8-2022

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Citation
SAMANIEGO, Roberto M. and SUN, Juliana Yu. The embodiment controversy: On the policy implications of vintage capital models. (2022). *Journal of Money, Credit and Banking*. 54, (5), 1175-1222.
Available at: https://ink.library.smu.edu.sg/soe_research/2560

Published in Journal of Money, Credit and Banking, 2022. DOI: 10.1111/jmcb.12912

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The Embodiment Controversy: On the Policy Implications of Vintage Capital Models

We explore the long-run impact of policy on the level of economic activity through changes in the vintage distribution of capital, in a model where different vintages coexist in production. Because firms can choose the vintage of capital in which they invest, investment subsidies do not affect the vintage structure of capital. In contrast, vintage-specific taxes or subsidies that target the newest vintages of capital can significantly affect output and welfare in the long run, mainly downward. Transition dynamics are rapid, so that steady-state comparisons give an accurate picture of the welfare impact of vintage tax wedges.

JEL codes: O11, O13, O16, O41, O47
Keywords: capital taxation, embodiment controversy, investment subsidies, transition dynamics,vintage capital

An extensive literature investigates whether productivity improvements are embodied in capital, in a debate known as the "embodiment controversy." On the one hand, recent developments in theory and in data have increased the popularity of vintage capital models due to their ability to account for U.S. growth and their implications for industry dynamics.[1]

At the same time, the *policy implications* of the hallmark of these models—that new vintages of capital are more productive than old vintages—have not been widely explored. This neglect is central to the evaluation of vintage capital models: Denison (1964) argues in an oft-quoted comment that the embodiment controversy is unimportant *precisely because it is not policy-relevant*. The argument is that policy would have to induce permanent and unrealistically large changes in investment rates to

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1. See, for example, Greenwood, Hercowitz, and Krusell (1997), Campbell (1998), Cummins and Violante (2002), Boucekkine, Del Río, and Licandro (2003), and Samaniego (2010).

significantly skew the productivity profile toward newer, more productive vintages of capital.

A key assumption underlying this argument is that all investment must take place in capital of the latest vintage. This assumption is a feature of most vintage capital models, see the survey in Jovanovic and Yatsenko (2012). However, allowing investment in vintages of capital other than the latest is essential for matching the gradual diffusion patterns widely observed in empirical studies on innovations—see Griliches (1957), Gort and Klepper (1982), Comín and Hobijn (2009), and Comín and Hobijn (2010), among others. If households may invest not only in the latest vintage of capital but also in capital of *earlier* vintages—either through the production of new capital goods of an older vintage, or through purchases or imports of used goods—then aggregate investment rates and the productivity distribution of capital become uncoupled. As a result, if households choose which vintage of capital to invest in, policy may affect aggregates through the productivity distribution of capital *even if investment rates are held constant*—as long as policy affects vintages of capital differently.

The interest in studying vintage-specific policies is empirical, as well as theoretical. As we show in Section 1, there exist policies in various countries that implicitly or explicitly differentiate by vintage. For example, government agencies in the People's Republic of China compile a list of new technologies to receive government support, with the list undergoing regular revisions. Subsidies to low-emission vehicles and machinery such as those studied in Adda and Cooper (2000) implicitly discriminate by vintage. On the other hand, Samaniego (2006a) surveys the literature on industrial support in Europe, finding that obsolescent firms and industries tend to obtain support. Thus, there exist policies that target newer vintages of capital, and some that target older vintages instead.

This paper explores the policy implications of capital-embodied technical progress in a model where investment is allowed in *any* current or past vintage of capital. Specifically, we study the impact of *vintage-specific* taxes and subsidies, which may distort the household's decision regarding the choice of vintage. The model is a version of the model of Jovanovic and Yatsenko (2012, henceforth JY12), extended to allow for such transfers. We select this model because it is a suitable workhorse for studying diffusion patterns: it allows different vintages of capital to coexist in the production function through imperfect substitution, and it displays the well-known feature of gradual diffusion curves for new capital goods, as identified in the empirical literature.

The key ingredient of the model, as in Chari and Hopenhayn (1991), is that there is a distinction between the date at which a particular capital good is produced and the *vintage of the technology embodied within*. For illustration, consider the example of operating system software, and assume for simplicity that all computers use Windows-operating systems. Windows 7 was introduced in 2009, but Microsoft continued to supply Windows *XP*, and firms could acquire newly produced copies of the older operating system for several years.[2] Moreover, firms that did purchase Windows 7 might do so without necessarily replacing their computer hardware of an older vintage, nor their furniture nor structures of an older vintage.[3] The reason why the ability to invest in capital of an older vintage is important for the policy implications of vintage capital models is that, as Denison (1964) observes, if all investment is only in the newest vintage, then the only channel through which policy can impact aggregates is through changes in net investment rates. In this example, this would amount to forcing firms that buy a new operating system in 2009 to buy Windows 7. In contrast, if investment does not necessarily have to be in the newest vintage, then expenditure on operating systems is no longer tied to expenditure on the *newest* operating systems. As a result, even without changes in investment rates, there could be a significant impact of policy on output and welfare in a vintage capital world if policy can skew the *vintage composition of investment*. [4] Of course, investment rates may also be affected by policy, so a contribution of the paper will be a *quantitative* assessment of the impact of policy on aggregates in general equilibrium, as well as an assessment that abstracts from changes in investment rates.

The policies we examine are taxes and subsidies that depend on the vintage of capital—or, more concisely, vintage-specific *tax wedges*. We show analytically that the vintage distribution is insensitive to transfers that are *not* vintage-specific, so that blanket investment subsidies have no impact on aggregates through the vintage distribution. Then, we analyze the impact of policies that differentially subsidize (or tax) the newest year of capital vintages—see Section 1 for some examples. We analyze the long-run impact of such policies on allocations, via their impact on the balanced growth path. To focus on the impact of changes in the vintage distribution on aggregates, we analyze two types of intervintage transfer schemes: (i) where there are no net subsidies to capital, and separately (ii) where transfers to capital are such that there is no impact on aggregate investment.[5] In this way, our paper also contributes to the literature on reallocation, such as Restuccia and Rogerson (2008), who assess the impact of policies that result in interfirm resource reallocation by means of firm-specific transfer schemes. In contrast, our focus is on the vintage distribution. Mulder, Groot, and Hofkesa (2003) use a multivintage model to study analytically the impact

2. The software example is also useful because Windows *XP* was subject to several free updates and improvements gradually over time, a feature captured in the model and interpreted as a component of "learning." It is this "learning" that leads to the gradual adoption of newer vintages. The presumption in a vintage capital model would be then that Windows 7 is more productive than Windows *XP*, conditional on similar updates and learning, but that Windows *XP* is more productive when Windows 7 is first introduced.

3. Stoneman and Kwon (1984) study the joint diffusion of numerically controlled machine tools and coated carbide tools through the UK metalworking and engineering industries, also finding that updating on one does not imply updating in the other.

4. Intuitively, if the technology for producing capital increases everywhere at a rate γ, but in one country, policy induces investment to occur on average in vintages of capital that are s years older than in another, yet the investment rate is the same, then GDP would be $s \times \gamma$ lower in the first country than in the second at all dates. Since, in principle, s is unbounded, factors that affect the average vintage of capital in use could lead countries to differ in terms of income by a significant amount.

5. It turns out that results are similar, because the net transfers required to keep aggregate investment constant are small.

of energy-saving technologies, whereas we provide a quantitative analysis of more general vintage-specific transfers.

We find that subsidies to new vintages financed out of taxing older vintages are detrimental to welfare and to GDP in the long run. In the benchmark calibration, a 20% subsidy to investment in the newest vintage leads to a decline in consumption in each period of 1%, a 50% subsidy lowers consumption by 5%, and a 100% subsidy lowers consumption by a full 18%. Moreover, this impact is not due to any inherent waste in the tax system: these results are for transfer schemes such that there are no net transfers to or from the capital goods sector. We obtain similar results when the transfer scheme is designed to ensure that there is no impact of aggregate investment: thus, these effects are entirely due to distortions in the vintage composition of investment.[6] The conclusion is that policy-induced distortions to the vintage distribution can have significant impact on welfare. Since this impact is potentially large, the paper identifies an as yet unexplored channel whereby policy, financing frictions, or other distortions might lead to differences in macroeconomic outcomes among developed and developing economies—a channel that can only be studied in a model where technical progress is at least partly embodied in capital.

One question is whether the impact of subsidies of this kind is large in steady state, but are smaller in transition. We perform simulations that show the transition to a new steady state after the removal of subsidies is rapid, taking about 2 years. When we compute the welfare impact of removing the policy in transition to the new steady state, we find that it is close to that measured by comparing steady states.

The paper also has a negative point. Subsidies to investment are sometimes justified by policymakers on the basis that they support new technologies. In fact, unless such subsidies are based on the age of the technology—not the date of the investment—then they have no impact on the vintage distribution at all.

Finally, a caveat. The model contains vintage-specific learning, a feature that ensures that adoption of new vintages of capital is gradual, as in the data. Some authors model learning as being based on some sort of externality, for example, learning by doing as in Spence (1981) or Lach and Jovanovic (1989). Our study without externalities serves as a natural benchmark, however. First, learning about new technologies is a *global* process, whereas policy is *local*. As a result, the learning curve is likely external to whatever is happening within a particular economy with or without tax wedges, and should not be significantly affected by the tax policy of any particular country. Learning about a technology is something that occurs through worldwide use, so the global mechanisms behind the learning function are exogenous to the conditions of any particular firm or country, especially if the country is small or not producing a lot of research and development (R&D) specifically in the field of application of that technology, which Eaton and Kortum (2001) argue is the empirically relevant case for much of the world. Thus, taking the learning curve as given is likely the

6. It is well known since at least Diamond and Mirrlees (1971) that production subsidies are inefficient absent other market failures. The point is that, in principle, distortions that affect specifically the vintage distribution can have significant aggregate impact. Blanket subsidies to investment may affect aggregates but, as we show, they have no impact on the vintage distribution.

empirically relevant approach. Second—assuming that there were a globally consistent tax wedge regime, or if some portion of the spillovers were purely local—our approach gives a sense of how large externalities would have to be to significantly affect the welfare results. Third, given that there are multiple models of knowledge externalities, it is not clear what particular model of learning externalities would serve as a reasonable alternative *benchmark*—whereas an environment without externalities is a natural benchmark. Fourth, as it stands the model poses computational challenges: adding learning externalities would increase them further. In any case, the goal of the paper is to establish that the vintage distribution is potentially sensitive to policy, so the exact form of the learning function is secondary. Still, it would be interesting to extend the model to allow for learning externalities in future work.

Section 1 discusses the literature and motivation. Section 2 describes the economic environment and solves for equilibrium. Section 3 calibrates the model and reports the results of quantitative policy experiments. Section 4 discusses transition dynamics in the model environment. Section 5 concludes with suggestions for future work.

1. MOTIVATION AND LITERATURE

Denison (1964) argues that the existence (or not) of capital embodied technical progress is not important for policymakers, because unreasonably large changes in the age structure of capital would be necessary for policy to significantly influence aggregates. Much of the related literature has focused on assessing whether productivity improvements in capital are an important factor of growth (e.g., Hulten 1992, or Greenwood, Hercowitz, and Krusell 1997), without addressing this key criticism: that an important factor in evaluating the usefulness of vintage capital models is the assessment of the *policy relevance of changes in the vintage distribution of capital*.

Assessing the impact of policy on the vintage distribution requires a model that accounts for basic properties of the vintage distribution of capital. First, different vintages must coexist in production. Second, the model should reproduce basic features of the vintage distribution—in particular, the slow diffusion of new capital goods. Third, as a result, the model should allow investment to occur not only in the latest vintage of capital, but also in older vintages too.

This feature requires a distinction between the age of a capital good and its *technological* vintage. For example, while technological progress implies that the most powerful computer available improves over time, computers of lesser power continue to be produced, using other than the latest processors. A consequence is that, even if investment rates do not change over time or are unresponsive to the policy environment, the technological vintage of the capital created via investment could be responsive to policy. The productivity distribution of capital could change significantly in terms of *technological* vintage, even in the absence of changes in investment rates.

Most vintage capital models are inadequate for performing this assessment. The reason is that most models either assume that all investment occurs in the newest vintage of capital, or they assume that there is a choice of vintage but the optimal choice is always the newest.[7] Instead, this paper adopts the framework recently introduced in JY12. In this framework, investment may, in principle, occur in *any* current or past vintage of capital. The reason households find it optimal to do so is that different vintages are imperfect substitutes in production. The gradual diffusion of new capital goods is achieved via the introduction of vintage-specific learning, which accumulates gradually over time. The model is simple and easily mapped into the data typically used in calibrating models in the related literature—a feature that will be important for generating quantitative results later on.

To underline the empirical relevance of our theoretical and quantitative work, we begin by asking (i) whether there is evidence of investment in old vintages of capital, (ii) whether there are any real-life policies that are more directed at capital goods of a certain vintage than others, and (iii) whether there is evidence that vintages of capital might vary across countries, which would provide support that vintage distributions can vary across time and space.

Regarding the first question, evidence of investment in technologies not of the most recent vintage abounds. Innovation is an ongoing process, whereas a given model of machinery may be in production for several years. In addition, the fact that machinery, in general, lasts for many years and may require periodic replacement of parts also means that parts compatible with any given vintage of machinery need to be produced for as long as that vintage of machinery remains in operation—and, to the extent that plants require wholesale restructuring when machinery is upgraded, vintage-specific parts may need to be produced for as long as the useful life of other machinery and structures that commonly share a plant.

An example is software: for instance, the operating system Windows 7 was introduced in 2009, yet Microsoft continued to sell copies of the older Windows XP into 2010[8] and Windows Vista into 2011.[9] Another example is computer hardware: for instance, the Sony Playstation 4 gaming console was introduced in 2003, yet production of its predecessor the Playstation 3 did not cease until 2007.[10] The autoindustry provides further examples. The Toyota Corolla—the best selling car nameplate in history—is redesigned every few years, but both a new line of Corollas and its predecessor are often produced and sold simultaneously. For instance, the E70 was produced over the period 1979–87, even though its successor the E80 was introduced in 1983. In turn, the E80 was produced until 1990, even though its

7. See the survey in JY12.

8. See https://web.archive.org/web/20080408004318/http://www.computerworld.com/action/article.do?command=viewArticleBasic&articleId=9074720, last checked 9/18/2020.

9. See https://news.softpedia.com/news/Slow-Death-for-Windows-Vista-Packaged-Software-End-of-Sales-Reached-in-October-170467.shtml, last checked 9/18/2020.

10. See https://www.cnet.com/news/at-long-last-end-of-the-line-for-the-sony-playstation-3/, last checked 9/19/2020.

successor (the E90) was introduced in 1987.[11] The Peugeot 405 was introduced in 1987 yet continues to be produced into 2020 (with minor modifications).[12]

Another example is the spread of industrial robots. We consider robots to be a cutting-edge technology, relative to similar nonautomated machinery that requires a human operator. According to the World Robotics database provided by the International Federation of Robotics, robot deliveries in the United States increased on average by 7.7% per year over the period 1993–2016, whereas the Bureau of Economic Analysis reports that U.S. investment in machinery and equipment overall rose by 3.6% on average over the same period. Nonetheless, spending on robots remains a small fraction of total investment in machinery: the U.S. census reports annual new machinery orders of around $300B per year, whereas global spending on robots is less than half of that.[13] It is straightforward to verify that manufacturers of heavy machinery continue to produce both automated and nonautomated versions of their products—for example, Summit Machine Tool continues to produce both manual and automated lathes.[14] Thus, there is increased diffusion of robots throughout the economy, but this diffusion is gradual and, at the same time, investment in nonautomated machinery continues.

