1-2022

Mechanism design by observant and informed planners

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Review of Economic Design
https://doi.org/10.1007/s10058-021-00281-4

Mechanism design by observant and informed planners

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Abstract
We study the mechanism design problem where the planner can observe ex-post the first-ranked alternatives or peaks of voter preferences. We contrast this with the design problem where the planner has ex-ante information regarding the peaks of voter preferences.

Keywords Local strategy-proofness · Strategy-proofness · Unanimity

JEL Classification D71

1 Introduction

The Gibbard–Satterthwaite Theorem (Gibbard 1973; Satterthwaite 1975), is a fundamental result in the theory of mechanism design. According to the Theorem, a planner can provide dominant strategy incentives for agents to reveal their private information or a social choice functions can be made *strategy-proof* only by allowing some agent (called the dictator) to always get his most preferred alternative. An assumption which is crucial for the result, is that the domain of preference orderings is complete. An extensive literature on restricted domains has emerged which shows that there are natural restrictions on the domain of preferences for which the strongly negative conclusion of the Gibbard–Satterthwaite Theorem can be avoided. For instance, the median voter rule is strategy-proof if preferences are single-peaked (Moulin 1980). In models with money and quasi-linear preferences, the rich theory of Vickrey–Groves–

This paper is dedicated to our friend Semih Koray with admiration and respect. We would also like to thank a reviewer for valuable comments and suggestions.

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Clarke mechanisms applies and numerous possibility results exist. Other examples include allocation models where agents have selfish preferences and models where outcomes are lotteries and agents' preferences satisfy von-Neumann–Morgenstern axioms (Gibbard 1977).

In this paper, we propose a model quite different in spirit to the one with restricted preference domains. We refer to it as one with a *partially observant* planner. The idea is that ex-ante, an agent can have any preference ordering. However, *after* realization, the planner is able to observe some feature of these preferences. For instance, in a model of voting by committee, the planner may be able to observe that voter 1's most preferred candidate is x, voter 2's least preferred candidate is y, voter 3 prefers w to z and so on. Thus, the planner has some (ex-post) information on preferences which could be based on commonly known ideological positions, personal dislikes etc. The mechanism, however, in keeping with the standard assumption has to be designed ex-ante, i.e. before the realization of preferences. On the other hand, in the restricted domains model, which we refer to as the partially informed planner model, the planner has some ex-ante information on the structure of preferences. We believe that the observant planner model is a realistic one and worthy of attention. It is particularly plausible in the standard voting model where it may be unnatural to impose structure such as single-peakedness, convexity or cardinal-valuedness.

The observant planner model is also related to the complete information implementation model (Maskin 1999). In the latter, agents know each others preferences perfectly but the planner is completely ignorant. The problem here is to design a mechanism which will allow the planner to collate reports from each agent to infer something about the true state of the world. In the observant planner model, *partial* information about each voters' preference is not only common knowledge amongst the other voters but is also known to the planner.

The analysis in the observant planner model differs in crucial respects from that in the informed planner model. In the former, the domain of preferences remains complete unlike that in the informed planner model. However, the incentive compatibility condition is weaker. In particular we require only that no voter can gain by misrepresenting his preferences only for those misrepresentations which are consistent with observed information. Suppose that the planner knows that voter i's peak is x. Then it must the case that the agent cannot do better than truth-telling than by announcing any other preference whose peak is x. The analysis, in the two models is thus independent of each other.

In the paper, we assume that the observant planner can observe the first-ranked alternative or peak of each voters' preference ordering. We provide a complete characterization of incentive compatible social choice functions under a range assumption. We contrast this case with that of a restricted domain model where the planner has some ex-ante information about peaks. In particular, we consider the case where it is known that each voter's peak lies in some pre-specified set which is a subset of the set of alternatives.

Our results are as follows. In the observant planner case we show that if a range condition is satisfied, a social choice condition is incentive-compatible if and only if, for every vector of peaks, there is a voter and a set of alternatives over which this voter is a dictator. The choice of the voter who dictates and the set over which he does so, could depend on what the planner observes. Our results in the informed planner model on the other hand, are less sharp. We show there are particular information structures regarding voter peaks where a dictatorship result emerges but we are unable to provide a general result. The dictatorship result is rather delicate in the following sense: we show by means of an example that if the planner has information on the alternatives which were ranked first and second, then non-dictatorial possibility results exist.

Although our results are quite intuitive they are not very easy to prove. There does not appear to be a way to apply the Gibbard–Satterthwaite Theorem directly. A special feature of these models is that the "effective" domain of preferences is voter specific. The induction technique of coalescing or cloning voters used in various proofs (for example, Sen (2001) can no longer be used. We develop a completely novel induction technique which can be used to provide yet another proof of the Gibbard–Satterthwaite Theorem.

This paper is organized as follows. Section 2 lays out the basic notation while the next two sections discuss the observant and informed planner models. The last section concludes.

2 Preliminaries

The set $I = \{1, \dots, N\}$ is the set of agents or voters. The set of alternatives is the set A with $|A| = m$. Elements of A will be denoted by a, b, c, d etc. Let $\mathbb{P}$ denote the set of strict orderings[1] of the elements of A. A typical preference ordering (or simply a preference) will be denoted by $P_i \in \mathbb{P}$ where aP_ib will signify that a is preferred (strictly) to b under P_i. A preference profile is an N-tuple whose ith component is the preference of voter i. Preference profiles will be denoted by P, $\bar{P}$, P' etc and their ith components by P_i, $\bar{P}_i$, P'_i respectively with $i = 1, \dots, N$. For any voter i, P_{-i} will denote a preference profile for the set of voters $N \backslash \{i\}$. Let $(\bar{P}_i, P_{-i})$ denote the preference profile where the ith component of the profile P is replaced by $\bar{P}_i$.

