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Abstract

Are trends in the price of capital technological in nature? First, we find that trends in the relative price of capital vary significantly across countries. We then show that a multi-industry growth model, calibrated to match differences in economic structure around the world and productivity growth rates across industries, accounts for this variation – mainly due to variation in the composition of capital. The finding indicates that the rate of change in the relative price of capital can be interpreted as investment-specific technical change – the extent to which productivity growth is relatively more rapid in the capital-producing sector. The model also accounts for the empirical dispersion in investment rates, but not in rates of economic growth.

Keywords: Investment-specific technical change, multi-sector growth models, structural transformation, capital goods prices.

JEL Codes: O11, O13, O33.

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I Introduction

Declines in the relative price of capital are viewed as an important factor of economic growth in the United States (US). See for example work by Hulten (1992), Greenwood et al. (1997), Cummins and Violante (2002) and Oulton (2007).[1] These studies typically identify the decline in the price of capital as being technological in nature, reflecting faster productivity growth in the production of new capital than in the production of consumption and services – a phenomenon known as *investment-specific technical change* (ISTC). However, the extent to which the relative price of capital declines *in other countries* is not known. In addition, it is not known whether trends in the price of capital around the world can be given a *technological* interpretation, such as ISTC. An alternative hypothesis is that these differences are due to the presence of barriers to capital accumulation, as proposed by Restuccia and Urrutia (2001) to account for differences in *levels* of the price of capital.

We begin by documenting that the rate at which the relative price of capital changes over time varies significantly across countries. We find that the median

[1] While there are differences across these studies in the approach to growth accounting and the approach to measuring the price of capital, they uniformly find a positive contribution of ISTC to post-War US growth, ranging from 20 to 60 percent.

growth rate of the price of capital is zero. In addition, the price of capital *increases* in as many places as it decreases. This indicates that, if there is a technological explanation for this phenomenon, technical progress in capital relative to other sectors must vary widely around the world.

If the explanation is indeed technological, however, one would expect such glaring differences in productivity to be evidence of draconian barriers to international technology transfer (or trade). The alternative possibility is that capital and consumption are themselves highly disaggregated, and that there are substantial differences in the *composition* of capital and consumption around the world that account for the aggregate differences in the trends in the relative price of capital.

We ask whether this variation can be accounted for by differences in *industry composition*. The reason we do this is as follows. It is well known that rates of technical progress in the US differ significantly not just between capital and non-capital, but also across *types* of consumption, services and capital. Thus, even if productivity growth rates are constant across countries for each industry, the rate of change in the relative price of capital may be different if the *composition* of capital – or the composition of consumption and services – is different. Indeed, we find that the composition of capital is skewed towards high-TFP growth capital types in countries where the price of capital declines rapidly. We therefore ask: to what extent can differences around the world in industry composition account for variation in the rate at which the relative price of capital changes?

To this end, we employ a canonical multi-industry growth model. In the model, the composition of the economy evolves as a result of changes in prices of different goods or services that agents consume, as well as changes in the prices of different capital goods.[2] In turn, these are determined by differences in productivity growth rates across industries. We calibrate the model using detailed productivity growth data from the US, as well as data on the initial composition of economies around the world in the year 1991. We use constant productivity growth rates for a given industry in all countries partly because of data limitations; however, as mentioned, significant barriers to technological transfer would have to exist to significantly devi-

[2] See for example Baumol (1967), Ngai and Pissarides (2007), and Samaniego and Sun (2016).

ate from this assumption. Composition is a key part of the "no barriers" hypothesis.

Strikingly, we find that the model delivers a close match to the rate of change in the relative price of capital, as measured using the Penn World Tables (PWT) version 7.1.[3] In a statistical sense, the model can account for the entirety of the magnitude of variation of the growth rate in the relative price of capital over the period from 1983 to 2011, simply based on industry TFP growth rate differences and on differences in industry composition across countries. Not only does the model match the extent of variation, but also the correlations between model-generated capital price growth rates and those in the data are highly significant. We conclude that differences in the relative price of capital around the world can be interpreted as a technological phenomenon – ISTC – and that a key factor behind these differences is industry composition.

The link between composition and the decline in the relative price of capital could be for two reasons: differences in the composition of capital, or in the composition of non-capital. We refer to these possibilities as the *capital hypothesis* and the *consumption hypothesis*, respectively. We study the importance of each hypothesis by removing productivity growth differences in the capital producing industries, and then separately removing them in the non-capital producing industries. We find that the *capital hypothesis* is mainly responsible for cross-country variation in ISTC: removing productivity growth in non-capital makes very little difference to the results, whereas removing productivity growth in capital-producing industries results in model-generated statistics that bear little relationship with the data.

Finally, we ask to what extent a growth model driven solely by these factors can account for differences in aggregate behavior across countries over the sample period. Specifically, we look at investment rates and rates of economic growth. This is a non-trivial task, as it requires solving for investment patterns in a model

[3] See Heston, Summers and Aten (2012). As discussed in the Appendix, price data from the PWT 7.1 are likely better suited for measuring trends in the relative price of capital than more recent editions because the latter use multiple benchmark years. The national accounts data reported alongside the latest edition (PWT 9.1, see Feenstra, Inklaar and Timmer (2015)) have only one benchmark year, but do not report capital and consumption price indices. We construct a relative price index using these data and examine it as well, finding that its link with the model is weaker.

where conditions for a balanced growth path do not hold in general. We find that the model generates investment rates that are strongly correlated with investment rates in the PWT 7.1 data and the PWT 9.1 data, although they underpredict the extent of empirical variation in investment rates. Thus, the model is able to capture cross-country variation in both ISTC and (to a lesser extent) investment rates, solely based on differences in industry composition. However, the model does not generate a good match to variation in rates of *economic growth* in the PWT 7.1 data, nor in the PWT 9.1 data. We conclude that there is widespread divergence in the rate of ISTC around the world, and that this accounts for variation in investment, but that economic growth rates are due to other factors. Interestingly, when we give each country an aggregate productivity trend that exactly matches its economic growth rate in the data, investment rates are no longer correlated with those in the data, suggesting that whatever factors do underlie rates of economic growth are not simply captured by a trend in productivity.

The results contribute to a long-standing debate regarding whether or not changes in the efficiency of investment are an important factor of growth. This debate goes back to Solow (1962), Abramovitz (1962) and Denison (1964). Greenwood et al. (1997) find that, in the US, more than half of economic growth can be accounted for by ISTC in a general equilibrium growth accounting framework. We provide a clear answer to the question about whether differences in the relative price of capital can be attributed to barriers or to technological factors, indicating that changes in the efficiency of investment are an important factor affecting growth rates. This is not to say that there is no scope for barriers to be important for the relative price of capital; however, their impact might not be direct, but rather *indirect*, through their influence on economic composition. More broadly, this suggests that future work on the manner in which factors of economic growth might be affected by policy through the channel of economic composition could be fruitful.[4]

Section II presents data on trends in the relative price of capital around the world,

[4]See Samaniego (2006) and Ilyina and Samaniego (2012) for studies of how employment protection and financial development respectively can impact growth rates through their impact on economic structure.

as well as evidence that variation in these trends is linked to variation in economic composition. Section III introduces the model economy. Section IV details the calibration procedure. Section V describes the results, and Section VI discusses whether there is any link between trends in the relative price of capital and country characteristics. Section VII concludes.

II Trends in the Price of Capital

We will use data on the price of capital and on economic aggregates (investment and GDP per capita) using the Penn World Tables (PWT) version 7.1, see Heston, Summers and Aten (2012). In the Appendix, we provide an extensive discussion for why we use this, rather than the latest version PWT 9.1, for our benchmark results. However, in the Appendix, we also repeat our analysis using data from version 9.1, finding generally similar results.

A Data

Define q_t as the inverse relative price of capital in a given country. An increase in q_t implies that capital has become cheaper, so that the opportunity cost of producing a unit of capital – measured in units of consumption and services – has declined. The related literature interprets q_t as an indicator of the efficiency of the investment process: it is the rate at which consumption can be converted into capital goods.

Define g_q as the average geometric growth factor of q_t over a period of time – so that $\log g_q$ is the average growth rate. The statistic $\log g_q$ is then the rate at which the efficiency of the investment process improves over time. If $\log g_q > 0$, then capital is getting cheaper over time, implying that the investment process is improving. Conversely, if $\log g_q < 0$, then capital is getting more expensive over time (relative to consumption), so that the amount of consumption that must be foregone in order to generate a unit of capital is increasing over time.

We measure the decline in the relative price of capital, g_q, using the average geometric factor by which the relative price of capital declines over the sample period

Table 1: Summary information for the change in the relative price of capital, PWT 7.1.

Statistic	$\log g_q$
Median	-0.0007
Mean	-0.0005
First decile	-0.0197
First quartile	-0.0068
Last quartile	0.0072
Last decile	0.0164
s.d.	0.0186
Share below zero	53.2%

from 1983 to 2011. We select our sample period to start in 1983 to compromise between the length of the time series and the country coverage of the panel, as well as avoiding the price instability of the 1970s and early 1980s. Our measurement strategy is similar to Hsieh and Klenow (2007), who use relative price of capital data in the PWT to look at cross-country differences in the *level* of the efficiency of investment: instead, we focus on *changes*.[5]

Table 1 reports some basic statistics about the rate $\log g_q$. Note that the median official value of $\log g_q$ is essentially zero. This implies that, on average, capital goods generally do not become cheaper over time, nor do they become more expensive. At the same time, there are several countries where the relative price of capital *rises* – so that $\log g_q < 0$. As seen in Figure 1, this occurs in 101 out of 190 countries. In the remainder, the relative price of capital declines, so that $\log g_q > 0$.

The correlation between $\log g_q$ and the initial level of q in the data is -0.62^{***}, as seen in Figure 1.[6] This implies that in countries where q is low (i.e. capital is expensive), capital tends to become disproportionately cheaper over time. This is consistent with Restuccia and Urrutia (2001), who find that the dispersion in relative prices of capital around the world has declined over time: in our data we find that the correlation between the annual standard deviation in q across countries and a time

[5] We also used the growth factor of the HP-filtered series for q, finding similar results.

[6] Throughout the paper one, two and three asterisks will indicate significance at the 10, 5 and 1 percent levels respectively.

trend is -0.51^{***}. As a result, the correlation between $\log g_q$ and the *average* level of q in the data is -0.26^{***} – lower than the initial level, but still highly significant.

Hsieh and Klenow (2007) find that the relative price of capital in *levels* tends to be higher in some less developed economies. This begs the question of whether $\log g_q$ is related to levels of economic development as well. Interestingly, Figure 1 shows that the correlation between $\log g_q$ and the natural logarithm of GDP per capita over the period is positive and significant (0.15^{**}). This implies that differences in q_t among developed and developing economies tend to widen over time. This fact is consistent with the fact that there is convergence in the level of q_t over time because this convergence must be due to factors that are unrelated to GDP per capita levels.[7] While, in general, there is convergence in the level of q_t, and while poorer countries tend to have lower levels of q_t, it is not the case that there is convergence in levels of q_t across countries at different levels of development.

Given the attention given to increases in q_t over time as a factor of economic growth in the US, two further questions arise. First, how important a contributor to international convergence in income levels are these increases in q_t? Second, are increases in q_t statistically related to rates of economic growth? Note that these are two distinct questions: the first is a growth accounting question, whereas the second is a statistical question. The first question will be addressed via a series of counterfactual experiments using the model economy to be developed in Section V. As for the second, even though $\log g_q$ is positively correlated with the *level* of economic development, the correlation between $\log g_q$ and GDP *growth rates* is 0.04 and not statistically significant, suggesting the likely dominance of other factors in accounting for rates of economic growth.[8]

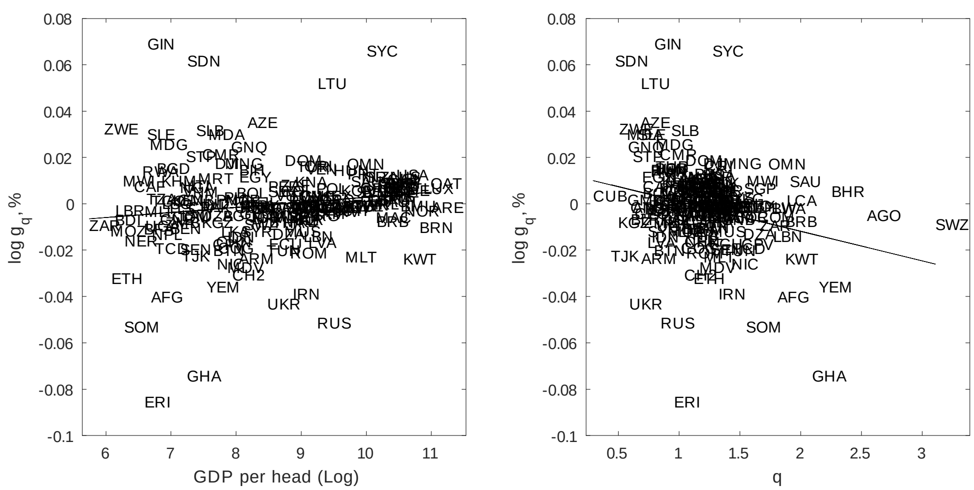
Thus, overall, the decline in the relative price of capital is largest in the US and other developed countries and smaller in less developed countries, although there

[7] We find that, when we look at countries separately by income quartile, there is convergence within all quartiles. However, when we compare the median q in each quartile, we see no convergence, suggesting there is convergence within (but not between) "clubs."

[8] The relationship with GDP measured in the PWT 9.1 appears weaker, the correlation is 9 percent. However it is due to outliers: the Spearman rank correlation is 17 percent and significant at the 5 percent level. The correlation with growth remains low at 1 percent.

are some poor countries with faster declines than the US. It is not clear that these are necessarily economies undergoing growth-promoting reforms, as there is no link between $\log g_q$ and GDP growth if we look at the full sample, nor if we focus on the countries in the top decile or the top quintile of $\log g_q$. Some of these outliers are oil producers, but we checked but found no significant link between $\log g_q$ and the share of oil revenues in GDP over the period. We conclude that a high value of $\log g_q$ may not have much to do with economic growth, and might have more to do with other factors that relate to the stage of development (such as economic composition),.

Figure 1 – The left panel displays the relationship between $\log g_q$ and real GDP per person in the data. The right panel displays the link between $\log g_q$ in the data and average q, $1983 - 2011$.



Notes: In the figure, $\log g_q$ is the rate of decline in the relative price of capital and q is its level. GDP per person (in logs) is the average over the period, drawn from the PWT 7.1.

B Economic Composition and Trends in the Price of Capital

A simple technological interpretation of the finding of significant variation in $\log g_q$ is that: in countries where $\log g_q < 0$, capital experiences slower productivity growth than consumption-services. This would apply to approximately half the countries in the data, suggesting striking divergences around the world in productivity growth in consumption or in capital. It is hard to think of such differences persisting year after year in the absence of significant barriers to technology transfer. It is also difficult to test this directly, as there is no comprehensive database of industry-level productivity growth over an adequate period of time that might be used to compute a measure of relative rates of technical progress inside and outside the capital goods sector to compare with g_q.[9] However, even if that were possible, it would not itself be evidence of widespread differences in rates of technological progress in different countries – or of barriers to trade or technology transfer being responsible for differences around the world in $\log g_q$.

The reason is that both capital and consumption-services *themselves* include several different types of goods and services, each of which is known to experience productivity growth at an idiosyncratic rate. As a result, the observation that $\log g_q$ varies dramatically around the world is not enough to discard a technological explanation for trends in the price of capital. We would need to analyze whether differences in the *composition* of capital and consumption might account for differences in g_q around the world, assuming a given rate of technical progress within each industry.

