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Abstract

We analyze the role of competition in information provision in random search markets. Multiple symmetric senders compete for the receiver's investment by disclosing information about their respective project qualities, and the receiver conducts random search to learn about the qualities of the projects. We show that in any symmetric Nash equilibrium, each sender chooses a strategy with the lowest possible reservation value. The receiver does not benefit from the competition of the senders, as the receiver's expected payoff does not change when the number of senders increases.

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1 Introduction

We consider a model of competitive information disclosure in random search markets. Multiple symmetric senders, each of whom endowed with a project, compete for the investment of a single receiver by disclosing information about their respective project qualities. The number of senders is small, and the receiver is assumed to observe the strategies of the senders. The search process is modeled to be random, to reflect the various exogenous factors that hinder the receiver's flexibility to conduct directed search. For example, consider the economics job market. Different universities choose what kind of information to disclosure about the match quality between the university and the job market candidates. Even though the job candidates understand the strategies of the universities, she might not be able to conduct directed search—dictating the order in which she visits the universities— due to scheduling concerns.

Formally, we consider a model in which multiple symmetric senders commit to information disclosure mechanisms. The receiver conducts random search, and incurs a search cost/ inspection cost to learn about the qualities of the senders' projects. We show that in any symmetric equilibrium, each sender chooses a strategy with the lowest possible reservation value. The receiver does not benefit from the competition of the senders, as the receiver's expected payoff does not change when the number of senders increases.[1]

Our paper is closely related to Au and Kawai (2020), Au and Whitmeyer (2021), and Whitmeyer (2021). Au and Kawai (2020) analyze competitive information disclosure when there is no search cost. They establish the unique symmetric equilibrium in this game. As the number of senders increases, each sender disclosures information more aggressively, and full disclosure by each sender arises in the limit of infinitely many senders. In contrast, we model a random search market with search frictions, and show that the receiver does not benefit from the competition of the senders. Au and Whitmeyer (2021) also study competitive information disclosure by multiple senders with search frictions. The main focus of their paper is the attraction motive, as the receiver

[1]While this result is reminiscent of the classical Diamond paradox (Diamond (1971)), our model is different; the receiver in our model observes the strategies of the senders, including any deviations from the equilibrium.

conducts directed search. They characterize the unique symmetric equilibrium—the receiver potentially benefits from the competition of the senders. They further consider the case of hidden signals—the firms' signals are not directly observable to the consumer at the outset of her search—and show that the consumer does not find it worthwhile to actively search. In settings in which the senders choose the information disclosure and also set a price, Whitmeyer (2021) shows that there is no symmetric equilibria in which consumers engage in active search, if neither the signal nor the price is observable until a consumer incurs the search cost.[2]

Board and Lu (2018) consider a search setting in which a receiver, at a positive search cost, sequentially samples senders who provide information concerning a common state. In contrast, in our setting, the senders have independent proposals, and they make disclosure simultaneously.

2 The model

There are n senders, each of whom is endowed with a project. They compete for the investment of a single receiver. The quality of sender i's project θ_i is either high (H) or low (L), and is independently and identically distributed across senders. The common prior is that each sender's project is of high quality with probability p. Each sender's objective is to maximize the probability that the receiver invests in his project. Without loss of generality, we normalize each sender's payoff to be 1 if the receiver invests in his project, and 0 otherwise. The receiver's valuation for a project is 1 if its quality is H, and 0 if its quality is L. The receiver invests in at most one project.

The timing of the game is as follows.

(1) At the beginning of the game, each sender i simultaneously commits to an information disclosure mechanism on the quality of his project, which consists of a message space M_i and a joint distribution on $\{H, L\} \times M_i$. It follows from standard Bayesian persuasion arguments (see Kamenica and Gentzkow (2011)) that

[2] Also see Au and Kawai (2019) that study a model of competition in which two senders vie for the patronge of a receiver by disclosing information about the qualities of their respective proposals, which are positively correlated.

each sender i chooses a distribution F_i on $[0,1]$ with mean p. Let $\mathcal{F}$ denote the collection of all such distributions. The chosen information disclosure mechanisms are publicly posted.

(2) The receiver learns about the qualities of the senders' projects through random search. At each stage of the search, the receiver can stop her search and invest in a project of any visited sender. Alternatively, she can incur a search cost of c, visit an unvisited sender, and observe the signal realization. To avoid triviality, we assume that $c < p$.

We focus on symmetric Nash equilibria in which all senders adopt the same strategy and the receiver adopts a tie-breaking rule that treats all senders identically.

3 Equilibrium analysis

3.1 Basics

For any $F \in \mathcal{F}$, let

$$H_F(z) = -c + \int_0^{z-} z \, \mathrm{d}F(x) + \int_z^1 x \, \mathrm{d}F(x).$$[3]

A solution to the equation $H_F(z) = z$ exists and is unique (see Weitzman (1979)). We denote the solution to this equation by v_F, and refer to it as the reservation value of F. It is convenient to use the following rearrangement of the equation $H_F(v_F) = v_F$:

$$c = \int_{v_F}^1 (x - v_F) \, \mathrm{d}F(x).$$

The reservation value plays an important role in our analysis. In particular, adopting similar arguments as in Weitzman (1979) to our setting, we have that for any strategy profile of the senders,

[3]Notation: we use $\int_a^b$ to denote the integral over the interval $[a,b]$, $\int_a^{b-}$ to denote the integral over the interval $[a,b)$, and $\int_{a+}^b$ to denote the integral over the interval $(a,b]$. We use $F(x)$ to denote the measure on the interval $[0,x]$, $F(x-)$ to denote the measure on the interval $[0,x)$, and $F(\{x\})$ to denote the measure of the point x.

