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Bubbly booms and welfare

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Inefficient Bubbly Booms?*

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Abstract

We show the competing effects of a housing bubble on the real economy by developing a two-sector dynamic model with housing production. On the one hand, firms can sell or collateralize their houses to obtain financing, so a housing bubble helps firms obtain credit to finance their investment and expand production. On the other hand, a boom in the housing sector crowds out labor in the non-housing sector. We show that the housing booms can generate static and dynamic inefficiencies (welfare losses) only when production externalities in the non-housing sector are sufficiently large. We quantitatively evaluate our model and demonstrate its robustness with model extensions that account for the fundamental values of housing and elastic labor supply. Policies that target labor, housing transactions and output generate different welfare implications.

Keywords: Housing Bubble; Credit Constraint; Collateral Effect; Crowd-out Effect; Housing Policies.

JEL: D92, E22, E44, E62, G1

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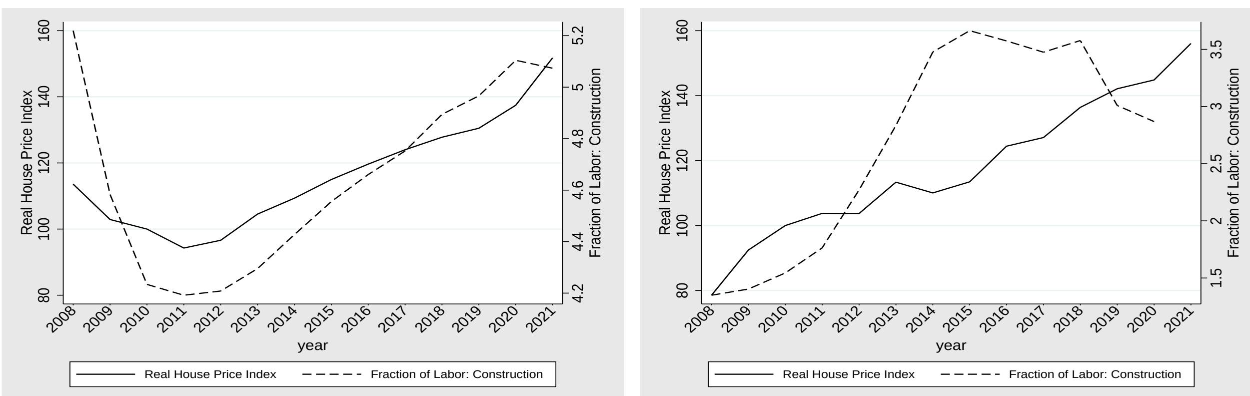
1 Introduction

In recent decades, many countries have witnessed housing booms and production expansions in the housing sector. It is well known that housing serves as a good candidate collateral good for borrowing that can relax firms' financial constraints. However, there are also increasing concerns that the expanding housing sector may "crowd-out" resources for other sectors: if labor and investment flow into the housing sector, non-housing sectors such as manufacturing production can be negatively affected.[1] Figure 1 shows that housing price booms are accompanied by labor flowing into the construction sector in the two largest economies in the world, the United States and China.[2] To this end, we model and quantify the economic and welfare consequences of housing booms with both collateral and crowding-out effects by integrating rational housing bubbles into a two-sector production economy. Then, we use the framework to conduct various policy analyses.



(a) US (b) China

Data Source: FRED (USA), National Bureau of Statistics (CHN). Real house price index of China is calculated using nominal house price index from NBS adjusted by CPI.

Figure 1: House Prices and Labor Force Flows: US and China

Non-housing sector firms can sell or collateralize their housing holdings to make more investment, so rising housing prices crowd in these firms' investment and lead to higher output (the *collateral* effect). However, housing sector booms also compete for labor with the non-housing sector for production, so a housing bubble that raises house prices can crowd out labor for the non-housing sector, leading to lower output (the *crowding-out* effect). We allow for production externalities in the non-housing sector with a free parameter, which captures the

[1]See, e.g., "How high property prices can damage the economy" in the Economist 2022 July edition on concerns over housing booms in both China and the developed. Shi (2018) provides empirical evidence that a booming housing sector can create misallocation of managerial talent.

[2]Housing sector-related employment can go beyond construction sector employment, e.g., workers in upstream industries or the real estate service sector.

extent of knowledge spillover between workers.[3] We find that with sufficiently large production externalities in the non-housing sector, in the steady state, when the loan-to-value (LTV) ratio is low, which means that the collateral effect is relatively weak, housing booms can reduce steady-state output and consumption, causing *static* inefficiency. Moreover, we examine the transition path with a housing boom (either from a bubbleless to a bubbly equilibrium or a relaxation of firms' borrowing constraint induced by financial development or financial policies), we find strong crowding out in the initial periods, which leads to reduced consumption. Interestingly, when the initial LTV ratio is not high, the transition reduces social welfare even if the ultimate steady state features higher consumption (that is, the housing boom is statically efficient), which implies that a housing bubble can cause *dynamic* inefficiency. However, when production externalities in the non-housing sector are low or nil, the collateral effect always dominates the crowding-out effect whereby housing booms are both statically and dynamically efficient.

We use U.S. data to discipline parameter values in our model and find that there are plausibly positive externalities in the U.S. non-housing sector. We show quantitatively that housing bubbles can be inefficient in that they reduce social welfare. While steady states feature higher consumption after the housing booms, along the transition path, consumption first decreases and then increases, and the overall effect is a lower lifetime welfare.

We experiment our framework with extensions. First, we introduce elastic labor supply. Although the aggregate labor supply increases with housing booms, which corresponds to the empirical results of Charles et al. (2018), the crowding-out effect persists whereby the non-housing sector's employment will decline. Quantitatively, the crowding-out effect remains large enough to dominate the collateral effect and results in welfare losses. Second, we add housing services into the households' utility function following Dong et al. (2022). Our extended model shares similar implications regarding the housing price and rent with Dong et al. (2022). Housing price booms still have competing collateral and crowding-out effects. With a credit expansion, housing prices increase while house rents rise mildly,[4] implying a higher price-to-rent ratio and a larger bubble component in the housing price. The transitional dynamics of the extended model are similar to those of baseline model, where consumption and output first decrease following financial development as a result of the strong crowding-out.

Last, we perform three policy analyses on the labor, housing transaction and output markets. The three policies we consider are a labor tax, housing transaction tax and subsidy for

[3] Davis and Dingel (2019) present a model that features positive knowledge spillovers between workers with an exchange of ideas in the non-housing tradable sector.

[4] In Dong et al. (2022), the rent does not change with the LTV ratio. This is because in their model output is given as endowment so a credit expansion will not affect consumption, and housing rent is a function of consumption in equilibrium.

firm revenues. We find that among the three policies, only the housing transaction tax helps to reduce the size of the housing bubble, but this is at the cost of lower consumption and social welfare. The labor tax and subsidy stimulate the demand for housing and lead to higher housing prices, non-housing output and social welfare. These results arise from the fact that housing is an important asset for non-housing firms to obtain external finance and expand production. When we use a tax to restrain house transactions and the housing sector, these firms cannot obtain enough finance to produce. Consequently, we obtain a lower social welfare.

Our paper is related to the vast literature on the role of land and housing in helping firms relax borrowing constraints when facing an incomplete financial market. Fisher (1933) proposes the "collateral channel" in the business cycle. After a bubble bursts, the collateral value decreases so that firms obtain less liquidity, and thus the shock to the asset market is transmitted to the real sector. Kiyotaki and Moore (1997) formalize the collateral channel and note that assets such as housing can act as both factors of production and collateral, and their price is affected by the credit constraint. The interaction between asset prices and credit constraints will reinforce the shocks in the economy and generate positive feedback between asset prices and the production economy. Empirically, Gan (2007), Chaney et al. (2012), Banerjee and Blickle (2016), Banerjee and Blickle (2016), Schmalz et al. (2017), and Bahaj et al. (2020) report extensive evidence of a collateral effect on the link between housing prices and firm investment using data from various countries.

Our paper is also related to a growing literature on rational bubbles. There are two major ways to introduce rational bubbles into an economy. Many models of rational bubbles adopt the overlapping generations (OLG) framework. Tirole (1985), Weil (1987) and Grossman and Yanagawa (1993) model a bubble asset as an asset without fundamentals that can transfer wealth between different generations and crowd out physical capital. They show that when an economy without a bubble is dynamically inefficient, a bubble asset with positive price may exist and crowd out savings, so bubbles can improve dynamic economic efficiency but retard capital accumulation and economic growth. Kocherlakota (2009), Arce and López-Salido (2011), Zhao (2015), Chen and Wen (2017) and Jiang et al. (2019) specifically use an OLG framework to analyze housing (or land) bubbles and the impacts of housing price cycles.[5] The second way of introducing rational bubbles is to consider an infinite-horizon framework, which is nontrivial due to the transversality condition. Kocherlakota (1992) notes that a constraint on debt accumu-

[5]There are also other studies that focus on the asset property of housing to explain the relationship between housing and the real economy. Dong et al. (2021a) regard housing as a safe asset to store value and show that agents tend to hold more housing when the economy faces increased financial market instability. Dong et al. (2021b) shows that an economic slowdown reduces the return of production capital, which increases firms' housing demand and generates a housing boom. Dong et al. (2022) provide a model with heterogeneous agents to explain reduced-form housing demand shocks. In their model, a credit expansion boosts house prices and does not affect rents, which explains the observed substantial volatility of house prices relative to rents.

lation is critical to the existence of bubbles, and Kocherlakota (2008) provides infinite-horizon endowment economy models with such features (solvency constraints on agents) where asset bubbles can emerge. Recently, more studies focus on introducing rational bubbles into a production economy. Kocherlakota (2009), Wang and Wen (2012), Aoki and Nikolov (2015), Miao et al. (2015b), Miao and Wang (2018), Hirano and Yanagawa (2016), and Biswas et al. (2020) introduce rational (sometimes stochastic) bubbles into an infinite-horizon production economy; Martin and Ventura (2012), Farhi and Tirole (2012) and Ikeda and Phan (2019) fit rational bubbles into the OLG framework. Most of these models have two key components: heterogeneity in productivity or investment and credit constraints. The credit constraint can be either exogenous or endogenous as in Miao et al. (2015b), Miao and Wang (2018) and Biswas et al. (2020). In these models, bubbles can exist because they relax the credit constraint and thus provide a liquidity premium. Our approach follows this strand of literature.

Our framework is closest to Miao et al. (2015a), but they have some stark differences. First, in our model, the supply of the bubbly housing asset is endogenous and produced with labor, while in most of the literature, the supply of the bubbly asset is exogenously given. Miao et al. (2015a) consider an extension with endogenous housing supply, but the housing suppliers in their model do not employ any labor, so the housing supply function is only used to pin down the steady-state housing stock and price, and the housing sector does not compete with the non-housing sector for inputs. As a result, their model is silent on the crowding-out effect of housing bubbles. Second, different from some early studies that consider the production of bubble assets such as Kocherlakota (2009), the housing sector is a separate sector in our model. In Kocherlakota (2009), bubble production is a decision made by non-housing firms, so they always have the option not to produce any bubbly assets, which means that as long as we observe a positive stock of the bubbly asset in the steady state, it must be beneficial for non-housing firms. However, in our model, the housing sector is independent of the non-housing sector, so the existence of a housing bubble or a housing boom does not necessarily benefit the non-housing sector. There is also an important strand of literature on inefficient booms (e.g., Lorenzoni (2008) and Bianchi (2011)). While these papers usually focus on the mechanism of pecuniary externalities, our framework features resource reallocation across sectors that can render inefficiencies. Lastly, we consider the production externality of labor in the non-housing sector, which is not mentioned in the exist bubble literature.

The remainder of the paper proceeds as follows. Section 2 presents the baseline model. Section 3 provides the model solution, including individual firms' investment decisions and a characterization of the equilibria. Section 4 uses a quantitative experiment to show the static and dynamic properties of the baseline model, including discussions on the efficiency of the housing bubble. Section 5 presents an extended model with elastic labor supply. Section 6 studies three different policies to improve social welfare or to reduce the size of bubbles. Section 7 concludes the paper. Technical proofs of some propositions and additional figures are relegated to the appendices.

2 The Model

Time is discrete and infinite ($t = 0, 1, \cdots, \infty$), and there are two production sectors: the housing sector and the non-housing sector. Households hold shares of firms from both sectors, provide labor and choose their consumption in each period to maximize their lifetime utility. The non-housing sector consists of a set of heterogeneous firms that rent capital and hire labor to produce a consumption good. The price of the consumption good is normalized to 1. The housing sector hires labor to produce houses that are purchased by the firms in the non-housing sector.

2.1 Production Sectors

2.1.1 Housing Sector

The housing sector hires labor L_t^H to produce housing asset H_t^n using the following decreasing-returns-to scale technology:

$$H_t^n = A_t^H \left(L_t^H\right)^{\sigma},$$

where $0 < \sigma < 1$ and A_t^H is the productivity in the housing sector.

Denote by W_t the wage rate for workers and by P_t the housing price. The optimization problem of firms in the housing sector is then

$$\max \left\{ P_t A_t^H \left(L_t^H\right)^{\sigma} - W_t L_t^H \right\}. \tag{1}$$

This yields the first-order condition

$$P_t A_t^H \sigma \left(L_t^H\right)^{\sigma-1} = W_t.$$

Denote by δ_h the depreciation rate of the housing asset. The law of motion of the aggregate housing stock H_t is then given by

$$H_{t+1} = (1 - \delta_h) H_t + H_t^n.$$

2.1.2 Non-housing Sector

There is a continuum of measure 1 of firms in the non-housing sector. A firm $i \in [0,1]$ combines capital and labor to produce

$$y_t^M(i) = A_t^M(L_t^M)k_t(i)^\alpha(l_t^M(i))^{1-\alpha},$$

where $A_t^M(L_t^M)$ is the aggregate productivity in the non-housing sector and $l_t^M(i)$ is the labor employed by non-housing sector firm i. To capture the production externality of labor in the non-housing sector, we assume that productivity is an increasing function of total labor in the non-housing sector:

$$A_t^M = A^M(L_t^M)^\gamma,$$

where A^M is the baseline productivity of the non-housing sector in the bubbleless equilibrium.

Firms in the non-housing sector can borrow and lend by trading one-period riskless bonds. They can also buy houses from the housing sector and trade houses among each other or use houses as collateral for borrowing. Each firm faces the following borrowing constraint[6]

$$\frac{b_{t+1}(i)}{R_{ft}} \leq \mu P_t h_{t+1}(i),$$

and no equity issuance constraint

$$d_t(i) \geq 0,$$

where $b_t(i)$ and $h_t(i)$ are bonds and the housing stock held by non-housing sector firm i, $\mu \in (0,1)$ is the maximum LTV ratio, $d_t(i)$ is the dividend to shareholders, R_{ft} is the (gross) interest rate and P_t is the housing price.

Following Kiyotaki and Moore (2008), we assume that the transactions of housing assets are illiquid. We impose the resaleability constraint

$$h_{t+1}(i) \geq \omega h_t(i). \tag{2}$$

Denote by δ_k the depreciation rate of physical capital. The law of motion for capital is

$$k_{t+1}(i) = (1-\delta_k)k_t(i) + i_t(i)\epsilon_t(i), \tag{3}$$

where $\epsilon_t(i)$ is the idiosyncratic investment efficiency shock, which is assumed to be indepen-

[6]Similar to our framework, Bayoumi and Zhao (2021) also study multisector economies with financial frictions, where they find that the existence of financial frictions (e.g., inadequate investment) is a crucial cause of housing booms in China.

dent and identically distributed (IID) across time and firms. The budget constraint of non-housing sector firm i is:

$$d_t(i) = A_t^M(L_t^M)k_t(i)^\alpha(l_t^M(i))^{1-\alpha} - W_t l_t^M(i) - P_t[h_{t+1}(i) - (1-\delta_h)h_t(i)] + \frac{b_{t+1}(i)}{R_{ft}} - b_t(i) - i_t(i).$$

As is standard in the literature, firms are owned by the representative households, and thus a firm's objective function is to maximize the sum of the discounted value of its dividends to households:

$$\sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} d_{t+s}(i), \tag{4}$$

where Λ_t is the marginal utility of consumption.

2.2 Households

There is a unit measure of identical households in the economy. Each household supplies 1 unit of labor inelastically in each period. A household chooses consumption C_t and holds firm shares $s_{t+1}(i)$ to maximize its expected lifetime utility

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t\right), \beta \in (0,1), \tag{5}$$

subject to the budget constraint

$$C_t + \int_0^1 s_{t+1}(i)\left[V_t(i) - d_t(i)\right] di = \int_0^1 s_t(i) V_t(i) di + W_t L_t + \Pi_t, \tag{6}$$

where $V_t(i)$ denotes non-housing sector firm i's market value and Π_t is the profit of the housing sector.

Defining λ_t as the Lagrangian multiplier of Equation (6), we obtain the first-order conditions, given by

$$V_t(i) = \beta\mathbb{E}\left\{\frac{\lambda_{t+1}}{\lambda_t} V_{t+1}(i)\right\} + d_t(i),$$

and

$$\frac{1}{C_t} = \lambda_t.$$

Therefore, $\Lambda_t = \lambda_t$ denotes the marginal utility of consumption.

2.3 Competitive Equilibrium

A competitive equilibrium is defined as a sequence of quantities $\{\{i_t(i), l_t^M(i), k_{t+1}(i), y_t(i), h_t(i), b_t(i), s_{t+1}(i)\}_{i\in[0,1]}, L_t^H, C_t\}_{t=0}^{\infty}$ and prices $\{W_t, P_t, \{V_t(i)\}_{i\in[0,1]}\}_{t=0}^{\infty}$ such that:

1. Given prices $\{W_t, P_t\}$, the sequence of quantities $\{i_t(i), l_t^M(i), k_{t+1}(i), h_{t+1}(i), b_{t+1}(i)\}$ and $\{L_t^H\}$ solve the non-housing sector firms' problem (4) and the housing firms' problem (1).

2. Given prices $\{W_t, V_t(i)\}$, the sequence $\{C_t, s_t(i)\}$ maximizes household utility (5) subject to (6).

3. All markets clear:

$$s_t(i) = 1, \tag{7}$$

$$\int_0^1 l_t^M(i)di + L_t^H = 1, \tag{8}$$

$$\int_0^1 h_{t+1}(i)di - \int_0^1 (1-\delta_h)h_t(i)di = H_t^n, \tag{9}$$

$$C_t + \int_0^1 i_t(i)di = \int_0^1 y_t(i)di, \tag{10}$$

$$\int_0^1 b_{t+1}(i)di = 0. \tag{11}$$

Note that Equation (7) is the market clearing condition of the stock market, Equation (8) is the market clearing condition of the labor market, Equation (9) is the market clearing condition of the housing market, Equation (10) is the market clearing condition of the goods market and Equation (11) is the market clearing condition for intertemporal bonds.

3 Model Solution

3.1 Decision Rules

In this section, we solve non-housing sector firms' problem and determine the investment decision rule and the pricing function. The first-order condition for labor of the non-housing sector firms is

$$A_t^M(L_t^M)(1-\alpha)k_t(i)^{\alpha}(l_t^M(i))^{1-\alpha} = W_t,$$

which implies

$$l_t^M(i) = \left[\frac{(1-\alpha)A_t^M(L_t^M)}{W_t}\right]^{\frac{1}{\alpha}} k_t(i).$$

Thus, we obtain the firms' capital return

$$R_t(i) = y_t(i) - W_t l_t^M(i) = R_{kt} k_t(i),$$

where

$$R_{kt} = \frac{\alpha W_t}{1-\alpha}\left[\frac{(1-\alpha)A_t^M(L_t^M)}{W_t}\right]^{\frac{1}{\alpha}}.$$

Note that for a single firm, total labor in the non-housing sector L_t^M is seen as exogenous, so it will only optimize $l_t^M(i)$ and regard A_t^M as a given variable.

We show that the firm's optimal decision rule is as follows using a "guess-and-verify" procedure.

Proposition 1. *A firm's optimal investment decision rule is given by a threshold strategy:*

$$i_t(i) = \begin{cases} R_{kt}k_t(i) + (1-\omega+\omega\mu-\delta_h)P_t h_t(i) - b_t(i), & \epsilon_t(i) > \epsilon_t^* \\ 0, & \epsilon_t(i) < \epsilon_t^* \end{cases}, \tag{12}$$

where ϵ_t^* is a time-varying cutoff independent of an individual firm i, satisfying $Q_t\epsilon_t^* = 1$. Q_t is Tobin's Q at time t. ϵ_t^* is determined by the following Euler equation :

$$\frac{1}{\epsilon_t^*} = \beta\int\frac{\Lambda_{t+1}}{\Lambda_t}\left[R_{k,t+1}L(\epsilon_{t+1}^*) + (1-\delta_k)\frac{1}{\epsilon_{t+1}^*}\right]dF(\epsilon), \tag{13}$$

where $L(\cdot) > 1$ captures the liquidity premium of one unit of net revenue and is determined by

$$L(\epsilon^*) = \int \max\left\{1, \frac{\epsilon(i)}{\epsilon^*}\right\} dF(\epsilon).$$

When the aggregate demand for housing $\Omega_{t+1} \equiv \int_0^1 h_{t+1}(i)di > 0$, the equilibrium price of housing is determined by

$$P_t = \beta\int\frac{\Lambda_{t+1}}{\Lambda_t}\left[1 + (L(\epsilon_{t+1}^*)-1)(1-\omega+\omega\mu) - L(\epsilon_{t+1}^*)\delta_h\right]P_{t+1}dF(\epsilon). \tag{14}$$

The gross interest rate follows

$$\frac{1}{R_{ft}} = \beta\int\frac{\Lambda_{t+1}}{\Lambda_t}L(\epsilon_{t+1}^*)dF(\epsilon). \tag{15}$$

PROOF

See Appendix A

3.2 General Equilibrium

Define aggregate variables $L_t^M = \int_0^1 l_t^M(i)di$, $L_t^H = \int_0^1 l_t^H(j)dj$, $I_t = \int_0^1 i_t(i)di$, $K_t = \int_0^1 k_t(i)di$, $Y_t = \int_0^1 y_t(i)di$ and $H_t(i) = \int_0^1 h_t(i)di$. By the factor demand functions of non-housing sector firms, we have $L_t^H = \left[\frac{W_t}{\sigma A_t^H}\right]^{\frac{1}{\sigma-1}}$, $L_t^M = \left[\frac{(1-\alpha)A_t^M(L_t^M)}{W_t}\right]^{\frac{1}{\alpha}}$ and $Y_t = A_t^M \left[\frac{(1-\alpha)A_t^M}{W_t}\right]^{\frac{1-\alpha}{\alpha}} K_t$. From the bond market clearing condition (11), we know that the aggregate bond in equilibrium equals 0, so we omit the bond term in the general equilibrium definition.

Proposition 2. *The general equilibrium paths of the model are characterized by 12 aggregate variables, $\{C_t, I_t, L_t^M, L_t^H, Y_t, K_{t+1}, H_{t+1}, P_t, \epsilon_t^*, Q_t, R_{ft}, W_t\}$, which are determined by the following nonlinear system of equations:*

$$
\begin{aligned}
Y_t &= A_t^M(L_t^M)K_t^\alpha(L_t^M)^{1-\alpha} && (16)\\
C_t + I_t &= Y_t && (17)\\
L_t^M + L_t^H &= 1 && (18)\\
I_t &= \left[\alpha Y_t + (1-\omega+\omega\mu-\delta_h)P_tH_t\right](1-F(\epsilon_t^*)) && (19)\\
K_{t+1} &= (1-\delta_k)K_t + \omega(\epsilon_t^*)I_t && (20)\\
H_{t+1} &= (1-\delta_h)H_t + A_t^H(L_t^H)^\sigma && (21)\\
W_t &= P_tA_t^H\sigma(L_t^H)^{\sigma-1} && (22)\\
W_t &= (1-\alpha)A_t^M(L_t^M)K_t^\alpha(L_t^M)^{-\alpha} && (23)\\
\frac{P_t}{C_t} &= \beta\int\frac{1}{C_{t+1}}\left[1+(L(\epsilon_{t+1}^*)-1)(1-\omega+\omega\mu)-L(\epsilon_{t+1}^*)\delta_h\right]P_{t+1}dF(\epsilon) && (24)\\
\frac{Q_t}{C_t} &= \beta\int\frac{1}{C_{t+1}}\left[\frac{\alpha Y_{t+1}}{K_{t+1}}L(\epsilon_{t+1}^*) + (1-\delta_k)Q_{t+1}\right]dF(\epsilon) && (25)\\
\frac{1}{C_tR_{ft}} &= \beta\int\frac{1}{C_{t+1}}L(\epsilon_{t+1}^*)dF(\epsilon) && (26)\\
Q_t\epsilon_t^* &= 1 && (27)
\end{aligned}
$$

where the coefficient $\omega(\epsilon_t^*) = \frac{\int_{\epsilon>\epsilon_t^*}\epsilon dF}{1-F(\epsilon_t^*)} > \epsilon_t^*$ measures the average marginal efficiency of aggregate investment.

