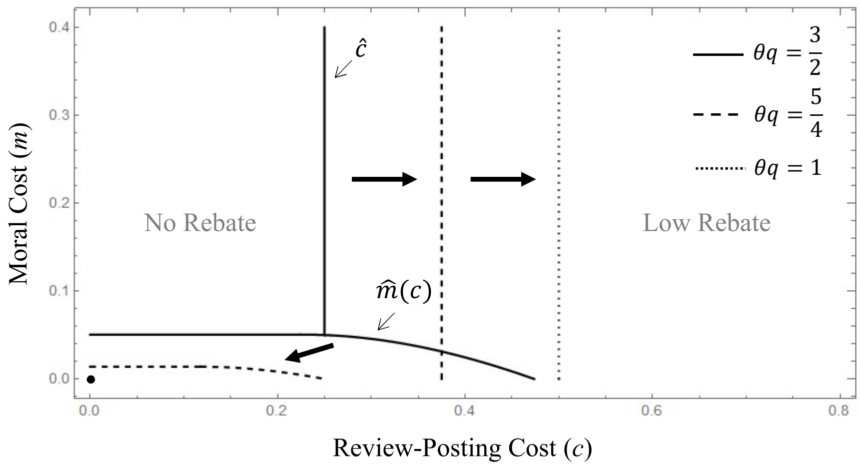

Figure 3: Effect of θq on Equilibrium Rebate Decisions

In sum, it is worth noting that the seller's conditional-rebate strategy does not necessarily result in fake reviews. Fake reviews come with a high cost for the firm. Not only must the firm provide enough monetary incentive to "bribe" unsatisfied consumers to lie, that monetary incentive will also be taken by satisfied consumers who would otherwise have posted positive reviews without the rebate. Only when the moral cost is low, review-posting cost is not too high, and the maximum value from nondigital attributes is high, would the firm prefer a high-rebate strategy and would fake reviews appear in equilibrium. Further, numerically, we find that offering a low rebate is more likely to arise as an equilibrium than a high rebate in the entire feasible parameter space we examine. Under the low-rebate equilibrium, the rebate is used to motivate more satisfied consumers to share their true opinion, rather than to bribe unsatisfied consumers to lie and post fake reviews. This finding sheds new light on the criticism and concern about the fake-review phenomenon induced by the conditional-rebate strategy.

5 Effect of Conditional Rebates

In this section, we examine the effect of conditional rebates on the firm's equilibrium profit and social welfare. We use the case in which no rebate is considered (i.e., the price-only strategy) as the benchmark to examine how the conditional-rebate strategy affects the equilibrium outcome.

5.1 Effect on the Firm's Profit

By substituting the equilibrium rebate and price into the profit function in Equation (3), we can derive the firm's equilibrium profit.

Proposition 2. *The firm's equilibrium profit is*

$$\Pi^* = \begin{cases} \frac{(2-\theta)^2 q}{16(1-\theta)} & \text{if } m \geq \hat{m}(c) \text{ and } c \leq \hat{c} \\ \frac{[4(1-c)^2+4q(8-5\theta+c\theta)+\theta^2 q^2]^2}{4096(1-\theta)q} & \text{if } m \geq \hat{m}(c) \text{ and } c > \hat{c} \\ \frac{[(1-m)^2+2q(4-m\theta-3\theta)+\theta^2 q^2]^2}{256(1-\theta)q} & \text{if } m < \hat{m}(c) \text{ and } m \leq \bar{m}(c) \\ \frac{[(2-\theta)q-c(1+c+m-\theta q)-m]^2}{16(1-\theta)q} & \text{otherwise.} \end{cases} \tag{10}$$

where $\hat{c}$, $\bar{m}(c)$, and $\hat{m}(c)$ are defined in Proposition 1.

To examine the effect of a conditional rebate on the firm's profit, we study a benchmark case in which no rebate is considered. In the benchmark, the firm maximizes its profit by optimally choosing the price only, which leads to the same solution as when the firm decides to offer no rebate under the conditional-rebate strategy (i.e., $s^* = 0$). As a result, the equilibrium price and profit in Propositions 1 and 2 when the firm chooses to offer no rebate are the equilibrium outcome for the benchmark case.

Notice that under the conditional-rebate strategy, the firm always has the option to offer zero rebate. The firm chooses to offer a positive conditional rebate only if doing so is more profitable than offering no rebate. Therefore, the option to offer a conditional rebate makes the firm (weakly) better off.

We next examine how the firm may gain from the conditional rebate. The conditional rebate is costly, because consumers who post positive reviews are paid by the firm. The firm can be better off only if the benefit from the rebate offsets the cost, and the benefit essentially stems from the consumers' boosted expected utilities. Compared with the price-only strategy under the benchmark case, we next analyze how the conditional-rebate strategy alters the equilibrium profit, demand, and price.

Proposition 3. *(a) The equilibrium profit under the conditional-rebate strategy is (weakly) greater than that under the benchmark case. (b) Both equilibrium demand and equilibrium price under the* *conditional-rebate strategy are higher than those under the benchmark case. Meanwhile,* $(p^* - s^*)$ *is lower than the price in the benchmark case.*

When consumers' perceived expected utilities are boosted by more positive reviews induced by the monetary incentive, the direct effect is that more consumers are willing to buy the product at the same price. Given the enhanced demand, the firm strategically raises its product price to balance the price increase and demand increase in maximizing its profit. As a result, the price and demand under the conditional-rebate strategy are higher than those under the price-only strategy.

Notice that, compared with the benchmark, the firm earns more profit from each consumer who does not redeem the rebate because of the increased product price. However, the firm earns profit $(p^* - s^*)$ from each purchasing consumer who redeems the rebate, which is lower than the price in the benchmark case. Therefore, even with an increased product price, offering a rebate involves a net cost for the firm.

