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Specification Tests for Temporal Heterogeneity in Spatial Panel Data Models with Fixed Effects*

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Abstract

We propose score type tests for testing the existence of temporal heterogeneity in slope and spatial parameters in spatial panel data (SPD) models, allowing for the presence of individual-specific and/or time-specific fixed effects (or in general intercept heterogeneity). The SPD model with spatial lag effect is treated in detail by first considering the model with individual-specific effects only, and then extending it to the model with both individual and time specific effects. Two types of tests (*naïve and robust*) are proposed, and their asymptotic properties are presented. These tests are then fully extended to an SPD model with both spatial lag and spatial error effects. Monte Carlo results show that the robust tests have much superior finite and large sample properties than the naive tests. Thus, the proposed robust tests provide reliable tools for identifying possible existence of temporal heterogeneity in regression and spatial coefficients. Empirical illustrations of the proposed tests are given.

Key Words: Spatial panels; Fixed effects; Time-Varying Covariate Effects; Time-Varying Spatial Effects; Change Points.

JEL Classification: C10, C13, C21, C23, C15

1. Introduction

Being able to control *unobserved heterogeneity* may be one of the most important features of a panel data (PD) model. Heterogeneity may occur on intercept, slope and error variance. In a spatial PD model (SDP), it may also occur on spatial parameters (Anselin, 1988). Heterogeneity in variance is often referred to as *heteroskedasticity*. Heterogeneity may occur in spatial and/or temporal dimension. When unobserved heterogeneity occurs on the intercept, it gives rise to individual-specific effects and/or time-specific effects, which may appear in the model *additively* or *interactively*. Change point or structural break may be considered as a special case of unobserved heterogeneity.

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Temporal heterogeneity is a common feature in an SPD model. It is an important issue but relatively unexplored in the spatial panel literature. Temporal heterogeneity may occur as a result of a credit crunch or debt, an oil price shock, a tax policy change, a fad or fashion in society, a discovery of a new medicine, and an enaction of new governmental program (Bai, 2010). Many economic processes, for example, housing decisions, technology adoption, unemployment, welfare participation, price decisions, crime rates, trade flows, etc., exhibit time heterogeneity patterns. Values observed at one location depend on the values of neighboring observations at nearby locations. Therefore, one may be interested in the question that does this dependence stay the same over time.

There is a sizable literature on temporal heterogeneity, mostly in change points, in regular panel data models, see, Bai (2010), Liao (2008), Feng et al. (2009), to name a few. In spatial models, previous literature has focused more on the spatial heterogeneity, see, e.g., Baltagi (2008). The literature on temporal heterogeneity in spatial panel data models is rather thin. We are only aware of the following two works, Sengupta (2017) who proposes tests for a structural break in a spatial panel model without fixed effects, and Li (2018) who study fixed effects SPD models with structural changes. SPD models with temporal heterogeneity also appear in finance literature, see, e.g., Blasques et al. (2016) and Catania and Billé (2017), but under a different setting where the time dimension is much larger than the spatial dimension.

In this paper, we extend the fixed effects SPD models of Lee and Yu (2010), see also Baltagi and Yang (2013) and Yang et al. (2016), to allow for temporal heterogeneity in regression as well as spatial coefficients. We focus on testing problems. A general method, *the adjusted quasi score* (AQS) method, is introduced for constructing tests for temporal homogeneity/heterogeneity on regression coefficients and spatial correlation coefficients, in spatial panel data (SPD) models, allowing the existence of spatial-temporal heterogeneity in the intercepts (or fixed effects). The SPD model with spatial lag effect is first treated in detail by first considering the model with individual-specific effects only, and then extended to the model with both individual and time specific effects. Two types of tests (*naïve and robust*) are proposed, and their asymptotic properties are presented. These tests are then fully extended to an SPD model with both spatial lag and spatial error effects. Monte Carlo results show that the robust tests have much superior finite and large sample properties than the naive tests. Thus, the proposed robust tests provide reliable tools for identifying possible existence of temporal heterogeneity in regression and spatial coefficients. Empirical illustrations of the proposed tests are given.

The rest of the paper is organized as follows. Section 2 presents AQS tests for the panel SL model with one-way and two-way fixed effects, along with the general method for constructing non-normality robust AQS tests. Section 3 generalizes these tests to a SPD model with both spatial lag and spatial error dependence. Section 4 presents Monte Carlo results. Section 5 presents some empirical applications to illustrate the proposed methods. Section 6 discuss possible extensions and concludes the paper.

2. Test for Temporal Heterogeneity in Panel SL Model

In this section, we consider specification tests for testing the existence of temporal heterogeneity in slope and spatial parameters in a spatial panel data (SPD) model where the spatial effects take the form of the so-called spatial lag (SL) dependence. We first derive the tests for a panel SL model with one-way fixed effects (1FE) (i.e., individual-specific fixed effects or unobserved *spatial heterogeneity* in the intercept), where a general principle is given for the construction of AQS tests, and then extend these tests to a panel SL model with two-way FEs (i.e., both individual and time specific effects or unobserved *spatiotemporal heterogeneity* in intercepts). Asymptotic properties of the proposed tests are presented. Some key quantities for calculating the test statistics, the Hessian matrix, expected Hessian matrix, and the variance-covariance (VC) matrix of the AQS function, are given in Appendix B, and Proofs are sketched in Appendix C.

2.1. Panel SL model with one-way FE

Consider the following panel SL model individual-specific FE:

$$Y_{nt} = \lambda_t W_n Y_{nt} + X_{nt}\beta_t + c_n + V_{nt}, \tag{2.1}$$

where Y_{nt} is an $n \times 1$ vector of observations on the dependent variable for $t = 1, 2, \ldots, T$; X_{nt} is an $n \times k$ matrix containing the values of k exogenous regressors, W_n is an $n \times n$ spatial weight matrix; V_{nt} is an $n \times 1$ vector of independent and identically distributed (iid) disturbances with mean zero and variance σ^2; λ_t is the *spatial lag parameters* and β_t is the $k \times 1$ vector of regression coefficients for the tth period; and c_n denotes the individual-specific fixed effects or the spatial heterogeneity in intercept.

We are primarily interested in testing for the temporal homogeneity of the regression coefficients and the spatial coefficients, i.e., the tests of the null hypothesis:

$$H_0 : \lambda_1 = \cdots = \lambda_T = \lambda \text{ and } \beta_1 = \cdots = \beta_T = \beta, \tag{2.2}$$

allowing the existence of the unobserved cross-sectional heterogeneity in the intercept, i.e., the individual specific fixed effects c_n. However, the methods developed in this paper can be applied to other type of tests as well. An interesting case would be tests for detecting *change points* as discussed latter. We develop score-type of tests as they require only the estimation of the null model. However, the construction of the score-type of tests requires the full quasi score (QS) function, based on the quasi Gaussian loglikelihood.

Denote $\boldsymbol{\beta} = (\beta_1', \ldots, \beta_T')'$, $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_T)'$, and $\boldsymbol{\theta} = (\boldsymbol{\beta}', \boldsymbol{\lambda}', \sigma^2)'$. Define $A_n(\lambda_t) = I_n - \lambda_t W_n$, $t = 1, \ldots, T$. The Gaussian loglikelihood function of the model is

$$\ell_{\texttt{SL1}}(\boldsymbol{\theta}, c_n) = -\frac{nT}{2}\ln(2\pi\sigma^2) + \sum_{t=1}^{T}\ln|A_n(\lambda_t)| - \frac{1}{2\sigma^2}\sum_{t=1}^{T} V_{nt}'(\lambda_t, \beta_t, c_n)V_{nt}(\lambda_t, \beta_t, c_n), \tag{2.3}$$

where $V_{nt}(\beta_t, \lambda_t, c_n) = A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t - c_n,\ T = 1, \ldots, T$.

Adjusted (quasi) score functions. As $\{\lambda_t\}$ and $\{\beta_t\}$ are allowed to change with t, the usual fixed-effects estimation method, such as first differencing or orthogonal transformation, cannot be applied. We propose an *adjusted score* (AS) or *adjusted quasi score* (AQS) method to estimate the model. This leads to a set of AS or AQS functions that are unbiased and hence a set of score-type of tests, reffered to as the **AQS tests** in this paper, for testing the homogeneity/heterogeneity of the spatial parameters and the regression coefficients. The method proceeds by first eliminating c_n through direct maximization of the loglikelihood function, given the other model parameters $\boldsymbol{\theta}$, and then adjusting the resulted concentrated (quasi) score function to eliminate the asymptotic bias or inconsistency. First, given $\boldsymbol{\theta}$, $\ell_{\texttt{SL1}}(\theta, c_n)$ is partially maximized at:

$$\tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda}) = \tfrac{1}{T}\textstyle\sum_{t=1}^T [A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t], \tag{2.4}$$

which gives the concentrated loglikelihood function of $\boldsymbol{\theta}$ upon substitution:

$$\ell^c_{\texttt{SL1}}(\boldsymbol{\theta}) = -\frac{nT}{2}\ln(2\pi\sigma^2) + \sum_{t=1}^T \ln|A_n(\lambda_t)| - \frac{1}{2\sigma^2}\sum_{t=1}^T \widetilde{V}'_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda})'\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}), \tag{2.5}$$

where $\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) = A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t - \tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda})$. Differentiating $\ell^c_{\texttt{SL1}}(\boldsymbol{\theta})$ gives the concentrated score (CS) or concentrated quasi score (CQS) function of $\boldsymbol{\theta}$:

$$S^c_{\texttt{SL1}}(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2}X'_{nt}\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}), \ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2}(W_nY_{nt})'\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) - \mathrm{tr}[G_n(\lambda_t)], \ \ t = 1, \ldots, T, \\ -\frac{nT}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^T \widetilde{V}'_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda})\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}), \end{cases} \tag{2.6}$$

where $G_n(\lambda_t) = W_nA_n^{-1}(\lambda_t), t = 1, \ldots, T$.

For the subsequent theoretical developments, we need to differentiate the general parameter vector $\boldsymbol{\theta} = (\boldsymbol{\beta}', \boldsymbol{\lambda}', \sigma^2)'$ and its true value $\boldsymbol{\theta}_0 = (\boldsymbol{\beta}'_0, \boldsymbol{\lambda}'_0, \sigma^2_0)'$. We view that the Model (2.1) holds only under the true values of the parameters. Furthermore, the usual expectation and variance operators correspond to $\boldsymbol{\theta}_0$.

At the true $\boldsymbol{\theta}_0$, $\tilde{c}_n(\boldsymbol{\beta}_0, \boldsymbol{\lambda}_0) = \overline{V}_n + c_n$ and thus $\widetilde{V}_{nt} \equiv \widetilde{V}_{nt}(\boldsymbol{\beta}_0, \boldsymbol{\lambda}_0) = V_{nt} - \overline{V}_n$, where $\overline{V}_n = \frac{1}{T}\sum_{t=1}^T V_{nt}$, and $W_nY_{nt} = G_n(\lambda_{t0})(X_{nt}\beta_{t0} + c_n + V_{nt})$. It is easy to show that,

$$\mathrm{E}[S^c_{\texttt{SL1}}(\boldsymbol{\theta}_0)] = \left\{0'_{Tk},\ -\tfrac{1}{T}\mathrm{tr}[G_n(\lambda_{t0})],\ t = 1, \ldots T,\ -\tfrac{n}{2\sigma_0^2}\right\}',$$

where 0_m denotes an $m \times 1$ vector of zeros.

Therefore, the direct approach does not yield consistent estimators unless T goes to large. Even if T goes large with n, there will be an asymptotic bias of order $O(\frac{1}{T^2})$ for the estimators of $\{\lambda_t\}$, and an asymptotic bias of order $O(\frac{1}{T})$ for the estimator of σ^2. Therefore, the concentrated (quasi) score function given in (2.6) should be adjusted by subtracting the above bias vector from it, leading to the *adjusted score* (AS) or *adjusted* *quasi score* (AQS) function as

$$S^{\star}_{\texttt{SL1}}(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2} X'_{nt} \widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}),\ t = 1, \ldots, T, \\ \frac{1}{\sigma^2} (W_n Y_{nt})' \widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) - \frac{T-1}{T} \text{tr}[G_n(\lambda_t)],\ t = 1, \ldots, T, \\ -\frac{n(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{t=1}^{T} \widetilde{V}'_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) \widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}). \end{cases} \tag{2.7}$$

It is easy to show that $\text{E}[S^{\star}_{\texttt{SL1}}(\boldsymbol{\theta})] = 0$, and that $\frac{1}{nT} S^{\star}_{\texttt{SL1}}(\boldsymbol{\theta}_0) \xrightarrow{p} 0$ as $n \to \infty$ alone, or both n and T go infinity. Thus, this AQS function gives a set of unbiased estimating functions, and paves the way for developing asymptotically valid score-type tests.[1] Simplifying this AQS function under the null hypothesis gives AQS function of the null model, leading to the constrained estimates of the null model parameters. See the end of section for details.

Construction of the AQS tests. Denote the constrained (under H_0) estimator of $\boldsymbol{\theta}$ as $\tilde{\boldsymbol{\theta}}_{\texttt{SL1}}$. In case of testing for temporal homogeneity, for example, we have $\tilde{\boldsymbol{\beta}}_{\texttt{SL1}} = 1_T \otimes \tilde{\beta}_{\texttt{SL1}}$, $\tilde{\boldsymbol{\lambda}}_{\texttt{SL1}} = 1_T \otimes \tilde{\lambda}_{\texttt{SL1}}$, and $\tilde{\boldsymbol{\theta}}_{\texttt{SL1}} = (\tilde{\boldsymbol{\beta}}'_{\texttt{SL1}}, \tilde{\boldsymbol{\lambda}}'_{\texttt{SL1}}, \tilde{\sigma}^2_{\texttt{SL1}})'$, where $\tilde{\beta}_{\texttt{SL1}}$ and $\tilde{\lambda}_{\texttt{SL1}}$ are the estimators of the common β and common λ under (2.2). Let $J_{\texttt{SL1}}(\boldsymbol{\theta}) = -\frac{\partial}{\partial \boldsymbol{\theta}'} S^{\star}_{\texttt{SL1}}(\boldsymbol{\theta})$ be the negative Hessian matrix of the AQS function with its expression given in Appendix B.1. The *usual* score test, treating $S^{\star}_{\texttt{SL1}}(\boldsymbol{\theta})$ as a genuine score vector, takes the form:

$$T_{\texttt{SL1}} = S^{\star}_{\texttt{SL1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}})' J^{-1}_{\texttt{SL1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}}) S^{\star}_{\texttt{SL1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}}). \tag{2.8}$$

However, $S^{\star}_{\texttt{SL1}}(\boldsymbol{\theta})$ is not a genuine score function even if the errors are normal, as it comes from the original score function after some adjustments. In this case, the well-known information matrix equality (IME) or the generalized IME (Cameron and Trivedy, 2005); Wooldridge, 2010)) does not hold. Hence, the $T_{\texttt{SL1}}$ constructed in this 'usual' way may not be a valid test statistic, even if the errors are normal.

To address these issues, denoting $k_q = \dim(\boldsymbol{\theta}) = (k+1)T + 1$, we put our testing problem in a general framework with null hypothesis being written as

$$H_0 : C\boldsymbol{\theta}_0 = 0, \tag{2.9}$$

where C is a $k_p \times k_q$ matrix generating k_p linear contrasts in the parameter vector $\boldsymbol{\theta}$.

For the null hypothesis defined in (2.2), $k_p = (T-1)(k+1)$, $C = \texttt{blkdiag}\{C_{T,k}, C_{T,1}, 0_{k_p}\}$,

$$C_{T,m} = \begin{pmatrix} I_m & -I_m & \mathbf{0}_m & \cdots & \mathbf{0}_m \\ I_m & \mathbf{0}_m & -I_m & \cdots & \mathbf{0}_m \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I_m & \mathbf{0}_m & \mathbf{0}_m & \cdots & I_m \end{pmatrix},$$

for $m = k$ and 1, where $\mathbf{0}_m$ denotes an $m \times m$ matrix of zeros to differentiate it from

[1] Solving the estimating equation, $S^{\star}_{\texttt{SL1}}(\boldsymbol{\theta}) = 0$, gives the AQS estimator of $\boldsymbol{\theta}$, which can also be referred to as an M-estimator. This equation solving process can be simplified by first solving the last equation for σ^2, leading the the constrained estimator of σ^2 given $(\boldsymbol{\beta}, \boldsymbol{\lambda})$, $\hat{\sigma}^{\star 2}_{\texttt{SL1}}(\boldsymbol{\beta}, \boldsymbol{\lambda}) = \frac{1}{n(T-1)} \sum_{t=1}^{T} \widetilde{V}'_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) \widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda})$, and then solving the resulted concentrated AQS equations for $(\boldsymbol{\beta}, \boldsymbol{\lambda})$.

the vector 0_m. Obviously, this set-up is not restricted to the null hypothesis defined in (2.2). This is particularly meaningful in the sense that when the null hypothesis defined in (2.2) is rejected, one would proceed to perform further tests to detect the 'true' temporal heterogeneity. An interesting case would be the test of the form, for $1 < b_0, \ell_0 < T$,

$$H_0 : \beta_1 = \cdots = \beta_{b_0} \neq \beta_{b_0+1} = \cdots = \beta_T \text{ and } \lambda_1 = \cdots = \lambda_{\ell_0} \neq \lambda_{\ell_0+1} = \cdots = \lambda_T, \quad (2.10)$$

in the spirit of change points detection (Bai, 2010, Li, 2018), where the change points b_0 and ℓ_0 for β_t and λ_t can be the same or different. In this case, we have $k_p = (T-2)(k+1)$ and the linear contrast matrix $C = \texttt{blkdiag}(C_{b_0,k}, C_{T-b_0,k}, C_{\ell_0,1}, C_{T-\ell_0,1}, 0_{k_p})$. In this lines, it would also be of interest to test (2.10) vs (2.2), and the test can be carried out repeatedly to detect the 'true' change points. In other interesting cases, k_p and C for the null hypothesis can all be easily written out.

The score-type test is based on the AQS function $S^\star_{\texttt{SL1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}})$ evaluated at the null estimate $\tilde{\boldsymbol{\theta}}_{\texttt{SL1}}$ of $\boldsymbol{\theta}$, and the asymptotic variance-covariance (VC) matrix of $S^\star_{\texttt{SL1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}})$. Let $I_{\texttt{SL1}}(\boldsymbol{\theta}_0) = \mathrm{E}[J_{\texttt{SL1}}(\boldsymbol{\theta}_0)]$ and $\Sigma_{\texttt{SL1}}(\boldsymbol{\theta}_0) = \mathrm{Var}[S^\star_{\texttt{SL1}}(\boldsymbol{\theta}_0)]$, with their analytical expressions being given in Appendix B.1. Denote by $N_0 = n(T-1)$ the *effective sample size* to differentiate from the overall sample size $N = nT$. Under mild regularity conditions, such as the $\sqrt{N_0}$-consistency of $\tilde{\boldsymbol{\theta}}_{\texttt{SL1}}$, we have by Taylor expansion:

$$\tfrac{1}{\sqrt{N_0}} S^\star_{\texttt{SL1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}}) = \tfrac{1}{\sqrt{N_0}} S^\star_{\texttt{SL1}}(\boldsymbol{\theta}_0) + \tfrac{1}{N_0} I_{\texttt{SL1}}(\boldsymbol{\theta}_0)\sqrt{N_0}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}} - \boldsymbol{\theta}_0) + o_p(1), \text{ and}$$

$$[\tfrac{1}{N_0} I_{\texttt{SL1}}(\boldsymbol{\theta}_0)]^{-1} \tfrac{1}{\sqrt{N_0}} S^\star_{\texttt{SL1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}}) = [\tfrac{1}{N_0} I_{\texttt{SL1}}(\boldsymbol{\theta}_0)]^{-1} \tfrac{1}{\sqrt{N_0}} S^\star_{\texttt{SL1}}(\boldsymbol{\theta}_0) + \sqrt{N_0}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}} - \boldsymbol{\theta}_0) + o_p(1).$$

As $C\boldsymbol{\theta}_0 = 0$ under H_0, we have $C\tilde{\boldsymbol{\theta}}_{\texttt{SL1}} = 0$. It follows that

$$C[\tfrac{1}{N_0} I_{\texttt{SL1}}(\boldsymbol{\theta}_0)]^{-1} \tfrac{1}{\sqrt{N_0}} S^\star_{\texttt{SL1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}}) = C[\tfrac{1}{N_0} I_{\texttt{SL1}}(\boldsymbol{\theta}_0)]^{-1} \tfrac{1}{\sqrt{N_0}} S^\star_{\texttt{SL1}}(\boldsymbol{\theta}_0) + o_p(1), \quad (2.11)$$

leading to the asymptotic VC matrix of $C[\frac{1}{N} I_{\texttt{SL1}}(\boldsymbol{\theta}_0)]^{-1} \frac{1}{\sqrt{N}} S^\star_{\texttt{SL1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}})$ as

$$\Xi_{\texttt{SL1}}(\boldsymbol{\theta}_0) = C[\tfrac{1}{N_0} I_{\texttt{SL1}}(\boldsymbol{\theta}_0)]^{-1}[\tfrac{1}{N_0}\Sigma_{\texttt{SL1}}(\boldsymbol{\theta}_0)][\tfrac{1}{N_0} I_{\texttt{SL1}}(\boldsymbol{\theta}_0)]^{-1}C'. \quad (2.12)$$

This gives an asymptotically valid and nonnormality robust AQS test:

$$T^r_{\texttt{SL1}} = \widetilde{S}^{\star\prime}_{\texttt{SL1}} \widetilde{I}^{-1}_{\texttt{SL1}} C' \big(C \widetilde{I}^{-1}_{\texttt{SL1}} \widetilde{\Sigma}_{\texttt{SL1}} \widetilde{I}^{-1}_{\texttt{SL1}} C'\big)^{-1} C \widetilde{I}^{-1}_{\texttt{SL1}} \widetilde{S}^\star_{\texttt{SL1}}, \quad (2.13)$$

where $\widetilde{S}^\star_{\texttt{SL1}} = S^\star_{\texttt{SL1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}})$, $\widetilde{I}_{\texttt{SL1}} = I_{\texttt{SL1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}})$, and $\widetilde{\Sigma}_{\texttt{SL1}} = \Sigma_{\texttt{SL1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}})$.

Although the AQS test given in (2.13) is developed based on the one-way FE panel SL model, the general principles behind apply to all models considered in this paper. It also applies to more complicated spatial models as well as many non-spatial models.

Asymptotic properties. In studying the asymptotic properties of the proposed tests, we focus on the tests of temporal homogeneity to ease the exposition. Therefore, some of the regularity conditions, i.e., Assumptions 2 and 4, correspond to the null model under H_0 in (2.2) only. However, these assumptions can be easily relaxed to cater a non-homogeneous null model. Denote $X^\circ_{nt} = X_{nt} - \bar{X}_n$, where $\bar{X}_n = \frac{1}{T}\sum_{t=1}^T X_{nt}$.