(1) the receiver should continue her search if every unvisited sender uses a strategy that has a weakly higher reservation value than the maximum sampled reward and at least one unvisited sender uses a strategy that has a strictly higher reservation value than the maximum sampled reward, and

(2) the receiver should stop search if every unvisited sender uses a strategy that has a weakly lower reservation value than the maximum sampled reward.

While these do not pin down the receiver's optimal search behavior, it suffices for our purpose.

Let F_F denote the full disclosure strategy, that is,

$$F_F(x) = \begin{cases} 1-p, & \text{if } x \in [0,1); \\ 1, & \text{if } x = 1. \end{cases}$$

Let F_N denote the null disclosure strategy, that is,

$$F_N(x) = \begin{cases} 0, & \text{if } x \in [0,p); \\ 1, & \text{if } x \in [p,1]. \end{cases}$$

It is straightforward to calculate that $v_{F_F} = 1 - \frac{c}{p}$ and $v_{F_N} = p - c$.

Lemma 1. *For any $F \in \mathcal{F}$, $0 < p - c \leq v_F \leq 1 - \frac{c}{p} < 1$.*

Proof. It is easy to show that if G is a mean-preserving spread of H, then $v_H \leq v_G$. Since any $F \in \mathcal{F}$ is a mean-preserving contraction of F_F and is a mean-preserving spread of F_N, we have the desired result. □

Lemma 2. *Suppose that $F \in \mathcal{F}$. Then $v_F = p - c$ if and only if $F(v_F-) = 0$.*

Proof. $F(v_F-) = 0 \iff \int_{v_F}^1 (x - v_F)\,\mathrm{d}F(x) = \int_0^1 (x - v_F)\,\mathrm{d}F(x) \iff v_F = p - c.$ □

Lemma 3. *Suppose that $F \in \mathcal{F}$. Then $v_F = 1 - \frac{c}{p}$ if and only if $F = F_F$.*

Proof. For the only if-part, suppose that $v_F = 1 - \frac{c}{p}$ but $F \neq F_F$. Then,

$$c = \int_{v_{F_F}}^{1} (x - v_{F_F})\,\mathrm{d}F_F(x) = \int_{v_F}^{1} (x - v_F)\,\mathrm{d}F_F(x) > \int_{v_F}^{1} (x - v_F)\,\mathrm{d}F(x) = c,$$

where the inequality follows from standard Bayesian persuasion arguments and that F_F is a mean-preserving spread of F. We have a contradiction. □

As a benchmark, suppose that there is a single sender. Regardless of the sender's strategy, the receiver incurs the search cost c to meet the sender, and invests in his project. The receiver's expected payoff is $p - c$.

3.2 Two senders

For the sake of clarity, we first consider the case in which there are two senders. Since each sender can always mimic the strategy of the other sender, in any equilibrium (if it exists), both senders get the same expected payoff of $\frac{1}{2}$. Theorem 1 characterizes the symmetric Nash equilibria when there are two senders.

Theorem 1. *Suppose that $F \in \mathcal{F}$. Then (F, F) is a symmetric equilibrium if and only if*

(1) $v_F = p - c$, and

(2) $1 + F(x) \leq \frac{x}{p-c}$ for all $x \in [p - c, 1 - \frac{c}{p})$.

Theorem 1 says that in any symmetric equilibrium (F, F), the reservation value v_F cannot be higher than $p - c$. Here, we briefly explain the intuition behind this result. If $v_F > p - c$, then either sender, say sender 1, could deviate to another strategy F' with a slightly lower reservation value and $F'([v_F, 1]) > F([v_F, 1])$. Such a deviation has two effects. On the one hand, if the receiver first visits sender 2 and has a posterior on $[v_{F'}, v_F)$, then she will not visit sender 1. On the other hand, if the receiver first visits sender 1 and has a posterior on $[v_F, 1]$ (which has a higher probability under F'), then she will stop search. We construct such an F' under which the second effect dominates the first one.

By Theorem 1, in any symmetric equilibrium (F, F), the receiver meets each sender i with equal probability $\frac{1}{2}$, and invests in his project regardless of the posterior q_i (since, by Lemma 2, $F(v_F-) = 0$). The receiver's expected payoff is $p - c$, the same as her payoff when there is a single sender. In other words, the receiver does not benefit from the competition of the two senders.

Proof of Theorem 1. We classify our analysis into two cases. We first consider the case in which $v_F > p - c$, and show that there is no symmetric Nash equilibrium in this case.[4] We then consider the case in which $v_F = p - c$.