Next, we examine how the moral cost and review-posting cost affect the firm's equilibrium profit. First, in the equilibrium when no rebate is offered, neither the consumers' nor the firm's decision involves the moral cost, and thus the firm's profit is independent of the moral cost. In addition, because the review-posting cost dictates consumers' review-posting decisions in the same way whether a consumer is satisfied or unsatisfied, it does not affect the proportion of positive reviews among purchasing consumers. Consequently, consumers' perceived expected utilities are independent of the review-posting cost, as is the firm's profit. Proposition 4 summarizes the other cases when a low or high rebate is offered.

Proposition 4. *(a) In the equilibrium when a low rebate is offered, the firm's profit increases in the review-posting cost and is independent of the moral cost (i.e.,* $\frac{\partial \Pi^*}{\partial c} > 0$ *and* $\frac{\partial \Pi^*}{\partial m} = 0$ *for* $m > \hat{m}(c)$ *and* $c > \hat{c}$*).*

(b) In the equilibrium when a high rebate is offered, the firm's profit (weakly) decreases in the review-posting cost and decreases in the moral cost (i.e., $\frac{\partial \Pi^*}{\partial c} \leq 0$ *and* $\frac{\partial \Pi^*}{\partial m} < 0$ *for* $m \leq \hat{m}(c)$*).*

In the equilibrium when a low rebate is offered, first, because the consumers' and the firm's decisions do not involve the moral cost, the firm's profit is independent of the moral cost in this case. Second, interestingly, we find that the firm's profit increases in the review-posting cost. The rationale behind this counterintuitive result is as follows. Notice that only satisfied consumers redeem the rebate in this case. There are two groups: those who are incentivized by the rebate to post positive reviews and those who claim the rebate as a free gift and would have written positive reviews without the rebate. The first group brings benefit by boosting positive reviews but the second group creates a pure cost to the firm because this group of consumers would have posted positive reviews in the absence of any monetary incentive. When the review-posting cost increases, the number of consumers in the second group decreases, which reduces the firm's pure cost and gives the firm more room to optimize its offering. In fact, due to the reduced marginal cost of the rebate, the firm is able to offer a higher rebate when the review-posting cost increases, as shown in Equation (8) of Proposition 1 (the second case), which in turn incentivizes more consumers in the first group to boost reviews. As a result, the firm's total profit increases.

In the equilibrium when a high rebate is offered, both satisfied and unsatisfied consumers redeem the rebate. In sharp contrast, the firm's profit (weakly) decreases in the review-posting cost. With the high rebate offering in this case, satisfied consumers take advantage of the rebate regardless of the amount. The equilibrium rebate amount is primarily determined by the incentive required to induce unsatisfied consumers to post fake reviews. When the review-posting cost is relatively high (i.e., $c \geq \bar{m}^{-1}(m)$ in Proposition 1), the firm optimally sets the rebate at $(c+m)$, the amount to cover unsatisfied consumers' review-posing and moral costs, which is just high enough to induce some unsatisfied consumers to post fake reviews. Therefore, the rebate increases but the firm's profit decreases in the review-posting cost. When the review-posting cost is low, the firm has incentive to rebate more than minimum amount required to induce fake reviews. In this case, unsatisfied consumers trade off the options of posting fake reviews (with payoff $s-m$) and posting true opinion (with payoff v). Subsequently, as also shown in Equation (8) of Proposition 1 (the third case), the review-posting cost becomes irrelevant. As a result, both the rebate and the firm's profit are independent of the review-posting cost.

In addition, in the high-rebate equilibrium, unsatisfied consumers are paid to lie and post fake reviews, which involves moral cost. The rebate needs to be high enough to cover the moral cost. In this case, the amount of rebate increases in the moral cost, as also shown in Equation (8) of Proposition 1 (the last two cases). Consequently, the equilibrium profit decreases in the moral cost.

5.2 Effect on Social Welfare

We next examine the effect of a conditional rebate on social welfare. The social value consists of two parts. First, the transaction between a consumer and the firm creates value $[(1-\theta)x + \theta y]q$, where $y \in \{0, 1\}$ represents low and high nondigital-attribute value, respectively. Second, consumers' review-posting behavior also creates value—posting their true opinions (either positive or negative) creates value $v - c$, and posting fake reviews creates value $(-c - m)$, where m is the moral cost of lying.

When a low rebate or no rebate is offered (i.e., when $m \geq \hat{m}(c)$), only a proportion of purchasing consumers post their true opinions. The social welfare can be formulated as

$$\int_{x^*}^{1}[(1-\theta)x + \tfrac{1}{2}\theta]q dx + \tfrac{1}{2}\int_{x^*}^{1}\int_{c-s}^{1}(v-c)dvdx + \tfrac{1}{2}\int_{x^*}^{1}\int_{c}^{1}(v-c)dvdx, \tag{11}$$

where x^* is defined in Equation (4). The first term captures the social value created by the transactions. The second and third terms represent the social value created by satisfied and unsatisfied consumers who post their true opinions, respectively. Notice that $s = 0$ corresponds to both the no-rebate equilibrium case and the benchmark case.

When a high rebate is offered (i.e., when $m < \hat{m}(c)$), some unsatisfied consumers are induced to post fake reviews. The social welfare can be formulated as

$$\begin{gathered}\int_{x^*}^{1}[(1-\theta)x + \tfrac{1}{2}\theta]q dx \\ + \tfrac{1}{2}\int_{x^*}^{1}\int_{0}^{1}(v-c)dvdx + \tfrac{1}{2}\int_{x^*}^{1}\int_{s-m}^{1}(v-c)dvdx + \tfrac{1}{2}\int_{x^*}^{1}\int_{0}^{s-m}(-c-m)dvdx\end{gathered} \tag{12}$$

Similar to Equation (11), the first term is the social value created by the transactions, and the second term represents the social value created by satisfied consumers who post their true opinions. Because some unsatisfied consumers with $v > s - m$ post true opinions and other unsatisfied ones with $v \leq s - m$ post fake reviews, the last two terms in Equation (12) capture the social value created by these unsatisfied consumers, respectively.

