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Worker selection, hiring, and vacancies

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Worker Selection, Hiring and Vacancies*

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Abstract

The ratio of hirings to vacancies in the U.S. has the following establishment level properties: (i) it steeply rises with employment growth rate; (ii) falls with establishment size; and (iii) rises with worker turnover rate. The standard Diamond-Mortensen-Pissarides (DMP) matching model is not compatible with these observations. This paper augments selection of workers prior to hiring into a random matching model with multi-worker firms. In the calibrated model, worker selection accounts for about 30% of the variation in the hiring-vacancy ratio observed in the data. Compared to the standard model, the worker selection model has both qualitative and quantitative policy implications. A hiring subsidy reduces the unemployment rate substantially in the worker selection model, whereas the reduction in the unemployment rate is very small in the standard model. The two models also differ regarding the impact of the hiring subsidy across firms. The worker selection model implies that firms that have initially high worker turnover rates experience proportionally higher worker turnover rates after the subsidy. In contrast, the standard model predicts that the worker turnover rate increases proportionally more at firms with initially lower worker turnover rates.

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1 Introduction

The standard Diamond-Mortensen-Pissarides (DMP) matching model treats vacancy posting as the sole instrument for hiring new workers and uses an aggregate matching function to summarize firms' hiring technology. While successful in many dimensions, the use of an aggregate matching function overlooks the recruitment process at firm level. It postulates that all the firms fill vacancies at a common rate, a feature inherited even in multi-worker extensions. Consequently, the standard DMP model implies that all the firms attain the same hiring-vacancy ratio, called the vacancy yield. This result is at odds with the data. In the U.S., the establishment level hiring-vacancy ratio: (i) rises steeply with employment growth rate; (ii) falls with employer size; (iii) rises with worker turnover rate.

In this paper, I account for these cross sectional patterns of the hiring-vacancy ratio and study its policy implications. I augment worker selection into a random matching model where multi-worker firms hire among a pool of applicants. In response to idiosyncratic productivity shocks, firms post vacancies and are randomly matched to unemployed workers according to an aggregate matching function. Unlike the standard DMP model, not all the matches are hired. Firms screen, interview and evaluate applicants before making a hiring decision. I model this worker selection process by allowing firms to choose a hiring standard and hire only the successful applicants. The model admits cross-sectional variation in the hiring-vacancy ratio as firms set different hiring standards depending on their hiring needs. In the calibrated model, worker selection accounts for about 30% of the cross sectional variation of the hiring-vacancy ratio observed in the data. On the policy side, accounting for these patterns of the hiring-vacancy ratio has both qualitative and quantitive implications. For example, when firms are subsidized for hiring new workers, the decline in unemployment rate is about 7 times larger in the worker selection model. Moreover, firms that have initially high worker turnover rates experience larger increases in the worker turnover rate after the employer subsidy in the worker selection model. In contrast, the standard model predicts that the employer subsidy increases worker turnover rate proportionally more at firms with initially lower worker turnover rates.

The cross sectional patterns of the hiring-vacancy ratio are documented in Davis, Faberman and Haltiwanger (2012), henceforth DFH, where the authors use the establishment-level Job Openings Labor Turnover Survey (JOLTS) data. Building upon these observations, they argue that firms heavily rely on other instruments in addition to vacancies as they hire new workers. This paper adds worker selection as a new tool for recruiting new workers. Such an extension is consistent with microeconomic evidence regarding firms' hiring practices. Barron and Bishop (1985) report from the 1982 Employment Opportunities Pilot Project

(EOPP) that company personnel spends on average 9.87 hours per hire to recruit, screen and interview the applicants. Using the same dataset, Silva and Toledo (2009) and Hagedorn and Manovskii (2008) estimate that per hire cost of recruitment is on average 4.3% of the quarterly wage a newly hired worker. The time spent for recruiting workers also varies across firms. The standard deviation of the time spent per hire for evaluating applicants is 17.16 hours. I interpret the variation in manhours spent for recruitment as evidence for the variation in firm's hiring standards obtained from the model.

I motivate the idea of worker selection by introducing worker heterogeneity in the form of unobserved match-specific quality shocks. Drawn upon matching, the match-specific quality shock determines the productivity of a worker at the hiring firm. When increasing employment, firms now face a trade-off between the quantity and the quality of the workers. Given the total number of vacancies, a firm would add fewer workers if it wanted to hire high quality workers. The optimal decision for the number of vacancies and the hiring standard depends on how the quality and quantity margins interact. Regarding the three patterns of the hiring-vacancy ratio explained above, I draw the following conclusions from the model: first, in a growing firm, the marginal cost of increasing the hiring standard is larger because a growing firm also posts more vacancies and contacts a larger group of applicants. Since it faces higher selection costs, a growing firm fills vacancies at a higher rate by being less picky about the new recruits and attains a higher hiring-vacancy ratio. Second, as the firm size increases, the employment growth rate decreases and the job filling rate increases. This generates a negative relationship between the firm size and the hiring-vacancy ratio. Third, a firm that has initially set a lower hiring standard lays off a larger fraction of the current recruits in near future. This implies a positive relationship between the hiring-vacancy ratio and the worker turnover rate.

This is the first paper to account for all the three cross sectional patterns of the hiring-vacancy ratio at the same time. Extending the standard DMP model to allow firms to post multiple vacancies do not generate cross-sectional variation in the hiring-vacancy ratio.[1] In these type of models, because all the workers are identical, firms indiscriminately hire all the workers they match. Then, for any firm, the hiring-vacancy ratio is, in effect, determined by the aggregate matching function. This result is obtained as long as matching is random. When matching is not random, this conclusion does not hold any more. Kaas and Kircher (2011) build a directed search model where firms attract workers by posting wages. Posting wages adds another competitive element into the labor: firms that want to grow faster post high wages in the market, attract more workers and fill vacancies at a higher rate.

[1] See, for example, Cahuc, Marque and Wasmer (2008), Elsby and Michael (2010), Acemoglu and Hawkins (2010) and Fujita and Nakajima (2013)

This generates a positive relationship between the hiring-vacancy ratio and the employment growth rate. The model in this paper is distinguished from theirs due to its ability to match the positive relationship between worker turnover and hiring-vacancy ratio. In Kaas and Kircher (2011), separation rate is constant at growing firms. In this paper, however, firms that fill vacancies at a faster rate proportionally hire more low quality workers. As a result, these firms separate from workers with a greater probability in near future. This implies that the separation rate is positively related to the employment growth rate in growing firms, which is indeed observed in JOLTS.

In the quantitative section, I use the calibrated model to examine the effects of a hiring subsidy on labor market outcomes. A hiring subsidy is given to firms for hiring new workers. To highlight the impact of worker selection, I compare the results obtained from the worker selection model to those obtained from the standard DMP model. I find that if employers are paid half of the average wage of a newly worker, unemployment rate in the worker selection model falls by half of a percentage point. In the standard model, the decline in unemployment rate in response to an equivalent subsidy is below 0.1 percentage point. Regarding the hiring-vacancy ratio, the two models produce qualitatively different result. In the standard model, hiring-vacancy ratio unambiguously goes down because increased number of total vacancies makes it harder for firms to find workers. However, in the worker selection, firms respond to the subsidy also by lowering their hiring standards. The effect on quality margin dominates the effect on the quantity margin at low levels of hiring subsidy.

Earlier works that have used matching models to analyze labor market policies are silent about the effects on firms with different characteristics as these papers assume that firms can hire only one worker.[2] Owing to the multi-worker setting in this paper, I give new insights about the effects of the subsidy across firms. The implications qualitatively differ between the worker selection model and the standard model. The worker selection model predicts that firms with high worker turnover rates experience proportionally larger worker turnover rates after the subsidy. The standard model, on the other hand, predicts that the effect of the subsidy on the worker turnover rate is larger at firms with initially lower worker turnover rates.

The paper is organized as follows. Section 2 describes the worker selection model and Section 3 characterizes the equilibrium. I calibrate the model in Section 4 and discuss my findings in Section 5. The next section presents the results from the counterfactual experiments with the hiring subsidy. The last section concludes.

[2] See Pries and Rogerson (2005) and Mortensen and Pissarides (2001).