PROOF
See Appendix B.

3.3 Characterization of Equilibria

In this section, we characterize the bubbleless and bubbly equilibria and calculate the aggregate variables in the steady states.

3.3.1 Bubbleless Steady State

We use subscript f to denote a variable in the equilibrium without bubbles, and we remove the time subscript for variables in steady state. In a bubbleless equilibrium, $P = 0$, so we must have $L_f^H = 0$ and $L_f^M = 1$, which means that the model acts as if we do not have a housing sector. Obviously, now we have $A_t^M = A^M$. Equation (19) yields $I_f = \alpha Y_f(1 - F(\epsilon_f^*))$, and hence, (20) becomes

$$K_f = (1-\delta_k)K_f + \omega(\epsilon_f^*)\left[1 - F(\epsilon_f^*)\right]\alpha Y_f = (1-\delta_k)K_f + \alpha Y_f \int_{\epsilon > \epsilon_f^*} \epsilon dF(\epsilon).$$

The return of capital is

$$R_{kf} = \frac{\alpha Y_f}{K_f} = \frac{\delta_k}{\int_{\epsilon > \epsilon_f^*} \epsilon dF(\epsilon)}, \tag{28}$$

and from Equation (25), we can solve for Tobin's Q and the cutoff value ϵ_f^*

$$1 - \beta(1-\delta_k) = \beta\delta_k \frac{\int \max\left\{\epsilon_f^*, \epsilon\right\} dF(\epsilon)}{\int_{\epsilon > \epsilon_f^*} \epsilon dF(\epsilon)}. \tag{29}$$

Since $L_f^H = 0$, we have $R_{kf} = \alpha \frac{Y_f}{K_f} = \alpha A^M K_f^{\alpha-1}$. Given ϵ_f^*, we can solve for R_{kf} from (28), and thus we can solve for K_f. To ensure that consumption is nonnegative in the steady state, we need $R_{kf} > \frac{\alpha\delta_k}{\omega(\epsilon_f^*)}$. The above analysis yields the following proposition:

Proposition 3. *Equation*

$$1 - \beta(1-\delta_k) = \beta\delta_k \frac{\int \max\left\{\epsilon_f^*, \epsilon\right\} dF(\epsilon)}{\int_{\epsilon > \epsilon_f^*} \epsilon dF(\epsilon)} \tag{30}$$

has a unique solution for $\epsilon_f^* \in (\epsilon_{min}, \epsilon_{max})$. If R_{kf} in Equation (28) satisfies:

$$R_{kf} > \frac{\alpha\delta_k}{\omega(\epsilon_f^*)}, \tag{31}$$

then $\frac{1}{\epsilon_f^*}$ is equal to Tobin's Q in the bubbleless steady state.

3.3.2 Bubbly Steady State

In this subsection, we assume that $P_t = P > 0$ for all t. We continue to remove the time subscript for variables in steady state, and we use subscript b to denote variables in the bubbly steady state.

In the bubbly equilibrium, Equation (24) suggests that the cutoff value ϵ_b^* satisfies

$$\frac{1}{\beta} - 1 = (1 - \omega + \omega\mu) \int_{\epsilon > \epsilon_b^*} \frac{\epsilon - \epsilon_b^*}{\epsilon_b^*} dF(\epsilon) - \delta_h \int \max\left\{\frac{\epsilon}{\epsilon_b^*}, 1\right\} dF(\epsilon). \tag{32}$$

By the intermediate value theorem, if the parameters satisfy the following condition

$$\frac{1}{\beta} - 1 < (1 - \omega + \omega\mu) \left(\frac{1}{\epsilon_{min}} \int \epsilon dF(\epsilon) - 1\right) - \frac{\delta_h}{\epsilon_{min}} \int \epsilon dF(\epsilon), \tag{33}$$

then Equation (32) has a unique solution $\epsilon_b^* \in (\epsilon_{min}, \epsilon_{max})$. After obtaining ϵ_b^*, we can solve for the labor allocation and housing price.

With ϵ_b^*, we can solve for the return on capital R_{kb} using Equation (25) and obtain

$$R_{kb} = \frac{1 - \beta(1 - \delta_k)}{\beta \int \max\{\epsilon, \epsilon_b^*\} dF(\epsilon)}. \tag{34}$$

Given the capital return, we can solve for the steady state:

Proposition 4. *Suppose that inequality (33) holds and that the return to capital R_{kb} in Equation (34) satisfies*

$$R_{kb} > \frac{\alpha\delta_k}{\omega(\epsilon_b^*)}. \tag{35}$$

Then, there exists a bubbly steady state.

Under a stricter condition,

$$R_{kb} > \frac{\alpha\delta_k}{\omega(\epsilon_f^*)}, \tag{36}$$

the bubbly and bubbleless steady states coexist if and only if

$$\frac{1}{\beta} - 1 < (1 - \omega + \omega\mu) \int_{\epsilon > \epsilon_f^*} \frac{\epsilon - \epsilon_f^*}{\epsilon_f^*} dF(\epsilon) - \delta_h \int \max\left\{\frac{\epsilon}{\epsilon_f^*}, 1\right\} dF(\epsilon), \tag{37}$$

where cutoffs ϵ_b^* and ϵ_f^* are determined by Equations (32) and (29), respectively, and the variables in the bubbly steady state $\{P_b, K_b, L_b^H\}$ are given by the following system:

$$P \frac{A^H}{\delta_h A^M(L^M)} \left(\frac{\alpha A^M(L^M)}{R_{kb}}\right)^{\frac{\alpha}{\alpha-1}} \frac{(L_b^H)^\sigma}{1 - L_b^H} = \frac{1}{1 - \omega + \omega\mu - \delta_h} \left[\frac{\alpha\beta\delta_k}{1 - \beta(1 - \delta_k)} \frac{\int \max\{\epsilon, \epsilon_b^*\} dF(\epsilon)}{\int_{\epsilon > \epsilon_b^*} \epsilon dF(\epsilon)} - \alpha\right], \tag{38}$$

$$R_{kb} = \alpha \frac{Y_b}{K_b} = \alpha (K_b)^{\alpha - 1} (L_b^M)^{1 + \gamma - \alpha}, \tag{39}$$

and

$$L_b^H = 1 - L_b^M = \left(\frac{W_t}{\sigma P A^H}\right)^{\frac{1}{\sigma-1}}. \tag{40}$$

Note that in the above proposition, condition (35) ensures that consumption is nonnegative in the steady state and condition (36) ensures that the bubbleless steady state can still exist when the bubbly steady state exists.

Comparing the two steady states, we obtain the following proposition.

Proposition 5. *If the bubbleless and bubbly steady states coexist, then $\epsilon_b^* > \epsilon_f^*$, $Q_b < Q_f$, $R_{kb} < R_{kf}$ and $R_{fb} > R_{ff}$*

PROOF

See Appendix C.

Proposition 5 states that the aggregate efficiency of investment is higher in the bubbly steady state since the existence of bubbles relaxes financial constraints for firms with high investment efficiencies. However, since in the bubbly equilibrium some labor is allocated to produce the bubbly housing asset, it is not immediately clear whether the bubbly steady state features a higher capital stock, output and consumption.

Since we already have $R_{kb} = \frac{1-\beta(1-\delta_k)}{\beta \int \max\left\{\epsilon,\epsilon_b^*\right\} dF(\epsilon)}$ and $R_{kf} = \frac{1-\beta(1-\delta_k)}{\beta \int \max\left\{\epsilon,\epsilon_f^*\right\} dF(\epsilon)}$, we can compare capital and output in the two steady states as follows:

$$\frac{K_b}{K_f} = \left(\frac{\int \max\left\{\epsilon,\epsilon_f^*\right\} dF(\epsilon)}{\int \max\left\{\epsilon,\epsilon_b^*\right\} dF(\epsilon)}\right)^{\frac{1}{\alpha-1}} \left[1 - L_b^H\right]^{\frac{\alpha-1-\gamma}{\alpha-1}}, \tag{41}$$

$$\frac{Y_b}{Y_f} = \left(\frac{\int \max\left\{\epsilon,\epsilon_f^*\right\} dF(\epsilon)}{\int \max\left\{\epsilon,\epsilon_b^*\right\} dF(\epsilon)}\right)^{\frac{\alpha}{\alpha-1}} \left[1 - L_b^H\right]^{\frac{1+\gamma-\alpha}{\alpha-1}}, \tag{42}$$

with $L_b^H = \left(\frac{W_t}{\sigma P A^H}\right)^{\frac{1}{\sigma-1}}$. Since $\epsilon_f^* < \epsilon_b^*$ and $L_b^H > 0$, the above two ratios can be either smaller or larger than 1.

4 Quantitative Analysis

In this section, we quantitatively analyze the properties of our model. In particular, we find that a housing bubble has both a collateral effect and crowding-out effect on the economy and that housing bubbles can lead to both static and dynamic inefficiencies (welfare losses). Note that in our model, a larger housing market can crowd out the non-housing sector in two ways:

first, a larger housing sector leads to lower labor supply for firms; second, the labor crowding-out effect leads to lower productivity in the non-housing sector. Comparing steady states, we find that it is possible that the crowding-out effect dominates the collateral effect in a bubbly boom, and the housing bubble will reduce the aggregate output and consumption, causing static inefficiency. During the transition path from one steady state to another with a bubbly boom, it is also likely that the crowding-out effect of bubbles will reduce consumption in initial periods, and the transition can cause dynamic inefficiency.

As is standard in the literature, we set the capital share $\alpha = 0.4$. Since we employ an annual calibration, we set the discount factor $\beta = 0.96$ and the depreciation rate $\delta_k = 0.1$. Following Wang and Wen (2012), we assume that investment efficiency $\epsilon(i)$ follows a Pareto distribution, $F(\epsilon) = 1 - \epsilon^{-\theta}$ with support $(1, \infty)$. To pin down the shape parameter θ, we follow the method of Dong and Xu (2022). Note that in the bubbleless steady state, we have $R_f = \frac{\alpha Y_f}{K_f} = \frac{1-\beta(1-\delta_k)}{\beta\Gamma(\epsilon_f^*)\epsilon_f^*}$, which equals the marginal productivity of capital $MPK = \frac{\alpha Y}{K}$, so we set $\theta = 2.5$ such that the marginal productivity of capital equals 0.08 as in Caselli and Feyrer (2007).

We then set the depreciation of housing assets δ_h and the resaleability constraint ω to target the value of commercial housing stock excluding apartments to the GDP ratio $\frac{PH_b}{PH_b^n+Y}$ and investment ratio $\frac{I_b}{Y_b}$. Specifically, we focus on the case where $\mu = 0.65$.[7] Note that in the bubbly steady state, we have:

$$\frac{I_b}{Y_b} = \frac{I_b}{K_b}\frac{K_b}{Y_b} = \frac{\alpha\beta\delta_k}{1-\beta(1-\delta_k)} \frac{\int \max\left\{\epsilon, \epsilon_b^*\right\} dF(\epsilon)}{\int_{\epsilon > \epsilon_b^*} \epsilon dF(\epsilon)}, \tag{43}$$

$$\frac{PH_b}{Y_b} = \frac{1}{1-\omega+\omega\mu-\delta_h}\left[\frac{\alpha\beta\delta_k}{1-\beta(1-\delta_k)} \frac{\int \max\left\{\epsilon, \epsilon_b^*\right\} dF(\epsilon)}{\int_{\epsilon > \epsilon_b^*} \epsilon dF(\epsilon)} - \alpha\right]. \tag{44}$$

and ϵ_b^* is given by:

$$\frac{1}{\beta} - 1 = (1-\omega+\omega\mu)\int_{\epsilon > \epsilon_b^*} \frac{\epsilon - \epsilon_b^*}{\epsilon_b^*} dF(\epsilon) - \delta_h \int \max\left\{\frac{\epsilon}{\epsilon_b^*}, 1\right\} dF(\epsilon), \tag{45}$$

Note that in the steady state $H_b^n = \delta_h H_b$; then, the investment ratio and real estate value to GDP ratio are only functions of $\{\delta_h, \omega\}$. We take $\delta_h = 0.1$ and $\omega = 0.5$, so $\frac{PH_b}{PH_b^n+Y}(\mu = 0.65) = 39\%$ in our model, as documented in Case et al. (2000) and Iacoviello and Neri (2010), and $\frac{I_b}{Y_b}(\mu = 0.65) = 20.2\%$.

[7] As a rule of thumb, the LTV ratio should not exceed 80%. According to the Comptroller's Handbook of OCC (Office of the Comptroller of the Currency), Version 2 2022, the supervisory LTV limit of raw land is 65% and the supervisory LTV limit of commercial construction is 80%. Since the banks' LTV limit should not exceed the SLTV limit, we focus on the case where $\mu = 0.65$.

Table 1: Parameters

Parameters	Meaning	Value	Target
β	Discount factor	0.96	Standard
α	Share of Capital income	0.4	Capital share
δ_k	Depreciation rate of capital	0.1	Standard
θ	Shape parameter	2.5	Bubbleless MPK
ϵ_{min}	Lowest investment efficiency	1	Wang and Wen (2012)
δ_h	Depreciation rate of housing	0.1	Investment ratio & real estate value
ω	Restriction on house selling	0.5	Investment ratio & real estate value
γ	Labor production externality	0.5	Estimation
A^M	TFP of non-housing sector: baseline	1	Normalization
A^H	TFP of housing sector	0.95	Output & labor shares
σ	House production function	0.8	Output & labor shares

For the labor externality in the non-housing sector γ, note that in steady state, we have:

$$\frac{Y}{L} = A^M(L^M)^\gamma \left(\frac{K}{L}\right)^\alpha \Rightarrow \underbrace{\log\left(\frac{Y}{L}\right)}_{\text{labor productivity}} = \alpha \cdot \underbrace{\log\left(\frac{K}{L}\right)}_{\text{capital intensity}} + \gamma \cdot \underbrace{\log(L^M)}_{\text{total hours}} + const. \tag{46}$$

Thus, we can estimate the value of γ using the manufacturing sector data. We obtain data on labor productivity, capital intensity, total employment and average working hours in the manufacturing sector from the NBER-CES Manufacturing Industry Database and regress the log of labor productivity (the ratio of value added to total working hours) on the log of total working hours, controlling for capital intensity and industry fixed effects, using the sample from 1970 to 2018, and the estimate of γ is 0.5146 with t-statistics $t = 45.41$, so we set $\gamma = 0.5$.

Finally, we choose the housing sector parameters, $\{\sigma, A^H\}$, to calibrate the labor share of the construction sector, $\frac{L^H}{L^M+L^H}$, and the output share of the construction sector, $\frac{PH^n}{PH^n+Y}$, where $H^n = A^H(L^H)^\sigma$ in the US economy, which are 5% and 4%, respectively. For simplicity, we normalize the baseline non-housing sector TFP $A^M = 1$. The results are $\sigma = 0.9$ and $A^H = 0.95$.

The parameters we use are summarized in Table 1

4.1 Comparison of Steady States

We compare steady states along two dimensions. One is for given parameter values, where we compare the bubbly steady state with the bubbleless steady state. The other setting is to compare the bubbly steady states under different μ (the borrowing multiplier against housing collateral), which is a proxy for financial development.

Figures 2(a) and 2(b) show the non-housing sector output and consumption in both the bubbleless and bubbly steady states under different levels of μ. When the financial market is relatively underdeveloped, consumption in the bubbly steady state is lower than that in the bubbleless steady state, while with μ increasing, consumption in the bubbly steady state increases and eventually exceed its counterpart in the bubbleless steady state. Regarding output, when μ is small, an increase in μ will lead to a decrease in L^M, which means lower productivity, so the output will decrease with μ. When μ is large, the effect on productivity is relatively small, and the collateral effect grows faster, so output increases with μ. In sum, in steady state, the existence of a housing bubble can generate *static* inefficiency when the financial market is underdeveloped. Figures 2(c) and 2(d) show that in the bubbly steady states, with the development of the financial system (increase in μ), the price of housing increases and more labor moves to the housing sector.

The intuition for the output and consumption comparison is as follows. We can show that in any steady state (bubbleless or bubbly), the output in the non-housing sector is given by:

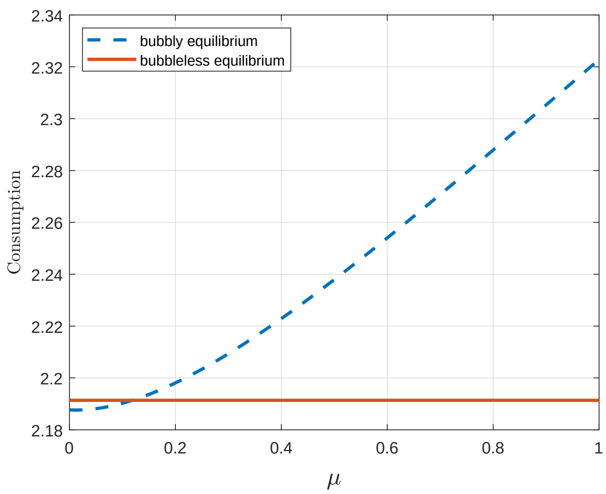
$$Y = \left[A^M(L^M) \left(\frac{R_k}{\alpha A^M(L^M)} \right)^{\frac{\alpha}{\alpha-1}} \right] L^M = A^M \left[\left(\frac{R_k}{\alpha A^M} \right)^{\frac{\alpha}{\alpha-1}} \right] (L^M)^{\frac{\alpha-1-\gamma}{\alpha-1}} \tag{47}$$

with the first term $A^M \left(\frac{R_k}{\alpha A^M} \right)^{\frac{\alpha}{\alpha-1}}$ increasing with ϵ^* (and thus in μ) and the second term $(L^M)^{\frac{\alpha-1-\gamma}{\alpha-1}}$ decreasing with ϵ^*. The first term denotes the collateral effect of the bubble. The existence of the housing bubble relaxes the credit constraint and allows non-housing firms to make more investment and accumulate more capital stock, which will increase non-housing output and consumption. The second term denotes the crowding-out effect of the bubble, which arises because the existence of a housing sector will crowd out labor for non-housing firms, reducing output and consumption. Here, due to the existence of γ, the crowding-out effect is exacerbated by the productivity channel, so output in the bubbly steady state is always lower.

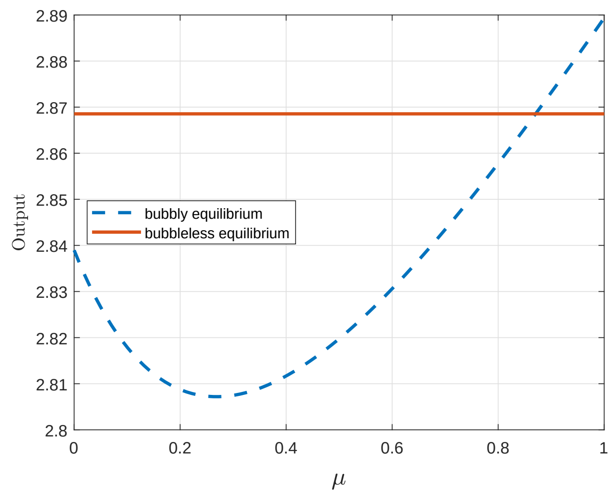
Similarly, steady-state consumption is given by

$$C = \left(\frac{R_k}{\alpha A^M} \right)^{\frac{\alpha}{\alpha-1}} \left[\left(\frac{\alpha A^M}{R_k} \right) \left(\frac{R_k}{\alpha} - \frac{\delta_h}{\omega(\epsilon^*)} \right) \right] (L^M)^{\frac{\alpha-1-\gamma}{\alpha-1}} \tag{48}$$

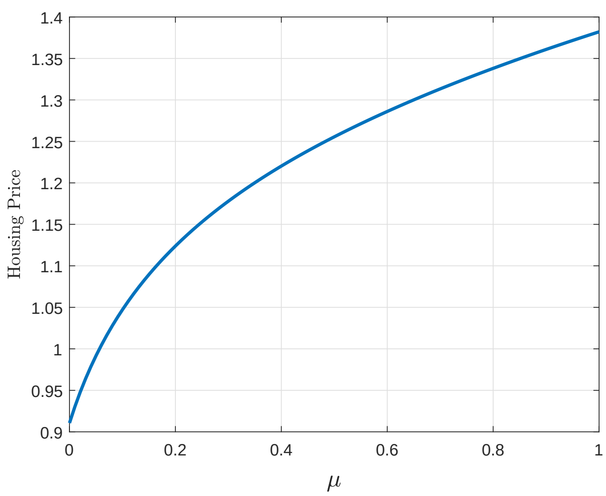
The first term of consumption, $\left(\frac{R_k}{\alpha A^M} \right)^{\frac{\alpha}{\alpha-1}}$, is larger in the bubbly steady state, which is the result of the collateral effect of the housing bubble, while the last term $(L^M)^{\frac{\alpha-1-\gamma}{\alpha-1}}$ is lower in the bubbly steady state, which shows the crowding-out effect of the housing bubble. The middle term, $\left(\frac{\alpha A^M}{R_k} \right) \left(\frac{R_k}{\alpha} - \frac{\delta_h}{\omega(\epsilon^*)} \right)$, denoting the consumption rates in the two steady states, is



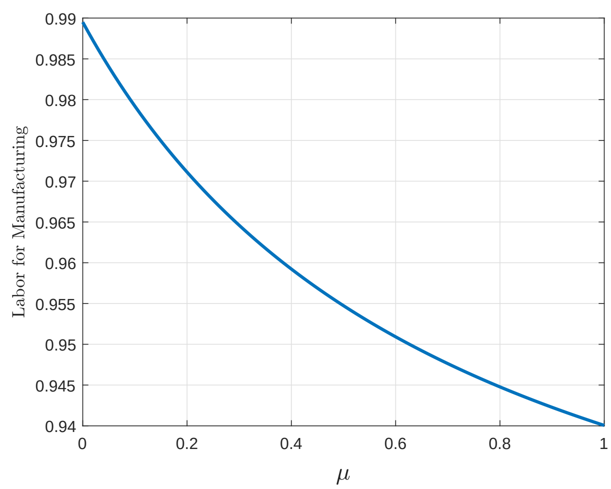
(a) Comparison of Consumption



(b) Comparison of Output



(c) Price of Housing



(d) Labor Allocation

Note: Figure 2(a) and 2(b) compare the output and consumption in the bubbleless and bubbly steady states, Figure 2(c) shows the house price in bubbly steady states with different levels of μ, and Figure 2(d) shows the fraction of labor in the non-housing sector in bubbly steady states with different μ.

Figure 2: Comparison of Steady States

an increasing function of μ. Intuitively, in the bubbly steady state, high-efficiency firms can raise more money to make more investment, so the aggregate investment efficiency is higher, which means that investment rate $\frac{I}{Y}$ is lower when μ is larger, leading to a higher consumption ratio, which is another effect of the collateral channel of the housing bubble.

The mathematical analysis above illustrates two effects from the housing bubble: the *crowding-out effect* and the *collateral effect*. The crowding-out effect means that the existence of a housing bubble raises housing prices, attracting more labor to the housing sector and crowding out labor for the non-housing sector. Moreover, due to the production externality of labor in the non-housing sector, a lower labor supply in the non-housing sector will also lead to lower productivity, exacerbating the crowding-out effect. The collateral effect exists because the housing bubble can provide liquidity and non-housing firms can use housing assets to finance their investment and expand their production. Given other parameters, when μ is quite low, firms can derive relatively little liquidity from housing assets, so the collateral effect is small. As a result, the bubbly steady state may have lower consumption and output despite that average investment efficiency is higher. When μ becomes larger, the collateral effect increases and eventually dominates the crowding-out effect, and bubbles become beneficial to social welfare. In summary, in steady state, housing bubbles can yield *static* inefficiency.

