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European floating strike lookback options: Alpha prediction and generation using unsupervised learning

Tristan LIM
Singapore Management University, tristan.lim.2018@mitb.smu.edu.sg

Aldy GUNAWAN
Singapore Management University, aldygunawan@smu.edu.sg

Chin Sin ONG
DBS Bank

European Floating Strike Lookback Options: *Alpha Prediction and Generation Using Unsupervised Learning*

TRISTAN LIM, CHIN SIN ONG, AND ALDY GUNAWAN

TRISTAN LIM
is a lecturer and researcher at the School of Business Management, Nanyang Polytechnic in Ang Mo Kio, Singapore.
tristan_lim@nyp.edu.sg; tris02@gmail.com

ONG CHIN SIN
is an assistant vice president at DBS Bank in Marina Bay, Singapore.
chinsinong@dbs.com

ALDY GUNAWAN
is an assistant professor (practice) at the School of Information Systems, Singapore Management University in Bras Basah, Singapore.
aldygunawan@smu.edu.sg

KEY FINDINGS

- Outperformance of mean Sharpe of forward European floating strike lookback call options selected using K-means clustering was statistically significant using ANOVA test at p value = 0.0002. Tukey–Kraner HSD test showed that Sharpe risk adjusted return improved by 4.94% to 7.04%.
- Evaluating Sharpe based on European floating strike lookback call option provided a clear choice of trading cluster of options, whereas this was not clearly apparent if the standard call option Sharpe was used as the only evaluating criteria for the selection of tradable option clusters.
- Consistency and stability of the predictive results implied that the European floating strike lookback call options acted as a useful evaluation criterion for option investment.

ABSTRACT: *This research utilized the intrinsic quality of European floating strike lookback call options, alongside selected return and volatility parameters, in a K-means clustering environment, to recommend an alpha generative trading strategy. The result is an elegant easy-to-use alpha strategy based on the option mechanisms which identifies investment assets with high degree of significance. In an upward trending market, the research had identified European floating strike lookback call option as an evaluative criterion and investable asset, which would both allow investors to predict and profit from alpha opportunities. The findings will be useful for (i) buy-side investors seeking alpha generation and/or hedging underlying assets, (ii) sell-side product manufacturers looking to structure the European floating strike lookback call options, and (iii) market trading platforms looking to introduce new products and enhance liquidity of the product.*

TOPICS: ***Options, volatility measures, statistical methods, simulations, machine learning****

*All articles are now categorized by topics and subtopics. **View at PM-Research.com.**

Goldman, Sosin, and Gatto (1979) introduced an exotic option contract, namely the lookback options, which are options at which the owners can "look back" at the point of contract expiration to determine the optimal point at time t between time $\{0, T\}$. At time *t*, the price differential between the strike price and the market price of the underlying asset at time t is the most optimal, thereby maximizing the option holders' profit. In this way, market timing of trading assets, such as commodities or stocks, is less important, providing investors with a new exotic class of tradable call and put assets, risk hedging instruments, while helping to expand the investable asset universe.

Lookback options are a lesser used investment instrument, largely because it is a non-exchanged traded structured product which lowers its market liquidity. Further, it is, by nature, more costly vis-à-vis normal call options due to its nature of profit maximization by utilizing the global minima price vis-à-vis a fixed strike price of the underlying asset.

In this study, the research will like to utilize the intrinsic quality of European floating strike lookback call options, alongside selected return and volatility parameters, in a K-means clustering environment, to recommend an alpha generative trading strategy. The result is an elegant easy-to-use alpha strategy based on the option mechanisms which identifies investment assets with high degree of significance.

LITERATURE REVIEW

In the face of continuing chase of yields and changing economic conditions for asset managers, client sophistication entails bespoke solutions which allow the capturing of alpha across price trends and volatility conditions. To address such needs, there exists increasing demand for alpha generation across differentiated structured financial products. There exists a dearth of studies applying lookback options for alpha generative strategies.

Lookback options come in two forms: (i) fixed strike, or (ii) floating strike; while this study is a proof-of-concept that focuses on the latter instrument, the study's result can be extrapolated, with further research undertaken in other exotic option classes. Floating strike lookback options (Goldman, Sosin, and Gatto 1979), in contrast with fixed strike lookback options, or options on extrema (Conze and Viswanathan 1991), are options with floating strike prices throughout the life of the option. At option maturity, the holder of the option can decide the point of time *t* during the life of the option when the price of the underlying asset is the most attractive and set this as the strike price.

For instance, given a European cash-settled floating strike call lookback option, and suppose the price of the option begins trading at the option initiation date at \$100, falls to \$50 and rises to \$120 during the life of the contract, and closes at \$90 at contract maturity. In the case of a standard European call option, if the strike price is fixed at option initiation date at \$100, the holder of the call option will have let the option expire worthless, as this holder will rather buy the asset at the market price of \$90 at contract expiry, rather than exercising the buy option of the option at the \$100 strike price. In comparison, the holder of this floating strike lookback option can "look back" and exercise the option at the global minina asset price when the price was \$50, thereby making a profitable trade of \$40 (or option maturity price of \$90 subtracting global minima price \$50). However, this profit opportunity comes at a cost – lookback options are priced at higher option premiums than the typical plain vanilla call and put options.

