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Actuarial Modeling and Analysis of the Hong Kong Life Annuity Scheme

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Abstract:
The Hong Kong Mortgage Corporation (HKMC) Limited, which was established in March 1997 and is wholly owned by the government of the Hong Kong Special Administrative Region, has a major mission to develop and provide different financial retirement instruments to Hong Kong residents to help address the income poverty of retirees. In June 2017, HKMC Annuity Limited, a wholly-owned subsidiary of the HKMC was incorporated to implement a new life annuity scheme which would be launched by mid-2018 to cater for the needs of cash-rich Hong Kong old age residents. The objective of the scheme is to provide an additional financial retirement planning option with minimum credit risk and a certain level of liquidity to the elderly by turning lump-sum premiums into lifetime streams of monthly income at a reasonable and stable return rate. In this paper, we establish an actuarial framework to model this life annuity scheme. The framework enables us to estimate the monthly annuity payments one might receive for a certain amount of investment. It also allows us to analyze the risk entailed in the product, thus shedding light on how the underlying risk can be managed through product design. Our findings will help potential subscribers to understand the scheme and decide whether this scheme should be included in their retirement investment portfolios when it is launched in 2018.

Keywords: life annuity, actuarial model, mortality, equivalence principle, inflation risk

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1 Introduction

Ensuring the constant incomes of old age residents through one or more pension schemes is critical in any ageing population to avoid poverty among retirees. Many Asia-Pacific countries have cautiously established various financially sustainable social-security programs for their old age populations (see, e. g. Choi and Kim 2016; Lee and Chou 2016). However, the ideal pension policies for retirees vary from country to country and are fundamentally determined by the prevailing social, political and economic structure. For instance, in the Hong Kong Special Administrative Region (HKSAR), which has been ranked as the world's freest economy for the past 24 consecutive years, the government has adopted a voluntary system of retirement funding for its residents, while the second freest economy, Singapore, has established a mandatory public pension scheme for its working population.[1] The voluntary strategy in Hong Kong has not only eased the pressure on the fiscal budget but has also allowed individual retirees to determine the optimal investment strategies according to their own risk appetite. In contrast, the mandatory approach in Singapore has provided a low risk and compulsory financial channel for the elderly to convert some of their lump-sum savings into a constant stream of retirement income at a reasonable and stable return rate with additional bequest benefits.[2]

Historically, the Hong Kong government has taken a *laissez-faire* attitude to retirement protection of its residents. The provision of pension plans has been left largely to the private sector. After the change of sovereignty from Britain to China in 1997, the new HKSAR government has been actively reforming its retirement security policy. In 2000, the Mandatory Provident Fund (MPF) scheme was introduced in Hong Kong. It is a defined contribution scheme, which covers all employees and self-employed persons between the ages of 18 and 65. The employee is required to contribute 5 % of his or her monthly income and the employer has to match this amount (Chan 2001). In context of the persistent inadequacy of the means-tested welfare programs in Hong Kong to

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tackle old age poverty, the introduction of the MPF scheme was aimed to alleviate the long-run reliance on government welfare provision (Commission on Poverty 2015). This development is analogous to pension reforms in the United States, several European and some developing countries in which the retrenchment of defined benefit (DB) pensions is coupled with an increasing preponderance of defined contribution (DC) pension provision (Ashcroft 2009). The MPF's accumulated assets reached HK$857 billion at the end of March 2018 with 2.596 million employees making compulsory contributions to the scheme (MPFA 2018).

In addition to the MPF system and retirement products offered by some private insurance companies in Hong Kong, the HKSAR government indirectly provides some instruments to boost the incomes of asset-rich old age residents after retirement. The Hong Kong Mortgage Corporation Limited (HKMC), which was established in March 1997 and is wholly owned by the HKSAR government, has a major mission to develop and provide financial retirement instruments to Hong Kong residents. In 2011, the HKMC launched a reverse mortgage program (RMP) for Hong Kong home owners aged 55 or above. Under the RMP, residential properties can be used as collateral for bank loans while the home owners are entitled to stay in their properties for life. The loans can be arranged as a stream of monthly payments either over a fixed period of 10, 15, 20 years or for the homeowner's entire life. However, lump-sum payout loans are also available if necessary. In general, home owners do not need to repay any of the principal or interest on the loans during their lifetime. After the death of a home owner, if the proceeds from the sale of the property exceed the outstanding loan balance, the surplus will be returned to the beneficiaries. However, if there is any shortfall, the beneficiaries will not be liable because of the RMP guarantees no negative equity. By the end of April 2018, a total of 2,529 RMP applications had been approved, with the average age of borrowers being 69 years and ranging from 55 to 103 years old. The average appraised property value was 5.1 million Hong Kong dollars with an average monthly payment of HK$15,000. Among the different monthly payout options, 44 % of the applicants chose the lifetime payment arrangement. As a result, the RMP provides an important option for asset-rich, cash-poor seniors who are able to use their properties to generate life-time monthly incomes.