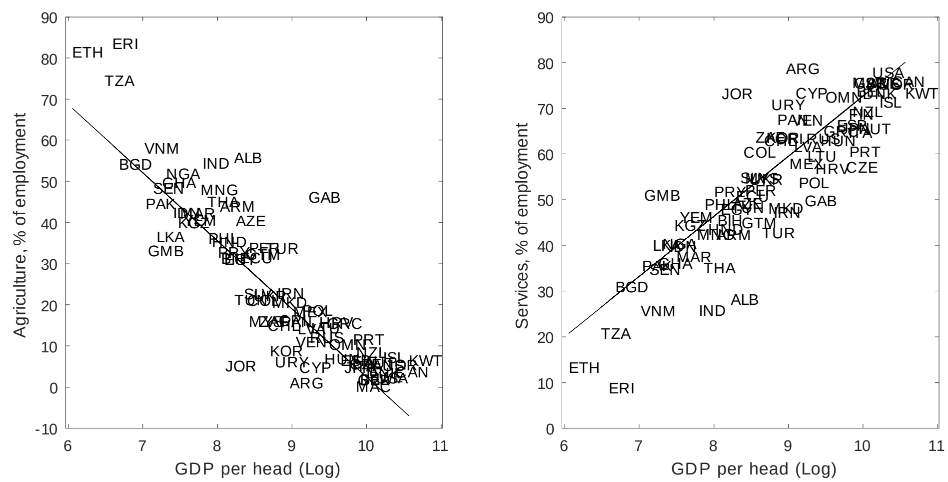
For example, it has been known since at least Kuznets (1966) that levels of economic development are positively associated with the share of the labor force engaged in the service sector, and negatively associated with the share engaged in the agricultural sector. Figure 2 shows that this is the case, comparing the share of agriculture and services to real GDP per capita in 2001. It is also well known since Baumol (1967) that productivity growth in services is generally slower than in all other sectors. If productivity growth in agriculture is more rapid than in

[9] One way to explore this hypothesis would be to examine comprehensive information on sector productivity growth rates around the world. The problem, however, is that such measures do not exist for a large set of countries. EUKLEMS reports disaggregated productivity data for only a few industries, and it is in terms of labor productivity, which is not suitable in our context. OECD.Stat reports multi factor productivity growth rates but the series are very short and again only available for very few countries (only 6 have data back to 1991). The same is true of STAN.

Instead, we impute differences in the productivity of capital and non-capital implied by differences in economic structure. This results in a well-defined experiment where the only factor varying from place to place is economic structure, not disaggregated productivity growth rates.

services, then the rate at which the relative price of capital declines in less developed economies would be slower than in more developed economies, simply because of the dominance of the slow-growing sector among non-capital producing industries. We refer to this possibility as the *consumption hypothesis*.

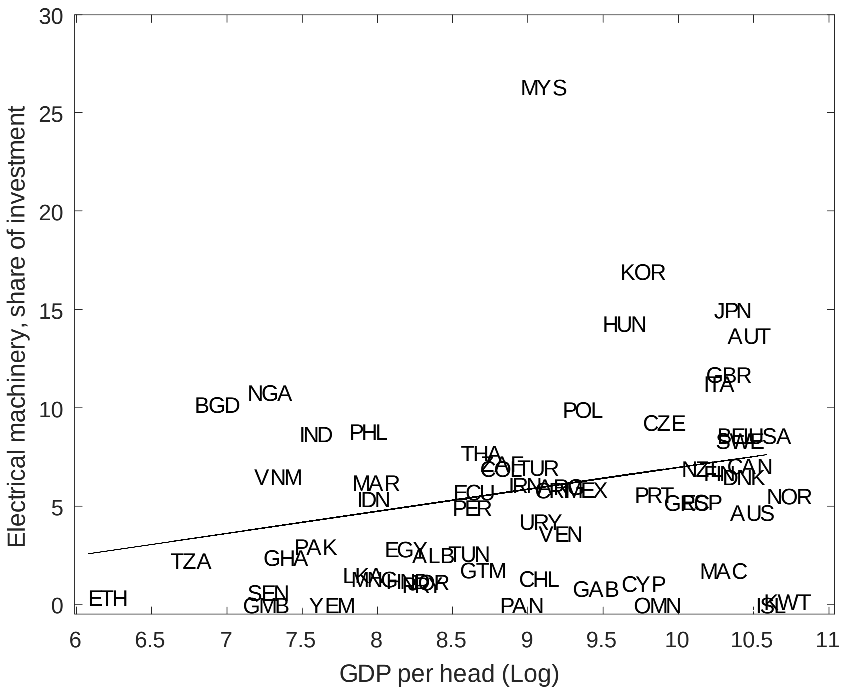
Figure 2 – Economic composition and income level.



Notes: The figure represents the share of agriculture or services in employment in 76 countries, drawn from the International Labor Organization (ILO) database, over the years 1991 – 2018, against real GDP as measured in the PWT 7.1. Results are similar using income data from the PWT 9.1.

Alternatively, it could be that the *composition of capital goods* differs significantly across countries. For example, information technology (IT) is well known to experience rapid productivity growth compared to other types of capital – see Cummins and Violante (2002). If some countries use IT less intensively than others, they would display slower productivity growth in capital overall, and would thus display lower values of g_q, *caeteris paribus*. IT capital falls in the industry category Electrical Machinery (ISIC 383), which has the highest productivity growth rate of any industry in our database. Indeed, Figure 3 indicates that more developed economies tend to have more intensive use of electrical machinery. The correlation is 0.27**, significant at the 5 percent level. We refer to the possibility that variation in the composition of capital around the world might account for variation in g_q as the *capital hypothesis*.

Figure 3 – Share of electrical machinery in investment against income level.



Notes: Investment is defined as the sum of value added in the industries that produce capital goods (ISIC codes 331, 332, 381, 382, 383, 384, 385 and 390) drawn from the UNIDO database, plus the share of construction drawn from the ILO database, in the initial year (1991). Real GDP per capita is the average from the PWT 7.1 over the years 1991 – 2011. The line is a regression line obtained using OLS. Source: UNIDO.

To sum up, assessing whether trends in the price of capital can be interpreted in terms of different rates of technical progress across sectors requires assessing whether or not differences in $\log g_q$ around the world can be linked to differences in economic composition – whether differences in the composition of capital, differences in the composition of non-capital, or both.

In what follows, we develop a multi-sector growth model. We will use the model to see whether differences in economic structure across countries at the available levels of disaggregation can account for the observed differences in g_q around the world. We will also use the model to see whether there is support for either the consumption hypothesis or the capital hypothesis

III Model Economy

The model is similar to the multisector growth model of Ngai and Pissarides (2007). There is a finite number S of sectors, where sector S produces capital goods and sectors $\{1, ..., S-1\}$ produce goods that are consumed by households. We extend the Ngai and Pissarides model (2007) in two ways. First, we do not restrict parameters to values that guarantee a balanced growth path. Second, we allow for the subdivision of each sector s into a set of industries I_s, where $|I_s| \geq 1$. We require this nested structure in order to map the model into disaggregated data. For example, Electrical Machinery (ISIC industry code 383) is an industry which is part of the capital goods sector S. The details of how we sort industries into sectors is outlined in Section IV.

A Preferences and technology

Time is discrete and there is a $[0,1]$ continuum of agents. Agents have isoelastic preferences over consumption c_t and discount the future using a factor $\beta \in (0,1)$, so that:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta} - 1}{1-\theta}. \tag{1}$$

Consumption c_t is an aggregate of the output of $S-1$ different consumption sectors:

$$c_t = \left[\sum_{s=1}^{S-1} \zeta_s y_{st}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \qquad \sum_{s=1}^{S-1} \zeta_s = 1 \tag{2}$$

where y_{st} is the output of each consumed sector $s \leq S-1$, ζ_s is the weight on good s and ε is the elasticity of substitution among them. There is an additional sector S, which produces capital goods.

Agents are endowed in period zero with capital K_0, and are endowed with one unit of labor every period which they supply inelastically. Let q_{st} be the price of the sector aggregate s, with r_t as the interest rate and w_t as the wage. Agents choose expenditure on each good y_{st}, $s \leq S$, so as to maximize (1) subject to the budget constraint:

$$\sum_{s=1}^{S} q_{st} y_{st} \leq r_t K_t + w_t n_t \tag{3}$$

and the capital accumulation equation:

$$K_{t+1} = y_{St} + (1-\delta) K_t \tag{4}$$

where y_{St} is the quantity of investment goods purchased, K_t is the capital stock at date t and n_t is labor input. It will be convenient to set the capital good as the numeraire in the derivations that follow.

Each sector $s \leq S$ is subdivided into industries. Let I_s be the set of industries that supplies sector s. We focus on the case in which each industry supplies only one sector,[10] so that $I_s \cap I_{s'} = \varnothing$, $\forall s \neq s'$.

Each sector $s \in \{1, ...S\}$ produces a CES aggregator of its component industries:

$$y_{st} = \left[\sum_{i \in I_s} \xi_{s,i} \times u_{sit}^{\frac{\varepsilon_S - 1}{\varepsilon_S}} \right]^{\frac{\varepsilon_S}{\varepsilon_S - 1}}, \quad \sum_{i \in I_s} \xi_{s,i} = 1, \qquad s = 1, ..., S \tag{5}$$

where y_{st} is sectorial output, u_{sit} is use of good i by sector s, $\xi_{s,i}$ is the weight on good i in the production function and ε_s is the elasticity of substitution among goods within sector s.

In turn, each industry produces output y_{it} using capital K_{it} and labor n_{it} using

[10] Note that this assumption is without loss of generality: one could easily generalize by having two industries identical in all ways that are distinguished by the fact that they provide a given good to different sectors. Also, in the case that we lack disaggregated data for a sector s, it is straightforward to set the cardinality of I_s to one.

a Cobb-Douglas production function:

$$y_{it} = A_{it} K_{it}^{\alpha} n_{it}^{1-\alpha}, \qquad A_{it} = A_{i0} g_i^t. \tag{6}$$

In equation (6), $g_i = A_{i,t+1}/A_{it}$ is the productivity growth factor of industry i, and A_{i0} is the given level of productivity at date zero.

Industry producers maximize profits so that:

$$\max_{n_{it}, K_{it}} \{p_{it} y_{it} - w_t n_{it} - r_t K_{it}\} \tag{7}$$

subject to (6), where p_{it} is the output price of industry i at time t. Capital and labor are freely mobile across sectors.

B Sectorial growth patterns and aggregation

The producers' first order conditions imply that the capital labor ratio is constant across industries, which in turn implies that $A_{it} p_{it} = A_{jt} p_{jt}$. Thus, goods that experience rapid productivity growth display a decline in their relative price compared to other goods in the same sector. This result, combined with the consumers' first order conditions, implies that the ratio of value added $p_{it} y_{it}$ in any two industries in the same sector s depends on preference parameters and productivity terms:

$$\frac{p_{it} y_{it}}{p_{jt} y_{jt}} = \left(\frac{\xi_{s,i}}{\xi_{s,j}}\right)^{\varepsilon_s} \left(\frac{A_{it}}{A_{jt}}\right)^{\varepsilon_s - 1} = \frac{n_{it}}{n_{jt}} \qquad \forall i, j \in I_s. \tag{8}$$

Notice that the same relationship holds for the ratio of employment because capital-labor shares are the same.

Define the growth factor of employment (or value added) in industry i as:

$$G_{it} \equiv \frac{n_{i,t+1}}{n_{i,t}} = \frac{p_{i,t+1} y_{i,t+1}}{p_{it} y_{it}}. \tag{9}$$

Then, the expression G_{it}/G_{jt} denotes the growth of employment (or value added) in industry i relative to industry j. Using (8), we have:

$$\frac{G_{it}}{G_{jt}} = \left(\frac{g_i}{g_j}\right)^{\varepsilon_s - 1} \qquad \forall i, j \in I_s. \tag{10}$$

Consequently, *within sectors*, structural change depends on relative TFP growth factors $\frac{g_i}{g_j}$ and on the elasticity of substitution ε_s.

As we shall see, comparing industries *across* sectors requires characterizing shifts in expenditure across sectors, as well as investment behavior if either of the sectors produces capital.

Notice that, in equilibrium, we can aggregate the industries in a given sector into a sectorial production function. To see this, define q_{st} as the price index for final goods in sector s, such that

$$q_{st} y_{st} \equiv \sum_{i \in I_s} p_{it} A_{it} k_t^{\alpha} n_{it} \tag{11}$$

where $k_t \equiv \frac{\alpha w_t}{(1-\alpha) r_t}$ is the equilibrium capital-labor ratio at date t, which is common across industries. Define a hypothetical sectorial production function:

$$y_{st} \equiv A_{st} K_{st}^{\alpha} n_{st}^{1-\alpha}, \; A_{st} = A_{s0} \bar{g}_{st} \tag{12}$$

where input use in sector s is $K_{st} = \sum_{i \in I_s} K_{it}$ and $n_{st} = \sum_{i \in I_s} n_{it}$.

Taking as given the capital labor ratio k_t, the problem of the sector firm and (11) can be combined as follows:

$$\max_{\substack{n_{it},\\ i \in I_s}} q_{st} \left[\sum_{i \in I_s} \xi_{s,i} \times \left(A_{it} k_t^{\alpha} n_{it} \right)^{\frac{\varepsilon_s - 1}{\varepsilon_s}} \right]^{\frac{\varepsilon_s}{\varepsilon_s - 1}} - r_t k_t \sum_{i \in I_s} n_{it} - w_t \sum_{i \in I_s} n_{it}. \tag{13}$$

Recall that the first order conditions from the industry problem imply that:

$$\frac{n_{jt}}{n_{it}} = \left(\frac{\xi_{s,j}}{\xi_{s,i}}\right)^{\varepsilon_s} \left(\frac{A_{it}}{A_{jt}}\right)^{1-\varepsilon_s} \tag{14}$$

Setting $n_{st} = \sum_i n_{it}$, we can use (14) to write n_{it} in terms of n_{st}. Substituting this back into the problem (13), we have simply that the sector firms' problem is:

$$\max_{n_{st}} \{q_{st}A_{st}k_t^{\alpha}n_{st} - r_t k_t n_{st} - w_t n_{st}\}$$

where

$$A_{st} = \left[\sum_{i\in I_s} \xi_{s,i}^{\varepsilon_s} \times A_{it}^{\varepsilon_s-1}\right]^{\frac{1}{\varepsilon_s-1}} = \left[\sum_{i\in I_s} \xi_{s,i}^{\varepsilon_s} \times A_{i0}^{\varepsilon_s-1} g_i^{(\varepsilon_s-1)t}\right]^{\frac{1}{\varepsilon_s-1}} \tag{15}$$

Defining the sectorial productivity growth factor $\bar{g}_{st} = \frac{A_{s,t+1}}{A_{s,t}}$, it can be shown that in equilibrium

$$\bar{g}_{st} = \prod_{i\in I_s} g_i^{x_{it}/X_{st}} \tag{16}$$

where the exponents are given by:

$$x_{it} = \xi_{s,i}^{\varepsilon_s} A_{it}^{\varepsilon_s-1},\ X_{st} = \sum_{i\in I_s} x_{it}.$$

In fact, x_{it}/X_{st} is also the share of expenditure on industry i within sector s.

