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Incorporating a Reverse Logistics Scheme in a Vehicle Routing Problem with Cross-Docking Network: a Modelling Approach

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Abstract—Reverse logistics has been implemented by various companies because of its ability to gain more profit and maintain the competitiveness of the company. However, extensive studies on the vehicle routing problem with cross-docking (VRPCD) only considered the forward flow instead of the reverse flow. Motivated by the ability of a VRPCD network to minimize the distribution cost in the forward flow, this research incorporates the reverse logistics scheme in a VRPCD network, namely the VRP with reverse cross-docking (VRP-RCD). We propose a VRP-RCD mathematical model for a four-level supply chain network that involves suppliers, cross-dock, customers, and outlets. The main objective is to minimize vehicle operational and transportation costs. Experimental results show that only small instances of newly generated VRP-RCD instances can be optimally solved by CPLEX. Furthermore, we present sensitivity analysis towards the cost structure.

Index Terms—vehicle routing problem; cross-docking; reverse logistics

I. INTRODUCTION

Many companies have been utilizing cross-dock as an intermediate facility to transfer products in bulk quantities from suppliers to receivers (e.g. customers). The products from suppliers are first sent to the cross-dock facility, sorted and consolidated inside the cross-dock facility, and then delivered to the customers. The advantages of this network are that the long origin-to-destination paths and the large number of vehicles occurring in the direct shipment can be eliminated [1]. The vehicle routing problem (VRP) plays an important role to assess the costs in distribution management and logistics [2]. Reference [3] was the first to address the integration of VRP and cross-docking (VRPCD). Reference [4] studied VRPCD by considering time windows. Reference [5] developed a new tabu search (TS) algorithm to solve VRPCD. However, most of research on VRPCD only addressed forward flow instead of reverse flow.

In recent decades, several companies have paid greater attention to the reverse logistics issue [6]. In reverse logistics, the products from the customers' sites are sent back to the suppliers' sites. This concept has been recognized as a source of profitability and competitiveness for companies [7].

According to [8], reverse logistics highly benefits companies with seasonal demand patterns, as long as the companies realize that the return policy could be a part of their business and look for ways to capture the value from it. In reverse logistics, [9] mentioned an activity called resell, in which the unsold products from the customers are directly sold to secondary customers as long as there is matching demand. For a business with seasonal demand patterns (e.g. fashion, books, or electronics), the unsold products are often commercialized through secondary channels (e.g. outlet stores) [10].

Motivated by the advantages of reverse logistics and VR-PCD, as well as the lack of research in this area, we aim to design a VRP with reverse cross-docking (VRP-RCD) that integrates both. To the best of our knowledge, the integration of VRP and forward/reverse cross-dock has only been addressed by [6]. However, they did not consider a reselling process to the secondary channels in the proposed network, therefore only addressing a three-level supply chain network, which involves suppliers, cross-dock, and customers. We summarize the following as our main contributions:

- we propose an extended model of VRP-RCD in which a four-level supply chain network (suppliers, cross-dock, customers, and outlets) is considered.
- we tackle the scenario when not all products supplied by the suppliers are perfect.
- we consider a more comprehensive cross-docking model in a supply chain network to deal with multiple-product flow.

II. PROPOSED MODEL

Consider three directed network graphs $G' = (C \cup 0, A')$, $G'' = (O \cup 0, A'')$, and $G''' = (S \cup 0, A''')$, where $C = \{1, 2, \ldots, |C|\}$ is the set of customer nodes, $O =$ $\{1, 2, ..., |O|\}$ is the set of outlet nodes, $S = \{1, 2, ..., |S|\}$ is the set of supplier nodes, and node 0 represents the cross-dock. $A' = \{(i, j) : i \neq j \in C \cup 0\}, A'' = \{(i, j) : i \neq j \in O \cup 0\},\$ and $A''' = \{(i, j) : i \neq j \in S \cup 0\}$ each refer to the set of arcs connecting two different nodes i and j . Each of the connected arc has a travel distance of e'_{ij}, e''_{ij} , and e'''_{ij} , and a travel time of

 t'_{ij} , t''_{ij} , and t'''_{ij} for connecting customer, outlet, and supplier nodes, respectively. Let c represents the transportation cost per unit distance. Fig. 1 illustrates the proposed VRP-RCD network.