4.2 Transitional Dynamics

We turn to the dynamic properties of our model. In particular, we will solve two transition dynamics from 1) the bubbleless steady state toward a bubbly steady state and 2) from the bubbly steady state with a low μ toward one with a higher μ. The latter can illustrate the dynamics of an economy with financial development or regulatory changes that allow more borrowing against certain housing collateral.

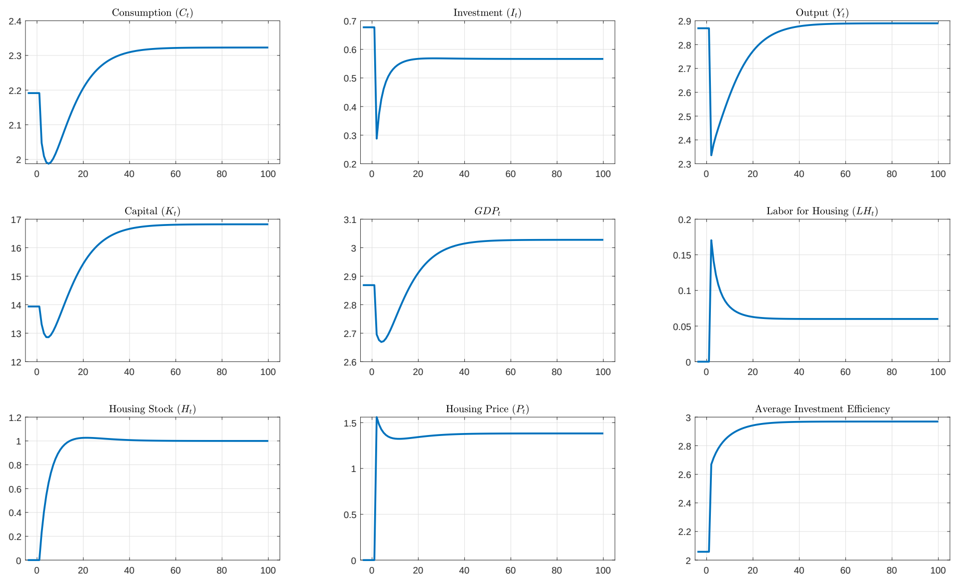
4.2.1 Transitions from Bubbleless to Bubbly Steady State

Suppose that before period 1, the economy remains in its bubbleless steady state, and at period 1, the economy starts to transit to a bubbly steady state. We solve the transitional dynamics under the parameter set given in Section 4.1 and $\mu = 1$.[8] The transitional dynamics are graphically shown in Figure 3.

The housing price jumps immediately since firms can use housing to refinance their investment, so the demand for houses increases. As a result, a fraction of labor moves into the housing sector. Due to the crowding-out effect, non-housing output decreases, and thus con-

[8]Changing the value of μ will only change the values of economic variables in the new steady state but will not affect the shape of the transition paths.

Figure 3: Transitional Dynamics from Bubbleless to Bubbly Steady States

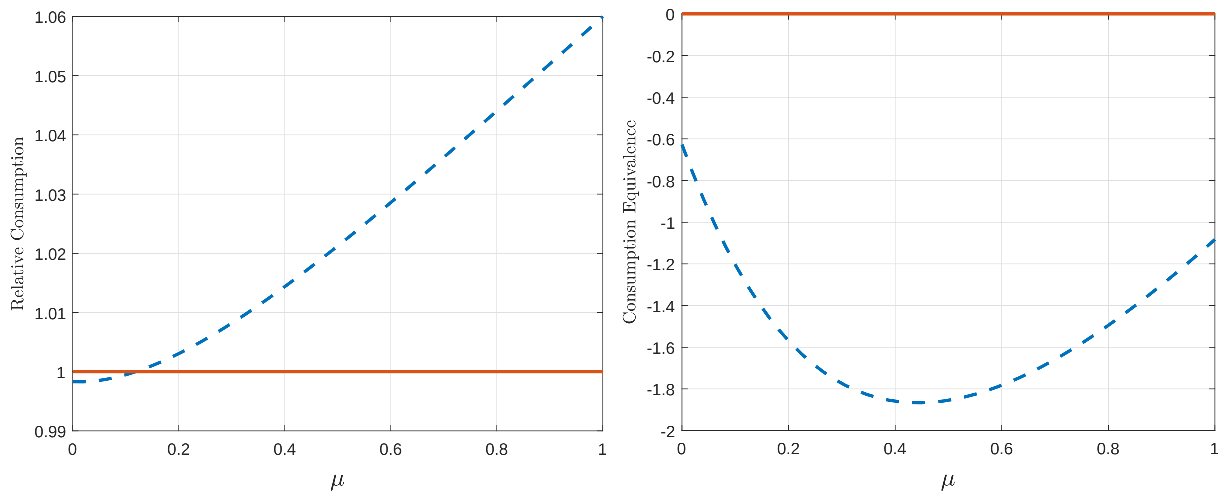


Note: In this figure, we show the transition path from the bubbleless steady state to the bubbly steady state with $\mu = 1$.

sumption and investment also decline (note that the housing stock is low in the first several periods, so firms can borrow little for investment using housing collateral), and the capital stock also initially decreases. However, with the accumulation of housing stock, the collateral effect becomes more important, so investment quickly increases and capital stock starts to accumulate. Some labor in the housing sector then returns to the non-housing sector, and output begins to recover. Consequently, consumption begins to rise and eventually exceeds that in the bubbleless steady state. With the outflow of labor into the housing sector and the accumulation of the housing stock, supply and demand for new housing decrease, so the housing price and housing stock return to their new bubbly steady state levels. During the transition path, GDP (the total output of the non-housing sector and the housing sector) first decreases and then rises.

As we have seen that on the transition path from a bubbleless to bubbly steady state, consumption first decreases and then increases to a new level, it is natural to ask whether this bubbly boom path is welfare-enhancing. Specifically, we can calculate the social welfare on the two paths: 1) one is to stay at the bubbleless steady state and 2) the other is to transit to the new bubbly steady state from the bubbleless steady state.[9] In Figure 4, we show the welfare

[9]To calculate welfare, we assume that the economy will converge to the new steady state after 1000 periods (that is, 1000 years with each period denoting a year).



(a) Relative Consumption (b) Welfare Comparison

Note: Figure 4(a) shows the ratios of consumption in different bubbly steady states and that in the bubbleless steady state, and Figure 4(b) shows the welfare effect of the transition using consumption equivalence.

Figure 4: Welfare Analysis: Bubbleless to Bubbly

comparisons under a given μ in each row. In particular, we compare social welfare in terms of consumption equivalence on the two paths following Lucas (1987). Figure 4(a) shows the ratio of consumption in the bubbly and bubbleless steady states, and Figure 4(b) shows the welfare gain or loss from the sudden transition from the bubbleless to the bubbly steady state. We define the welfare gain or loss of the transition as the permanent percentage increase (positive) or decrease (negative) in consumption in the bubbleless steady state that is required for the representative household to remain indifferent between living in the bubbleless and bubbly economies.

From Figure 4, we see that although the bubbly steady state may be desirable, which means that it features higher steady-state consumption, the transition from the bubbleless to the bubbly steady state may decrease total social welfare. This is because in the first few periods during the transition path, the jump in house prices will attract a large fraction of labor to the housing sector, and this will lead to a strong crowding-out effect, through both labor supply and productivity. As a result, output and consumption will decrease, and the larger μ is, the larger the initial decline. Due to the initial decrease in consumption, the transition from a bubbleless to a bubbly steady state may not be welfare-enhancing.

4.2.2 Transitions after an Increase in μ

A housing boom can also be driven by an increase in μ, which represents financial development or policies that relax firms' borrowing constraint. We focus on the bubbly equilibria when discussing the transitions after an increase in μ since in the bubbleless equilibria, the housing price is 0 and changes in μ have no impacts.

As before, we can calculate the transition path from a low-μ steady state toward a high-μ steady state. In particular, Figure 5 shows the transition path from a bubbly steady state with $\mu = 0$ toward a bubbly steady state with $\mu = 1$.[10] We can see that the shape of the transition path is quite similar to that in the previous subsection, where we calculate the transition path from a bubbleless to a bubbly steady state.

We can also conduct a welfare analysis for the transition from a low-μ bubbly steady state to a high-μ steady state. We start the bubbly economy with $\mu = 0$ and increase it permanently to higher values. Following the same procedures as for the transitions from the bubbleless to the bubbly steady state, we can calculate the welfare impact of housing booms. Figure 6 shows the numerical results. Figure 6(a) provides a comparison of consumption in the steady states, and obviously a higher μ leads to higher steady-state consumption. Figure 6(b) shows the welfare effect of the transition, and we see that due to the initial crowding-out effect, the transition toward a statically efficient steady state may be dynamically inefficient.

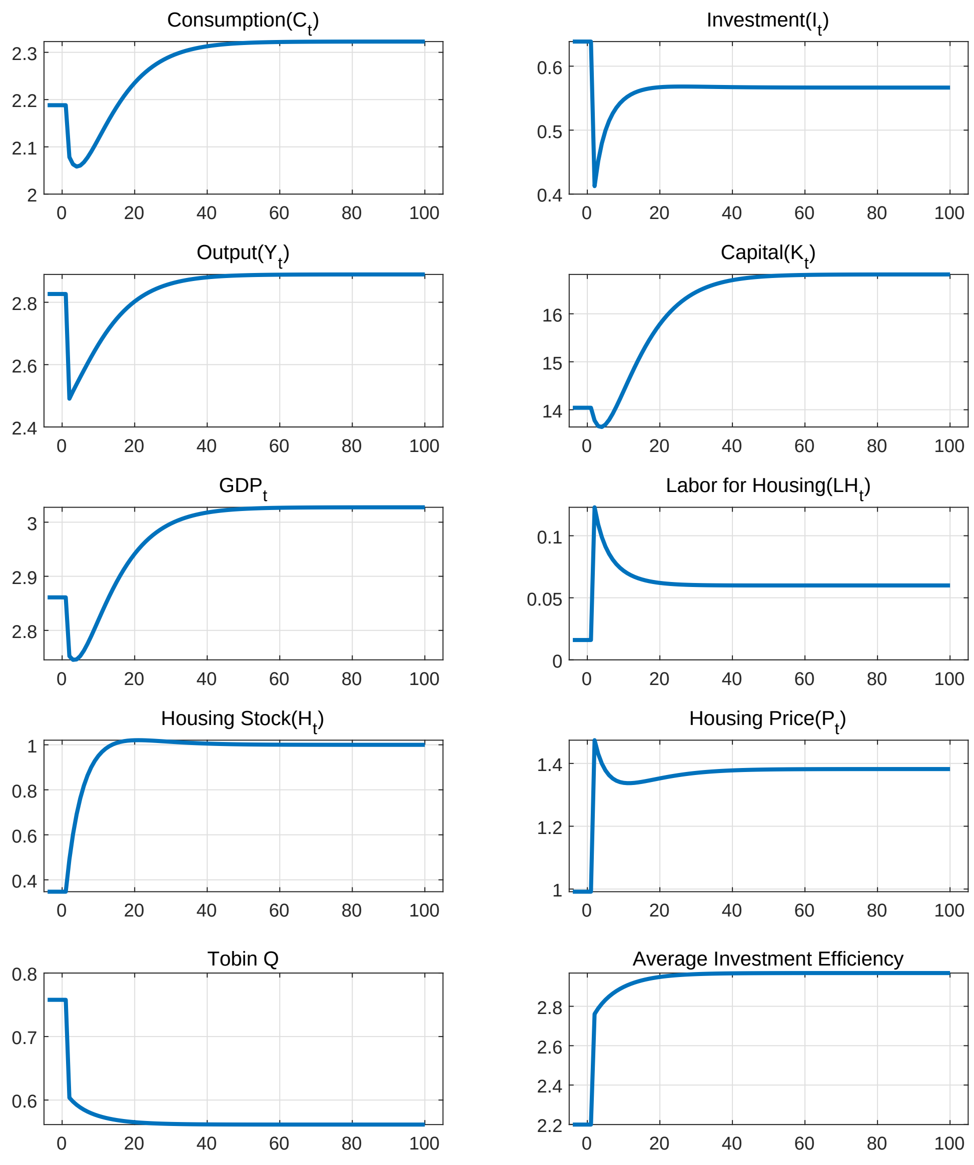
Figure 6 illustrates that a sudden increase in μ is not necessarily welfare-improving. An increase in μ leads to a jump in the housing price, so labor flows into the housing sector, which constitutes a crowding-out effect. As a result, consumption first decreases on the transition path. Due to the initial decline in consumption, the transition toward a statically efficient steady state can be dynamically inefficient if the ultimate level of consumption is not high enough under low levels of μ^H.

4.3 The Role of Labor Externality

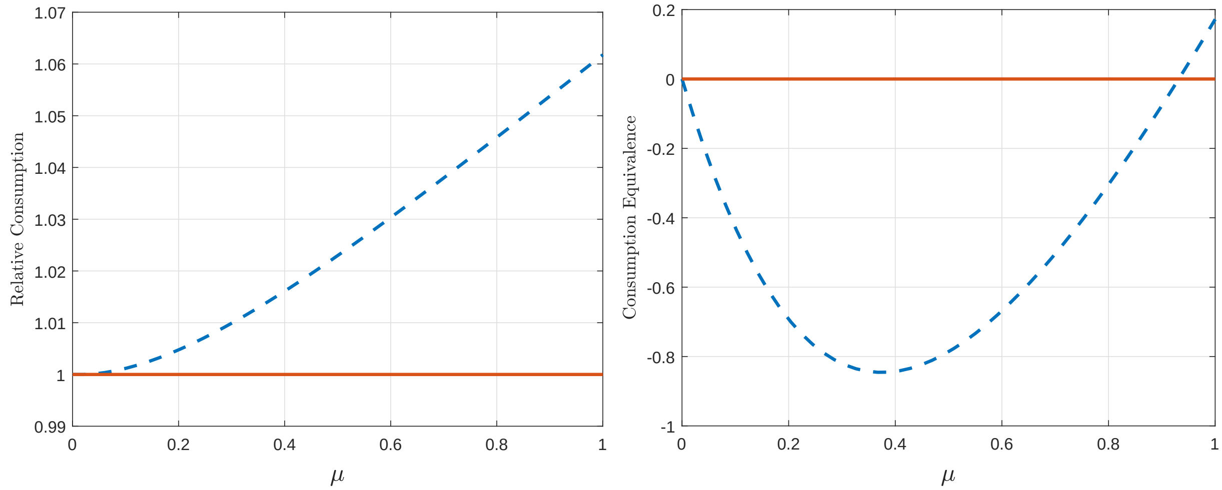
To illustrate the role of a productive externality of labor, we shut down the externality (that is, set $\gamma = 0$) and conduct the static and dynamic analysis. We report the results in Appendix F.1. The comparative statics results are shown in Figure F.1, the transitional dynamics are shown in Figures F.2 and F.3, and the welfare analysis is shown in Figures F.4 and F.5. Clearly, when there is no externality, both static inefficiency and dynamic inefficiency disappear. The collateral effect always dominates, and the existence of a bubble will increase social welfare. Due to the initial decline in consumption on the transition path, the welfare gain from the transition is lower than that from the comparative statics, but the transition toward a bubbly steady state is always dynamically efficient.

[10] Changing the values of μ will only change the values of economic variables in the new steady state but will not affect the shape of the transition paths. Note that since houses can be sold to raise money, there may be a bubbly equilibrium even with $\mu = 0$.

Figure 5: Transitional Dynamics from Low-μ to High-μ Steady States



Note: In this figure, we show the transition path from the bubbly steady state with $\mu = 0$ to the bubbly steady state with $\mu = 1$.



(a) Relative Consumption (b) Welfare Comparison

Note: Figure 6(a) shows the ratios of consumption in different bubbly steady states and that in the bubbly steady state with $\mu = 0$, and Figure 6(b) shows the welfare effect of the transition using consumption equivalence.

Figure 6: Welfare Analysis: Low-μ to High-μ

5 Model Extension

One concern regarding our key results on an inefficient housing boom is that the aggregate labor supply is fixed, so the housing boom will crowd out labor in the non-housing sector. If labor supply is instead elastic, in theory, labor in the non-housing sector does not have to decrease after the boom in the housing sector. In this section, we study an extension of the baseline model to illustrate the robustness of the results of our baseline model. In Appendix E, we consider another extension where we introduce housing services into the households' utility function and show that the main results are similar to those of our baseline model, except that there only exists one "bubbly" equilibrium due to the endogenous fundamental house value.

5.1 Elastic Labor Supply

We introduce a labor-leisure tradeoff in the households' utility function by assuming that households' per period utility function has the form:

$$u(C_t, N_t) = \log(C_t) - \psi \frac{N_t^{1+\eta}}{1+\eta}, \tag{49}$$

where N_t denotes labor supply and η is a parameter. Each household is endowed with one unit of time, and it allocates the time between labor and leisure.

$$\max_{\{C_t, s_{t+1}(i), N_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \psi \frac{N_t^{1+\eta}}{1+\eta}\right], \tag{50}$$

subject to

$$C_t + \int_0^1 s_{t+1}(i)\left[V_t(i) - d_t(i)\right] di = \int_0^1 s_t(i) V_t(i) di + W_t N_t + \Pi_t. \tag{51}$$

Denote by λ_t the Lagrangian multiplier of the budget constraint, and we have the first-order conditions as follows:

$$\frac{1}{C_t} = \lambda_t, \tag{52}$$

$$\psi N_t^{\eta} = \lambda_t W_t, \tag{53}$$

$$V_t(i) = \beta \mathbb{E}_t \left\{\frac{\lambda_{t+1}}{\lambda_t} V_{t+1}(i)\right\} + d_t(i). \tag{54}$$

From the above system, we can obtain the following *intratemporal* optimization rule

$$\psi N_t^{\eta} = \frac{W_t}{C_t}, \tag{55}$$

and the Bellman equation for non-housing firms:

$$V_t(i) = \beta \mathbb{E}_t \left\{\frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(i)\right\} + d_t(i), \tag{56}$$

where $\Lambda_t = \frac{1}{C_t}$ denotes the marginal utility of consumption for households. Clearly, the problems for the non-housing firms and housing firms remain the same, and their decision rules are unchanged.

The way to solve this model is similar to our previous approach, and we present the details in Appendix D. Importantly, we can show that there are still two equilibria, one bubbleless and one bubbly.

To parameterize the extended model, we use the same parameter set as in Section 4.1. Furthermore, following Chetty et al. (2011)'s advice, we calibrate our model such that the Frisch labor elasticity in our model equals $1/\eta = 0.75$, and we then set $\psi = 0.7854$ to ensure that the aggregate labor supply is 1 in the bubbleless steady state, which corresponds to our baseline model. We find that the main results are quite similar to those of our baseline model. The housing bubble still has a collateral effect and crowding-out effect, and as μ increases, the collateral effect increases faster. As a result, aggregate consumption, aggregate output, steady state utility and house prices all increase with μ, as shown in Figures 7(a), 7(b), 7(c) and 7(d).

An interesting result is that when we introduce the labor-leisure tradeoff, we find that the housing bubble will stimulate labor supply. From Figures 8(a), 8(b) and 8(c), we see that labor in the non-housing sector is still crowded out but the expansion of the housing sector and the bubbly boom attract more labor supply overall, which is consistent with the empirical findings in Charles et al. (2018).

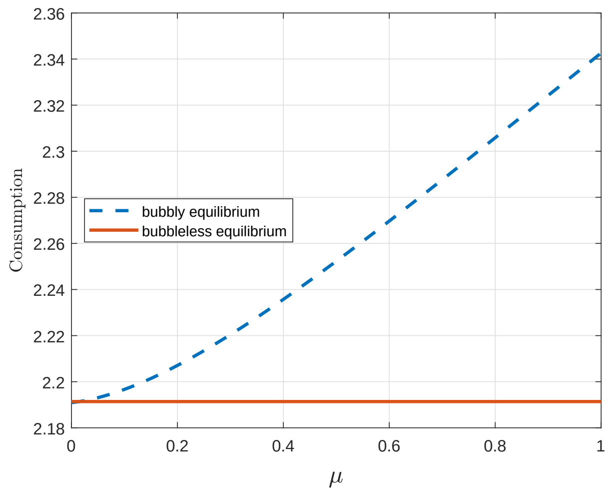
Regarding the transitional dynamics, we calculate the transition path from a bubbleless steady state to a bubbly steady state, which is shown in Figure 9. We find the dynamic path quite similar to that in the baseline model. The house price jumps to a higher level, inducing some labor into the housing sector, which crowds out labor supply for the non-housing sector. Due to the crowding-out effect during the initial periods, consumption and output decline for a few periods and then rise to their new steady-state levels. Note that after the initial decrease, non-housing sector labor begins to increase with the accumulation of its capital stock. Finally, GDP has an initial jump followed by a decrease and then increases to the new steady-state level.

Using the transitional path, we can calculate the welfare effect of the transition from a bubbleless steady state to a bubbly steady state, reported in Figure 10.

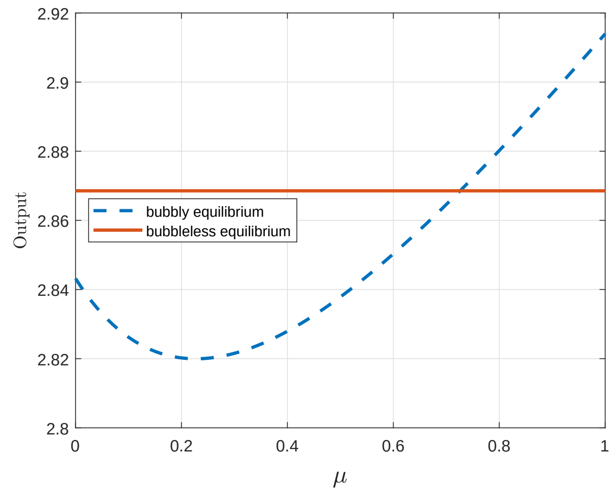
Following the previously employed procedure, we then calculate the transition path from a low-μ to a high-μ bubbly steady state. The results are shown in Figure 11 and are quite similar to those of the baseline model. The results of the welfare analysis from a low-μ steady state to a high-μ steady state are shown in Figure 12, where we find that the housing bubble can generate dynamic inefficiency even with elastic labor.

Note that "consumption equivalence" denotes *holding the labor allocation at the initial level*, the level that consumption should change permanently to obtain utility over the initial path equal to that over the new path. Similar to the results in the baseline model, due to the strong crowding-out effect during the first few periods of transition, the transition from a bubbleless to a bubbly steady state will decrease welfare, while the transition toward a statically more efficient bubbly steady state may be dynamically inefficient. Furthermore, we find that when we introduce elastic labor supply, the crowding-out effect of the housing bubble is partly offset by the increase in total employment. As a result, the welfare loss from the transition from a bubbleless to a bubbly steady state is smaller than that in the inelastic-labor case.

