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STATISTICS



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Local dominance*

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Abstract

We define notions of dominance between two actions in a dynamic game. Local dominance considers players who have a blurred view of the future and compare the two actions by first focusing on the outcomes that may realize at the current stage. When considering the possibility that the game may continue, they can only check that the local comparison is not overturned under the assumption of "continuing in the same way" after the two actions (in a newly defined sense). Despite the lack of forward planning, local dominance solves dynamic mechanisms that were found easy to play and implements social choice functions that cannot be implemented in obviously-dominant strategies.

Keywords: weak dominance, obvious dominance, strategy-proofness.

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1 Introduction

The point of view under discussion may be symbolized by the proverb “Look before you leap”, and the one to which it is opposed by the proverb “You can cross that bridge when you come to it”. [...] To cross one’s bridges when one comes to them means to attack relatively simple problems of decisions by artificially confining attention to so small a world that the principle of “Look before you leap” can be applied there. I am unable to formulate criteria for selecting these small worlds and indeed believe that their selection may be a matter of judgement and experience about which it is impossible to enunciate complete and sharply defined general principles. (Savage, 1954)

Traditionally, game theory has treated any given game as a small world in which players can scrutinize all possible contingencies and anticipate all their future decisions. But empirical and experimental evidence have shown that this accurate analysis of the game may be too hard for real players. Li (2017) addressed players’ difficulties with contingent reasoning by introducing *obvious dominance*: a strategy obviously dominates another strategy if, at each decision node from which the two strategies depart, the worst outcome that is still possible under the first strategy is not worse than the best outcome that is still possible under the second strategy. With this, obvious dominance explores the idea that dynamic mechanisms may be easier to play because they allow to compare strategies under smaller outcome sets. Pycia and Troyan (2022) further address players’ difficulties with planning by introducing *strong obvious dominance*: an action is strongly obviously dominant at a decision node if the worst outcome that may follow the first action is not worse than the best outcome that may follow any other action. Thus, strong obvious dominance explores the idea that players compare actions without a forward plan.

In this paper, we take the following perspective on players’ choices in dynamic games. Instead of planning at the outset, players tackle one choice problem at a time — they “cross a bridge when they come to it” — and compare

the available actions by first focusing on their *possible immediate consequences*. At the same time, players realize that, under some actions, the game may continue — they “look before they leap” —, but their view of the continuation game is blurred, so they only look for confirmation of the local comparison through simple considerations about the future. We introduce a notion of *local dominance* between two actions that captures this perspective on the game.

To illustrate, consider the “Japanese” version of a single-unit, ascending-price auction with a discrete clock. At each stage, players simultaneously choose between “leaving” and “bidding”. The object is assigned at the current price either when one player bids, and then she is the winner, or when no player bids, in which case the winner is determined at random. Take the viewpoint of a player who values the object above the current price. Bidding may immediately yield the object for sure, if no one else bids, or let the auction continue, otherwise. Before considering the latter scenario, which requires foresight, it may be natural to compare the possible, immediate outcomes of the two actions in the former scenario. Since leaving only yields the object with some probability, bidding beats leaving. This comparison is simple because it only involves the few outcomes associated with the current price. By contrast, in a sealed-bid, second-price auction, each bid can result in winning the object at many different prices. In general, comparing actions in a static mechanism may be difficult because each action can immediately induce a plethora of outcomes. Dynamic mechanisms, instead, draw players’ attention to the few outcomes that can realize at each stage. Local dominance captures this simplicity factor of dynamic mechanisms. By contrast, obvious and strong obvious dominance drag into the same picture present and future outcomes of a strategy/action. As a consequence, in the ascending auction, bidding until the price reaches one’s valuation is not obviously dominant, because it may result in eventually losing the object, while leaving earlier may result in winning the object at the lottery. Local dominance, instead, focuses first on the possible *immediate* outcomes of *both* actions. This is a coarse form of contingent reasoning, arising from the separate treatment of present and future of the game in the player’s mind.

Players might base their choices *only* on their possible immediate conse-

quences and just ignore the continuation game as too complicated to analyze.¹ However, they may also be concerned that the local comparison would be overturned if the game continues, in ways they cannot fully scrutinize. Because of this concern, such players may even settle an action that ranks clearly below another action in terms of possible immediate outcomes.² Local dominance ensures that players can find confirmation of the local comparison without forward planning, and yet without careful scrutiny of *all* possible future outcomes.

Consider first the possibility that the game continues after the candidate dominating action but not after the alternative. In the auction, this occurs when not just our player but also some opponent bids. In this scenario, while leaving implies losing the object, the final outcome after bidding will depend on the future moves. However, our player does not reason about how she will actually play. Instead, she only entertains the idea of leaving at the next stage, without coming up with any creative alternative. By doing so, she excludes the possibility of a loss after bidding, confirming its superiority to leaving. But what makes “leaving at the next stage” a salient continuation strategy? In our view, the fact that it “mimics” the alternative under consideration (in a sense we will make precise). Local dominance will endogenize in this way what continuation strategy is simple for the player, requiring that it is available and that it confirms the local comparison.

In other situations, the game may continue after both actions under comparison. To give an example, add action “wait” to the ascending-price auction. Waiting differs from leaving only in that, if the auction continues, a player who waited can still move. The opponents will not observe whether our player waited or bid. But then, which of the two actions she chose makes no difference whatsoever for the future: she can play in the same way after the two actions,

¹Our positive results for dynamic mechanisms would obviously hold through if local dominance was defined with just the comparison of the outcomes that can realise when both actions terminate the game.

²For instance, in the dynamic translation of the Top Trading Cycles allocation rule where players demand one of the still-available objects at a time, a player may be tempted to ask for a less-preferred object if she fears that the opportunity to obtain it will fade and the opportunity to obtain a more-preferred one will never arise. We will address this concern with specific game rules and local dominance in Section 5.2.

and then she will also get the same outcome. Local dominance will thus require that the scenario in which the game continues after both actions is irrelevant for the choice and can be rightfully ignored. Realising that the present choice cannot have any impact on the continuation game does not truly require to scrutinize all possible ways in which the game may continue, but just a rough understanding of the game pattern. Recognizing the invariance of future outcomes could also be easier than finding best and worst outcomes across two heterogenous outcome sets, because it does not require to understand the details of a possibly complicated outcome rule.³

In static game, local and (strong) obvious dominance coincide. But in a dynamic game, even a strongly obviously dominant action need not be locally dominant. Yet, local dominance provides a possible explanation of why some dynamic mechanisms that are not obviously strategy-proof were found easy to play, and it yields positive implementation results for relevant social choice functions that cannot be implemented in obviously dominant strategies. The “Japanese” ascending-price auction (with the random tie-breaking rule) was found easy to play in the experimental work of Kagel et al. (1987). Moreover, we construct a dynamic mechanism that implements the Top Trading Cycles allocation rule in locally dominant actions, although (as shown by Li, 2017) no obviously strategy-proof mechanism can implement it.⁴

Local dominance builds on a more general approach that we develop, whereby players compare actions rather than strategies, under a partition of the contingencies and without a plan for the future. We start with an exogenously given partition of the possible states of nature and opponents’ strategies, and we call each partition element a “scenario”. We say that action a *dominates* action b given the partition if, for every continuation strategy after b , there is a con-

³In Section 6.3, we consider an example of a maze in which, at the first bifurcation, after going right the player may or may not find the exit, whereas after going left she will certainly not find it, but discovering this requires detailed scrutiny of two very complicated subtrees.

⁴Our mechanism is a special case of the class of “menu mechanisms” defined by Mackenzie and Zhou (2022). In particular, it is very similar to Bo and Hakimov’s (2022) “pick-an-object mechanism” for the implementation of the TTC rule, the difference being in players’ information flow. Bo and Hakimov (2022) also provide experimental evidence for the simplicity of their mechanism. See Section 6.3 for details.

continuation strategy after a that guarantees a better outcome in every scenario. A distinctive feature of our approach is that our player does not associate the candidate dominating action with *one* continuation strategy — continuation strategies are only used as mental checks and hence can change with the alternative under consideration. Coherently with this idea, we further allow the flexibility of tailoring the continuation strategy on the scenario under analysis. We say that action a *scenario-by-scenario dominates* (s-dominates) action b if, in each scenario, for every continuation strategy after b , there is a continuation strategy after a that can only do better. Thus, to establish s-dominance, each scenario is analyzed in isolation. Because of this, in each scenario, s-dominance does not truly require to scrutinize the possible continuation strategies after b ; one can just look for the best possible outcome after b and compare it with the worst possible outcome after a under some good-enough continuation strategy.

Dominance spans between the case of perfect contingent reasoning (with the finest partition of the space of uncertainty) and the case of no contingent reasoning (with the coarsest partition). In the first case, we talk of *weak dominance* between actions, in the second case of *obvious dominance*. Because of the lack of global planning, by choosing weakly/obviously undominated actions a player may follow a weakly/obviously dominated strategy — a form of dynamic inconsistency. Yet, we show that a strategy is weakly/obviously dominant if and only if it prescribes a weakly/obviously dominant action at every information sets that can be reached when playing that strategy. Thus, in a (obviously) strategy-proof dynamic game, a player does not have to recognize the existence of the dominant strategy in advance; by just spotting one dominant action at a time, she will — perhaps unknowingly — carry it out. In other words, the flexibility in the use of continuation strategies can be exploited without making mistakes.

Since a dominant action is s-dominant (given the same partitions), a player can also discover it without reasoning across scenarios, hence using the most convenient continuation strategy in each scenario. Perhaps surprisingly, we show that the following converse implication also holds: when there is an s-dominant action at every information set, such actions are also dominant. Thus,

the simpler approach of s-dominance is equally effective to determine whether there is a dominant action at each decision node. With perfect contingent reasoning, s-dominance boils down to comparing the best possible outcomes after the two actions in each contingency. This is a form of wishful thinking (we call it *wishful dominance*) that eliminates the need to even conceive continuation strategies. Seen from this angle, if players are capable of perfect contingent reasoning, dynamic strategy-proof mechanisms do not require any sort of planning, and this could be an explanation for their simplicity.

While a (s-)dominance relation between actions may be easy to spot, a priori there is no guarantee of this. First, even the coarsest scenarios that allow to establish dominance may be hard to identify. Second, the continuation strategies that allow to establish dominance may be hard to identify as well; the optimal continuation strategy may even be the only one that does the job. Therefore, with local dominance, we endogenize the scenarios from the local viewpoint, and we impose a simple use of continuation strategies.

The partition associated with local dominance, which we call “local partition”, is driven by the natural separation in a player’s mind between the scenario in which an action terminates the game, and hence yields an immediate consequence, and the scenario in which it does not, and hence one must look ahead. With this, our player identifies the four scenarios in which each of the two actions either terminates her game or not.⁵ These scenarios are “local”, in the sense that they only depend on the moves of the opponents before the next decision nodes of our player, avoiding any consideration on their future moves. In terms of continuation strategies, in the scenarios where the candidate dominating action a terminates the game, a player does not actually need to entertain any continuation strategy. In the scenarios where action a does not terminate the game, compared to s-dominance, local dominance restricts the use of continuation strategies by only allowing to compare action a with

⁵Both in the ascending-price auction (without waiting action) and in our TTC mechanism, local dominance only requires to distinguish the two scenarios in which the *dominant* action terminates the game or not. This is particularly convenient in the TTC mechanism, because it allows to use the same unique partition of the contingencies across all comparisons with many alternatives.

action b under the hypothesis of *continuing in the same way*. By doing so, our player defers any consideration about the optimality of her future moves. In the scenario where b terminates the game, “continuing in the same way” after a translates into playing b or an action that terminates the game — in the auction, “leaving”. In the more complicated scenario where the game continues after both actions, local dominance departs from s-dominance by requiring that the final outcome is invariant to the present choice.

