2-2024

What difference do the new factor models make in portfolio allocation?

Frank J. Fabozzi
EDHEC

Dashan HUANG
Singapore Management University, DASHANHUANG@smu.edu.sg

Fuwei Jiang
Central University of Finance and Economics

Jiexun WANG
P&G Singapore Innovation Centre

What Difference Do New Factor Models Make in Portfolio Allocation?[*]

Frank J. Fabozzi[†] **Dashan Huang**[‡] **Fuwei Jiang**[§] **Jiexun Wang**[¶]

November 2023

[*]We are grateful to Raymond Kan, Jay Shanken, Avanidhar Subrahmanyam, and participants at the 2015 Singapore Management University Finance Summer Camp for helpful comments. We thank Kenneth French, Robert Novy-Marx, and Chen Xue for sharing their data, and Michael Grant and Stephen Boyd for making their CVX MatLab Toolbox available.

[†]EDHEC Business School, 393, Promenade des Anglais BP3116, 06202 Nice Cedex 3–France. E-mail: fabozzi321@aol.com.

[‡]Lee Kong Chian School of Business, Singapore Management University, Singapore, 178899; E-mail: dashanhuang@smu.edu.sg.

[§]School of Finance, Central University of Finance and Economics, Beijing, China, 100018. E-mail: jfuwei@gmail.com.

[¶]P&G Singapore Innovation Centre, 70 Biopolis St., Singapore, 138547. E-mail: wangjiexun@gmail.com.

What Difference Do New Factor Models Make in Portfolio Allocation?

Abstract

This paper compares the Hou-Xue-Zhang four-factor model with the Fama-French five-factor model from an investing perspective both in- and out-of-sample. Without margin requirements and model uncertainty, the Hou-Xue-Zhang model outperforms the Fama-French model. However, the outperformance could become negligible if an investor is subject to margin requirements and model uncertainty. The Hou-Xue-Zhang model shows similar power as the Fama-French model in describing the covariance matrix of asset returns. Overall, the two models do not make a difference for investing in a realistic setting.

JEL Classification: G11; G12; C11

Keywords: Portfolio allocation; Mean-variance analysis; Factor model; Asset pricing

1 Introduction

Much of asset pricing research involves searching for factors to improve the understanding of the cross-section of stock returns. Based on the neoclassical Tobin q-theory, Hou, Xue, and Zhang (HXZ, 2015) propose a four-factor model, which can explain 29 out of 36 significant anomalies. Concurrently, motivated by the dividend discount model, Fama and French (FF5, 2015a, 2016) propose a five-factor model that explains anomalies such as the low market beta, share repurchases, low stock return volatility, etc. In a follow-up study, Hou, Xue, and Zhang (2019) compare the *pricing* power of HXZ with FF5 and conclude that the HXZ model largely subsumes FF5 and is able to explain more anomalies.

This paper asks whether HXZ is a better model for *investing* relative to FF5. There are three reasons for this question. First, as shown by Pástor and Stambaugh (2000), a model that is better for pricing is not necessarily better for investing, because investors are usually subject to margin requirements and model uncertainty that prevent them from implementing certain extreme investment strategies suggested by asset pricing models. Second, investing involves both the mean and covariance of asset returns. A model that is worse for pricing is not necessarily worse for investing. There could be factors that account for substantial return comovements, but they are not priced or have very low risk premiums (Constantidides, 1980). Although these factors do not improve the description of average asset returns, they are important for an investor to control portfolio risk (Lewellen, 2023). Finally, pricing errors may not be a desirable criterion to assess a model, because, in-sample, "it is not difficult to data mine factor models that explain a large cross-section of anomalies" (Tian, 2021). Instead, investing provides an economic criterion for model comparison that accommodates pricing errors and allows one to compare asset pricing models out-of-sample, which is advocated by MacKinlay (1995) and Ang (2014), among others.

This paper compares the investing performance of HXZ with FF5 assuming the investor is subject to margin requirement and model uncertainty. Specifically, we focus on the portfolio allocation problem in the standard mean-variance framework. To make the optimal portfolio implementable, the investor faces a margin requirement ranging from 0% to 50% (Pástor and Stambaugh, 2000). Fama and French (2015b) show that, without a short selling constraint, it is easy for an investor who is investing in two anomalies to have a leverage ratio of more than 300, which is apparently unrealistic in practice. Moreover, because the distribution of asset returns is unknown, the investor imposes a factor model, either HXZ or FF5, to reduce the dimension of the estimation problem and to allocate her wealth among the factors. Although this restriction often improves the portfolio performance (MacKinlay and Pástor, 2000), the investor faces uncertainty regarding the model's pricing ability. Following Pástor and Stambaugh (2000) and Wang (2005), we assume that the investor has a prior belief, specified with varying degrees of confidence in the factor model, and computes the optimal portfolio with her posterior belief, which is updated by the data.

We consider 15 anomaly portfolios in Novy-Marx and Velikov (2016) as the non-benchmark risky assets to invest.[1] We classify the anomalies into two groups. The first group consists of five anomalies that can be explained by HXZ but not FF5, i.e., the alpha of each anomaly is insignificant with HXZ but significant with FF5. The average alphas are 0.13% ($t = 0.63$) and 0.67% ($t = 3.45$) when using the two models. The second group consists of 10 anomalies that cannot be explained by either HXZ or FF5. In this case, the average alphas are 0.72% ($t = 4.11$) and 0.75% ($t = 4.44$). These two groups of anomalies are intentionally classified to explore whether HXZ is better for investing when it performs better than or the same as FF5 for pricing.

We first compare the in-sample investing performances between the two models. When an asset pricing model cannot explain the average returns of risky assets with significant alphas, imposing the model on the return-generating process can lead to biased estimates for the predictive mean and covariance matrix of asset returns, and therefore, results in certainty-equivalent return (CER) losses relative to the case without imposing any model. As such, an asset pricing model is better for investing if it generates smaller CER losses. Without a margin requirement, we find that HXZ uniformly outperforms FF5. For example, the CER loss for an investor with a perfect confidence in HXZ is over 13% per year less than the CER loss for an investor with a perfect confidence in FF5. However, the outperformance of HXZ is shrunk dramatically when there is a margin requirement. For instance, a 50% margin requirement shrinks the HXZ outperformance to be less than 3%. In this case, the investor who uses HXZ is even suggested to invest in redundant assets, assets that are explained by HXZ with insiginficant alphas.

We then compare the out-of-sample investing performances. To this goal, we perform two exercises. The first exercise assumes that asset returns are independent and identically distributed (i.i.d) over time, and uses a bootstrap simulation to compare the out-of-sample CERs between the two models. The second exercise relaxes the i.i.d assumption and compares the out-of-sample CERs with real time data. We use an expanding window approach. At the end of each month, we estimate the predictive mean and covariance matrix with

[1] Novy-Marx and Velikov (2016) consider 32 anomalies in total, but only 15 are significant when using the FF5 as the benchmark. The data can be downloaded from https://sites.psu.edu/assayinganomalies/code/overview/.

the most up-to-date data, and apply the resulting optimal portfolio to the next month's returns. Then, we calculate the out-of-sample CERs with the realized portfolio returns. In either exercise, because the investor has to estimate the model parameters, the estimation error or model uncertainty reduces the outperformance of HXZ further. With a 50% margin requirement, the two models perform virtually the same. Without margin requirements, FF5 even performs better when the investor does not have high confidence in HXZ.

Finally, we explore how HXZ and FF5 describe the covariance matrix of asset returns. A better model for investing could be the result of its superior ability to describe the mean, the covariance matrix, or both. We compare the performances of the global-minimum-variance portfolios between the two models, which solely use the predictive covariance information for portfolio allocation. The result shows that the two models perform virtually the same, no matter which group of anomalies is used as the non-benchmark assets.

The studies that are most closely related to this paper are Pástor and Stambaugh (2000) and Wang (2005), which incorporate model uncertainty, measured by investors' varying beliefs about asset pricing models, into the framework of portfolio allocation. Focusing on the Fama-French three-factor model and the Daniel and Titman (1997) characteristic model, Pástor and Stambaugh (2000) find that the two models generate indistinguishable performances under model uncertainty and margin requirements. This paper concentrates on the two most recent competing factor models and finds that HXZ and FF5 perform similarly for investing, even in the case when they have different degrees of pricing ability on the non-benchmark assets.[2] Wang (2005) focuses on model uncertainty and does not consider the effect of margin requirements. Moreover, despite its importance for investing, these two papers do not consider out-of-sample performance.

This paper is also related to the literature in portfolio allocation with factor-based asset pricing models. To evaluate the performance of different factor models for the covariance structure of individual stock returns, Chan, Karceski, and Lakonishok (1998, 1999) show that the Fama-French three-factor model does a fair job constructing the global-minimum-variance portfolio. Also focusing on the estimation of the covariance structure, Briner and Connor (2008) explore the trade-off between estimation errors and model specification errors.[3]

[2] After our paper was firstly posted at SSRN on March 16, 2016, Kan, Wang and Zheng (2022) explore the effect of estimation errors on the mean-variance frontiers spanned by the factors of the two models. Detzel, Novy-Marx and Velikov (2023) and Li, DeMiguel and Marín-Rtrera (2023) examine how linear and nonlinear transaction costs affect the factor performance of the two models, respectively. Interestingly, all the three papers reach a similar conclusion as ours.

[3] Excellent books that consider portfolio allocation with factor models include Brandt (2010), Connor, Goldberg, and Korajczyk (2010), and Ang (2014).

The remainder of the paper is organized as follows. Section 2 reviews the HXZ and FF5 factor models and discusses the importance of comparing them from the perspective of investing. Section 3 presents a framework for making portfolio allocation and shows that HXZ and FF5 perform similarly for investing if the investor is subject to margin rquirement and model uncertainty.

2 New Factor Models

This section reviews the HXZ and FF5 factor models and discusses the importance of comparing them from an investing perspective. We focus on these two models because they are representative and are motivated by two different theories.[4]

The HXZ model is motivated by the neoclassical q-theory of investment and consists of four factors: a market factor (MKT), a size factor (ME), an investment factor (I/A), and a profitability factor (return on equity, ROE). The first factor is the market excess return and the last three factors are constructed from a triple $(2 \times 3 \times 3)$ sort on size, investment-to-assets, and return-on-equity. More specifically, size is the market equity, which is stock price per share times shares outstanding from the Center for Research in Security Prices, I/A is the annual change in total assets divided by one-year-lagged total assets, and ROE is income before extraordinary items divided by one-quarter-lagged book equity.

The FF5 model is based on the dividend discount valuation theory and adds an investment (conservative-minus-aggressive, CMA) factor and a profitability (robust-minus-weak, RMW) factor to the Fama-French three-factor model, which consists of market, size (small-minus-big, SMB), and value (high-minus-low, HML) factors. More specifically, CMA is defined as the difference between the returns on diversified portfolios of low and high investment stocks and RMW is defined as the difference between the returns on diversified portfolios of stocks with robust and weak profitability.

Table 1 presents summary statistics of the HXZ and FF5 factors in the sample period of 1972:01–2013:12. Panel A reports the average return (mean), t-statistic from the test that the average return of the factor is zero, standard deviation, skewness, kurtosis, first-order autocorrelation, and annualized Sharpe ratio. Among the seven descriptive statistics, mean and standard deviation are reported in percent per month. The average monthly returns on the factors are all more than two standard errors above zero, except for the

[4] Subsequent models, such as Stambaugh and Yuan (2017) and Daniel, Hirshleifer, and Sun (2020), are variations of these two models. The MGMT factor in Stambaugh and Yuan (2017) and the FIN factor in Daniel, Hirshleifer, and Sun (2020) are variations of the investment factor, and the PERF and PEAD factors in the two papers are variations of the profitability factor.

FF5 size factor, which has an average monthly return of 0.23% ($t = 1.71$). The HXZ size factor has a higher average monthly return of 0.31%, with a t-statistic of 2.20.

Although both HXZ and FF5 use annual asset growth as the proxy for investment, the HXZ investment factor I/A has a higher average monthly return (0.44% versus 0.37%) and a lower standard deviation (1.87% versus 2.00%) than the FF5 investment factor CMA. As a result, I/A has a higher annualized Sharpe ratio than CMA (0.82 versus 0.66). Moreover, I/A is less persistent than CMA and their first-order autocorrelations are 0.06 and 0.12, respectively.

The most striking difference between HXZ and FF5 is the profitability factor. First, the HXZ profitability factor ROE uses monthly earnings data, whereas the FF5 profitability factor RMW uses annual operating profitability data. Hou, Xue, and Zhang (2015) argue that the ROE factor is designed to capture anomalies, such as price momentum, earnings surprise, and financial distress, which are all studied at a monthly frequency. Next, since the ROE factor contains the most up-to-date information about future ROE, its standard deviation is slightly higher than the CMW factor (2.62% versus 2.25%), and its average monthly return almost doubles (0.57% versus 0.29%). This is reflected directly in their annualized Sharpe ratios, 0.75 and 0.44. Finally, ROE has a more negative skewness (-0.75 versus -0.44) and a smaller kurtosis (8.01 versus 14.4) than RMW.

Panel B of Table 1 reports the contemporaneous correlations of all of the factors. The market factors in the HXZ and the FF5 models have a perfect correlation of 1, and the size factors have a correlation of 0.98. Together with the descriptive statistics in Panel A, we assume that the market and size factors in the two models are indistinguishable and use the returns of MKT and SMB in the FF5 model for portfolio allocation throughout the paper.[5] The negative correlations of MKT with the investment and profitability factors suggest the necessity of new factors that can hedge the market risk. The two investment factors have a correlation of 0.90, and the two profitability factors have a correlation of 0.67. An interesting observation is that the FF5 value factor has high correlations with the investment factors and low correlations with the profitability factors, suggesting that the redundancy of HML for pricing, shown in FF5 (2015a), is mainly due to the investment factor.

Table 1 raises a question about the main difference in the two models in explaining the cross-section of stock returns. Using factor spanning regressions, Hou, Xue, and Zhang (2019) compare the two models and

[5] In portfolio allocation, when assets i and j are highly correlated, the estimation of the covariance is highly volatile with extreme entries on $(i,i),(i,j),(j,i)$ and (j,j), resulting in extreme portfolio positions in assets i and j that swing dramatically over time.

find that HXZ can fully describe FF5 in terms of alpha. As such, they conclude that FF5 is in essence a noisy version of the *q*-factor model.

