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Government support for SMEs in response to COVID-19: theoretical model using Wang transform

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Abstract

Purpose – Many governments are taking measures in support of small and medium-sized enterprises (SMEs) to mitigate the economic impact of the COVID-19 outbreak. This paper presents a theoretical model for evaluating various government measures, including insurance for bank loans, interest rate subsidy, bridge loans and relief of tax burdens.

Design/methodology/approach – This paper distinguishes a firm's intrinsic value and book value, where a firm can lose its intrinsic value when it encounters cash-flow crunch. Wang transform is applied to (1) calculating the appropriate level of interest rate subsidy payable to incentivize banks to issue more loans to SMEs and to extend the loan maturity of current debt to the SMEs, (2) describing the frailty distribution for SMEs and (3) defining banks' underwriting capability and overlap index in risk selection.

Findings – Government support for SMEs can be in the form of an appropriate level of interest rate subsidy payable to incentivize banks to issue more loans to SMEs and to extend the loan maturity of current debt to the SMEs.

Research limitations/implications – More available data on bank loans would have helped strengthen the empirical studies.

Practical implications – This paper makes policy recommendations of establishing policy-oriented banks or investment funds dedicated to supporting SMEs, developing risk indices for SMEs to facilitate refined risk underwriting, providing SMEs with long-term tax relief and early-stage equity-type investments.

Social implications – The model highlights the importance of providing bridge loans to SMEs during the COVID-19 disruption to prevent massive business closures.

Originality/value – This paper provides an analytical framework using Wang transform for analyzing the most effective form of government support for SMEs.

Keywords COVID-19, SME, Bank loan, Government subsidy, Wang transform

Paper type Research paper

1. Introduction

Small and medium-sized enterprises (SMEs) have an integral role within the economy, where they account for the majority of registered entities and employ a good majority of the country's workforce. However, SMEs have found it more challenging to obtain bank loans, relative to other corporations such as state-owned enterprises (SOEs). Currently, with the ongoing and widespread COVID-19 situation coupled with the global economic downturn, it

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is critical for SMEs to receive more support, such as gaining better access to bank loans, to ensure survivability for the SMEs and economic stability in the country.

Many governments around the world are taking measures in support of SMEs to mitigate the economic impact of the COVID-19 outbreak. This paper presents a theoretical model for evaluating various government measures, including bridge loans, insurance for bank loans, interest rate subsidy and relief of tax burdens. Traditionally, governments are reluctant to be a direct lender of funds to SMEs as the government is not in the business of risk underwriting and debt-collection. Therefore, it is of critical importance to design novel methods to ensure that government funds can be effectively utilized to provide adequate support to SMEs.

In this paper, we make several contributions to the important topic of government support for SMEs. First, we provide a theoretical model that demonstrated the economic benefits of providing emergency bridge loan to SMEs and helping them cope with the COVID-19. Our model shows how the optimal amount of emergency bridge loan depends on the duration of the economic disruption of the COVID-19. Moreover, due to limited resource, it is of significant importance that the bridge loan is extended to good firms rather than the bad firms, thus justifying the importance of underwriting capability.

Second, we have analyzed government-funded insurance pool for bank loans to SMEs, which has some but limited effect in encouraging banks in issuing more loans to SMEs. Instead, we propose that governments provide an interest rate subsidy to bring the level of interest rate to adequately compensate for increased risk of bank loans to SMEs. Similarly, the interest rate subsidy can be paid to banks to encourage them to extend the loan maturity of current debt to the SMEs. In the face of COVID-19, extending loan maturity of current debt to the SMEs has the similar effect of issuing a new loan. We apply the Wang transform to calculate the appropriate level of interest rate subsidy. This Wang transform formula for interest rate subsidy differs from current practice today, where interest rate subsidy is used to help SMEs to reduce their interest rate burden, without increasing the level of loan interest payable to banks.

Third, we apply the Wang transform to define a frailty model for SMEs and to describe a bank's underwriting capability. We illustrate the importance of underwriting capability by the bank and provide numerical examples on the difference in underwriting outcome between banks with strong underwriting capability and banks with weak underwriting capability. We also introduce a metric of *overlap* index as a novel proxy of the bank's underwriting capability.

Fourth, we make policy recommendation for other government support measures. To enhance the leverage effect in encouraging banks to make more loans to SMEs, a case is made for (1) establishing policy-oriented banks or investment funds dedicated to supporting SMEs and (2) encouraging research and development of risk indices for SMEs that can facilitate refined risk underwriting. With a long and broad view of boosting the economy post-COVID-19, the paper makes a case for government support of SMEs in the form of providing long-term tax relief and early-stage equity-type investments.

The rest of this paper is organized as follows. Section 2 reviews the literature. Section 3 provides a brief introduction to the Wang transform. Section 4 lays out the firm's intrinsic value versus book value, and the double process relating to an SME. Section 5 sets up the structural model that relates the loan default risks to a firm's intrinsic value. Sections 6, 7, 8, and 9 apply the theoretical models developed thus far to the economic disruption of COVID-19 and explore potential solutions. Specifically, section 6 explores the idea of extending bridge loans to SMEs; section 7 analyzes the proposition of an insurance pool for bank loans to SMEs; section 8 examines the notion of an interest rate subsidy for bank loans and section 9 studies the proposal of tax relief for SMEs. Section 10 recognizes the limited information setting underlying the default risk models and provides the associated confidence intervals in estimating the drift terms. Section 11 provides an extension by considering the heterogeneity of SMEs using the Wang transform frailty distribution, and utilizes this concept of heterogeneity and employs the Wang transform to investigate the importance of the bank's underwriting risk selection capability. Section 12 presents some empirical evidence of bank loan default rates across different bank types. Section 13 presents a discussion and concludes.

2. Literature review

SMEs are receiving increased attention as they are the most dynamic group of firms, especially so in emerging economies (Pissarides, 1999). Ayyagari *et al.* (2011) found that SMEs with no more than 250 employees were the fuel of growth for many countries. Furthermore, Beck *et al.* (2005) documented that SMEs employed more than 60% of employees in the manufacturing industry within most developing economies. It was also well-documented that SMEs play an integral role in generating technical advancements and innovation in the economy (Ács and Audretsch, 1999; Radas and Bozic, 2009). In differentiation to their large-firm counterparts, SMEs are documented to be able to react quickly to the needs of the economy as they are more flexible and agile in structure (Alpkan *et al.*, 2007). SMEs are said to be the backbone of the European Union's (EU) economy (European Commission, 2016). In China, more than 98.64% of all companies are considered small businesses with no more than 300 employees (OECD, 2019). Given the relative importance of the SMEs, and the cash-flow crunch that the recent COVID-19 situation has brought about for many SMEs, it is of critical importance to provide support to the SMEs.

In terms of debt financing for SMEs by banks, an important bank function would be the underwriting capability. A strong underwriting capability ensures an effective differentiation between good and bad firms and allows for an efficient allocation of resource. The importance of underwriting capability within the finance industry is not only important for debt issuance (Gande *et al.*, 1997) but it is also critical for equity issuance (Corwin and Schultz, 2005; Chao *et al.*, 2014). Therefore, the importance of underwriting is of critical importance, and our paper seeks to extend this importance under a COVID-19 setting.

SMEs, as well as their large-firm counterparts, can essentially be financed by both debt and equity. Due to the different claim priorities on the firm's asset value, this led Black and Scholes (1973) to formally consider equity as a type of call option on the company's asset value. Merton (1974) took it a step further with the corporate bond pricing model that resulted in the structural risk approach in modeling corporate debt. Subsequently, the KMV model (Vasicek, 1984; Kealhofer, 1993; McQuown, 1993), which was largely derived from Merton's model, was launched as an industry-based model by the KMV Corporation. From a theoretical standpoint, the KMV model focuses on the probability of default, while Merton's model focuses on the valuation of the debt, both using Black-Scholes-Merton option pricing formula as the underlying engine. The main difference is that the KMV model can account for multiple classes of liabilities, thus much better representing the real-life situation facing a firm. This has allowed the KMV model to be calibrated based on an extensive proprietary database which made it a popular subscription by financial institutions around the world. Our paper provides an extension based on their models where we seek to incorporate the COVID-19's situation and generate useful insights accordingly.

There is a vast literature on risk-adjustment using probability transforms, in particular the Wang transform as a universal pricing formula. In general, estimated (i.e. "subjective") probability of various events is different from the actual (i.e. "objective") probabilities of these events. The difference is exceptionally significant for events with probabilities that are close to either 0 or 1. Kahneman and Tversky (1979) showed that there exists a one-to-one functional relationship, denoted as… between the objective and subjective probabilities. This implies that, given the objective probability, denoted as $\mathrm{Prob}_{\mathrm{obj}}(E)$, we can pass it through $g(y)$ to yield the associated subjective probability, $\mathrm{Prob}_{\mathrm{subj}}(E) = g(\mathrm{Prob}_{\mathrm{obj}}(E))$. The Wang transform represents such a function $g(y)$ and it is applied to the decumulative distribution. Wang transform has been widely cited as a benchmark pricing model (see Wang, 2000, 2002, 2003). There are many papers that provide justification of the Wang transform in pricing risks (Kijima and Muromachi, 2008; Glasserman, 2007; Sornette *et al.*, 2000; Kreinovich *et al.*, 2009). It was shown that the Wang transform formula for pricing default risk is consistent with the Black-Scholes-Merton structural model. At the same time, Wang transform differentiates by being applicable to any-shaped probability curves, without the prior assumption of any structural model. In the next section, we provide a brief introduction of the Wang transform.

3. Wang Transform for pricing loan default risk

In the financial markets, a general principle underlying risk pricing is that investors require that high risks should be matched with high returns. This is because the market requires a "fair" return for taking on risks. However, the market pricing of the risk-neutral probability measure tends to be different from the real probability measure of the risk. The Wang transform is a risk pricing formula that has been invented in and is widely applied in the field of insurance and actuarial science. The Wang transform is consistent with the credit risk pricing under the Merton's model and is equivalent to the Black-Scholes option pricing formula, but the Wang's transform has a wider range of applications and has no restrictions on the probability distribution of the risk.