However, only a relatively small number of asset-rich elderly have benefited from the RMP and the scheme is not really an option for residents who do not own any property. Unfortunately, the under-developed (due to low demand and limited supply)[3] private life annuity market in Hong Kong is not able to provide an attractive alternative for retirees wishing to fund their retirement incomes. It is well documented that the low demand for life annuity products is due to most retirees, especially in Asia, being economically short-sighted, bequest motives, and a lack of familiarity with the products. Moreover, most of the private life annuity products are considered expensive in today's low interest rate environment and consumers tend to perceive the product as having high-costs, low liquidity, and large credit risk.[4] In such small life annuity markets with high operational costs and possible adverse selection of annuitants, insurance companies have to set relatively high premiums on their annuities to account for the increased longevity of annuitants.[5] Babbel and Merrill (2007) find that a small default risk can have a very large impact on consumers' annuity purchase decisions, while the low liquidity of life annuities with a high liquidity risk premium leads to poor value life annuity products in terms of the money's worth ratios.[6] As a result, it is unlikely that the private life annuity market will be able to sufficiently and effectively meet the demands of retirement financing and address the income poverty of retirees in Hong Kong.

In April 2017, the HKMC announced that the Board of Directors had approved in principle to introduce a new life annuity scheme to cater for the needs of cash-rich Hong Kong old age residents. HKMC Annuity (HKMCA) Limited, a wholly-owned subsidiary of the HKMC was incorporated in June 2017 to implement a new life annuity scheme, called HKMC Annuity Plan, which would provide an additional financial retirement planning option for the elderly in which lump-sum premiums can be converted into lifetime streams of fixed monthly income[7]. With the commitment to provide a lifetime monthly income to annuitants at a reasonable and stable long-term investment return, the plan is designed to be financially viable and sustainable.

To reasonably forecast the demand and impact of the HKMC Annuity Plan in Hong Kong, it is essential to establish an actuarial pricing model of the plan and conduct a thorough financial and actuarial analysis of its underlying features so that potential plan subscribers can understand the tradeoff between the costs and benefits. Policymakers can also consider how to enhance the initial plan to cover different age groups or provide different benefit options by modifying our proposed actuarial pricing model when necessary. In Section 2, we discuss the actuarial framework of the pricing model for this new life annuity plan. In Section 3, we derive projected cohort life tables for the population of Hong Kong by considering a wide range of stochastic mortality models in the literature. The projected cohort life tables as well as some specific return rates are then fed into the pricing model to produce various payout examples, which are presented in Section 4. Then, in Section 5, we analyze the features of the plan that may affect an individual's decision to apply for this plan and the longevity risk of the HKMC Annuity Plan portfolio. A brief summary of our findings and suggestions for further enhancing the plan are presented in Section 6.

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2 Actuarial Framework of the Pricing Model

In this section, we establish the actuarial framework of the pricing model for the HKMC Annuity Plan based on the information given in the HKMC's two press releases.[8] The life annuity plan, which will only be available to Hong Kong residents aged 65 or above, will provide lifetime monthly payments after a retiree has made a lump-sum premium contribution of between HK$50,000 and HK$1 million. The plan also guarantees that each annuitant will receive a total payout amounting to 105 % of the premium contribution no matter how long he or lives. This implies that if an annuitant dies before receiving 105 % of the premium contribution, his or her beneficiaries will continue to receive the remaining unpaid monthly payments until the total payout amount reaches 105 % of the premium contribution or a lump-sum amount of the difference. In case of emergency needs, the annuitant will also be able to exercise a surrender option to receive a lump-sum amount that equals the present value of the remaining unpaid installments if they have received less than 105 % of the premium contribution.[9]

In the actuarial framework, which includes assumptions on the investment return and mortality, the actuarial equivalence principle is included in the pricing model to justify the relationship between the premium amount and the monthly payments. For any given fixed internal rate of return i per year and a specified lifetime monthly amount P payable to a specific annuitant, the pricing model determines the actuarial present value of this stream of monthly payments, which is then set as the premium contribution C.

Consider a Hong Kong annuitant aged x for $x \geq 65$. Let v be the discount factor (*i.e.*,$v = (1+i)^{-1}$). Denote the monthly curtate future lifetime random variable by $K_x^{(12)}$, which is the future lifetime of an annuitant at age x, rounded to the lower month in that year. From the perspective of the HKMC, the present value of the future loss random variable is denoted as L. Assuming that the bequest is paid as a lump-sum amount at the end of the month of death and ignoring surrenders, the present value of the future loss random variable[10] is

$$L = -C + 12P \times \ddot{a}^{(12)}_{\overline{K_x^{(12)}+\frac{1}{12}}|} + \max\left[1.05C - P \times (12K_x^{(12)}+1),\ 0\right] \times v^{K_x^{(12)}+\frac{1}{12}} \tag{1}$$

for $K_x^{(12)} = 0, \frac{1}{12}, \frac{2}{12}, \cdots$.

In the equation above, $12P\times\ddot{a}^{(12)}_{\overline{K_x^{(12)}+\frac{1}{12}}|}$ represents the present value of the monthly annuity payments, whereas $\max[1.05C - P\times(12K_x^{(12)}+1),\ 0]\times v^{K_x^{(12)}+\frac{1}{12}}$ represents the present value of the benefit payable to the beneficiary if the total amount of the annuity payments paid to the annuitant has not yet reached 1.05C when the annuitant dies. Note that the latter is equivalent to a monthly decreasing term life insurance. In what follows, we use t to represent the term (in years) of the monthly decreasing term insurance embedded in the product. Given P and C, t can be calculated as $1.05C/12P$ rounded down to the nearest 1/12.