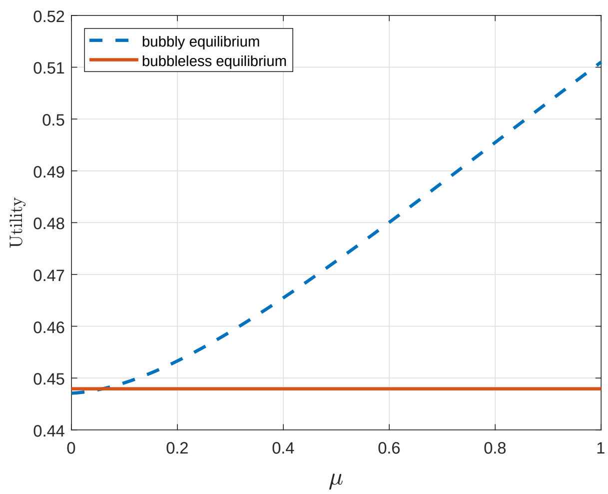
Similarly, we shut down the labor externality to illustrate its role in generating inefficiency, and the result is similar to those obtained previously: with no externality, there is no inefficiency. We show the results with no externality in Appendix F.2. The comparative statics results are shown in Figures F.6 and F.7, and the results of welfare analysis are shown in Figures



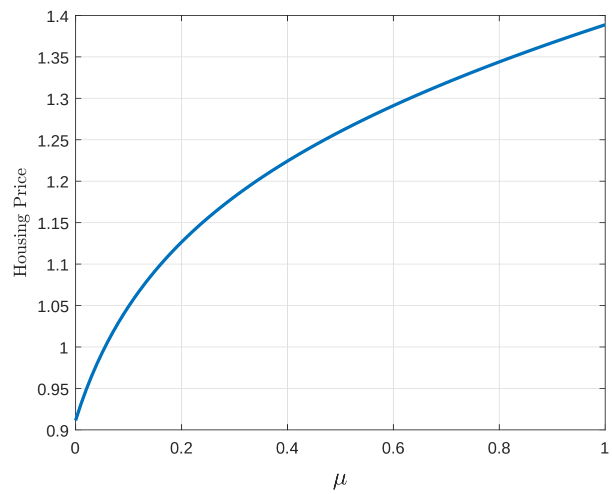
(a) Comparison of Consumption



(b) Comparison of Output



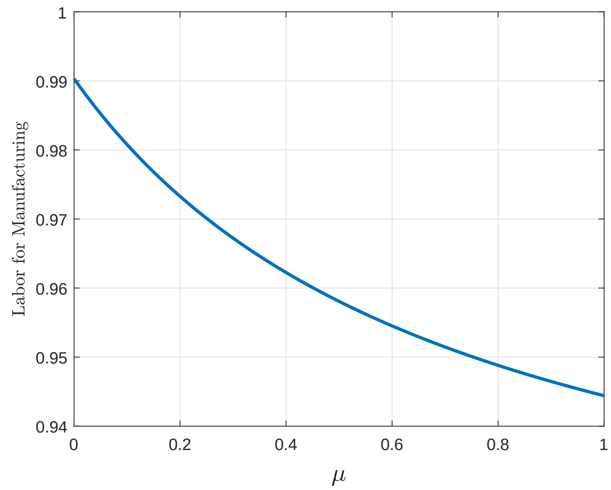
(c) Comparison of Utility



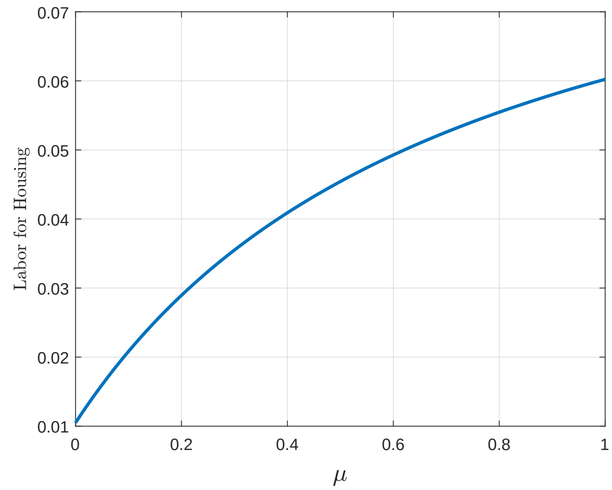
(d) Housing Price

Note: Figures 7(a) and 7(b) show the comparison of output and consumption between bubbly steady states and the bubbleless steady state, Figure 7(c) shows the comparison of steady-state utility, and Figure 7(d) shows the house price in different bubbly steady states.

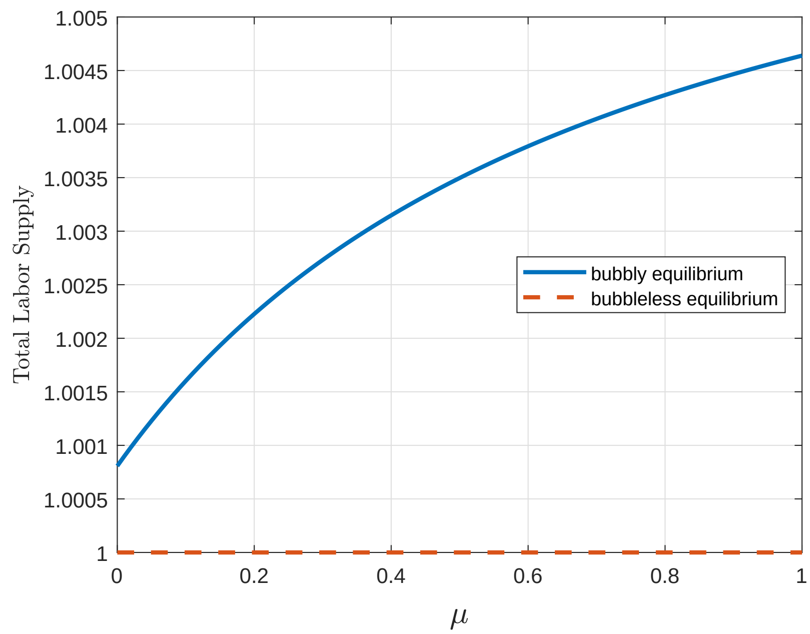
Figure 7: Comparison of Steady States: Elastic Labor



(a) Labor for the Non-housing Sector



(b) Labor for the Housing Sector

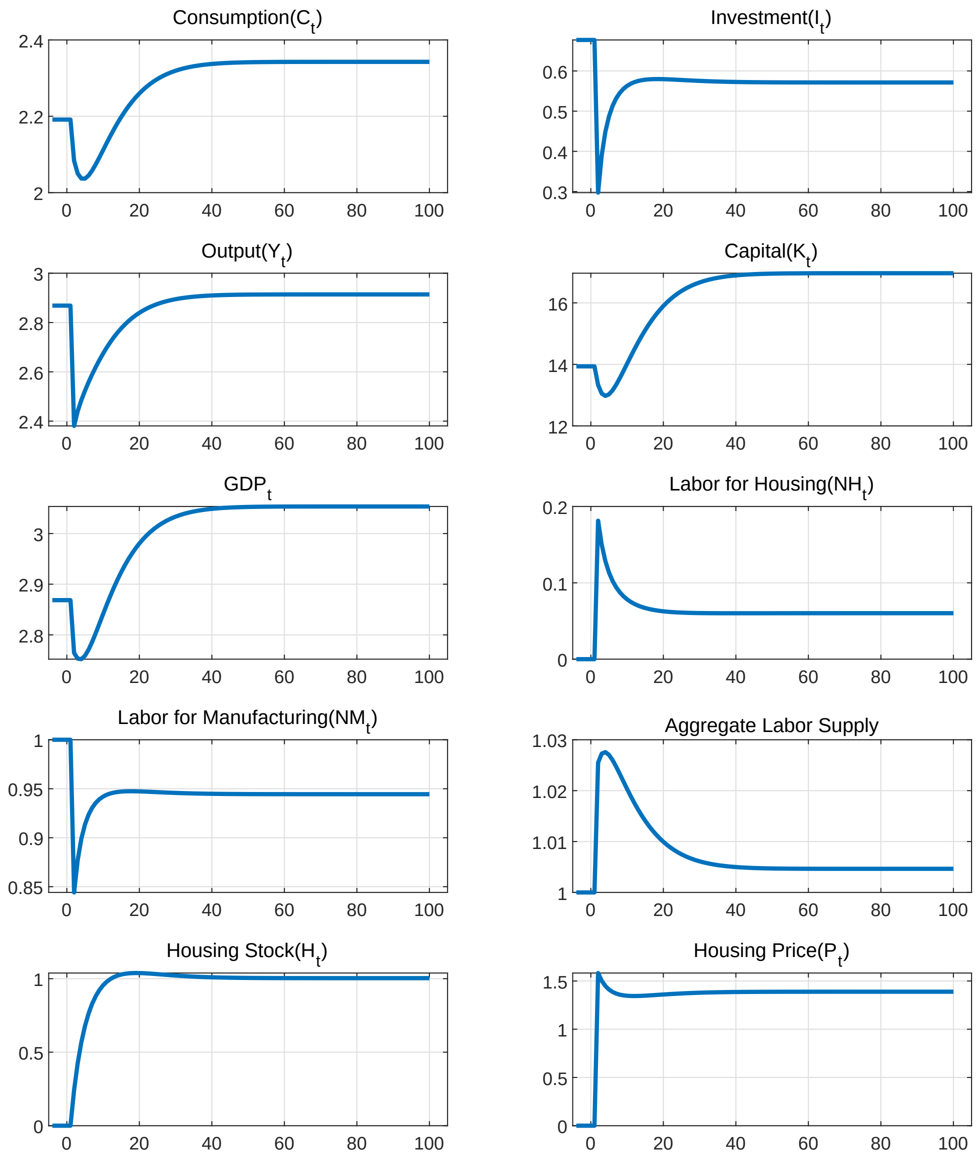


(c) Aggregate Labor Supply

Note: Figures 8(a) and 8(b) show the labor supply in the non-housing and housing sector in different bubbly steady states, respectively, and Figure 8(c) shows the aggregate labor supply.

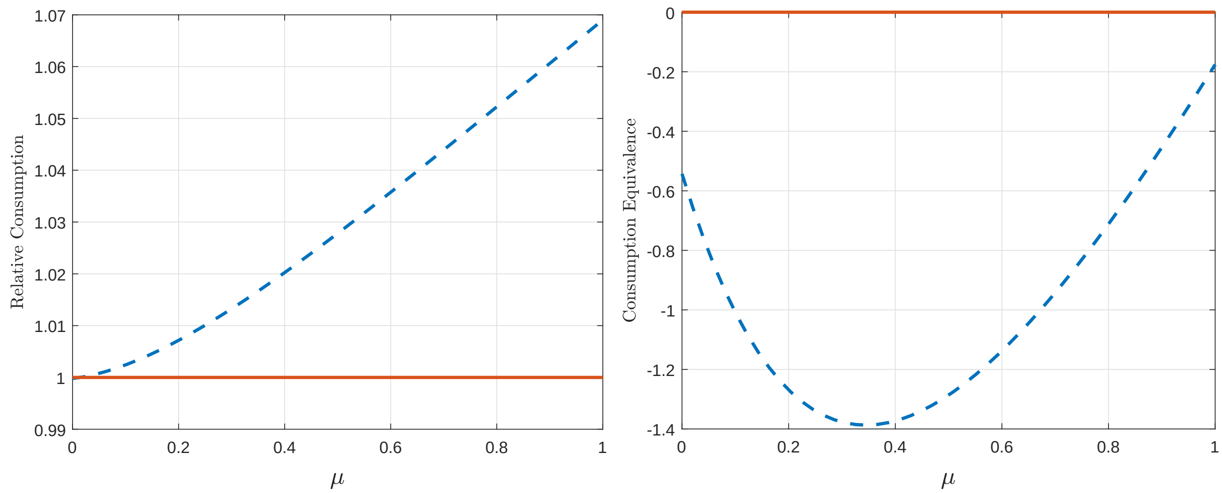
Figure 8: Effect of Housing Bubble on Labor Supply

Figure 9: Transitional Dynamics from Bubbleless to Bubbly Steady State with Elastic Labor



Note: In this figure, we show the transition path from the bubbleless steady state to the bubbly steady state with $\mu = 1$ for the extended model with elastic labor supply.

Figure 10: Welfare Analysis: Bubbleless to Bubbly, Elastic Labor



(a) Relative Consumption (b) Welfare Comparison

Note: Figure 11(a) shows the ratios of consumption in different bubbly steady states and that in the bubbleless steady state, and Figure 11(b) shows the welfare effect of the transition using consumption equivalence.

F.8 and F.9.

6 Policy Analysis

In this section, we use the baseline model to analyze the effects of government policies. In particular, we focus on the bubbly equilibrium in which housing price dynamics matter for the economy.

6.1 Labor Tax and Subsidy

In this subsection, we investigate the effect of a tax on hiring labor in the housing sector. In particular, we assume that when a firm in the housing sector hires 1 unit of labor, it pays a tax of $\tau_h W_t$ so that the unit labor cost for the housing sector becomes $(1+\tau_h)W_t$. The government, which runs a balanced budget, uses the tax revenue to subsidize the hiring of labor in the non-housing sector, so the unit labor cost for the non-housing sector becomes $(1-\tau_m)W_t$ where τ_m is the subsidy rate. The government budget constraint indicates that $\tau_h L_t^H = \tau_m L_t^M$. It is easy to check that the equation to solve for ϵ_t^* remains unchanged, so we only need to adjust the price-calculation part. In steady state, the labor hiring decisions for the housing and non-housing sectors imply

$$\begin{aligned} (1+\tau_h)W_t &= P_t A_t^H \sigma (L_t^H)^{\sigma-1}, \quad (57) \\ (1-\tau_m)W_t &= (1-\alpha)A_t^M K_t^\alpha (L_t^M)^{1-\alpha} = (1-\alpha)A^M K_t^\alpha (L_t^M)^{1+\gamma-\alpha}, \quad (58) \end{aligned}$$

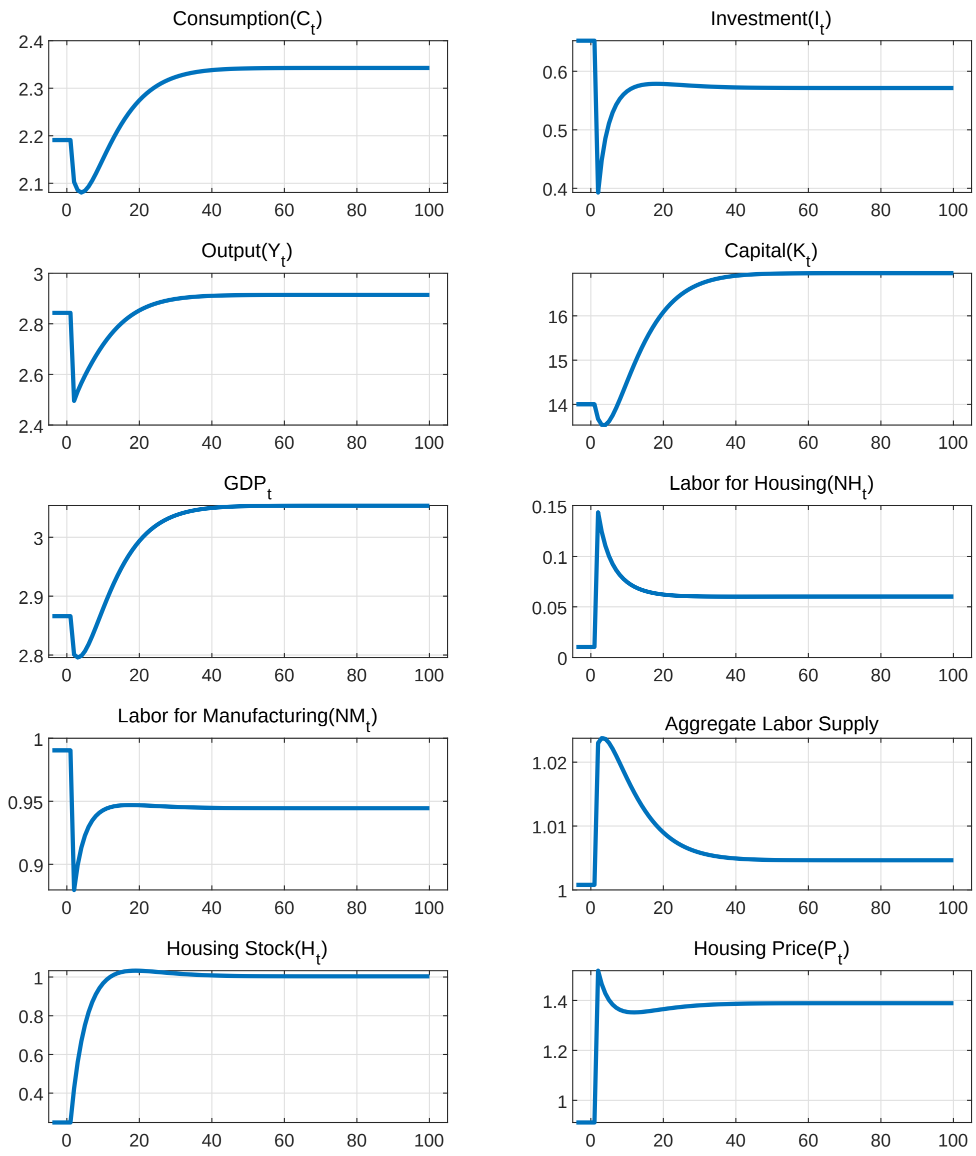
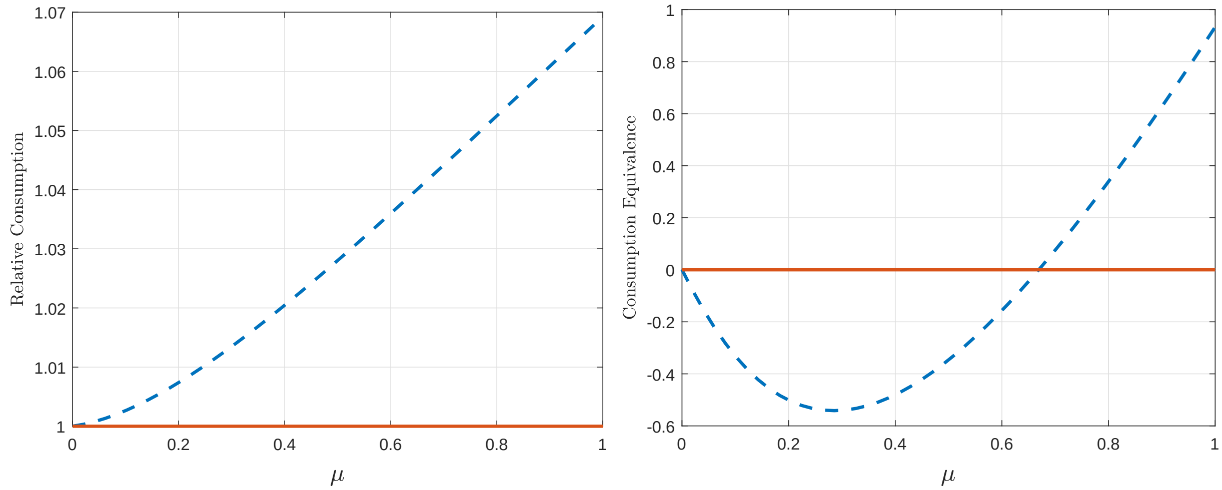


Figure 11: Transitional Dynamics from Low-μ to High-μ State with Elastic Labor



(a) Relative Consumption

(b) Welfare Comparison

Figure 12: Welfare Analysis: Low-μ to High-μ, Elastic Labor

and the government budget constraint yields

$$\tau_h = \frac{1 - L_t^H}{L_t^H} \tau_m. \tag{59}$$

Following the same derivations as before, we have $R_k = \frac{1-\beta(1-\delta_k)}{\beta \int \max\{\epsilon,\epsilon^*\} dF(\epsilon)}$, so in steady state we have

$$R_k = \alpha A^M K^{\alpha-1} (L^M)^{1+\gamma-\alpha} \tag{60}$$

$$W = \frac{1-\alpha}{1-\tau_m} A^M K^{\alpha} (L^M)^{\gamma-\alpha} \tag{61}$$

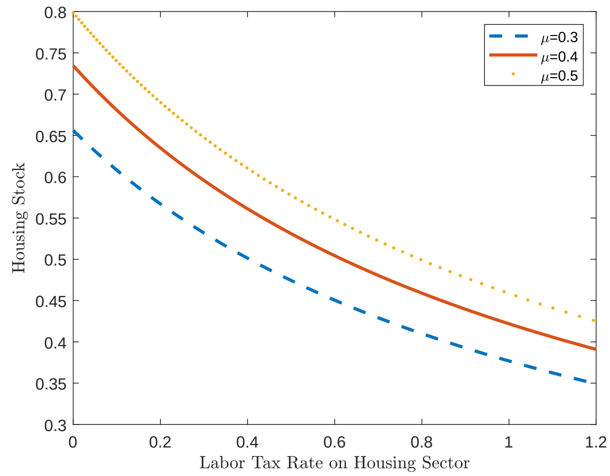
Then, given the housing price P, the steady-state labor in the housing sector satisfies

$$\left(1 + \frac{L^M}{1 - L^M} \tau_m\right) W = P A^H \sigma (1 - L_M)^{\sigma - 1}. \tag{62}$$

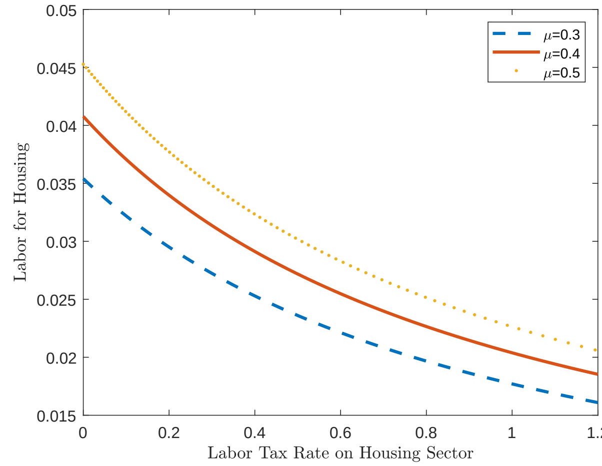
As before, in steady state, housing price P satisfies:

$$\frac{1}{1-\omega+\omega\mu-\delta_h}\left[\frac{\alpha\beta\delta_k}{1-\beta(1-\delta_k)} \frac{\int \max\{\epsilon,\epsilon^*\} dF(\epsilon)}{\int_{\epsilon>\epsilon^*} \epsilon dF(\epsilon)} - \alpha\right] = P \frac{A^H}{\delta_h A^M} \left(\frac{\alpha A^M}{R_k}\right)^{\frac{\alpha}{\alpha-1}} \frac{(1-L^M)^{\sigma}}{(L^M)^{\frac{\alpha-1-\gamma}{\alpha-1}}}. \tag{63}$$

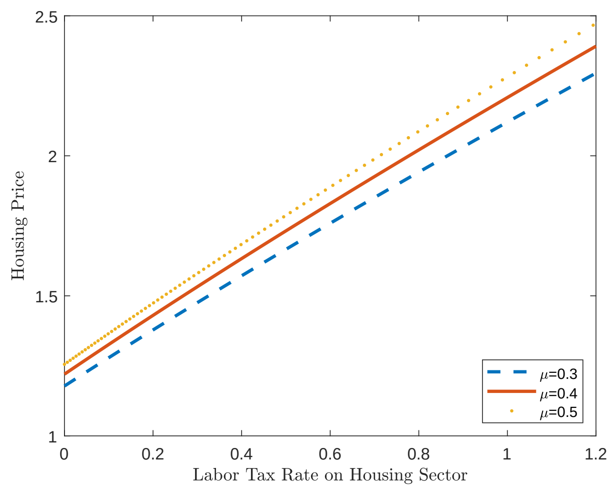
However, the expressions for W and L^H have changed. From the above four equations, we can simultaneously solve for price P and labor in the housing sector L^H. We hold μ at different levels and change τ_m and solve for the bubbly steady state of the model with labor tax and subsidy, and the results are as shown in Figure 13.



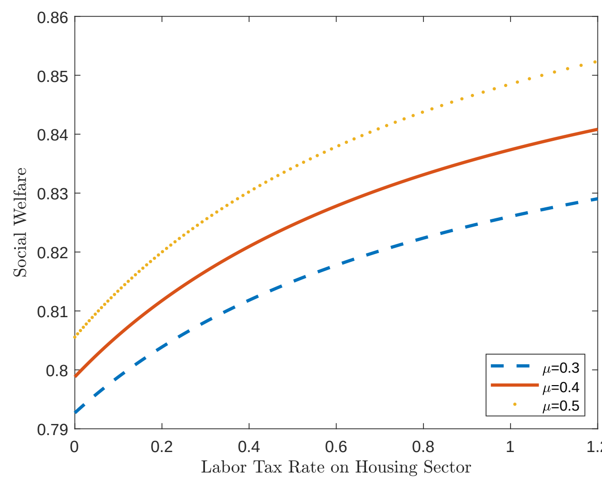
(a) House Stock



(b) Labor in the Housing Sector



(c) House Price



(d) Social Welfare

Note: Figures 13(a) and 13(b) show the housing stock and labor in the housing sector under labor taxes, Figure 13(c) shows the effect on house price and Figure 13(d) shows the effect of the labor tax on steady-state welfare.

Figure 13: Effects of Labor Tax

Figures 13(a) and 13(b) show the housing stock and labor in the housing sector under labor taxes. We see that when we impose the tax on hiring labor in the housing sector, its labor cost increases, which leads to a lower hiring in the housing sector and a lower house supply. As a result, the house stock in steady state decreases. We use the tax revenue to subsidize hiring in the non-housing sector, so the demand for houses increases. Consequently, the tax on the housing sector will lead to a higher house price. The labor subsidy to the non-housing sector stimulates production, so consumption and social welfare increase with the labor tax.

