A rapid algorithm for reliability optimization of parallel redundant systems

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A Rapid Algorithm For Reliability Optimisation Of Parallel Redundant Systems

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Key Words—**Optimization, Parallel redundant system, Algorithm**

Readers Aids—
Purpose: Widen state of art
Special math needed: None
Results useful to: Reliability theoreticians

Summary & Conclusions—**A rapid method is proposed for optimization of reliability of multiconstraint parallel redundant systems. The constraints need not be linear. This method provides good starting values, which are close to the boundary of the feasible region, for the number of redundant units in each subsystem. No proof has been presented to establish the optimality obtained by this method. Yet for examples tried out this method provides optimal or near optimal solutions.**

1. INTRODUCTION

The reliability of a system with series subsystems can be increased by introducing redundancy in one or more of the subsystems. Very often there are constraints on cost, weight, volume, etc., which limit the extent of redundancy.

There have been solutions based on variational method [1], computational method [2,3], dynamic programming [4], discrete maximum principle [5], integer programming [6], and branch and bound procedure [7]. All the above methods become unwieldy as the number of subsystems becomes large. Sharma and Venkateswaran [8] have proposed a heuristic method which is very simple. The method presented in this paper calculates starting values for each subsystem proportional to the ratios of their unreliabilities. This results in better starting values for the subsystems as compared to the earlier methods [8,9].

2. NOTATION AND DEFINITIONS

n_i	number of redundant units in subsystem i, treated as a continuous number.
N	number of subsystems in series.
q_i	probability of failure of each unit in subsystem i.
n	$(n_1, n_2, \ldots, n_n)$
$Q(n)$	unreliability of the system.
r	number of constraints.
ns_i	normalised units of subsystem i used in calculating the initial values of n_i; $ns_i \geqslant 1$
$[n_i]^+$	minimum integer which is not less than n_i.

3. PROBLEM FORMULATION

Assumptions.

1. The system has N subsystems in series (i-out-of-N:F) with n_i i.i.d. units in parallel. ($n_i \geqslant 1$)
2. All units are s-independent.
3. All l.h.s. of the constraint equations have positive coefficients.

The problem of maximizing system reliability is stated as follows: Select **n** such that $Q(\mathbf{n})$ is minimised subject to the constraints

$$\sum_{i=1}^{N} G_{ji}(\mathbf{n}) \leqslant B_j, j = 1,2, \ldots, r, \quad (1)$$

where

$$Q(\mathbf{n}) = 1 - \prod_{i=1}^{N} 1 - q_i^{n_i})$$

For Q_i small enough

$$Q(\mathbf{n}) \approx q(\mathbf{n}) \equiv \sum_{i=1}^{N} q_i^{n_i} \quad (2)$$

and hence the problem reduces approximately to minimizing $q(\mathbf{n})$. The proposed method makes use of approximation (2) to minimize $q(\mathbf{n})$.

4. OPTIMISATION PROCEDURE

To start with, initial values for the number of units in each subsystem are calculated. Now all the subsystems are under consideration for adjustment. Let us now consider the three possibilities.

a) If one or more constraints are exactly satisfied the algorithm ends.

b) If one or more constraints are violated, exactly one unit is removed from the currently most reliable subsystem. This process is repeated until no constraint is violated. The procedure mentioned in c) is then followed.

c) If no constraint is violated, a unit is considered for addition to the currently most unreliable subsystem. If such an addition is likely to violate one or more constraints, the unit is not added and this subsystem is removed from further consideration. If however, the addition is not likely to violate constraints, the number of units in the subsystem is increased by one. This procedure is repeated till one of the following conditions is met.

i) One or more of the constraints are exactly satisfied.

ii) There are no more subsystems under consideration for adjustment.

The number of units of the subsystem in the final step is used to calculate the system unreliability $Q(\mathbf{n})$ according to the approximation (2). The system reliability is $1 - Q(\mathbf{n})$.

