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Citation

YANG, Hai; SHAO, Chaoyi; Hai WANG; and YE, Jieping. Integrated reward scheme and surge pricing in a ride-sourcing market. (2018). *Transportation Systems in the Connected Era: Proceedings of the 23rd International Conference of Hong Kong Society for Transportation Studies HKSTS 2018: Hong Kong, December 8-10*. 209-218.

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Integrated Reward Scheme and Surge Pricing in a Ride-sourcing Market

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Surge pricing is commonly used in on-demand ride-sourcing platforms (e.g., Uber, Lyft and Didi) to dynamically balance demand and supply. However, since the price for ride service cannot be unlimited, there is usually a reasonable or legitimate range of prices in practice. Such a constrained surge pricing strategy fails to balance demand and supply in certain cases, e.g., even adopting the maximum allowed price cannot reduce the demand to an affordable level during peak hours. In addition, the practice of surge pricing is controversial and has stimulated long debate regarding its pros and cons. In this paper, to address the limitation of current surge pricing practice, we propose a novel reward scheme integrated with surge pricing: users can pay an additional amount on top of the regular surge price to a reward account during peak hours, and then use the balance in the reward account to compensate for their trips during off-peak hours. The integrated mechanism is valuable for both transportation and operations management research community. It also proposes another important practical tool to balance demand and supply in ride-sourcing platforms. Specifically, we build up an optimization model to determine number of travel requests in the platforms on demand side of the market, an equilibrium model to characterize number of active drivers on supply side of the market, and an optimization model on platforms decision to maximize platform profit. We compare scenarios with and without reward scheme and explore them from three perspectives: user utility, driver income, and platform revenue and profit. We find that, in some situations, all the three stakeholders, i.e., users, drivers, and the platform, will be better off under the reward scheme integrated with surge pricing. It shows that the integrated reward scheme is a potentially powerful tool for the on-demand ride-sharing market.

Key words: Ride-sourcing, Surge Pricing, Reward Scheme, User Utility, Driver Income, Platform Revenue

1. Introduction

Recent development and penetration of mobile internet technologies has enabled the introduction of various innovative services under the sharing economy concept in our daily lives. Ride-sourcing

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transportation platforms such as Uber and Didi provide pioneering on-demand ride-sharing services to users in the sharing economy context . Such platforms provide intermediary means for connecting demand (e.g., users) and supply (e.g., drivers) in real time. These companies create mobile apps for ride-sourcing which dispatch vehicles to serve users based on their real-time locations. Users are informed of the coming vehicle information, estimated pick-up and arrival time, and estimated fare. The platforms charge a particular fare per trip to users and pay certain wage to the drivers. The difference between the fare and the wage is the commission withheld by the platform. The commission is normally between 15% and 25%, depending on regions and companies. Such ride-sourcing transportation platforms provide flexible working opportunities for private car owners who are additional source of service providers that can satisfy on-demand travel requests.

The ride-sourcing sharing transportation platform provides a typical two-sided market. It is a meeting place for two groups of agents (users and drivers) who interact and provide each other with network benefits. Rochet and Tirole (2003) firstly pointed out the commonality across seemingly different businesses/markets with a clear characterization of the two-sided market. In the shared transportation context, users and drivers are sensitive to the prices and wages of the service, which are critical decisions that the platform makes in coordinating and balancing demand and supply. It leads to the introduction of the so-called “surge pricing” strategy, where the platform adjusts the prices and wages dynamically based on real-time information of the demand and supply, taking both social welfare (including user utility and driver income) and platform revenue and profit into consideration. Surge pricing is very common, especially during demand peak hours, when Uber and Didi usually increase prices/wages to reduce travel requests from users and attract more drivers.

However, surge pricing is controversial and has been questioned by service users, scholars, and policy makers. The literature includes much debate on the potential harm of surge pricing on the long term performance of sharing service platforms due to the strategic behavior of users, e.g., Chen and Hu (2017) and Banerjee et al. (2015) showed that surge pricing underperforms static pricing when the market environment is stable and the market size is large. In particular, Chen and Hu (2017) showed that this result is true in the presence of forward-looking riders and drivers. Additionally, in some extreme cases, the high prices determined by certain surge pricing algorithms even created severe criticism to the platform. We refer the readers to the reports about slam on Uber for its surge pricing after the terrorist attacks in Sydney¹ and London². In fact, under pressure from the general public and regulators, and also considering the potential long term impacts, platforms often adopt certain upper limits on the surge pricing. For example, Uber caps

¹ <https://newrepublic.com/article/120564/during-terrorist-attack-sydney-uber-imposing-surge-pricing>

² <http://www.dailymail.co.uk/news/article-4570020/Uber-slammed-price-surges-London-terror-attack.html>

its fares at a price that matches fourth highest price in a particular area over the preceding two months³ during states of emergency in the U.S. However, from the perspective of meeting demand with supply, any pre-determined range of price may result an imbalance between demand and supply.

In this paper, we propose a novel reward scheme to reduce the potential negative impact of surge pricing. Rewarding customers with the companies' own products and services has become an increasingly popular practice across a spectrum of industries, such as airlines, hotels, and telecommunication. To address the limitation of surge pricing, we propose an integrated reward scheme. When a user takes a trip during demand peak hours (i.e., the period with high user travel utility, hence high demand and surge price), on top of the surge price that is legitimate for the platform to charge within a pre-determined range, the user can opt to pay a certain additional money for priority dispatch of the service. This additional money does not go to the platform and drivers directly, instead, it goes to the user's so-called "reward account". During demand off-peak hours (e.g., the period with low user travel utility, hence low demand and normal price), the user can use some of the balance in the reward account to compensate for his/her fare.

We explore the integrated reward scheme and surge price and find its benefits/advantages from different perspectives:

- **Prioritized Urgent Users and Increased Utility:** The optional additional amount paid by users during peak hours can indicate the travel urgency and thereby help users with urgent needs to receive priority service. User utility may also increase in some cases.
- **Increased Drivers Income:** Users may take more trips during off-peak hours because of the trip compensation, so the number of total realized trips and hence the total income of drivers would increase.
- **Increased Platform Revenue and Profit:** The platform's total revenue and profit may increase due to more trips being realized during off-peak hours.
- **Improved User Loyalty:** Since users maintain a certain balance in the reward account, they will have more incentive and hence loyalty in using the ride-sourcing services.
- **Improved Public Image:** Since the additional money paid by users during peak hours is optional and does not go to the platform directly, it helps to reduce any criticism of the platform on surcharges, hence improving its public image.

This study makes several contributions to the literature on balance supply and demand in the ride-sourcing market: First, we explore, to our knowledge, the first study to leverage a reward scheme with surge pricing in a shared transportation platform. We propose a model to describe the

³ <http://time.com/3633469/uber-surge-pricing/>

behavior of users and drivers facing surge pricing integrated with a reward scheme. Second, we make comparisons between scenarios with and without a reward scheme. We evaluate the performance of the integrated mechanism in terms of social welfare and platform revenue/profit. Third, we find that, using the reward scheme integrated with surge pricing, we can arrive at a win-win-win situation for all the three stakeholders in the ride-sourcing business, namely users, drivers and the platform.

The rest of the paper is organized as follows. The relevant literature is reviewed in Section 2, And we formally present the model formulation and analysis in Section 3. In Section 4, comparisons between performance with and without a reward scheme are conducted from the perspectives of three stakeholders: users, drivers, and the platform. Section 5 concludes this paper.

2. Literature Review

The two-sided market has spawned a surge of interest and research in academia. Rochet and Tirole (2003) provided a clear characterization of a two-sided market: by holding constant the sum of the prices faced by two sides, any change in price structure (or price distribution) affects the volume of transactions on the platform. They proposed a canonical model of two-sided markets with one platform encompassing usage and membership externalities. Caillaud and Jullien (2003) determined the equilibrium of two-sided market structures and characterized the efficiency properties. Armstrong (2006) proposed three models to illustrate platform competition in terms of pricing strategy in two-sided markets. Diverse issues related to two-sided markets were reviewed and addressed by Roson (2005), with various assumptions on timing, price instruments, and externalities.