2 The Worker Selection Model

2.1 Overview

The economy is populated by risk-neutral workers, the measure of which is normalized to 1, and a large number of risk-neutral entrepreneurs. Time is discrete and the discount factor is β. Each entrepreneur runs a firm which produces a single good. Hereafter, I refer to firms and entrepreneurs interchangeably. To avoid negative consumption, I also allow entrepreneurs to hold a portfolio of all firms.

In any period, a worker is either employed or unemployed. An employed worker receives a wage income, but cannot search for a new job. An unemployed worker searches for a job; if he cannot find a job, he is engaged in home production and receives b. Savings are disallowed, so workers consume all of their income in the current period.

In any period, a firm can be either active or inactive. An active firm employs a measure of workers denoted by n. Firm productivity has an idiosyncratic component, ε. It evolves according to a Markov process, $F(\varepsilon'|\varepsilon)$, where I adopt prime notation to denote variables in the next period. The productivity process is common to all of the firms. An inactive firm can become active at the beginning of each period by paying a fixed entry cost, c_e. Upon entry, it draws its initial idiosyncratic productivity from the unconditional distribution of the same Markov process, $F_0(\varepsilon)$. Active firms become inactive with probability δ. Productivity shocks are large enough to ensure that none of the firms optimally chooses to become inactive.

2.2 Recruiting New Workers

Recruiting new workers consists of three stages: vacancy posting, worker selection and wage bargaining. The first and the last stages are common to the standard matching model. The innovation of this paper is the introduction of the interim stage where firms selectively hire among a pool of applicants.[3]

In the first stage, firms post vacancies, v, to attract unemployed workers and pays c_v per vacancy. There are matching frictions in the labor market. Total number of matches in the economy is determined via an aggregate matching function, which has a CES form:

$$M(U,V) = (U^{-\zeta} + V^{-\zeta})^{-\frac{1}{\zeta}} \tag{1}$$

U and V are total number of unemployed workers and vacancies, respectively. $\zeta > 0$ governs

[3] The idea of selecting workers prior to hiring is not new in the literature. Pries and Rogerson (2005), Villena-Roldan (2008) and Merkl and van Rens (2013) are examples of selection models where firms hire only one worker. Helpman, Itskhoki and Redding (2008) examine worker selection in a multi-worker setting.

the degree of elasticity of substitution. Let $\theta = V/U$ be the market tightness. Then, a firm that posts v number of vacancies meets $q(\theta)v$ workers, where $q(\theta)$ is the probability that a vacancy meets a worker. $q(\theta)$ is derived from the matching function as follows:

$$q(\theta) = \frac{M(U,V)}{V} = (1+\theta^{\zeta})^{-\frac{1}{\zeta}} \tag{2}$$

Similarly, $M(U,V)/U = \theta q(\theta)$ is the probability a worker meets a vacancy.[4] In the sequel, I drop θ and simply write q for notational purposes.

In the second stage, each worker matched with a firm draws an unobserved match-specific quality shock, x_i, from a uniform distribution between 0 and 1. The match-specific quality shock determines whether the worker will be productive or unproductive at the hiring firm conditional on being hired. Specifically, a worker with a match-specific quality x_i becomes productive at the hiring firm with probability $x_i^{\gamma-1}$, where $\gamma > 1$. Otherwise, the worker becomes unproductive. Both the firm and the worker learn the true productivity of the worker only after one period of employment. If a worker turns out to be unproductive, he leaves the firm. Although the match-specific quality shock is unobserved at the time of hiring, the firm can engage in a costly process where it evaluates the applicants and infer about their true match-specific quality. I model this process by allowing firms to choose a hiring standard, $p \in [0,1]$, and observe the match-specific quality of an applicant up to p. If an applicant's match-specific quality is greater than p, it is still unobserved, but known to be greater than p.

Total selection costs have the following quadratic form:

$$C_s(p, qv) = \frac{c_s}{2} \exp\left(\frac{c_p}{2}p^2\right)(qv)^2 \tag{3}$$

This functional form assumes that selection technology exhibits decreasing returns to scale in the number of applicants captured by the quadratic term: given p, the marginal cost of selection is increasing in the number of applicants. It also assumes that this marginal cost is increasing in the hiring standard set by the firm. The interpretation is that worker selection is time-consuming and as the hiring standard goes up the interviewer spends more time to distinguish high quality workers from low quality ones. As I will show later, the quadratic form ensures that all the hiring firms set a positive hiring standard.[5]

[4]Note that $\zeta > 0$ guarantees that both of the meeting probabilities lie in the interval [0,1].

[5]The exponential form in the total selection cost satisfy three conditions. First, it is greater than zero when evaluated at $p = 0$. This prevents firms to choose p close to zero, post enormous amount of vacancies and converge to their long-run employment level in a short period of time. Second, the derivative of this function with respective p is zero when $p = 0$. This guarantees interior solution for p. Finally, it is log-convex, which guarantees that the dynamic programming problem of a hiring firm is concave.

Due to matching frictions in the labor market, a firm's current match with its workers generates bilateral monopoly rents. In the third stage, firms bargain over the wage with their existing workers and the workers in their applicant pool to split these rents. I describe wage bargaining formally below.

I refer to the second stage above as worker selection, because the wage bargaining process implies that a firm does not hire all the workers it matches. To see that, consider the worker with $x_i = 0$. His contribution to output is zero in this period and he leaves the firm at the end of the period. However, the firm has to compensate for his outside option, i.e. value of finding a job with a higher match quality. The total value of this match is negative and both parties mutually agree not to form an employment relationship. Furthermore, the value of a worker to the firm increases with x_i. Hence, there exists a reservation match-specific quality below which workers have negative value to the firm.[6] The selection technology enables the firm to identify these workers.

2.3 Firms' Problem

Since firms are subject to idiosyncratic productivity shocks, large firms that receive an adverse productivity shock may find it optimal to reduce employment. However, such a firm would never find it optimal to hire from the unemployment pool, because an existing worker is more productive than any potential new worker and adjustment is costly. Moreover, the problem of a hiring firm includes an additional control variable, p. Therefore, I write down the dynamic problem of a hiring and firing firm separately. If a firm neither hires nor fires workers, I interpret that as a boundary solution to the firing firm problem and do not write it separately.

Let $\Pi^h(n, \varepsilon)$ and $\Pi^f(n, \varepsilon)$ denote the value of a hiring and firing firm, respectively. Let also $\Pi(n, \varepsilon) = \max(\Pi^h(n, \varepsilon), \Pi^f(n, \varepsilon))$. Given the timing of events in a period, total number of hires at a firm posting v vacancies are equal to $qv(1 - p)$, but only $qv(1 - p^\gamma)/\gamma$ will become productive and remain at the firm next period. Hence, employment at a hiring firm evolves over time according to the following equation:

$$n' = (1 - \lambda)n + qv(1 - p^\gamma)/\gamma \tag{4}$$

I assume that incumbent workers lose their jobs at the beginning of the period with probability λ and can search for a new job in the current period.

An active firm has access to a Cobb-Douglas production function which depends on

[6] Some of the newly hired workers will have negative value to the firm. However, due to costly selection, the firm does not attempt to find those workers in the applicant pool.

the number of *productive* workers employed in the current period: $A\varepsilon n'^{\alpha}$. Note that the production function accounts for the fact that new recruits are employed in the current period.

Let $w^n(n', \varepsilon)$ and $w^p(n', \varepsilon, p)$ denote wages paid to existing workers and new recruits, respectively. I conjecture that both wages depend on the total number of productive workers and firm productivity. In addition, the hiring standard affects the wage payment to the new workers as it determines the expected productivity of a randomly selected new hire. This specification is later verified in the wage bargaining section. The dynamic programing problem of a hiring firm is as follows:

$$\begin{aligned}\Pi^h(n, \varepsilon) = \max_{n',p,v} \quad & -c_v v - \frac{c_s}{2}\exp\{\frac{c_p}{2}p^2\}\,(qv)^2 + A\varepsilon n'^{\alpha} - qv(1-p)w^p(n', \varepsilon, p) \\ & -(1-\lambda)nw^n(n', \varepsilon) + \beta(1-\delta)E_{\varepsilon'|\varepsilon}\Pi(n', \varepsilon') \end{aligned} \quad (5)$$

subject to 4.