Algorithm

I. Find Initial Values

Step 1. Calculate ns as follows

Let $Q_j = \min\{Q_i\}$, $i = 1,2, ..., N$

now, $ns_i = Q_i/Q_j$, $i = 1,2, ..., N$

Step 2. For all j, $1 \leq j \leq r$, calculate D_j

$$= B_j \Big/ \sum_{i=1}^{N} G_{ji}(ns_i)$$

Step 3. Find $D = \min\{D_1, D_2, ..., D_r\}$

Step 4. For all i, $1 \leq i \leq N$, calculate $n_i = [ns_i * D]^+$

II. Adjust Initial Values

S is the set of subsystems which are being considered for adjustment; R is the element of the set which is to be removed; ϕ is a null element.

Step 5. Set $R = \phi$, $S = \{1,2, ..., N\}$, GO TO STEP 9.

Step 6. Remove R from S. If $S = \phi$, GO TO STEP 12.

Step 7. Find subsystem i which has the maximum unreliability among the subsystems in S.

Step 8. Set: $n_i \leftarrow n_i + 1$

Step 9. Check for constraints. If exactly satisfied GO TO STEP 12. If no constraint is violated GO TO STEP 7. If one or more constraints are violated GO TO STEP 10.

Step 10. Find the subsystem i which has the highest reliability. Set: $n_i \leftarrow n_i - 1$

Step 11. Check for constraints. If none of the constraints are violated, Set $R \leftarrow \{i\}$. GO TO STEP 6. Otherwise GO TO STEP 10.

Step 12. STOP.

Steps 7–9 of this algorithm are same as that presented in [8,9].

5. EXAMPLE

The example is taken from [8] to illustrate the speed and elegance of the proposed method.

Problem:

Maximize the reliability subject to two linear constraints. The system is shown in TABLE I. The problem is to find $n_1 - n_5$ to maximize system reliability.

TABLE I

Subsystem	1	2	3	4	5
Element Reliability	0.9	0.75	0.65	0.8	0.85
Cost	5	4	9	7	7
Weight	8	9	6	7	8

Constraints: Cost ≤ 132 Weight ≤ 142

Solution:

The ratio of $q_1 : q_2 : q_3 : q_4 : q_5 = 0.1 : 0.25 : 0.35 : 0.2 : 0.15$

$Q_j = \min\{Q_i\}$, $i=1,2, ..., 5$
$= 0.1$

$ns_i = Q_i/Q_j$, $i=1,2, ..., 5$

Hence $\{ns_1, ns_2, ns_3, ns_4, ns_5\} = \{1.0, 2.5, 3.5, 2.0, 1.5\}$. The constraints are

$$5n_1 + 4n_2 + 9n_3 + 7n_4 + 7n_5 \leq 132 \quad (3)$$

$$8n_1 + 9n_2 + 6n_3 + 7n_4 + 8n_5 \leq 142 \quad (4)$$

Substituting ns_i into (3) and (4), we obtain

$F_1 = 5 + 10 + 31.5 + 14 + 10.5 = 71$

$F_2 = 8 + 22.5 + 21 + 14 + 12 = 77.5$

$D_1 = B_1 / F_1 = 132 / 71 = 1.8874$

$D_2 = B_2 / F_2 = 142 / 77.5 = 1.8322$.

D_2 is smaller than D_1 and hence

$n_i = [ns_i * 1.8322]^+$

This results in $n_1 = [1.8322]$, $n_2 = [4.5805]^+$, $n_3 = [6.4127]^+$, $n_4 = [3.6644]^+$, $n_5 = [2.7483]^+$

$\mathbf{n} = \{2,5,7,4,3\}$.

Refer to TABLE II for further computations.

From Table II it can be seen that the final solution is reached in fewer steps due to the good starting values provided by this method.

The optimum reliability of the system is 0.985.

TABLE II

Number of units n	Unreliability of subsystem (10^{-3}) 1	2	3	4	5	Total cost	Total weight
2 5 7 4 3	10.0	0.9766	0.6434	1.6	3.375	142	155
2 5 6 4 3	10.0	0.9766	1.8380	1.6	3.375	133	149
2 5 5 4 3	10.0	0.9766	5.2520	1.6	3.375	124	143
2 4 5 4 3	10.0	3.9060	5.2520	1.6	3.375	120	134
3 4 5 4 3	1.0	3.9060	5.2520	1.6	3.375	125	142

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