However, there are two concerns about the conclusion. First, the comparison in Hou, Xue, and Zhang (2019) is *ex post*. The conclusion may change if we compare them ex ante. At the end of each month, we regress individual stock returns on the factors of the two models using the past 60-month observations, and examine how many stocks are mispriced. Figure 1 plots the proportions of firms that are mispriced by HXZ and FF5, respectively. Surprisingly, there are a lot of months where HXZ has worse pricing performance, say 2005 and 2013. Second, alpha is not an appropriate metric for model comparison and can generate counterintuitive results.[6] For example, over the sample period of 1972:01–2013:12, the monthly alpha of the momentum factor is 0.78% ($t = 3.94$) for the CAPM model but is 0.95% ($t = 4.82$) for the Fama-French three-factor model, which is in stark contrast to most studies, if not all, that the three-factor model is a better pricing model. Another example is from Fama and French (2016) who find that the FF5 model exaggerates, instead of shrinking, the accrual anomaly. The FF5 alpha is 0.31% ($t = 2.27$) and the Fama-French three-factor alpha is 0.27% ($t = 1.96$) over the sample period of this paper. In general, Barillas and Shanken (2017) show that zero alpha for a non-benchmark asset is neither a sufficient nor a necessary condition for model comparison.

In terms of investing, Figure 2 plots the mean-variance frontiers for investing in the factors of HXZ, FF5, or both, where the market and size factors in the two models are assumed to be the same and refer to the MKT and SMB factors in FF5. Three observations follow the figure immediately. First, the frontier of FF5 does not lie inside or overlaps that of HXZ. Second, the global minimum-variance of investing in the HXZ is different from that of the FF5. Specifically, the standard deviation and mean of the global minimum-variance for investing in HXZ are 1.03% and 0.44%, which are in contrast to 0.97% and 0.34% for investing in FF5. This difference suggests that the HXZ factors cannot mimic the global minimum-variance portfolio of FF5. Third and lastly, if one invests in both the HXZ and FF5 factors (FF5's five factors plus HXZ's I/A and ROE factors), the standard deviation and mean of the global minimum-variance are 0.94% and 0.38%, respectively. Therefore, this strategy can reduce the minimum-variance and improve its expected return, relative to investing in HXZ or FF5 alone.

According to Barillas and Shanken (2017, 2018), a factor model is better for pricing if it can price the factors in the competing model with zero alphas. Similarly, a factor model is better for investing if it can

[6] Levy and Roll (2016) also show that with an optimization framework alpha is a bad target for investing.

mimic the performance of the competing model, i.e., it outperforms *any* portfolio spanned by the competing model in terms of the Sharpe ratio. As such, we turn to Huberman and Kandel (1987) and run a mean-variance *spanning* test on the hypothesis that whether the competing factors' returns can be spanned or replicated in the mean-variance space of the factor model.

Following Kan and Zhou (2012), we carry out six spanning tests: Wald test under conditional homoscedasticity, Wald test under independent and identically distributed (i.i.d.) elliptical distribution, Wald test under conditional heteroscedasticity, Bekerart-Urias spanning test with errors-in-variables (EIV) adjustment, Bekerart-Urias spanning test without the EIV adjustment and DeSantis spanning test. The six spanning test results reported in Table 2 strongly reject the hypothesis that the FF5 factors are inside the mean-variance frontier of the HXZ factors. Delving deeper, we also test whether the FF5's investment and profitability factors can be replicated by the HXZ factors, and find that the answer is negative. Hence, it not clear whether the HXZ model is better for investing, which is the focus of this paper. For comparison, in the last column of Table 2, we conduct the Gibbons-Ross-Shanken (GRS) test and show an opposite result: The HXZ model largely subsumes the FF5 factors.

3 Comparing Factor Models in Portfolio Allocation

This section presents the mean-variance portfolio allocation problem under model uncertainty and margin requirements. The objective is to compare asset pricing models from the perspective of investing. For a given investment universe, we calculate the portfolio that is selected by an investor who bases her prior belief in HXZ or FF5, and compare the performances of the two models in- and out-of-sample.

3.1 Portfolio allocation under model uncertainty and margin requirements

Consider the portfolio allocation problem in a universe with a risk-free asset and n risky assets. Without loss of generality, we assume that the risk-free rate r_f is constant over time throughout the paper. Let $r_t = (r'_{1t}, r'_{2t})'$ be the long-short spread returns, as in Pástor and Stambaugh (2000), where the long side is a risky asset and the short side is either a risky asset or a risk-free asset, r_{1t} is the first m non-benchmark assets, and r_{2t} is the last k $(= n - m)$ benchmark assets. For example, when HXZ is used as the benchmark model, r_{2t} is the HXZ four-factor returns and the HML, CMA, and RMW factor returns are included in r_{1t}, which then has $n - 4$ elements. Similarly, when FF5 is used as the benchmark model, r_{2t} is the FF5 five-factor returns and the I/A and ROE factor returns are simply included in r_{1t}, which then has $n-5$ elements.

Suppose r_t follows a multivariate normal distribution and is i.i.d. over time. The true mean and covariance matrix are denoted as follows corresponding to the m assets and k factors:

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \qquad V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, \tag{1}$$

which can be summarized in a regression model:

$$r_{1t} = \alpha + Br_{2t} + u_t, \tag{2}$$

where u follows a multivariate normal distribution with mean zero and covariance matrix equal to Σ. With this factor structure, one can write the mean and covariance matrix of the risky assets as

$$\mu = \begin{bmatrix} \alpha + B\mu_2 \\ \mu_2 \end{bmatrix}, \qquad V = \begin{bmatrix} BV_{22}B' + \Sigma & BV_{22} \\ V_{22}B' & V_{22} \end{bmatrix}. \tag{3}$$

The asset pricing model is true if and only if $\alpha = 0_{m\times 1}$, where $0_{m\times 1}$ is an $m \times 1$ vector of zeros.

In the portfolio allocation framework using asset pricing models, the mean-variance investor chooses to believe or not to believe the asset pricing model. If the investor does not believe the asset pricing model at all, she estimates μ and V without restricting α to zero. The maximum likelihood estimates of α, B, and Σ are denoted by $\hat{\alpha}, \hat{B}$, and $\hat{\Sigma}$, respectively. The investor estimates μ and V in (3) as:

$$\hat{\mu} = \begin{bmatrix} \hat{\mu}_1 \\ \hat{\mu}_2 \end{bmatrix} = \begin{bmatrix} \hat{\alpha} + \hat{B}\hat{\mu}_2 \\ \hat{\mu}_2 \end{bmatrix}, \qquad \hat{V} = \begin{bmatrix} \hat{B}\hat{V}_{22}\hat{B}' + \hat{\Sigma} & \hat{B}\hat{V}_{22} \\ \hat{V}_{22}\hat{B}' & \hat{V}_{22} \end{bmatrix}, \tag{4}$$

where $\hat{\mu}_2$ and $\hat{V}_{22}$ are the sample mean and covariance matrix of r_{2t}.

When the investor has a dogmatic belief about the asset pricing model, μ and V can be estimated by by imposing $\alpha = 0_{m\times 1}$. Let $\bar{B}$ and $\bar{\Sigma}$ be the maximum likelihood estimates of B and Σ with the restriction. The estimates of μ and V are

$$\bar{\mu} = \begin{bmatrix} \bar{B}\hat{\mu}_2 \\ \hat{\mu}_2 \end{bmatrix}, \qquad \bar{V} = \begin{bmatrix} \bar{B}\hat{V}_{22}\bar{B}' + \bar{\Sigma} & \bar{B}\hat{V}_{22} \\ \hat{V}_{22}\bar{B}' & \hat{V}_{22} \end{bmatrix}. \tag{5}$$

In this paper, we assume that the investor places a confidence level of ω in the asset pricing model. Let $R = \{r_t, t = 1, \cdots, T\}$ and $\hat{S}^2 = \hat{\mu}_2' \hat{V}_{22}^{-1} \hat{\mu}_2$ be the squared Sharpe ratio of the ex ante tangency portfolio with the same mean and covariance matrix of the k factors. With a Bayes updating, Wang (2005) shows that the investor estimates the predictive mean and covariance matrix as:

$$\tilde{\mu} = \mathrm{E}(r_{T+1}|R,\omega) = \begin{bmatrix} \hat{\mu}_1 + \omega(\bar{B}\hat{\mu}_2 - \hat{\mu}_1) \\ \hat{\mu}_2 \end{bmatrix}, \tag{6}$$

$$\tilde{V} = \mathrm{Var}(r_{T+1}|R,\omega) = \begin{bmatrix} \psi_0 + \omega\psi_1 + \omega^2\psi_2 & b[\omega\bar{B} + (1-\omega)\hat{B}]\hat{V}_{22} \\ b\hat{V}_{22}[\omega\bar{B} + (1-\omega)\hat{B}]' & b\hat{V}_{22} \end{bmatrix}, \tag{7}$$

where

$$\psi_0 = b\hat{B}\hat{V}_{22}p\hat{B}' + h\hat{\delta}\hat{\Sigma}, \tag{8}$$

$$\psi_1 = b(\bar{B} - \hat{B})\hat{V}_{22}\hat{B}' + b\hat{B}\hat{V}_{22}(\bar{B} - \hat{B})' + h(\bar{\delta} - \hat{\delta})\hat{\Sigma} + h\hat{\delta}(\bar{\Sigma} - \hat{\Sigma}), \tag{9}$$

$$\psi_2 = b(\bar{B} - \hat{B})\hat{V}_{22}(\bar{B} - \hat{B})' + h(\bar{\delta} - \hat{\delta})(\bar{\Sigma} - \hat{\Sigma}), \tag{10}$$

and where $\bar{\delta}, \hat{\delta}, b$, and h are scalars and are defined as follows:

$$\bar{\delta} = \frac{T(T-2)+k}{T(T-k-2)} - \frac{k+3}{T(T-k-2)} \cdot \frac{\hat{S}^2}{1+\hat{S}^2}, \tag{11}$$

$$\hat{\delta} = \frac{(T-2)(T+1)}{T(T-k-2)}, \tag{12}$$

$$b = \frac{T+1}{T-k-2}, \tag{13}$$

$$h = \frac{T}{T-m-k-1}. \tag{14}$$

From (6) and (7), the HXZ and FF5 models imply different restrictions on α and yield different predictive means and covariance matrices. As a result, their optimal portfolios are different. When $\omega = 0$, the predictive mean and covariance are the sample mean and covariance matrix, which are unbiasedly estimated without the restriction on α. When $\omega = 1$, the predictive mean and covariance matrix are fully determined by the estimates that restrict α to zero.

Let x denote the n-vector with the ith element x_i. With $\tilde{\mu}$ and $\tilde{\Sigma}$, the Bayesian investor is assumed to choose x to maximize the mean-variance objective function:

$$\max_{x} \quad x'\tilde{\mu} - \frac{\gamma}{2} x'\tilde{V}x, \tag{15}$$

where γ is the coefficient of relative risk aversion. For simplicity, we assume that γ is equal to three throughout the paper. Without any constraint, the optimal portfolio weight $\tilde{x}$ is

$$\tilde{x} = \frac{1}{\gamma}\tilde{V}^{-1}\tilde{\mu} = \frac{1}{\gamma}\begin{bmatrix} \tilde{\Sigma}^{-1}\tilde{\alpha} \\ \tilde{V}_{22}^{-1}\tilde{\mu}_2 - \tilde{B}'\tilde{\Sigma}^{-1}\tilde{\alpha} \end{bmatrix}. \tag{16}$$

One important property in (16) is that if some of the non-benchmark assets in r_1 have non-zero alphas, the investor with benchmark assets of r_2 should improve her portfolio Sharpe ratio by changing her portfolio weights on the non-benchmark assets in proportion to their alphas. The alpha of a non-benchmark asset, calculated with respect to a given asset pricing model, measures the change in the portfolio's Sharpe ratio that is driven by a marginal increase in the asset weight of the portfolio. Thus, the sign of alpha is the direction of the marginal adjustment in portfolio weight space that yields the maximal increase in the portfolio's Sharpe ratio. Therefore, alphas explain the optimal way to marginally adjust the portfolio relative to the benchmark: increase the weights of non-benchmark assets with positive alphas, and decrease the weights with negative alphas.

In the framework of asset pricing, by the mathematical definition, the adjustment to the portfolio weight can be *infinitesimal*. However, in the framework of investing, the adjustment is actually *finite* as the investor is usually subject to portfolio constraints (Almazan et al., 2004). Certain risky assets are not tradable because the investor cannot sell short with full use of the proceeds. Fama and French (2015b) show that, without a short selling constraint, it is easy for an investor who is investing in two anomalies to have a leverage ratio of more than 300, which is apparently unrealistic in practice.

Following Pástor and Stambaugh (2000), we assume that the mean-variance investor in the optimization problem (15) is subject to the following margin requirements:

$$\sum_{j\in\Lambda} 2|x_j| + \sum_{j\notin\Lambda} |x_j| \le c, \tag{17}$$

where Λ denotes the set of positions in which the short position for the spread return j is risky, and c is the maximum permitted total value of risky long and short positions per dollar of the investor's wealth. For example, $c = 2$ corresponds to a margin requirement of 50% and $c = 10$ corresponds to a margin requirement of 10%. When $c = \infty$, there is no margin requirement and the investor will invest in the capital market line with the maximum Sharpe ratio that she can obtain.

Another reason for using constraint (17) is that it has a good statistical property when controlling for estimation risk (Fan, Zhang, and Yu, 2012). For simplicity, suppose the short position of any spread return j in constraint (17) is a risk-free asset. Then, (17) reduces to

$$\sum_{j=1}^{n} |x_j| = \|x\|_1 \leq c. \tag{18}$$

Given the true mean μ and and covariance matrix V, the utility loss of the optimal portfolio $\tilde{x}$ from using the predictive $\tilde{\mu}$ and covariance $\tilde{V}$ has an upper bound as:

$$\begin{aligned} \left|(\tilde{x}'\tilde{\mu} - \frac{\gamma}{2}\tilde{x}'\tilde{V}\tilde{x}) - (\tilde{x}'\mu - \frac{\gamma}{2}\tilde{x}'V\tilde{x})\right| &\leq \left|\tilde{x}'\tilde{\mu} - \tilde{x}'\mu\right| + \frac{\gamma}{2}\left|\tilde{x}'\tilde{V}\tilde{x} - \tilde{x}'V\tilde{x}\right| \\ &\leq \|\tilde{\mu} - \mu\|_\infty \|\tilde{x}\|_1 + \frac{\gamma}{2}\|\tilde{V} - V\|_\infty \|\tilde{x}\|_1^2, \end{aligned} \tag{19}$$

where $\|\tilde{x}\|_1$ is the L_1 norm of vector $\tilde{x}$, and $\|\hat{\mu} - \mu\|_\infty$ and $\|\hat{V} - V\|_\infty$ are the maximum component-wise estimation errors.[7] Therefore, if $\|\tilde{x}\|_1$ is bounded above (economically, it is a margin requirement), the utility loss resulting from estimation errors is controlled by the largest component-wise errors of $\|\tilde{\mu} - \mu\|_\infty$ and $\|\tilde{V} - V\|_\infty$. As long as each element is estimated well, the overall utility is approximated well without the accumulation of estimation errors.