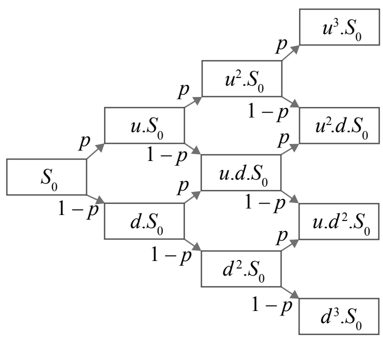
There are many different methods to price a European cash-settled floating strike call lookback option. The key differences lie on whether the sub-interval of the said option is of discrete-time, or of continuous-time interval.

Discrete-time Binomial Option Pricing Model

Introduced by Cox, Ross, and Rubinstein (1979), and further expounded by Kat (1995) and Cheuk and Vorst (1997), among others, binomial option pricing model provides a simple and accurate method to numerically price options. Binomial pricing model is a discrete-time path dependent model, calculated over a prespecified number of observation frequency fixings, for instance, daily, weekly, quarterly. Valuation is performed from the valuation date at time t_0, iteratively across fixed observation frequencies (or discrete time points N at time points t_0, t_1, t_2, … t_N), for the time to maturity T, with t_i = iT/N.

Exhibit 1 illustrates a visualization of a binomial valuation tree diagram. From Exhibit 1, asset price S_0 at t_0 is recomputed for every node at each discrete time point until t_N. Assuming volatility of the underlying asset σ, and Δt = T/N, the option value for each node step is assumed to move up or down by a factor of $u = e^{\sigma\sqrt{t}}$ or $d = e^{-\sigma\sqrt{t}} = \frac{1}{u}$, such that for instance, option value at the upper node for time t_1 is given by $S_u = u.S_0$. In addition, let r be one plus domestic market risk-free rate over discrete time period, and let μ be the no arbitrage growth rate $\mu = e^{\bar{r}\Delta t}$. Price of a European floating strike call lookback option, C_{float} at time point t_j for underlying asset value S is given by Equation 1.

E X H I B I T 1
Binomial Valuation Lattice



$$C_{float} = \frac{\left[\sum_{m,n} \max(S_0 u^m d^n - \min_{0 \le j \le N} S_j,\ 0) \cdot p^m \cdot (1-p)^n\right]}{r^N}, \tag{1}$$

where p and $(1 - p)$ are risk neutral probabilities, $p = \frac{(\mu - d)}{(u - d)}$, and m, n are the number of up and down states at each t_N node.

Monte Carle Simulation on Binominal Option Pricing Model

With Monte Carlo simulation, binomial option pricing model can be extended beyond small discrete time points N. Lookback options are path dependent options which do not have analytic price solutions; solutions that depend not only on asset prices at option maturity, but also the path of the prices at each discrete time point.

In Exhibit 2 (Benninga, 2008), the variable Runs in the VBA algorithm indicate the number of randomized price paths created, which are averaged to compute the Monte Carlo simulated price of the call option. Price paths are generated based on price neutral probabilities. In Exhibit 2, the price of the asset increases when the random number generator rand() is greater than price neutral probability p, and decreases when the random number generator is less than or equal to p. Henceforth, risk neutral probabilities of each price path are built into the price path itself (Benninga, 2008).

Black Scholes Option Pricing Model

Introduced by Black and Scholes (1973), and modified by Goldman, Sosin, and Gatto (1979) under the Black Scholes framework, Black Scholes model provides a lookback option with continuous and non-discrete monitoring of asset price across the lifetime of the option.

Assuming S is the price of the underlying asset, S_{min} is the global minima asset price, r is the risk free rate, q is the continuous payout, σ is the volatility of the underlying asset, T is the time to maturity and $N(\cdot)$ is the cumulative normal distribution, price of a European floating strike call lookback option, C_{float} is given by Equation 2.

$$C_{float} = Se^{-qT}N(a_1) - S_{min}e^{-rT}N(a_2) + Se^{-rT}\frac{\sigma^2}{2(r-q)} \times \left[\left(\frac{S}{S_{min}}\right)^{-\frac{2(r-q)}{\sigma^2}} N\left(-a_1 + \frac{2(r-q)}{\sigma}\sqrt{T}\right) - e^{-qT}N(-a_1)\right], \tag{2}$$

where $a_1 = \frac{\ln(S/S_{\min}) + (r - q + 0.5\sigma^2)T}{\sigma\sqrt{T}}$ and $a_2 = a_1 - \sigma\sqrt{T}$.

Exhibit 3 (Benninga, 2008) illustrates the VBA functions used to price a European floating strike call lookback option.

METHODOLOGY

The aim of this study is to identify predictive alpha strategies using lookback option mechanism based on unsupervised clustering techniques. Specifically, the research will like to use common industry used variables, such as daily historical equity return and volatility variables, alongside K-means clustering as a technique to cluster-identify and predict suitable long call candidates in an upward trending market. Post-clustering, investors can use the equities identified in the clustering technique to generate alpha by investing in its associated call options.