By deriving the social welfare in Equations (11) and (12) and comparing the equilibrium outcome with the benchmark case, Proposition 5 summarizes the effect of the conditional rebate.

Proposition 5. *(a) In the equilibrium when a low rebate is offered, the social welfare under a*

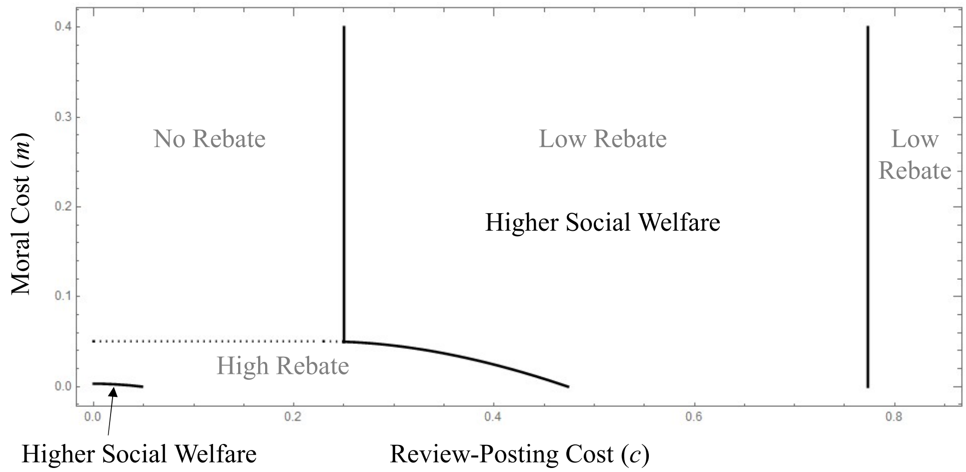

Figure 4: Effect of a Conditional Rebate on Social Welfare

conditional rebate is greater than that under the benchmark case if and only if the review-posting cost is not too high (i.e., $c \leq 1 - \frac{(4\sqrt{3}-3)\theta q}{26}$*).*

(b) In the equilibrium when a high rebate is offered, the social welfare under a conditional rebate is greater than that under the benchmark case if and only if both the review-posting cost and the moral cost are very small; that is, if and only if $c \leq \frac{8m-2(\theta q-1-m)^2+\sqrt{\psi}}{8(2-\theta)q}$*, where* ψ *is defined in Equation (26) in the appendix.*

The social gain associated with a conditional rebate comes from the additional transactions resulting from consumers' boosted expected utilities because of the additional positive reviews induced by the monetary incentive. The social loss associated with a conditional rebate stems from consumers who post reviews when their review-sharing benefits are below the review-posting cost and from some consumers being induced to post fake reviews, incurring both review-posting cost and moral cost. In the absence of a rebate, consumers post reviews if and only if the review-sharing benefit is greater than or equal to the review-posting cost, which is socially efficient. The rebate distorts the consumers' review-posting behavior, and the degree of distortion depends on the rebate amount.

Proposition 5 and Figure 4 show that, when a low rebate is offered, the social welfare is higher than that in the price-only benchmark case if the review-posting cost is not too high. Notice that the amount of rebate in equilibrium increases in the review-posting cost. When the review-posting cost is very high, the rebate becomes large and social loss due to the distortion in consumers' review-posting behavior is too significant to be offset by the social gain from the additional transactions.

When a high rebate is offered, many unsatisfied consumers are induced to post fake reviews. In this case, the social loss is considerable because unsatisfied consumers do not derive any review-sharing benefit and they incur moral cost in addition to the review-posting cost (as seen in the last integral in Equation (12)). As a result, only when both the moral and review-posting costs are small enough does the social gain outweigh the social loss and the conditional rebate increases social welfare. Numerically, we can show this parameter space to be very small, as demonstrated in Figure 4.

6 Extension: The Case with Sophisticated Consumers

In our baseline model, we assume that all consumers are naive in a sense that they do not factor in the effect of the rebate on the reviews in their expectation. In this extension, we consider that some consumers may know the likelihood of the nondigital-attribute value being high or low before purchase because, for example, they might have used similar products before. As a result, these consumers' perceived likelihood that they will be satisfied with the nondigital attributes is not affected by the incentivized reviews. We call these consumers sophisticated consumers. We next show that our main results continue to hold qualitatively in the presence of sophisticated consumers.

We denote α as the proportion of naive consumers and $1-\alpha$ as the proportion of sophisticated consumers. A sophisticated consumer's expected utility at the point of purchase is $[(1-\theta)x + \theta/2]q - p$. Sophisticated consumers with $x \geq \frac{p-\theta q/2}{(1-\theta)q}$ purchase the product. Notice the proportion of naive consumers who purchase the product might differ from that of sophisticated consumers, because the former is affected by the rebate offering whereas the latter is not. Everything else stays the same as in the baseline model.

Similar to the baseline model, Proposition 6 derives the firm's equilibrium choice (the proof is similar to that of Proposition 1).