3.2 Anomalies that are considered for investing

We consider the anomalies in Novy-Marx and Velikov (2016) as the non-benchmark risky assets. After excluding those that are insignificant with FF5, 15 are left, including return-on-book equity, value-momentum combo, idiosyncratic volatility, momentum, size, accruals, net issuance, investment, gross margins, value-momentum-profitability combo, industry momentum, industry relative reversals, high-frequency combo, seasonality, and industry low volatility. Among these 15 anomalies, the first five are explained by HXZ but not FF5, and the remaining 10 are not explained by either HXZ and FF5.

[7] If x is an N-dimensional vector, $\|x\|_1 = \sum_{j=1}^{N} |x_j|$ and $\|x\|_\infty = \max_j |x_j|$. If X is an $M \times N$-dimensional matrix, $\|X\|_\infty = \max_i \sum_{j=1}^{N} |x_{i,j}|$, where $x_{i,j}$ is the element in row i and column j of X.

Table 3 reports the summary statistics of the anomalies. Panel A consists of those unexplained by FF5 but HXZ. The average alpha is 0.13% ($t = 0.63$) with HXZ and 0.67% ($t = 3.45$) with FF5. An interesting observation is that the average R^2 statistics of these two models are 50% and 53%, respectively, implying that although HXZ outperforms FF5 in terms of alpha, there are half variations in the average anomaly returns unexplained by both of them. Panel B consists of the 10 anomalies unexplained by either HXZ or FF5. The average alphas are 0.72% ($t = 4.11$) and 0.75% ($t = 4.44$), respectively. The average regression R^2s of the two models are 18% and 22%, suggesting that there are more than three quarters of variations in the average anomaly returns left unexplained by both models. Compared with Table 1, the high average annualized Sharpe ratio, 0.71, is slightly smaller than that of the investment and profitability factors of HXZ (0.82 and 0.75), but it is larger than any other factors.

3.3 Predictive means and standard deviations

According to (6) and (7), imposing an asset pricing model on the return-generating process with a confidence of ω has a first-order effect on the predictive mean and a second-order effect on the predictive variance. Hence, we examine the predictive means and standard deviations of risky assets before analyzing the portfolio decisions. If there are no substantial differences in these parameter estimates, it is unlikely that there are dramatic differences in portfolio allocations.

Panels A and B of Table 4 report the predictive means. It is apparent that by varying the confidence ω and asset pricing model, the predictive mean of each asset is dramatically changed. For example, the predictive mean of the return-on-book equity anomaly return is 0.71% per month if the investor is agnostic about HXZ or FF5 by setting $\omega = 0$ (the sample mean in this case). In contrast, if the investor believes dogmatically in one of them ($\omega = 1$), the predictive mean is 0.70% with HXZ and 0.25% FF5. The dramatic estimation bias from using FF5 is due to the fact that FF5 cannot explain the return-on-book equity anomaly. When one imposes a constraint by setting the alpha equal to zero when estimating the predictive mean, the estimate is dramatically biased. Instead, since HXZ is able to describe the anomaly with an insignificant alpha, the model is likely to capture the true mean of the return-generating process and the potential estimation bias is negligible, as the imposed constraint is nearly slack. This argument is supported by Panel B of Table 4. Since each anomaly in this panel cannot be explained by the two models, the estimates with them for the predictive mean dramatically deviate from the unbiased estimate, the sample mean by setting $\omega = 0$. For instance, the unbiased predictive mean of the accrual anomaly is 0.26% per month, whereas it is -0.07%

when using HXZ and -0.02% when using FF5.

Panels C and D of Table 4 report the predictive standard deviations. In contrast with the results in Panels A and B, the difference in estimates between the two models is relatively small. This result is consistent with Chan, Karceski, and Lakonishok (1999) who show that various models for forecasting covariances generally perform quite similarly. In Panel C, although each anomaly can be explained by HXZ but not FF5, the estimates of the predictive standard deviations with the two models are virtually the same. When $\omega = 1$, the biggest difference in the estimated predictive standard deviations between the two models is 0.09% in the momentum anomaly, and amounts to only 1% of the estimate when $\omega = 0$ (the case with an unbiased predictive variance estimate). In Panel D, when $\omega = 1$, the biggest difference is 0.08% in the ROE factor, which amounts to approximately 3% of the estimate of $\omega = 0$.

An interesting observation in Panel D is that the differences between $\omega = 1$ and $\omega = 0$ for both models are generally larger than that in Panel C. For example, regarding the high-frequency combo anomaly, the predictive standard deviations with the two models are 4.04% and 4.05% when $\omega = 1$, and are both 3.77% when $\omega = 0$. The biases, 0.27% and 0.28%, amount to approximately 7% of 3.77%. We attribute this larger bias between $\omega = 1$ and $\omega = 0$ to the larger proportion of variations in the average anomaly returns that are left unexplained by the two factor models, as shown by the lower regression R^2s in Table 3. Therefore, when there is a significant mispricing error, imposing an asset pricing model leads to a large bias in the estimation of the predictive standard deviation. To some extent, our finding is consistent with MacKinlay and Pástor (2000) that when a risk factor is missing from an asset pricing model, the resulting mispricing is embedded within the residual covariance matrix.

3.4 In-sample comparison

Table 5 reports optimal allocations per \$100 of wealth when prior beliefs are centered on either of the two asset pricing models, with varying degrees of confidence ω. The risky assets include the five anomalies that can be explained by HXZ but not FF5 (see details in Panel A of Table 3), five factors in FF5, and the investment and profitability factors of HXZ. Hence, the investment universe consists of 12 risky assets and one risk-free asset. When the investor employs the HXZ model, the non-benchmark assets are the five anomalies plus the HML, CMA, and RMW factors in FF5. Similarly, when FF5 is the asset pricing model, the non-benchmark assets are the five anomalies plus the HXZ I/A and ROE factors.

For a given c, let $\tilde{x}$ be the optimal portfolio under the predictive mean $\tilde{\mu}$ and covariance $\tilde{V}$. We compute the in-sample expected utility or certainty-equivalent return (CER) as:

$$\text{CER} = \tilde{x}'\tilde{\mu} - \frac{\gamma}{2}\tilde{x}'\tilde{V}\tilde{x}. \tag{20}$$

Moreover, we calculate the Sharpe ratio as:

$$\text{Sharpe ratio} = \frac{\tilde{\mathrm{x}}'\tilde{\mu}}{\sqrt{\tilde{\mathrm{x}}'\tilde{\mathrm{V}}\tilde{\mathrm{x}}}}. \tag{21}$$

The investing problem (15) treats the risky assets on an individual basis, ignoring the fact that they are constructed as portfolios of individual stocks, and a given stock can appear in a non-benchmark portfolio and in each of the benchmark factors. For this reason, one can argue that the returns on the risky assets are correlated; large differences in position-by-position allocations need not necessarily produce economically significant differences in the overall portfolio characteristics. As a result, the CER and SR are more sensible measures for model comparison.

In addition to portfolio weights, the last two rows of each panel in Table 5 report the in-sample CER and Sharpe ratio. To facilitate understanding, we multiply the monthly CER by 1,200 to express it as percent per year and multiply the monthly Sharpe ratio by $\sqrt{12}$ for an annual value. As confidence decreases, the optimal portfolio converges to the portfolio based on the sample mean and covariance matrix of anomaly returns (the case of $\omega = 0$), regardless of the asset pricing model. The aim here is to explore the extent to which this behavior occurs at interesting confidence levels of ω. The results in Table 5 are reported for $\omega = 0.75$ and 0.5 as well as the limiting cases $\omega = 1$ (exact pricing) and $\omega = 0$ (no use of a pricing model). We consider two levels of margin requirements, $c = \infty$ and 2, which correspond to without a margin requirement and with a 50% margin requirement.

Panel A of Table 5 is the case in which there is no margin requirement, i.e., $c = \infty$. When $\omega = 1$, each asset pricing model estimates the parameters of the non-benchmark assets with zero alphas, the portfolio weights for these assets are zero. As a result, the investor allocates her wealth among the factors of the model that she employs. However, when the investor does not have a dogmatic belief but places a confidence of $\omega = 0.75$, she will allocate across all of the risky assets, regardless of the asset pricing model. An interesting result with this panel is that, while the portfolio is more diversified when the investor's confidence level decreases, the CER and Sharpe ratio do not improve substantially. Instead, the allocations on the non-benchmark assets are generally much smaller than the investments in the factors. In terms of the alternative interpretation of the margin requirements in (19), the small portfolio weights on the non-benchmark assets are partially due to estimation errors.

When the investor knows the mean and covariance, the CER is 42.6%. When she does not know the mean and covariance and assumes a factor structure to estimate, the CER from using HXZ is at least 40.7%, close to 42.6%. In contrast, the CER from using FF5 is at most 31.0%, far less than 42.6%. The performance difference is mainly due to the fact that HXZ can price all the non-benchmark assets and imposing a factor structure does not make the estimation biased. Instead, FF5 cannot price all the non-benchmark assets and imposing a factor structure makes the estimation biased. As a result, HXZ outpreforms FF5.

Panel B reports the results of $c = 2$, which is a constraint with a 50% margin requirement. In this case, the CER reduces dramatically. Even if the investor knows the true mean and covariance ($\omega = 0$), the CER is only 9.5%, much smaller than the counterpart 42.6% in Panel A that does not impose a margin requirement. The first two columns of Panel B, with $\omega = 1$, display the allocations corresponding to the dogmatic beliefs in each of the two asset pricing models. In this case, each asset pricing model estimates the parameters of the non-benchmark assets by restricting alphas equal to zero. According to Table 3 and Hou, Xue, and Zhang (2019), ex post, HXZ explains the average returns of the five anomalies and the HML, CMA, and RMW factors. Hence, the zero alpha constraint when using HXZ is nearly slack and innocuous. With this tight margin requirement constraint, the optimal portfolio under the HXZ model includes four assets: return-on-book equity (19.8), value-momentum combo (37.3), momentum (12.4), and MKT (61.0), where only MKT is a benchmark asset. This result seems counterintuitive. HXZ can explain all the non-benchmark assets, so the investor should allocate all his investment among the HXZ factors. However, because of margin requirements, the investor cannot invest as much as Panel A in the HXZ factors. For this reason, she switches to non-benchmark assets for risk diversification. FF5 cannot explain the non-benchmark assets, so it is not surprising that the investor who imposes an FF5 factor structure invests in these non-benchmark assets. Because of model misspecification, FF5 underperforms HXZ in terms of CER. However, the underperformance is much less pronounced than the case without a margin requirement.

Table 6 reports the optimal portfolio choices, CERs, and Sharpe ratios when the investment universe of risky assets are the 10 anomalies, five FF5 factors, and HXZ's I/A and ROE factors. The key difference between Tables 5 and 6 is that each of the anomaly returns in Tables 6 cannot be explained by HXZ or FF5. From Table 3, both models have similar degrees of mispricing errors for the 10 anomalies. In Panel

A, when there is no margin requirement, HXZ underperforms the case when the investor knows the mean and covariance, but it still outperforms FF5. Hence, the model misspecification for imposing the HXZ factor structure is less severe than imposing the FF5 factor structure. However, when including a margin requirement in Panel B, the outperformance of HXZ become negligible.

As a mean-variance investor, if the distribution of asset returns is known, imposing an asset pricing model by setting alphas equal to zero can lead to biased estimates of the portfolio parameters, which gives rise to a CER loss. A better asset pricing model is the one that yields smaller biases in the predictive mean and covariance matrix. As a result, it should have a smaller CER loss. Suppose the true mean and covariance matrix are μ and V, and x_o is the resulting optimal portfolio for a given c. We calculate the CER as $\mathrm{CER}_o = x_o'\mu - \frac{\gamma}{2}x_o'Vx_o$. Then we calculate the CER of a suboptimal allocation x_s as $\mathrm{CER}_s = x_s'\mu - \frac{\gamma}{2}x_s'Vx_s$, where x_s is an allocation that is optimal for the same c and ω under the predictive distribution from imposing an asset pricing model. For example, if an asset pricing model is imposed, x_s is the optimal allocation from using the predictive mean and covariance matrix, $\tilde{\mu}$ and $\tilde{V}$, that are estimated according to (6) and (7). The difference $\mathrm{CER}_o - \mathrm{CER}_s$ provides an economic measure of CER loss from imposing a pricing constraint on the distribution of asset returns.

Figure 3 displays the annualized CER losses for an investor who believes in the sample mean and covariance matrix that are estimated without imposing any asset pricing model, but is forced to hold a portfolio chosen by another investor with a confidence ω in HXZ or FF5. The risky assets are the same as Table 5, including five anomalies that can be explained by HXZ but not FF5, five factors in FF5, and the investment and profitability factors in HXZ. For each of four values of c, the figure plots the CER loss versus confidence ω. Losses are calculated for portfolios from HXZ and from FF5. The goal is to explore whether HXZ is a better model for investing when it is a better model for pricing, as shown in Panel A of Table 3.

Figure 3 makes three statements. First, the CER loss increases monotonically with respect to the confidence level ω. When $\omega = 0$, the investor does not believe in the asset pricing model and the predictive mean and covariance matrix are the same as the sample mean and covariance matrix. In this case, there is no CER loss. When ω increases, the investor places a larger weight on the mean and covariance matrix that are estimated by setting the alphas of non-benchmark assets equal to zero. The resulting predictive mean and covariance matrix are more likely to be biased, and therefore, the CER loss is more likely to increase. Second, the CER loss increases with respect to c. As c increases, the constraint of margin requirements becomes less likely to be binding and the optimal portfolio is closer to the one suggested by the predictive mean and covariance matrix. Hence, the CER loss increases. Third and finally, HXZ outperforms FF5 when the investor has a dogmatic belief in HXZ and does not suffer from a margin requirement. In a more realistic setting, the investor does not have a dogmatic belief in HXZ and suffers from a margin requirement, HXZ performs similarly as FF5.

Figure 4 displays the annualized CER losses for the case in which 10 anomaly spread returns cannot be explained by either HXZ or FF5. As in Figure 3, the investor is assumed to believe in the sample mean and covariance matrix but is forced to hold the portfolio chosen by another investor with confidence ω in HXZ or FF5. The risky assets are those in Table 6. The goal here is to explore whether HXZ is a better model for investing even when it performs similarly to FF5 for pricing the anomalies in Panel B of Table 3. Overall, the performance difference between the two models is similar as Figure 3.