As a preparation, we calculate the payoffs of the senders at every posterior when sender i chooses F_i and sender j chooses F_j. Without loss of generality, assume that $v_{F_i} \le v_{F_j}$. Sender i's payoff at the posterior q_i is

$$\begin{cases} \frac{1}{2}\Big\{F_j(q_i-) + \frac{1}{2}F_j(\{q_i\})\Big\} + \frac{1}{2}\Big\{F_j(q_i-) + \frac{1}{2}F_j(\{q_i\})\Big\}, & \text{if } q_i \in [0, v_{F_i}); \\ \frac{1}{2}\Big\{F_j(q_i-) + \frac{1}{2}F_j(\{q_i\})\Big\} + \frac{1}{2}F_j(v_{F_i}-), & \text{if } q_i \in [v_{F_i}, v_{F_j}); \\ \frac{1}{2} + \frac{1}{2}F_j(v_{F_i}-), & \text{if } q_i \in [v_{F_j}, 1], \end{cases}$$

and sender j's payoff at the posterior q_j is

$$\begin{cases} \frac{1}{2}\Big\{F_i(q_j-) + \frac{1}{2}F_i(\{q_j\})\Big\} + \frac{1}{2}\Big\{F_i(q_j-) + \frac{1}{2}F_i(\{q_j\})\Big\}, & \text{if } q_j \in [0, v_{F_i}); \\ \frac{1}{2} + \frac{1}{2}\Big\{F_i(q_j-) + \frac{1}{2}F_i(\{q_j\})\Big\}, & \text{if } q_j \in [v_{F_i}, v_{F_j}); \\ \frac{1}{2} + \frac{1}{2}F_i(v_{F_j}-), & \text{if } q_j \in [v_{F_j}, 1]. \end{cases}$$

Case I: $p - c < v_F \le 1 - \frac{c}{p}$. We show that there does not exist a symmetric Nash equilibrium in this case. Suppose to the contrary, there exists a Nash equilibrium (F, F) with $p - c < v_F \le 1 - \frac{c}{p}$. Step 1 - Step 4 below establish properties that F necessarily satisfies. Step 5 shows that sender 1 has a profitable deviation, which contradicts that

[4] This further implies that there is no (symmetric or asymmetric) equilibrium in which the reservation value of some sender's strategy is greater than $p - c$. To see this, suppose that (F_1, F_2) is a Nash equilibrium where $v_{F_i} > p - c$ for some i. By symmetry, (F_2, F_1) is also a Nash equilibrium. The inter-changeability property of zero-sum games (see Osborne and Rubinstein (1994, Proposition 22.2)) implies that (F_i, F_i) is also a Nash equilibria.

(F, F) is a Nash equilibrium.

Step 1. By Lemma 2, $F(v_F-) \neq 0$.

Step 2. F has no jumps on $(0, v_F)$.

Suppose that F has a jump at some $z \in (0, v_F)$. We show that there exists a profitable deviation for sender 1, which contradicts that (F, F) is a Nash equilibrium. Let $G = G_1 + G_2$, where

1. G_1 is a finite measure with total measure $1 - F(\{z\})$ such that $G_1(A) = F(A)$ for any $A \subseteq [0, z) \cup (z, 1]$, and
2. G_2 is a finite measure with total measure $F(\{z\})$ such that $G_2(\{z - 2\epsilon\}) = \frac{1}{3}F(\{z\})$ and $G_2(\{z + \epsilon\}) = \frac{2}{3}F(\{z\})$, where $\epsilon > 0$ is sufficiently small such that $0 < z - 2\epsilon < z < z + \epsilon < v_F$.

Clearly, G is a probability measure, and has the same mean and reservation value as F. The difference of sender 1's payoff under G and F is

$$\frac{1}{3}F(\{z\})\big(\frac{1}{2}F((z-2\epsilon)-) + \frac{1}{2}F(z-2\epsilon)\big) + \frac{2}{3}F(\{z\})\big(\frac{1}{2}F((z+\epsilon)-) + \frac{1}{2}F(z+\epsilon)\big)$$
$$- F(\{z\})\big(\frac{1}{2}F(z-) + \frac{1}{2}F(z)\big),$$

which converges to $\frac{1}{6}F(\{z\})\big(F(z) - F(z-)\big) > 0$ as $\epsilon \to 0$. Thus, there exists a profitable deviation for sender 1.

Step 3. $F(0) = 0$.

Suppose that $F(0) > 0$. We show that there exists a profitable deviation for sender 1. For sufficiently small $\epsilon > 0$, let χ and ϵ' be such that

$$\chi = F(0) + \frac{c}{1 - v_F} - \frac{c}{1 - \epsilon - v_F} \text{ and } \chi \cdot \epsilon' + \frac{c}{1 - \epsilon - v_F}(1 - \sqrt{\epsilon}) = \frac{c}{1 - v_F}.$$

Since $\epsilon' \to 0$ as $\epsilon \to 0$, we can choose ϵ such that $0 < \chi < F(0)$ and $0 < \epsilon' < v_F <$

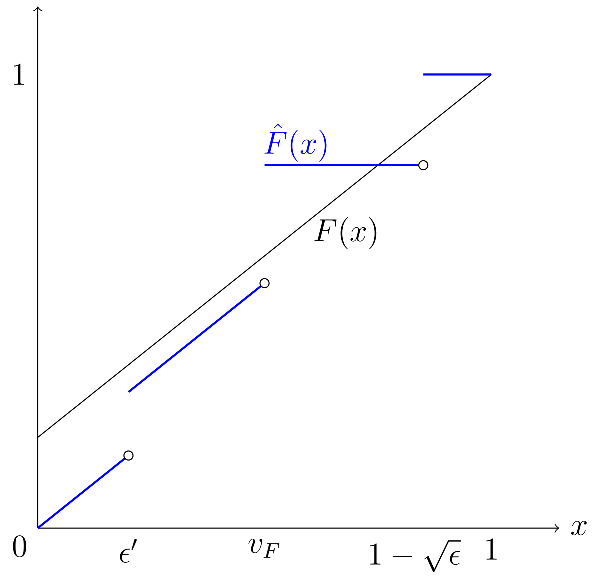

Figure 1: F and $\hat{F}$ in Step 2 of the proof (for illustration purposes only).