6.2 Transaction Tax

The government can also intervene in the housing market through transaction taxes. In particular, we consider a policy whereby when a firm buys one unit of housing, it must pay an ad valorem tax τ, and the tax revenue, $\tau(1-\omega)P_tH_t(1-F(\epsilon_t^*))$, is used to subsidize households in a lump sum fashion. In this scenario, the pricing function of houses becomes

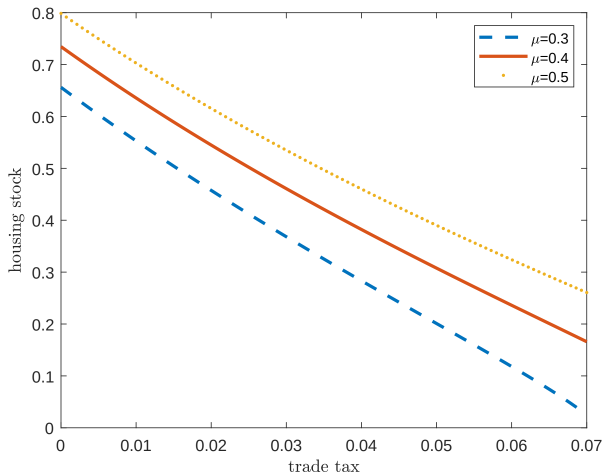
$$\frac{(1+\tau)P_t}{C_t} = \beta \int \frac{1}{C_{t+1}} \left\{ [1 + (L(\epsilon_{t+1}^*) - 1)(1 - \omega + \omega\mu) - L(\epsilon_{t+1}^*)\delta_h] P_{t+1} \right\} dF(\epsilon). \tag{64}$$

Note that now the investment efficiency cutoff in the bubbly steady state ϵ_b^* will change with the transaction tax rate, τ. The calculation of price and labor are the same as in the baseline model. As before, we hold μ at different levels and change τ and solve the bubbly steady state. The results are as shown in Figure 14. We see that the tax on housing transactions reduces the demand for housing, leading to lower house prices and a smaller housing sector in steady state. The transaction tax has two effects on social welfare: on the one hand, a higher transaction tax will lead to a smaller housing sector, which means more labor supply and higher productivity for the real sector; on the other hand, having a smaller housing sector will make it more difficult for high-efficiency firms to securing financing with housing assets and thus retard real production. In equilibrium, the crowding-out effect dominates the crowding-in effect, and under our parameter settings, a higher transaction tax will lead to lower steady-state social welfare.[11]

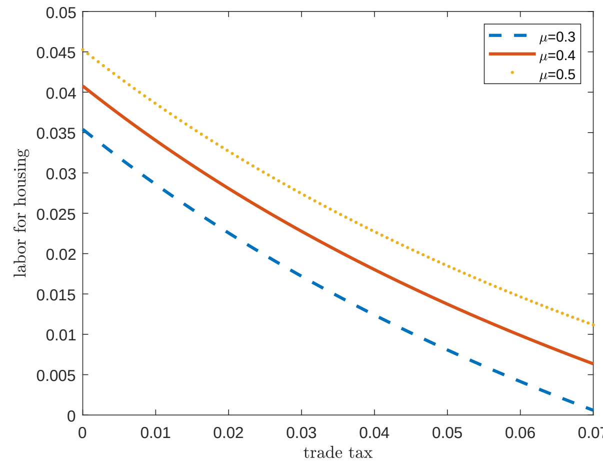
6.3 Subsidy for the Non-housing Sector's Revenue

We consider a subsidy to the non-housing sector in this subsection and assume that the government grants a subsidy of τ_m to non-housing firms, so now the "revenue" of non-housing firm i becomes $(1+\tau_m)A_t^M k_t(i)^\alpha (l_t^M(i))^{1-\alpha}$. To balance the budget, the government collects corresponding tax revenue τ_h from the housing sector, so the revenue of the housing sector becomes $(1-\tau_h)A^H(L_t^H)^\sigma$. It is straightforward to show that the equilibrium system now becomes

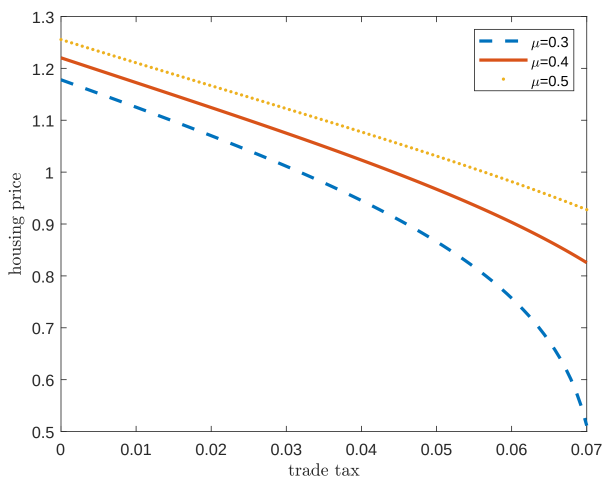
[11] The result of the welfare analysis is sensitive to parameter values; under another set of parameters, there exists an optimal tax rate to maximize steady-state social welfare.



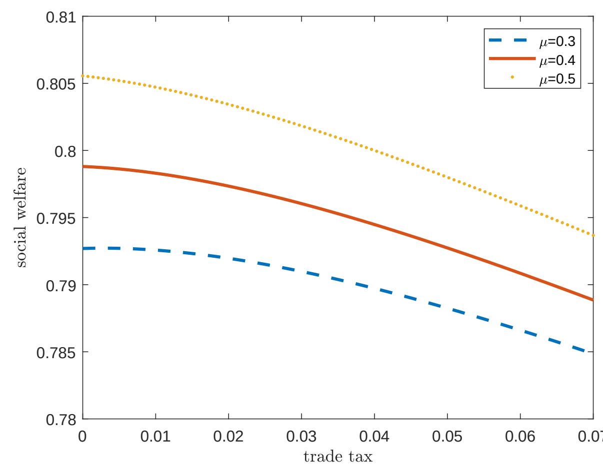
(a) House Stock



(b) Labor in the Housing Sector



(c) House Price



(d) Social Welfare

Note: Figures 14(a) and 14(b) show the housing stock and labor in the housing sector under trading taxes, Figure 14(c) shows the effect on house prices and Figure 14(d) shows the effect of the trading tax on steady-state welfare.

Figure 14: Effects of Housing Transaction Tax

$$
\begin{aligned}
Y_t &= A_t^M(L_t^M)K_t^\alpha(L_t^M)^{1-\alpha} = A^M K_t^\alpha (L_t^M)^{1+\gamma-\alpha} && (65)\\
C_t + I_t &= Y_t && (66)\\
L_t^M + L_t^H &= 1 && (67)\\
I_t &= [\alpha(1+\tau_m)Y_t + (1-\omega+\omega\mu-\delta_h)P_tH_t](1-F(\epsilon_t^*)) && (68)\\
K_{t+1} &= (1-\delta_k)K_t + \omega(\epsilon_t^*)I_t && (69)\\
H_{t+1} &= (1-\delta_h)H_t + A_t^H(L_t^H)^\sigma && (70)\\
W_t &= (1-\tau_h)P_tA_t^H\sigma(L_t^H)^{\sigma-1} && (71)\\
W_t &= (1+\tau_m)(1-\alpha)A_t^MK_t^\alpha(L_t^M)^{\gamma-\alpha} && (72)\\
\frac{P_t}{C_t} &= \beta\int\frac{1}{C_{t+1}}\left\{[1+(L(\epsilon_{t+1}^*)-1)(1-\omega+\omega\mu)-L(\epsilon_{t+1}^*)\delta_h]P_{t+1}\right\}dF(\epsilon) && (73)\\
\frac{Q_t}{C_t} &= \beta\int\frac{1}{C_{t+1}}\left[\alpha\frac{(1+\tau_m)Y_{t+1}}{K_{t+1}}L(\epsilon_{t+1}^*)+(1-\delta_k)Q_{t+1}\right]dF(\epsilon) && (74)\\
\frac{1}{R_{ft}C_t} &= \beta\int\frac{1}{C_{t+1}}L(\epsilon_{t+1}^*)dF(\epsilon) && (75)\\
Q_t\epsilon_t^* &= 1 && (76)\\
\tau_mY_t &= \tau_hP_tA^H(L_t^H)^\sigma && (77)
\end{aligned}
$$

Note that Equation (66), the resource constraint, remains unchanged. This is because although the subsidy increases the revenue of the non-housing sector and thus increases wages and dividends to households, this effect is offset by the taxation on the housing sector, which means a lower profit passing from the housing sector to households. If we regard the economy as a whole, the total resource produced every period remains Y_t, and the subsidy only distorts its allocation.

Here, the pricing function of housing remains the same, so the cutoff efficiency ϵ_b^* in steady state is not affected by the subsidy system. With the cutoff ϵ^*, we can calculate the relative value of housing assets to total output:

$$
\frac{PH}{Y} = \frac{1}{1-\omega+\omega\mu-\delta_h}\left[\frac{(1+\tau_m)\alpha\beta\delta_k}{1-\beta(1-\delta_k)}\frac{\int\max\{\epsilon,\epsilon^*\}dF(\epsilon)}{\int_{\epsilon>\epsilon^*}\epsilon dF(\epsilon)} - \alpha(1+\tau_m)\right]. \qquad (78)
$$

As before, we can also calculate from the production functions that

$$
\frac{PH}{Y} = P\frac{A^H}{\delta_hA^M}\left(\frac{(1+\tau_m)\alpha A^M}{R_k}\right)^{\frac{\alpha}{\alpha-1}}\frac{(1-L^M)^\sigma}{(L^M)^{\frac{\alpha-\gamma-1}{\alpha-1}}}, \qquad (79)
$$

where $L^H = \left(\frac{W}{(1-\tau_h)PA^H\sigma}\right)^{\frac{1}{\sigma-1}}$ and $R_k = \frac{1-\beta(1-\delta_k)}{\beta\int\max\{\epsilon,\epsilon^*\}dF(\epsilon)} = \alpha(1+\tau_m)A^M K^{\alpha-1}(L^M)^{1+\gamma-\alpha}$.

Moreover, the government budget constraint implies

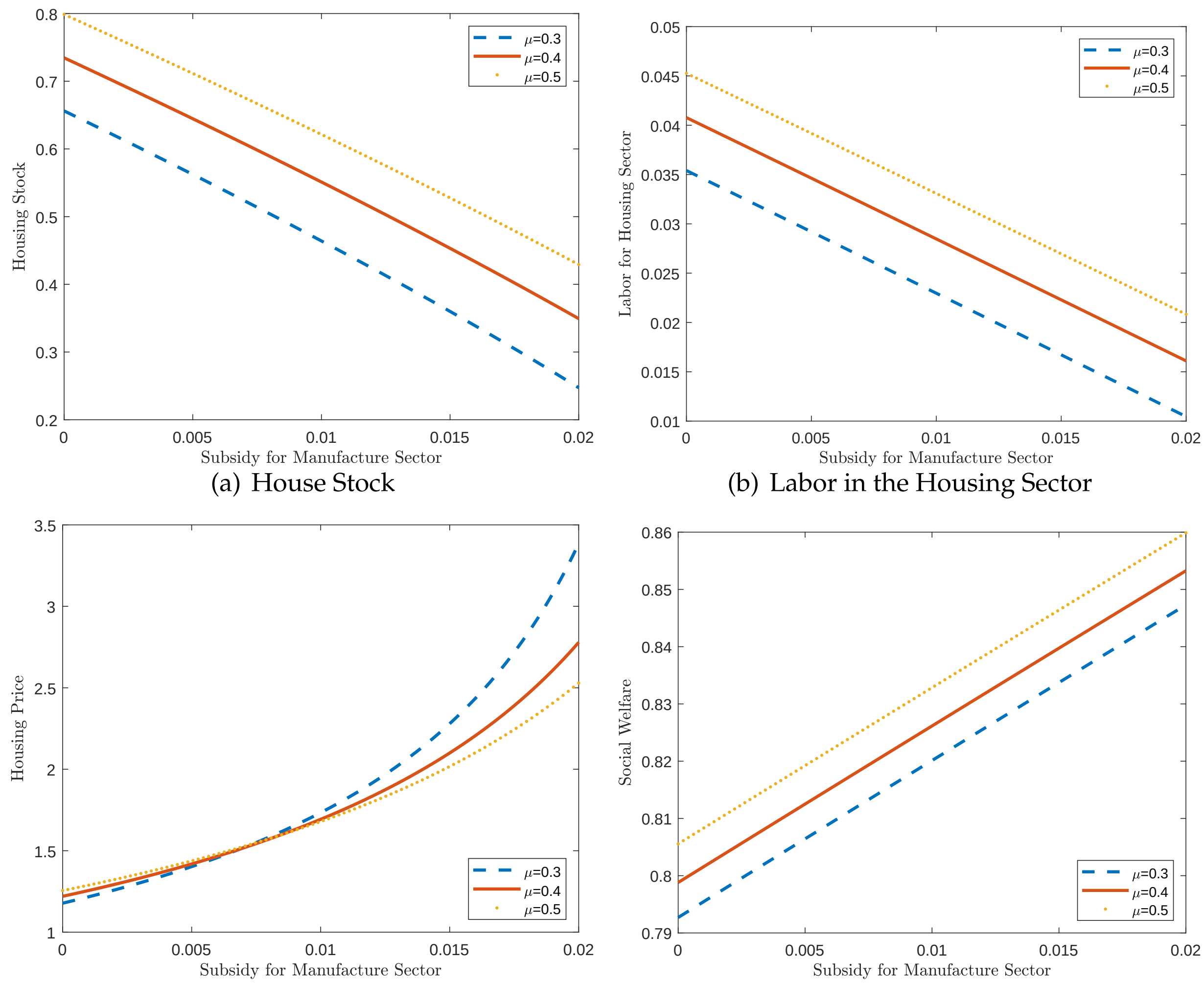
$$\tau_h PA^H(L^H)^\sigma = \tau_m A^M \left(\frac{R_k}{\alpha(1+\tau_m)A^M}\right)^{\frac{\alpha}{\alpha-1}} (L^M)^{\frac{\alpha-\gamma-1}{\alpha-1}}. \tag{80}$$

Given subsidy rate τ_m, from the above equations, we can solve for the price P, labor L^H and tax rate in the housing sector τ_h. Similarly to before, we hold μ at different levels and change the subsidy rate τ_m and solve the bubbly steady state. The results are shown in Figure 15. We find that the taxation-subsidy system lowers the revenue of the housing sector, leading to less labor hiring and less housing stock in steady state. However, due to the subsidy, the production of the non-housing sector is stimulated, so housing demand increases. As a result, the system will lead to a higher housing price and a larger housing bubble. Finally, a larger non-housing sector will lead to higher consumption and thus higher social welfare.

7 Concluding Remarks

In this paper, we set up a multisector model with heterogeneous non-housing firms to study the impact of housing bubbles on the economy when the bubble asset is produced by an independent economic sector. A housing bubble can emerge because it provides a liquidity premium. In our model, the bubbly housing asset is produced by the housing sector using labor, so the housing sector and the non-housing sector compete for labor.

We find that the housing bubble has two effects: a collateral effect and a crowding-out effect. On the one hand, non-housing firms can sell their houses or use them as collateral to finance their investment and expand production, thus increasing output and consumption in the steady state and raising social welfare. On the other hand, the housing boom increases demand for labor in the housing sector, crowding out labor supply for the non-housing sector, which will decrease output and consumption. In steady state, the effect of the housing bubble on the real economy depends on the relative strength of the two effects: when credit market imperfections are severe and housing prices and the LTV ratio are low, the collateral effect is relatively weak, which means that the housing bubble will decrease output, consumption and social welfare in steady state, thus creating static inefficiency. From the intertemporal equilibrium system, we can calculate the transition path from a bubbleless to a bubbly steady state. During the initial few periods along the transition path, the housing price jumps such that a fraction of labor flows into the housing sector, leading to a sudden decrease in consumption and output. Furthermore, on the transition path, the housing stock rises above its new steady-



(a) House Stock
(b) Labor in the Housing Sector
(c) House Price
(d) Social Welfare

Note: Figures 15(a) and 15(b) show the housing stock and labor in the housing sector under the subsidy, Figure 15(c) shows the effect on house prices and Figure 15(d) shows the effect of the subsidy on steady-state welfare.

Figure 15: Effects of Revenue Subsidies

state value, at the expense of labor as inputs. By calculating the total social welfare on the bubbleless path and the transition path to a bubbly steady state, we find that when credit market imperfections are strong, even if the housing bubble is statically efficient, the transition to a bubbly steady state can be dynamically inefficient. This result implies that the government should be prudent in promoting financial development. We also investigate tax policies that the government can use to intervene in the housing market. We find that policies that can reduce housing prices will restrain production in the non-housing sector and may decrease social welfare; however, policies that can stimulate production and social welfare will lead to higher demand for houses and a larger housing bubble.

We consider an extended version of our baseline model where we endogenize the labor supply decision to study the effect of the housing bubble and housing boom on labor supply. We find that the existence of a housing bubble leads to higher labor supply, and a larger housing boom will attract more agents into the labor market, which is consistent with the literature. The other comparative statics and transitional dynamics of the extended model are similar to those of our baseline model.

There are several directions for future work. First, in our model, we assume that the only resource that the housing sector and the non-housing sector compete for is labor, while in reality we observe that the housing sector absorbs not only labor resources but also a large fraction of financial resources. It would be interesting to model the competition for both labor and financial resources and study how the competition for the two resources interact. Second, in our model, we assume that the worker flow between the housing sector and the non-housing sector is frictionless. Introducing costly migration across sectors would be a fruitful research direction. Finally, in our model, there is only one bubble asset (housing bubble). Miao and Wang (2014) and Dong et al. (2021c) discuss the case of multiple bubble assets. It would be interesting to examine the interactions between multiple bubble booms.

References

Aoki, Kosuke and Kalin Nikolov, "Bubbles, banks and financial stability," *Journal of Monetary Economics*, 2015, *74*, 33–51.

Arce, Óscar and David López-Salido, "Housing bubbles," *American Economic Journal: Macroeconomics*, 2011, *3* (1), 212–41.

Bahaj, Saleem, Angus Foulis, and Gabor Pinter, "Home values and firm behavior," *American Economic Review*, 2020, *110* (7), 2225–70.

Banerjee, Ryan and Kristian Blickle, "Housing collateral and small firm activity in Europe," 2016.

Bayoumi, Tamim and Yunhui Zhao, "Incomplete financial markets and the booming housing sector in China," 2021.

Bianchi, Javier, "Overborrowing and systemic externalities in the business cycle," *American Economic Review*, 2011, *101* (7), 3400–3426.

Biswas, Siddhartha, Andrew Hanson, and Toan Phan, "Bubbly recessions," *American Economic Journal: Macroeconomics*, 2020, *12* (4), 33–70.

Case, Karl E, Edward L Glaeser, and Jonathan A Parker, "Real estate and the macroeconomy," *Brookings Papers on Economic Activity*, 2000, *2000* (2), 119–162.

Caselli, Francesco and James Feyrer, "The marginal product of capital," *The quarterly journal of economics*, 2007, *122* (2), 535–568.

Chaney, Thomas, David Sraer, and David Thesmar, "The collateral channel: How real estate shocks affect corporate investment," *American Economic Review*, 2012, *102* (6), 2381–2409.

Charles, Kerwin Kofi, Erik Hurst, and Matthew J Notowidigdo, "Housing booms and busts, labor market opportunities, and college attendance," *American Economic Review*, 2018, *108* (10), 2947–94.

Chen, Kaiji and Yi Wen, "The great housing boom of China," *American Economic Journal: Macroeconomics*, 2017, *9* (2), 73–114.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber, "Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins," *American Economic Review*, 2011, *101* (3), 471–75.

Davis, Donald R and Jonathan I Dingel, "A spatial knowledge economy," *American Economic Review*, 2019, *109* (1), 153–170.

Dong, Ding, Zheng Liu, Pengfei Wang, and Tao Zha, "A Theory of Housing Demand Shocks," *Journal of Economic Theory*, 2022, p. 105484.

Dong, Feng and Zhiwei Xu, "Bubbly bailout," *Journal of Economic Theory*, 2022, *202*, 105460.

— , **Jianfeng Liu, Zhiwei Xu, and Bo Zhao**, "Flight to housing in China," *Journal of Economic Dynamics and Control*, 2021, *130*, 104189.

— , **Yumei Guo, Yuchao Peng, and Zhiwei Xu**, "Economic slowdown and housing dynamics in China: A tale of two investments by firms," *Journal of Money, Credit and Banking*, 2021.

— , **Zhen Huo, and Yi Wen**, "Contagious Bubbles," *Available at SSRN*, 2021.

Farhi, Emmanuel and Jean Tirole, "Bubbly liquidity," *The Review of economic studies*, 2012, *79* (2), 678–706.

Fisher, Irving, "The debt-deflation theory of great depressions," *Econometrica: Journal of the Econometric Society*, 1933, pp. 337–357.

Gan, Jie, "Collateral, debt capacity, and corporate investment: Evidence from a natural experiment," *Journal of Financial Economics*, 2007, *85* (3), 709–734.

Grossman, Gene M and Noriyuki Yanagawa, "Asset bubbles and endogenous growth," *Journal of Monetary Economics*, 1993, *31* (1), 3–19.

Hirano, Tomohiro and Noriyuki Yanagawa, "Asset bubbles, endogenous growth, and financial frictions," *The Review of Economic Studies*, 2016, *84* (1), 406–443.

Iacoviello, Matteo and Stefano Neri, "Housing market spillovers: evidence from an estimated DSGE model," *American Economic Journal: Macroeconomics*, 2010, *2* (2), 125–164.

Ikeda, Daisuke and Toan Phan, "Asset bubbles and global imbalances," *American Economic Journal: Macroeconomics*, 2019, *11* (3), 209–51.

Jiang, Shenzhe, Jianjun Miao, and Yuzhe Zhang, "China's Housing Bubble, Infrastructure Investment, and Economic Growth," *International Economic Review*, 2019.

Kiyotaki, Nobuhiro and John Moore, "Credit cycles," *Journal of political economy*, 1997, *105* (2), 211–248.

— **and** — , "Liquidity, monetary policy and business cycles," Technical Report 2008.

Kocherlakota, Narayana, "Injecting rational bubbles," *Journal of Economic Theory*, 2008, *142* (1), 218–232.

— , "Bursting bubbles: Consequences and cures," *Unpublished manuscript, Federal Reserve Bank of Minneapolis*, 2009, *84.*

Kocherlakota, Narayana R, "Bubbles and constraints on debt accumulation," *Journal of Economic theory*, 1992, *57* (1), 245–256.

Lorenzoni, Guido, "Inefficient credit booms," *The Review of Economic Studies*, 2008, *75* (3), 809–833.

Lucas, Robert EB, "Emigration to South Africa's mines," *The American Economic Review*, 1987, pp. 313–330.

Martin, Alberto and Jaume Ventura, "Economic growth with bubbles," *American Economic Review*, 2012, *102* (6), 3033–58.

Miao, Jianjun and Pengfei Wang, "Sectoral bubbles, misallocation, and endogenous growth," *Journal of Mathematical Economics*, 2014, *53*, 153–163.

_ **and** _ , "Asset bubbles and credit constraints," *American Economic Review*, 2018, *108* (9), 2590–2628.

_ **,** _ **, and Jing Zhou**, "Asset bubbles, collateral, and policy analysis," *Journal of Monetary Economics*, 2015, *76*, S57–S70.

_ **,** _ **, and Zhiwei Xu**, "A Bayesian dynamic stochastic general equilibrium model of stock market bubbles and business cycles," *Quantitative Economics*, 2015, *6* (3), 599–635.

Schmalz, Martin C, David A Sraer, and David Thesmar, "Housing collateral and entrepreneurship," *The Journal of Finance*, 2017, *72* (1), 99–132.

Shi, Ms Yu, *Sectoral booms and misallocation of managerial talent: Evidence from the Chinese real estate boom*, International Monetary Fund, 2018.

Tirole, Jean, "Asset bubbles and overlapping generations," *Econometrica: Journal of the Econometric Society*, 1985, pp. 1499–1528.

Wang, Pengfei and Yi Wen, "Speculative bubbles and financial crises," *American Economic Journal: Macroeconomics*, 2012, *4* (3), 184–221.

Weil, Philippe, "Confidence and the real value of money in an overlapping generations economy," *The Quarterly Journal of Economics*, 1987, *102* (1), 1–22.

Zhao, Bo, "Rational housing bubble," *Economic Theory*, 2015, *60* (1), 141–201.