In general, a bank loan has a random loss X due to default, which can be described by a loss curve $S_X(w) = \Pr\{X > wL\}$, where w is percentage of the outstanding loan amount L.

As shown in Figure 1, from the loan default loss curve $S_X(w)$, one can calculate the expected value of bank loan default loss as the total area underneath the loss curve $S_X(w)$:

$$E[X] = \int_0^\infty S_X(x)dx = L\int_0^1 F_R(1-w)dw \tag{1}$$

The Wang transform is a mathematical formula that converts a *loss curve* $S_X(w)$ to a *price curve* $S_X^*(w)$ such that

$$S_X^*(w) = \Phi\left[\Phi^{-1}(S_X(w)) - \lambda\right].\# \tag{2}$$

where λ is defined as the market price of risk, which reflects the underlying systematic risk and Φ is the cumulative distribution function of the normal (or Gaussian) law.

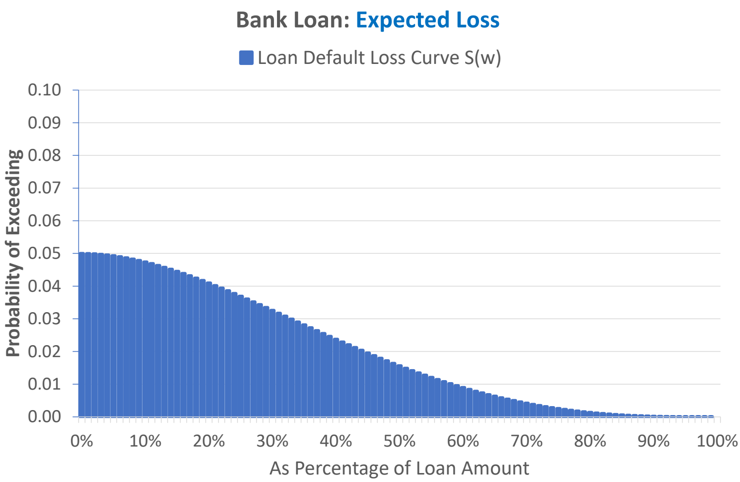

Figure 1.
Expected loss of a bank loan equal the area underneath the loan default loss curve

The price of bank loan default loss is the expected value under the risk-neutral probability measure $S_X^*(w)$:

$$E^*[X] = L\int_0^1 S_X^*(w)dw\# \quad (3)$$

As shown in Figure 2, the risk margin is the area between the price curve $S_X^*(w)$ and loss curve $S_X(w)$:

$$\text{Risk Margin} = E^*[X] - E[X] = L\cdot\int_0^1 [S_X^*(w) - S_X(w)]dw\# \quad (4)$$

The Wang transform (2) is unique in that it converts a Gaussian distribution to another Gaussian distribution, and at the same time, it converts a lognormal distribution to another lognormal distribution.

The Wang transform (2) converts a given loss probability distribution to a price distribution, without reflecting an increasing sampling errors at the tails of the distribution (extreme events with rare probability). The two-factor Wang transform is introduced by Wang (2002) to incorporate increasing sampling errors at the tails of the distribution. Assuming that we have k independent observations from a given population that follows a Gaussian distribution with parameter (μ, σ^2), where the best estimate is given by the sample mean $\tilde{\mu}$ and sample SD $\tilde{\sigma}$. As we are now making probability assessments concerning a future outcome, we would need to use the Student-t distribution with k degrees of freedom [1]. Therefore, the two-factor Wang transform is given as follows:

$$S^*(x) = Q\left[\Phi^{-1}(S(x)) + \lambda\right].\# \quad (5)$$

where Q follows a Student-t distribution with k degrees of freedom.

From an empirical perspective, the two-factor Wang transform (5) has also been shown to perform quite well in explaining real-world risk pricing data. Wang (2004) studied empirical data containing loss curves coupled with risk premiums across the twenty-eight catastrophe bond market transactions during 1999 and 2000 [2] and found that the two-factor Wang

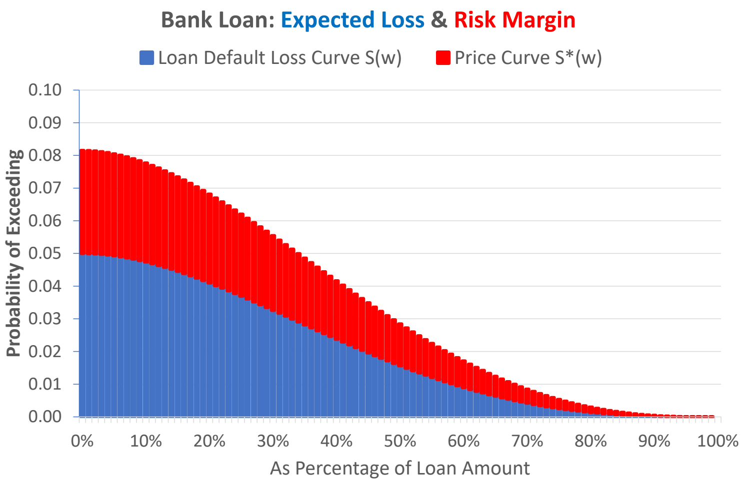

Figure 2.
An illustration of Wang transform from loss curve S to price curve S^*, where the plot is loss exceedance probability against the loss as % of loan value. The red-shaded region represents the risk premium

transform method with two parameters provided a good explanation of the empirical data. In the next section, we define the firm's intrinsic value and book value, and the double process that an SME is subjected to.

Subsequently, in section 8, we apply this concept and use the two-factor Wang transform (5) to calculate the corresponding "fair" interest rate under different levels of default probabilities, which would help the government in estimating the interest rate margin and subsidies required.

4. Firm's intrinsic value versus book value

We now consider the accounting balance sheet of the SME. We define the book value, asset value and liability value, at time t for the ith SME as $\mathrm{BV}(\theta_i)|_t$, $\mathrm{Asset}(\theta_i)|_t$ and $\mathrm{Liability}(\theta_i)|_t$, respectively. Thus, the book value is the difference between the asset value and the liability value, given as follows:

$$\mathrm{BV}(\theta_i)|_t = \mathrm{Asset}(\theta_i)|_t - \mathrm{Liability}(\theta_i)|_t \ \# \tag{6}$$

We define $\mathrm{Asset}(\theta_i)|_t$ as the assets held by SME (θ_i) at time t. A non-exhaustive list includes line items such as cash holdings, account receivables and financial investments. More significantly, we assume that the $\mathrm{Liability}(\theta_i)|_t$ is equal to the outstanding debt of the firm, where:

$$\mathrm{Liability}(\theta_i)|_t = \mathrm{Debt}(\theta_i)|_t \# \tag{7}$$

Therefore, we define an enterprise as being in the state of an accounting default at a given point in time t, when the book value drops below zero, as follows:

$$\mathrm{BV}(\theta_i)|_t < 0$$

Despite being insolvent from an accounting balance sheet perspective, we propose that, more often than not, it is the economic balance sheet of the SME that defines the future growth potential of a given SME. Specifically, it is widely known that the SME's intrinsic value, denoted as $\mathrm{IV}(\theta_i)|_t$ may differ from its book value. The difference can be justified by the firm's franchise value, denoted as $\mathrm{FranchiseValue}(\theta_i)|_t$. This yields the following expression:

$$\mathrm{IV}(\theta_i)|_t = \mathrm{BV}(\theta_i)|_t + \mathrm{Franchise\ Value}(\theta_i)|_t \# \tag{8}$$

Thus, we define an enterprise as being in the state of an economic default at a given point in time t, when the intrinsic value drops below zero, as follows:

$$\mathrm{IV}(\theta_i)|_t < 0.$$

To better understand the concept of a firm's Franchise Value, one can look toward the future accounting earnings of the SME. Specifically, we define the income, revenue and expenditure during a given accounting year $[t, t+1]$, as $\mathrm{Income}[t, t+1]$, $\mathrm{Revenue}[t, t+1]$ and $\mathrm{Expenditure}[t, t+1]$, respectively. The revenue comes from normal business operations, while a non-exhaustive list of expenditure would include office cost, salary and employee benefits. Thus, the SME can expect to have the following expected income:

$$\mathrm{Income}[t, t+1] = \mathrm{Revenue}[t, t+1] - \mathrm{Expenditure}[t, t+1] \# \tag{9}$$

Thus, the Franchise Value can be viewed as the discounted value of future (after-tax) income, given as follows:

$$\text{Franchise Value}(\theta_i)|_t = \sum_{j=0}^{\infty} \frac{(1-\tau(\theta_i))\cdot \text{Income}[t+j,\ t+j+1]}{(1+r(\theta_i))^j}.\#\qquad(10)$$

where $\tau(\theta_i)$ is the effective corporate tax rate [3], $r(\theta_i)$ is the discount rate for future cash flows. Furthermore, as the discount rate increases as the uncertainty associated with future cash flows increases, we define $r(\theta_i) = r_0 + \lambda \cdot \sigma(\theta_i)$, where r_0 is the risk-free rate, λ is the market price of risk and $\sigma(\theta_i)$ is the volatility of future earnings for the SME (θ_i).

4.1 Double process

A firm SME (θ_i) is subject to two processes: (1) intrinsic value which follows a geometric Brownian motion and (2) book value liquidity crunch due to unforeseen events such as COVID-19. In order words, a firm SME (θ_i) may experience a liquidity crunch, despite the fact that the firm has a positive intrinsic value (which would disappear when the firm closes its business due to a liquidity crunch). In the next section, we set up the structural model that associates the bank loan default risk to the SME's IV.