Pricing and wage optimization of on-demand service, especially on-demand ride-sourcing transportation service, has attracted much attention in recent years. For example, Bai et al. (2017) presented a queuing model with endogenous supply (number of participating agents) and endogenous demand (customer requests) to describe an on-demand service platform. They showed that it is optimal for the platform to charge a higher price, pay a higher wage, and offer a higher commission when potential customer demand increases. Bikhchandani and Sushil (2016) showed that charging a fixed commission reduces intermediary profits and may also magnify a surge in buyer prices and attenuate the surge in seller prices during high demand periods. Hu and Zhou (2017) analyzed the price strategy of an on-demand platform under market uncertainty and found that for a given realized market condition, the joint price and wage optimization can be reduced to a one-dimensional problem of solving for the optimal matching quantity, and the optimal price has a U-shaped relationship with the wage. Cachon et al. (2017) found that both users and drivers benefit from surge pricing on a platform with self-scheduling capacity. Taylor (2017) examined the impacts of delay sensitivity and agent independence on price and wage of the platform.

There have also been some studies on reward schemes and loyalty programs. By introducing loyalty programs in diverse formats, agents or retailers attract a huge number of regular members who keep using their services, see Lal and Bell (2003), Caminal and Claici (2007), Singh et al. (2008), Caminal (2012), Gandomi and Zolfaghari (2013). For example, Kim et al. (2001) and Kim et al. (2004) investigated the design and adoption of loyalty and reward programs in the context of capacity management and found that reward programs impose additional incentives for firms to set higher prices. Yang and Tang (2018) proposed a new fare-reward scheme for railway transit aiming to shift central peak period commuters to the shoulder peak period, which can reduce commuter queuing time at stations without reducing transit operators revenue. In this study, we explore, to our knowledge, for the first time, integrating a reward scheme with surge pricing in a ride-sourcing market.

3. Models and Analysis

In this section, we propose models to describe the demand side, supply side, and the platform's decision making in a ride-sourcing market.

Demand Side of a Ride-sourcing Market

Assume that the total number of potential users (i.e., all the users of the ride-sourcing platform) in a service district is N . Each individual user i , needs to travel during both off-peak and peak hours, and can travel either using service on the ride-sourcing platform or by public transit (e.g., bus and metro service). We assume that all individuals are rational and make travel decisions with the objective of maximizing their utilities from the trips. In this paper, we consider two types of periods for the trips—off-peak hours and peak hours. Off-peak hours are defined as the periods with relatively low travel utility to the users, hence normally have low demand. During off-peak hours, by market regulation, platform can only set the trip fare at the basic fare and no surge pricing is involved. Peak hours are defined as the periods with relatively high travel utility to the users, hence normally have high demand. During peak hours, the platform can set the trip fare to a higher level, known as the surge price.

We use p_L and p_H to denote the fare of each trip that the platform charges to users during off-peak hours (with low user travel utility) and peak hours (with high user travel utility), respectively. We use w_L and w_H to denote the wage of each trip that the platform pays to drivers during off-peak and peak hours, respectively. We also assume that a trip by transit has a constant fare, p_T , during both off-peak and peak hours. Let n_L , n_H and n_T denote a user's decision on the number of trips using the ride-sourcing platform during off-peak hours, the number of trips using the platform during peak hours, and the total number of trips by transit, respectively. Table 1 in Appendix provides a glossary of notation.

In this paper, we ignore the difference in distance of each trip and assume that the utility of travel for any user follows a specific Cobb–Douglas type of production function (see, e.g., Varian (1992) and Yang and Yang (2011)). Then, the travel decisions of a representative user will be the optimal solutions of the following constrained optimization problem:

$$\max_{n_H^i, n_L^i, n_T^i, s_M^i} U = (n_H^i)^{\alpha_H} (n_L^i)^{\alpha_L} (n_T^i)^{\alpha_T} (s_M^i)^{\alpha_M} \quad (1)$$

$$s.t. \ p_H n_H^i + p_L n_L^i + p_T n_T^i + s_M^i = B^i \quad (2)$$

$$n_H^i, n_L^i, n_T^i, s_M^i \geq 0 \quad (3)$$

$\alpha_L, \alpha_H, \alpha_T$ denote the elasticity of utility with respect to trips using the platform during off-peak hours and peak hours, and trips by transit, respectively. We assume $\alpha_H > \alpha_L > \alpha_T$, which means that considering the urgency, time saving and comfort, trips using the platform during peak hours have a higher utility compared to trips using the platform during off-peak hours, and trips by transit contribute the least utility. We set α to 0 when the corresponding number of trips is 0, e.g., we set $\alpha_H = 0$ when $n_H^i = 0$, which can be regarded as the corresponding trip mode not being available. B^i is monetary limit that a user can afford to spend on travel in a decision period, e.g., in one week. Equation (2) represents a budget constraint and s_M captures the money saved. α_M in the objective function denotes the elasticity of utility with respect to the saved money. We assume B^i is a constant for a specific user.

The corresponding Lagrangian problem is:

$$L = (n_H^i)^{\alpha_H} (n_L^i)^{\alpha_L} (n_T^i)^{\alpha_T} (s_M^i)^{\alpha_M} - \mu(p_H n_H^i + p_L n_L^i + p_T n_T^i + s_M^i - B^i) \quad (4)$$

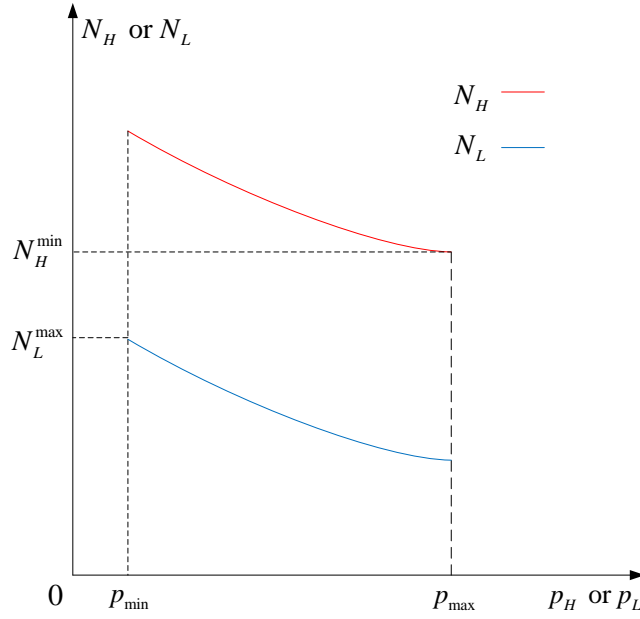
The first order necessary conditions yield

$$\begin{aligned} \alpha_H (n_H^i)^{\alpha_H-1} (n_L^i)^{\alpha_L} (n_T^i)^{\alpha_T} (s_M^i)^{\alpha_M} - \mu p_H &= 0 \\ \alpha_L (n_H^i)^{\alpha_H} (n_L^i)^{\alpha_L-1} (n_T^i)^{\alpha_T} (s_M^i)^{\alpha_M} - \mu p_L &= 0 \\ \alpha_T (n_H^i)^{\alpha_H} (n_L^i)^{\alpha_L} (n_T^i)^{\alpha_T-1} (s_M^i)^{\alpha_M} - \mu p_T &= 0 \\ \alpha_M (n_H^i)^{\alpha_H} (n_L^i)^{\alpha_L} (n_T^i)^{\alpha_T} (s_M^i)^{\alpha_M-1} - \mu &= 0 \\ p_H n_H^i + p_L n_L^i + p_T n_T^i + s_M^i - B^i &= 0 \end{aligned} \quad (5)$$

We can now solve this problem as below:

$$\begin{aligned} n_H^i &= \frac{\alpha_H}{\alpha_H + \alpha_L + \alpha_T + \alpha_M} \frac{B^i}{p_H} \\ n_L^i &= \frac{\alpha_L}{\alpha_H + \alpha_L + \alpha_T + \alpha_M} \frac{B^i}{p_L} \\ n_T^i &= \frac{\alpha_T}{\alpha_H + \alpha_L + \alpha_T + \alpha_M} \frac{B^i}{p_T} \\ s_M^i &= \frac{\alpha_M}{\alpha_H + \alpha_L + \alpha_T + \alpha_M} B^i \end{aligned} \quad (6)$$

Figure 1 Demand Curve During Peak and Off-peak Hours



As introduced in Section 1, there is usually a pre-determined range of prices that is acceptable or legitimate in practice. We assume that the range is $[p_{min}, p_{max}]$ and the platform cannot set fares beyond this range. Particularly, p_{min} is the basic fare during off-peak hours, and the platform adopts a surge price strategy and can increase the fare up to p_{max} during peak hours.