$1-\sqrt{\epsilon} < 1$. Consider the following distribution $\hat{F}$ (see Figure 1):

$$\hat{F}(x) = \begin{cases} F(x) - F(0), & \text{if } x \in [0, \epsilon'); \\ F(x) - F(0) + \chi, & \text{if } x \in [\epsilon', v_F); \\ 1 - \frac{c}{1-v_F} - F(0) + \chi, & \text{if } x \in [v_F, 1-\sqrt{\epsilon}); \\ 1, & \text{if } x \in [1-\sqrt{\epsilon}, 1]. \end{cases}$$

$\hat{F}$ and F have the same mean, since

$$\begin{aligned} &\int_0^1 x \, \mathrm{d}\hat{F}(x) - \int_0^1 x \, \mathrm{d}F(x) \\ &= \chi \cdot \epsilon' + \big(1 - \frac{c}{1-v_F} - F(v_F-)\big) \cdot v_F + (\frac{c}{1-v_F} + F(0) - \chi) \cdot (1-\sqrt{\epsilon}) - \int_{v_F}^1 x \, \mathrm{d}F(x) \\ &= \Big\{\chi \cdot \epsilon' + \frac{c}{1-\epsilon-v_F} \cdot (1-\sqrt{\epsilon}) - \frac{c}{1-v_F}\Big\} + \Big\{c + \big(1 - F(v_F-)\big) \cdot v_F - \int_{v_F}^1 x \, \mathrm{d}F(x)\Big\} \\ &= 0, \end{aligned}$$

where the last line uses the definition of the reservation value.

Since

$$\int_{v_F}^{1} (x - v_F)\,\mathrm{d}\hat{F}(x) - c = \frac{c}{1-\epsilon-v_F} \cdot (1 - \sqrt{\epsilon} - v_F) - c < 0$$

and

$$\int_{v_F+\epsilon-\sqrt{\epsilon}}^{1} \left(x - (v_F + \epsilon - \sqrt{\epsilon})\right) \mathrm{d}\hat{F}(x) - c = \int_{v_F+\epsilon-\sqrt{\epsilon}}^{v_F} \left(x - (v_F + \epsilon - \sqrt{\epsilon})\right) \mathrm{d}\hat{F}(x) > 0,$$

we have $v_F+\epsilon-\sqrt{\epsilon} < v_{\hat{F}} < v_F$. As $\epsilon \to 0$, $v_{\hat{F}} \to v_F$ and $F(v_{\hat{F}}) \to F(v_F-)$. Furthermore, $0 < \epsilon' < v_{\hat{F}} < v_F < 1 - \sqrt{\epsilon} < 1$ for $\epsilon > 0$ sufficiently small.

The difference of sender 1's payoff under $\hat{F}$ and F is

$$\begin{aligned} &\chi \cdot F(\epsilon') - F(0) \cdot \frac{1}{2}F(0) \\ &+ \int_{v_{\hat{F}}}^{v_F-} \left[\frac{1}{2}F(x) + \frac{1}{2}F(v_{\hat{F}})\right] \mathrm{d}\hat{F}(x) - \int_{v_{\hat{F}}}^{v_F-} F(x)\,\mathrm{d}F(x) \\ &+ \left(1 - \hat{F}(v_F-)\right) \left[\frac{1}{2} + \frac{1}{2}F(v_{\hat{F}})\right] - \left(1 - F(v_F-)\right) \left[\frac{1}{2} + \frac{1}{2}F(v_F-)\right], \end{aligned}$$

where the first line captures the payoff difference on the interval $[0, v_{\hat{F}})$, the second line captures the payoff difference on the interval $[v_{\hat{F}}, v_F)$, and the third line captures the payoff difference on the interval $[v_F, 1]$. The total difference converges to $F(0)\cdot\frac{1}{2}F(0) > 0$ as $\epsilon \to 0$, since $\chi \cdot F(\epsilon')$ converges to $F(0) \cdot F(0)$, both the two terms in the second line converge to 0, and the two terms in the third line converge to the same value. Thus, there exists a profitable deviation for sender 1.

Step 4. F is linear on $[0, x_F]$ for some $0 < x_F < v_F$ and flat on $[x_F, v_F)$.

Since $F(0) = 0$ and F has no jumps on $(0, v_F)$, the payoff of sender 1 at any posterior $q_1 \in [0, v_F)$ is $F(q_1)$. Thus, F has to be linear on $[0, x_F)$ for some $0 < x_F \leq v_F$ and flat on $[x_F, v_F)$. Otherwise, sender 1 could do a mean-preserving spread or a mean-preserving contraction on $[0, v_F)$ without changing the reservation value to obtain a higher payoff. Next, we show that $x_F < v_F$. Since sender 1's payoff at the posterior v_F is $\frac{1}{2} + \frac{1}{2}F(v_F-) > F(v_F-)$, the payoff of sender 1 has a jump at the posterior v_F. Thus, if F is linear on $[0, v_F)$, sender 1 could do a mean-preserving spread on $[0, v_F]$ without changing the reservation value to obtain a higher payoff.