With the idea of “continuing in the same way”, our player compares the two actions *ceteris paribus* with respect to all other decisions she will make. This can be seen as an adaptation of the classical one-shot deviation check for a player who does not have a plan. More importantly, the idea of continuing in the same way allows to apply the *sure thing principle* (STP) to the scenario in which the game continues after both actions. We find this application of STP simple because such a scenario is already isolated in our player’s mind — we contrast this with a classical violation of STP in Section 4.

The paper is organized as follows. In Section 2 we describe the game. In Section 3 we introduce and analyze our baseline notion of dominance and the notion of s-dominance. Section 4 is devoted to local dominance. Section 5 is devoted to applications of local dominance: we solve the ascending auctions and design our dynamic TTC mechanism. Section 6 concludes with a final comparison with Li (2017) and Pycia and Troyan (2022), along with a discussion of other related literature and an avenue for future research. The Appendix collects the formal proofs of some of the results and an example of a relaxation of local strategy-proofness (*on-path strategy-proofness*) that could be natural and more appropriate in some contexts.

2 Framework

We consider a finite multistage game in which players possess payoff-relevant private information, which we represent as the asymmetric observation of an initial move of nature. Nature has a finite set Θ of possible moves, and each player $i \in I$ has a finite set A_i of actions that are available at some point of the

game; let $A = \times_{i \in I} A_i$. Without loss of generality, we assume that, after the move of nature, the game always lasts exactly T stages, and that all players simultaneously choose an action at every stage. (When a player is not truly active, only a dummy action will be available.) Thus, the set X^1 of stage-1 histories is Θ , and for each $t = 2, \dots, T$, the set X^t of stage- t histories is a subset of $\Theta \times A^{t-1}$; the set of terminal histories Z is a subset of $\Theta \times A^T$. Let $X = \cup_{t=1}^T X^t$ and $\bar{X} = X \cup Z$; \bar{X} is endowed with the “prefix of” partial order, denoted by “ \prec ”. An information set of player i collects histories of the *same* stage that player i cannot distinguish (players know the stage); the set of i ’s information sets H_i partitions X and satisfies perfect recall, and thus it inherits the precedence order \prec from \bar{X} . For each $h \in H_i$, let $A_i^h \subseteq A_i$ denote the set of actions available to player i at h . Let H_i^* collect the information sets $h \in H_i$ where player i is active, that is, $|A_i^h| > 1$.

For each player i , there is a set of possible outcomes Y_i . Let $g_i : Z \rightarrow Y_i$ denote the function that associates each terminal history with i ’s final outcome, and let $u_i : Z \rightarrow \mathbb{R}$ denote player i ’s payoff function.

A reduced strategy of player i (henceforth, just “strategy”) is a map s_i that assigns an action $a_i \in A_i^h$ to each information set $h \in H_i$ that can be reached given the actions assigned to the previous information sets. Note that a strategy prescribes a dummy action also at every information set where player i is not active. Let S_i denote the set of strategies of player i , and let $S_{-i} = \Theta \times (\times_{j \in I \setminus \{i\}} S_j)$. For each $(s_i, s_{-i}) \in S_i \times S_{-i}$, let $\zeta(s_i, s_{-i})$ denote the induced terminal history. For each $h \in H_i$, let $S_i(h)$ and $S_{-i}(h)$ denote, respectively, the elements of S_i and S_{-i} that allow to reach h . For each $a_i \in A_i^h$, let $S_i(h, a_i)$ denote the set of strategies $s_i \in S_i(h)$ such that $s_i(h) = a_i$. Although, formally, an element of $S_i(h, a_i)$ is a strategy for the entire game, we will use $S_i(h, a_i)$ to describe the *continuation strategies* of i after choosing a_i at h . Let $H_i^*(s_i)$ denote the set of active information sets of i that are consistent with strategy s_i , i.e., that can be reached by playing s_i .

3 Dominance between actions

3.1 Baseline notion of dominance

We take the viewpoint of a player who compares two available actions \bar{a}_i, a_i at an information set $h \in H_i^*$. As we are mainly interested in the existence of a dominant action, we only consider comparisons between pure actions. The relevant state for player i 's choice is s_{-i} , which includes the move of nature θ and the strategies of the opponents $(s_j)_{j \neq i}$.⁶ Given the current information set h , the set $S_{-i}(h)$ is thus the set of relevant states that are still possible at h . A *scenario* is a subset of $S_{-i}(h)$ under which player i compares the two actions. We assume that player i considers a collection of scenarios that partitions $S_{-i}(h)$. For example, a player could ask herself “will my opponent go left or right at the current stage?”, and compare actions under each of the two scenarios of the corresponding bipartition. Our baseline notion of dominance postulates how a player compares the two actions under a given partition.

Definition 1 Fix an information set $h \in H_i^*$, an action pair $(\bar{a}_i, a_i) \in A_i^h \times A_i^h$, and a partition \mathcal{S} of $S_{-i}(h)$. Action \bar{a}_i **dominates** action a_i given \mathcal{S} if for every $s_i \in S_i(h, a_i)$, there exists $\bar{s}_i \in S_i(h, \bar{a}_i)$ such that

$$\forall \bar{S}_{-i} \in \mathcal{S}, \quad \min_{s_{-i} \in \bar{S}_{-i}} u_i(\zeta(\bar{s}_i, s_{-i})) \geq \max_{s_{-i} \in \bar{S}_{-i}} u_i(\zeta(s_i, s_{-i})). \quad (1)$$

Action \bar{a}_i is *dominant (at h)* if it dominates every other $a_i \in A_i^h$ given the partition associated with (\bar{a}_i, a_i) .

Action \bar{a}_i dominates action a_i if for every possible continuation strategy after a_i , there exists a continuation strategy after \bar{a}_i that does better in every scenario. Local dominance does not require player i to find *one* continuation strategy after \bar{a}_i that beats *all* continuation strategies after a_i — such a continuation strategy may not even exist. Rather, the continuation strategy after

⁶For coherence with the view that player i does not formulate a global plan, the strategies of the opponents are best interpreted not as plans either, but just as sets of conditional statements about behavior: how a player would behave upon reaching an information set.

\bar{a}_i conceived by player i can change with the continuation strategy after a_i that player i is considering. As we will see in Section 4, this is particularly convenient when for every continuation strategy after a_i , there is a “mimicking” continuation strategy after \bar{a}_i which does the job. Our baseline notion of dominance does not allow, instead, to change the continuation strategy after \bar{a}_i depending on the scenario under consideration — we will formalize a notion of dominance with this additional flexibility in the next subsection.

To establish that a continuation strategy after \bar{a}_i “does better” than a continuation strategy after a_i within a scenario, we employ the comparison of the worst and the best outcome that can realize under, respectively, the former and the latter continuation strategies. This conservative approach, borrowed from obvious dominance, makes our notions of dominance robust to possible ways in which players compare actions within a scenario.

It is easy to see that the finer is the partition into scenarios, the weaker is the notion of dominance associated with the partition.

Remark 1 *Fix two partitions $\mathcal{S}, \tilde{\mathcal{S}}$ of $S_{-i}(h)$ where $\tilde{\mathcal{S}}$ refines \mathcal{S} . If \bar{a}_i dominates a_i given \mathcal{S} , then \bar{a}_i dominates a_i given $\tilde{\mathcal{S}}$.*

Consider now the finest and the coarsest partitions of $S_{-i}(h)$: the singleton partition $\mathcal{S} = \{\{s_{-i}\} \mid s_{-i} \in S_{-i}(h)\}$, and the trivial partition $\mathcal{S} = \{S_{-i}(h)\}$. Under these two partitions, in light of Remark 1, we obtain the weakest and the strongest notions of dominance we can establish by just varying the level of detail in contingent reasoning. We formalize these two extreme notions of dominance between actions and call them “weak dominance” and “obvious dominance” because in static games they coincide with the corresponding notions of dominance between strategies.⁷

Definition 2 *Fix an information set $h \in H_i^*$.*

⁷As common in mechanism design but not in game theory, we define weak dominance without requiring strict inequality under some contingency — in game theory, this notion is called *very weak dominance* (Marx and Swinkels, 1997).

Action $\bar{a}_i \in A_i^h$ **weakly dominates** action $a_i \in A_i^h$ if for every $s_i \in S_i(h, a_i)$, there exists $\bar{s}_i \in S_i(h, \bar{a}_i)$ such that

$$\forall s_{-i} \in S_{-i}(h), \quad u_i(\zeta(\bar{s}_i, s_{-i})) \geq u_i(\zeta(s_i, s_{-i})). \quad (2)$$

Action $\bar{a}_i \in A_i^h$ **obviously dominates** action $a_i \in A_i^h$ if for every $s_i \in S_i(h, a_i)$, there exists $\bar{s}_i \in S_i(h, \bar{a}_i)$ such that

$$\min_{s_{-i} \in S_{-i}(h)} u_i(\zeta(\bar{s}_i, s_{-i})) \geq \max_{s_{-i} \in S_{-i}(h)} u_i(\zeta(s_i, s_{-i})). \quad (3)$$

Action \bar{a}_i is weakly/obviously dominant if it weakly/obviously dominates every other $a_i \in A_i^h$.

We now compare weak/obvious dominance between actions with weak/obvious dominance between strategies (in dynamic games). We first report the definitions of the latter. Given two strategies $\bar{s}_i, s_i \in S_i$, let $\mathcal{D}(\bar{s}_i, s_i)$ be the set of their *points of departure*, that is, the information sets $h \in H_i^*(\bar{s}_i) \cap H_i^*(s_i)$ such that $\bar{s}_i(h) \neq s_i(h)$.⁸

Definition 3 Strategy \bar{s}_i weakly dominates strategy s_i if

$$\forall s_{-i} \in S_{-i}, \quad u_i(\zeta(\bar{s}_i, s_{-i})) \geq u_i(\zeta(s_i, s_{-i})). \quad (4)$$

Strategy \bar{s}_i obviously dominates strategy s_i if

$$\forall h \in \mathcal{D}(\bar{s}_i, s_i), \quad \min_{s_{-i} \in S_{-i}(h)} u_i(\zeta(\bar{s}_i, s_{-i})) \geq \max_{s_{-i} \in S_{-i}(h)} u_i(\zeta(s_i, s_{-i})). \quad (5)$$

A strategy is weakly/obviously dominant if it weakly/obviously dominates every other strategy.

Weak/obvious dominance between two actions at an information set is equivalent to the following relationship between strategies that reach the in-

⁸Given our focus on reduced strategies, and thanks to perfect recall, every point of departure is an *earliest* point of departure in the sense of Li (2017).

formation set: every strategy that prescribes the dominated action is weakly/obviously dominated by some strategy that prescribes the dominating action.⁹

Proposition 1 *Action $\bar{a}_i \in A_i^h$ weakly/obviously dominates action $a_i \in A_i^h$ if and only if every $s_i \in S_i(h, a_i)$ is weakly/obviously dominated by some $\bar{s}_i \in S_i(h, \bar{a}_i)$.*

The “only if” part of Proposition 1 implies that weak dominance between actions has the following foundation: if action a_i is weakly dominated by some action \bar{a}_i (but not the other way round), then choosing a_i is not optimal under any full-support belief over $S_{-i}(h)$. Moreover, the “if” part guarantees that action a_i is (weakly/obviously) dominated by action \bar{a}_i if *every* strategy that prescribes a_i is dominated by a strategy that prescribes \bar{a}_i . Note however that a dominated strategy need not prescribe a dominated action. It can even be the case that all the actions prescribed by a dominated strategy are undominated. This is a consequence of the lack of global planning. As an example, consider the following perfect information game:

$$\begin{array}{ccccccc}
 \text{Ann} & \longrightarrow & \text{Bob} & \longrightarrow & \text{Ann} & \longrightarrow & \text{Bob} & \longrightarrow & (3, 3) \\
 \downarrow & & \downarrow & & \downarrow & & \downarrow & & \longrightarrow = \text{across} \\
 (2, 0) & & (0, 1) & & (1, 0) & & (0, 1) & & \downarrow = \text{down}
 \end{array}$$

The strategy of Ann that prescribes *across* at the initial history and *down* at history $(across, across)$ is weakly/obviously dominated by strategy *down*. Nonetheless, it is easy to see that both action *across* at the initial history and action *down* at history $(across, across)$ are not weakly/obviously dominated.