We assume that each user has an individual total budget B^i , so the demand of trips on the platform is $N_L = \sum_{i=1}^N n_L^i$ and $N_H = \sum_{i=1}^N n_H^i$ during off-peak hours and peak hours respectively. The total demand for the public transit is $N_T = \sum_{i=1}^N n_T^i$. Figure 1 illustrates the demand curve of trips for the platform.

Supply Side of a Ride-sourcing Market

Let M be the total number of potential earning sensitive drivers who may decide to participate in the platform, e.g., M is the number of registered drivers. We use w_L and w_H to denote the wage of each trip that the platform pays to drivers during off-peak and peak hours, respectively and assume they can not less than the minimum wage w_{min} determined by market. Let c_L and c_H denote the operation cost of each vehicle during off-peak hours and peak hours, respectively. Let e_L and e_H denote the earnings of drivers during the off-peak hours and the peak hours, respectively. Let m_L and m_H denote the realized number of drivers participating in the platform during off-peak hours and peak hours, respectively. We have $m_L \leq M$ and $m_H \leq M$. If the number of realized trips is N_L^r and N_H^r , then we have

$$e_L = \frac{w_L N_L^r}{m_L} - c_L \quad (7)$$

$$e_H = \frac{w_H N_H^r}{m_H} - c_H \quad (8)$$

To model the earning-sensitivity of drivers, we assume that each potential driver has a reservation earning q (i.e., corresponding to his/her outside option), where q varies across different drivers (e.g., see Bai et al. (2017)). To model the heterogeneity among drivers, we assume that there is a continuum of driver types so that the reservation earning q spreads over a range $[q_{min}, q_{max}]$ according to a cumulative distribution function $F(\cdot)$, where $F(\cdot)$ is a strictly increasing function with $F(q_{min}) = 0$ and $F(q_{max}) = 1$.

For a driver with reservation earning q , he/she will participate to offer service only if his/her earning e_L and e_H is at least equal to q during off-peak hours and peak hours, respectively. For simplicity, if we assume $F(\cdot)$ is uniformly distributed, the probability of drivers participating in the ride-sourcing platform during off-peak hours and peak hours are:

$$F(e_L) = \frac{e_L - q_{min}}{q_{max} - q_{min}} \quad (9)$$

$$F(e_H) = \frac{e_H - q_{min}}{q_{max} - q_{min}} \quad (10)$$

Hence, the total number of drivers participating in the platform are:

$$m_L = MF(e_L) = M \frac{e_L - q_{min}}{q_{max} - q_{min}} \quad (11)$$

$$m_H = MF(e_H) = M \frac{e_H - q_{min}}{q_{max} - q_{min}} \quad (12)$$

Let k_L and k_H denote the maximum number of trips that a driver/vehicle can serve during off-peak and peak hours, respectively. We also have

$$m_L \geq N_L^r / k_L \quad (13)$$

$$m_H \geq N_H^r / k_H \quad (14)$$

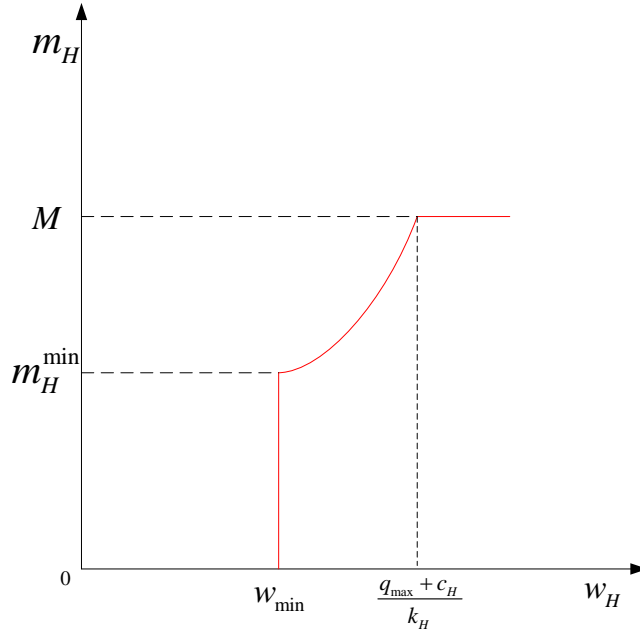
Figure 2 illustrates the supply curve of trips for the platform during peak hours, which is quite similar to the case during off-peak hours.

Market Assumptions

In this paper, we explore the integrated reward scheme with surge price. We expect that the reward scheme will be particularly valuable if the relationship between supply and demand is very different during off-peak and peak hours. In such case, manipulation of the surge price in a pre-determined range will fail to balance the supply and demand.

Let N_H^{min} denote the minimum possible demand during peak hours and N_L^{max} denote the maximum possible demand during off-peak hours (see Figure 1). Let $n_H(p)$ and $n_L(p)$ denote the functions n_H and n_L , depending on price p . Let $m(w)$ denote the function of m depending on wage w . We make following three assumptions on the supply and demand in the market to better analyze the impacts of the reward scheme.

Figure 2 Supply Curve During Peak Hours



ASSUMPTION 1. *Demand during peak hours is always higher than demand during off-peak hours in the pre-determined range of prices. Specifically, we have*

$$\frac{\alpha_H}{p_{max}} \geq \frac{\alpha_L}{p_{min}}, \quad (15)$$

then

$$N_H^{min} = \frac{\alpha_H}{\alpha_H + \alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i}{p_{max}} \geq N_L^{max} = \frac{\alpha_L}{\alpha_H + \alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i}{p_{min}}. \quad (16)$$

Under Assumption 1, the demand during peak hours under the maximum legitimate price $N_H^{min} = \sum_{i=1}^N n_H^i(p_{max})$ is still higher than demand during off-peak hours under the basic minimum price $N_L^{max} = \sum_{i=1}^N n_L^i(p_{min})$.

ASSUMPTION 2. *The market has over demand during peak hours. Specifically, we have*

$$k_H M < \frac{\alpha_H}{\alpha_H + \alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i}{p_{max}}. \quad (17)$$

All participating drivers are fully utilized during peak hours, so we have

$$k_H m_H = N_H^T. \quad (18)$$

Under Assumption 2, during peak hours, even if we set the price to the maximum legitimate value p_{max} to reduce demand, and set the driver's wage to obtain the maximum earning q_{max} to attract all the drivers, the demand is still more than the supply. In addition, all drivers who participate to the platform are fully occupied during peak hours.

ASSUMPTION 3. *The platform profit increases with the increase of the number of participating drivers during peak hours. Specifically, we have*

$$p_{max} \geq \frac{2q_{max} + c_H - q_{min}}{k_H} \frac{(\alpha_L + \alpha_T + \alpha_M)p_{min}}{(\alpha_T + \alpha_M)p_{min} + \alpha_L w_{min}}. \quad (19)$$

ASSUMPTION 4. *The market has over supply during off-peak hours. Specifically, we have*

$$\frac{\alpha_L}{\alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i - p_{max} k_H M}{p_{min}} < \frac{c_L + q_{max}}{w_{min}} M. \quad (20)$$

Not all participating drivers are fully occupied during off-peak hours, so we have

$$w_{min} \geq \frac{\alpha_L}{\alpha_H + \alpha_L + \alpha_T + \alpha_M} \frac{(q_{max} - q_{min}) \sum_{i=1}^N B^i}{k_L^2 M p_{min}} + \frac{c_L + q_{min}}{k_L}, \quad (21)$$

then

$$m_L(w_{min}) > \sum_{i=1}^N n_L^i(p_{min})/k_L. \quad (22)$$

Under Assumption 4, during off-peak hours, even if we set the price to the basic minimum price p_{min} to attract more users, and a minimum wage guarantee w_{min} to reduce the supply, the supply is still more than the demand.