Step 5. Sender 1 has a profitable deviation.

Let q denote the slope of F on $[0, x_F]$. Consider the following strategy F' (see Figure 2):

$$F'(x) = \begin{cases} 0, & \text{if } x \in [0, y); \\ q(x-y), & \text{if } x \in [y, x_F); \\ q(x_F - y), & \text{if } x \in [x_F, v_F); \\ q(x_F - y) + \kappa, & \text{if } x \in [v_F, 1); \\ 1, & \text{if } x = 1, \end{cases}$$

where $y > 0$ is sufficiently small, and

$$\kappa = \frac{\frac{1}{2}q(x_F^2 - y^2) + 1 - q(x_F - y) - p}{1 - v_F}$$

such that the mean of F' is p. We claim that

$$v_{F'} = \frac{p - c - \frac{1}{2}q(x_F^2 - y^2)}{1 - q(x_F - y)}$$

for $y > 0$ sufficiently small. Indeed,

$$v_{F'} = \frac{p - c - \frac{1}{2}q(x_F^2 - y^2)}{1 - q(x_F - y)} < \frac{p - c - \frac{1}{2}qx_F^2}{1 - qx_F} = v_F{}^{[5]}$$

for $y > 0$ sufficiently small, $v_{F'} \to v_F$ as $y \to 0$, and $v_{F'}$ satisfies that

$$\kappa \cdot (v_F - v_{F'}) + \big(1 - q(x_F - y) - \kappa\big) \cdot (1 - v_{F'}) = c.$$

Pick y sufficiently small such that $0 < y < x_F < v_{F'} < v_F$. Sender 1's payoff by using

[5]Since F is linear on $[0, x_F]$ with slope q and flat on $[x_F, v_F)$,

$$\int_0^1 x \, \mathrm{d}F(x) = \int_0^{x_F} x \, \mathrm{d}F(x) + \int_{v_F}^1 x \, \mathrm{d}F(x) = \frac{1}{2}qx_F^2 + v_F \cdot (1 - qx_F) + c = p.$$

The last equality follows.

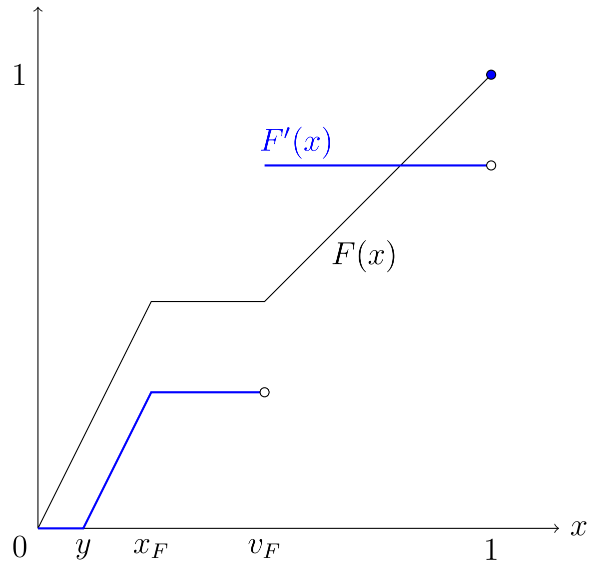

Figure 2: F and F' in Step 5 of the proof (for illustration purposes only).

F' when sender 2 uses F is

$$\begin{aligned}
&\int_y^{x_F} F(x)\,\mathrm{d}F'(x) + \big(1 - F'(v_F-)\big)\big(\frac{1}{2} + \frac{1}{2}F(v_{F'})\big) \\
&= \frac{1}{2}q^2(x_F^2 - y^2) + \big(1 - q(x_F - y)\big)\big(\frac{1}{2} + \frac{1}{2}qx_F\big) \\
&= \frac{1}{2} + \frac{1}{2}qy\big(1 + q(x_F - y)\big) \\
&> \frac{1}{2}.
\end{aligned}$$

Thus, sender 1 has a profitable deviation. This completes the analysis of Case I.

Case II: $v_F = p - c$**.** We now consider the case in which $v_F = p - c$.

Step 6. If $1 + F(x) \le \frac{x}{p-c}$ *for all* $x \in [p - c, 1 - \frac{c}{p})$*, then* (F, F) *is a Nash equilibrium.*

We show that neither sender has a profitable deviation. By symmetry, we only show this for sender 1. Since $v_F = p - c$, by Lemma 2, we have $F(v_F-) = 0$. If sender 1 deviates to some F' with $v_{F'} \in [p - c, 1 - \frac{c}{p})$, then his payoff is

$$\int_{v_F}^{v_{F'}-} \left[\frac{1}{2} + \frac{1}{2}\left[F(x-) + \frac{1}{2}F(\{x\})\right]\right] \mathrm{d}F'(x) + \int_{v_{F'}}^{1} \left[\frac{1}{2} + \frac{1}{2}F(v_{F'}-)\right] \mathrm{d}F'(x)$$

$$
\begin{aligned}
&\leq \int_{v_F}^{v_{F'}-} \frac{x}{2(p-c)} \,\mathrm{d}F'(x) + \int_{v_{F'}}^{1} \frac{v_{F'}}{2(p-c)} \,\mathrm{d}F'(x) \\
&\leq \frac{1}{2(p-c)} \left[p - \int_{v_{F'}}^{1} x \,\mathrm{d}F'(x) + \int_{v_{F'}}^{1} v_{F'} \,\mathrm{d}F'(x) \right] \\
&= \frac{1}{2},
\end{aligned}
$$

where the last line uses the definition of the reservation value. If sender 1 deviates to F'' with $v_{F''} = 1 - \frac{c}{p}$ (by Lemma 3, $F'' = F_F$), then his payoff is

$$
p\left[\frac{1}{2} + \frac{1}{2}F(v_{F''}-)\right] \leq p\frac{v_{F''}}{2(p-c)} = \frac{1}{2}.
$$

Thus, sender 1 does not have a profitable deviation.