For all $P_i \in \mathbb{P}$ and $k = 1, \dots, m$, let $r_k(P_i)$ denote the kth ranked alternative in P_i, i.e., $r_k(P_i) = a$ implies that $|\{b \neq a | bP_ia\}| = k - 1$. For all P_i, the alternative $r_1(P_i)$ will be referred to as the peak of P_i. For any $a \in A$, $\mathbb{P}^a = \{P_i \in \mathbb{P} : r_1(P_i) = a\}$, i.e. it is the set of preference ordering with a as the peak. Let $\bar{a} = (\bar{a}_1, \dots, \bar{a}_N) \in A^N$ be an N-tuple of alternatives. Then $\mathbb{P}^{\bar{a}}$ is the set of profiles P such that $r_1(P_i) = \bar{a}_i$ for all $i = 1, \dots N$. For any $\bar{a} \in A^N$ and $i = 1, \dots, N$, let $\bar{a}_{-i} = (\bar{a}_1, \dots, \bar{a}_{i-1}, \bar{a}_{i+1}, \dots, \bar{a}_N) \in A^{N-1}$. Also let $\mathbb{P}^{\bar{a}_{-i}}$ be the set of profiles P_{-i} such that $r_1(P_j) = \bar{a}_j$ for all $j \neq i$. For all $P_i \in \mathbb{P}$ and $B \subset A$, $\max(P_i, B)$ will denote the maximal element in B according to P_i.

A *social choice function* (SCF) is a mapping $f : \overbrace{\mathbb{P} \times \cdots \times \mathbb{P}}^{\text{N times}} \to A$. The range of the SCF f denoted by R^f is the set $\{a \in A : f(P) = a \text{ for some } P \in \mathbb{D}\}$. The SCF f is *dictatorial* if there exists $i \in I$ such that $f(P) = \max(P_i, R^f)$ for all $P \in \mathbb{D}$. The SCF f is *manipulable* by voter i at profile $P \in \mathbb{D}$ if there exists $P'_i \in \mathbb{D}_i$ such that $f(P'_i, P_{-i})P_i f(P)$. The SCF is *strategy-proof* if it is not manipulable by

[1] A strict ordering is a complete, transitive and antisymmetric binary relation.

any voter at any profile. A strategy-proof SCF has the property that no voter can gain by misrepresenting her preference ordering irrespective of the announcements of the voters. A fundamental result for strategy-proof SCFs is the Gibbard–Satterthwaite Theorem.

Theorem 1 (Gibbard-Satterthawite) *Let $f : \mathbb{P}^N \rightarrow A$ be a SCF satisfying $|R^f| \geq 3$. Then f is strategy-proof if and only if it is dictatorial.*

3 The partially observant planner

In this model, it is assumed that each voter i's preference P_i is drawn from the set $\mathbb{P}$ and is private information for i. However, once it has been realized, each voter i's peak $r_1(P_i)$ can be observed by the planner. Since the outcome of an SCF at a preference profile can depend on more than voters' peaks at that profile, preferences still have to be elicited from voters. The appropriate notion of incentive-compatibility is that a voter cannot manipulate by a preference consistent with the information observed by the planner.

Definition 1 The SCF f is constrained strategy-proof (CSP) if, for all $i \in I$, for all $a \in A$ and $P_i \in \mathbb{P}^a$, there does not exist $P'_i \in \mathbb{P}^{a_i}$, and an $N \backslash \{i\}$ profile P_{-i} such that $f(P'_i, P_{-i}) P_i f(P_i, P_{-i})$.

The notion of CSP differs from strategy-proofness insofar as the former imposes the additional consistency requirement that voter i can manipulate using a preference P'_i only if $P'_i \in \mathbb{P}^{a_i}$. It is a clear that CSP is a weaker requirement than strategy-proofness.

The next example shows that the conclusion of the Gibbard–Satterthwaite Theorem does not hold in this model.

Example 1 For each $a \in A$, let $B^a \subset A$. Define the SCF f as follows: for all profiles P, $f(P) = \max(P_2, B^{r_1(P_1)})$. Voter 1 "offers" a set of outcomes for voter 2 to choose from. This set depends on voter 1's observable peak. This SCF satisfies the CSP property. It can also be specified so as to be non-dictatorial. In fact, by choosing the set B^a to be either a singleton or the whole set A, the set of preference profiles can be partitioned arbitrarily into two sets, one over which voter 1 gets his maximum in R^f and the other over which 2 gets her maximum in R^f.

We now present a more general result. Let f be a SCF. Let $\bar{a} \in A^N$ denote an N-tuple of alternatives whose ith component is $\bar{a}_i \in A$. Let $R^f(\bar{a}) = \{f(P) | P \in \mathbb{P}^{\bar{a}}\}$, i.e. $R^f(\bar{a})$ is the range of f when voter's peaks are constrained to be $\bar{a}$.

Theorem 2 *Let f be a SCF satisfying CSP. Let $\bar{a} \in A^N$ be such that $|R^f(\bar{a})| \geq 3$. There exists $j \in I$ such that for all $P \in \mathbb{P}^{\bar{a}}$, we have $f(P) = \max(P_j, R^f(\bar{a}))$.*

Suppose f is a SCF that satisfies the CSP requirement and the range condition. Then f must have a particular structure. A voter and a subset of alternatives is specified depending on the N-tuple of first-ranked alternatives at a preference profile. The outcome at the profile is the specified voter's maximal alternative in the specified set. The SCF in Example 1 clearly belongs to the class of SCFs described in the Theorem.

Proof We will prove the result by induction on N. Suppose there is only one voter, say voter 1. Let f be a SCF satisfying CSP. Let $\bar{a}_1$ be such that $|R^f(\bar{a}_1)| \geq 3$. Suppose there exists $P_1 \in \mathbb{P}_1^{\bar{a}_1}$ and $a, b \in A$ such that $f(P_1) = a$, $b \in R^f(\bar{a}_1)$ and bP_1a. By assumption, there exists $P_1' \in \mathbb{P}^{\bar{a}_1}$ such that $f(P_1') = b$. This violates the assumption that f satisfies CSP. Hence $f(P) = \max(P_1, R^f(\bar{a}_1))$ establishing the result for $N = 1$.