Thus, when players do not follow a global plan, by choosing undominated actions they might still end up playing a dominated strategy. As a consequence, fewer patterns of behavior can be ruled out with dominance between actions in place of dominance between strategies. In light of this, it seems harder to achieve “(obvious) strategy-proofness” without assuming global planning.

⁹Shimoji and Watson (1998) introduce the notion of conditional (strict) dominance between strategies at an information set. Proposition 1 essentially relates weak and obvious dominance between actions to the weak and obvious counterparts of conditional dominance.

But this is not true: when a player has a weakly/obviously dominant strategy, all the actions it prescribes are weakly/obviously dominant, so weak/obvious dominance between actions does rule out any other behavior. Thus, we obtain the following characterization.

Theorem 1 *A strategy \bar{s}_i is weakly/obviously dominant if and only if $\bar{s}_i(h)$ is weakly/obviously dominant at every $h \in H_i^*(\bar{s}_i)$.*

Theorem 1 implies that (obvious) strategy-proofness can be broken down into a collection of local conditions, each requiring the existence of a weakly/obviously dominant action. This local property may be key to understand why some (obviously) strategy-proof dynamic mechanisms are easier to play than their static, direct counterparts. Dynamic mechanisms decompose the problem of revealing players' entire preferences into a series of smaller partial-revelation problems, but this advantage is lost if all such problems must be jointly solved at the outset — to what extent can players really tackle them in isolation? Theorem 1 says that players do not necessarily have to plan globally if the decomposition preserves the existence of a dominant strategy; instead, they can find one dominant action at a time. We build our simplicity theory precisely on this ground: with local dominance we will identify a natural way of tackling each choice problem in isolation.

3.2 Scenario-by-scenario dominance

Now we explore the possibility that, for the comparison of two actions, players analyze each scenario in isolation. This means that players may use different continuation strategies while considering different scenarios.

Definition 4 *Fix an information set $h \in H_i^*$, an action pair $(\bar{a}_i, a_i) \in A_i^h \times A_i^h$, and a partition \mathcal{S} of $S_{-i}(h)$. Action \bar{a}_i scenario-by-scenario dominates (s-dominates) action a_i given \mathcal{S} if, for every $\bar{S}_{-i} \in \mathcal{S}$,*

$$\forall s_i \in S_i(h, a_i), \exists \bar{s}_i \in S_i(h, \bar{a}_i), \min_{s_{-i} \in \bar{S}_{-i}} u_i(\zeta(\bar{s}_i, s_{-i})) \geq \max_{s_{-i} \in \bar{S}_{-i}} u_i(\zeta(s_i, s_{-i})). \quad (6)$$

Action \bar{a}_i is s-dominant if it s-dominates every other $a_i \in A_i^h$ given the partition associated with (\bar{a}_i, a_i) .

As for the baseline notion of dominance, the finer the partition, the weaker s-dominance. Given the same partition, s-dominance is weaker than dominance, because the continuation strategy after the s-dominating action can change with the scenario. When the partition is trivial s-dominance and dominance coincide. The following remark summarizes these observations.

- Remark 2**
1. For any two partitions $\mathcal{S}, \tilde{\mathcal{S}}$ of $S_{-i}(h)$ such that $\tilde{\mathcal{S}}$ refines \mathcal{S} , if \bar{a}_i s-dominates a_i given \mathcal{S} , then \bar{a}_i s-dominates a_i given $\tilde{\mathcal{S}}$.
 2. If action \bar{a}_i dominates action a_i given \mathcal{S} , then \bar{a}_i s-dominates a_i given \mathcal{S} .
 3. Action \bar{a}_i obviously dominates a_i if and only if it s-dominates a_i given the trivial partition $\mathcal{S} = \{S_{-i}(h)\}$.

Since a dominant action is also s-dominant, to discover a dominant action a player can also analyze every scenario in isolation. Typically, this simplifies her task, because it allows to tailor the continuation strategy after the dominant action on the scenario under consideration.¹⁰ Perhaps surprisingly, when there are s-dominant actions everywhere, the converse implication holds as well: the s-dominant actions are also dominant. Thus, the simpler approach of s-dominance is equally effective to determine whether there is a dominant action at each decision node.

Theorem 2 For each strategy $\bar{s}_i \in S_i$, $\bar{s}_i(h)$ is dominant at every $h \in H_i^*(\bar{s}_i)$ if and only if $\bar{s}_i(h)$ is s-dominant at every $h \in H_i^*(\bar{s}_i)$.

Of course, Theorem 2 assumes that, for every information set and every alternative action, dominance and s-dominance are established under the same partition. Instead, surprisingly, Theorem 2 does not require any discipline across the partitions used at different information sets. One could expect that,

¹⁰Whether players actually reason in this way is a question for experimental/empirical research.

to guarantee that an s-dominant action is dominant, the existence of a dominant action at every future information set must be established under sufficiently coarse partitions. This is not the case.

Definition 4 highlights the key difference between s-dominance and dominance by simply changing the order of quantifiers: first the scenario is fixed, then different continuation strategies are considered. But s-dominance can be checked without actually scrutinizing all possible continuation strategies after the dominated action. Given a scenario, among the continuation strategies after \bar{a}_i that verify condition (6) against the continuation strategies after a_i , at least one works against *all* the continuation strategies after a_i : the one that gives the highest worst-case payoff within the scenario. But then, condition (6) boils down to checking that such a payoff is larger than the best possible payoff after a_i . This yields a convenient operational definition of s-dominance. To formalize, let $Z(h, a_i, \bar{S}_{-i})$ denote the terminal histories that can be reached under $\bar{S}_{-i} \subseteq S_{-i}(h)$ if i chooses a_i at h .

Remark 3 *Action \bar{a}_i s-dominates action a_i given \mathcal{S} if and only if for every $\bar{S}_{-i} \in \mathcal{S}$, there exists $\bar{s}_i \in S_i(h, \bar{a}_i)$ such that*

$$\min_{s_{-i} \in \bar{S}_{-i}} u_i(\zeta(\bar{s}_i, s_{-i})) \geq \max_{z \in Z(h, a_i, \bar{S}_{-i})} u_i(z).$$

In the case of perfect contingent reasoning, s-dominance does not even require to come up with a continuation strategy after the dominating action: tailoring the continuation strategy to each contingency is equivalent to just identifying the best possible payoff given each contingency. The following definition formalizes this special case.

Definition 5 *Action $\bar{a}_i \in A_i^h$ wishfully dominates action $a_i \in A_i^h$ if*

$$\forall s_{-i} \in S_{-i}(h), \quad \max_{z \in Z(h, \bar{a}_i, s_{-i})} u_i(z) \geq \max_{z \in Z(h, a_i, s_{-i})} u_i(z), \quad (7)$$

Action \bar{a}_i is wishfully dominant if it wishfully dominates every other $a_i \in A_i^h$.

Remark 4 *Action \bar{a}_i wishfully dominates a_i if and only if \bar{a}_i s-dominates a_i given the singleton partition $\mathcal{S} = \{\{s_{-i}\} \mid s_{-i} \in S_{-i}(h)\}$.*

We will illustrate wishful dominance by example in our dynamic TTC mechanism of Section 5.2. The term “wishful” is justified by the fact that the player looks at the best payoffs she could obtain under each contingency, although, typically, no single continuation strategy can achieve the best outcome in all contingencies. This observation also highlights a fundamental inconsistency between planning and wishful dominance: a player cannot, at the same time, be a planner and reason according to wishful dominance.¹¹

Wishful dominance allows us to provide a characterization of strategy-proofness that does not involve continuation strategies. By theorem 1, strategy-proofness is equivalent to the existence of a weakly dominant action at every information set that is consistent with the dominant strategy. Thus, by Theorem 2, the same equivalence holds with wishful in place of weak dominance.

Theorem 3 *A game is strategy-proof if and only if, for each player i , there exists a strategy \bar{s}_i such that $\bar{s}_i(h)$ is wishfully dominant at every $h \in H_i^*(\bar{s}_i)$.*

Theorem 3 says that the search for a weakly dominant strategy can be decomposed into local problems that do not even require to conceive continuation strategies. In other words, strategy-proofness is robust to the inability to plan forward, if players analyze each contingency in isolation. The possibility to discover the dominant actions without even entertaining continuation strategies strengthens our argument for the simplicity of strategy-proof dynamic mechanisms — in our dynamic TTC mechanism, searching for wishfully dominant actions will be particularly easy.

4 Local Dominance

The notions of dominance we introduced so far are silent as to whether the two actions can be ranked using a partition and continuation strategies that are truly easy to identify. In this section, we endogenize the partition of the

¹¹Games with perfect and complete information are an exception, in that looking for the best outcome in each contingency pins down a well-defined plan. We conjecture that in this class of games, wishful dominance, weak dominance, and even obvious dominance coincide.

contingencies and restrict the use of continuation strategies according to the simplicity principles outlined in the introduction. While doing so, as in s-dominance, we maintain the idea that players consider each scenario separately. This is a natural choice given that, with local dominance, we aim to capture the separate treatment of present and future of the game in the player's mind.

Endogenizing the partition Fix an information set $h \in H_i$. For each action $a_i \in A_i^h$, let $S_{-i}^{a_i, h}$ denote the set of contingencies in which the game will end for player i after choosing a_i at h , i.e., there is no further information set where i is active after playing a_i at h :

$$S_{-i}^{a_i, h} = \{s_{-i} \in S_{-i}(h) : \forall z \in Z(h, a_i, s_{-i}), \nexists h' \in H_i^*, h \prec h' \prec z\}.$$

Given an action pair $(\bar{a}_i, a_i) \in A_i^h \times A_i^h$, let

$$\begin{aligned} S_{\bar{a}_i, a_i}^{\bar{a}_i, h}(h) &= S_{-i}^{a_i, h} \cap S_{-i}^{\bar{a}_i, h} & S_{\bar{a}_i}^{\bar{a}_i}(h) &= S_{-i}^{\bar{a}_i, h} \setminus S_{-i}^{a_i, h} \\ S_{\bar{a}_i}^{a_i}(h) &= S_{-i}^{a_i, h} \setminus S_{-i}^{\bar{a}_i, h} & S_{\bar{a}_i, a_i}(h) &= S_{-i}(h) \setminus (S_{-i}^{\bar{a}_i, h} \cup S_{-i}^{a_i, h}). \end{aligned}$$

In words, each of these sets contains the strategies of the opponents that terminate our player's game if she chooses the action(s) at the superscript, and do not terminate it if she chooses the action(s) at the subscript. Let

$$\mathcal{S}^\ell(h, \bar{a}_i, a_i) = \{S_{\bar{a}_i, a_i}^{\bar{a}_i, h}(h), S_{\bar{a}_i}^{a_i}(h), S_{\bar{a}_i}^{\bar{a}_i}(h), S_{\bar{a}_i, a_i}(h)\}.$$

We call $\mathcal{S}^\ell(h, \bar{a}_i, a_i)$ the **local partition**.