Step 7. If (F, F) is a Nash equilibrium, then $1 + F(x) \leq \frac{x}{p-c}$ for all $x \in [p-c, 1-\frac{c}{p})$.

Suppose to the contrary, there is some $x^* \in [p-c, 1-\frac{c}{p})$ such that $1 + F(x^*) > \frac{x^*}{p-c}$. Since F is right continuous, there exists some $y \in [x^*, 1-\frac{c}{p})$ such that F is continuous at y and $1 + F(y) > \frac{y}{p-c}$. Let

$$
F'(x) = \begin{cases} 1 - \frac{c}{1-y} - \frac{1}{y}(p - \frac{c}{1-y}) & \text{if } x \in [0, y); \\ 1 - \frac{c}{1-y} & \text{if } x \in [y, 1); \\ 1 & \text{if } x = 1. \end{cases}
$$

Clearly, $F' \in \mathcal{F}$ and $v_{F'} = y$. Sender 1's payoff by using the strategy F' when sender 2 uses the strategy F is

$$
\left[\frac{c}{1-y} + \frac{1}{y}(p - \frac{c}{1-y})\right]\left[\frac{1}{2} + \frac{1}{2}F(y)\right] > \left[\frac{p}{y} - \frac{c}{y}\right]\frac{y}{2(p-c)} = \frac{1}{2}.
$$

Thus, sender 1 has a profitable deviation, and (F, F) is not a Nash equilibrium. We have a contradiction.

This completes the proof of Theorem 1. $\square$

3.3 More than two senders

Suppose that $n \geq 3$. Clearly, in any symmetric equilibrium (if it exists), all senders get the same expected payoff of $\frac{1}{n}$.

Theorem 2. *Suppose that $F \in \mathcal{F}$.*

(1) If $(F, F, \dots, F)$ is a symmetric equilibrium, then

$$v_F = p - c.$$

(2) If $v_F = p - c$ and $\sum_{k=0}^{n-1} F^k(x) \leq \frac{x}{p-c}$ for all $x \in [p-c, 1-\frac{c}{p})$, then $(F, F, \dots, F)$ is a symmetric equilibrium.

By Theorem 2, in any symmetric equilibrium, the receiver meets each sender i with equal probability $\frac{1}{n}$, and invests in his project regardless of the posterior q_i (since, by Lemma 2, $F(v_F-) = 0$). The receiver's expected payoff is $p - c$, the same as her payoff when there is a single sender. In other words, the receiver does not benefit from the competition of the senders.

Proof of Theorem 2. We first consider the case in which $v_F > p-c$, and show that there is no such symmetric equilibrium. We then consider the case in which $v_F = p - c$.

Case I*: $p - c < v_F \leq 1 - \frac{c}{p}$. Step 1* - Step 4* below establish properties that F necessarily satisfies. Step 5* shows that sender 1 has a profitable deviation, which contradicts that $(F, F, \dots, F)$ is a Nash equilibrium.

Step 1*. By Lemma 2, $F(v_F-) \neq 0$.

Step 2*. F has no jumps on $(0, v_F)$.

Suppose that F has a jump at some $z \in (0, v_F)$. We show that sender 1 has a profitable deviation. Let $G = G_1 + G_2$ where

1. G_1 is a finite measure with total measure $1 - F(\{z\})$ such that $G_1(A) = F(A)$ for any $A \subseteq [0, z) \cup (z, 1]$, and

2. G_2 is a finite measure with total measure $F(\{z\})$ such that $G_2(\{z-(n-1)\epsilon\}) = \frac{1}{n}F(\{z\})$ and $G_2(\{z+\epsilon\}) = \frac{n-1}{n}F(\{z\})$, where $\epsilon > 0$ is sufficiently small such that $0 < z-(n-1)\epsilon < z < z+\epsilon < v_F$.

Clearly, G is a probability measure, and has the same mean and reservation value as F. The difference of sender 1's expected payoff under G and F is at least (in the first line below we break ties against sender 1 for the calculation of sender 1's payoff at the posteriors $z-(n-1)\epsilon$ and $z+\epsilon$)

$$\begin{aligned}
&\frac{1}{n}F(\{z\})F^{n-1}((z-(n-1)\epsilon)-) + \frac{n-1}{n}F(\{z\})F^{n-1}((z+\epsilon)-) \\
&- F(\{z\}) \sum_{0\le j\le n-1} \left[\frac{1}{j+1}\frac{(n-1)!}{j!(n-j-1)!}F^{n-j-1}(z-)F^{j}(\{z\})\right] \\
&= \frac{1}{n}F(\{z\})F^{n-1}((z-(n-1)\epsilon)-) + \frac{n-1}{n}F(\{z\})F^{n-1}((z+\epsilon)-) \\
&- F(\{z\})\frac{F^n(z)-F^n(z-)}{n(F(z)-F(z-))},
\end{aligned}$$

which converges to

$$\begin{aligned}
&\frac{F(\{z\})}{n}\left[F^{n-1}(z-) + (n-1)F^{n-1}(z) - \frac{F^n(z)-F^n(z-)}{F(z)-F(z-)}\right] \\
=&\frac{F(\{z\})}{n}\left[F^{n-1}(z-) + (n-1)F^{n-1}(z) - \sum_{0\le j\le n-1} F^{n-j-1}(z)F^{j}(z-)\right] > 0
\end{aligned}$$

as $\epsilon \to 0$.