We can aggregate the consumption sectors in the same way, since the aggregator $c_t = \left[\sum_{s=1}^{S-1} \zeta_s y_{st}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$ has the same form as each of its component sectors. Defining the problem of a consumption-producing firm, analogous to (13), we have that the consumption sector can be represented as follows:

$$c_t = A_{ct}K_{ct}^{\alpha}n_{ct}^{1-\alpha},\ A_{ct} = \left[\sum_{s=1}^{S-1} \zeta_s^{\varepsilon} \times A_{st}^{\varepsilon-1}\right]^{\frac{1}{\varepsilon-1}} \tag{17}$$

As a result, the aggregate behavior of the model economy with many sectors is the same as that of a two-sector economy that produces c_t using technology (17) and produces capital goods using technology (12). In the consumption goods sector, firms maximize

$$\max_{K_{ct},n_{ct}} \left\{p_{ct}A_{ct}K_{ct}^{\alpha}n_{ct}^{1-\alpha} - r_t K_{ct} - w_t n_{ct}\right\}$$

where

$$A_{ct} = \left[\sum_{s=1}^{S-1} \zeta_s^{\varepsilon} \times A_{st}^{\varepsilon-1}\right]^{\frac{1}{\varepsilon-1}}.$$

In the capital goods sector:

$$\max_{K_{St},n_{St}} \left\{ p_{St} A_{St} K_{St}^{\alpha} n_{St}^{1-\alpha} - r_t K_{St} - w_t n_{St} \right\}$$

where

$$A_{St} = \left[\sum_{i \in I_S} \xi_{S,i}^{\varepsilon_S} \times A_{it}^{\varepsilon_S - 1} \right]^{\frac{1}{\varepsilon_S - 1}}.$$

To sum up, according to the simplified problem, consumers choose consumption c_t and investment y_{St} to solve:

$$\begin{aligned} & & \max_{c_t, y_{St}} \left\{ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta} - 1}{1-\theta} \right\} & \quad (18) \\ s.t. \quad & p_{ct} c_t + p_{St} y_{St} \leq & r_t K_t + w_t & \quad (19) \\ & K_{t+1} = & K_t (1-\delta) + y_{St} & \quad (20) \\ & & K_0 \text{ given.} & \quad (21) \end{aligned}$$

In equilibrium, capital and labor markets must clear at all dates, so

$$\begin{aligned} c_t &= A_{ct} K_{ct}^{\alpha} n_{ct}^{1-\alpha} & (22) \\ y_{St} &= A_{St} K_{St}^{\alpha} n_{St}^{1-\alpha} & \\ K_t &= K_{St} + K_{ct} & (23) \\ n_{ct} + n_{St} &= 1 & (24) \end{aligned}$$

where the productivity terms A_{ct} and A_{St} are taken as given at date zero, and computed as above in all subsequent periods.

C Changes in the relative price of capital

As in Ngai and Pissarides (2007), it will be convenient to set capital as the numeraire so that $p_{St} = 1 \ \forall t$. In this way, consumption goods prices p_{ct} are expressed relative to the price of capital goods. The term $1/p_{ct}$ is then the relative price of capital in units of consumption. Define aggregate output in units of capital as $y_t = y_{St} + p_{ct} c_t$.

Measuring output in units of capital is just a matter of convenience when solving the model. When we compare model-generated GDP to GDP in the data, we will use an appropriate measure.

Notice that changes in the relative price of capital in this model are technologically determined: they reflect changes in productivity in the capital producing sector relative to the rest of the economy, as in the ISTC literature. In turn, these productivity indices reflect technical progress at the more disaggregated industry level, as well as differences in composition.

Solving the reduced 2-sector problem and using the equilibrium conditions, we obtain expressions for labor shares in the capital goods sector n_{St} and the consumption goods sector $n_{ct} = 1 - n_{St}$. These turn out to be functions only of the productivity growth rates g_i, parameters, and of the equilibrium growth rate of aggregate consumption $g_{ct} = \frac{p_{c,t+1}c_{t+1}}{p_{ct}c_t}$, which is endogenous. This will be true at all dates except at date zero, where n_{St} is determined by the initial condition K_0.

Define q_t as the marginal rate of transformation between consumption and capital. which is also the (inverse) relative price of capital. It follows from the above derivations that:

Proposition 1 *The inverse relative price of capital is given by the following expression:*

$$q_t \equiv \frac{A_{St}}{A_{ct}} = \frac{\left[\sum_{i \in I_S} \xi_{S,i}^{\varepsilon_S} \times A_{it}^{\varepsilon_S - 1}\right]^{\frac{1}{\varepsilon_S - 1}}}{\left[\sum_{s=1}^{S-1} \zeta_s^{\varepsilon} \times A_{st}^{\varepsilon - 1}\right]^{\frac{1}{\varepsilon - 1}}}$$

Thus, the growth factor of the marginal rate of transformation – in other words, the growth factor of ISTC – is:

$$g_{qt} = \frac{A_{St+1}/A_{St}}{A_{ct+1}/A_{ct}} = \frac{\prod_{i \in I_S} g_i^{x_{it}/X_{st}}}{\prod_{s=1}^{S-1} \overline{g}_{st}^{x_{st}/X_{ct}}} \tag{25}$$

where $\bar{g}_{st} = \prod_{i \in I_s} g_i^{x_{it}/X_{st}}$ *for sector* 1 *to* $S-1$ *as defined above,* $x_{st} = \xi_s^{\varepsilon} A_{st}^{\varepsilon - 1}$, *and*

$X_{ct} = \sum_{s=1}^{S-1} x_{st}$.

Since the terms x_{it}/X_{st} are not in general constant over time, g_{qt} will not be constant over time either. However, it can be computed over time given an appropriate starting point.

Notice that equation (25) is all we need in order to compute model-generated values of g_{qt}. Given values for g_i in all industries, and given initial shares of GDP or of the labor force in each industry, we can compute g_{qt} at all dates by the repeated application of equations (6), (10) and (25). In particular, we do not need to solve for the general equilibrium of the model: the data inputs we require are initial conditions on the composition of the economy, estimates of industry productivity growth rates, and estimates of the sectorial elasticities to compute future paths of economic composition inside and outside the capital goods sector. However, solving for the full general equilibrium of the model is needed in order to compute investment rates and rates of economic growth.

One assumption of our quantitative exercise is that industry TFP growth rates are shared across countries. It is worth pointing out that model-generated values of g_q do not change if each country has an additional factor of productivity growth that affects all industries equally – for example, if developing economies have more rapid productivity growth in all industries because of technological catch-up. If all productivity growth factors g_i were multiplied by some country-wide factor g_z, or even a country-date specific factor g_{zt}, equation (25) that derives g_q for a given country based on composition and on structural change would not be affected, as the factor g_z would affect both the numerator and the denominator equally.

IV Calibration

In what remains of the paper we calibrate the model economy separately for each country in our data, and compare the behavior of the model countries to the data. We will assume that all countries share the same parameters, which we calibrate using US data. The only exceptions will be those parameters that affect economic composition, which we calibrate to match initial economic composition in each country.

We will focus on a particular type of equilibrium. Observe that the capital stock will be efficient at all dates except date $t = 0$, since it is given at that date. In other words, the investment share of the model economy will generally evolve smoothly over time – except between dates $t = 0$ and $t = 1$, where it may jump depending on the initial condition for the capital stock. The model will be calibrated to the available data and, since the initial year in which data for a given country became available has no economic content, it is difficult to justify why the first year we have data for (generally 1991) happens to be the only date when the intertemporal optimization condition is not satisfied. For this reason, we focus on an equilibrium where this does not occur.

Definition 1 *An Euler Growth Path (EGP) is an equilibrium and an initial condition K_0 such that intertemporal optimization holds at date zero.*

The EGP is a generalization of a balanced growth path, which may exist in models that do not exhibit balanced growth. For the benchmark results, we calibrate the model to match an EGP by matching the *composition* of manufacturing but not necessarily its *size*. Nonetheless, it is important to underline that the results concerning g_q do not to hinge on whether we focus on an EGP because (as discussed) the series for g_q can be computed without computing investment rates and aggregate growth rates. See Appendix D for further details on the computation of the equilibrium.

Calibrating the model economy requires a choice of industries and sectors, as well as a correspondence between industries and sectors in the model and the data. We begin by discussing this correspondence, and then turn to the choice of parameter values.

Data on economic composition are available at different levels of aggregation. Within manufacturing, our main sources are the INDSTAT 2 and INDSTAT 4 databases distributed by UNIDO, which report the value added volume for different manufacturing industries, and can thus be used to compute relative shares of value added. INDSTAT 4 data are more disaggregated, but are available for fewer countries, so we use the INDSTAT 2 classification and, when only INDSTAT 4 data are available, we aggregate them to the classification in INDSTAT 2. These data include data for capital goods producing industries, and also for non-capital manufactured goods, and we treat the two groups separately as discussed below, since the distinction between capital and non-capital is central to the paper.

We measure industry composition *outside* manufacturing using data from the International Labor Organization (ILO). These data report labor shares, not shares of value added. As shown earlier, however, the two are the same in the model economy. This is because we have assumed that the labor income share of the Cobb-Douglass production function is the same across industries, and that there are no barriers to labor mobility across industries, as in Ngai and Pissarides (2007).[11] ILO data report composition for 14 sectors, and are available for the years $1991 - 2011$.

We exclude countries that were formerly in the USSR or Yugoslavia, as well as Eritrea. These countries had just become independent and undergone dramatic institutional transition around 1991, in many cases having experienced war shortly thereafter. As a result, they likely have distorted initial conditions and are undergoing structural change for reasons related to their disintegration that would not be captured adequately by a growth model based on long run trends.[12] In addition, the political transition after itself likely indicates lack of institutional capacity to provide accurate economic statistics, at least for a period. This leaves a data set with 64 countries.

The first issue in calibration is the correspondence between the various industries in the model and the data. The most basic distinction in our context is between capital and non-capital goods. The literature defines capital as machinery/equipment plus structures: we follow this definition. In practice, this requires distinguishing between industries in the UNIDO manufacturing data and sectors in the ILO data that produce capital, thus defined.

In the UNIDO manufacturing data, we define capital goods-producing industries as in Table 2. In addition, we include the construction sector from the ILO database as the industry that produces structures.[13] The remaining UNIDO manufacturing

[11] Indeed, labor shares and value added shares in manufacturing in INDSTAT 2 are very highly correlated.

[12] Indeed these countries are mostly outliers in terms of g_q.

[13] Herrendorf et al (2014) argue that software should also be considered part of investment. We

Table 2: Capital good-producing manufacturing industries

Industry	ISIC code
Wood products	331
Furniture, except metal	332
Fabricated metal products	381
Machinery, except electrical	382
Machinery, electric	383
Transport equipment	384
Prof. & sci. equip.	385
Other manufactured prod.	390

industries and ILO industries are considered non-capital, alongside the 12 sectors in the ILO data excluding construction and manufacturing.

The next step in mapping these industries into the model relates to whether any of these should be grouped together into "sectors", exploiting the nesting structure in the model economy. We group the manufacturing industries not included in Table 2 into the non-capital manufacturing sector.

Regarding non-manufacturing sectors, although the ILO data reports 14 sectors (one of which is manufacturing), recent developments suggest that the service sectors can be divided into two groups. For example, Duarte and Restuccia (forthcoming) distinguish between "Traditional" and "Non-traditional" industries, based on whether their relative prices are found to rise or to fall with income per capita in cross section. This suggests that the data on economic composition as it relates to development favor grouping services into 2 groups.

We assemble the groups following their criterion. We define the Non-traditional services to include Financial and Insurance activities; Real estate and Business Services; transport and communication; accommodation and food services; and other services. Traditional services are: Education, Human health, and Public administra-

were unable to find any data on stand-alone software expenditures for many countries. On the other hand, we note that software that is incorporated into equipment would be counted (e.g. copies of Windows 10 installed on computers). See the Appendix for further discussion.

tion.

We treat the remainder of the non-manufacturing sectors as single-industry sectors. These are Agriculture, Mining, Utilities and Wholesale/Retail trade. This leaves a total of $S = 8$ sectors: Agriculture, Mining, Utilities, Wholesale/retail trade, Traditional services, Non-traditional services, Non-capital manufacturing, and capital manufacturing. See Table 3 for a depiction of the industry nesting structure in the calibrated model economy.

Now that we have a correspondence between industry data and industries in the model, we require the values of industry productivity initial conditions A_{i0} in each country and industry. These will be chosen so as to match the industry composition of each economy in the initial year. Before going further, note that in order to execute our main quantitative exercise – the comparison of a model-generated series for g_q with the data – we do not require a vector of productivity values $A_{i0.}$ for all industries and sectors per se. All we need are shares within the capital goods sector (relative to each other), and shares for the non-capital goods sectors and industries (relative to each other), along with the elasticity parameters that enter equation (25).

For the initial productivity values of the capital and consumption sectors, we initially set $A_{c0} = 1$. This is the productivity index for aggregate consumption, not an index that corresponds to any particular industry: this is essentially normalization. Then we set the aggregate capital index $A_{S0} = q_0$ for each country.[14] Then, using (8) and (15), for the each capital sector industries $i \in I_S$, we set initial TFP to equal $A_{i0} = \left[\frac{n_{i0}}{\sum_{i \in I_S} \xi_{S,i}^{\varepsilon_S} n_{i0}}\right]^{\frac{1}{\varepsilon_S - 1}}$, thus matching the initial share of capital industries in each country n_{i0}. For the sectors and industries that make up consumption, set A_{s0} so as to match the initial share of that sector in each country: $A_{s0} = \left[\frac{n_{s0}}{\sum_{s=1}^{S-1} \zeta_s^{\varepsilon} n_{s0}}\right]^{\frac{1}{\varepsilon - 1}}$. Finally, for industry productivity in non-capital manufacturing or in any other disaggregated sector, we have again $A_{i0} = \left[\frac{n_{i0} A_{s0}}{\sum_{i \in I_s} \xi_{s,i}^{\varepsilon_s} n_i}\right]^{\frac{1}{\varepsilon_s - 1}}$.

The data sources for all these industry shares are the UNIDO data and the ILO

[14] That said, the choice of A_{S0} is without loss of generality because the size of the non-investment sectors is independent of the level of these parameters: it is equivalent to a normalization of the units in which we measure capital along the assumed Euler growth path.

Table 3: Industry and sector hierarchy in consumption

Sectors	Industries
Agriculture	Agriculture
Mining	Mining
Whole/Ret. Trade	Whole/Ret. Trade
Utilities	Utilities
Traditional Services	Education Health Public services
Non-trad. Services	Finance and insurance Business services Transport services Accommodation Other services
Non-cap. Manuf.	UNIDO industries not included in capital.

data described earlier. We use the shares in 1991, the earliest year for which we have ILO data.

Recall that we calibrate the model to match an EGP. In practice, this requires matching the shares of capital-goods producing industries so that, relative to each other, they match the data. Then, separately, we do the same for the industries that do not produce capital. Finally, we set the initial size of the sector S (capital goods) relative to all other sectors so as to satisfy the Euler equation at date zero. The result is that the series for g_q generated by the model economy will follow equation (25) at all dates.[15]

It remains to calibrate the following parameters, which we draw from US data:

1. Technological parameters α, δ.
2. Multiplicative preference parameters $\xi_{s,i}$, ζ_s and β.
3. Elasticities of substitution ε_s for $s \leq S$, and ε, the elasticity across consumption (i.e. non-capital) sectors
4. The intertemporal elasticity parameter θ.
5. Industry productivity growth values g_i.

We select them as follows:

1. We assume that $\delta = 0.06$ as in Greenwood et al. (1997): this is a standard values in models in which the productivity of the investment technology exceeds that in the consumption sector. We use a standard value for the capital share, $\alpha = 0.3$.
2. To calibrate the utility weights $\xi_{s,i}$, it should be noted that in a sense these weights are arbitrary, as they depend on the exact unit of measurement for good

[15] Finally, we could undo the initial normalization by multiplying A_{i0} in all industries by a country-specific constant so that the country GDP per head relative to US GDP per head in the initial year is the same as in the data. We do not do this in practice as our quantitative experiments do not concern levels of economic activity.

i.[16] Thus, without loss of generality, We set $\xi_{s,i} = \frac{1}{|I_s|}$, where $|I_s|$ is the number of industries in sector s. The same applies to the sector weights ζ_s, so $\zeta_s = \frac{1}{S-1}$. This is not true, however, in the case that for a given sector $\varepsilon_s = 1$. In that case, $\xi_{s,i}$ is set independently in each country so that it matches correctly the share of expenditure of each industry in sector s in any given country in the initial year.