The local partition seems natural from a local viewpoint, for various reasons. For a player who compares the current actions with a blurred view of the future, it is natural to first focus on their possible immediate consequences. This requires the player to understand in what circumstances each action will be the last action she plays and hence will directly yield the final outcome. This way of partitioning is indeed "local", in that the scenarios only depend on the moves of the opponents before our player's next active stage, and can be identified as long as she can conceive her next moves. Formally, the local partition is measurable with respect to the information our player receives at

the next decision nodes. For each $\tilde{a}_i = \bar{a}_i, a_i$, call $\mathcal{H}^*(h, \tilde{a}_i)$ the set of the first *active* information sets of i after choosing \tilde{a}_i at h , and let $\mathcal{S}^*(h, \tilde{a}_i)$ denote the collection $\{S_{-i}(h') | h' \in \mathcal{H}^*(h, \tilde{a}_i)\} \cup \{S_{-i}^{\tilde{a}_i, h}\}$. The local partition, $\mathcal{S}^\ell(h, \bar{a}_i, a_i)$, is weakly coarser than the meet of the partitions $\mathcal{S}^*(h, \bar{a}_i)$ and $\mathcal{S}^*(h, a_i)$.

Mimicking strategies Next, we formalize the idea of comparing two actions “*ceteris paribus*” with respect to the future moves, that is, under the hypothesis of “continuing in the same way” after the two. Coherently with analysing each scenario in isolation, we are going to talk of continuing in the same way conditional on the particular scenario under consideration.

Let $\tau(\tilde{h})$ denote the stage of an information set \tilde{h} . Fix $\bar{s}_i, s_i \in S_i(h)$ and a scenario $\bar{S}_{-i} \subseteq S_{-i}(h)$. We say that \bar{s}_i **mimics** s_i given h and \bar{S}_{-i} when, for every $s_{-i} \in \bar{S}_{-i}$, the following condition holds: For every $\bar{h} \in \mathcal{H}^*(h, \bar{s}_i(h))$ and for every $\bar{h}', h' \in H_i$ such that $\bar{h} \preceq \bar{h}' \prec \zeta(\bar{s}_i, s_{-i})$, $h' \prec \zeta(s_i, s_{-i})$, and $\tau(\bar{h}') = \tau(h')$, we have $\bar{s}_i(\bar{h}') = s_i(h')$. In words, under every element of \bar{S}_{-i} , once player i becomes active again after choosing $\bar{s}_i(h)$ at h , the mimicking strategy prescribes the same action as the mimicked strategy at each subsequent stage. Note that mimicking only starts at the next active information set because, before that, a player is forced to play a dummy action.

Mimicking is a strong requirement because it is formulated “*ex-post*”: for each realization of s_{-i} in \bar{S}_{-i} , the actual sequence of moves of player i (from the first active stage after \bar{a}_i) must be the same under s_i and its mimicker \bar{s}_i .

Irrelevance Now we introduce our version of “*sure thing principle*” for the problem at hand, that is, the idea that a scenario is “*irrelevant*” for the comparison of two actions. Our player will conclude that a scenario is irrelevant when, by continuing in the same way after the two actions, she would obtain the same outcome. This observation is different in nature from the comparison of best and worst outcomes we considered so far. Typically, it follows from a symmetric structure of the game that is easy to spot without actual scrutiny of all the possible ways she and the opponents may continue playing, and of the consequent outcomes.

Formally, we say that a scenario \bar{S}_{-i} is **irrelevant** for (\bar{a}_i, a_i) at h when, for every $s_i \in S_i(h, a_i)$, there exists $\bar{s}_i \in S_i(h, \bar{a}_i)$ that mimics s_i given h and \bar{S}_{-i} such that

$$\forall s_{-i} \in \bar{S}_{-i}, \quad g_i(\zeta(\bar{s}_i, s_{-i})) = g_i(\zeta(s_i, s_{-i})). \quad (8)$$

We define irrelevance for an *ordered* pair of actions because we have in mind a player who has already concluded that \bar{a}_i is better than a_i in terms of possible immediate consequences. To check that this ranking is not overturned when looking at the possible future consequences, it is enough to conclude that for any outcome that she can obtain after a_i , she can also obtain it after \bar{a}_i .

Ignoring an irrelevant scenario is an application of the sure thing principle in which the irrelevant states are obtained under the hypothesis of continuing in the same way. The sure thing principle is sometimes violated in other contexts, and one of the reasons could be that, in the problem at hand, the irrelevant states are naturally pooled with relevant ones into a non-irrelevant scenario in the decision-maker's mind. For instance, in Ellsberg's problem with 30 red balls and 60 yellow or blue balls, the state "blue" is irrelevant for the comparison of "bet on red" and "bet on yellow", but it is naturally pooled with the other losing state "yellow" under the first bet and with the other losing state "red" under the second bet. In our context, we expect a player to identify the scenario "the game continues after both actions" before she recognizes its irrelevance, for the reasons we outlined before, which have nothing to do with the outcome function.

Local dominance We are now ready to formalize the idea of comparing actions under the local partition and the hypothesis of "continuing in the same way".

Definition 6 Fix an information set $h \in H_i^*$ and an action pair $(\bar{a}_i, a_i) \in A_i^h \times A_i^h$. Action \bar{a}_i **locally dominates** action a_i if for each non-empty $\bar{S}_{-i} \in \mathcal{S}^\ell(h, \bar{a}_i, a_i)$, for each $s_i \in S_i(h, a_i)$, there exists $\bar{s}_i \in S_i(h, \bar{a}_i)$ that mimics s_i

given h and \bar{S}_{-i} such that:

$$\begin{aligned} \text{if } \bar{S}_{-i} \neq S_{\bar{a}_i, a_i}(h), \quad & \min_{s_{-i} \in \bar{S}_{-i}} u_i(\zeta(\bar{s}_i, s_{-i})) \geq \max_{s_{-i} \in \bar{S}_{-i}} u_i(\zeta(s_i, s_{-i})); \\ \text{if } \bar{S}_{-i} = S_{\bar{a}_i, a_i}(h), \quad & g_i(\zeta(\bar{s}_i, \cdot))|_{\bar{S}_{-i}} = g_i(\zeta(s_i, \cdot))|_{\bar{S}_{-i}} \end{aligned} \quad (9)$$

(i.e., $S_{\bar{a}_i, a_i}(h)$ is irrelevant).

Action \bar{a}_i is locally dominant if it locally dominates every other $a_i \in A_i^h$.

Local dominance builds on s-dominance by considering each scenario of the local partition in isolation. While s-dominance assumes that, in each scenario, a player can always find a good-enough continuation strategy after the dominating action, local dominance does not assume this ability; it only requires to entertain the salient continuation strategy that “continues in the same way” as after the dominated action, i.e., the mimicking strategy. In the way outcomes are compared, local dominance departs from (s-)dominance in the scenario where the game continues after both actions: instead of assuming that a player can find worst and best outcomes after the two actions, it assumes that a player can realize that the current choice has no impact on the continuation game. Of course, a cognitively limited player could also ignore this scenario simply because it seems too complicated — local dominance is robust to these considerations.

Local dominance can be interpreted as a new kind of one-shot deviation check. In particular, local dominance evaluates a switch from the dominating to the dominated action not under a fixed continuation strategy, which may prescribe different future moves after the two actions, but under the idea of “continuing in the same way” after the two actions, no matter how. In this sense, local dominance is a game theoretical translation of the “ceteris paribus” principle. For the scenario in which the game continues after both actions, the invariance of final outcomes typically requires the continuation play of the opponents to be the same as well. In turn, this typically requires that the opponents do not observe (and hence cannot condition their choices on) whether our player chose one action or the other. Indeed, like weak dominance and unlike obvious dominance, local dominance is sensitive to how informed the

opponents are of a player's moves: roughly speaking, the more they observe, the more the opportunities to strategize, the more difficult for the player to identify a dominant choice.

A peculiarity of local dominance is not being transitive. The reason is that the local partition changes with the action pair under comparison. We do not find this intransitivity undesirable for a notion of dominance that aims to capture an idea of simplicity: "similar" enough alternatives may be easy to rank, too "dissimilar" ones may not.

Definition 6 formalizes the idea of comparing actions under the view of continuing in the same way. In particular, it matches each continuation strategy after the dominated action with a mimicking continuation strategy after the dominating action, mirroring Definition 4 of s-dominance. But just like s-dominance, a player can assess local dominance without scrutinizing the continuation strategies after the dominated action (see Remark 3); moreover, under the local partition, in the scenarios where the dominating action terminates the game, there is clearly no need to entertain any continuation strategy after the dominating action either. Thus, by the very nature of the local partition, our player can analyze each scenario as follows.

Remark 5 *Action \bar{a}_i locally dominates action a_i if and only if the following conditions hold (when the respective scenario is non-empty):*

1. (comparison of immediate consequences)

$$\min_{z \in Z(h, \bar{a}_i, S_{\bar{a}_i, a_i}^{\bar{a}_i}(h))} u_i(z) \geq \max_{z \in Z(h, a_i, S_{a_i, a_i}^{a_i}(h))} u_i(z); \quad (10)$$

- 2.

$$\min_{z \in Z(h, \bar{a}_i, S_{\bar{a}_i}^{\bar{a}_i}(h))} u_i(z) \geq \max_{z \in Z(h, a_i, S_{a_i}^{a_i}(h))} u_i(z); \quad (11)$$

3. there exists $\bar{s}_i \in S_i(h, \bar{a}_i)$ that mimics any $s_i \in S_i(h, a_i)$ given h and $S_{\bar{a}_i}^{a_i}(h)$ such that

$$\min_{s_{-i} \in S_{\bar{a}_i}^{a_i}(h)} u_i(\zeta(\bar{s}_i, s_{-i})) \geq \max_{z \in Z(h, a_i, S_{\bar{a}_i}^{a_i}(h))} u_i(z); \quad (12)$$

4. $S_{\bar{a}_i, a_i}(h)$ is irrelevant.

To see the equivalence between conditions (10),(11) and condition (9), note that in scenarios $S^{\bar{a}_i, a_i}(h)$ and $S_{a_i}^{\bar{a}_i}(h)$ there is no active information set of player i after \bar{a}_i . Therefore, the only “continuation strategy” after \bar{a}_i (the one that prescribes the dummy actions until stage T) trivially mimics any continuation strategy after a_i — recall that mimicking is only required to start from the next active information sets onwards. As for scenario $S_{\bar{a}_i}^{a_i}(h)$, there is just one mimicking strategy to consider, because, within this scenario, all $s_i \in S_i(h, a_i)$ induce the same dummy continuation strategy after a_i .

We say that a game is **locally strategy-proof** when each player has a locally dominant action at every information set that can be reached if she always chooses her locally dominant actions. In Section 5.1, we show that the ascending auctions we considered in the introduction are locally strategy-proof, and we construct a locally strategy-proof mechanism that implements the TTC allocation rule.

Comparison with other notions of dominance between actions We start by comparing local dominance with the weakest notion of dominance we introduced, wishful dominance. Suppose that action \bar{a}_i locally dominates action a_i ; then, under each contingency s_{-i} , compared to the best outcome after a_i , after \bar{a}_i one can achieve an outcome that is at least as good (in particular, the *same* outcome if $s_{-i} \in S_{\bar{a}_i, a_i}(h)$). Therefore, local dominance refines wishful dominance. With this, Theorem 3 implies that local strategy-proofness refines strategy-proofness.