We briefly outline the main idea behind the proof of the induction step. We demonstrate the existence of two voters i, j and a preference $P_i \in \mathbb{P}^{\bar{a}_i}$ such that $f(P_i, P_{-i}) = \max(P_j, R^f(P_i))$. Here $R^f(P_i)$ denotes the range of f fixing voter $i's$ preference at P_i. Next we show that $f(P_i', P_{-i}) = \max(P_j, R^f(P_i'))$ continues to hold for all $P_i' \in \mathbb{P}^{\bar{a}_i}$. Finally we show that $R^f(P_i) = R^f(P_i')$ for all $P_i \in \mathbb{P}^{\bar{a}_i}$, so that $R^f(\bar{a}) = \bigcup_{P_i \in \mathbb{P}^{\bar{a}_i}} R^f(P_i) = R^f(P_i)$. This establishes $f(P) = \max(P_j, R^f(\bar{a}))$ as required.

In order to complete the induction step, we fix an integer $N \geq 2$ and assume the following Induction Hypothesis (IH):

IH: Let $h : \overbrace{\mathbb{P} \times \cdots \times \mathbb{P}}^{\text{N-1 times}} \to A$ be an arbitrary SCF satisfying CSP. Let $\bar{b} \in A^{N-1}$ be such that $|R^h(\bar{b})| \geq 3$. There exists voter i such that for all $P \in \mathbb{P}^{\bar{b}}$, we have $h(P) = \max(P_i, R^h(\bar{b}))$.

Let $f : \overbrace{\mathbb{P} \times \cdots \times \mathbb{P}}^{\text{N times}} \to A$ be a SCF satisfying CSP. Let $\bar{a} \in A^N$ be such that $|R^f(\bar{a})| \geq 3$. Lemmas 1, 2, 3 and 4 will enable us to identify a voter i and to construct a SCF h defined over a set of voters $N - 1$ which satisfies CSP and $|R^h(\bar{a}_{-i})| \geq 3$. This will then allow us to apply IH and complete the proof.

Define the function $g : \mathbb{P}^{\bar{a}} \to A$ as follows: for all $P \in \mathbb{P}^{\bar{a}}$, $g(P) = f(P)$. Thus g can be interpreted as a social choice function defined on the restricted domain $\mathbb{P}^{\bar{a}}$. Since f satisfies CSP, it follows that g is strategy-proof in the conventional sense, i.e. for all voters i and $P_i \in \mathbb{P}^{\bar{a}_i}$, there does not exist an $N \backslash \{i\}$ profile $P_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$ and $P_i' \in \mathbb{P}^{\bar{a}_i}$ such that $g(P_i, P_{-i}) P_i g(P)$.

Let $R^g = \{a \in A | g(P) = a \text{ for some } P \in \mathbb{P}^{\bar{a}}\}$, i.e. R^g is the range of g. For convenience, we shall denote R^g simply as B for the remainder of the proof. Note that $B = R^f(\bar{a})$ and so $|B| \geq 3$ by assumption.

Fix $i \in I$ and $P_i \in \mathbb{P}^{\bar{a}_i}$. Let $R^g(P_i) = \{x \in A | x = g(P_i, P_{-i}) \text{ for some } P_{-i} \in \mathbb{P}^{\bar{a}_{-i}}\}$. Thus $R^g(P_i)$ is the range of g when voter i's preference is fixed at P_i. We will show that there exists i and P_i such that $|R^g(P_i)| \geq 3$.

Lemma 1 *Pick* $x \in B$ *and* $P_i \in \mathbb{P}_i^{\bar{a}_i}$ *such that* $x = \max(P_i, B)$. *Let* $P_i \in \mathbb{P}_i^{\bar{a}_i}$ *be such that* $x = r_2(P_j)$ *whenever* $x \neq \bar{a}_j$. *Then* $g(P_i, P_{-i}) = x$.

Proof Since $x \in B$ there exists $\bar{P} \in \mathbb{P}^{\bar{a}}$ such that $g(\bar{P}) = x$. Pick $j \neq i$ and switch her preference from $\bar{P}_j$ to P_j. Note that $r_1(\bar{P}_j) = r_1(P_j) = \bar{a}_j$. Let $g(P_j, \bar{P}_{-j}) = w \neq x$. If xP_jw, then the CSP property of f (or strategy-proofness of g) is violated since j can manipulate by announcing $\bar{P}_j$ when her true preference is P_j. If wP_jx, then $w = \bar{a}_j$ and $w\bar{P}_jx$. Again, CSP is violated since j can manipulate by announcing P_j when her true preference is $\bar{P}_j$. Hence $w = x$. Repeating this argument for all voters $j \neq i$, we conclude $g(\bar{P}_i, P_{-i}) = x$. Let $g(P_i, P_{-i}) = w$ and suppose $w \neq x$. Since

$x = \max(P_i, B)$, it must be true that xP_iw. Since $r_1(P_i) = r_1(\bar{P}_i) = \bar{a}_i$, CSP is violated because i can manipulate by announcing $\bar{P}_i$ when her true preference is P_i. Therefore $g(P_i, P_{-i}) = x$. □

An immediate consequence of Lemma 1 is that $x \in R^g(P_i)$ whenever $x = \max(P_i, B)$.

Lemma 2 *Let* $P_i, \bar{P}_i \in \mathbb{P}^{\bar{a}_i}$ *be such that* $\max(P_i, B) = \max(\bar{P}_i, B)$. *Then* $R^g(P_i) = R^g(\bar{P}_i)$.

Proof Suppose the Lemma is false. Let $\max(P_i, B) = \max(\bar{P}_i, B) = x$. Assume without loss of generality that there exists $y \neq x$ such that $y \in R^g(P_i)$ but $y \notin R^g(\bar{P}_i)$. Choose $P_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$ satisfying the following properties: for all $j \neq i$ (i) $r_2(P_j) = y$ if $x = \bar{a}_j$ (ii) $r_2(P_j) = x$ if $y = \bar{a}_j$ and (iii) $r_2(P_j) = y$ and $r_3(P_j) = x$ if $x, y \neq \bar{a}_j$. Thus y and x are ranked "as high as possible" with y always ranked above x unless $x = \bar{a}_j$.

We claim that $g(P_i, P_{-i}) = y$. Since $y \in R^g(P_i)$, there exists $P'_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$ such that $y = g(P_i, P'_{-i})$. Pick $j \neq i$ and switch her preference from P'_j to P_j. Suppose the outcome of g at the resultant profile is $w \neq y$. If yP_jw, then CSP is violated because j can gain by manipulating to P'_j when her true preference is P_j. If wP_jy, then $w = \bar{a}_j$, so that wP'_jy holds. Now j can gain by manipulating to P_j when her true preference is P'_j violating CSP. Hence $w = y$. Changing the preferences of all voters other than i in sequence, we conclude $g(P_i, P_{-i}) = y$.