Turning to the scientific literature, it is widely known that new inventions diffuse slowly, implying that investment must continue in older vintages of capital for some time—see Griliches (1957), Gort and Klepper (1982), Comín and Hobijn (2009), and Comín and Hobijn (2010). In particular, Comín and Hobijn (2009) develop a list of inventions that have close but less productive "predecessor technologies." For instance, they view newspapers as being a predecessor of radios, and radios as being a predecessor of television. In their data, it is very common that investment in predecessor technologies continues after the introduction of the newer technology, or that production using a predecessor technology peaks after the introduction of a new one. For instance, the open hearth (OH) process preceded blast oxygen (BO) steel manufacturing. Peak output with OH steel is in 1964, 10 years after the introduction of its superior competitor. Figure 1 displays the case of telephones, which precedes cellphones and is preceded by telegraphs. Observe that investment in telegraphs peaks long after the introduction of the telephone, and investment in telephones does not decline as soon as cellphones are introduced in the mid-1980s. This is direct evidence that investment in vintages of capital other than the latest occurs.

Various industry studies also find evidence consistent with investment continuing in old vintages of capital. For example, Gort and Boddy (1967) find that plant output is affected by a nontrivial interaction among vintages of capital used by in the U.S. electric utilities industry, indicating that they are not perfect substitutes. Colombo

11. See http://https://www.toyota-global.com/ and https://en.wikipedia.org/wiki/Toyota_Corolla, last checked 9/19/2020.

12. See https://www.carthrottle.com/post/peugeot-is-selling-brand-new-405s-for-7800/, last checked 9/19/2020.

13. See https://www.idc.com/getdoc.jsp?containerId=prUS45800320, last checked 9/19/2020.

14. See https://www.summitmt.com/product-category/manual-lathes/, last checked 9/19/2020.

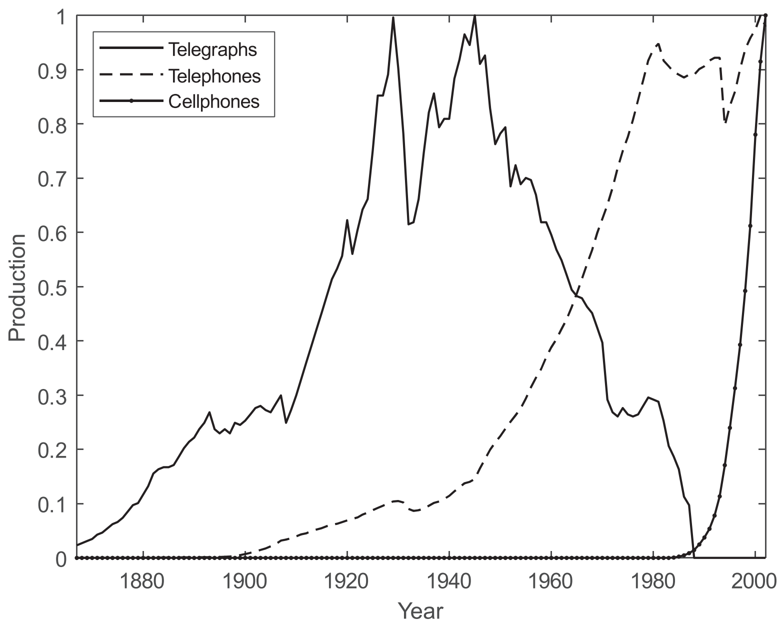

Fig 1. Telegraphs, Telephones, and Cellphones Purchased in the United States, 1867–2002.

NOTES: Values are normalized by peak output over the period for comparability.

and Mosconi (1995) perform a detailed study of the adoption of flexible automation in the Italian metalworking industry, finding that adoption patterns vary across firms depending on the makeup of their existing equipment. Moreover, firms that did not adopt that leading edge technology did nonetheless continue investing in other forms of machinery—again indicative of investment in a nonfrontier technology.

Regarding the second question, it turns out that it is not uncommon for the tax/subsidy system to target capital goods based on vintage.[15] However, this is not done in a systematic way based on a measure of vintage: rather it is often done by subsidizing specific types of capital that did not exist or that were very underdeveloped until recently. Whether or not it is the objective of the policy, there are policies that may favor capital of a particular age or vintage. In addition, other policies exist that support specific types of capital that are on the path to obsolescence—see the survey in Samaniego (2006a).

Aghion et al. (2015) observe that industrial policy in developing economies tends to be designed so as to encourage recent technologies. For example, the government of the People's Republic of China has an explicit program for doing so. Government agencies compile a list of "Major technical equipment and product catalogues

15. In general, arguments in support of industrial subsidies tend to focus on the promotion of recent technologies, see, for example, the survey of Pack and Saggi (2006).

supported by the State" almost every year, with revisions of the items in the list from year to year.[16] Equipment types on the list are entitled to import tariff subsidies and to tax relief, according to the general administration of Customs, general administration of taxation and other related ministries. Changes in the items in the catalogue are explicitly based on the stages of technological development of the industry in China relative to the rest of the world: in other words, it subsidizes capital goods of recent vintage that the country does not yet produce. Industries in which China is still at early stage or developing path are newly added to the catalogues, while industries in which China has already well developed capacity are removed. The list is quite specific.[17] For example, if we compare the lists in 2017 and 2010, garbage burning generators, polyethylene cycle compressors, reciprocating coal water slurry diaphragm pumps, and coal liquefaction hydrogenation reactors (among others) are removed, while gas and oil drill equipment, methanol to olefin equipment, and absorbent systems are newly added. We can infer from the changes in the catalog that the Chinese government treats capital good related to new technologies differently from older technologies, because it is more likely that China is at the early stage of development regarding new technologies. Note that these are not R&D subsidies: these are subsidies to the use of specific, relatively new types of capital goods.

Similarly, the government of Singapore encourages the adoption of new capital goods in specific areas, such as data analytics and robotics. For example, the Productivity and Innovation Credit (PIC) program provides 400% tax deductions/allowances up to S$400,000[18] of spending per year for qualifying expenditures on IT and automation equipment by small and medium enterprises. The list of qualified equipment types includes image or graphics processing equipment, data processing and information technology equipment, and data communications and networking equipment, among others.[19] The government of Hong Kong recently launched the Re-industrialisation Funding Scheme (RFS) that subsidizes firms' adoption of "smart" production lines (i.e., production lines that make use of artificial intelligence, automation, or robotics) by up to a third of the cost. The government of India has a variety of programs for subsidizing the adoption of advanced machinery in agriculture at rates ranging from 80% to 100%, through the National Bank for Agricultural and Rural Development or the Department of Agriculture, Cooperation and Farmers Welfare.[20]

16. The list is jointed determined by Ministry of Finance, the Development and Reform Commission, the Ministry of Industry and Information Technology, the General Administration of Customs, the General Administration of Taxation, and the Energy Bureau.

17. See the following government websites, last checked August 15, 2018. http://www.miit.gov.cn/n1146285/n1146352/n3054355/n3057278/n3057290/c6007926/part/6007930.pdf
http://www.chinatax.gov.cn/n810341/n810765/n812161/n812569/c1085706/part/1085707.pdf
http://www.gov.cn/guowuyuan/2018-01/09/content_5254556.htm
http://gss.mof.gov.cn/zhengwuxinxi/zhengcefabu/201801/t20180105_2793555.html

18. Roughly $300,000 at time of writing.

19. See the following links, last checked August 15, 2018: https://www.techinasia.com/singapore-give-local-companies-money-adopt-tech https://www.iras.gov.sg/irashome/uploadedFiles/IRASHome/Quick_Links/PIC%20Automation%20Equipment%20List%20(as%20at%20270911).pdf

20. See https://www.grainmart.in/news/subsidies-for-farmers-in-india-for-selected-machinery/,last checked 9/22/2020.

A different way in which new and old vintage capital is treated differently in taxation involves the treatment of imported capital. Eaton and Kortum (2001) argue that most countries, in fact, import much of their capital stock from a few advanced economies. In those cases, there exists a way for developing economies to differentially tax new and old capital: by treating new and used capital differently when it is imported. For example, if it takes a year for capital to significantly enter the used capital market, then differential tariffs on imports of new or old capital are equivalent to taxes that affect capital differently depending on its vintage. In practice, used machinery tends to experience higher trade barriers than new machinery, including outright prohibition, see the surveys by Soloaga, Navaretti, and Takacs (1999) and the United States Department of Commerce (2015). This suggests that tariff systems around the world implicitly favor newer capital over older vintages.

In addition, many developing economies have various policies such as tax holidays to subsidize foreign direct investment—see the survey of Blomström, Kokko, and Mucchielli (2003). Branstetter, Fisman, and Foley (2006) find that firms who perform foreign direct investment tend to transfer newer technologies only in the event that the receiving country has a strong intellectual property rights enforcement regime, so as to maintain control over their intellectual property of more recent vintage. In the absence of IPR enforcement, any subsidy to FDI is then effectively a subsidy to investment in technologies of older vintage.

Further examples among developed economies are directed toward controlling emissions from the use of fossil fuels, implicitly discriminating among vintages of transportation technology or power generation technology. Adda and Cooper (2000) study a tax reform in France that was designed to encourage households in two provinces to replace their old vehicles with new ones. They find that the long-run impact of the policy is, in fact, to depress output in the vehicle sector. Licandro and Sampayo (2006) find that a similar scheme in Spain had little long run impact. One possibility is that households simply replaced their old vehicles with new ones of similar vintage.

As another example, in the European Union, countries impose different tax incentives to foster electronic vehicles (EVs) compared to conventional vehicles. EVs are generally exempt from fuel consumption/pollution tax, ownership tax, and company car tax. Detailed policies vary across countries. [21] For instance, in Belgium, EVs pay the lowest rate of tax under the annual circulation tax. Ireland taxes vehicles based on their emissions of nitrogen oxides. Norway provides a more telling example that these incentives are directed toward EVs because they are new, and only while they are new: while EVs have been exempt from all vehicle taxes in Norway, these incentives began to be phased out in 2018, whereas there were plans to extend such tax incentives to the adoption of electric *aircraft*. By evolving over time, these policies attempt to target the newest vintages of transportation capital by focusing on emissions.

21. The European Commission-funded European Alternative Fuels Observatory tracks these policies: see http://www.eafo.eu, last checked 8/23/2018.

The examples so far involve support for relatively new technologies. However, there are also support programs for old technologies, often justified in the name of job protection. Burton (1983), Ford and Suyker (1989), Samaniego (2006a), and Dang and Samaniego (2020) survey the industrial support programs of many developed economies, finding that industrial support in developed economies historically tends to be directed toward firms in industries that face technological obsolescence—mainly in Europe. Leonard and Audenrode (1993) provide a detailed study of industrial support in Belgium, arguing that industrial support toward firms in obsolescent industries could potentially have important macroeconomic consequences.

In addition, there exist other policies that work as implicit wedges between technologies of different vintages that indirectly favor older technologies. For example, Samaniego (2010) and Bergoeing et al. (2016) find that barriers to the entry of startups can lead to low investment in new technologies as these often require setting up new establishments, thus indirectly transferring resources from newer to older vintages of capital. Samaniego (2006b) finds that the same may be true of employment protection programs that impose firing costs. Parente and Prescott (2000) and Saint-Paul (2002) show that interests associated with older vintages of technology have an incentive to push for support through the political system. Thus, we conclude that there exist various policies that implicitly discriminate by vintage—some toward the new, some toward the old.

The United States stands out among developed economies as not having a history of any consistent industrial policy: see Krugman (1983) and Ford and Suyker (1989). For example, in the United States, investment tax credits are sometimes formulated to promote particular new technologies or relatively recent vintage,[22] for example, various "cash for clunkers" programs introduced periodically such as several state-level programs in the 1990s[23] and the more recent 2009 Car Allowance Rebate System, the Production Tax Credit adopted in 1992, and the more recent Solar Investment Tax Credit, which are directed at the promotion of new wind and solar energy investments, respectively. All of these are recent technologies, whereas older systems for transportation or power generation would not benefit from these policies.[24] However, some of these programs were temporary: the CARS program lasted just a few months. While the Solar Investment Tax Credit is ongoing, older fossil fuel industries also benefit from support. The Environmental and Energy Study Institute estimates that the coal industry receives US$4 billion in subsidies from the U.S. government every year, which indirectly supports coal-using utilities.[25] Moreover, among utilities,

22. In general, the formulation of the Investment Credit (IRS Form 3468) is targeted toward investments of recent vintage.

23. See Hahn (1995).

24. This is true even though the purpose of these policies may have to do with reducing pollution or other policy objectives: their impact on the vintage distribution might be an unintended consequence. If so, an implication of our paper is that more direct policies that do not target the vintage distribution such as carbon taxes might be preferable.

25. See https://www.eesi.org/papers/view/fact-sheet-fossil-fuel-subsidies-a-closer-look-at-tax-breaks-and-societal-costs, last checked 9/17/2020.

there are plants of differing vintage and efficiency even across plants of the same type, so these policies are only loosely related to the vintage of a technology. Thus, in the case of the United States, it is not clear that there is any obvious bias toward or away from subsidizing capital of any particular vintage, even though specific policies might potentially discriminate by vintage. Later, when we calibrate our model economy, we will use this observation to calibrate the model assuming no distortions as a baseline case.

Finally, regarding the third question, we ask: is there any evidence that there do exist differences in the vintage distribution of capital around the world, even at similar levels of development? This is hard to determine if we take seriously the distinction between the age of capital and the vintage of the technology used to make it. However, the motor vehicle industry stands out as one where this distinction may not be so critical. Motor vehicles are often produced with a vintage attached to them, and in all countries, the existence of an active secondary market for motor vehicles implies that there is a choice between new and old vehicles, including to some extent imported used vehicles.[26] There is, of course, significant heterogeneity among vehicles of similar vintage: for example, the quality differences between a 2017 Toyota Corolla and a 2017 Ferrari F12 are not just related to their vintage, and this heterogeneity could hamper inference about the productivity of capital based solely on measured vintages.[27] However, this should be less the case for vehicles used in *public transport*. This is what we focus on, using data provided by the United Nations Economic Commission for Europe (UNECE). The advantage of using European data is that the existence of open markets implies that the second hand market among the countries in the data is relatively fluid, so there is easy access to vehicles of older vintage in these countries. Figure 2 shows that the age distribution is generally tipped toward older vintages of public transport vehicles in lower income countries, where income is measured using GDP per capita (PPP adjusted) in 2012, as reported by the World Bank. The relationship is strong: a 100% increase in GDP per capita is associated with a full 11 percentage point decrease in the share of public transport motor vehicles older than 10 years. We do not infer from this anything about the particular policies that might either lead to these outcomes or that might be used to overcome them, although this would be interesting to study: the observation is simply that vintage distributions do appear to vary around the world.

In the remainder of the paper, we focus on intervintage policy experiments in the calibrated economy.

26. Most countries impose some limits on the ability to import used vehicles, although these tend to be weaker for commercial vehicles, see United States Department of Commerce (2015).

27. This need not be a problem in principle, since the differences we are interested are between a 2017 Toyota Corolla and a 2007 Toyota Corolla, but, in practice, we want to know that differences in age distributions are likely due to difference in vintage composition, not to other sources of quality difference.