The corresponding number of realized trips during peak and off-peak hours on the ride-sourcing platform, and the number of realized trips by public transit can be formulated as follows:

Objective function(1)

s.t. Equations and Inequalities (2), (3) and (18)

By solving the corresponding Langrangian problems, we obtain the realized trips in each mode, as below:

$$\begin{aligned} N_H^r &= k_H m_H; \\ N_L^r &= \frac{\alpha_L}{\alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i - p_H k_H m_H}{p_L}; \\ N_T^r &= \frac{\alpha_T}{\alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i - p_H k_H m_H}{p_T}. \end{aligned} \quad (23)$$

Platform Decision Considering both the demand side and the supply side, the ride-sourcing platform determines the price (p_L , p_H) and wage (w_L , w_H) during both off-peak and peak hours, with the objective of maximizing its profit:

$$\max_{p_H, p_L, w_H, w_L} Pro = (p_H - w_H) N_H^r + (p_L - w_L) N_L^r \quad (24)$$

s.t. (19),(23).

Under Assumption 2, during peak hours, we need to set price $p_H^* = p_{max}$ to reduce the demand as much as possible. Under Assumption 3, we set wage w_H such that the earning e_H will reach q_{max} to attract all the drivers, i.e., $e_H = q_{max}$ and $m_H = M$. Combining equations (8) and (14), we can obtain the corresponding wage w_H as follows:

$$w_H^* = \frac{q_{max} + c_H}{k_H}. \quad (25)$$

Hence, the realized trips in each mode yield:

$$\begin{aligned} N_H^r &= k_H M; \\ N_L^r &= \frac{\alpha_L}{\alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i - p_H k_H M}{p_L}; \\ N_T^r &= \frac{\alpha_T}{\alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i - p_H k_H M}{p_T}. \end{aligned} \quad (26)$$

Under Assumption 4, during off-peak hours, due to market regulation, the platform sets the price to the basic price $p_L^* = p_{min}$ and hence $N_L^r = N_L^{max}$. To maximize the profit, we have $w_L^* = w_{min}$. Combining equations (7) and (11), we can obtain the corresponding m_L and e_L as follows:

$$m_L = \frac{\sqrt{M^2(c_L + q_{min})^2 + 4Mw_{min}N_L^r(q_{max} - q_{min})} - M(c_L + q_{min})}{2(q_{max} - q_{min})}, \quad (27)$$

$$e_L = \frac{2(q_{max} - q_{min})w_{min}N_L^r}{\sqrt{M^2(c_L + q_{min})^2 + 4Mw_{min}N_L^r(q_{max} - q_{min})} - M(c_L + q_{min})} - c_L. \quad (28)$$

Therefore, under the market assumptions, the platform sets the price as $p_H^* = p_{max}$ and $p_L^* = p_{min}$, and set the wage as $w_H^* = \frac{q_{max} + c_p}{k_p}$ and $w_L^* = w_{min}$.

4. Comparisons of Scenarios with and without a Reward Scheme

In this section, we compare the scenarios with and without a reward scheme from three perspectives— platform revenue and profit, driver income, and user utility.

We elaborate the details of reward scheme. As discussed in Assumption 2 in Section 3, demand is still higher than supply during peak hours and hence some users cannot receive service even if they are willing to pay the maximum legitimate price p_{max} . In such cases, for a user with real urgent travel need or who simply does not want to wait, he/she can voluntarily choose to pay an additional amount of money s to obtain service priority during peak hours. This additional payment does not go to the platform directly, instead, it goes to the user's reward account (which can be easily checked on the mobile app). During off-peak hours, the user can use any balance in his reward account to compensate for any trip made, e.g., using r dollars of the travel cost per trip.

If we set the additional payment per trip s during peak hours and the compensation per trip r during off-peak hours to satisfy following two constraints:

$$\begin{aligned} 0 &\leq s \leq s_{max} \\ s &\geq V(r - p_{min}) + Z, \end{aligned} \quad (29)$$

where $s_{max} = \frac{\alpha_H}{\alpha_H + \alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i}{k_H M} - p_{max}$, $Z = \frac{\sum_{i=1}^N B^i}{k_H M} - p_{max}$, and $V = \frac{c_L + q_{max}}{w_{min} k_H} \frac{\alpha_L + \alpha_T + \alpha_M}{\alpha_L}$, then market assumptions 2, 3, and 4 with a reward scheme still hold. If r is set as $r = p_{min}$, then the constraints can be written as $s \geq Z$ and $Z - V p_{min} < 0$. Constraint (29) can also provide a range of $r \leq r_{max}$ where $r_{max} = p_{min} - \frac{\alpha_L}{\alpha_H + \alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i}{M} \frac{w_{min}}{c_L + q_{max}}$.

Depending on the values of s and r , we have the following two situations: (1) users have a balance left in the reward account under their optimal number of trips in each mode, and (2) users have used up the balance in the reward account. Next, we discuss the two situations.

Situation I: Users have Balance Left in the Reward Account

Let $Y = \frac{\alpha_L}{\alpha_L + \alpha_T + \alpha_M} \frac{r}{p_{min} - r}$, on top of condition (29), if s and r further satisfy

$$\max\left\{\frac{YZ}{Y+1}, V(r - p_{min}) + Z\right\} \leq s \leq s_{max} \quad (30)$$

we have $sN_H^{rR} \geq rN_L^{rR}$ and $sn_H^{irR} \geq rn_L^{irR}$ for each user i . In such a case, users cannot use up their balance in the reward account.

Then, given (30), we can obtain the realized number of trips under the reward scheme by solving

Objective function(1)

s.t. Equations and inequalities (2), (3), (18), (19) and (30)

Let p_H^R and p_L^R denote the price during peak and off-peak hours under the reward scheme. Specifically, by replacing p_H with $p_H^R + s$ and p_L with $p_L^R - r$ in equation (26) as follows:

$$\begin{aligned} N_H^{rR} &= k_H M \\ N_L^{rR} &= \frac{\alpha_L}{\alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i - (p_H^R + s)k_H M}{p_L^R - r} \\ N_T^{rR} &= \frac{\alpha_T}{\alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i - (p_H^R + s)k_H M}{p_T} \end{aligned} \quad (31)$$

where the superscript R represents the number of trips under the reward scheme.

Users' balance in the reward account can be taken as so-called "breakage" from the perspective of accountancy⁴. In this paper, we investigate both the platform's profit (denoted as Pro) and the

⁴ "Breakage" is a term used in accountancy to indicate any type of service which is unused by the customer; the monetary value of breakage can be accounted as revenue or as a reduction of an expense, e.g., see Feinson (2008).

revenue (i.e., profit + breakage, denoted as Rev) under the reward scheme.

Revenue: If we keep the price (p_H^* and p_L^*) and wage (w_H^* and w_L^*) the same as in the case without a reward scheme, and set s and r to satisfy conditions (29) and (30), then the platform's revenue under reward scheme is:

$$Rev^R = (p_H^* + s - w_H^*)N_H^{rR} + (p_L^* - r - w_L^*)N_L^{rR} \quad (32)$$

Compared with the case without a reward scheme, we have the change of revenue as:

$$Rev^R - Rev = sN_H^{rR} + (p_{min} - w_{min} - r)N_L^{rR} - (p_{min} - w_{min})N_L^r \quad (33)$$

Let $X_1 = \frac{\alpha_T + \alpha_M}{\alpha_L + \alpha_T + \alpha_M} \frac{p_{min}}{w_{min}} + \frac{\alpha_L}{\alpha_L + \alpha_T + \alpha_M}$, and if

$$s \geq \frac{YZ}{Y + X_1}, \quad (34)$$

the change of revenue (33) will be positive and the platform's revenue will increase under a reward scheme. In fact, since it is commonly to have that $p_{min}/w_{min} \geq 1$, we have $X_1 \geq 1$. Therefore, given (30), (34) will always be true, indicating that the platform's revenue always increases when users cannot use up their rewards and have a balance in their reward account.