Note that if the annuitant dies d years (rounded down to the nearest 1/12 year) later, the lump-sum bequest will be $1.05C - 12dP$ for $d \leq t$. There will be no bequest for $d > t$ because the decreasing term insurance matures in t years and the total amount of monthly payments is more than 105 % of premium contribution. Within t years after the issue of the annuity product, the annuitants can exercise the surrender option when necessary.

In the case of $d \leq t$, if a beneficiary chooses the remaining unpaid monthly payments option, the HKMC Annuity Plan is equivalent to the ordinary life annuity with a t-year period certain or a t-year guaranteed life annuity. Due to the time value of money, it is unlikely that the remaining payments option will be chosen, instead a lump sum amount will be withdrawn. Therefore, our pricing model is constructed and analyzed based on the lump-sum bequest arrangement.

Based on the principle of actuarial equivalence, in which the actuarial present value of cash inflows is equal to the actuarial present value of cash outflows, i.e. $E(L) = 0$, we take the expectations in both sides of equation (1) and then derive the monthly payment amount as

$$P = \frac{C\times\left[1-1.05A^{\ \ (12)}_{\overset{1}{x}:\overline{t}|}\right]}{12\times\left[\ddot{a}_x^{(12)}-(I^{(12)}A)^{\ \ (12)}_{\overset{1}{x}:\overline{t}|}\right]} \tag{2}$$

3 Mortality Projection

The pricing framework described in the previous section is implemented with projected gender-specific cohort mortality rates. In this section, we explain the projected rates are obtained.

To begin with, let us define the following notation:

– $m_{x,t}$ is the central death rate at age x and in year t;

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- $q_{x,t}$ is the conditional probability of death at age x and in year t;[11]
- $[x_0, x_1]$ is the sample age range to which the candidate stochastic mortality models are fitted;
- $[t_0, t_1]$ is the sample period to which the candidate stochastic mortality models are fitted;
- $n_a = x_1 - x_0 + 1$ is the number of ages covered by the sample age range;
- $n_y = t_1 - t_0 + 1$ is the length of the sample period;
- $c = t - x$ is the year of birth; note that within the data sample c ranges from $t_0 - x_1$ to $t_1 - x_0$;
- n_c is the number of data points associated with year-of-birth c.
- $\bar{x} = (x_0 + x_1)/2$ is the mid-point of the sample age range;
- $(\bar{x} - x)_+ = \max(\bar{x} - x, 0)$ is the maximum of $\bar{x} - x$ and 0.

Following Cairns et al. (2009) and Li, Zhou, and Liu (2018a, b); Li et al. (2019), we consider the following collection of nine stochastic mortality models. All of the stochastic mortality models are estimated using the method of Poisson maximum likelihood, in which the actual death count for each age x and year t is assumed to be a realization of a Poisson random variable with a mean that equals to the product of the corresponding exposure count and the value of $m_{x,t}$ implied by the model structure.[12]

- The Lee-Carter model (Lee and Carter 1992)

$$\ln m_{x,t} = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)}$$

- The Renshaw-Haberman Model (Renshaw and Haberman 2006)

$$\ln m_{x,t} = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \beta_x^{(3)}\gamma_c$$

- The age-period-cohort model (Osmond 1985)

$$\ln m_{x,t} = \beta_x^{(1)} + n_a^{-1}\kappa_t^{(1)} + n_a^{-1}\gamma_c$$

- The original Cairns-Blake-Dowd (CBD) model (Cairns, Blake, and Dowd 2006)

$$\ln \frac{q_{x,t}}{1 - q_{x,t}} = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x})$$

- The CBD model with a cohort effect (Cairns et al. 2009)

$$\ln \frac{q_{x,t}}{1 - q_{x,t}} = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \gamma_c$$

- The CBD Model with quadratic age and cohort effects (Cairns et al. 2009)

$$\ln \frac{q_{x,t}}{1 - q_{x,t}} = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \kappa_t^{(3)}((x - \bar{x})^2 - \hat{\sigma}_x^2) + \gamma_c$$

- The CBD Model with an age-modulated cohort effect (Cairns et al. 2009)

$$\ln \frac{q_{x,t}}{1 - q_{x,t}} = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \gamma_c(x_c - x)$$

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– The full Plat model (Plat 2009)

$$\ln m_{x,t} = \beta_x^{(1)} + \kappa_t^{(1)} + \kappa_t^{(2)}(\bar{x} - x) + \kappa_t^{(3)}(\bar{x} - x)_+ + \gamma_c$$

– The simplified Plat model (Plat 2009):

$$\ln m_{x,t} = \beta_x^{(1)} + \kappa_t^{(1)} + \kappa_t^{(2)}(\bar{x} - x) + \gamma_c$$

The parameters in the above mentioned models can be interpreted as follows:

- $\beta_x^{(i)}$, $i = 1, 2, 3$, are age-specific parameters; specifically,
  - $\beta_x^{(1)}$ is a stand-alone age-specific parameter,
  - $\beta_x^{(2)}$ is an age-specific parameter that interacts with a time-varying index, and
  - $\beta_x^{(3)}$ is an age-specific parameter that interacts with a cohort-related index;
- $\kappa_t^{(i)}$, $i = 1, 2, 3$, are time-varying indexes; specifically,
  - $\kappa_t^{(1)}$ is a stand-alone time-varying index;
  - $\kappa_t^{(2)}$ is a time-varying index that interacts with an age-specific parameter or a linear function of age;
  - $\kappa_t^{(3)}$ is a time-varying index that interacts with a non-linear function of age;
  - γ_c is a cohort-related index.