Proposition 2 *If \bar{a}_i locally dominates a_i , then it wishfully dominates a_i . Therefore, if a game is locally strategy-proof, it is strategy-proof.*

Local dominance is neither weaker nor stronger than s-dominance if we fix the local partition. The reason why it is not stronger, despite the restrictions on the use of continuation strategies, is that, as we observed, irrelevance is just a different criterion from the comparison of worst and best payoffs.

Similarly, local dominance is not weaker or stronger than obvious dominance between actions. In static games, the two notions actually coincide, because the only non-empty scenario is the one in which the game ends after every action. In our framework, a game is static if $T = 1$, so there is stage 0 for the move of nature and only stage 1 for players' moves.

Proposition 3 *Fix a static game, an information set $h \in H_i^*$, and an action pair $(\bar{a}_i, a_i) \in A_i^h \times A_i^h$. Then, action \bar{a}_i locally dominates action a_i if and only if \bar{a}_i obviously dominates a_i .*

Proposition 3 allows to establish a connection between local and obvious strategy-proofness, which we discuss in Section 6.1.

5 Applications

5.1 Ascending-price auctions

We analyze with local dominance the ascending-price auction we outlined in the introduction, and its variant with the waiting option. The one without the waiting option is the auction format that was shown easy to play by the experiment of Kagel et al. (1987). There are n bidders with private valuations for the object, which for simplicity we assume to be integer.¹² At each stage $t = 1, \dots, T$ (with T larger than any possible valuation), player only observe whether the object was already assigned or not, and if not, the players who have not left before simultaneously choose between bid (b) and leave (ℓ); in presence of the additional waiting option, they can also wait (w). Formally, the players who left before stage t are forced to play ℓ , and all players must play ℓ once the object is assigned. The “stage- t outcome rule” is the following: if no player bids, the object is assigned at random at price $p = t$ among the bidders who have not left before stage t ; if only one player bids, she wins the object at price $p = t$; if more than one player bids, the auction moves to round $t + 1$.

¹²With non-integer valuations, the valuation of a bidder may be higher than the current price but lower than the next price, and then bidding would not locally dominate leaving. Nonetheless, we could still claim that players will bid *at least* up to that price.

If waiting is allowed, the stage- t outcome rule makes no distinction between waiting and leaving. Thus, the only difference between waiting and leaving is that, if the auction continues, a player who waited can move at the next stages, while a player who left must play ℓ forever.

Proposition 4 *The ascending-price auction is locally strategy-proof (also with the waiting option). In particular, for every player, it is locally dominant to bid as long as the price is below her valuation and then leave when the price reaches her valuation.*

Here we provide a sketch of the proof, which is formalized in the Appendix.

Take first the viewpoint of a player at a stage where the price is still below her valuation. What makes the decision to bid easy to take? We start from the comparison with leaving. It is probably natural for a player to first focus on the scenario in which bidding terminates the game and immediately yields the final outcome. This occurs when no other player bids. In this scenario, the outcome of bidding is winning the object, the outcome of leaving is the lottery. Thus, in terms of possible immediate consequences, bidding is better than leaving. Then, our player also realizes that if she bids and the game continues, her outcome will be determined in the future. To restrict the realm of the possible future outcome after bidding, it is probably natural to entertain the idea of leaving at the next stage. With this continuation strategy, our player cannot incur a loss after bidding at the current stage, therefore the comparison of local outcomes is not overturned. So, bidding locally dominates leaving.

Now we move to the comparison between bidding and waiting. In the scenario where bidding terminates the game (i.e., no one else bids), waiting is equivalent to leaving, and thus bidding is better than waiting. When bidding does not terminate the game, waiting may or may not terminate the game. When only one opponent bids, waiting terminates the game. Then, also for the comparison between bidding and waiting, it is probably natural to entertain the idea of leaving (and thus terminating the game) at the next stage. So, for the same argument of the comparison with leaving, bidding is better than waiting. When more than one opponent bids, the game continues also after

waiting. In this scenario, it is easy to recognize that the choice between bidding and leaving has no impact whatsoever on the final outcome. In greater detail, bidding or waiting does not alter what the opponents observe, and hence what they will do, nor what our player will observe and be allowed to do. Note also that the outcome of a stage only depends on players' moves at that stage. Thus, the scenario in which the auction continues after both bidding and leaving is irrelevant for the comparison. Hence, bidding locally dominates waiting.

Take now the viewpoint of a bidder at the stage where the price reaches her valuation. What makes the decision to leave easy to take? If the auction ends, not matter our player's action, her surplus will be zero. Leaving can only have this immediate consequence, while bidding and waiting can also entail that our player's outcome will be determined in the future. However, no matter what this outcome will be, it cannot bring a positive payoff to our player. Thus, leaving locally dominates bidding and, if available, waiting.

5.2 Top Trading Cycles

Consider the classical object allocation problem without monetary transfers. There are n agents and N objects. Each agent initially owns an object and has a strict preference ranking over all objects. A rule specifies for every preference profile a reallocation of the objects to the agents such that each agent gets exactly one object. A prominent rule that has been extensively studied in the literature is the TTC rule. It adopts an iterative algorithm proposed by Gale. At every iteration, the algorithm generates a directed graph in which the nodes are the agents who are yet to be assigned an object and the arrows link each agent to the owner of her highest-ranked object that is still available. Such a graph always has at least one cycle, and the agents that end up in a cycle are assigned the object they are pointing to.

The TTC rule is known to be strategy-proof: in the direct mechanism, reporting the true preferences is weakly dominant. However, the direct mechanism is known to be difficult to play in practice. In particular, a player might be tempted to rank an object b above an object a despite preferring a to b , because she fears that she might miss her chance to get object b while the algo-

rithm keeps pointing her to object a unsuccessfully. To address these concerns, we translate Gale’s TTC algorithm into a dynamic mechanism with three simplicity features. First, at each stage, players are only asked to name one of the still-available objects. In this way, we decompose the problem of revealing your own preferences into a sequence of smaller partial-revelation problems. Second, players cannot move until the last object they named is assigned to someone else. This reassures players that whenever an opportunity for trade pops up (i.e., some other player points to her directly or indirectly),¹³ it remains intact through time and can be exploited later. Third, our mechanism carefully releases information to players so to reassure them that, if the game continues after both a and b , they can continue in the same way, and then will also get the same outcome. To allow players to continue in the same way, we let them observe the set of available outcomes also when they do not move, so that their available information does not depend on how long they have to wait.¹⁴ To guarantee the same final outcome, we do not reveal players’ choices to the opponents, so that one does not have to worry that her choices may affect the opponents’ future choices, in a way that negatively affects her final outcome. With this, we show that in our mechanism naming the favorite object is always locally dominant, whereas in the direct mechanism submitting the true preference ranking is not locally dominant (i.e., by Proposition 5, obviously dominant). Therefore, we obtain the following positive result, which contrasts with Li’s (2017) impossibility of obvious strategy-proof implementation of the TTC rule.

Theorem 4 *The TTC rule can be implemented with a locally strategy-proof mechanism.*

To prove the theorem, we explicitly construct such a mechanism, which

¹³We say that player j points to the object of player i indirectly if there is a sequence of players starting with j and ending with i such that each player in the sequence names the object owned by the next player.

¹⁴Formally, as we give the traditional representation of information flows just at the information sets where players are active, they observe the past history of available objects once they get to move again. The observability of such history, and not just of the current menu of available objects, distinguishes our mechanism from that of Bo and Hakimov (2022).

we call *dynamic TTC mechanism*. At stage 1, every player names one object. The players who end up in a cycle are assigned the object they named. At each stage $t = 2, \dots, N$, *players only observe which objects are still available*, i.e., which objects have not been assigned in the previous stages. If the last object a player named is still available, then the player cannot modify her choice — to comply with our formalism, she is obliged to play the dummy action of renaming the same object. Analogously, a player who has already been assigned an object keeps naming that object. If instead the last object a player named becomes unavailable at the current stage, then she must name one of the available objects. Again, the players who end up in a cycle trade their objects. Since at least one cycle occurs at every stage, by stage N the assignment is complete and players leave with their assigned object.

In the dynamic TTC mechanism, naming your favorite available object is locally dominant at every information set. In the next paragraph we provide a sketch of the proof, which we formalize in the Appendix.

Take the point of view of a player at an information set where she must name a new object. Let a be her favorite available object and b another available object. Is naming a better than naming b ? If naming a immediately yields a — without a doubt. Now suppose that after naming a our player must move again at some stage t , because a was assigned to someone else. Then, she may wonder whether she will still be in time to catch up with b or any other object she would name after b at stage t , and then continue in the same way, so to obtain the same object (the object one gets is always the final object one names). The answer is yes: whatever object our player would be naming at stage t after b will still be available after a . As long as our player is not assigned an object, her moves do not affect the set of available objects, and thus do not affect the moves of the opponents. Therefore, if after naming b our player is not assigned an object before stage t , she will face the same set of available objects at stage t after naming a and after naming b . If instead after b our player ends up in a cycle and is assigned an object at a stage $t^* < t$, that object must remain available until stage t also after a , because the other members of the cycle cannot modify their choices between stages t^* and t .

Note that our dynamic TTC mechanism actually requires a simpler form of contingent reasoning than the local partition. The above argument only requires our player to distinguish the scenario in which the locally dominant action a terminates the game from the scenario in which it does not, and deem the latter scenario irrelevant, i.e., she can get the same object by continuing in the same way. This bipartition is entirely driven by the dominant action, which is very easy to recognize as the right candidate choice based on the comparison of the possible immediate consequences. Then, our player can use the same (bi)partition to finalize the comparison with all the alternatives.

If players are capable of perfect contingent reasoning, they can also find their optimal actions by reasoning according to wishful dominance. In our dynamic TTC game, player i can ask herself: “Suppose that after playing b the best object I can obtain is c . Can I get something at least as good after playing a ?” The answer is yes. If player i does not get a (which is the best she can get), she will have the opportunity of naming c next, and then she will actually get c . The reason is that, as long as player i does not get an object, she does not affect what the opponents observe, and hence she does not affect their moves. So, in a contingency where she can get c after naming b , all the opponents in the cycle that gives her c make the same moves also when she names a in place of b , so she can close that cycle by naming c after naming a , if she does not get a .

Proving that naming the favorite object is wishfully dominant is somewhat simpler than proving that it is locally dominant, because it does not require to check that the object c obtained after naming b can also be obtained after naming a *by continuing in the same way*. That proving local dominance requires more work is natural, since it is a stronger notion than wishful dominance (see Proposition 2); nonetheless, the two notions capture the same intuition that we expect players to have in this game: there is nothing to lose from naming the favorite object. In other games, there could be wishfully dominant actions that are not locally dominant. In such a case, local dominance, which is a stronger simplicity standard, deems the intuitions captured by wishful dominance too difficult for real players.

6 Comparison with the literature

6.1 Local dominance versus obvious dominance

In static games, local strategy-proofness and obvious strategy-proofness coincide. This is a consequence of Proposition 3 and Theorem 1: by Proposition 3, at an information set of a static game (i.e., given a “type”), an action is locally dominant if and only if it is obviously dominant, and hence, by Theorem 1, a strategy is obviously dominant if and only if it prescribes an obviously dominant action at every information set.

Proposition 5 *A static game is obviously strategy-proof if and only if it is locally strategy-proof.*

Thus, like obvious dominance, local dominance rules out most direct mechanisms as too complicated.

In dynamic games, in one dimension, local dominance adopts a stricter simplicity standard than obvious dominance, in that it imposes the use of mimicking continuation strategies, rather than admitting the use of the optimal continuation strategy. In the dimension of contingent reasoning, instead, local dominance is more permissive than obvious dominance, in that it introduces the local partition. For this reason, local dominance is not stronger than obvious dominance, as shown by our dynamic TTC mechanism. Similarly, because of the complete lack of contingent reasoning, obvious dominance cannot explain why the ascending-price auction with simultaneous moves was found easy to play by Kagel et al. (1987), as we show next.