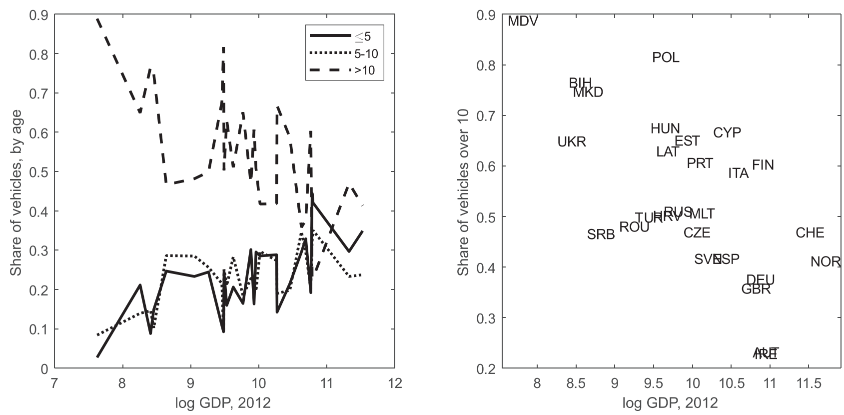

Fig 2. Share of Motor Coaches, Buses, and Trolleybuses at End 2012, by Age, in Selected Countries Indicated by ISO Codes.

NOTES: The correlation in the lower panel is −0.67. Sources: UNECE, World Bank, and own calculations.

2. ECONOMIC ENVIRONMENT

We extend the framework of JY12 to allow for intervintage transfer schemes. Consider a continuous time market economy with a population of unit mass. Each household is endowed with a unit flow of labor each date t which they may supply to the labor market. Utility is defined over streams of consumption:

$$U(c) = \int_0^{\infty} e^{-\rho t} \frac{c_t^{1-\eta} - 1}{1-\eta} dt, \, c : \mathbb{R}^+ \to \mathbb{R}^+. \tag{1}$$

There is a production technology that produces a final good y_t, which can be used for consumption or for investment. The production function is:

$$y_t = K_t^{\alpha} N_t^{1-\alpha}, \, \alpha \in (0, 1), \tag{2}$$

where N_t is labor used at date t and K_t is aggregate capital services, defined below.

Each date t a new investment technology becomes available, referred to as a *vintage*. Households may invest u_{vt} units of the final good in producing capital of any vintage $v \leq t$, and there is a stock of capital of any vintage k_{vt}. Aggregate capital is defined as

$$K_t = \left[\int_{-\infty}^{t} A_{t-v} (z_v k_{vt})^{\beta} dv \right]^{\frac{1}{\beta}}, \tag{3}$$

where z_v is a productivity level embodied in capital of vintage v, and A_{t-v} is a *learning function* associated with capital of age $t - v$. Parameter β is related to the elasticity of substitution among vintages: if σ is the elasticity of substitution between vintages, then $\beta \equiv \frac{\sigma-1}{\sigma}$, or $\sigma = \frac{1}{1-\beta}$. The purpose of the learning function (see JY12) is to capture the empirical fact that new products (including capital goods) tend to diffuse slowly, so the peak in use of new goods is not until several years after their introduction, for example, see Gort and Klepper (1982). If A_{t-v} were a constant, and if z_v were strictly increasing in v (the hallmark of a vintage capital model), then we would have that $k_{vt} > k_{v-1,t}\ \forall v, t$. We assume $\sigma \geq 1$ ($\beta \geq 0$). This implies that different vintages of capital are imperfect substitutes.

In what remains of the paper (with the exception of Proposition 6 below), we will assume that $z_v = e^{\gamma v}$ and that A_s is an increasing and concave function ($\frac{dA_s}{ds} > 0$, $\frac{d^2A_s}{ds_2} \leq 0$) such that

$$\lim_{s\to\infty} \frac{\dot{A}_s}{A_s} < \beta\gamma. \tag{4}$$

This assumption ensures that all vintages of capital eventually obsolesce.

The stock of physical capital of any particular vintage k_{vt} accumulates according to:

$$\frac{\partial k_{vt}}{\partial t} = u_{vt} - \delta k_{vt}, \tag{5}$$

where δ is the depreciation rate and $u_{vt} \geq 0$ is investment in capital of vintage v, determined by the households. Thus, the resource constraint for the economy is

$$y_t \geq c_t + \int_{-\infty}^{t} u_{vt} dv. \tag{6}$$

At date 0, the quantity k_{v0} is given for all $v \leq 0$ at date zero. We assume that endowments of capital are symmetric across households. It is then straightforward to show that (5) implies that

$$k_{vt} = e^{-\delta(t-v)} x_v + \int_{v}^{t} e^{-\delta(t-s)} u_{vs} ds,\ v > 0, \tag{7}$$

$$k_{vt} = e^{-\delta(t-v)} k_{v0} + \int_{0}^{t} e^{-\delta(t-s)} u_{vs} ds,\ v \leq 0, \tag{8}$$

where x_v is investment in new capital at the moment it was new.[28]

At each date t, firms solve:

$$\max_{KN_t} \{y_t - r_t K_t - w_t N_t\} \tag{9}$$

28. This formulation splits investment into an initial mass pulse x_v and a subsequent density u_{vt}.

subject to the production function (2).

Example 1. *Before closing the model, we can use the production technology to ask: what is the difference in the productivity of two economies with a different vintage structure? Imposing the fact that $N_t = 1$ in equilibrium, we have that output is given by:*

$$y_t = \left[\int_{-\infty}^{t} A_{t-v} (e^{\gamma v} k_{vt})^{\beta} dv \right]^{\frac{\alpha}{\beta}}. \tag{10}$$

Consider any continuous k_{vt}. If there is another economy with a distribution $\tilde{k}_{vt}$ such that $\int_{-\infty}^{t} k_{vt} dv = \int_{-\infty}^{t} \tilde{k}_{vt} dv$, where $\tilde{k}_{vt}$ first-order stochastically dominates k_{vt} in terms of the age of capital, then the economy with $\tilde{k}_{vt}$ will have higher output $\tilde{y}_t$ than the other, even though the total physical units of capital are the same in both economies. Indeed, if the stochastic dominance is sufficiently significant, differences in the distribution of vintages could make differences in output across the two economies arbitrarily large, even if the savings rate in both economies is the same. To see this, suppose that the distributions $k_{v,t}$ and $\tilde{k}_{v+\bar{s},t}$ are given, such that $k_{v+\bar{s},t}$ is a downward translation of the distribution $\tilde{k}_{v+\bar{s},t}$:

$$k_{v,t} = \begin{cases} 0 & t - v < \bar{s} \\ \tilde{k}_{v+\bar{s},t} & t - v \geq \bar{s}. \end{cases}$$

Here, the distribution of $k_{v,t}$ is the same as $\tilde{k}_{v+\bar{s},t}$, so the entire distribution is shifted down by $\bar{s}$. Again, the two have identical mass in terms of raw units of capital. However, it is straightforward to show that

$$\frac{y_t}{\tilde{y}_t} = e^{-\alpha\gamma\bar{s}}$$

so that, as $\bar{s} \to \infty$, $\frac{y_t}{\tilde{y}_t} \to 0$. The example shows that distortions to the vintage distribution can have arbitrarily large impact on aggregates even with constant savings rates.

2.1 Vintage-Specific Transfers

Assume that there is a tax $\tau(t-v) - 1$ on investment of age $t - v$, u_{vt}. Thus, instead of paying 1 for a unit of investment, they pay $\tau(t-v)$, where $\tau : \mathbb{R}^+ \to \mathbb{R}^+$ is twice continuously differentiable. Thus, τ is a multiplicative price wedge on investment. The revenues are distributed lump sum to consumers T_t, leading to the following government budget constraint:

$$T_t = \int_{-\infty}^{t} [\tau(t-v) - 1] u_{vt} dv. \tag{11}$$

If τ is increasing, then newer vintages of capital are favored by the tax system, through lower taxes, tax rebates, or subsidies. The household's budget constraint is then

$$c_t + x_t\tau(0) + \int_{-\infty}^{t} \tau(t-v)u_{vt}dv \leq r_tK_t + w_tN_t + T_t. \tag{12}$$

2.2 Model Solution

DEFINITION 1. *An equilibrium of the model is a set of prices $\{r_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$, allocations $\{c_t\}_{t=0}^{\infty}$, and capital for each vintage $v \leq t$ $\{k_{vt}\}_{t=0}^{\infty}$ such that, given the initial condition k_{v0} $(v \leq 0)$, the household chooses investment u_{vt} and consumption c_t at each date so as to maximize* (1) *subject to* (3), (7), (8), *and* (12), *firms maximize* (9) *and the government satisfies* (11) *for all t.*

DEFINITION 2. *A balanced growth path is an equilibrium and an initial condition k_{v0} $(v \leq 0)$ such that the growth rate of consumption g is constant over time, and the age distribution of capital is constant over time, so $\frac{k_{vt}}{k_{wt}} = \frac{k_{v-\Delta,t-\Delta}}{k_{w-\Delta,t-\Delta}}$ for any $\Delta \in \mathbb{R}$ and any two vintages v and w.*

We now study some properties of the equilibrium and of the balanced growth path. All proofs are in the Appendix.

PROPOSITION 1. *There exists a unique balanced growth path.*

First of all, in a model without taxes ($\tau = 1$), the user cost of capital of any vintage v is equivalent to $r + \delta$, and investment will be chosen optimally so as to equalize this and the marginal product of capital of each vintage v. However, with a nontrivial tax scheme, the user cost of capital and therefore the marginal product is affected by the tax scheme in two ways. First, the level of $\tau(v)$ affects the level of the user cost of capital. Second, marginal variation in the tax scheme by vintage $\tau'(v)$ also affects the optimal investment rate in each vintage. For example, if there is a range of v over which the tax rate $\tau(v)$ is flat and then rises rapidly, the marginal cost of new capital will be constant and then rise. As the acceleration begins, households find it preferable to invest in the vintage with the low tax rather than the otherwise very similar vintage with the higher tax rate (or lower subsidy rate). Given the tax scheme, this investment profile is optimal.

PROPOSITION 2. *The household's optimal investment decision satisfies:*

$$\alpha z_v^{\beta} y_t^{\frac{\alpha-\beta}{\alpha}} A_{t-v} k_{vt}^{\beta-1} \leq (\rho+\delta)\tau(t-v) - \tau'(t-v) \quad \forall t,\ v \leq t \tag{13}$$

with equality when $u_{vt} > 0$.

An alternative statement of this equation that lends itself is simply:

$$\frac{MPK_{vt} + \tau'(t-v)}{\tau(t-v)} \leq \rho + \delta,$$

where MPK_{vt} is the marginal product of vintage v capital at date t. The user cost of capital $r+\delta$ must be equal to the marginal product of capital from each vintage plus the effective price change of capital induced by a distortive tax system, all discounted by the tax rate, unless the investment constraint that $u_{vt} \geq 0$ binds.

PROPOSITION 3. *$k_{vt} = e^{gt}\chi_{t-v}$, where $\chi_{t-v} : \mathbb{R}^+ \to \mathbb{R}^+$ depends neither on t nor v independently.*

PROPOSITION 4. *$u_{vt} = e^{gt}\kappa_{t-v}$, where $\kappa_{t-v} : \mathbb{R}^+ \to \mathbb{R}^+$ depends neither on t nor v independently.*

The closed-form solutions for $\chi_{t-v} : \mathbb{R}^+ \to \mathbb{R}^+$ and $\kappa_{t-v} : \mathbb{R}^+ \to \mathbb{R}^+$ are given by equations $(A11)$ and $(A13)$, respectively, in the Appendix. A consequence of these results is that capital and investment of any given age s are a constant share of GDP over time, even if the share of any particular vintage rises and then falls over time.

DEFINITION 3. *The age distribution at date t is defined as the density function:*

$$\hat{k}_{st} \equiv \frac{k_{st}}{\int_0^\infty k_{ut}du} = \frac{\chi_{st}}{\int_0^\infty \chi_{ut}du}. \tag{14}$$

Having defined this density, we can make two observations about the model economy.

PROPOSITION 5. *A vintage-independent tax or subsidy $\tau(s) = \bar{\tau}$ does not affect the age distribution.*

Proposition 5 has important implications. When the choice of technological vintage is distinct from the date of production, investment taxes (or subsidies) cannot be justified as policies that stimulate investment in new technology. Since investment can occur in capital goods produced using a variety of capital producing technologies, both new and not-so-new, a tax on investment in itself does nothing to skew the vintage structure of capital.

Furthermore, there is a sense in which the overall amount of taxes and subsidies toward or away from capital does not affect the age distribution of capital, the subject of this paper. Rather, only the *relative sensitivity* of transfers to the vintage does. Consider that any vintage-specific transfer scheme $\tau(s)$ can be formulated as a profile $\tau(s) \equiv \bar{\tau}\eta(s)$, where $\eta(\cdot)$ is a relative sensitivity to vintage and $\bar{\tau}$ is a constant related to revenue generation.

COROLLARY 1. *Considering that any vintage-specific transfer scheme $\tau(s)$ can be formulated as $\tau(s) \equiv \bar{\tau}\eta(s)$, the value of the constant $\bar{\tau}$ does not affect the age distribution.*

For learning and taxation profiles that can be interpreted in terms of rates of sensitivity to age, we can deliver a further result about the age distribution of capital.

To do so, we consider a special case. Assume that $A_s = e^{\theta s}$. Furthermore, assume that $\tau(s) = \bar{\tau}e^{\omega s}$. Parameter $\bar{\tau}$ captures the overall size of the transfer scheme, and parameter $\omega \lesseqgtr 0$ captures the rate at which the transfer scheme favors capital of different vintages, so that higher ω implies relatively higher taxation of old capital (and relative higher subsidization of new capital). Under this parameterization, taxing new capital relatively less than the old (i.e., higher ω) can be shown analytically to skew the age distribution of capital toward newer vintages.

PROPOSITION 6. *Assume that* $A_s = e^{\theta s}$ *(*$\theta > 0$ *). Consider economies* $i \in \{1, 2\}$*, such that* $\tau(s) = \bar{\tau} e^{\omega_i s}$*, and assume that* $\beta\gamma - \theta + \omega_i(1 - \beta) > 0 \, \forall i$*. The age distribution of capital* $\hat{k}_s$ *in economy 1 first-order stochastically dominates that in 2 iff* $\omega_1 < \omega_2$*.*

Remark 1. When there is no taxation, Jovanovic and Yatsenko (2012) show that when $A_s = e^{\theta s}$, if economy 1 has higher β, higher γ, or lower θ than economy 2, then the vintage distribution in 1 first order stochastically dominates that in 2. The same holds true in our environment with taxation when $\tau(s) = \bar{\tau} e^{\omega_i s}$. However, since the current paper is concerned with the impact of policy $\tau(\cdot)$ on the vintage distribution, we keep constant technological parameters such as β, γ, and θ in our thought experiments and numerical experiments.

3. QUANTITATIVE EVALUATION

3.1 Calibration

We now calibrate the model economy to data from the United States, in order to perform quantitative policy experiments. Although the model is formulated in continuous time, we need a unit for measuring time in order to calibrate parameters in a consistent manner. We measure time in years. Details of the computational procedure are in the Appendix.

We require functional forms for the tax function $\tau(\cdot)$ and for the learning function $A(\cdot)$. Section 1 earlier outlined the absence of any systematic bias toward any particular vintage of capital in U.S. industrial policy. More generally, Krugman (1983) underlines in a series of examples how systematic industrial support programs present in various countries have been mostly absent in the United States, and the survey of Ford and Suyker (1989) finds that subsidies to industry in the United States are far smaller than in any other advanced economy. As a result, for the calibration process, we set $\tau(s) = 1$, so there are no intervintage transfers.[29] Later, we discuss the intervintage transfer schemes we consider for the policy experiments.