3. We set $\beta = 0.95$, a standard value.

4. For each sector, equation (10) is equivalent to $\log G_i = \alpha + (\varepsilon_s - 1)\log g_i + \epsilon_i$ where $\alpha = \log G_j - (\varepsilon_s - 1)\log g_j$ for some arbitrary industry j and ϵ_i is any unmodelled noise in the relationship. We regress a panel of US value added growth rates on TFP growth rates for capital and non-capital manufacturing goods, respectively, over the period $1991 - 2011$. Based on these estimates, we set $\varepsilon_s = 1$ for the non-capital manufacturing sector, $\varepsilon_s = 1$ for Non-traditional services, $\varepsilon_s = 0.87$ for Traditional services and $\varepsilon_S = 1.67$ for the capital-producing sector.[17] Across consumption sectors, we use the value $\varepsilon = 0$ as found in Herrendorf et al. (2013). Notice that these estimates have the following implications. As in Ngai and Pissarides (2007), the composition of the consumption sector shifts over time to favor the subsector with the lowest TFP growth rate. Within these subsectors the same thing occurs: either $\varepsilon_s = 1$ so there is no structural change within each sector, or $\varepsilon < 1$ so that Traditional services itself comes to be dominated by the lowest-tech Traditional service. In the capital goods sector, however, $\varepsilon_S > 1$, so that capital gradually becomes dominated by the most high-tech capital. This implies that, over time, g_q will tend to accelerate, as TFP growth in the capital goods sector accelerates, whereas it decelerates in the consumption sector.

5. The preference parameter θ is calibrated so that in the long run the investment share of GDP converges to 17 percent, which is the share in the US over the

[16]For example, if I measure apples and get $\xi_{s,apples} = 2$ (and $A_{apples,0} = 3$), I could choose to measure apples in units of "half an apple" and then $\xi_{s,apples} = 1$ (and $A_{apples,0} = 1.5$).

[17]Standard deviations are 0.09, 0.026, 0.026 and 0.14 respectively.

relevant period (and also in post-war data, according to the FRED database). This implies that $\theta = 3$: typical values used in calibration in the growth or finance literature fall in the range $\theta \in [1, 5]$,[18] so it is encouraging that our value falls in the middle.

6. Productivity growth values g_i are average values over the period $1991 - 2011$. Manufacturing industry TFP growth data are computed using the NBER-CES Manufacturing Industry Database. Note that the NBER industry classification is at the 4-digit SIC level of disaggregation. We use Domar weights to aggregate these TFP growth rates from the SIC classification to the classification in the ISIC revision 2 data, following Bartelsman and Gray (1996). For sectors outside manufacturing, the sector TFP growth data are computed using the price decline for each sector relative to manufacturing sector from the Bureau of Economic Analysis (BEA) price index table over the same time period. This approach is standard in the related literature, see for example Ngai and Pissarides (2007) or Samaniego and Sun (2016). Since the BEA industry classification is more disaggregated than ILO, we compute the weighted average price index first using output shares as the weight in the BEA database to match the ILO industry categories. Then we are able to retrieve the relative price growth relative to the manufacturing sector for all other sectors. For this purpose, the average TFP growth rate of manufacturing sector is computed from the NBER database using Domar weights as before. See the Appendix for the values of g_i we use.

V Quantitative Findings

In this section we simulate the behavior of the model economy starting from 1991 until 2011. For each country we use common parameters, the only exception being the parameters that match the initial economic composition in 1991, and simulate its

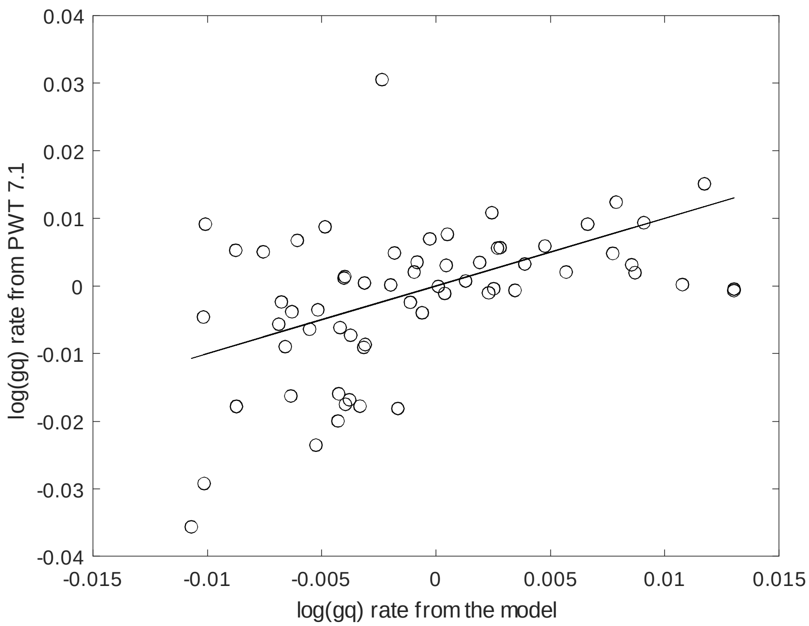
[18] Growth models tend to use $\theta = 1$, whereas asset pricing studies tend to use larger values. For an example with $\theta = 5$, see for example Jermann (1998).

aggregate behavior using the algorithm in Section B. We then compare the model-generated statistics to aggregate data, primarily the data on $\log g_q$ presented earlier in Section 2. For the time being, we measure $\log g_q$ using the PWT 7.1 data. Later on, we compare model behavior to the PWT 9.1 data. We remind the reader that the model itself uses neither in the calibration process: all the parameters are drawn from other sources.

A Baseline results: Trends in the price of capital

We find that the model-generated rate of decline in the relative price of capital is strongly related to the same statistic in the data: see Figure 4. Two facts stand out. First, the two are statistically significantly correlated: the correlation is 0.36^{***} and significance is at the one-percent level. This indicates that the model accounts for the statistical variation in the rate of decline in the relative price of capital around the world. Second, the regression coefficient on the model generated ISTC growth is 0.95^{***} – a coefficient that is not statistically significantly different from 1. The estimate of the vertical intercept, on the other hand, is -0.001, very small and not statistically significant from zero. This implies that, in addition, the model is able to account for the extent of variation in g_q in the data and also for the general average in the data of g_q. Thus, albeit with some noise, the model accounts for the values of g_q in the data in terms of magnitude and variation. This is a very strong result, considering that no country-level information on g_q is used at all in calibrating the model. The model-generated values of g_q are entirely based on initial composition and on structural change induced by industry-level productivity growth.

Figure 4 –Correlation of $\log g_q$ in the PWT 7.1 database and the model-generated values of $\log g_q$.

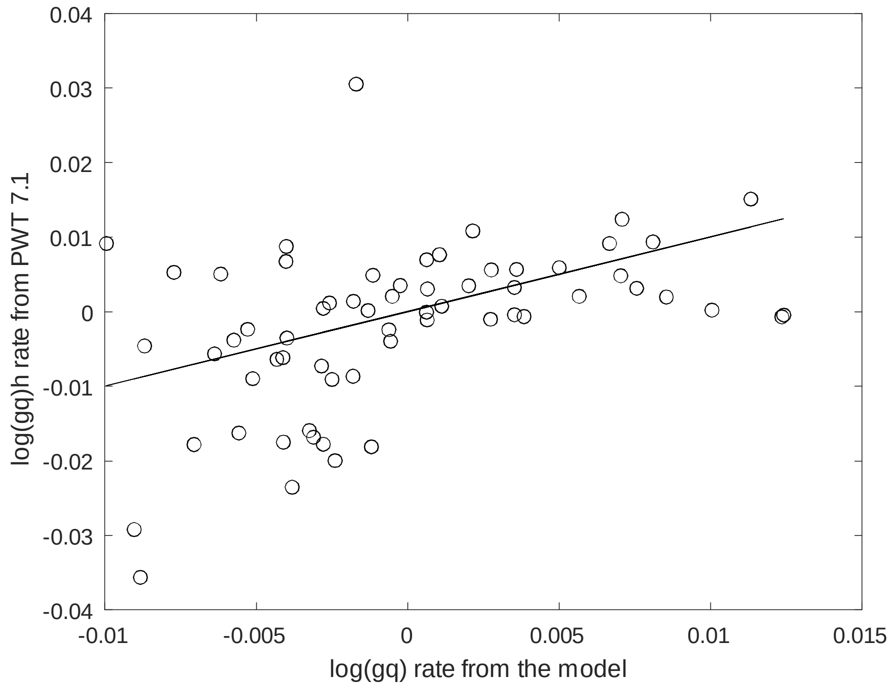


Notes: Model values are computed using simulated changes in economic composition $1991 - 2011$, using the benchmark calibration. The line is a 45 degree line.

B Counterfactual experiment: the Nesting Structure

Is grouping services into Traditional and Non-traditional sectors important for this result? To verify the robustness of our results, we do not group service industries into Traditional and Non-traditional groups. Instead, we assume they have the same elasticity of substitution among other consumption industries ($\varepsilon = 0$). This means that all service industries are separate and at the same level of aggregation as agriculture (as in Samaniego and Sun (2016)). When we regress the log of g_q from the PWT 7.1 average over the sample period against the model generated average log of g_q, we find that the coefficient is 1.06 and significant at 1%. As before, the coefficient itself is not statistically significant from one. In addition, the constant of $-.0003$ is not significantly different from 0. Thus, in a sense, except for some noise, the model is able to generally account for the extent of variation in trends in the relative price of capital in the data just as before. See Figure 5. We continue to use the original nesting structure as a benchmark in the following subsections, given recent developments in the literature that indicate it is appropriate, but note that it is not necessary to reproduce the broad patterns of $\log g_q$ observed in the data.

Figure 5 – Correlation of $\log g_q$ in the PWT 7.1 database and the model-generated values.

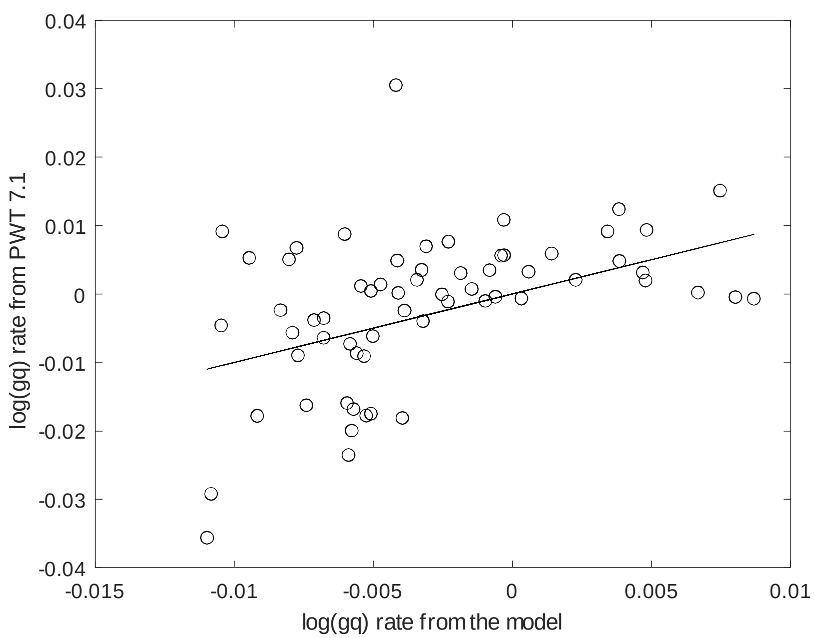


Notes: Model values are computed using simulated changes in economic composition $1991 - 2011$, assuming an alternative industry structure where services are not nested, i.e. all services are treated as separate sectors. The line is a 45 degree line.

C Counterfactual experiment: the impact of structural change

Does the process of structural change play an important role in accounting for the above results concerning the distribution of g_q around the world? To answer this question, we calibrate the model economy as described earlier, except that we set $\varepsilon = \varepsilon_s = 1 \; \forall s \leq S$ in the model. With all elasticities set to unity, there is no structural change in the model economy: economic composition remains what it was in the initial period. We then compute $\log g_q$ for this counterfactual experiment: see Figure 6. We find that the model-generated values of g_q remain highly correlated with those in the data. We find that the regression coefficient rises to 1.14*** – still not significantly different from unity – and that the intercept remains indistinguishable from zero. We conclude that structural change over the period in question is too slow to impact the results.

Figure 6 – Correlation of $\log g_q$ in the PWT 7.1 database and the model-generated values of $\log g_q$.

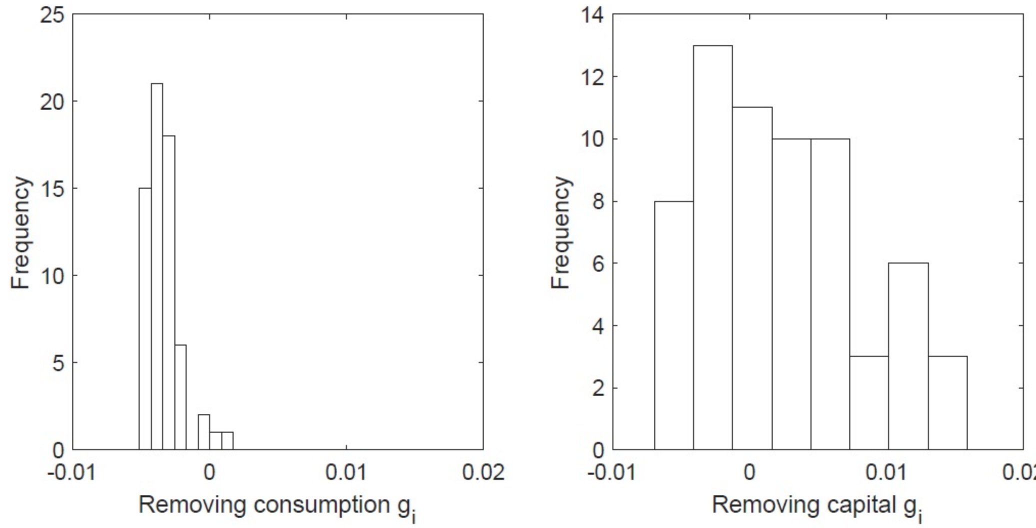


Notes: Model values are computed using simulated changes in economic composition $1991 - 2011$ assuming that all elasticities ε and ε_s are set to unity, so there is no structural change in the model. The line is a 45 degree line.

D Counterfactual experiment: Capital and Consumption hypotheses

Unlike in the related literature on ISTC (e.g. GHK), the decline in the relative price of capital is not due to a single technological process, but is rather the outcome of technological progress in several different sectors, as well as structural change induced by technological progress. However, we can use our model to determine whether ISTC is primarily due to technological progress in the capital goods industries, or to technological progress in the consumption sector. In one counterfactual experiment, we assume that there is no technological progress in non-capital goods, keeping g_i in other industries the same as the baseline model. In another experiment, we assume that there is no technological progress in capital goods industries, while keeping other g_i constant in the non-capital goods producing sectors. Then, in each case we compute the change in the value of $\log g_q$ in the model economy.

Figure 7 – Contributions of capital sector vs consumption sector TFP growth to g_q.



Notes: The left panel is the difference between baseline model-generated $\log g_q$ and the value generated by the model in an experiment where g_i is set to equal one in all non-capital producing industries. The right panel is the difference between baseline model-generated $\log g_q$ and the value generated by the model in an experiment where g_i is set to equal one in all capital producing industries. The vertical axis is frequency.