Next, we claim that $g(\bar{P}_i, P_{-i}) = x$. For all $j \neq i$, define $\bar{P}_j \in \mathbb{P}^{\bar{a}_j}$ by reversing the ranking of x and y in P_j leaving the ranking of all other alternatives undisturbed. If either x or y coincide with $\bar{a}_j$, we let $P_j = \bar{P}_j$. Note that we have $x = r_2(P_j)$ whenever $x \neq \bar{a}_j$. Since $x = \max(\bar{P}_i, B)$, Lemma 1 applies allowing us to conclude that $g(\bar{P}_i, \bar{P}_{-i}) = x$. Note that $r_1(P_j) = r_1(\bar{P}_j)$ for all $j \neq i$. Now, consider a sequence of profiles obtained by progressively switching the preferences of all voters $j \neq i$ from $\bar{P}_j$ to P_j. Since $g(\bar{P}_i, \bar{P}_{-i}) = x$ and the only alternatives whose rankings differ between $\bar{P}_j$ and P_j are x and y, CSP implies that the outcomes of g along this sequence can only be either x or y. But the outcome cannot be y since $y \notin R^g(\bar{P}_i)$ by assumption. Hence $g(\bar{P}_i, P_{-i}) = x$.

Observe that $r_1(P_i) = r_1(\bar{P}_i)$ and xP_iy (since $x, y \in B$ and $x = \max(P_i, B)$). We have argued that $g(P_i, P_{-i}) = y$ and $g(\bar{P}_i, P_{-i}) = x$. Therefore we have a contradiction to CSP. □

Lemma 3 *Either* $|R^g(P_i)| = 1$ *or* $|R^g(P_i)| \geq 3$.

Proof Suppose the Lemma is false. Since $|R^g| \geq 3$, we can assume without loss of generality that $x, y, z \in R^g$ with $x, y \in R^g(P_i)$ but $z \notin R^g(P_i)$. In view of the Observation following Lemma 1, we can assume without loss of generality that $x = \max(P_i, B)$. According to Lemma 2, $R^g(P_i)$ depends only on $\max(P_i, B)$. Therefore we can assume $r_2(P_i) = x$, $r_3(P_i) = z$ and $r_4(P_i) = y$ if $x \neq \bar{a}_i$ and $r_2(P_i) = z$, $r_3(P_i) = y$ if $x = \bar{a}_i$.

Choose $P_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$ satisfying the following properties: for all $j \neq i$ (i) $r_2(P_j) = z$ if $y = \bar{a}_j$ (ii) $r_2(P_j) = y$ if $z = \bar{a}_j$ and (iii) $r_2(P_j) = z$ and $r_3(P_j) = y$ if $y, z \neq \bar{a}_j$. We claim that $g(P_i, P_{-i}) = y$.

Let $\bar{P}_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$ be the profile of preferences of voters $j \neq i$ obtained by switching the ranking of y and z in P_j unless $\bar{a}_j = z$ or $\bar{a}_j = y$. Thus y is ranked "as high as possible" in $\bar{P}_j$ unless $\bar{a}_j = z$. The arguments in Lemma 2 can be applied to conclude that $g(P_i, \bar{P}_{-i}) = y$. Note that $r_1(P_j) = r_1(\bar{P}_j)$. Consider the sequence of profiles where voters $j \neq i$ progressively switch preferences from $\bar{P}_j$ to P_j. Since g satisfies CSP, the outcome can switch only from y to z along the sequence. Since $z \notin R^g(P_i)$, the outcome cannot be z anywhere along the sequence. Hence $g(P_i, P_{-i}) = y$.

Since $z \in R^g$, there exists $P_i' \in \mathbb{P}^{\bar{a}_i}$ and $\hat{P}_{-i} \in \mathbb{D}^{\bar{a}_{-i}}$ such that $g(P_i', \hat{P}_{-i}) = z$. Note that z is as "high as possible" in P_{-i}. Therefore the arguments in Lemma 2 apply and $g(P_i', P_{-i}) = z$. Since $g(P_i, P_{-i}) = y$, $r_1(P_i) = r_1(P_i')$ and $z P_i y$, we have a violation of CSP. □

Lemma 4 *There exists $i \in I$ and $P_i \in \mathbb{P}^{\bar{a}_i}$ such that $|R^g(P_i)| \neq 1$.*

Proof Suppose the Lemma is false, i.e. $|R^g(P_i)| = 1$ for all $i \in I$ and all $P_i \in \mathbb{P}^{\bar{a}_i}$. Pick an arbitrary $i \in I$ and $\bar{P}_i \in \mathbb{P}^{\bar{a}_i}$ and let $R^g(\bar{P}_i) = \{x\}$. We will show that for an arbitrary profile $P \in \mathbb{P}^{\bar{a}}$, we have $g(P) = x$ contradicting our assumption that $|B| \geq 3$.

Choose the profile $\bar{P}_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$ as follows: for all $j \neq i$, $\bar{P}_j = P_j$ if $\bar{a}_j = x$; otherwise x is bottom-ranked in $\bar{P}_j$. Since $R^g(\bar{P}_i) = \{x\}$, we have $g(\bar{P}_i, \bar{P}_{-i}) = x$. Let $\hat{P}_i \in \mathbb{P}^{\bar{a}_i}$ be such that $\hat{P}_i = P_i$ if $\bar{a}_i = x$; otherwise x is bottom-ranked in $\hat{P}_i$. For any $j \neq i$, $|R^g(\bar{P}_j)| = 1$ by assumption. Since $g(\bar{P}_i, \bar{P}_{-i}) = x$, it must be true that $g(\hat{P}_i, \bar{P}_{-i}) = x$. At the profile $(\hat{P}_i, \bar{P}_{-i})$, x is bottom-ranked for all agents k for whose preferences are different from P_k. Chose such a voter k and change her preference to P_k. If the outcome of g changes from x, there will be a violation of CSP since x is bottom-ranked for k in the profile $(\hat{P}_i, \bar{P}_{-i})$. Switching the preferences of all such voters k, we have $g(P) = x$ as claimed. □