We set $A_s = 1 - e^{-\phi s}$ where $\phi > 0$. In this way, learning about any particular vintage is bounded, so sooner or later all vintages become for obsolete for any $\gamma > 0$. This implies that investment in the newest vintage u_{tt} equals zero, consistent with the observation that new capital diffuses gradually rather than exhibiting "jumps." Later, we assess the sensitivity of results to this assumption.

29. If we were to assume a benchmark transfer scheme other than $\tau(s) = 1$, the surveys of Samaniego (2006a) and Dang and Samaniego (2020) suggest that the likely alternative benchmark for a developed economy would be a transfer scheme that relatively favors older technologies. As shown later, the aggregate behavior of such a model is very similar to the economy with the $\tau(s) = 1$ benchmark.

Given these choices of functional form, the parameters to be calibrated are γ, α, δ, σ, ρ, η, and ϕ.

We set $\alpha = 0.33$, a standard value for the capital share of income. This is consistent with the idea that the learning is not embodied in the physical capital itself but in some other resource—for example, in the labor that uses the capital, or in the productivity of the firm that uses the capital as in Samaniego (2010).[30] However, for robustness later, we allow for larger values of α, which is equivalent to interpreting "capital" as including other accumulable resources that might embody the learning. Note that assuming a small value of α is a conservative assumption in the sense that it limits the impact of changes in the vintage distribution on aggregates. If $\alpha = 0$, then capital and the vintage distribution are irrelevant for aggregate outcomes.

Since $g = \frac{\alpha}{1-\alpha}\gamma$, a 1.5% annual GDP growth as is typically found in U.S. data would imply that $\gamma = 0.0350$. However, this number is very elevated compared to empirical estimates. The reason is that such an approach to calibration assumes that all growth is due to capital-embodied growth, as in Solow (1960). If instead, we view γ as reflecting improvements in the marginal rate of transformation between consumption and new capital goods (including quality improvements to capital) as in Greenwood, Hercowitz, and Krusell (1997) and Cummins and Violante (2002) among others, then we can match γ using the growth rate of the quality-adjusted relative price of capital. Using the values from Greenwood, Hercowitz, and Krusell (1997), we have that[31] $\gamma = 0.018$, so that $g = 0.0077$. The remainder of growth is due to unexplained technical progress that is outside the model.[32]

An important parameter is the elasticity of substitution among vintages σ. We set $\sigma = 2$. JY12 argue that $\sigma \approx 2$ based on the estimates of Bahk and Gort (1993) for just two types of capital, new and old. Independently, Edgerton (2011) finds estimates based on looking at the substitutability between new and old capital, ranging from 1.7 to 10.5 depending on the type of capital, with the estimates clustered toward the lower range. This implies that $\beta = 0.5$. We also examine the impact of larger values of σ.

Another key model parameter is the speed of learning ϕ. JY12 set $\phi = 0.6$ based on the finding of Bahk and Gort (1993) that most vintage-specific learning appears to be complete after 6 years.[33]

Finally, we set $\delta = 0.06$, $\rho = 0.03$, and $\eta = 1$, all of which are standard values in a growth accounting context.[34] Based on these assumptions, the average age of capital

30. Profit that accrues to entrepreneurs who use their labor to create firms is not capital income, see Gollin (2002).

31. This is the average rate across equipment and structures used in that paper.

32. If overall growth is 1.5% as in JY12, this value of γ accounts for 51% or about half of growth. If overall growth is 1.24% as in Greenwood, Hercowitz, and Krusell (1997), then γ accounts for about 60% of growth, as they find.

33. The value $\phi = 0.6$ stems from assuming that exactly 95% of the learning is complete by the sixth year.

34. $\eta = 1$ implies that $U(c) = \int_0^\infty e^{-\rho t} \ln c_t dt$.

TABLE 1

Calibration Statistics

Parameter	Interpretation	Value
γ	Rate of capital embodied tech. prog.	0.018
α	Capital share of income	0.33
δ	Physical depreciation rate	0.06
σ	Cross-vintage substitution elasticity	2
ρ	Discount rate	0.03
η	Intertemporal elasticity of substitution	1
ϕ	Speed of learning	0.6

Note: Calibration parameters for the benchmark economy. Calibration assumes that there are no net intervintage transfers.

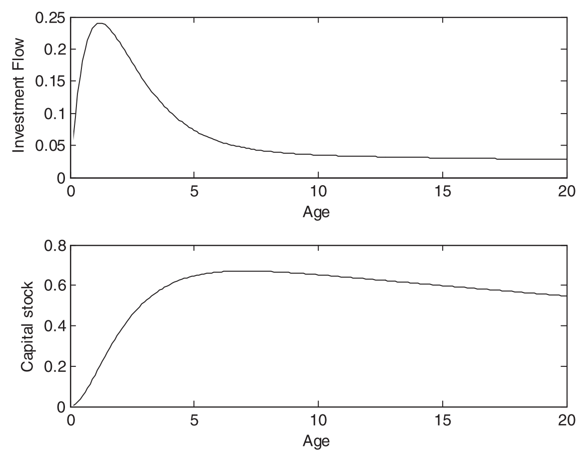

Fig 3. Investment $u_{v,t}$ and Capital Stock $k_{v,t}$ Based on Age t-v in the Calibrated Model Balanced Growth Path, in Model Units.

in the model economy is 14.5 years, not very different from the 12 year average found in U.S. data. [35] See Table 1 for all parameter values.

The calibrated model displays reasonable investment behavior. First, in the calibrated economy, we find that the investment share of GDP is 18.5%. This is very close to the value in U.S. data, even though this parameter was not directly calibrated. Also, Figure 3 shows that the diffusion pattern is an *S*-shape followed by a gentle decline, as found by Gort and Klepper (1982) for a variety of capital goods. This is due to the initially gradual adoption of each vintage of capital due to learning, followed by a gentle decline as investment shifts toward newer vintages and the older capital depreciates. The peak in usage is when the capital is about 7 years of age—although the peak in *investment* is much earlier, between the first and second years of introduction. This reflects the finding of Bahk and Gort (1993) that vintage-specific learning

35. See https://apps.bea.gov/iTable/iTable.cfm?ReqID=10&step=2#, last checked 9/22/2020.

is in general quite rapid, along with the fact that, in a relative sense, the learning is counteracted by the advance in the productivity of newly introduced vintages.

3.2 Policy Experiments

In the remainder of the paper, we focus on intervintage policy experiments in the calibrated economy.

For these experiments, we must choose a specific intervintage transfer scheme $\tau(\cdot)$. We examine the impact of transfers either too or from capital of the "newest vintage," interpreted as capital produced using technology introduced during the most recent *year*.

We choose this transfer scheme for the following reasons. First, Denison's (1956) criticism conceives of the policy implications of vintage capital models in this fashion, shifting resources toward the technology of the latest vintage. Second, it is not unusual in policy circles to discuss investment tax credits (i.e., subsidies) as being useful for targeting new technologies. As shown by Proposition 5, an investment tax credit or subsidy that is not vintage-specific will not affect the vintage distribution, as firms could write off the tax credit for investment in new capital of *any* vintage. However, in practice, in the United States, investment tax credits are formulated to promote particular new technologies (even if they are offset by other subsidies to older technologies, as discussed in Section 1). Our experiments will explore the impact of ramping up those programs. In addition, differential treatment of capital imports depending on whether they are new or used is equivalent to a tax directed at older vintages.

We examine two types of tax schemes:

- schemes with no net transfers to or from capital;
- schemes where net transfers to or from capital are enough to keep aggregate investment constant.

In our benchmark results, we look at tax schemes such that there are *no net transfers to or from capital*, that is, $T_t = 0$. The reason we focus on policies with no net transfers to capital is to focus on strictly *intervintage* transfers: the results using any policy that allows $T_t \neq 0$ would conflict the impact of policy through the vintage distribution with its redistributive impact.[36]

We do so as follows. Let τ_0 be the tax rate for firms below 1 year of age. Let $\tau_0 + \tau_{diff}$ be the tax rate for firms above 1 year of age. Then let τ_{diff} reflects (in levels) the preferential tax treatment given to newer vintages. The government budget balance condition would then become

$$[\tau_0 - 1] \int_0^1 u_{vt} dv + \left[\tau_{diff} + \tau_0 - 1\right] \int_1^\infty u_{vt} dv = 0. \tag{15}$$

36. The review in Samaniego (2006a) finds that, at least among OECD countries, there are no net transfers to or from firms.

Given a value of τ_{diff}, we can raise or lower τ_0 so as to ensure that condition is met.

Two important technical notes are in order regarding this transfer scheme.

1. As specified, the tax scheme is not continuously differentiable, whereas to solve the model, we require it to be at least twice continuously differentiable, because the second derivative of $\tau(\cdot)$ enters the optimal decision rule for investment u_{vt}.[37] As a result, we use a smooth approximation to the above "jumping" transfer scheme. In practice, we use the following:

$$\tau(s) = \tau_0 + \tau_{diff}\Phi(s|1, \varsigma), \tag{16}$$

where $\Phi(s|1, \varsigma)$ is the cumulative distribution function of the normal distribution with mean 1 and standard deviation ς. The balanced budget condition 15 must be modified accordingly:

$$\int_0^\infty \left[\tau_0 + \tau_{diff}\Phi(s|1, \varsigma) - 1\right] u_{vt} dv = 0. \tag{17}$$

The key to ensuring that (15) and (17) are similar is to set ς to a small value, so that the transition between tax rates τ_0 and $\tau_0 + \tau_{diff}$ is rapid. We set $\varsigma = 0.001$, which implies that capital of vintage 2 days less than a year is taxed at a rate negligibly different from τ_0, and that capital of vintage 2 days more than a year is taxed at a rate negligibly different from $\tau_0 + \tau_{diff}$.
2. For a given value of $\tau_{diff} \neq 0$, it is not necessarily the case that there exists a value of τ_0 that satisfies the balanced budget condition (17). For example, if the relative subsidy τ_{diff} is very large, the subsidy on young capital may be so much that it cannot be financed only through taxing old capital—of which there may be little, especially if σ is high, so capital of different vintages are very good substitutes. As vintages become perfect substitutes ($\sigma \to \infty$), then small tax differentials between different vintages will result in huge differences in investment patterns, so that practically, all investment is directed toward the subsidized vintages, so that government budget balance is not possible for sufficiently large values of τ_{diff}. Still, we are interested in schemes that do satisfy these properties of interest because they allow us to understand the impact of distortions to the vintage distribution in a controlled environment.

What is the impact of such a tax scheme on diffusion patterns? Proposition 2 indicates that optimal investment u_{vt} is affected by the structure of the tax system. With the tax system defined by equation (16), when investment tax rates jump up or jump down around age 1, investment patterns may change suddenly. When $\tau_{diff} > 0$ (so new vintages are taxed less), Figure 4 shows that investment drops off in general for vintages older than 1. Close to 1, there is a spike as the tax rate accelerates from

37. We could allow tax systems with jumps, which would result in infinite investment rates for certain vintages and which could be analyzed using the methodology of Vind (1967). We choose not to do so for expositional simplicity.

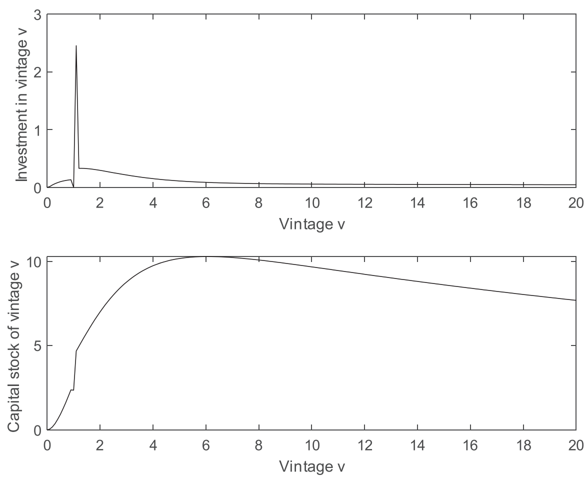

Fig 4. Investment u_{vt} by Age When Capital with V under 1 is Taxed Differentially from Capital with V above 1, on the Balanced Growth Path.

NOTES: In the top panel, old capital is taxed more ($\tau_{diff} = 0.3$). In the lower panel, new capital is taxed more ($\tau_{diff} = -0.3$).

τ_0 toward $\tau_0 + \tau_{diff}$, as it is more profitable to invest in those vintages than in other vintages that are similar technologically but very different for tax purposes. This is followed by a sharp drop as the tax rate slows down and approaches $\tau_0 + \tau_{diff}$. In contrast, when $\tau_{diff} < 0$ (so new vintages are taxed more), Figure 4 shows that investment *rises* in general for vintages older than 1. Close to 1, there is a sharp drop as the tax rate declines from τ_0 toward $\tau_0 + \tau_{diff}$, followed by a sharp pulse as the tax rate settles down and approaches $\tau_0 + \tau_{diff}$.

As mentioned, separately, we also examine policies that are designed to keep investment constant. In these experiments given a value of τ_{diff}, we select τ_0 so that investment equals 18.5% of GDP as in the benchmark economy. In this case, we will have that

$$\int_0^{\infty} \left[\tau_0 + \tau_{diff}\Phi(s|1, \varsigma) - 1\right] u_{vt} dv = T_t, T_t \gtrless 0. \tag{18}$$

The point of this experiment is to demonstrate that results for the other type of policy are not primarily due to the aggregate impact of changes in investment rates: they are due to distortions in the vintage structure. Indeed, results turn out to be very similar as the required values of T_t are small. Again, it is not the case that for any particular value of τ_{diff}, it is possible to find a transfer scheme that satisfies these properties.

3.3 Results

Before anything, it is worth pointing out that the usual welfare theorems apply to the model economy. In particular, in the Appendix, we prove that, regardless of whether or not we are in a steady state, all vintage-specific taxes are distorting and thus reduce welfare. Assume first that there are some vintage-specific taxes in the range of vintages where there is some investment; otherwise, vintage-specific taxes would be irrelevant.

Condition 1. There is an age a such $\tau'(a) \neq 0$ in a neighborhood of a and the planner's problem has nonzero investment in capital in a neighborhood of vintage $v = t - a$ at some date t.

Another case we discussed earlier does not create vintage-specific wedges, but instead simply taxes or subsidizes investment in general.

Condition 2. $\tau(a) = \bar{\tau}\ \forall a$.

PROPOSITION 7. *For any initial conditions $k_{v,0}$, $v \leq 0$, if either Condition* 1 *or Condition* 2 *applies, then equilibrium allocations are inefficient.*

At the same time, it is not necessarily the case that the *steady-state* impact of taxes and transfers is negative. The reason is simple: we are comparing across steady-state economies, which have different initial values of $k_{v,0}$, $v \leq 0$. The welfare theorems apply to the model economy with a *given* initial condition $k_{v,0}$. Nonetheless, as we shall see, there does not appear to be much scope for increasing long-run welfare through intravintage transfers even comparing across steady states. In general, it is not clear whether subsidizing the new is going to increase or decrease long-run welfare, since new capital is less productive in the sense of learning but more productive in terms of γ, and learning is rapid. We measure welfare changes using the percentage change (relative to the calibrated benchmark) in the level of consumption in each period, similar to a dynamic compensating variation. Later on, we look at transition dynamics.