Profit: If we keep price (p_H^* and p_L^*) and wage (w_H^* and w_L^*) as in the case without a reward scheme, and set s and r to satisfy conditions (29) and (30), then the platform's profit under a reward scheme is:

$$Pro^R = (p_H^* - w_H^*)N_H^{rR} + (p_L^* - w_L^*)N_L^{rR} \quad (35)$$

Compared with the case without a reward scheme, we have the change of profit as:

$$Pro^R - Pro = (p_{min} - w_{min})(N_L^{rR} - N_L^r) \quad (36)$$

Further, if

$$s \leq \frac{Z}{p_{min}}r, \quad (37)$$

the change of profit (36) will be positive and the platform's profit will increase under reward scheme.

Drivers Income: From (26) and (31), the total number of trips on the ride-sourcing platform during peak hours remains unchanged under the reward scheme, i.e., $N_H^{rR} = N_H^r$. Under condition (37), the total number of trips on the ride-sourcing platform during off-peak hours will increase

under the reward scheme, i.e., $N_L^{rR} > N_L^r$. Since the wage paid to drivers remains unchanged and $de_L/dN_L^r > 0$, from (28), we find that drivers will be better off with a higher earning rate e_L during off-peak hours under the reward scheme.

Number of Users' Trips: The total number of users' trips (including trips on the ride-sourcing platform and trips by transit, denoted as Trp) is the sum of N_H^r , N_L^r and N_T^r . Compared with the case without a reward scheme, we have the change in the number of users' trips as:

$$Trp^R - Trp = N_L^{rR} + N_T^{rR} - N_L^r - N_T^r \quad (38)$$

Let $X_2 = \frac{\alpha_T}{\alpha_L + \alpha_T + \alpha_M} \frac{p_{min}}{p_T} + \frac{\alpha_L}{\alpha_L + \alpha_T + \alpha_M}$, and if

$$s \leq \frac{YZ}{Y + X_2}, \quad (39)$$

the change in the number of trips (38) will be positive and the number of trips will increase under the reward scheme.

Combining (39) with (30), we find that when

$$\frac{p_{min}}{p_T} < \frac{\alpha_T + \alpha_M}{\alpha_T}, \quad (40)$$

there is an intersection of conditions (30) and (39), i.e., there exists a value of s such that $\frac{YZ}{Y+1} \leq s \leq \frac{YZ}{Y+X_2}$.

User Utility: Compared with the case without a reward scheme, we have the change of user's utility (denoted as Uti) as:

$$Uti^R - Uti = (n_H^{irR})^{\alpha_H} (n_L^{irR})^{\alpha_L} (n_T^{irR})^{\alpha_T} (s_M^{irR})^{\alpha_M} - (n_H^{ir})^{\alpha_H} (n_L^{ir})^{\alpha_L} (n_T^{ir})^{\alpha_T} (s_M^{ir})^{\alpha_M} \quad (41)$$

To compare Uti^R and Uti , we can also consider $Uti^R/Uti = \frac{(n_H^{irR})^{\alpha_H} (n_L^{irR})^{\alpha_L} (n_T^{irR})^{\alpha_T} (s_M^{irR})^{\alpha_M}}{(n_H^{ir})^{\alpha_H} (n_L^{ir})^{\alpha_L} (n_T^{ir})^{\alpha_T} (s_M^{ir})^{\alpha_M}}$. We next show $Uti^R/Uti < 1$.

We substitute (26) and (31) into $Uti^R/Uti < 1$, giving $(\frac{Z-s}{Z})^{\alpha_L + \alpha_T + \alpha_M} (\frac{p_{min}}{p_{min} - r})^{\alpha_L} \leq 1$. Taking logarithms, the inequality comes to

$$\frac{\ln \frac{Z}{Z-s}}{\ln \frac{p_{min}}{p_{min} - r}} \geq Q \quad (42)$$

where $Q = \frac{\alpha_L}{\alpha_L + \alpha_T + \alpha_M}$ for simplicity.

The LHS of (42) is increasing with s , and $s \geq \frac{YZ}{Y+1}$ under condition (30), thus $\frac{\ln \frac{Z}{Z-s}}{\ln \frac{p_{min}}{p_{min} - r}} \geq \frac{\ln(Y+1)}{\ln(\frac{Y}{Q}+1)}$. Since $\frac{\ln(Y+1)}{\ln(\frac{Y}{Q}+1)}$ is increasing with Y , and $Y \geq 0$, then we have $\frac{\ln \frac{Z}{Z-s}}{\ln \frac{p_{min}}{p_{min} - r}} \geq \lim_{Y=0} \frac{\ln(Y+1)}{\ln(\frac{Y}{Q}+1)} = Q$. Therefore, (42) always holds and so does $Uti^R/Uti < 1$. The change of utility (41) is always negative,

which indicates that the user's utility always decreases when user has a balance left in the reward account.

We summarize the results for the situation in which users have a balance left in the reward account in the following theorem.

THEOREM 1 (Users have Balance Left). *If $\max\{\frac{YZ}{Y+1}, V(r - p_{min}) + Z\} \leq s \leq s_{max}$, users cannot use up their reward and will have balance left in their reward account. Additionally,*

(1) *User's utility always decreases.*

(2) *When $\frac{p_{min}}{p_T} \geq \frac{\alpha_T + \alpha_M}{\alpha_T}$:*

- *If $\frac{Z}{p_{min}}r < s \leq s_{max}$, then the user makes fewer trips, the driver has a lower income, and the platform has a higher revenue and lower profit (area (1) in Fig. 4(a),(c),(e)).*
- *If $\max\{\frac{YZ}{Y+1}, V(r - p_{min}) + Z\} \leq s \leq \min\{\frac{Z}{p_{min}}r, s_{max}\}$, then the user makes fewer trips, the driver has a higher income, and the platform has a higher revenue and profit (area (2) in Fig. 4(a),(c),(e)).*

(3) *When $\frac{p_{min}}{p_T} < \frac{\alpha_T + \alpha_M}{\alpha_T}$:*

- *If $\frac{Z}{p_{min}}r < s \leq s_{max}$, then the user makes fewer trips, the driver has a lower income, and the platform has a higher revenue and lower profit (area (5) in Fig. 4(b),(d),(f)).*
- *If $\max\{\frac{YZ}{Y+X_2}, V(r - p_{min}) + Z\} \leq s \leq \min\{\frac{Z}{p_{min}}r, s_{max}\}$, then the user makes fewer trips, the driver has a higher income, and the platform has a higher revenue and profit (area (6) in Fig. 4(b),(d),(f)).*
- *If $\max\{\frac{YZ}{Y+1}, V(r - p_{min}) + Z\} \leq s < \max\{\frac{YZ}{Y+X_2}, V(r - p_{min}) + Z\}$, then the user makes more trips, the driver has a higher income, and the platform has a higher revenue and profit (area (7) in Fig. 4(b),(d),(f)).*

Situation II: Users Use Up Reward

ASSUMPTION 5. *Users will use the balance in the reward account, if any, to compensate for their trips during off-peak hours.*

Under Assumption 5, if s and r satisfy

$$\max\{0, V(r - p_{min}) + Z\} \leq s < \frac{YZ}{Y+1}, \quad (43)$$

Then, we have $sN_H^R < rN_L^R$ and $sn_H^{irR} < rn_L^{irR}$ for each individual user i . In such cases, users will use up their balance in the reward account.

In situation II, some of the trips during off-peak hours are compensated while others are not. Let $n_L^{ir^c}$ and $n_L^{ir^n}$ denote the number of realized trips of user i on the platform during off-peak hours, with and without compensation, respectively. We have $n_L^{ir^R} = n_L^{ir^c} + n_L^{ir^n}$.