Applying Lemmas 3 and 4 , we conclude that there exists $i \in I$ and $P_i \in \mathbb{P}^{\bar{a}_i}$ such that $|R^g(P_i)| \geq 3$. Define the map $h : \mathbb{P}^{\bar{a}_{-i}} \to A$ as follows: for all $P_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$, $h(P_{-i}) = g(P_i, P_{-i})$. Clearly h is a SCF defined over a society of $N - 1$ voters. It is trivial to check that h satisfies CSP. Since $|R^g(P_i)| \geq 3$, we can appeal to IH to conclude that there exists a voter $j \in N \backslash \{i\}$ such that, for all $P_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$ we have $g(P_i, P_{-i}) = h(P_{-i}) = \max(P_j, R^g(P_i))$. We will conclude the proof by showing that the identity of voter j cannot depend on P_i and that $R^g(P_i)$ is in fact independent of P_i and is completely determined by $\bar{a}_i$.

Pick an arbitrary $\bar{P}_i \in \mathbb{P}^{\bar{a}_i}$. We know from Lemma 3 that $|R^g(\bar{P}_i)| = 1$ or $|R^g(\bar{P}_i)| \geq 3$ must hold. We show that $|R^g(\bar{P}_i)| = 1$ cannot hold.

Suppose $R^g(\bar{P}_i) = \{z\}$. Since $|R^g(P_i)| \geq 3$ we can pick distinct x, y and z and $x, y \in R^g(P_i)$. Suppose $z \neq \max(P_i, R^g(P_i))$. Assume without loss of generality that $x = \max(P_i, R^g(P_i))$. Pick $\hat{P}_i \in \mathbb{P}^{\bar{a}_i}$ such that $r_2(\hat{P}_i) = x$ and $r_3(\hat{P}_i) = z$ if $\bar{a}_i \neq x$ and $r_2(\hat{P}_i) = z$ if $\bar{a}_i = x$. According to the observation following Lemma 1, we have $x = \max(P_i, B)$. By construction, $x = \max(\hat{P}_i, B)$. Applying Lemma 2, we have $R^g(\hat{P}_i) = R^g(P_i)$; in particular $y \in R^g(\hat{P}_i)$. Note that $x \neq \bar{a}_j$. In that case $|R^g(P_i)| = 1$ contradicting our assumption that $|R^g(P_i)| \geq 3$. Choose $P_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$ such that $r_2(P_j) = y$. Also $r_2(P_k) = y$ for all $k \neq j, i$ unless $\bar{a}_k = y$. Since

$y \in R^g(\hat{P}_i)$, we can apply the arguments in Lemma 2 to conclude that $g(\hat{P}_i, P_{-i}) = y$. However $g(\bar{P}_i, P_{-i}) = z$. Since $z\hat{P}_i y$, we have a contradiction to CSP.

The remaining case is $z = \max(P_i, R^g(P_i))$, i.e. zP_ix holds. Choose $P_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$ such that $r_2(P_j) = x$. Since j is a dictator in h, we have $g(P_i, P_{-i}) = x$ while $g(\bar{P}_i, P_{-i}) = z$. Once again CSP is violated.

We have deduced that $|R^g(\bar{P}_i)| \geq 3$. The IH implies the existence of a voter $k \in N \backslash \{i\}$ such that, for all $P_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$, we have $g(\bar{P}_i, P_{-i}) = \max(P_k, R^g(\bar{P}_i))$. The only case of interest is when $k \neq j$. We consider this case in detail below.

Suppose there exists $x \in R^g(P_i)$ and $y \in R^g(\bar{P}_i)$ such that yP_ix. Choose $P_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$ such that $r_2(P_j) = x$ and $r_2(P_k) = y$. (Note, once again that $\bar{a}_j \neq x$ and $\bar{a}_k \neq y$; otherwise $R^g(P_i) = \{x\}$ and $R^g(\bar{P}_i) = \{y\}$ contradicting the assumption that $|R^g(P_i)|, |R^g(\bar{P}_i)| \geq 3$. Therefore $g(P_i, P_{-i}) = x$ while $g(\bar{P}_i, P_i) = y$ which is a violation of CSP.

Suppose the hypothesis in the earlier paragraph fails to hold. Let $x = \max(P_i, R^f(\bar{P}_i))$. By assumption, there exists $y \in R^g(\bar{P}_i)$ and xP_iy. Choose $P_{-i} \in \mathbb{P}^{\bar{a}_{-i}}$ such that $r_2(P_j) = y$ and $r_2(P_k) = x$. Then $g(P_i, P_{-i}) = y$ while $g(\bar{P}_i, P_{-i}) = x$. Once again, CSP is violated.

In conclusion, the identity of voter j only depends on the observable peak of voter i, $\bar{a}_i$. Similarly $R^g(P_i)$ also depends only on $\bar{a}_i$. To verify this step, consider $P_i, \bar{P}_i \in \mathbb{P}^{\bar{a}_i}$ and assume without loss of generality that $z \in R^g(P_i) \backslash R^g(\bar{P}_i)$. Then it follows from Lemma 2 that $\max(P_i, B) = x \neq y = \max(\bar{P}_i, B)$. (Note, once again that $\bar{a_j} \neq x, z$ and $\bar{a_j} \neq y$; otherwise $R^g(P_i) = \{x\}$ and $R^g(\bar{P}_i) = \{y\}$ contradicting the assumption that $|R^g(P_i)|, |R^g(\bar{P}_i)| \geq 3$). Now consider P_{-i} where P_j has $r_2(P_j) = z$, $r_3(P_j) = y$ and consider P_i' which is derived from P_i by placing y immediately below x and z immediately below y in P_i. Then by Lemma 2 we have $R^g(P_i) = R^g(P_i')$, so that $g(P_i', P_{-i}) = z$ while $g(\bar{P}_i, P_{-i}) = y$, and consequently i manipulates at P_i'. Thus $R^g(P_i)$ depends only on $\bar{a}_i$. To sum up, we have shown that $R^f(\bar{a}) = R^g(P_i)$, and we have identified voter j as the voter specified in the statement of the Theorem. □

The proof of Theorem 2 has a special feature. The usual induction proofs of such propositions, such as the proof of the Gibbard–Satterthwaite Theorem in Sen (2001), employ the technique of coalescing or cloning voters in the induction step. This is done in order to define a SCF on a society of lower cardinality with the appropriate properties (strategy-proofness and unanimity). This makes the induction step relatively straightforward but entails the additional cost of having to establish the Theorem in the non-trivial case of $N = 2$. In the current setting, the cloning technique does not work because the peaks of all voters in the function g may be different. In order to define a SCF in a society of $N - 1$ voters we use a projection technique. Most of the effort in proving the result goes into showing that there exists a SCF induced on a $N - 1$ voter society which satisfies the range requirement.

