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Standardized LM tests for spatial error dependence in linear or panel regressions

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Summary The robustness of the Lagrange Multiplier (LM) tests for spatial error dependence of Burridge (1980) and Born and Breitung (2011) for the linear regression model, and Anselin (1988) and Debarsy and Etur (2010) for the panel regression model with random or fixed effects are examined. While all tests are asymptotically robust against distributional mis-specification, their finite sample behaviour may be sensitive to the spatial layout. To overcome this shortcoming, standardized LM tests are suggested. Monte Carlo results show that the new tests possess good finite sample properties. An important observation made throughout this study is that the LM tests for spatial dependence need to be both mean- and variance-adjusted for good finite sample performance to be achieved. The former is, however, often neglected in the literature.

Keywords: *Bootstrap*, *Distributional mis-specification*, *Group interaction*, *LM test*, *Moran's I test*, *Robustness*, *Spatial layout*, *Spatial panel models*.

1. INTRODUCTION

LM tests for spatial error correlation in the linear regression model (Burridge, 1980, and Born and Breitung, 2011) and the panel regression model (Anselin, 1988, Baltagi et al., 2003, and Debarsy and Ertur, 2010) are developed under the assumption that the model errors are normally distributed. This leads to a natural question on how robust these tests are against mis-specification of the error distribution. While these tests are robust asymptotically against distributional mis-specification, as can be inferred from the results of Kelejian and Prucha (2001) for Moran's I test in the linear regression model, and proved in this article for the panel regression model, their finite sample behaviour may not be so; it can also be sensitive to the spatial layout. The main reason, as shown in this paper, is the lack of standardization of these tests, that is, subtracting the mean and dividing by the standard deviation.[1] In particular, when each spatial unit has many

[1] Honda (1985) shows that the LM test for random individual effects in the panel data regression model is uniformly most powerful and is robust against non-normality. Moulton and Randolph (1989) show that this test can perform poorly when the number of regressors is large or the interclass correlation of some of the regressors is high. They suggest a standardized LM test by centring and scaling Honda's LM test. They show that the standardized LM test performs better in small samples when asymptotic critical values from the normal distribution are used. However these papers do not deal with spatial correlation.

neighbours (the number of neighbours grows with the number of spatial units), the mean of these tests can be far below zero even when the sample size is fairly large (e.g. 1000), causing severe size distortion of these tests.

Standardized LM (SLM) tests are recommended, which correct both the mean and variance of the existing LM tests under more relaxed assumptions on the error distributions. It is shown that these LM tests are not only robust against distributional mis-specification, but are also quite robust against changes in the spatial layout. Monte Carlo simulations show that SLM tests have excellent finite sample properties and significantly outperform their non-standardized counterparts. The Monte Carlo simulations also show that SLM tests are comparable to the bootstrap counterparts (when they are available) in terms of size. Once size-adjusted, LM and SLM tests have similar power.

It is well known in the statistics and econometrics literature that standardizing an LM test improves its performance especially if asymptotic critical values are used. Moulton and Randolph (1989) emphasized this for the panel data regression model with random individual effects, see also Honda (1991), Baltagi et al. (1992), and Baltagi (2008). Koenker (1981) showed that the standardization (or studentization in his terminology) leads to a robustified LM test for heteroscedasticity. This point, however, is not emphasized in the spatial econometrics literature, except for Anselin (1988), Kelejian and Prucha (2001), and Florax and de Graaff (2004), where the authors mainly stressed variance correction but not mean correction. Recently, Robinson (2008) proposed a general chi-squared test for non-spherical disturbances, including spatial error dependence (SED), in a linear regression model. He pointed out that this test has an LM interpretation and may not provide a satisfactory approximation in small samples. He then introduced a couple of modifications directly on the chi-squared statistic. Our approach of standardization is more in line with that of Koenker (1981). It works on the 'standard normal' version of an LM test, and thus is simpler. More importantly, our approach allows the errors to be *non-normal* and is not restricted to linear regression models of non-spherical disturbances.

Our Monte Carlo simulation shows that the mean correction as well as variance correction are *both* essential to attain good size and power. Section 2 deals with tests for SED in a linear regression model. Section 3 deals with tests for SED in a panel data regression model with random or fixed space-specific effects. Section 4 presents the Monte Carlo results, while Section 5 concludes the paper. Proofs of all results are given in the Appendix.

2. TESTS FOR SED IN A LINEAR REGRESSION MODEL

This section studies LM-type tests for zero SED in a linear regression model. Moran's (1950) I tests, Burridge's (1980) LM test based on the expected information (EI) and Born and Breitung's (2011) LM test based on the outer product of gradients (OPG) are considered. The standardized versions of these two LM tests are proposed for improving their finite sample performance. A bootstrap version of Burridge's LM test, discussed in Lin et al. (2007), is used as a benchmark for comparisons.

2.1. Moran's I tests

The original form of Moran's I test (Moran, 1950) is based a sample of observations $Y = \{Y_1, Y_2, \ldots, Y_n\}'$ on a variable of interest Y, which takes the form

$$I = \frac{\sum_i \sum_j w_{ij}(Y_i - \bar{Y})(Y_j - \bar{Y})}{\sum_i (Y_i - \bar{Y})^2}, \tag{2.1}$$

where w_{ij} is an element of an $N \times N$ spatial weight matrix W with $w_{ii} = 0$ and $\sum_{j=1}^{N} w_{ij} = 1, i = 1, \ldots, N$, and $\bar{Y}$ is the average of the Y_is. If the observations are normal, then the null distribution of Moran's I test statistic is shown to be asymptotic normal. Cliff and Ord (1972) extended Moran's I test to the case of a spatial linear regression model:

$$Y = X\beta + u, \tag{2.2}$$

where Y is an $N \times 1$ vector of observations on the response variable, X is an $N \times k$ matrix containing the values of explanatory (exogenous) variables and u is an $n \times 1$ vector of disturbances with mean zero and variance σ_u^2. The extended Moran's I test takes the form

$$I = \frac{\tilde{u}' W \tilde{u}}{\tilde{u}'\tilde{u}}, \tag{2.3}$$

where $\tilde{u}$ is a vector of OLS residuals obtained from regressing Y on X. If u is normal, then the distribution of I under the null hypothesis of no SED is asymptotically normal with mean and variance given by:

$$\mathrm{E}(I) = \frac{1}{N-k} \mathrm{tr}(MW),$$

$$\mathrm{Var}(I) = \frac{\mathrm{tr}(MWMW') + \mathrm{tr}((MW)^2) - \frac{2}{N-k}[\mathrm{tr}(MW)]^2}{(N-k)(N-k+2)}.$$

Here $M = I_N - X(X'X)^{-1}X'$ and I_N is an N-dimensional identity matrix. In empirical applications, the test should be carried out based on $I^* = (I - \mathrm{E}\, I)/\mathrm{Var}^{\frac{1}{2}}(I)$, and referred to the standard normal distribution (see Anselin and Bera, 1998). However, most of the literature suggested or hinted at the use of $I^\circ = I/\mathrm{Var}^{\frac{1}{2}}(I)$; see, for example, Anselin (2001), Kelejian and Prucha (2001), and Florax and de Graaff (2004). The reason may be that the mean correction is asymptotically negligible or may be that $I^\circ = I/\mathrm{Var}^{\frac{1}{2}}(I)$ corresponds directly to the Burridge (1980) LM test described later.

2.2. LM and standardized LM tests based on EI

Consider the case where u in (2.2) follows either a spatial autoregressive (SAR) process $u = \lambda W u + \varepsilon$ or a spatial moving average (SMA) process $u = \lambda W \varepsilon + \varepsilon$, where W is defined, λ is the spatial parameter and ε is a vector of independent and identically distributed (iid) normal innovations with mean zero and variance σ_ε^2. The hypothesis of no spatial error correlation can be expressed explicitly as $H_0 : \lambda = 0$ vs $H_a : \lambda \neq 0$. For this model specification, Burridge (1980) derived an LM test for H_0 based on the EI:

$$LM_{EI} = \frac{N}{\sqrt{S_0}} \frac{\tilde{u}' W \tilde{u}}{\tilde{u}'\tilde{u}}, \tag{2.4}$$

where $S_0 = \text{tr}\,(W'W + W^2)$. Under the null hypothesis of no spatial error correlation, $LM_{EI} \xrightarrow{d} N(0, 1)$. LM_{EI} resembles I° except for a scale factor. Our Monte Carlo simulations show that it is important to standardize it if one is using asymptotic critical values, especially for certain spatial layouts. Some discussion on this is given after Theorem 2.1.

The three test statistics (I^*, I° and LM_{EI}) are derived under the assumption that the errors are normally distributed. Theorem 2.1 shows that all three tests behave well asymptotically under non-normality. But how do they behave in finite samples? We first present a modified version of these tests allowing the error distributions to be *non-normal*, and then give some discussion answering why the finite sample performance of I° and LM_{EI} can be poor. The following regularity conditions are necessary for studying the asymptotic behaviour of these test statistics.

ASSUMPTION 2.1. *The innovations $\{\varepsilon_i\}$ are iid with mean zero, variance σ_ε^2 and excess kurtosis κ_ε. Also, the moment $E|\varepsilon_i|^{4+\eta}$ exists for some $\eta > 0$.*

ASSUMPTION 2.2. *The elements $\{w_{ij}\}$ of W are at most of order h_N^{-1} uniformly for all i, j, with the rate sequence $\{h_N\}$, bounded or divergent, satisfying $h_N/N \to 0$ as N goes to infinity. The $N \times N$ matrices $\{W\}$ are uniformly bounded in both row and column sums with $w_{ii} = 0$ and $\sum_j w_{ij} = 1$ for all i.*

ASSUMPTION 2.3. *The elements of the $N \times k$ matrix X are uniformly bounded for all N, and $\lim_{N\to\infty} \frac{1}{N} X'X$ exists and is non-singular.*

Assumptions 2.1–2.3 are essentially the same as those in Yang (2010) for a spatial error components (SEC) model where the disturbance vector u has two independent components with the first being spatially correlated, i.e., $u = Wv + \varepsilon$, in contrast to the SED model considered in this paper where $u = \lambda W u + \varepsilon$ or $u = \lambda W \varepsilon + \varepsilon$.

THEOREM 2.1. *Under Assumptions 2.1–2.3, the standardized LM_{EI} test for testing $H_0 : \lambda = 0$ versus $H_a : \lambda \neq 0$ (or $\lambda < 0$, or $\lambda > 0$) takes the form*

$$SLM_{EI} = \frac{N\tilde{u}'(W - S_1 I_N)\tilde{u}}{(\tilde{\kappa}_\varepsilon S_2 + S_3)^{\frac{1}{2}}\,\tilde{u}'\tilde{u}}, \tag{2.5}$$

where $S_1 = \frac{1}{N-k}\text{tr}\,(WM)$, $S_2 = \sum_{i=1}^N a_{ii}^2$ and $S_3 = \text{tr}\,(AA' + A^2)$, $A = MWM - S_1 M$, a_{ii} are the diagonal elements of A, and $\tilde{\kappa}_\varepsilon$ is the excess sample kurtosis of $\tilde{u}$. Under H_0, we have (a) $SLM_{EI} \xrightarrow{d} N(0, 1)$; (b) the four test statistics, I^*, I°, LM_{EI} and SLM_{EI} are asymptotically equivalent.

To help in understanding the theory, we outline the key steps leading to the standardization given in (2.5). First note that $\tilde{u}'W\tilde{u}$, the key quantity appearing in the numerators of (2.3)–(2.5), is not centred because $\text{E}(\tilde{u}'W\tilde{u}) = \sigma_\varepsilon^2\text{tr}\,(WM) \neq 0$. This motivates us to consider $\tilde{u}'W\tilde{u} - \sigma_\varepsilon^2\text{tr}\,(WM)$, or its feasible version $\tilde{u}'W\tilde{u} - \frac{1}{n-k}(\tilde{u}'\tilde{u})\text{tr}\,(WM) = u'Au$. Upon finding the variance of $u'Au$ and replacing σ_ε^2 in the variance expression by its MLE, we obtain (2.5). Some remarks follow.

The SLM statistic given in Theorem 2.1 has an identical form as that for the SEC model given in Yang (2010). The difference is that in Yang (2010) W is replaced by WW'. As a result, LM_{EI}, and SLM_{EI} are asymptotically equivalent due to the fact that W has zero diagonal elements.

In contrast, the LM and SLM statistics for the SEC model are not asymptotically equivalent in general due to the fact that the diagonal elements of WW' are not zero. See the proofs for the two sets of results for details.

It is important to note that the standardization of Moran's I in earlier work based on $\tilde{u}'W\tilde{u}/\tilde{u}'\tilde{u}$ and its mean and variance are derived under the assumption that $u \sim N(0, \sigma_\varepsilon^2 I_N)$. Robinson's (2008) approach works on LM_{EI}^2 or $(\tilde{u}'W\tilde{u}/\tilde{u}'\tilde{u})^2$. Again, the derivations of the mean and variance depend on the normality assumption. Our approach works on the quadratic form $u'Au$ with its mean and variance readily available as long as the first four moments of the elements of u exist. Thus, our approach is simpler and does not depend on the normality assumption. It is applicable to other models with more complicated structure.

Although both Moran's I and the LM_{EI} test statistics are derived under the assumption that the innovations are normally distributed, Theorem 2.1 shows that they are asymptotically equivalent to the SLM test derived under relaxed conditions on the error distribution. This means that all four tests are robust against distributional mis-specification when the sample size is large. But will the four tests behave similarly under finite samples? The following discussion points out that their finite sample performance may be different.

The major difference between LM_{EI} and SLM_{EI} lies in the mean correction of the statistic $\tilde{u}'W\tilde{u}/\tilde{u}'\tilde{u}$. This correction may quickly become negligible as the sample size increases under certain spatial layouts, but not necessarily under other spatial layouts. From (A.1) in the Appendix, we see that this mean correction factor is of the magnitude

$$\frac{NS_1}{(\tilde{\kappa}_\varepsilon S_2 + S_3)^{\frac{1}{2}}} = O_p\big((h_N/N)^{\frac{1}{2}}\big),$$

which shows that the magnitude of the mean correction depends on the ratio $(h_N/N)^{\frac{1}{2}}$. For example, when $h_N = N^{0.8}$, $(h_N/N)^{\frac{1}{2}} = N^{-0.1}$. Thus, if $N = 20, 100$ and 1000, $N^{-0.1} = 0.74, 0.63$ and 0.50. This shows that the means of LM_{EI} and I° can differ from the means of LM_{EI}^* and I^* by 0.74 when $N = 20$, 0.63 when $N = 100$ and 0.50 when $N = 1000$. Note that situations leading to $h_N = N^{0.8}$ may be the spatial layouts constructed under large group interactions, where the group sizes are large and the number of groups is small.[2] Our results show that in this situation, the non-standardized LM test or Moran's I test without the mean correction may be misleading. Monte Carlo simulations presented in Section 4 confirm these findings.

2.3. LM and standardized LM tests based on OPG

Recently, Born and Breitung (2011) derived an OPG variant of Burridge's LM test based on an elegant idea: decomposing the score into a sum of uncorrelated components making use of the fact that the diagonal elements of the W matrix are zero, so that the variance of the score can be estimated by the OPG method. The test can be expressed simply as follows:

$$LM_{OPG} = \frac{\tilde{u}'W\tilde{u}}{\sqrt{(\tilde{u} \odot \tilde{u})'(\tilde{\xi} \odot \tilde{\xi})}}, \tag{2.6}$$

where $\odot$ denotes the Hadamard product, $\tilde{\xi} = (W_l + W_u')\tilde{u}$, W_l and W_u are the lower and upper triangular matrices such that $W_l + W_u = W$, and $LM_{\mathrm{OPG}}|_{H_0} \xrightarrow{d} N(0, 1)$. An important feature

[2] See Lee (2007) for a detailed discussion of spatial models with group interactions.

of this test is that it is robust against heteroscedasticity of unknown form. However, the test statistic is not centred and thus is expected to suffer from the same problem as Burridge's LM test even when the innovations are homoscedastic.

Combining the idea leading to SLM_{EI} and the idea leading to LM_{OPG}, we obtain a standardized OPG-based LM test. Decompose the matrix A defined in Theorem 2.1 as $A = A_l + A_u + A_d$, where $A_d = diag(A)$, $A_l = tri_l(A) - A_d$ and $A_u = tri_u(A) - A_d$, with $diag(A)$, $tri_l(A)$ and $tri_u(A)$ denoting, respectively, the diagonal, lower triangular and upper triangular matrices of a square matrix A.