During peak hours, the maximum number of trips for all users is $\sum_{i=1}^N n_H^{ir^R} = k_H M$. Thus, the number of realized trips with compensation during off-peak hours is $n_L^{ir^c} = n_H^{ir^R} \frac{s}{r}$ as the balance can be used up. Users then maximize their utility as follows:

$$\max_{n_H^{ir^R}, n_L^{ir^c}, n_L^{ir^n}, n_T^{ir^R}, s_M^{ir^R}} U = (n_H^{ir^R})^{\alpha_H} (n_L^{ir^c})^{\alpha_L} (n_L^{ir^n})^{\alpha_L} (n_T^{ir^R})^{\alpha_T} (s_M^{ir^R})^{\alpha_M} \quad (44)$$

$$s.t. (p_H^R + s)n_H^{ir^R} + (p_L^R - r)n_L^{ir^c} + p_L^R n_L^{ir^n} + p_T n_T^{ir^R} + s_M^{ir^R} = B^i \quad (45)$$

$$\sum_{i=1}^N n_H^{ir^R} = k_H m_H \quad (46)$$

$$n_L^{ir^c} = n_H^{ir^R} \frac{s}{r} \quad (47)$$

$$n_H^{ir^R}, n_L^{ir^c}, n_L^{ir^n}, n_T^{ir^R}, s_M^{ir^R} \geq 0 \quad (48)$$

Comparing objective function (44) with (1), we can interpret that there is an additional choice for users in making decisions under the reward scheme. We can also simply add a corresponding term $(n_L^{ir^c})^0$ in the objective function (1) to indicate that trips with compensation during off-peak hours are not applicable when the reward scheme is not adopted.

By solving the associated Lagrangian problem and combining with Assumption 3, we determine the number of realized trips as follows:

$$\begin{aligned} N_H^{r^R} &= k_H M \\ N_L^{r^c} &= k_H M \frac{s}{r} \\ N_L^{r^n} &= \frac{\alpha_L}{\alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i - (p_H^R + \frac{s}{r} p_L^R) k_H M}{p_L^R} \\ N_T^{r^R} &= \frac{\alpha_T}{\alpha_L + \alpha_T + \alpha_M} \frac{\sum_{i=1}^N B^i - (p_H^R + \frac{s}{r} p_L^R) k_H M}{p_T} \end{aligned} \quad (49)$$

Revenue/Profit: In situation II, since users use up their reward balance (i.e., breakage is zero), the revenue and the profit of the platform are identical. If we keep the price (p_H^* and p_L^*) and wage (w_H^* and w_L^*) the same as in the case without reward scheme, and set s and r to satisfy conditions (29) and (43), then the platform's revenue and profit under the reward scheme is:

$$Rev^R = Pro^R = (p_H^* + s - w_H^*) N_H^{r^R} + (p_L^* - r - w_L^*) N_L^{r^R} \quad (50)$$

Compared with the case without the reward scheme, we have the change of revenue/profit as:

$$Rev^R - Rev = Pro^R - Pro = (p_{min} - w_{min})(N_L^{r^c} + N_L^{r^n} - N_L^r) \quad (51)$$

If (43) holds, the change of revenue (51) will be positive and the platform's revenue and profit will increase under the reward scheme.

Drivers Income:

From (26) and (49), the total number of trips on the ride-sourcing platform during peak hours remains unchanged under the reward scheme, i.e., $N_H^{rR} = N_H^r$. Under condition (43), the total number of trips on the ride-sourcing platform during off-peak hours will increase under the reward scheme, i.e., $N_L^{rR} > N_L^r$. Since the wage paid to drivers remains unchanged and $de_L/dN_L^r > 0$ from (28), we find that drivers will be better off with a higher earning rate e_L during off-peak hours under the reward scheme.

Number of Users' Trips: Compared with the case without a reward scheme, we have the change of the total number of users' trips as:

$$Trp^R - Trp = N_L^{rc} + N_L^{rn} + N_T^{rR} - N_L^r - N_T^r \quad (52)$$

When

$$\frac{p_{min}}{p_T} \leq \frac{\alpha_T + \alpha_M}{\alpha_T}, \quad (53)$$

the change (52) will be positive and the total number of trips will increase under the reward scheme.

User Utility: Compared with the case without the reward scheme, we have the change of user's utility as:

$$Uti^R - Uti = (n_H^{irR})^{\alpha_H} (n_L^{ir^c})^{\alpha_L} (n_L^{ir^n})^{\alpha_L} (n_T^{ir^R})^{\alpha_T} (s_M^{ir^R})^{\alpha_M} - (n_H^{ir})^{\alpha_H} (n_L^{ir})^{\alpha_L} (n_T^{ir})^{\alpha_T} (s_M^{ir})^{\alpha_M} \quad (54)$$

The change of utility will be positive when

$$Q \ln(k_H M \frac{s}{r}) - \ln \frac{Z}{Z - \frac{s}{r} p_{min}} \geq 0, \quad (55)$$

where $Q = \frac{\alpha_L}{\alpha_L + \alpha_T + \alpha_M}$.

When $\frac{s}{r} = 0$, the LHS of (55) is negative.

Further, when

$$\frac{s}{r} \leq \frac{Q}{Q+1} \frac{Z}{p_{min}}, \quad (56)$$

the LHS of (55) increases with $\frac{s}{r}$.

The maximum value of $\frac{s}{r}$ is obtained at the intersection point of the upper bound and lower bound in (43), which is $(s, r) = (\frac{Q(Vp_{min}-Z)}{1-Q}, \frac{Vp_{min}-Z}{V(1-Q)})$. At this point, $\frac{s}{r} = QV$. Combining (20) in

assumption 4, we have $QV > \frac{Q}{Q+1} \frac{Z}{p_{min}}$, which means the LHS of (55) increases in $(0, \frac{Q}{Q+1} \frac{Z}{p_{min}})$ and decreases in $(\frac{Q}{Q+1} \frac{Z}{p_{min}}, QV)$.

When $\frac{s}{r} = \frac{Q}{Q+1} \frac{Z}{p_{min}}$, the LHS of (55) reaches the maximum value of $Q \ln(\frac{Q}{Q+1} \frac{k_H M Z}{p_{min}}) - \ln(Q+1)$, which is greater than 0 when

$$\ln(\frac{k_H M Z}{p_{min}}) \geq \frac{Q+1}{Q} \ln(Q+1) - \ln Q. \quad (57)$$

When $\frac{s}{r} = QV$, the LHS of (55) reaches a value of $Q \ln(Q k_H M V) - \ln \frac{Z}{Z - QV p_{min}}$, which is greater than 0 when

$$\ln(k_H M V) \geq -\frac{\ln(1-Q)}{Q} - \ln Q. \quad (58)$$

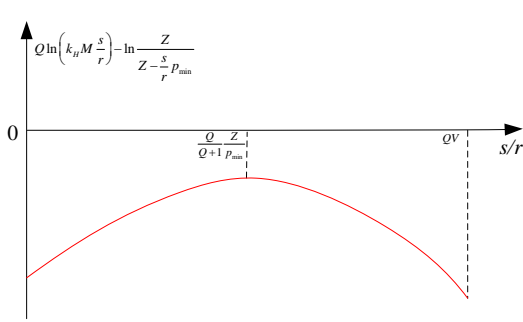
Therefore, as shown in Figure 3, when $\ln(\frac{k_H M Z}{p_{min}}) < \frac{Q+1}{Q} \ln(Q+1) - \ln Q$, the LHS of (55) is always negative and user's utility always decreases (see Figure 3(a)); when $\ln(\frac{k_H M Z}{p_{min}}) \geq \frac{Q+1}{Q} \ln(Q+1) - \ln Q$ and $\ln(k_H M V) < -\frac{\ln(1-Q)}{Q} - \ln Q$, the LHS of (55) is negative when $\frac{s}{r} = 0$ and $\frac{s}{r} = QV$ and positive when $\frac{s}{r} = \frac{Q}{Q+1} \frac{Z}{p_{min}}$, so there is an interval of $\frac{s}{r}$ denoted as (B_1, B_2) in which user's utility increases (see Figure 3(b)); when $\ln(k_H M V) \geq -\frac{\ln(1-Q)}{Q} - \ln Q$, the LHS of (55) is negative when $\frac{s}{r} = 0$ and positive when $\frac{s}{r} = \frac{Q}{Q+1} \frac{Z}{p_{min}}$ and $\frac{s}{r} = QV$, so there is an interval of $\frac{s}{r}$ denoted as (B_1, QV) in which user's utility increases (see Figure 3(c)).