Some of the methods here are similar to the arguments developed in Barberà and Peleg (1990). In fact the object $R^g(P_i)$ can be interpreted as the option set offered by voter i to the voters $I \backslash \{i\}$. However, an attractive aspect of our approach is that the induction can begin at $N = 1$ which is a trivial case.

Another noteworthy aspect of our result is that it does not invoke the axiom of *unanimity*. According to the axiom, the outcome at a profile P is a whenever $r_1(P_i) = a$

for all $i \in I$. Unanimity is an assumption frequently made in the context of characterization results for strategy-proof SCFs. It is a convenient way of ensuring that SCFs under consideration have full range. It can be incorporated in our framework without difficulty. Consider the special case where the profile P and the alternative a are such that $r_i(P_i) = a$ for all $i \in I$. Unanimity would imply that for any f satisfying CSP, we would have $f(P) = a$ for all $P \in \mathbb{P}^a$, i.e. $R^f(a) = \{a\}$. On the other hand, if P is such that all voters do not have a common first-ranked alternative, unanimity does not impose any restrictions on f.

4 The partially informed planner

In this section, we consider the case where the planner has some *ex-ante* information about the peaks of individual preferences. We refer to this situation as one where the planner is partially informed. This is the standard case of restricted domains. Our objective is to contrast both the formulation and the results with those in the partially observant planner model.

For all $i \in I$, let $\bar{A}_i \subset A$ denote the set of admissible peaks for voter i. In other words, the planner knows ex-ante that i's peak must belong to the set $\bar{A}_i$. The N-tuple $(\bar{A}_1, \ldots \bar{A}_N) \in [2^N \backslash \emptyset]^N$ will be denoted by $\bar{A}$, i.e. the ith component of $\bar{A}$ is $\bar{A}_i$. We let $\mathbb{P}^{\bar{A}_i} \subset \mathbb{P}$ denote the set of preferences whose peaks are restricted to belong to $\bar{A}_i$. Also $\mathbb{P}^{\bar{A}}$ will denote the set of profiles whose i component is a preference in $\mathbb{P}^{\bar{A}_i}$. Fix an N-tuple $\bar{A}$. A SCF is a map $f : \mathbb{P}^{\bar{A}} \to A$. The notion of strategy-proofness in this model is standard and is the one defined in Sect. 2.

A general analysis of the structure of strategy-proof SCFs $f : \mathbb{D}(\bar{A}) \to A$ is difficult because it may depend on the sets $\bar{A}_i$. We are however, able to prove that it is dictatorial in some special cases.

Proposition 1 *Fix an N-tuple $\bar{A} \in [2^A \setminus \emptyset]^N$. Let $f : \mathbb{D}(\bar{A}) \to A$ be a strategy-proof SCF with $|R^f| \geq 3$. Then f is dictatorial if either of the two cases below occur.*

(i) $\bar{A}_i$ is a singleton set for each $i \in I$.
(ii) $R^f \cap (\cup_{i \in I} \bar{A}_i) = \emptyset$.

Proof Case (i) follows immediately from an application of Theorem 2. For case (ii), we use the well-known fact that the value of a SCF at any profile can only depend on voter preferences over the range of f. Since $R^f \cap (\cup_{i \in I} \bar{A}_i) = \emptyset$, preferences over R^f are in effect, unrestricted and the Gibbard–Satterthwaite Theorem applies. □

The analysis in the case where $\bar{A}_i \cap R^f \neq \emptyset$ for some voters i, is far more subtle. We prove a result in the special case of two voters.

Proposition 2 *Let $N = \{i, j\}$. Fix an N-tuple $\bar{A} \in [2^N \backslash \emptyset]^2$. Let $f : \mathbb{D}(\bar{A}) \to A$ be a strategy-proof SCF with $|R^f| \geq 3$. Suppose $|(R^f \cap \bar{A}_i) \cap (R^f \cap \bar{A}_j)| \geq 3$. Then f is dictatorial.*

Proof Let f be a strategy-proof SCF with $|R^f| \geq 3$ and $|(R^f \cap \bar{A}_i) \cap (R^f \cap \bar{A}_j)| \geq 3$. We say that i dictates over alternative x if, for all profiles (P_i, P_j) such that

$\max(P_i, R^f) = x$, we have $f(P_i, P_j) = x$. In order to show that f is dictatorial we will show that there exists a voter i who dictates over all alternatives $x \in R^f$. We will proceed in three steps. In the first, we will identify a voter i who dictates over all $x \in (R^f \cap \bar{A}_i) \cap (R^f \cap \bar{A}_j)$. In the next step, we will show that i dictates over all $x \in (R^f \cap \bar{A}_i)$. Finally, we will show that i dictates over all $x \in R^f \backslash \bar{A}_i$.

The following observations will be used frequently in the proof.

Observation 1 (i) *Suppose* $x \in (R^f \cap \bar{A}_i) \cap (R^f \cap \bar{A}_j)$. *Then* $f(P_i, P_j) = x$ *whenever* $r_1(P_i) = r_1(P_j) = x$.
(ii) *Let* i *be an arbitrary voter. Suppose* $x \in (R^f \cap \bar{A}_i)$ *and let* P_i *be such that* $r_1(P_i) = x$. *Then there exists* P_j *such that* $f(P_i, P_j) = x$.