Let f denote total firings. Then, employment at a firing firm evolves according to:

$$n' = (1-\lambda)n - f \quad (6)$$

The dynamic programing problem of a firing firm is as follows:

$$\Pi^f(n, \varepsilon) = \max_{n',f} \quad A\varepsilon n'^{\alpha} - n'w^n(n', \varepsilon) + \beta(1-\delta)E_{\varepsilon'|\varepsilon}\Pi(n', \varepsilon') \quad (7)$$

subject to 6.

2.4 Worker's Value Functions

Let $\tilde{V}^u$ and V^u denote the value of unemployment at the beginning of the period and after the labor market closes, respectively. I describe how they are related to each other further below. The value function of an existing worker employed at a firm with n workers and productivity ε is:

$$V^n(n, \varepsilon) = w^n(n'(n, \varepsilon), \varepsilon) + \beta((1-\delta)((1-\lambda)E_{\varepsilon'|\varepsilon}V^n(n', \varepsilon') + \lambda\tilde{V}^u) + \delta\tilde{V}^u) \quad (8)$$

where $n'(n, \varepsilon)$ is the firm's optimal decision for employment. The interpretation is standard: an existing worker receives $w^n(n', \varepsilon)$ this period. With probability $(1-\delta)(1-\lambda)$, he is employed at the same firm and enjoys the expected value of employment. Otherwise, he receives $\tilde{V}^u$. Note that the expected value of employment is over the productivity shocks and accounts for the change in firm's employment.

Let $g(p) = \frac{1-p^\gamma}{\gamma(1-p)}$ be the probability that a randomly selected new hire is productive. Then, the value function of a newly hired worker is:

$$\begin{aligned} V^p(n,\varepsilon) = \; & w^p(n'(n,\varepsilon),\varepsilon) + \beta(g(p(n,\varepsilon))((1-\delta)((1-\lambda)E_{\varepsilon'|\varepsilon}V^n(n',\varepsilon') + \lambda V^u) \\ & +\delta V^u) + (1-g(p(n,\varepsilon)))V^u) \end{aligned} \quad (9)$$

Note that the continuation value of a newly hired worker depends on the hiring standard set this period, $p(n,\varepsilon)$. The functional equation above is otherwise same with (8).[7] Finally, V^u and $\tilde{V}^u$ are related according to:

$$V^u = \; b + \beta \tilde{V}^u \quad (10)$$

$$\tilde{V}^u = \; \theta q \int_\varepsilon \int_n \frac{v(n,\varepsilon)\left((1-p(n,\varepsilon))V^p(n,\varepsilon) + p(n,\varepsilon)V^u\right)}{\int_\varepsilon \int_n v(n,\varepsilon) dG(n,\varepsilon)} dG(n,\varepsilon) + (1-\theta q)V^u \quad (11)$$

where $G(n,\varepsilon)$ is the steady state stationary distribution of firm size and productivity. At the beginning of the period, an unemployed worker matches with a vacancy with probability θq. Conditional on a match, he receives the expected value of the outcome of the selection process: with probability $(1-p(n,\varepsilon))$ he is employed and enjoys the value of being employed at a firm with n workers and productivity ε. Otherwise, he is unemployed and receives V^u. The probability that he matches with a firm of size n and productivity ε is weighted by the firm's share of vacancies in total vacancies. Finally, with probability $(1-\theta q)$, he does not find a match and receives V^u.

Before moving to the next section, let me formally define the stationary distribution. Given the firms' optimal decision for employment, $G(n,\varepsilon)$ must satisfy:

$$G(n,\varepsilon) = \; \sum_{\tilde{\varepsilon}} F(\varepsilon|\tilde{\varepsilon}) \int_{\tilde{n}} \mathcal{I}(n'(\tilde{n},\tilde{\varepsilon}) \leq n) dG(\tilde{n},\tilde{\varepsilon}) \quad (12)$$

where $\mathcal{I}$ is an indicator function which is 1 if the condition is satisfied and 0 otherwise.

2.5 Wage Bargaining

To determine wages, I adopt the bargaining solution in Stole and Zwiebel (1996). They describe a dynamic game where the firm negotiates the wage payment in pairwise bargaining sessions with its employees in an arbitrary order. If an agreement is reached between the worker and the firm during a bargaining session, the firm continues bargaining with the next worker. Otherwise, the worker leaves the firm and the bargaining process resumes with *all*

[7] In fact, (8) can be thought as the limiting case of (9) as $p \to 1$.

the remaining workers. Each bargaining session is the limiting case of the offer-counteroffer game between the firm and the worker described in Binmore et al. (1986). In this offer-counteroffer bargaining game, each time the worker rejects an offer, there is an exogenous probability, $(1-\phi)d$, that the negotiations break down. Similarly, each time the firm rejects an offer, the negotiations break down with probability ϕd. As $d \to 0$, they split the joint surplus net of outside options such that the worker receives ϕ fraction of it. If there is only one worker, the solution is the Nash bargaining solution with ϕ being the workers' bargaining power. For the firm, the surplus is continuing the bargaining process with one less worker. When labor is continuous, the solution to the wage function implies a split of the *marginal* surplus and outside option of the worker according to bargaining powers.

The bargaining game above assumes workers are the same with respect to their productivities. In my model, due to worker selection, existing and new workers differ in size and productivity and are potentially paid different wages. The firm negotiates with $(1-\lambda)n$ existing workers and $qv(1-p)$ potential workers. The productivity of existing workers is 1 and the productivity of selected applicants is $g(p)$. Let me define *total* surplus to the firm at the bargaining stage as $D(\tilde{n}, r, p, \varepsilon)$, where $\tilde{n} = (1-\lambda)n$ and $r = qv(1-p)$. At the bargaining stage, vacancy posting and selection costs are sunk. Hence, from the firms problem, I obtain total surplus as follows:

$$\begin{aligned} D(\tilde{n}, r, p, \varepsilon) = \quad & A\varepsilon(\tilde{n} + g(p)r)^{\alpha} - w^n(\tilde{n} + g(p)r, \varepsilon)\tilde{n} - w^p(\tilde{n} + g(p)r, \varepsilon)r \\ & +\beta(1-\delta)E_{\varepsilon'|\varepsilon}\Pi(\tilde{n} + g(p)r, \varepsilon') \end{aligned} \tag{13}$$

Note that n' is equal to $\tilde{n} + g(p)r$. The marginal surplus to the firm from an existing worker is the partial derivative of the total surplus with respect to $\tilde{n}$, $D_{\tilde{n}}(\tilde{n}, r, p, \varepsilon)$. The marginal surplus to the firm from a potential worker is similarly given by $D_r(\tilde{n}, r, p, \varepsilon)$. Then, the solution to the bargaining problem satisfies the following conditions:

$$\phi D_{\tilde{n}}(\tilde{n}, r, p, \varepsilon) = (1-\phi)(V^n(n', \varepsilon) - V^u) \tag{14}$$

$$\phi D_{\tilde{r}}(\tilde{n}, r, p, \varepsilon) = (1-\phi)(V^p(n', \varepsilon) - V^u) \tag{15}$$

Using these two conditions along with the firm's problem and workers' value functions, I obtained closed form solutions to the wage functions for each group as follows:

$$w^n(n', \varepsilon) = \quad \frac{\alpha\phi}{1-\phi+\alpha\phi} A\varepsilon n'^{\alpha-1} + (1-\phi)\Omega \tag{16}$$

$$w^p(n', \varepsilon, p) = \quad g(p)\frac{\alpha\phi}{1-\phi+\alpha\phi} A\varepsilon n'^{\alpha-1} + (1-\phi)\Omega \tag{17}$$

where I define Ω as:

$$\Omega = (1-\beta)V^u - (\lambda + \delta - \lambda\delta)((1-\beta)V^u - b) \tag{18}$$

The derivations are available in Appendix A. The wage functions are similar to ones obtained in other papers featuring random matching with multi-worker firms, e.g., Acemoglu and Hawkins(2010), Elsby and Michaels(2012), Cahuc, Marque and Wasmer (2008). The solution for the wage equation in Stole and Zwiebel (1996) implies sharing of the worker's outside option and the weighted average of infra-marginal products of labor. The solution above preserves this property except that it now includes additional terms to the worker's outside option due to timing assumption. The wages at a non-hiring firm (firing or no-action) is the same as $w^n(n', \varepsilon)$. Further, wages at a firing firm are such that $V^n(n', \varepsilon) = V^u$. This is implied by equation (14). Finally, as I conjectured, both wage functions depend only on the total number of productive workers and not on the number of workers in each group separately. This result is not surprising given that both groups enter the production function linearly.