In Figure 7 (left panel) we can see that removing productivity growth in the non-capital goods industries lowers g_q in most countries. This is because, although productivity growth in non-capital goods is in the denominator of g_q, in many of them $g_i < 1$, so setting $g_i = 1$ raises the denominator and lowers g_q overall. However, the decrease is not very large: on the order of 0.5 percent or less. In this sense, the contribution of productivity change in the *capital goods* industries to g_q is large and mainly positive. On the other hand, removing productivity growth in the capital goods industries (Figure 7, right panel) makes a large difference to the values of g_q in the model – sometimes positive, and sometimes negative, reflecting the fact that g_i within capital goods is positive for most but not all industries. This suggests that productivity growth in the *non-capital industries* likely contributes little to g_q in the model, since removing it makes little difference, whereas removing its alternative does.

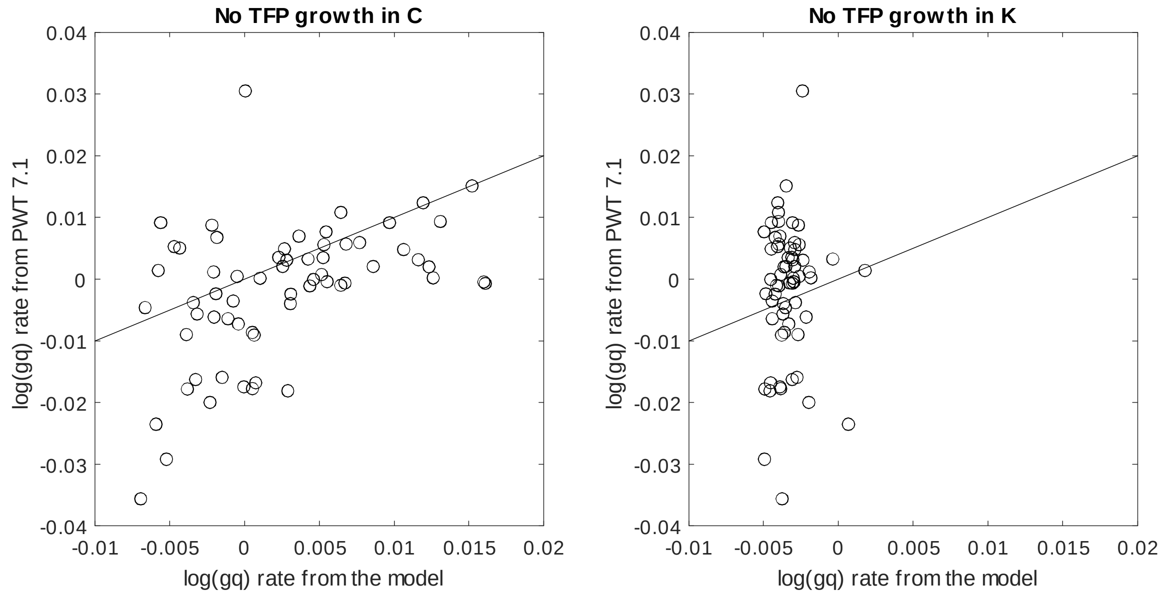
Another way to assess the contribution of productivity growth in capital-producing as opposed to non-capital producing industries is as follows: suppose we shut down productivity growth *outside of capital*, it remains the case that $\log g_q$ in the model is related to $\log g_q$ in the data and the correlation remains significant at the one percent level. The coefficient declines to 0.84^{***}, but remains not significantly different from unity. The value of the intercept is -0.004^{***}, however, and is significantly different from zero. This indicates that the magnitudes of the model-generated $\log g_q$ values are slightly overstated when we remove productivity growth outside of capital. See the left panel in Figure 8.

On the other hand, suppose we shut down productivity growth in the *capital-producing industries only*. It is no longer the case that $\log g_q$ in the model is related to $\log g_q$ in the data. The coefficient declines to 0.31 and is not significantly different from zero. The value of the intercept is not significantly different from zero either. This indicates that the model-generated values are now totally unrelated to those in the data. See the right panel in Figure 8.

Recalling the capital and consumption hypotheses introduced in Section II, we conclude that the model indicates that the *capital hypothesis* mainly accounts for variation around the world in the relative price of capital. It alone accounts for the correlation and much of the extent of variation in rates of $\log g_q$ around the world. This does not mean that productivity growth outside of capital-producing industries is irrelevant in accounting for $\log g_q$. In particular, the significant intercept of -0.4 percent in the regression of model-generated $\log g_q$ without such productivity growth indicates that, without it, the model overshoots $\log g_q$ on average. Setting productivity growth g_i to equal 0.4 percent in all the non-capital producing sectors brings the value of g_q in line with the data in terms of magnitudes, and setting productivity growth g_i in non-capital to as low as 0.2 percent turns out to be enough to make the intercept no longer statistically significant. However, *variation* in productivity growth among non-capital producing sectors plays no role in accounting for $\log g_q$:

indeed Figure 8 suggests that this variation acts more like noise. Thus the *consumption hypothesis* plays no role in accounting for variation in $\log g_q$ around the world.[19]

Figure 8 – Correlation of $\log g_q$ in the PWT 7.1 database and the model-generated values of $\log g$



Notes: In the left panel, only the capital goods sector experiences productivity change, and g_i is set to one in all non-capital producing industries. In the right panel, only the non-capital goods industries experiences productivity change, and g_i is set to one in all capital-producing industries. The lines are 45 degree lines.

E Baseline results: Economic Aggregates

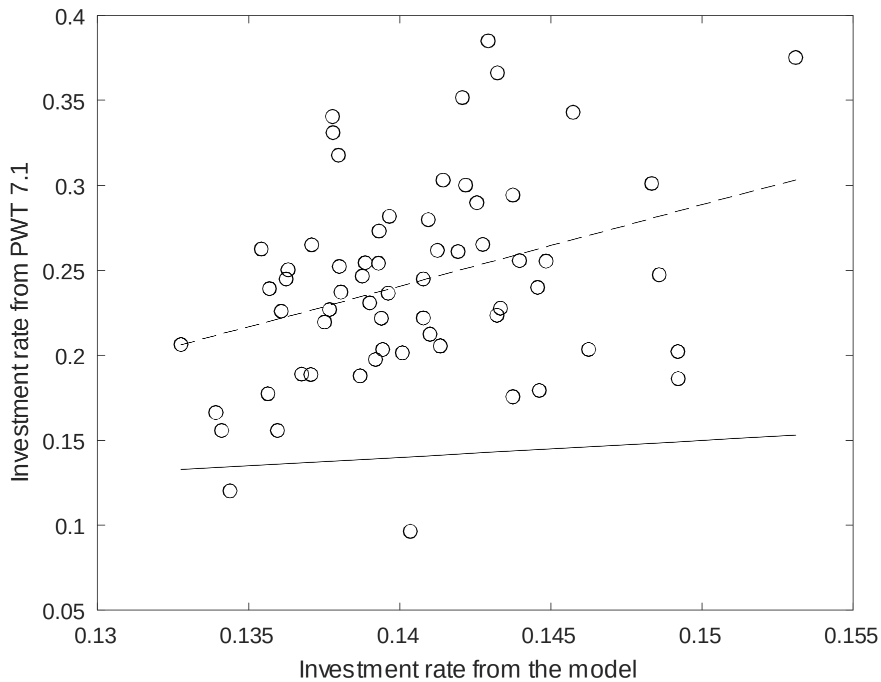
Having examined the ability of the model to account for variation in g_q around the world, we examine the extent to which a model in the neoclassical growth framework

[19]Later we look at investment rates and it is worth noting that the same is true of model-generated investment rates: shutting down productivity growth in consumption doesn't make much difference, but shutting down productivity growth in capital disables the model from accounting for any empirical patterns.

that accounts for g_q is able to account for aggregate behavior. In particular, since g_q is a key element of the return to capital in equation (29), it should be an important determinant of investment rates – see equation (32). To examine this issue, we return to the benchmark calibration that allows for structural change and the nesting of services and apply the simulation algorithm described in section IV — although it is worth noting that the following results hold for the cases where services are not nested and where structural change is deactivated in the model.

Figure 9 shows that model-generated investment rates are also strongly correlated with those in the data. Again, there is nothing in the calibration procedure that might match investment rates: in particular, our focus on EGPs implies that we do not explicitly match initial investment rates when calibrating the model economy. The correlation is 0.34^{***} and significant at the one percent level, and the regression coefficient is 4.8^{***}. The fact that the regression coefficient is larger than one implies that the model variation in investment rates is less than the empirical variation in investment rates. The intercept of -0.43 is not statistically significant from zero. In this sense, the model accounts for the correlation and the magnitudes of investment rates, although with insufficient dispersion.

Figure 9 –Correlation of investment rates from the PWT 7.1 database and the model-generated investment rates.



Notes: Model values are computed using simulated changes in economic composition $1991 - 2011$, using the benchmark calibration. The dashed line is a regression line obtained using OLS. The full line is the 45 degree line.

If we examine the equation for investment (32), we see there are two possible effects. One is the effect of transition dynamics in r_t. Another is what we might call the direct impact of ISTC on investment rates – the impact from the fact that $\overline{g}_{A_{St}}$ is an argument of (32).

Suppose that r_t is constant over time. Then investment rates reduce to the expression:

$$\frac{y_{St}}{y_t} = \frac{\alpha}{r_t}\left[\overline{g}_{A_{St}}^{\frac{1}{1-\alpha}} - (1-\delta)\right] \tag{26}$$

Now, since we find the consumption hypothesis is not important, this implies that $\overline{g}_{A_{St}} \approx g_q$. Thus, we compute counterfactual investment rates assuming that $\overline{g}_{A_{St}}$ takes the model-generated average value of g_q over the period, and apply equation (26).This is the investment rate if ISTC rather than transition dynamics in r_t were the main factor behind investment rates. Note that we do not need a value of r_t to do this since it enters multiplicatively and is assumed to be constant over time in this counterfactual. When we do this, we find that the correlation with investment rates in the benchmark model is 0.66^{***}, which is quite high. However, the correlation with investment rates in the data declines to only 0.09. We conclude that the transition dynamics of r_t are key for the ability of the model to account for empirical variation in investment rates.

Next, we return to the benchmark model and compare model generated GDP growth rates with those from the data.[20] Figure 10 shows that the correlation between model and data is only 0.07. Thus the model is not able to account for GDP growth rates in the data. As we show in the Appendix, the result concerning investment rates is robust to using investment rates computed using the PWT 9.1 data, and the result concerning growth rates is similar.

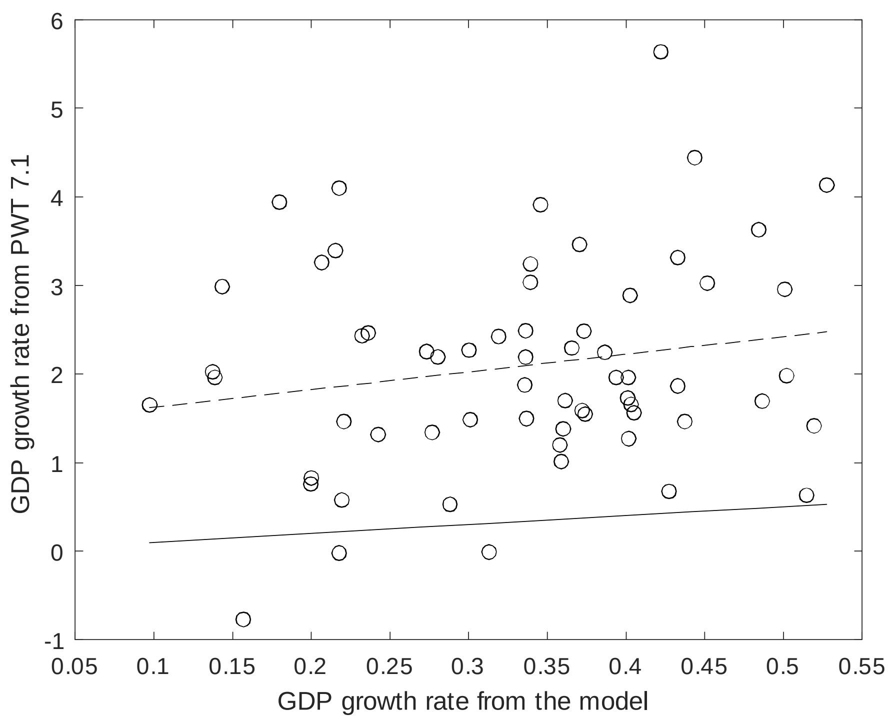
Figure 10 suggests that model-generated GDP growth rates are lower than those in the data. For example, the US GDP growth rate in the data is 1.4 percent, whereas it is about 1 percent in the model.[21] We repeated the exercise multiplying all industry and sector TFP growth rates by a common factor g_z for all countries to ensure that we exactly match US GDP growth rates over the period. This makes sure that the general magnitude of productivity growth rates matches that in the

[20] For these purposes, we compute GDP growth in the model using chain-weighting, so as to generate a measure of GDP growth that is comparable to the data.

[21] We are unsure of the specific source of the discrepancy. However there are several abstractions we make that could be responsible, including the uniformity of capital shares and the absence of input-output linkages. Our model also assumes perfect competition, whereas recent work by Baqaee and Farhi (2019) argues a substantial share of US growth is attributable to improvements in allocative efficiency.

data for the US; however, as before, the variation in GDP growth rates is too small and the correlation is not significant. This alternative calibration does not change the other predictions of the model: model-generated values of $\log g_q$ are identical and, regarding investment, the correlation between investment rates in model and data is high at 0.41^{***}.

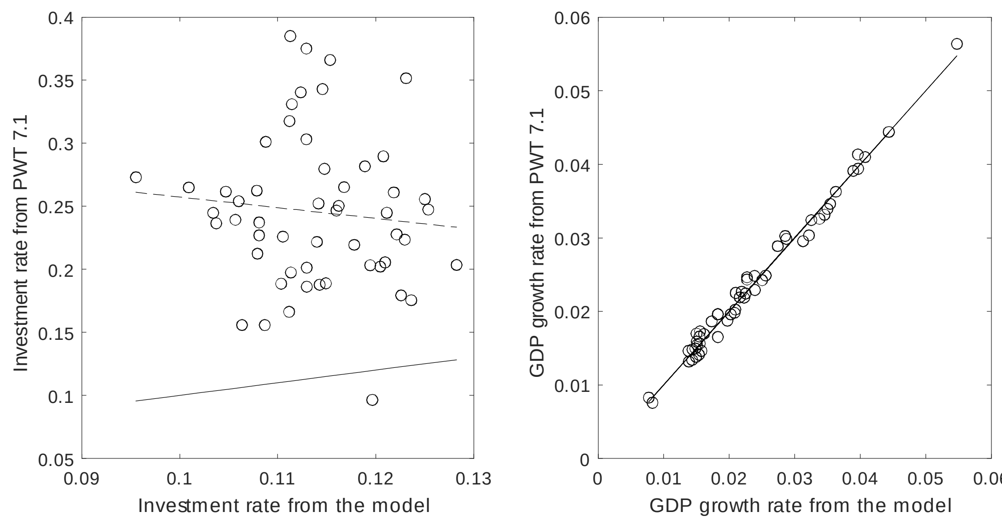
Figure 10 – Correlation of aggregate growth rates in the PWT 7.1 database and the model-generated values of GDP growth rate.



Notes: Model values are computed using simulated changes in economic composition 1991 – 2011, using the benchmark calibration. Units are in percentages. The dashed line is a regression line obtained using OLS. The full line is the 45 degree line.

We perform one last exercise. For each country, we multiply all industries by a country-specific growth factor g_z chosen so as to match the economic growth rate in the data for all countries over the period in question. Since this raises both the numerator and denominator of g_q by the same factor, this does not affect the model's ability to account for g_q. However, it does turn out to reduce the correlation between investment rates in the model and the data to -10 percent. See Figure 11.

Figure 11 – Comparison between investment rates and economic growth in the PWT 7.1 compared to their model-generated counterparts.



Notes: Model values use simulated changes in economic composition $1991 - 2011$. In this experiment, productivity growth rates in each country were multiplied by a country-specific factor g_z, selected so as to closely match GDP growth rates in each country. The dashed line is a regression line obtained using OLS. The full line is the 45 degree line.