The observations follow from the assumption that $x \in R^f$ and the familiar "lifting" lemma for strategy-proof SCF. We omit the details of the proof.

We closely follow arguments in Sen (2001) in order to establish the first step in the argument. Pick $x, y \in (R^f \cap \bar{A}_i) \cap (R^f \cap \bar{A}_j)$. Let (P_i, P_j) be a profile such that $r_1(P_i) = x$ and $r_1(P_j) = y$. We claim that $f(P_i, P_j) = \{x, y\}$. Suppose this is false and assume $f(P_i, P_j) = a$ where $a \neq x, y$. Let $\bar{P}_i$ and $\bar{P}_j$ be such that $r_1(\bar{P}_i) = x$, $r_2(\bar{P}_i) = y$, $r_1(\bar{P}_j) = y$ and $r_2(\bar{P}_j) = x$. Consider the profile $(\bar{P}_i, P_j)$. The outcome here cannot be x; otherwise i would manipulate at (P_i, P_j) by announcing $\bar{P}_i$. It cannot also be an alternative worse than y at P_i since i can obtain y by announcing y as her first-ranked alternative (Observation 1 part (i)). Therefore $f(\bar{P}_i, P_j) = y$. Furthermore $f(\bar{P}_i, \bar{P}_j) = y$; otherwise j would manipulate at $(\bar{P}_i, \bar{P}_j)$ by announcing P_j. On the other hand, a symmetric argument leads to the conclusion that $f(P_i, \bar{P}_j) = f(\bar{P}_i, \bar{P}_j) = x$. In order to avoid a contradiction, we must have $f(P_i, P_j) \in \{x, y\}$. Assume without loss of generality that $f(P_i, P_j) = x$. The rest of the proof consists in showing that i is a dictator in f.

Starting from the profile (P_i, P_j) consider all preferences for i where x is ranked first. Strategy-proofness implies that the outcomes at all these profiles is x. Now change the preferences of voter j keeping y as the first-ranked alternative. By the argument in the previous paragraph, the outcome at all these profiles must either be x or y. But if it is ever y, then j would manipulate. We conclude that at a profile where i ranks x first and j ranks y first and x last, the outcome must be x. But the lifting argument implies that the outcome must be x if i ranks x as the top ranked alternative, irrespective of j's preferences. Consider a profile (P_i, P_j) where $x = \max(P_i, R^f)$. If $f(P_i, P_j) \neq x$, i will manipulate by announcing an ordering where x is first-ranked thereby obtaining x. We have shown that i dictates for alternative x.

Pick an arbitrary alternative z distinct from x and y such that $z \in (R^f \cap \bar{A}_i) \cap (R^f \cap \bar{A}_j)$. Such an alternative exists in view of our assumption on the cardinality of the set $(R^f \cap \bar{A}_i) \cap (R^f \cap \bar{A}_j)$. Let (P_i, P_j) be a profile such that $r_1(P_i) = x$, $r_2(P_i) = z$ and $r_1(P_j) = y$. We know that $f(P_i, P_j) = x$. Let $\bar{P}_i$ be an ordering where $r_1(\bar{P}_i) = z$ and $r_2(\bar{P}_i) = x$. Strategy-proofness implies $f(\bar{P}_i, P_j) \in \{x, z\}$. However, we also have $f(\bar{P}_i, P_j) \in \{z, y\}$. Clearly $f(\bar{P}_i, P_j) = z$. Replicating our earlier arguments, we conclude that i dictates over z.

Once again, let z be an alternative distinct from x and y such that $z \in (R^f \cap \bar{A}_i) \cap (R^f \cap \bar{A}_j)$. Let (P_i, P_j) be a profile such that $r_1(P_i) = z$, $r_2(P_i) = y$, $r_1(P_j) = x$ and y is ranked last in P_j. Let $\bar{P}_i$ be an ordering where $r_1(\bar{P}_i) = y$ and $r_2(\bar{P}_i) = z$.

Our earlier arguments imply $f(\bar{P}_i, P_j) = y$. Since y is ranked last in P_j, it follows that i dictates over y. We have therefore shown that i dictates over all alternatives in $(R^f \cap \bar{A}_i) \cap (R^f \cap \bar{A}_j)$ completing the first step of the proof.

Pick an arbitrary alternative $a \in (R^f \cap \bar{A}_i)\backslash \bar{A}_j$. Let $x, y \in (R^f \cap \bar{A}_i) \cap (R^f \cap \bar{A}_j)$. Let (P_i, P_j) be a profile such that $r_1(P_i) = a$, $r_2(P_i) = x$, $r_1(P_j) = y$ and a and x are ranked second-last and last respectively in P_j. Since i dictates over x, i can always obtain x by announcing a preference where x is first-ranked, it follows that $f(P_i, P_j) \in \{a, x\}$. Suppose $f(P_i, P_j) = x$. Since $a \in R^f$ and $a = r_1(P_i)$, Observation 1, part (ii) implies the existence of a preference P'_j such that $f(P_i, P'_j) = a$. Since aP_jx, j will manipulate at (P_i, P_j) via P'_i. Hence $f(P_i, P_j) = a$. Let $\bar{P}_i$ be a preference where a and x are ranked first and last respectively. Strategy-proofness implies $f(\bar{P}_i, P_j) = a$. Let $\bar{P}_j$ be a preference where $r_1(\bar{P}_j) = y$ and x and a are ranked second-last and last respectively. Strategy-proofness implies $f(\bar{P}_i, \bar{P}_j) \in \{a, x\}$. Suppose $f(\bar{P}_i, \bar{P}_j) = x$. Since $y\bar{P}_ix$ and i dictates over y, i will manipulate at $(\bar{P}_i, \bar{P}_j)$ by announcing a preference where y is first-ranked thereby obtaining y. Hence $f(\bar{P}_i, \bar{P}_j) = a$. Since a is ranked first and last at $\bar{P}_i$ and $\bar{P}_j$ respectively, it follows that i dictates over a. This concludes the second step.