2.6 Recursive Stationary Equilibrium

The final condition needed to characterize the recursive stationary equilibrium is the free entry condition:

$$E_\varepsilon(\Pi(0, \varepsilon)) = c_e \tag{19}$$

This simply states that the expected value of an entrant firm is equal to c_e.

A formal definition of recursive stationary equilibrium is available in Appendix B. Importantly, two equilibrium outcomes are not in the definition of the recursive competitive equilibrium: the measure of firms and the total number of unemployed workers seeking for jobs. Both of the outcomes are endogenous and can be calculated from other endogenous variables. Let μ denote the measure of firms in equilibrium. Then, total vacancies and total unemployed workers are:

$$V = \mu \int_n \int_\varepsilon v(n, \varepsilon) dG(n, \varepsilon)$$
$$U = 1 - (1 - \lambda - \delta)\mu \int_n \int_\varepsilon n dG(n, \varepsilon) - \mu \int_n \int_\varepsilon f(n, \varepsilon) dG(n, \varepsilon)$$

Recall market tightness is $\theta = V/U$. Using the equilibrium value of θ and the calculated decision rule for firings, one can obtain the equilibrium value of μ. Plugging μ in the second equation above, equilibrium unemployment is determined.

3 Characterization of Equilibrium

In this section, I analyze the problem of a hiring firm since heterogeneity in firms' recruiting practices is the main focus of this paper. The problem of a firing firm is rather standard. Inserting the wage functions in the hiring firm's optimization problem, the dynamic programming problem becomes:

$$\begin{aligned}\Pi^h(n,\varepsilon) = \max{}_{n',p,v} \quad & -\frac{c_v}{v} - \frac{c_s}{2}\exp\{\frac{c_p}{2}p^2\}(qv)^2 + \frac{1-\phi}{1-\phi+\alpha\phi}A\varepsilon n'^{\alpha} \\ & -(1-\phi)\Omega\left((1-\lambda)n + (1-p)qv\right) \\ & +\beta(1-\delta)E_{\varepsilon'|\varepsilon}\Pi(n',\varepsilon') \end{aligned} \tag{20}$$

subject to (4).

3.1 Optimal Decision for the Hiring Standard

In (20), when $\gamma = 1$, any worker from the unemployment pool is productive. Hence, firms optimally choose to hire every worker they match, i.e. $p = 0$. In this case, the model is standard MP model with multi-worker firms. In general, replacing qv from (4) into the firm's problem in (20) and taking the derivative with respect to p implies:

$$\begin{aligned} & (1+(\gamma-1)p^{\gamma} - \gamma p^{\gamma-1})((1-\phi)\Omega \\ & -\gamma p^{\gamma-1}cv/q - \frac{c_s}{2}\exp\{\frac{c_p}{2}p^2\}\left(\frac{\gamma\Delta}{1-p^{\gamma}}\right)\left(\frac{1}{1-p^{\gamma}}\right)^{z-1}(c_p p(1-p^{\gamma}) + 2\gamma p^{\gamma-1}) \\ & \leq 0 \end{aligned} \tag{21}$$

where $\Delta = n' - (1-\lambda)n$ is the absolute change in employment.[8] The first term in (21) is strictly decreasing in p and is equal to 0 when $p = 1$. I interpret this term as the marginal benefit from increasing the hiring standard: as a firm increases the hiring standard, it avoids paying wages to the workers who are more likely to be unproductive in the next period. However, this gain diminishes with p as the firm has to post more vacancies to satisfy a given level of Δ. The second term, on the other hand, is strictly increasing in p and equal

[8] The common term $\frac{\gamma\Delta}{(1-p^{\gamma})^2}$ is factored out.

to 0 when $p = 0$. I interpret this term as marginal cost of increasing the hiring standard: as a hiring firm increases the hiring standard, the marginal cost of selection increases not only because the selection costs are larger when p is larger, but also because the firm has to post more vacancies to satisfy a given level of Δ. I plotted these curves in Figure 1 using the calibrated parameter values. Evidently from Figure 1, the solution to p is interior and unique.

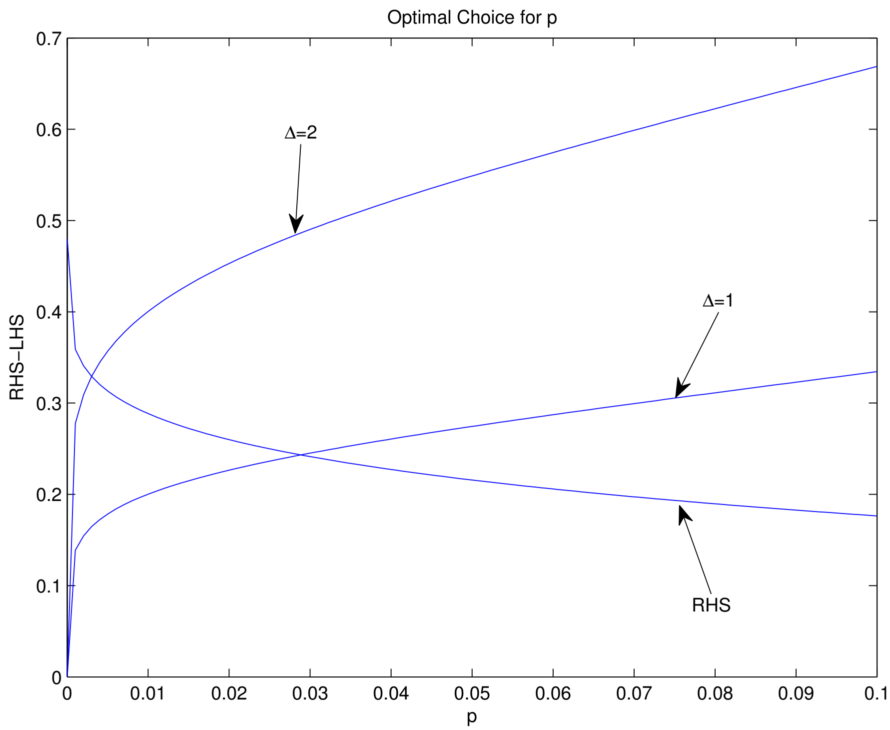

Figure 1: Optimal Choice for p

Now, consider an increase in Δ, i.e. the firm grows faster. This shifts the marginal cost curve up and leaves the marginal benefit unchanged. Such a change is depicted in Figure 1. The optimal solution for p becomes smaller. Hence, the firm fills vacancies at a higher rate, which is consistent with the data. Furthermore, as the firm decreases p, it is more likely to separate from newly hired workers in the next period. This generates a positive correlation between the job filling rate and the worker turnover rate at firm level, which is also consistent with the data. The relationship between firm size and job filling rate depends on the optimal decision on employment, which I analyze in the next section.

3.2 Optimal Decision for Employment

In the previous section, I obtained the optimal solution for p. An important observation is that when Δ is given, the decision for p is independent of the production in the current period and the continuation value of the firm. This allows me to characterize the adjustment cost function in terms of Δ given that p is optimally chosen. Let $C(\Delta)$ be the total cost to the firm from changing the employment from n to n'. It is the value function of the following minimization problem:

$$C(\Delta) = \min_{p} \frac{c_v}{q}\frac{\gamma\Delta}{1-p^\gamma} + \frac{c_s}{2}\exp\{\frac{c_p}{2}p^2\}\left(\frac{\gamma\Delta}{1-p^\gamma}\right)^2 + (1-\phi)\Omega\gamma\Delta\frac{1-p}{1-p^\gamma} \tag{22}$$

I obtained the following results regarding the problem in (22). A detailed analysis is available in the Appendix C.