Proposition 6. *In the presence of sophisticated consumers, the firm's optimal price and rebate are*

$$(\tilde{p}^*, \tilde{s}^*) = \begin{cases} \left(\frac{(2-\theta)q}{4}, 0\right) & \textit{if } m \geq \hat{m}(c) \textit{ and } c \leq \hat{c} \\ \left(\frac{4q(8-4\theta-\alpha\theta+\alpha c\theta)+3\alpha^2\theta^2q^2-4(1-c)^2}{64}, \frac{\alpha\theta q-2(1-c)}{4}\right) & \textit{if } m \geq \hat{m}(c) \textit{ and } c > \hat{c} \\ \left(\frac{2q(4-2\theta-\alpha\theta-\alpha m\theta)+3\alpha^2\theta^2q^2-(1-m)^2}{16}, \frac{\alpha\theta q-(1-m)}{2}\right) & \textit{if } m < \hat{m}(c) \textit{ and } m \leq \bar{m}(c) \\ \left(\frac{q(2-\theta+\alpha c\theta)+(1+c)(c+m)}{4}, c+m\right) & \textit{otherwise,} \end{cases}$$

where $\hat{c} = \frac{2-\alpha\theta q}{2}$, $\bar{m}(c) = \alpha\theta q - 1 - 2c$, *and*

$$\hat{m}(c) = \begin{cases} (\sqrt{\alpha\theta q} - 1)^2 & \textit{if } c \leq \sqrt{\alpha\theta q} - 1 \\ \frac{(\alpha\theta q - c - 1)c}{1+c} & \textit{if } \sqrt{\alpha\theta q} - 1 < c \leq \hat{c} \\ \frac{4c(3\alpha\theta q-2)-20c^2-(2-\alpha\theta q)^2}{16(c+1)} & \textit{if } \hat{c} < c. \end{cases}$$

Notice that the firm's optimal price and rebate decisions and the corresponding boundary conditions take the same form as in Proposition 1 in the baseline model. The only difference is that now the firm's decisions depend on the proportion of naive consumers. When $\alpha = 1$, the extension reduces to the baseline model.

As long as there are some naive consumers, as in the baseline model, the above result shows that the firm's optimal price and rebate decisions critically depend on both the moral and review-posting costs. There are three equilibrium rebate strategies (no rebate, low rebate, and high rebate), corresponding to three review outcomes (authenticate reviews, boosted positive reviews, and fake reviews). Only when the moral cost is low and the review-posting cost is not too high would the firm offer a high rebate, which results in fake positive reviews. When the review-posting cost is high or when it is intermediate and the moral cost is high, the firm prefers to offer a low rebate, which leads to boosted positive reviews. When the review-posting cost is low but the moral cost is high, the firm prefers to offer no rebate, yielding authentic reviews. The intuition is the same as in the baseline model.

As in the baseline model, the firm's equilibrium rebate offering, or, equivalently, the segmentation curves $\hat{c}$ and $\hat{m}(c)$, are affected by θq (the valuation associated with nondigital attributes). The difference is that now the proportion of naive consumers also plays a role in the firm's equilibrium rebate offering. Intuitively, this is because sophisticated consumers know the true likelihood of being satisfied and the firm can only fool naive consumers by manipulating reviews with a monetary incentive. Similar to Corollary 2, we find that the segmentation curve $\hat{c}$ decreases and $\hat{m}(c)$ increases in $\alpha\theta q$ for $c \in [0, 1]$. When $\alpha\theta q$ decreases, the firm is less likely to offer a rebate (either low or high). When $\alpha\theta q \leq 1$, the firm never offers a high rebate, regardless of the moral or review-posting cost. These results suggest that the firm becomes less likely to offer a rebate when the proportion of naive consumers gets smaller.

In sum, in the presence of sophisticated consumers, we show that the firm is less likely to offer a rebate than in the baseline model. Qualitatively, the underlying driving forces and the rebate-offering patterns continue to hold, and our main insights are robust.

7 Discussion and Conclusion

Today, fake reviews have become a common problem and major concern on many online platforms. Leading platforms such as Google and Amazon have developed sophisticated algorithms to combat bots and click farms that are often used to produce fake likes and views. While advanced technologies are increasingly capable of detecting accounts with unusual activities or fake users, strategic sellers turn to bribing legitimate users into leaving positive reviews, for example, by providing underhanded conditional-rebate offerings offline in the form of mail-in rebates. This study analyzes the seller's optimal pricing and rebate strategies, its impact on consumers' review-posting behavior and social welfare implications. Our findings provide important new insights to inform future platform review management and policies.

First, it is not always profitable for strategic sellers to adopt the conditional-rebate strategy. The rebates may motivate consumers who would otherwise not provide feedback to post positive reviews. On the other hand, they might also be redeemed by satisfied consumers who would have voluntarily posted positive reviews, which could be costly for the seller. The rule of thumb is that sellers must consider consumers' review-posting and moral costs, as well as the product nondigital-attribute valuation to determine whether to offer a conditional rebate and in what amount. While positive reviews are valuable, blindly offering incentives may not help achieve the goal of review manipulation. In particular, when nondigital-attribute valuation (the part of product valuation that can be influenced by reviews) is small, the cost of offering a conditional rebate might not be compensated by its benefit and thus sellers should not offer rebates. Even in the presence of high nondigital-attribute valuation, if the review-posting cost is low but the moral cost is high, the seller's optimal strategy is to offer no rebate.

Second, the conditional-rebate strategy does not always result in fake reviews. Fake reviews occur only if the moral cost is low and the review-posting cost is not too high. Therefore, in addition to developing effective information policies or countermeasures to fight against incentivized reviews, it is equally (if not more) important to educate consumers and retailers. When consumers have high moral standards, care about truth-telling, and are willing to keep feedback instructive, it would become very costly for sellers to game the review system and execute the conditional-rebate strategy.

When the moral cost is low and moral standards cannot be raised in a short term, sellers with high nondigital-attribute-valuation products might offer high rebates, leading to fake reviews. Because the sellers essentially pay consumers to buy positive fake reviews, information is distorted, which might be harmful to society. In this case, the seller gains from increased sales due to the inflated positive product reviews at the consumers' expense. This finding helps explain the reason why many e-commerce platforms combat such practices using policy guidelines and legal tools. To keep the online platform a safe and trusted place for shopping, platforms should carefully design their review policies, implement methods to monitor and identify fake reviews (especially associated with high nondigital-attribute-valuation products), and penalize any underhanded tactics to artificially manipulate reviews that go against the platforms' review policies.