Assumption 1. *The disturbances* $\{v_{it}\}$ *are iid across* i *and* t *with mean zero, variance* σ_0^2*, and* $E|v_{it}|^{4+\epsilon_0} < \infty$ *for some* $\epsilon_0 > 0$.

Assumption 2. *Under* H_0*, the parameter space* Λ *of the common* λ *is compact, and the true value* λ_0 *is in the interior of* Λ*. The matrix* $A_n(\lambda)$ *is invertible for all* $\lambda \in \Lambda$.

Assumption 3. *The elements of* X_{nt} *are non-stochastic, and are bounded uniformly in* n *and* t*, such that* $\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{T} X_{nt}^{\circ\prime}X_{nt}^{\circ}$ *exists and nonsingular. The elements of* c_n *are uniformly bounded.*

Assumption 4. W_n *has zero diagonal elements, and is uniformly bounded in both row and column sums in absolute value.* $A_n^{-1}(\lambda)$ *is also uniformly bounded in both row and column sums in absolute value for* λ *in a neighborhood of* λ_0.

Theorem 2.1. *Under Assumptions 1-4, if further, (i)* $\tilde{\boldsymbol{\theta}}_{\texttt{SL1}}$ *is* $\sqrt{N_0}$*-consistent for* $\boldsymbol{\theta}_0$ *under* H_0*, and (ii)* $I_{\texttt{SL1}}(\boldsymbol{\theta})$ *and* $\boldsymbol{\Xi}_{\texttt{SL1}}(\boldsymbol{\theta})$ *are positive definite for* $\boldsymbol{\theta}$ *in a neighborhood of* $\boldsymbol{\theta}_0$ *when* N_0 *is large enough, then we have, under* H_0, $T_{\texttt{SL1}}^{r} \xrightarrow{D} \chi^2_{k_p}$*, as* $n \to \infty$.

Note that in case of testing for temporal homogeneity, $k_p = (T-1)(k+1)$, and that in case of testing for a 'single change' of points, $k_p = (T-2)(k+1)$.

Remark 2.1. *It can easily be seen that* $T_{\texttt{SL1}}$ *is not an asymptotic pivotal quantity due to the violation of IME (see Appendix B1).*

Remark 2.2. *When* $T \to \infty$ *as* $n \to \infty$*, the degrees of freedom (d.f) of the chi-square statistic increase with* n*. In this case, one may apply the arguments for 'double asymptotics' (see, e.g., Rempala and Wesolowski, 2016) to show that* $(T_{\texttt{SL1}}^{r} - k_p)/\sqrt{2k_p} \xrightarrow{D} N(0,1)$ *as* $n/\sqrt{T} \to \infty$*. This sample size requirement (*n *goes large faster than* $\sqrt{T}$*) is rather weak as it is typical in spatial panels that* n *is at least as large as* T.

Estimation of the null models. As the proposed tests are based on the estimation of the null model, a detailed discussion on this is necessary for the implementation of the tests. Consider the null model under H_0 given in (2.2). Let $\theta = (\beta', \lambda, \sigma^2)'$. The constrained estimate of c_n given (β, λ) becomes $\tilde{c}_n^{\circ}(\beta,\lambda) = A_n(\lambda)\bar{Y}_n - \bar{X}_n\beta$ where $\bar{Y}_n$ and $\bar{X}_n$ are the averages of $\{Y_{nt}\}$ and $\{X_{nt}\}$, respectively. Along the same line leading to (2.7), one can easily show that AQS function for the null model takes the form:

$$S_{\texttt{SL1}}^{\circ}(\theta) = \begin{cases} \frac{1}{\sigma^2}\sum_{t-1}^{T} X_{nt}^{\circ\prime}\widetilde{V}_{nt}^{\circ}(\beta,\lambda), \\ \frac{1}{\sigma^2}\sum_{t-1}^{T}(W_nY_{nt}^{\circ})'\widetilde{V}_{nt}^{\circ}(\beta,\lambda) - (T-1)\mathrm{tr}[G_n(\lambda)], \\ -\frac{n(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^{T}\widetilde{V}_{nt}^{\circ\prime}(\beta,\lambda)\widetilde{V}_{nt}^{\circ}(\beta,\lambda), \end{cases} \tag{2.14}$$

$\widetilde{V}_{nt}^{\circ}(\beta,\lambda) = A_n(\lambda)Y_{nt} - X_{nt}\beta - \tilde{c}_n^{\circ}(\beta,\lambda) = A_n(\lambda)Y_{nt}^{\circ} - X_{nt}^{\circ}\beta$, where $Y_{nt}^{\circ} = Y_{nt} - \bar{Y}_n$ and $X_{nt}^{\circ} = X_{nt} - \bar{X}_n$. Solving the estimating equations, $S_{\texttt{SL1}}^{\circ}(\theta) = 0$, gives the null estimator $\tilde{\theta}_{\texttt{SL1}}$ of θ. The process can be simplified by first solving the first set of equations and the last equation of (2.14), giving the constrained estimators of β and σ^2 (for a given λ) as

$$\tilde{\beta}_{\texttt{SL1}}(\lambda) = (\textstyle\sum_{t=1}^{T} X_{nt}^{\circ\prime}X_{nt}^{\circ})^{-1}\sum_{t=1}^{T} X_{nt}^{\circ}A_n(\lambda)Y_{nt}^{\circ},$$
$$\tilde{\sigma}_{\texttt{SL1}}^2(\lambda) = \tfrac{1}{n(T-1)}\textstyle\sum_{t=1}^{T}\widetilde{V}_{nt}^{\circ\prime}(\hat{\beta}_{\texttt{SL1}}(\lambda),\lambda)\widetilde{V}_{nt}^{\circ}(\hat{\beta}_{\texttt{SL1}}(\lambda),\lambda).$$

Substituting $\tilde{\beta}_{\texttt{SL1}}(\lambda)$ and $\tilde{\sigma}^2_{\texttt{SL1}}(\lambda)$ into the middle equation of (2.14) and solving the resulted concentrated estimating equation lead to the AQS estimator $\tilde{\lambda}_{\texttt{SL1}}$ of the common λ, which in turn gives the AQS estimator $\tilde{\beta}_{\texttt{SL1}} = \tilde{\beta}_{\texttt{SL1}}(\tilde{\lambda}_{\texttt{SL1}})$ of the common β, and the AQS estimator $\tilde{\sigma}^2_{\texttt{SL1}} = \tilde{\sigma}^2_{\texttt{SL1}}(\tilde{\lambda}_{\texttt{SL1}})$ of σ^2. Finally, the AQS estimator of θ is $\tilde{\theta}_{\texttt{SL1}} = (\tilde{\beta}'_{\texttt{SL1}}, \tilde{\lambda}_{\texttt{SL1}}, \tilde{\sigma}^2_{\texttt{SL1}})'$. The proposed null estimator based on the AQS function provides an alternative to the direct and transformation approaches of Lee and Yu (2010). It can be shown to be asymptotically equivalent to the estimator based on an orthogonal transformation given in Lee and Yu (2010). Thus, $\tilde{\theta}_{\texttt{SL1}}$ is $\sqrt{n(T-1)}$-consistent for θ.

To estimate a non-homogeneous null model, e.g., the model specified by H_0 given in (2.10), the estimating functions for the null model can easily be obtained by simplifying the general AQS function given in (2.7). Thus, the proposed AQS approach offers a more general method to estimate the null model than the transformation approach of Lee and Yu (2010) which works only for a homogeneous model.

Finally, from the expressions of $I_{\texttt{SL1}}(\boldsymbol{\theta}_0)$ and $\Sigma_{\texttt{SL1}}(\boldsymbol{\theta}_0)$ given in Appendix B1, we see that they both contain c_n, which is estimated by plugging the null estimates $\tilde{\boldsymbol{\beta}}_{\texttt{SL1}}$ and $\tilde{\boldsymbol{\lambda}}_{\texttt{SL1}}$ into $\tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda})$. Furthermore, in case of nonnormality, the VC matrix $\Sigma_{\texttt{SL1}}(\boldsymbol{\theta}_0)$ contains two additional parameters, the skewness γ and excess kurtosis κ of the idiosyncratic errors $V_{n,it}$, and their estimates are obtained by applying Lemma 4.1 (a) of Yang et al. (2016).

2.2. Panel SL model with two-way FE

While the unit-specific fixed effects are important to the spatial panel data models, the time-specific effects often cannot be neglected. In this section, we extend our tests to panel SL model with two-way FEs. The model takes the following form:

$$Y_{nt} = \lambda_t W_n Y_{nt} + X_{nt}\beta_t + c_n + \alpha_t l_n + V_{nt}, \tag{2.15}$$

where $\{\alpha_t\}$ are the unobserved time-specific effects or the unobserved temporal heterogeneity in the intercept, and l_n is an $n \times 1$ vector of ones. As the spatial parameters and regression coefficients change only with time. One can apply transformation method to eliminate the time-specific effects as is widely applied in the literature, see, e.g., Lee and Yu (2010), Baltagi and Yang (2013a) and Liu and Yang (2016). Define $J_n = I_n - \frac{1}{n}l_n l'_n$. Assume W_n is row-normalized (i.e., row sums are one). Then, $J_n W_n = J_n W_n J_n$. Let $(F_{n,n-1}, \frac{1}{\sqrt{n}}l_n)$ be the orthonormal eigenvector matrix of J_n, where $F_{n,n-1}$ is the $n \times (n-1)$ sub-matrix corresponding to the eigenvalues of one. By *Spectral Theorem*, $J_n = F_{n,n-1}F'_{n,n-1}$. It follows that $F'_{n,n-1}W_n = F'_{n,n-1}W_n F_{n,n-1}F'_{n,n-1}$. Premultiplying $F'_{n,n-1}$ on both sides of (2.15), we have the following transformed model:

$$Y^*_{nt} = \lambda_t W^*_n Y^*_{nt} + X^*_{nt}\beta_t + c^*_n + V^*_{nt},\ t = 1, \ldots, T, \tag{2.16}$$

where $Y^*_{nt} = F'_{n,n-1}Y_{nt}$, and similarly are X^*_{nt}, c^*_n and V^*_{nt} defined; $W^*_n = F'_{n,n-1}W_n F_{n,n-1}$. After the transformation, the effective sample size is $(n-1)T$. Model (2.16) takes an identical form as Model (2.1). Furthermore, $V^*_{nt} \sim (0, \sigma_0^2 I_{n-1})$, which is normal if V^*_{nt} is, and is independent of V^*_{ns}, $s \neq t$.[2] Hence, the steps leading to the score-type tests and the consistent estimation of the null model are similar to those for the SL one-way FE model.

Define $A^*_n(\lambda_t) = I_{n-1} - \lambda_t W^*_n$, $t = 1, \ldots, T$. The quasi Gaussian loglikelihood function of $\boldsymbol{\theta} = (\boldsymbol{\beta}', \boldsymbol{\lambda}', \sigma^2)'$ and c^*_n of Model (2.16) is

$$\begin{aligned} \ell_{\mathtt{SL2}}(\boldsymbol{\theta}, c^*_n) = & -\tfrac{(n-1)T}{2} \ln(2\pi\sigma^2) + \textstyle\sum_{t=1}^T \ln |A^*_n(\lambda_t)| \\ & - \tfrac{1}{2\sigma^2} \textstyle\sum_{t=1}^T V^{*\prime}_{nt}(\lambda_t, \beta_t, c^*_n) V^*_{nt}(\lambda_t, \beta_t, c^*_n), \end{aligned} \tag{2.17}$$

where $V^*_{nt}(\beta_t, \lambda_t, c^*_n) = A^*_n(\lambda_t) Y^*_{nt} - X^*_{nt}\beta_t - c^*_n$. Given $\boldsymbol{\theta}$, $\ell_{\mathtt{SL2}}(\boldsymbol{\theta}, c^*_n)$ is maximized at:

$$\tilde{c}^*_n(\boldsymbol{\beta}, \boldsymbol{\lambda}) = \tfrac{1}{T} \textstyle\sum_{t=1}^T [A^*_n(\lambda_t) Y^*_{nt} - X^*_{nt}\beta_t], \tag{2.18}$$

which gives the concentrated loglikelihood function of $\boldsymbol{\theta}$ upon substitution:

$$\ell^c_{\mathtt{SL2}}(\boldsymbol{\theta}) = -\tfrac{(n-1)T}{2} \ln(2\pi\sigma^2) + \textstyle\sum_{t=1}^T \ln |A^*_n(\lambda_t)| - \tfrac{1}{2\sigma^2} \textstyle\sum_{t=1}^T \widetilde{V}^{*\prime}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda})' \widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}), \tag{2.19}$$

where $\widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) = A^*_n(\lambda_t) Y^*_{nt} - X^*_{nt}\beta_t - \tilde{c}^*_n(\boldsymbol{\beta}, \boldsymbol{\lambda})$. Now, define $G^*_n(\lambda_t) = W^*_n A^{*-1}_n(\lambda_t)$. Differentiating $\ell^c_{\mathtt{SL2}}(\boldsymbol{\theta})$ gives the CS or CQS function of $\boldsymbol{\theta}$ of Model (2.16):

$$S^c_{\mathtt{SL2}}(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2} X^{*\prime}_{nt} \widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}), \ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2} (W^*_n Y^*_{nt})' \widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) - \mathrm{tr}[G^*_n(\lambda_t)], \ \ t = 1, \ldots, T, \\ -\frac{(n-1)T}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{t=1}^T \widetilde{V}^{*\prime}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) \widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}). \end{cases} \tag{2.20}$$

Takes the expectation of the above score, we have,

$$\mathrm{E}[S^c_{\mathtt{SL2}}(\boldsymbol{\theta}_0)] = \big\{0'_{Tk}, \ -\tfrac{1}{T}\mathrm{tr}[G^*_n(\lambda_{t0})], \ t = 1, \ldots T, \ -\tfrac{n-1}{2\sigma_0^2}\big\}',$$

which again shows that model estimation based on maximizing the quasi loglikelihood would not lead to consistent estimates of the model parameters. The CQS function given in (2.20) should be adjusted by subtracting the above bias vector from it, leading to the AQS function of Model (2.16) as

$$S^\star_{\mathtt{SL2}}(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2} X^{*\prime}_{nt} \widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}), \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2} (W^*_n Y^*_{nt})' \widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) - \frac{T-1}{T}\mathrm{tr}[G^*_n(\lambda_t)], \ t = 1, \ldots, T, \\ -\frac{(n-1)(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{t=1}^T \widetilde{V}^{*\prime}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) \widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}). \end{cases} \tag{2.21}$$

It is easy to show that $\mathrm{E}[S^\star_{\mathtt{SL2}}(\boldsymbol{\theta})] = 0$, and that $\frac{1}{nT} S^\star_{\mathtt{SL2}}(\boldsymbol{\theta}_0) \xrightarrow{p} 0$ as $n \to \infty$ alone, or both n and T go infinity. Thus, this AQS function gives a set of unbiased estimating functions, and paves the way for developing asymptotic valid score-type tests.[3] Again, simplifying this AQS function under various null hypotheses gives AQS functions of the null models, leading to the constrained estimates of the model parameters $\boldsymbol{\theta}$.

[2] The time-specific effects can also be eliminated by pre-multiplying J_n on both sides of (2.15). However, the resulted disturbances $J_n V_{nt}$ would not be linearly independent over the cross-section dimension.

[3] Solving the estimating equation, $S^\star_{\mathtt{SL2}}(\boldsymbol{\theta}) = 0$, gives the full AQS estimator of $\boldsymbol{\theta}$, which can besimplified by first solving the last equation for σ^2, leading to $\hat{\sigma}^{2*}_{\mathtt{SL2}}(\boldsymbol{\beta}, \boldsymbol{\lambda}) = \frac{1}{n(T-1)} \sum_{t=1}^T \widetilde{V}'_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) \widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda})$, and then solving the resulted concentrated AQS equations for $(\boldsymbol{\beta}, \boldsymbol{\lambda})$.

Now the test of H_0 defined in (2.2) becomes a test of temporal homogeneity of the regression and the spatial coefficients in the panel SL model, allowing the existence of both unobserved cross-sectional and time-specific heterogeneity in the intercept, i.e., the existence of both individual specific fixed effects and the time specific fixed effects. As the transformed two-way FE (2FE) panl SL model takes an identical form as the one-way FE (1FE) panel SL model, the tests developed for 1FE panel SL model extends directly to give tests for the 2FE panel SL model. The AS test takes the form:

$$T_{\texttt{SL2}} = S^{\star}_{\texttt{SL2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL2}})' J^{-1}_{\texttt{SL2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL2}}) S^{\star}_{\texttt{SL2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL2}}), \tag{2.22}$$

where $\tilde{\boldsymbol{\theta}}_{\texttt{SL2}}$ is a consistent estimate of θ_0, and $J_{\texttt{SL2}}(\boldsymbol{\theta}) = -\frac{\partial}{\partial \boldsymbol{\theta}'} S^{\star}_{\texttt{SL2}}(\boldsymbol{\theta})$ with its expression given in Appendix B.2. Furthermore, let C be defined as in Sec. 2.2, $I_{\texttt{SL2}}(\boldsymbol{\theta}_0) = \mathrm{E}[J_{\texttt{SL2}}(\boldsymbol{\theta}_0)]$, and $\Sigma_{\texttt{SL2}}(\boldsymbol{\theta}_0) = \mathrm{Var}[S^{\star}_{\texttt{SL2}}(\boldsymbol{\theta}_0)]$, with their analytical expressions being given in Appendix B.2. The AQS test robust against nonnormality takes the form:

$$T^{r}_{\texttt{SL2}} = \widetilde{S}^{\star\prime}_{\texttt{SL2}} \widetilde{I}^{-1}_{\texttt{SL2}} C' \big(C \widetilde{I}^{-1}_{\texttt{SL2}} \widetilde{\Sigma}_{\texttt{SL2}} \widetilde{I}^{-1}_{\texttt{SL2}} C'\big)^{-1} C \widetilde{I}^{-1}_{\texttt{SL2}} \widetilde{S}^{\star}_{\texttt{SL2}}, \tag{2.23}$$

where $\widetilde{S}^{\star}_{\texttt{SL2}} = S^{\star}_{\texttt{SL2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL2}})$, $\widetilde{I}_{\texttt{SL2}} = I_{\texttt{SL2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL2}})$, and $\widetilde{\Sigma}_{\texttt{SL2}} = \Sigma_{\texttt{SL2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL2}})$.

Asymptotic properties of these tests can be studied along the same line of the tests for 1FE panel SL model, with Assumption 3 being replaced by Assumption $3'$ given below to take into account the involvement of the projection J_n. For the 2FE panel SL model, the effective sample size becomes $N_0 = (n-1)(T-1)$ due to the 'estimation' of both individual- and time-specific FEs. Let $\boldsymbol{\Xi}_{\texttt{SL2}}(\boldsymbol{\theta})$ be defined as $\boldsymbol{\Xi}_{\texttt{SL1}}(\boldsymbol{\theta})$ in (2.12).

Assumption $3'$: The elements of X_{nt} are nonstochastic, and are bounded uniformly in n and t, such that $\lim_{N_0 \to \infty} \frac{1}{N_0} \sum_{t=1}^{T} X^{*\circ\prime}_{nt} X^{*\circ}_{nt}$ exists and is nonsingular.

Theorem 2.2. *Under Assumptions 1-2, $3'$, and 4, if further, (i) $\tilde{\boldsymbol{\theta}}_{\texttt{SL2}}$ is $\sqrt{N_0}$-consistent for $\boldsymbol{\theta}_0$ under H_0, and (ii) $I_{\texttt{SL2}}(\boldsymbol{\theta})$ and $\boldsymbol{\Xi}_{\texttt{SL2}}(\boldsymbol{\theta})$ are positive definite for $\boldsymbol{\theta}$ in a neighborhood of $\boldsymbol{\theta}_0$ when N_0 is large enough, then we have, under H_0, $T^{r}_{\texttt{SL2}} \xrightarrow{D} \chi^2_{k_p}$, as $n \to \infty$.*

Note that while the effective sample size for the 2FE-SL model is smaller than that of the 1FE-SL model, the d.f. associated with the test statistics remain the same. As in the Remarks 2.1 and 2.2, it can be shown that $T_{\texttt{SL2}}$ is not an asymptotic pivotal quantity, and $(T^{r}_{\texttt{SL2}} - k_p)/\sqrt{2k_p} \xrightarrow{D} N(0,1)$, as $n/\sqrt{T} \to \infty$.

Estimation of the null model. Let $\theta = (\beta', \lambda, \sigma^2)'$. Under H_0, the constrained estimate of c^{*}_n given (β, λ) becomes $\tilde{c}^{*\circ}_n(\beta, \lambda) = A^{*}_n(\lambda)\bar{Y}^{*}_n - \bar{X}^{*}_n \beta$ where $\bar{Y}^{*}_n$ and $\bar{X}^{*}_n$ are the averages of $\{Y^{*}_{nt}\}$ and $\{X^{*}_{nt}\}$, respectively. Along the same line leading to (2.21), one can easily show that the AS or AQS function for the null model takes the form:

$$S^{\circ}_{\texttt{SL2}}(\theta) = \begin{cases} \frac{1}{\sigma^2} \sum_{t-1}^{T} X^{*\circ\prime}_{nt} \widetilde{V}^{*\circ}_{nt}(\beta, \lambda), \\ \frac{1}{\sigma^2} \sum_{t-1}^{T} (W^{*}_n Y^{*\circ}_{nt})' \widetilde{V}^{*\circ}_{nt}(\beta, \lambda) - (T-1)\mathrm{tr}[G^{*}_n(\lambda)], \\ -\frac{(n-1)(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{t=1}^{T} \widetilde{V}^{*\circ}_{nt}{}'(\beta, \lambda) \widetilde{V}^{*\circ}_{nt}(\beta, \lambda), \end{cases} \tag{2.24}$$

where $\widetilde{V}_{nt}^{*\circ}(\beta,\lambda) = A_n^*(\lambda)Y_{nt}^* - X_{nt}^*\beta - \tilde{c}_n^{*\circ}(\beta,\lambda) = A_n(\lambda)Y_{nt}^{*\circ} - X_{nt}^{*\circ}\beta$, $Y_{nt}^{*\circ} = Y_{nt}^* - \bar{Y}_n^*$ and $X_{nt}^{*\circ} = X_{nt}^* - \bar{X}_n^*$. Solving the estimating equations, $S_{\texttt{SL2}}^{\circ}(\theta) = 0$, gives the null estimator $\tilde{\theta}_{\texttt{SL2}}$ of θ. Denote the AQS estimator of θ as $\tilde{\theta}_{\texttt{SL2}} = (\tilde{\beta}_{\texttt{SL2}}', \tilde{\lambda}_{\texttt{SL2}}, \tilde{\sigma}_{\texttt{SL2}}^2)'$. The proposed null estimator based on the AQS function provides an alternative to the transformation approaches of Lee and Yu (2010). It can be shown to be asymptotically equivalent to the estimator based on an orthogonal transformation given in Lee and Yu (2010). Thus, $\tilde{\theta}_{\texttt{SL2}}$ is $\sqrt{(n-1)(T-1)}$-consistent for θ. As pointed out in the discussions at the end of Sec. 2.1, the AQS approach is more general as it allows the estimation of a non-homogeneous null model by simplifying the general estimation functions (2.21) accordingly. Finally, the estimation of c_n and γ and κ caontained in $I_{\texttt{SL2}}(\boldsymbol{\theta}_0)$, and $\Sigma_{\texttt{SL2}}(\boldsymbol{\theta}_0$ proceed similarly.