Data

This research focused on European floating strike lookback call options and standard European call options utilizing the top equities by market capitalization listed on the Singapore Exchange as underlying assets. In this research, the top 120 equities were selected. Post-data processing, such as the accounting for lack of data due

Exhibit 2

VBA Algorithm of Monte Carlo Simulation on Binomial Option Pricing Model

```
Function ELBCall(initial, Up, Down, _
  Interest, Periods, Runs)
  Dim PricePath() As Double
  ReDim PricePath(Periods)

  'Risk-neutral probabilities
  piup = (Interest – Down)/(Up – Down)
  pidown = 1 – piup

  Temp = 0

  For Index = 1 To Runs
    'Generate path
    For i = 1 To Periods
      PricePath(0) = initial
      pathprob = 1
      If Rnd > pidown Then
        PricePath(i) = PricePath(i – 1) * Up

        Else:
        PricePath(i) = PricePath(i – 1) * _
        Down
      End If
    Next i

    callpayoff = Application.Max _
    (PricePath(Periods) – Application.Min(PricePath), 0)
    Temp = Temp + callpayoff

  Next Index

  ELBCall = (Temp/Interest ^ Periods)/_
    Runs

End Function
```

Exhibit 3

VBA Algorithm of Black Scholes Option Pricing Model

```
Function aOne(Stock, Smin, Time, Interest, Sigma)
aOne = (Log(Stock/Smin) + ((Interest + (Sigma ^ 2)/2) * Time))/_
  (Sigma * Sqr(Time))
End Function
```

```
Function aThree(Stock, Smin, Time, Interest, Sigma)
aThree = (Log(Stock/Smin) + ((–Interest + (Sigma ^ 2)/2) * Time))/_
  (Sigma * Sqr(Time))
End Function
```

```
Function yOne(Stock, Smin, Interest, Sigma)
yOne = –1 * (2 * (Interest – ((Sigma ^ 2)/2)) * _
  (Log(Stock/Smin)))/(Sigma ^ 2)
End Function
```

```
Function BSLBCall(Stock, Smin, Time, Interest, Sigma)
BSLBCall = Stock * Application.NormSDist(aOne(Stock, _
  Smin, Time, Interest, Sigma)) – Stock * _
  ((Sigma ^ 2)/(2 * Interest)) * _
  Application.NormSDist(–1 * aOne(Stock, _
  Smin, Time, Interest, Sigma)) – Smin * _
  Exp(–Time * Interest) * (Application.NormSDist _
  (aOne(Stock, Smin, Time, Interest, Sigma) _
  – Sigma * Sqr(Time)) – ((Sigma ^ 2)/(2 * Interest)) _
  * Exp(yOne(Stock, Smin, Interest, Sigma)) * _
  Application.NormSDist(–1 * aThree(Stock, Smin, _
  Time, Interest, Sigma)))
End Function
```

to the launch of initial public offering late in the period of selection, 92 securities were selected.

In order to improve rigority of test results, research utilized four period historical two-year rolling windows as follows: (i) 1 January 2015 to 31 December 2016, (ii) 1 April 2015 to 31 March 2017, (iii) 1 July 2015 to 30 June 2017, and (iv) 1 October 2015 to 30 September 2017. For each rolling window period, the option prices were obtained for (i) continuous (Black-Scholes framework), (ii) daily (binomial model), (iii) weekly (binomial model), and (iv) monthly (binomial model) security price intervals, for forward three month, six month, nine month and twelve month maturities for both European floating strike lookback call options and standard European call options. With these parameters, the study attains 11,776 option price observations, inclusive of four rolling window periods, each computing four forward maturity observations for 92 securities, for four different continuous-discrete price intervals, for the two aforementioned types of options.

Security prices between 2015 to 2018 were extracted from Bloomberg. Public holiday that fell on a weekday will result in blank data. These were imputed with the last closing price. Use of Bloomberg was consistent as a widely used data source for portfolio management investment practitioners.

Research utilized the computation of logarithmic returns, which conveniently represented the calculation of compounded returns, while allowing the distribution to be transformed for normalization, adjusting for trend and seasonality effects (Dhamija and Bhalla 2010). This also brought about an added convenience of time additivity of a running sequence of *n* trades. Equations 3 and 4 were used to compute the geometric return (r_i) and annualized standard deviation (σ_i) respectively.

$$r_i = \ln\left(\frac{p_i}{p_{i-1}}\right) \tag{3}$$

$$\sigma_i = \sqrt{\frac{1}{T}\Sigma_{i=1}^{T}(r_i - \overline{r}) * Trading\ \ days} \quad (4)$$

Historical and forward holding period returns and lookback drawdown returns were computed as input variables; historical returns for test of predictive qualities, and forward returns for payoff and Sharpe computation. Computation of holding period returns is trivial. Computation of lookback drawdown returns is illustrated in Equation 5. Lookback drawdown returns r_{LB} equation parameters are the option maturity stock price S_{Final} and option global minima price S_{min}.

$$r_{LB} = \frac{S_{Final} - S_{min}}{S_{min}} \quad (5)$$

Sharpe ratio was utilized to evaluate the performance of trading results. According to Harvey and Liu (2015), it is a routine industry practice to discount the reported Sharpe ratios by 50%, among many other proposed haircut methods, in trading backtests. In this research, we would discount any improvements or declines in Sharpe ratio, δS, when comparing two portfolio Sharpe by 50% as a simple penalization hurdle, based on Equation 6.

$$\delta S = \frac{1}{2}\left(\frac{S_i}{S_{i-1}} - 1\right) \text{ where } S_i = \left(\frac{r_i - r_f}{\sigma_i}\right) \quad (6)$$

Since the dataset comprised of Singapore equities, annualized risk-free rate (r_f) was assumed at 2%, approximating the short-term annual interest returns of Singapore Government Securities.