1. Let $p(\Delta)$ be the policy function in (22). Then, $p'(\Delta) < 0$, verifying the analysis in the previous section.

2. $C'(\Delta) > 0$, i.e. the adjustment cost function is increasing.

3. $C''(\Delta) > 0$, i.e. the adjustment cost function is strictly convex. This result uses the fact that $\exp\{\frac{c_p}{2}p^2\}$ is log-convex in p.

The last result implies that the dynamic programming problem of a hiring firm is concave and the first order condition with respect to n' is necessary and sufficient for optimal decision. For completeness, I write the first order condition for n' from (20) here:

$$\begin{aligned}&-\frac{c_v}{q}\frac{\gamma}{1-p^\gamma} - c_s\exp\{\frac{c_p}{2}p^2\}\left(\frac{\gamma}{1-p^\gamma}\right)^2(n'-(1-\lambda)n) + A\varepsilon\alpha n'^{\alpha-1}\\ &-\frac{\gamma(1-p)}{1-p^\gamma}\Omega + \beta(1-\delta)E_{\varepsilon'|\varepsilon}\Pi(n',\varepsilon') \leq 0\end{aligned} \tag{23}$$

Importantly, convex adjustment costs imply that an entrant firm converges to its long-run size gradually while the employment growth rate becomes smaller as it approaches to the long-run employment. I have already established the positive relationship between the growth rate and the job filling rate. Hence, at firm level, there is a negative relationship between firm size and job filling rate. Whether this relationship holds in the cross-section depends on the stationary distribution of firms. In the calibration exercise, I verify that the negative relationship holds, which is also consistent with the data.

4 Calibration

In this section, I calibrate the model to match the salient features of JOLTS documented in DFH. Unless otherwise stated, all the targeted moments are taken from their work. The parameter estimates are presented in Table 1.

Parameter	**Meaning**	**Value**
β	Discount factor	0.9996
ϕ	Bargaining power	0.5000
α	Production function curvature	0.6777
ρ	Persistence of idiosyncratic shocks	0.5900
σ	Dispersion of idiosyncratic shocks	0.1730
γ	Success probability parameter	4.0944
λ	Exogenous separation probability	0.0006
δ	Exogenous exit probability	0.00075
b	Value of leisure	0.9220
c_v	Flow cost of vacancy	0.0058
ζ	Matching function parameter	1.6783
c_s	Selection cost- quantity margin	0.0465
c_p	Selection cost- quality margin	1.6404
A	Aggregate productivity	3.3070
c_e	Fixed entry cost	3154.6083

Table 1: Calibrated Parameters (Weekly Model)

I choose a period to be equal to one week. The discount factor is set to match the quarterly interest rate of 1.12%. As in Acemoglu and Hawkins (2010) and Fujita and Nakajima (2013), I use 0.67 for the curvature of the production function. For worker's bargaining power, Shimer(2005) and Hagedorn and Manovskii (2008) use 0.72 and 0.052, respectively. I use an intermediate value and assume equal bargaining power between the firm and its workers.

The idiosyncratic productivity process approximates an AR(1) process:

$$\log \varepsilon_{t+1} = \rho \log \varepsilon_t + \eta_t \tag{24}$$

with $\eta_t \sim N(0, \sigma^2)$. For the persistence parameter, ρ, I use the estimate in Abraham and White (2006). They find the persistence of the idiosyncratic shocks as 0.59 on an annual basis.[9] To represent this process on a weekly basis, I impose that firms draw a productivity shock with probability 1/48 in a given week. I choose the variance of the shocks to match a hiring rate of 3.4%.

There are three sources of separation in the model. First, firms fire productive workers if they are hit by a low productivity shock. Separations due to firings are driven by the productivity process. Since separations are equal to hirings in a stationary equilibrium, I account for this type separation by setting σ to match the hiring rate. Second, some of the newly hired workers leave the firm next period if they turn out to be unproductive. The probability that a worker with the average match-quality will be productive next period depends on γ. All else equal, when γ becomes larger, a larger fraction of the newly hired workers leaves the firm next period. This implies a larger difference between the worker turnover rate, the sum of hiring and separation rates, and the job turnover rate, the sum of *net* job creation and destruction rates. In JOLTS, monthly job turnover rate is 3.0%, less than half of the worker turnover rate. Since the hiring rate is already targeted, I choose γ to match the monthly job turnover rate. Finally, separations occur exogenously with probability λ or due to firm exit with probability δ. In JOLTS, separations due to reasons other than quits and lay-offs is 0.24%. I set λ to this value. Consistently with the evidence from Davis, Haltiwanger, and Schuh (1996), I choose δ so that one-sixth of job destruction is due to firm exit. JOLTS excludes exiting firms. Accordingly, I set $\delta = 0.015/5 = 0.003$.

The value of b relative to the average worker productivity plays an important role in the context of the volatility puzzle. A higher value of this ratio tends to amplify the effects of productivity shocks in the standard MP model. The values used in the literature lies between 0.4 and 0.955.[10] Following Motensen and Nagypal (2007), I set the ratio of b to average worker productivity (Y/N) to 0.72. I later verify that a smaller target value attenuates the responses of labor market outcomes to aggregate productivity shocks. Furthermore, I normalize the equilibrium value Ω to 1 and choose A to match this value.

Note from equation (21) that, as the number of vacancies posted approaches to zero, the optimal hiring standard approaches to a value that is strictly less than 1. Given γ and V^u, the magnitude of c_v determines this distance. In the lowest worker turnover quintile, the daily job filling probability is equal to 0.011. A similar value is observed around zero growth rate. In weekly terms, this is equal to 0.0745. I choose c_v so that the job filling probability is equal to this value in the model when total vacancies are equal to zero.

The daily job filling rate in the data is 0.05. This implies that the probability of filling

[9] This is the estimate without the fixed effects. To be consistent with the specification in (24), I use this value.

[10] See Shimer (2005) and Hagedorn and Manovskii (2008).

a vacancy in a week is 0.3017. The model counterpart of this value is $q(1 - \bar{p})$, where $\bar{p}$ is the average hiring standard set by the firms. Shimer (2005) estimates that the average job finding probability of a worker in a month is 0.45. In weekly terms, this is equal to 0.1388. In the model, this is given by $\theta q(1 - \bar{p})$. Dividing the latter by the former, I obtain $\theta = 0.4601$. To determine q, I use the fact from Roldan-Vilena (2008) that firms interview, on average, with 5 applicants before filling an open position. This implies, conditional on being matched, the daily probability that a firm hires a worker is 0.20. This is simply $(1-\bar{p})$ in daily terms. Then, the daily probability that a firm meets a worker is $0.05/0.20 = 0.25$. On a weekly basis, this is equivalent to setting $q = 0.8665$. Using the calibrated values of θ and q, I find $\zeta = 1.6783$.

There are two parameters in the selection cost function to be estimated: c_s and c_p. They determine how the quantity and quality margins are related. Accordingly, I choose these parameters value to match average job filling rate and the average firm size. The job filling probability is calculated above as 0.3017. The average firm size in Business and Employment Dynamics (BED) is 21.6. I choose c_s and c_p to match these figures.

The last parameter to be calibrated is the fixed entry cost. I choose this value so that the expected value of an entrant is equal to zero in equilibrium.

5 Discussion

In this section, I present the results from the model regarding the cross sectional behavior of the job filling rate. My main finding is that the model qualitatively matches the cross-sectional properties of the job filling rate regarding employment growth, worker turnover and establishment size.

Figure 2 plots the daily job filling rate against the monthly establishment employment growth rate bins. I constructed the growth rate bins so that the share of vacancies are equal in each bin. This produces narrower bins near zero growth rate and progressively wider bins as the employment growth rate becomes larger. DFH constructs the growth rate bins in a similar fashion. The job filling rate near zero percent is around 3% and reaches to 5.5% as the growth rate becomes about 5%. After this point, the response of job filling rate to growth becomes weaker, reaching up to 6.5% at 20% growth rate. The corresponding figures in the data are stronger: the job filling rate at 20% growth rate is about 18%.