We summarize the results for the situation in which users use up reward in the following theorem.

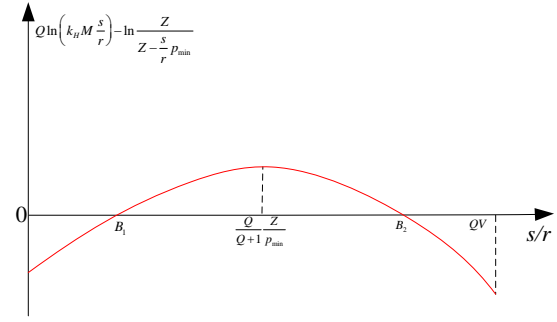
THEOREM 2 (Users Use Up Reward). *If $\max\{0, V(r - p_{min}) + Z\} \leq s < \max\{\frac{YZ}{Y+1}, V(r - p_{min}) + Z\}$, users will exhaust their reward and will have zero balance left in their reward account. Additionally,*

- (1) *Driver has higher income.*
- (2) *Platform has higher revenue and profit.*
- (3) *When $\frac{p_{min}}{p_T} \geq \frac{\alpha_T + \alpha_M}{\alpha_T}$: user makes fewer trips (area (3) in Fig. 4(a),(c),(e) and area (4) in Fig. 4(c),(e)).*
- (4) *When $\frac{p_{min}}{p_T} < \frac{\alpha_T + \alpha_M}{\alpha_T}$: user makes more trips (area (8) in Fig. 4(b),(d),(f) and area (9) in Fig. 4(d),(f)).*
- (5) *When $\ln(\frac{k_H M Z}{p_{min}}) < \frac{Q+1}{Q} \ln(Q+1) - \ln Q$: user's utility decreases (area (3) in Fig. 4(a) and area (8) in Fig. 4(b)).*
- (6) *When $p_{min}/p_T \geq (\alpha_T + \alpha_M)/\alpha_T$, $\ln(\frac{k_H M Z}{p_{min}}) \geq \frac{Q+1}{Q} \ln(Q+1) - \ln Q$, and $\ln(k_H M V) < -\frac{\ln(1-Q)}{Q} - \ln Q$: user's utility decreases in area (3) in Fig. 4(c) and area (8) in Fig. 4(d); user's utility increases in area (4) in Fig. 4(c) and area (9) in Fig. 4(d).*
- (7) *When $p_{min}/p_T < (\alpha_T + \alpha_M)/\alpha_T$ and $\ln(k_H M V) \geq -\frac{\ln(1-Q)}{Q} - \ln Q$: user's utility decreases in area (3) in Fig. 4(e) and area (8) in Fig. 4(f); user's utility increases in area (4) in Fig. 4(e) and area (9) in Fig. 4(f).*

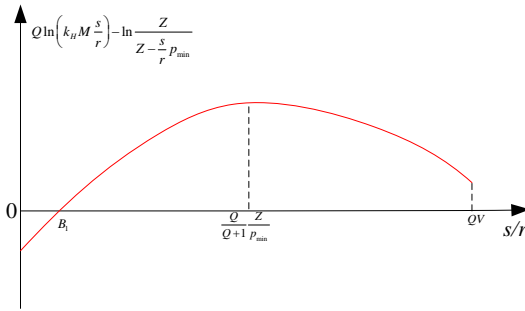
Figure 3 Curves of LHS of (55) $Q \ln(k_H M \frac{s}{r}) - \ln \frac{Z}{Z - \frac{s}{r} p_{min}}$ with Respect to $\frac{s}{r}$



(a) When $\ln(\frac{k_H M Z}{p_{min}}) < \frac{Q+1}{Q} \ln(Q+1) - \ln Q$



(b) When $\ln(\frac{k_H M Z}{p_{min}}) \geq \frac{Q+1}{Q} \ln(Q+1) - \ln Q$ and $\ln(k_H M V) < -\frac{\ln(1-Q)}{Q} - \ln Q$



(c) When $\ln(k_H M V) \geq -\frac{\ln(1-Q)}{Q} - \ln Q$

Combining situation I and situation II described above, we illustrate the impacts of the reward scheme in Figure 4. Figure 4(a),(c),(e) shows the case when $p_{min}/p_T \geq (\alpha_T + \alpha_M)/\alpha_T$ and Figure 4(b),(d),(f) shows the case when $p_{min}/p_T < (\alpha_T + \alpha_M)/\alpha_T$.

The black bold lines represent the boundary given in (29), which guarantee the market assumptions 2 and 4 under the reward scheme.

The red curve represents the boundary given in (30) and (43) that indicates whether users will use up the balance in the reward account: the area above the red curve is the domain of s and r in which users cannot use up their reward and thus have a balance left in the reward account, and vice versa.

The green line is the boundary given in (37) to indicate the change of both the driver's income and the platform's profit: the area below the green line is the domain of s and r in which both driver's income and platform's profit increase under reward scheme, and vice versa. The platform's revenue always increases under the reward scheme in the entire region, which is the area within the black bold lines.

The blue curve is the boundary given in (39) that indicates the change in the total number of trips that the user makes: the area below the blue curve is the domain of s and r in which user makes totally more trips under the reward scheme, and vice versa. It depends on p_{min} and p_T : When $p_{min}/p_T \geq (\alpha_T + \alpha_M)/\alpha_T$ as in Figure 4(a), number of trips always decrease; when $p_{min}/p_T < (\alpha_T + \alpha_M)/\alpha_T$ as in Figure 4(b), number of trips increases, as below the blue curve.

The purple lines are the boundaries given in (55) to indicate the change of the user's utility: the area above (or between) the purple line(s) is the domain of s and r in which user's utility increases under the reward scheme.

Particularly, the shadow area (area (4) and area (9)) demonstrates the domain of s and r generating a win-win-win case for all the three stakeholders—users with more utility, drivers with higher income, and the platform with higher revenue and profit. In area (9), the total number of trips also increases.

5. Conclusions

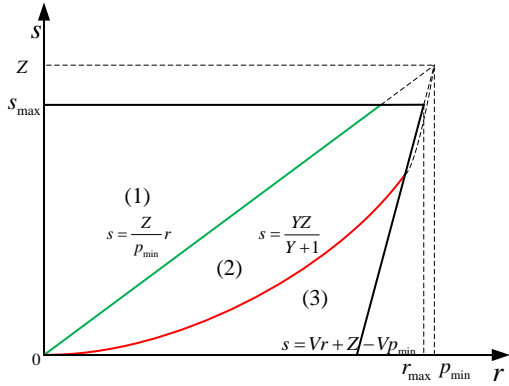
In this paper, we propose an innovative mechanism with an integrated reward scheme and surge pricing for a ride-sourcing market. Currently, platforms use surge pricing to balance demand and supply. However, since the price for ride service cannot be unlimited, usually there is a reasonable or legitimate range of prices in practice. Such constrained surge pricing strategy does, however, fail to balance demand and supply in some cases.

We propose a novel reward scheme integrated with surge pricing: users can pay an additional amount on top of the regular surge price to a reward account during peak hours, and then use the balance in the reward account to compensate for their trips during off-peak hours. This reward scheme can be more acceptable to the public since the additional money paid by users during peak hours does not go to the platform directly.