We conclude that the simple model of economic composition accounts well for the data on g_q, and reasonably well for variation in investment rates. On the other hand, there is a sort of "growth dilemma" in that a simple multi-industry extension of the neoclassical growth framework that accounts for variation around the world in the relative price of capital cannot account for investment rates and growth rates at the same time. Given that our model is a simple extension of the standard workhorse of growth theory, the neoclassical growth model, that allows for many industries and various rates of technical progress, the minimum extension to allow for structural change, this poses a challenge for the growth literature and the literature on structural transformation. Equation (26) tells us that if we multiply all productivity growth rates by a country-specific productivity growth term $g_z > 1$ it should raise the investment rate, since it raises $\bar{g}_{A_{St}}$. This implies that growth is underestimated by the model in countries that have low investment rates empirically. When we raise g_z to match those growth rates, we make the investment rates too high. Alternatively, growth is overpredicted by the model in many places that have high investment rates. The implication is that investment rates may be high in some countries due to distortions of some sort that do not contribute to growth. We leave an investigation of these factors for future work.

F Robustness: on the measurement of capital

Herrendorf et al. (2014) suggest that some of the output of different industries ends up incorporated into capital, including even to some extent service industries. To see whether this might affect our findings, for robustness, we repeat our quantitative exercise using measures of $\log g_q$ computed in a different fashion. Instead of directly using the productivity growth rates measured as described above to measure $\log g_i$ in the production of each capital good, we instead use input-output tables to create a measure by types of capital goods corresponding to the above categories. For each capital good, we use the input-output "make" tables from 1997 (a benchmark year in the middle of the period of interest) to compute a weighted average measure of $\log g_i$, based on each of the different industries that contribute to its production. Then, we repeat our quantitative experiments. For consistency, we use this procedure to measure all industry values $\log g_i$, not just capital goods.

First, the correlation between model-generated $\log g_q$ using these values and the data remains high, 0.36***. However, what is more telling is that the regression coefficient is 0.94***, almost exactly unity and almost exactly the same as before. The intercept is close to zero and not significant, as before. Thus, our results are robust to measuring productivity growth rates with input-output tables in mind.

We define capital as including the manufacturing industries in Table 2. However, an alternative definition would include software, which has comprised an increasing share of investment in developed economies; for the case of the US see Cummins and Violante (2002). It is not clear that software makes up a significant share of investment in developing economies over the period in question, which make up much of our data and, to the extent that software is sold pre-installed on equipment, it is already counted in investment spending. However it is difficult to tell whether stand-alone software is an important part of investment around the world, as international software data are hard to come by. One way to solve this is to note that software is counted as part of the industry Printing and Publishing (ISIC 342). Thus, we could create an alternative definition of capital in the model that includes ISIC 342 as well. However, only a small part of ISIC 342 likely comprises software, so it is not clear that this is a useful approach.

In any case we tried this procedure. Since the value of g_i for ISIC 342 is considerably lower than for the capital sectors in Table 2, we contend that including it leads to a model-generated value of g_q that is far lower than that produced in the benchmark calibration, even if the *correlation* is still fairly strong with g_q in the data. Indeed the correlation remains high and significant, 0.33***. The regression coefficient is 1.44***, highly significant and not statistically different from one, as before. The difference with the older results is that the intercept 0.0079** is positive and significant, as conjectured.

Greenwood et al. (1997) observe that the rate of ISTC in structures is lower than in equipment and machinery. This begs the question of whether differences in g_q might be primarily driven by differences in the share of structures in the capital stock. To address this, we constructed the average share of investment in structures using the supplementary data provided alongside the PWT 9.1 – these data were not available for the 7.1 edition. We found that the correlation between the share of investment in structures over the period and our benchmark measure of $\log g_q$ based on the PWT 7.1 was −0.06 and not significant. Similarly, when we compute this share using the procedure in the calibration,[22] we find that the correlation is 0.05.[23] We conclude that is the more detailed composition of capital that accounts for our findings, not just shares of machinery or of structures.

We also adjust our measure for robustness to exclude investment in residential structures. We could not find comprehensive international data on the share of residential structures in new investment, but we found that in the US this is about 40 percent according to the breakdown in the 1997 IO tables. We reconstructed our measures of $\log g_q$ excluding 40 percent of construction for the exercise, finding little change in the results.

VI Discussion

A Comments on institutions

We find that the model without barriers accounts well for the empirical magnitude and variation in $\log g_q$, solely on the basis of economic composition. On the other hand, our findings do leave the door open for institutional factors or other barriers to influence $\log g_q$ *indirectly*, through any impact they might have on economic composition.

What might these determinants be? There is a precedent in the literature for the idea that policy or institutional factors may affect composition. For example, Samaniego (2006) shows in an open-economy context that *labor market regulation* can affect comparative advantage in industries depending on their rate of ISTC, skewing industrial composition towards industries that use capital types with low values of g_i (an effect termed *high-tech aversion*). Also, Ilyina and Samaniego (2012) show that

[22] We define capital as the value added of the industries that produce capital goods (ISIC codes 331,332,381,382,383,384,385 and 390) drawn from the UNIDO database, plus the share of construction drawn from the ILO database, in the initial year (1991).

[23] The correlations of the structures share with $\log g_q$ as measured in the PWT 9.1 is 0.07, also not significant.

when technology adoption requires external financing, *financial underdevelopment* also skews industrial composition towards low-tech industries. This begs the question as to whether any policy or institutional indicators might be statistically related to our findings. Of course, there is a question of reverse causality: political economy considerations imply that countries that depend on technological transfer rather than *de novo* innovation for growth might adopt particular kinds of institutions, see for example Boldrin and Levine (2004). Given this, we briefly explore whether there is suggestive evidence of a link between $\log g_q$ in the data and institutions, without taking a stand on the direction of causality.

Following Samaniego (2006) we look at firing costs (drawn from the World Bank, firing costs paid by workers with at least one year's tenure, FC). We also look at other forms of regulation that have been found to be important for aggregate outcomes – namely product market regulation, measured using entry costs paid as a share of GDP, EC, as reported by the World Bank. See Moscoso-Boedo and Mukoyama (2012). Another possibility suggested by Ilyina and Samaniego (2012) is financial development, which we measure using FD, the credit-to-GDP ratio, as in King and Levine (1993). Data on FC, EC and FD are from the World Bank $1960 - 2010$.

In addition, Acemoglu and Johnson (2005) and others argue that financial development is ultimately derived from the state of contracting institutions and property rights institutions. We measure the strength of contracting institutions using the negative of the index of legal system formality from Djankov et al. (2003), which we call $CONT$. We measure property rights enforcement using the index developed by the Property Rights Alliance (2008), $PROP$, averaged over the available period $2007 - 2013$. Finally, we also look at *intellectual* property rights, which have been related to the generation and diffusion of technology, see Samaniego (2013) for a survey. We measure intellectual property rights IPR, using the patent enforcement method developed in Ginarte and Park (1997), as reported by the World Bank, averaging over the available sample. Ilyina and Samaniego (2011) find that copyright enforcement specifically is a form of IPR enforcement that bears the strongest relationship to financial development – see also Samaniego (2013). The BSA (Software

Alliance) publishes the rate at which unlicensed software is used in different countries. Following the Property Rights Alliance (2008), we take this measure (times -1) as an indicator of copyright enforcement. Finally, we also look at human capital, HC, using the standard Barro and Lee (2010) schooling-based measure averaged over the period. While this is not an institutional measure as such it is an important country characteristic which could be related to the need or ability to produce or import high-tech capital goods.

We compare these institutional measures to three measures of $\log g_q$:

1. the value of $\log g_q$ from the data;

2. the value of $\log g_q$ generated by the baseline calibration of the model, based solely on economic composition and productivity growth differences; and

3. the difference between the data value and the model value. We interpret this as the part of $\log g_q$ in the data that is not accounted for by economic composition, at least at the level of aggregation in our data.

See Table 4 for the results. Several institutional measures stand out, as hypothesized, including those related to property rights enforcement and financial development as well as human capital depth. We conclude that a variety of institutional failures appear statistically related to g_q. What is more interesting is that the same is true of the value of $\log g_q$ generated by the model solely on the basis of economic composition, but not the difference between data and model (for the most part). This suggests that, if these institutional indicators are related to $\log g_q$, it is primarily through their impact on economic composition, rather than through other channels, as hypothesized. This suggests that it would be interesting in future work to study the ways in which policies and institutions may affect macroeconomic behavior specifically through the channel of economic composition.

B Comments on Trade

The model abstracts from international trade. Eaton and Kortum (2001) find that machinery is often imported by developing countries, which might suggest that the

Table 4: Correlations between ISTC and different institutional indicators.

		Dep var.		
	Indicator	1. $\log g_q$ (data)	2. $\log g_q$ (model)	3. Diff. b/w model and data
Property rights	$PROP$	0.50^{**}	0.46^{**}	0.38^{**}
	IPR	0.40^{**}	0.55^{**}	0.11
	$COPY$	0.38^{**}	0.64^{**}	-0.10
Financial dev.	$CONT$	0.30^{**}	0.48^{**}	0.08
	FD	0.26^{**}	0.45^{**}	0.07
Market frictions	EC	-0.16	-0.43^{**}	0.04
	FC	-0.30^{**}	-0.31^{**}	-0.18
Trade costs	TC	-0.00	-0.53^{***}	0.26
	TCg	-0.11	-0.29	0.02
Other	HC	0.39^{**}	0.63^{**}	0.12

price of capital could be significantly affected by trade rather than by domestic output, and that domestic output shares might not be indicative of the composition of capital. However their data is for 1985, so it is not clear that their findings are relevant for the relative price of capital in more recent data. Indeed, using data for 1995, Caselli and Wilson (2004) find that the composition of imported machinery is very highly correlated with the composition of domestically-produced capital, both in developed and developing countries. Nonetheless, it would certainly be interesting to explore the extent to which trends in the price of capital might be affected either by trade flows or by changes in trade costs. In particular, Mutreja et al. (2018) argue that reductions in trade costs may lower the relative price of capital by allowing countries to more easily access capital from countries that might produce them more efficiently. This suggests that one factor that might contribute to the dispersion in investment rates could be trade costs.

To check this, we measure trade costs using the ESCAP World Bank International Trade Costs dataset. We compute the weighted average trade cost (TC) for each destination country using the shares of imports from origin countries with UNCTAD-Trade Analysis Information System (TRAINS) import dataset over the available years $1995-2014$. However, Table 4 does not indicate that $\log g_q$ is clearly related to the level of trade costs.

One might ask whether *trends* in trade costs are behind our findings. If capital is largely imported in developing countries and trade costs decline, this could lead to a higher value of g_q as this declining trade cost might disproportionately lower the price of capital. However declining trade costs cannot be responsible for secular increases in g_q according to a typical trade model. For example, in the framework of Eaton and Kortum (2001), trade costs are defined relative to the efficiency of intra-country trade, so they are bounded below by unity. Thus, trends in trade costs can only be *detrimental* in the long run, i.e. they should apply to countries with secular *increases* in trade costs. Increasing trade costs could then be responsible for *low* values of g_q, but considering that trade costs have largely been in decline over the period of interest this seems an unlikely explanation.[24] In any case, Table 4 does not suggest any clear link between g_q and levels nor growth rates of trade costs (TC_g). Thus, changes in trade costs are unlikely to account for the trends in the relative price of capital. This is consistent with Święcki (2017), who compares several determinants of structural change, finding that sector productivity growth rates are an important mechanism – whereas trade costs are important only in selected economies, not across the board.

Interestingly, high levels of trade costs are negatively related to $\log g_q$ as measured in the *model*. Since differences in model-generated $\log g_q$ are driven solely by differences in composition, this suggests that one channel through which trade costs *might* affect $\log g_q$ is through composition itself. We leave this question for future work.

VII Concluding remarks

We document extensive differences in the rate of change in the relative price of capital around the world. We then show that these differences can be accounted for on the basis of differences around the world in economic composition, without recourse to any barriers or frictions. We also find that a general equilibrium model economy accounts for a significant portion of the variation in the rate of change

[24] The World Trade Organization was established on January 1, 1995.

in the relative price of capital and for differences in investment rates around the world, although not for differences in rates of economic growth. We conclude that these differences can be given a technological interpretation, based on differences in composition among industries with different rates of technical progress. As a result, the term "investment-specific technical progress," which the literature widely identifies with declines in the relative price of capital, is appropriate. Given the key role played by industry composition in this phenomenon it seems important to understand what are the deep determinants of industry composition. Is it due to comparative advantage or other trade-theoretic mechanisms? Is due to policy distortions, as suggested by Samaniego (2006) or Ilyina and Samaniego (2012)? Or does it result form hysteresis, for example, based on the date at which the process of development began in earnest and the speed of transition, as in Ngai (2004)? These are likely useful questions for further research.

VIII References

Abramovitz, Moses. 1962. Economic Growth in the United States: A Review Article. American Economic Review 52, 762-82.

Acemoglu, Daron and Simon Johnson. 2005. Unbundling Institutions. Journal of Political Economy 113(5), pp. 949-995.

Baqaee, David Rezza and Emmanuel Farhi. 2019. Productivity and Misallocation in General Equilibrium. NBER Working Paper No. 24007.

Bartelsman, Eric J. and Wayne Gray. 1996. The NBER Manufacturing Productivity Database. NBER Technical Working Paper No. 205.

Baumol, William J. 1967. Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis. American Economic Review 57 (3): 415–26.

Cummins, Jason G. and Giovanni L. Violante. 2002. Investment-Specific Technical Change in the US (1947-2000): Measurement and Macroeconomic Consequences. Review of Economic Dynamics 5(2), 243-284.

Denison, Edward F. 1964. The Unimportance of the Embodied Question. American Economic Review, 54(2), 90-93.

Djankov, Simeon, Rafael LaPorta, Florencio Lopez-de-Silanes, and Andrei Shleifer. 2003. Courts. Quarterly Journal of Economics 118 (2): 453-517.

Duarte, Margarida and Diego Restuccia. Forthcoming. Relative Prices and Sectoral Productivity. Journal of the European Economic Association.

Feenstra, Robert C., Robert Inklaar and Marcel P. Timmer. 2015. The Next Generation of the Penn World Table. American Economic Review, 105(10), 3150-3182, available for download at www.ggdc.net/pwt

Ginarte, Juan Carlos, and Walter G. Park. 1997. Determinants of Patent Rights: A Cross-National Study. Research Policy,26, 283–301.

Gordon, Robert. 1990. The Measurement of Durable Goods Prices. University of Chicago Press.

Greenwood, Jeremy and Boyan Jovanovic. 2001. Accounting for Growth. In "New Developments in Productivity Analysis," edited by Charles R. Hulten, Edwin R. Dean and Michael J. Harper. Chicago: University of Chicago Press (for NBER), 179-222.

Greenwood, Jeremy, Hercowitz, Zvi and Krusell, Per, 1997. Long-Run Implications of Investment-Specific Technological Change. American Economic Review 87(3), 342-62.

Herrendorf, Berthold, Richard Rogerson and Ákos Valentinyi. 2013. Two Perspectives on Preferences and Structural Transformation. American Economic Review 103(7), 2752-89.

Herrendorf, Berthold, Richard Rogerson and Ákos Valentinyi. 2014. Growth and Structural Transformation. In Philippe Aghion and Steven N. Durlauf, Handbook of Economic Growth Volume 2, 855–941.

Heston, Alan, Robert Summers and Bettina Aten. 2012. Penn World Table Version 7.1, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania.

Hsieh, Chang-Tai and Peter Klenow. 2007. Relative Prices and Relative Prosperity. American Economic Review 97, 562-585.

Hulten, Charles R. 1992. Growth Accounting When Technical Change is Embodied in Capital. American Economic Review 82(4), 964-980.