Appendix

A Proof of Proposition 1

First, consider the intratemporal decision for labor hiring; the FOC for labor yields

$$A_t^M(1-\alpha)k_t(i)^\alpha(l_t^M(i))^{1-\alpha} = W_t \Rightarrow l_t^M(i) = \left[\frac{(1-\alpha)A_t^M}{W_t}\right]^{\frac{1}{\alpha}} k_t(i).$$

Here, note that $A_t^M(L_t^M)$ is an aggregate variable, so it is taken as given by the individual firms. Thus, we obtain

$$R_t(i) = y_t(i) - W_t l_t^M(i) = R_{kt}k_t(i),$$

where

$$R_{kt} = \frac{\alpha W_t}{1-\alpha}\left[\frac{(1-\alpha)A_t^M}{W_t}\right]^{\frac{1}{\alpha}}.$$

Then, the firm's problem can be rewritten as

$$V(k_t(i), h_t(i), b_t(i)) =$$
$$\max \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \frac{\Lambda_{t+1}}{\Lambda_t}\left\{R_{kt}k_t(i) - P_t\left[h_{t+1}(i) - (1-\delta_h)h_t(i)\right] + \frac{b_{t+1}(i)}{R_{ft}} - b_t(i) - i_t(i)\right\},$$

subject to

$$d_t(i) \geq 0, \tag{A.1}$$
$$i_t(i) \geq 0, \tag{A.2}$$
$$h_{t+1}(i) \geq \omega h_t(i), \tag{A.3}$$
$$\frac{b_{t+1}(i)}{R_{ft}} \leq \mu P_t h_{t+1}(i), \tag{A.4}$$
$$k_{t+1}(i) = (1-\delta_k)k_t(i) + \epsilon_t(i)i_t(i), \tag{A.5}$$

The Bellman equation can be written as

$$V(k_t(i), h_t(i), b_t(i)) = \max_{k_{t+1}(i), h_{t+1}(i), b_{t+1}(i), i_t(i)} \left\{d_t(i) + \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}V(k_{t+1}(i), h_{t+1}(i), b_{t+1}(i))\right\} \tag{A.6}$$

To solve the problem, we suppose that $V(k_t(i), h_t(i), b_t(i)) = v(\epsilon_t(i))k_t(i) + p(\epsilon_t(i))h_t(i) -$

$\phi(\epsilon_t(i))b_t(i)$ with Tobin's Q, the aggregate house price and the gross interest rate defined as:

$$Q_t = \beta \int \frac{\Lambda_{t+1}}{\Lambda_t} v(\epsilon_{t+1}(i))dF(\epsilon) \tag{A.7}$$
$$P_t = \beta \int \frac{\Lambda_{t+1}}{\Lambda_t} p(\epsilon_{t+1}(i))dF(\epsilon) \tag{A.8}$$
$$\frac{1}{R_{ft}} = \beta \int \frac{\Lambda_{t+1}}{\Lambda_t} \phi(\epsilon_{t+1}(i))dF(\epsilon) \tag{A.9}$$

By plugging the assumption back into Bellman equation (A.6), we obtain

$$v(\epsilon_t)k_t + p(\epsilon_t)h_t - \phi(\epsilon_t)b_t = \max_{h_{t+1},b_{t+1},i_t} \left\{[R_{kt} + Q_t(1-\delta_k)]k_t + (1-\delta_h)P_t h_t - b_t + (Q_t\epsilon_t - 1)i_t\right\}.$$

Obviously, there exists a cutoff $\epsilon_t^* = \frac{1}{Q_t}$; when $\epsilon_t(i) > \epsilon_t^*$, the firm will make use of all the resource to make investment, which means

$$\begin{aligned} h_{t+1}(i) &= \omega h_t(i) \\ \frac{b_{t+1}(i)}{R_{ft}} &= \mu P_t h_{t+1}(i) \\ i_t(i) &= R_{kt}k_t(i) + (1-\omega+\omega\mu-\delta_h)P_t h_t(i) - b_t(i) \end{aligned},$$

otherwise, the firm will not invest and is indifferent between holding bubbly housing and debts, which means that $i_t(i) = 0$ and any choices of $b_{t+1}(i)$ and $h_{t+1}(i)$ are possible.

Combining the two cases discussed above, we obtain the investment decision as in Equation 12. The rate of return on housing depends on the expected value of liquidity, which is denoted by

$$L(\epsilon^*) = \mathbb{E}[Q_t\epsilon_t] = \int_0^1 \max\left\{1, \frac{\epsilon(i)}{\epsilon^*}\right\} dF(\epsilon) > 1. \tag{A.10}$$

We then consider the pricing equations (13), (14) and (15). When $\epsilon_t(i) > \epsilon_t^*$, we plug the investment decision back into the value function, and by comparing the coefficients we obtain

$$\begin{aligned} v_t &= (1-\delta_k)Q_t + Q_t\epsilon_t R_{kt} \\ p_t &= [1 + (Q_t\epsilon_t - 1)(1-\omega+\omega\mu) - Q_t\epsilon_t\delta_h]\, P_t. \\ \phi_t &= Q_t\epsilon_t \end{aligned}$$

By combining the equations above and definitions of prices (A.7), (A.8) and (A.9), we directly obtain (13), (14) and (15).

B Proof of Proposition 2

Because $R_{kt} = \alpha \frac{Y_t}{K_t}$, Equation (13) can be written as

$$\frac{1}{\epsilon_t^*} = \beta \int \frac{\Lambda_{t+1}}{\Lambda_t} \left[\frac{\alpha Y_{t+1}}{K_{t+1}} L(\epsilon_{t+1}^*) + (1-\delta_k) \frac{1}{\epsilon_{t+1}^*} \right] dF(\epsilon).$$

Aggregate effective aggregate investment is

$$\int_0^1 \epsilon_t(i) i_t(i) di = \omega(\epsilon_t^*) I_t,$$

where

$$\omega(\epsilon_t^*) = \frac{\int_{\epsilon > \epsilon_t^*} \epsilon dF}{1 - F(\epsilon_t^*)} > \epsilon_t^*.$$

From the individual firm's investment decision, we obtain aggregate investment (19)

$$I_t = \int_0^1 i_t(i) di = [\alpha Y_t + (1 - \omega + \omega \mu - \delta_h) P_t H_t] \left(1 - F(\epsilon_t^*)\right).$$

Equation (16) is just the aggregate production function for the non-housing sector. (17) is the aggregate resource constraint. Labor market clearing yields (18). Equations (20) and (21) are the laws of motion for the non-housing sector and housing sector, where we make use of the housing sector production function $H_t^n = A_t^H (L_t^H) \sigma$. Equations (23) and (22) are wage determination functions for the non-housing and housing sector, and wages in the two sectors should be the same. Equations (25), (24) and (26) are aggregate versions of pricing functions (13), (14) and (15), where we use the conclusion from the consumer sector that $\Lambda_t = u'(C_t) = \frac{1}{C_t}$. Equation (27) is just the determination rule for cutoff value ϵ_t^*.

C Proof of Propositions 4 and 5

We first show how to calculate the variables $\{K_b, P_b, L_b^H\}$ in the bubbly steady state:

First, from the definition of capital return, we obtain

$$R_{kb} = \alpha \frac{Y_b}{K_b} = \alpha (K_b)^{\alpha - 1} (L_b^M)^{1 + \gamma - \alpha}. \tag{C.1}$$

Moreover, from the optimization of the non-housing sector, we can solve for the wage rate as

$$W_t = (1 - \alpha)(K_b)^{\alpha} (L_b^M)^{\gamma - \alpha} \tag{C.2}$$

Combine the above equation with Equations (22) and (21), we can solve for both L_b^H and H_b

$$L_b^H = 1 - L_b^M = \left(\frac{W_t}{\sigma P A^H}\right)^{\frac{1}{\sigma-1}} \tag{C.3}$$

$$H_b = \frac{1}{\delta_h} A^H (L_b^H)^\sigma = \frac{A^H}{\delta_h}\left(\frac{W_t}{\sigma P A^H}\right)^{\frac{\sigma}{\sigma-1}}, \tag{C.4}$$

Combined with $\frac{\alpha Y_b}{K_b} = R_{kb}$ and $I_b = \frac{\delta_k}{\omega(\epsilon_b^*)} K_b$, Equation (19) shows that

$$\frac{PH_b}{Y_b} = \frac{1}{1-\omega+\omega\mu-\delta_h}\left[\frac{\alpha\beta\delta_k}{1-\beta(1-\delta_k)}\frac{\int \max\{\epsilon,\epsilon_b^*\}\, dF(\epsilon)}{\int_{\epsilon>\epsilon_b^*}\epsilon dF(\epsilon)} - \alpha\right]. \tag{C.5}$$

Finally, we combine Equations (C.4) and (C.5) to obtain the equation to solve for housing price P:

$$P\frac{A^H}{\delta_h A^M(L^M)}\left(\frac{\alpha A^M(L^M)}{R_{kb}}\right)^{\frac{\alpha}{\alpha-1}}\frac{(L_b^H)^\sigma}{1-L_b^H} = \frac{1}{1-\omega+\omega\mu-\delta_h}\left[\frac{\alpha\beta\delta_k}{1-\beta(1-\delta_k)}\frac{\int \max\{\epsilon,\epsilon_b^*\}\, dF(\epsilon)}{\int_{\epsilon>\epsilon_b^*}\epsilon dF(\epsilon)} - \alpha\right] \tag{C.6}$$

where $L_b^H = \left(\frac{W_t}{\sigma P A^H}\right)^{\frac{1}{\sigma-1}}$, $A^M(L^M) = A^M(L^M)^\gamma$. Then, from Equations (C.1), (C.3) and (C.6), we can solve for K_b, L_b^M and P.

Next, we assume that the two equilibria coexist and prove Proposition 5 and the necessity of condition (37).

By Equation (25), we obtain

$$R_{kb} = \frac{1-\beta(1-\delta_k)}{\beta\int \max\{\epsilon,\epsilon_b^*\}\, dF(\epsilon)}.$$

Then, from Equation (C.5) and the condition that $PH_b > 0$, we obtain

$$\frac{\alpha\delta_k}{\int_{\epsilon>\epsilon_b^*}\epsilon dF(\epsilon)}\frac{1}{R_{kb}} > \alpha,$$

which yields

$$R_{kb} < \frac{\delta_k}{\int_{\epsilon>\epsilon_b^*}\epsilon dF(\epsilon)}.$$

Combining the above result and Equation (25) yields

$$1-\beta(1-\delta_k) = \beta R_{kb}\int \max\{\epsilon,\epsilon_b^*\}\, dF(\epsilon) < \beta\delta_k\frac{\int \max\{\epsilon,\epsilon_b^*\}\, dF(\epsilon)}{\int_{\epsilon>\epsilon_b^*}\epsilon dF(\epsilon)}.$$

Moreover, Equation (28) implies

$$1-\beta(1-\delta_k)=\beta\delta_k\frac{\int\max\left\{\epsilon_f^*,\epsilon\right\}dF(\epsilon)}{\int_{\epsilon>\epsilon_f^*}\epsilon dF(\epsilon)}.$$

By comparing the above two equations, we directly obtain $\epsilon_b^*>\epsilon_f^*$ and accordingly $R_{kb}<R_{kf}$, which means that the average investment efficiency is higher in the bubbly equilibrium. Additionally, $\epsilon_b^*>\epsilon_f^*$ directly implies (37), which proves the necessity.

Since $\epsilon_b^*>\epsilon_f^*$, then (26) yields $R_{fb}>R_{ff}$, so the bubbly equilibrium has a higher gross interest rate.

We then suppose that (33), (36) and (37) hold and prove that the two equilibria coexist.

First, since (33) holds, Equation (32) provides a unique solution $\epsilon_b^*\in(\epsilon_{min},\epsilon_{max})$, and we just need to show that this ϵ_b^* determines a bubbly equilibrium, which means that we just need to show that $C_b>0$.

We have already derived that $K_b=\left(\frac{R_{kb}}{\alpha A^M(L_b^M)}\right)^{\frac{1}{\alpha-1}}L_b^M=\left(\frac{R_{kb}}{\alpha A^M}\right)^{\frac{1}{\alpha-1}}(L_b^M)^{\frac{\alpha-1-\gamma}{\alpha-1}}$ and $I_b=\frac{\delta_k}{\omega(\epsilon_b^*)}K_b$ so $Y_b=A^M(L_b^M)K_b^\alpha(L_b^M)^{1-\alpha}=A^M\left(\frac{R_{kb}}{\alpha A^M}\right)^{\frac{\alpha}{\alpha-1}}(L_b^M)^{\frac{\alpha-1-\gamma}{\alpha-1}}$. Then, consumption is obtained as

$$C_b=Y_b-I_b=\left(\frac{R_{kb}}{\alpha A^M}\right)^{\frac{1}{\alpha-1}}(L_b^M)^{\frac{\alpha-1-\gamma}{\alpha-1}}\left(\frac{R_{kb}}{\alpha}-\frac{\delta_k}{\omega(\epsilon_b^*)}\right)>0 \tag{C.7}$$

which is nonnegative because (36) holds and $\omega(\epsilon_b^*)>\omega(\epsilon_f^*)$. Now, we have proved that the bubbly equilibrium exists. Then, we show that bubbleless equilibrium also exists. From Proposition 3 we just need to show that $R_{kf}>\frac{\alpha\delta_k}{\omega(\epsilon_f^*)}$. Since $\epsilon_b^*>\epsilon_f^*$ and $R_{kf}>R_{kb}$, the condition is obviously satisfied, so under (33), (36) and (37) the two equilibria coexist.

D Solving the Model with Elastic Labor

Denote by N_t^M and N_t^H the labor hired in the non-housing sector and the housing sector, respectively. From the analysis in Section 5.1, we can easily obtain the equilibrium system in the extended model as follows:

Proposition D.1. *The general equilibrium paths of the model are characterized by 12 aggregate variables, $\{C_t, I_t, N_t^M, N_t^H, Y_t, K_{t+1}, H_{t+1}, P_t, \epsilon_t^*, Q_t, R_{ft}, W_t\}$, which are determined by the following non-*

linear system of equations:

$$Y_t = A_t^M(N_t^M)K_t^\alpha(N_t^M)^{1-\alpha} \tag{D.1}$$

$$C_t + I_t = Y_t \tag{D.2}$$

$$\psi N_t^\eta = \frac{W_t}{C_t} \tag{D.3}$$

$$I_t = [\alpha Y_t + (1-\omega+\omega\mu-\delta_h)P_tH_t](1-F(\epsilon_t^*)) \tag{D.4}$$

$$K_{t+1} = (1-\delta_k)K_t + \omega(\epsilon_t^*)I_t \tag{D.5}$$

$$H_{t+1} = (1-\delta_h)H_t + A_t^H(N_t^H)^\sigma \tag{D.6}$$

$$W_t = P_tA_t^H\sigma(N_t^H)^{\sigma-1} \tag{D.7}$$

$$W_t = (1-\alpha)A_t^M(L_t^M)K_t^\alpha(N_t^M)^{-\alpha} \tag{D.8}$$

$$\frac{P_t}{C_t} = \beta\int\frac{1}{C_{t+1}}\left[1+(L(\epsilon_{t+1}^*)-1)(1-\omega+\omega\mu)-L(\epsilon_{t+1}^*)\delta_h\right]P_{t+1}dF(\epsilon) \tag{D.9}$$

$$\frac{Q_t}{C_t} = \beta\int\frac{1}{C_{t+1}}\left[\frac{\alpha Y_{t+1}}{K_{t+1}}L(\epsilon_{t+1}^*)+(1-\delta_k)Q_{t+1}\right]dF(\epsilon) \tag{D.10}$$

$$\frac{1}{C_tR_{ft}} = \beta\int\frac{1}{C_{t+1}}L(\epsilon_{t+1}^*)dF(\epsilon) \tag{D.11}$$

$$Q_t\epsilon_t^* = 1 \tag{D.12}$$

where the coefficient $\omega(\epsilon_t^*) = \frac{\int_{\epsilon>\epsilon_t^*}\epsilon dF}{1-F(\epsilon_t^*)} > \epsilon_t^*$ *measures the average marginal efficiency of aggregate investment.*

D.0.1 Bubbleless Steady State

We first consider a bubbleless steady state where $P = 0$ and so $N^H = 0$. Then, Equations (D.4) and (D.5) imply that

$$R_{kf} = \frac{\alpha Y_f}{K_f} = \frac{\delta_k}{\int_{\epsilon>\epsilon_f^*}\epsilon dF(\epsilon)} = \alpha A^M(K_f)^{\alpha-1}(L_f^M)^{\gamma+1-\alpha},$$

which is the same as in the baseline model.

As in the baseline model, the wage in steady state is given by:

$$W_f = (1-\alpha)A^M(N_f^M)\left(\frac{K_f}{N_f^M}\right)^\alpha = (1-\alpha)A^M(K_f)^\alpha(N_f^M)^{\gamma-\alpha} = (1-\alpha)A^M\left(\frac{R_{kf}}{\alpha A^M}\right)^{\frac{\alpha}{\alpha-1}}(N_f^M)^{-\frac{\gamma}{\alpha-1}}.$$

From Equation (D.10), in steady state, we can solve for the cutoff value ϵ_f^* in the steady state, which satisfies the same equation as in our baseline model:

$$1 - \beta(1-\delta_k) = \beta\delta_k \frac{\int \max\left\{\epsilon_f^*, \epsilon\right\} dF(\epsilon)}{\int_{\epsilon > \epsilon_f^*} \epsilon dF(\epsilon)}. \tag{D.13}$$

From the investment equations (D.2) and (D.5), we can solve for investment and consumption:

$$I_f = \alpha(1 - F(\epsilon_f^*))Y_f,$$

and

$$\begin{aligned} C_f &= Y_f - I_f, \\ &= (1 - \alpha + \alpha F(\epsilon_f^*))Y_f, \\ &= (1 - \alpha + \alpha F(\epsilon_f^*))A^M (K_f)^\alpha (N_f^M)^{1+\gamma-\alpha}, \\ &= (1 - \alpha + \alpha F(\epsilon_f^*))A^M \left(\frac{R_{kf}}{\alpha A^M}\right)^{\frac{\alpha}{\alpha-1}} (N_f^M)^{\frac{\alpha-1-\gamma}{\alpha-1}}. \end{aligned}$$

Then, from the intratemporal decision rule, we obtain the equation that N_f^M satisfies:

$$\psi(N_f^M)^\eta = \frac{1-\alpha}{(1 - \alpha + \alpha F(\epsilon_f^*))N_f^M}. \tag{D.14}$$

D.0.2 Bubbly Steady State

We then consider the bubbly steady state where $P_t = P$ for all t. Now, the pricing equation (D.9) yields the same equation that ϵ_b^* satisfies in our baseline model:

$$\frac{1}{\beta} - 1 = (1 - \omega + \omega\mu) \int_{\epsilon > \epsilon_b^*} \frac{\epsilon - \epsilon_b^*}{\epsilon_b^*} dF(\epsilon) - \delta_h \int \max\left\{\frac{\epsilon}{\epsilon_b^*}, 1\right\} dF(\epsilon). \tag{D.15}$$

Following the same procedure as before, we can solve for the return of capital in the bubbly steady state:

$$R_{kb} = \frac{1 - \beta(1-\delta_k)}{\beta \int \max\left\{\epsilon, \epsilon_b^*\right\} dF(\epsilon)}.$$

Since $R_{kb} = \alpha\frac{Y_b}{K_b} = \alpha A^M (K_b)^{\alpha-1}(N_b^M)^{1+\gamma-\alpha}$, we have:

$$W_t = (1-\alpha)A^M(K_b)^\alpha(N_b^M)^{\gamma-\alpha} = (1-\alpha)A^M\left(\frac{R_{kb}}{\alpha A^M}\right)^{\frac{\alpha}{\alpha-1}} (N_b^M)^{-\frac{\gamma}{\alpha-1}},$$

so we obtain an equation that P and N_b^H satisfy:

$$PA^H\sigma(N_b^H)^{\sigma-1} = W_t = (1-\alpha)A^M\left(\frac{R_{kb}}{\alpha A^M}\right)^{\frac{\alpha}{\alpha-1}}(N_b^M)^{-\frac{\gamma}{\alpha-1}}. \tag{D.16}$$

Following a similar procedure as before, we can solve for the relative value of houses to output PH/Y in two ways and obtain another equation that is similar to that in our baseline model:

$$\frac{1}{1-\omega+\omega\mu-\delta_h}\left[\frac{\alpha\beta\delta_k}{1-\beta(1-\delta_k)}\frac{\int\max\{\epsilon,\epsilon_b^*\}dF(\epsilon)}{\int_{\epsilon>\epsilon_b^*}\epsilon dF(\epsilon)}-\alpha\right] = P\frac{A^H}{\delta_h A^M}\left(\frac{\alpha A^M}{R_{kb}}\right)^{\frac{\alpha}{\alpha-1}}\frac{(N_b^H)^\sigma}{(N_b^M)^{\frac{\alpha-1-\gamma}{\alpha-1}}}. \tag{D.17}$$

We now consider consumption. As we have shown previously, the investment ratio in the bubbly steady state is:

$$\frac{I_b}{Y_b} = \frac{I_b}{K_b}\frac{K_b}{Y_b} = \frac{\delta_k}{\omega(\epsilon_b^*)}\frac{\alpha}{R_{kb}} = \frac{\alpha\beta\delta_k}{1-\beta(1-\delta_k)}\frac{\int\max\{\epsilon,\epsilon_b^*\}dF(\epsilon)}{\int_{\epsilon>\epsilon_b^*}\epsilon dF(\epsilon)}(1-F(\epsilon_b^*)),$$

and then from the resource constraint, we obtain

$$C_b = \left(1-\frac{I_b}{Y_b}\right)Y_b = \left(1-\frac{I_b}{Y_b}\right)A^M(K_b)^\alpha(N_b^M)^{1+\gamma-\alpha} = \left(1-\frac{I_b}{Y_b}\right)A^M\left(\frac{R_{kb}}{\alpha A^M}\right)^{\frac{\alpha}{\alpha-1}}(N_b^M)^{\frac{\alpha-\gamma-1}{\alpha-1}}. \tag{D.18}$$

Thus, the intratemporal labor supply decision yields the third equation:

$$\psi(N_b^H+N_b^M)^\eta = \frac{W_t}{C_t}. \tag{D.19}$$

From Equations (D.16), (D.17) and (D.19), we can solve for $\{P, N_b^M, N_b^H\}$ in the bubbly steady state, and all other variables are solvable.

E Model Extension with Household Housing Demand

E.1 Introduce Housing Demand of Households

Households choose consumption C_t, the share of non-housing firms $s_{t+1}(i)$ and rent of housing H_t^C to maximize expected lifetime utility:

$$\max \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[log(C_t)+\chi log(H_t^C)\right],$$

subject to budget constraint

$$C_t + r_{ht}H_t^C + \int s_{t+1}(i)[V_t(i) - d_t(i)]di = \int s_t(i)V_t(i)di + W_tL_t + \Pi_t,$$

where χ captures the households' taste for housing services.

The FOCs of the households are

$$\begin{aligned}
\frac{1}{C_t} &= \Lambda_t, \\
\frac{\chi}{H_t^C} &= \Lambda_t r_{ht}, \\
V_t(i) &= \beta \mathbb{E}_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(i) + d_t(i) \right\}.
\end{aligned}$$

The housing sector is the same as before, and we do not repeat here.

The non-housing sector is similar to that before, except that in every period, firms can first rent their housing to households and obtain rent r_{ht}, so the firms' budget constraint is:

$$d_t(i) = R_{kt}k_t(i) - P_t[h_{t+1}(i) - (1-\delta_h)h_t(i)] + \frac{b_{t+1}(i)}{R_{ft}} - b_t(i) + r_{ht}h_t^C(i) - i_t(i),$$

where $h_t^C(i)$ is the house rented to households by firm i, and the house-renting constraint is

$$h_t^C(i) \leq h_t(i).$$

Note that here we assume that renting houses to households will not affect the firms' house sales and collateralization.

The way to solve this extended model is similar to that of the baseline model, and we show the details in the Appendix E.2. We find that in this model, there is only one equilibrium, and in this equilibrium, the housing price is higher than its fundamental value, which is defined as the discounted value of rent flow. As stated previously, the high price comes from the liquidity premium on houses.