THEOREM 2.2. *Under Assumptions 2.1–2.3, the standardized LM_{OPG} test for testing $H_0 : \lambda = 0$ versus $H_a : \lambda \neq 0$ (or $\lambda < 0$, or $\lambda > 0$) takes the form*

$$SLM_{OPG} = \frac{\tilde{u}'(W - S_1 I_N)\tilde{u}}{\sqrt{(\tilde{u} \odot \tilde{u})'[\tilde{\zeta} \odot \tilde{\zeta} + (A_d\tilde{u}) \odot (A_d\tilde{u})]}}, \tag{2.7}$$

where $\tilde{\zeta} = (A_l + A'_u)\tilde{u}$. Under H_0, (a) $SLM_{OPG} \xrightarrow{d} N(0, 1)$, and (b) $SLM_{OPG} \sim LM_{OPG}$.

Like LM_{OPG}, SLM_{OPG} is also asymptotically robust against heteroscedasticity. However, the finite sample mean correction is derived under the assumption that the errors are homoscedastic. Monte Carlo results presented in Section 4 show that SLM_{OPG} improves LM_{OPG} significantly in terms of the finite sample null distribution, and that it is generally comparable, in terms of the tail probabilities, to the bootstrap LM test suggested below.

2.3.1. Tests based on bootstrap P values. We end this section by describing the bootstrap LM test that serves as the benchmark for the finite sample performance of our SLM tests. Essentially, each of the tests presented above has a bootstrap counterpart in the spirit of Lin et al. (2007). One of the simplest is that based on LM_{EI}, denoted as BLM_{EI}. Note that $LM_{EI} = \frac{N}{\sqrt{S_0}}\frac{u'MWMu}{u'Mu}$. Our suggested bootstrap procedure is as follows:

Step 1. Draw a bootstrap sample $\tilde{u}^b$ from the OLS residuals $\tilde{u}$;
Step 2. Compute the bootstrap value of LM_{EI} as $BLM^b_{EI} = \frac{N}{\sqrt{S_0}}\frac{\tilde{u}^{b\prime}MWM\tilde{u}^b}{\tilde{u}^{b\prime}M\tilde{u}^b}$;
Step 3. Repeat (a)–(b) B times to give $\{BLM^b_{EI}\}_{b=1}^B$, and thus the bootstrap P-value.

The suggested bootstrap procedure is simpler than that of Lin et al. (2007) in that each bootstrap value of the tests statistic is based on a bootstrap sample of the OLS residuals, and thus the re-estimation of the spatial parameter in each bootstrap sample is avoided.[3]

3. TESTS FOR SED IN A PANEL LINEAR REGRESSION MODEL

This section studies the LM and standardized LM tests for zero SED in a panel linear regression with random or fixed effects. When repeated observations are made on the same set of N spatial

[3] We thank an anonymous referee for suggesting the bootstrap test. By noting that $LM_{EI}|_{H_0}$ is free of the parameters and is asymptotically robust against error distribution, the validity of the suggested bootstrap procedure can be inferred from the work of Hall and Horowitz (1996).

units over time, Model (2.2) becomes

$$Y_t = X_t\beta + u_t, \quad t = 1, \ldots, T, \tag{3.1}$$

resulting in a panel data regression model, where $\{Y_t, X_t\}$ denote the data collected at the tth time period. A defining feature of a panel data model is that the error vector u_t is allowed to possess a general structure of the form

$$u_{it} = \mu_i + \varepsilon_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T, \tag{3.2}$$

where μ_i denotes the unobservable space-specific effect, due to aspects of regional structure, firm's specific feature, etc. Spatial units may be dependent. To allow for such a possibility, Anselin (1988) introduced an SAR process into the disturbance vector $\varepsilon_t = \{\varepsilon_{1t}, \ldots, \varepsilon_{Nt}\}'$,

$$\varepsilon_t = \lambda W \varepsilon_t + v_t, \quad t = 1, \ldots, T, \tag{3.3}$$

where the spatial weight matrix W is defined similarly to that in model (2.2), and v_t is an $N \times 1$ vector of iid remainder disturbances with mean zero and variance σ_v^2.

We are interested in testing the hypothesis $H_0 : \lambda = 0$. We consider the scenario where the time dimension T is small and the 'space' dimension N is large. This is the typical feature for many micro-level panel data sets. The space-specific effects μ_i can be random or fixed. As T is small, the time-specific effects can be directly built into the model.

3.1. Panel linear regression with random effects

Let $B = I_N - \lambda W$. Stacking the vectors (Y_t, u_t, v_t) and the matrix X_t, the model can be written in matrix form:

$$Y = X\beta + u, \quad u = (\iota_T \otimes I_N)\mu + (I_T \otimes B^{-1})v, \tag{3.4}$$

where ι_m represents an $m \times 1$ vector of ones, I_m represents an $m \times m$ identity matrix.

Assume (a) the elements of μ are iid with mean zero and variance σ_μ^2, (b) the elements of v are iid with mean zero and variance σ_v^2 and (c) μ and v are independent. The log-likelihood function, assuming μ and v are both normally distributed, is given by:

$$\ell\big(\beta, \sigma_v^2, \sigma_\mu^2, \lambda\big) = -\frac{NT}{2}\log\big(2\pi\sigma_v^2\big) - \frac{1}{2}\log|\Sigma| - \frac{1}{2\sigma_v^2}u'\Sigma^{-1}u, \tag{3.5}$$

where $\Sigma = \frac{1}{\sigma_v^2}\mathrm{E}(uu') = \phi(J_T \otimes I_N) + I_T \otimes (B'B)^{-1}$, $\Sigma^{-1} = \bar{J}_T \otimes (T\phi I_N + (B'B)^{-1})^{-1} + E_T \otimes (B'B)$, $\phi = \sigma_\mu^2/\sigma_v^2$, $J_T = \iota_T\iota_T'$, $\bar{J}_T = \frac{1}{T}J_T$ and $E_T = I_T - \bar{J}_T$. See Anselin (1988) and Baltagi et al. (2003) for details. Maximizing (3.3) gives the MLE of the model parameters if the error components are normally distributed, otherwise it gives a quasi-maximum likelihood estimator (QMLE).

Anselin (1988, p. 155) presents an LM test of $H_0 : \lambda = 0$ for Model (3.4) in the presence of random space-specific effects, which can be written in the form

$$LM_{RE} = \frac{\tilde{u}'[\tilde{\rho}^2(\bar{J}_T \otimes W) + E_T \otimes W]\tilde{u}}{\tilde{\sigma}_v^2[(T - 1 + \tilde{\rho}^2)S_0]^{\frac{1}{2}}}, \tag{3.6}$$

where $S_0 = \text{tr}(W'W) + W^2)$, $\tilde{\rho}$ and $\tilde{\sigma}_v^2$ are the constrained QMLEs of $\rho = \sigma_v^2/(T\sigma_\mu^2 + \sigma_v^2)$ and σ_v^2 under H_0, and $\tilde{u}$ is the vector of constrained QMLE residuals.[4]

A nice feature of the LM test is that it requires only the estimates of the model under H_0. However, even under H_0, the constrained QMLE of ρ (or ϕ) does not posses an explicit expression, meaning that $\tilde{\rho}$ has to be obtained via numerical optimization. In fact, under H_0, the partially maximized log-likelihood (with respect to β and σ_v^2) is given by:

$$\ell_{\max}(\rho) = \text{constant} - \frac{NT}{2}\log\,\tilde{\sigma}_v^2(\rho) + \frac{N}{2}\log\,\rho, \tag{3.7}$$

where $\tilde{\sigma}_v^2(\rho) = \frac{1}{NT}\tilde{u}'(\rho)\Sigma_0^{-1}\tilde{u}(\rho)$, $\tilde{u}(\rho) = Y - X\tilde{\beta}(\rho)$, $\tilde{\beta}(\rho) = (X'\Sigma_0^{-1}X)^{-1}X'\Sigma_0^{-1}Y$ and $\Sigma_0^{-1} = \Sigma^{-1}|_{\lambda=0} = \rho\bar{J}_T \otimes I_N + E_T \otimes I_N$. Maximizing (3.7) gives the constrained QMLE (under H_0) $\tilde{\rho}$ of ρ, which in turn gives the constrained QMLEs $\tilde{\beta} \equiv \tilde{\beta}(\tilde{\rho})$, $\tilde{\sigma}_v^2 \equiv \tilde{\sigma}_v^2(\tilde{\rho})$, $\tilde{\Sigma}_0^{-1} \equiv \tilde{\rho}\bar{J}_T \otimes I_N + E_T \otimes I_N$, and $\tilde{u} \equiv \tilde{u}(\tilde{\rho})$, for β, σ_v^2, Σ_0^{-1} and $u(\rho)$, respectively.

Similar to the LM test in the linear regression model, the numerator of LM_{RE} given in (3.6) is again a quadratic form in the disturbance vector u, but now u contains two independent components. The large sample mean of this quadratic form is zero, but its finite sample mean is not necessarily zero. This may distort the finite sample distribution of the test statistic, in particular the tail probability. We now present a standardized version of the LM_{RE} test, which corrects both the mean and the variance and has a better finite sample performance in the situation where each spatial unit has 'many' neighbours. Lemma A.3 given in the Appendix is essential in deriving the modified test statistics. Some basic regularity conditions are listed later.

ASSUMPTION 3.1. *The random effects* $\{\mu_i\}$ *are iid with mean zero, variance* σ_μ^2 *and excess kurtosis* κ_μ. *The idiosyncratic errors* $\{v_{it}\}$ *are iid with mean zero, variance* σ_v^2 *and excess kurtosis* κ_v. *Also, the moments* $E|\mu_i|^{4+\eta_1}$ *and* $E|v_{it}|^{4+\eta_2}$ *exist for some* $\eta_1, \eta_2 > 0$.

ASSUMPTION 3.2. *The elements* $\{w_{ij}\}$ *of* W *are at most of order* h_N^{-1} *uniformly for all i, j, with the rate sequence* $\{h_N\}$, *bounded or divergent, satisfying* $h_N/N \to 0$ *as* N *goes to infinity. The* $N \times N$ *matrices* $\{W\}$ *are uniformly bounded in both row and column sums with* $w_{ii} = 0$ *and* $\sum_j w_{ij} = 1$ *for all* i.

ASSUMPTION 3.3. *The elements of the* $NT \times k$ *matrix* X *are uniformly bounded for all* N *and* $\lim_{N\to\infty}\frac{1}{N}X'X$ *exists and is non-singular.*

Now, define $A(\rho) = \rho^2(\bar{J}_T \otimes W) + E_T \otimes W$, $M(\rho) = I_{NT} - X(X'\Sigma_0^{-1}X)^{-1}X'\Sigma_0^{-1}$, $C(\rho) = M'(\rho)[A(\rho) - a_0(\rho)\Sigma_0^{-1}]M(\rho)$ and $a_0(\rho) = \frac{1}{NT-k}\text{tr}\,[\Sigma_0 M'(\rho)A(\rho)M(\rho)]$. Let diagv$(A)$ be a column vector formed by the diagonal elements of a square matrix A. We have the following theorem.

THEOREM 3.1. *Assume that the constrained QMLE* $\tilde{\rho}$ *under* H_0 *is a consistent estimator of* ρ. *Under Assumptions 3.1–3.3, for testing* $H_0 : \lambda = 0$, *the standardized LM test which corrects*

[4] Baltagi et al. (2003) considered the joint, marginal and conditional LM tests for λ and/or σ_μ^2, which includes (3.6) as a special case, and presented Monte Carlo results under spatial layouts with a fixed number of neighbours. Apparently, the LM test given in (3.6) does not fit into the framework of Robinson (2008), but it does if the test concerns $H_0 : \lambda = 0, \sigma_\mu = 0$. We note that our approach is applicable to all scenarios similar to (3.6), that is, testing spatial effect allowing other type of effects (such as random effects, heteroscedasticity, etc.) to exist in the model.

both the mean and variance takes the form:

$$SLM_{RE} = \frac{\tilde{u}'\big(\tilde{A} - \tilde{a}_0\tilde{\Sigma}_0^{-1}\big)\tilde{u}}{[\tilde{\phi}^2\tilde{\kappa}_\mu\tilde{a}_1'\tilde{a}_1 + \tilde{\kappa}_v\tilde{a}_2'\tilde{a}_2 + \mathrm{tr}\,(\tilde{\Sigma}(\tilde{C}' + \tilde{C})\tilde{\Sigma}\tilde{C})]^{\frac{1}{2}}\tilde{\sigma}_v^2}, \tag{3.8}$$

where $\tilde{A} = A(\tilde{\rho})$, $\tilde{C} = C(\tilde{\rho})$, $\tilde{a}_0 = a_0(\tilde{\rho})$, $\tilde{\kappa}_\mu$ *is the sample excess kurtosis of* $\tilde{\mu} = (\bar{J}_T \otimes I_N)\tilde{u}$, $\tilde{\kappa}_v$ *is the sample excess kurtosis of* $\tilde{v} = \tilde{u} - (\iota \otimes I_N)\tilde{\mu}$, $\tilde{a}_1 = diagv[(\iota_T' \otimes I_N)\tilde{C}(\iota_T \otimes I_N)]$ *and* $\tilde{a}_2 = diagv(\tilde{C})$. *Under* H_0, *we have (a)* $SLM_{RE} \xrightarrow{d} N(0, 1)$ *and (b)* $SLM_{RE} \sim LM_{RE}$.

Similar to the results of Theorem 2.1, the results of Theorem 3.1 show that the mean correction factor for the standardized LM test is also of the order $O_p((h_N/N)^{\frac{1}{2}})$. Thus, the LM_{RE} test can have large mean bias when h_N is large.[5]

3.2. Panel linear regression with fixed effects

When the space-specific effects $\{\mu_i\}$ are treated as fixed, the incidental parameters problem occurs. The standard practice is to remove these fixed effects by some kind of transformation. Recently, Lee and Yu (2010) studied the asymptotic properties of QML estimation of spatial panel models with fixed effects, which contain the above model as a special case. They used an orthogonal transformation to the model specified by (3.1)–(3.3) to obtain

$$Y_t^* = X_t^*\beta + \varepsilon_t^*, \varepsilon_t^* = \lambda W\varepsilon_t^* + v_t^*, \quad t = 1, \ldots, T-1,$$

where $(Y_1^*, Y_2^*, \ldots, Y_{T-1}^*) = (Y_1, Y_2, \ldots, Y_T)F_{T,T-1}$, $F_{T,T-1}$ is a $T \times (T-1)$ matrix whose columns are the eigenvectors of $I_T - \frac{1}{T}\iota_T\iota_T'$ corresponding to the eigenvalues of one, and similarly ε_t^*, v_t^* and the columns of X_t^* are defined.

Debarsy and Ertur (2010) followed up with LM tests for spatial dependence. In case of a spatial error panel model with fixed space-specific effects, the LM test takes the form:

$$LM_{FE} = \frac{N(T-1)}{\sqrt{\mathbb{S}_0}}\frac{\tilde{\varepsilon}^{*\prime}\mathbb{W}\tilde{\varepsilon}^*}{\tilde{\varepsilon}^{*\prime}\tilde{\varepsilon}^*}, \tag{3.9}$$

where $\tilde{\varepsilon}^*$ is OLS residuals from regressing Y^* on X^* with Y^* being the stacked $\{Y_t^*\}$ and X^* the stacked $\{X_t^*\}$, $\mathbb{S}_0 = (T-1)S_0$ and $\mathbb{W} = I_{T-1} \otimes W$. With the fixed effects specification, the model wipes out time-invariant regressors.

ASSUMPTION 3.4. *The idiosyncratic errors* $\{v_{it}\}$ *are iid with mean zero, variance* σ_v^2 *and excess kurtosis* κ_v. *Also, the moment* $E\,|v_{it}|^{4+\eta}$ *exists for some* $\eta > 0$.

ASSUMPTION 3.5. *The elements of the* $NT \times k$ *matrix X are uniformly bounded for all N and* $\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^T(X_t - \bar{X})'(X_t - \bar{X})$ *exists and is non-singular, where* $\bar{X} = \frac{1}{T}\sum_{t=1}^T X_t$.

Define $\mathbb{M} = I_{N(T-1)} - X^*(X^{*\prime}X^*)^{-1}X^{*\prime}$, $\mathbb{A} = (F_{T,T-1} \otimes I_N)(\mathbb{M}\mathbb{W}\mathbb{M} - \mathbb{S}_1\mathbb{M})(F_{T,T-1}' \otimes I_N)$ and $\mathbf{a}_{ii}$ as the diagonal elements of $\mathbb{A}$. We have the following theorem:

[5] The condition in Theorem 3.1 may be relaxed to allow $\tilde{\rho}$ to be an arbitrary consistent estimator of ρ.