Ilyina, Anna O. and Roberto M. Samaniego. 2011. Technology and Financial Development. Journal of Money, Credit and Banking 43(5), 899-921.

Ilyina, Anna O. and Roberto M. Samaniego. 2012. Structural Change and Financing Constraints. Journal of Monetary Economics 59(2), 166-179.

King, Robert G., and Ross Levine. 1993. Finance and Growth: Schumpeter Might be Right. Quarterly Journal of Economics 108, 717–37.

Kuznets, Simon. 1966. Modern Economic Growth: Rate, Structure, and Spread. New Haven: Yale University Press.

Moscoso-Boedo, Hernan J. and Toshihiko Mukoyama. 2012. Evaluating the effects of entry regulations and firing costs on international income differences. Journal of Economic Growth 17(2), 143–170.

Mutreja, Piyusha, B. Ravikumar and Michael Sposi. 2018. Capital Goods Trade, Relative Prices, and Economic Development. Review of Economic Dynamics 27, 101-122.

Ngai, L. Rachel. 2004. Barriers and the Transition to Modern Growth. Journal of Monetary Economics 51, 1353–1383.

Ngai, L. Rachel and Christopher A. Pissarides. 2007. Structural Change in a Multisector Model of Growth. American Economic Review 97(1), 429-443.

Oulton, Nicholas. 2007. Investment-specific technological change and growth accounting. Journal of Monetary Economics 54(4), 1290-1299.

Pinkovskiy, Maxim and Xavier Sala-i-Martin. 2016. Newer Need Not Be Better: Evaluating the Penn World Tables and the World Development Indicators Using Nighttime Lights. New York Fed Stall Report 778.

Property Rights Alliance. 2008. International Property Rights Index Report. Washington,DC: Property Rights Alliance.

Restuccia, Diego and Carlos Urrutia. 2001. Relative prices and investment rates. Journal of Monetary Economics 47(1), 93-121.

Samaniego, Roberto M. 2006. Employment Protection and High-Tech Aversion. Review of Economic Dynamics 9(2), 224-241.

Samaniego, Roberto M. 2013. Knowledge Spillovers and Intellectual Property Rights. International Journal of Industrial Organization 31(1), 50-63.

Samaniego, Roberto M. and Juliana Sun. 2016. Productivity Differences and Structural Transformation. Review of Economic Dynamics 21, 266–285.

Solow, Robert M. 1962. Technical Progress, Capital Formation, and Economic Growth. American Economic Review 52(2) Papers and Proceedings of the Seventy-Fourth Annual Meeting of the American Economic Association, 76-86.

Święcki, Tomasz. 2017. Determinants of Structural Change. Review of Economic Dynamics 24, 95-131.

Whelan, Karl. 2003. A Two-Sector Approach to Modeling U.S. NIPA Data. Journal of Money, Credit and Banking, 35(4), 627-56.

A Data on industry productivity

Table 5 and Table 6 report the values of g_i used in the paper.

B On the use of PWT 7.1 and PWT 9.1

For the purposes of this paper, it is important to underline that the data in the latest Penn World Tables version 9.1 (Feenstra et al. (2015)) are *not* suitable for measuring changes over time in the relative price of capital. In versions of the PWT prior to 8.0, the database has one benchmark year for which goods prices were measured in a comparable way across countries to establish purchasing power parity (PPP), and data for other years were extrapolated using price indices reported in the national accounts. Thus, in the PWT 7.1, the *change over time* in the price level relative to the price index of new capital is exactly our notion of g_q: we are interested in the *growth* of relative prices, not in *levels* at any particular date. Furthermore, this procedure implies that the price of capital measured using the PWT 7.1 reflects differences in the composition of capital that might exist across countries. In contrast, PWT versions 8.0 and above use several benchmark years for the price data, which renders them unsuitable for our purposes. In benchmark years, a set of comparable products is priced in each country, and the geometric mean is taken between the

Table 5: NBER TFP Growth Factors for the ISIC revision 2 industry classification. Source: NBER productivity database and author's calculations.

Industry	ISIC code	NBER TFP Growth factor
Food products	311	1.002
Beverages	313	1.017
Tobacco	314	0.987
Textiles	321	1.005
Apparel	322	0.993
Leather	323	1.031
Footwear	324	1.001
Wood products	331	0.997
Furniture, except metal	332	1.009
Paper and products	341	0.999
Printing and publishing	342	0.984
Industrial chemicals	351	1.021
Other chemicals	352	1.006
Petroleum refineries	353	0.998
Misc. pet. and coal products	354	0.993
Rubber products	355	1.018
Plastic products	356	1.008
Pottery, china, earthenware	361	1.001
Glass and products	362	1.012
Other non-metallic mineral prod.	369	0.996
Iron and steel	371	1.015
Non-ferrous metals	372	1.009
Fabricated metal products	381	1.007
Machinery, except electrical	382	1.037
Machinery, electric	383	1.053
Transport equipment	384	1.005
Prof. & sci. equip.	385	0.998
Other manufactured prod.	390	0.998

Table 6: Sector TFP Growth Factors. Source:BEA database and author's calculations.

ILO 1-Digit Classification	TFP Growth factor
Agriculture; forestry and fishing	1.0050
Mining and quarrying	0.9717
Manufacturing	1.0119
Utilities	1.0003
Construction	0.9952
Wholesale and retail trade; repair of motor	1.0125
Transport; storage and communication	1.0160
Accommodation and food service activities	1.0014
Financial and insurance activities	1.0066
Real estate; business and administrative	1.0037
Public administration and defence	0.9965
Education	0.9960
Human health and social work activities	0.9995
Other services	1.0003

PPPs measured using weights based on expenditure shares in *comparable* countries in order to compute the exchange rate that would make a basket of such products *have equal value* in different countries.[25] Thus, rather than measuring the price of goods in each country at any date, the benchmark prices for any given country are actually measured using expenditure shares on various goods in *other* countries, and the sampling method focuses on goods that are comparable across countries, instead of being representative of goods purchases in any given country. As a result, the price indices in versions 8.0 and above are unsuitable for our purposes of measuring g_q in a manner that reflects, among other factors, compositional differences across countries.

The PWT version 9.1 do come with a separate database (the National Accounts data) which do have only one benchmark. Unlike the PWT 7.1, however, they do not contain measures of the price of capital. It is possible in principle to create a proxy for the price of capital by means of the following procedure:

1. compute an implicit deflator p_{ct} for consumption[26] by dividing local currency consumption expenditure by the "real" measure: $p_{ct} = v\_c \div q\_c$;
2. compute an implicit deflator p_{it} for investment by dividing local currency investment expenditure by the "real" measure: $p_{it} = v\_i \div q\_i$;
3. compute $q_t = p_{ct}/p_{it}$.

A concern with this measure is that it is indirect, so errors in any of the four component series could compound. In addition, the PWT 9.1 use 2011 as a base year. According to National Accounting guidelines,[27] a base year "should represent normal operation of the economy – it should be a year without major shocks or distortions." The year 2011 is close to the Great Recession of 2009 and the subsequent global financial crisis, suggesting it is one in which many countries were likely subject to

[25] See the ICP 2003-2006 Handbook from the World Bank for a detailed discussion.

[26] Alternatively the price index for GDP could be used: results are the same.

[27] See https://datahelpdesk.worldbank.org/knowledgebase/articles/680284-why-do-countries-revise-their-national-accounts
Last checked 7/30/2019.

price or other distortions. In contrast, the PWT 7.1 use 2005 as a base year, arguably a more stable period.

It is less clear whether this is a concern for measuring aggregates such as GDP or investment rates in the PWT 9.1, since the use of several benchmarks should deal with this issue. As a result, we do measure economic aggregates using the PWT 9.1. We draw values on levels of GDP per person, rates of growth in GDP per person and investment rates from this database. However, Pinkovskiy and Sala-i-Martin (2016) argue that the PWT 7.1 real GDP data concur more closely with economic activity as measured using nightime lights than more recent editions – although the 9.1 version was not available at the time its construction is generaly similar to the 8.1 version, particularly in that GDP values are not chain-weighted. Thus, we report results using aggregates measured using the PWT 7.1 as well as the PWT 9.1.

C Quantitative results with PWT 9.1 data

We now repeat the above exercises comparing the key model-generated statistics to the same data generated by the PWT 9.1.

First, we compute $\log g_q$ using the national accounts data reported in the PWT 9.1. We find that the mean and median values are now larger than one, and that the share of countries reporting sub-zero values of $\log g_q$ is now a little larger. The standard deviation is larger than that of the measure computed using PWT 7.1 data. Thus, this measure of g_q tends to be a bit higher, and has even more dispersion than the data we used earlier as a benchmark. See Table 7. The correlation with the earlier measure is 0.42^{***}, significant at the 1 percent level.

The correlation between $\log g_q$ and the initial level of q in the data is -0.69^{***} and significant at the 1 percent level. Thus, these data also indicate a convergence over time in levels of q. Again, we check whether $\log g_q$ is related to levels of economic development as well. We find that the correlation between $\log g_q$ and log GDP per capita over the period is 0.13^{*}, significant at the 10 percent level. The correlation between $\log g_q$ and GDP growth rates, on the other hand, is 0.03 and not significant. This is similar to what we found earlier with PWT 7.1 data.

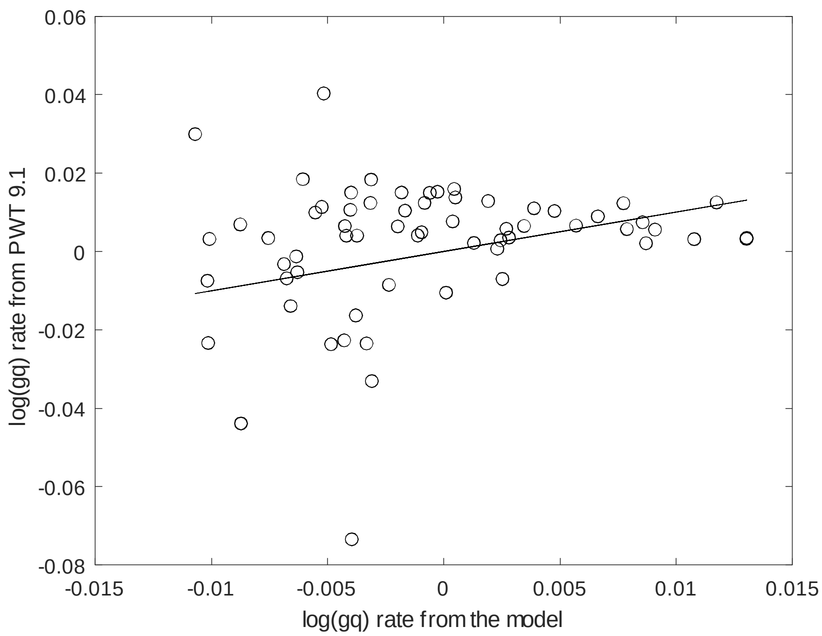
Table 7: Summary information for the change in the relative price of capital, PWT 9.1.

Statistic	$\log g_q$
Median	0.0010
Mean	0.0007
s.d.	0.0226
Share below zero	42.0%

When we compare the PWT 9.1 version of $\log g_q$ to that generated by the model, the findings are much weaker than before. The correlation is 0.22^{*}, positive as before, but significant at the 10 percent level. The regession coefficient on the model series is 0.61^{*}. This value is statistically different from one, and is statistically different from zero at the 10 percent level. Thus, while the model accounts well for $\log g_q$ as measured in the PWT 7.1, there is more noise in its relationship with the PWT 9.1

measure. See Figure 12.

Figure 12 – Correlation of $\log g_q$ in the PWT9.1 database and the model-generated values of $\log g_q$.

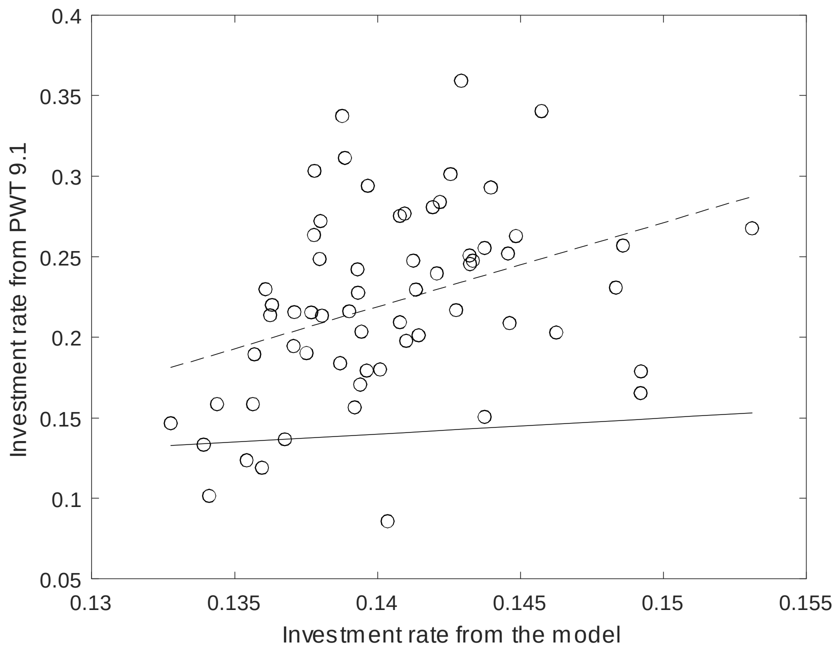


Notes: Model values are computed using simulated changes in economic structure $1991 - 2011$ assuming that all elasticities ε and ε_s are set to unity, so there is no structural change in the model.

The line is a regression line obtained using OLS.

Regarding investment rates as measured in the PWT 9.1, Figure 13 shows that model-generated investment rates remain strongly correlated with those in the data. The correlation is 0.38^{***} and significant at the 1 percent level, and the regression coefficient is about 5.1^{***}, slightly higher than with the PWT 7.1 data. The fact that the regression coefficient is larger than 1 implies that the model variation in investment rates is less than empirical variation in investment rates, as before. However, the intercept of -0.51^{**} is also significant, albeit at the 5 percent level. Thus, the model accounts for the correlation well, although not the magnitudes of investment rates as measured in the PWT 9.1.

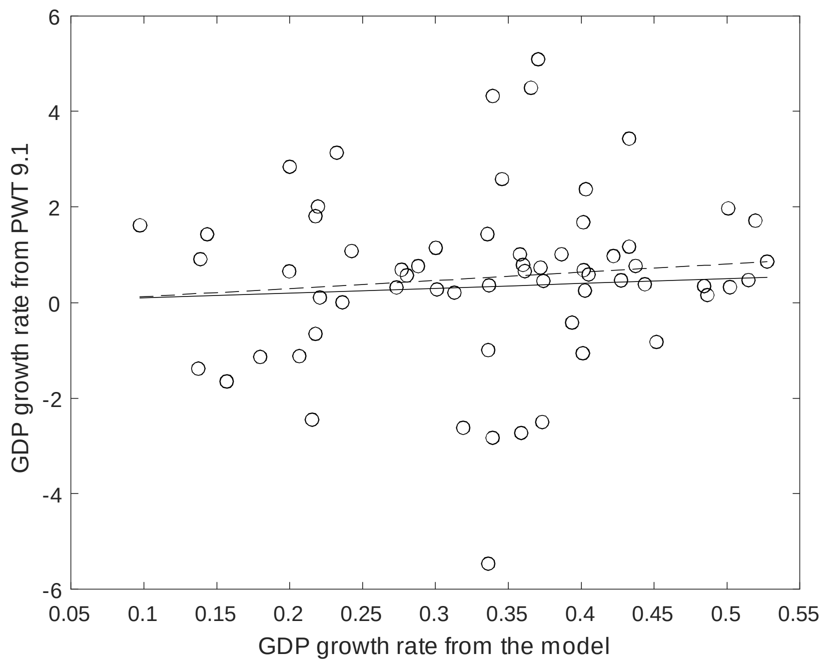
Figure 13 –Correlation of investment rates from the PWT 9.1 database and the model-generated investment rates.