Fig. 1. The proposed VRP-RCD network

A set of homogeneous vehicles $V = \{1, 2, \ldots, |V|\}$, all having the same capacity q , is available at the cross-dock and can be used to perform one of the three processes:

- customer pickup process: pickup unsold and defective products from customers (if any).
- outlet delivery and pickup process: deliver outlets' demand and simultaneously pickup their unsold and defective products (if any).
- supplier delivery process: deliver unsold and defective products to the supplier that supplied those products (if any).

An operational cost H is charged once a vehicle is utilized to perform any of those three processes. Due to the nature of the cross-dock where no storages are allowed to be kept inside, those three processes must be done within T_{max} time horizon. Each customer i having demand for product k as much as d'_{ik} may not be able to sell all of those products with percentage f_{ik} , and each product k has a percentage of p_k defects. The returned products from customers are then inspected inside the cross-dock facility. The non-defective products are consolidated according to outlets' demand. Each outlet *i* having demand of product *k* as much as $d_{ik}^{''}$ may not be able to receive all of its demand, depends on the total returned products from the customers. However, each outlet i may not be able to sell all of its received products with percentage $f_{ik}^{''}$. Therefore, the returned products from outlets, together with several defective products from customers (if any), and the returned products from customers which are not sent to any outlets during the second process are then consolidated inside the cross-dock and will be sent back to the supplier that supplied those products. Here, each supplier k is assumed to supply one type of product, namely product k .

The decision variables and the mathematical model are as follows:

- $x_{ij}^{'v}$ is a binary variable with the value of 1 if vehicle v moves from node i to j in the customer pickup process; 0 otherwise $(i, j \in C \cup 0, v \in V)$
- $x_{ij}^{''v}$ is a binary variable with the value of 1 if vehicle v moves from node i to j in the outlet delivery and pickup process; 0 otherwise $(i, j \in O \cup 0, v \in V)$
- $x_{ij}^{'''}$ is a binary variable with the value of 1 if vehicle v moves from node i to j in the supplier delivery process; 0 otherwise $(i, j \in S \cup 0, v \in V)$
- y_k is a binary variable with the value of 1 if the demand of product k from all outlets is less than the amount of non-defective unsold of product k obtained from all customers; 0 otherwise ($k \in S$)
- A_i' ^v is the amount of products picked up from node *i* by vehicle v in the customer pickup process ($i \in C, v \in V$)
- $A_{ik}^{"v}$ is the amount of product k delivered to node i by vehicle v in the outlet delivery and pickup process ($i \in$ $O, k \in S, v \in V$
- $A_i^{i'v}$ is the amount of products delivered to node *i* by vehicle v in the supplier delivery process ($i \in S, v \in V$)
- $q_0^{'v}$ is the initial load of vehicle v upon leaving the crossdock in the customer pickup process ($v \in V$)
- \bullet $q_0^{''v}$ is the initial load of vehicle v upon leaving the crossdock in the outlet delivery and pickup process ($v \in V$)
- \bullet $q_0^{'''v}$ is the initial load of vehicle v upon leaving the crossdock in the supplier delivery process $(v \in V)$
- \bullet q'_i is the amount of load remaining in the vehicle upon visiting node i in the customer pickup process ($i \in C$)
- \bullet q_i'' is the amount of load remaining in the vehicle upon visiting node i in the outlet delivery and pickup process $(i \in O)$
- \bullet $\alpha_i^{''''}$ is the amount of load remaining in the vehicle upon visiting node i in the supplier delivery process ($i \in S$)
- Tcp_{max} records the maximum traveling duration time for the customer pickup process
- T^{odp}_{max} records the maximum traveling duration time for the outlet delivery and pickup process
- Tsd_{max} records the maximum traveling duration time for the supplier delivery process
- u'_i defines the order in which node i is visited on a tour in the customer pickup process ($i \in C$)
- \bullet u_i'' defines the order in which node *i* is visited on a tour in the outlet delivery and pickup process ($i \in O$)
- \bullet $u_i^{'''}$ defines the order in which node *i* is visited on a tour in the supplier delivery process ($i \in S$)