Differently from local dominance, obvious dominance assumes that the bidder can formulate at the outset the plan of bidding until the price reaches her valuation. Yet, according to obvious dominance, she cannot establish the superiority of this “sincere strategy” over a “stingy strategy” that leaves at a lower price p . This is because, when she compares the two strategies, she considers at the same time the chance of winning the object with the stingy strategy (when all the opponents leave at price p), and the possibility of not winning the object with the sincere strategy (when some opponent bids up to a higher

price than our bidder’s valuation). According to local dominance, instead, the chance of winning after leaving is liquidated in the primary scenario where the auction ends also after bidding, and in this scenario bidding is clearly better than leaving, as it guarantees winning the object. The experimental findings of Kagel et al. (1987) suggest that players do not mix up this “local” scenario with the alternative scenario in which bidding does not terminate the auction.

6.2 Comparison with Pycia and Troyan (2022)

The paper that is closest to ours is Pycia and Troyan (2022), who introduce the notion of *simple dominance*. At each decision node, players can only plan for a given set of “simple” future nodes and consider a “strategic plan” for those nodes. A strategic plan is simply dominant when the worst outcome that is consistent with it is not worse than the best outcome that the player may obtain after any alternative action at the current node. Thus, compared to obvious dominance, simple dominance considers the larger set of outcomes that are consistent not only with all the possible future moves of the opponents, but also with all possible own moves at the non-simple future nodes.

Among our notions of dominance, the closest to simple dominance is obvious dominance between actions, because, like simple dominance, it does not rely on any form of contingent reasoning. To facilitate the comparison, we provide the following characterization of our notion.

Remark 6 *Fix an information set $h \in H_i^*$. Action $\bar{a}_i \in A_i^h$ is obviously dominant if and only if, for every $a_i \in A_i^h \setminus \{\bar{a}_i\}$, there exists $\bar{s}_i \in S_i(h, \bar{a}_i)$ such that*

$$\min_{s_{-i} \in S_{-i}(h)} u_i(\zeta(\bar{s}_i, s_{-i})) \geq \max_{z \in Z(h, a_i, \bar{s}_{-i})} u_i(z).$$

Remark 6 is a corollary of Remark 3, as obvious dominance between actions coincides with s-dominance under the trivial partition.

The difference between the notion of simply dominant strategic plan and the notion of obviously dominant action is that the first specifies not just the current action but also the actions at the future decision nodes the player can

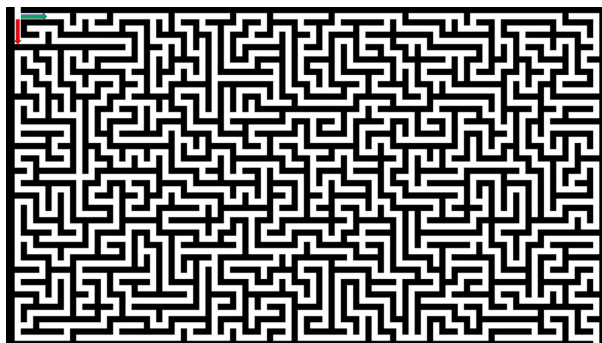
plan for. This (partial) continuation plan has to beat all the alternatives at the current node. Instead, an obviously dominant action is not associated with one *fixed* continuation strategy: as shown by Remark 6, our player can entertain different continuation strategies for the comparison with different alternatives. But even if we allow for flexibility in the use of continuation strategies, the issue with both notions is that finding the optimal (partial) continuation strategy may be the only way to establish dominance.

Pycia and Troyan (2022) eradicate this problem by focusing on *strong obvious dominance*, the special case of simple dominance in which the player does not perceive any future decision as simple, and thus does not plan at all. Thus, an action \bar{a}_i is strongly obviously dominant when the worst outcome that follows it is not worse than the best outcome that follows any alternative action. Such outcomes are computed across all the possible future moves of the player herself, because she has no clue of how she herself will play.

With local dominance, we let our player compare first the possible outcomes in case the game immediately ends, which do not depend on the future moves. This separation between present and future outcomes is a coarse form of contingent reasoning. When it comes to comparing the possible future outcomes, we introduce some simple considerations about the continuation game that ease the task. For the scenario in which the game continues after the candidate dominating action a but not after the alternative action b , our player simplifies her view of the possible future outcomes after a with a salient “mimicking strategy”, such as terminating the game at the next stage, or reverting to b . While any continuation strategy is a legitimate way of restricting the realm of possible outcomes after the candidate dominating action, by using the mimicking strategy we endogenize which continuation strategy is simple to conceive for the player, without requiring any consideration on the optimality of future moves. For the most complicated scenario in which the game continues after both actions, our player only checks whether the current choice may matter at all for the final outcome — the notion of irrelevance. Overall, compared to strong obvious dominance, we also consider a player who has no clue of how she will play in the future, but we radically differ in the way the player tackles

this problem.

Because of irrelevance, local dominance is not weaker than strong obvious dominance. Indeed, irrelevance is just a different, not a weaker criterion than the comparison of the best and the worst possible future outcomes. The following example illustrates this point.¹⁵



In this one-player game (called maze), the player has to enter at the top-left corner and exit at the bottom-right corner. Suppose that the player can never turn back, and getting stuck yields a payoff of 0, whereas reaching the exit yields a payoff of 1. At the first decision node, the player can go left (green arrow) or right (red arrow). Going right strongly obviously dominates going left. This is because, after going right, the player may reach the exit or not, depending on her future moves, whereas after going left the player always gets stuck. However, going right does not locally dominate going left. This is because, in the unique scenario in which the game continues after both actions, the continuation game is completely different after the two actions. Indeed, the choice between the two actions is not irrelevant for the final outcome.

6.3 Other related literature

Our theory of simplicity is related to the literature on limited foresight and coarse contingent reasoning. While we tackle the issue of foresight in a very different way than in the literature, our representation of contingent reasoning is in line with recent contributions on this topic. Like us, Zhang and Levin

¹⁵This image is borrowed from <https://www.youtube.com/watch?v=yuf8cd1eypA>.

(2021) model coarse contingent reasoning in games with a partition of the contingencies. However, Zhang and Levin take a global view of the game and fix a partition for the entire game exogenously at the outset, whereas our partitions depend on the information set, and even on the action pair under comparison. Chew and Wang (2022) only fix a cardinality k for the partition and stipulate that a strategy k -dominates another strategy when there exists a partition of cardinality k such that the usual “worst vs best” outcome comparison goes through within each partition element. Their notion of dominance is motivated by the sure thing principle as first described by Savage (1954) in his motivating example: if an agent finds that under each of two complementary events one option is better than the other, then she should find the former option better before the resolution of the uncertainty. In contrast, our notion of irrelevance of a scenario is inspired by Savage’s (1954) formal axiom, which says that a decision-maker should ignore a scenario under which all the alternatives yield the same outcome. Saponara (2022) considers a decision maker who evaluates each available act under a specific partition of the contingencies. For each element of the partition, the act is assigned the minimum attained utility. The decision maker then computes the expectation of these utilities with respect to a belief over the partition elements. Following Li (2017), instead, we adopt a belief-free approach and we require dominance to hold when the dominated action is assigned the best possible outcome in each non-irrelevant scenario. Karni and Viero (2013) consider a decision maker who progressively constructs a state space from the feasible acts she encounters. In particular, if two acts f and g can give the same two consequences c and d , she will recognize the existence of (at most) four possible states: the state where f and g give c , the state where they give d , the state where f gives c and g gives d , and vice versa. Let f and g be our two actions under comparison and let c and d be the consequence that the game ends or not: the induced state space coincides with our partition of the true state space.

Going in an opposite direction compared to our work and the literature on obvious dominance, Borgers and Li (2019) define a more permissive simplicity standard than strategy-proofness. In particular, they identify a class of “sim-

ple mechanisms” that only depend on the first-order beliefs about the types of the opponents. Dworzak and Li (2021) relax (obvious) strategy-proofness by allowing for multiplicity of (obviously) undominated strategies, provided that they all induce the desired outcome. In this way, they can implement social choice functions that cannot be implemented in (obviously) dominant strategies. This relaxation of (obvious) strategy-proofness allows, for each profile of players’ types, a multiplicity of undominated behaviors outside of the path that leads to the desired outcome. This is akin to requiring the existence of dominant actions for all players only along a path, a notion of “on-path strategy-proofness” which is natural in our framework and has some desirable properties: we illustrate it by example in the Appendix.

In simultaneous and independent works, Bo and Hakimov (2022) and Mackenzie and Zhou (2022) introduce two classes of mechanisms (respectively, “pick an object mechanisms” and the more general “menu mechanisms”) that encompass our dynamic TTC mechanism as a special case. Bo and Hakimov (2022) provide experimental evidence of the simplicity of a version of their mechanism (very similar to ours) precisely for the TTC rule, and justify it theoretically with a notion of “robust truthful equilibrium”; Mackenzie and Zhou (2022) prove the existence of versions of dominant-strategy equilibrium in their mechanisms. It would be interesting to investigate whether some of their mechanisms yield locally strategy-proof implementation of other social choice functions than the TTC rule. This is far from guaranteed, because a player’s moves may affect her future menus and the opponents’ menus; in this case, one cannot guarantee to a player that her current choice, if it does not yield an immediate outcome, will not affect the final outcome, making it impossible to satisfy local dominance.

7 Appendix

Proof of Proposition 1. Weak dominance. If. Fix $s_i \in S_i(h, a_i)$. By assumption, there exists $\bar{s}_i \in S_i(h, \bar{a}_i)$ that weakly dominates s_i . Condition (4) implies condition (2). Hence, a_i is weakly dominated by \bar{a}_i .

Only if. Fix $s_i \in S_i(h, a_i)$. By assumption, at h , \bar{a}_i weakly dominates a_i ,

and thus by Definition 2 there exists $\bar{s}_i \in S_i(h, \bar{a}_i)$ such that

$$\forall s_{-i} \in S_{-i}(h), \quad u_i(\zeta(\bar{s}_i, s_{-i})) \geq u_i(\zeta(s_i, s_{-i})). \quad (13)$$

Define \bar{s}'_i as $\bar{s}'_i(h') = \bar{s}_i(h')$ if $h' \succeq h$ and $\bar{s}'_i(h') = s_i(h')$ if $h' \not\succeq h$. Thus,

$$\forall s_{-i} \in S_{-i}(h), \quad \zeta(\bar{s}'_i, s_{-i}) = \zeta(\bar{s}_i, s_{-i}), \quad (14)$$

$$\forall s_{-i} \notin S_{-i}(h), \quad \zeta(\bar{s}'_i, s_{-i}) = \zeta(s_i, s_{-i}). \quad (15)$$

By (14), (13), and (15), we have

$$\forall s_{-i} \in S_{-i}, \quad u_i(\zeta(\bar{s}'_i, s_{-i})) \geq u_i(\zeta(s_i, s_{-i})),$$

i.e., condition (4): s_i is weakly dominated by \bar{s}'_i .

Obvious dominance. If. Fix $s_i \in S_i(h, a_i)$. By assumption, there exists $\bar{s}_i \in S_i(h, \bar{a}_i)$ that obviously dominates s_i . Since $s_i(h) \neq \bar{s}_i(h)$, $h \in \mathcal{D}(\bar{s}_i, s_i)$. Thus, condition (5) implies condition (3). So, a_i is obviously dominated by \bar{a}_i .