5. Structural model of bank loan default risk in relation to the SME's intrinsic value

We consider the intrinsic value of SME(θ_i) at time t, denoted as $\text{IV}_t(\theta_i)$. We propose that the $\text{IV}_t(\theta_i)$ varies across time, following that of a geometric Brownian process defined with drift $\mu(\theta_i)$ and volatility $\sigma(\theta_i)$. The drift parameter ("skill") is an indicator of the prospects of the firm, and can vary due to both external and internal factors. From an external perspective, the drift $\mu(\theta_i)$ could be relatively larger when the firm's sector is growing and there exists significant growth potential; from an internal perspective, the drift $\mu(\theta_i)$ could be relatively larger when the firm has proprietary processes and/or intellectual property. The volatility parameter $\sigma(\theta_i)$ accounts for noise and variation, which helps to explain observed difference in firms' performance over short time horizons (less than a few years), despite them having similar drift parameter values. In a single-period model [t = 0, t = T], the terminal intrinsic value, denoted by $\text{IV}_{t=T}(\theta_i)$, would have a lognormal distribution. The interplay between the drift parameter and the volatility determines entirely the possibility to distinguish good firms from bad firms and their probability of default. At relatively short times up to several years, luck can dominate over skill (Sornette *et al.*, 2019) and this needs to be properly accounted for.

Recall that a firm is economically in default, if its intrinsic value at t drops below the outstanding debt $\text{Debt}(\theta_i)|_t$. We employ the distance to default model that initiates a relationship between the drift parameter and the default probability, given by the following expression and illustrated in Figure 3:

$$\ln(\text{IV}_{t=T}(\theta_i)) \sim \text{Normal}\left(\mu(\theta_i)T + \ln(\text{IV}_{t=0}(\theta_i)),\ \sigma(\theta_i)^2 T\right)\#\qquad(11)$$

Using the Merton model, one can derive the default probability from the projected intrinsic value as follows:

$$q_{t=0,\ t=T}(\theta_i) = \Phi\left(\frac{\ln(\text{Debt}(\theta_i)|_{t=T}) - \ln(\text{IV}_{t=T}(\theta_i))}{\sigma(\theta_i)\sqrt{T}}\right).\#\qquad(12)$$

We have *a structural equation:*

$$\Phi^{-1}\left(q_{t=0,t=T}(\theta_i)\right) = \frac{\ln(\text{Debt}(\theta_i)|_{t=T}) - \mu(\theta_i)T - \sigma^2(\theta_i)T/2 - \ln(\text{IV}_{t=0}(\theta_i))}{\sigma(\theta_i)\sqrt{T}}\#\qquad(13)$$

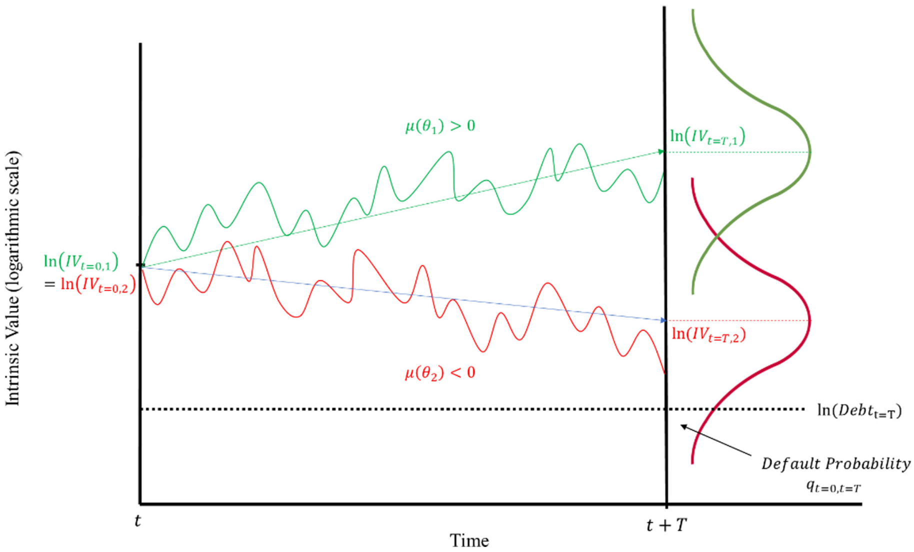

Figure 3.
Illustration of the distance to default model for one firm with different drift terms

Remark. In a different context of pricing European put options, the intrinsic value $\mathrm{IV}_{t=T}(\theta_i)$ is replaced by the underlying stock price at time $t = T$, and the debt is replaced by the strike price. The risk-neutral probability measure for the stock price at $t = T$ has a lognormal distribution with the drift term equal to the risk-free interest rate r_0, and we get the celebrated Black and Scholes (1973) price of an European option as the total area underneath the risk-neutral distribution:

$$\text{Price of Put Option} = e^{-r_0 T}\int_0^D \Phi\left(\frac{\ln(x) - r_0 T - \sigma^2(\theta_i)T/2 - \ln(\mathrm{IV}_{t=0}(\theta_i))}{\sigma(\theta_i)\sqrt{T}}\right)dx\# \quad (14)$$

Thus, the objective default probability q and the risk-neutral default probability q^* is connected by the Wang transform formula:

$$q^*_{t=0,t=T}(\theta_i) = \Phi\left[\Phi^{-1}\left(q_{t=0,t=T}(\theta_i)\right) - \lambda_T\right]\# \quad (15)$$

where $\lambda_T = \frac{\mu(\theta_i) - r}{\sigma(\theta_i)}\sqrt{T}$.

6. Economic disruption of COVID-19 and bridge loans for SMEs

The COVID-19 outbreak has triggered major disruptions of economic activities around the world. The international supply chain has been greatly hampered, while the level of economic uncertainty has significantly increased. Given the harsh economic impact of the COVID-19, we use our model to provide insights on four potential measures that could be taken to mitigate the negative effect of COVID-19. In particular, we explore the idea of extending bridge loans to SMEs in this section. Moving forward, we analyze the proposition of an insurance pool for bank loans to SMEs in section 7; we examine the notion of an interest rate subsidy for bank loans in section 8; and we study the proposal of tax relief for SMEs in section 9.

We note that an enterprise is required to maintain a given level of cash holdings in a bid to support its day-to-day operations. A non-exhaustive list of these operations includes payment of employees' salaries, general office operational expenses and interest expenses on outstanding loans. Specifically, we denote the company's cash runway as the number of months the company can maintain daily operations until it runs out of cash. Thus, we represent the cash level of the ith SME at a given time t as $C_t(\theta_i)$, expressed as the company's cash runway.

For an SME (θ_i), we define a *liquidity crunch* occurs at time $t = T$ when the cash-based accounting book value drops to zero:

$$\mathrm{BV}(\theta_i)|_{t=T} < 0$$

We consider the random variable, T, as the number of months of economic interruption arising from COVID-19. We assume that the variable T follows a Weibull distribution with shape parameter k and scale parameter β, where $k > 0$ and $\beta > 0$:

$$1 - F_T(t) = \Pr\{T > t\} = \exp\left\{-\left(\frac{t}{\beta}\right)^k\right\}, \quad \text{for } t > 0\# \tag{16}$$

The parameters of the Weibull distribution depend on the length and effectiveness of the various "containment" measures implemented. These measures could include government strategies, as well as international containment efforts on COVID-19. From the government's perspective, the policy objectives can be largely attributed as: (1) stabilizing SMEs to mitigate unemployment and (2) encouraging more bank loans made to SMEs via the leverage effect of government funding.

We estimate the probability of a *liquidity crunch* for the ith SME, defined as $q(\theta_i)$, as follows:

$$q(\theta_i) = 1 - F_T(C_0(\theta_i)) = \exp\left(-\left(\frac{C_0(\theta_i)}{\beta}\right)^k\right)\# \tag{17}$$

Based on our prior assumption, upon suffering from a liquidity crunch, the SME would proceed to declare bankruptcy.

In the event of business closure of an SME(θ_i) due to liquidity crunch, the SME(θ_i) loses all its future after-tax earnings (Franchise Value), the bank may not be able to collect its outstanding loan to SME(θ_i), the government loses all future tax revenues from the SME(θ_i) and the economy may incur indirect losses due to unemployment and reduced consumptions. The combined loss to the whole community due to business closure of the SME(θ_i) is the community value, or Comm(θ_i) which can be a large multiple "M" of a firm's Franchise Value. The expected loss in the event of business closure is given as follows:

$$\text{Expected Loss}(\theta_i) = \exp\left(-\left(\frac{C_0(\theta_i)}{\beta}\right)^k\right) \cdot \neq \mathrm{Comm}(\theta_i)|_{t=1}.\# \tag{18}$$

6.1 Government emergency support by providing bridge loans to SMEs

In such a scenario, consider a government emergency support measure by providing a bridge loan of amount E, which is expressed as the additional cash runway for the SME(θ_i). Therefore, with the bridge loan amount E, the probability of business closure for SME(θ_i) is reduced, denoted as $q(\theta_i)^*$, expressed as follows:

$$q(\theta_i)^* = 1 - F_T(C_0(\theta_i) + E) = \exp\left(- \left(\frac{C_0(\theta_i) + E}{\beta} \right)^k \right) \# \tag{19}$$

From the government's perspective, the expected loss in the event of business closure is reduced, denoted as $\text{Expected Loss}(\theta_i)^*$, expressed as follows:

$$\text{Expected Loss}(\theta_i)^* = \exp\left(- \left(\frac{C_0(\theta_i) + E}{\beta} \right)^k \right) \cdot \text{Comm}(\theta_i)|_{t=1} \# \tag{20}$$

Therefore, the value creation brought about by providing the bridge loan is denoted as $\text{Value Creation}(\theta_i)$ and is given as follows:

$$\text{Value Creation}(\theta_i) = \text{Expected Loss}(\theta_i) - \text{Expected Loss}(\theta_i)^* \# \tag{21}$$

$$\text{ValueCreation}(\theta_i) = \left\{ \exp\left(- \left(\frac{C_0(\theta_i)}{\beta} \right)^k \right) - \exp\left(- \left(\frac{C_0(\theta_i) + E}{\beta} \right)^k \right) \right\} \cdot \text{Comm}(\theta_i)|_{t=1} \# \tag{22}$$