$$
Min \ c(\sum_{v \in V} \sum_{i \in C \cup 0} \sum_{j \in C \cup 0} x_{ij}^{'v} e_{ij}^{'} + \sum_{v \in V} \sum_{i \in C \cup 0} \sum_{j \in C \cup 0} x_{ij}^{'v} e_{ij}^{'} + \sum_{v \in V} \sum_{i \in S \cup 0} \sum_{j \in S \cup 0} x_{ij}^{''v} e_{ij}^{''v} + \sum_{v \in V} \sum_{j \in C} x_{0j}^{''v} + \sum_{v \in V} \sum_{j \in S} x_{0j}^{''v} \right)
$$
(1)

$$
\sum_{j \in C} x_{0j}^{'v} + \sum_{j \in O} x_{0j}^{''v} + \sum_{j \in S} x_{0j}^{'''v} \le 1 \ \forall v \in V \tag{2}
$$

$$
\sum_{i \in C \cup 0} \sum_{j \in C \cup 0, j \neq i} x_{ij}^{'v} t_{ij}' \leq Tcp_{max} \ \forall v \in V \tag{3}
$$

$$
\sum_{i \in O \cup 0} \sum_{j \in O \cup 0, j \neq i} x_{ij}^{''v} t_{ij}^{''} \leq Todp_{max} \ \forall v \in V \tag{4}
$$

$$
\sum_{i \in S \cup 0} \sum_{j \in S \cup 0, j \neq i} x_{ij}^{'''v} t_{ij}^{'''} \leq Tsd_{max} \ \forall v \in V \tag{5}
$$

$$
Tcp_{max} + Todp_{max} + Tsd_{max} \le T_{max} \tag{6}
$$

Objective function (1) minimizes the transportation and operational costs. Constraint (2) ensures that each vehicle can only be utilized for only one of the three processes. Constraints (3) to (5) record the maximum time for the three processes, each. Constraint (6) ensures all processes are done before a time limit.

$$
L\sum_{v\in V}\sum_{i\in C\cup 0, i\neq j}x_{ij}^{'v}\geq \sum_{k\in S}f_{jk}^{'}d_{jk}^{'}\ \forall j\in C\tag{7}
$$

$$
\sum_{i \in C} \sum_{j \in C, j \neq i} x_{ij}^{'v} \le L \sum_{j \in C} x_{0j}^{'v} \quad \forall v \in V \tag{8}
$$

$$
\sum_{i \in C \cup 0, i \neq l} x_{il}^{'v} = \sum_{j \in C \cup 0, j \neq l} x_{lj}^{'v} \quad \forall l \in C, \forall v \in V \tag{9}
$$

$$
\sum_{i \in C} x_{0i}^{'v} \le 1 \quad \forall v \in V \tag{10}
$$

$$
\sum_{v \in V} \sum_{i \in C \cup 0} x_{ij}^{'v} \le 1 \ \forall j \in C \tag{11}
$$

$$
A_j^{'v} = \sum_{k \in S} f_{jk}' d_{jk}' \sum_{i \in C \cup 0} x_{ij}^{'v} \quad \forall j \in C, \forall v \in V \qquad (12)
$$

$$
q_0^{'v} = 0 \quad \forall v \in V \tag{13}
$$

$$
q_i' \ge q_0^{'v} + A_i^{'v} - L(1 - x_{0i}^{'v}) \quad \forall i \in C, \forall v \in V \tag{14}
$$

$$
q_i^{'} \le q_0^{'v} + A_i^{'v} + L(1 - x_{0i}^{'v}) \quad \forall i \in C, \forall v \in V \tag{15}
$$

$$
q'_{j} \ge q'_{i} + A'^{v}_{j} - L(1 - x'^{v}_{ij}) \quad \forall i, j \in C, \forall v \in V \tag{16}
$$

$$
q'_{j} \le q'_{i} + A'_{j}^{v} + L(1 - x'_{ij}) \quad \forall i, j \in C, \forall v \in V \tag{17}
$$

$$
q_0^{'v} \le q \quad \forall v \in V \tag{18}
$$

$$
q_j^{'} \le q \ \forall j \in C \tag{19}
$$

$$
u'_{j} \ge u'_{i} + 1 - |C|(1 - \sum_{v \in V} x'_{ij}) \quad \forall i, j \in C \tag{20}
$$

Constraint (7) ensures that if there are returned products that need to be picked up from a customer, then a vehicle will serve that customer. L refers to a constant large number. Constraint (8) ensures that if a vehicle serves a customer, then the vehicle needs to start its trip from the cross-dock.