Interestingly, we find that sellers' profits (weakly) decrease in the review-posting cost in the high-rebate, fake-review equilibrium, whereas they increase in the low-rebate, boosted-review equilibrium. Because the high-rebate equilibrium outcome causes serious concerns about fake reviews, this finding implies that the platform can manage to mitigate the occurrence of fake reviews by increasing consumers' review-posting cost. For example, the platform can set more strict review-posting rules such as imposing a minimum number of words in reviews or requiring attachment of photos or videos. The increased review-posting cost will decrease the seller's profit of offering a high rebate and increase its profit of offering a low rebate, making the seller more likely to choose the low-rebate strategy over the high-rebate strategy. Instead of purely relying on costly technical methods to detect fake reviews, our research suggests economic means to combat fake reviews.

Finally, our results show that offering a conditional rebate might be socially beneficial. Even though social-welfare improvement can rarely occur in the high-rebate equilibrium, it can occur in the low-rebate equilibrium if the review-posting cost is not too high. In a relatively wide range of parameter values, platform owners or policy designers may control the platform review-posting cost at an appropriate level to induce the low-rebate equilibrium, under which the seller has an incentive to offer a small monetary incentive to boost its positive reviews without attempting to entice consumers to lie. In addition to the social-welfare gain, the platform can benefit from more sales and greater seller success. Our result thus sheds new light on the debate about whether platforms should completely ban incentivized reviews. Overall, our findings offer new insights into the fake-review phenomenon induced by the conditional-rebate strategy on many e-commerce platforms.

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A Appendix

A.1 Proof of Lemma 1

Proof. A satisfied consumer posts a positive review if $v + s \geq c$; otherwise, the benefit cannot compensate for the review-posting cost and she does not post.

An unsatisfied consumer posts a negative review only if the review-sharing value, v, is greater than the value of posting a fake positive review, $s - m$, and greater than review-posting cost c; that is, $v \geq \max\{s - m, c\}$. An unsatisfied consumer posts a fake positive review only if the benefit of doing so, $s - m$, is greater than the value of sharing a true opinion, v, and greater than review-posting cost c; that is, $s - m \geq \max\{v, c\}$. In other cases, unsatisfied consumers post no reviews. $\square$

A.2 Proof of Proposition 1

Proof. First, we notice that the intermediate-rebate case with $c \leq s < c + m$ cannot arise in equilibrium. When $s \in [c, c + m)$, by Equation (6), n_g, n_b, and n_o remain the same for any s, and so do consumers' perceived expected utilities. Therefore, any $s \in (c, c + m)$ is dominated by $s = c$ to maximize the firm's profit. We next consider the low-rebate and high-rebate cases.

Low-Rebate Case ($0 \leq s < c$). By substituting n_g, n_b, and n_o in Equation (7) into Equation (3), we have the firm's profit function:

$$\Pi_l(p, s) = \tfrac{1}{2}(1 - x^*)(p - s)[1 - (c - s)] + \tfrac{1}{2}(1 - x^*)p[(1 - c) + (c - s) + c], \tag{13}$$

where x^* is specified in Equation (4). The first-order partial derivatives of $\Pi_l(p, s)$ with respect to p and s can be derived as

$$\begin{aligned} \frac{\partial \Pi_l(p,s)}{\partial p} &= \frac{-8p+2s(1-c+s)+(4-2\theta+s\theta)q}{4(1-\theta)q} \\ \frac{\partial \Pi_l(p,s)}{\partial s} &= \frac{4(1+2s-c)(p-q)+\theta q[2+2p+2c(s-1)+s(2-3s)]}{8(1-\theta)q}. \end{aligned} \tag{14}$$

By letting $\frac{\partial \Pi_l(p,s)}{\partial p} = 0$ and $\frac{\partial \Pi_l(p,s)}{\partial s} = 0$, we have

$$p_l^* = \frac{4q(8-5\theta+c\theta)+3\theta^2q^2-4(1-c)^2}{64} \text{ and } s_l^* = \frac{\theta q-2(1-c)}{4}. \tag{15}$$

We can verify that the Hessian matrix is negative and $\Pi_l(p,s)$ is concave around (p_l^*, s_l^*). Therefore, (p_l^*, s_l^*) is the optimal price–rebate pair for the unconstrained maximization problem $\max_{p,s} \Pi_l(p,s)$.

Next, we check the constraints $0 \le s < c$. Under the assumption $\theta q \le \frac{3}{2}$, we notice that $s_l^* < c$, and $s_l^* \ge 0$ requires that $c \ge \hat{c} \equiv 1 - \frac{\theta q}{2}$. When $c \ge \hat{c}$, (p_l^*, s_l^*) in Equation (15) is the optimal solution to the constrained optimization problem for the low-rebate case. By substituting (p_l^*, s_l^*) into Equation (13), we derive the resulting profit as

$$\Pi_l^* = \frac{[4(1-c)^2 + 4q(8-5\theta+c\theta)+\theta^2 q^2]^2}{4096(1-\theta)q}. \tag{16}$$

When $c < \hat{c}$, because of the concavity of $\Pi_l(p,s)$, the optimal rebate is 0. The resulting optimal price is the one that satisfies $\frac{\partial \Pi_l(p,s)}{\partial p} = 0$ with $s = 0$ in Equation (14), leading to

$$p_n^* = \frac{(2-\theta)q}{4} \text{ and } \Pi_n^* = \frac{(2-\theta)^2 q}{16(1-\theta)}. \tag{17}$$