There are at least two possible explanations for this discrepancy between the model and the data. First, there can be micro level randomness in the data. Some firms are lucky to find good candidates and therefore fill vacancies at a higher rate and growth faster. The maximum value that the daily job filling rate can take in the model is 25%, which happens

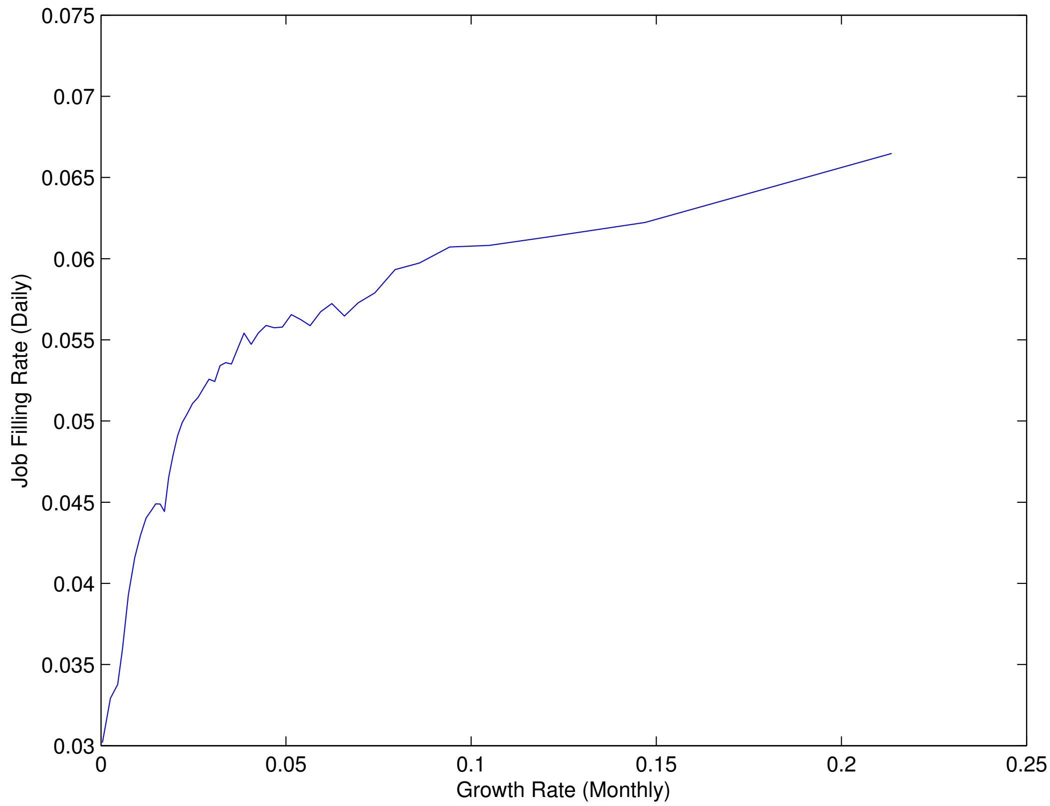

Figure 2: The cross sectional relationship between growth rate and daily job filling rate.

when the hiring standard is set equal to zero. This natural bound constrains the firms to achieve a higher job filling rate. Second, there might be increasing return at the establishment level. For example, it may be easier to attract more workers when the firm has more open positions. Hence, firms that are posting more vacancies meet proportionally more workers. Such a feature is absent in the model.

To quantitatively evaluate the model, I plotted log of the daily job filling rate against log of the monthly hiring rate in Figure 3. The slope gives an estimate of the elasticity of the job filling rate with respect to the hiring rate. DFH estimate for this elasticity is 0.82. I estimated the slope in the steeper region of the plot in Figure 3 as 0.312. This implies that the model alone can account for 38% of the variation in the growth rates. Further, DFH find that 0.04 of 0.82 is due to increasing returns at the establishment level. In a simulation exercise, they also estimate that the luck effect accounts for about 10% of the variation across the growth rate bins. These together imply that the worker selection can account for about 45% of the slope after controlling for scale and luck effects.

Figure 4 plots the daily job filling rate against monthly worker turnover quintiles. Moving

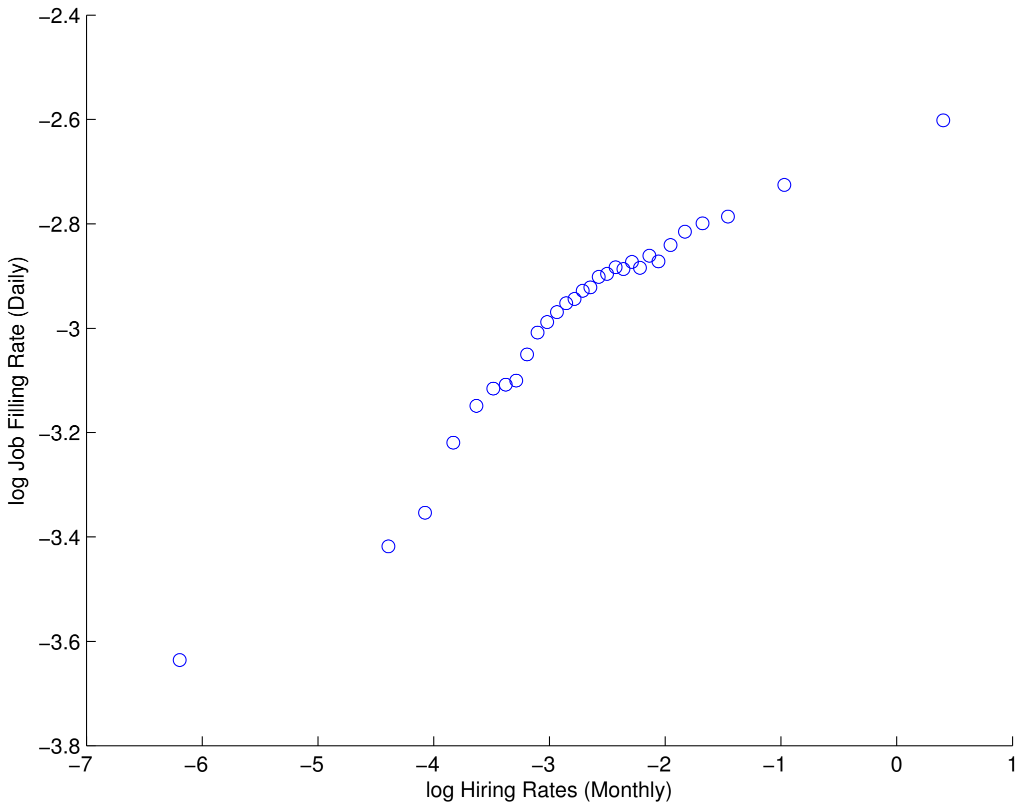

Figure 3: The relationship between log hiring rate and log daily job filling rate across the growth rate bins.

from the first quintile to the fourth quintile, the job filling rate rises from 2.5% to 5.2%, whereas the worker turnover rate becomes about 9%. Similar figures are present in the data as well. However, the job filling rate in the fifth quintile shoots up to 11.4% in the data, which is almost as twice as the corresponding number calculated from the model.