We now consider the opportunity cost of providing the bridge loan. As resources are scarce, when capital is deployed to serve as a bridge loan to the firm, this implies that this capital will not be deployed somewhere else, thus incurring an opportunity cost. Therefore, the associated opportunity cost, denoted as $\text{Opportunity Cost}(\theta_i)$, is given as follows:

$$\text{Opportunity Cost}(\theta_i) = \omega \cdot V(E). \# \tag{23}$$

where ω is the opportunity cost factor, which is a percentage of the amount of bridge loan rendered. $V(E)$ is the monetary value of the bridge loan, noting that E is expressed as the additional cash runway for the company. For simplicity of analysis, we assume that the bridge loan is used to cover the expenditure of the company, which is incurred at a constant rate across a time period of a year. Therefore, the monetary value of bridge loan per month of is given by $\frac{\text{Expenditure}[t, t+1]}{12}$. Therefore, an alternative expression for $\text{Opportunity Cost}(\theta_i)$ is given as follows:

$$\text{Opportunity Cost}(\theta_i) = \omega \cdot E \cdot \frac{\text{Expenditure}[t,\ t+1]}{12} \# \tag{24}$$

Hence, the net value creation brought about by providing the bridge loan is denoted as $\text{Net Value Creation}(\theta_i)$ and is given as follows:

$$\text{Net Value Creation}(\theta_i) = \text{Value Creation}(\theta_i) - \text{Opportunity Cost}(\theta_i) \# \tag{25}$$

Finally, we provide a numerical example to demonstrate the optimal bridge loan, defined as E^*. We provide the following numerical figures over a one-year period model in our example and the graphical plot is shown in Figure 4:

(1) $0 \leq E \leq 12$

(2) $\text{Revenue}[0,\ 1] = \$\ 1{,}000{,}000$

(3) $\text{Expenditure}[0,\ 1] = \$\ 800{,}000$

(4) $R(\theta_i) = 5\%$

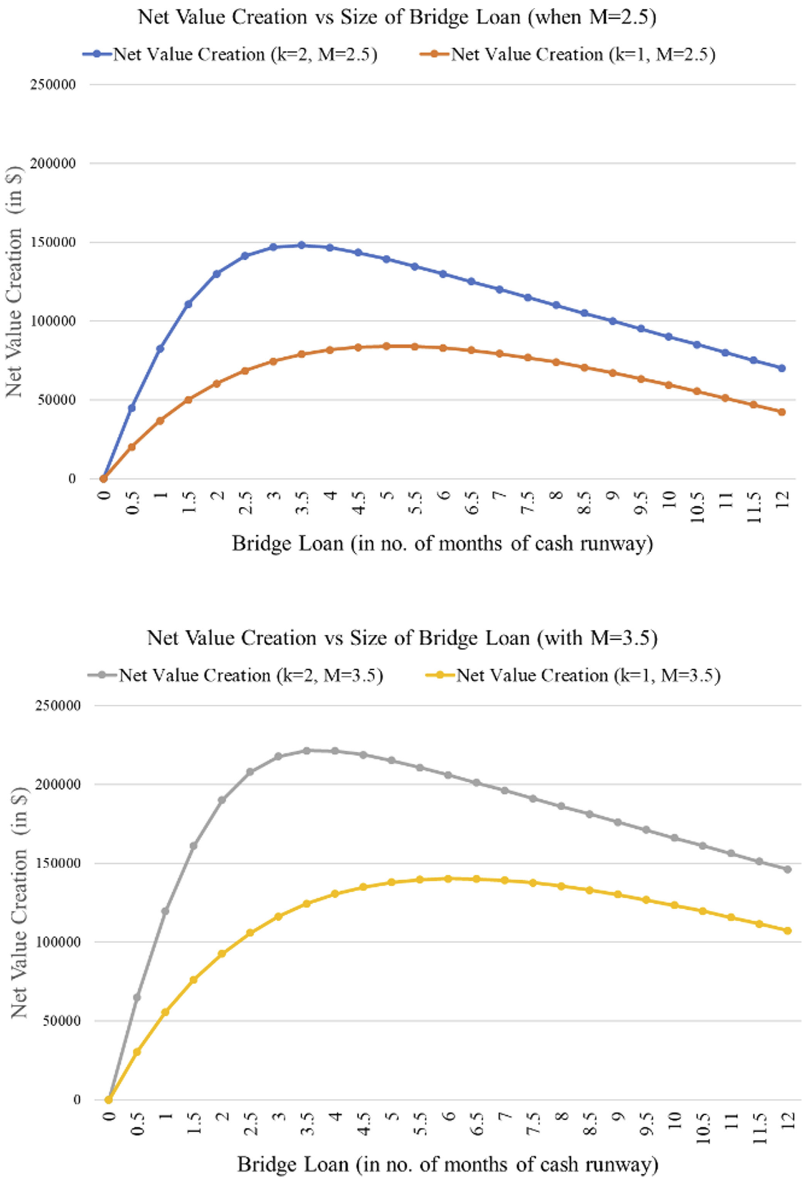

Figure 4.
Net value creation vs bridge loan (with different shape parameters), and with $M = 2.5$ (left panel) and $M = 3.5$ (right panel)

(5) $C_0(\theta_i) = 2.5\,\text{months}$

(6) $\beta = 3$

(7) $k = 1,\ 2$

(8) $\omega = 15\%$

(9) $M = 2.5, 3.5$

(10) $\tau(\theta_i) = 20\%$

As shown in Figure 4, there exists an optimal E^* that maximizes the corresponding Net Value Creation(θ_i) for different values of the shape parameters and multiplier values. As shown in Table 1, when $M = 2.5$, the E^* is 3.4 and 5.1 months of cash runway for k values of 2 and 1, respectively; and when $M = 3.5$, the E^* is 3.7 and 6.1 months of cash runway for k values of 2 and 1, respectively. This implies that the optimal E^* increases in the multiplier value. As a note, the optimal bridge loans might not always exist for some parameter combinations (e.g., for a very small ω). However, this does not change our general observations from the model.

An interesting thing to note is that there exists an exceedance threshold such that, when exceeded, the net value creation turns negative. In our example, when $M = 2.5$, the net value creation turns negative when the bridge loan exceeds 19.0 and 16.5 months of cash runway, for k values of 2 and 1, respectively; and when $M = 3.5$, the net value creation turns negative when the bridge loan exceeds 26.6 and 23.2 months of cash runway, for k values of 2 and 1, respectively. This important implication suggests that simply dumping bank loans onto SMEs do not necessarily ensure that the net value creation is increased but rather this could even lead to negative net value creation in the ecosystem.

Currently, at the time of writing the paper, there is still considerable amount of uncertainty surrounding the speed of economic recovery from COVID-19. As pointed out by economists (e.g. Stephen Roach), although the supply side can recover relatively soon, the consumer demand may take much longer to recover. Such uncertainty can be translated to estimated Weibull distribution parameters and the resulting optimal amount of bridge loan to be adequate to sustain SMEs.

By providing SMEs with emergency bridge loans can prevent massive business closures. This has additional benefits of preventing large accumulated credit losses from the insurance pool.

7. Insurance pool for bank loans to SMEs

We provide context for the ecosystem in relation to bank loans to SMEs.

(1) A population of SMEs where the ith SME is identified as SME (θ_i), where $i = 1, 2, \ldots, n$

(2) A number of banks $\{\text{Bank}_1, \text{Bank}_2, \ldots \text{Bank}_m\}$ choose to issue loans to the SMEs, and participate in a government-funded pool.

(3) Government-funded pool that provides insurance for loans made to SMEs by participating banks.

Consider that Bank_j issues a bank loan to an SME (θ_i). We denote the face amount of the loan as $L(\theta_i)$, the annualized coupon rate as $c(\theta_i)$ and a future loan maturity date of $D(\theta_i)$.

Table 1. The optimal bridge loan and exceedance threshold for different values of shape parameter and multiplier

	$M = 2.5$		$M = 3.5$	
	Optimal bridge loan E^*	Exceedance threshold	Optimal bridge loan E^*	Exceedance threshold
$k = 2$	3.4	19.0	3.7	26.6
$k = 1$	5.1	16.5	6.1	23.2

We denote $q_{t,t+T}(\theta_i)$ as the probability of loan default during the time period $[t, t+T]$. For simplicity, we use q to denote the probability of bank loan default when the time period $[t, t+T]$ is dropped.

In the event of default, let R be the recovery rate (as ratio of loan amount "L") with a distribution function: $F_R(w) = \Pr\{R \le w\}$, where $0 < w < 1$.

The random loss X due to bank loan default can be described by a default frequency and recovery rate

$$X = \begin{cases} 0, & \text{with probability } 1 - q \\ L(1 - R), & \text{with probability } q \end{cases}$$

The bank loan's random loss X has a loss curve or decumulative distribution function of exceeding w percentage of the loan value:

$$S_X(w) = q \cdot \Pr\{R \le 1 - w\} = q \cdot F_R(1 - w)$$

with $S(0) = q$ being the probability of loan default.

Assume that the insurance pool provides a proportional 50% of bank loan default loss X, with a payout:

$$Z = 0.5X$$

The insurance payout Z has a loss exceedance curve:

$S_Z(z) = \Pr\{Z > z\} = S_X(2z)$ for all z.

After insurance, the net retained loss to the bank is:

$$Y = X - Z = 0.5X$$

The bank's retained loan default loss Y has a loss exceedance curve (see Figure 5):

$S_Y(y) = \Pr\{Y > y\} = S_X(2y)$ for all y.

As shown in Figure 5, we make the following observations:

Proportional insurance of bank loan to SME does not reduce the default probability as the insurance comes into play only after the bank loan default is triggered. Local branch bank managers generally are evaluated based on the non-performing loan ratio (default ratio).