Figure 5 shows that subsidizing the new (a negative tax differential) actually *decreases* welfare in the calibrated economy. A 20% subsidy on investment in the newest vintages (conditional on overall transfers to capital being zero) leads to a decline in consumption in each period of 1%. The impact of such transfers is nonlinear: a 50% subsidy lowers consumption by 5% and a 100% subsidy lowers consumption by a full 18%. In contrast, a 50% *tax* on the new (which is equivalent to a small subsidy to older capital) increases consumption in each period by about 1%. These long-run gains increase with greater taxation of the new, peaking around 1.5% when $\tau_{diff} = 300\%$ (not shown in Figure 5) and then fading gradually: at this point, there is very little investment in new capital because of the onerous taxation.

Interestingly, Figure 5 shows that the impact of vintage-specific taxation on *overall* investment is not significant. Varying τ_{diff} between -100% and $+100\%$ decreases investment from about 21% of GDP down to about 18% (the baseline value is 18.5%). On the other hand, the share of investment devoted to investment of the newest vintage

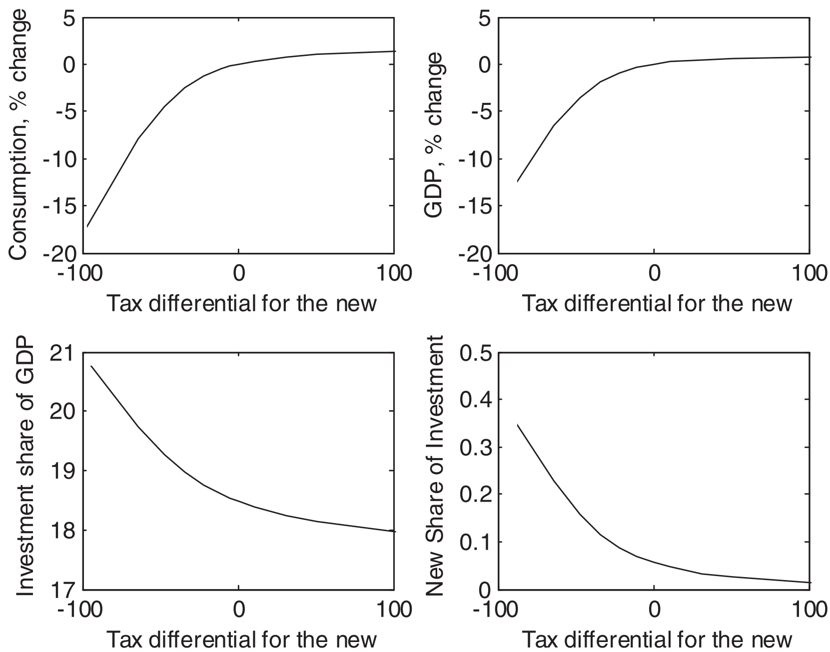

Fig 5. Impact of Intervintage Transfer Systems along the Balanced Growth Path.

NOTES: The x-axis in each case is the percent tax premium on capital of vintage over 1 year, τ_{diff}. The figure and all figures below assume that there are no net transfers to capital unless otherwise indicated. "New share of investment" refers to investment of vintage under 1 year.

(again, defined as the newest *year* of vintages) varies significantly, from about 40% down to almost zero (compared to the baseline value of 5.7%). This suggests that it is the distortions to the vintage structure—not changes in aggregate investment—that are responsible for the results.

How do these policies compare to those surveyed in Section 1? The results vary. For example, Adda and Cooper (2000) document that some provinces in France introduced subsidies of up to 9% for the replacement of old cars with new ones. While this is a substantial subsidy that could tip the marginal potential buyer's decision, if such a subsidy toward new capital were generalized (tax differential of −9%), it would only lower GDP by about 0.3% in the calibrated model. However, other policies are more extensive. The Solar Investment Tax Credit takes the form of a 30% tax credit, and the Hong Kongese RFS scheme for subsidizing automated production lines is also a 30% subsidy. If this were applied to innovative capital in general, it would correspond to a tax differential of −30% that would lower output in the calibrated model by a fairly substantial 1.5%. The Production tax credit, which applies to wind farms, involves a

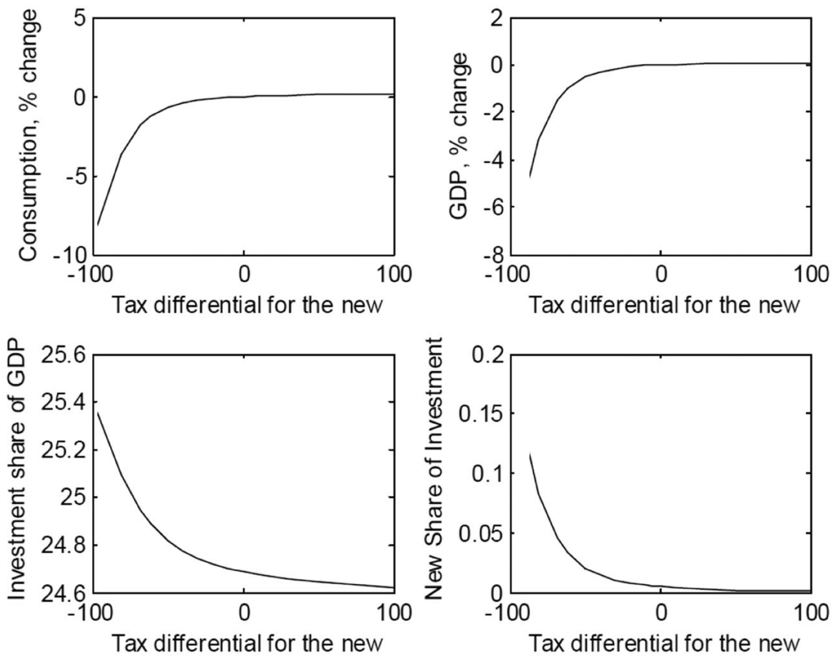

Fig 6. Impact of Intervintage Transfer Systems along the Balanced Growth Path.

NOTES: The x-axis in each case is the percent tax premium on capital of vintage over 1 year, τ_{diff}. This figure assumes that transfers are set so that investment is constant.

much larger tax differential, close to half the cost of the initial investment itself. [38] If generalized to other forms of investment, this would lower GDP in the calibrated model by up to 3.9% if applied to advanced technologies with a steep learning curve. In contrast, the kinds of support for ageing technologies that Ford and Suyker (1989) find in many European countries have little long-run aggregate impact, raising GDP by at most 0.7% (around $\tau = 100$). Thus, in the long run, the impact of these policies is not very significant. Note that, because we are comparing across steady states, it is not the case that this increased capacity necessarily means that agents are better off than in the undistorted economy. As we show later, agents would prefer the removal of these policies: if the policies were removed, the economy would transition to an new undistorted steady state with the agents preferring that transition to the continuation of the policy, suggesting that the main reason for the existence of such policies is political.

This is confirmed in Figure 6. Figure 6 reports results for transfer systems where τ_0 and therefore T_t are chosen so as to keep aggregate investment constant. The

38. A utility-scale windfarm costs around $2M and produces about 2 MWh. The tax credit is $0.012 per kWh over the first 10 years of operation: thus, the discounted tax rebate is almost as large as the cost. See http://www.windustry.org/how_much_do_wind_turbines_cost, last checked 9/21/2020.

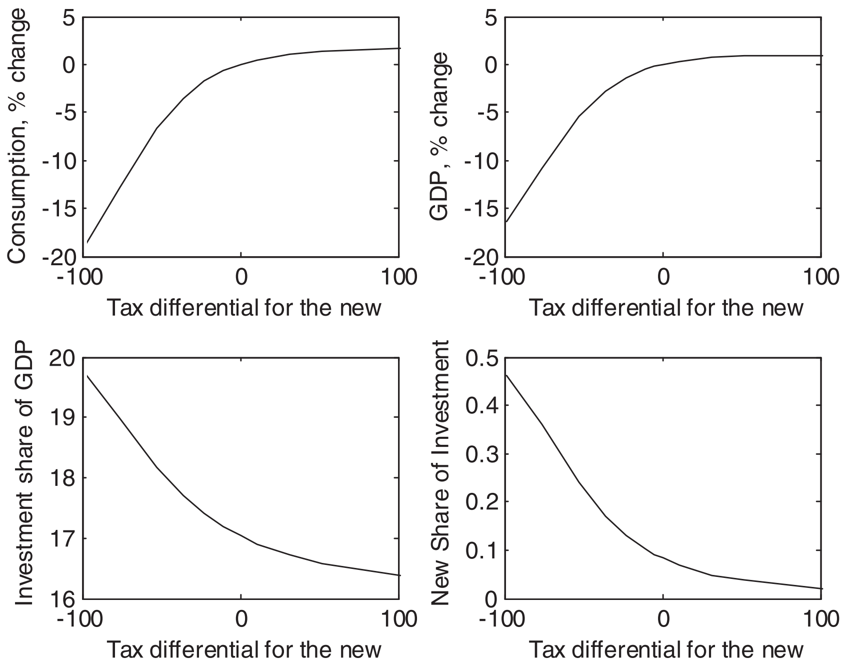

Fig 7. Impact of Intervintage Transfer Systems along the Balanced Growth Path.

NOTES: The x-axis in each case is the percent tax premium on capital of vintage over 1 year, τ_{diff}. Assume that initial learningA_0 is positive, so that $A_s = 1 - e^{-\phi(s+\bar{s})}$ and $\bar{s} \geq 0$.

results concerning welfare as measured by detrended consumption, as well as GDP and the share of investment in new vintages, are very similar. In addition, varying τ_{diff} between -100 and $+100\%$ only entails net transfers to consumers from the capital sector of $+2\%$ to -0.5%.

3.4 Robustness: The Impact of Learning

One might ask whether the negative impact of new vintage subsidies in Figure 5 is because of the assumption that initial productivity of new capital is zero. To examine this question, we modify the learning function so that:

$$A_s = 1 - e^{-\phi(s+\bar{s})}, \bar{s} \geq 0.$$

Allowing the parameter $\bar{s} > 0$ is equivalent to assuming that investment in any vintage of capital jumps from zero to a positive value when $v = t$. We set $\bar{s} = 1.4$, which is about the age that maximizes the investment flow in the baseline calibration. Figure 7 shows that allowing $\bar{s} > 0$ can actually *increase* the macroeconomic impact of vintage-specific transfers, although the difference is not very large compared to the baseline with $\bar{s} = 0$. The reason is that, since new capital is not as unproductive as

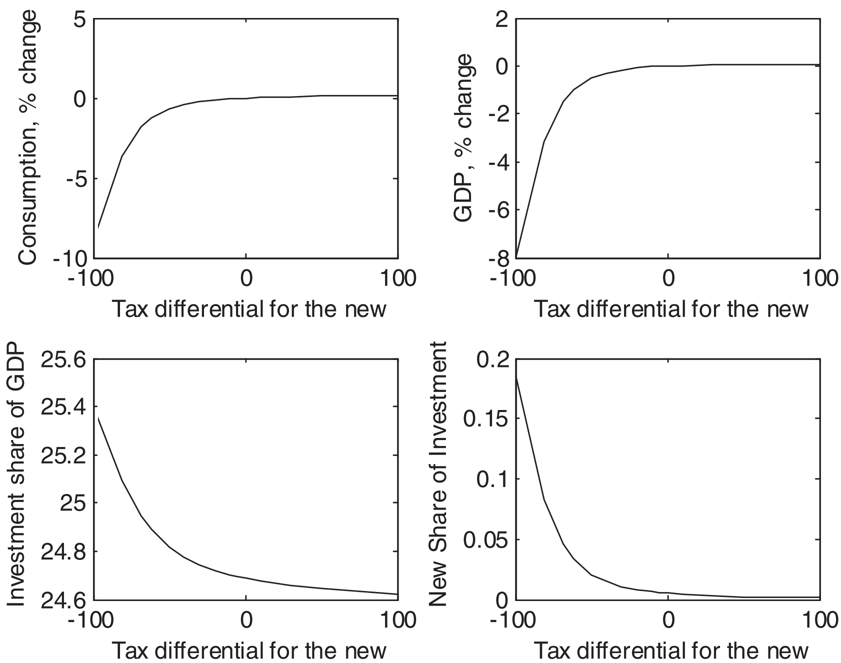

Fig 8. Impact of Intervintage Transfer Systems along the Balanced Growth Path.

NOTES: The x-axis in each case is the percent tax premium on capital of vintage over 1 year, τ_{diff}. Assume that γ equals 0.001.

when $\bar{s} = 0$, the distortion to the capital distribution is greater (as seen in that investment is more strongly skewed toward new investment, see the SE panel in Figure 7. Thus, it is the substitution effect that makes these subsidies (slightly) more harmful in this case.

The upside remains small in the long run (around 2% of consumption) but the downside can be even larger than before. This suggests that the shape of the learning profile, while important for matching diffusion curves, is not critical for the policy implications of intervintage transfers: instead, the productivity differences between vintages, and the difficulty of substituting between different vintages, are important.

3.5 Robustness: The Impact of Embodiment

In the model, there are two reasons why intervintage transfers might have aggregate impact. One is the fact that the vintages have different productivity. The other is that they are simply imperfect substitutes. To see whether embodiment (rather than substitution alone) is important, we perform two exercises. First, in Figure 8, we repeat the experiments with a low value of γ. Second, in Figure 9, we raise the elasticity of substitution σ to a larger value.

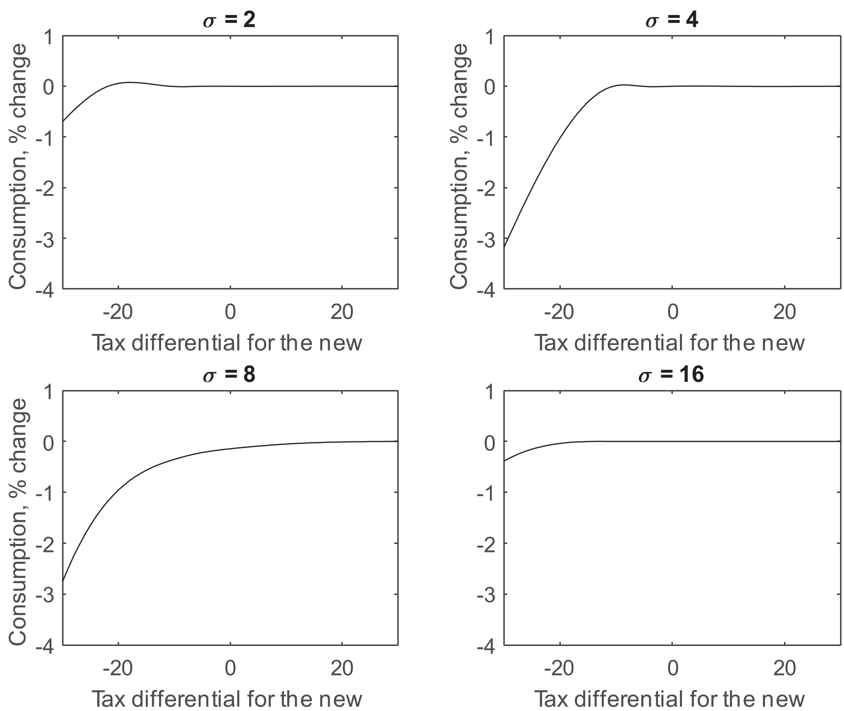

Fig 9. Impact of Intervintage Transfer Systems along the Balanced Growth Path, for Different Values of σ.

NOTES: Tax differentials τ_{diff} below -30% do not satisfy the balanced budget condition (17) when σ is large so that they are not displayed.