Modelling

Modelling interactivity interface was generated using Microsoft Excel, supported by Python for computationally intensive iterative operations, for instance, high run-rate binomial option pricing modelling. SAS® Enterprise Miner™ version 14.1 ("EM") was also utilized during the results validation stage for its fast unsupervised clustering computation capabilities for large datasets, and SAS® JMP Pro 14.0.0 was utilized for One-way ANOVA and post-hoc Tukey Honestly Significant Difference (HSD) tests. Exhibit 4 illustrates the research modelling utilized to generate the research output.

K-means clustering was utilized to cluster-identify investable assets for its extensivity of use and simplicity as an unsupervised machine learning technique, for the purposes of industry adoption. According to Hartigan (1975), K-means clustering divides data observations into like-clusters, by breaking up *M* points in *N* dimensions into K clusters, through the minimization of within-cluster sum of squares (WSS). Repeated iterations are conducted until the stopping criterion is met. In this research, Euclidean distance is utilized for similarity distance and the Aligned Box Criterion is the method used to identify the optimal clusters generated. The latter improves on the gap statistic method (Tibshirani, Walther and Hastie 2001) through a high-performance machine-learning based analysis structure (SAS Institute 2016). Historical equity holding period return, historical equity lookback drawdown return and historical equity volatility were the input variables for K-means clustering; Equity names acted as the input ID.

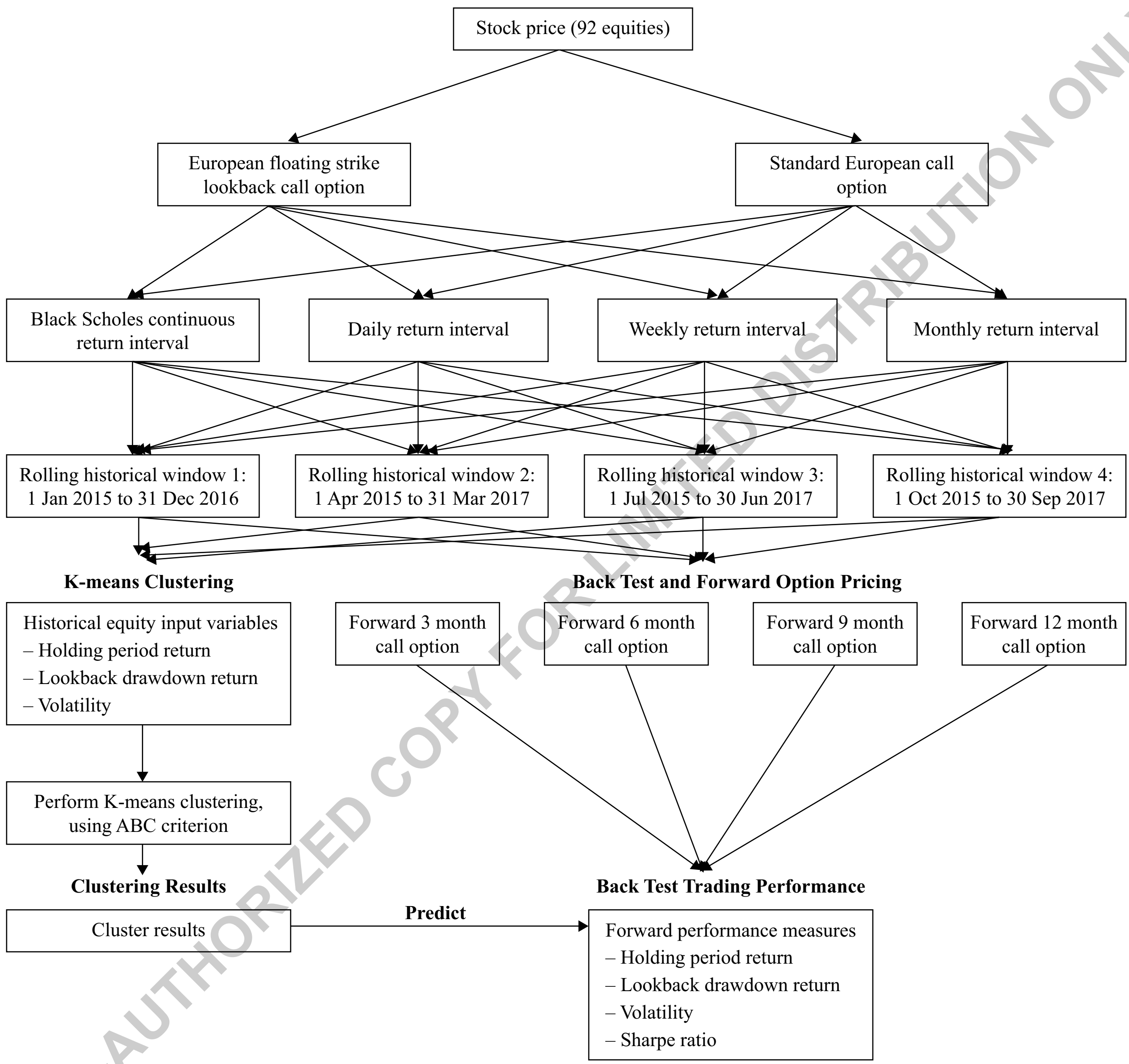
Microsoft Excel and Python were utilized for the pricing of European floating strike lookback call options and standard European call options based on algorithms in Exhibits 2 and 3 respectively. Computation of binomial option pricing model required averaging binomial option price paths of 15,000 runs for each Monte Carlo simulation, at a total of 48 Monte Carlo simulations. Due to the computational intensiveness of the high number of iterations, Python was used for generation of binomial option pricing and Sharpe, among others.