Notes: Model values are computed using simulated changes in economic structure 1991 – 2011, using the benchmark calibration. The line is a regression line obtained using OLS.

Finally, we return to the benchmark model and compare model generated GDP growth rates with those from the data. Figure 15 shows that the correlation between model and data is not statistically significant. We also find that the values tend to be too small. For example, the US growth rate in the model is around 1 percent, whereas it is 1.5 percent in the data. See Figure 14.

Figure 14 – Correlation of aggregate growth rates in the PWT9.1 database and the model-generated values of GDP growth.

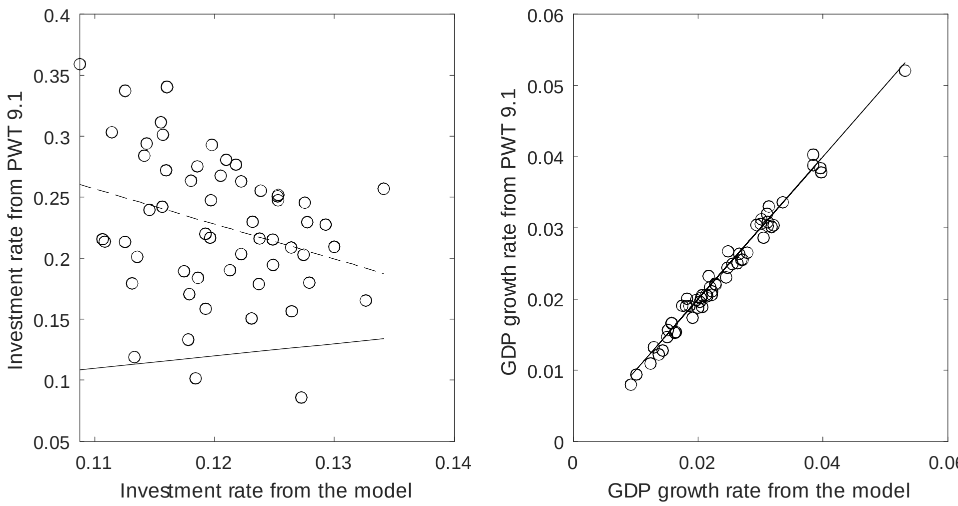


Notes: Model values are computed using simulated changes in economic structure 1991 − 2011, using the benchmark calibration.

The line is a regression line obtained using OLS.

To deal with the question of overall magnitudes, we repeated the exercise multiplying all industry and sector TFP growth rates by a factor g_z to ensure that we exactly match US GDP growth rates over the period. However, the variation in growth rates remains too small, and the correlation is not significant either. Regarding investment, the correlation between investment rates in model and data in this calibration remains high at 0.41***, but the regression coefficient is too large and the intercept is still significant as before. Finally, when we set g_z separately for each country so as to match its growth rate in the data, it throws off the link between model and emprical investment rates, as before. See Figure 15.

Figure 15 – Comparison between investment rates and economic growth in the PWT 7.1 compared to their model-generated counterparts.



Notes: Model values use simulated changes in economic structure 1991 − 2011. In this experiment, productivity growth rates in each country were multiplied by a country- specific factor g_z, selected so as to closely match GDP growth rates in each country. The lines are regressions line obtained using OLS.

D Computing equilibrium

A Investment patterns and aggregate growth

It follows from the Euler equations of the household's problem that:

Proposition 2 *In equilibrium, the growth factors of total capital* K_t,*and total output* y_t *depend on the growth factors of TFP in the consumption and capital sectors and* *on the growth factor of consumption sector (as well as parameters):*

$$g_{K_t} = \frac{K_{t+1}}{K_t} = \overline{g}_{A_{St}}^{\frac{1}{1-\alpha}} \left(\frac{r_t}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} \tag{27}$$

and

$$g_{yt} = \frac{y_{t+1}}{y_t} = \overline{g}_{A_{St}}^{\frac{1}{1-\alpha}} \left(\frac{r_t}{r_{t+1}}\right)^{\frac{\alpha}{1-\alpha}} \tag{28}$$

where $\overline{g}_{A_{St}} = \frac{A_{St+1}}{A_{St}}$*, GDP is defined as* $y_t = y_{St} + p_{ct}c_t$ *and the equilibrium interest rate is*

$$r_t = \frac{g_{qt-1}^{1-\theta} g_{ct-1}^{\theta}}{\beta} - 1 + \delta \text{ for } t > 0. \tag{29}$$

At date zero, r_0 *is determined by market clearing given* K_0*.*

Furthermore, from equation (15) it follows that:

Proposition 3 *The model economy converges to a balanced growth path where in each sector*

$$\lim_{t\to\infty} A_{st} = A_{jt} \text{ where } j = \begin{cases} \arg\max_{i\in I_s} \{g_i\} & \text{if } \varepsilon_s > 1 \\ \arg\min_{i\in I_s} \{g_i\} & \text{if } \varepsilon_s < 1 \end{cases},$$

and in the consumption goods sector

$$\lim_{t\to\infty} A_{ct} = A_{st} \text{ where } s = \begin{cases} \arg\max_{s<S} \{g_s\} & \text{if } \varepsilon > 1 \\ \arg\min_{s<S} \{g_s\} & \text{if } \varepsilon < 1 \end{cases}.$$

If $\varepsilon_s = 1$ *in any sector, then* A_{st} *is constant over time.*

Proposition 3 predicts that for each sector, the sector productivity will converge to the productivity of industry with highest (lowest) TFP growth rate if the sector elasticity of substitution > (<) 1. The aggregate TFP of the consumption goods sector will converge to the productivity of the sector with highest (lowest) TFP growth rate if the elasticity of substitution of the consumption sector is > (<) 1.

Recalling that the only endogenous variable that affects r_t for $t > 0$ is g_{ct},[28] Proposition 2 implies that we can compute the equilibrium for the multi-industry model economy in transition, provided we can derive the series for g_{ct}. The economy with many consumption goods sectors will asymptotically converge to an economy with one consumption sector which has either the highest or lowest TFP growth rate depending on the elasticity of substitution. The same occurs within the capital goods sector. As a result, the expression r_t converges to some constant r and, although in general the model does not possess a balanced growth path (see Ngai and Pissarides (2007)), it converges to one. This suggests that the equilibrium may be computed by finding a sufficiently good *approximation* to the series for g_{ct}. In the limit, if $\varepsilon_s \neq 1$ for some sector s, one industry will end up dominating that sector. However, we wish to study the behavior of the model economy in transition, where sectors are relatively diversified. Again, this is not necessary for computing a series for g_{qt} for different countries. However it is necessary for computing series for investment rates and for GDP.

B A computational algorithm

Equilibrium investment is given by the expression

$$y_{St} = (\alpha A_{St})^{\frac{1}{1-\alpha}} \left[\left(\frac{\overline{g}_{A_{St}}}{r_{t+1}} \right)^{\frac{1}{1-\alpha}} - (1-\delta) \left(\frac{1}{r_t} \right)^{\frac{1}{1-\alpha}} \right]. \tag{30}$$

In turn, equation 15 implies that aggregate output is given by

$$\begin{aligned} y_t &= y_{St} + p_{ct} c_t \\ &= A_{St} k_t^{\frac{\alpha}{1-\alpha}} = \left(\frac{\alpha}{r_t} \right)^{\frac{\alpha}{1-\alpha}} A_{St}^{\frac{1}{1-\alpha}}. \end{aligned} \tag{31}$$

[28] In general, at $t = 0$, the value of r_0 is determined by market clearing and the value of K_0.

As a result, the equilibrium investment rate is

$$\frac{y_{St}}{y_t} = \frac{(\alpha A_{S_t})^{\frac{1}{1-\alpha}}\left[\left(\frac{\bar{g}_{A_{St}}}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\left(\frac{1}{r_t}\right)^{\frac{1}{1-\alpha}}\right]}{\left(\frac{\alpha}{r_t}\right)^{\frac{\alpha}{1-\alpha}} A_{St}^{\frac{1}{1-\alpha}}} = \frac{\alpha}{r_t}\left[\left(\frac{\bar{g}_{A_{St}} r_t}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\right] \tag{32}$$

Since in equilibrium r_t is a function of g_q, the investment rate is also linked to g_q, and thus to economic composition.

All of these expressions are straightforward to evaluate in the limit as $\bar{g}_{A_{St}}$ and r_t converge to constants. However, Ngai and Pissarides (2007) show that for generic parameters this will not be the case in transition. As a result, we require a computational procedure for computing investment rates and output levels in transition. However, the above Propositions essentially tell us how to do this.

First, we choose a date T far into the future and assume long run relationships hold from then on. This should be very far out (say $T = 1000$) so that the model behavior will be indistinguishable from long-run behavior for a while before T – i.e. the expressions in Proposition 3 are assumed to hold for $t \geq T$, and T is so large that behavior for t not far below T will likely be indistinguishable. Then, for dates $t \in \{0, ..., T-1\}$ we can compute A_{St} and A_{ct} using Proposition 1, given initial conditions for A_{i0} in all industries i. Then, we can make a guess for the series g_{ct} – a guess that must converge to the long-run value at date T. This generates a series for r_t, which can be used to compute investment and GDP growth rates, using the equations in Proposition 2. It can also be used to compute a new series for g_{ct}, one implied by the model behavior, which can then be used to generate a new candidate series for r_t, and the process is repeated until the series for g_{ct} used to generate a series for r_t equals the series generated by model behavior.

E Proofs

In this section we outline some of the derivations required for the Propositions in the paper.

Proof of proposition 2. Solving the 2 sector problem and using the equilibrium conditions, we have:

$$A_{St} = p_{ct}A_{ct} \tag{33}$$

$$r_t = \frac{\frac{p_{ct}c_t^{\theta}}{p_{c,t-1}c_{t-1}^{\theta}}}{\beta} - 1 + \delta = \frac{\left(\frac{\overline{g}_{A_{S,t-1}}}{\overline{g}_{A_{c,t-1}}}\right)^{1-\theta} g_{ct-1}^{\theta}}{\beta} - 1 + \delta \text{ if } \beta \neq 0$$

where $g_{c,t-1} \equiv \frac{p_{ct}c_t}{p_{c,t-1}c_{t-1}}$ is the growth factor of aggregate consumption and $\overline{g}_{A_{c,t-1}} = \frac{A_{ct}}{A_{c,t-1}}, \overline{g}_{A_{S,t-1}} = \frac{A_{St}}{A_{S,t-1}}$ are known

Let $\phi_t = \alpha^{-1} r_t^{\frac{-\alpha}{1-\alpha}} - \overline{g}_{A_{St}}^{\frac{1}{1-\alpha}} r_{t+1}^{\frac{-1}{1-\alpha}} + (1-\delta)\, r_t^{\frac{-1}{1-\alpha}}$

$$\begin{aligned} k_t &= \frac{K_{St}}{n_{St}} = \frac{K_{ct}}{n_{ct}} = \left(\frac{\alpha A_{S_t}}{r_t}\right)^{\frac{1}{1-\alpha}} \\ K_t &= k_t \end{aligned}$$

The growth factor of capital per capita in each sector is:

$$g_{kt} = \frac{k_{t+1}}{k_t} = \overline{g}_{A_{St}}^{\frac{1}{1-\alpha}} \left(\frac{r_t}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} \tag{34}$$

Using (33) and (20), we derive capital sector output, i.e., investment:

$$\begin{aligned} y_{St} &= \left(\frac{\alpha A_{S,t+1}}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\left(\frac{\alpha A_{S_t}}{r_t}\right)^{\frac{1}{1-\alpha}} \\ &= (\alpha A_{S_t})^{\frac{1}{1-\alpha}} \left[\left(\frac{\overline{g}_{A_{St}}}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\left(\frac{1}{r_t}\right)^{\frac{1}{1-\alpha}}\right] \end{aligned} \tag{35}$$

and the growth factor of investment y_{St} becomes:

$$g_{St} = \frac{y_{S,t+1}}{y_{St}} = \overline{g}_{A_{St}}^{\frac{1}{1-\alpha}} \frac{\left(\frac{\overline{g}_{A_{S,t+1}}}{r_{t+2}}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\left(\frac{1}{r_{t+1}}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{\overline{g}_{A_{St}}}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\left(\frac{1}{r_t}\right)^{\frac{1}{1-\alpha}}}$$

so that the labor in capital sector is:

$$n_{St} = \alpha \left[\frac{1}{r_t} \left(\frac{\overline{g}_{A_{St}} r_t}{r_{t+1}} \right)^{\frac{1}{1-\alpha}} - \frac{(1-\delta)}{r_t} \right] \tag{36}$$

and the growth factor of n_{St} is:

$$g_{n_{St}} = \frac{n_{S,t+1}}{n_{St}} = \frac{\frac{1}{r_{t+1}} \left(\frac{\overline{g}_{A_{S,t+1}} r_{t+1}}{r_{t+2}} \right)^{\frac{1}{1-\alpha}} - \frac{(1-\delta)}{r_{t+1}}}{\frac{1}{r_t} \left(\frac{\overline{g}_{A_{St}} r_t}{r_{t+1}} \right)^{\frac{1}{1-\alpha}} - \frac{(1-\delta)}{r_t}} \tag{37}$$

Notice that n_{St} (and hence $n_{ct} = 1 - n_{St}$) is independent of the level of technology in c and S as long as the interest rate is too. We obtain capital in capital sector:

$$K_{St} = \alpha \left[\frac{1}{r_t} \left(\frac{\overline{g}_{A_{St}} r_t}{r_{t+1}} \right)^{\frac{1}{1-\alpha}} - \frac{(1-\delta)}{r_{t-1}} \right] \left(\frac{\alpha A_{S_t}}{r_t} \right)^{\frac{1}{1-\alpha}} \tag{38}$$

Define the aggregate output per capita as $y_t = y_{St} + p_{ct}c_t$. Since $K_{ct} = K_t - K_{St}$ and $n_{ct} = 1 - n_{St}$,

$$\begin{aligned} y_t &= y_{St} + p_{ct}c_t \\ &= A_{St}K_{St}^{\alpha}n_{St}^{1-\alpha} + p_{ct}A_{ct}K_{ct}^{\alpha}n_{ct}^{1-\alpha} \\ &= A_{St}k_t^{\frac{\alpha}{1-\alpha}} = \left(\frac{\alpha}{r_t}\right)^{\frac{\alpha}{1-\alpha}} A_{St}^{\frac{1}{1-\alpha}} \end{aligned} \tag{39}$$

and its growth factor is:

$$g_{yt} = \frac{y_{t+1}}{y_t} = \overline{g}_{A_{St}}^{\frac{1}{1-\alpha}} \left(\frac{r_t}{r_{t+1}} \right)^{\frac{\alpha}{1-\alpha}} \tag{40}$$

Aggregate consumption is:

$$C_t = p_{ct}c_t = y_t - y_{St} \tag{41}$$

$$= \left(\frac{\alpha}{r_t}\right)^{\frac{\alpha}{1-\alpha}} A_{St}^{\frac{1}{1-\alpha}} - (\alpha A_{S_t})^{\frac{1}{1-\alpha}} \left[\left(\frac{\overline{g}_{A_{St}}}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\left(\frac{1}{r_t}\right)^{\frac{1}{1-\alpha}}\right] \tag{42}$$

The growth factor of consumption is:

$$g_{Ct} = \frac{C_{t+1}}{C_t} = \overline{g}_{A_{St}}^{\frac{1}{1-\alpha}} \frac{\phi_{t+1}}{\phi_t}. \tag{43}$$

Notice that as $t \to \infty$ the expressions for $\overline{g}_{A_{St}}$ and $\overline{g}_{Act}$ converge to constants. ■

Proof of Proposition 3. Corollary of the proof of Proposition 1 and (10). ■