Figure 8 distinguishes between the impact of embodiment and the impact of substitution among vintages by assuming that γ is small. When $\gamma = 0.001$, compared to the calibrated value of $\gamma = 0.018$, the impact of taxation on long-run consumption declines by more than half. For example, while a 100% subsidy to new vintages lowers consumption by 18%, when γ is small consumption declines by only about 8%. Thus, it is not just the fact that old and new capitals are not perfect substitutes that affect the results: the rate of capital-embodied technical progress is a key determinant of the results.

3.6 Robustness: The Impact of σ

Next, we explore the impact of varying σ. The impact of varying σ is unclear. As σ rises, two things happen. On the one hand, different vintages of capital become closer substitutes. As $\sigma \to \infty$, we would then expect the impact of tax wedges to fade: if different types of capital are perfect substitutes, the vintage distribution should not matter. On the other hand, as $\sigma \to \infty$, taxes distort equilibrium investment rates more severely. As a result, the welfare impact of vintage tax wedges may be nonmonotonic.

To see this, consider that as $\sigma \rightarrow 1$ investment share should become relatively insensitive to tax wedges, since different vintages are no longer substitutes. In other words, as σ increases, tax wedges create greater distortions in the vintage distribution on the one hand, while, on the other hand, these distortions become less costly.

Figure 9 shows the results for $\sigma = 4$, to examine the sensitivity of results to the intervintage elasticity of substitution within the empirically relevant range. When $\tau_{diff} = 30\%$, consumption drops relative to the untaxed economy by about 3%. In contrast, in the baseline scenario with $\sigma = 2$, a tax differential $\tau_{diff} = 30\%$ leads consumption to drop relative to the untaxed economy by about 1%. Thus, the higher value of σ is associated a *larger* impact of vintage tax wedges on welfare. In this sense, our choice of benchmark elasticity $\sigma = 2$ is conservative within the likely empirically relevant range of $\sigma \in [2, 10]$. As σ continues to rise in this range, the impact of tax wedges weakens and continues to exceed that when $\sigma = 2$, but eventually begins to decline.

3.7 Robustness: The Impact of α

Finally, we also check the sensitivity of results to interpreting capital K_t as including not only physical capital but also whatever resource embodies the learning A_s. See Figure 10, where we assume that $\bar{s} = 0$ as in the baseline economy, but raise the capital share to $\alpha = 0.5$. In this case, the impact of intervintage transfers is larger, as might be expected, since capital (and hence distortions to capital) are more important for output and consumption when α is large. While $\tau_{diff} = -100\%$ led to a decline in long-run consumption of 18% in the baseline economy with $\alpha = 0.33$, when $\alpha = 0.5$, consumption declines by 34%.

4. TRANSITION DYNAMICS

One might ask whether the welfare impact of policy is very different in transition. This could be important because what might appear a large difference in welfare between steady states could be small once transition dynamics are taken into account, particularly if the transition is slow.

To characterize transition dynamics in the model economy, we require an approximation procedure. First, we simulate a discrete time version of the model economy, where investment follows the equation

$$k_{vt+1} = u_{vt} + (1 - \delta)k_{vt}. \tag{19}$$

All other features of the model, including parameter values, are held constant. We can then compute the steady state of the model under a particular policy regime, as well as computing the undistorted steady state. Finally, we use the steady-state distribution of capital across vintages under the policy as an initial condition. We then simulate the transition dynamics of the model economy over time, as it converges to the efficient steady state without the policy. If the transition is rapid,

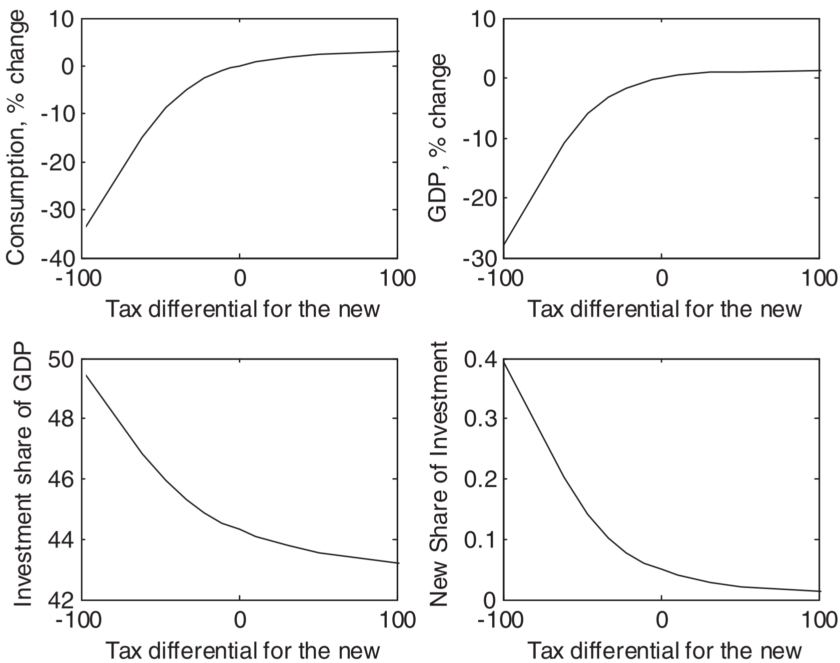

Fig 10. Impact of Intervintage Transfer Systems along the Balanced Growth Path.

NOTES: The x-axis in each case is the percent tax premium on capital of vintage over 1 year, τ_{diff}. Assume that α equals 0.5.

it indicates that the steady-state welfare comparisons are adequate for considering transition dynamics as well. A more precise welfare computation can also be made by comparing the difference between consumption in the undistorted steady state and the consumption stream in transition.

The procedure is as follows, and involves a series of nested loops. First, at date zero, k_{v0} is given. Let time increase in increments of Δ. For date Δ, given a guess for r_Δ and y_Δ, we use the discrete time analog of equation (13) to determine the unconstrained capital stock $k_{v\Delta}$. Then, we can compute u_{v0} based on $k_{v\Delta}$ and k_{v0}. If we find that $u_{v\Delta} < 0$ computed in this way, it implies the nonzero investment constraint binds and we set $u_{v0} = 0$. Using this corrected value for u_{v0}, we can use equation (19) to determine $k_{v,\Delta}$. This, in turn, gives us a prediction for what y_Δ should be, using the production function. We iterate on y_Δ until it converges.

We can then repeat this procedure for date $t = 2\Delta$, $t = 3\Delta$, and so on, to produce a series for all the variables given the initial guess for the series r_t.

The problem then is how to compute a new guess for the series $\{r_t\}_{t=0}^{\infty}$. We consider only "smooth" paths for r_t that converge to the steady state of the model economy in finite time. We ensure that they are smooth by using the Hodrick–Prescott filter, as in Samaniego (2008).

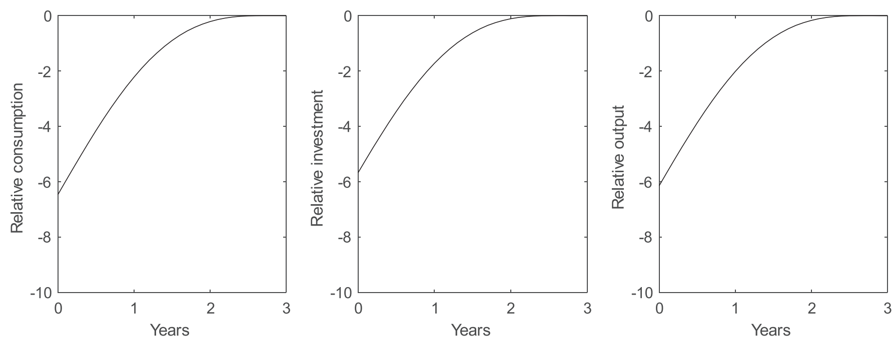

Fig 11. Transition Dynamics from the Steady State with τ_{diff} Set to -1 to the Undistorted Steady State.

NOTES: The computational procedure forces a smooth path as described in the text.

Given a guess $\{r_t\}_{t=0}^{T}$, we can compute C_t for all dates, and generate a new predicted r_t sequence using the relationship:

$$r_t = (C_{t+1}/C_t)^{\eta}/\beta - 1 + \delta;$$

where β is the household's discount factor. We smooth the series using the Hodrick-Prescot filter, and use the smoothed series as a new guess. Our initial guess assumes that r_t converges to its steady-state value by date T.

Computing transition dynamics in this manner poses several challenges. The first is that we need to determine a size for the interval between periods. This time interval should be small, but not so small as to raise the computational cost excessively. We assume that there are 10 periods in a year ($\Delta = 0.1$), and measure all rates and parameters with this in mind.

The second is that the state variable of the model economy is a large object, a value of k_{vt} at each date t and for all vintages $v \leq t$. Thus, the simulation requires assuming that above some sufficiently large age of capital $\bar{a} = t - v$, capital is zero, that is, $k_{vt} = 0$ for all $t - v > \bar{a}$. We set this at 1,000 years, so there are 10,000 vintages of capital active at any point in time.

Finally, we need to compute transition dynamics over a finite horizon T. This horizon should be long enough that the transition to the new steady state is essentially complete. We tried various horizons for convergence, finding similar results. We report results with $T = 10$ (10 years, or $10/\Delta = 100$ periods). See the Appendix for further details.

We begin by computing the transition from a regime where new capital is subsidized. We set $\tau_{diff} = 1$, so there was a 100% subsidy over older capital prior to date zero. At date zero, the state variable of the economy is the steady-state vintage distribution when $\tau_{diff} = 1$. However, from date zero onward, $\tau_{diff} = 0$.

Figure 11 displays paths of consumption, investment, and output relative to their finishing point. Output under the policy is depressed relative to the undistorted steady

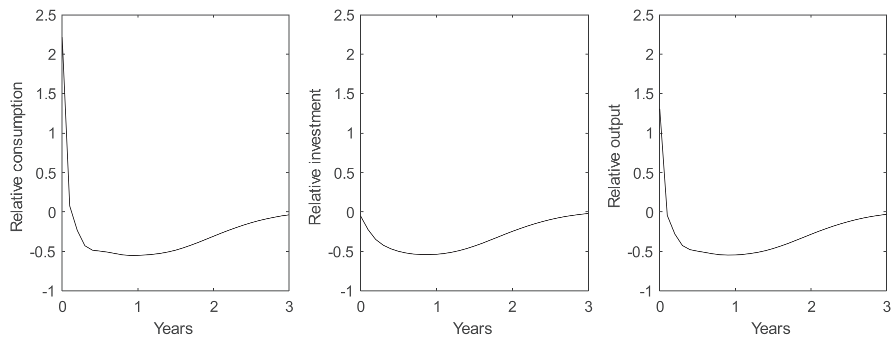

Fig 12. Transition Dynamics from the Steady State with τ_{diff} Set to $+1$ to the Undistorted Steady State.
NOTES: The computational procedure forces a smooth path as described in the text.

state, so both consumption and investment start below the new steady-state level. However, the most notable feature of the transition to the new steady state is that it is essentially complete in about 2 years. We find that this is the case regardless of how long we assume that it takes for the economy to converge to its new steady state.

As a result, the welfare computation from the steady states should accurately reflect the welfare difference if the policy were to be suddenly removed. To see this, we compute the percentage of consumption in the undistorted steady states that the households would require each period along the transition path to make them indifferent between the steady state and the transition. We find that this compensating variation is the equivalent of just 0.17% of steady-state consumption every period. While households are worse off in transition than if they had started off at the undistorted steady state, the welfare difference is small. Thus, unless there are substantial investment adjustment costs of some sort, the steady-state welfare comparisons accurately reflect the welfare comparisons in transition as well.

We repeat this computation for the transition from a regime where new capital is taxed. We set $\tau_{diff} = -1$, so there is a 100% tax on new capital prior to date zero. At date 0, the subsidy and the taxes funding it are removed, and there is no vintage tax wedge thereon.

Figure 12 displays paths of consumption, investment, and output relative to their finishing point. As discussed earlier, in this steady-state output, it is slightly higher than in the undistorted steady state because the subsidy favors capital for which there has already been a lot of learning. However, once the subsidy is removed, it is efficient to let the older capital depreciate and to invest in the newer (rarer) new capital that was previously taxed, as it is scarce initially. Since this is a relatively unproductive investment, investment declines and consumption increases at first. Again, within 2 years, the economy has converged back to its steady state: the transition is rapid.

Finally, we compute the percentage of the steady-state level of consumption that would have to go to the households along the transition path each period to make them indifferent between the steady state and the transition, as before. We find that this steady-state compensating variation is the equivalent of 0.02% of steady-state consumption every period. Notice that this is still positive: households are worse off starting with the distorted capital structure and transitioning to the undistorted steady state than if they had started off in the steady state in the first place—even though the difference is small.

5. CONCLUDING REMARKS

The paper finds that policy-based distortions to the vintage distribution of capital can have significant aggregate and welfare impact. The results provide a clear response to a central argument in the embodiment controversy that the usefulness of models where technical progress is embodied in capital hinges on their policy relevance. They also indicate a new channel for policy, market frictions, or other distortions to affect the wealth of nations—a channel that can only be addressed using a model where technical progress is embodied in capital.

The paper strictly focuses on the impact of policy through the vintage distribution. We do not mean to suggest that there are not channels other than the vintage distribution through which policy might affect aggregates that relate to environments where technology is embodied in capital. One possibility is the fact, documented in Cummins and Violante (2002), that differences in rates of technology improvement vary across capital goods. Thus, changes in the *composition* of capital—not the *vintage* distribution, but the *type* distribution—could matter too. There could also be interactions between regulation and vintage capital through *firm dynamics*, as suggested by Samaniego (2006a), Samaniego (2006b), and Samaniego (2010), [39] which could be propagated through the choice of vintage. These questions remain for future work.

Another channel from which we abstract is a potential interaction with vintage-specific *human* capital, or with the skill composition of the economy, as suggested by Chari and Hopenhayn (1991). Extending the model to allow vintage physical and vintage human capital accumulation to interact would likely amplify the results of the paper.

We do not study the distinction between used and new capital of a given vintage. This distinction could matter in an environment where there is a concept of reallocation among production units, and where there might be costs of reallocation. Lanteri (2018) studies such reallocation but in an environment without a vintage model. Eisfeldt and Rampini (2007) find that used capital is important for the operation of credit-constrained firms, suggesting that changes in the vintage distribution could be important for the aggregate impact of financing frictions. It could also be interesting to study the costs and benefits of restrictions on imports of used equipment, restrictions that are common in developing economies.

The model we used has no externalities: in particular, the learning function is exogenous. Some authors suggest that past adoption of technologies may lead to

39. These studies analyze the impact of industrial subsidies, firing costs and entry costs, respectively.

learning for future adopters through either learning-by-doing or through knowledge spillovers. In the former case, the dynamics of learning would be endogenous and thus harder to compute, and in the latter case, the normal welfare theorems might not apply so that identifying the optimal transfer scheme would be an interesting (and nontrivial) question. In any case, neither of these extensions is necessary to bring out the main points of the paper. It would be interesting to study the optimal transfer scheme in an environment with externalities. Naturally, the optimal scheme would depend on the details of any externalities or other sources of market failure. Our model provides a suitable benchmark against which to compare findings in any environment with externalities. Our results indicate that externalities might have to be substantial to overturn the welfare findings. Also, since externalities could be modeled in many ways, we believe that the parsimony of our approach is valuable, whereas it is not clear what criteria, empirical or otherwise, could be used to justify one approach to modeling externalities over another as an alternative benchmark, nor the presence of externalities in the first place. We think that it is possible that learning externalities might be important where a new general purpose technology is introduced, for example, see Greenwood and Jovanovic (1999). However, this paper is about gradual improvements in existing forms of capital, where we think that learning externalities are much less likely since the change is incremental. Market inefficiencies could exist for many other reasons, such as financing constraints that might distort investment decisions, but exploring the potential interaction of subsidies with financing constraints is beyond the scope of the paper. At the same time, it is not clear that financing constraints are important for the impact of industrial subsidies in general: for example, Dang and Samaniego (2020) find no evidence of an interaction between industrial subsidies and financing constraints in firm-level data from Vietnam.