Figure 5 displays precisely the same analysis except that the CER losses are computed for an investor who believes in HXZ with confidence ω but is forced to hold the portfolio chosen by another investor with the same degree of confidence in FF5 (left two panels), and vice versa (right two panels). The upper two panels correspond to the risky assets in Figure 3 and the lower two panels correspond to the risky assets in Figure 4. When $\omega = 1$ and $c = 2$, the CER loss for the investor who believes in one model but is forced to use another model is always less than 2% per year. When $c = 10$, the CER losses for the investor who believes HXZ but is forced to use FF5 are more than 7% per year, which is an economically large magnitude, regardless of the risky assets. Similarly, the CER losses for the investor who believes in FF5 but is forced to use HXZ are about 6% per year. When $c = \infty$, all of the CER losses are more than doubled, in comparison to $c = 10$.

3.5 Out-of-sample comparison

A model with better in-sample performance for investing does not necessarily mean it has better out-of-sample performance because of estimation errors. For example, DeMiguel, Garlappi, and Uppal (2009) report that the Sharpe ratio is 0.219 for the mean-variance model with assets MKT, SMB, and HML, whereas the Sharpe ratio is 0.096 with assets MKT, SMB, HML, momentum, 10 book-to-market portfolios, and 10 size portfolios. This example suggests that comparing models out-of-sample is important in that adding more assets could reduce portfolio performance if the estimation errors are not controlled. Kan and Wang

(2019) explicitly consider the out-of-sample utility loss using the sample mean and covariance matrix.[8]

3.5.1 Pseudo out-of-sample analysis

We follow Kozak, Nagel, and Santosh (2018) and perform a bootstrap simulation for out-of-sample comparison, which maintains the i.i.d property over time, an assumption made in the main framework. We randomly sample (with replacement) $T+300$ returns on the risky assets and use the first T to calculate the portfolio weights, which are used for the remaining 300 observations to calculate the out-of-sample CER (CER_{OS}),

$$\text{CER}_{\text{OS}} = \hat{\mu}_{\tilde{x}} - \frac{\gamma}{2}\hat{\sigma}^2_{\tilde{x}}, \tag{22}$$

where $\hat{\mu}_x$ and $\hat{\sigma}^2_{\tilde{x}}$ are the sample mean and variance of the 300 out-of-sample excess returns of portfolio $\tilde{x}$ that is based on the first T observations. We repeat the procedure 1,000 times and report the average CER_{OS}. As the sample size is important for out-of-sample performance, we consider five values of T: 60, 120, 240, 360, and 600.

Table 7 reports the annualized CER_{OS} for investing in the anomalies that can be explained by HXZ but not FF5. There are four observations. First, for given values of c and ω, the CER_{OS} increases as the sample size T increases. For example, when $c = 10$ and $\omega = 1$, the annualized CER_{OS} of HXZ is 18.1% when $T = 60$, and it increases monotonically to 26.4% when $T = 600$. Similarly, the annualized CER_{OS} of HXZ increases from 12.2% to 19.8% in this case. Second, when $c = 2$, since HXZ is a "correct" pricing model and explains all of the anomalies, its CER_{OS} is flat and does not change as ω decreases. This pattern holds true regardless of the sample size T. In contrast, since FF5 cannot explain the anomalies and its estimates for the predictive mean and covariance matrix are biased, the CER_{OS} increases in general when ω decreases.

Third, fixing ω, the CER_{OS} does not necessarily increase when the margin requirement is relaxed (increasing c). When $\omega = 1$ and $T = 60$, the annualized CER_{OS} values at $c = 2, 10$, and ∞ are 6.3%, 18.1%, and 14.8% for the HXZ model, and they are 4.6%, 12.2%, and −4.0% for the FF5 model. Finally, the outperformance of HXZ becomes smaller when estimation errors are larger (e.g., estimating the model with a small sample size), which is further confirmed in Table 8 if the investor looks at the annualized

[8] In the literature, there is a large number of papers that explain why the out-of-sample performance could be poor and how to improve it, such as MacKinlay and Pástor (2000), Jagannathan and Ma (2003), Siegel and Woodgate (2007), Kan and Zhou (2007), Garlappi, Uppal, and Wang (2007), DeMiguel et al. (2009), Tu and Zhou (2011), and DeMiguel et al. (2013), among others.

SR_{OS}s. This conclusion continues to be true when the non-benchmark assets are the anomalies that cannot be explained by either HXZ or FF5 (Tables 9 and 10).

3.5.2 Real-time out-of-sample analysis

The previous analysis assumes that the risky returns are i.i.d over time. In practice, however, this assumption does not hold and expected returns are varying over time,[9] which means that the expected returns are moving targets and can never be estimated accurately (Gârleanu and Pedersen, 2013). As such, the outperformance of the HXZ model in the previous subsection may not exist in real time.

We use an expanding window approach to compare their out-of-sample performances. With an initial window of 120 months, in each month t, we use data from month 1 to month t to compute the various portfolio rules, and apply them to determine the investments in the next month. For instance, let $\tilde{x}_t$ be the estimated optimal portfolio in month t and r_{t+1} be the excess return on the risky assets realized in month $t+1$. The realized excess return on the portfolio is $r_{\tilde{x},t+1} = \tilde{x}_t' r_{t+1}$. We then compute the average value of the realized returns, $\hat{\mu}_{\tilde{x}}$, and the variance, $\hat{\sigma}^2_{\tilde{x}}$. The out-of-sample CER can be calculated accordingly.

Figure 6 plots the CER_{OS}s for investing in anomalies that can be explained by HXZ but not FF5. As shown in Table 7, when ω is high, the difference in CER_{OS}s between HXZ and FF5 is economically significant. For example, when $\omega = 1$, the CER_{OS} values for the two models are 22.2% and 16.3% at $c = 10$, and 28.3% and 20.6% at $c = 20$, respectively. The difference in CER_{OS}s suggests that the HXZ outperforms the FF5 by 5.8% or 6.7% per year when the investor is subject to a 10% or 5% margin requirement. However, when ω is low, the investor faces more model uncertainty, HXZ performs similarly as FF5. In the case of no margin requirement, it could even underperforms FF5. Figure 7 plots the CER_{OS} values for investing in the anomalies that cannot be explained by either HXZ or FF5, and delivers a similar pattern as Figure 6.

4 Source of Difference between HXZ and FF5

In the mean-variance framework, the only two parameters are the predictive mean and covariance matrix of risky assets. The better investing performance of HXZ, if there is any, must come from its better ability to capture the mean, the covariance matrix, or both.

[9] We keep the normality assumption as it works well in evaluating portfolio performance in a mean-variance framework (Tu and Zhou, 2004).

This subsection considers the global-minimum-variance portfolio allocation. That is, the investor chooses portfolio x to minimize $x'\tilde{V}x$ with the margin requirement (17), where $\tilde{V}$ is the predictive covariance matrix and is given in (7). Mathematically, this is an extreme case of (15) with $\gamma = \infty$. The goal here is to show that whether the estimates of $\tilde{V}$ with HXZ and FF5 are different enough to yield different portfolios.

Figures 8 and 9 plot the in-sample annualized Sharpe ratio losses from the perspective of a global-minimum-variance investor, who knows the true mean and covariance matrix but is forced to hold the global-minimum-variance portfolio chosen by another investor who places a confidence of ω in HXZ or FF5, where the sample mean and covariance matrix are assumed to be the true mean and covariance matrix. In addition to the five factors in FF5 and investment and profitability factors in HXZ, the risky assets also include five anomalies that can be explained by the HXZ but not the FF5 model (Figure 8), or 10 anomalies that cannot be explained by the HXZ or the FF5 (Figure 9).

A striking pattern in Figure 8 is that the Sharpe ratio loss is negligible and is always less than 0.1 for both models, regardless of c and ω. In fact, the Sharpe ratio losses are virtually the same when c exceeds 10. This suggests that imposing one of the two asset pricing models does not lead to a significant bias for estimating the risky assets' covariance matrix. In Figure 9, the Shapre ratio loss is large; it is at least 0.2 for both models. However, the losses for the HXZ and FF5 are similar for given c and ω, suggesting that the predictive means and covariance matrix with the two pricing models are biased and similar. The evidence in these two figures is similar to Chan, Karceski, and Lakonishok (1999), who focus on individual stocks and find that the Fama-French three-factor model performs the same as a nine-factor model under the global-minimum-variance criterion. With this exercise, one can conclude that HXZ does not outperform FF5 in describing the covariance matrix of asset returns.

5 Conclusion

There are two schools of thought in empirical asset pricing. One school tries to identify new factors that generate abnormal returns relative to commonly used asset pricing models, and the other school attempts to choose a couple of factors from existing ones to construct a new asset pricing model. Both schools have been successful in the past four decades. The former has identified more than 300 factors (see, e.g., Harvey, Liu, and Zhu, 2016), and the latter has proposed the HXZ, FF5, Stambaugh and Yuan (2017) mispricing-factor model, Daniel, Hirshleifer, Sun (2020) behavioral-factor model, etc. When introducing a new model, it is typically examined for its superior pricing capabilities compared to current ones, encapsulated by the saying "it takes a model to beat a model."

Instead of exploring the pricing ability, in this paper we are interested in the investing implications of the two representative models, HXZ and FF5. We argue that although HXZ has better pricing power, documented in Hou, Xue, and Zhang (2019), its suggested strategies ignore margin requirements and model uncertainty and, therefore, are hard to implement in practice. We show that if an investor is subject to margin requirements and model uncertainty, she would not benefit much from using HXZ, relative to FF5. That is, the difference between the two models for investing is not economically significant, if there is any.

There are several limitations or directions for future research. It is interesting to examine how transaction costs affect the investing performance in our framework, in a similar spirit of Detzel, Novy-Marx, and Velikov (2023) and Li, DeMiguel, and Martín-Utrera (2023). This paper assumes that asset returns are i.i.d over time. It is of interest to relax this assumption and explore how conditional information affects the investing performance of the two models. While the investment framework does incorporate the investor's varying beliefs in the asset pricing models, it does not address the linkage between the priors and the economic objectives, which can significantly improve the investing performance (Tu and Zhou, 2010).

References

Almazan, A., Brown, K.C., Carlson, M., Chapman, D.A., 2004. Why constrain your mutual fund manager? Journal of Financial Economics 73, 289–321.

Ang, A., 2014. Asset Management: A Systematic Approach to Factor Investing. Oxford University Press; 1st ed.

Barillas, F., Shanken, J., 2017. Which alpha? Review of Financial Studies 30, 1316–1338.

Barillas, F., Shanken, J., 2018. Comparing asset pricing models. Journal of Finance 73, 715–754.

Brandt, M.W., 2010. Portfolio choice problems. In: Ait-Sahalia, Y., Hansen, L.P. (Ed.), Handbook of Financial Econometrics, Volume I: Tools and Techniques. Amsterdam, Netherlands: North-Holland, pp. 269–336.

Briner, B.G., Connor, G., 2008. How much structure is best? A comparison of market model, factor model and unstructured equity covariance matrices. Journal of Risk 10, 3–30.

Chan, L.K.C., Karceski, J., Lakonishok, J., 1998. The risk and return from factors. Journal of Financial and Quantitative Analysis 33, 159–188.

Chan, L.K.C., Karceski, J., Lakonishok, J., 1999. On portfolio optimization: Forecasting covariances and choosing the risk model. Review of Financial Studies 12, 937–974.

Connor, G., Goldber, L.R., Korajczyk, R.A., 2010. Portfolio Risk Analysis. Princeton, NJ: Princeton University Press.

Constantinides, G.M., 1980. Admissible uncertainty in the intertemporal asset pricing model. Journal of Financial Economics 8, 71–86.

Daniel, K., Titman, S., 1997. Evidence on the characteristics of cross sectional variation in stock returns. Journal of Finance 52, 1–33.

DeMiguel, V., Plyakha, Y., Uppal, R., Vilkov, G., 2013. Improving portfolio selection using option-implied volatility and skewness. Journal of Financial and Quantitative Analysis 48, 1813–1845.

DeMiguel, V., Garlappi, L., Nogales, F.J., Uppal, R., 2009. A generalized approach to portfolio optimization: Improving performance by constraining portfolio norms. Management Science 55, 798–812.

DeMiguel, V., Garlappi, L., Uppal, R., 2009. Optimal versus naive diversification: How inefficient is the $1/N$ portfolio strategy? Review of Financial Studies 22, 1915–53.

Detzel, A.L., Novy-Marx, R., Velikov, M., 2023. Model comparison with transaction costs. Journal of Finance 78, 1743–1775.

Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33, 3–56.

Fama, E.F., French, K.R., 1996. Multifactor explanations of asset pricing anomalies. Journal of Finance 51, 55–84.

Fama, E.F., French, K.R., 2015a. A five-factor pricing model. Journal of Financial Economics 116, 1–22.

Fama, E.F., French, K.R., 2015b. Incremental variables and the investment opportunity set. Journal of Financial Economics 117, 470–488.

Fama, E.F., French, K.R., 2016. Dissecting anomalies with a five-factor model. Review of Financial Studies 29, 69–103.

Fan, J., Zhang, J., Yu, K., 2012. Vast portfolio selection with gross-exposure constraints. Journal of American Statistical Association 107, 592–606.

Garlappi, L., Uppal, R., Wang, T., 2007. Portfolio selection with parameter and model uncertainty: A multi-prior approach. Review of Financial Studies 20, 41–81.

Gârleanu, N., Pedersen, L.H., 2013. Dynamic trading with predictable returns and transaction costs. Journal of Finance 68, 2309–2340.

Hou, K., Xue, C., Zhang, L., 2019. Which factors? Review of Finance 23, 1–35.

Hou, K., Xue, C., Zhang, L., 2015. Digesting anomalies: An investment approach. Review of Financial Studies 28, 650–705.

Hou, K., Xue, C., Zhang, L., 2020. Replicating anomalies. Review of Financial Studies 33, 2019–2133.

Huberman, G., Kandel, S., 1987. Mean-variance spanning. Journal of Finance 42, 873–888.

Jagannathan, R., Ma, T., 2003. Risk reduction in large portfolios: Why imposing the wrong constraints helps. Journal of Finance 58, 1651–1684.

Kan, R., Zhou, G., 2007. Optimal portfolio choice with parameter uncertainty. Journal of Financial and Quantitative Analysis 42, 621–656.

Kan, R., Zhou, G., 2012. Tests of mean-variance spanning. Annals of Economics and Finance 13, 145–193.

Kan, R., Wang, X., 2019. On the economic value of alphas. Available at SSRN 1785161.