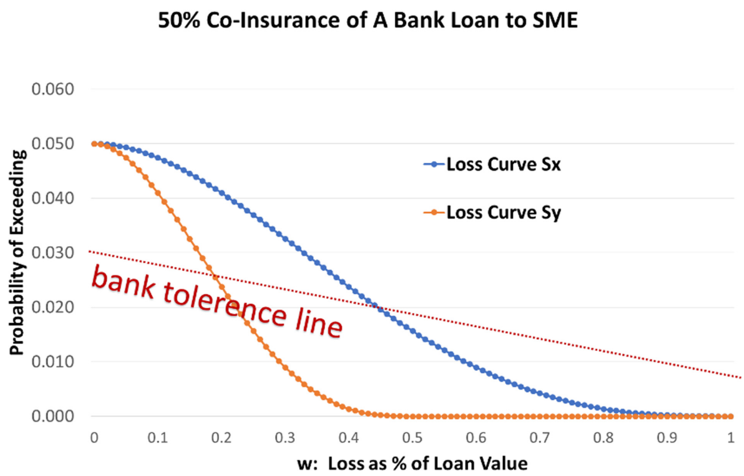

Figure 5.
Illustration of the probability of exceedance against the loss as % of loan value, under a 50% co-insurance of a bank loan to SME

Currently, as conditions for banks to participate in the insurance pool, there are requirements for banks to ensure that the overall loan portfolio default ratio does not exceed a given threshold (e.g. 3%). Within some banks, there are internal underwriting criteria for bank loan's default probability to be within a risk tolerance level. In general, proportional insurance for bank loan defaults does not help reduce the individual loan default probability or the aggregate loan portfolio default ratio. Bank managers would still be penalized if the percentage of bad debt occurrence exceeds a given threshold (e.g. 3%).

Traditionally, the concept of insurance is based on law of large numbers, and thus does not work well for systematic risk such as COVID-19 pandemics. Proportional insurance of bank loans provides some but somehow weak incentive for banks to lend to SMEs. Generally speaking, the default risk for loans to SMEs can be higher than loans to large corporations or SOEs. Banks would not have an economic incentive to issue loans to the SMEs unless the default risks are properly priced within the interest rate margins. We propose that government subsidy can be used to pay banks an adequate level of interest rate commensurate with the increased risk level of loans to SMEs.

A natural question is: what is a benchmark price of default risk for bank loans to SMEs? We seek to answer this question in the next section using the Wang transform.

8. Interest rate subsidy for bank loans

As an alternative to using government funds as an insurance pool for bank loans, our paper proposes that government funds can be used to pay banks an extra interest (i.e. additional coupon rate). This can be viewed as an interest rate subsidy, and the purpose is to increase the potential profits (risk margins) for the banks and generate increased incentives for them to issue more bank loans to the SMEs and to extend the loan maturity of current debt to the SMEs. By issuing more bank loans to SMEs or extending loan maturity of current debt to SMEs will increase the risks faced by banks. Thus, an interest rate subsidy seeks to incentivize and adequately compensate the banks for taking on the higher risks.

We seek to identify the appropriate level of interest rate subsidy by using the universal pricing method given by the Wang transform. Specifically, for a given default loss curve (with probability q and loss recovery rate R), we can use the Wang transform model to calculate the price curve (with implied risk-neutral default probability q^*). The Wang transform (5) converts the real-world estimated loan default probability, q, to the risk-neutral loan default probability, q^*:

$$q^* = Q\left[\Phi^{-1}(q) - \lambda\right]$$

As shown in Figure 6, the associated risk premium would be given by the area between the two curves.

For simplicity, we assume that, in the event of a default, the recovery rate is zero. It is straightforward to derive that the key results of our analysis will remain robust to relaxing this assumption. Under our assumption of zero recovery rate in the event of default, the associated risk premium would then be given by $(q^* - q)$. Therefore, in line with our assumption, the appropriate interest rate margin would then be given by the following:

$$\text{Interest rate margin} = \int_0^1 \left[S_Y^*(y) - S_Y(y)\right]dy = \int_0^1 (q^* - q)dy = (q^* - q)\# \quad (26)$$

One method for governments to support SMEs is to provide interest rate subsidy payable to banks as an incentive for banks to issue loans to SMEs, extend the loan maturity of current

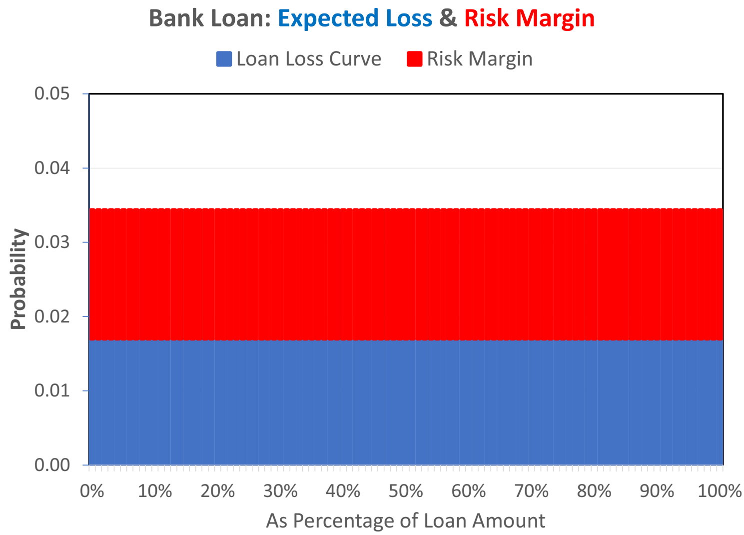

Figure 6.
An illustration of Wang transform from loss curve S to price curve S^*, where the plot is loss exceedance probability against the loss as % of loan value, assuming that in the event of default, the recovery rate is zero

debt to the SMEs and to perform risk underwriting selection. One question is what level of interest rate subsidy would be appropriate and how the level of interest rate subsidy varies with the underlying risk across sectors or firms.

We apply the Wang transform to calculate an appropriate (benchmark) level of interest rate subsidy payable to banks in order to incentivize them to issue more loans to SMEs while performing sharper risk selection.

We now apply the Wang transform to calculate the appropriate level of interest rate subsidy. We demonstrate the application of our two-factor Wang transform model using the following numerical example and our results are presented in Figure 7:

(1) $0\% <$ Default probability $< 4.5\%$

(2) 2-factor Wang transform parameters: $\lambda = 0.3$, $k = 9$

Therefore, the Wang transform can be used to calculate the benchmark risk-based pricing of the interest rate subsidy. This provides a tractable method for the government to decide on the appropriate interest rate subsidy to grant the banks so as to incentivize them to issue more bank loans to SMEs and to extend the loan maturity of current debt to the SMEs.

As shown in Figure 7, when the loan default probability for an SME is lower than 1.7%, the risk-neutral interest rate would not exceed 5.0% (i.e. the prevailing bank loan's market interest rate), thus the bank would be willing to issue a loan to the corresponding SME. However, when the loan default probability exceeds 1.7%, the risk-neutral interest rate would exceed 5.0%, which results in the loans not being issued. In this regard, if the government is able to provide an interest rate subsidy that covers extra risk margin such that the bank receives the risk-neutral interest rate, this would provide sufficient incentive to the bank to issue the loans. One simple breakdown of the interest cost is that the government pays for no less than the risk margin portion, while asking the SME to pay for no more than the expected loss portion of the risk-neutral interest rate. Taken together, this would achieve the government's primary objective in encouraging more bank loans to be issued to the SMEs.

It is noted that, currently in China, a type of government support for SMEs provide interest subsidy to SMEs rather than paying the interest subsidy to banks to compensate for

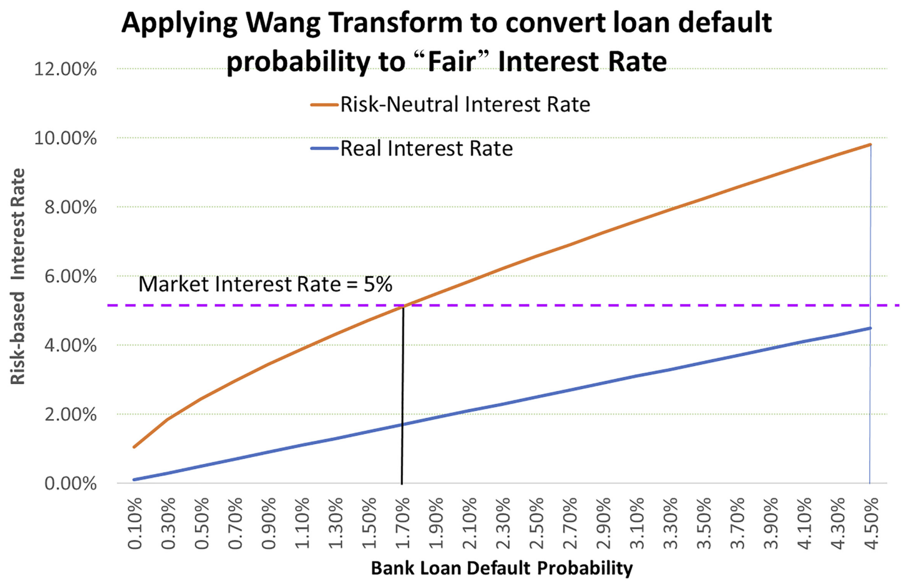

Figure 7. Illustration of the loan default probability and the associated real interest rate and risk-neutral interest rate, calculated using the Wang transform

assuming the default risks of bank loans to SMEs. We note that providing interest subsidy directly to banks will create greater incentive for banks to issue more loans to SMEs, thus have a leverage effect on increasing the number of bank loans to SMEs. At the same time, an interest subsidy can incentivize banks to extend the loan maturity of current debt to the SMEs, which can prove to be significant in helping the SMEs tide over the economic downturn brought about by the COVID-19. In addition, a key input to the model is the estimated default probability. This highlights the need and importance of underwriting for the banks, and this is discussed in greater detail in section 12.