High-Rebate Case ($s \ge c + m$). By substituting n_g, n_b, and n_o in Equation (5) into Equation (3), we have the firm's profit function:

$$\Pi_h(p,s) = \frac{1}{2}(1-x^*)(p-s)[1+(s-m)] + \frac{1}{2}(1-x^*)p[1-(s-m)], \tag{18}$$

where x^* is specified in Equation (4). The first-order partial derivatives of profit $\Pi_h(p,s)$ with respect to p and s can be derived as

$$\begin{aligned} \frac{\partial \Pi_h(p,s)}{\partial p} &= \frac{-4p+q(2+\theta s-\theta-\theta m)+s(1+s-m)}{2(1-\theta)q} \\ \frac{\partial \Pi_h(p,s)}{\partial s} &= \frac{\theta q(1-m^2+4ms+2p-3s^2)+2(1+2s-m)(p-q)}{4(1-\theta)q}, \end{aligned} \tag{19}$$

By letting $\frac{\partial \Pi_h(p,s)}{\partial p} = 0$ and $\frac{\partial \Pi_h(p,s)}{\partial s} = 0$, we have

$$p_h^* = \frac{2q(4-3\theta-m\theta)+3\theta^2 q^2-(1-m)^2}{16} \text{ and } s_h^* = \frac{\theta q+m-1}{2}, \tag{20}$$

We can verify that the Hessian matrix is negative and $\Pi_h(p,s)$ is concave around (p_h^*, s_h^*). Therefore, (p_h^*, s_h^*) is the optimal price–rebate pair for the unconstrained maximization problem $\max_{p,s} \Pi_h(p,s)$.

Next, we check the constraint $s \ge c + m$. Under the assumption $\theta q \le \frac{3}{2}$, first, we notice that

$s_h^* - m = \frac{\theta q - m - 1}{2} \leq 1$, so that n_g, n_b, and n_o in Equation (5) are well behaved. Further, $s_h^* \geq c + m$ if $m \leq \bar{m}(c) \equiv \theta q - 1 - 2c$. Accordingly, when $m \leq \bar{m}(c)$, (p_h^*, s_h^*) in Equation (20) is the optimal solution to the constrained optimization problem for the high-rebate case. By substituting (p_h^*, s_h^*) into Equation (18), we derive the resulting profit as

$$\Pi_h^* = \frac{[(1-m)^2 + 2q(4 - m\theta - 3\theta) + \theta^2 q^2]^2}{256(1-\theta)q}, \tag{21}$$

When $m > \bar{m}(c)$, because of the concavity of $\Pi_h(p, s)$, the optimal rebate is $c + m$. The resulting optimal price is the one that satisfies $\frac{\partial \Pi_h(p,s)}{\partial p} = 0$ with $s = c + m$ in Equation (19), leading to the optimal price and profit:

$$p_{hc}^* = \frac{q(2 - \theta + c\theta) + (1+c)(c+m)}{4} \text{ and } \Pi_{hc}^* = \frac{[(2-\theta)q - c(1 + c + m - \theta q) - m]^2}{16(1-\theta)q}, \tag{22}$$

Globally Optimal Price and Rebate Decisions. We next compare the optimal outcomes in the low-rebate and high-rebate cases to derive the globally optimal price and rebate. Based on Equations (16), (17), (21), and (22), we can derive $\Pi_n^* \geq \Pi_h^*$ if and only if $m \geq (\sqrt{\theta q} - 1)^2$, $\Pi_n^* \geq \Pi_{hc}^*$ if and only if $m \geq \frac{(\theta q - c - 1)c}{1+c}$, and $\Pi_l^* \geq \Pi_{hc}^*$ if and only if $m \geq \frac{4c(3\theta q - 2) - 20c^2 - (2 - \theta q)^2}{16(c+1)}$. We notice that $\bar{m}^{-1}((\sqrt{\theta q} - 1)^2) = \sqrt{\theta q} - 1 < \hat{c}$ under the assumption $\theta q \leq \frac{3}{2}$. Therefore, when $c < \sqrt{\theta q} - 1$ and $m \geq (\sqrt{\theta q} - 1)^2$ or when $c \in [\sqrt{\theta q} - 1, \hat{c}]$ and $m \geq \frac{(\theta q - c - 1)c}{1+c}$, $(p_n^*, 0)$ is optimal. When $c > \hat{c}$ and $m \geq \frac{4c(3\theta q - 2) - 20c^2 - (2 - \theta q)^2}{16(c+1)}$, (p_l^*, s_l^*) is optimal. In other areas (i.e., if $m < \hat{m}(c)$ in Proposition 1), if $m \leq \bar{m}(c)$, (p_h^*, s_h^*) is optimal by the definition of $\bar{m}(c)$, and (p_{hc}^*, s_{hc}^*) is optimal otherwise. Altogether, the optimal solution can be summarized as in Proposition 1. □

A.3 Proof of Corollary 1

Proof. By the definition in Equation (9), $\hat{m}(c)$ (weakly) decreases in c because, for $c \in (\sqrt{\theta q} - 1, \hat{c}]$, $\frac{d\hat{m}(c)}{dc} = \frac{\theta q}{(1+c)^2} - 1 < 0$, and $\frac{d\hat{m}(c)}{dc} = \frac{-20c(c+2) + \theta q(\theta q + 8) - 4}{16(c+1)^2} < 0$ for $c > \hat{c}$. If $\theta q \leq 1$, $\hat{m}(c) \leq 0$ for all $c \in [0, 1]$, and thus the firm does not offer a high rebate. If $\theta q > 1$, because $\hat{m}(0) > 0$ and $\hat{m}(1) < 0$, $\hat{m}(c)$ crosses zero from the positive to the negative side once over $c \in (0, 1)$. Notice that $\frac{(\theta q - c - 1)c}{1+c}$ might cross zero at $\theta q - 1$ over its support. Therefore, if $\theta q - 1 \leq \hat{c} = \frac{2 - \theta q}{2}$, or, equivalently, if $\theta q \leq \frac{4}{3}$, $\hat{m}(c)$ intersects zero at $\theta q - 1$, beyond which the firm does not offer a high rebate. Otherwise, $\hat{m}(c)$ intersects zero at the point satisfying $\frac{4c(3\theta q-2)-20c^2-(2-\theta q)^2}{16(c+1)} = 0$, leading to $\bar{c}$ in the corollary, beyond which the firm does not offer a high rebate. □