In Exhibit 5, it can be observed that at about 15,000 runs and above, the standard deviations of 48 Monte Carlo binomial option pricing model simulations were reduced and plateaued at just below 2%. Above 15,000 runs, the marginal benefit of additional runs was insignificant. Hence, the research model utilized 15,000 runs of binomial option price paths for each Monte Carlo simulation.

RESULTS AND DISCUSSION

Exhibit 6 illustrates that in a relatively high positive Sharpe environment for the underlying stock, trading European floating strike lookback call options and standard European call options typically outperform its underlying equity trade. However, the option cost will offset payoff generation for option trades, to the

Exhibit 4
Research Modelling



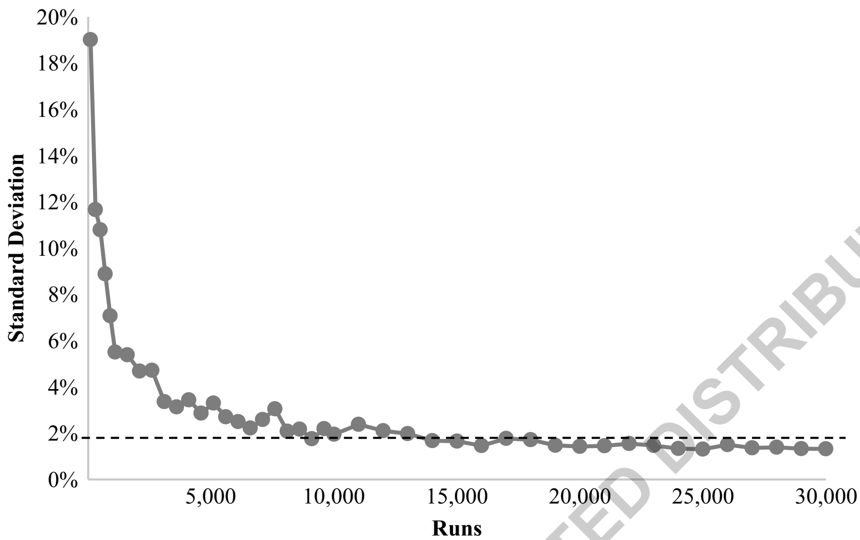
extent weak positive Sharpe positions in the underlying equity trades correspond to negative option trading performances.

Weak performances for both European floating strike lookback call options and standard European call options valued using the Black Scholes framework across all historical rolling windows and forward option maturity periods preclude the use of a Black–Scholes–priced option as a viable option trading strategy.

K-means clustering results are listed in Exhibit 7. Variables with suffix of (1) are clustering inputs, and variables with suffix of (2) are the forward actual performance of equity and options. K-means clustering identified Cluster 3 as a consistent outlier performer in

EXHIBIT 5

Standard Deviation of Binomial Option Pricing on 48 Monte Carlo Simulations



all daily, weekly, and monthly option pricing intervals, across all rolling window periods and option maturity periods. Cluster 3 comprised one underlying equity with exceptional return performance, and high volatility.

From a product manufacturing standpoint, weekly and monthly fixings due to its innate higher intervals would likely suffer from poor liquidity and stale industry pricing vis-à-vis daily fixings. Consistency of the clustering results between the daily, weekly, and monthly fixing options meant that performance of clustering based on daily fixings was sufficient. Should a fund manager decide to long a weekly or monthly option, he or she can do so by extrapolating the daily clustering results.

We investigate the daily option pricing interval performance. Results in Exhibits 7, 8, and 9 yielded a few observations:

(i) *Identification of outperforming equity options*: With reference to Exhibits 8 and 9, outperformance of mean Sharpe of forward European floating strike lookback call options was statistically significant using ANOVA test at p value = 0.0002. Tukey-Kraner HSD test showed that Sharpe risk adjusted return improved by 4.94% to 7.04% in Cluster 3 vis-à-vis other clusters. As a test of strategy, applying a haircut of 50%, the improvement of the Sharpe risk-adjusted return was robust at 247 basis points to 352 basis points. While the difference in mean between Cluster 3 and 2 was not statistically significant, the wide variance and large number of equities in Cluster 2 precludes Cluster 2 as a choice of trading cluster. Evaluating Sharpe based on the European floating strike lookback call option was able to provide a clear choice of positive trading cluster, whereas this was not clearly apparent if the standard call option Sharpe was used as the only evaluating criteria for the selection of an option tradable cluster.

(ii) *Viability as a trading strategy*: With reference to Exhibits 7, 8, and 9, Cluster 3 had one identified equity. Investing in its associated European floating strike lookback call option, even across rolling periods, was more executable as a trading strategy vis-à-vis Cluster 1 and 2, which had 58 and 33 equities respectively. Further, the spread of Sharpe in Cluster 1 and 2 were wide, which implied that Cluster 1 and 2 were not viable trading strategies.