We use the parameter values in Section 4.1 and calibrate $\chi = 0.1$ such that the price-to-rent ratio when $\mu = 0.65$ in our model is approximately 16, close to that in the U.S. We change the value of μ from near zero to one and calculate the values of variables in steady state. We show the results in Figures 1(a), 1(b), 1(c), 1(d), 1(e) and 1(f). Similar to the empirical results of Dong et al. (2022), financial development will increase house prices but does not significantly impact house rent, causing a relatively larger liquidity premium. Moreover, since firms can obtain more credit from house collateralization, production is stimulated, and both output and consumption increase. As a result, social welfare increases with the development of the financial market. Furthermore, a more developed financial system leads to a boom in the housing sector: the house stock in steady state increases with μ, and more labor goes into the housing sector. The main findings are consistent with those of our baseline model.

We can also calculate the transition path from a low-μ steady state to a high-μ steady state and illustrate it in Figure E.2. We see that the transition path is similar to that of our baseline model: after the sudden development of the financial market, the price of housing jumps and a substantial amount of labor goes into the housing sector, causing a decline in consumption and output. Different from that in the baseline model, investment jumps (rather than drops as in the baseline model) since the house stock is now positive (note that in the baseline model, the house stock is zero before the transition), so the crowding-in effect of increasing housing prices exceeds the crowding-out effect of reduced output. After the initial boom, labor returns to the non-housing sector, and the house stock decreases to its new steady-state value. On the transition path, the housing rent first decreases due to the drop in consumption and then rises to its new steady state.

E.2 Solving the Model with Household Housing Demand

Obviously, the best rental decision for firms is $h_t^C(i) = h_t(i)$. We can show that firms' investment decision also follows a trigger policy:

Proposition E.1. *The firms' investment decision follows a trigger policy:*

$$i_t(i) = \begin{cases} R_{kt}k_t(i) + [(1-\omega+\omega\mu-\delta_h)P_t + r_{ht}]h_t(i) - b_t(i), & \text{if } \epsilon_t(i) \geq \epsilon_t^* \\ 0, & \text{if } \epsilon_t(i) < \epsilon_t^* \end{cases} \tag{E.1}$$

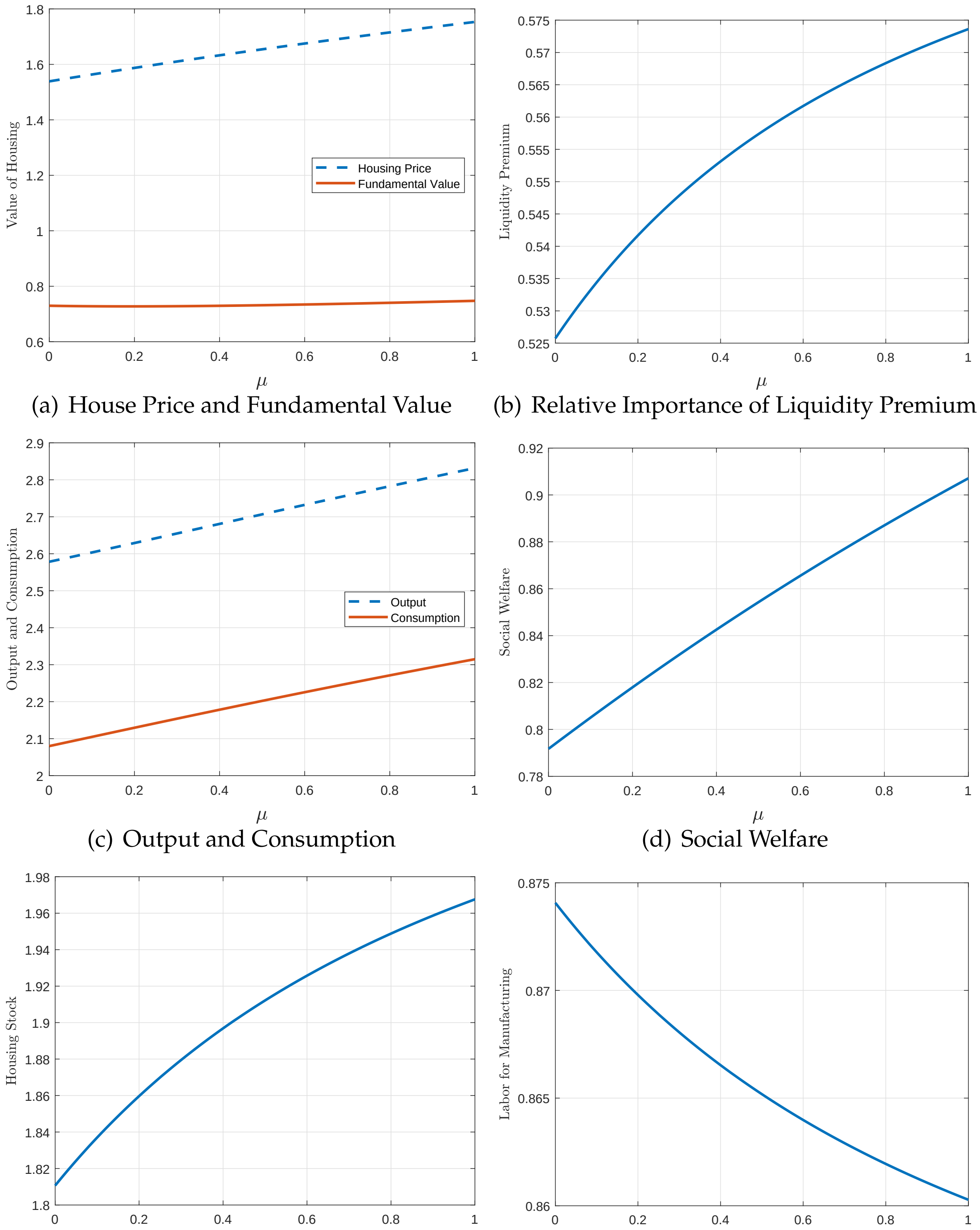
Where the cutoff value ϵ_t^* is determined by the Euler equation:

$$\frac{1}{\epsilon_t^*} = \beta \int \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ R_{k,t+1}L(\epsilon_{t+1}^*) + (1-\delta_k)\frac{1}{\epsilon_{t+1}^*} \right\} dF(\epsilon) \tag{E.2}$$

When housing demand is positive, the housing price is determined by:

$$P_t = \beta \int \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ L(\epsilon_{t+1}^*) r_{h,t+1} + [1 + (L(\epsilon_{t+1}^*) - 1)(1-\omega+\omega\mu) - L(\epsilon_{t+1}^*)\delta_h] P_{t+1} \right\} dF(\epsilon) \tag{E.3}$$

The proof is obtained by the "guess-and-verify" method and is the same as before so we



(a) House Price and Fundamental Value

(b) Relative Importance of Liquidity Premium

(c) Output and Consumption

(d) Social Welfare

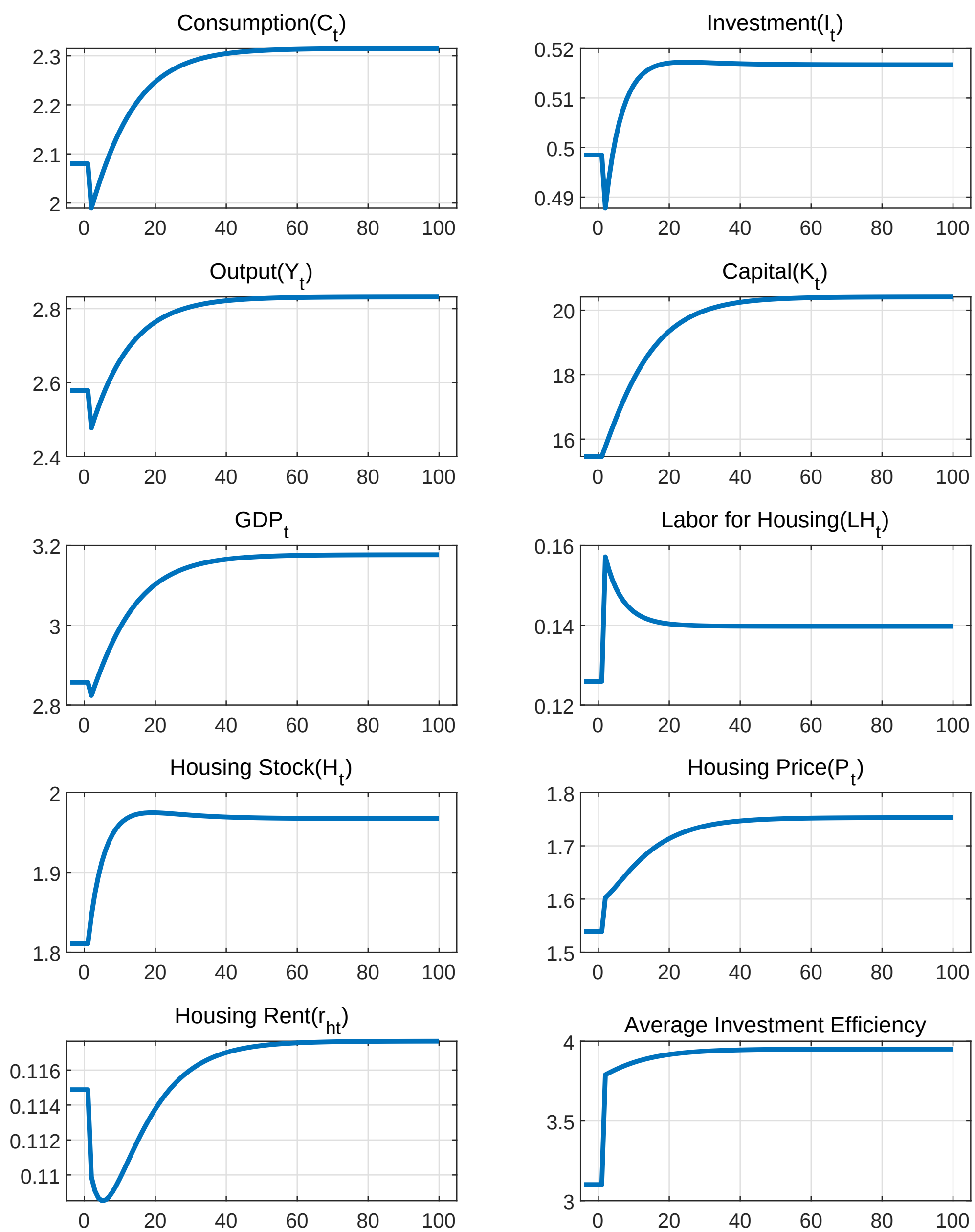
(e) House Stock

(f) Labor Allocation

Note: Figure 1(a) shows the house price and fundamental value (discounted value of rent flow) under different levels of μ, and Figure 1(b) shows the relative importance, or the fraction of the "bubble" in the house price. Figure 1(c) shows output and consumption under different levels of μ, and Figure 1(d) compares the steady-state social welfare. Figures 1(e) and 1(f) show the house stock and labor supply in the non-housing sector, respectively.

Figure E.1: Comparative Statics: Endogenous Fundamentals

Figure E.2: Transition Path



Note: Here, we calculate the transition path from the steady state where $\mu = 0$ to that with $\mu = 1$.

omit the proof here. As before, the general equilibrium system is (note that house rental market clearing yields $\int h_t^C(i) = H_t^C$) characterized by the following proposition.

Proposition E.2. *The general equilibrium system is given by the following nonlinear system:*

$$Y_t = A_t^M(L_t^M)K_t^\alpha(L_t^M)^{1-\alpha} = A^M K_t^\alpha (L_t^M)^{1+\gamma-\alpha} \tag{E.4}$$

$$C_t + I_t = Y_t \tag{E.5}$$

$$L_t^M + L_t^H = 1 \tag{E.6}$$

$$I_t = [\alpha Y_t + (1-\omega+\omega\mu-\delta_h)P_t H_t + r_{ht}H_t](1-F(\epsilon_t^*)) \tag{E.7}$$

$$K_{t+1} = (1-\delta_k)K_t + \omega(\epsilon_t^*)I_t \tag{E.8}$$

$$H_{t+1} = (1-\delta_h)H_t + A_t^H(L_t^H)^\sigma \tag{E.9}$$

$$W_t = P_t A_t^H \sigma (L_t^H)^{\sigma-1} \tag{E.10}$$

$$W_t = (1-\alpha)A^M K_t^\alpha (L_t^M)^{\gamma-\alpha} \tag{E.11}$$

$$\frac{\chi}{H_t} = \frac{r_{ht}}{C_t} \tag{E.12}$$

$$\frac{P_t}{C_t} = \beta \int \frac{1}{C_{t+1}} \left\{ L(\epsilon_{t+1}^*) r_{h,t+1} + [1 + (L(\epsilon_{t+1}^*) - 1)(1-\omega+\omega\mu) - L(\epsilon_{t+1}^*)\delta_h] P_{t+1} \right\} dF(\epsilon) \tag{E.13}$$

$$\frac{Q_t}{C_t} = \beta \int \frac{1}{C_{t+1}} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} L(\epsilon_{t+1}^*) + (1-\delta_k) Q_{t+1} \right] dF(\epsilon) \tag{E.14}$$

$$\frac{1}{R_{ft}C_t} = \beta \int \frac{1}{C_{t+1}} L(\epsilon_{t+1}^*) dF(\epsilon) \tag{E.15}$$

$$Q_t \epsilon_t^* = 1 \tag{E.16}$$

Note that the "liquidity premium" term before the housing rent and housing price are different: the liquidity premium of housing rent is just $L(\epsilon_{t+1}^*)$, since by holding a unit of housing, the firm can always obtain a cash flow of $r_{h,t+1}$, and the firm can either hold the money or use it for investment without any financial friction. However, the collateral value of houses is not simply P_t. If a firm holds a unit of housing, when it wants to invest and needs credit, it can only sell $1-\omega$ units of housing and use the remainder as collateral, so the net cash flow is $(1-\omega+\omega\mu)P_{t+1}$, and the corresponding liquidity premium for house holding is $1 + (L(\epsilon_{t+1}^*) - 1)(1-\omega+\omega\mu)$. Here, we assume that the depreciation rate for housing is zero. When there are "no" financial frictions, which means that either $\mu = 1$ or $\omega = 0$, the cash flow from housing rent and house holding are the same, and thus the liquidity premia of the two are the same. When $\omega = 0$, firms can sell all of their house assets; when $\mu = 1$, the LTV ratio equals 1, so selling houses and using them as collateral yield the same cash flow.

We now solve the extended model. In the steady state, Equation (E.13) yields

$$L(\epsilon^*)r_h = \left[\frac{1}{\beta} + L(\epsilon^*)\delta_h - (L(\epsilon^*)-1)(1-\omega+\omega\mu) - 1\right] P,$$

so that we have $\frac{r_h}{P} = \lambda(\epsilon^*)$, where

$$\begin{aligned}\lambda(\epsilon^*) &= \frac{1}{L(\epsilon^*)}\left[\frac{1}{\beta} + L(\epsilon^*)\delta_h - (L(\epsilon^*)-1)(1-\omega+\omega\mu) - 1\right] \\ &= \frac{1}{L(\epsilon^*)}\left(\frac{1}{\beta} - \omega(1-\mu)\right) - (1-\omega+\omega\mu-\delta_h).\end{aligned}$$

If we assume that $\frac{1}{\beta} - \omega(1-\mu) > 0$ (which is easy to satisfy if we assume that agents are patient enough), then obviously $\lambda(\epsilon^*)$ increases with ϵ^*.

We then can rewrite equation (E.7) as $I = [\alpha Y + (1-\omega+\omega\mu-\delta_h+\lambda(\epsilon^*))PH](1-F(\epsilon^*))$. Following the same process as before, we obtain $\frac{I}{Y} = \frac{\alpha\delta_k}{\omega(\epsilon^*)R_k}$ so that

$$\frac{PH}{I} = \frac{1}{1-\omega+\omega\mu-\delta_h+\lambda(\epsilon^*)}\left[\frac{1}{1-F(\epsilon^*)} - \frac{\omega(\epsilon^*)R_k}{\delta_k}\right],$$

where $R_k = \frac{1-\beta(1-\delta_k)}{\beta\int\max\{\epsilon,\epsilon^*\}dF(\epsilon)}$ as in Section 3. We can then obtain

$$\frac{PH}{C} = \frac{PH}{I}\frac{I}{C} = \frac{PH}{I}\frac{1}{\frac{Y}{I}-1} = \frac{1}{1-\omega+\omega\mu-\delta_h+\lambda(\epsilon^*)}\left[\frac{1}{1-F(\epsilon^*)} - \frac{\omega(\epsilon^*)R_k}{\delta_k}\right]\frac{\alpha\delta_k}{\omega(\epsilon^*)R_k-\alpha\delta_k}. \tag{E.17}$$

From Equation (E.12) we directly obtain

$$\frac{PH}{C} = \frac{\chi}{\lambda(\epsilon^*)}. \tag{E.18}$$

Combining Equations (E.17) and (E.18), we can solve for ϵ^* in steady state:

$$\frac{\chi}{\lambda(\epsilon^*)} = \frac{1}{1-\omega+\omega\mu-\delta_h+\lambda(\epsilon^*)}\frac{\beta\delta_k\epsilon^*F(\epsilon^*) - (1-\beta)\int_{\epsilon>\epsilon^*}\epsilon dF(\epsilon)}{\beta\delta_k(1-F(\epsilon^*))\int\max\{\epsilon,\epsilon^*\}dF(\epsilon)}\frac{\alpha\delta_k}{\omega(\epsilon^*)R_k-\alpha\delta_k}. \tag{E.19}$$

After obtaining ϵ^*, we follow a similar procedure as before to solve for price P. From the analysis above, we know that the relative value of housing to output is:

$$\frac{PH}{Y} = \frac{1}{1-\omega+\omega\mu-\delta_h+\lambda(\epsilon^*)}\left[\frac{\alpha\beta\delta_k}{1-\beta(1-\delta_k)}\frac{\int\max\{\epsilon,\epsilon^*\}dF(\epsilon)}{\int_{\epsilon>\epsilon^*}\epsilon dF(\epsilon)} - \alpha\right]. \tag{E.20}$$

As before, we can also calculate the following from the production functions of the non-housing and housing sectors:

$$\frac{PH}{Y} = P\frac{A^H}{\delta_h A^M}\left(\frac{\alpha A^M}{R_k}\right)^{\frac{\alpha}{\alpha-1}}\frac{(L^H)^\sigma}{(1-L^H)^{\frac{\alpha-\gamma-1}{\alpha-1}}}, \tag{E.21}$$

where

$$R_k = \alpha A^M K^{\alpha-1}(L^M)^{1+\gamma-\alpha},$$

and

$$W = (1-\alpha)A^M K^\alpha (L^M)^{\gamma-\alpha} = \sigma P (L^H)^{\sigma-1}.$$

We find that there is only one solution for ϵ^* in the reasonable range, which means that there is only one steady state in our extended model. The disappearance of multiple steady states stems from the fact that in our model the "fundamental value" of houses is *endogenous*. However, in our extended model, the house price may be higher than its fundamental value, which means that the housing price may still contain a liquidity premium term.

From the above four equations, we can solve for price and labor in the housing sector in steady state, and then we can calculate all other variables in steady state from the equilibrium system. Moreover, note that for the non-housing firms, the fundamental value of houses is the discount value of the rent flow, that is,

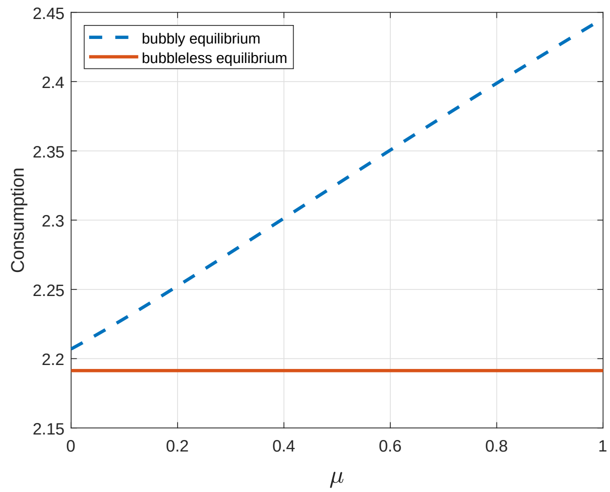
$$P_t^f = \sum_{\tau=1}^{\infty}(\beta(1-\delta_h))^\tau r_{h,t+\tau}.$$

In the steady state, we obviously have $P^f = \frac{r_h}{1-\beta(1-\delta_h)}$.

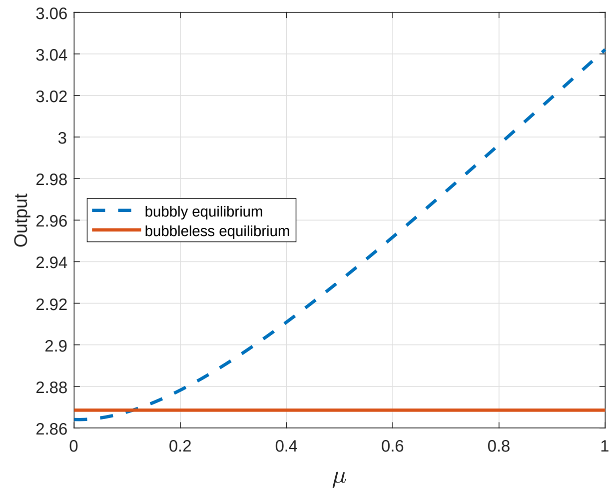
F Results without Externality

F.1 Baseline Model without Externality

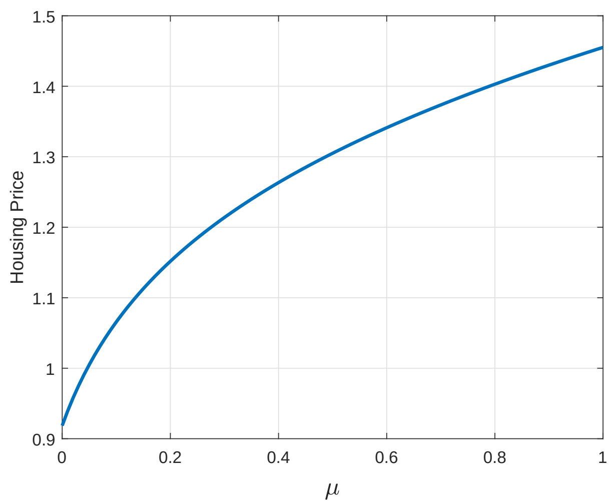
F.2 Extended Model without Externality



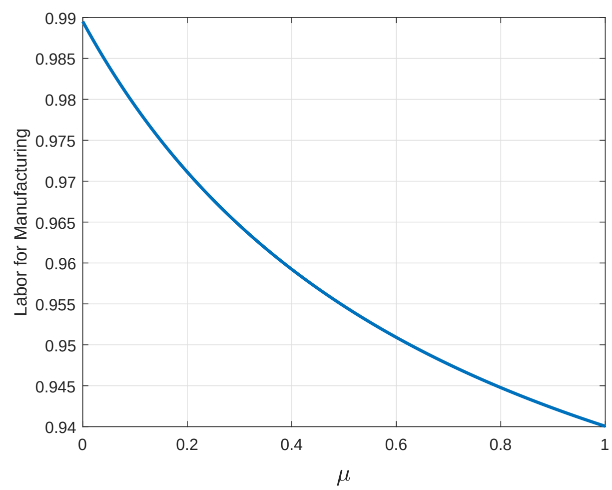
(a) Comparison of Consumption



(b) Comparison of Output



(c) Price of Housing

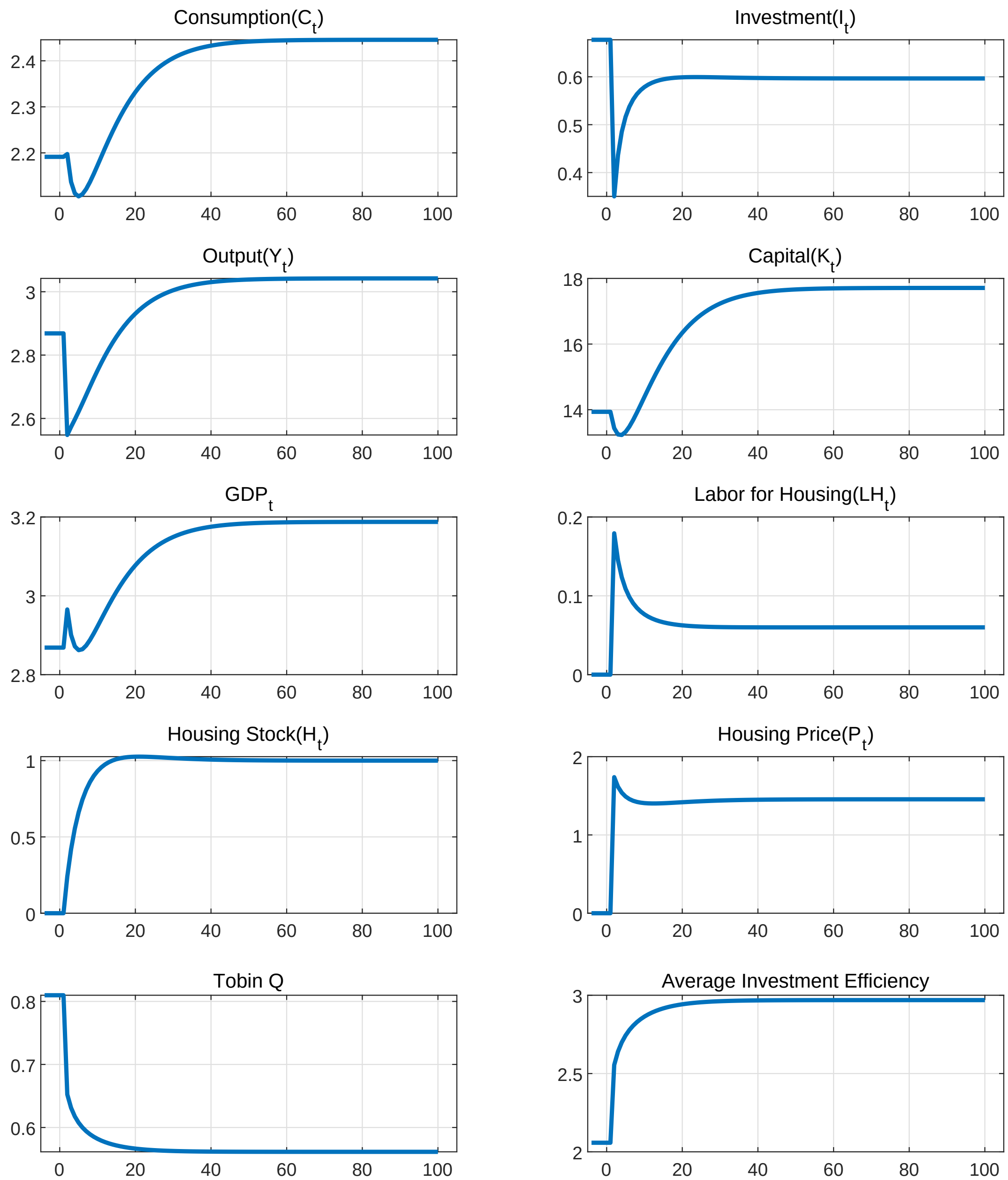


(d) Labor Allocation

Note: Here, we show the comparative statics of the baseline model with no externality. Figures 1(a) and 1(b) compare the output and consumption in the bubbleless and bubbly steady states, Figure 1(c) shows the house price in bubbly steady states with different levels of μ, and Figure 1(d) shows the fraction of labor in the non-housing sector in bubbly steady states with different μ.