THEOREM 3.2. *Under Assumptions 3.2, 3.4 and 3.5, for testing H_0 : $\lambda = 0$, the standardized LM test which corrects both the mean and variance takes the form*:

$$SLM_{FE} = \frac{N(T-1)}{\sqrt{\tilde{\kappa}_v \mathbb{S}_2 + \mathbb{S}_3}} \frac{\tilde{\varepsilon}^{*\prime}(\mathbb{W} - \mathbb{S}_1 I_{N(T-1)})\tilde{\varepsilon}^*}{\tilde{\varepsilon}^{*\prime}\tilde{\varepsilon}^*}, \tag{3.10}$$

where $\mathbb{S}_1 = \frac{1}{N(T-1)-k}\mathrm{tr}\,(\mathbb{W}\mathbb{M})$, $\mathbb{S}_2 = \sum_{i=1}^{N(T-1)} \mathbf{a}_{ii}^2$, $\mathbb{S}_3 = \mathrm{tr}\,(\mathbb{A}\mathbb{A}' + \mathbb{A}^2)$ *and* $\tilde{\kappa}_v$ *is a consistent estimator of* κ_v. *Under* H_0, *we have (a)* $SLM_{FE} \xrightarrow{d} N(0, 1)$ *and (b)* $SLM_{FE} \sim LM_{FE}$.

For practical applications of the above theorem, one needs a consistent estimator of κ_v. While the elements of ε^* are uncorrelated, they may not be independent and thus the sample kurtosis of $\tilde{\varepsilon}^*$ may not provide a consistent estimator for κ_v as in the case of a linear regression model. The following corollary provides the needed result.

COROLLARY 3.1. *Under the assumptions of Theorem 3.2, a method of moments type estimator for* κ_v *that is consistent under* H_0 *takes the form*:

$$\tilde{\kappa}_v = \frac{(T-1)^2}{\sum_{t=1}^{T} c_t^4} \left(\frac{\sum_{i=1}^{N} (1'_{T-1}\tilde{\varepsilon}^*_{i\cdot})^4}{N^{-1}(\tilde{\varepsilon}^{*\prime}\tilde{\varepsilon}^*)^2} - 3 \right), \tag{3.11}$$

where c_t *is the t-th element of* $F_{T,T-1}1_{T-1}$, *and* $\tilde{\varepsilon}^{*\prime}_{i\cdot}$ *is the i-th row of* $(\tilde{\varepsilon}^*_1, \tilde{\varepsilon}^*_2, \ldots, \tilde{\varepsilon}^*_{T-1})$.

3.2.1. Tests based on bootstrap P-values. Again, for each of the LM tests presented, one may construct a bootstrap counterpart by extending the procedure given at the end of Section 2. This is typically the case for the fixed effects model as seen later, but for the random effects model there are two complications: one is the existence of error components that makes it unclear on the way of re-sampling, and the other is that the parameter σ_μ^2 has to be estimated in each bootstrap sample, making the bootstrap procedure computationally more demanding. We thus present a bootstrap version only for $\mathrm{LM_{FE}}$, denoted as BLM_{FE}. Noting that $LM_{FE} = \frac{N(T-1)}{\sqrt{\mathbb{S}_0}} \frac{\varepsilon^{*\prime}\mathbb{M}\mathbb{W}\mathbb{M}\varepsilon^*}{\varepsilon^{*\prime}\mathbb{M}\varepsilon^*}$, a bootstrap procedure similar to that for LM_{EI} can be obtained as follows:

Step 1. Draw a bootstrap sample $\tilde{\varepsilon}^{*b}$ from the OLS residuals $\tilde{\varepsilon}^*$;
Step 2. Compute the bootstrap value of LM_{FE} as $BLM_{FE}^b = \frac{N(T-1)}{\sqrt{\mathbb{S}_0}} \frac{\tilde{\varepsilon}^{*b\prime}\mathbb{M}\mathbb{W}\mathbb{M}\tilde{\varepsilon}^{*b}}{\tilde{\varepsilon}^{*b\prime}\mathbb{M}\tilde{\varepsilon}^{*b}}$;
Step 3. Repeat (a) and (b) B times to give $\{BLM_{FE}^b\}_{b=1}^B$, and thus the bootstrap P-value.

The suggested BLM_{FE} test can be used as a benchmark for the finite sample performance of the SLM tests. Its validity can again be inferred from Hall and Horowitz (1996).

4. MONTE CARLO RESULTS

The finite sample performance of the test statistics introduced in this paper are evaluated based on a series of Monte Carlo experiments. These experiments involve a number of different error distributions and a number of different spatial layouts. Comparisons are made between the standardized tests and their non-standardized counterparts to see the effects of the error distributions, the spatial layouts and the design of the regression model. In cases of a linear regression and panel linear regression with fixed effects, LM tests referring to bootstrap P-values are also implemented to serve as benchmarks for the comparison.

4.1. Spatial layouts and error distributions

Three general spatial layouts are considered in the Monte Carlo experiments and they are applied to all the test statistics involved in the experiments. The first is based on the Rook contiguity, the second is based on Queen contiguity and the third is based on the notion of group or social interactions with the number of groups $G = N^d$ where $0 < d < 1$. In the first two cases, the number of neighbours for each spatial unit stays the same (2–4 for Rook and 3–8 for Queen) and does not change when sample size N increases. However, in the last case, the number of neighbours for each spatial unit increases with the sample size but at a slower rate, and changes from group to group.

The details for generating the W matrix under Rook contiguity is as follows: (a) index the N spatial units by $(1, 2, \ldots, N)$, randomly permute these indices and then allocate them into a lattice of $r \times m(\geq N)$ squares, (b) let $W_{ij} = 1$ if the index j is in a square which is on the immediate left, or right, or above, or below the square which contains the index i, otherwise $W_{ij} = 0$ and (c) divide each element of W by its row sum. The W matrix under Queen contiguity is generated in a similar way, but with additional neighbours which share a common vertex with the unit of interest.

To generate the W matrix according to the group interaction scheme, (a) calculate the number of groups according to $G = \text{Round}(N^d)$, and the approximate average group size $m = N/G$, (b) generate the group sizes $(n_1, n_2, \ldots, n_G)$ according to a discrete uniform distribution from $m/2$ to $3m/2$, (c) adjust the group sizes so that $\sum_{i=1}^{G} n_i = N$ and (d) define $W = \text{diag}\{W_i/(n_i - 1), i = 1, \ldots, G\}$, a matrix formed by placing the sub-matrices W_i along the diagonal direction, where W_i is an $n_i \times n_i$ matrix with ones on the off-diagonal positions and zeros on the diagonal positions. In our Monte Carlo experiments, we choose $d = 0.2$, 0.5 and 0.8, representing, respectively, the situations where (a) there are few groups and many spatial units in a group, (b) the number of groups and the sizes of the groups are of the same magnitude and (c) there are many groups with few elements in each. Under Rook or Queen contiguity, h_N defined in the theorems is bounded, whereas under group interaction h_N is divergent with rate N^{1-d}. This spatial layout covers the scenario considered in Case (1991). Lee (2007) shows that the group size variation plays an important role in the identification and estimation of econometric models with group interactions, contextual factors and fixed effects. Yang (2010) shows that it also plays an important role in the robustness of the LM test of SEC.

The reported Monte Carlo results correspond to the following three error distributions: (a) standard normal, (b) mixture normal, standardized to have mean zero and variance 1 and (c) log-normal, also standardized to have mean zero and variance one. The standardized normal-mixture variates are generated according to

$$u_i = [(1 - \xi_i)Z_i + \xi_i \tau Z_i]/(1 - p + p * \tau^2)^{0.5},$$

where ξ is a Bernoulli random variable with probability of success p and Z_i is standard normal independent of ξ. The parameter p in this case also represents the proportion of mixing the two normal populations. In our experiments, we choose $p = 0.05$, meaning that 95% of the random variates are from standard normal and the remaining 5% are from another normal population with standard deviation τ. We choose $\tau = 10$ to simulate the situation where there are gross errors in the data. The standardized log-normal random variates are generated according to

$$u_i = [\exp(Z_i) - \exp(0.5)]/[\exp(2) - \exp(1)]^{0.5}.$$

This gives an error distribution that is both skewed and leptokurtic. The normal mixture gives an error distribution that is still symmetric like normal but leptokurtic. Other non-normal distributions, such as normal-gamma mixture and chi-squared, are also considered and the results are available from the authors upon request. All the Monte Carlo experiments are based on 10,000 replications.

4.2. *Performance of the tests for the linear regression model*

The finite sample performance of seven LM-type test statistics are investigated and compared: LM_{EI}, SLM_{EI}, BLM_{EI} which is LM_{EI} referring to the bootstrap P-values (Lin et al., 2007), I°, I^{*}, LM_{OPG} and SLM_{OPG}. The Monte Carlo experiments are carried out based on the following data generating process:

$$Y_i = \beta_0 + X_{1i}\beta_1 + X_{2i}\beta_2 + u_i.$$

The design of the experiment, or the way the regressors are generated also matters. We thus consider two scenarios: (a) IID scheme: $X_{1i} \overset{iid}{\sim} \sqrt{6}U(0, 1)$ and $X_{2i} \overset{iid}{\sim} N(0, 1)/\sqrt{2}$; and (b) Non-IID scheme: the ith pair of X values in the gth group are generated according to $X_{1,ig} = (2z_g + z_{ig})/\sqrt{7}$ and $X_{2,ig} = (v_g + v_{ig})/\sqrt{7}$, where $\{z_g, z_{ig}, v_g, v_{ig}\}$ are iid $N(0, 1)$ across all i and g (see Lee, 2004a). Both X_1 and X_2 are treated as fixed in the experiments. The parameters $\beta = \{5, 1, 1\}'$ and $\sigma = 1$, resulting in a signal-to-noise ratio of 1. Five different sample sizes are considered, that is, $N = 50$, 100, 200, 500 and 1000.

4.2.1. Null behaviour of the tests. Tables 1(a–c) reports the (null) empirical mean, standard deviation and the tail probabilities (10%, 5% and 1%) for the seven test statistics. From the results (reported and unreported), the general observations are as follows: (a) in terms of closeness to $N(0, 1)$, the standardized tests (SLM_{EI}, I^{*} and SLM_{OPG}) improve significantly over their non-standardized counterparts (LM_{EI}, I^{0} and LM_{OPG}); (b) the finite sample null distributions of LM_{EI}, I^{0} and LM_{OPG} can be altered greatly by the spatial layout, and they can also be affected by the error distributions and the way the regressors are generated; and (c) in general, SLM_{EI} and in particular SLM_{OPG}, perform comparably with BLM_{EI}.

Some details are as follows: all tests including BLM_{EI} perform better under (a) light spatial dependence compared with heavy spatial dependence, (b) normal errors rather than non-normal errors, (c) IID regressors rather than Non-IID regressors. The tests LM_{EI}, I^{0} and LM_{OPG} have a downward mean shift, which can be sizable even when N is quite large. Besides the mean shift, LM_{EI} also has a downward SD shift, which can be sizable as well when N is not large, but goes to zero as N increases. In contrast, SLM_{EI}, I^{*} and SLM_{OPG} have mean close to zero and SD close to 1 which explain why they have better size in all experiments. Recalling that LM_{EI} corrects neither mean nor SD, and that I^{0} and LM_{OPG} correct only for SD, it is clear now why I^{0} and LM_{OPG} have size distortions, and why LM_{EI} is more severely undersized than I^{0}. Thus, the LM tests of spatial dependence need to be *both* mean- and variance-adjusted for good finite sample performance.

The results in Table 1 show that one of the major factors affecting the null distribution of LM_{EI}, I^{0} and LM_{OPG} is the spatial layout, or rather the degree of spatial dependence. In

Table 1a. Empirical means, SDs and tail probabilities: linear regression, normal errors.

	Group: $G = N^{0.5}$					Queen contiguity				
Test	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
					$N = 50$					
1	−0.5270	0.8408	0.0615	0.0144	0.0038	−0.2268	0.9312	0.0784	0.0363	0.0058
2	−0.0047	1.0367	0.0836	0.0507	0.0214	−0.0132	1.0336	0.1034	0.0545	0.0143
3	−0.5270	0.8408	0.0977	0.0485	0.0113	−0.2268	0.9312	0.0946	0.0493	0.0110
4	−0.6235	0.9948	0.1583	0.0583	0.0085	−0.2416	0.9919	0.0994	0.0493	0.0092
5	−0.0045	0.9948	0.0743	0.0445	0.0184	−0.0127	0.9919	0.0901	0.0441	0.0115
6	−0.7146	0.9806	0.1843	0.0972	0.0163	−0.2946	0.9950	0.1116	0.0518	0.0071
7	−0.1840	1.0669	0.1315	0.0641	0.0094	−0.0748	1.0320	0.1100	0.0531	0.0070
					$N = 100$					
1	−0.5027	0.8892	0.0859	0.0244	0.0048	−0.1811	0.9585	0.0906	0.0396	0.0070
2	−0.0035	1.0334	0.0884	0.0477	0.0193	0.0091	1.0154	0.1010	0.0515	0.0130
3	−0.5027	0.8892	0.1018	0.0519	0.0107	−0.1811	0.9585	0.0988	0.0498	0.0097
4	−0.5725	1.0126	0.1553	0.0677	0.0086	−0.1880	0.9950	0.1030	0.0476	0.0095
5	−0.0034	1.0126	0.0828	0.0453	0.0174	0.0089	0.9950	0.0936	0.0459	0.0109
6	−0.6701	1.0085	0.1825	0.1001	0.0221	−0.2271	0.9920	0.1080	0.0492	0.0077
7	−0.1576	1.0667	0.1280	0.0659	0.0119	−0.0361	1.0124	0.1044	0.0508	0.0084
					$N = 200$					
1	−0.4032	0.9200	0.0920	0.0323	0.0045	−0.1246	0.9781	0.0962	0.0446	0.0088
2	0.0168	1.0199	0.0924	0.0498	0.0177	−0.0057	1.0040	0.1031	0.0515	0.0100
3	−0.4032	0.9200	0.1074	0.0557	0.0110	−0.1246	0.9781	0.1029	0.0493	0.0111
4	−0.4425	1.0097	0.1362	0.0591	0.0076	−0.1266	0.9939	0.1017	0.0478	0.0095
5	0.0167	1.0097	0.0887	0.0475	0.0172	−0.0056	0.9939	0.0991	0.0490	0.0094
6	−0.5403	1.0171	0.1597	0.0878	0.0223	−0.1571	0.9962	0.1050	0.0496	0.0088
7	−0.1133	1.0485	0.1192	0.0638	0.0141	−0.0371	1.0052	0.1052	0.0491	0.0079
					$N = 500$					
1	−0.3315	0.9401	0.0865	0.0368	0.0059	−0.0844	0.9869	0.0961	0.0462	0.0078
2	0.0045	1.0010	0.0888	0.0434	0.0139	−0.0017	0.9975	0.0988	0.0480	0.0087
3	−0.3315	0.9401	0.0925	0.0501	0.0117	−0.0844	0.9869	0.0998	0.0482	0.0088
4	−0.3516	0.9970	0.1123	0.0516	0.0086	−0.0850	0.9935	0.0986	0.0480	0.0082
5	0.0045	0.9970	0.0876	0.0426	0.0137	−0.0017	0.9935	0.0971	0.0469	0.0084
6	−0.4395	1.0082	0.1380	0.0703	0.0187	−0.1064	0.9947	0.0999	0.0484	0.0088
7	−0.0976	1.0235	0.1029	0.0527	0.0120	−0.0236	0.9980	0.1010	0.0468	0.0085
					$N = 1000$					
1	−0.2929	0.9654	0.0956	0.0427	0.0076	−0.0591	0.9884	0.0946	0.0454	0.0101
2	−0.0060	1.0089	0.0965	0.0473	0.0145	−0.0011	0.9937	0.0954	0.0467	0.0108
3	−0.2929	0.9654	0.1035	0.0525	0.0109	−0.0591	0.9884	0.0969	0.0487	0.0119
4	−0.3055	1.0069	0.1118	0.0540	0.0103	−0.0593	0.9918	0.0959	0.0462	0.0103
5	−0.0059	1.0069	0.0958	0.0470	0.0145	−0.0011	0.9918	0.0950	0.0460	0.0106
6	−0.3829	1.0143	0.1256	0.0715	0.0166	−0.0743	0.9931	0.0978	0.0478	0.0108
7	−0.0923	1.0234	0.1112	0.0575	0.0121	−0.0162	0.9947	0.0969	0.0477	0.0111

Notes: Test: 1 = LM_{EI}, 2 = SLM_{EI}, 3 = bootstrap LM_{EI}, 4 = I°, 5 = I^{*}, 6 = LM_{OPG} and 7 = SLM_{OPG}. X-Value: Non-IID for group interaction scheme, and IID for Queen contiguity. True parameter values: $\beta = \{5, 1, 1\}'$, and $\sigma = 1$.

Continued

Table 1b. Empirical means, SDs and tail probabilities: linear regression, normal mixtures.