Constraint (9) ensures the outflow and inflow of a vehicle. Constraint (10) ensures that each vehicle can only leave the cross-dock at maximum once. Constraint (11) ensures that each customer is visited at maximum once. The amount of products to be picked up from each customer is calculated in constraint (12). Constraints (13) to (17) track the total load inside a vehicle. Constraints (18) and (19) limit the vehicle capacity. Constraint (20) is the sub-tour elimination.

$$
\sum_{i \in O} d'_{ik} - (1 - p_k) \sum_{i \in C} f'_{ik} d'_{ik} \ge -Ly_k \quad \forall k \in S \tag{21}
$$

$$
\sum_{i \in O} d_{ik}^{''} - (1 - p_k) \sum_{i \in C} f_{ik}^{'} d_{ik}^{'} \le L(1 - y_k) \ \forall k \in S \quad (22)
$$

$$
\sum_{v \in V} \sum_{i \in O} A_{ik}^{''v} \ge (1 - p_k) \sum_{i \in C} f_{ik}^{'} d_{ik}^{'} - Ly_k \ \forall k \in S \tag{23}
$$

$$
\sum_{v \in V} \sum_{i \in O} A_{ik}^{''v} \le (1 - p_k) \sum_{i \in C} f_{ik}^{'} d_{ik}^{'} + L y_k \ \forall k \in S \tag{24}
$$

$$
\sum_{v \in V} \sum_{i \in O} A_{ik}^{''v} \ge \sum_{i \in O} d_{ik}^{''} - L(1 - y_k) \ \forall k \in S \qquad (25)
$$

$$
\sum_{v \in V} \sum_{i \in O} A_{ik}^{''v} \le \sum_{i \in O} d_{ik}^{''} + L(1 - y_k) \ \forall k \in S \qquad (26)
$$

$$
\sum_{v \in V} A_{ik}^{''v} \le d_{ik}^{''} \quad \forall i \in O, \forall k \in S \tag{27}
$$

$$
L \sum_{i \in O \cup 0, i \neq j} x_{ij}^{''v} \ge \sum_{k \in S} A_{jk}^{''v} \quad \forall j \in O, \forall v \in V \qquad (28)
$$

$$
\sum_{i \in O} \sum_{j \in O, j \neq i} x_{ij}^{''v} \le L \sum_{j \in O} x_{0j}^{''v} \quad \forall v \in V \tag{29}
$$

$$
\sum_{i \in O \cup 0, i \neq l} x_{il}^{"v} = \sum_{j \in O \cup 0, j \neq l} x_{lj}^{"v} \quad \forall l \in O, \forall v \in V \tag{30}
$$

$$
\sum_{i \in O} x_{0i}^{''v} \le 1 \quad \forall v \in V \tag{31}
$$

$$
q_0''^v = \sum_{j \in O} \sum_{k \in S} A_{jk}''^v \quad \forall v \in V \tag{32}
$$

$$
q_i'' \ge q_0''^v - \sum_{k \in S} A_{ik}''^v + \sum_{k \in S} f_{ik}'' A_{ik}''^v - L(1 - x_{0i}''^v)
$$

$$
\forall i \in O, \forall v \in V
$$
 (33)

$$
q_i'' \le q_0''^v - \sum_{k \in S} A_{ik}''^v + \sum_{k \in S} f_{ik}'' A_{ik}''^v + L(1 - x_{0i}''^v)
$$

$$
\forall i \in O, \forall v \in V
$$
 (34)

$$
q_j'' \ge q_i'' - \sum_{v \in V} \sum_{k \in S} A_{jk}''^v + \sum_{v \in V} \sum_{k \in S} f_{jk}'' A_{jk}''^v - L(1 - \sum_{v \in V} x_{ij}''^v) \quad \forall i, j \in O
$$
 (35)

$$
q_j'' \le q_i'' - \sum_{v \in V} \sum_{k \in S} A_{jk}''^v + \sum_{v \in V} \sum_{k \in S} f_{jk}'' A_{jk}''^v + L(1 - \sum_{v \in V} x_{ij}''^v) \ \forall i, j \in O \tag{36}
$$

$$
q_0^{''v} \le q \ \forall v \in V \tag{37}
$$

$$
q_j^{''} \le q \ \forall j \in O \tag{38}
$$

$$
u_j'' \ge u_i'' + 1 - |O|(1 - \sum_{v \in V} x_{ij}''v) \ \forall i, j \in O \tag{39}
$$