(iii) *European floating strike lookback call options as an evaluative predictive criterion for option trading*: In Exhibit 7, the consistency and stability of the predictive results also implied that it was viable to invest in the corresponding standard call option alternative

Exhibit 6

Average and Median Sharpe Performance

Historical Period Start Date	Historical Period End Date	Option Start Date	Option End Date	No. of Observations	Equity Sharpe (if no option traded)	Black Scholes ELB Call	Black Scholes Std Call	Daily ELB Call	Daily Std Call	Weekly ELB Call	Weekly Std Call	Monthly ELB Call	Monthly Std Call
Panel A: Average Back Test Sharpe Performance													
January 1, 2015	December 30, 2016	January 2, 2017	March 31, 2017	1,472	1.80	–6.90	–7.10	5.30	26.00	10.20	28.50	46.20	78.60
January 1, 2015	December 30, 2016	January 2, 2017	June 30, 2017	1,472	1.20	–2.10	–3.10	2.40	10.90	4.30	15.20	11.90	27.30
January 1, 2015	December 30, 2016	January 2, 2017	September 29, 2017	1,472	0.80	–1.80	–1.60	0.50	7.30	1.70	8.00	5.30	12.60
January 1, 2015	December 30, 2016	January 2, 2017	December 29, 2017	1,472	0.80	–0.30	–0.20	1.30	7.10	2.30	7.70	5.40	12.30
April 1, 2015	March 31, 2017	April 3, 2017	June 30, 2017	1,472	0.30	–18.00	–20.60	–7.00	–2.90	–7.60	–0.80	–16.30	–5.80
April 1, 2015	March 31, 2017	April 3, 2017	September 29, 2017	1,472	0.10	–8.30	–8.90	–4.40	0.70	–4.30	–0.50	–4.30	2.50
April 1, 2015	March 31, 2017	April 3, 2017	December 29, 2017	1,472	0.30	–4.00	–4.40	–1.10	3.40	–0.70	2.80	1.50	6.00
April 1, 2015	March 31, 2017	April 3, 2017	March 30, 2018	1,472	0.10	–3.40	–4.10	–1.90	–0.70	–2.10	–0.70	–1.70	0.30
July 1, 2015	June 30, 2017	July 3, 2017	September 29, 2017	1,472	–0.20	–21.40	–22.20	–13.10	–5.90	–16.20	–15.40	–30.80	–26.60
July 1, 2015	June 30, 2017	July 3, 2017	December 29, 2017	1,472	0.30	–8.10	–8.70	–2.80	3.00	–3.30	0.30	–0.90	4.50
July 1, 2015	June 30, 2017	July 3, 2017	March 30, 2018	1,472	0.00	–5.50	–6.30	–3.40	–2.50	–4.00	–3.00	–3.60	–2.10
July 1, 2015	June 30, 2017	July 3, 2017	June 29, 2018	1,472	–0.30	–4.60	–5.30	–3.80	–4.30	–4.80	–4.80	–5.50	–5.30
October 1, 2015	September 29, 2017	October 2, 2017	December 29, 2017	1,472	0.60	–18.90	–19.20	–5.70	9.60	–4.20	6.30	6.70	20.60
October 1, 2015	September 29, 2017	October 2, 2017	March 30, 2018	1,472	–0.10	–8.80	–9.80	–5.50	–4.30	–6.00	–4.60	–6.20	–3.60
October 1, 2015	September 29, 2017	October 2, 2017	June 29, 2018,	1,472	–0.40	–6.20	–7.00	–5.10	–6.00	–6.30	–6.50	–7.40	–7.20
October 1, 2015	September 29, 2017	October 2, 2017	September 28, 2018	1,472	–0.30	–3.80	–4.90	–2.80	–4.00	–3.10	–4.00	–3.80	–4.10
Median Back Test Sharpe Performance													
January 1, 2015	December 30, 2016	January 2, 2017	March 31, 2017	1,472	1.82	–9.42	–9.92	1.72	21.93	7.22	29.49	19.33	30.79
January 1, 2015	December 30, 2016	January 2, 2017	June 30, 2017	1,472	1.25	–2.94	–3.67	2.98	10.32	4.39	16.33	10.38	20.72
January 1, 2015	December 30, 2016	January 2, 2017	September 29, 2017	1,472	0.83	–2.28	–2.31	0.42	4.75	1.27	5.00	2.28	7.26
January 1, 2015	December 30, 2016	January 2, 2017	December 29, 2017	1,472	0.88	–0.93	–0.68	0.63	4.37	0.97	5.32	2.49	6.51
April 1, 2015	March 31, 2017	April 3, 2017	June 30, 2017	1,472	0.48	–17.63	–20.56	9.27	–9.47	–10.17	–11.19	–8.60	–9.45
April 1, 2015	March 31, 2017	April 3, 2017	September 29, 2017	1,472	0.10	–8.08	–9.01	–5.77	–4.77	–5.93	–6.45	–7.31	–6.72
April 1, 2015	March 31, 2017	April 3, 2017	December 29, 2017	1,472	0.25	–4.36	–5.33	–2.34	–1.64	–2.77	–3.17	–2.31	–2.68
April 1, 2015	March 31, 2017	April 3, 2017	March 30, 2018	1,472	0.12	–3.84	–4.82	–3.08	–3.45	–3.50	–3.42	–3.61	–3.64
July 1, 2015	June 30, 2017	July 3, 2017	September 29, 2017	1,472	–0.21	–22.08	–22.46	–15.76	–14.71	–18.83	–20.70	–22.95	–25.68
July 1, 2015	June 30, 2017	July 3, 2017	December 29, 2017	1,472	0.38	–8.39	–8.68	–4.33	–2.04	–5.26	–5.77	–3.63	–5.87
July 1, 2015	June 30, 2017	July 3, 2017	March 30, 2018	1,472	–0.06	–5.68	–6.88	–4.68	–5.77	–5.12	–5.78	–5.30	–5.62
July 1, 2015	June 30, 2017	July 3, 2017	June 29, 2018	1,472	–0.28	–4.86	–5.46	–4.17	–5.12	–5.33	–5.84	–5.77	–5.70
October 1, 2015	September 29, 2017	October 2, 2017	December 29, 2017	1,472	0.53	–18.08	–17.79	–7.40	–2.83	–6.15	–8.92	–2.27	–7.86
October 1, 2015	September 29, 2017	October 2, 2017	March 30, 2018	1,472	–0.14	–8.82	–10.01	–7.04	–7.94	–7.48	–8.67	–8.02	–8.20
October 1, 2015	September 29, 2017	October 2, 2017	June 29, 2018	1,472	–0.44	–6.28	–7.24	–5.51	–7.01	–6.98	–7.70	–7.53	–7.65
October 1, 2015	September 29, 2017	October 2, 2017	September 28, 2018	1,472	–0.25	–4.00	–5.24	–3.06	–4.50	–3.30	–4.80	–3.81	–4.81