	Group: $G = N^{0.5}$					Queen contiguity				
Test	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
					$N = 50$					
1	−0.5280	0.7350	0.0414	0.0112	0.0031	−0.2253	0.8103	0.0510	0.0230	0.0036
2	−0.0060	0.8916	0.0559	0.0324	0.0112	−0.0114	0.8927	0.0669	0.0328	0.0076
3	−0.5280	0.7350	0.0838	0.0407	0.0098	−0.2253	0.8103	0.0825	0.0410	0.0088
4	−0.6247	0.8696	0.1115	0.0395	0.0058	−0.2400	0.8631	0.0659	0.0315	0.0059
5	−0.0057	0.8696	0.0520	0.0288	0.0099	−0.0111	0.8631	0.0588	0.0283	0.0059
6	−0.6874	0.8907	0.1288	0.0543	0.0052	−0.3028	0.9863	0.0967	0.0392	0.0025
7	−0.0920	1.0212	0.1017	0.0415	0.0044	−0.0456	1.0136	0.0954	0.0363	0.0028
					$N = 100$					
1	−0.4969	0.7844	0.0552	0.0227	0.0057	−0.1875	0.8417	0.0588	0.0283	0.0062
2	0.0032	0.8946	0.0601	0.0341	0.0139	0.0023	0.8882	0.0667	0.0342	0.0094
3	−0.4969	0.7844	0.0873	0.0437	0.0114	−0.1875	0.8417	0.0875	0.0433	0.0100
4	−0.5658	0.8932	0.1000	0.0436	0.0105	−0.1946	0.8738	0.0669	0.0331	0.0074
5	0.0033	0.8932	0.0597	0.0346	0.0137	0.0023	0.8738	0.0635	0.0314	0.0087
6	−0.6550	0.9357	0.1408	0.0624	0.0094	−0.2491	0.9764	0.0870	0.0327	0.0039
7	−0.0598	1.0185	0.1028	0.0421	0.0059	−0.0091	0.9913	0.0815	0.0308	0.0033
					$N = 200$					
1	−0.4273	0.8505	0.0722	0.0259	0.0040	−0.0994	0.9181	0.0754	0.0412	0.0111
2	−0.0098	0.9295	0.0693	0.0335	0.0097	0.0201	0.9413	0.0811	0.0442	0.0139
3	−0.4273	0.8505	0.1001	0.0499	0.0108	−0.0994	0.9181	0.1008	0.0522	0.0139
4	−0.4690	0.9334	0.1092	0.0451	0.0074	−0.1010	0.9330	0.0799	0.0438	0.0127
5	−0.0098	0.9334	0.0703	0.0343	0.0097	0.0200	0.9330	0.0786	0.0425	0.0134
6	−0.5547	0.9754	0.1338	0.0609	0.0104	−0.1472	0.9982	0.0904	0.0368	0.0038
7	−0.0783	1.0256	0.1014	0.0434	0.0052	0.0012	1.0033	0.0871	0.0351	0.0043
					$N = 500$					
1	−0.3249	0.9106	0.0791	0.0340	0.0065	−0.0890	0.9471	0.0840	0.0459	0.0116
2	0.0115	0.9678	0.0826	0.0409	0.0117	−0.0063	0.9570	0.0856	0.0471	0.0118
3	−0.3249	0.9106	0.0995	0.0517	0.0113	−0.0890	0.9471	0.0987	0.0514	0.0103
4	−0.3446	0.9657	0.1024	0.0475	0.0085	−0.0896	0.9534	0.0863	0.0474	0.0123
5	0.0115	0.9657	0.0822	0.0407	0.0116	−0.0063	0.9534	0.0830	0.0466	0.0115
6	−0.4241	0.9924	0.1273	0.0594	0.0079	−0.1127	0.9897	0.0919	0.0373	0.0045
7	−0.0582	1.0187	0.1068	0.0471	0.0043	−0.0145	0.9917	0.0874	0.0371	0.0043
					$N = 1000$					
1	−0.2814	0.9555	0.0901	0.0404	0.0079	−0.0640	0.9819	0.0948	0.0494	0.0115
2	0.0061	0.9973	0.0921	0.0467	0.0133	−0.0060	0.9872	0.0957	0.0486	0.0122
3	−0.2814	0.9555	0.1042	0.0522	0.0120	−0.0640	0.9819	0.1031	0.0507	0.0107
4	−0.2935	0.9966	0.1069	0.0495	0.0105	−0.0642	0.9853	0.0958	0.0500	0.0116
5	0.0061	0.9966	0.0918	0.0465	0.0132	−0.0060	0.9853	0.0951	0.0483	0.0120
6	−0.3642	1.0036	0.1268	0.0597	0.0089	−0.0847	1.0027	0.1010	0.0387	0.0039
7	−0.0619	1.0189	0.1058	0.0499	0.0079	−0.0192	1.0035	0.0963	0.0393	0.0038

Notes: Test: $1 = LM_{\mathrm{EI}}$, $2 = SLM_{\mathrm{EI}}$, 3 = bootstrap LM_{EI}, $4 = I^{\circ}$, $5 = I^{*}$, $6 = LM_{\mathrm{OPG}}$ and $7 = SLM_{\mathrm{OPG}}$. X-Value: Non-IID for group interaction scheme, and IID for Queen contiguity. True parameter values: $\beta = \{5, 1, 1\}'$, and $\sigma = 1$. For normal mixture, $P = 0.1$ and $\tau = 5$.

Continued

Table 1c. Empirical means, SDs and tail probabilities: linear regression, log-normal errors.

	Group: $G = N^{0.5}$					Queen contiguity				
Test	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
					$N = 50$					
1	−0.5198	0.7849	0.0441	0.0137	0.0042	−0.2105	0.8586	0.0563	0.0241	0.0064
2	0.0041	0.9573	0.0674	0.0394	0.0178	0.0050	0.9483	0.0789	0.0430	0.0137
3	−0.5198	0.7849	0.0861	0.0446	0.0120	−0.2105	0.8586	0.0898	0.0438	0.0085
4	−0.6151	0.9287	0.1222	0.0424	0.0082	−0.2242	0.9145	0.0745	0.0328	0.0091
5	0.0040	0.9287	0.0610	0.0366	0.0162	0.0047	0.9145	0.0691	0.0359	0.0115
6	−0.7387	0.9345	0.1623	0.0842	0.0170	−0.3630	0.9778	0.1083	0.0461	0.0057
7	−0.1662	1.0361	0.1116	0.0532	0.0070	−0.1177	1.0033	0.0963	0.0392	0.0036
					$N = 100$					
1	−0.5076	0.8186	0.0631	0.0223	0.0050	−0.2058	0.8854	0.0653	0.0283	0.0056
2	−0.0093	0.9403	0.0643	0.0392	0.0164	−0.0171	0.9358	0.0752	0.0382	0.0095
3	−0.5076	0.8186	0.0908	0.0450	0.0112	−0.2058	0.8854	0.0926	0.0453	0.0094
4	−0.5780	0.9321	0.1151	0.0486	0.0101	−0.2137	0.9192	0.0751	0.0358	0.0074
5	−0.0090	0.9321	0.0630	0.0386	0.0158	−0.0168	0.9192	0.0695	0.0354	0.0082
6	−0.7197	0.9460	0.1667	0.0865	0.0169	−0.3548	0.9971	0.1176	0.0580	0.0099
7	−0.1581	1.0032	0.0996	0.0465	0.0059	−0.1436	1.0117	0.1027	0.0486	0.0054
					$N = 200$					
1	−0.3979	0.8718	0.0660	0.0242	0.0055	−0.1305	0.9350	0.0754	0.0337	0.0096
2	0.0225	0.9573	0.0718	0.0367	0.0138	−0.0117	0.9590	0.0783	0.0406	0.0136
3	−0.3979	0.8718	0.0947	0.0487	0.0116	−0.1305	0.9350	0.1007	0.0451	0.0108
4	−0.4367	0.9568	0.1024	0.0421	0.0084	−0.1326	0.9502	0.0796	0.0360	0.0105
5	0.0225	0.9568	0.0715	0.0371	0.0137	−0.0116	0.9502	0.0748	0.0387	0.0128
6	−0.5879	0.9876	0.1496	0.0811	0.0199	−0.2868	1.0210	0.1188	0.0579	0.0102
7	−0.1343	1.0185	0.1031	0.0507	0.0096	−0.1552	1.0237	0.1082	0.0515	0.0077
					$N = 500$					
1	−0.3326	0.9001	0.0710	0.0276	0.0061	−0.0895	0.9524	0.0803	0.0386	0.0078
2	0.0034	0.9569	0.0736	0.0369	0.0136	−0.0068	0.9624	0.0840	0.0408	0.0093
3	−0.3326	0.9001	0.0905	0.0426	0.0090	−0.0895	0.9524	0.0968	0.0480	0.0089
4	−0.3527	0.9545	0.0937	0.0380	0.0084	−0.0901	0.9588	0.0828	0.0399	0.0078
5	0.0034	0.9545	0.0730	0.0365	0.0135	−0.0068	0.9588	0.0825	0.0400	0.0089
6	−0.4923	0.9882	0.1385	0.0760	0.0132	−0.2348	1.0151	0.1129	0.0560	0.0118
7	−0.1320	1.0019	0.1010	0.0471	0.0086	−0.1448	1.0122	0.1050	0.0500	0.0094
					$N = 1000$					
1	−0.2946	0.9392	0.0877	0.0367	0.0069	−0.0712	0.9479	0.0827	0.0399	0.0077
2	−0.0077	0.9801	0.0830	0.0414	0.0131	−0.0132	0.9530	0.0835	0.0417	0.0086
3	−0.2946	0.9392	0.1022	0.0497	0.0114	−0.0712	0.9479	0.0950	0.0447	0.0073
4	−0.3073	0.9796	0.1020	0.0466	0.0084	−0.0714	0.9512	0.0839	0.0405	0.0080
5	−0.0077	0.9796	0.0827	0.0414	0.0131	−0.0132	0.9512	0.0830	0.0411	0.0086
6	−0.4311	1.0054	0.1336	0.0718	0.0162	−0.1974	1.0057	0.1076	0.0528	0.0129
7	−0.1305	1.0110	0.1057	0.0504	0.0087	−0.1348	1.0025	0.1023	0.0494	0.0105

Notes: Test: 1 = LM_{EI}, 2 = SLM_{EI}, 3 = bootstrap LM_{EI}, 4 = I°, 5 = I^{*}, 6 = LM_{OPG} and 7 = SLM_{OPG}. X-Value: Non-IID for group interaction scheme, and IID for Queen contiguity. True parameter values: $\beta = \{5, 1, 1\}'$, and $\sigma = 1$.

situations of a large group interaction, for example, $G = Round(N^{0.2})$ (results not reported to conserve space), the number of groups ranges from 2 to 4 for N ranging from 50 to 1000. Thus, there are only a few groups, each containing many spatial units which are all neighbours of each other. This 'heavy' spatial dependence distorts severely the null distributions of LM_{EI}, I^0 and LM_{OPG}, and combined with Non-IID regressors these tests fail completely. In a sharp contrast, SLM_{EI} still performs reasonably under these extreme situations. In contrast, in situations of small group interaction, for example, $G = Round(N^{0.8})$ (results not reported to conserve space), the number of groups ranges from 23 to 251 for N ranging from 50 to 1000. In this case, there are many groups each having only 2 to 4 units, giving a spatial layout with very weak spatial dependence. As a result, the null distributions of LM_{EI}, I^0 and LM_{OPG} are much closer to $N(0, 1)$ though still not as close as those of the null distributions of SLM_{EI}, I^* and SLM_{OPG}. These observations are consistent with the discussion following Theorem 2.1. Another factor affecting the null distribution of LM_{EI}, I^0 and LM_{OPG} is the way the regressors were generated (or the design of the model). Under the group interaction spatial layout, the null distributions of LM_{EI}, I^0 and LM_{OPG} are much closer to $N(0, 1)$ when the regressors are generated under the IID scheme than under the Non-IID scheme.

4.2.2. Power of the tests. Empirical frequencies of rejection of the seven tests are plotted in Figures 1(a)–(c) against the values of λ (horizontal line). Simulated critical values for each test are used, which means that the reported powers of the tests are size-adjusted. In each plot of Figures 1(a)–(c), the power lines for LM_{EI}, SLM_{EI}, I^0 and I^*(tests 1, 2, 4 and 5) overlap. This means that once size-adjusted, these four tests have almost identical power. This is not surprising as all four tests share the same term $\tilde{u}'W\tilde{u}/\tilde{u}'\tilde{u}$. The four tests differ mainly in their locations and scales, and thus have different sizes or null behaviours in general when referred to the standard normal distribution. If, however, the exact critical values are used, they become essentially the same test. However, in empirical applications, asymptotic critical values are often used. In this case, it is important to do the mean and variance corrections to the test statistics so that the asymptotic critical values give a better approximation. An alternative way is to bootstrap. The power of BLM_{EI} is generally very close to that of the four tests, but the power of the two OPG-based LM tests can be noticeably different from that of the four tests.

Figure 1 further reveals that the spatial layout and the sample size are the two important factors affecting the power of these tests. With less neighbours (plots on the right) or with a larger sample, the tests become more powerful. It is interesting to note that when the spatial dependence is strong, it is harder to detect the spatial dependence when the spatial parameter is negative than when it is positive (see the plots on the left). Another factor affecting the power of the tests is the way that the regressors are generated. The results (not reported to conserve space) show that the tests under IID regressors are more powerful than tests under Non-IID regressors, although the signal-to-noise ratios are the same. The error distribution also affects the power of the tests, but to a lesser degree.

4.3. Performance of the tests for the random effects panel model

The LM and SLM tests (LM_{RE} and SLM_{RE}) introduced in Section 3.1 are compared by Monte Carlo simulation based on the following DGP (Data generating process):

$$Y_t = \beta_0 + X_{1t}\beta_1 + X_{2t}\beta_2 + u_t, \quad \text{with} \quad u_t = \mu + \varepsilon_t, \quad t = 1, \ldots, T,$$

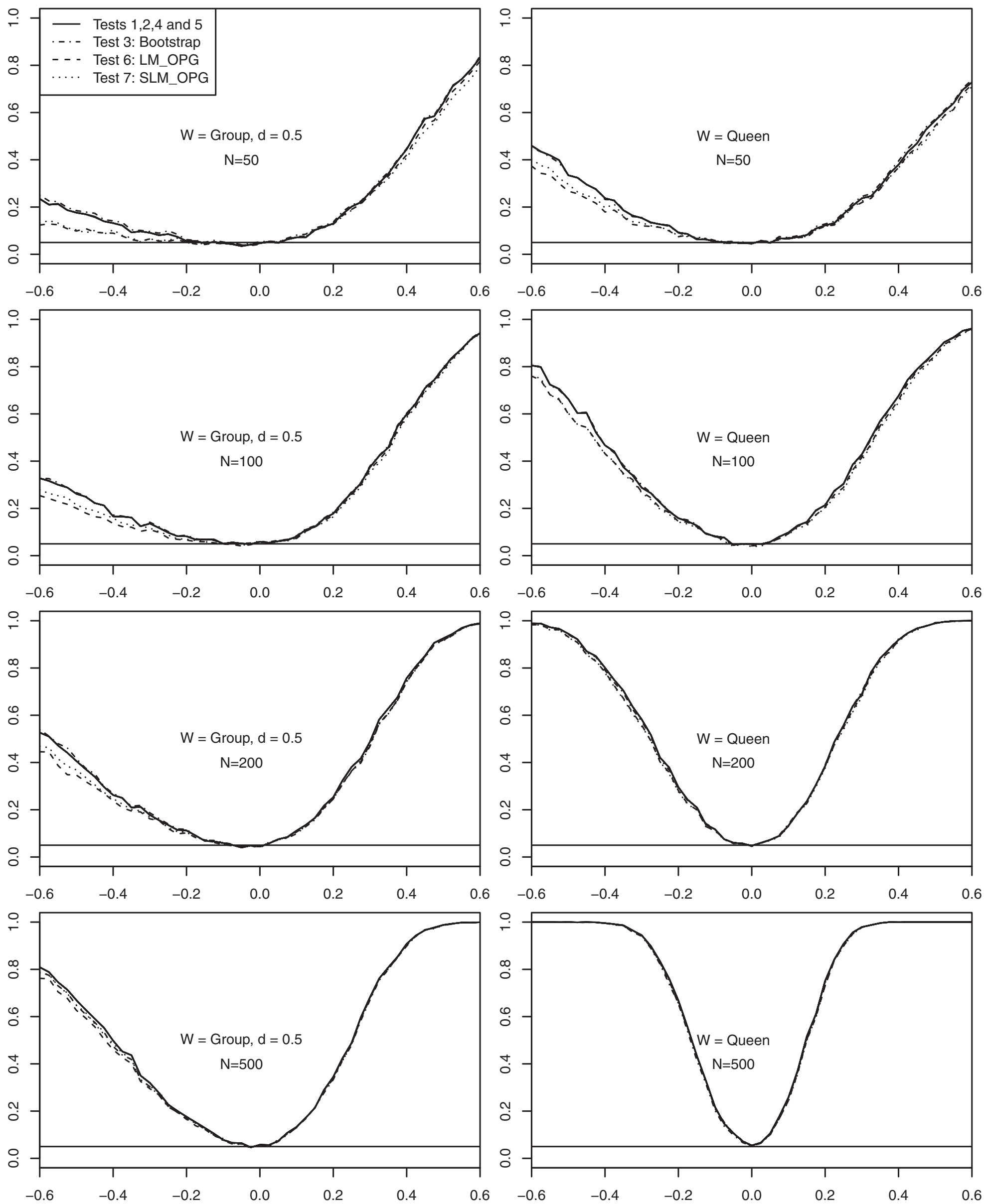

Figure 1a. Size-adjusted empirical power of tests 1–7: normal error.