9. Tax relief for SMEs

The SME contributes to the government via the channel of tax revenue, which is proportional to the corresponding Franchise Value. The government can redeploy the tax revenue toward public goods which can increase the community value. At the same time, the SME employs individuals, which is valuable to the government as a higher employment rate implies a more stable and robust economy, as well as provides additional tax revenues. Thus, from the government's perspective, the community value of an SME, denoted as $\text{Comm Value}(\theta_i)|_{t=0}$ can be estimated by:

$$\text{Comm Value}(\theta_i)|_t = \left[\frac{\tau(\theta_i)}{1-\tau(\theta_i)}\right]\text{FranchiseValue}(\theta_i)|_t = \sum_{j=0}^{\infty}\frac{\tau(\theta_i)\cdot\text{Income}[t+j,t+j+1]}{(1+r(\theta_i))^j}\# \tag{27}$$

It is widely known in the corporate finance literature that a firm's market value (for public firms) and intrinsic value (for private firms) can differ from its book value. The former relates to the growth potential of the firm, and the latter relates to the firm's accounting assets minus liabilities. In this paper, we utilize this differentiation and define that an accounting default occurs when book value is negative; an economic default occurs when the intrinsic value is negative. The accounting default is more closely related to the cash level. Specifically, a firm with positive book value might appear to be healthy, but if the future outlook of the firm is poor, this would diminish the firm's intrinsic value. Alternatively, a firm with negative book value might have short-term cash-flow difficulty but, if the future outlook for the firm is good, this would imply a high intrinsic value.

9.1 Optimal effective tax rate

Firms prefer a lower effective tax rate to maximize their residual earnings. However, if the effective tax rate is too low, the government will not have sufficient tax revenue to provide a basic level of public infrastructure and firms might find the general business environment less conducive. At the same time, if the effective tax rate is too high, it might act as a disincentive for companies to engage in profitable ventures. Therefore, it is important for the government to choose an effective tax rate that allows the government to fund public infrastructure, while at the same time, be able to encourage sufficient market participation by SMEs.

As shown in equation (10), a firm's Franchise Value represents its future discounted after-tax earnings. Thus, a firm's Franchise Value is affected by not only the effective tax rate but also by the broad business environment, where a more positive and stable business environment creates better prospect for future income and thus increases the firm's Franchise Value. In light of the massive economic disruption of COVID-19, effective government support for SMEs shall require a systemic and long-term strategy in improving the broader business environment.

10. Confidence intervals in estimating the drift term

In our discussion of default risk for banks loans to SMEs, we do not make the strong assumption of efficient market. Instead, our working assumption is that there are limited information and inherent uncertainty in estimating the true drift term for a firm. We assume that, due to limited information, a bank j can estimate the drift term $\widehat{\mu}(\theta_i)$ with the confidence interval $\left(\mu^{\text{lower}}(\theta_i),\ \mu^{\text{upper}}(\theta_i)\right)$.

Using the structural equation (13), a confidence interval $(\mu^{\text{lower}}(\theta_i), \mu^{\text{upper}}(\theta_i))$ for the drift term $\mu(\theta_i)$ can be translated to a range of default probability (in reverse order of the drift term):

$$\left(q^{\text{lower}}_{t=0,\,t=T}(\theta_i),\ q^{\text{upper}}_{t=0,\,t=T}(\theta_i)\right)\#$$

Where the default probabilities satisfy:

$$\Phi^{-1}\left(q^{\text{lower}}_{t=0,\,t=T}(\theta_i)\right) = \frac{\ln(\text{Debt}(\theta_i)|_{t=T}) - \mu^{\text{upper}}(\theta_i)T - \sigma^2(\theta_i)T/2 - \ln(\text{IV}_{t=0}(\theta_i))}{\sigma(\theta_i)\sqrt{T}} \tag{28}$$

$$\Phi^{-1}\left(q^{\text{upper}}_{t=0,\,t=T}(\theta_i)\right) = \frac{\ln(\text{Debt}(\theta_i)|_{t=T}) - \mu^{\text{lower}}(\theta_i)T - \sigma^2(\theta_i)T/2 - \ln(\text{IV}_{t=0}(\theta_i))}{\sigma(\theta_i)\sqrt{T}}$$

In terms of a structural equation (13), a bank's underwriting capability can be viewed as its ability in estimating the drift term. A bank can differentiate a good firm from a bad firm as defined by their respective drift parameters. For ease of comparison, we assume that two firms have the same intrinsic value at time $t = 0$ and have the same amount of bank loan, but the two firms have different drift terms:

$$\text{IV}_{t=0}(\theta_{\text{Good}}) = \text{IV}_{t=0}(\theta_{\text{Bad}})$$

$$\mu(\theta_{\text{Good}}) > \mu(\theta_{\text{Bad}})$$

Under normal circumstances, both firm's intrinsic values follow geometric Brownian motions. At the end of the single-time period, the intrinsic value of the good firm is stochastically dominant (Levy, 1992) relative to the intrinsic value of the bad firm, where for every value of x:

$\Pr\{\text{IV}_{t=1}(\theta_{\text{Good}}) > x\} \geq \Pr\{\text{IV}_{t=1}(\theta_{\text{Bad}}) > x\}$, or equivalently

$$q_{t=0,\,t=T}(\theta_{\text{Good}}) \leq q_{t=0,\,t=T}(\theta_{\text{Bad}})$$

Conceptually, we need to differentiate the two types of variations:

(1) $\sigma(\theta_i)$ is the *inherent process variation* of the logarithm of IV for SME (θ_i) over time and

(2) ε_j is the standard error of Bank$_j$'s estimate of the drift term $\mu(\theta_i)$, and more intuitively, can be interpreted as Bank$_j$'s ability to differentiate between good firms and bad firms.

Using the structural equation (13), the z-score $\Phi^{-1}(q_{t=0,t=T}(\theta_i))$ follows a normal distribution with mean and SD, respectively.

$$E\left[\Phi^{-1}\left(q_{t=0,t=T}(\theta_i)\right)\right] = \frac{\ln(\text{Debt}(\theta_i)|_{t=T}) - \widehat{\mu}(\theta_i)T - \sigma^2(\theta_i)T/2 - \ln(\text{IV}_{t=0}(\theta_i))}{\sigma(\theta_i)\sqrt{T}} \# \quad (29)$$

$$\text{Stdev}\left[\Phi^{-1}\left(q_{t=0,t=T}(\theta_i)\right)\right] = \frac{\varepsilon_j}{\sigma(\theta_i)}\sqrt{T} \# \quad (30)$$

We now provide an extension to our model by considering a population of firms with heterogeneous default probabilities. The median default probability among all firms in the population is q_0, and the variation of default probability across firms is described by a parameter δ, such that, for any value $0 < q < 1$, the proportion of firms with default probability not exceeding q is given by the Wang Transform formula.

We define a statistical model of variation across firms as the **Wang transform frailty distribution** with median default rate q_0 and variation δ, where the proportion of firms with default probability not exceeding q equals:

$$F(q;\, q_0,\, \delta) = \Phi\left[\frac{\Phi^{-1}(q) - \Phi^{-1}(q_0)}{\delta}\right], \quad \text{for } 0 < q < 1 \# \quad (31)$$

with boundary values of. $F(0;\, q_0,\, \delta) = 0$ and $F(1;\, q_0,\, \delta) = 1$.

This is a novel two-parameter distribution for default rates of a heterogeneous population, anchored at the median default rate q_0 and the variation δ. When compared to the well-known beta distribution, the Wang transform frailty distribution (31) has the advantage where the two parameters are more intuitive.

For instance, assume that the population of SMEs has a median default rate of $q_0 = 10\%$ and variation of $\delta = 0.5$. We have the following cumulative proportion of firms with default probability not exceeding q in Table 2, and the cumulative and density graphs are plotted in Figure 8:

If the underwriting criteria for the bank is to select SMEs with default probability not exceeding 3%, only a proportion of 11.54% of the SMEs would qualify for a bank loan.

However, if the underwriting criteria can be relaxed to SMEs with default probability not exceeding 5%, then 23.37% of the SMEs would qualify for a bank loan, thus potentially doubling the number of loans issued to SMEs.

11. Bank's underwriting risk selection capability

Similar to the frailty model, the Wang transform can be used to describe a bank's underwriting capability. In a similar way, a bank's underwriting capability in selecting risks can be described by the variation parameter δ.