3. Test for Temporal Heterogeneity in Panel SLE Model

The tests introduced in the earlier section can be easily extended to a more general SPD model where the the disturbances are also subject to spatial interactions, giving an SPD model with both spatial lag and error (SLE) dependence. Again, we first present results for the one-way FE model, and then the results for the two-way FE model.

3.1. Panel SLE model with one-way FE

The SLE model with one-way fixed effects has the form:

$$Y_{nt} = \lambda_t W_n Y_{nt} + X_{nt}\beta_t + c_n + U_{nt}, \quad U_{nt} = \rho_t M_n U_{nt} + V_{nt}, \tag{3.1}$$

where M_n is another spatial weight matrix capturing the spatial interactions among the disturbances, which can be the same as W_n, and $\{\rho_t\}$ are the spatial error parameters, possibly different in different time periods. Again, we are primarily interested in the test for temporal homogeneity, which now corresponds to a test of the following null hypothesis:

$$H_0: \beta_1 = \cdots = \beta_T = \beta, \;\; \lambda_1 = \cdots = \lambda_T = \lambda, \;\; \text{and} \;\; \rho_1 = \cdots = \rho_T = \rho. \tag{3.2}$$

If this test is rejected, one would be interested in testing the hypothesis of the form in (2.10) extended to include the ρ-component, or some other form of temporal heterogeneity.

Following the same set of notation as in the earlier section, and further denoting $\boldsymbol{\rho} = (\rho_1, \ldots, \rho_T)'$, $\boldsymbol{\theta} = (\boldsymbol{\beta}', \boldsymbol{\lambda}', \boldsymbol{\rho}', \sigma^2)'$, and $B_n(\rho_t) = I_n - \rho_t M_n, t = 1, \ldots, T$, we have the (quasi) Gaussian loglikelihood for $(\boldsymbol{\theta}, c_n)$:

$$\begin{aligned}\ell_{\texttt{SLE1}}(\boldsymbol{\theta}, c_n) = &-\tfrac{nT}{2}\ln(2\pi\sigma^2) + \textstyle\sum_{t=1}^T \ln|A_n(\lambda_t)| + \sum_{t=1}^T \ln|B_n(\rho_t)| \\ &- \tfrac{1}{2\sigma^2}\textstyle\sum_{t=1}^T V_{nt}'(\beta_t, \lambda_t, \rho_t, c_n)V_{nt}(\beta_t, \lambda_t, \rho_t, c_n),\end{aligned} \tag{3.3}$$

where $V_{nt}(\beta_t, \lambda_t, \rho_t, c_n) = B_n(\rho_t)[A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t - c_n]$, $t = 1, \ldots, T$.

Similarly to the developments in the previous section, we first eliminate c_n through a direct maximization of the loglikelihood function, given the other model parameters $\boldsymbol{\theta}$, and then adjust the resulted CS or CQS function to eliminate the asymptotic bias or inconsistency. Given $\boldsymbol{\theta}$, $\ell_{\texttt{SLE1}}(\boldsymbol{\theta}, c_n)$ is maximized at

$$\tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) = \big[\textstyle\sum_{t=1}^T B_n'(\rho_t)B_n(\rho_t)\big]^{-1} \sum_{t=1}^T \big[B_n'(\rho_t)B_n(\rho_t)\big(A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t\big)\big], \quad (3.4)$$

leading to the concentrated (quasi) Gaussian loglikelihood function of $\boldsymbol{\theta}$ upon substitution:

$$\begin{aligned}\ell_{\texttt{SLE1}}^c(\boldsymbol{\theta}) = &- \tfrac{nT}{2}\ln(2\pi\sigma^2) + \textstyle\sum_{t=1}^T \ln|A_n(\lambda_t)| + \sum_{t=1}^T \ln|B_n(\rho_t)| \\ &- \tfrac{1}{2\sigma^2}\textstyle\sum_{t=1}^T \widetilde{V}_{nt}'(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}),\end{aligned} \quad (3.5)$$

where $\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) = V_{nt}(\beta_t, \lambda_t, \rho_t, \tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})) = B_n(\rho_t)[A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t - \tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})]$.

To facilitate the subsequent derivations, denote $U_{nt}^{\circ}(\beta_t, \lambda_t) = A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t$, $D_n(\rho_t) = B_n'(\rho_t)B_n(\rho_t)$ and $\mathbb{D}_n(\boldsymbol{\rho}) = \sum_{t=1}^T D_n(\rho_t)$. Then,

$$\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) = B_n(\rho_t)U_{nt}^{\circ}(\beta_t, \lambda_t) - B_n(\rho_t)\tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}),$$

$\tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) = \mathbb{D}_n^{-1}(\boldsymbol{\rho})\sum_{t=1}^T D_n(\rho_t)U_{nt}^{\circ}(\beta_t, \lambda_t)$, and the key term in (3.5):

$$\begin{aligned}\textstyle\sum_{t=1}^T \widetilde{V}_{nt}'(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) &= \textstyle\sum_{t=1}^T U_{nt}^{\circ\prime}(\beta_t, \lambda_t)D_n(\rho_t)U_{nt}^{\circ}(\beta_t, \lambda_t) \\ &- \big(\textstyle\sum_{t=1}^T D_n(\rho_t)U_{nt}^{\circ}(\beta_t, \lambda_t)\big)'\mathbb{D}_n^{-1}(\boldsymbol{\rho})\big(\sum_{t=1}^T D_n(\rho_t)U_{nt}^{\circ}(\beta_t, \lambda_t)\big).\end{aligned}$$

Differentiating $\ell_{\texttt{SLE1}}^c(\boldsymbol{\theta})$ gives the CS or CQS function of $\boldsymbol{\theta}$:

$$S_{\texttt{SLE1}}^c(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2}X_{nt}'B_n'(\rho_t)\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}), \ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2}(W_nY_{nt})'B_n'(\rho_t)\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) - \mathrm{tr}[G_n(\lambda_t)], \ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2}\widetilde{V}_{nt}'(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})H_n(\rho_t)\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) - \mathrm{tr}[H_n(\rho_t)], \ \ t = 1, \ldots, T, \\ -\frac{nT}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^T \widetilde{V}_{nt}'(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}), \end{cases} \quad (3.6)$$

where $H_n(\rho_t) = M_nB_n^{-1}(\rho_t), \ \ t = 1, \ldots, T$.

At the true $\boldsymbol{\theta}_0$, we have, $\tilde{c}_n(\boldsymbol{\beta}_0, \boldsymbol{\lambda}_0, \boldsymbol{\rho}_0) = c_n + \mathbb{D}_n^{-1}\sum_{s=1}^T B_{ns}'V_{ns}$ and hence $\widetilde{V}_{nt} \equiv \widetilde{V}_{nt}(\boldsymbol{\beta}_0, \boldsymbol{\lambda}_0, \boldsymbol{\rho}_0) = V_{nt} - B_{nt}\mathbb{D}_n^{-1}\sum_{s=1}^T B_{ns}'V_{ns}$, and $W_nY_{nt} = G_{nt}(X_{nt}\beta_0 + c_n + B_{nt}^{-1}V_{nt})$, where $B_{nt} = B_n(\rho_{t0})$, $G_{nt} = G_n(\lambda_{t0})$, and $\mathbb{D}_n = \mathbb{D}_n(\boldsymbol{\rho}_0)$. It is easy to show that,

$$\mathrm{E}[S_{\texttt{SLE1}}^c(\boldsymbol{\theta}_0)] = \begin{cases} \mathbf{0}_{Tk}, \\ -\mathrm{tr}[\mathbb{D}_n^{-1}(\boldsymbol{\rho}_0)B_n'(\rho_{t0})B_n(\rho_{t0})G_n(\lambda_{t0})], \ t = 1, \ldots T, \\ -\mathrm{tr}[B_n(\rho_{t0})\mathbb{D}_n^{-1}(\boldsymbol{\rho}_0)B_n'(\rho_{t0})H_n(\rho_t0)], \ t = 1, \ldots T, \\ -\frac{n}{2\sigma_0^2}. \end{cases}$$

Therefore, the AS or AQS function of $\boldsymbol{\theta}$ for Model (3.1) takes the form:

$$S_{\texttt{SLE1}}^*(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2}X_{nt}'B_n'(\rho_t)\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}), \ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2}(W_nY_{nt})'B_n'(\rho_t)\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) - \mathrm{tr}[R_{nt}(\boldsymbol{\rho})G_n(\lambda_t)], \ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2}\widetilde{V}_{nt}'(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})H_n(\rho_t)\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) - \mathrm{tr}[S_{nt}(\boldsymbol{\rho})H_n(\rho_t)], \ \ t = 1, \ldots, T, \\ -\frac{n(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^T \widetilde{V}_{nt}'(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}), \end{cases} \quad (3.7)$$

where $R_{nt}(\boldsymbol{\rho}) = I_n - \mathbb{D}_n^{-1}(\boldsymbol{\rho})B_n'(\rho_t)B_n(\rho_t)$ and $S_{nt}(\boldsymbol{\rho}) = I_n - B_n(\rho_t)\mathbb{D}_n^{-1}(\boldsymbol{\rho})B_n'(\rho_t)$.

It is easy to show that $\mathrm{E}[S^{\star}_{\mathtt{SLE1}}(\boldsymbol{\theta})] = 0$, and that $\frac{1}{nT}S^{\star}_{\mathtt{SLE1}}(\boldsymbol{\theta}_0) \xrightarrow{p} 0$ as $n \to \infty$ alone, or both n and T go infinity. Thus, this AQS function gives a set of unbiased estimating functions, and paves the way for developing asymptotic valid score-type tests.[4]

Construction of AQS tests. Denote the constrained estimator (under H_0) of $\boldsymbol{\theta}$ by $\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}}$. In case of testing for temporal homogeneity, i.e., test of H_0 given in (3.2), the constrained estimators of $\boldsymbol{\beta}$, $\boldsymbol{\lambda}$ and $\boldsymbol{\rho}$ are, respectively, $\tilde{\boldsymbol{\beta}}_{\mathtt{SLE1}} = 1_T \otimes \tilde{\beta}_{\mathtt{SLE1}}$, $\tilde{\boldsymbol{\lambda}}_{\mathtt{SLE1}} = 1_T \otimes \tilde{\lambda}_{\mathtt{SLE1}}$, and $\tilde{\boldsymbol{\rho}}_{\mathtt{SLE1}} = 1_T \otimes \tilde{\rho}_{\mathtt{SLE1}}$, where $\tilde{\beta}_{\mathtt{SLE1}}$, $\tilde{\lambda}_{\mathtt{SLE1}}$ and $\tilde{\rho}_{\mathtt{SLE1}}$ are the estimators of the common β, λ and ρ, leading to the constrained estimator of $\boldsymbol{\theta}$ as $\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}} = (\tilde{\boldsymbol{\beta}}'_{\mathtt{SLE1}}, \tilde{\boldsymbol{\lambda}}'_{\mathtt{SLE1}}, \tilde{\boldsymbol{\rho}}'_{\mathtt{SLE1}}, \tilde{\sigma}^2_{\mathtt{SLE1}})'$. Let $J_{\mathtt{SLE1}}(\boldsymbol{\theta}) = -\frac{\partial}{\partial \boldsymbol{\theta}'}S^{\star}_{\mathtt{SLE1}}(\boldsymbol{\theta})$ with its expression given in Appendix B.3. The *usual or naïve* score-type test, treating $S^{\star}_{\mathtt{SLE1}}(\boldsymbol{\theta})$ as a genuine score function, takes the form:

$$T_{\mathtt{SLE1}} = S^{\star}_{\mathtt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}})' J^{-1}_{\mathtt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}}) S^{\star}_{\mathtt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}}). \tag{3.8}$$

Again, $S^{\star}_{\mathtt{SLE1}}(\boldsymbol{\theta})$ is not a genuine score function and the IME or generolized IME does not hold (see the relevant expressions given in Appendix B2). Hence, the test constructed in the usual way may not be a valid test statistic, even if the errors are normal.

As in the previous section, to address both issues, we again put our testing problem in a general framework with null hypothesis being written as H_0: $C\boldsymbol{\theta}_0 = 0$, with some modifications on C to include the ρ parameters. Now, C is a $k_p \times k_q$ matrix generating k_p linear contrasts on the parameter vector $\boldsymbol{\theta}$ of dimension $k_q = (k+2)T + 1$. For the null hypothesis defined in (3.2), we have $k_p = (T-1)(k+2)$ and $C = \mathtt{blkdiag}\{C_{T,k}, C_{T,1}, C_{T,1}, 0_{k_p}\}$, where C_m is defined in the previous section. Tests for change of points can also be carried out based on the following hypothesis:

$$\begin{aligned} H_0: \beta_1 = \cdots = \beta_{b_0} \neq \beta_{b_0+1} \cdots = \beta_T, \quad \lambda_1 = \cdots = \lambda_{\ell_0} \neq \lambda_{\ell_0+1} \cdots = \lambda_T, \\ \rho_1 = \cdots = \rho_{r_0} \neq \rho_{r_0+1} \cdots = \rho_T, \end{aligned} \tag{3.9}$$

for the set of specified values $1 < b_0, \ell_0, r_0 < T$. Again, with a different C matrix, our test can repeatedly be carried out to identify a relatively more parsimonious model instead of the full model with the regression and spatial coefficients changing at every time point.

Similarly, the score-type test is based on the AQS function $S^{\star}_{\mathtt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}})$ evaluated at the null estimate $\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}}$ of $\boldsymbol{\theta}$, and the asymptotic VC matrix of $S^{\star}_{\mathtt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}})$. Let $I_{\mathtt{SLE1}}(\boldsymbol{\theta}_0) = \mathrm{E}[J_{\mathtt{SLE1}}(\boldsymbol{\theta}_0)]$ and $\Sigma_{\mathtt{SLE1}}(\boldsymbol{\theta}_0) = \mathrm{Var}[S^{\star}_{\mathtt{SLE1}}(\boldsymbol{\theta}_0)]$ with their expressions being given in Appendix B.3. Now, the effective sample size is back to $N_0 = n(T-1)$ as for the 1FE panel SL model. Under mild regularity conditions, such as the $\sqrt{N_0}$-consistency of $\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}}$, we have an asymptotically valid and nonnormality robust AQS test:

$$T^{r}_{\mathtt{SLE1}} = \widetilde{S}^{\star\prime}_{\mathtt{SLE1}} \widetilde{I}^{-1}_{\mathtt{SLE1}} C' \big(C \widetilde{I}^{-1}_{\mathtt{SLE1}} \widetilde{\Sigma}_{\mathtt{SLE1}} \widetilde{I}^{-1}_{\mathtt{SLE1}} C'\big)^{-1} C \widetilde{I}^{-1}_{\mathtt{SLE1}} \widetilde{S}^{\star}_{\mathtt{SLE1}}, \tag{3.10}$$

[4]Solving the estimating equation, $S^{\star}_{\mathtt{SLE1}}(\boldsymbol{\theta}) = 0$, gives the AQS or M estimator of $\boldsymbol{\theta}$, which is obtained by first solving the last equation for σ^2 given $(\boldsymbol{\beta}, \boldsymbol{\lambda})$, to give $\hat{\sigma}^{\star 2}_{\mathtt{SLE1}}(\boldsymbol{\beta}, \boldsymbol{\lambda}) = \frac{1}{n(T-1)} \sum_{t=1}^{T} \widetilde{V}'_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) \widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda})$, and then solving the resulted concentrated AQS equations for $(\boldsymbol{\beta}, \boldsymbol{\lambda})$.

where $\widetilde{S}^{\star}_{\texttt{SLE1}} = S^{\star}_{\texttt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE1}})$, $\widetilde{I}_{\texttt{SLE1}} = I_{\texttt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE1}})$, and $\widetilde{\Sigma}_{\texttt{SLE1}} = \Sigma_{\texttt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE1}})$.

Asymptotic properties of the proposed tests are established based on Assumptions 1-4 in Sec. 2, and the following additional conditions on M_n and $B_n(\rho)$.

Assumption 5. *Under H_0, the parameter space $\mathbb{P}$ of the common ρ is compact. The true value ρ_0 is in the interior of $\mathbb{P}$. The matrix $B_n(\rho)$ is invertible for all $\rho \in \mathbb{P}$. M_n has zero diagonal elements, and are uniformly bounded in both row and column sums in absolute value. $B_n^{-1}(\rho)$ is uniformly bounded in both row and column sums in absolute value for ρ in a neighborhood of ρ_0.*

Furthermore, the existence and consistency of the constrained estimator $\tilde{\beta}_{\texttt{SLE1}}$ depends on the existence and nonsingularity of $\lim_{n\to\infty}\frac{1}{nT}\sum_{t=1}^{T} X^{\circ\prime}_{nt}B'_nB_nX^{\circ}_{nt}$, which follows from Assumption 2 and the positive definiteness of B'_nB_n. Denoting $\boldsymbol{\Xi}_{\texttt{SLE1}}(\boldsymbol{\theta}) = CI^{-1}_{\texttt{SLE1}}(\boldsymbol{\theta})\Sigma_{\texttt{SLE1}}(\boldsymbol{\theta})I^{-1}_{\texttt{SLE1}}(\boldsymbol{\theta})C'$, we have the following theorem.

Theorem 3.1. *Under Assumptions 1-5, if further, (i) $\tilde{\boldsymbol{\theta}}_{\texttt{SEL1}}$ is $\sqrt{N}$-consistent for $\boldsymbol{\theta}_0$ under H_0, and (ii) $I_{\texttt{SLE1}}(\boldsymbol{\theta})$ and $\boldsymbol{\Xi}_{\texttt{SLE1}}(\boldsymbol{\theta})$ are positive definite for $\boldsymbol{\theta}$ in a neighborhood of $\boldsymbol{\theta}_0$ when N_0 is large enough, then we have, under H_0, $T^{r}_{\texttt{SLE1}} \xrightarrow{D} \chi^2_{k_p}$, as $n \to \infty$.*

Note that the d.f. associated with the test statistics is $k_p = (T-1)(k+2)$ for testing for temporal homogeneity, and $k_p = (T-2)(k+2)$ for testing for a 'single change'. Similarly, it can be shown that $T_{\texttt{SLE1}}$ is not an asymptotic pivotal quantity, and that $(T^{r}_{\texttt{SLE1}} - k_p)/\sqrt{2k_p} \xrightarrow{D} N(0,1)$, as $n/\sqrt{T} \to \infty$.

Estimation of the null model. Let $\theta = (\beta', \lambda, \rho, \sigma^2)'$. Under H_0, the constrained estimate of c_n given (β, λ) becomes $\tilde{c}^{\circ}_n(\beta,\lambda) = A_n(\lambda)\bar{Y}_n - \bar{X}_n\beta$, and the error vector becomes $\widetilde{V}^{\circ}_{nt}(\beta,\lambda,\rho) = B_n(\rho)[A_n(\lambda)Y^{\circ}_{nt} - X^{\circ}_{nt}\beta]$, where $Y^{\circ}_{nt} = Y_{nt} - \bar{Y}_n$, $X^{\circ}_{nt} = X_{nt} - \bar{X}_n$, and $\bar{Y}_n = \frac{1}{T}\sum_{t=1}^{T} Y_{nt}$ and $\bar{X}_n = \frac{1}{T}\sum_{t=1}^{T} X_{nt}$. Along the same line leading to (3.7), one can easily show that AQS function for the null model takes the form:

$$S^{\circ}_{\texttt{SLE1}}(\theta) = \begin{cases} \frac{1}{\sigma^2}\sum_{t=1}^{T} X^{\circ}_{nt}B'_n(\rho)\widetilde{V}^{\circ}_{nt}(\beta,\lambda,\rho), \\ \frac{1}{\sigma^2}\sum_{t=1}^{T}(W_nY^{\circ}_{nt})'B'_n(\rho)\widetilde{V}^{\circ}_{nt}(\beta,\lambda,\rho) - (T-1)\mathrm{tr}[G_n(\lambda)], \\ \frac{1}{\sigma^2}\sum_{t=1}^{T}\widetilde{V}^{\circ\prime}_{nt}(\beta,\lambda,\rho)H_n(\rho)\widetilde{V}^{\circ}_{nt}(\beta,\lambda,\rho) - (T-1)\mathrm{tr}[H_n(\rho)], \\ -\frac{n(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^{T}\widetilde{V}^{\circ\prime}_{nt}(\beta,\lambda)\widetilde{V}^{\circ}_{nt}(\beta,\lambda). \end{cases} \tag{3.11}$$

Solving the estimating equations, $S^{\circ}_{\texttt{SLE1}}(\theta) = 0$, gives the null estimator $\tilde{\theta}_{\texttt{SLE1}}$ of θ. The process can be simplified by first solving the first set of equations and the last equation of (3.11), giving the constrained estimators of β and σ^2 (for given λ and ρ) as

$$\tilde{\beta}_{\texttt{SLE1}}(\lambda,\rho) = (\textstyle\sum_{t=1}^{T} X^{\circ\prime}_{nt}D_n(\rho)X^{\circ}_{nt})^{-1}\sum_{t=1}^{T} X^{\circ}_{nt}D_n(\rho)A_n(\lambda)Y^{\circ}_{nt},$$
$$\tilde{\sigma}^2_{\texttt{SLE1}}(\lambda,\rho) = \tfrac{1}{n(T-1)}\textstyle\sum_{t=1}^{T}\widetilde{V}^{\circ\prime}_{nt}(\hat{\beta}_{\texttt{SLE1}}(\lambda,\rho),\lambda,\rho)\widetilde{V}^{\circ}_{nt}(\hat{\beta}_{\texttt{SLE1}}(\lambda,\rho),\lambda,\rho).$$

Substituting $\tilde{\beta}_{\texttt{SLE1}}(\lambda,\rho)$ and $\tilde{\sigma}^2_{\texttt{SLE1}}(\lambda,\rho)$ into the middle two equations of (3.11) and solving the resulted concentrated estimating equations lead to the AQS estimators $(\tilde{\lambda}_{\texttt{SLE1}}, \tilde{\rho}_{\texttt{SLE1}})$

of the common (λ, ρ), which in turn give the AQS estimator $\tilde{\beta}_{\mathtt{SLE1}} = \tilde{\beta}_{\mathtt{SLE1}}(\tilde{\lambda}_{\mathtt{SLE1}}, \tilde{\rho}_{\mathtt{SLE1}})$ of the common β, and the AQS estimator $\tilde{\sigma}^2_{\mathtt{SLE1}} = \tilde{\sigma}^2_{\mathtt{SLE1}}(\tilde{\lambda}_{\mathtt{SLE1}}, \tilde{\rho}_{\mathtt{SLE1}})$ of σ^2. Finally, the AQS estimator of θ is $\tilde{\theta}_{\mathtt{SLE1}} = (\tilde{\beta}'_{\mathtt{SLE1}}, \tilde{\lambda}_{\mathtt{SLE1}}, \tilde{\sigma}^2_{\mathtt{SLE1}})'$. It can be shown to be asymptotically equivalent to the estimator based on an orthogonal transformation given in Lee and Yu (2010), and thus is $\sqrt{n(T-1)}$-consistent. To estimate the 'other' type of null models, simplify the general AQS function (3.7) and then solve the resulted estimation equations. To estimate c_n, γ and κ, refer to the discussions at the end of Section 2.1.