The candidate models are estimated to data over a sample age range of [65, 99] and a sample period of [1971, 2017].[13] All of the required data are provided by the Statistics and Census Department (2018) of the HKSAR.

The specific model we use to obtain projected cohort mortality rates is chosen on the basis of the Bayes Information Criterion, which, in the context of this analysis, is defined as

$$\text{BIC} = -\hat{l} + \frac{1}{2} \times n_p \times \ln(n_a n_y),$$

where $\hat{l}$ represents the maximized log-likelihood value, n_p denotes the number of model parameters, and $n_a n_y$ is the number of data points (i.e. age-time cells covered by the data sample). The value of the BIC is lower when the model gives a better goodness-of-fit (measured by $\hat{l}$) and is more parsimonious (when n_p is smaller). We choose the model that yields the lowest BIC value.

The BIC values produced by the nine candidate models are reported in Table 1. For both genders, the full Plat model is the best performing and is therefore chosen for projection purposes. To obtain projected mortality rates with the estimated full Plat model, the three time-varying indexes are extrapolated by a trivariate random walk with drift, whereas the cohort-related index is extrapolated with a first-order autoregressive process. The projected mortality rates are fed into the pricing model described in Section 2 to calculate payouts. For instance, when evaluating L for annuitants aged 65 in 2018 (when the annuities are sold), the following projected values are used:

$$q_{65,2018}, \; q_{66,2019}, \; q_{67,2020}, \; \cdots.$$

Table 1: The BIC values produced by the nine stochastic mortality models under consideration.

	BIC	
Model	**Male**	**Female**
Lee-Carter	41641.9	60388.6
Renshaw-Haberman	23488.1	22902.6
Age-period-cohort	31056.3	33522.8
Original CBD	25728.5	50153.0
CBD with a cohort effect	19401.1	19897.9
CBD with cohort and quadratic age effects	15257.6	15048.9
CBD with an age-modulated cohort effect	19075.4	21489.9
Full Plat	14031.5	13883.3

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4 Illustrative Results of the Pricing Model

To compute the payouts of the life annuity plan, we need to specify the return rate of the annuity fund and the mortality rates of the annuitants. After having an independent consultant verify and validate the plan, the HKMC confirmed that the annual internal rate of return can be expected to be at the higher end of the indicated range of 3 % to 4 %. For illustrative purposes, the pricing model is assumed to have a fixed investment return rate of 3 % to 4 % per annum.

The illustrative results are based on the projected cohort mortality rates obtained in Section 3.[14] While future mortality improvement is taken into account, the mortality assumptions we make may still be conservative as residents in good health are more likely to purchase life annuity products compared with residents in poor health. That being said, the projected mortality rates (which are applicable to the general population of Hong Kong) may overstate the mortality rates for those residents who actually purchase the life annuity.[15]

Now consider a premium contribution of $100,000 for both male and female annuitants at entry ages ranging from 65 to 80 under an internal rate of return of either 3 % or 4 %. Based on the proposed pricing model, the payment amounts are given in Table 2.[16]

Because they have higher mortality rates, male annuitants receive around 10 % to 12 % more in monthly payments than female annuitants at any entry age. For both genders, the monthly payments increase by about 2 % – 3 % for each one year increase in entry age regardless the interest rate being 3 % or 4 %. This simple relationship enables the provider to easily communicate the pricing information to the public. When the interest rate changes from 3 % to 4 %, the monthly payments in general increase by about 11 % – 14 % for all the considered cases with slightly higher impact on the lower entry age.

Table 2: Illustrative examples of monthly payments in the pricing model.

	$i = 4\,\%$		$i = 3\,\%$	
Age	**Male**	**Female**	**Male**	**Female**
65	534	488	469	427
66	545	497	480	435
67	556	506	491	445
68	569	516	503	454
69	582	526	515	464
70	595	537	528	475
71	610	549	541	486
72	625	562	556	498
73	641	575	571	511
74	658	589	587	525
75	676	604	603	539
76	695	620	621	555
77	715	637	640	571
78	737	656	660	589
79	759	676	681	608
80	784	697	703	628

Notes: The premium contribution is assumed to be 100, 000.

5 Analysis of the HKMC Annuity Plan

5.1 Individual Contracts

By rewriting the pricing equation (2), we can show that the contribution amount C is used to cover the costs of two insurance products: a whole-life annuity and a t-year decreasing term insurance (reflects the guaranteed return of premium):

$$C = 12P \times \ddot{a}_x^{(12)} + \left[1.05C \times A^{1\ \ (12)}_{x:\ \overline{t}|} - 12P \times (I^{(12)}A)^{1\ \ (12)}_{x:\ \overline{t}|}\right].$$

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These two components respond to the future lifetime random variable in opposite directions, thereby creating a natural hedging effect which makes the product as a whole less risky. This natural hedging effect is explored in more detail later in this section.