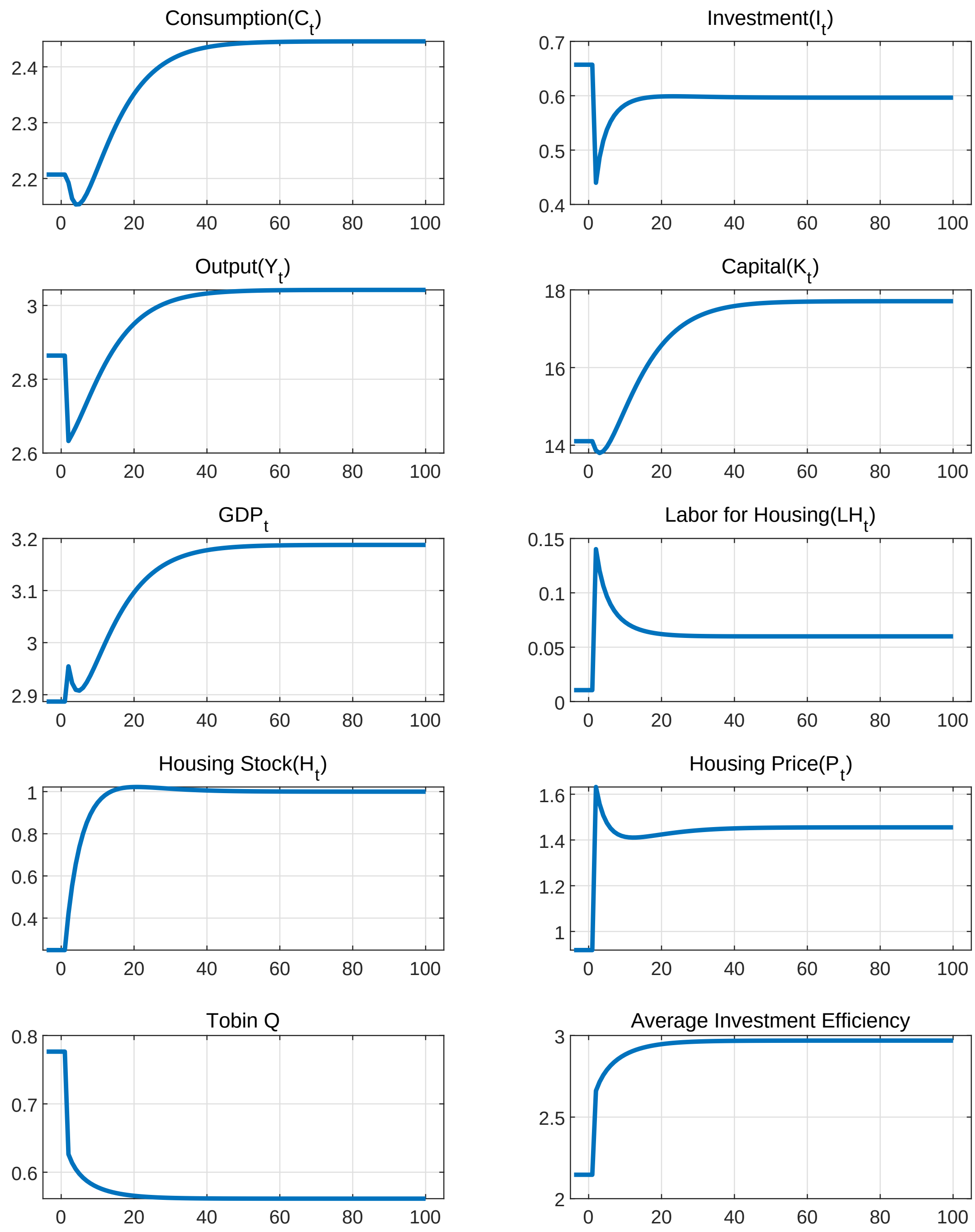
Figure F.1: Comparison of Steady States: Without Externality

Figure F.2: Transitional Dynamics from Bubbleless to Bubbly Steady States, $\gamma = 0$



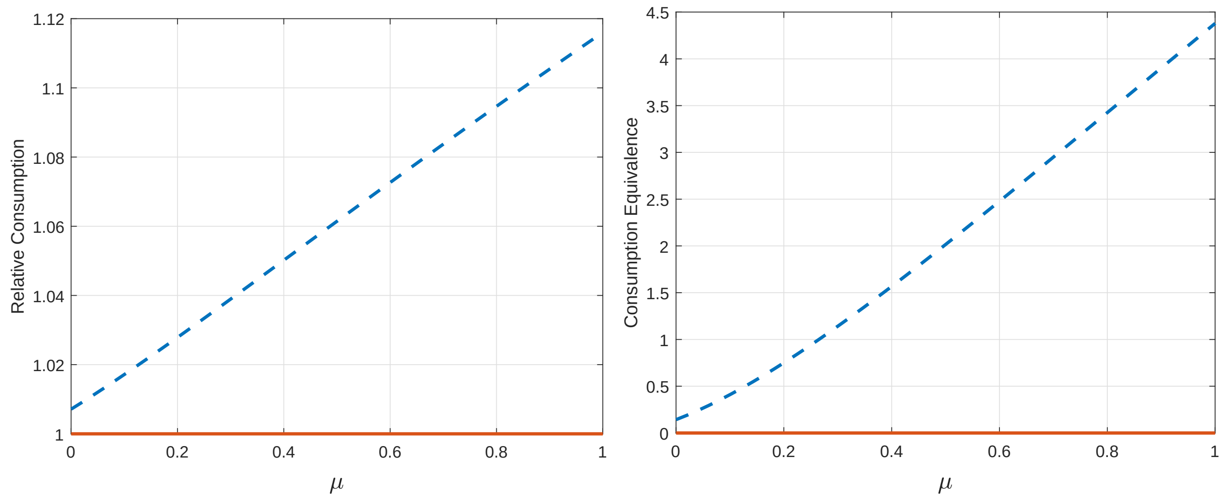
Note: Here, we calculate the transition path of the baseline model with no externality from the bubbleless steady state to the bubbly steady state with $\mu = 1$.

Figure F.3: Transitional Dynamics from Low-μ to High-μ Steady States, $\gamma = 0$



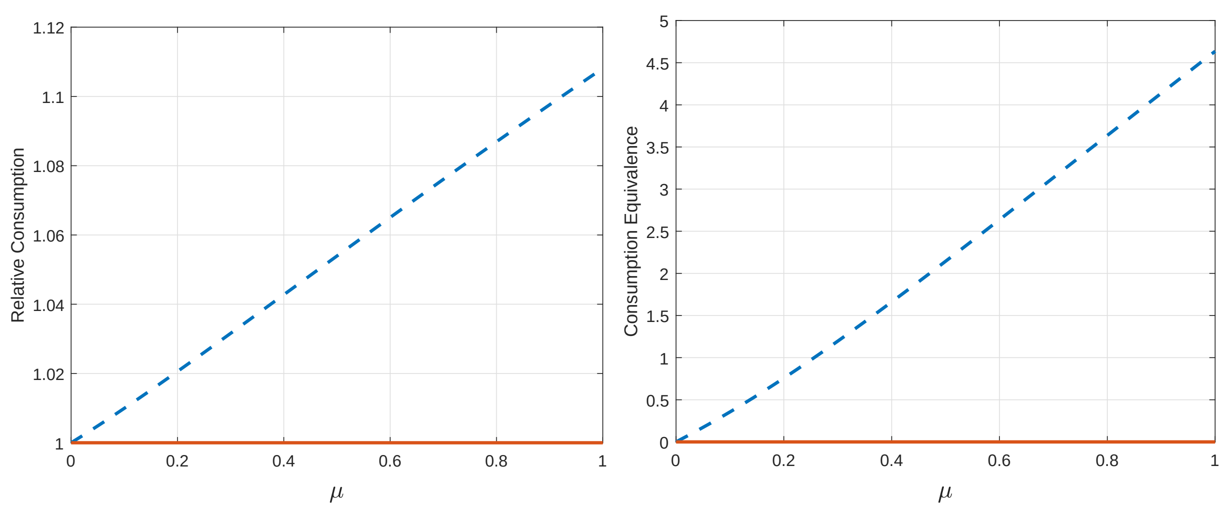
Note: Here, we calculate the transition path of the baseline model with no externality from the bubbly steady state with $\mu = 0$ to that with $\mu = 1$.

Figure F.4: Welfare Analysis: Bubbleless to Bubbly, $\gamma = 0$



(a) Relative Consumption (b) Welfare Comparison

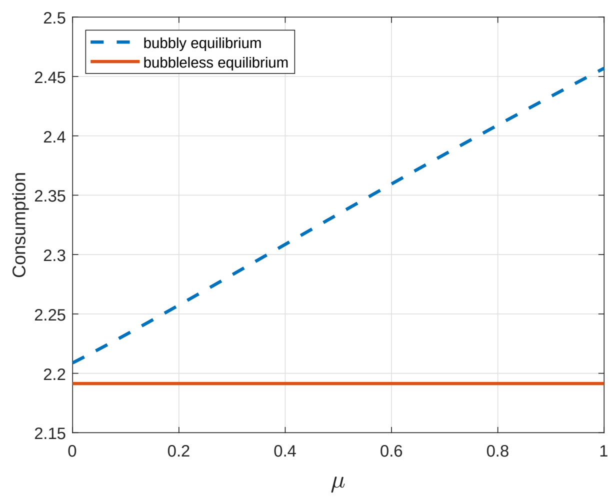
Note: Here, we perform the welfare analysis of the baseline model with no externality. Figure 5(a) shows the ratios of consumption in different bubbly steady states and that in the bubbleless steady state, and Figure 5(b) shows the welfare effect of the transition using consumption equivalence.



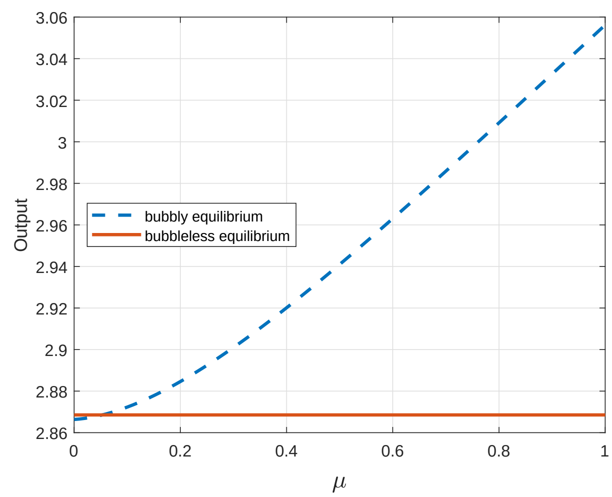
(c) Relative Consumption (d) Welfare Comparison

Note: Figure 5(c) shows the ratios of consumption in different bubbly steady states, and Figure 5(d) shows the welfare effect of the transition using consumption equivalence.

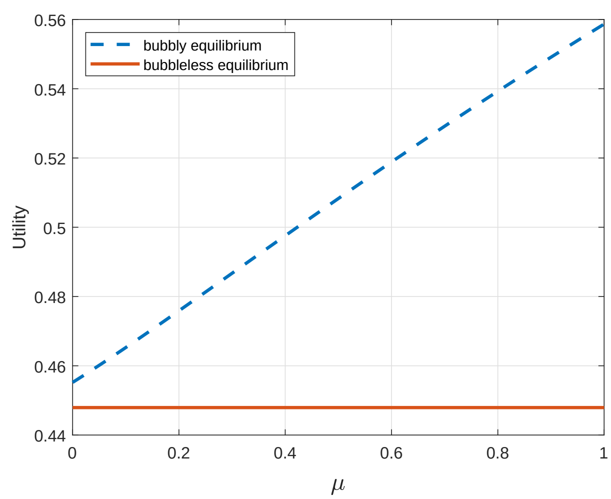
Figure F.5: Welfare Analysis: Low-μ to High-μ, $\gamma = 0$



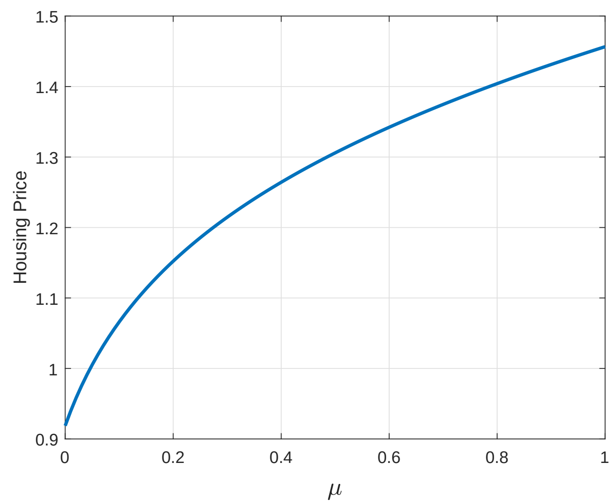
(a) Comparison of Consumption



(b) Comparison of Output



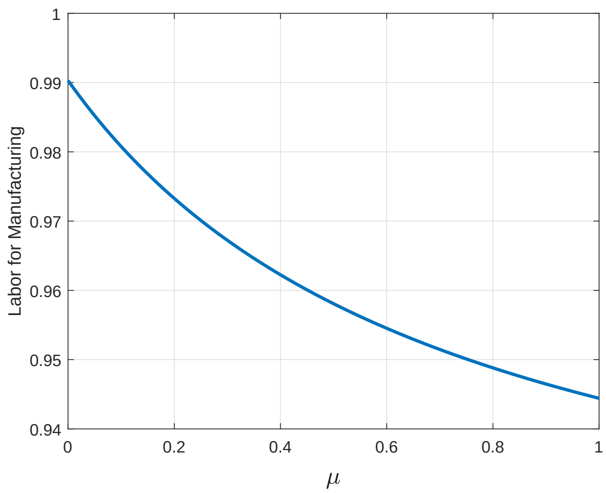
(c) Comparison of Utility



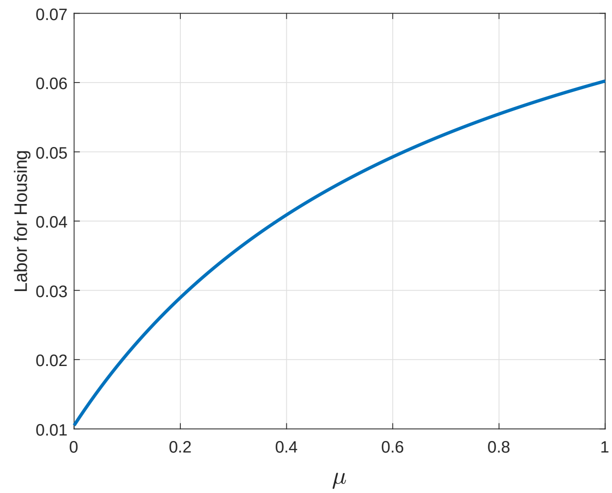
(d) Housing Price

Note: Figures 6(a) and 6(b) show the comparison of output and consumption between bubbly steady states and the bubbleless steady state, Figure 6(c) shows the comparison of steady state utility, and Figure 6(d) shows the house price in different bubbly steady states.

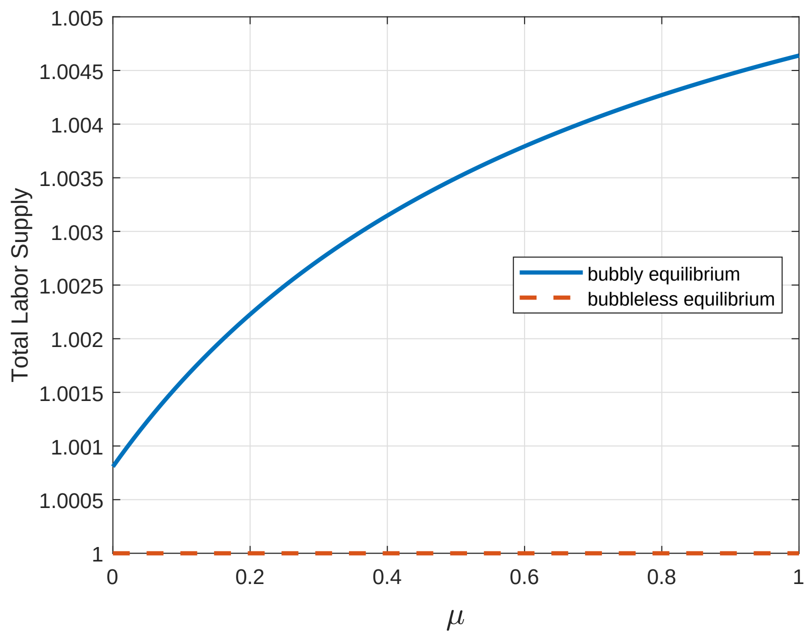
Figure F.6: Comparison of Steady States: Elastic Labor, Without Externality



(a) Labor for the Non-housing Sector



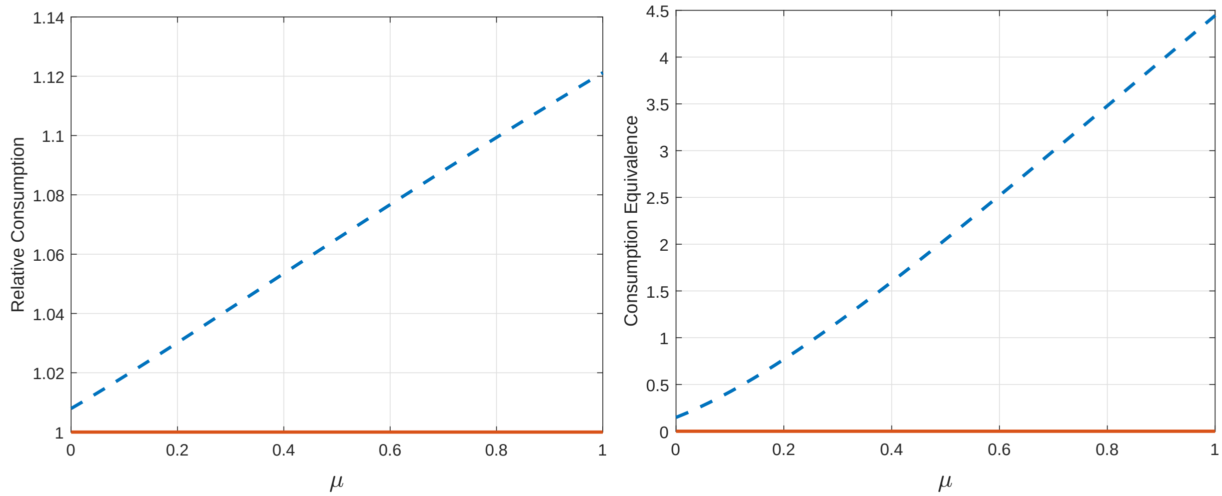
(b) Labor for the Housing Sector



(c) Aggregate Labor Supply

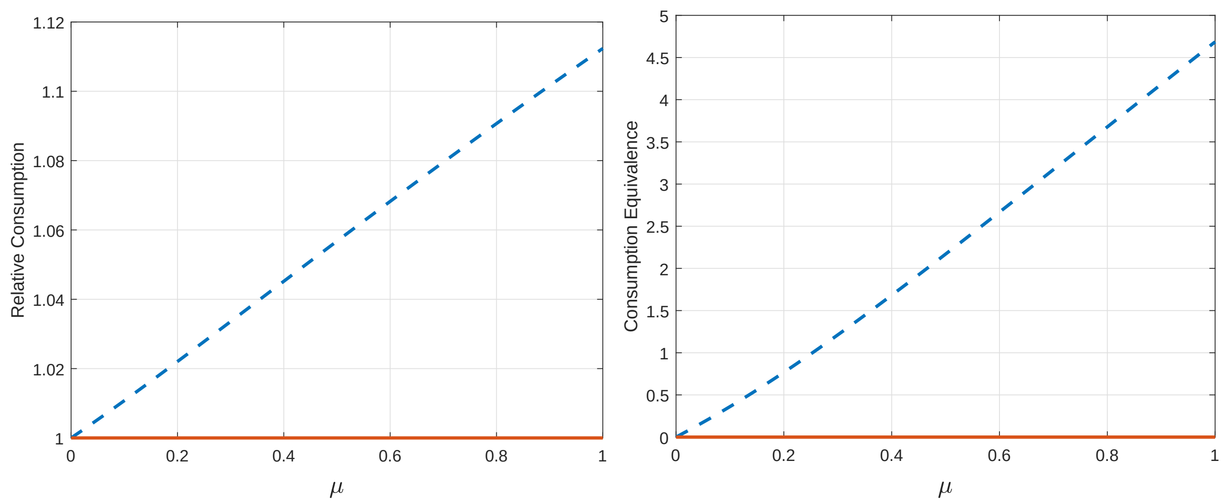
Note: Figures 7(a) and 7(b) show the labor supply in the non-housing and housing sectors in different bubbly steady states, respectively, and Figure 7(c) shows the aggregate labor supply.

Figure F.7: Effect of Housing Bubble on Labor Supply: Without Externality



(a) Relative Consumption (b) Welfare Comparison

Figure F.8: Welfare Analysis: Bubbleless to Bubbly, Elastic Labor, $\gamma = 0$



(a) Relative Consumption (b) Welfare Comparison

Figure F.9: Welfare Analysis: Low-μ to High-μ, Elastic Labor, $\gamma = 0$