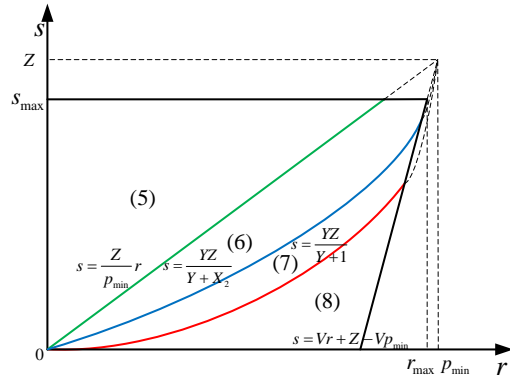
We propose a model to describe the behavior of users and drivers facing surge pricing integrated with a reward scheme. We then compare the scenarios with and without the reward scheme from three perspectives: user's utility, driver income, and platform revenue and profit. We find that, in some cases, all the three stakeholders, i.e., users, drivers, and the platform, will be better off under the integrated reward scheme with surge pricing.

There are some avenues for future work. One is to consider the additional money s that users pay during peak hours and the compensation r that users employ during off-peak hours as platform's or users' decision variables, which may further improve the benefits of the reward scheme. Another area is to numerically estimate and calibrate parameters in the model using real data from a ride-sourcing company, and implement this novel reward scheme in practice.

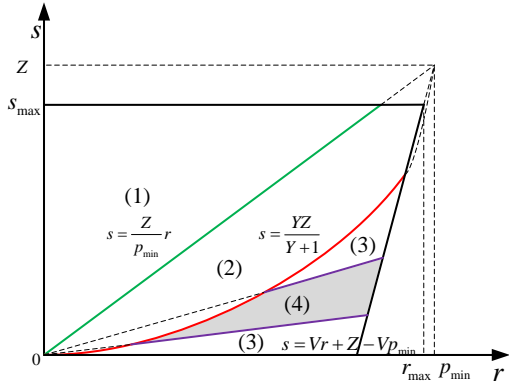
Figure 4 Comparisons between Cases with and without Reward Scheme



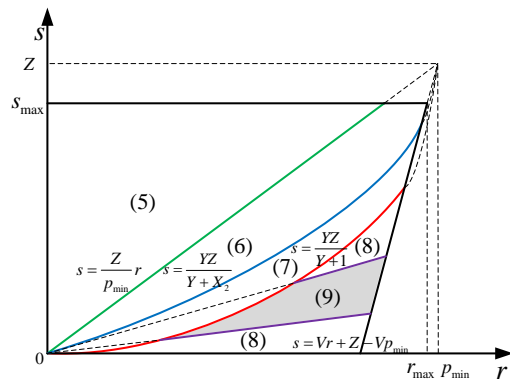
(a) When $p_{min}/p_T \geq (\alpha_T + \alpha_M)/\alpha_T$ and $\ln(\frac{k_H M Z}{p_{min}}) < \frac{Q+1}{Q} \ln(Q+1) - \ln Q$



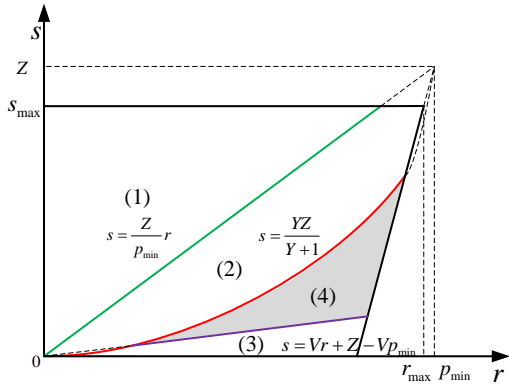
(b) When $p_{min}/p_T < (\alpha_T + \alpha_M)/\alpha_T$ and $\ln(\frac{k_H M Z}{p_{min}}) < \frac{Q+1}{Q} \ln(Q+1) - \ln Q$



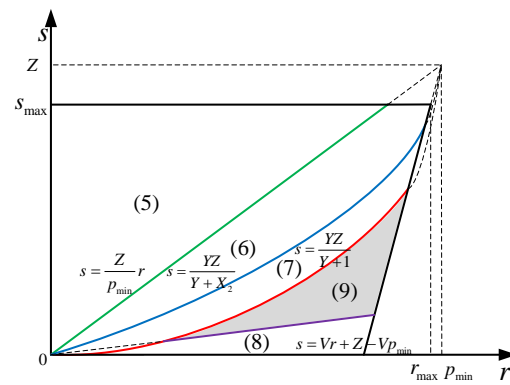
(c) When $p_{min}/p_T \geq (\alpha_T + \alpha_M)/\alpha_T$, $\ln(\frac{k_H M Z}{p_{min}}) \geq \frac{Q+1}{Q} \ln(Q+1) - \ln Q$ and $\ln(k_H M V) < -\frac{\ln(1-Q)}{Q} - \ln Q$



(d) When $p_{min}/p_T < (\alpha_T + \alpha_M)/\alpha_T$, $\ln(\frac{k_H M Z}{p_{min}}) \geq \frac{Q+1}{Q} \ln(Q+1) - \ln Q$ and $\ln(k_H M V) < -\frac{\ln(1-Q)}{Q} - \ln Q$



(e) When $p_{min}/p_T \geq (\alpha_T + \alpha_M)/\alpha_T$ and $\ln(k_H M V) \geq -\frac{\ln(1-Q)}{Q} - \ln Q$



(f) When $p_{min}/p_T < (\alpha_T + \alpha_M)/\alpha_T$ and $\ln(k_H M V) \geq -\frac{\ln(1-Q)}{Q} - \ln Q$

Acknowledgment

The work described in this paper was supported by grants from Hong Kongs Research Grants Council under Project No. HKUST16222916, and the Singapore Ministry of Education (MOE) Academic Research Fund (AcRF) Tier 1 grant.

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Parameters	Definitions
N	Total number of users
$\alpha_H, \alpha_L, \alpha_T, \alpha_M$	Elasticity of utility with respect to the corresponding variables
p_T	Trip fare by transit
B^i	Total money budget of user i
p_{max}, p_{min}	Maximum/minimum trip fare that the platform can set
M	Total number of drivers
q_{max}, q_{min}	Maximum/minimum reservation earning rate of drivers
c_H, c_L	Operation cost of drivers during peak/off-peak hours
k_H, k_L	Maximum number of trips that a driver/vehicle can serve during peak/off-peak hours
w_{min}	Minimum wage that the platform can set
s	Additional money per trip paid to the reward account during peak hours
r	Compensation per trip using reward balance during off-peak hours
Users Decisions	
n_H^i, n_L^i, n_T^i	Number of trips on platform during peak/off-peak hours and by transit, user i
$n_H^{i,r}, n_L^{i,r}, n_T^{i,r}$	Number of trips users realized in each mode without reward scheme, user i
$n_H^{ir,R}, n_L^{ir,R}, n_T^{ir,R}$	Number of trips users realized in each mode with reward scheme, user i
$n_L^{ir,c}$	Number of realized trips with fare compensation during off-peak hours, user i
$n_L^{ir,n}$	Number of realized trips without fare compensation during off-peak hours, user i
s_M^i	Saved money, user i
$s_M^{i,r}, s_M^{ir,R}$	Realized saved money without/with reward scheme, user i
Platform Decisions	
p_H, p_L	Trip fare during peak/off-peak hours without reward scheme
w_H, w_L	Wage to drivers during peak/off-peak hours without reward scheme
p_H^R, p_L^R	Trip fare during peak/off-peak hours with reward scheme
w_H^R, w_L^R	Wage to drivers during peak/off-peak hours with reward scheme
$p_H^*, p_L^*, w_H^*, w_L^*$	Optimal decisions of platform without reward scheme
Intermediate Variables	
N_H^{min}, N_L^{max}	Minimum/maximum possible number of trips during peak/off-peak hours
m_H, m_L	Number of participating drivers during peak/off-peak hours
e_H, e_L	Drivers earning rate during peak/off-peak hours
Rev, Rev^R	Platform revenue without/with reward scheme
Pro, Pro^R	Platform profit without/with reward scheme
Trp, Trp^R	Total number of trips users make without/with reward scheme
Uti, Uti^R	User utility of a representative user

Table 1 Summary of Major Notation.

Appendix