A.4 Proof of Corollary 2

Proof. (a) For $\hat{c}$ in Proposition 1, we have $\frac{d\hat{c}}{d(\theta q)} = -\frac{1}{2} < 0$. For $\hat{m}(c)$ in Equation (9), we have $\frac{(\theta q-c-1)c}{1+c}$ increases in θq. We notice that

$$\frac{4c(3\theta q-2)-20c^2-(2-\theta q)^2}{16(c+1)} = \frac{-[(2+6c)-\theta q]^2+12c-20c^2}{16(c+1)} ,$$

which increases in θq for $c \in [0,1]$ and $\theta q < \frac{3}{2}$. $(\sqrt{\theta q}-1)^2$ increases in θq for $\theta q \geq 1$. When $\theta q < 1$, $\sqrt{\theta q} - 1 < 0$ and, thus, the area $c \leq \sqrt{\theta q} - 1$ is irrelevant. Altogether, we conclude that $\hat{m}(c)$ increases in θq for $c \in [0,1]$.

(b) Notice that the firm does not offer a rebate when $m > \hat{m}(c)$ and $c < \hat{c}$. When θq decreases, $\hat{m}(c)$ decreases and c increases, and thus the firm is more likely not to offer a rebate, or, equivalently, less likely to offer a rebate.

(c) As in the proof of Corollary 1, if $\theta q \leq 1$, $\hat{m}(c) \leq 0$ for all $c \in [0,1]$, so the firm does not offer a high rebate. □

A.5 Proof of Proposition 2

Proof. The equilibrium profits are derived in the proof of Proposition 1 as in Equations (16), (17), (21), and (22). □

A.6 Proof of Proposition 3

Proof. By Equations (8) and (10), $p_b^* = \frac{(2-\theta)q}{4}$ and $\Pi_b^* = \frac{(2-\theta)^2 q}{16(1-\theta)}$ in the benchmark case.

(a) The proof of Proposition 1 guarantees that the equilibrium profit under the conditional rebate is higher than that under benchmark case.

(b) By Equations (5) and (7), we can derive $\hat{\lambda}$ to be $\frac{1+s-m}{2}$ and $\frac{2+s}{4}$ for the high- and low-rebate cases, which is greater than the $\frac{1}{2}$ in the benchmark. By Equation (4), the demand $(1-x^*)$ is higher under the conditional rebate than under the benchmark case.

Under a low rebate, by Equation (14), we have $p^*(s) = \frac{1}{8}[2s^2 + (2 - 2c + \theta q)s + (4q - 2\theta q)]$. Therefore, $p^*(s^*) > p^*(0) = p_b^*$, and $p^*(s^*) - s^* < p^*(0) - 0 = p_b^*$ because $s < c$.

Under a high rebate (with $\theta q > 1$), by Equation (8), if $m \geq \bar{m}(c)$,

$$p^* - p_b^* = \frac{c\theta q + (1+c)(c+m)}{4} > 0 \text{ and } (p^* - s^*) - p_b^* = \frac{c\theta q - (3-c)(c+m)}{4} < 0 \ .$$

If $m < \bar{m}(c)$, we have

$$p^* - p_b^* = \frac{-[m + (\theta q - 1)]^2 + 4\theta q(\theta q - 1)}{16} > \frac{-[(\sqrt{\theta q} - 1)^2 + (\theta q - 1)]^2 + 4\theta q(\theta q - 1)}{16} = \frac{\theta q(\sqrt{\theta q} - 1)}{2} > 0 \ ,$$

where the first inequality is because $\hat{m}(c)$ weakly decreases in c by Corollary 1 and, thus, $\bar{m}(c) \leq (\sqrt{\theta q} - 1)^2$ by Equation (9). Meanwhile, we have

$$(p^* - s^*) - p_b^* = \frac{-m^2 - 2\theta q(m+5) - 6m + 3\theta^2 q^2 + 7}{16} < 0.$$

This concludes the proof of Proposition 3. □

A.7 Proof of Proposition 4

Proof. (a) By Equation (10), $\frac{\partial \Pi^*}{\partial m} = 0$, and

$$\frac{\partial \Pi^*}{\partial c} = \frac{8[4(1-c)^2 + 4q(8 - 5\theta + c\theta) + \theta^2 q^2][-2(1-c) + \theta q]}{4096(1-\theta)q} > 0$$

because $c > \hat{c} = \frac{2 - \theta q}{2}$.

(b) By Equation (10), for $m \leq \bar{m}(c)$, $\frac{\partial \Pi^*}{\partial c} = 0$, and

$$\frac{\partial \Pi^*}{\partial m} = \frac{2[(1-m)^2 + 2q(4 - m\theta - 3\theta) + \theta^2 q^2][-2(1-m) - 2\theta q]}{256(1-\theta)q} < 0.$$

For $m > \bar{m}(c)$,

$$\frac{\partial \Pi^*}{\partial m} = \frac{2[(2-\theta)q - c(1 + c + m - \theta q) - m][-c-1]}{16(1-\theta)q} < 0$$

and

$$\frac{\partial \Pi^*}{\partial c} = \frac{2[(2-\theta)q - c(1 + c + m - \theta q) - m][-(1 + 2c + m - \theta q)]}{16(1-\theta)q} < 0$$

because $m > \bar{m}(c) = \theta q - 1 - 2c$. □

A.8 Proof of Proposition 5

Proof. By substituting (p^*, s^*) from Equation (8) into Equations (11) and (12) and by simple algebra, we can derive social welfare as follows.