Constraints (21) and (22) determine whether the demand of all outlets for a particular product is greater than the amount of non-defective pickup products from all customers. Constraints (23) to (26) determine the amount of a particular product to be delivered to all outlets. Constraint (27) guarantees that the amount of a particular product to be delivered to an outlet is less than the demand of the product that is needed by the outlet. Constraint (28) ensures that if there is a delivery process to an outlet, then a vehicle will serve the outlet. Constraint (29) ensures that if a vehicle serves an outlet, then the vehicle needs to start its trip from the cross-dock. Constraint (30) ensures the outflow and inflow of a vehicle. Constraint (31) ensures that each vehicle can only leave the cross-dock at maximum once. Constraints (32) to (36) track the total load inside a vehicle. Constraints (37) and (38) limit the vehicle capacity. Constraint (39) is the sub-tour elimination.

$$
L \sum_{v \in V} \sum_{i \in S \cup 0, i \neq j} x_{ij}^{'''v} \ge \sum_{v \in V} \sum_{i \in O} f_{ij}^{''} A_{ij}^{''v} + \sum_{i \in C} f_{ij}^{'} d_{ij}^{'} - \sum_{v \in V} \sum_{i \in O} A_{ij}^{''v} \quad \forall j \in S
$$
\n(40)

$$
\sum_{i \in S} \sum_{j \in S, j \neq i} x_{ij}^{\prime \prime \prime} v \le L \sum_{j \in S} x_{0j}^{\prime \prime \prime} v \quad \forall v \in V \tag{41}
$$

$$
\sum_{i \in S \cup 0, i \neq l} x_{il}^{m'} = \sum_{j \in S \cup 0, j \neq l} x_{lj}^{m'} \quad \forall l \in S, \forall v \in V \tag{42}
$$

$$
\sum_{i \in S} x_{0i}^{'''v} \le 1 \quad \forall v \in V \tag{43}
$$

$$
\sum_{v \in V} \sum_{i \in S \cup 0} x_{ij}^{m_v} \le 1 \ \forall j \in S \tag{44}
$$

$$
\sum_{v \in V} A_{j}^{"v} \geq \left(\sum_{v \in V} \sum_{i \in O} f_{ij}^{''} A_{ij}^{"v} + \sum_{i \in C} f_{ij}^{'} d_{ij}^{'} - \sum_{v \in V} \sum_{i \in O} A_{ij}^{"v} \right) - L \left(1 - \sum_{v \in V} \sum_{i \in S \cup 0} x_{ij}^{"v} \right) \ \forall j \in S \tag{45}
$$

$$
\sum_{v \in V} A_{j}^{"v} \leq (\sum_{v \in V} \sum_{i \in O} f_{ij}^{''} A_{ij}^{"v} + \sum_{i \in C} f_{ij}^{'} d_{ij} - \sum_{v \in V} \sum_{i \in O} A_{ij}^{"v} + L(1 - \sum_{v \in V} \sum_{i \in S \cup 0} x_{ij}^{"v}) \ \forall j \in S
$$
\n(46)

$$
L \sum_{i \in S \cup 0, i \neq j} x_{ij}^{m'v} \ge A_j^{m'v} \quad \forall j \in S, \forall v \in V \tag{47}
$$

$$
q_0^{\prime\prime\prime} v = \sum_{j \in S} A_j^{\prime\prime\prime} v \quad \forall v \in V \tag{48}
$$

$$
q_i^{'''} \ge q_0^{'''} - A_i^{'''} - L(1 - x_{0i}^{''''}) \quad \forall i \in S, \forall v \in V \tag{49}
$$

$$
q_i^{'''} \le q_0^{'''} - A_i^{'''} v + L(1 - x_{0i}^{''v}) \quad \forall i \in S, \forall v \in V \tag{50}
$$

$$
q_j''' \ge q_i''' - \sum_{v \in V} A_j'''^v - L(1 - \sum_{v \in V} x_{ij}''^v) \ \forall i, j \in S \quad (51)
$$

$$
q_j^{'''} \le q_i^{'''} - \sum_{v \in V} A_j^{'''} + L(1 - \sum_{v \in V} x_{ij}^{'''v}) \ \forall i, j \in S \quad (52)
$$

$$
q_0^{'''v} \le q \ \forall v \in V \tag{53}
$$

$$
q_j^{'''} \le q \ \forall j \in S \tag{54}
$$

$$
u_j''' \ge u_i''' + 1 - |S|(1 - \sum_{v \in V} x_{ij}''^v) \ \forall i, j \in S \qquad (55)
$$