Only if. Fix $s_i \in S_i(h, a_i)$. By assumption, at h , \bar{a}_i obviously dominates a_i , and thus by Definition 2 there exists $\bar{s}_i \in S_i(h, \bar{a}_i)$ such that

$$\min_{s_{-i} \in S_{-i}(h)} u_i(\zeta(\bar{s}_i, s_{-i})) \geq \max_{s_{-i} \in S_{-i}(h)} u_i(\zeta(s_i, s_{-i})). \quad (16)$$

Define \bar{s}'_i as $\bar{s}'_i(h') = \bar{s}_i(h')$ if $h' \succeq h$ and $\bar{s}'_i(h') = s_i(h')$ if $h' \not\succeq h$. Thus,

$$\forall s_{-i} \in S_{-i}(h), \quad \zeta(\bar{s}'_i, s_{-i}) = \zeta(\bar{s}_i, s_{-i}), \quad (17)$$

$$\forall s_{-i} \notin S_{-i}(h), \quad \zeta(\bar{s}'_i, s_{-i}) = \zeta(s_i, s_{-i}). \quad (18)$$

By (17) and (16), we have

$$\min_{s_{-i} \in S_{-i}(h)} u_i(\zeta(\bar{s}'_i, s_{-i})) \geq \max_{s_{-i} \in S_{-i}(h)} u_i(\zeta(s_i, s_{-i})),$$

that is, \bar{s}'_i and s_i satisfy condition (5) at h . Since $s_i, \bar{s}'_i \in S_i(h)$ but $\bar{s}'_i(h) \neq s_i(h)$, we have $h \in \mathcal{D}(\bar{s}'_i, s_i)$. By (18), there is no point of departure between s_i and \bar{s}'_i along any path that does not go through h . So, $\mathcal{D}(\bar{s}'_i, s_i) = \{h\}$. Thus, s_i is

obviously dominated by \bar{s}'_i . ■

Proof of Theorem 1. Weak dominance. Only if. Fix $\bar{h} \in H_i^*(\bar{s}_i)$. Fix $a_i \in A_i^{\bar{h}} \setminus \{\bar{s}_i(\bar{h})\}$. For every $s_i \in S_i(\bar{h}, a_i)$, since $\bar{s}_i \in S_i(\bar{h}, \bar{s}_i(\bar{h}))$ is weakly dominant, it weakly dominates s_i . Hence, by Proposition 1 (if part) $\bar{s}_i(\bar{h})$ weakly dominates a_i .

If. Let \bar{s}_i be a strategy that prescribes a weakly dominant action at every $h \in H_i^*(\bar{s}_i)$. Fix $s_{-i} \in S_{-i}$. Fix $\bar{h} \prec \zeta(\bar{s}_i, s_{-i})$ and suppose by way of induction that, for every $h' \in H_i^*$ such that $\bar{h} \prec h' \prec \zeta(\bar{s}_i, s_{-i})$,¹⁶ for every s'_i that departs from \bar{s}_i at h' ,

$$u_i(\zeta(\bar{s}_i, s_{-i})) \geq u_i(\zeta(s'_i, s_{-i})).$$

Fix s_i that departs from \bar{s}_i at \bar{h} . By Proposition 1 (only if part) there exists $\bar{s}'_i \in S_i(\bar{h}, \bar{s}_i(\bar{h}))$ that weakly dominates s_i , and thus

$$u_i(\zeta(\bar{s}'_i, s_{-i})) \geq u_i(\zeta(s_i, s_{-i})). \quad (19)$$

Either $\zeta(\bar{s}_i, s_{-i}) = \zeta(\bar{s}'_i, s_{-i})$, or \bar{s}'_i departs from \bar{s}_i at some $h' \in H_i^*$ such that $\bar{h} \prec h' \prec \zeta(\bar{s}_i, s_{-i})$, and also in this second case, by the induction hypothesis,

$$u_i(\zeta(\bar{s}_i, s_{-i})) \geq u_i(\zeta(\bar{s}'_i, s_{-i})). \quad (20)$$

Inequalities (20) and (19) yield $u_i(\zeta(\bar{s}_i, s_{-i})) \geq u_i(\zeta(s_i, s_{-i}))$. Clearly, the same holds (with equality) also for all $s_i \in S_i$ that do not depart from \bar{s}_i at any $h \prec \zeta(\bar{s}_i, s_{-i})$. Thus, since s_{-i} was arbitrary, \bar{s}_i weakly dominates every $s_i \in S_i$.

Obvious dominance. Only if. Fix $\bar{h} \in H_i^*(\bar{s}_i)$. Fix $a_i \in A_i^{\bar{h}} \setminus \{\bar{s}_i(\bar{h})\}$. For every $s_i \in S_i(\bar{h}, a_i)$, since $\bar{s}_i \in S_i(\bar{h}, \bar{s}_i(\bar{h}))$ is obviously dominant, it obviously dominates s_i . Hence by Proposition 1 (if part) $\bar{s}_i(\bar{h})$ obviously dominates a_i .

If. Let \bar{s}_i be a strategy that prescribes an obviously dominant action at every $h \in H_i^*(\bar{s}_i)$. Fix $\bar{h} \in H_i^*(\bar{s}_i)$ and suppose by way of induction that for

¹⁶If \bar{h} is a stage- T information set, or anyway the last active information set along path $\zeta(\bar{s}_i, s_{-i})$, the induction hypothesis is vacuously satisfied.

every $h' \in H_i^*(\bar{s}_i)$ that follows \bar{h} ,¹⁷ for every s'_i that departs from \bar{s}_i at h' ,

$$\min_{s_{-i} \in S_{-i}(h')} u_i(\zeta(\bar{s}_i, s_{-i})) \geq \max_{s_{-i} \in S_{-i}(h')} u_i(\zeta(s'_i, s_{-i})).$$

Fix s_i that departs from \bar{s}_i at \bar{h} . By Proposition 1 (only if part) there exists $\bar{s}'_i \in S_i(\bar{h}, \bar{s}_i(\bar{h}))$ that obviously dominates s_i , and thus

$$\min_{s_{-i} \in S_{-i}(\bar{h})} u_i(\zeta(\bar{s}'_i, s_{-i})) \geq \max_{s_{-i} \in S_{-i}(\bar{h})} u_i(\zeta(s_i, s_{-i})). \quad (21)$$

For each $s_{-i} \in S_{-i}(\bar{h})$, either $\zeta(\bar{s}_i, s_{-i}) = \zeta(\bar{s}'_i, s_{-i})$, or \bar{s}'_i departs from \bar{s}_i at some $h' \in H_i^*$ such that $\bar{h} \prec h' \prec \zeta(\bar{s}_i, s_{-i})$, and also in this second case, by the induction hypothesis,

$$u_i(\zeta(\bar{s}_i, s_{-i})) \geq u_i(\zeta(\bar{s}'_i, s_{-i})). \quad (22)$$

Inequality (22) for all $s_{-i} \in S_{-i}(\bar{h})$, along with inequality (21), yield

$$\min_{s_{-i} \in S_{-i}(\bar{h})} u_i(\zeta(\bar{s}_i, s_{-i})) \geq \max_{s_{-i} \in S_{-i}(\bar{h})} u_i(\zeta(s_i, s_{-i})).$$

Since this holds for all $\bar{h} \in \mathcal{D}(\bar{s}_i, s_i)$, \bar{s}_i obviously dominates s_i . ■

Proof of Theorem 2. Only if: by inspection of the definitions.

If. Let \bar{s}_i be a strategy that prescribes an s-dominant action at every $h \in H_i^*(\bar{s}_i)$. First we prove that \bar{s}_i is weakly dominant. Fix $\bar{h} \in H_i^*(\bar{s}_i)$ and suppose by way of induction that for every $h \in H_i^*(\bar{s}_i)$ that follows \bar{h} ,³⁵

$$\forall s_i \in S_i(h), \forall s_{-i} \in S_{-i}(h), \quad u_i(\zeta(\bar{s}_i, s_{-i})) \geq u_i(\zeta(s_i, s_{-i})). \quad (23)$$

Fix $s_i \in S_i(\bar{h})$ and $s_{-i} \in S_{-i}(\bar{h})$. Since $\bar{s}_i(\bar{h})$ is s-dominant, it is wishfully dominant, and thus there exists $\bar{s}'_i \in S_i(\bar{h}, \bar{s}_i(\bar{h}))$ such that

$$u_i(\zeta(\bar{s}'_i, s_{-i})) \geq u_i(\zeta(s_i, s_{-i})).$$

¹⁷When no such h' exists, the induction hypothesis is vacuously satisfied.

Since $\bar{s}'_i(\bar{h}) = \bar{s}_i(\bar{h})$, either $\zeta(\bar{s}_i, s_{-i}) = \zeta(\bar{s}'_i, s_{-i})$, or there exists $h' \in H_i^*(\bar{s}_i)$ that follows \bar{h} such that $\bar{s}'_i \in S_i(h')$ and $s_{-i} \in S_{-i}(h')$, and also in this second case, by the induction hypothesis,

$$u_i(\zeta(\bar{s}_i, s_{-i})) \geq u_i(\zeta(\bar{s}'_i, s_{-i})).$$

So we have $u_i(\zeta(\bar{s}_i, s_{-i})) \geq u_i(\zeta(s_i, s_{-i}))$, as desired. Thus, for every $s_i \in S_i$, for each $s_{-i} \in S_{-i}$, if there is $h \in H_i^*(\bar{s}_i)$ such that $s_i \in S_i(h)$ and $s_{-i} \in S_{-i}(h)$, then we have just shown that $u_i(\zeta(\bar{s}_i, s_{-i})) \geq u_i(\zeta(s_i, s_{-i}))$, otherwise, $\zeta(\bar{s}_i, s_{-i}) = \zeta(s_i, s_{-i})$. Hence, \bar{s}_i weakly dominates s_i .

Now, we prove that, for each $h \in H_i^*(\bar{s}_i)$ and $a_i \in A_i(h) \setminus \{\bar{s}_i(h)\}$, $\bar{s}_i(h)$ dominates a_i given the partition \mathcal{S}^h of $S_{-i}(h)$ that player i uses for the comparison (both with s-dominance and dominance). For every $s_i \in S_i(h, a_i)$ and $\bar{s}_{-i} \in \mathcal{S}^h$, by condition (6), there exists $\bar{s}'_i \in S_i(h)$ such that

$$\min_{s_{-i} \in \bar{S}_{-i}} u_i(\zeta(\bar{s}'_i, s_{-i})) \geq \max_{s_{-i} \in \bar{S}_{-i}} u_i(\zeta(s_i, s_{-i})).$$

Since \bar{s}_i is weakly dominant,

$$\min_{s_{-i} \in \bar{S}_{-i}} u_i(\zeta(\bar{s}_i, s_{-i})) \geq \min_{s_{-i} \in \bar{S}_{-i}} u_i(\zeta(\bar{s}'_i, s_{-i})).$$

The last two inequalities combined yield condition (1): $\bar{s}_i(h)$ dominates a_i . ■

Proof of Proposition 4. Fix an information set h where bidder i has not left the auction yet. First, we compare b and ℓ , regardless of the presence of w . The partition $\mathcal{S}^\ell(h, b, \ell)$ features only two non-empty scenarios: the scenario $S^{b, \ell}(h)$ where none of the opponents bid at round t , and the scenario $S_b^\ell(h)$ where at least one does — $S_{b, \ell}(h)$ and $S_\ell^b(h)$ are empty because after ℓ there is no active information set of player i . We now check local dominance. In scenario $S^{b, \ell}(h)$, the only possible outcome after b is that i wins the auction, therefore either b , if $t \leq v - 1$, or ℓ , if $t \geq v$, satisfies condition (10) (with the role of \bar{a}_i). For the scenario $S_b^\ell(h)$, consider first $t \geq v$. In this case, ℓ terminates the game with payoff 0, while after b the payoff cannot be strictly

positive, therefore ℓ satisfies condition (11) (with $\bar{a}_i = \ell$). Thus, ℓ locally dominates b . Consider now $t \leq v - 1$. The strategy that prescribes ℓ at every information set after b mimicks the dummy continuation strategy that prescribes ℓ at every information set after ℓ . With the mimicking continuation strategy (in the scenario under consideration), i 's payoff will be determined at stage $t + 1$, and since $t + 1 \leq v$, it cannot be negative. Player i ' payoff after ℓ is always 0 in scenario $S_\ell^b(h)$. Therefore, condition (12) holds. Thus, b locally dominates ℓ .