where the error components μ and ε_t can be drawn from any of the three distributions used in the previous two subsections, or the combination of any two distributions. For example, μ and ε_t can both be drawn from the normal mixture, or μ from the normal mixture but ε_t from the normal or log-normal distribution. The beta parameters are set at the same

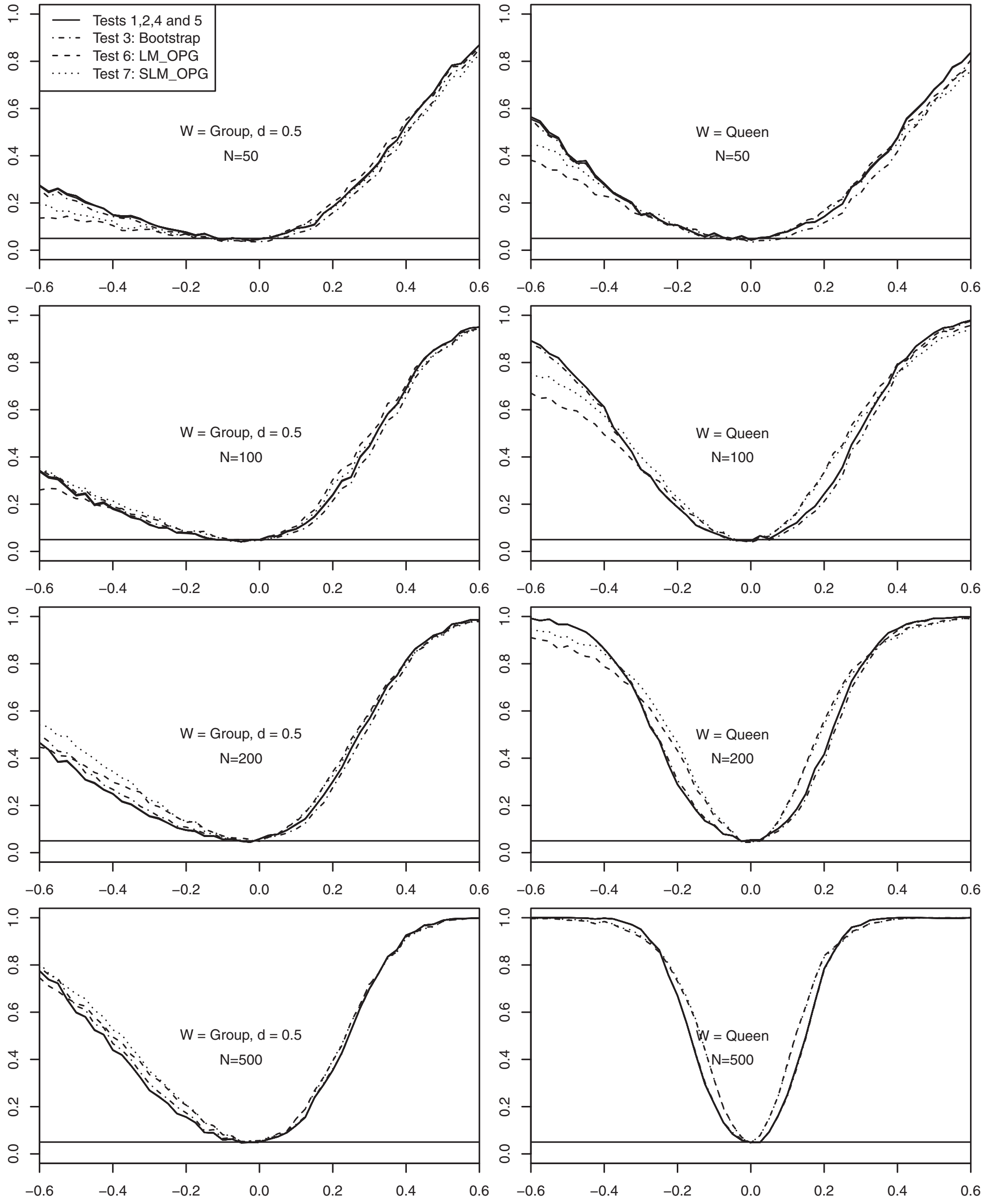

Figure 1b. Size-adjusted empirical power of tests 1–7: normal mixture errors.

values as before, $\sigma_\mu = \sigma_v = 1$. For sample sizes, $T = 3, 10$; and $N = 20, 50, 100, 200, 500$. The same spatial layouts are used as described above. The two regressors follow either the IID scheme where $\{X_{1,it}\} \overset{iid}{\sim} \sqrt{12}(U(0,1) - 0.5)$, and $\{X_{2,it}\} \overset{iid}{\sim} N(0,1)$, or the Non-IID scheme for group interaction layout: $X_{1,itg} = (2z_{tg} + z_{itg})/\sqrt{7}$ and $X_{2,itg} = (v_{tg} + v_{itg})/\sqrt{7}$,

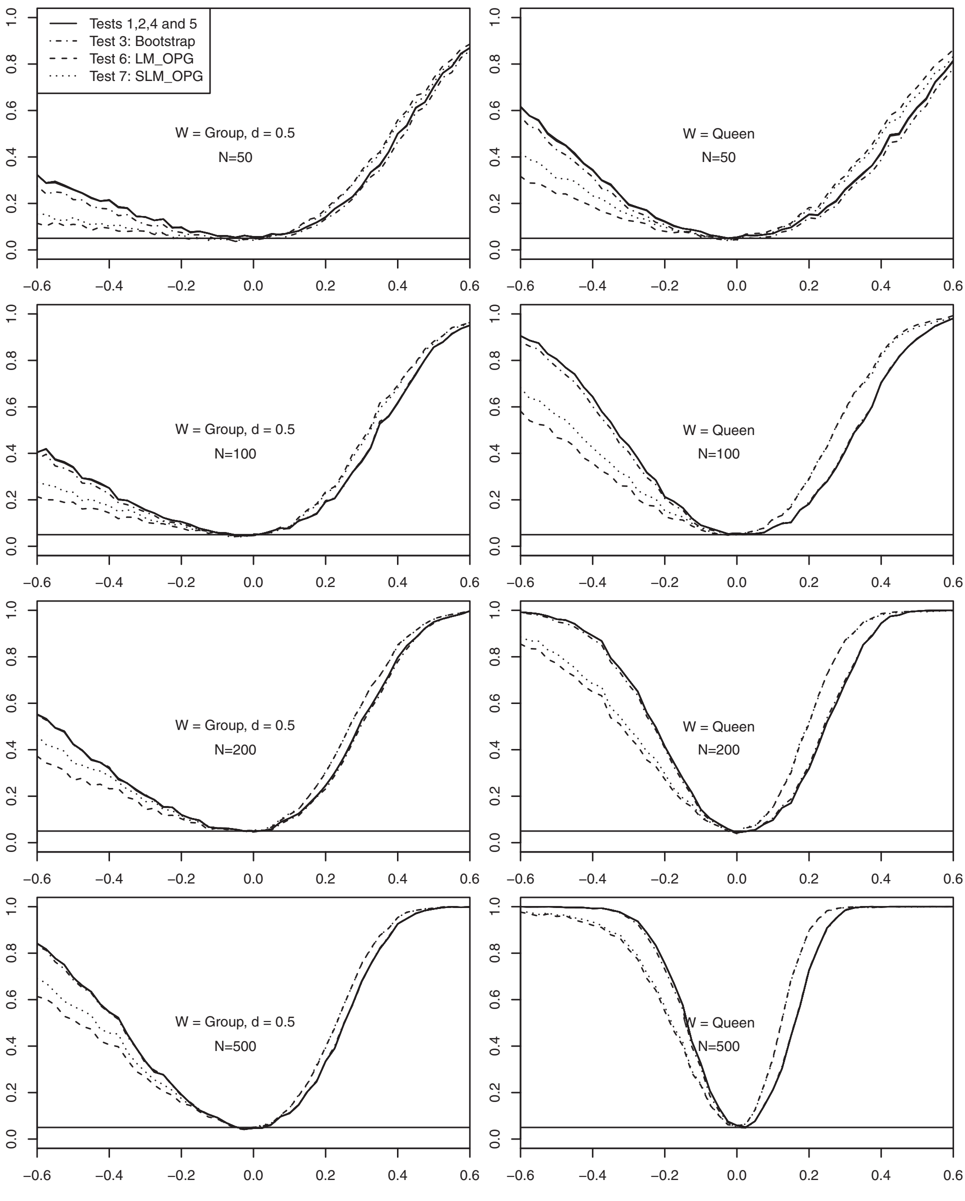

Figure 1c. Size-adjusted empirical power of tests 1–7: log-normal errors.

with $\{z_{tg}, z_{itg}, v_{tg}, v_{itg}\}$ being iid $N(0, 1)$ across all i, t and g. For this model, we are unable to implement the bootstrap method due to the extra complication in the error structure.

4.3.1. Null behaviour of the tests. The results presented in Tables 2a and 2b correspond to cases where both μ and v_t are normal, both are normal mixture, and both are log-normal. Essentially, the same conclusions hold as in the case of the spatial linear regression model.

Table 2a. Empirical means, SDs and tail probabilities: panel with random effects, $T = 3$.

	Group: $G = N^{0.5}$					Queen contiguity				
N	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
					Normal errors					
20	−0.4242	0.9192	0.0915	0.0308	0.0036	−0.0776	0.9854	0.0889	0.0434	0.0090
	0.0040	1.0275	0.0960	0.0514	0.0184	0.0054	1.0635	0.1143	0.0602	0.0154
50	−0.2383	0.9491	0.0873	0.0334	0.0063	−0.0382	0.9904	0.0965	0.0490	0.0102
	0.0044	1.0042	0.0905	0.0493	0.0135	−0.0090	1.0272	0.1083	0.0562	0.0131
100	−0.2456	0.9627	0.0919	0.0414	0.0071	−0.0211	0.9878	0.0943	0.0459	0.0105
	−0.0049	1.0045	0.0943	0.0485	0.0131	0.0211	1.0152	0.1032	0.0537	0.0136
200	−0.2132	0.9866	0.0959	0.0433	0.0091	−0.0278	0.9938	0.0970	0.0474	0.0097
	0.0045	1.0187	0.0971	0.0520	0.0143	−0.0042	1.0122	0.1037	0.0518	0.0108
500	−0.1808	0.9789	0.0910	0.0415	0.0073	−0.0139	1.0018	0.1025	0.0510	0.0105
	0.0004	0.9993	0.0932	0.0463	0.0102	0.0001	1.0160	0.1063	0.0547	0.0118
					Normal mixture error					
20	−0.4533	0.8550	0.0715	0.0272	0.0046	−0.1006	0.9109	0.0698	0.0305	0.0061
	−0.0193	0.9520	0.0726	0.0398	0.0125	−0.0075	0.9832	0.0927	0.0463	0.0100
50	−0.2483	0.9255	0.0780	0.0335	0.0070	−0.0473	0.9579	0.0848	0.0423	0.0080
	−0.0020	0.9774	0.0823	0.0416	0.0131	−0.0131	0.9932	0.0959	0.0508	0.0104
100	−0.2501	0.9399	0.0815	0.0357	0.0075	−0.0361	0.9764	0.0931	0.0443	0.0086
	−0.0076	0.9800	0.0826	0.0432	0.0117	0.0074	1.0035	0.1033	0.0511	0.0109
200	−0.2293	0.9515	0.0885	0.0385	0.0067	−0.0335	0.9867	0.0950	0.0474	0.0092
	−0.0113	0.9822	0.0879	0.0440	0.0107	−0.0093	1.0048	0.1013	0.0508	0.0103
500	−0.1779	0.9830	0.0947	0.0461	0.0087	−0.0231	0.9973	0.1004	0.0484	0.0096
	0.0035	1.0033	0.0947	0.0478	0.0110	−0.0090	1.0114	0.1050	0.0517	0.0103
					Log-normal errors					
20	−0.4311	0.8532	0.0691	0.0270	0.0041	−0.0997	0.8820	0.0583	0.0249	0.0062
	0.0079	0.9501	0.0786	0.0418	0.0147	−0.0008	0.9510	0.0782	0.0373	0.0098
50	−0.2535	0.9076	0.0682	0.0311	0.0071	−0.0329	0.9338	0.0746	0.0366	0.0090
	−0.0034	0.9572	0.0735	0.0380	0.0125	0.0063	0.9681	0.0846	0.0426	0.0120
100	−0.2377	0.9272	0.0761	0.0347	0.0080	−0.0443	0.9555	0.0826	0.0430	0.0090
	0.0077	0.9663	0.0809	0.0435	0.0126	0.0011	0.9818	0.0898	0.0481	0.0114
200	−0.2014	0.9424	0.0798	0.0343	0.0070	−0.0293	0.9641	0.0864	0.0448	0.0094
	0.0186	0.9724	0.0814	0.0409	0.0112	−0.0039	0.9819	0.0917	0.0491	0.0105
500	−0.1779	0.9741	0.0936	0.0426	0.0089	−0.0039	0.9776	0.0888	0.0449	0.0117
	0.0042	0.9939	0.0944	0.0473	0.0134	0.0110	0.9914	0.0931	0.0478	0.0125

Notes: Under each N, the first row corresponds to LM_{RE} and the second corresponds to SLM_{RE}. X-Value: Non-IID for group interaction scheme, and IID for Queen contiguity. True parameter values: $\beta = \{5, 1, 1\}'$, and $\sigma_\mu = \sigma_v = 1$.

Table 2b. Empirical means, SDs and tail probabilities: panel with random effects, $T = 10$.

	Group: $G = N^{0.5}$					Queen contiguity				
N	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
					Normal errors					
20	−0.1770	0.9675	0.0926	0.0441	0.0062	0.0045	1.0036	0.1011	0.0516	0.0126
	−0.0032	0.9902	0.0956	0.0477	0.0093	0.0097	1.0431	0.1136	0.0606	0.0155
50	−0.1474	0.9849	0.1017	0.0448	0.0074	−0.0062	0.9968	0.0982	0.0495	0.0111
	−0.0127	0.9988	0.1000	0.0463	0.0087	0.0035	1.0210	0.1082	0.0555	0.0128
100	−0.1255	0.9903	0.0943	0.0461	0.0089	0.0020	1.0096	0.1013	0.0501	0.0119
	−0.0058	1.0003	0.0956	0.0453	0.0108	0.0076	1.0276	0.1075	0.0554	0.0125
200	−0.0942	0.9967	0.1005	0.0491	0.0089	0.0016	0.9987	0.1006	0.0467	0.0089
	0.0063	1.0041	0.1010	0.0509	0.0102	0.0050	1.0134	0.1053	0.0513	0.0096
500	−0.0685	0.9882	0.0983	0.0464	0.0085	0.0048	1.0048	0.0981	0.0492	0.0094
	0.0131	0.9928	0.0955	0.0472	0.0099	0.0072	1.0176	0.1029	0.0523	0.0103
					Normal mixture error					
20	−0.1698	0.9571	0.0849	0.0410	0.0096	−0.0079	0.9645	0.0896	0.0432	0.0089
	0.0087	0.9788	0.0852	0.0443	0.0123	0.0035	1.0023	0.1030	0.0518	0.0117
50	−0.1247	0.9790	0.0905	0.0442	0.0102	−0.0132	0.9922	0.0965	0.0481	0.0098
	0.0119	0.9925	0.0927	0.0460	0.0128	−0.0019	1.0162	0.1051	0.0534	0.0111
100	−0.1203	0.9924	0.0960	0.0478	0.0102	0.0076	0.9932	0.0997	0.0496	0.0095
	0.0001	1.0022	0.0972	0.0498	0.0113	0.0140	1.0110	0.1046	0.0544	0.0113
200	−0.1027	0.9919	0.0966	0.0454	0.0090	−0.0027	0.9973	0.0980	0.0509	0.0100
	−0.0020	0.9992	0.0974	0.0482	0.0102	0.0008	1.0119	0.1034	0.0548	0.0111
500	−0.0652	0.9984	0.1017	0.0490	0.0104	0.0024	0.9974	0.0985	0.0505	0.0100
	0.0165	1.0031	0.1001	0.0509	0.0097	0.0048	1.0100	0.1024	0.0531	0.0106
					Log-normal errors					
20	−0.1893	0.9216	0.0711	0.0334	0.0081	−0.0204	0.9433	0.0753	0.0380	0.0098
	−0.0081	0.9417	0.0721	0.0395	0.0118	−0.0061	0.9801	0.0859	0.0455	0.0127
50	−0.1255	0.9560	0.0800	0.0382	0.0107	−0.0200	0.9553	0.0805	0.0393	0.0090
	0.0128	0.9688	0.0789	0.0439	0.0135	−0.0068	0.9783	0.0869	0.0449	0.0107
100	−0.1267	0.9615	0.0876	0.0397	0.0098	0.0004	0.9760	0.0865	0.0452	0.0115
	−0.0054	0.9709	0.0864	0.0429	0.0116	0.0076	0.9934	0.0937	0.0481	0.0128
200	−0.0987	0.9755	0.0919	0.0444	0.0101	0.0023	0.9845	0.0922	0.0467	0.0109
	0.0027	0.9825	0.0907	0.0472	0.0108	0.0064	0.9989	0.0961	0.0495	0.0123
500	−0.0750	0.9778	0.0948	0.0446	0.0074	0.0048	0.9928	0.0954	0.0467	0.0113
	0.0068	0.9823	0.0946	0.0466	0.0090	0.0074	1.0054	0.0991	0.0496	0.0125

Notes: Under each N, the first row corresponds to LM_{RE} and the second corresponds to SLM_{RE}. X-Value: Non-IID for group interaction scheme, and IID for Queen contiguity. True parameter values: $\beta = \{5, 1, 1\}'$, and $\sigma_\mu = \sigma_v = 1$.