EXHIBIT 7
K-Means Clustering Results—Historical Mean-Variance and Forward Back Test Performance

K-Means Cluster	No. of Observations	No. of Underlying Equity	Historical Equity Holding Period Return (1)	Historical Equity Lookback Drawdown Return (1)	Historical Equity Volatility (1)	Forward Equity Holding Period Return (2)	Forward Equity Lookback Drawdown Return (2)	Forward Equity Volatility (2)	Forward ELB Call Sharpe (2)	Forward Std Call Sharpe (2)
Monthly Option Pricing Interval										
1	555	35	0.11	0.31	0.18	0.11	0.19	0.14	–0.65	10.75
2	365	23	–0.23	0.08	0.20	0.09	0.19	0.17	2.77	10.31
3	16	1	12.03	13.41	0.61	0.61	0.76	0.49	3.70	5.73
4	536	34	0.12	0.30	0.18	0.03	0.10	0.16	–1.85	0.61
Weekly Option Pricing Interval										
1	392	25	0.34	0.58	0.23	0.07	0.20	0.19	–1.74	1.54
2	232	15	–0.30	0.12	0.26	0.08	0.24	0.24	–2.42	0.22
3	16	1	12.23	13.61	0.55	0.61	0.84	0.49	3.15	5.23
4	832	52	–0.03	0.18	0.18	0.08	0.15	0.13	–3.45	2.26
Daily Option Pricing Interval										
1	932	58	0.16	0.39	0.21	0.07	0.19	0.18	–3.77	0.85
2	524	33	–0.20	0.12	0.22	0.12	0.23	0.19	–1.67	4.93
3	16	1	12.03	13.61	0.56	0.59	0.95	0.50	3.27	4.68

EXHIBIT 8
ANOVA Output Using SAS® JMP Pro 14.0.0

⊿Oneway ANOVA

⊿Summary of Fit

Rsquare	0.011447
Adj Rsquare	0.010101
Root Mean Square Error	11.10464
Mean of Response	–2.94799
Observations (or Sum Wgts)	1472

⊿Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
H Cluster D	2	2097.63	1048.82	8.5053	0.0002*
Error	1469	181146.76	123.31		
C. Total	1471	183244.39			

⊿Means for Oneway ANOVA

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
1.00	932	–3.7705	0.3637	–4.484	–3.057
2.00	524	–1.6748	0.4851	–2.626	–0.723
3.00	16	3.2653	2.7762	–2.180	8.711

Std Error uses a pooled estimate of error variance.

rather than the European floating strike lookback call options, as the standard call option alternative would generally be of a lower option premium, and a higher payoff. Hence, in this case, the European floating strike lookback call options acted as a useful evaluation criterion for option investment.

In Exhibit 6, it is also observed that in an upward trending high positive Sharpe environment, investing in a call option would generate a significantly higher alpha vis-à-vis the underlying equity due to the latter's higher capital requirement.

From a trading implementation standpoint, the study recommends the trading process as illustrated in Exhibit 10. Post-trade, it is recommended to continually monitor trading performance vis-à-vis changes in the systematic and non-systematic environment of the underlying equity, and making necessary adjustments to the trading model to sharpen alpha generation.

EXHIBIT 9

Tukey-Kramer HSD Output Using SAS® JMP Pro 14.0.0

Means Comparisons

Comparisons for all pairs using Tukey-kramer HSD

Confidence Quantile

q*	Alpha
2.34608	0.05

HSD Threshold Matrix

Abs(Dif)-HSD

	3.00	2.00	1.00
3.00	–9.2109	–1.6718	0.4670
2.00	–1.6718	–1.6095	0.6732
1.00	0.4670	0.6732	–1.2069

Positive values show pairs of means that are significantly different.

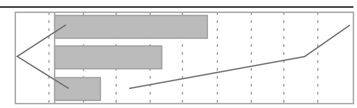
Connecting Letters Report

Level			Mean
3.00	A		3.265262
2.00	A		–1.674779
1.00		B	–3.770502

Levels not connected by same letter are significantly different.