Finally, Figure 5 shows the relationship between the job filling rate and firm size. Firm size is calculated as the average of the employment at the firm at the beginning and the end of the period. Since I do not directly target the firm size distribution, the firm sizes from the model are smaller than the firm sizes observed in the data. To make the size groups comparable to the data, I construct size bins such that log difference of average size in two consecutive bins are equal. When the firms are small, the job filling rate is about 7.5%. Then, it goes down to 4.5% in intermediate size groups before reaching up to about 5.3% in the largest size group. The job filling rate in the data goes from 6.6% down to 2.6% moving from small establishments to large establishments. The job filling rate at large firms in the model stays high compared to the data. Large firms might have cost advantages in hiring,

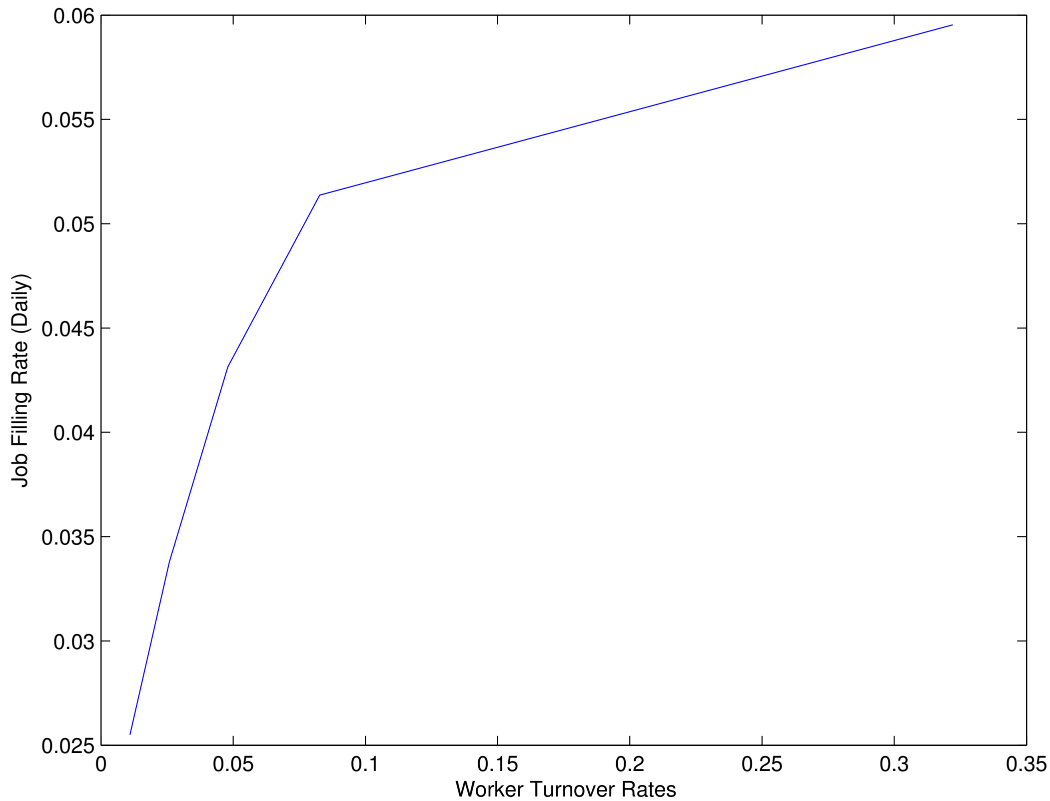

Figure 4: The cross sectional relationship between worker turnover rate and daily job filling rate.

e.g. an advanced human resources department. Introducing cost advantages to large firms could improve the model outcome.

6 Effects of a Hiring Subsidy

In this section, I examine the effects of a hiring subsidy on labor market outcomes. A hiring subsidy is a one-time payment to the firms for hiring new workers. To highlight the effect of worker selection, I also calibrate the standard model in this section. I make some modifications to the standard model to make the two models comparable, which I discuss below.

6.1 Calibrating the Standard Model

Recall that firms contact workers with probability q, but fill with probability $(1-p)q$, where p varies across firms. In the standard model, the contact probability coincides with the job

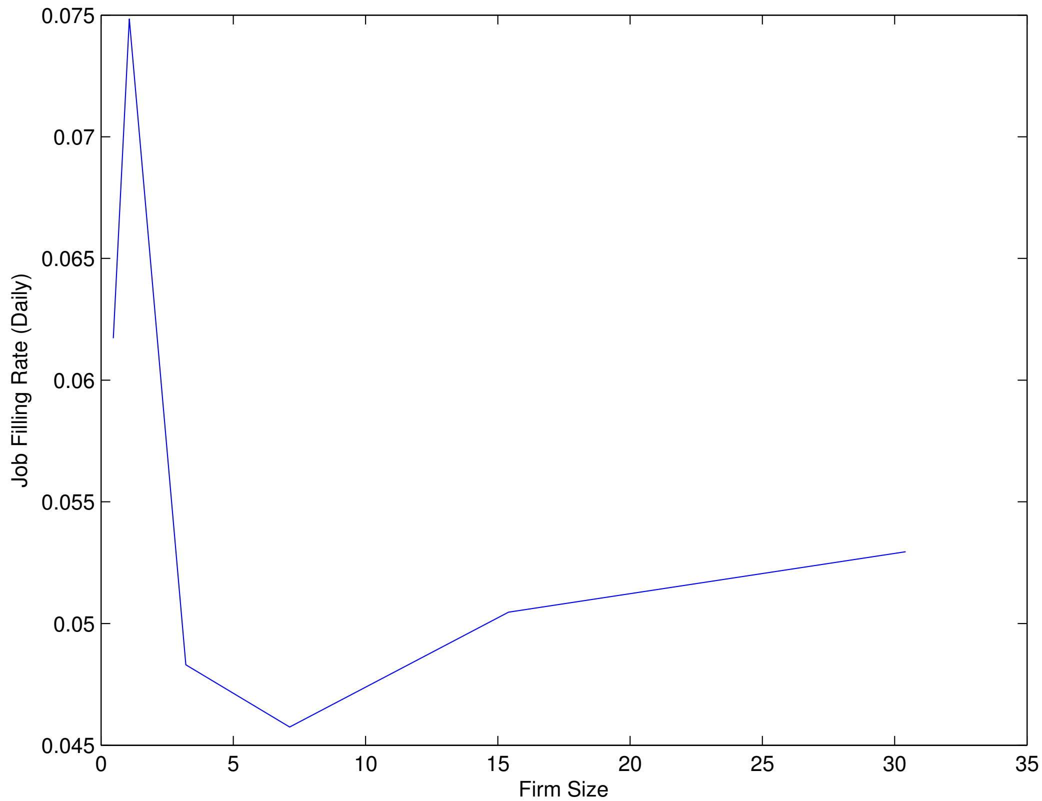

Figure 5: The cross sectional relationship between firm size and daily job filling rate.

filling probability. Accordingly, I define $\bar{p}$ and set it equal to the average value of the hiring standard obtained from the calibration of the worker selection model. This modification makes the average job filling probability the same across the two models. However, there will be no variation in hiring-vacancy ratio in the standard model.

The choice of hiring standard also affects the probability of separation in the next period through the value of γ. Let's define a new parameter p_γ in the standard model such that the law of motion becomes:

$$n' = (1 - \lambda)n + (1 - \bar{p})qv(1 - p_\gamma) \tag{25}$$

The parameter p_γ now determines a common separation probability for new workers. Recall that I targeted the job turnover rate in Section 4 to calibrate γ in the worker selection model. Similarly, I set p_γ in the standard model to match the job turnover rate in JOLTS. While I search for the value of p_γ, I also change the value of c_s, A, σ and c_e. As in Section 4, c_s targets average firm size; A targets an equilibrium value of unemployment equal to

1; σ is set to match the hiring rate; and c_e satisfies the free entry condition. I maintain the restriction that $\frac{b}{Y/N} = 0.72$. I also drop the parameter c_p from the selection cost while preserving the quadratic form in the the number of applicants. I set the value of c_v equal to the value obtained from the calibration of worker selection model. The precise treatment of this parameter does not change the conclusions in this paper. Finally, I set all the remaining parameters equal to the corresponding values presented in Table 4. The values of the newly calibrated parameters are presented in Table 6.1.

Parameter	**Meaning**	**Value**
$\bar{p}$	Hiring probability parameter	0.6519
p_γ	Success probability parameter	0.4540
σ	Dispersion of idiosyncratic shocks	0.1714
b	Value of leisure	0.9222
c_s	Selection cost	0.0481
A	Aggregate productivity	3.3094
c_e	Fixed entry cost	3155.7578

Table 2: Calibrated Parameters from the Standard Model (Weekly)

6.2 Introducing Hiring Subsidy

The introduction of the hiring subsidy affects the wage bargaining outcome as the hiring subsidy changes the outside option of the firm. Let s denote a per hire subsidy payment to the hiring firms measured in terms of the consumption good. Following the same steps in the derivation of wages detailed in Appendix A, one can show that the wage equation for new workers becomes:

$$w^p(n', \varepsilon, p) = \quad g(p)\frac{\alpha\phi}{1-\phi+\alpha\phi}A\varepsilon n'^{\alpha-1} + (1-\phi)\Omega + \phi s \tag{26}$$

where Ω is defined as in 18. The wage equation for the standard model is obtained after replacing $g(p)$ with $1 - p_\gamma$. The wage equation for existing workers stays the same though the equilibrium wages changes through changes in the value of unemployment.