$$\begin{cases} \frac{[4(1-c)^2+3(2-\theta)q](2-\theta)}{32(1-\theta)} & \text{if } m \geq \hat{m}(c) \text{ and } c \leq \hat{c} \\ \frac{[4(1-c)^2+32q-4(5-c)\theta q+\theta^2q^2][52(1-c)^2+96q-12(3+c)\theta q-3\theta^2q^2]}{8192(1-\theta)q} & \text{if } m \geq \hat{m}(c) \text{ and } c > \hat{c} \\ \frac{[(1-m)^2-2m\theta q+8q-(6-\theta q)\theta q][13-32c+5m^2+2m(3-\theta q)+24q-3(2+\theta q)\theta q]}{512(1-\theta)q} & \text{if } m < \hat{m}(c) \text{ and } m \leq \bar{m}(c) \\ \frac{[(2-\theta)q-m-c(1+c+m-\theta q)][4+m+3(2-\theta)q-c(7+c+3m+\theta q)]}{32(1-\theta)q} & \text{if } m < \hat{m}(c) \text{ and } m > \bar{m}(c) \end{cases}$$

Notice the first case ($s^* = 0$) is equivalent to the benchmark case. We denote SW_b as the social welfare under the benchmark case, and denote SW_l, SW_h, and SW_{hc} as the social welfare for the last three cases above, respectively.

(a) We notice that

$$SW_l - SW_b = \frac{(\theta q+2c-2)^2[52(1-c)^2+12(1-c)\theta q-3\theta^2q^2]}{8192(1-\theta)q}.$$

We can show that if and only if $c \leq 1 + \frac{(3-4\sqrt{3})\theta q}{26}$, $SW_b < SW_l$.

(b) When $\theta q \leq 1$, a high rebate does not arise as an equilibrium. We next consider $\theta q > 1$. We notice that when $m \leq \bar{m}(c)$,

$$(SW_h - SW_b)|_{\{m=0,c=0\}} = \frac{(\theta q-1)^2[13+3(2-\theta q)\theta q]}{512(1-\theta)q} > 0 \tag{23}$$

and

$$(SW_h - SW_b)|_{\{m=0,c=\bar{m}^{-1}(0)\}} = \frac{(\theta q-1)^2[29-q(32-3\theta(2-\theta q))]}{512(1-\theta)q} < 0, \tag{24}$$

where $\bar{m}^{-1}(0) = \frac{1}{2}(\theta q - 1)$. Meanwhile,

$$(SW_h - SW_b)|_{m=\bar{m}(c\}} = \frac{-(8c+1)\theta^2q^2+4\theta q(c(c(c+4)+2q+1)+q+1)+c(c(4-c(5c+4))-8(c+2)q)-4q-3}{32(\theta-1)q}$$

and

$$\frac{\partial\left[(SW_h - SW_b)|_{m=\bar{m}(c)}\right]}{\partial c} = \frac{(c+1)[4q-c(2-5c)]-[c(3c+8)+2q+1]\theta q+2\theta^2 q^2}{8(1-\theta)q} > 0 \tag{25}$$

because $1 < \theta q \leq \frac{3}{2}$ and $\theta \leq \frac{1}{2}$.

By Equations (24) and (25), we conclude $SW_h - SW_b < 0$ on $\bar{m}(c)$, and by Equation (23), we conclude $SW_h - SW_b > 0$ when $m = c = 0$. Therefore, there exists a curve $\tilde{c}(m)$ in this parameter space which makes $SW_h - SW_b = 0$, and when $c \leq \tilde{c}(m)$, $SW_h - SW_b > 0$. By solving $SW_h - SW_b = 0$, we can derive $\tilde{c}(m) = \frac{8m-2(\theta q-1-m)^2+\sqrt{\psi}}{8(2-\theta)q}$, where

$$\begin{aligned} \psi = \; & q(2-\theta)[13-20m+m^2(6-m(4-5m)+64q)-4q\theta(1+m)^2(5+3m)-64m\theta q^2] \\ & +2(18m+3m^2-1)q^2\theta^2+4(3+m)q^3\theta^3-3q^4\theta^4+4[(\theta q-1-m)^2-4m]^2. \end{aligned} \tag{26}$$

When $m > \bar{m}(c)$, we have

$$\frac{\partial(SW_{hc}-SW_b)}{\partial m} = -\frac{(2-c-5c^2-2c^3)+(1-2c-3c^2)m-[1+(4-c)c]\theta q+6cq+2q}{16(1-\theta)q} < 0 \tag{27}$$

and

$$(SW_{hc} - SW_b)|_{m=0} = \frac{c\left[-(c+2)\theta^2 q^2+2\theta q(c+2q+3)+c(c(c+8)-16q+3)-4(q+1)\right]}{32(1-\theta)q} \equiv \frac{cT(c)}{32(1-\theta)q}.$$

Notice that $\frac{dT(c)}{dc} = 3 + c(3c+16) - q[16-\theta(2-\theta q)] < 0$, and

$$T(c)|_{c=\bar{m}^{-1}(0)} = -\frac{1}{8}(\theta q-1)[32q-29-3(2-\theta q)\theta q] \; < 0.$$

Therefore, $T(c) < 0$ and $(SW_{hc} - SW_b)|_{m=0} < 0$ for all $c \in [\bar{m}^{-1}(0), \hat{m}^{-1}(0)]$. In addition, $(SW_{hc} - SW_b)|_{m=\bar{m}(c)} < 0$ because we have established $(SW_h - SW_b)|_{m=\bar{m}(c)} < 0$ and $(SW_h - SW_b)$ and $(SW_{hc} - SW_b)$ have the same value on the curve $\bar{m}(c)$. By Equation (27), we conclude $SW_{hc} - SW_b < 0$. □