Now we compare b with w for $t \leq v - 1$. If only one opponent is still in the auction, w is equivalent to ℓ , because it always terminates the auction. Otherwise, the partition $\mathcal{S}^\ell(h, b, w)$ features three non-empty scenarios: the scenario $S^{b,w}(h)$ where none of the opponents bid at round t , the scenario $S_b^w(h)$ where only one bids, and the scenario $S_{b,w}(h)$ where more than one bids — $S_w^b(h)$ is empty because if the auction ends with b , so it does with w . For the first two scenarios, the comparison between b and w is identical to the comparison between b and ℓ . Note in particular that in scenario $S_b^w(h)$, player i is forced to play ℓ after w , therefore the mimicking continuation strategy after b prescribes ℓ as before. For scenario $S_{b,w}(h)$, we show that it is irrelevant, so that, as for the comparison with ℓ , b locally dominates w . Thus, for $t < v$ let $(\bar{a}_i, a_i) = (b, w)$, for $t \geq v$ let $(\bar{a}_i, a_i) = (w, b)$. Fix $s_i \in S_i(h, a_i)$. Construct the mimicking $\bar{s}_i \in S_i(h, \bar{a}_i)$ as follows. Fix a stage $t' > t$ and suppose by way of induction that, for each $s_{-i} \in S_{b,w}(h)$, either the object was assigned before stage t' under both s_i and the \bar{s}_i under construction, or it was not assigned before stage t' under both s_i and the \bar{s}_i under construction. Under all $s_{-i} \in S_{b,w}(h)$ that fall into the first category, s_i prescribes ℓ at the stage- t' information set $h' \prec \zeta(s_i, s_{-i})$, and we can let \bar{s}_i prescribe ℓ as well, at every stage- t' information set that is consistent with \bar{s}_i and with the fact that the auction ended. Under all $s_{-i} \in S_{b,w}(h)$ that fall into the second category, player i reaches the same stage- t' information set h' with s_i , and the same stage- t' information set \bar{h}' with \bar{s}_i . This is because all that player i learnt in the previous stages is that the auction did not end.¹⁸ Since the auction is still

¹⁸To be precise, regarding the behavior of the opponents at stage t , after b player i has less

ongoing both at h' and \bar{h}' , all actions are available and we can let $\bar{s}_i(\bar{h}') = s_i(h')$. Moreover, at stage t' , the opponents reach the same information set regardless of s_i or \bar{s}_i , because all they observe is that the auction is still ongoing.¹⁹ Hence, just like player i , they make the same move at stage t' regardless of s_i or \bar{s}_i . So, at stage t' , if the auction ends under s_i it also ends under \bar{s}_i , and if it continues under s_i , it also continues under \bar{s}_i : the induction hypothesis for stage $t' + 1$ is proven. Moreover, by the same token, if the auction ends at stage t' , the outcome is the same under s_i and \bar{s}_i . Thus, irrelevance holds.

To conclude the proof, we need to show that ℓ locally dominates w when $t = v$. There two non-empty scenarios: $S^{\ell,w}(h)$ and $S_w^\ell(h)$. In the first scenario, since $t = v$, both actions always yield payoff 0, thus ℓ satisfies condition (10) (with the role of \bar{a}_i). In the second scenario, ℓ terminates the game with payoff 0, while after w the payoff cannot be strictly positive, therefore ℓ satisfies condition (11) (with $\bar{a}_i = \ell$). ■

Proof of Theorem 4. Fix a player $i \in I$, a stage $t \in \{1, \dots, N\}$, and a stage- t information set $h \in H_i^*$. Let a be i 's favorite still-available object, and let b be another available objects. Thus, $a, b \in A_i^h$. We are going to show that a locally dominates b .

Note that the object assigned to a player coincides with the last object she names. Therefore, in the scenarios $S^{a,b}(h)$ and $S_b^a(h)$, player i gets a after naming a , so Conditions (10) and (11) are satisfied.

We will show that, for every $s_i \in S_i(h, b)$, there exists $\bar{s}_i \in S_i(h, a)$ that mimics s_i under $S_a^b(h) \cup S_{a,b}(h)$. For every $s_{-i} \in S_a^b(h) \cup S_{a,b}(h)$, player i moves again after choosing a at h , therefore \bar{s}_i and s_i prescribe the same last move along the paths induced by (s_i, s_{-i}) and by (\bar{s}_i, s_{-i}) , and hence yield the same outcome. Thus, $S_{a,b}(h)$ is irrelevant. Moreover, given that s_i yields b under all $s_{-i} \in S_a^b(h)$, so does \bar{s}_i , and condition (12) is satisfied as an equality.

accurate information than after w , because for the auction to continue after waiting it takes at least two opponents to bid at round t instead of one. Nonetheless, *in the scenario where the auction continues after both actions*, there is just one information set per stage also after w .

¹⁹Given that under s_{-i} the choice of i at h is not pivotal to determine whether the auction continues, it means that i 's opponents cannot infer her choice at h from the fact that the auction continued.

Now we construct the mimicking \bar{s}_i . We will repeatedly use the fact that players only observe the history of available objects, therefore as long as player i does not obtain an object, her moves cannot affect the moves of the opponents, and hence the history of available items. Fix $h' \in \mathcal{H}^*(h, a)$. Let ψ denote the history of available objects that player i observes at h' . First we prove the following fact

Claim 5 *For each $s_{-i} \in S_{-i}(h')$, if under s_i player i is assigned an object by stage $\tau(h')$, then that object is available at h' .*

Proof of the claim. Suppose that, under s_{-i} and s_i , player i is assigned an object c at some stage $t' \leq \tau(h')$. Then, until stage t' , under s_{-i} , the moves of the opponents are the same under s_i and after choosing a at h . Therefore, also after choosing a at h , at stage t' the owner of c is pointing, directly or indirectly, to player i . Since this player and possibly the other players in the chain that links them to player i cannot move as long as player i is not assigned an object, c remains available at h' . \square

Now determine $\bar{s}_i(h')$ as follows. For all $s_{-i} \in S_{-i}(h')$, as long as player i does not get an object, the history of available objects follows ψ also under s_i . Thus, if b is available at h' , under s_i and all $s_{-i} \in S_{-i}(h')$ either player i gets b by stage $\tau(h')$, or she cannot move until stage $\tau(h')$; either way, she will be naming b at stage $\tau(h')$, so we can let $\bar{s}_i(h') = b$. If instead object b stops being available along ψ at some stage $t' \leq \tau(h')$, by Claim 5 player i did not obtain b by stage t' under any $s_{-i} \in S_{-i}(h')$. But then, using s_i and ψ , we can determine the *unique* object c player i names at stage t' under s_i and *all* $s_{-i} \in S_{-i}(h')$. Repeating the reasoning we made for b , we can determine if player i names c at stage $\tau(h')$, or will switch to another object d at some stage $t'' \leq \tau(h')$. Going on in this fashion, we can determine the *unique* object e player i names at stage $\tau(h')$ under s_i and *all* $s_{-i} \in S_{-i}(h')$, and let $\bar{s}_i(h') = e$.

Now fix a stage $t'' > \tau(h')$, a stage- t'' information set $h'' \succ h'$, and suppose by way of induction that \bar{s}_i was defined and mimics s_i at every $\tilde{h} \prec h''$. Thus, for all $s_{-i} \in S_{-i}(h'')$, under both s_i and \bar{s}_i , either before stage $\tau(h'')$ player i has already obtained the same object e , and then we can let $\bar{s}_i(h'') = e$, or player

i has not obtained any object. In this second case, the history of available objects at h'' coincides with the one at the *unique* stage- $\tau(h'')$ information set \tilde{h} that is consistent with s_i and *all* $s_{-i} \in S_{-i}(h'')$, and this also implies that $h'' \in H_i^*$ if and only if $\tilde{h} \in H_i^*$. So, we can let $\bar{s}_i(h'') = s_i(\tilde{h})$.

To conclude, note that $\cup_{h' \in \mathcal{H}^*(h,a)} S_{-i}(h') = S_a^b(h) \cup S_{a,b}(h)$, so \bar{s}_i mimics s_i under $S_a^b(h) \cup S_{a,b}(h)$. ■

Example of on-path strategy proofness

A shepherd dog has to recall the sheep from the top of the hill for the night. The dog's goal is to maximize the number of sheep that make it all the way down to the sheepfold before falling asleep. Then, by contract, the dog has to guard the sheep from 2/3 of their average sleeping altitude.²⁰ A sheep's payoff is the distance from the dog during the sleep. Because of the fog, the sheep cannot see where they are going and typically sleep scattered on the slope. To solve this problem, the dog comes up with the following idea. The dog first stands at altitude 67 (the top of the hill is at altitude 100) and shines a light. The sheep can see the light through the fog and walk down towards it. Those who stop along the way (at an altitude between 68 and 100) cannot help falling asleep. Those who reach the dog at 67 get to see each other and manage to stay awake. Then the dog moves to altitude 45 and shines a light again for the sheep that are still awake. The game continues in this fashion until the dog reaches the sheepfold at altitude 1. Then, if some sheep has not arrived, the dog looks for his prescribed guarding position and stops there.

Take the viewpoint of a sheep at the initial history or at an information set where it has reached the dog and sees that all other sheep have reached the dog as well. Reaching the dog at the next, lower altitude is locally dominant and obviously dominant. Given any alternative action of stopping at a higher

²⁰This game is a dynamic transformation of "guess 2/3 of the average", which obviates the possible distrust in the opponents' rationality by letting players *observe* whether the opponents have played rationally. Glazer and Rubinstein (1996) provide general rules to transform a dominance-solvable static game into a strategy-proof dynamic one.

altitude, the local partition coincides with the trivial partition, as there is only one scenario: the one in which the alternative action terminates the game (the sheep then falls asleep) and reaching the dog does not (unless the dog has reached the sheepfold, in which case the game ends also after reaching the dog). The sheep can entertain the continuation strategy (after reaching the dog) of not moving forward and sleep there. Such a continuation strategy mimicks the dummy continuation strategy after the alternative action. If the sheep reaches the dog and sleeps there, the payoff will certainly be higher than after the alternative action: given that all sheep have reached the dog at its old altitude, the dog will not sleep below its new altitude, which is $2/3$ of the old one.

Note that the sheep do not have an obviously dominant strategy, or a strategy entirely made of locally dominant actions. If at some point a sheep observes that not all others have reached the dog, walking down to the next dog's altitude might not be optimal, because the dog might have to finally guard the sheep from a higher altitude. Therefore, the game is not obviously strategy proof or locally strategy-proof. However, it is *on-path strategy proof* with both obvious and local dominance, in the sense that reaching the dog is dominant when all sheep have reached the dog, i.e., when they have *all* played their dominant action at the previous information sets. The off-path information sets, though, could be eliminated from the game: after observing that not all sheep have arrived, the dog could quit the game and move directly to its guarding position, instead of trying to drag the remaining sheep down to the sheepfold. Quitting the game in this way, however, requires the ability of the dog to commit to a suboptimal behavior given its objective function.

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