Kan, R., Wang, X., Zheng, X., 2022. In-sample and out-of-sample Sharpe Ratios of multi-factor asset pricing models. Available at SSRN 3454628.

Kozak, S., Nagel, S., Santosh, S., 2018. Interpreting factor models. Journal of Finance 73, 1183–1223.

Levy, M., Roll, R., 2016. Seeking alpha? It's a bad guideline for portfolio optimization. Journal of Portfolio Management 42, 107–112.

Lewellen, J., 2023. How many factors? Working paper, Dartmouth College.

Li, S., DeMiguel, V., Martín-Utrera, A., 2023. Comparing factor models with price-impact costs. Available at SSRN 3688484.

MacKinlay, A.C., 1995. Multifactor models do not explain deviations from the CAPM. Journal of Financial Economics 38, 3–28.

MacKinlay, A.C., Pástor, Ľ, 2000. Asset pricing models: Implications for expected returns and portfolio selection. Review of Financial Studies 13, 883–916.

Novy-Marx, R., Velikov, M., 2016. A taxonomy of anomalies and their trading costs. Review of Financial Studies 29, 104–147.

Pástor, Ľ., Stambaugh, R.F., 2000. Comparing asset pricing models: An investment perspective. Journal of Financial Economics 56, 335–381.

Stambaugh, R.F., Yuan, Y., 2017. Mispricing factors. Review of Financial Studies 30, 1270–1315.

Tian, M., 2021. Firm characteristics and empirical factor models: A data-mining experiment. Review of Financial Studies 34, 6087–6125.

Tu, J., Zhou, G., 2004. Data-generating process uncertainty: What difference does it make in portfolio decisions? Journal of Financial Economics 72, 385–421.

Tu, J., Zhou, G., 2010. Incorporating economic objectives into Bayesian priors: Portfolio choice under parameter uncertainty. Journal of Financial and Quantitative Analysis 45, 959–986.

Tu, J., Zhou, G., 2011. Markowitz meets Talmud: A combination of sophisticated and naive diversification strategies. Journal of Financial Economics 99, 204–215.

Siegel, A.F., Woodgate, A., 2007. Performance of portfolios optimized with estimation errors. Management Science 53, 1005–1015.

Wang, Z., 2005. A shrinkage approach to model uncertainty and asset allocation. Review of Financial Studies 18, 673–705.

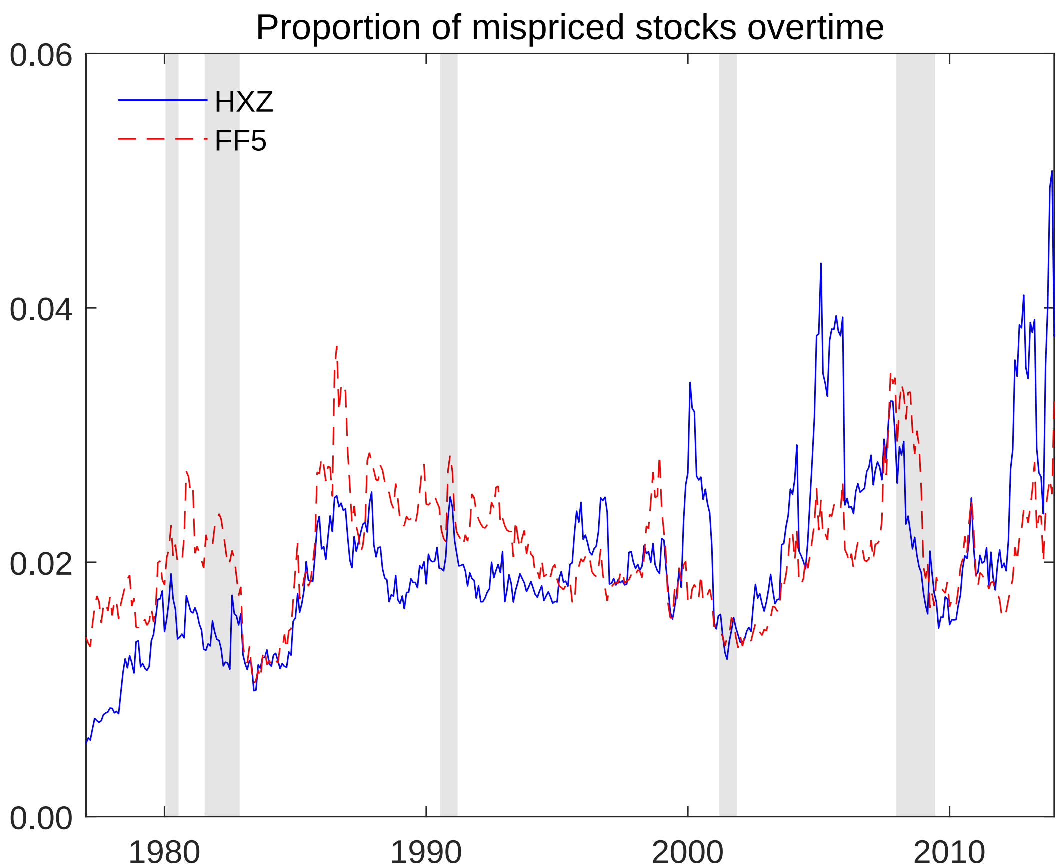

Figure 1 Proportion of mispriced stocks over time
This figure plots the proportion of firms with significant alpha (at the 5% level) for each factor model over time. The alpha of each firm is calculated with a rolling window of 60 months, with a requirement of at least 36 observations. Gray bars indicate NBER recessions. The sample period is 1972:01–2013:12.

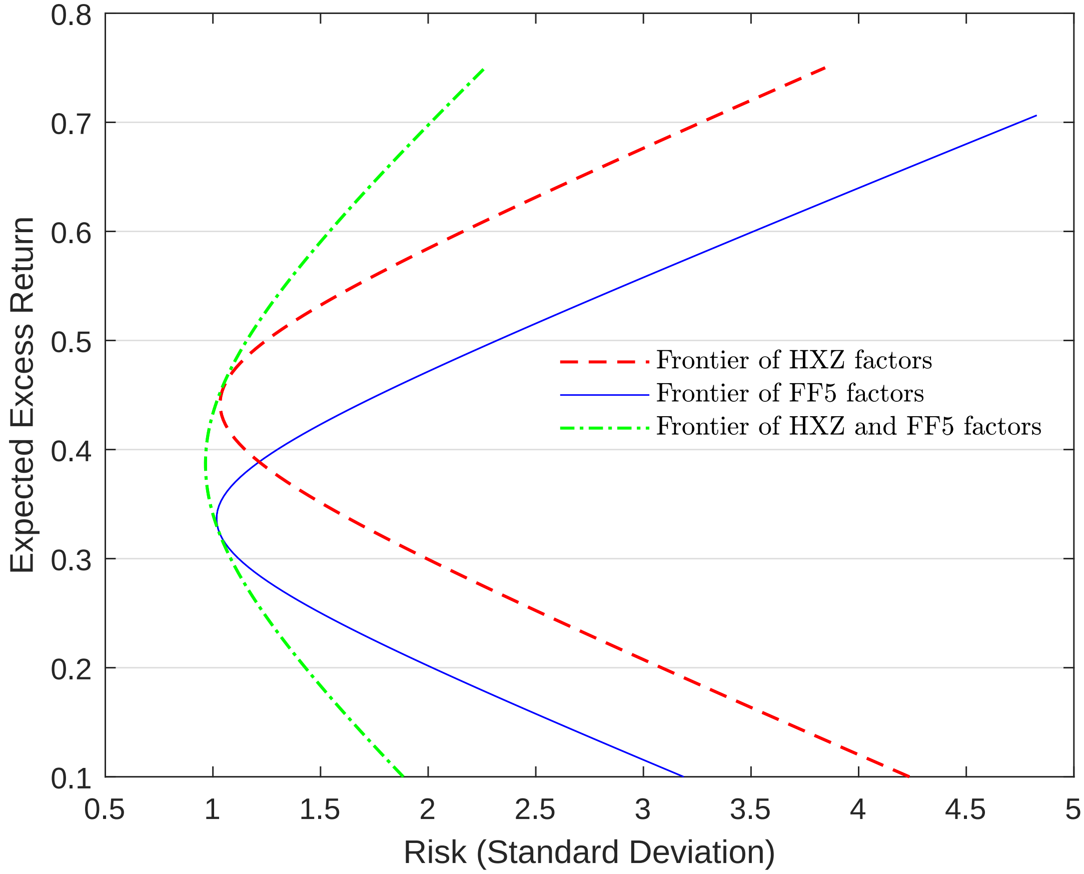

Figure 2 In-sample mean-variance frontiers
This figure plots the mean-variance frontiers (in % per month) for investing in the HXZ four factors, the FF5 five factors, or the joint factors of HXZ and FF5.

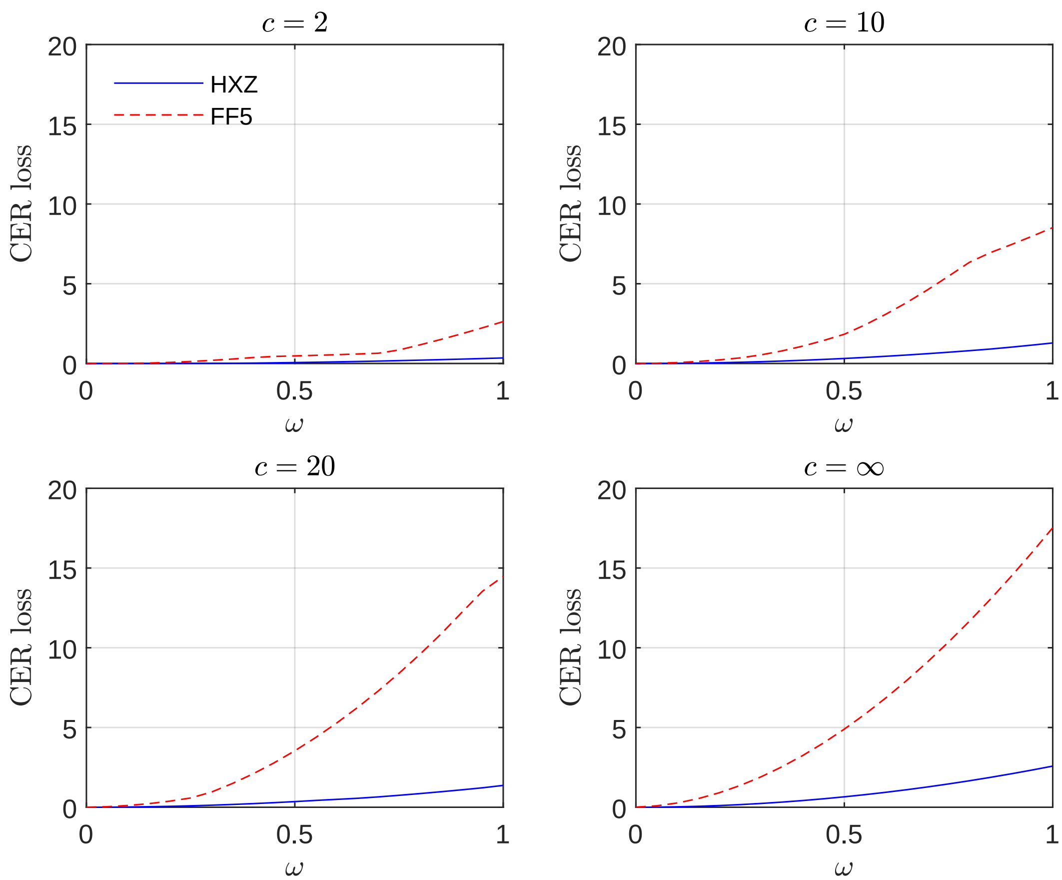

Figure 3 In-sample certainty-equivalent return loss
This figure plots the in-sample certainty-equivalent return (CER, in % per year) loss from the perspective of a mean-variance investor, who knows the true mean and covariance but is forced to hold the portfolio chosen by another investor who places a confidence ω in HXZ or FF5. The portfolio consists of five anomalies that can be explained by HXZ but FF5, five factors in FF5, and investment and profitability factors in HXZ. The sample mean and covariance are assumed to be the true mean and covariance. c is the maximum value of risky positions that can be established per dollar of wealth. The investor's risk aversion is set to 3.

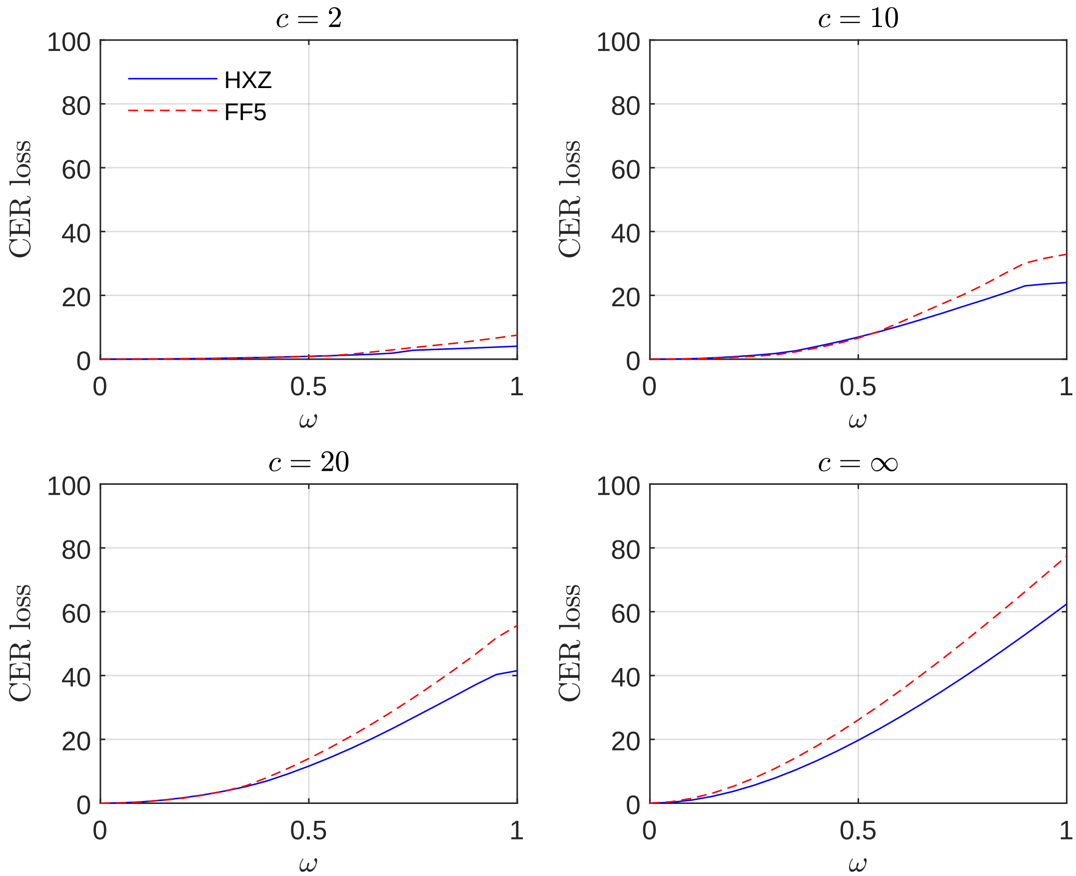

Figure 4 In-sample certainty-equivalent return loss
This figure plots the in-sample certainty-equivalent return (CER, in % per year) loss from the perspective of a mean-variance investor, who knows the true mean and covariance but is forced to hold the portfolio chosen by another investor who places a confidence ω in HXZ or FF5. The portfolio consists of 10 anomalies that cannot be explained by either HXZ or FF5, five factors in FF5, and investment and profitability factors in HXZ. c is the maximum value of risky positions that can be established per dollar of wealth. The sample mean and covariance are assumed to be the true mean and covariance. The investor's risk aversion is set to 3.