Constraint (40) ensures that when either of the two cases – the existence of outlets' returned products or the existence of returned products from customers that are not sent to any outlet (including the defective products) – occurs, then the related supplier will be visited. Constraint (41) ensures that if a vehicle visits suppliers, then that vehicle needs to start its trip from the cross-dock. Constraint (42) ensures the outflow and inflow of a vehicle. Constraint (43) ensures that each vehicle can only leave the cross-dock at maximum once. Constraint (44) ensures that each supplier is visited at maximum once. The amount of products to be delivered to each supplier is calculated in constraints (45) and (46). Constraint (47) ensures split delivery does not occur. Constraints (48) to (52) track the total load inside a vehicle. Constraints (53) and (54) limit the vehicle capacity. Constraint (55) is the sub-tour elimination.

III. MODEL IMPLEMENTATION

To the best of our knowledge, benchmark instances for VRP-RCD in a four-level supply chain model are not available. Therefore, we modify the benchmark VRPCD instances [3] and introduce our VRP-RCD instances. We differentiate the VRP-RCD instances into small instance (15 nodes) and large instance (40 nodes), each consisting of 30 problems. The detailed parameters are summarized in Table I. The mathematical model was solved by CPLEX 12.8.0.0 within a time limit of two hours. The results are presented in Table II.

All problems in small instances can be solved optimally on average in less than one minute. However, as the number of nodes are increased slightly to more than double, the problem becomes significantly harder to solve. None of the problems

TABLE I VRP-RCD PARAMETER VALUES

	Small instance	Large instance
S		
	6	23
	5	10
	10	20
\boldsymbol{q}	70	150
ϵ		
H	1000	1000
${\cal T}_{max}$	16 hrs	16 hrs
$^{\prime\prime\prime}$	$U \sim (48, 560)$	$U\sim(48, 480)$
$\begin{array}{c} e_{ij},\,e_{ij},\,e_{ij}\\ t_{ij}^{'} ,\,t_{ij}^{''},\,t_{ij}^{'''} \end{array}$	$U \sim (20,200)$	$U\sim(20,100)$
$\sum_{k \in S} d'_{ik}, \sum_{k \in S} d''_{ik}$	$U\sim(5,50)$	$U \sim (5,20)$
p_k	$U\sim(0.0.05)$	$U\sim(0,0.05)$
l_{ik} ,	$U \sim (0,1) * U \sim (0,1)$	$U\sim(0,1)*U\sim(0,1)$

TABLE II COMPUTATIONAL RESULTS

can be solved optimally within two hours, and therefore we only report the best found solutions so far. Moreover, for some problems such as 40-8, 40-18, 40-20, 40-22, 40-25, and 40-28, CPLEX cannot even get any feasible solutions.

We then conduct sensitivity analysis for analyzing the impact of the time horizon, T_{max} , towards the total obtained cost value. It turns out that increasing the time horizon could reduce the total cost in the VRP-RCD network because each vehicle may serve more nodes, and therefore the number of vehicles used decreases. However, from the practical point of view, the time horizon cannot be set too high (e.g. exceeding 24 hours) as this may lead to keeping storages inside the crossdock, which contradicts the main idea of a cross-dock [11].

IV. CONCLUSION

The integration of VRP and cross-dock, namely VRPCD, has been extensively studied in the literature. However, most research only focused on forward flow instead of reverse flow. We proposed a VRP-RCD network in a four-level supply chain, which consists of suppliers, cross-dock, customers, and outlets (i.e. secondary channels). The customers' unsold and defective products are picked up in the first process. The unsold products are then distributed to the outlets for the reselling process in the second process. Outlets' unsold and defective products are picked up simultaneously in this process. Finally, all the unsold and defective products are returned to every supplier for further treatments, such as repairing or remanufacturing.

We formulated the VRP-RCD network as a mathematical model and solved modified benchmark VRP-RCD instances by an optimization solver, CPLEX. All 30 small instances can be solved to optimality within less than one minute, on average. However, none of the problems in large instances can be solved to optimality within two hours of running time. Therefore, future work may consider to design an algorithm to solve the proposed VRP-RCD network.

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