Ordered Differences Report

Level	– Level	Difference	Std Err Dif	Lower CL	Upper CL	*P*-Value
3.00	1.00	7.035764	2.799888	0.46699	13.60453	0.0324*
3.00	2.00	4.940040	2.818225	–1.67175	11.55183	0.1861
2.00	1.00	2.095724	0.606333	0.67321	3.51823	0.0016*



CONCLUSION AND FUTURE WORKS

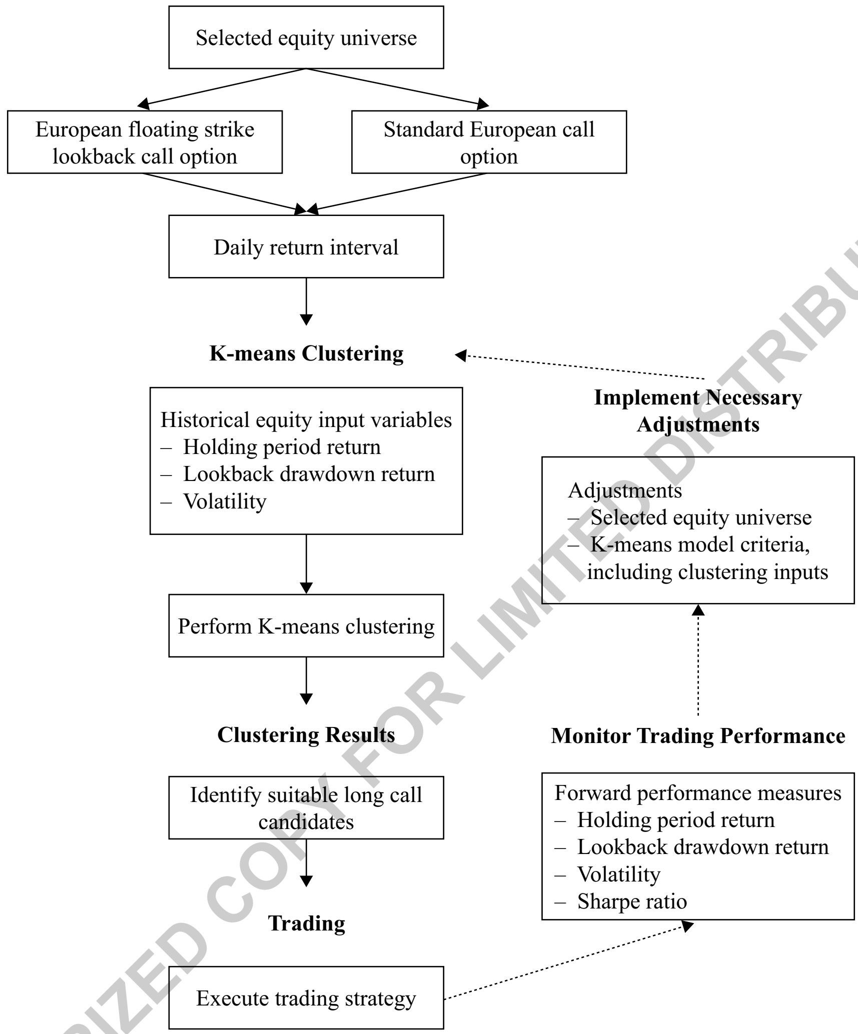
The research had successfully utilized a simple industry-utilizable K-means as a clustering technique, with common industry used variables – daily historical equity holding period return, daily historical equity lookback drawdown return and historical volatility – to identify long equity call option candidates. Post-clustering, investors can use the equities identified in the clustering to generate alpha by investing in its associated call options.

In an upward trending market, the research had identified European floating strike lookback call option as an evaluative criterion and investable asset, which would both allow investors to predict and profit from alpha opportunities.

The findings will be useful for (i) buy-side investors seeking alpha generation and/or hedging underlying assets, (ii) sell-side product manufacturers looking to structure the European floating strike lookback call options, and (iii) market trading platforms looking to introduce new products and enhance liquidity of the product.

Future works can be applied across geographical markets, business cycles and asset classes to validate and extent the findings of this research.

EXHIBIT 10
Trading Process



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ADDITIONAL READING

Backtesting

CAMPBELL R. HARVEY AND YAN LIU
The Journal of Portfolio Management
https://jpm.pm-research.com/content/42/1/13

ABSTRACT: *When evaluating a trading strategy, it is routine to discount the Sharpe ratio from a historical backtest. The reason is simple according to the authors: there is inevitable data mining by both the researcher and by other researchers in the past. In this article, the authors provide a statistical framework that systematically accounts for these multiple tests. They propose a method to determine the appropriate haircut for any given reported Sharpe ratio. They also provide a profit hurdle that any strategy needs to achieve in order to be deemed "significant."*

A Backtesting Protocol in the Era of Machine Learning

ROB ARNOTT, CAMPBELL R. HARVEY, AND HARRY MARKOWITZ
The Journal of Financial Data Science
https://jfds.pm-research.com/content/1/1/64

ABSTRACT: *Machine learning offers a set of powerful tools that holds considerable promise for investment management. As with most quantitative applications in finance, the danger of misapplying these techniques can lead to disappointment. One crucial limitation involves data availability. Many of machine learning's early successes originated in the physical and biological sciences, in which truly vast amounts of data are available. Machine learning applications often require far more data than are available in finance, which is of particular concern in longer-horizon investing. Hence, choosing the right applications before applying the tools is important. In addition, capital markets reflect the actions of people, who may be influenced by the actions of others and by the findings of past research. In many ways, the challenges that affect machine learning are merely a continuation of the long-standing issues researchers have always faced in quantitative finance. Although investors need to be cautious—indeed, more cautious than in past applications of quantitative methods—these new tools offer many potential applications in finance. In this article, the authors develop a research protocol that pertains both to the application of machine learning techniques and to quantitative finance in general.*