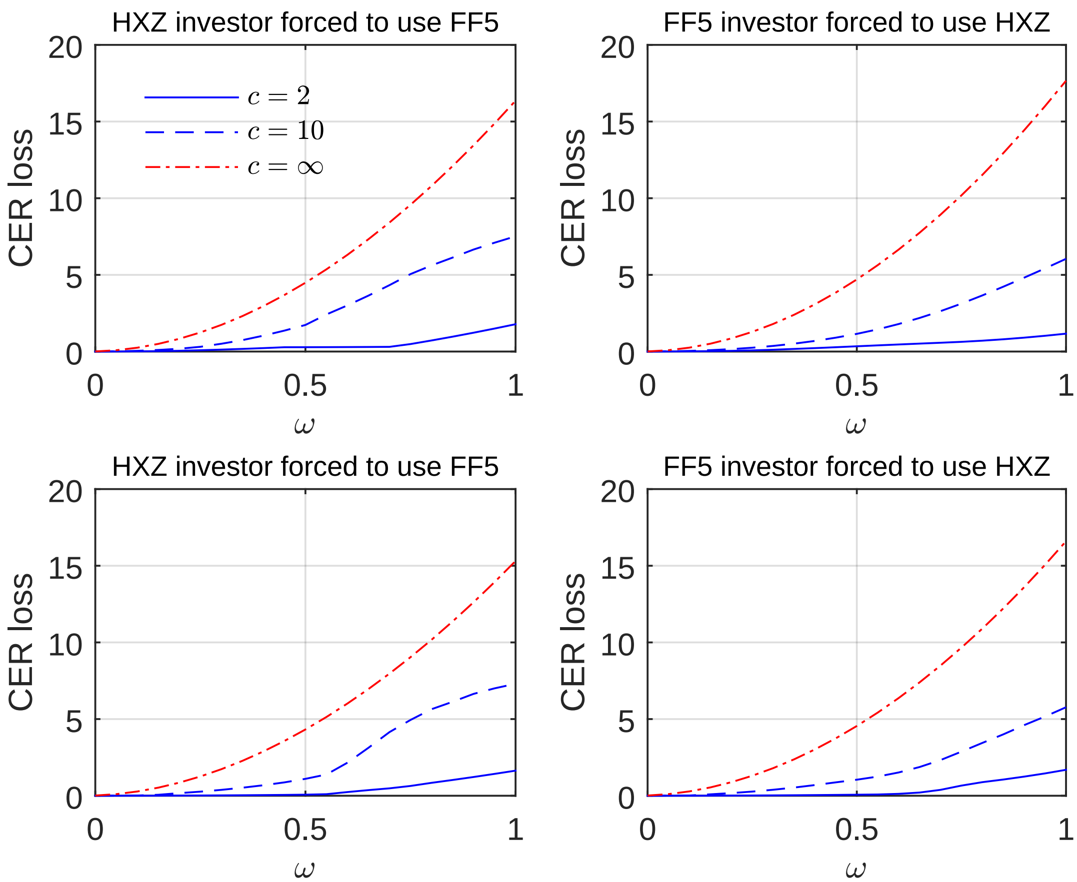

Figure 5 In-sample certainty-equivalent return loss
This figure plots the in-sample certainty-equivalent return (CER, in % per year) loss from the perspective of a mean-variance investor, who believes in HXZ with a confidence ω but is forced to hold the portfolio chosen by another investor with the same degree of confidence in FF5 (left two panels), and vice versa (right two panels). In addition to the five factors in FF5 and the investment and profitability factors in HXZ, the risky assets are five anomalies that can be explained by HXZ but not FF5 (upper two panels), or 10 anomalies that cannot be explained by either HXZ or FF5. c is the maximum value of risky positions that can be established per dollar of wealth. The investor's risk aversion is set to 3.

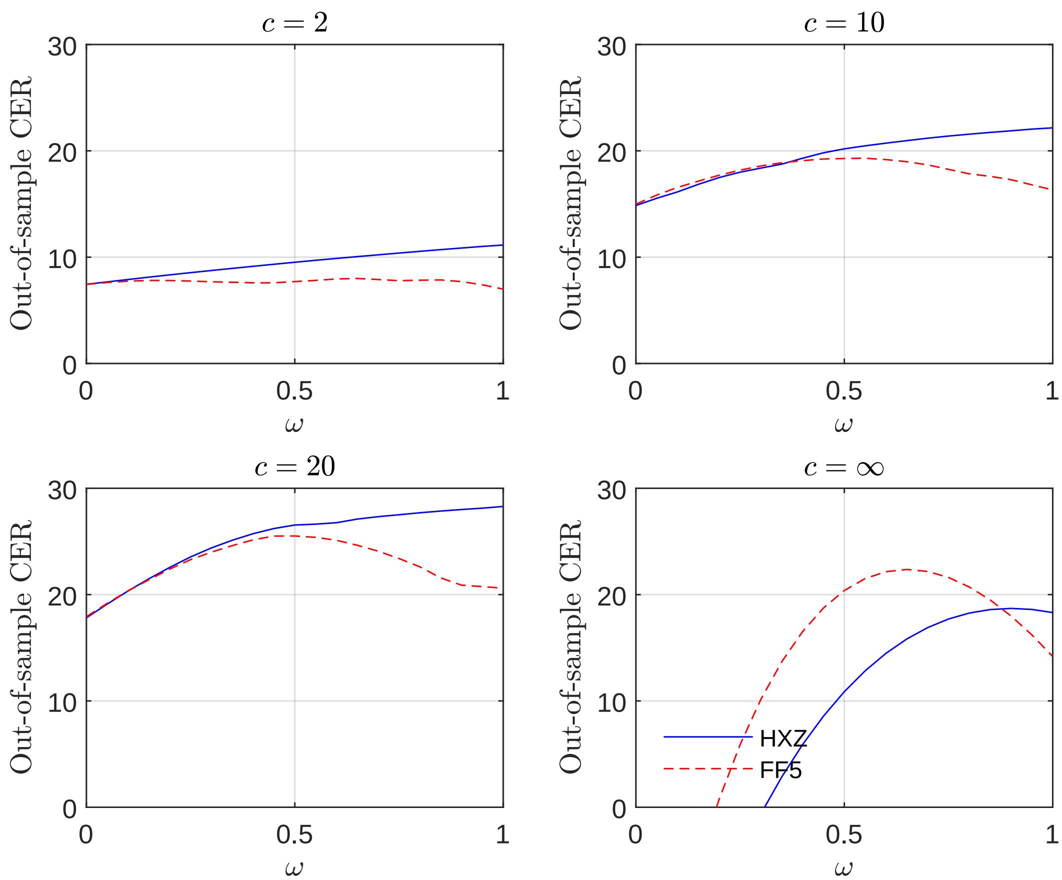

Figure 6 Out-of-sample certainty-equivalent return
This figure plots the out-of-sample certainty-equivalent return (CER, in % per year) for a mean-variance investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in HXZ or FF5. The risky assets include five anomalies that can be explained by HXZ but not FF5, five factors in FF5, and investment and profitability factors in HXZ. We use an expanding window approach in calculating the out-of-sample CER, where the initial window is 120 months. In each month t, we use data from month 1 to month t to compute the various portfolio rules, and apply them to determine the investments in the next month. For instance, let $\tilde{x}_t$ be the estimated optimal portfolio in month t and r_{t+1} be the excess return on the risky assets realized in month $t+1$. The realized excess return on the portfolio is $r_{\tilde{x},t+1} = \tilde{x}_t' r_{t+1}$. We then compute the mean and variance of the realized returns as $\hat{\mu}_{\tilde{x}}$ and $\hat{\sigma}^2_{\tilde{x}}$. The out-of-sample CER is thus given by $\text{CER}_{\text{OS}} = \hat{\mu}_{\tilde{x}} - \gamma\hat{\sigma}^2_{\tilde{x}}/2$.

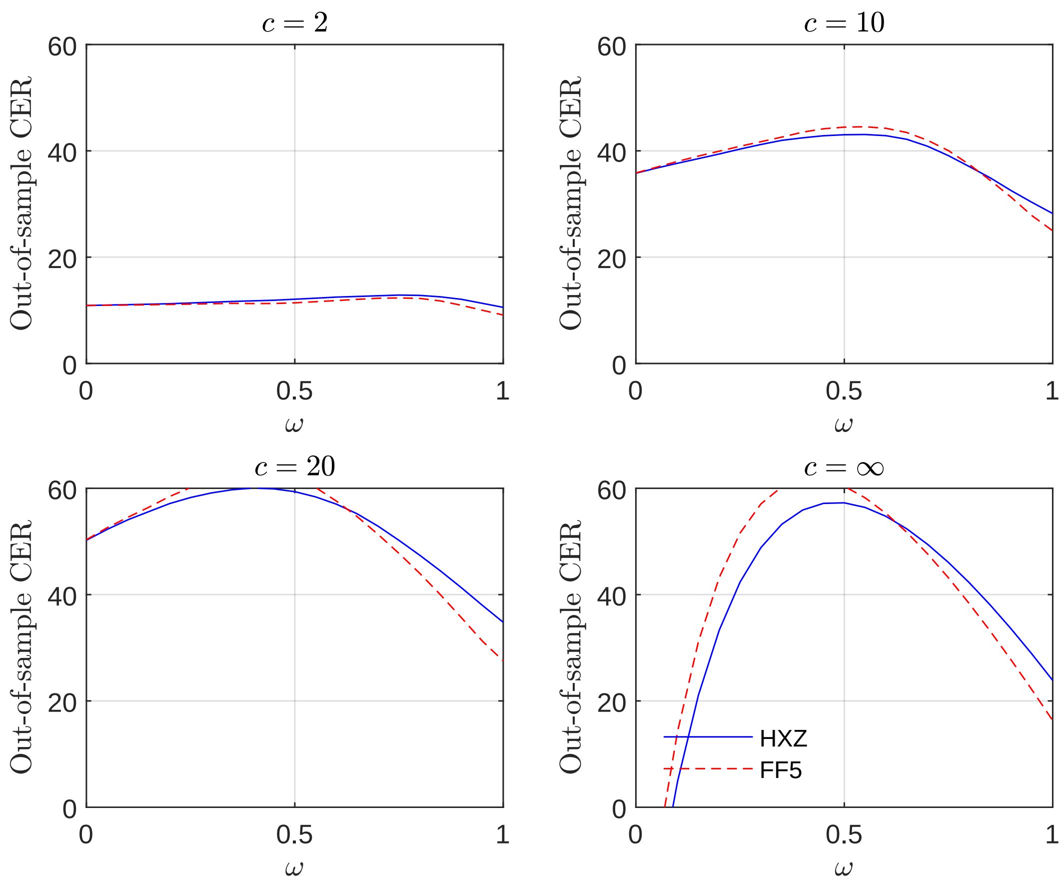

Figure 7 Out-of-sample certainty-equivalent return
This figure plots the out-of-sample certainty-equivalent return (CER, in % per year) for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in HXZ or FF5. The risky assets include 10 anomalies that cannot be explained by either HXZ or FF5, five factors in FF5, and investment and profitability factors in HXZ. We use an expanding window approach in calculating the out-of-sample CER, where the initial window is 120 months. In each month t, we use data from month 1 to month t to compute the various portfolio rules, and apply them to determine the investments in the next month. For instance, let $\tilde{x}_t$ be the estimated optimal portfolio in month t and r_{t+1} be the excess return on the risky assets realized in month $t+1$. The realized excess return on the portfolio is $r_{\tilde{x},t+1} = \tilde{x}_t' r_{t+1}$. We then compute the mean and variance of the realized returns as $\hat{\mu}_{\tilde{x}}$ and $\hat{\sigma}^2_{\tilde{x}}$. The out-of-sample CER is thus given by $\mathrm{CER}_{\mathrm{OS}} = \hat{\mu}_{\tilde{x}} - \gamma \hat{\sigma}^2_{\tilde{x}}/2$.