3.2. Panel SLE model with two-way FE

The panel SLE model with two-way fixed effects has the form:

$$Y_{nt} = \lambda_t W_n Y_{nt} + X_{nt}\beta_t + c_n + \alpha_t l_n + U_{nt}, \quad U_{nt} = \rho_t M_n U_{nt} + V_{nt}, \tag{3.12}$$

which extends Model (2.15) by adding the spatial error dependence term. Applying the same orthonormal transformation as that for Model (2.15), i.e., premultiplying $F'_{n,n-1}$ on both sides of (3.12), we have the following transformed model:

$$Y^*_{nt} = \lambda_t W^*_n Y^*_{nt} + X^*_{nt}\beta_t + c^*_n + U^*_{nt}, \quad U^*_{nt} = \rho_t M^*_n U^*_{nt} + V^*_{nt}, \tag{3.13}$$

where Y^*_{nt}, X^*_{nt}, c^*_n, W^*_n and V^*_{nt} are defined as in Model (2.16), and $M^*_n = F'_{n,n-1} M_n F_{n,n-1}$. After the transformation, the effective sample size becomes $(n-1)(T-1)$ as for the 2FE panel SL model. As Model (3.13) takes an identical form as Model (3.1) and the elements of V^*_{nt} are iid normal if the original errors are normal, the steps leading to the score-type test and the steps leading to consistent estimation of the null model are similar.

Define $A^*_n(\rho_t) = I_{n-1} - \lambda_t W^*_n$ and $B^*_n(\rho_t) = I_{n-1} - \rho_t M^*_n$, $t = 1, \ldots, T$. Similar to the previous section, we eliminate c^*_n through a direct maximization of the loglikelihood function to give the concentrated loglikelihood function of $\boldsymbol{\theta}$:

$$\begin{aligned} \ell^c_{\mathtt{SLE2}}(\boldsymbol{\theta}) = &- \tfrac{nT}{2}\ln(2\pi\sigma^2) + \textstyle\sum_{t=1}^T \ln|A^*_n(\lambda_t)| + \sum_{t=1}^T \ln|B^*_n(\rho_t)| \\ &- \tfrac{1}{2\sigma^2}\textstyle\sum_{t=1}^T \widetilde{V}^{*\prime}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})\widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) \end{aligned} \tag{3.14}$$

where $\widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) = B^*_n(\rho_t)U^{\circ *}_{nt}(\beta_t, \lambda_t) - B^*_n(\rho_t)\mathbb{D}^{*-1}_n(\boldsymbol{\rho})\sum_{s=1}^T D^*_n(\rho_s)U^{\circ *}_{ns}(\beta_s, \lambda_s)$, $\mathbb{D}^*_n(\boldsymbol{\rho}) = \sum_{t=1}^T D^*_n(\rho_t)$, $D^*_n(\rho_t) = B^{*\prime}_n(\rho_t)B^*_n(\rho_t)$, and $U^{\circ *}_{nt}(\beta_t, \lambda_t) = A^*_n(\lambda_t)Y^*_{nt} - X^*_{nt}\beta_t$. As in the previous subsection, we can obtain the AS or AQS function of $\boldsymbol{\theta}$ for Model (3.12) as

$$S^\star_{\mathtt{SLE2}}(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2} X^{*\prime}_{nt} B^{*\prime}_n(\rho_t)\widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}), \ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2}(W^*_n Y^*_{nt})' B^{*\prime}_n(\rho_t)\widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) - \mathrm{tr}[R^*_{nt}(\boldsymbol{\rho})G^*_n(\lambda_t)], \ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2}\widetilde{V}^{*\prime}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})H^*_n(\rho_t)\widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) - \mathrm{tr}[S^*_{nt}(\boldsymbol{\rho})H^*_n(\rho_t)], \ \ t = 1, \ldots, T, \\ -\frac{(n-1)(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^T \widetilde{V}^{*\prime}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})\widetilde{V}^*_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}), \end{cases} \tag{3.15}$$

where $R^*_{nt}(\boldsymbol{\rho}) = I_{n-1} - \mathbb{D}^{*-1}_n(\boldsymbol{\rho})D^*_{nt}(\rho_t)$, and $S^*_{nt}(\boldsymbol{\rho}) = I_{n-1} - B^*_{nt}(\rho_t)\mathbb{D}^{*-1}_n(\boldsymbol{\rho})B^{*\prime}_{nt}(\rho_t)$.

Denote the null estimator of $\boldsymbol{\theta}$ by $\tilde{\boldsymbol{\theta}}_{\mathtt{SLE2}}$. Let $J_{\mathtt{SLE2}}(\boldsymbol{\theta}) = -\frac{\partial}{\partial \boldsymbol{\theta}'}S^\star_{\mathtt{SLE2}}(\boldsymbol{\theta})$, $I_{\mathtt{SLE2}}(\boldsymbol{\theta}_0) =$

$\mathrm{E}[J_{\mathtt{SLE2}}(\boldsymbol{\theta}_0)]$ and $\Sigma_{\mathtt{SLE2}}(\boldsymbol{\theta}_0) = \mathrm{Var}[S^{\star}_{\mathtt{SLE2}}(\boldsymbol{\theta}_0)]$ with their expressions given in Appendix B.4. The *usual* score-type test and the robust version have the forms:

$$T_{\mathtt{SLE2}} = S^{\star}_{\mathtt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE2}})' J^{-1}_{\mathtt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE2}}) S^{\star}_{\mathtt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE2}}), \quad \text{and} \tag{3.16}$$

$$T^{r}_{\mathtt{SLE2}} = \widetilde{S}^{\star\prime}_{\mathtt{SLE2}} \widetilde{I}^{-1}_{\mathtt{SLE2}} C' \big(C \widetilde{I}^{-1}_{\mathtt{SLE2}} \widetilde{\Sigma}_{\mathtt{SLE2}} \widetilde{I}^{-1}_{\mathtt{SLE2}} C'\big)^{-1} C \widetilde{I}^{-1}_{\mathtt{SLE2}} \widetilde{S}^{\star}_{\mathtt{SLE2}}, \tag{3.17}$$

respectively, where $\widetilde{S}^{\star}_{\mathtt{SLE2}} = S^{\star}_{\mathtt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE2}})$, $\widetilde{I}_{\mathtt{SLE2}} = I_{\mathtt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE2}})$, $\widetilde{\Sigma}_{\mathtt{SLE2}} = \Sigma_{\mathtt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE2}})$, and the linear contrast matrix C has the same form as that for the 1FE panel SLE model. Let $\boldsymbol{\Xi}_{\mathtt{SLE2}}(\boldsymbol{\theta})$ be defined similarly as $\boldsymbol{\Xi}_{\mathtt{SLE1}}(\boldsymbol{\theta})$ for the 1FE panel SLE model.

Theorem 3.2. *Under Assumptions 1-2, 3′, and 4-5, if (i) $\tilde{\boldsymbol{\theta}}_{\mathtt{SLE2}}$ is $\sqrt{N}$-consistent for $\boldsymbol{\theta}_0$ under H_0, and (ii) $I_{\mathtt{SLE2}}(\boldsymbol{\theta})$ and $\boldsymbol{\Xi}_{\mathtt{SLE2}}(\boldsymbol{\theta})$ are positive definite for $\boldsymbol{\theta}$ in a neighborhood of $\boldsymbol{\theta}_0$ when N_0 is large enough, then we have, under H_0, $T^{r}_{\mathtt{SLE2}} \xrightarrow{D} \chi^2_{k_p}$, as $n \to \infty$.*

The d.f. k_p associated with these tests remain the same as that in Theorem 3.1. Similarly, it can be shown that $T_{\mathtt{SLE2}}$ is not an asymptotic pivotal quantity, and that $(T^{r}_{\mathtt{SLE2}} - k_p)/\sqrt{2k_p} \xrightarrow{D} N(0,1)$, as $n/\sqrt{T} \to \infty$.

Estimation of the null model. Let $\theta = (\beta', \lambda, \rho, \sigma^2)'$. Under H_0, the constrained estimate of c_n given (β, λ) becomes $\tilde{c}^{\circ *}_n(\beta, \lambda) = A^*_n(\lambda)\bar{Y}^*_n - \bar{X}^*_n\beta$ where $\bar{Y}^*_n$ and $\bar{X}^*_n$ are the averages of $\{Y^*_{nt}\}$ and $\{X^*_{nt}\}$, respectively. Along the same line leading to (3.15), one can easily show that AQS function for the null model of (3.13) takes the form:

$$S^{\circ *}_{\mathtt{SLE2}}(\theta) = \begin{cases} \frac{1}{\sigma^2}\sum_{t=1}^{T} X^{\circ *\prime}_{nt} B^{*\prime}_n(\rho) \widetilde{V}^{\circ *}_{nt}(\beta, \lambda, \rho), \\ \frac{1}{\sigma^2}\sum_{t=1}^{T} (W^*_n Y^{\circ *}_{nt})' B^{*\prime}_n(\rho) \widetilde{V}^{\circ *}_{nt}(\beta, \lambda, \rho) - (T-1)\mathrm{tr}[G^*_n(\lambda)], \\ \frac{1}{\sigma^2}\sum_{t=1}^{T} \widetilde{V}^{\circ *\prime}_{nt}(\beta, \lambda, \rho) H^*_n(\rho) \widetilde{V}^{\circ *}_{nt}(\beta, \lambda, \rho) - (T-1)\mathrm{tr}[H^*_n(\lambda)], \\ -\frac{(n-1)(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^{T} \widetilde{V}^{\circ *\prime}_{nt}(\beta, \lambda, \rho) \widetilde{V}^{\circ *}_{nt}(\beta, \lambda, \rho), \end{cases} \tag{3.18}$$

$\widetilde{V}^{\circ *}_{nt}(\beta, \lambda, \rho) = B^*_n(\rho)[A^*_n(\lambda)Y^*_{nt} - X^*_{nt}\beta - \tilde{c}^{\circ *}_n(\beta, \lambda)] = B^*_n(\rho)[A^*_n(\lambda)Y^{\circ *}_{nt} - X^{\circ *}_{nt}\beta]$, where $Y^{\circ *}_{nt} = Y^*_{nt} - \bar{Y}^*_n$ and $X^{\circ *}_{nt} = X^*_{nt} - \bar{X}^*_n$. Solving the estimating equations, $S^{\circ *}_{\mathtt{SLE2}}(\theta) = 0$, gives the null estimator $\tilde{\theta}_{\mathtt{SLE2}}$ of θ, which is obtained by first solving the first and last sets of equations of (3.18) to give the constrained estimators of β and σ^2, given λ and ρ, as

$$\tilde{\beta}_{\mathtt{SLE2}}(\lambda, \rho) = \big(\textstyle\sum_{t=1}^{T} X^{\circ *\prime}_{nt} B^{*\prime}_n(\rho) B^*_n(\rho) X^{\circ *}_{nt}\big)^{-1} \sum_{t=1}^{T} X^{\circ *}_{nt} B^{*\prime}_n(\rho) B^*_n(\rho) A^*_n(\lambda) Y^{\circ *}_{nt},$$
$$\tilde{\sigma}^2_{\mathtt{SLE2}}(\lambda, \rho) = \tfrac{1}{n(T-1)} \textstyle\sum_{t=1}^{T} \widetilde{V}^{\circ *\prime}_{nt}(\hat{\beta}_{\mathtt{SLE2}}(\lambda, \rho), \lambda, \rho) \widetilde{V}^{\circ *}_{nt}(\hat{\beta}_{\mathtt{SLE2}}(\lambda, \rho), \lambda, \rho),$$

and then substituting $\tilde{\beta}_{\mathtt{SLE2}}(\lambda, \rho)$ and $\tilde{\sigma}^2_{\mathtt{SLE2}}(\lambda, \rho)$ into the middle equations of (3.18) and solving the resulted concentrated estimating equation to give the null AQS estimators $\tilde{\lambda}_{\mathtt{SLE2}}$ of the common λ and $\tilde{\rho}_{\mathtt{SLE2}}$ of the common ρ, which in turn gives the AQS estimator $\tilde{\beta}_{\mathtt{SLE2}} = \tilde{\beta}_{\mathtt{SLE2}}(\tilde{\lambda}_{\mathtt{SLE2}}, \tilde{\rho}_{\mathtt{SLE2}})$ of the common β, and the AQS estimator $\tilde{\sigma}^2_{\mathtt{SLE2}} = \tilde{\sigma}^2_{\mathtt{SLE2}}(\tilde{\lambda}_{\mathtt{SLE2}}, \tilde{\rho}_{\mathtt{SLE2}})$ of σ^2. Finally, the AQS estimator of θ is $\tilde{\theta}_{\mathtt{SLE2}} = (\tilde{\beta}'_{\mathtt{SLE2}}, \tilde{\lambda}_{\mathtt{SLE2}}, \tilde{\rho}_{\mathtt{SLE2}}, \tilde{\sigma}^2_{\mathtt{SLE2}})'$. The proposed null estimator based on the AQS function provides an alternative to the direct and transformation approaches of Lee and Yu (2010). It can be shown to be asymptotically equivalent to the estimator based on an orthogonal transformation given in Lee and Yu (2010). Thus, $\tilde{\theta}_{\texttt{SLE2}}$ is $\sqrt{(n-1)(T-1)}$-consistent for θ. As in the 1FE panel SLE model, to estimate the null model under a different hypothesis, we simplify the general AQS function (3.15) and solve the resulted estimating functions. Estimation of c_n, γ and κ proceeds similarly.

4. Monte Carlo Study

Extensive Monte Carlo experiments are conducted to investigate the finite sample performance of the proposed tests, based on the following four data generation processes (DGPs), corresponding to the SDP models with, respectively, 1FE-SL, 2FE-SL, 1FE-SLE and 2FE-SLE:

$$\begin{aligned}
\texttt{DGP1}: \; & Y_{nt} = \lambda_{t0} W_n Y_{nt} + X_{1nt}\beta_{1t0} + X_{2nt}\beta_{2t0} + c_{n0} + V_{nt}, \; t = 1, 2, \ldots, T, \\
\texttt{DGP2}: \; & Y_{nt} = \lambda_{t0} W_n Y_{nt} + X_{1nt}\beta_{1t0} + X_{2nt}\beta_{2t0} + c_{n0} + \alpha_{t0}\ell_n + V_{nt}, \; t = 1, 2, \ldots, T. \\
\texttt{DGP3}: \; & Y_{nt} = \lambda_{t0} W_n Y_{nt} + X_{1nt}\beta_{1t0} + X_{2nt}\beta_{2t0} + c_{n0} + U_{nt}, \\
& U_{nt} = \rho_{t0} M_n U_{nt} + V_{nt}, \; t = 1, 2, \ldots, T. \\
\texttt{DGP4}: \; & Y_{nt} = \lambda_{t0} W_n Y_{nt} + X_{1nt}\beta_{1t0} + X_{2nt}\beta_{2t0} + c_{n0} + \alpha_{t0}\ell_n + U_{nt}, \\
& U_{nt} = \rho_{t0} M_n U_{nt} + V_{nt}, \; t = 1, 2, \ldots, T.
\end{aligned}$$

For all the Monte Carlo experiments, $\beta_{t0} = (\beta_{1t0}, \beta_{2t0})'$ is set to $(1, 1)'$ for all $t = 1, \ldots, T$, $\sigma_0^2 = 1$ and $\lambda_0 = \{0.5, 0, -0.5\}$, $\rho_0 = \{0.5, 0, -0.5\}$, $n = \{50, 100, 200, 500\}$, and $T = \{3, 6\}$. Each set of Monte Carlo results is based on 10,000 Monte Carlo samples for the two SL models, and 5,000 for the two SLE models.

The **weight matrices** are generated based on three different methods: (*i*) `Rook Contiguity`, (*ii*) `Queen Contiguity`, and (*iii*) `Group Interaction`, with details given in Yang (2015). In spatial layouts (*i*)-(*ii*), the degree of spatial interactions (number of neighbors each unit has) is fixed, while in (*iii*) it may grow with the sample size. This is attained by allowing for the number of groups, k, for each sample to be directly related to n. We have considered $k = n^{0.5}$, where k is the number of groups for each n and hence the degree of spatial dependence indicated by the average group size is $m = n/k$. The actual sizes of the groups are generated from a discrete uniform distribution from $.5m$ to $1.5m$.

The two **exogenous regressors** are generated according to `REG1:` $X_{knt} \overset{iid}{\sim} N(0, I_n)$ for $k = 1, 2$ and $t = 1, \ldots, T$; and `REG2:` the ith value of the kth regressor in the gth group is such that $X_{kt,ig} \overset{iid}{\sim} (2z_g + z_{ig})/\sqrt{10}$, where $(z_g, z_{i,g}) \overset{iid}{\sim} N(0, 1)$ when group interaction scheme is followed; $\{X_{kt,ig}\}$ are thus independent across k and t, but not across i.

The **errors**, $v_{it} = \sigma_0 e_{it}$, are generated according to `err1:` $\{e_{n,i}\}$ are iid standard normal; `err2:` $\{e_{n,i}\}$ are iid normal mixture with 10% of values from $N(0, 4)$ and the remaining from $N(0, 1)$, standardized to have mean 0 and variance 1; and `err3:` $\{e_{n,i}\}$ iid log-normal (i.e., $\log e_{it} \overset{iid}{\sim} N(0, 1)$) standardized to have mean 0 and variance 1.

Partial Monte Carlo results are reported in Tables 1 & 2 for the panel SL models, and

Tables 3 & 4 for the panel SLE models. The results in Tables 1 & 2 show the following.

(i) The proposed robust test performs very well in general with empirical coverage probabilities all very close to their nominal levels, except that in cases of heavy spatial dependence (`Group Interaction`) and not-so-large n, it can be slightly undersized. As sample size increases, the empirical sizes quickly converge to their nominal levels.

(ii) In contrast, the naïve test can perform quite badly, with empirical sizes being as high as 35% for tests of 10% nominal level, when the erorrs are fairly non-normal (e.g., log-normal). It is interesting to note that the size distortions for the naïve tests also drop as sample size increase.

(iii) A larger T seems lead to a worsened performance for the naïve tests under `Queen Contiguity` but not under `Group Interaction`.

(iv) The finite sample performance of the tests for 1FE panel SL model do not seem to differ much from those for 2FE panel SL model.

From the results for the panel SLE model, reported (in Tables 3 & 4) and unreported (available from the authors upon request), similar patterns are observed for the finite sample performance of the proposed tests. In summary, the proposed robust tests are reliable and easy to apply, and hence are recommended for the applied researchers.

5. Empirical Applications

The specification tests of temporal homogeneity in spatial panel data models given in this paper are demonstrated in a empirical settings using two well known data sets: *Public Capital Productivity* (Munnell, 1990) and *Cigarette Demand* (Baltagi and Levin, 1986).

Public Capital Productivity. The data set gives indicators related to public capital productivity for 48 US states observed over 17 years (1970-1986). In Munnel (1990), the empirical model specifies a Cobb-Douglas production function of the form:

$$\lg(\texttt{gsp}) = \beta_0 + \beta_1 \lg(\texttt{pcap}) + \beta_2 \lg(\texttt{pc}) + \beta_3 \lg(\texttt{emp}) + \beta_4 \texttt{unemp} + \epsilon,$$

with possibly one-way or two-way fixed effects, where '`gsp`' is the gross social product of a given state, '`pcap`', '`pc`' and '`emp`' are the inputs of public capital, private capital, and labor respectively. In order to capture business cycle effects, an additional variable '`unemp`' is also added which indicates the state unemployment rate. The model now is extended by adding the spatial effects. The spatial weight matrix (W_n) is specified using a contiguity form where (i, j)th element is indicated as 1 if state i and j share a common border, otherwise 0. The final W_n is row normalized.

Table below summarize the values of the test statistics and their p-values, for the *usual or naïve* score-type test and the nonnormality robust AQS test for temporal homogeneity based on both the full dataset and a subset of data corresponding to years 1970-75, fitted using the four models: 1FE-SL, 2FE-SL, 1FE-SLE and 2FE-SLE.

Tests for Temporal Homogeneity: Public Capital Productivity

	T_{SL1}	T^r_{SL1}	T_{SL2}	T^r_{SL2}	T^r_{SLE1}	T^r_{SLE1}	T_{SLE2}	T^r_{SLE2}
$T = 17$	1621.23	320.67	3189.55	328.02	1970.79	289.20	1555.99	326.51
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$T = 6$	263.88	83.40	30.32	24.11	93.52	44.87	16.36	26.82
p-value	0.00	0.00	0.21	0.51	0.00	0.04	0.98	0.63

Note that when $T = 6$, df = 25 for SL model and 30 for SLE model, with 10%, 5%, and 1% critical values being 34.38, 37.65, and 44.31 for SL model, and 40.2560, 43.77, and 50.89 for SLE model. When $T = 17$, df = 80 for SL model and 96 for SLE model, with 10%, 5%, and 1% critical values being 96.58, 101.88, and 112.33 for SL model, and 114.13, 119.87, and 131.14 for SLE model.

Overall, the results based on the full data show strong evidence against temporal homogeneity. For the results based on first six periods data, the tests are significant for the models with 1FE, but not for the models with 2FE. The latter is perhaps due to control of time-specific effects. Comparing the results based on full data of $T = 17$ periods with the results based on first six periods data, there seems to be a strong indication for the existence of change point(s) in the model, and hence it would be interesting to carry out further tests in 'dig' out the change point(s). The relatively much bigger values of the usual or naïve tests show that are rather unreliable, in line with the Monte Carlo results.