To analyze the components of the HKMC Annuity Plan for both genders at two different interest rates, the relative amounts of contribution paid for the decreasing term insurance component versus entry ages are presented in Figure 1. The lower the interest rate, the higher the proportion of the term insurance component for both genders. Because higher mortality rates imply higher costs for the term insurance and lower costs for the life annuity, it is not surprising to see that the proportions of the term insurance component are higher for older annuitants. For example, at an interest rate of 4 % for males, the proportion of the insurance component is about 9 % at an entry age of 65 but it increases to more than 25 % at an entry age of 80. In other words, if the male annuitants aged 80 are willing to give up the bequest benefit, the monthly payments can increase by more than 25 %. Figure 1 provides a useful comparison of the HKMC Annuity Plan with the ordinary whole-life annuity without any guaranteed pay-back amount at death.

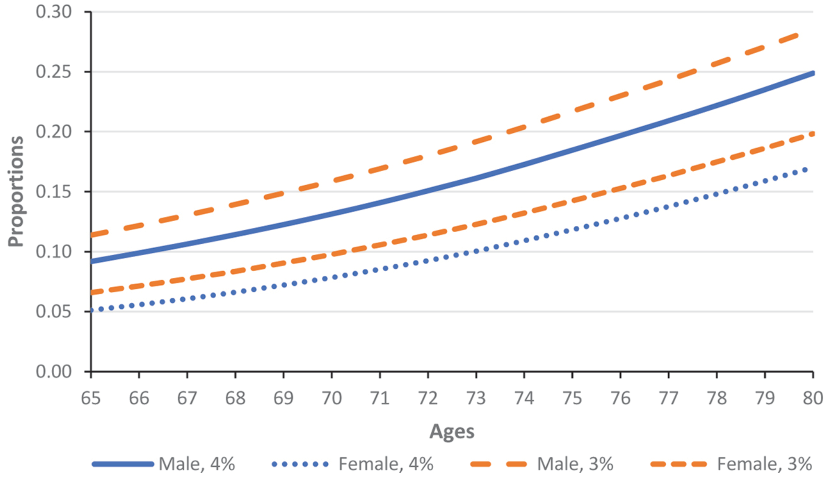

Figure 1: Proportion of the decreasing term insurance component versus the entry age in the pricing model.

To understand how long the annuitants are covered by the decreasing-term insurance or how long the surrender option is valid, Figure 2 presents the terms of decreasing-term insurance versus the entry ages for both genders at two different interest rates. Note that t does not depend on the amount of premium contribution. Although the older annuitants pay a higher proportion of premium for the term insurance, they enjoy a shorter term of insurance coverage because the older annuitants receive higher payments and have a higher mortality rate. For example, a male annuitant aged 65 will receive more than 24 years of decreasing term insurance protection at 4 % interest rate while a male annuitant aged 80 will be covered by less than 14 years. In general, for every one year increase in the entry age, the term of insurance is reduced by 7-10 months and 9-11 months for male and female annuitants, respectively.

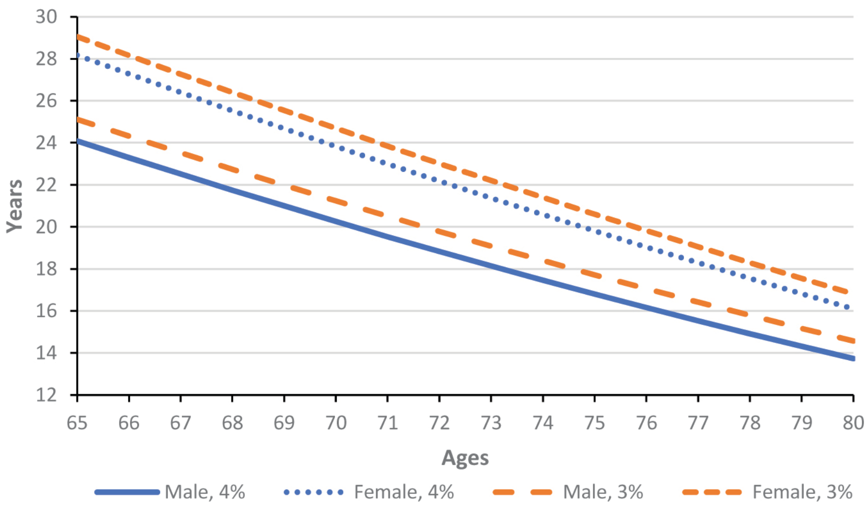

Figure 2: The term of decreasing-term insurance versus the entry age.

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It is easier for most lay people to understand the monthly payment benefits for a given premium amount in terms of the annuity rates, which are defined as the total annual cash payout divided by the premium contribution, i.e. $12P/C \times 100\,\%$. Figure 3 presents the annuity rates versus entry ages for both genders at two interest rates. For interest rates ranging from 3 % – 4 %, the annuity rates for a male annuitant at the entry ages of 65 and 80 are around 5 % – 6 % and 8 % – 9 %, respectively. Similar gradual increases in annuity rates are also observed for female annuitants as the entry age increases. The curves of the annuity rates for male annuitants at the 3 % rate and for female annuitants at the 4 % rate almost overlap which implies that the impact of the mortality rate difference between the genders on the annuity rate is roughly equivalent to the impact of the difference between the credited interest rates of 3 % and 4 %.