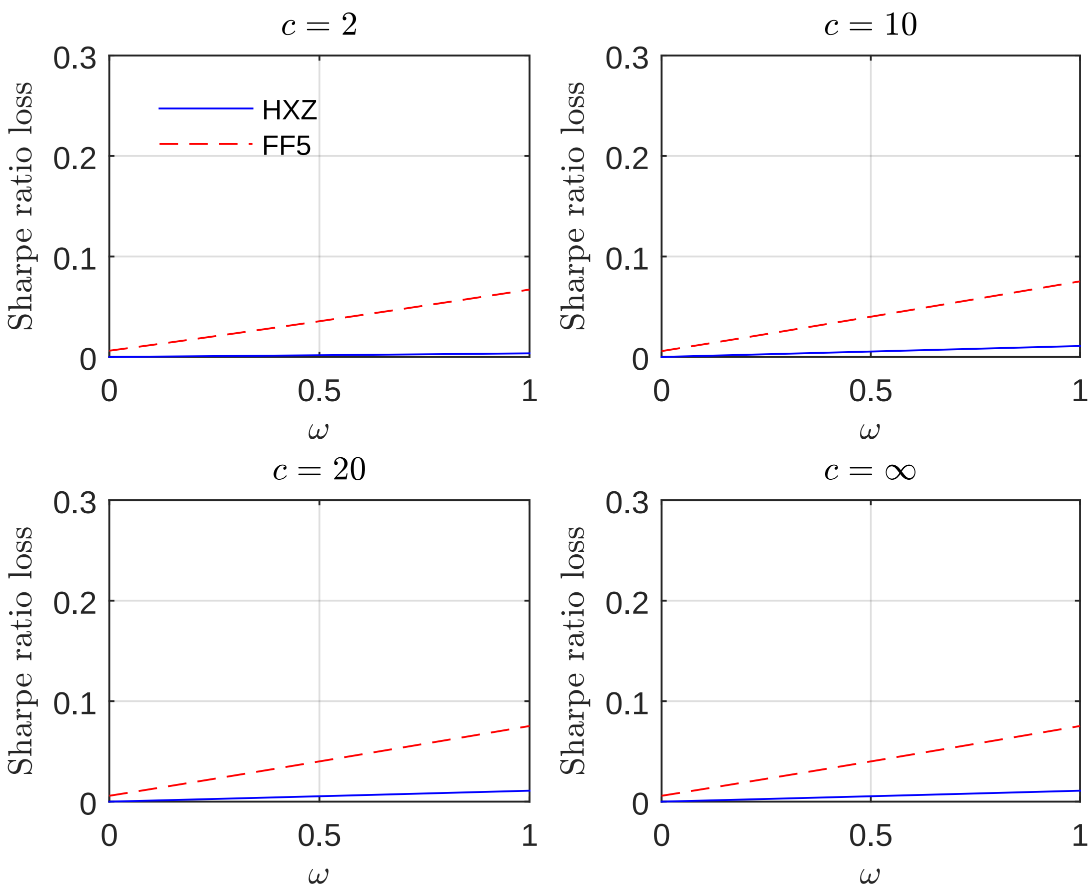

Figure 8 In-sample annualized Sharpe ratio loss
This figure plots the in-sample annualized Sharpe ratio loss from the perspective of a global-minimum-variance investor, who knows the true mean and covariance but is forced to hold the global-minimum-variance portfolio chosen by another investor who places a confidence ω in HXZ or FF5. The risky assets include five anomalies that can be explained by HXZ but not FF5, five factors in FF5, and investment and profitability factors in HXZ. c is the maximum value of risky positions that can be established per dollar of wealth. The sample mean and covariance are assumed to be the true mean and covariance.

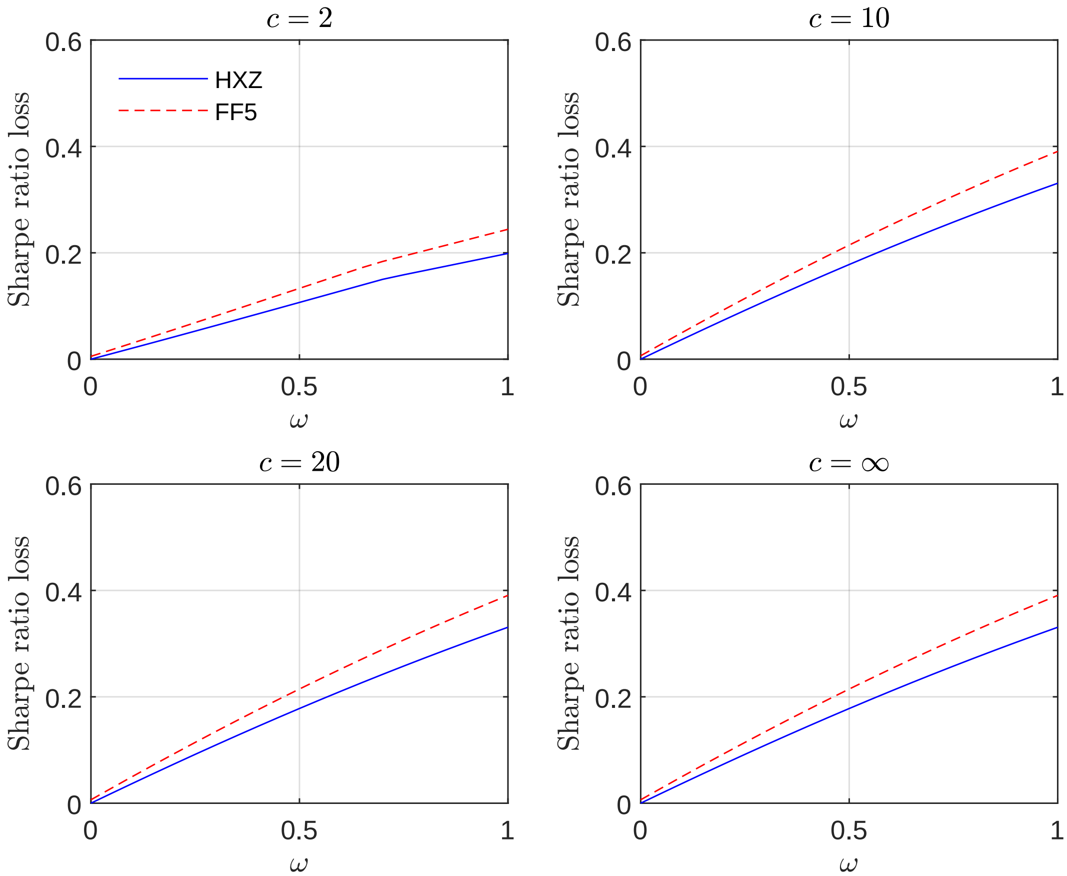

Figure 9 In-sample annualized Sharpe ratio loss
This figure plots the in-sample annualized Sharpe ratio loss from the perspective of a global-minimum-variance investor, who knows the true mean and covariance but is forced to hold global-minimum-variance portfolio chosen by another investor who places a confidence ω in HXZ or FF5. The risky assets include 10 anomalies that cannot be explained by either HXZ or FF5, five factors in FF5, and investment and profitability factors in HXZ. c is the maximum value of risky positions that can be established per dollar of wealth. The sample mean and covariance are assumed to be the true mean and covariance.

Table 1
Summary statistics of factors.

Panel A reports the summary statistics of asset pricing factors, where MKT^{HXZ}, ME, I/A, and ROE are the market, size, investment, and profitability factors in HXZ, and MKT, SMB, HML, CMA, and RMW are the market, size, value, investment, and profitability factors in FF5, respectively. t-stat is the t-statistic from the test that the average return of the factor is zero, and is calculated using White heteroskedasticity robust standard error. AC(1) represents the first-order autocorrelation. Annualized Sharpe ratio (SR) for each individual factor is calculated as the mean return divided by its standard deviation and multiplied by $\sqrt{12}$. Panel B reports the cross-sectional correlations of the factors.

Panel A: Summary statistics

	Mean	t-stat	Std	Skew	Kurt	AC(1)	SR
MKT^{HXZ}	0.51	2.51	4.61	−0.58	5.11	0.08	0.39
ME	0.31	2.20	3.14	0.64	9.02	0.03	0.34
I/A	0.44	5.33	1.87	0.12	4.91	0.06	0.82
ROE	0.57	4.85	2.62	−0.75	8.01	0.10	0.75
MKT	0.53	2.57	4.61	−0.55	5.00	0.08	0.40
SMB	0.23	1.71	3.08	0.43	7.48	0.03	0.26
HML	0.39	2.96	3.01	−0.03	5.54	0.14	0.46
CMA	0.37	4.25	2.00	0.35	4.74	0.12	0.66
RMW	0.29	2.87	2.25	−0.44	14.4	0.18	0.44

Panel B: Correlation matrix

	MKT^{HXZ}	ME	I/A	ROE	MKT	SMB	HML	CMA	RMW
MKT^{HXZ}	1.00	0.25	−0.37	−0.19	1.00	0.25	−0.32	−0.39	−0.24
ME		1.00	−0.12	−0.31	0.25	0.98	−0.07	−0.01	−0.38
I/A			1.00	0.06	−0.36	−0.15	0.69	0.90	0.10
ROE				1.00	−0.18	−0.38	−0.09	−0.09	0.67
MKT					1.00	0.25	−0.32	−0.39	−0.23
SMB						1.00	−0.11	−0.05	−0.39
HML							1.00	0.70	0.15
CMA								1.00	−0.03
RMW									1.00

Table 2
Mean-variance spanning tests

This table reports the results of testing whether factors in FF5 can be spanned by the HXZ four factors. W is the Wald test under conditional homoskedasticity, W_e is the Wald test under the i.i.d. elliptical distribution, W_a is the Wald test under the conditional heteroskedasticity, J_1 is the Bekerart-Urias test with the Errors-in-Variables (EIV) adjustment, J_2 is the Bekerart-Urias test without the EIV adjustment, and J_3 is the DeSantis test. All of the six tests have an asymptotic Chi-Squared distribution with $2m$ degrees of freedom, where m is the number of non-benchmark factors. As a comparison, the last column reports the GRS statistics of Gibbons, Ross, and Shanken (1989). The p-values are reported in the parentheses.

	W	W_e	W_a	J_1	J_2	J_3	GRS
FF5	224.6	103.2	126.5	75.7	73.3	173.6	0.62
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.69)
CMA	23.2	12.9	15.4	12.4	12.4	16.7	0.13
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.72)
RMW	97.2	41.1	49.9	22.1	21.9	37.9	0.23
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.63)

Table 3
Summary statistics of anomalies.

This table reports the summary statistics of anomaly long-short spread returns, which are from Novy-Marx and Velikov (2016) and used in this paper as non-benchmark risky assets in portfolio allocation. Sharpe ratios (SRs) are reported in annualized terms. α, t-stat, and R^2 are based on HXZ and FF5, respectively. Average represents the average absolute value of the statistics in the same column. Panel A includes anomalies of return-on-book equity (Return-on-book equity), Value-momentum combo, idiosyncratic volatility (Idiosyncratic volatility), momentum, and return-on-market equity (Size). Panel B includes anomalies of accruals, net issuance (rebal.:A), investment, gross margins, Value-momentum-profitability combo, industry momentum (Industry momentum), industry relative reversals (Industry relative reversals), high-frequency combo (High-frequency combo), seasonality, and industry low volatility (Industry low volatility).

		HXZ			FF5		
	SR	α	t-stat	R^2	α	t-stat	R^2
Panel A: Anomalies that can be explained by HXZ but not FF5							
Return-on-book equity	0.47	0.00	0.01	0.76	0.56	3.91	0.68
Value-momentum combo	0.61	0.03	0.14	0.29	0.46	2.32	0.31
Idiosyncratic volatility	0.29	0.19	1.02	0.73	0.47	2.71	0.77
Momentum	0.56	0.18	0.59	0.29	1.17	4.86	0.52
Size	0.73	0.27	1.39	0.45	0.69	3.43	0.37
Average	0.53	0.13	0.63	0.50	0.67	3.45	0.53
Panel B: Anomalies that cannot be explained by HXZ or FF5							
Accruals	0.28	0.38	2.80	0.23	0.31	2.27	0.19
Net issuance	0.88	0.45	3.53	0.25	0.41	3.55	0.35
Investment	0.61	0.30	2.31	0.27	0.33	2.79	0.33
Gross margins	0.02	0.34	2.69	0.34	0.33	2.78	0.38
Value-momentum-profitability combo	1.01	0.89	4.35	0.27	1.23	6.66	0.36
Industry momentum	0.45	0.74	2.59	0.05	0.95	3.47	0.08
Industry relative reversals	0.70	0.95	4.80	0.16	0.74	4.16	0.28
High-frequency combo	1.36	1.32	7.44	0.02	1.38	7.96	0.01
Seasonality	0.69	0.88	4.70	0.09	0.88	4.84	0.09
Industry low volatility	1.07	0.92	5.90	0.13	0.89	5.93	0.16
Average	0.71	0.72	4.11	0.18	0.75	4.44	0.22

Table 4
Predictive means and standard deviations of anomalies.

This table reports the predictive means (in percentage) and standard deviations of the anomaly long-short spread returns from Novy-Marx and Velikov (2016) and different benchmark factors in HXZ and FF5, respectively. ω is the confidence level the investor places in the HXZ or the FF5 model.

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.5$		$\omega = 0$
	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
Panel A: Predictive means of anomalies that can be explained by HXZ but not FF5							
Return-on-book equity	0.70	0.25	0.70	0.36	0.70	0.48	0.71
Value-momentum combo	0.87	0.45	0.87	0.55	0.87	0.66	0.88
Idiosyncratic volatility	0.41	0.19	0.46	0.30	0.51	0.41	0.62
Momentum	1.04	0.24	1.07	0.47	1.10	0.70	1.16
Size	0.86	0.46	0.92	0.63	0.98	0.79	1.11
HML	0.33	0.37	0.34	0.37	0.35	0.37	0.37
CMA	0.36	0.38	0.36	0.38	0.37	0.38	0.38
RMW	0.26	0.29	0.27	0.29	0.27	0.29	0.29
I/A	0.44	0.35	0.44	0.37	0.44	0.39	0.44
ROE	0.58	0.19	0.58	0.29	0.58	0.39	0.58
SMB	0.31	0.31	0.31	0.31	0.31	0.31	0.31
MKT	0.56	0.56	0.56	0.56	0.56	0.56	0.56
Panel B: Predictive means of anomalies that cannot be explained by HXZ or FF5							
Accruals	−0.07	−0.02	0.01	0.05	0.09	0.12	0.26
Net issuance	0.40	0.41	0.49	0.49	0.58	0.58	0.76
Investment	0.31	0.26	0.37	0.33	0.43	0.40	0.55
Gross margins	−0.27	−0.26	−0.20	−0.19	−0.13	−0.12	0.02
Value-momentum-profitability combo	0.71	0.37	0.89	0.64	1.07	0.90	1.43
Industry momentum	0.18	−0.04	0.33	0.17	0.48	0.37	0.78
Industry relative reversals	0.10	0.21	0.30	0.38	0.50	0.55	0.90
High-frequency combo	0.35	0.21	0.63	0.52	0.90	0.83	1.45
Seasonality	0.06	0.03	0.25	0.22	0.43	0.41	0.80
Industry low volatility	0.29	0.26	0.48	0.46	0.68	0.66	1.07
HML	0.33	0.39	0.35	0.39	0.36	0.39	0.39
CMA	0.36	0.37	0.36	0.37	0.36	0.37	0.37
RMW	0.26	0.29	0.27	0.29	0.27	0.29	0.29
I/A	0.44	0.34	0.44	0.37	0.44	0.39	0.44
ROE	0.57	0.18	0.57	0.28	0.57	0.37	0.57
SMB	0.23	0.23	0.23	0.23	0.23	0.23	0.23
MKT	0.53	0.53	0.53	0.53	0.53	0.53	0.53

Table 4 (continued)