Step 3.* $F(0) = 0$.

Suppose that $F(0) > 0$. We show that there exists a profitable deviation for sender 1. For sufficiently small $\epsilon > 0$, let χ and ϵ' be such that

$$\chi = F(0) + \frac{c}{1-v_F} - \frac{c}{1-\epsilon-v_F} \text{ and } \chi\cdot\epsilon' + \frac{c}{1-\epsilon-v_F}(1-\sqrt{\epsilon}) = \frac{c}{1-v_F}.$$

As $\epsilon' \to 0$ when $\epsilon \to 0$, we can choose ϵ such that $0 < \chi < F(0)$ and $0 < \epsilon' < v_F <$

$1-\sqrt{\epsilon}<1$. Consider the following distribution $\hat{F}$:

$$\hat{F}(x)=\begin{cases} F(x)-F(0), & \text{if } x\in[0,\epsilon'); \\ F(x)-F(0)+\chi, & \text{if } x\in[\epsilon',v_F); \\ 1-\frac{c}{1-v_F}-F(0)+\chi, & \text{if } x\in[v_F,1-\sqrt{\epsilon}); \\ 1, & \text{if } x\in[1-\sqrt{\epsilon},1]. \end{cases}$$

By the analysis in the two-sender case, $\hat{F}$ and F have the same mean, $v_{\hat{F}}\to v_F$ and $F(v_{\hat{F}})\to F(v_F-)$ as $\epsilon\to 0$, and $0<\epsilon'<v_{\hat{F}}<v_F<1-\sqrt{\epsilon}<1$ for $\epsilon>0$ sufficiently small.

The difference of sender 1's payoff under $\hat{F}$ and F is at least

$$\begin{aligned} &\chi\cdot F^{n-1}(\epsilon')-F(0)\cdot\frac{1}{n}F^{n-1}(0) \\ &+\int_{v_{\hat{F}}}^{v_F-}\frac{1}{n}F^{n-1}(x)\,\mathrm{d}\hat{F}(x)-\int_{v_{\hat{F}}}^{v_F-}F^{n-1}(x)\,\mathrm{d}F(x) \\ &+\big(1-\hat{F}(v_F-)\big)\frac{1}{n}\sum_{k=0}^{n-1}F^k(v_{\hat{F}})-\big(1-F(v_F-)\big)\frac{1}{n}\sum_{k=0}^{n-1}F^k(v_F-), \end{aligned}$$

since

(1) in the first term of the second line, for the calculation of sender 1's payoff at any posterior $q_1\in[v_{\hat{F}},v_F-)$, we only include the scenario in which the receiver visits sender 1 first (which happens with probability $\frac{1}{n}$) and all senders other than sender 1 have posteriors less than q_1, and

(2) in the first term of the third line, for the calculation of sender 1's payoff at any posterior $q_1\in[v_F,1]$, we only include the scenarios in which all senders other than sender 1 have posteriors weakly less than $v_{\hat{F}}$.

This lower bound converges to $\frac{n-1}{n}F^n(0)>0$ as $\epsilon\to 0$. Thus, sender 1 has a profitable deviation.

Step 4. F^{n-1} is linear on $[0,x_F]$ for some $0<x_F<v_F$ and flat on $[x_F,v_F)$.*

Since $F(0) = 0$ and F has no jumps on $(0, v_F)$, the payoff of sender 1 at any posterior $q_1 \in [0, v_F)$ is $F^{n-1}(q_1)$. The argument is similar to that in the case of two senders.

Step 5. Sender* 1 *has a profitable deviation.*

Let q denote the slope of F on $[0, x_F]$, and let $\hat{q} = q^{\frac{1}{n-1}}$. Consider the following strategy F':

$$F'(x) = \begin{cases} 0, & \text{if } x \in [0, y); \\ \hat{q}(x-y)^{\frac{1}{n-1}}, & \text{if } x \in [y, x_F); \\ \hat{q}(x_F-y)^{\frac{1}{n-1}}, & \text{if } x \in [x_F, v_F); \\ \hat{q}(x_F-y)^{\frac{1}{n-1}} + \kappa, & \text{if } x \in [v_F, 1); \\ 1, & \text{if } x = 1, \end{cases}$$

where $y > 0$ is sufficiently small, and

$$\kappa = \frac{\frac{\hat{q}}{n}(x_F-y)^{\frac{n}{n-1}} + \hat{q}y(x_F-y)^{\frac{1}{n-1}} + 1 - \hat{q}(x_F-y)^{\frac{1}{n-1}} - p}{1 - v_F}$$