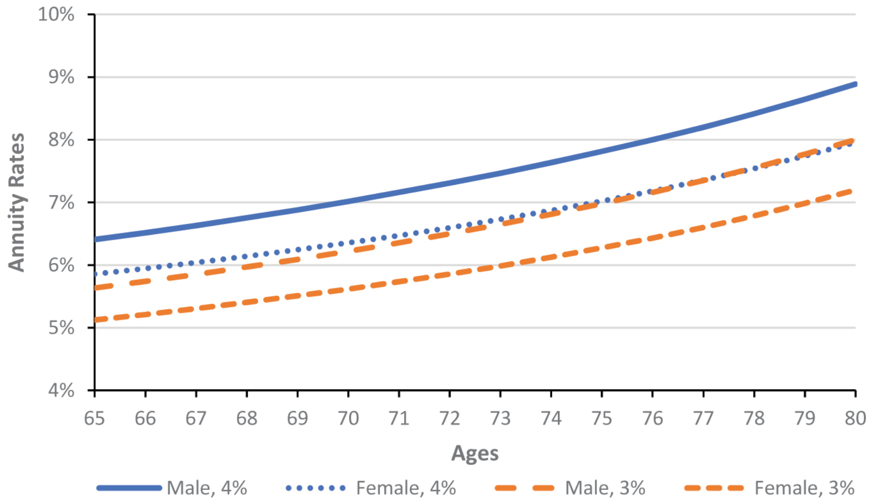

Figure 3: Annuity rates versus entry ages.

To highlight the individual financial risk faced by the provider (HKMC) of the annuity plan for both genders at two different interest rates, Figure 4 shows the standard deviation of the present value of loss L for a premium contribution of 100,000 versus the entry age. As the general rule observed in the model, a lower interest rate environment leads to less volatility of loss.

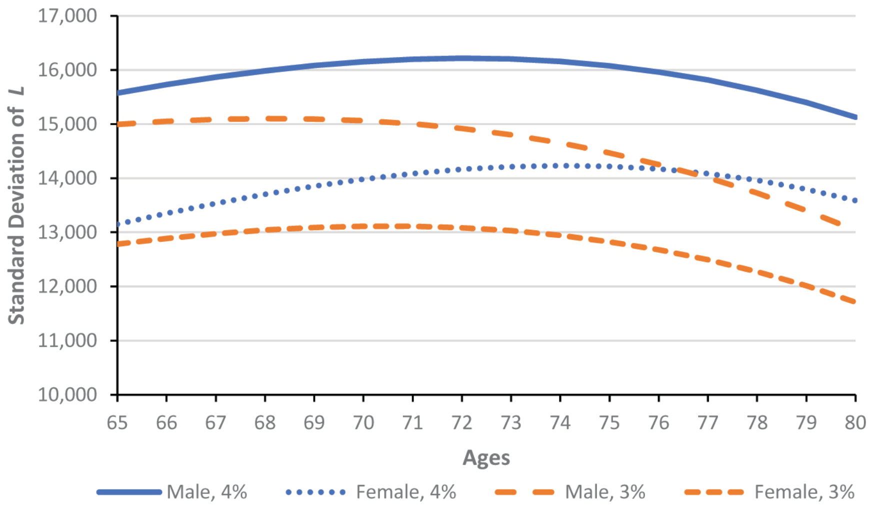

Figure 4: Standard deviations of L with 100,000 premium versus the entry age.

Under the same interest rate, male annuitants have a higher standard deviation of present value of loss compared with female annuitants. This outcome can be attributed to the empirical fact that $\mathrm{Var}(K_x^{(12)})$ for males is higher. It may also be explained by the empirical fact that the survival function for females is more "rectangular" so that the deaths of females are more concentrated at advanced ages instead of being spread evenly over a wider age range.

Due to the significant natural hedging effects between the components of decreasing term insurance and life annuity for older annuitants, the volatility of loss at an interest rate of 4 % is a concave downward curve, with the maximum risk at around entry ages 72 for male and 74 for female. This implies that the financial risks of the plan for entry ages of around 65 and 80 are relatively low. As a result, to minimize the portfolio risk of the

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plan, the HKMC may consider including more annuitants aged around 65 and 80 into the portfolio. Also, the natural hedge is under-utilized at a 105 % guaranteed return, and therefore from a risk management viewpoint, the product will be less risky to the provider if the guaranteed return of the premium is higher than 105 %. The HKMC is encouraged to take this point into consideration in future revisions of the product. Finally, we remark here that the natural hedging between the two components in the product applies to not only the idiosyncratic mortality risk but also the systematic longevity trend risk.

5.2 Aggregate Loss

We now analyze the potential loss of the HKMC Annuity Plan portfolio as a whole. According to the HKMCA[17], the first tranche of the HKMC Annuity Plan records a total of 9,410 registrations of Subscription Intention. The total subscription amount and the average subscription amount are approximately HK$ 4.94 billion and HK$ 525,000, respectively. Around 30 % of the applicants registered their intent to subscribe for HK$ 1 million, whereas around 54 % of them registered their intent to subscribe for HK$ 500,000 or above. As for the age distribution of the applicants, around 62 % are aged 65 to 69, around 23 % are aged 70 to 74, and the rest are aged 75 or above.