The SLM test outperforms its LM counterpart in all the experiments considered. Increasing the value of T from 3 to 10 significantly improves both tests. Another interesting phenomenon is that the null behaviour of LM_{RE} also depends upon the relative magnitude of the variance components σ_μ^2 and σ_v^2. The larger the ratio σ_v^2/σ_μ^2, the worse is the performance of the LM_{RE} test. In contrast, the performance of SLM_{RE} is very robust.

4.3.2. Power of the tests. Empirical frequencies of rejection, based on the simulated critical values, of the two tests are plotted in Figure 2 against the values of λ (horizontal line). Now each line we see from each plot of Figure 2 is in fact an overlap of two lines, one for LM_{RE} and the other for SLM_{RE}. Similar to the case of the linear regression model, the two tests have almost identical power once they are size-adjusted. The power of the tests depend heavily on the degree of spatial dependence and on the sample size. It also depends on the error distributions and the type of regressors, though to a lesser degree.

Some interesting details are as follows. The two plots in the first row of Figure 2 show that the two tests possess very low power and that the power does not seem to increase as N increase from 20 to 50 (with T fixed at 3). This is because the underlying spatial layout generates very strong spatial dependence. When N is increased from 20 to 50, the number of groups stays at $G = Round(N^{0.2}) = 2$. This means that under this spatial layout, the degree of spatial dependence at $N = 50$ is bigger than that at $N = 20$. As a result, the power does not go up, and might even go down slightly.

4.4. Performance of the tests for the fixed effects panel model

The LM and SLM tests (LM_{FE} and SLM_{FE}) introduced in Section 3.2 are compared by Monte Carlo simulation based on the following DGP:

$$Y_t = X_{1t}\beta_1 + X_{2t}\beta_2 + X_{3t}\beta_3 + u_t, \quad \text{with} \quad u_t = \mu + \varepsilon_t, \quad t = 1, \ldots, T.$$

As this model (after the transformation) and the corresponding test LM_{FE} are quite similar to the model and the test LM_{EI} given in Section 2, a bootstrap version of LM_{FE}, denoted as BLM_{FE}, is also implemented to serve as a benchmark for the finite sample performance of the proposed test SLM_{FE}. The fixed effects are generated either according to $\mu = \frac{1}{T}\sum_{t=1}^{T} X_{3t}$, or as a vector of iid $N(0, 1)$ random numbers independent of the X-values. The regressors are generated according to either the IID scheme: $X_{1,it} \overset{iid}{\sim} 2U(0, 1)$, $X_{2,it} \overset{iid}{\sim} N(0, 1)/\sqrt{3}$ and $X_{3,it} \overset{iid}{\sim} [\exp(N(0, 1) - \exp(0.5))]/(3\exp(2) - 3\exp(1))^{0.5}$, or the Non-IID scheme for group interaction layout: $X_{1,itg} = (2z_{tg} + z_{itg})/\sqrt{15}$, $X_{2,itg} = (2v_{tg} + v_{itg})/\sqrt{15}$ and $X_{3,itg} = (2e_{tg} + e_{itg})/\sqrt{15}$ with $\{z_{tg}, z_{itg}, v_{tg}, v_{itg}\}$ being iid $N(0, 1)$ and $\{e_{tg}, e_{itg}\}$ iid $[\exp(N(0, 1) - \exp(0.5))]/(3\exp(2) - 3\exp(1))^{0.5}$ across all i, t and g.

4.4.1. Null behaviour of the tests. The results reported in Tables 3a and 3b provide even stronger evidence for the effectiveness of centring and re-scaling in improving the finite sample performance of an LM test, compared with the case of the random effects model. General observations made from the Monte Carlo results for the earlier two models still hold. Our SLM test is generally comparable to the BLM test in terms of tail probabilities.

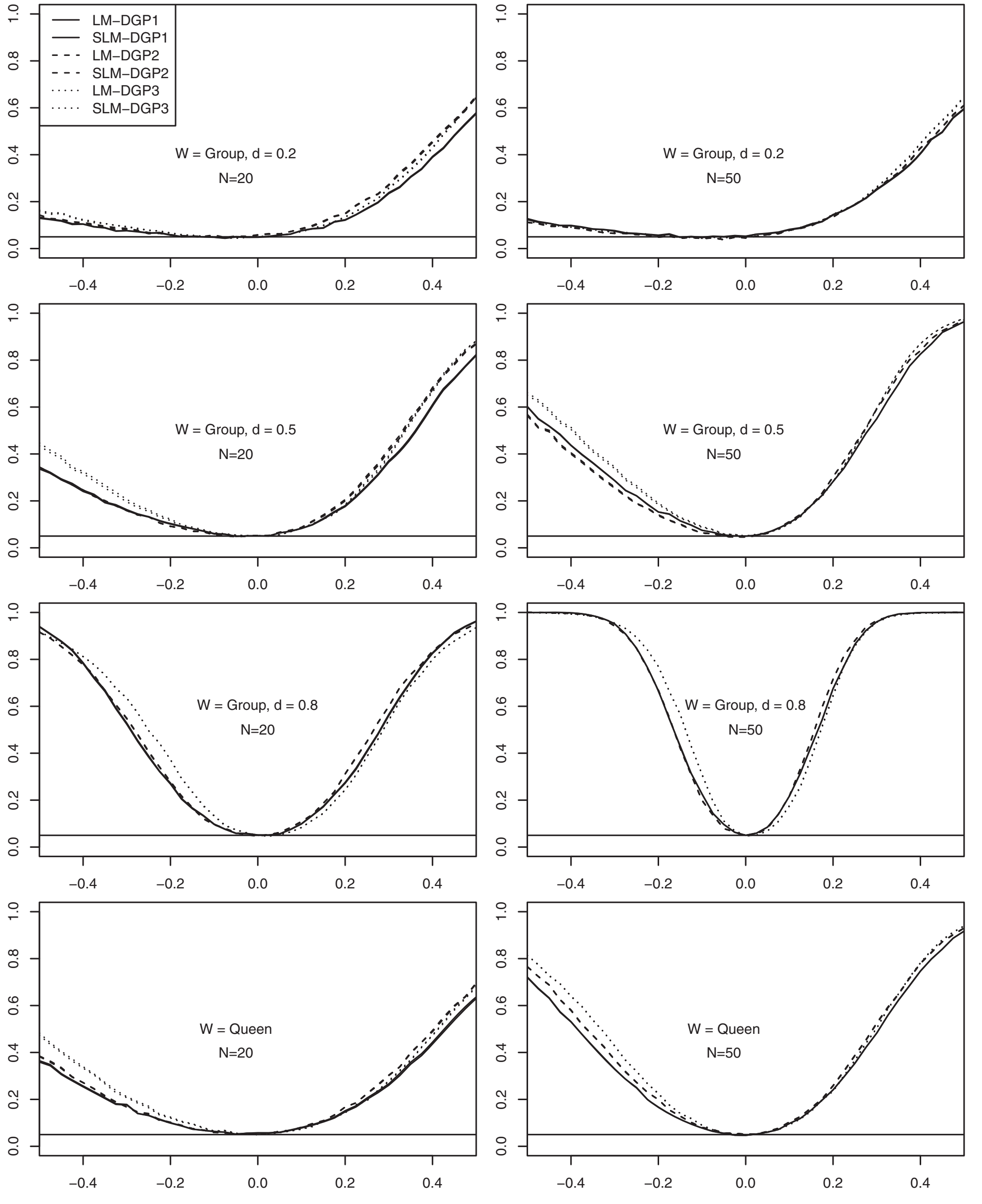

Figure 2. Size-adjusted empirical power of LM and SLM tests for random effects panel model.

4.4.2. Power of the tests. Selected results on size-adjusted power of the tests under the nominal size 5% are plotted in Figure 3. Again, LM_{FE} and SLM_{FE} have almost identical size-adjusted power. The power of BLM_{FE} (based on bootstrap size) can be lower than the other two tests when the error distribution is skewed.

Table 3a. Empirical means, SDs and tail probabilities: panel with fixed effects, $T = 3$.

	Group: $G = N^{0.5}$					Queen contiguity				
N	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
					Normal errors					
20	−0.5108	0.8690	0.0831	0.0254	0.0026	−0.0393	0.9691	0.0830	0.0394	0.0105
	−0.0124	1.0487	0.0988	0.0549	0.0193	−0.0061	1.0405	0.1060	0.0577	0.0150
	−0.5108	0.8690	0.1031	0.0531	0.0113	−0.0393	0.9691	0.0929	0.0470	0.0101
100	−0.2865	0.9491	0.0916	0.0392	0.0063	0.0098	0.9947	0.0976	0.0465	0.0102
	−0.0013	1.0110	0.0977	0.0504	0.0142	0.0051	1.0089	0.1016	0.0498	0.0111
	−0.2865	0.9491	0.1035	0.0521	0.0126	0.0098	0.9947	0.0982	0.0503	0.0119
200	−0.2776	0.9483	0.0896	0.0378	0.0058	−0.0102	1.0048	0.1024	0.0530	0.0117
	0.0020	0.9970	0.0917	0.0455	0.0119	−0.0101	1.0122	0.1054	0.0547	0.0121
	−0.2776	0.9483	0.0983	0.0495	0.0112	−0.0102	1.0048	0.1061	0.0556	0.0119
500	−0.2439	0.9751	0.0980	0.0440	0.0078	0.0160	0.9914	0.0970	0.0477	0.0084
	0.0027	1.0076	0.0973	0.0473	0.0122	0.0134	0.9943	0.0980	0.0481	0.0086
	−0.2439	0.9751	0.1024	0.0507	0.0118	0.0160	0.9914	0.0992	0.0485	0.0092
					Normal mixture errors					
20	−0.5117	0.8234	0.0645	0.0224	0.0041	−0.0215	0.9317	0.0745	0.0358	0.0068
	−0.0129	0.9825	0.0804	0.0467	0.0171	0.0128	0.9936	0.0938	0.0492	0.0120
	−0.5117	0.8234	0.0866	0.0434	0.0109	−0.0215	0.9317	0.0923	0.0447	0.0095
100	−0.2592	0.9455	0.0869	0.0358	0.0078	−0.0032	0.9657	0.0891	0.0422	0.0070
	0.0278	1.0049	0.0908	0.0484	0.0152	−0.0080	0.9789	0.0929	0.0447	0.0075
	−0.2592	0.9455	0.1030	0.0514	0.0130	−0.0032	0.9657	0.0957	0.0462	0.0092
200	−0.2816	0.9314	0.0842	0.0359	0.0050	0.0100	0.9920	0.0964	0.0498	0.0099
	−0.0022	0.9782	0.0853	0.0435	0.0108	0.0103	0.9991	0.0990	0.0518	0.0105
	−0.2816	0.9314	0.0939	0.0473	0.0098	0.0100	0.9920	0.1004	0.0527	0.0119
500	−0.2533	0.9644	0.0938	0.0415	0.0087	−0.0020	0.9972	0.0990	0.0482	0.0101
	−0.0070	0.9960	0.0920	0.0459	0.0129	−0.0047	1.0001	0.0992	0.0489	0.0101
	−0.2533	0.9644	0.0971	0.0497	0.0118	−0.0020	0.9972	0.1016	0.0505	0.0105
					Log-normal errors					
20	−0.4935	0.8121	0.0583	0.0217	0.0046	−0.0300	0.8875	0.0617	0.0273	0.0063
	0.0095	0.9678	0.0782	0.0448	0.0178	0.0033	0.9449	0.0784	0.0360	0.0097
	−0.4935	0.8121	0.0822	0.0422	0.0095	−0.0300	0.8875	0.0728	0.0341	0.0050
100	−0.2813	0.9135	0.0761	0.0341	0.0077	−0.0053	0.9501	0.0789	0.0405	0.0105
	0.0045	0.9686	0.0816	0.0434	0.0135	−0.0101	0.9626	0.0820	0.0429	0.0113
	−0.2813	0.9135	0.0923	0.0495	0.0115	−0.0053	0.9501	0.0917	0.0459	0.0103
200	−0.2842	0.9061	0.0705	0.0317	0.0049	−0.0129	0.9654	0.0883	0.0431	0.0083
	−0.0049	0.9498	0.0744	0.0369	0.0105	−0.0128	0.9722	0.0899	0.0448	0.0087
	−0.2842	0.9061	0.0832	0.0420	0.0087	−0.0129	0.9654	0.0955	0.0470	0.0088
500	−0.2550	0.9530	0.0902	0.0399	0.0077	−0.0043	0.9905	0.0964	0.0481	0.0088
	−0.0088	0.9830	0.0884	0.0439	0.0112	−0.0070	0.9933	0.0975	0.0484	0.0089
	−0.2550	0.9530	0.0962	0.0497	0.0109	−0.0043	0.9905	0.1008	0.0500	0.0089

Notes: Under each N, first row: LM_{FE}, second row: SLM_{FE} and third row: bootstrap LM_{FE}. X-Value: Non-IID for group interaction, IID for Queen contiguity; $\beta = \{1, 1, 1\}'$, $\sigma_v = 1$.

Continued

Table 3b. Empirical means, SDs and tail probabilities: panel with fixed effects, $T = 10$.