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.5$		$\omega = 0$
	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
Panel C: Predictive standard deviations of anomalies that can be explained by HXZ but not FF5							
Return-on-book equity	5.30	5.36	5.30	5.35	5.30	5.33	5.30
Value-momentum combo	5.01	5.07	5.01	5.06	5.01	5.04	5.00
Idiosyncratic volatility	7.53	7.55	7.53	7.54	7.52	7.53	7.52
Momentum	7.30	7.39	7.29	7.37	7.29	7.34	7.27
Size	5.36	5.42	5.35	5.40	5.34	5.37	5.31
HML	3.06	3.05	3.06	3.05	3.06	3.05	3.06
CMA	2.03	2.02	2.03	2.02	2.03	2.02	2.02
RMW	2.30	2.28	2.30	2.28	2.29	2.28	2.29
I/A	1.90	1.92	1.90	1.92	1.90	1.91	1.90
ROE	2.64	2.71	2.64	2.70	2.64	2.68	2.64
SMB	3.09	3.09	3.09	3.09	3.09	3.09	3.09
MKT	4.69	4.70	4.69	4.70	4.69	4.70	4.69
Panel D: Predictive standard deviations of anomalies that cannot be explained by HXZ or FF5							
Accruals	3.27	3.27	3.26	3.27	3.26	3.26	3.25
Net issuance	3.12	3.12	3.10	3.10	3.08	3.08	3.05
Investment	3.19	3.19	3.18	3.18	3.17	3.17	3.15
Gross margins	3.26	3.26	3.26	3.26	3.26	3.27	3.27
Value-momentum-profitability combo	5.17	5.22	5.13	5.16	5.09	5.11	5.01
Industry momentum	6.24	6.25	6.22	6.23	6.21	6.22	6.18
Industry relative reversals	4.63	4.63	4.60	4.60	4.58	4.58	4.53
High-frequency combo	4.04	4.05	3.97	3.98	3.90	3.91	3.77
Seasonality	4.21	4.21	4.18	4.19	4.16	4.17	4.12
Industry low volatility	3.67	3.67	3.63	3.63	3.59	3.59	3.51
HML	3.06	3.03	3.06	3.03	3.05	3.03	3.05
CMA	2.02	2.01	2.02	2.01	2.01	2.01	2.01
RMW	2.28	2.26	2.28	2.26	2.28	2.26	2.28
I/A	1.88	1.91	1.88	1.91	1.88	1.90	1.88
ROE	2.63	2.71	2.63	2.70	2.63	2.68	2.63
SMB	3.09	3.10	3.09	3.10	3.09	3.10	3.09
MKT	4.64	4.64	4.64	4.64	4.64	4.64	4.64

Table 5
Optimal allocations (in-sample) for investing in anomalies that can be explained by HXZ but not FF5.

This table reports optimal allocations (position sizes) per $100 of wealth for a mean-variance Bayesian investor with relative risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in HXZ or FF5. The risky assets include five anomalies that can be explained by HXZ but not FF5, five factors in FF5, and investment and profitability factors in HXZ. Also reported are the certainty-equivalent return (CER, in % per year), and annualized Sharpe ratio of the portfolio's return with respect to the given predictive distribution.

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.5$		$\omega = 0$
	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
			Panel A: $c = \infty$ (without margin requirement)				
Return-on-book equity	0.0	0.0	−16.8	−16.1	−33.7	−32.6	−67.7
Value-momentum combo	0.0	0.0	−15.4	−14.7	−30.9	−29.9	−62.1
Idiosyncratic volatility	0.0	0.0	13.7	13.1	27.5	26.6	55.2
Momentum	0.0	0.0	10.3	9.9	20.7	20.1	41.6
Size	0.0	0.0	17.7	16.9	35.5	34.4	71.3
HML	0.0	−70.9	12.8	−41.3	25.7	-11.2	51.7
CMA	0.0	577.8	28.7	463.7	57.5	350.3	115.6
RMW	0.0	376.7	−2.9	281.7	−5.7	186.5	−11.5
I/A	597.3	0.0	560.9	105.0	524.9	213.6	454.1
ROE	416.5	0.0	403.3	84.8	390.4	172.5	365.6
SMB	238.2	161.3	255.6	194.2	273.4	229.4	309.8
MKT	180.6	187.6	189.2	192.2	198.2	198.5	216.6
CER	40.7	27.4	40.8	28.3	41.2	31.0	42.6
Sharpe ratio	1.56	1.28	1.57	1.30	1.57	1.36	1.60

Table 5 (continued)

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.5$		$\omega = 0$
	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
			Panel B: $c = 2$ (with margin requirement)				
Return-on-book equity	19.8	0.0	18.7	0.0	17.6	6.8	3.5
Value-momentum combo	37.3	12.6	34.3	29.6	31.3	40.2	19.8
Idiosyncratic volatility	0.0	6.5	0.0	14.5	0.0	14.4	12.1
Momentum	12.4	0.0	16.4	0.0	20.4	0.0	31.0
Size	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HML	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CMA	0.0	44.7	0.0	16.6	0.0	0.0	0.0
RMW	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I/A	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ROE	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SMB	0.0	0.0	0.0	0.0	0.0	0.0	0.0
MKT	61.0	72.5	61.2	78.5	61.4	77.1	67.2
CER	9.0	6.2	9.1	6.6	9.2	7.3	9.5
Sharpe ratio	0.93	0.75	0.93	0.75	0.92	0.79	0.91

Table 6
Optimal allocations (in-sample) for investing in anomalies that cannot be explained by the HXZ or FF5.

This table reports optimal allocations (position sizes) per \$100 of wealth for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in HXZ or FF5. The risky assets include 10 anomalies from Novy-Marx and Velikov (2016) that cannot be explained by either HXZ or FF5, five factors in FF5, and investment and profitability factors in HXZ. Also reported are the certainty-equivalent return (CER, in % per year), and annualized Sharpe ratio of the portfolio's return with respect to the given predictive distribution.

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.5$		$\omega = 0$
	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
Panel A: $c = \infty$ (without margin requirement)							
Accruals	0.0	0.0	23.9	22.8	50.9	49.2	117.1
Net issuance	0.0	0.0	34.9	33.3	74.4	71.9	171.1
Investment	0.0	0.0	15.4	14.7	32.8	31.6	75.3
Gross margins	0.0	0.0	20.3	19.4	43.3	41.8	99.6
Value-momentum-profitability combo	0.0	0.0	28.5	27.1	60.6	58.5	139.3
Industry momentum	0.0	0.0	6.0	5.7	12.8	12.4	29.5
Industry relative reversals	0.0	0.0	34.0	32.4	72.4	69.9	166.4
High-frequency combo	0.0	0.0	50.8	48.4	108.3	104.5	248.8
Seasonality	0.0	0.0	27.8	26.5	59.2	57.1	136.0
Industry low volatility	0.0	0.0	44.8	42.7	95.4	92.1	219.3
HML	0.0	−39.2	3.4	−26.5	7.2	−13.2	16.5
CMA	0.0	531.3	19.7	421.1	41.9	313.8	96.3
RMW	0.0	357.2	−5.6	265.2	−11.9	172.3	−27.4
I/A	598.0	0.0	504.5	49.2	417.6	106.2	269.5
ROE	396.6	0.0	379.0	74.9	371.4	161.6	395.8
SMB	201.5	127.2	184.9	127.1	172.4	131.8	163.7
MKT	178.2	182.7	157.7	160.0	140.1	140.7	116.1
CER	37.8	24.5	41.3	28.4	52.5	41.4	105.4
Sharpe ratio	1.51	1.21	1.57	1.31	1.78	1.58	2.52

Table 6 (continued)

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.5$		$\omega = 0$
	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
Panel B: $c = 2$ (with margin requirement)							
Accruals	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Net issuance	0.0	26.6	0.0	34.3	0.0	0.0	0.0
Investment	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Gross margins	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Value-momentum-profitability combo	33.1	3.6	54.8	30.6	51.7	42.3	33.7
Industry momentum	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Industry relative reversals	0.0	0.0	0.0	0.0	0.0	0.0	0.0
High-frequency combo	0.0	0.0	4.3	3.1	30.7	37.5	66.3
Seasonality	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Industry low volatility	0.0	0.0	0.0	0.0	0.0	0.4	0.0
HML	0.0	25.6	0.0	1.2	0.0	0.0	0.0
CMA	0.0	11.3	0.0	0.0	0.0	0.0	0.0
RMW	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I/A	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ROE	39.4	0.0	13.8	0.0	0.0	0.0	0.0
SMB	0.0	0.0	0.0	0.0	0.0	0.0	0.0
MKT	55.0	65.8	54.3	61.6	35.3	39.7	0.0
CER	7.4	5.9	8.4	6.8	10.3	9.1	15.6
Sharpe ratio	0.88	0.75	0.89	0.80	1.10	0.99	1.62

Table 7
Out-of-sample CER for investing in anomalies that can be explained by HXZ but not FF5.

This table reports the out-of-sample CER (in % per year) for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in HXZ or FF5. The risky assets include five anomalies from Novy-Marx and Velikov (2016) that can be explained by HXZ but not FF5, five factors in FF5, and investment and profitability factors in HXZ. We randomly sample (with replacement) $T+300$ returns of the risky assets and use the first T to calculate the portfolio weights, which are used to the remaining 300 observations for calculating the out-of-sample CER. The procedure is repeated 1,000 times; average CERs are shown.

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.50$		$\omega = 0$
T	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
			Panel A: $c = \infty$ (without margin requirement)				
60	14.8	−4.0	10.9	−0.7	−3.6	−8.0	−90.1
120	29.3	14.0	27.9	18.9	22.5	18.9	−4.6
240	35.2	21.1	34.8	26.7	32.6	29.3	21.8
360	37.3	23.3	37.2	29.2	35.9	32.5	29.6
600	38.7	24.9	38.8	30.8	38.2	34.7	34.8
			Panel B: $c = 2$ (with margin requirement)				
60	6.3	4.6	6.4	5.4	6.5	5.9	6.2
120	7.5	5.6	7.7	6.5	7.7	7.1	7.5
240	8.7	6.5	8.9	7.6	8.9	8.4	8.7
360	9.2	6.8	9.4	8.1	9.4	8.9	9.2
600	9.6	7.3	9.8	8.8	9.9	9.5	9.8

Table 8
Out-of-sample annualized Share ratio for investing in anomalies that can be explained by HXZ but not FF5.

This table reports the out-of-sample Sharpe ratio for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in HXZ or FF5. The risky assets include five anomalies that can be explained by HXZbut not FF5, five factors in FF5, and investment and profitability factors in HXZ. We randomly sample (with replacement) $T+300$ returns of the risky assets and use the first T to calculate the portfolio weights, which are used to the remaining 300 observations for calculating the out-of-sample Sharpe ratio. The procedure is repeated 1,000 times; average Sharpe ratios (annualized by multiplying $\sqrt{12}$) are shown.

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.50$		$\omega = 0$
T	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
			Panel A: $c = \infty$ (without margin requirement)				
60	1.34	0.98	1.32	1.07	1.24	1.08	1.06
120	1.45	1.11	1.44	1.22	1.38	1.26	1.24
240	1.51	1.19	1.51	1.32	1.48	1.38	1.38
360	1.54	1.22	1.54	1.36	1.52	1.43	1.44
600	1.55	1.25	1.56	1.39	1.55	1.47	1.50
			Panel B: $c = 2$ (with margin requirement)				
60	0.71	0.63	0.72	0.67	0.71	0.69	0.68
120	0.79	0.72	0.80	0.76	0.80	0.77	0.77
240	0.88	0.79	0.89	0.85	0.88	0.87	0.86
360	0.92	0.84	0.93	0.90	0.93	0.92	0.90
600	0.97	0.88	0.98	0.95	0.98	0.97	0.96

Table 9
Out-of-sample CER for investing in anomalies that cannot be explained by either HXZ or FF5.

This table reports the out-of-sample CER (in % per year) for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in HXZ or FF5. The risky assets include 10 anomalies from Novy-Marx and Velikov (2016) that cannot be explained by eithr HXZ or FF5, five factors in FF5, and investment and profitability factors in HXZ. We randomly sample (with replacement) $T+300$ returns of the risky assets and use the first T to calculate the portfolio weights, which are used to the remaining 300 observations for calculating the out-of-sample CER. The procedure is repeated 1,000 times; average CERs are shown.

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.50$		$\omega = 0$
T	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
Panel A: $c = \infty$ (without margin requirement)							
60	11.3	−7.1	31.3	17.8	35.7	29.2	−146
120	25.9	10.8	49.5	38.8	63.8	58.5	13.8
240	31.7	17.7	56.6	46.6	75.1	69.8	66.3
360	33.9	20.0	58.9	48.9	78.3	72.9	80.7
600	35.4	21.6	60.5	50.7	81.0	75.5	91.6
Panel B: $c = 2$ (with margin requirement)							
60	6.7	5.1	8.6	7.6	10.4	10.0	11.7
120	7.9	5.7	10.1	9.0	12.0	11.6	13.1
240	8.7	6.3	11.0	9.8	13.0	12.6	14.0
360	9.0	6.5	11.4	10.2	13.4	13.0	14.4
600	9.3	6.7	11.7	10.2	13.8	13.5	14.9

Table 10
Out-of-sample annualized Share ratio for investing in anomalies that cannot be explained by either HXZ or FF5.

This table reports the out-of-sample Sharpe ratio for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in HXZ or FF5. The risky assets include 10 anomalies from Novy-Marx and Velikov (2016) that cannot be explained by either HXZ or FF5, five factors in FF5, and investment and profitability factors in HXZ. We randomly sample (with replacement) $T+300$ returns of the risky assets and use the first T to calculate the portfolio weights, which are used to the remaining 300 observations for calculating the out-of-sample Sharpe ratio. The procedure is repeated 1,000 times; average Sharpe ratios (annualized by multiplying $\sqrt{12}$) are shown.

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.50$		$\omega = 0$
T	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
			Panel C: $c = \infty$				
60	1.28	0.91	1.62	1.38	1.78	1.65	1.76
120	1.39	1.04	1.81	1.62	2.03	1.94	2.06
240	1.45	1.11	1.91	1.76	2.18	2.13	2.25
360	1.47	1.14	1.94	1.81	2.24	2.20	2.33
600	1.49	1.17	1.98	1.85	2.29	2.26	2.41
			Panel A: $c = 2$				
60	0.85	0.72	0.99	0.94	1.13	1.11	1.21
120	0.97	0.79	1.11	1.07	1.25	1.26	1.35
240	1.04	0.84	1.18	1.15	1.33	1.35	1.46
360	1.08	0.87	1.21	1.19	1.38	1.41	1.52
600	1.12	0.89	1.23	1.22	1.43	1.48	1.58