Pick an arbitrary alternative $a \in R^f \backslash \bar{A}_i$. By assumption, there exists a profile, say (P_i^*, P_j^*) such that $f(P_i^*, P_j^*) = a$. Let $r_1(P_i^*) = b$. Note that $b \notin R^f$. If $b \in R^f$, the second step applies leading to the conclusion that $f(P_i^*, P_j^*) = b$. Let $x \in (R^f \cap \bar{A}_i) \cap (R^f \cap \bar{A}_j)$. Let P_i be a preferences such that $r_1(P_i) = b$, $r_2(P_i) = a$, $r_3(P_i) = x$. Let P_j be a preference where a is ranked second-last and x last respectively. Let $f(P_i, P_j) = c$. Clearly $c \neq b$ since $b \notin R^f$. If c is ranked below x in P_i, then i will manipulate in (P_i, P_j) by announcing x as her first-ranked alternative. Here we are using the fact that i dictates over x. Therefore $c \in \{a, x\}$. Suppose $c = x$. Let us return to the profile (P_i^*, P_j^*) where the outcome is a. Since $r_1(P_i^*) = r_1(P_i) = b$, the lifting argument implies $f(P_i, P_j^*) = a$. Since aP_jx and $f(P_i, P_j) = x$, j will manipulate at (P_i, P_j) by announcing P_j^*. Hence $c = x$ is impossible, i.e. $f(P_i, P_j) = a$.

Let $\bar{P}_i$ be a preference where $r_1(\bar{P}_i) = b$, $r_2(\bar{P}_i) = a$ and x is ranked last. Strategy-proofness implies $f(\bar{P}_i, P_j) = a$. Let $\bar{P}_j$ be a preference which is obtained by switching the ranks of a and x in P_j, i.e. x is ranked second-last and a last in $\bar{P}_j$. By strategy-proofness, $f(\bar{P}_i, \bar{P}_j) \in \{a, x\}$. Suppose $f(\bar{P}_i, \bar{P}_j) = x$. Consider $y \neq x$ and $y \in (R^f \cap \bar{A}_i) \cap (R^f \cap \bar{A}_j)$. By construction yP_ix. Moreover i dictates over y. Consequently i will manipulate at $(\bar{P}_i, \bar{P}_j)$. Therefore $f(\bar{P}_i, \bar{P}_j) = a$.

We can summarize our conclusions for the third step of the proof as follows: there exists $b \notin R^f$ such that for all profiles (P_i, P_j) where $r_1(P_i) = b$ and $a = \max(P_i, R^f)$, we have $f(P_i, P_j) = a$. Suppose there exists P'_i where $r_1(P'_i) = c \neq b$, $a = \max(P_i, R^f)$ and $f(P'_i, P_j) = d \neq a$. Clearly aP'_id holds. Then i will manipulate at (P'_i, P_j) by announcing P_i where $r_1(P_i) = b$ and $r_2(P_i) = a$. Therefore i dictates over a and the third step is established. This also completes the proof. □

We note that existing results on dictatorial domains do not readily apply to our model because of the inherent asymmetry in voter preferences that we allow for. Consider the notion of a linked introduced in Aswal et al. (2003). A strategy-proof SCF defined on a linked domain satisfying the additional property of unanimity is shown to be dictatorial. An assumption that is central in this approach is that if the domain contains a preference where alternatives a and b are ranked first and second respectively, the domain also includes a preference where b is first and a second. This assumption is clearly violated in our model.

There are non-trivial difficulties associated with extending Proposition 2 to the case of an arbitrary number of voters.[2] In Aswal et al. (2003), it is assumed that all voters have the same preference domain. Moreover, this common domain has the property of minimal richness - for every alternative a, there is a preference in the domain where a is first-ranked. Any domain satisfying these properties satisfies a fundamental reduction property: if every two-voter strategy-proof SCF satisfying unanimity defined on the domain, is strategy-proof, then every n, ($n \geq 3$) voter strategy-proof SCF satisfying unanimity defined on the domain, is strategy-proof. Clearly we cannot rely on such a result because minimal richness is too strong a property in our model. We could attempt to formulate n-voter version of the condition in Proposition 1. We choose not to because it is likely to be cumbersome; in any case we are unable to show whether conditions of this sort are necessary.

The dictatorship result of Proposition 1 does not hold if the planner has more information than just the top-ranked alternatives of the voters. Consider the following example.

Example 2 Let $I = \{1, 2\}$ and $A = \{a, b, c, d\}$. Suppose the planner has the following ex ante information regarding the preferences of voter 1. He knows that if voter 1 ranks a first, then voter 1 ranks b second. There are no other restrictions regarding the ranking of alternatives. In other words, among the 12 possible pairs of first and second alternatives, exactly two, viz. a is first and c second and a is first and d second are infeasible. There are no other restrictions on the preferences of other voters.

We claim that there exists a non-dictatorial SCF in this setting which is defined as follows. The outcome at any profile is voter 1's top ranked alternative if this alternative is b, c or d. If it is a then the outcome is the alternative in the set $\{a, b\}$ which is higher ranked in voter 2's preferences. It is easy to check that this SCF is strategy-proof. Voter 1 does not get his peak only in the case where his peak is a. In this case he may get his second ranked alternative b. However, since 2 prefers b to a, there is no way for 1 to do better and get a. □

Example 2 appeared originally in Aswal et al. (2003) where it was employed for a different purpose. We note that there are other ways to extend the spirit of the domain restrictions from the tops case analysed in this section to more general cases. For instance, the planner may be aware that an alternative is never ranked among the top k alternatives and so on. The restricted domain settings that arise from such information structures could be interesting to explore.

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[2] There are also difficulties in verifying whether Conditions such as (i) and (ii) are necessary for dictatorship in the two-voter case. At present, we are unable to answer this question.

5 Conclusion

We have introduced a formulation of mechanism design called the observant planner model where a planner receives *ex post* information on the realization of voters preferences. Specifically, we study the case where the planner observes the peak of each voters preferences. We provide a characterization of all social choice functions that satisfy an appropriate version of strategy-proofness, under a requirement on the range of the social choice function. We contrast this model with the partially informed planner model, which is one where the planner possesses *ex ante* information on the peaks of voters preferences. This latter model differs from the standard restricted domain formulation in that the planner's information may vary across voters. Standard techniques for proving characterization results do not apply to either model. It would be of interest to explore the consequences of the planner obtaining other forms of information on the preferences rather than just information of the peaks.

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