	Group: $G = N^{0.5}$					Queen contiguity				
N	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
					Normal errors					
20	−0.2167	0.9780	0.0972	0.0508	0.0064	0.0193	1.0151	0.1036	0.0538	0.0126
	0.0048	1.0133	0.1034	0.0540	0.0116	0.0197	1.0322	0.1084	0.0594	0.0146
	−0.2167	0.9780	0.1048	0.0574	0.0114	0.0193	1.0151	0.1104	0.0580	0.0134
50	−0.1363	1.0044	0.1044	0.0472	0.0090	−0.0243	0.9919	0.1006	0.0462	0.0084
	0.0189	1.0230	0.1058	0.0532	0.0122	−0.0279	0.9983	0.1026	0.0472	0.0084
	−0.1363	1.0044	0.1110	0.0532	0.0116	−0.0243	0.9919	0.1038	0.0510	0.0106
100	−0.1658	0.9835	0.0994	0.0450	0.0070	−0.0349	0.9819	0.0896	0.0460	0.0088
	−0.0057	0.9982	0.1000	0.0514	0.0088	−0.0329	0.9852	0.0908	0.0464	0.0088
	−0.1658	0.9835	0.1014	0.0498	0.0100	−0.0349	0.9819	0.0934	0.0466	0.0114
200	−0.1345	0.9909	0.1026	0.0470	0.0074	−0.0054	1.0029	0.1026	0.0470	0.0086
	−0.0016	1.0014	0.0980	0.0494	0.0094	−0.0068	1.0046	0.1032	0.0472	0.0088
	−0.1345	0.9909	0.1036	0.0512	0.0088	−0.0054	1.0029	0.1042	0.0486	0.0104
					Normal mixture errors					
20	−0.2345	0.9639	0.0944	0.0440	0.0096	0.0282	0.9654	0.0850	0.0434	0.0090
	−0.0136	0.9953	0.0930	0.0514	0.0134	0.0288	0.9810	0.0896	0.0468	0.0100
	−0.2345	0.9639	0.0970	0.0520	0.0140	0.0282	0.9654	0.0904	0.0510	0.0120
50	−0.1518	0.9726	0.0904	0.0422	0.0112	0.0080	0.9845	0.0944	0.0478	0.0088
	0.0031	0.9898	0.0948	0.0458	0.0108	0.0047	0.9907	0.0968	0.0494	0.0088
	−0.1518	0.9726	0.0980	0.0474	0.0150	0.0080	0.9845	0.1028	0.0508	0.0134
100	−0.1409	0.9941	0.0986	0.0510	0.0090	−0.0362	0.9922	0.0992	0.0460	0.0086
	0.0196	1.0077	0.1054	0.0536	0.0120	−0.0342	0.9955	0.0996	0.0466	0.0088
	−0.1409	0.9941	0.1062	0.0562	0.0122	−0.0362	0.9922	0.1028	0.0464	0.0108
200	−0.1529	0.9825	0.0966	0.0498	0.0096	−0.0067	0.9767	0.0896	0.0422	0.0072
	−0.0202	0.9929	0.0958	0.0472	0.0100	−0.0081	0.9783	0.0900	0.0424	0.0072
	−0.1529	0.9825	0.1000	0.0532	0.0112	−0.0067	0.9767	0.0940	0.0452	0.0102
					Log-normal errors					
20	−0.2386	0.9202	0.0780	0.0348	0.0088	−0.0008	0.9355	0.0702	0.0348	0.0096
	−0.0181	0.9481	0.0764	0.0432	0.0128	−0.0007	0.9500	0.0752	0.0362	0.0100
	−0.2386	0.9202	0.0818	0.0424	0.0108	−0.0008	0.9355	0.0808	0.0398	0.0104
50	−0.1520	0.9623	0.0806	0.0396	0.0108	0.0065	0.9628	0.0840	0.0438	0.0118
	0.0030	0.9777	0.0772	0.0424	0.0138	0.0031	0.9687	0.0854	0.0442	0.0118
	−0.1520	0.9623	0.0870	0.0452	0.0134	0.0065	0.9628	0.0912	0.0464	0.0128
100	−0.1487	0.9747	0.0868	0.0428	0.0122	−0.0028	0.9622	0.0816	0.0426	0.0094
	0.0117	0.9853	0.0868	0.0450	0.0156	−0.0006	0.9653	0.0834	0.0434	0.0096
	−0.1487	0.9747	0.0936	0.0486	0.0140	−0.0028	0.9622	0.0858	0.0424	0.0102
200	−0.1288	0.9845	0.0938	0.0470	0.0090	−0.0021	0.9953	0.0922	0.0522	0.0154
	0.0042	0.9944	0.0984	0.0502	0.0126	−0.0035	0.9969	0.0924	0.0528	0.0154
	−0.1288	0.9845	0.1004	0.0524	0.0112	−0.0021	0.9953	0.0982	0.0534	0.0158

Notes: Under each N, first row: LM_{FE}, second row: SLM_{FE} and third row: bootstrap LM_{FE}. X-Value: Non-IID for group interaction, IID for queen contiguity; $\beta = \{1, 1, 1\}'$, $\sigma_v = 1$.

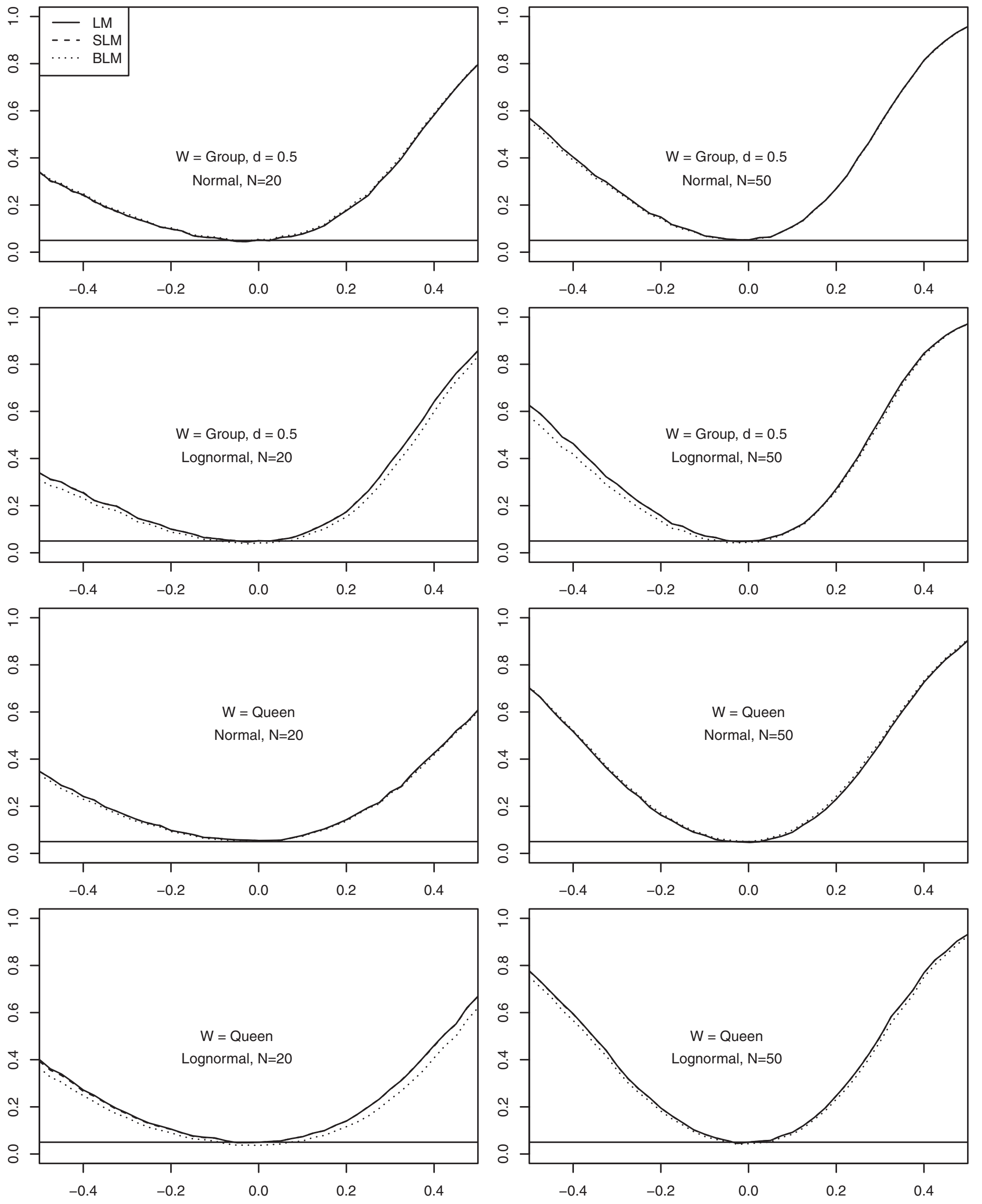

Figure 3. Size-adjusted empirical power of LM, SLM and bootstrap LM tests for fixed effects panel model.

5. CONCLUSION AND DISCUSSION

This paper recommends *standardized* LM tests of SED for the linear as well as the panel regression model. We showed that when standardizing the LM tests for spatial effects, it is important to adjust for *both* the mean and variance of the LM statistics. The mean adjustment is, however, often neglected in the literature. One important reason for the mean adjustment of the LM tests for spatial effects is that the degree of spatial dependence may grow with the sample size. This slows down the convergence speed of the MLEs (Lee, 2004a), making the concentrated score function (the key element of the LM test) more biased.

There are other LM tests for other spatial models that are derived under normal assumptions such as Baltagi et al. (2003), and the LM test for spatial lag effect in the linear SAR models (Anselin, 1988, 2001) and panel linear SAR models (Debarsy and Ertur, 2010), which can be studied in a similar manner. This paper recommends the standardized version of these LM tests because it offers improvements in their finite sample performance, in addition to preserving the simplicity of the original LM tests so that they can be easily adopted by applied researchers.

Two related and important issues, *bootstrap* and *heteroscedasticity*, deserve some further discussions as both are of potential interest for future research.[6] The two bootstrap tests and the corresponding Monte Carlo results presented in this paper are rather encouraging. The questions are whether similar results can be obtained for more complicated models, and whether a formal justification for the validity of the bootstrap methods can be given in a more general framework. A detailed study of these issues is beyond the scope of this paper. We plan to pursue them in future research. Another important issue in testing SED is the possible existence of heteroscedasticity. Our tests are developed under the assumption that the idiosyncratic errors are homoscedastic. By extending the idea of Born and Breitung (2011), we have successfully obtained SLM_{OPG}, which greatly improves upon their LM_{OPG} in case of homoscedasticity. However, the finite sample mean correction in SLM_{OPG} is still subject to the homoscedasticity assumption. Nevertheless, the derivation of SLM_{OPG} sheds much light on a possible solution to the general issue of standardizing spatial LM tests so that they are robust against unknown heteroscedasticity in both large and finite samples.

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REFERENCES

Anselin, L. (1988). *Spatial Econometrics: Methods and Models*. Dordrecht: Kluwer Academic.

Anselin, L. (2001). Rao's score test in spatial econometrics. *Journal of Statistical Planning and Inference 97*, 113–39.

Anselin, L. and A. K. Bera (1998). Spatial dependence in linear regression models with an introduction to spatial econometrics. In A. Ullah and D. E. A. Giles (Eds.), *Handbook of Applied Economic Statistics, Volume 155*, 237–89. New York: Marcel Dekker.

Baltagi, B. H. (2008). *Econometric Analysis of Panel Data* (4th ed.). New York: John Wiley.

[6] We are very grateful to an anonymous referee for raising these two important issues.

Baltagi, B. H., Y. J. Chang and Q. Li (1992). Monte Carlo results on several new and existing tests for the error component model. *Journal of Econometrics 54*, 95–120.

Baltagi, B. H., S. H. Song and W. Koh (2003). Testing panel data regression models with spatial error correlation. *Journal of Econometrics 117*, 123–50.

Born, B. and J. Breitung (2011). Simple regression based tests for spatial dependence. *Econometrics Journal 14*, 330–42.

Burridge, P. (1980). On the Cliff-Ord test for spatial correlation. *Journal of the Royal Statistical Society*, Series B *42*, 107–08.

Case, A. C. (1991). Spatial patterns in household demand. *Econometrica 59*, 953–65.

Cliff, A. and J. K. Ord (1972). Testing for spatial autocorrelation among regression residuals. *Geographical Analysis 4*, 267–84.

Debarsy, N. and C. Ertur (2010). Testing for spatial autocorrelation in a fixed effects panel data model. *Regional Science and Urban Economics 40*, 453–70.

Florax, R. J. G. M. and T. de Graaff (2004). The performance of diagnostic tests for spatial dependence in linear regression models: a meta-analysis of simulation studies. In L. Anselin, R. J. G. M. Florax and S. J. Rey (Eds.) *Advances in Spatial Econometrics*, 29–63. Berlin: Springer.

Hall, P. and J. L. Horowitz (1996). Bootstrap critical values for tests on generalized-method-of-moments estimators. *Econometrica 64*, 891–916.

Honda, Y. (1985). Testing the error components model with non-normal disturbances. *Review of Economic Studies 52*, 681–90.

Honda, Y. (1991). A standardized test for the error components model with the two-way layout. *Economics Letters 37*, 125–28.

Kelejian, H. H. and I. R. Prucha (2001). On the asymptotic distribution of the Moran *I* test statistic with applications. *Journal of Econometrics 104*, 219–57.

Koenker, R. (1981). A note on studentising a test for heteroscedasticity. *Journal of Econometrics 17*, 107–12.

Lee, L. F. (2004a). Asymptotic distributions of quasi-maximum likelihood estimators for spatial autoregressive models. *Econometrica 72*, 1899–925.

Lee, L. F. (2004b). A supplement to ‘Asymptotic distributions of quasi-maximum likelihood estimators for spatial autoregressive models’. Working paper, Department of Economics, Ohio State University.

Lee, L. F. (2007). Identification and estimation of econometric models with group interaction, contextual factors and fixed effects. *Journal of Econometrics 140*, 333–74.

Lee, L. F. and J. Yu (2010). Estimation of spatial autoregressive panel data models with fixed effects. *Journal of Econometrics 154*, 165–85.

Lin, K.-P., Z. Long and M. Wu (2007). Bootstrap test statistics for spatial econometric models. Working paper, Department of Economics, Portland State University. Available at: http://wise.xmu.edu.cn/panel2007/paper/LIN(Kuan-Pin).pdf.

Moran, P. A. P. (1950). Notes on continuous stochastic phenomena. *Biometrika 37*, 17–33.

Moulton, B. R. and W. C. Randolph (1989). Alternative tests of the error components model. *Econometrica 57*, 685–93.

Robinson, P. M. (2008). Correlation testing in time series, spatial and cross-sectional data. *Journal of Econometrics 147*, 5–16.

Yang, Z. L. (2010). A robust LM test for spatial error components. *Regional Science and Urban Economics 40*, 299–310.

APPENDIX: PROOFS OF RESULTS

To prove the theorems, we need the following lemmas.

LEMMA A.1. (Lee, 2004a) *Let v be an $N \times 1$ random vector of iid elements with mean zero, variance σ^2 and finite excess kurtosis κ. Let A be an $N \times N$ matrix with its elements denoted by $\{a_{ij}\}$. Then* $\mathrm{E}(v'Av) = \sigma^2 \mathrm{tr}(A)$ *and* $\mathrm{Var}(v'Av) = \sigma^4\kappa \sum_{i=1}^N a_{ii}^2 + \sigma^4 \mathrm{tr}(AA' + A^2)$.

LEMMA A.2. Lemma A.9, Lee, 2004b: *Suppose that A represents a sequence of $N \times N$ matrices that are uniformly bounded in both row and column sums. Elements of the $N \times k$ matrix X are uniformly bounded; and* $\lim_{n\to\infty} \frac{1}{N} X'X$ *exists and is non-singular. Let $M = I_N - X(X'X)^{-1}X'$. Then (a)* $\mathrm{tr}(MA) = \mathrm{tr}(A) + O(1)$; *(b)* $\mathrm{tr}(A'MA) = \mathrm{tr}(A'A) + O(1)$; *(c)* $\mathrm{tr}[(MA)^2] = \mathrm{tr}(A^2) + O(1)$ *and (d)* $\mathrm{tr}[(A'MA)^2] = \mathrm{tr}[(MA'A)^2] = \mathrm{tr}[A'A)^2] + O(1)$. *Furthermore, if the elements of A are such that $a_{ij} = O(h_N^{-1})$ for all i and j, then (e)* $\mathrm{tr}^2(MA) = \mathrm{tr}^2(A) + O(\frac{N}{h_N})$ *and (f)* $\sum_{i=1}^N [(MA)_{ii}]^2 = \sum_{i=1}^N (a_{ii})^2 + O(h_N^{-1})$, *where $(MA)_{ii}$ are the diagonal elements of MA, and a_{ii} are the diagonal elements of A.*

LEMMA A.3. *Let $u = G_1\mu + G_2 v$, where u and v are two independent random vectors not necessarily of the same length containing, respectively, iid elements of means zero, variances σ_μ^2 and σ_v^2, skewness α_μ and α_v, and excess kurtosis κ_μ and κ_v, and G_1 and G_2 are two conformable non-stochastic matrices. Let A be a conformable square matrix. Then, (a)* $\mathrm{E}(u'Au) = \sigma_v^2 \mathrm{tr}(\Sigma A)$ *and (b)* $\mathrm{Var}(u'Au) = \sigma_\mu^4 \kappa_\mu a_1' a_1 + \sigma_v^4 \kappa_v a_2' a_2 + \sigma_v^4 \mathrm{tr}[\Sigma(A' + A)\Sigma A]$, *where* $\Sigma = \sigma_v^{-2}\mathrm{E}(uu') = \frac{\sigma_\mu^2}{\sigma_v^2} G_1 G_1' + G_2 G_2'$, $a_1 = \mathrm{diagv}(G_1' A G_1)$ *and* $a_2 = \mathrm{diagv}(G_2' A G_2)$.