On the basis of the information provided by the HKMCA and the following simplifying assumptions, a proxy portfolio that mimics the first tranche of the HKMC Annuity Plan portfolio is constructed:

1. Applicants falling into the age bands of 65-69, 70-74, and 75+ are aged 67, 72, and 77 exact.
2. The amount of subscription for each applicant is HK$525,000 exact.
3. Of the 9,410 applicants, 5,646 (or 60 %) are males and 3,764 are females.
4. The age distribution of applicants for both genders are identical.

The characteristics of the resulting proxy portfolio are summarized in Table 3.

Table 3: The age-gender distribution of the 9,410 contracts in the proxy portfolio that mimics the first tranche of the HKMC Annuity Plan portfolio.

Age	Male	Female	Both Genders
67	3,496	2,331	5,827
72	1,296	864	2,160
77	854	569	1,423
Total	5,646	3,764	9,410

Notes: The amount of premium for each contract is HK$ 525,000 exact.

Assuming that the bequest is paid as a lump-sum amount, the aggregate loss variable of the proxy portfolio can be expressed as follows:

$$L_1^* = \sum_{i=1}^{3,496} L_1^M(67,i) + \sum_{i=1}^{1,296} L_1^M(72,i) + \sum_{i=1}^{854} L_1^M(77,i) + \sum_{i=1}^{2,331} L_1^F(67,i) + \sum_{i=1}^{864} L_1^F(72,i) + \sum_{i=1}^{569} L_1^F(77,i),$$

where $L_1^g(x_0,i)$ represents the future loss random variable for an individual contract that is sold to the ith person with gender g in the age group of x_0; that is, $L_1^g(x_0,i)$ is L (defined in equation (1)) evaluated at $x = x_0$, $C = 525{,}000$, and a value of P that is calculated in Section 4. For simplicity, it is assumed that the lifetimes of all individuals in the proxy portfolio are independent; that is, $L_1^g(x_0,i)$ and $L_1^{g'}(x_0',i')$ are independent if $g \neq g', x_0 \neq x_0'$ and /or $i \neq i'$. For comparison purpose, we also assume the proportion of male applications in the proxy portfolio to be 75 % and its corresponding aggregate loss denoted as L_2^*.

The standard deviations and the 99.5 % Value-at-Risk (i. e. the 99.5th percentile of the aggregate loss distribution) for L_1^* and L_2^* are calculated (see Table 4) at interest rate of 4 %.[18] It is not surprising to see that the portfolio with higher proportion of male applicants has higher Value-at-Risk as male applicants has higher volatility of cash flows.

As the HKMCA is wholly-owned by the HKSAR government, the risk inherent in the HKMC Annuity Plan portfolio is borne by the HKSAR government (i.e. the general tax-payers). Compared to the 2017 Hong Kong

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fiscal surplus of $138 billion, the 99.5 % Value-at-Risk of the proxy portfolio is relatively small, suggesting that the impact of the annuity plan on government finances is minimal.

Table 4: The calculated standard deviations and 99.5 % Value-at-Risk of the aggregate loss variables L_1^* and L_2^*.

	L_1^*	L_2^*
Standard deviation	7.69 million	7.86 million
Value-at-Risk	19.82 million	20.23 million

6 Summary

We construct an actuarial framework to analyze the features of the HKMC Annuity Plan. Based on the principle of actuarial equivalence, the monthly payments are derived for a given premium contribution under the options of with and without lump-sum bequest amounts payable at death. After specifying the assumptions of mortality and the credited rate of interest in the pricing model, we present several numerical examples of monthly payout amounts under these two options. With the pricing model under the option of bequest payable at death, we then analyze the features and benefits of the plan in terms of the proportion of the decreasing term insurance component, the maturity of the term insurance, the annuity rate and the volatility of the present value of loss random variable. With the backing of the HKSAR government and surrender option of the plan, the HKMC Annuity Plan provides an attractive annuity life product with a minimum credit risk and a certain level of liquidity at a reasonable and stable return rate. Because more potential subscribers will be able to understand the benefits of the plan compared with private annuity products, we are very optimistic about the success of the plan on its inauguration in mid-2018.

The HKMC Annuity Plan provides fixed lifetime payments and bequest benefits for Hong Kong residents aged 65 or above. The plan clearly cannot meet the retirement funding needs of the whole society, especially the unqualified younger population (aged 64 or below) preparing for retirement. However, to increase the impact and effectiveness of the retirement funding plan, an enhanced plan that includes some options of specific plan benefits for a wider age group of Hong Kong residents should be in the pipeline and possibly launched in the near future. For example, the enhanced plan could cover different age groups, such as ages 40 to 64, so that the younger age groups can also benefit from the deferred life annuity by saving for retirement during their working lives. The enhanced plan may include other options, such as flexible payout benefits to tackle the inflation risk or flexible premium contribution guarantees for different bequest needs. Furthermore, the HKMCA may wish to consider launching joint-life annuities that cover both (instead of just one) members of a couple (see, e.g. Ji and Zhou 2018). In future work we will seek to extend the initial plan to a more comprehensive plan by using the actuarial framework presented in this paper.