For a bank loan to firm i with default probability $q(\theta_i)$ over the time interval $[0,1]$, we apply the Wang transform method and define the corresponding z-score as follows:

$$\Phi^{-1}(q(\theta_i)) \# \tag{32}$$

We further assume that the estimated z-score, $\Phi^{-1}(\widehat{q(\theta_i)})$ is unbiased and follows a Gaussian distribution with a mean z-score:

$$E\left[\Phi^{-1}(\widehat{q(\theta_i)})\right] = \Phi^{-1}(q(\theta_i)) \# \tag{33}$$

In addition, we assume that, due to varying underwriting capability of different banks, arising from the inherent variation in the class of bank loans as well as limited available information, the estimated z-score includes margin of error with a SD denoted by δ_j:

$$\text{Stdev}\left[\Phi^{-1}[\widehat{q(\theta_i)}]\right] = \delta_j, \quad \text{for Bank } j\# \tag{34}$$

It is straightforward to note that, when more information is available, or when the bank has greater risk underwriting capability, the corresponding δ_j will be lower, given as follows:

$$\delta_{\text{strong underwriting}} < \delta_{\text{weak underwriting}}$$

However, in practical applications, the δ_j may be bounded by a floor, and this arises from the inherent unknown-unknowns (or limitation of available information)

11.1 Illustrative example

Assume that the population of SMEs have two types of firms, 1,000 good quality firms and 1,000 bad quality firms, with the following default probabilities:

$$q(\theta_{\text{Good}}) = 0.02 < 0.10 = q(\theta_{\text{Bad}})$$

Assume that there are two banks, one with strong underwriting capability and one with weak underwriting capability.

$$\delta_{\text{strong bank}} = 0.25 < 0.5 = \delta_{\text{weak bank}}$$

Table 2. Some tabular values of the Wang transform frailty distribution, with median $q_0 = 10\%$ and variation of $\delta = 0.5$

Probability	Probatility · not exceeding
0	–
0.01	0.0183
0.02	0.0612
0.03	0.1154
0.04	0.1741
0.05	0.2337
0.06	0.2924
0.07	0.3488
0.05	0.4024
0.09	0.4529
0.1	0.5000

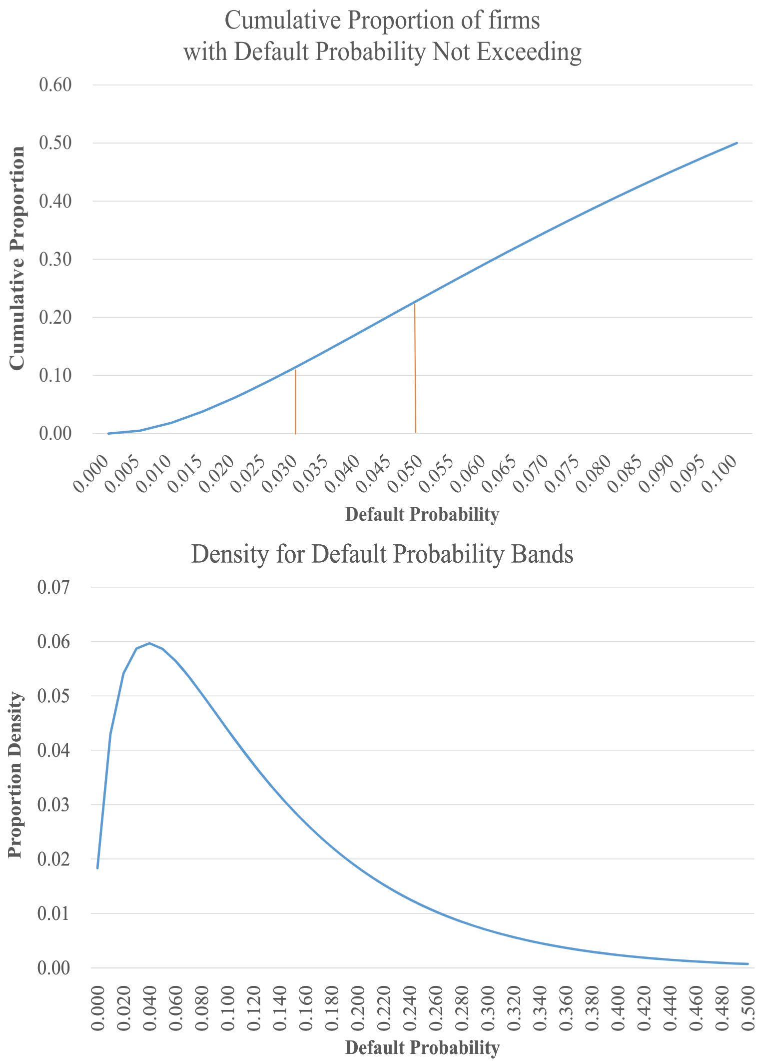

Figure 8.
Graphic illustration of the Wang transform frailty distribution, median $q_0 = 10\%$ and variation of $\delta = 0.5$, with cumulative proportion of firms (left panel) and density of default probability bands (right panel) plotted against the default probability

We further assume that, when the bank's estimate of the firm's default probability exceeds 3%, the bank would not issue the loan to the firm.

We define the following:

(1) TP (true positive): as number of loans issued to a good firm,

(2) TN (true negative): as number of loans not issued to a bad firm,

(3) FP (false positive or type I error) as number of loans issued to a bad firm,

(4) FN (false negative or type II error) as number of loans not issued to a good firm.

As shown in Table 3, the bank with weak underwriting capability would issue a total of 743 loans with 630 TP and 113 FP, corresponding to a sensitivity TP/(TP + FN) of 63% and a specificity TP/(TP + FP) of 84.8%. The expected default probability of approximately is 3.22%.

As shown in Table 4, the bank with stronger underwriting capability would issue a total of 754 loans with 746 TP and 8 FP, corresponding to a sensitivity TP/(TP + FN) of 74.6% and a specificity TP/(TP + FP) of 98.9%. The expected default probability of approximately is 2.08%.

Thus, the main difference between a weak bank and a strong bank is found in their specificities, more than in their sensitivities.

Furthermore, as illustrated in Figure 9, the red region + green region is the estimated default probability density of the good firms by the weak bank; the green region + blue region is the estimated default probability density of the bad firms by the weak bank. We define the differentiation index, which measures the cumulative probability of good firms and

Table 3. Bank with weak underwriting capability, where $\delta_{\text{weak bank}}(\theta_i) = 0.5$

	Sample size	# Of loans made	# Of loans not made	Expected # of default	Expected default rate
Good firm ($q(\theta_{\text{Good}}) = 0.02$)	1,000	630 (TP)	370 (FN)	12.6	0.02
Bad firm ($q(\theta_{\text{Bad}}) = 0.10$)	1,000	113 (FP)	887 (TN)	11.3	0.10
Total	2,000	*743*	*1,257*	*23.9*	*0.0322*

Table 4. Bank with strong underwriting capability, where $\delta_{\text{strong bank}} = 0.25$

	Sample size	# Of loans made	# Of loans not made	Expected # of default	Observed default rate
Good firm ($q(\theta_{\text{Good}}) = 0.02$)	1,000	746 (TP)	254 (FN)	14.92	0.02
Bad firm ($q(\theta_{\text{Bad}}) = 0.10$)	1,000	8 (FP)	992 (TN)	0.80	0.10
Total	2,000	*754*	*1,246*	*15.72*	*0.0208*

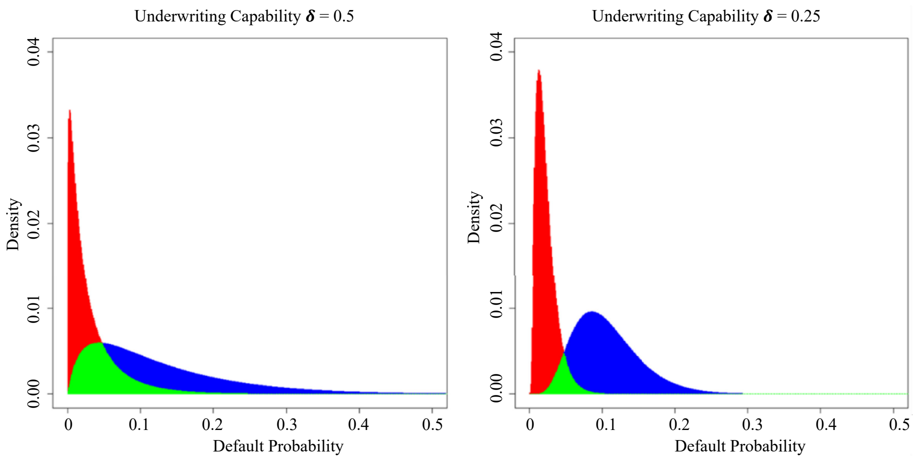

Figure 9. Overlapping regions in loans to good firms and bad firms by banks with different underwriting capability. Left panel: bank with weak underwriting capability; right panel: bank with strong underwriting capability

bad firms with distinct estimated default probability, as follows:

$$\text{Differentiation Index} = \frac{1}{2}(\text{Red Area} + \text{Blue Area})\,\# \tag{35}$$

We also define the overlap index, which measures the cumulative probability of good firms and bad firms with the same estimated default probability. This is a novel proxy proposed by our paper that captures the difficulty the bank has in differentiating between good firms and bad firms. The overlap index is defined as follows:

$$\text{Overlap Index} = 1 - \text{Differentiation Index}\,\# \tag{36}$$

Per our example, we calculate that the differentiation index for the weak and strong bank is 56.00 and 87.75%, respectively, while the overlap index for the weak and strong bank is 44.00 and 12.25%, respectively. The strong bank's differentiation (overlap) index is higher (lower) than that of the weak bank's, which is in line with our expectations. This is because we expect the strong bank to experience less difficulty in differentiating between good firms and bad firms. The authors would also like to highlight that a key differentiation between the differentiation/overlap index and other earlier conventional statistical measures is that the latter captures information ex-post (i.e. after some form of classification has been done); whereas the former captures information *ex ante* (i.e. the proportion of differentiation/overlap between various classes even before any classification had been done). It is thus natural to see that having a lower (higher) differentiation index would lead to poorer (better) scores for the other earlier conventional statistical measures; similarly, having a higher (lower) overlap index would result in poorer (better) scores for the other earlier conventional statistical measures.

Our model exemplifies the importance of underwriting capabilities of the bank. Furthermore, there are companies that already possess domain expertise based on their prior information and knowledge. These are factors that can potentially improve the underwriting capabilities and ensure that bank loans are issued to good firms, achieving an efficient allocation of resource within the economy.

It is also important to note that the value of underwriting highlighted in this paper is not in contradiction with traditional financial concepts such as the efficient market hypothesis and the no-arbitrage principle (Fama, 1970). Specifically, an ideal world assumes that full information is available. However, in the real-world, information is often in short supply and there are plenty of misinformation (as well as information asymmetry) (Akerlof, 1970). Therefore, it is challenging to estimate the firm's default probability. Prior knowledge and experience from underwriters are valuable, and these are the factors that impact the underwriting capability. Taken together, a case can also be made to nurture and grow the risk underwriting industry as it plays a key role in making the market more efficient. Kim *et al.* (2008) studied empirical underwriting spreads by commercial banks and investment banks, providing further support for the value of risk underwriting.