Proof: The result (a) is trivial. For (b), we have,

$$u'Au = \mu' G_1' A G_1 \mu + v' G_2' A G_2 v + \mu' G_1'(A + A') G_2 v.$$

It is easy to see that the three terms are uncorrelated. Thus,

$$\mathrm{Var}(u'Au) = \mathrm{Var}(\mu' G_1' A G_1 \mu) + \mathrm{Var}(v' G_2' A G_2 v) + \mathrm{Var}[\mu' G_1'(A' + A) G_2 v].$$

From Lemma A.1, we obtain $\mathrm{Var}(\mu' G_1' A G_1 \mu) = \sigma_\mu^4 \kappa_\mu a_1' a_1 + \sigma_\mu^4 \mathrm{tr}[A G_1 G_1'(A' + A) G_1 G_1']$, and $\mathrm{Var}(v' G_2' A G_2 v) = \sigma_v^4 \kappa_v a_2' a_2 + \sigma_v^4 \mathrm{tr}[A G_2 G_2'(A' + A) G_2 G_2']$. It is easy to show that $\mathrm{Var}(\mu' G_1'(A' + A) G_2 v) = \sigma_\mu^2 \sigma_v^2 \mathrm{tr}[(A' + A) G_2 G_2' (A' + A) G_1 G_1']$. Putting these three expressions together leads to (b). □

Proof of Theorem 2.1: First, we note that

$$\tilde{u}' W \tilde{u} - S_1 \tilde{u}'\tilde{u} = \tilde{u}'(W - S_1 I_N)\tilde{u} = u' M (W - S_1 I_N) M u = u' A u.$$

Under H_0 and Assumption 2.1, Lemma A.1 is applicable to $u'Au$, which gives $\mathrm{E}u'Au = \sigma_\varepsilon^2 \mathrm{tr}\, A = 0$ and $\mathrm{Var}(u'Au) = \sigma_\varepsilon^4 \kappa_\varepsilon \sum_{i=1}^n a_{ii}^2 + \sigma_\varepsilon^4[\mathrm{tr}(AA') + \mathrm{tr}(A^2)]$. Letting $W^* = W - S_1 I_N$, we have $A = M W^* M$. By Lemma A.2(a) and Assumption 2.2, $\mathrm{tr}(WM) = O(1)$ which gives $S_1 = O(N^{-1})$. Hence, the elements of W^* are of uniform order $O(h_N^{-1})$. Under Assumption 2.3, M is uniformly bounded in both row and column sums (Lee, 2004a, Appendix A). It follows that the elements of A are of uniform order $O(1/h_N)$, and that the row and column sums of the matrix A are uniformly bounded. Thus, the generalized central limit theorem for linear-quadratic forms of Lee (2004a, Appendix A) is applicable, which shows that $u'Au$ is asymptotically normal, or equivalently,[7]

$$\frac{u'Au}{\sigma_\varepsilon^2(\kappa_\varepsilon S_2 + S_3)^{\frac{1}{2}}} = \frac{\tilde{u}'W\tilde{u} - S_1\tilde{u}'\tilde{u}}{\sigma_\varepsilon^2(\kappa_\varepsilon S_2 + S_3)^{\frac{1}{2}}} \xrightarrow{d} N(0,1).$$

Now, it is easy to show that under H_0 $\tilde{\sigma}_\varepsilon^2 \equiv \tilde{u}'\tilde{u}/N \xrightarrow{p} \sigma_\varepsilon^2$ and $\tilde{\kappa}_\varepsilon \equiv \frac{1}{n\tilde{\sigma}_\varepsilon^4}\sum_{i=1}^n \tilde{u}_i^4 - 3 \xrightarrow{p} \kappa_\varepsilon$ (see Yang, 2010, for the proof of a similar result). The result (a) thus follows from Slutsky's theorem by replacing σ_ε by $\tilde{\sigma}_\varepsilon$ and κ_ε by $\tilde{\kappa}_\varepsilon$.

To prove the asymptotic equivalence of LM_{EI} and SLM_{EI}, we note that

$$SLM_{EI} = \left(\frac{S_0}{\tilde{\kappa}_\varepsilon S_2 + S_3}\right)^{\frac{1}{2}} LM_{EI} - \frac{NS_1}{(\tilde{\kappa}_\varepsilon S_2 + S_3)^{\frac{1}{2}}}. \quad \text{(A.1)}$$

Thus, it is sufficient to show that the factor in front of LM_{EI} is $O_p(1)$ and the second term is $o_p(1)$. As the elements $\{w_{ij}^*\}$ of W^* are uniformly $O(h_N^{-1})$, Lemma A.2(e) and Assumption 2.2 ($w_{ii} = 0$) lead to $S_2 = \sum_{i=1}^n a_{ii}^2 = \sum_{i=1}^N (w_{ii}^*)^2 + O(h_N^{-1}) = O(h_N^{-1})$. Lemma A.2(b) and (c) lead to $S_3 = S_0 + O(1)$. Since the elements of W are uniformly $O(h_N^{-1})$ and the row sums of W are uniformly bounded, it follows that the elements of WW' and W^2 are uniformly $O(h_N^{-1})$. Hence, S_0 is $O(N/h_N)$, and so is S_3. Furthermore, $\tilde{\kappa}_\varepsilon = O_p(1)$. These lead to $(S_0/(\tilde{\kappa}_\varepsilon S_2 + S_3))^{\frac{1}{2}} = O_p(1)$ and $NS_1/(\tilde{\kappa}_\varepsilon S_2 + S_3)^{\frac{1}{2}} = O_p((h_N/N)^{\frac{1}{2}}) = o_p(1)$, showing that $\text{LM}_B \sim \text{LM}_B^*$. Similarly, one can show that $\text{Var}(I) \sim S_0$, and hence $\text{LM}_B \sim I^*$. Finally, it is evident that $I^o \sim I^*$. □

Proof of Theorem 2.2: To show (a), we have, $\tilde{u}'(W - S_1 I_N)\tilde{u} = u'Au = u'(A_l + A_u')u + u'A_d u = u'\zeta + u'A_d u$. It can be shown that $u'\zeta = \sum_{i=1}^n u_i\zeta_i$ and $u'A_d u = \sum_{i=}^n a_{ii}u_i^2$ are uncorrelated, and $u_i\zeta_i$ and $u_j\zeta_j$, $i \neq j$, are uncorrelated, where $\{a_{ii}\}$ are the diagonal elements of A. These lead to a natural estimator of Var $(u'Au)$:

$$\sum_{i=1}^n \tilde{u}_i^2\tilde{\zeta}_i^2 + \sum_{i=1}^n a_{ii}^2\tilde{u}_i^4.$$

The result (a) thus follows from $\frac{1}{N}(\sum_{i=1}^N \tilde{u}_i^2\tilde{\zeta}_i^2 + \sum_{i=1}^n a_{ii}^2\tilde{u}_i^4) - \frac{1}{N}\sigma_\varepsilon^4(\kappa_\varepsilon S_2 + S_3) \xrightarrow{p} 0$, and the result (b) follows from $\frac{1}{\sqrt{N}}\tilde{u}'(W - S_1 I_N)\tilde{u} - \frac{1}{\sqrt{N}}\tilde{u}'W\tilde{u} \xrightarrow{p} 0$, and $\frac{1}{N}(\sum_{i=1}^N \tilde{u}_i^2\tilde{\zeta}_i^2 + \sum_{i=1}^n a_{ii}^2\tilde{u}_i^4) - \frac{1}{N}\sum_{i=1}^N \tilde{u}_i^2\tilde{\xi}_i^2 \xrightarrow{p} 0$, which are all trivial. □

Proof of Theorem 3.1: We have $\tilde{u} = Y - X\tilde{\beta} = Y - X(X'\tilde{\Sigma}^{-1}X)^{-1}X'\tilde{\Sigma}^{-1}Y \equiv M(\tilde{\rho})Y$. The numerator of LM_{RE} becomes $\tilde{u}'A(\tilde{\rho})\tilde{u} = Y'M'(\tilde{\rho})A(\tilde{\rho})M(\tilde{\rho})Y = u'M'(\tilde{\rho})A(\tilde{\rho})M(\tilde{\rho})u \equiv u'C_0(\tilde{\rho})u$. By the mean value theorem,

$$u'C_0(\tilde{\rho})u = u'C_0(\rho)u + u'\dot{C}_0(\bar{\rho})u\ (\tilde{\rho} - \rho),$$

where $\bar{\rho}$ lies between $\tilde{\rho}$ and ρ, $\dot{C}_0(\rho) = \frac{\partial C_0(\rho)}{\partial\rho} = 2M'(\rho)[\rho(\bar{J}_T \otimes W) - (\bar{J}_T \otimes I_N)P(\rho)A(\rho)]M(\rho)$, and $P(\rho) = X(X'\Sigma_0^{-1}X)^{-1}X'$. From the results of Lee (2004a, Appendix), it is easy to see the elements of $C_0(\rho)$ are of uniform order $O(1/h_N)$ uniformly in ρ, and so are the elements of $\dot{C}_0(\bar{\rho})$. As $\tilde{\rho}$ is consistent, it follows that $\text{E}[u'C_0(\tilde{\rho})u] \sim \text{E}\,[u'C_0(\rho)u] = \sigma_v^2\text{tr}\,[\Sigma_0 C_0(\rho)]$. This leads to a centred quantity $\tilde{u}'A(\tilde{\rho})\tilde{u} - \sigma_v^2\text{tr}\,[\Sigma_0 C_0(\rho)]$, or its feasible version:

$$\tilde{u}'A(\tilde{\rho})\tilde{u} - \frac{1}{NT-k}\text{tr}\,[\tilde{\Sigma}_0 C_0(\tilde{\rho})]\ \tilde{u}'\tilde{\Sigma}_0^{-1}\tilde{u} = \tilde{u}'\left(\tilde{A} - \tilde{s}_0\tilde{\Sigma}_0^{-1}\right)\tilde{u},$$

which gives the numerator of SLM_{SE}.

[7] Lee (2004a) generalized the results of Kelejian and Prucha (2001) to cover the case where h_N is unbounded. Lee's results require the matrix A to be symmetric. If it is not, it can be replaced by $\frac{1}{2}(A + A')$.

As $\tilde{u}'(\tilde{A} - \tilde{s}_0 \tilde{\Sigma}_0^{-1})\tilde{u} = u'M(\tilde{\rho})'(\tilde{A} - \tilde{s}_0 \tilde{\Sigma}_0^{-1})M(\tilde{\rho})u = u'C(\tilde{\rho})u$, applying the mean value theorem again leads to $u'C(\tilde{\rho})u \sim u'C(\rho)u$. It follows that $\text{Var}\,[u'C(\tilde{\rho})u] \sim \text{Var}\,[u'C(\rho)u]$. Now, $u'C(\rho)u$ can be decomposed into the following three terms,

$$\mu'(\iota_T' \otimes I_N)C(\rho)(\iota_T \otimes I_N)\mu + v'C(\rho)v + \mu'(\iota_T' \otimes I_N)C(\rho)v,$$

which are either independent or asymptotically independent. Thus, the asymptotic normality of the first two terms on the right-hand side of the above equation follow from the generalized central limit theorem for linear-quadratic forms of Lee (2004a, Appendix A). The asymptotic normality of the last term follows from the fact that the two random vectors involved are independent. The mean and variance of $u'C(\rho)u$ can be easily obtained from Lemma A.3. In fact, $E(u'C(\rho)u) = 0$, and

$$\text{Var}\,(u'C(\rho)u) = \sigma_v^4\{\phi^2\kappa_\mu a_1'a_1 + \kappa_v a_2'a_2 + \text{tr}\,[\Sigma_0(C(\rho)' + C(\rho))\Sigma_0 C(\rho)]\}.$$

Thus the result in (a) follows and $SLM_{RE} \xrightarrow{d} N(0, 1)$.

To prove the result in (b), let $X(\rho) = \Sigma_0^{-\frac{1}{2}} X$ and $M^*(\rho) = I_{NT} - X(\rho)[X'(\rho)X(\rho)]^{-1}X'(\rho)$. Assumption 3.3 and the structure of $\Sigma_0^{-\frac{1}{2}}$ guarantee that the elements of $X(\rho)$ are bounded uniformly in both N and ρ. Thus, Lemma A.2 is applicable on $M^*(\rho)$ for each ρ. We have $C_0(\rho) = M'(\rho)A(\rho)M(\rho) = \Sigma_0^{-\frac{1}{2}} M^*(\rho)A(\rho)M^*(\rho)\Sigma_0^{-\frac{1}{2}}$. Thus,

$$\begin{aligned}
\text{tr}\,[\Sigma_0 C_0(\rho)] &= \text{tr}\,[M^*(\rho)A(\rho)M^*(\rho)\Sigma_0] \\
&= \text{tr}\,[A(\rho)M^*(\rho)\Sigma_0] + O(1) \qquad \text{(by Lemma A.2)} \\
&= \text{tr}\,[M^*(\rho)\Sigma_0 A(\rho)] + O(1) \\
&= \text{tr}\,[\Sigma_0 A(\rho)] + O(1) \qquad \text{(by Lemma A.2)} \\
&= O(1).
\end{aligned}$$

Thus, $a_0\,(\rho) = \frac{1}{NT-k}\text{tr}\,(\Sigma_0 C_0(\rho)] = O(\frac{1}{N})$. Similarly, by successively applying Lemma A.2, one shows that

$$\begin{aligned}
\text{tr}\,[\Sigma_0(C_0(\rho)' + C_0(\rho))\Sigma_0 C_0(\rho)] &= \text{tr}\,[M^*(\rho)(A(\rho)' + A(\rho))M^*(\rho)\Sigma_0 M^*(\rho)A(\rho)M^*(\rho)\Sigma_0] \\
&= \text{tr}\,[(A(\rho)' + A(\rho))\Sigma_0 A(\rho)\Sigma_0] + O(1) \\
&= (T - 1 + \rho^2)S_0 + O(1).
\end{aligned}$$

It follows that $\text{tr}\,[\Sigma_0(C(\rho)' + C(\rho))\Sigma_0 C(\rho)] = (T - 1 + \rho^2)S_0 + O(1)$ as $C(\rho) = C_0(\rho) - a_0(\rho)\Sigma_0^{-1}M(\rho)0$. Under Assumption 3.2, the elements of W^2 and WW' are of uniform order $O(1/h_N)$. It follows that S_0, the quantity in LM_{RE}, is $O(N/h_N)$. Hence,

$$\text{tr}\,[\Sigma_0 C(\rho)\Sigma_0 C(\rho)] \sim (T - 1 + \rho^2)S_0 = O(N/h_N).$$

Finally, Lemma A.2(f) leads to $a_1'a_1 = O(1/h_N)$ and $a_2'a_2 = O(1/h_N)$. The result in (b) thus follows and the two LM tests given in (3.6) and (3.8) are asymptotically equivalent. □

Proof of Theorem 3.2: The proof of this theorem parallels that of Theorem 2.1. □

Proof of Corollary 3.1: Note that $(\varepsilon_1^*, \varepsilon_2^*, \ldots, \varepsilon_{T-1}^*) = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_T)F_{T,T-1}$. With $\varepsilon_{i\cdot}^{*\prime}$ denoting the ith row of the $N \times (T-1)$ matrix $(\varepsilon_1^*, \varepsilon_2^*, \ldots, \varepsilon_{T-1}^*)$ and $\varepsilon_{i\cdot}'$ the ith row of the $N \times T$ matrix $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_T)$, we have

$$\text{Var}\,(1_{T-1}'\varepsilon_{i\cdot}^*) = \text{Var}\,(1_{T-1}'F_{T,T-1}'\varepsilon_{i\cdot}) = 1_{T-1}'F_{T,T-1}'\text{Var}\,(\varepsilon_{i\cdot})F_{T,T-1}1_{T-1} = (T-1)\sigma_\varepsilon^2.$$

Denoting $c = F_{T,T-1}1_{T-1}$, and applying Lemma A.1, we have

$$\mathrm{Var}\,[(1'_{T-1}\varepsilon^*_{i\cdot})^2] = \mathrm{Var}\,[(c'\varepsilon_{i\cdot})^2] = \mathrm{Var}\,[\varepsilon'_{i\cdot}(cc')\varepsilon_{i\cdot}] = \sigma_v^4\kappa_v \sum_{t=1}^{T} c_t^4 + 2(T-1)^2\sigma_v^4.$$

It follows that $\mathrm{E}[(1'_{T-1}\varepsilon^*_{i\cdot})^4] = \mathrm{E}\,[(c'\varepsilon_{i\cdot})^4] = \sigma_v^4\kappa_v \sum_{t=1}^{T} c_t^4 + 3(T-1)^2\sigma_v^4$. As $c'\varepsilon_{i\cdot}$ are iid, Kolmogorov's law of large numbers ensures that

$$\frac{1}{N}\sum_{i=1}^{N}(1'_{T-1}\varepsilon^*_{i\cdot})^4 \to \sigma_v^4\kappa_v \sum_{t=1}^{T} c_t^4 + 3(T-1)^2\sigma_v^4.$$

The result follows by moving the terms other than κ_v to the left and then replacing $\varepsilon^*_{i\cdot}$ by $\tilde{\varepsilon}^*_{i\cdot}$, and σ_v^2 by $\frac{1}{N(T-1)}\tilde{\varepsilon}^{*\prime}\tilde{\varepsilon}^*$. □