Finally, the model implies that vintage-specific taxation could influence capital prices or investment patterns. For example, trading turnover in certain capital goods is nonmonotonic in vintage, as shown by Stolyarov (2002). It would be interesting to explore whether the tax treatment of goods of different vintages, for example, differences between the tax treatment of depreciation and actual physical or economic depreciation patterns, could be responsible for nonmonotonicity in resale or pricing patterns of used or of old-vintage capital.

APPENDIX A: PROOFS

Below are proofs for the results reported in the main text.

PROOF OF PROPOSITION 1. The proof is a consequence of the results below, which construct a unique balanced growth path for the model economy. □

PROOF OF PROPOSITION 2. In equilibrium $N_t = 1$ since labor does not enter the utility function, and $w_t = (1 - \alpha)y_t$. Thus, the Lagrangian for the investment problem is

$$L = \int_0^\infty e^{-\rho t} \left\{ K_t^\alpha - x_t \tau(0) - \int_{-\infty}^t \tau(t - v) u_{vt} dv + \right.$$

$$+\int_0^t \lambda_{vt}\left[e^{-\delta(t-v)}x_v + \int_v^t e^{-\delta(t-s)}u_{v,s}ds - k_{vt}\right]dv$$

$$+\int_{-\infty}^0 \lambda_{vt}\left[e^{-\delta(t-v)}k_{v0} + \int_0^t e^{-\delta(t-s)}u_{v,s}ds - k_{vt}\right]dv\Bigg\}dt. \tag{A1}$$

Using small variations of the controls δu, δx, δy, δk, we have

$$\delta L \approx \int_0^\infty e^{-\rho t}\Bigg\{\delta y_t - \delta x_t\tau(0) - \int_{-\infty}^t \tau(t-v)\delta u_{vt}dv$$

$$+\int_0^t \lambda_{vt}\left[e^{-\delta(t-v)}\delta x_v + \int_v^t e^{-\delta(t-s)}\delta u_{v,s}ds - \delta k_{vt}\right]dv$$

$$+\int_{-\infty}^0 \lambda_{vt}\left[\int_0^t e^{-\delta(t-s)}\delta u_{v,s}ds - \delta k_{vt}\right]dv\Bigg\}dt, \tag{A2}$$

where since $y_t = [\int_{v\in\{-\infty,t\}} A_{t-v}(z_v k_{vt})^\beta dv]^{\frac{\alpha}{\beta}}$ we have that:

$$\delta y_t = \alpha\left[\int_{v\in\{-\infty,t\}} A_{t-v}(z_v k_{vt})^\beta dv\right]^{\frac{\alpha-\beta}{\beta}} \int_{-\infty}^t A_{t-v}z_v^\beta k_{vt}^{\beta-1}\delta k_{vt}dv.$$

Then,

$$\delta L \approx \int_0^\infty e^{-\rho t}\left\{\delta y_t - \delta x_t\tau(0) - \int_{-\infty}^t \tau(t-v)\delta u_{vt}dv\right\}dt$$

$$+\int_0^\infty e^{-\rho t}\int_0^t \lambda_{vt}\left[e^{-\delta(t-v)}\delta x_v + \int_v^t e^{-\delta(t-s)}\delta u_{v,s}ds - \delta k_{vt}\right]dvdt$$

$$+\int_0^\infty e^{-\rho t}\int_{-\infty}^0 \lambda_{vt}\left[\int_0^t e^{-\delta(t-s)}\delta u_{v,s}ds - \delta k_{vt}\right]dvdt. \tag{A3}$$

Now we bring together the terms in δx, δv, δk to obtain

$$\delta L \approx \int_0^\infty \int_0^v \lambda_{tv}e^{-\rho v-\delta(v-t)}\delta x_t dtdv - \int_0^\infty e^{-\rho t}\{\delta x_t\tau(0)\}dt$$

$$+\int_0^\infty e^{-\rho t}\left\{-\int_{-\infty}^t \tau(t-v)\delta u_{vt}dv\right\}dt$$

$$+\int_0^\infty e^{-\rho t}\int_{-\infty}^t \lambda_{vt}\left[\int_v^t e^{-\delta(t-s)}\delta u_{v,s}ds\right]dvdt$$

$$+\int_0^\infty e^{-\rho t}\left\{\alpha y_t^{\frac{\alpha-\beta}{\alpha}}\int_{-\infty}^t A_{t-v}z_v^\beta k_{vt}^{\beta-1}\delta k_{vt}dv\right\}dt$$

$$+\int_0^\infty e^{-\rho t}\int_{-\infty}^t \lambda_{vt}[-\delta k_{vt}]dvdt. \tag{A4}$$

Switching the integrals and rearranging (see Hritonenko and Yatsenko 1996, Hritonenko and Yatsenko 2005),

$$\begin{aligned}\delta L \approx & \int_0^\infty \left[\int_t^\infty e^{-\rho s-\delta(s-t)}\lambda_{ts}ds - e^{-\rho t}\tau(0)\right]\delta x_t dt \\ & + \int_0^\infty \int_{-\infty}^t \left[\int_t^\infty e^{-\rho s-\delta(s-t)}\lambda_{vs}ds - e^{-\rho t}\tau(t-v)\right]\delta u_{vt}dvdt \\ & + \int_0^\infty \int_{-\infty}^t e^{-\rho t}\left\{\alpha y_t^{\frac{\alpha-\beta}{\alpha}} A_{t-v} z_v^\beta k_{vt}^{\beta-1}\delta - \lambda_{vt}\right\}\delta k_{vt}dvdt. \end{aligned} \tag{A5}$$

Setting the coefficients of δ to zero yields the following optimality conditions for the case when the solution is interior:

$$\int_t^\infty e^{-rs-\delta(s-t)}\lambda_{ts}ds = e^{-rt}\tau(0), \forall t \tag{A6}$$

$$\int_t^\infty e^{-rs-\delta(s-t)}\lambda_{vs}ds = e^{-rt}\tau(t-v), \forall v \le t \tag{A7}$$

$$\alpha y_t^{\frac{\alpha-\beta}{\alpha}} A_{t-v} z_v^\beta k_{vt}^{\beta-1} = \lambda_{vt}, \forall v \le t. \tag{A8}$$

Thus, if the solution is interior, optimally investment is chosen so that

$$\alpha z_v^\beta \int_t^\infty e^{-(\rho+\delta)s} y_s^{\frac{\alpha-\beta}{\alpha}} A_{s-v} k_{vs}^{\beta-1}ds = e^{-(\rho+\delta)t}\tau(t-v)\forall v \le t, t \ge 0. \tag{A9}$$

Notice that this is the same as the solution for JY12 except that $\widehat{z_v^\beta} = z_v^\beta/\tau(t-v)$. Differentiating this condition with respect to t yields

$$\alpha z_v^\beta y_t^{\frac{\alpha-\beta}{\alpha}} A_{t-v} k_{vt}^{\beta-1} = (\rho+\delta)\tau(t-v) - \tau'(t-v),$$

where $\tau'(s)$ is the derivative of the tax function, and $\tau'(0) \equiv \lim_{s\to 0^+}\tau'(s)$. On the other hand, when the solution is not interior for given v, it is because satisfying this condition would require negative investment. This means that the value of k_{vt} required to satisfy this expression with the constraint binding is too large, that is, the left-hand side expression must be smaller. □

PROOF OF PROPOSITIONS 3 and 4. We use some proportionality relationships regarding how aggregate variables must grow in a balanced growth path, in order to derive further results regarding optimal investment. Recall that:

$$z_v = e^{\gamma v}.$$

The optimal growth rate of consumption given the utility function (1) and the production function (2) is:

$$g = \frac{r - \rho}{\eta}.$$

The fraction of investment in GDP is constant (since consumption cannot grow at a rate different from GDP), and K_t grows at the rate $\gamma + g$. This implies as in typical models with capital-embodied technical progress that:

$$g = \frac{\alpha}{1 - \alpha}\gamma.$$

Now consider that $y_t^{\frac{\alpha-\beta}{\alpha}} \propto e^{gt\frac{\alpha-\beta}{\alpha}}$, and $z_v^{\beta} \propto e^{\beta\gamma v} = e^{\beta\frac{1-\alpha}{\alpha}gv}$. Since $gt\frac{\alpha-\beta}{\alpha} + \beta\frac{1-\alpha}{\alpha}gv = (1-\beta)gt - \beta\gamma(t-v)$, we have that, using Proposition 2,

$$\begin{aligned} k_{vt} &= \left(\frac{\alpha z_v^{\beta} y_t^{\frac{\alpha-\beta}{\alpha}} A_{t-v}}{(\rho+\delta)\tau(t-v) - \tau'(t-v)}\right)^{\frac{1}{1-\beta}} \qquad \text{(A10)} \\ &= \left(\frac{\alpha z_0^{\beta} y_0^{\frac{\alpha-\beta}{\alpha}} e^{(1-\beta)gt - \beta\gamma(t-v)} A_{t-v}}{(\rho+\delta)\tau(t-v) - \tau'(t-v)}\right)^{\frac{1}{1-\beta}} \\ &= e^{gt}\chi_{t-v}, \end{aligned}$$

where

$$\begin{aligned} \chi_s &= \left(\frac{\alpha z_0^{\beta} y_0^{\frac{\alpha-\beta}{\alpha}} e^{-\beta\gamma s} A_s}{(\rho+\delta)\tau(s) - \tau'(s)}\right)^{\frac{1}{1-\beta}} \qquad \text{(A11)} \\ &= \bar{k}\left(\frac{e^{-\beta\gamma s} A_s}{\tau(s) - \frac{\tau'(s)}{(\rho+\delta)}}\right)^{\frac{1}{1-\beta}} \end{aligned}$$

and

$$\bar{k} = \left((\rho+\delta)^{-1}\alpha z_0^{\beta} y_0^{\frac{\alpha-\beta}{\alpha}}\right)^{\frac{1}{1-\beta}}, z_0 = 1.$$

Since

$$y_t = \left[\int_{-\infty}^{t} A_{t-v}(z_v k_{vt})^{\beta} dv\right]^{\frac{1}{\beta}\alpha}, \tag{A12}$$

this becomes

$$\bar{k} = \left((\rho+\delta)^{-1}\alpha y_0^{\frac{\alpha-\beta}{\alpha}}\right)^{\frac{1}{1-\beta}} = \left((\rho+\delta)^{-1}\alpha\left[\int_{-\infty}^{0} A_{-v}k_{v0}^{\beta}dv\right]^{\frac{\alpha-\beta}{\beta}}\right)^{\frac{1}{1-\beta}},$$

and then,

$$\bar{k} = \left((\rho+\delta)^{-1}\alpha\left[\int_{-\infty}^{0} A_{-v}\left[e^{g0}\chi_{0-v}\right]^{\beta}dv\right]^{\frac{\alpha-\beta}{\beta}}\right)^{\frac{1}{1-\beta}}$$

$$= \left((\rho+\delta)^{-1}\alpha\left[\int_{-\infty}^{0} A_{-v}\left[\bar{k}\left(\frac{e^{\beta\gamma v}A_{-v}}{\tau(-v)-\frac{\tau'(-v)}{\rho+\delta}}\right)^{\frac{1}{1-\beta}}\right]^{\beta}dv\right]^{\frac{\alpha-\beta}{\beta}}\right)^{\frac{1}{1-\beta}}.$$

Rearranging we end up with:

$$\bar{k} = \left[(\rho+\delta)^{-1}\alpha\right]^{\frac{1}{1-\alpha}}\left[\int_{0}^{\infty} A_s\left[\left(\frac{e^{-\beta\gamma s}A_s}{\tau(s)-\frac{\tau'(s)}{\rho+\delta}}\right)^{\frac{1}{1-\beta}}\right]^{\beta}ds\right]^{\frac{\alpha-\beta}{\beta(1-\alpha)}}.$$

Next, we turn to the calculation of optimal investment. For *new capital*,

$$x_t = k_{tt} = e^{gt}\chi_0 = e^{gt}\bar{k}\left(\frac{A_0}{\tau(0)-\frac{\tau'(0)}{(\rho+\delta)}}\right)^{\frac{1}{1-\beta}},$$

which will be a constant fraction of GDP. For *old capital*, the capital accumulation equation (5) and the above derivations imply that

$$u_{vt} = ge^{gt}\chi_{t-v} + e^{gt}\frac{\partial\chi_{t-v}}{\partial t} + \delta k_{vt} = e^{gt}\left[(g+\delta)\chi_{t-v} + \frac{\partial\chi_{t-v}}{\partial t}\right].$$

Since

$$\frac{\partial\chi_{t-v}}{\partial t} = \bar{k}\frac{\left[e^{-\frac{\beta\gamma(t-v)}{1-\beta}}\frac{dA_{t-v}^{\frac{1}{1-\beta}}}{dt}\right]}{\left[\tau(t-v)-\frac{\tau'(t-v)}{(\rho+\delta)}\right]^{\frac{1}{1-\beta}}} - \frac{\beta\gamma}{1-\beta}\chi_{t-v}$$

$$-\bar{k}\frac{\frac{1}{1-\beta}e^{-\frac{\beta\gamma(t-v)}{1-\beta}}A_{t-v}^{\frac{1}{1-\beta}}\left[\tau(t-v)-\frac{\tau'(t-v)}{(\rho+\delta)}\right]^{\frac{1}{1-\beta}-1}\left[\tau'(t-v)-\frac{\tau''(t-v)}{(\rho+\delta)}\right]}{\left[\tau(t-v)-\frac{\tau'(t-v)}{(\rho+\delta)}\right]^{\frac{2}{1-\beta}}},$$

we have that

$$\frac{\partial\chi_{t-v}}{\partial t}=\bar{k}\left[e^{-\frac{\beta\gamma(t-v)}{1-\beta}}\frac{dA_{t-v}^{\frac{1}{1-\beta}}}{dt}-\frac{\beta\gamma}{1-\beta}e^{-\frac{\beta\gamma(t-v)}{1-\beta}}A_{t-v}^{\frac{1}{1-\beta}}\right]\left[\tau(t-v)-\frac{\tau'(t-v)}{(\rho+\delta)}\right]^{\frac{-1}{1-\beta}}$$
$$-\left(\frac{1}{1-\beta}\right)\bar{k}e^{-\frac{\beta\gamma(t-v)}{1-\beta}}A_{t-v}^{\frac{1}{1-\beta}}\left[\tau(t-v)-\frac{\tau'(t-v)}{(\rho+\delta)}\right]^{\frac{-1}{1-\beta}-1}$$
$$\left[\tau'(t-v)-\frac{\tau''(t-v)}{(\rho+\delta)}\right].$$