Cigarette Demand. Second application of the proposed tests uses another well known data set, *the Cigarettes Demand for the United States*. It contains a panel of 46 states over 30 time periods (1963-1992) and is listed as `CIGAR.TXT` on the Wiley web site associated with Baltagi (2008) with the response variable Y = Cigarette sales in packs per capita; and the covariates X_1 = Price per pack of cigarettes; X_2 = Population (Pop); X_3 = Population above the age of 16; X_4 = Per capita disposable income; X_5 = Minimum price in adjoining states per pack of cigarettes. Earlier studies include Hamilton (1972), McGuiness and Cowling (1975), Baltagi and Levin (1986, 1992), Baltagi et al. (2000), and Yang et al. (2006), all under homogeneity assumption and in log-log form except in Yang et al. (2006) who estimated the Box-Cox functional form. The spatial weight matrix (W_n) is specified using a contiguity form where (i, j)th element is indicated as 1 if state i and j share a common border, otherwise 0. The final W_n is row normalized.

Tests for temporal homogeneity/heterogeneity is of particular interest in cigarette demand, due to government's policy interventions (in 1965, 1967, 1971) in attempting reducing the consumptions of cigarettes, and the reports from medial journals as well as Surgeon General warning (in 1983) about the health hazards of smoking (see Baltagi and Levin, 1986). The table below summarize the values of the test statistics and their p-values, for tests of homogeneity based on, respectively, the first 17 periods of data and the first 6 periods of data, and using the log-log form.

Tests for Temporal Homogeneity: Cigarette Demand

	T_{SL1}	T^r_{SL1}	T_{SL2}	T^r_{SL2}	T_{SLE1}	T^r_{SLE1}	T_{SLE2}	T^r_{SLE2}
$T = 17$	2828.62	327.73	4265.07	323.37	913.31	323.20	3723.99	342.54
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$T = 6$	79.25	45.66	59.40	38.00	328.38	50.06	72.51	45.34
p-value	0.00	0.03	0.001	0.15	0.00	0.048	0.00	0.11

Note that when $T = 6$, df = 30 for SL model and 35 for SLE model, with 10%, 5%, and 1% critical values being 40.26, 43.77, and 50.89 for SL model and 46.06, 49.80, and 57.34 for SLE model. When $T = 17$, df = 96 for SL model and 112 for SLE model, with 10%, 5%, and 1% critical values being 114.13, 119.87, 131.14 for SL model, and 131.56, 137.70, 149.73 for SLE model.

Again, the larger data set provide strong evidence against temporal homogeneity as the p-values for all the tests are very small. The results corresponding to the small data are mixed: the tests are significant based on the 1FE models but not based on the 2FE models, indicating that once the time-specific effects are controlled, the regression and spatial coefficients remain stable in the first six years. These suggest the existence of structure breaks, and further tests can be carried out to identify a 'parsimonious model' if one is not willing to go for the largest model with full temporal heterogeneity on the regression and spatial coefficients. Again, the relatively much bigger values of the usual or naïve tests show that they are rather unreliable, as the Monte Carlo results indicate that they over reject the null hypothesis.

6. Conclusion and Discussion

We introduce *adjusted quasi score tests* for temporal homogeneity/heterogeneity in regression and spatial coefficients in spatial panel data models allowing the existence of spatial and temporal heterogeneity in the intercepts of the model. The proposed tests are robust against nonnormality, they are simple and reliable as shown by the Monte Carlo results, and can be repeatedly applied to identity a 'parsimonious model' instead of the model with full temporal heterogeneity. That is, once the null hypothesis of homogeneity is rejected (as in the two empirical applications), one may proceed with further tests of hypotheses with known change points suggested by the data (as in Cigarette Demand application). Thus, the proposed tests provide useful tools for the applied researchers.

The tests can be extended by (*i*) adding higher-order spatial terms and spatial Durbin terms in the model, (*ii*) treating individual- and time-specific effects as random effects, or correlated random effects, (*iii*) allowing spatial-temporal heterogeneity in error variance (i.e., heteroskedasticity), (*iv*) allowing interactive fixed effects, and (*v*) by allowing dynamic effects in the model. These extensions are interesting but clearly beyond the scope of the current paper, which will be in our future research agenda.

Appendix A: Some Basic Lemmas

This section presents some basic lemmas that greatly facilitate the derivations and proofs of theoretical results given in the subsequent appendices.

Lemma A.1 (Kelejian and Prucha, 1999; Lee, 2002): *Let* $\{A_n\}$ *and* $\{B_n\}$ *be two sequences of* $n \times n$ *matrices that are uniformly bounded in both row and column sums. Let* C_n *be a sequence of conformable matrices whose elements are uniformly bounded. Then*

(i) the sequence $\{A_n B_n\}$ *are uniformly bounded in both row and column sums,*

(ii) the elements of A_n *are uniformly bounded and* $tr(A_n) = O(n)$*, and*

(iii) the elements of $A_n C_n$ *and* $C_n A_n$ *are uniformly bounded.*

Lemma A.2 (Yang, 2015b, Lemma A.1, extended). *For* $t = 1, 2$*, let* A_{nt} *be* $n \times n$ *matrices and* c_{nt} *be an* $n \times 1$ *vectors. Let* ε_n *be an* $n \times 1$ *vector of iid elements with mean zero, variance* σ^2*, and finite 3rd and 4th cumulants* μ_3 *and* μ_4*. Let* a_{nt} *be the vector of diagonal elements of* A_{nt}*. Define* $Q_{nt} = c'_{nt}\varepsilon_n + \varepsilon'_n A_{nt}\varepsilon_n$*,* $t = 1, 2$*. Then, for* $t, s = 1, 2$*,*

$$\begin{aligned}&\mathrm{Cov}(Q_{nt}, Q_{ns}) \equiv f(A_{nt}, c_{nt}; A_{ns}, c_{ns}) \\ =&\sigma^4 \mathrm{tr}[(A'_{nt} + A_{nt})A_{ns})] + \mu_3 a'_{nt}c_{ns} + \mu_3 c'_{nt}a_{ns} + \mu_4 a'_{nt}a_{ns} + \sigma^2 c'_{nt}c_{ns}. \end{aligned} \tag{A.1}$$

Various useful special cases of (A.1) are as follows:

(i) $\mathrm{Cov}(c'_{n1}\varepsilon_n, Q_{n2}) = f(\mathbf{0}, c_{n1}; A_{n2}, c_{n2}) = \mu_3 c'_{n1}a_{n2} + \sigma^2 c'_{n1}c_{n2}$,
where c_{n1} can be an $n \times k$ matrix with $k \geq 1$;

(ii) $\mathrm{Var}(Q_{n1}) = f(A_{n1}, c_{n1}; A_{n1}, c_{n1}) = \sigma^4 \mathrm{tr}[(A'_{n1} + A_{n1})A_{n1})] + 2\mu_3 a'_{n1}c_{n1}$
$+\mu_4 a'_{n1}a_{n1} + \sigma^2 c'_{n1}c_{n1}$;

(iii) $\mathrm{Var}(\varepsilon'_n A_{n1}\varepsilon_n) = f(A_{n1}, \mathbf{0}; A_{n1}, \mathbf{0}) = \sigma^4 \mathrm{tr}[(A'_{n1} + A_{n1})A_{n1})] + \mu_4 a'_{n1}a_{n1}$.

Lemma A.3 (CLT for Linear-Quadratic Forms, Kelejian and Prucha, 2001). *Let* A_n, a_n, c_n *and* ε_n *be as in Lemma A.2. Assume (i)* A_n *is bounded uniformly in row and column sums, (ii)* $n^{-1}\sum_{i=1}^n |c_{n,i}^{2+\eta_1}| < \infty, \eta_1 > 0$*, and (iii)* $E|\varepsilon_{n,i}^{4+\eta_2}| < \infty$*,* $\eta_2 > 0$*. Then,*

$$\frac{\varepsilon'_n A_n \varepsilon_n + c'_n \varepsilon_n - \sigma^2 \mathrm{tr}(A_n)}{\{\sigma^4 \mathrm{tr}(A'_n A_n + A_n^2) + \mu_4 a'_n a_n + \sigma^2 c'_n c_n + 2\mu_3 a'_n c_n\}^{\frac{1}{2}}} \xrightarrow{D} N(0, 1).$$

Appendix B: Hessian, Expected Hessian and VC Matrices

Notation. For $t, s = 1, \ldots, T$, `blkdiag`$\{A_t\}$ forms a block-diagonal matrix by placing A_t diagonally, $\{A_t\}$ forms a matrix by stacking A_t horizontally, and $\{B_{ts}\}$ forms a matrix by the component matrices B_{ts}. The negative Hessian matrix $J_\varpi(\boldsymbol{\theta}_0)$, its expectation $I_\varpi(\boldsymbol{\theta}_0)$, and the VC matrix $\Sigma_\varpi(\boldsymbol{\theta}_0)$ of the AQS function, ϖ=`SL1`, `SL2`, `SLE1`, `SLE2`, are all partitioned according to the slope parameters $\boldsymbol{\beta}$, the spatial lag parameters $\boldsymbol{\lambda}$, spatial error parameters $\boldsymbol{\rho}$ (if existing in the model), and the error variance σ^2, with the sub-matrices denoted by, e.g., $J_{\boldsymbol{\beta\beta}}$, $J_{\boldsymbol{\beta\lambda}}$, $I_{\boldsymbol{\beta\beta}}$, $I_{\boldsymbol{\beta\lambda}}$, $\Sigma_{\boldsymbol{\beta\beta}}$, $\Sigma_{\boldsymbol{\beta\lambda}}$. Furthermore, `diag`$(\cdot)$ forms a diagonal matrix and `diagv`$(\cdot)$ a column vector, based on the diagonal elements of a square matrix.

B.1. Panel SL model with one-way FE. Letting $\eta_{nt} = G_{nt}(X_{nt}\beta_t + c_n)$ and $g_{nt} = \texttt{diagv}(G_{nt})$, the negative Hessian matrix, $J_{\texttt{SL1}}(\boldsymbol{\theta}_0)$, has the components:

$$
\begin{aligned}
J_{\boldsymbol{\beta\beta}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2}X'_{nt}X_{nt}\big\} - \big\{\tfrac{1}{T\sigma_0^2}X'_{nt}X_{ns}\big\},\\
J_{\boldsymbol{\lambda\beta}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2}(W_nY_{nt})'X_{nt}\big\} - \big\{\tfrac{1}{T\sigma_0^2}(W_nY_{nt})'X_{ns}\big\},\\
J_{\boldsymbol{\lambda\lambda}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2}(W_nY_{nt})'(W_nY_{nt}) + \tfrac{T-1}{T}\mathrm{tr}(G_{nt}^2)\big\} - \big\{\tfrac{1}{T\sigma_0^2}(W_nY_{nt})'(W_nY_{ns})\big\},\\
J_{\sigma^2\boldsymbol{\beta}} &= \big\{\tfrac{1}{\sigma_0^4}\tilde{V}'_{nt}X_{nt}\big\},\ J_{\sigma^2\boldsymbol{\lambda}} = \big\{\tfrac{1}{\sigma_0^4}(W_nY_{nt})'\tilde{V}_{nt}\big\},\ J_{\sigma^2\sigma^2} = -\tfrac{n(T-1)}{2\sigma_0^4} + \tfrac{1}{\sigma_0^6}\textstyle\sum_{t=1}^T \tilde{V}'_{nt}\tilde{V}_{nt}.
\end{aligned}
$$

The expected negative Hessian matrix, $I_{\texttt{SL1}}(\boldsymbol{\theta}_0)$, has the components:

$$
\begin{aligned}
I_{\boldsymbol{\beta\beta}} &= J_{\boldsymbol{\beta\beta}},\ I_{\boldsymbol{\lambda\beta}} = \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2}\eta'_{nt}X_{nt}\big\} - \big\{\tfrac{1}{T\sigma_0^2}\eta'_{nt}X_{ns}\big\},\\
I_{\boldsymbol{\lambda\lambda}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2}\eta'_{nt}\eta_{nt} + \tfrac{T-1}{T}\mathrm{tr}(G_{nt}^sG_{nt})\big\} - \big\{\tfrac{1}{T\sigma_0^2}\eta'_{nt}\eta_{ns}\big\},\\
I_{\sigma^2\boldsymbol{\beta}} &= 0'_{tk},\ I_{\sigma^2\lambda} = \big\{\tfrac{T-1}{T\sigma_0^2}\mathrm{tr}(G_{nt})\big\},\ I_{\sigma^2\sigma^2} = \tfrac{n(T-1)}{2\sigma_0^4}.
\end{aligned}
$$

The VC matrix $\Sigma_{\texttt{SL1}}(\boldsymbol{\theta}_0) = I_{\texttt{SL1}}(\boldsymbol{\theta}_0) + \Omega_{\texttt{SL1}}(\boldsymbol{\theta}_0)$, where $\Omega_{\texttt{SL1}}(\boldsymbol{\theta}_0)$ has components:

$$
\begin{aligned}
\Omega_{\boldsymbol{\beta\beta}} &= 0_{tk\times tk},\ \Omega_{\boldsymbol{\lambda\beta}} = \texttt{bikdiag}\big\{\tfrac{T-1}{T\sigma_0}\gamma g'_{nt}X_{nt}\big\} - \big\{\tfrac{T-1}{T^2\sigma_0}\gamma g'_{nt}X_{ns}\big\},\\
\Omega_{\boldsymbol{\lambda\lambda}} &= \texttt{blkdiag}\big\{\tfrac{2(T-1)}{T\sigma_0}\gamma\eta'_{nt}g_{nt} + (\tfrac{T-1}{T})^2\kappa g'_{nt}g_{nt} - \tfrac{1}{T}\mathrm{tr}(G_{nt}G_{nt})\big\}\\
&\quad -\big\{\tfrac{T-1}{T^2\sigma_0}\gamma(\eta'_{nt}g_{ns} + g'_{nt}\eta_{ns}) - \tfrac{1}{T^2}\mathrm{tr}(G_{nt}G_{ns})\big\},\\
\Omega_{\sigma^2\boldsymbol{\beta}} &= \big\{0'_{tk}\big\},\ \Omega_{\sigma^2\boldsymbol{\lambda}} = \big\{\tfrac{(T-1)^2}{2T^2\sigma_0^2}\kappa\,\mathrm{tr}(G_{nt})\big\},\ \Omega_{\sigma^2\sigma^2} = \tfrac{n(T-1)^2}{4T\sigma_0^4}\kappa.
\end{aligned}
$$

where γ and κ are, respectively, the measures of skewness and excess kurtosis of $v_{i,t}$.

Alternatively, we can find the VC matrix $\Sigma_{n,T}(\boldsymbol{\theta}_0)$ by first expressing the AQS function (2.7) at $\boldsymbol{\theta}_0$ in terms of $\mathbb{V}_N = (V'_{n1}, \ldots, V'_{nT})'$, where $N = nT$, and then applying Lemma A.2. Let z_t be a $T \times 1$ of element 1 in the tth position and 0 elsewhere, and define $Z_{Nt} = z_t \otimes I_n$, $\bar{Z}_N = \frac{1}{T}(l_T \otimes I_n)$, and $Z^{\circ}_{Nt} = Z_{Nt} - \bar{Z}_N$. Thus, $V_{nt} = Z'_{Nt}\mathbb{V}_N$ and $\widetilde{V}_{nt} = V_{nt} - \overline{V}_n = Z^{\circ\prime}_{Nt}\mathbb{V}_N$. The AQS function (2.7) at $\boldsymbol{\theta}_0$ takes the form:

$$
S^{\star}_{\texttt{SL1}}(\boldsymbol{\theta}_0) = \begin{cases} \Pi'_{1t}\mathbb{V}_N,\ t = 1, \ldots, T, \\ \Pi'_{2t}\mathbb{V}_N + \mathbb{V}'_N\Phi_t\mathbb{V}_N - \frac{T-1}{T}\mathrm{tr}(G_{nt}),\ t = 1, \ldots, T, \\ \mathbb{V}'_N\Psi\mathbb{V}_N - \frac{n(T-1)}{2\sigma^2}, \end{cases} \tag{B.1}
$$

where $\Pi_{1t} = \frac{1}{\sigma_0^2}Z^{\circ}_{Nt}X_{nt}$, $\Pi_{2t} = \frac{1}{\sigma_0^2}Z^{\circ}_{Nt}\eta_{nt}$, $\Phi_t = \frac{1}{\sigma_0^2}Z_{Nt}G'_{nt}Z^{\circ\prime}_{Nt}$, and $\Psi = \frac{1}{2\sigma^4}\sum_{t=1}^T Z^{\circ}_{Nt}Z^{\circ\prime}_{Nt}$.

Applying Lemma A.2 with ε, c_{nt} and A_{nt} replaced by $\mathbb{V}_N$, Π_{1t} and Π_{2t}, Φ_t, and Ψ, we obtain the VC matrix of the AQS function:

$$
\Sigma_{\texttt{SL1}}(\boldsymbol{\theta}_0) = \begin{pmatrix} \{f(\mathbf{0}, \Pi_{1t}; \mathbf{0}, \Pi_{1s})\}, & \{f(\mathbf{0}, \Pi_{1t}; \Phi_s, \Pi_{2s})\}, & \{f(\mathbf{0}, \Pi_{1t}; \Psi, \mathbf{0})\} \\ \sim, & \{f(\Phi_t, \Pi_{2t}; \Phi_s, \Pi_{2s})\}, & \{f(\Phi_t, \Pi_{2t}; \Psi, \mathbf{0})\} \\ \sim, & \sim, & f(\Psi, \mathbf{0}; \Psi, \mathbf{0}) \end{pmatrix}.
$$

This expression can be reduced to that given above, but it greatly simplifies the calculation.

B.2. Panel SL model with two-way FE. Letting $\eta^*_{nt} = G^*_{nt}(X^*_{nt}\beta_t + c^*_n)$ and $g^*_{nt} = \mathtt{diagv}(G^*_{nt})$, as the AQS function takes a similar form as that for 1FE panel SL model, the negative Hessian, $J_{\mathtt{SL2}}(\boldsymbol{\theta}_0)$, also takes a similar form:

$$J_{\boldsymbol{\beta\beta}} = \mathtt{blkdiag}\big\{\tfrac{1}{\sigma_0^2}X^{*\prime}_{nt}X^*_{nt}\big\} - \big\{\tfrac{1}{T\sigma_0^2}X^{*\prime}_{nt}X^*_{ns}\big\},$$

$$J_{\boldsymbol{\lambda\beta}} = \mathtt{blkdiag}\big\{\tfrac{1}{\sigma_0^2}(W^*_nY^*_{nt})'X^*_{nt}\big\} - \big\{\tfrac{1}{T\sigma_0^2}(W^*_nY^*_{nt})'X^*_{ns}\big\},$$

$$J_{\boldsymbol{\lambda\lambda}} = \mathtt{blkdiag}\big\{\tfrac{1}{\sigma_0^2}(W^*_nY^*_{nt})'(W^*_nY^*_{nt}) + \tfrac{T-1}{T}\mathrm{tr}(G^{*2}_{nt})\big\} - \big\{\tfrac{1}{T\sigma_0^2}(W^*_nY^*_{nt})'(W^*_nY^*_{ns})\big\},$$

$$J_{\sigma^2\boldsymbol{\beta}} = \big\{\tfrac{1}{\sigma_0^4}\tilde{V}^{*\prime}_{nt}X^*_{nt}\big\},\ J_{\sigma^2\boldsymbol{\lambda}} = \big\{\tfrac{1}{\sigma_0^4}(W^*_nY^*_{nt})'\tilde{V}^*_{nt}\big\},\ J_{\sigma^2\sigma^2} = -\tfrac{(n-1)(T-1)}{2\sigma_0^4} + \tfrac{1}{\sigma_0^6}\textstyle\sum_{t=1}^T \tilde{V}^{*\prime}_{nt}\tilde{V}^*_{nt}.$$

As the derivation of the expected negative Hessian matrix involves only the first two moments of the transformed errors which are the same as the first two moments of the original error, the expected negative Hessian matrix, $I_{\mathtt{SL2}}(\boldsymbol{\theta}_0)$, also takes a similar form as that of 1FE panel SL model and contains the following components:

$$I_{\boldsymbol{\beta\beta}} = J_{\boldsymbol{\beta\beta}},\ I_{\boldsymbol{\lambda\beta}} = \mathtt{blkdiag}\big\{\tfrac{1}{\sigma_0^2}\eta^{*\prime}_{nt}X^*_{nt}\big\} - \big\{\tfrac{1}{T\sigma_0^2}\eta^{*\prime}_{nt}X^*_{ns}\big\},$$

$$I_{\boldsymbol{\lambda\lambda}} = \mathtt{blkdiag}\big\{\tfrac{1}{\sigma_0^2}\eta^{*\prime}_{nt}\eta^*_{nt} + \tfrac{T-1}{T}\mathrm{tr}(G^{*s}_{nt}G^*_{nt})\big\} - \big\{\tfrac{1}{T\sigma_0^2}\eta^{*\prime}_{nt}\eta^*_{ns}\big\},$$

$$I_{\sigma^2\boldsymbol{\beta}} = 0'_{tk},\ I_{\sigma^2\lambda} = \big\{\tfrac{T-1}{T\sigma_0^2}\mathrm{tr}(G^*_{nt})\big\},\ I_{\sigma^2\sigma^2} = \tfrac{(n-1)(T-1)}{2\sigma_0^4}.$$