Finally, we conclude this paper with a comparison between the public annuity plans offered in Hong Kong and Singapore. Launched in 2009, Singapore's public annuity plan, known as the Lifelong Income for the Elderly (LIFE) scheme, provides a channel to decumulate retirement income savings with a possible refund of premium as a bequest upon death. Despite both being public annuity schemes, the HKMC Annuity Plan and the LIFE scheme have several notable differences, a summary of which is provided below.

– Participation: Participation in the HKMC Annuity Plan is voluntary; however, all members of the Central Provident Fund (CPF; Singapore's mandatory public pension plan) who are aged 55 or above with sufficient funds in their CPF accounts are required to join the LIFE scheme.

– Linkage to the corresponding mandatory public pension plan: The HKMC Annuity Plan is independent of the MPF scheme in Hong Kong. However, the LIFE scheme is closely linked the the CPF, primarily through the funds in the Retirement Account that is created when a CPF member reaches age 55.

– Deferment: The HKMC Annuity Plan has no deferment period: the subscriber starts to receive monthly annuity payouts the month when the lump-sum premium is paid. In contrast, the annuities provided by the LIFE scheme are deferred, in the sense that some premium must be paid at age 55 but annuity payouts do not begin until at least age 65.

– Protection against inflation: Under the LIFE scheme, a participant may choose the 'escalating plan' which provide monthly payouts that increase by 2 % yearly. The 2 % annual increase in payout may defray the erosion in purchasing power due to inflation. However, such an option is not offered under the HKMC Annuity Plan.

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The comparison above sheds some light on how the HKMC Annuity Plan may be further improved. First, to boost the participation rate, connections between the life annuity plan and the MPF scheme may be developed. While it may be too extreme to require all MPF members to participate in the life annuity plan, financial incentives may be created to encourage participation. Second, the payment of premium can be made more flexible, allowing subscribers to pay at least part of the premium when they are young and have good disposable incomes. Finally, to alleviate potential subscribers' concerns about inflation risk, subscribers may be given an option to choose monthly payouts that start lower but increase yearly (possibly according to the realized values of the consumer price index).

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Notes

1 The 2018 Index of Economic Freedom Report, which was released by the Heritage Foundation in February 2018, ranks Hong Kong and Singapore as the world's freest economies with scores of 90.2 and 88.8, respectively, on a scale of 0 to 100. Retrieved 4 May 2018, from https://www.heritage.org/index/
2 See Kwong, Tse, and Chan (2017) for the details of Singapore public pension scheme.
3 See Hui and Chan (2004) for a discussion.
4 See Blake (1999) for details.
5 See Friedman and Warshawsky (1990) for a discussion of unpopular life annuity products due to adverse selection in the annuity market.
6 See Mitchell, Poterba, and Warshawsky (1999) for the definition and estimation of the money's worth ratios for pension annuities.
7 In May 2018, HKMCA was officially granted the formal authorization to carry on Life and Annuity business in or from Hong Kong.
8 In April 2017, the HKMC announced the introduction of the new life annuity plan and highlighted its main features. In the second press release two months later, the HKMC confirmed that it had completed the verification and validation of the life annuity plan and the plan would be launched by mid-2018. (Retrieved 4 May 2018, from http://www.hkmc.com.hk/eng/information_centre/press_releases.html#type=hkmc&year=2017).
9 A surrender charge may have to be paid. However, the HKMC's press releases do not provide an estimate of it.
10 Commonly used actuarial notations are not defined in this paper. Readers may refer to Bowers et al. (1997) for details.
11 In our calculations, it is assumed that $q_{x,t} = m_{x,t}/(1 + 0.5m_{x,t})$ whenever necessary. This assumption holds exact when deaths are uniformly distributed between consecutive integer ages.
12 When estimating some of the candidate stochastic morality models, identifiability constraints are needed to stipulate parameter uniqueness. The identifiability constraints we use are the same as those used in the baseline results of Cairns et al. (2009) and Li, Zhou, and Liu (2018a, b); Li et al. (2019).
13 While data from age 0 to 59 are available, we choose to use data from age 60 to 99 only in part because we need only projected mortality rates from age 65 and in part because some of the candidate models are unsuitable for modeling mortality rates at younger ages.
14 When computing death/survival probabilities involving fractional ages, we assume that deaths are uniformly distributed between two successive integer ages with the maximum attainable age of 100.
15 It will take time for the HKMCA to collect enough data to establish its own mortality table of the annuitants for pricing the new annuity product. As the numerical outcomes are only used to illustrate the plan's features, it is not our intention to argue that the projected mortality rates we produce are the most appropriate for the pricing the HKMC Annuity Plan.
16 All monetary amounts in this paper are in Hong Kong dollars. For simplicity, the dollar signs are suppressed from now on.
17 https://www.hkma.gov.hk/eng/key-information/press-releases/2018/20180810-4.shtml
18 We remark that both L_1^* and L_2^* have a mean of zero, because all premiums are calculated using the equivalence principle. Further, the 99.5 % Value-at-Risk is calculated using a normal approximation.

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