11.2 Some empirical evidence

From publicly reported default rates figures of Chinese Banks by the CCIRC, there are large differences in loan default rates between different types of banks. As described in Table 5, for the first quarter of 2020, rural community banks have the highest default rates (4.09%), followed by city community banks (2.45%), both of which are higher than the commercial banks (1.64%) and large commercial banks (1.39%). On the one hand, these large differences may be due to the fact that rural and city community banks having weaker underwriting capability than commercial banks. On the other hand, one may argue that rural and city community banks have higher concentration of loans to SMEs than commercial banks, in terms of total amount of loans to SMEs as percentage of total asset, as shown in Table 6.

Traditionally, governments have had some but limited success in stimulating large commercial banks to issue more loans to SMEs, in part this is due to the perception that loans to most SMEs have a higher default risk than what large commercial banks would be accustomed to underwrite (e.g. relative to loans to State-Owned Enterprises). As a result, only a selective sub-sample of SMEs would meet the underwriting criteria of large commercial banks.

More interestingly, per our strategy in section 8, we also apply the Wang transform and convert the corresponding default rates figures of Chinese Banks to the risk-neutral interest rate, with the results presented in Table 7. On the one hand, we document that the risk-neutral interest rate for large commercial banks and commercial banks are 4.49 and 4.99%, which are both lower than the prevailing interest rate of 5%. On the other hand, the risk-neutral interest rate for city community banks and rural community banks are 6.55 and 9.18%, which are both higher than the prevailing interest rate of 5%. The rural community banks have a higher default rate than other banks, this may be due to that rural community banks have a higher concentration of loans made to SMEs, or that rural community banks have a weaker

Table 5. Reported bank loan default rates for the 1st Quarter 2020

Type of banks	Default rate of all bank loans	Loans to SME s (billion RMB)
Large commercial banks	1.39%	3,751.79
Commercial banks	1.64%	2,233.54
City community banks	2.45%	1,840.08
Rural community banks	4.09%	4,547.00
Overall	2.04%	12,372.41

Note(s): Data retrieved http://www.cbirc.gov.cn/cn/view/pages/ItemDetail.html?docId=903311&itemId=954

Table 6. Bank loans made to SMEs as compared to bank asset for the 1st Quarter 2020

Type of banks	Total asset (billion RMB)	Loans to SMEs (billion RMB)	Loans to SMZs as % of Rank asset
Large commercial banks	124,030.50	3,751.79	3.02
Commercial banks	54,242.40	2,2335.54	4.12
City community banks	38,139.30	1,840.08	4.82
Rural community banks	39,0900.60	4,547.00	11.63
Total combined	255,502.80	12,372.41	4.84

Note(s): Data retrieved http://www.cbirc.gov.cn/cn/view/pages/ItemDetail.html?docId=903308&itemId=954&generaltype=0 and http://www.cbirc.gov.cn/cn/view/pages/ItemDetail.html?docId=903303&itemId=954

Table 7. Reported bank loan default rates for the 1st Quarter 2020 and the risk-neutral interest rate calculated using Wang transform

Type of banks	Default rate of all bank loans	Real interest rate	Risk-neutral interest rate
Large commercial banks	1.39%	1.39%	4.49%
Commercial banks	1.64%	1.64%	4.99%
City community banks	2.49%	2.49%	6.55%
Rural community banks	4.09%	4.09%	9.18%

underwriting capacity. Taken together, this provides evidence on the applicability of the Wang transform.

In addition, we observe that some of our proposed measures have been implemented in Japan, and we document the empirical successes of these measures [4]. The Japan Federation of Credit Guarantee Corporations (JFG) has introduced a guarantee program for the SMEs in response to the COVID-19 disruptions. JFG has received 90,000 applications, of which 74,000 applications were approved (i.e. an approval rate of 82.2%). In addition, the aid relief by the Japanese Government is not limited to just bank loans but they also provide direct support to the enterprise. The Japan Finance Corporation (JFC), a national bank, has introduced special low-interest loans to SMEs to help them cope with the changes in operating environment brought about by the COVID-19 disruptions. Upon implementation of the policy, JFC has received 246,6000 loan applications (i.e. more than 10,000 loan applications per day), of which JFC has approved 125,400 loan applications (i.e. an approval rate of 50.8%). We observe that Japan's broader industry coverage and higher success rate in government support for the SMEs can be attributed to having organizations in place that can efficiently execute these government support programs.

12. Conclusion and discussion

The economic impacts of COVID-19 are far reaching in terms of both national boundaries and time span. Many countries have rolled out measures to support SMEs which are struggling to survive amidst the economic downturns due to COVID-19.

Generally speaking, the effectiveness of government support for SMEs can be evaluated by how such measures help increase SMEs' current cash liquidity and franchise value.

Government supported bridge loans are effective emergency measures in helping SMEs to cope with the impact of COVID-19. It is essential for governments to support SMEs and ensure their survivability as that would affect the economic stability in the country. Taking into account the opportunity cost of financial support (as they could have been deployed for other purposes), there is an optimal amount of bridge loan to be granted to SMEs depending on the duration of economic disruption of COVID-19.

Government-funded insurance pool for bank loans provide some but weak incentives for banks to issue loans to SMEs as such insurance does not reduce the probability of loan default in the first place.

As a more effective measure, governments can provide an interest rate subsidy to banks to incentivize them to issue more bank loans to SMEs and to have them extend the loan maturity of current debt to the SMEs. As common practice, government interest subsidy is used to help SMEs to pay a portion of the interest cost; however, banks do not have the flexibility to charge market risk-based interest rates for loans to SMEs. Thus, we propose that government can provide interest subsidy payable to banks to bring the total interest rate to a risk-based level. This would adequately compensate the banks for the increased risks when the banks issue more bank loans to SMEs or extend the loan maturity of current debt to SMEs. The paper applies the Wang transform to calculate the appropriate level of interest rate for SMEs with a given level of default risk, and thus quantifies the level of interest rate subsidy needed to incentivize banks to lend to SMEs.

We recognize the limitation of existing banking channel in lending to SMEs, which require a different underwriting approach due to the relatively small size of loan amounts and large number of SMEs (where some SMEs have viable business and many others do not have a viable business). To enhance the underwriting efficiency for issuing loans to a large number of SMEs, a case is made for establishing policy-oriented banks or investment funds dedicated to supporting SMEs.

Other measures of government support can come in the form of tax relief. A key concept used in our theoretical model is a firm's franchise value in equation (10) as the sum of discounted future after-tax earnings. Based on equation (10), when the effective tax rate $\tau(\theta_i)$ for SMEs is reduced, this would increase the firm's franchise value. Currently, a common type of government support measure is to provide temporary tax reliefs (i.e. three months, 6 months tax relief). This can indeed serve as a type of incentive for companies and increase the amount of available working capital for the companies. However, we note that a key issue faced by companies is a lack of demand for their products and services. Therefore, stimulating confidence in the economy for the consumers is also essential in boosting consumption and aggregate demand. Therefore, reducing tax burdens should be done in conjunction with providing a more predictable business environment. We propose that by providing support from both the demand- and supply side, this would have a more profound effect in creating tangible value for the SMEs. Having many new short-term initiatives do not necessarily help to instill confidence within the market participants.

Government support for bank loans should be steered toward SMEs with higher intrinsic value. As the government is not in the business of risk underwriting or serving the loans, it is essential for government support to leverage on the underwriting capabilities of financial institutions, capital markets or Internet-based risk exchange platforms. Our paper showed how the bank's underwriting capability affects the proportion of bank loans issued to good firms versus bad firms. The Wang transform method is applied to describe the underwriting capability of banks. This led to the concept of the overlap index which is a novel proxy of the bank's underwriting capability in selecting risks.

It is advisable for the government to support the development of the risk management industry equipped with strong underwriting capability. Government can provide support for innovative risk information services for SMEs and online risk exchange platforms for the SME community to facilitate better information about the business prospects of SMEs. One specific suggestion is for government to support the development of risk indices for SMEs, including both downside risk and upside potential; such risk indices for SMEs would need to incorporate macro-level, sector-level and firm-level economic and business information, thus quite an enormous undertaking and may require collaboration between universities and various industry participants.

Government can achieve greater leverage effect by helping SMEs with high franchise value to tap into the private funds through securitization and early-stage angel investing. Collateralizing commercial loans and bank financing by granting a security interest in IP is a growing practice in Internet-based SMEs and in technology sectors. Currently, the markets for intellectual property asset-based securities are still in an infant stage as the universe of knowledgeable buyers and sellers is limited; however, it is growing with more interests from a variety of investors. For firms with high IP content, government support for SMEs can be in the form of early-stage start-up loan facilitated by private sector angel investors or asset management firms, where government support can be turned into equity stake in later stage series-A funding.

We hope that our paper can lay the groundwork for other researchers to conduct additional research on government support for SMEs, especially within the context of the COVID-19 situation.

Notes

1. Student-t with k degrees of freedom has density

$$f(t;k) = \frac{1}{\sqrt{2\pi}} \cdot c_k \cdot \left[1 + \frac{t^2}{k}\right]^{-\left(\frac{k+1}{2}\right)} \text{with } c_k = \sqrt{\frac{2}{k}} \cdot \frac{\Gamma\left(\frac{k+1}{2}\right)}{\Gamma\left(\frac{k}{2}\right)}$$

2. Market transactions data provided by Lane Financial LLC, http://lanefinancialllc.com/.
3. The local government in Shenzhen has a standard corporate tax rate of 25%. In a bid to spur innovation and nurture a high-tech ecosystem, the local government has reduced the corporate tax rate for high-tech companies to 15% (i.e. a reduction of 40%), (Retrieved May 18, 2020, from http://www.sckjlt.com/gaoxin/1511.html).
4. Data retrieved http://shingetsunewsagency.com/2020/05/12/japans-covid-19-subsidy-and-relief-programs-for-smes/ and https://www.thepaper.cn/newsDetail_forward_7260924

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