The derivation of the VC matrix of the AQS function, however, is different from that of one-way panel SL model due to the involvement of 3rd and 4th moments of the errors. The elements of the transformed errors V^*_{nt} may not be totally independent unless the original errors are normal and their 3rd and 4th moments may not be constant. Thus, one needs to work with the original error vector V_{nt} through $V^*_{nt} = F'_{n,n-1}V_{nt}$. The VC matrix $\Sigma_{\mathtt{SL2}}(\boldsymbol{\theta}_0) = I_{\mathtt{SL2}}(\boldsymbol{\theta}_0) + \Omega_{\mathtt{SL2}}(\boldsymbol{\theta}_0)$, where $\Omega_{\mathtt{SL2}}(\boldsymbol{\theta}_0)$ has components:[5]

$$\Omega_{\boldsymbol{\beta\beta}} = 0_{tk\times tk},\ \Omega_{\boldsymbol{\lambda\beta}} = \mathtt{bikdiag}\big\{\tfrac{T-1}{T\sigma_0}\gamma g^{*\prime}_{nt}F_{n,n-1}X^*_{nt}\big\} - \big\{\tfrac{T-1}{T^2\sigma_0}\gamma g^{*\prime}_{nt}F_{n,n-1}X^*_{ns}\big\},$$

$$\begin{aligned}\Omega_{\boldsymbol{\lambda\lambda}} &= \mathtt{blkdiag}\big\{\tfrac{2(T-1)}{T\sigma_0}\gamma\eta^{*\prime}_{nt}F'_{n,n-1}g^*_{nt} + (\tfrac{T-1}{T})^2\kappa g^{*\prime}_{nt}g^*_{nt} - \tfrac{1}{T}\mathrm{tr}(G^*_{nt}G^*_{nt})\big\}\\ &\quad -\big\{\tfrac{T-1}{T^2\sigma_0}\gamma(\eta^{*\prime}_{nt}F'_{n,n-1}g^*_{ns} + g^{*\prime}_{nt}F_{n,n-1}\eta^*_{ns}) - \tfrac{1}{T^2}\mathrm{tr}(G^*_{nt}G^*_{ns})\big\},\end{aligned}$$

$$\Omega_{\sigma^2\boldsymbol{\beta}} = \big\{0'_{tk}\big\},\ \Omega_{\sigma^2\boldsymbol{\lambda}} = \big\{\tfrac{(T-1)^2}{2T^2\sigma_0^2}\kappa\mathrm{diag}(J_n)\mathtt{diag}(F_{n,n-1}G^*_{nt}F'_{n,n-1})\big\},\ \Omega_{\sigma^2\sigma^2} = \tfrac{n(T-1)^2}{4T\sigma_0^4}\kappa.$$

Similarly, $\Sigma_{\mathtt{SL2}}(\boldsymbol{\theta}_0)$ can be obtained by first expressing $S^\star_{\mathtt{SL2}}(\boldsymbol{\theta}_0)$ in $\mathbb{V}_N$, through $V^*_{nt} = F'_{n,n-1}Z'_{Nt}\mathbb{V}_N$ and $\widetilde{V}^*_{nt} = V^*_{nt} - \overline{V}^*_n = F'_{n,n-1}Z^{\circ\prime}_{Nt}\mathbb{V}_N$:

$$S^\star_{\mathtt{SL2}}(\boldsymbol{\theta}_0) = \begin{cases}\Pi'_{1t}\mathbb{V}_N,\ t = 1,\ldots,T,\\ \Pi'_{2t}\mathbb{V}_N + \mathbb{V}'_N\Phi_t\mathbb{V}_N - \frac{T-1}{T}\mathrm{tr}(G_{nt}),\ t = 1,\ldots,T,\\ \mathbb{V}'_N\Psi\mathbb{V}_N - \frac{(n-1)(T-1)}{2\sigma^2},\end{cases} \tag{B.2}$$

where $\Pi_{1t} = \frac{1}{\sigma_0^2}Z^\circ_{Nt}F_{n,n-1}X^*_{nt}$, $\Pi_{2t} = \frac{1}{\sigma_0^2}Z^\circ_{Nt}F_{n,n-1}\eta^*_{nt}$, $\Phi_t = \frac{1}{\sigma_0^2}Z_{Nt}F_{n,n-1}G^{*\prime}_{nt}F'_{n,n-1}Z^{\circ\prime}_{Nt}$, and $\Psi = \frac{1}{2\sigma^4}\sum_{t=1}^T Z^\circ_{Nt}F_{n,n-1}F'_{n,n-1}Z^{\circ\prime}_{Nt}$. Then, applying Lemma A.2 with ε, c_{nt} and A_{nt} replaced by $\mathbb{V}_N$, Π_{1t} and Π_{2t}, Φ_t, and Ψ to give $\Sigma_{\mathtt{SL2}}(\boldsymbol{\theta}_0)$ in an identical form as $\Sigma_{\mathtt{SL1}}(\boldsymbol{\theta}_0)$.

[5]In the derivations, we have used: (*i*) $(I_{n-1} - \lambda_tF'_{n,n-1}W_nF_{n,n-1})^{-1} = F'_{n,n-1}(I_n - \lambda_tW_n)^{-1}F_{n,n-1}$ (Lee and Yu (2010, Lemma A.2), and (*ii*) for a row normalized W_n, $F'_{n,n-1}W_nJ_n = F'_{n,n-1}W_n$ and $G^*_n(\lambda_t) = F'_{n,n-1}G_n(\lambda_t)F_{n,n-1}$ and $g^*_{nt} = \mathtt{diag}(F_{n,n-1}G^*_n(\lambda_t)F'_{n,n-1})$.

B.3. Panel SLE model with one-way FE. Let $\dot{D}_{nt} = -\frac{d}{d\rho_{t0}} D_{nt} = M_n' B_{nt} + B_{nt}' M_n$. We have the components of the negative Hessian matrix $J_{\mathtt{SEL1}}(\boldsymbol{\theta}_0)$:

$$
\begin{aligned}
J_{\boldsymbol{\beta}\boldsymbol{\beta}} &= \mathtt{blkdiag}\{\tfrac{1}{\sigma_0^2} X_{nt}' D_{nt} X_{nt}\} - \{\tfrac{1}{\sigma_0^2} X_{nt}' D_{nt} \mathbb{D}_n^{-1} D_{ns} X_{ns}\};\\
J_{\boldsymbol{\beta}\boldsymbol{\lambda}} &= \mathtt{blkdiag}\{\tfrac{1}{\sigma_0^2} X_{nt}' D_{nt} W_n Y_{nt}\} - \{\tfrac{1}{\sigma_0^2} X_{nt}' D_{nt} \mathbb{D}_n^{-1} D_{ns} W_n Y_{ns}\};\\
J_{\boldsymbol{\beta}\boldsymbol{\rho}} &= \mathtt{blkdiag}\{\tfrac{1}{\sigma_0^2} X_{nt}' \dot{D}_{nt} B_{nt}^{-1} \widetilde{V}_{nt}\} - \{\tfrac{1}{\sigma_0^2} X_{nt}' D_{nt} \mathbb{D}_n^{-1} \dot{D}_{ns} B_{ns}^{-1} \widetilde{V}_{ns}\};\\
J_{\boldsymbol{\lambda}\boldsymbol{\lambda}} &= \mathtt{blkdiag}\{\tfrac{1}{\sigma_0^2} (W_n Y_{nt})' D_{nt} (W_n Y_{nt}) + \mathrm{tr}(R_{nt} G_{nt}^2)\}\\
&\quad - \{\tfrac{1}{\sigma_0^2} (W_n Y_{nt})' D_{nt} \mathbb{D}_n^{-1} D_{ns} (W_n Y_{ns})\};\\
J_{\boldsymbol{\lambda}\boldsymbol{\rho}} &= \mathtt{blkdiag}\{\tfrac{1}{\sigma_0^2} (W_n Y_{nt})' \dot{D}_{nt} B_{nt}^{-1} \widetilde{V}_{nt} + \mathrm{tr}[\mathbb{D}_n^{-1} \dot{D}_{nt} G_{nt}]\}\\
&\quad - \{\tfrac{1}{\sigma_0^2} (W_n Y_{nt})' D_{nt} \mathbb{D}_n^{-1} \dot{D}_{ns} B_{ns}^{-1} \widetilde{V}_{ns} + \mathrm{tr}[\mathbb{D}_n^{-1} D_{nt} G_{nt} \mathbb{D}_n^{-1} \dot{D}_{ns}]\};\\
J_{\boldsymbol{\rho}\boldsymbol{\lambda}} &= \mathtt{blkdiag}\{\tfrac{1}{\sigma_0^2} (W_n Y_{nt})' \dot{D}_{nt} B_{nt}^{-1} \widetilde{V}_{nt}\} - \{\tfrac{1}{\sigma_0^2} (W_n Y_{ns})' D_{ns} \mathbb{D}_n^{-1} \dot{D}_{nt} B_{nt}^{-1} \widetilde{V}_{nt}\};\\
J_{\boldsymbol{\rho}\boldsymbol{\rho}} &= \mathtt{blkdiag}\{\tfrac{1}{\sigma^2} \widetilde{V}_{nt}' H_{nt}' H_{nt} \widetilde{V}_{nt} + \mathrm{tr}(H_{nt}^2 + \mathbb{D}_n^{-1} M_n' M_n)\}\\
&\quad - \{\tfrac{1}{\sigma^2} \widetilde{V}_{nt}' H_{nt}^s B_{nt} \mathbb{D}_n^{-1} B_{ns}' H_{ns}^s \widetilde{V}_{ns} + \mathrm{tr}(\mathbb{D}_n^{-1} B_{nt}' M_n \mathbb{D}_n^{-1} \dot{D}_{ns})\};\\
J_{\sigma^2\boldsymbol{\beta}} &= \{\tfrac{1}{\sigma_0^4} X_{nt}' B_{nt}' \widetilde{V}_{nt}\};\;\; J_{\sigma^2\boldsymbol{\lambda}} = \{\tfrac{1}{\sigma_0^4} (W_n Y_{nt})' B_{nt}' \widetilde{V}_{nt}\};\\
J_{\sigma^2\boldsymbol{\rho}} &= \{\tfrac{1}{\sigma_0^4} \widetilde{V}_{nt}' H_{nt} \widetilde{V}_{nt}\};\;\; J_{\sigma^2\sigma^2} = -\tfrac{n(T-1)}{2\sigma^4} + \tfrac{1}{\sigma^6} \textstyle\sum_{t=1}^T \widetilde{V}_{nt}' \widetilde{V}_{nt}.
\end{aligned}
$$

and the components of the expected negative Hessian matrix $I_{\mathtt{SEL1}}(\boldsymbol{\theta}_0)$:

$$
\begin{aligned}
& I_{\boldsymbol{\beta}\boldsymbol{\beta}} = J_{\boldsymbol{\beta}\boldsymbol{\beta}},\;\; I_{\boldsymbol{\lambda}\boldsymbol{\beta}} = \mathtt{blkdiag}\{\tfrac{1}{\sigma_0^2} \eta_t' D_{nt} X_{nt}\} - \{\tfrac{1}{\sigma_0^2} \eta_t' D_{nt} \mathbb{D}_n^{-1} D_{ns} X_{ns}\},\;\; I_{\boldsymbol{\rho}\boldsymbol{\beta}} = 0_{Tk}\\
& I_{\boldsymbol{\lambda}\boldsymbol{\lambda}} = \mathtt{blkdiag}\{\tfrac{1}{\sigma_0^2} \eta_{nt}' D_{nt} \eta_{nt} + \mathrm{tr}[S_{nt}(\rho) \dot{G}_{nt}^s \dot{G}_{nt}]\} - \{\tfrac{1}{\sigma_0^2} \eta_{nt}' D_{nt} \mathbb{D}_n^{-1} D_{ns} \eta_{ns}\},\\
& I_{\boldsymbol{\lambda}\boldsymbol{\rho}} = \mathtt{blkdiag}\{\mathrm{tr}[\dot{G}_{nt}' S_{nt}(\boldsymbol{\rho}) H_{nt}^s]\};\;\; I_{\sigma^2\sigma^2} = -\tfrac{n(T-1)}{2\sigma_0^4} + \tfrac{1}{\sigma_0^4} \textstyle\sum_{t=1}^T \mathrm{tr} S_{nt}(\boldsymbol{\rho})\\
& I_{\boldsymbol{\rho}\boldsymbol{\lambda}} = \mathtt{blkdiag}\{\mathrm{tr}[\dot{G}_{nt}' S_{nt}(\boldsymbol{\rho}) H_{nt}^s S_{nt}(\boldsymbol{\rho})]\} - \{\mathrm{tr}[G_{ns}' D_{ns} \mathbb{D}_n^{-1} \dot{D}_{nt} \mathbb{D}_n^{-1}]\}\\
& I_{\boldsymbol{\rho}\boldsymbol{\rho}} = \mathtt{blkdiag}\{\mathrm{tr}[H_{nt}^s S_{nt}(\boldsymbol{\rho}) H_{nt} - B_{nt} \mathbb{D}_n^{-1} \dot{D}_{nt} B_{nt}^{-1} H_{nt}]\} + \{\mathrm{tr}[B_{nt} \mathbb{D}_n^{-1} \dot{D}_{ns} \mathbb{D}_n^{-1} B_{nt}' H_{nt}]\}\\
& I_{\sigma^2\boldsymbol{\beta}} = 0_{tk}',\;\; I_{\sigma^2\boldsymbol{\lambda}} = \{\tfrac{1}{\sigma_0^2} \mathrm{tr}[R_{nt}(\boldsymbol{\rho}) G_{nt}]\},\;\; I_{\sigma^2\boldsymbol{\rho}} = \tfrac{1}{\sigma_0^2} \mathrm{tr}(S_{nt}(\boldsymbol{\rho}) H_{nt}).
\end{aligned}
$$

To derive $\Sigma_{\mathtt{SLE1}}(\boldsymbol{\theta}_0)$, we have, $\widetilde{V}_{nt} \equiv \widetilde{V}_{nt}(\boldsymbol{\beta}_0, \boldsymbol{\lambda}_0, \boldsymbol{\rho}_0) = V_{nt} - B_{nt} \mathbb{D}_n^{-1} \sum_{s=1}^T B_{ns}' V_{ns} = Z_{Nt}^{\circ\prime} \mathbb{V}_N$, where $Z_{Nt}^{\circ\prime} = [Z_{Nt}' - B_{nt} \mathbb{D}_n^{-1} (l_T' \otimes I_n) \mathbb{B}_N]$ and $\mathbb{B}_N = \mathtt{blkdiag}(B_{n1}, \ldots, B_{nT})$, and $W_n Y_{nt} = G_{nt}(X_{nt}\beta_0 + c_n + B_{nt}^{-1} V_{nt}) = \eta_{nt} + G_{nt} B_{nt}^{-1} Z_{Nt}' \mathbb{V}_N$. These lead to,

$$
S_{\mathtt{SLE1}}^*(\boldsymbol{\theta}_0) = \begin{cases} \Pi_{1t}' \mathbb{V}_N, \;\; t = 1, \ldots, T, \\ \Pi_{2t}' \mathbb{V}_N + \mathbb{V}_N' \Phi_{1t} \mathbb{V}_N - \mathrm{tr}(R_{nt} G_{nt}), \;\; t = 1, \ldots, T, \\ \mathbb{V}_N' \Phi_{2t} \mathbb{V}_N - \mathrm{tr}(S_{nt} H_{nt}), \;\; t = 1, \ldots, T, \\ \mathbb{V}_N' \Psi \mathbb{V}_N - \frac{n(T-1)}{2\sigma^2}, \end{cases} \tag{B.3}
$$

where $\Pi_{1t} = \frac{1}{\sigma_0^2} Z_{Nt}^{\circ} B_{nt} X_{nt}$, $\Pi_{2t} = \frac{1}{\sigma_0^2} Z_{Nt}^{\circ} B_{nt} \eta_{nt}$, $\Phi_{1t} = \frac{1}{\sigma_0^2} Z_{Nt} B_{nt}^{-1\prime} G_{nt}' B_{nt}' Z_{Nt}^{\circ\prime}$, $\Phi_{2t} = \frac{1}{\sigma_0^2} Z_{Nt}^{\circ} H_{nt} Z_{Nt}^{\circ\prime}$, and $\Psi = \frac{1}{2\sigma^4} \sum_{t=1}^T Z_{Nt}^{\circ} Z_{Nt}^{\circ\prime}$. Applying Lemma A.2 gives:

$$
\Sigma_{\mathtt{SLE1}}(\boldsymbol{\theta}_0) = \begin{pmatrix} \{f(\mathbf{0}, \Pi_{1t}; \mathbf{0}, \Pi_{1s})\}, & \{f(\mathbf{0}, \Pi_{1t}; \Phi_{1s}, \Pi_{2s})\}, & \{f(\mathbf{0}, \Pi_{1t}; \Phi_{2s}, \mathbf{0})\}, & \{f(\mathbf{0}, \Pi_{1t}; \Psi, \mathbf{0})\} \\ \sim, & \{f(\Phi_{1t}, \Pi_{2t}; \Phi_{1s}, \Pi_{2s})\}, & \{f(\Phi_{1t}, \Pi_{2t}; \Phi_{2s}, \mathbf{0})\}, & \{f(\Phi_{1t}, \Pi_{2t}; \Psi, \mathbf{0})\} \\ \sim, & \sim, & \{f(\Phi_{2t}, \mathbf{0}; \Phi_{2s}, \mathbf{0})\}, & \{f(\Phi_{2t}, \mathbf{0}; \Psi, \mathbf{0})\} \\ \sim, & \sim, & \sim, & \{f(\Psi, \mathbf{0}; \Psi, \mathbf{0})\} \end{pmatrix}.
$$

B.4. Panel SLE model with two-way FE. Let $\dot{D}_{nt} = -\frac{d}{d\rho_{t0}} D^*_{nt} = M^{*\prime}_n B^*_{nt} + B^{*\prime}_{nt} M^*_n$. We have the components of the negative Hessian matrix $J_{\texttt{SEL2}}(\boldsymbol{\theta}_0)$:

$$
\begin{aligned}
J_{\boldsymbol{\beta\beta}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} X^{*\prime}_{nt} D^*_{nt} X^*_{nt}\big\} - \big\{\tfrac{1}{\sigma_0^2} X{*\prime}_{nt} D^*_{nt} \mathbb{D}^{*-1}_n D^*_{ns} X^*_{ns}\big\};\\
J_{\boldsymbol{\beta\lambda}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} X^{*\prime}_{nt} D^*_{nt} W^*_n Y^*_{nt}\big\} - \big\{\tfrac{1}{\sigma_0^2} X^{*\prime}_{nt} D^*_{nt} \mathbb{D}^{*-1}_n D^*_{ns} W^*_n Y^*_{ns}\big\};\\
J_{\boldsymbol{\beta\rho}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} X^{*\prime}_{nt} \dot{D}_{nt} B^{*-1}_{nt} \widetilde{V}^*_{nt}\big\} - \big\{\tfrac{1}{\sigma_0^2} X^{*\prime}_{nt} D^*_{nt} \mathbb{D}^{*-1}_n \dot{D}_{ns} B^{*-1}_{ns} \widetilde{V}^*_{ns}\big\};\\
J_{\boldsymbol{\lambda\lambda}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} (W^*_n Y^*_{nt})' D^*_{nt} (W^*_n Y^*_{nt}) + \mathrm{tr}(R^*_{nt} G^{*2}_{nt})\big\}\\
&\quad - \big\{\tfrac{1}{\sigma_0^2} (W^*_n Y^*_{nt})' D^*_{nt} \mathbb{D}^{*-1}_n D_{ns} (W^*_n Y^*_{ns})\big\};\\
J_{\boldsymbol{\lambda\rho}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} (W^*_n Y^*_{nt})' \dot{D}_{nt} B^{*-1}_{nt} \widetilde{V}^*_{nt} + \mathrm{tr}[\mathbb{D}^{*-1}_n \dot{D}_{nt} G^*_{nt}]\big\}\\
&\quad - \big\{\tfrac{1}{\sigma_0^2} (W^*_n Y^*_{nt})' D^*_{nt} \mathbb{D}^{*-1}_n \dot{D}_{ns} B^{*-1}_{ns} \widetilde{V}^*_{ns} + \mathrm{tr}[\mathbb{D}^{*-1}_n D^*_{nt} G^*_{nt} \mathbb{D}^{*-1}_n \dot{D}_{ns}]\big\};\\
J_{\boldsymbol{\rho\lambda}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} (W^*_n Y^*_{nt})' \dot{D}_{nt} B^{*-1}_{nt} \widetilde{V}^*_{nt}\big\} - \big\{\tfrac{1}{\sigma_0^2} (W^*_n Y^*_{ns})' D^*_{ns} \mathbb{D}^{*-1}_n \dot{D}_{nt} B^{*-1}_{nt} \widetilde{V}^*_{nt}\big\};\\
J_{\boldsymbol{\rho\rho}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma^2} \widetilde{V}^{*\prime}_{nt} H^{*\prime}_{nt} H^*_{nt} \widetilde{V}^*_{nt} + \mathrm{tr}(H^{*2}_{nt} + \mathbb{D}^{*-1}_n M^{*\prime}_n M^*_n)\big\};\\
&\quad - \big\{\tfrac{1}{\sigma^2} \widetilde{V}^{*\prime}_{nt} H^{*s}_{nt} B^*_{nt} \mathbb{D}^{*-1}_n B^{*\prime}_{ns} H^{*s}_{ns} \widetilde{V}^*_{ns} + \mathrm{tr}(\mathbb{D}^{*-1}_n B^{*\prime}_{nt} M^*_n \mathbb{D}^{*-1}_n \dot{D}_{ns})\big\}\\
J_{\sigma^2\boldsymbol{\beta}} &= \big\{\tfrac{1}{\sigma_0^4} X^{*\prime}_{nt} B^{*\prime}_{nt} \widetilde{V}^*_{nt}\big\};\;\; J_{\sigma^2\boldsymbol{\lambda}} = \big\{\tfrac{1}{\sigma_0^4} (W^*_n Y^*_{nt})' B^{*\prime}_{nt} \widetilde{V}^*_{nt}\big\};\\
J_{\sigma^2\boldsymbol{\rho}} &= \big\{\tfrac{1}{\sigma_0^4} \widetilde{V}^{*\prime}_{nt} H^*_{nt} \widetilde{V}^*_{nt}\big\};\;\; J_{\sigma^2\sigma^2} = -\tfrac{(n-1)(T-1)}{2\sigma^4} + \tfrac{1}{\sigma^6} \textstyle\sum_{t=1}^T \widetilde{V}^{*\prime}_{nt} \widetilde{V}^*_{nt}.
\end{aligned}
$$

and the expected negative Hessian matrix, $I_{\texttt{SEL2}}(\boldsymbol{\theta}_0)$, with components:

$$
\begin{aligned}
&I_{\boldsymbol{\beta\beta}} = J_{\boldsymbol{\beta\beta}},\;\; I_{\boldsymbol{\lambda\beta}} = \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} \eta^{*\prime}_t D^*_{nt} X^*_{nt}\big\} - \big\{\tfrac{1}{\sigma_0^2} \eta^{*\prime}_t D^*_{nt} \mathbb{D}^{*-1}_n D^*_{ns} X^*_{ns}\big\};\;\; I_{\boldsymbol{\rho\beta}} = 0_{Tk};\\
&I_{\boldsymbol{\lambda\lambda}} = \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} \eta^{*\prime}_{nt} D^*_{nt} \eta^*_{nt} + \mathrm{tr}[S^*_{nt}(\rho) \dot{G}^{*s}_{nt} \dot{G}^*_{nt}]\big\} - \big\{\tfrac{1}{\sigma_0^2} \eta^{*\prime}_{nt} D^*_{nt} \mathbb{D}^{*-1}_n D^*_{ns} \eta^*_{ns}\big\};\\
&I_{\boldsymbol{\lambda\rho}} = \texttt{blkdiag}\big\{\mathrm{tr}[\dot{G}{*\prime}_{nt} S^*_{nt}(\boldsymbol{\rho}) H^{*s}_{nt}]\big\};\;\; I_{\sigma^2\sigma^2} = -\tfrac{(n-1)(T-1)}{2\sigma_0^4} + \tfrac{1}{\sigma_0^4} \textstyle\sum_{t=1}^T \mathrm{tr} S^*_{nt}(\boldsymbol{\rho});\\
&I_{\boldsymbol{\rho\lambda}} = \texttt{blkdiag}\big\{\mathrm{tr}[\dot{G}^{*'}_{nt} S^*_{nt}(\boldsymbol{\rho}) H^{*s}_{nt} S^*_{nt}(\boldsymbol{\rho})]\big\} - \big\{\mathrm{tr}[G^{*'}_{ns} D^*_{ns} \mathbb{D}^{*-1}_n \dot{D}_{nt} \mathbb{D}^{*-1}_n]\big\};\\
&I_{\boldsymbol{\rho\rho}} = \texttt{blkdiag}\big\{\mathrm{tr}[H^{*s}_{nt} S^*_{nt}(\boldsymbol{\rho}) H^*_{nt} - B^*_{nt} \mathbb{D}^{*-1}_n \dot{D}_{nt} B^{*-1}_{nt} H^*_{nt}]\big\} + \big\{\mathrm{tr}[B^*_{nt} \mathbb{D}^{*-1}_n \dot{D}_{ns} \mathbb{D}^{*-1}_n B^{*'}_{nt} H^*_{nt}]\big\};\\
&I_{\sigma^2\boldsymbol{\beta}} = 0'_{tk};\;\; I_{\sigma^2\boldsymbol{\lambda}} = \big\{\tfrac{1}{\sigma_0^2} \mathrm{tr}[R^*_{nt}(\boldsymbol{\rho}) G^*_{nt}]\big\};\;\; I_{\sigma^2\boldsymbol{\rho}} = \big\{\tfrac{1}{\sigma_0^2} \mathrm{tr}(S^*_{nt}(\boldsymbol{\rho}) H^*_{nt})\big\}.
\end{aligned}
$$