So $u_{vt}=e^{gt}\kappa_{t-v}$ where

$$\kappa_{t-v}=\frac{\bar{k}e^{-\frac{\beta\gamma(t-v)}{1-\beta}}}{\left[\tau(t-v)-\frac{\tau'(t-v)}{(\rho+\delta)}\right]^{\frac{1}{1-\beta}}}\left[(g+\delta)A_{t-v}^{\frac{1}{1-\beta}}+\left[\frac{dA_{t-v}^{\frac{1}{1-\beta}}}{dt}-\frac{\beta\gamma}{1-\beta}A_{t-v}^{\frac{1}{1-\beta}}\right]\right.\tag{A13}$$
$$\left.-\left(\frac{1}{1-\beta}\right)A_{t-v}^{\frac{1}{1-\beta}}\frac{\left[\tau'(t-v)-\frac{\tau''(t-v)}{(\rho+\delta)}\right]}{\left[\tau(t-v)-\frac{\tau'(t-v)}{(\rho+\delta)}\right]}\right].$$

□

PROOF OF PROPOSITION 5. When $\tau(s)=\bar{\tau}$, the tax and its derivative cancel out of both the numerator and denominator of the vintage distribution in equation (14), using equation $(A11)$:

$$\hat{k}_s=\frac{\frac{e^{-\frac{\beta}{1-\beta}\gamma s}A_s^{\frac{1}{1-\beta}}}{\tau(s)-\frac{\tau'(s)}{(\rho+\delta)}}}{\int_0^\infty\left[\frac{e^{-\frac{\beta}{1-\beta}\gamma u}A_u^{\frac{1}{1-\beta}}}{\tau(u)-\frac{\tau'(u)}{(\rho+\delta)}}\right]du}=\frac{\frac{e^{-\frac{\beta}{1-\beta}\gamma s}A_s^{\frac{1}{1-\beta}}}{\bar{\tau}}}{\int_0^\infty\left[\frac{e^{-\frac{\beta}{1-\beta}\gamma u}A_u^{\frac{1}{1-\beta}}}{\bar{\tau}}\right]du},\tag{A14}$$

where the integrals in the denominator converge because of the assumption in equation (4). □

PROOF OF COROLLARY 1. It is straightforward to show that

$$\hat{k}_s=\frac{e^{-\frac{\beta}{1-\beta}\gamma s}A_s^{\frac{1}{1-\beta}}}{\eta(s)-\frac{\eta'(s)}{(\rho+\delta)}}\left[\int_0^\infty\left[\frac{e^{-\frac{\beta}{1-\beta}\gamma u}A_u^{\frac{1}{1-\beta}}}{\eta(u)-\frac{\eta'(u)}{(\rho+\delta)}}\right]du\right]^{-1},$$

so the value of $\bar{\tau}$ is irrelevant for $\hat{k}_s$.

PROOF OF PROPOSITION 6. The distribution of capital vintages is

$$\hat{k}_s = \frac{e^{-\frac{\beta}{1-\beta}\gamma s} e^{\frac{\theta s}{1-\beta}} e^{-\omega s}}{\int_0^\infty \left[e^{-\frac{\beta}{1-\beta}\gamma u} e^{\frac{\theta u}{1-\beta}} e^{-\omega u} \right] du}.$$

Then assuming $\beta\gamma - \theta + \omega(1-\beta) > 0$, this reduces to

$$\hat{k}_s = \left[\frac{\beta}{1-\beta}\gamma - \frac{\theta}{1-\beta} + \omega \right] e^{-\left[\frac{\beta}{1-\beta}\gamma - \frac{\theta}{1-\beta} + \omega\right] s}.$$

□

APPENDIX B: PLANNER'S PROBLEM

The Planner's problem is to maximize utility function (1) subject to the resource constraint (6), with production function (2) and the capital accumulation functions (5), and assuming $n_t \in [0, 1]$ for all t. The resource constraint for the planner is then:

$$\int_{-\infty}^{t} u_{vt} dv + c_t \leq \left[\int_{-\infty}^{t} A_{t-v} (z_v k_{vt})^\beta dv \right]^{\frac{\alpha}{\beta}} n^{1-\alpha} - \int_{-\infty}^{t} \delta k_{vt} dv.$$

PROOF OF PROPOSITION 7. First, local nonsatiation of U plus the fact that n does not enter U implies that $n_t = 1$ $\forall t$ in the Planner's solution. Then, the Hamiltonian for the Planner's problem is

$$H(k, c, \mu, \lambda, t) = e^{-\rho t} u(c) + \int \mu(v, t)[u_{vt} - \delta k_{vt}] dv + \lambda$$

$$(t) \left[\int_{-\infty}^{t} u_{vt} dv - \left[\int_{-\infty}^{t} A_{t-v} (z_v k_{vt})^\beta dv \right]^{\frac{\alpha}{\beta}} + \int_{-\infty}^{t} \delta k_{vt} dv - c_t \right].$$

The necessary conditions for a maximum are that, almost everywhere,

$$e^{-\rho t} u'(c) = \lambda(t), \; \mu(v, t) + \lambda(t) = 0 \tag{B1}$$

from the derivative of the Hamiltonian with respect to consumption and investment, and

$$-\mu(v, t)\delta + \lambda(t)\alpha \left[\int_{-\infty}^{t} A_{t-v} (z_v k_{vt})^\beta dv \right]^{\frac{\alpha}{\beta}-1} A_{t-v} (z_v)^\beta k_{vt}^{\beta-1} = -\dot{\mu}_{vt} \tag{B2}$$

from the derivative of the Hamiltonian with respect to k_{vt}. Combining $(B1)$ and $(B2)$, we obtain:

$$\delta + \alpha \left[\int_{-\infty}^{t} A_{t-s}(z_v k_{st})^{\beta} ds \right]^{\frac{\alpha}{\beta}-1} A_{t-v}(z_v)^{\beta} k_{vt}^{\beta-1} = \frac{\dot{\lambda}(t)}{\lambda(t)},$$

which implies that a necessary condition for the Planner's solution is that for any two vintages v, $w \leq t$, it must be that, at all dates t:

$$\frac{k_{vt}}{k_{wt}} = \left(\frac{A_{t-v}(z_v)^{\beta}}{A_{t-w}(z_w)^{\beta}} \right)^{\frac{1}{1-\beta}}, \tag{B3}$$

which ensures that the marginal product of capita is equalized across vintages. Now consider the competitive equilibrium. Households maximize $u(x)$ subject to

$$c_t + \int_{-\infty}^{t} \tau(t-v) u_{vt} dv \leq T_t + w_t + r_t K_t \ \forall t.$$

Firms maximize

$$\max_{K_t, n_t} \left\{ K_t^{\alpha} n_t^{1-\alpha} - w_t n_t - r_t K_t \right\} \forall t.$$

On the production side, we have that all the output goes to the consumers, so the budget constraint along with the tax redistribution and nonsatiation means that the budget constraint equals the resource constraint in equilibrium:

$$w_t + r_t K = y_t = \left[\int_{-\infty}^{t} A_{t-v}(z_v k_{vt})^{\beta} dv \right]^{\frac{\alpha}{\beta}}. \tag{B4}$$

On the demand side, we have the Hamiltonian

$$H(k, c, \mu, \lambda, t) = e^{-\rho t} u(c) + \int \mu(v,t)[u_{vt} - \delta k_{vt}] dv + \lambda(t) \left[\int_{-\infty}^{t} \tau(t-v) u_{vt} dv - w_t - r_t K_t + \int_{-\infty}^{t} \delta k_{vt} dv - c_t \right].$$

The necessary conditions for a maximum include that, almost everywhere,

$$e^{-\rho t} u'(c) = \lambda(t), \ \mu(v,t) + \tau(t-v)\lambda(t) = 0 \tag{B5}$$

and

$$-\mu(v,t)\delta + \lambda(t) \left[\int_{-\infty}^{t} A_{t-v}(z_v k_{vt})^{\beta} dv \right]^{\frac{1}{\beta}-1} A_{t-v}(z_v)^{\beta} k_{vt}^{\beta-1} r = -\dot{\mu}_{vt}. \tag{B6}$$

Combining ($B5$) and ($B6$), we obtain that for any two vintages $v, w \leq t$ for which there is positive investment, it must be that, at all dates t:

$$\begin{aligned}
&\tau(t-v)\delta + \left[\int_{-\infty}^{t} A_{t-v}(z_v k_{vt})^{\beta} dv\right]^{\frac{1}{\beta}-1} A_{t-v}(z_v)^{\beta} k_{vt}^{\beta-1} r_t - \frac{\dot{\lambda}(t)\tau(t-v)}{\lambda(t)} \\
&+ \tau'(t-v) \\
= &\tau(t-w)\delta + \left[\int_{-\infty}^{t} A_{t-v}(z_v k_{vt})^{\beta} dv\right]^{\frac{1}{\beta}-1} A_{t-w}(z_w)^{\beta} k_{wt}^{\beta-1} r_t - \frac{\dot{\lambda}(t)\tau(t-w)}{\lambda(t)} \\
&+\tau'(t-w)
\end{aligned}$$

or

$$\begin{aligned}
&\tau(t-v)\delta + \left[\int_{-\infty}^{t} A_{t-v}(z_v k_{vt})^{\beta} dv\right]^{\frac{1}{\beta}-1} A_{t-v}(z_v)^{\beta} k_{vt}^{\beta-1} r_t + \tau'(t-v) \\
&+ \frac{\left[\rho e^{-\rho t} u'(c) - e^{-\rho t} u''(c)\dot{c}\right]\tau(t-v)}{-e^{-\rho t} u'(c)} \\
= &\tau(t-w)\delta + \left[\int_{-\infty}^{t} A_{t-v}(z_v k_{vt})^{\beta} dv\right]^{\frac{1}{\beta}-1} A_{t-w}(z_w)^{\beta} k_{wt}^{\beta-1} r_t \\
&+ \tau'(t-w) + \frac{\left[\rho e^{-\rho t} u'(c) - e^{-\rho t} u''(c)\dot{c}\right]\tau(t-w)}{e^{-\rho t} u'(c)}
\end{aligned}$$

since the terms $\tau'(t-v) + \frac{[\rho e^{-\rho t} u'(c) - e^{-\rho t} u''(c)\dot{c}]\tau(t-v)}{-e^{-\rho t} u'(c)}$ and $\tau'(t-w) + \frac{[\rho e^{-\rho t} u'(c) - e^{-\rho t} u''(c)\dot{c}]\tau(t-w)}{e^{-\rho t} u'(c)}$ are nonzero and nonequal, the only way to reduce this expression to the necessary condition for the Planner's problem ($B3$) is to have $\tau(t-v) = \bar{\tau}$. Turning to this case, combining ($B5$) and ($B6$), we obtain that:

$$\begin{aligned}
&-\tau e^{-\rho t} u'(c)\delta - e^{-\rho t} u'(c)\left[\int_{-\infty}^{t} A_{t-v}(z_v k_{vt})^{\beta} dv\right]^{\frac{1}{\beta}-1} A_{t-v}(z_v)^{\beta} k_{vt}^{\beta-1} r \\
&= \rho\tau e^{-\rho t} u'(c) - \tau e^{-\rho t} u''(c)\dot{c}, \qquad \text{(B7)}
\end{aligned}$$

which only equals the corresponding equation in the planner's problem if $\tau = 1$. □

APPENDIX C: COMPUTATIONAL PROCEDURE

C.1 Steady-State Computations

We compute the model economy with and without taxation by means of quadrature approximation. Functions such as u_{vt} and k_{vt} are defined continuously. Then each integral required to compute K_t or y_t is evaluated using quadrature approximation by evaluating these functions at small time intervals up to some age T. The age $T =$

1, 000 years was chosen so that vintages $v \geq T$ would be negligible in the production function. The time interval was chosen to be small. Results are reported using 100 time intervals per year: using 1, 000 time intervals per year did not change the results at a precision of six significant figures.

C.2 Transition Dynamics

The procedure is as follows, and involves a series of nested loops. First, at date zero, k_{v0} is given. It equals the steady-state distribution under the policy regime out of which we wish to compute the transition.

Let time increase in increments of Δ. For date Δ, given a guess for r_Δ and y_Δ, we use the discrete time analog of equation (13) to determine the unconstrained capital stock $k_{v\Delta}$, which in the absence of tax wedges is:

$$A_{t+\Delta-v} z_v^\beta \left(\alpha y_{t+\Delta}^{\frac{\alpha-\beta}{\alpha}} - r_{t+\Delta} y_{t+\Delta}^{\frac{1-\beta}{\alpha}} \right) k_{v,t+\Delta}^{\beta-1} = \rho + \delta, t \geq 0.$$

Then, we can compute u_{v0} based on $k_{v\Delta}$ and k_{v0}. If we find that $u_{v\Delta} < 0$ computed in this way, it implies the nonzero investment constraint binds and we set $u_{v0} = 0$. Using this corrected value for u_{v0}, we can use equation (19) to determine $k_{v,\Delta}$. This, in turn, gives us a prediction for what y_Δ should be, using the production function. We can then iterate on y_Δ until it converges.

We can then repeat this procedure for date $t = 2\Delta$, $t = 3\Delta$, and so on, to produce a series for all the variables given the initial guess for the series r_t.

The problem then is how to compute a new guess for the series $\{r_t\}_{t=0}^T$, as well as how to determine the date T at which we assume that convergence is complete. We chose T as 10 years. However, we found that values of T down to 3 did not affect the findings, indicating that convergence was complete within 2 or 3 years.

We consider only "smooth" paths for r_t that converge to the steady state of the model economy in finite time. We ensure that they are smooth by using the Hodrick–Prescott filter, as in Samaniego (2006c) and Samaniego (2008).

There are several decisions to be made for implementing this procedure. The key decision is about the smoothing parameter λ to be used in filtering. We do not want to use a large number because then we would be artificially inserting persistence into the model. What we did was start with a relatively high value, $\lambda = 20$. Then once this had converged, we lowered the value to $\lambda = 10$, and repeated the procedure using the r_t series from the previous iteration. We repeated this procedure for progressively lower values of λ, stopping when $\lambda = 0.2$. At this point, the filtering procedure provides little smoothing. The procedure is similar to simulated annealing (Bertsimas and Tsitsilkis 1993), applied to a dynamic series rather than a set of parameters.

Given a guess $\{r_t\}_{t=0}^T$, we can compute C_t for all dates. We do so by first smoothing the series for output y_t (except at date zero, which is given by k_{v0}) and for investment. The reason for the smoothing is that for an arbitrary $\{r_t\}_{t=0}^T$ series, the resulting series for y_t was often jagged, as was the series for u_{vt}. This led to series for C_t that would then lead to a jagged new guess for r_t. In other words, restricting ourselves to smooth paths is required for convergence. We also limited aggregates from fluctuating within 10% of their long-run values in transition to ensure that our guesses could not diverge arbitrarily from reasonable behavior. As displayed in Figures 10 and 11, these bounds do not bind once the series have converged.

Given a smooth series for C_t, we generate a new predicted $\{r_t\}_{t=0}^{T}$ sequence using the relationship that $r_t = (C_{t+1}/C_t)^{\eta}/\beta - 1 + \delta$, where β is the household's discount factor. We Hodrick–Prescott filter the new r_t series to ensure that it is smooth, and use it as a new guess. We continue until the smoothed sequence r_t no longer changes.

Computing transition dynamics in this manner poses several challenges. The first is that we need to determine a size for the interval between periods. This time interval should be small, but not so small that the simulation takes too long. We assume that there are 10 periods in a year, and measure all rates and parameters with this in mind.

The second is that the state variable of the model economy is a large object, a value of k_{vt} at each date t and for all vintages $v \leq t$. Thus, the simulation requires assuming that for some sufficiently large value of $t - v$, k_{vt} becomes negligible. We set this at 1,000 years, so there are 10,000 vintages active at any point in time.

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