such that the mean of F' is p. Using similar arguments as in the case of two senders, we can show that (1) $v_{F'} < v_F$ for $y > 0$ sufficiently small, and (2) $v_{F'} \to F$ as $y \to 0$. Pick y sufficiently small such that $0 < y < x_F < v_{F'} < v_F$. Thus, both F and F' have zero measure on $[v_{F'}, v_F)$, and sender 1's payoff by using F' when all the other senders use F is

$$\begin{aligned} &\int_y^{x_F} F^{n-1}(x)\, \mathrm{d}F'(x) + \big(1 - F'(v_F-)\big) \left[\frac{1}{n}\sum_{k=0}^{n-1} F^k(v_{F'})\right] \\ &= \frac{1}{n}\hat{q}^n(x_F-y)^{\frac{n}{n-1}} + \hat{q}^n y(x_F-y)^{\frac{1}{n-1}} + (1 - \hat{q}(x_F-y)^{\frac{1}{n-1}}) \left[\frac{1}{n}\sum_{k=0}^{n-1} \hat{q}^k x_F^{\frac{k}{n-1}}\right] \\ &= \frac{1}{n} + \frac{n-1}{n}\hat{q}^n y(x_F-y)^{\frac{1}{n-1}} + \frac{1}{n}\hat{q}(x_F^{\frac{1}{n-1}} - (x_F-y)^{\frac{1}{n-1}}) \sum_{k=0}^{n-2} \hat{q}^k x_F^{\frac{k}{n-1}} \\ &> \frac{1}{n}. \end{aligned}$$

Thus, sender 1 has a profitable deviation. This completes the analysis of Case I*.

Case II*: $v_F = p - c$**.** We now consider the case in which $v_F = p - c$.

Step 6. If* $\sum_{k=0}^{n-1} F^k(x) \leq \frac{x}{p-c}$ *for all* $x \in [p-c, 1-\frac{c}{p})$*, then* $(F, F, \ldots, F)$ *is a Nash equilibrium.*

We show that none of the senders has a profitable deviation. By symmetry, we only show this for sender 1. Since $v_F = p - c$, by Lemma 2, we have $F(v_F-) = 0$. If sender 1 deviates to some F' with $v_{F'} \in [p-c, 1-\frac{c}{p})$, then his payoff is at most

$$\begin{aligned}
&\int_{v_F}^{v_{F'}-} \frac{1}{n} \sum_{k=0}^{n-1} F^k(x)\, \mathrm{d}F'(x) + \int_{v_{F'}}^{1} \frac{1}{n} \sum_{k=0}^{n-1} F^k(v_{F'})\, \mathrm{d}F'(x) \\
\leq &\int_{v_F}^{v_{F'}-} \frac{x}{n(p-c)}\, \mathrm{d}F'(x) + \int_{v_{F'}}^{1} \frac{v_{F'}}{n(p-c)}\, \mathrm{d}F'(x) \\
\leq &\frac{1}{n(p-c)} \left[p - \int_{v_{F'}}^{1} x\, \mathrm{d}F'(x) + \int_{v_{F'}}^{1} v_{F'}\, \mathrm{d}F'(x) \right] \\
= &\frac{1}{n},
\end{aligned}$$

since for both terms in the first line we relax the calculations in two aspects: (a) the receiver always continues searching if sender 1 has not been visited and the maximum sampled reward so far is weakly less than $v_{\hat{F}}$; (b) the receiver always invests in sender 1's project whenever there is a tie. If sender 1 deviates to F'' with $v_{F''} = 1 - \frac{c}{p}$ (that is, $F'' = F_F$), then sender 1's payoff is at most

$$p \left[\frac{1}{n} \sum_{k=0}^{n-1} F^k(v_{F''}) \right] \leq p \frac{v_{F''}}{n(p-c)} = \frac{1}{n}.$$

Thus, sender 1 does not have a profitable deviation.

This completes the proof of Theorem 2. □

4 Outside option

Our analysis can be readily extended to the case in which the receiver has an outside option u_0. Clearly,

(1) if $u_0 < p - c$, then our analysis in Section 3 remains unchanged; and

(2) if $u_0 > 1 - \frac{c}{p}$, then the receiver will not search at all.

In what follows, we consider the case in which $u_0 \in [p - c, 1 - \frac{c}{p}]$.

Suppose that there is only one sender. As in the Bayesian persuasion literature, we break ties in favor of the sender; that is, the receiver continues search if the reservation value of the sender's strategy equals u_0, and invests in the sender's project if the realized posterior is u_0. Without loss of generality, the sender uses a strategy with a reservation value that is at least u_0. It is easy to verify that, if the sender uses a strategy with a reservation value $v \geq u_0$, then the highest payoff of the sender is

$$\frac{c}{1-v} + \frac{p - \frac{c}{1-v}}{u_0},$$

which is decreasing in v, by using a strategy that places probability $\frac{c}{1-v}$ on 1, $\frac{p-\frac{c}{1-v}}{u_0}$ on u_0, and the remaining probability on 0. Thus, the sender would choose a strategy with the reservation value u_0. The receiver incurs the search cost c to meet the sender, and invests in his project if and only if the realized posterior is weakly higher than u_0. The receiver's expected payoff is u_0.

Now suppose that there are $n \geq 2$ senders. Using similar arguments as in the proof of Theorem 2, we can show that in any symmetric Nash equilibrium $(F, F, \ldots, F)$, $v_F = u_0$. Thus, the receiver's expected payoff is u_0, the same as her payoff when there is a single sender. The receiver does not benefit from the competition of the senders.

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