To derive $\Sigma_{\texttt{SLE2}}(\boldsymbol{\theta}_0)$, $\widetilde{V}^*_{nt} \equiv \widetilde{V}^*_{nt}(\boldsymbol{\beta}_0, \boldsymbol{\lambda}_0, \boldsymbol{\rho}_0) = V^*_{nt} - B^*_{nt} \mathbb{D}^{*-1}_n \sum_{s=1}^T B^{*\prime}_{ns} V^*_{ns} = F'_{n,n-1} Z^{\circ\prime}_{Nt} \mathbb{V}_N$, and $W^*_n Y^*_{nt} = G^*_{nt}(X^*_{nt}\beta_0 + c^*_n + B^{-*1}_{nt} V^*_{nt}) = \eta^*_{nt} + G^*_{nt} B^{*-1}_{nt} F'_{n,n-1} Z'_{Nt} \mathbb{V}_N$, leading to,

$$
S^*_{\texttt{SLE2}}(\boldsymbol{\theta}_0) = \begin{cases} \Pi'_{1t} \mathbb{V}_N, \;\; t = 1, \ldots, T, \\ \Pi'_{2t} \mathbb{V}_N + \mathbb{V}'_N \Phi_{1t} \mathbb{V}_N - \mathrm{tr}(R^*_{nt} G^*_{nt}), \;\; t = 1, \ldots, T, \\ \mathbb{V}'_N \Phi_{2t} \mathbb{V}_N - \mathrm{tr}(S^*_{nt} H^*_{nt}), \;\; t = 1, \ldots, T, \\ \mathbb{V}'_N \Psi \mathbb{V}_N - \frac{(n-1)(T-1)}{2\sigma^2}, \end{cases} \tag{B.4}
$$

where $\Pi_{1t} = \frac{1}{\sigma_0^2} Z^{\circ *}_{Nt} B^*_{nt} X^*_{nt}$, $\Pi_{2t} = \frac{1}{\sigma_0^2} Z^{\circ *}_{Nt} B^*_{nt} \eta^*_{nt}$, $\Phi_{1t} = \frac{1}{\sigma_0^2} Z^*_{Nt} B^{*-1\prime}_{nt} G^{*'}_{nt} B^{*\prime}_{nt} Z^{\circ *\prime}_{Nt}$, $\Phi_{2t} = \frac{1}{\sigma_0^2} Z^{\circ *}_{Nt} H^*_{nt} Z^{\circ *\prime}_{Nt}$, and $\Psi = \frac{1}{2\sigma^4} \sum_{t=1}^T Z^{\circ *}_{Nt} Z^{\circ *\prime}_{Nt}$, with $Z^*_{Nt} = Z_{Nt} F_{n,n-1}$ and $Z^{\circ *}_{Nt} = Z^{\circ}_{Nt} F_{n,n-1}$. Applying Lemma A.2 with ε, c_{nt} and A_{nt} replaced by $\mathbb{V}_N$, Π_{1t} and Π_{2t}, and Φ_{1t}, Φ_{2t} and Ψ, we obtain the VC matrix $\Sigma_{\texttt{SLE2}}(\boldsymbol{\theta}_0)$ taking identical form as $\Sigma_{\texttt{SLE1}}(\boldsymbol{\theta}_0)$ given above.

Appendix C: Proof of the Theorems

Proof of Theorem 2.1. From (B.1), we see that the AQS function at the true parameters contains both linear and quadratic forms in the vector of original errors $\mathbb{V}_N$,

$$S^{\star}_{\text{SL1}}(\boldsymbol{\theta}_0) = \begin{cases} \Pi'_{1t}\mathbb{V}_N, \ t = 1, \dots, T, \\ \Pi'_{2t}\mathbb{V}_N + \mathbb{V}'_N\Phi_t\mathbb{V}_N - \frac{T-1}{T}\text{tr}(G_{nt}), \ t = 1, \dots, T, \\ \mathbb{V}'_N\Psi\mathbb{V}_N - \frac{n(T-1)}{2\sigma^2}, \end{cases}$$

where $\Pi_{1t} = \frac{1}{\sigma_0^2}Z^{\circ}_{Nt}X_{nt}$, $\Pi_{2t} = \frac{1}{\sigma_0^2}Z^{\circ}_{Nt}\eta_{nt}$, $\Phi_t = \frac{1}{\sigma_0^2}Z_{Nt}G'_{nt}Z^{\circ\prime}_{Nt}$, $\Psi = \frac{1}{2\sigma^4}\sum_{t=1}^{T} Z^{\circ}_{Nt}Z^{\circ\prime}_{Nt}$, $Z_{Nt} = z_t \otimes I_n$, $Z^{\circ}_{Nt} = Z_{Nt} - \bar{Z}_N$, $\bar{Z}_N = \frac{1}{T}(l_T \otimes I_n)$, and z_t is a $T \times 1$ vector with tth element being 1 and other elements being zero.

First, as the elements of X_{nt} are non-stochastic and uniformly bounded (by Assumption 3), it is easy to see that the elements of Π_{1t} are uniformly bounded. By Assumption A.4 and Lemma A.1(i), G_{nt} is uniformly bounded in both row and column sums. Thus, the elements of $\eta_{nt} = G_{nt}(X_{nt}\beta_{t0} + c_n)$ are uniformly bounded by Assumption A3 and Lemma A.1(iii). It follows that the elements of Π_{2t} are uniformly bounded. Now, from the definition of Z_{Nt} and Z°_{Nt}, it is easy to see that $\Phi_t$and Ψ are uniformly bounded in both row and column sums. Thus, under Assumptions 1-4 the central limit theorem (CLT) of linear-quadratic (LQ) form of Kelejian and Prucha (2001) or its simplified version (under iid errors) given in Lemma A.3 can be applied to the elements of $S^{\star}_{\text{SL1}}(\boldsymbol{\theta}_0)$. Therefore, an application of Cramér-Wold device under a finite T leads to, as $N_0 \to \infty$, $\frac{1}{\sqrt{N_0}}S^{\star}_{\text{SL1}}(\boldsymbol{\theta}_0) \xrightarrow{D} N\big(\mathbf{0}, \ \lim_{N_0\to\infty}\frac{1}{N_0}\Sigma_{\text{SL1}}(\boldsymbol{\theta}_0)\big)$. It follows that by (2.11) and (2.12),

$$C[\tfrac{1}{N_0}I_{\text{SL1}}(\boldsymbol{\theta}_0)]^{-1}\tfrac{1}{\sqrt{N_0}}S^{\star}_{\text{SL1}}(\tilde{\boldsymbol{\theta}}_{\text{SL1}}) \xrightarrow{D} N\big(\mathbf{0}, \ \lim_{N_0\to\infty}\Xi_{\text{SL1}}(\boldsymbol{\theta}_0)\big).$$

It left to show that $\frac{1}{N_0}[I_{\text{SL1}}(\tilde{\boldsymbol{\theta}}_{\text{SL1}}) - I_{\text{SL1}}(\boldsymbol{\theta}_0)] \xrightarrow{p} \mathbf{0}$ and $\frac{1}{N_0}[\Sigma_{\text{SL1}}(\tilde{\boldsymbol{\theta}}_{\text{SL1}}) - \Sigma_{\text{SL1}}(\boldsymbol{\theta}_0)] \xrightarrow{p} \mathbf{0}$. Under the $\sqrt{N_0}$-consistency of $\tilde{\boldsymbol{\theta}}_{\text{SL1}}$ and with the analytical expressions of $I_{\text{SL1}}(\boldsymbol{\theta}_0)$ and $\Sigma_{\text{SL1}}(\boldsymbol{\theta}_0)$ given in Appendix B1, the proofs of these results are repeated applications of the mean value theorem (MVT) to each component of $\frac{1}{N_0}[I_{\text{SL1}}(\tilde{\boldsymbol{\theta}}_{\text{SL1}}) - I_{\text{SL1}}(\boldsymbol{\theta}_0)]$ and each component of $\frac{1}{N_0}[\Sigma_{\text{SL1}}(\tilde{\boldsymbol{\theta}}_{\text{SL1}}) - \Sigma_{\text{SL1}}(\boldsymbol{\theta}_0)]$, with the key results to note:

$$\tfrac{1}{N_0}(\tilde{c}_n\tilde{G}_{nt}\tilde{c}_n - c_nG_{nt}c_n) \xrightarrow{p} 0; \ \ \tilde{\gamma} - \gamma \xrightarrow{p} 0; \ \ \tilde{\kappa} - \kappa \xrightarrow{p} 0. \tag{C.1}$$

See the end of Section 2.1 for details ■

Proof of Theorem 2.2. From the derivations in Section 2.2 and further results in Appendix B2, we see that all the quantities in the 2FE panel SL model relate to the corresponding quantities in the 1FE panel SL model through the orthonormal transformation matrix $F_{n,n-1}$. Thus, the proof of Theorem 2.2 is carried out in a similar manner as that for 1FE panel SL model. For the results similar to those in (C.1), see the end of Section 2.2 for details. ■

Proof of Theorem 3.1. Again the AQS function at the true parameters can be expressed in terms of linear and quadratic forms in $\mathbb{V}_N$ as shown in (B.3),

$$S^*_{\mathtt{SLE1}}(\boldsymbol{\theta}_0) = \begin{cases} \Pi'_{1t}\mathbb{V}_N, \quad t = 1, \ldots, T, \\ \Pi'_{2t}\mathbb{V}_N + \mathbb{V}'_N\Phi_{1t}\mathbb{V}_N - \text{tr}(R_{nt}G_{nt}), \quad t = 1, \ldots, T, \\ \mathbb{V}'_N\Phi_{2t}\mathbb{V}_N - \text{tr}(S_{nt}H_{nt}), \quad t = 1, \ldots, T, \\ \mathbb{V}'_N\Psi\mathbb{V}_N - \frac{n(T-1)}{2\sigma^2}, \end{cases}$$

where $\Pi_{1t} = \frac{1}{\sigma_0^2}Z^\circ_{Nt}B_{nt}X_{nt}$, $\Pi_{2t} = \frac{1}{\sigma_0^2}Z^\circ_{Nt}B_{nt}\eta_{nt}$, $\Phi_{1t} = \frac{1}{\sigma_0^2}Z_{Nt}B^{-1\prime}_{nt}G'_{nt}B'_{nt}Z^{\circ\prime}_{Nt}$, $\Phi_{2t} = \frac{1}{\sigma_0^2}Z^\circ_{Nt}H_{nt}Z^{\circ\prime}_{Nt}$, $\Psi = \frac{1}{2\sigma^4}\sum_{t=1}^T Z^\circ_{Nt}Z^{\circ\prime}_{Nt}$, $Z^{\circ\prime}_{Nt} = [Z'_{Nt} - B_{nt}\mathbb{D}_n^{-1}(l'_T \otimes I_n)\mathbb{B}_N]$ and $\mathbb{B}_N = \mathtt{blkdiag}(B_{n1}, \ldots, B_{nT})$. Under Assumptions 1-5, it is easy to verify that each component of $S^*_{\mathtt{SLE1}}(\boldsymbol{\theta}_0)$ or a linear combination of the components of $S^*_{\mathtt{SLE1}}(\boldsymbol{\theta}_0)$ satisfies the conditions of Lemma A.3, leading to the asymptotic normality result:

$$C[\tfrac{1}{N_0}I_{\mathtt{SLE1}}(\boldsymbol{\theta}_0)]^{-1}\tfrac{1}{\sqrt{N_0}}S^\star_{\mathtt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}}) \xrightarrow{D} N\big(\mathbf{0},\ \lim_{N_0\to\infty}\Xi_{\mathtt{SLE1}}(\boldsymbol{\theta}_0)\big).$$

The proofs of $\frac{1}{N_0}[I_{\mathtt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}}) - I_{\mathtt{SLE1}}(\boldsymbol{\theta}_0)] \xrightarrow{p} \mathbf{0}$ and $\frac{1}{N_0}[\Sigma_{\mathtt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}}) - \Sigma_{\mathtt{SLE1}}(\boldsymbol{\theta}_0)] \xrightarrow{p} \mathbf{0}$ are again carried out by repeated applications of MVT under the $\sqrt{N_0}$-consistency of $\tilde{\boldsymbol{\theta}}_{\mathtt{SLE1}}$. For details on the estimation of c_n, the skewness γ and excess kurtosis κ for the 1FE panel SLE model, and the consistency of these estimates, see the end of Section 3.1. ■

Proof of Theorem 3.2. The proof is similar to that for the 1FE panel SLE model, as the quantities in the 2FE panel SLE model relate to those in the 1FE panel SLE model through the orthonormal transformation matrix $F_{n,n-1}$. ■

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Table 1a. Empirical Sizes of Tests for Temporal Homogeneity in Panel SL Model
One-Way Fixed Effects, `Queen Contiguity`

		T = 3						T = 6					
λ	n	T_{SL1}			T^r_{SL1}			T_{SL1}			T^r_{SL1}		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
Normal Error													
.5	50	.208	.135	.052	.096	.045	.007	.216	.138	.050	.095	.044	.008
	100	.150	.086	.024	.098	.046	.009	.161	.097	.028	.103	.050	.009
	200	.128	.068	.015	.103	.049	.008	.129	.069	.018	.099	.051	.010
	500	.107	.054	.010	.097	.046	.007	.110	.054	.011	.098	.049	.009
0	50	.204	.135	.053	.102	.048	.008	.214	.137	.050	.095	.046	.009
	100	.147	.086	.025	.099	.048	.008	.160	.096	.027	.105	.051	.009
	200	.127	.069	.015	.104	.049	.009	.127	.068	.018	.100	.049	.010
	500	.111	.056	.011	.100	.048	.008	.109	.056	.012	.099	.050	.010
-.5	50	.204	.133	.055	.102	.048	.008	.212	.136	.051	.097	.046	.009
	100	.147	.086	.025	.099	.049	.008	.160	.097	.027	.103	.050	.009
	200	.129	.068	.015	.103	.048	.009	.127	.070	.017	.100	.050	.010
	500	.108	.055	.012	.101	.048	.009	.110	.056	.012	.100	.049	.010
Normal Mixture Error													
.5	50	.201	.129	.053	.096	.047	.006	.229	.154	.061	.121	.070	.023
	100	.149	.088	.027	.100	.048	.009	.163	.096	.029	.099	.050	.010
	200	.130	.073	.019	.105	.052	.011	.133	.073	.018	.103	.054	.010
	500	.112	.058	.012	.102	.051	.009	.118	.061	.012	.102	.051	.010
0	50	.197	.126	.052	.099	.047	.007	.229	.150	.061	.103	.053	.011
	100	.149	.087	.028	.102	.049	.010	.161	.094	.029	.099	.048	.010
	200	.129	.073	.019	.105	.052	.010	.132	.073	.018	.104	.054	.011
	500	.111	.059	.012	.103	.051	.010	.120	.061	.012	.102	.053	.009
-.5	50	.193	.129	.052	.097	.048	.008	.231	.151	.062	.103	.053	.012
	100	.150	.088	.028	.101	.050	.010	.162	.094	.030	.101	.050	.010
	200	.130	.073	.019	.104	.052	.011	.132	.073	.018	.103	.053	.011
	500	.113	.059	.013	.102	.051	.010	.118	.062	.013	.101	.052	.010
Log-normal Error													
.5	50	.180	.119	.045	.089	.043	.008	.211	.145	.060	.100	.054	.017
	100	.149	.087	.027	.097	.047	.009	.164	.102	.032	.101	.057	.012
	200	.133	.071	.018	.097	.045	.009	.147	.087	.030	.101	.055	.014
	500	.127	.071	.018	.100	.051	.011	.142	.078	.030	.101	.050	.011
0	50	.180	.118	.046	.093	.044	.008	.193	.130	.056	.099	.054	.015
	100	.132	.078	.023	.094	.047	.009	.146	.086	.024	.100	.052	.010
	200	.109	.057	.013	.089	.042	.008	.114	.064	.017	.094	.051	.012
	500	.099	.052	.012	.010	.050	.010	.110	.058	.013	.102	.053	.011
-.5	50	.194	.128	.049	.097	.045	.008	.225	.154	.072	.106	.058	.016
	100	.142	.083	.024	.096	.047	.010	.191	.118	.042	.104	.057	.013
	200	.120	.067	.017	.095	.046	.009	.166	.102	.032	.102	.054	.012
	500	.118	.065	.016	.098	.050	.011	.151	.102	.032	.102	.050	.010

Table 1b. Empirical Sizes of Tests for Temporal Homogeneity in Panel SL Model
One-Way Fixed Effects, `Group Interaction`

		$T=3$						$T=6$					
λ	n	$T_{\texttt{SL1}}$			$T^r_{\texttt{SL1}}$			$T_{\texttt{SL1}}$			$T^r_{\texttt{SL1}}$		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
Normal Error													
.5	50	.222	.144	.057	.086	.034	.004	.219	.136	.048	.085	.039	.007
	100	.150	.089	.025	.088	.039	.006	.165	.094	.028	.089	.042	.007
	200	.124	.067	.018	.092	.042	.008	.128	.070	.016	.094	.045	.008
	500	.110	.059	.014	.097	.049	.011	.113	.057	.012	.095	.048	.009
0	50	.232	.157	.065	.087	.036	.005	.232	.151	.056	.084	.040	.007
	100	.155	.091	.027	.089	.040	.006	.173	.099	.030	.091	.044	.008
	200	.124	.068	.020	.090	.042	.008	.131	.071	.016	.095	.044	.008
	500	.110	.060	.015	.098	.049	.010	.114	.058	.013	.096	.048	.009
-.5	50	.238	.163	.071	.086	.038	.004	.239	.159	.063	.085	.038	.007
	100	.157	.092	.029	.088	.040	.005	.178	.102	.033	.089	.043	.008
	200	.126	.069	.020	.091	.043	.008	.133	.072	.016	.096	.043	.008
	500	.111	.061	.014	.098	.049	.010	.115	.059	.012	.096	.048	.009
Normal Mixture Error													
.5	50	.230	.151	.056	.087	.033	.004	.215	.143	.051	.088	.046	.009
	100	.154	.088	.025	.087	.041	.006	.165	.094	.025	.087	.041	.009
	200	.131	.070	.017	.095	.043	.008	.133	.071	.018	.093	.043	.009
	500	.114	.061	.013	.100	.048	.009	.116	.059	.011	.096	.048	.008
0	50	.241	.163	.068	.088	.036	.005	.231	.155	.061	.088	.046	.008
	100	.157	.092	.029	.089	.041	.006	.170	.098	.029	.089	.041	.008
	200	.133	.070	.018	.095	.044	.008	.133	.072	.019	.094	.042	.009
	500	.114	.059	.014	.099	.048	.010	.133	.072	.019	.094	.042	.009
-.5	50	.259	.181	.081	.093	.043	.007	.270	.186	.083	.096	.050	.010
	100	.168	.103	.033	.096	.046	.007	.193	.118	.040	.093	.046	.010
	200	.136	.075	.020	.097	.045	.009	.142	.079	.023	.094	.045	.010
	500	.116	.060	.015	.098	.048	.009	.117	.059	.012	.097	.048	.008
Log-normal Error													
.5	50	.218	.143	.054	.081	.035	.005	.206	.137	.050	.079	.040	.009
	100	.151	.088	.026	.084	.037	.005	.176	.107	.034	.091	.048	.012
	200	.130	.069	.018	.091	.043	.006	.142	.081	.022	.095	.051	.012
	500	.108	.057	.012	.094	.045	.008	.126	.066	.016	.101	.049	.010
0	50	.227	.151	.064	.084	.036	.006	.243	.166	.075	.087	.045	.010
	100	.152	.091	.029	.088	.040	.006	.185	.122	.046	.097	.049	.013
	200	.137	.077	.019	.096	.047	.008	.136	.078	.025	.097	.052	.011
	500	.107	.059	.014	.098	.048	.009	.115	.057	.014	.098	.048	.010
-.5	50	.263	.188	.086	.093	.043	.008	.350	.259	.139	.106	.057	.015
	100	.179	.114	.042	.101	.049	.010	.260	.186	.090	.105	.054	.014
	200	.161	.096	.029	.107	.056	.010	.185	.114	.043	.103	.052	.013
	500	.123	.067	.018	.100	.051	.010	.131	.072	.021	.101	.051	.010

Table 2a. Empirical Sizes of Tests for Temporal Homogeneity in Panel SL Model
Two-Way Fixed Effects, `Queen Contiguity`

		$T=3$						$T=6$					
λ	n	T_{SL2}			T^r_{SL2}			T_{SL2}			T^r_{SL2}		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
Normal Error													
.5	50	.192	.123	.047	.093	.045	.007	.228	.148	.059	.100	.050	.010
	100	.140	.080	.023	.096	.048	.009	.157	.094	.029	.102	.050	.011
	200	.120	.064	.015	.098	.049	.009	.128	.068	.017	.101	.052	.009
	500	.103	.051	.013	.098	.048	.011	.105	.056	.012	.095	.049	.010
0	50	.194	.123	.048	.094	.046	.008	.224	.147	.059	.101	.049	.010
	100	.138	.082	.023	.095	.050	.009	.126	.069	.017	.099	.051	.009
	200	.115	.064	.016	.096	.049	.009	.157	.095	.027	.101	.049	.010
	500	.101	.052	.012	.098	.048	.009	.126	.069	.017	.099	.051	.009
-.5	50	.192	.123	.047	.093	.045	.009	.225	.148	.058	.100	.049	.009
	100	.138	.081	.023	.096	.049	.008	.157	.092	.027	.101	.048	.010
	200	.116	.063	.015	.096	.049	.009	.125	.069	.016	.102	.050	.009
	500	.105	.055	.011	.096	.048	.009	.108	.056	.013	.097	.051	.011
Normal Mixture Error													
.5	50	.198	.131	.052	.100	.048	.008	.232	.155	.063	.106	.054	.013
	100	.140	.080	.025	.096	.047	.010	.165	.100	.030	.107	.055	.012
	200	.124	.067	.016	.101	.051	.009	.132	.071	.019	.104	.051	.013
	500	.110	.055	.013	.100	.050	.010	.106	.056	.012	.097	.051	.010
0	50	.199	.132	.052	.102	.048	.009	.234	.154	.064	.110	.055	.013
	100	.139	.080	.024	.097	.047	.009	.166	.100	.031	.109	.054	.011
	200	.124	.067	.017	.102	.051	.010	.129	.072	.019	.102	.051	.013
	500	.110	.055	.012	.102	.050	.010	.106	.055	.013	.096	.049	.010
-.5	50	.199	.130	.053	.101	.049	.009	.234	.157	.066	.112	.057	.013
	100	.143	.084	.025	.101	.048	.009	.164	.097	.031	.107	.053	.012
	200	.123	.069	.016	.103	.051	.010	.133	.073	.020	.105	.053	.012
	500	.109	.056	.012	.101	.050	.009	.107	.056	.014	.096	.048	.012
Log-normal Error													
.5	50	.196	.131	.055	.100	.050	.009	.242	.171	.079	.107	.067	.018
	100	.139	.081	.027	.095	.050	.011	.171	.112	.041	.105	.055	.015
	200	.128	.070	.018	.106	.053	.010	.141	.081	.026	.104	.052	.013
	500	.109	.060	.014	.101	.052	.011	.123	.068	.019	.101	.051	.010
0	50	.196	.133	.059	.106	.055	.010	.239	.167	.081	.110	.055	.021
	100	.137	.078	.024	.095	.048	.010	.166	.110	.039	.107	.054	.018
	200	.126	.070	.018	.104	.052	.010	.133	.079	.025	.105	.049	.015
	500	.107	.056	.013	.100	.051	.010	.116	.061	.016	.102	.051	.013
-.5	50	.205	.141	.066	.112	.062	.011	.249	.177	.083	.108	.055	.026
	100	.154	.089	.028	.106	.052	.012	.172	.110	.042	.099	.048	.019
	200	.129	.074	.019	.107	.056	.012	.145	.088	.030	.098	.049	.020
	500	.110	.058	.014	.103	.052	.010	.122	.068	.018	.100	.049	.014

Table 2b. Empirical Sizes of Tests for Temporal Homogeneity in Panel SL Model
Two-Way Fixed Effects, `Group Interaction`

		$T=3$						$T=6$					
λ	n	T_{SL2}			T^r_{SL2}			T_{SL2}			T^r_{SL2}		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
							Normal Error						
.5	50	.226	.148	.059	.086	.038	.005	.223	.142	.052	.087	.040	.007
	100	.155	.090	.025	.090	.036	.006	.166	.095	.029	.089	.043	.007
	200	.124	.070	.018	.091	.044	.006	.131	.073	.016	.093	.045	.008
	500	.112	.060	.015	.097	.050	.010	.114	.057	.013	.096	.047	.010
0	50	.240	.159	.068	.088	.039	.005	.237	.154	.059	.086	.040	.007
	100	.159	.094	.025	.090	.037	.006	.174	.102	.031	.088	.042	.007
	200	.127	.072	.018	.091	.044	.007	.133	.074	.016	.094	.046	.008
	500	.112	.060	.014	.097	.050	.010	.116	.059	.013	.097	.046	.010
-.5	50	.244	.167	.075	.088	.039	.005	.249	.164	.065	.086	.040	.007
	100	.163	.096	.028	.089	.038	.006	.179	.104	.033	.085	.043	.007
	200	.127	.073	.019	.092	.045	.007	.134	.076	.017	.094	.045	.008
	500	.113	.059	.014	.098	.049	.010	.117	.059	.013	.097	.046	.010
							Normal Mixture Error						
.5	50	.232	.150	.058	.080	.034	.005	.222	.144	.055	.082	.041	.008
	100	.159	.090	.024	.088	.039	.006	.164	.095	.027	.083	.041	.008
	200	.130	.072	.018	.095	.045	.007	.133	.071	.017	.089	.043	.010
	500	.114	.059	.014	.097	.048	.009	.118	.060	.012	.098	.047	.009
0	50	.245	.167	.069	.085	.038	.006	.247	.165	.071	.083	.039	.007
	100	.164	.098	.027	.089	.040	.006	.175	.103	.032	.080	.038	.007
	200	.131	.072	.018	.094	.043	.008	.132	.072	.019	.089	.041	.009
	500	.115	.059	.014	.096	.048	.009	.119	.060	.012	.096	.047	.009
-.5	50	.269	.185	.085	.097	.047	.009	.298	.209	.100	.101	.052	.012
	100	.177	.110	.035	.099	.046	.007	.205	.127	.045	.094	.046	.008
	200	.138	.077	.020	.096	.045	.008	.145	.082	.023	.095	.045	.010
	500	.115	.059	.014	.096	.047	.009	.122	.063	.013	.099	.049	.009
							Log-normal Error						
.5	50	.217	.143	.057	.078	.036	.005	.215	.142	.055	.076	.036	.008
	100	.152	.088	.025	.079	.034	.005	.176	.111	.036	.082	.041	.009
	200	.132	.073	.018	.089	.044	.006	.141	.080	.023	.088	.046	.010
	500	.113	.057	.013	.094	.047	.008	.119	.062	.014	.096	.048	.009
0	50	.240	.165	.073	.085	.040	.006	.246	.174	.079	.085	.038	.008
	100	.164	.099	.034	.086	.041	.006	.191	.129	.051	.091	.040	.008
	200	.135	.076	.020	.092	.043	.007	.143	.083	.027	.095	.044	.009
	500	.111	.057	.014	.092	.045	.008	.113	.060	.013	.097	.045	.010
-.5	50	.287	.207	.104	.112	.060	.013	.347	.269	.151	.119	.068	.022
	100	.201	.131	.054	.109	.057	.012	.270	.195	.099	.119	.065	.019
	200	.156	.095	.028	.105	.054	.010	.191	.122	.049	.105	.056	.014
	500	.120	.067	.017	.098	.050	.009	.141	.081	.021	.103	.052	.010

Table 3a. Empirical Sizes of Tests for Temporal Homogeneity in Panel SLE Model
One-Way Fixed Effects, `Queen Contiguity`, $\lambda = 0.5$.

		$T=3$						$T=6$					
ρ	n	$T_{\texttt{SLE1}}$			$T^r_{\texttt{SLE1}}$			$T_{\texttt{SLE1}}$			$T^r_{\texttt{SLE1}}$		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
		Normal Error											
.5	50	.199	.142	.075	.082	.039	.005	.161	.099	.036	.090	.042	.011
	100	.123	.068	.025	.094	.043	.009	.097	.050	.012	.092	.043	.006
	200	.084	.044	.009	.099	.046	.007	.079	.038	.008	.102	.049	.011
	500	.070	.034	.006	.104	.049	.009	.064	.030	.005	.102	.054	.009
0	50	.223	.164	.093	.090	.042	.006	.171	.104	.041	.093	.047	.010
	100	.132	.076	.029	.095	.046	.012	.105	.058	.014	.097	.047	.007
	200	.087	.046	.011	.103	.050	.010	.082	.039	.008	.104	.050	.011
	500	.069	.036	.006	.102	.050	.011	.063	.028	.005	.103	.054	.010
-.5	50	.232	.174	.098	.093	.042	.006	.181	.120	.048	.096	.047	.010
	100	.134	.083	.033	.097	.045	.011	.118	.064	.014	.098	.048	.008
	200	.097	.047	.013	.105	.050	.012	.079	.039	.008	.102	.052	.011
	500	.070	.035	.006	.102	.052	.009	.061	.028	.005	.102	.049	.011
		Normal Mixture Error											
.5	50	.196	.139	.072	.081	.037	.004	.168	.106	.044	.092	.047	.008
	100	.121	.070	.025	.087	.040	.008	.107	.057	.017	.096	.053	.012
	200	.084	.043	.011	.092	.046	.006	.082	.044	.010	.101	.052	.013
	500	.071	.035	.008	.099	.052	.012	.070	.036	.009	.097	.046	.014
0	50	.212	.151	.080	.087	.042	.005	.167	.110	.044	.089	.045	.010
	100	.131	.076	.028	.089	.041	.009	.105	.054	.015	.097	.046	.011
	200	.085	.046	.011	.095	.046	.008	.078	.039	.009	.100	.047	.012
	500	.071	.036	.007	.097	.050	.010	.064	.032	.006	.104	.054	.012
-.5	50	.226	.164	.090	.093	.040	.006	.197	.131	.057	.104	.056	.013
	100	.140	.083	.030	.094	.043	.009	.126	.073	.023	.104	.055	.013
	200	.094	.050	.013	.102	.051	.010	.086	.048	.013	.103	.055	.014
	500	.073	.038	.009	.101	.051	.012	.074	.034	.005	.102	.055	.011
		Log-normal Error											
.5	50	.150	.102	.046	.083	.038	.006	.169	.108	.044	.092	.048	.010
	100	.115	.075	.035	.091	.044	.010	.106	.058	.015	.098	.051	.010
	200	.109	.067	.027	.095	.046	.009	.073	.036	.008	.090	.046	.010
	500	.089	.050	.016	.100	.049	.011	.064	.032	.006	.104	.052	.012
0	50	.217	.160	.090	.082	.041	.009	.179	.118	.045	.092	.048	.011
	100	.126	.077	.031	.087	.042	.009	.108	.062	.017	.100	.055	.008
	200	.101	.055	.015	.103	.048	.010	.074	.035	.007	.095	.044	.008
	500	.071	.035	.008	.096	.048	.010	.059	.031	.006	.099	.050	.011
-.5	50	.192	.138	.069	.090	.045	.006	.202	.136	.054	.098	.050	.011
	100	.137	.087	.038	.092	.048	.010	.128	.074	.019	.108	.057	.010
	200	.094	.045	.014	.101	.048	.011	.081	.041	.008	.099	.049	.009
	500	.078	.040	.010	.102	.051	.010	.064	.030	.005	.105	.050	.012

Table 3b. Empirical Sizes of Tests for Temporal Homogeneity in Panel SLE Model
One-Way Fixed Effects, `Queen Contiguity`, $\lambda = -0.5$.

		$T=3$						$T=6$					
ρ	n	$T_{\mathtt{SLE1}}$			$T^r_{\mathtt{SLE1}}$			$T_{\mathtt{SLE1}}$			$T^r_{\mathtt{SLE1}}$		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
		Normal Error											
.5	50	.190	.131	.058	.088	.037	.007	.167	.102	.036	.088	.042	.010
	100	.116	.068	.022	.093	.044	.009	.098	.050	.013	.091	.044	.007
	200	.079	.042	.009	.094	.046	.007	.078	.040	.010	.100	.050	.012
	500	.071	.033	.007	.101	.050	.009	.060	.029	.005	.102	.053	.009
0	50	.209	.149	.073	.091	.040	.006	.169	.104	.040	.094	.043	.010
	100	.125	.073	.027	.099	.050	.011	.102	.056	.013	.093	.047	.006
	200	.084	.043	.010	.098	.048	.010	.079	.040	.008	.104	.051	.010
	500	.072	.033	.007	.103	.050	.011	.059	.029	.005	.096	.054	.010
-.5	50	.225	.162	.085	.095	.040	.006	.172	.111	.044	.094	.043	.010
	100	.131	.081	.031	.101	.050	.011	.109	.059	.013	.099	.047	.008
	200	.089	.044	.013	.105	.049	.011	.082	.039	.009	.104	.052	.010
	500	.069	.032	.007	.100	.049	.010	.057	.030	.005	.096	.049	.012
		Normal Mixture Error											
.5	50	.187	.129	.061	.079	.034	.004	.176	.111	.043	.092	.047	.008
	100	.111	.068	.022	.086	.042	.008	.105	.054	.016	.097	.051	.013
	200	.083	.044	.009	.091	.047	.006	.085	.046	.010	.102	.056	.012
	500	.072	.033	.008	.102	.049	.011	.074	.036	.008	.099	.053	.010
0	50	.200	.140	.071	.086	.039	.006	.166	.105	.041	.090	.047	.010
	100	.126	.074	.027	.092	.042	.009	.103	.056	.016	.095	.049	.011
	200	.079	.045	.009	.095	.047	.008	.076	.041	.010	.098	.050	.012
	500	.071	.035	.008	.100	.049	.010	.064	.031	.007	.101	.050	.012
-.5	50	.218	.156	.080	.088	.041	.007	.191	.124	.052	.100	.054	.013
	100	.136	.079	.031	.096	.045	.008	.119	.068	.021	.105	.055	.013
	200	.087	.048	.013	.098	.048	.009	.088	.048	.014	.106	.057	.014
	500	.073	.037	.009	.103	.053	.011	.075	.034	.007	.104	.053	.011
		Log-normal Error											
.5	50	.175	.125	.063	.084	.036	.009	.174	.110	.043	.092	.046	.010
	100	.138	.087	.038	.089	.042	.010	.099	.055	.016	.098	.050	.011
	200	.096	.048	.014	.096	.045	.008	.075	.037	.008	.098	.046	.011
	500	.075	.038	.009	.101	.052	.011	.066	.028	.006	.100	.053	.013
0	50	.207	.145	.081	.086	.042	.011	.173	.111	.044	.093	.046	.010
	100	.122	.078	.029	.090	.044	.009	.105	.056	.013	.096	.048	.009
	200	.091	.047	.010	.095	.047	.008	.076	.037	.007	.099	.046	.009
	500	.071	.035	.008	.099	.049	.011	.057	.027	.006	.101	.047	.011
-.5	50	.201	.138	.072	.093	.043	.008	.191	.125	.051	.097	.049	.012
	100	.141	.092	.039	.096	.048	.010	.118	.067	.017	.104	.053	.010
	200	.089	.045	.012	.104	.050	.009	.084	.041	.008	.104	.051	.010
	500	.072	.034	.007	.104	.049	.010	.062	.029	.006	.103	.046	.012

Table 4a. Empirical Sizes of Tests for Temporal Homogeneity in Panel SLE Model
Two-Way Fixed Effects, `Queen Contiguity`, $\lambda = 0.5$.

		$T=3$						$T=6$					
ρ	n	T_{SLE2}			T^r_{SLE2}			T_{SLE2}			T^r_{SLE2}		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
Normal Error													
.5	50	.235	.181	.105	.083	.038	.006	.310	.226	.115	.087	.044	.008
	100	.212	.151	.086	.093	.045	.008	.190	.111	.036	.090	.041	.007
	200	.182	.121	.054	.098	.044	.006	.139	.079	.021	.101	.049	.011
	500	.134	.073	.022	.100	.048	.010	.121	.064	.014	.102	.055	.009
0	50	.272	.208	.117	.088	.043	.007	.314	.224	.111	.094	.045	.010
	100	.217	.143	.070	.094	.046	.011	.197	.116	.036	.093	.043	.008
	200	.161	.097	.032	.100	.051	.008	.142	.083	.022	.103	.050	.011
	500	.125	.065	.017	.105	.049	.011	.119	.064	.014	.102	.053	.010
-.5	50	.302	.233	.136	.094	.042	.005	.321	.239	.114	.092	.045	.009
	100	.209	.142	.062	.095	.046	.011	.205	.128	.042	.096	.047	.009
	200	.153	.090	.029	.102	.050	.009	.151	.081	.023	.098	.052	.010
	500	.119	.064	.015	.102	.054	.009	.115	.061	.014	.103	.051	.010
Normal Mixture Error													
.5	50	.221	.159	.090	.083	.037	.004	.315	.242	.127	.090	.044	.008
	100	.212	.154	.085	.085	.044	.008	.201	.128	.050	.097	.053	.010
	200	.183	.122	.059	.092	.046	.008	.150	.090	.029	.101	.052	.009
	500	.137	.082	.028	.100	.053	.012	.139	.079	.022	.100	.053	.010
0	50	.269	.201	.114	.089	.043	.005	.315	.235	.124	.092	.052	.012
	100	.212	.149	.075	.089	.045	.009	.189	.118	.043	.096	.047	.010
	200	.158	.098	.033	.096	.048	.008	.143	.078	.025	.099	.050	.013
	500	.121	.070	.016	.099	.050	.010	.120	.063	.016	.102	.053	.012
-.5	50	.285	.225	.137	.093	.046	.008	.380	.286	.164	.103	.056	.011
	100	.229	.161	.083	.100	.047	.010	.229	.152	.061	.108	.060	.012
	200	.166	.102	.036	.101	.053	.009	.176	.106	.034	.104	.058	.012
	500	.132	.070	.018	.106	.054	.012	.136	.075	.020	.097	.050	.010
Log-normal Error													
.5	50	.239	.181	.105	.085	.039	.006	.314	.232	.123	.091	.043	.008
	100	.222	.154	.086	.090	.043	.007	.196	.117	.041	.095	.047	.009
	200	.185	.126	.056	.096	.047	.008	.138	.079	.020	.097	.047	.009
	500	.138	.074	.024	.102	.049	.011	.123	.064	.016	.105	.052	.010
0	50	.246	.188	.108	.085	.042	.010	.319	.235	.115	.095	.047	.011
	100	.204	.141	.074	.090	.045	.007	.194	.115	.040	.095	.051	.008
	200	.180	.114	.047	.095	.047	.009	.142	.076	.021	.095	.048	.009
	500	.129	.075	.022	.097	.048	.010	.115	.060	.014	.100	.050	.011
-.5	50	.300	.235	.146	.093	.044	.008	.344	.246	.126	.097	.050	.011
	100	.214	.145	.064	.094	.045	.010	.208	.133	.050	.101	.055	.010
	200	.156	.092	.028	.099	.046	.008	.154	.086	.023	.101	.049	.011
	500	.123	.066	.015	.104	.051	.010	.121	.061	.014	.102	.050	.010

Table 4b. Empirical Sizes of Tests for Temporal Homogeneity in Panel SLE Model
Two-Way Fixed Effects, `Queen Contiguity`, $\lambda = -0.5$.

		$T=3$						$T=6$					
ρ	n	T_{SLE2}			T^r_{SLE2}			T_{SLE2}			T^r_{SLE2}		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
		Normal Error											
.5	50	.235	.173	.105	.086	.039	.007	.313	.225	.117	.089	.044	.009
	100	.216	.158	.086	.093	.046	.009	.189	.113	.037	.088	.044	.006
	200	.180	.117	.054	.093	.047	.007	.143	.079	.023	.100	.049	.012
	500	.134	.076	.021	.103	.048	.010	.118	.062	.014	.100	.053	.010
0	50	.271	.206	.116	.089	.040	.007	.315	.226	.109	.093	.044	.009
	100	.220	.149	.072	.098	.048	.011	.197	.115	.038	.092	.047	.008
	200	.160	.096	.032	.100	.051	.009	.146	.085	.024	.104	.052	.011
	500	.127	.062	.017	.103	.049	.011	.111	.059	.015	.094	.050	.010
-.5	50	.301	.233	.130	.095	.038	.007	.325	.232	.112	.092	.044	.009
	100	.214	.146	.065	.101	.048	.011	.206	.127	.039	.096	.046	.008
	200	.158	.092	.029	.103	.050	.011	.152	.087	.022	.102	.053	.010
	500	.117	.065	.014	.100	.050	.010	.111	.057	.013	.096	.048	.011
		Normal Mixture Error											
.5	50	.220	.161	.088	.080	.035	.005	.316	.243	.129	.093	.047	.009
	100	.213	.153	.085	.088	.043	.009	.204	.129	.048	.103	.051	.012
	200	.182	.121	.059	.096	.047	.006	.153	.089	.032	.106	.058	.013
	500	.139	.083	.030	.104	.049	.010	.137	.080	.022	.101	.051	.010
0	50	.256	.194	.113	.084	.043	.006	.321	.242	.124	.093	.049	.011
	100	.214	.151	.079	.091	.046	.008	.189	.121	.042	.098	.046	.011
	200	.155	.100	.033	.097	.048	.009	.146	.079	.028	.095	.051	.013
	500	.124	.068	.018	.099	.049	.011	.118	.064	.017	.102	.053	.012
-.5	50	.279	.219	.138	.089	.043	.007	.378	.288	.162	.111	.059	.016
	100	.232	.157	.082	.097	.049	.010	.234	.151	.058	.110	.057	.013
	200	.166	.103	.035	.102	.050	.010	.170	.104	.035	.106	.052	.014
	500	.128	.072	.019	.103	.054	.011	.134	.078	.018	.098	.047	.010
		Log-normal Error											
.5	50	.230	.178	.105	.086	.039	.008	.317	.232	.125	.089	.043	.009
	100	.218	.156	.087	.093	.045	.008	.197	.116	.042	.093	.049	.009
	200	.184	.122	.055	.093	.044	.008	.143	.080	.022	.100	.047	.010
	500	.139	.077	.024	.101	.052	.010	.119	.063	.015	.102	.053	.011
0	50	.242	.184	.107	.087	.043	.011	.315	.230	.113	.095	.046	.010
	100	.202	.142	.074	.091	.043	.010	.196	.115	.039	.093	.047	.008
	200	.176	.114	.046	.098	.046	.010	.141	.082	.023	.099	.049	.010
	500	.128	.074	.024	.098	.050	.011	.110	.055	.013	.102	.050	.010
-.5	50	.298	.230	.138	.095	.042	.008	.332	.245	.127	.097	.050	.010
	100	.220	.146	.067	.100	.045	.011	.212	.129	.048	.100	.052	.010
	200	.156	.092	.029	.100	.046	.009	.154	.089	.022	.105	.051	.011
	500	.123	.065	.015	.104	.051	.009	.115	.060	.015	.100	.048	.011