Consider incremental increases in the hiring subsidy.[11] Tables 3 and 4 report equilibrium labor market outcomes, total output net of adjustment cost and total amount of subsidy

[11] I assume that government finances the hiring subsidy through a lump-sum tax on workers.

from the worker selection selection and the standard models, respectively. I also report the amount of subsidy as a fraction of the average wage of a newly hired worker for each model to compare the relative size of the subsidy between the two models.

	The Worker Selection Model					
Subsidy	0.000	0.100	0.200	0.300	0.400	0.500
Subsidy (% of average w^p)	0.00%	11.54%	21.99%	31.50%	40.22%	48.14%
Unemployment Rate	5.35%	5.25%	5.12%	4.98%	4.87%	4.79%
Market Tightness	0.460	0.469	0.481	0.498	0.526	0.620
Hiring-vacancy ratio	0.050	0.052	0.054	0.056	0.056	0.055
Output (net of adjustment costs)	1.1403	1.1405	1.1408	1.1410	1.1412	1.1410
Total subsidy	0.0000	0.0189	0.0392	0.0611	0.0860	0.1213

Table 3: The Effect of the Hiring Subsidy on Equilibrium in the Worker Selection Model.

	The Standard Model					
Subsidy	0.000	0.100	0.200	0.300	0.400	0.500
Subsidy (% of average w^p)	0.00%	11.49%	21.72%	30.90%	39.18%	46.70%
Unemployment Rate	5.36%	5.34%	5.32%	5.31%	5.29%	5.28%
Market Tightness	0.460	0.467	0.474	0.482	0.491	0.506
Hiring-vacancy ratio	0.050	0.0498	0.0496	0.0494	0.0492	0.0488
Output (net of adjustment costs)	1.1404	1.1406	1.1407	1.1408	1.1409	1.1410
Total subsidy	0.0000	0.0185	0.0374	0.0567	0.0765	0.0977

Table 4: The Effect of the Hiring Subsidy on Equilibrium in the Standard Model.

If the policymaker assessed the hiring subsidy using the standard model, he would not be optimistic about the hiring subsidy in combating unemployment rate. When firms are subsidized about the half of the average wage of newly hired worker, the decline in unemployment is only 0.08 percentage points. On the other hand, the worker selection model predicts that the same policy is a powerful tool to reduce unemployment. A hiring subsidy that is equal to the half of the average wage of a newly hired worker reduces the unemployment rate by 0.5 percentage point.[12]

The hiring-vacancy ratio responses are qualitatively different between the two models. In the standard model, the hiring-vacancy ratio decreases with the hiring subsidy. Because the aggregate level of vacancies increases with the subsidy, the probability that a firm contacts a worker goes down due to increased market tightness. If firms can select workers, however, there is an additional effect on the hiring-vacancy ratio coming from the optimal choice of the hiring standard. When firms are subsidized for hiring new workers, they become less

[12]Increasing the subsidy amount beyond that 60% worsens the situation: firms start replacing existing productive workers with new workers the unemployment rate starts rising again.

picky about the workers as they are compensated for the loss due to hiring an unproductive worker. The combined effect on the hiring vacancy-ratio is ambiguous. Table 3 shows that the effect of the latter is greater than the former when the subsidy is small. When the subsidy becomes large, the hiring-vacancy ratio starts declining.

The output net of adjustment costs initially increases with the hiring subsidy. When the subsidy exceed 0.4, output net of the adjustment costs starts declining in either model. The total amount of subsidy is, however, larger than the increases in net output even if the subsidy is small. For example, when subsidy is equal to 0.1, the total of the amount of the subsidy is 3 times larger than the increase in output.

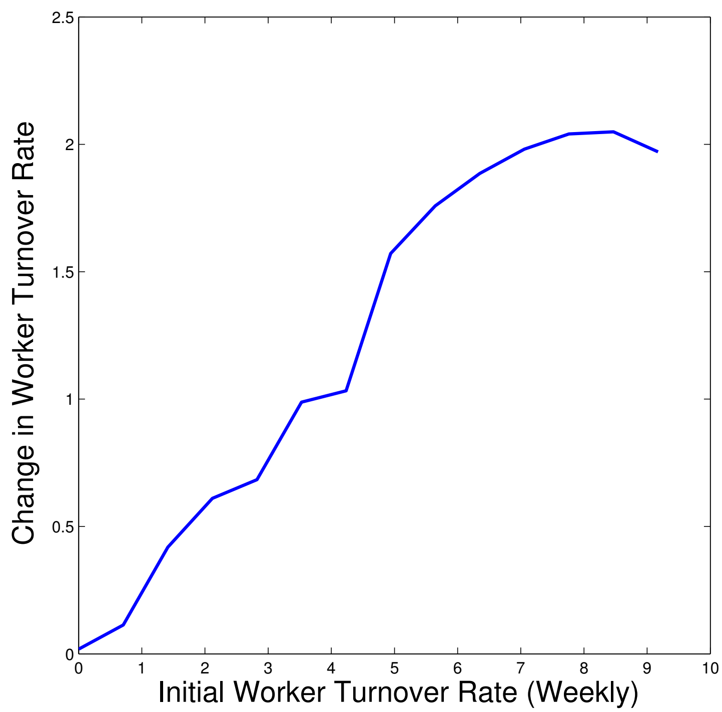

Figure 6: The change in the employment growth rate in the worker selection model after the subsidy across different firm sizes.

The effect of the hiring subsidy differs across the firms and the worker selection and the standard models produce qualitatively different results. Below I plot the changes in worker turnover rates after a hiring subsidy equal to 0.5 across different worker turnover groups. Figure 6 and 7 correspond to the worker selection and the standard models, respectively. In each figure, firms are grouped into 15 equally spaced worker turnover bins. Firms in each bin are weighted by their size and measure in the stationary distribution at the initial equilibrium. The increase in the worker turnover rate is larger at firms with initially higher worker turnover rate in the worker selection model. In contrast, the increase in the worker

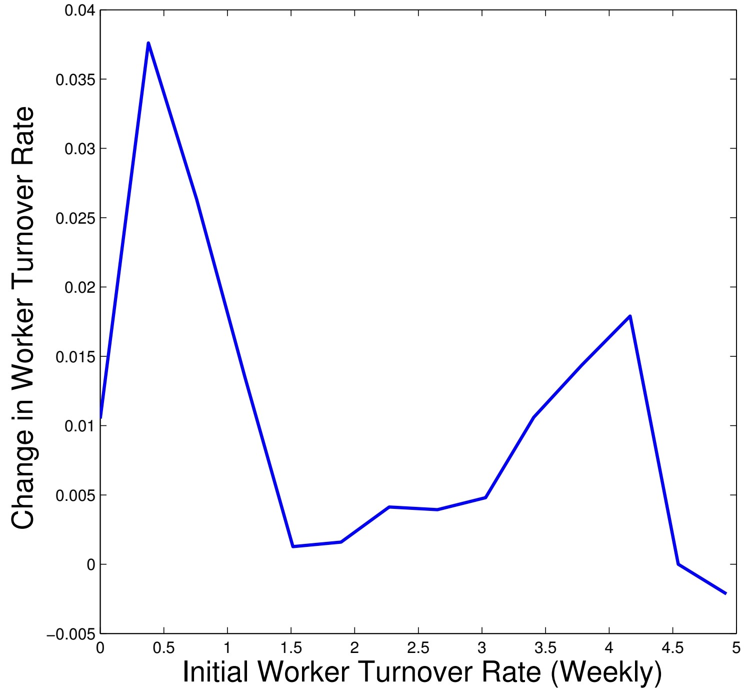

Figure 7: The change in the employment growth rate in the standard model after the subsidy across different firm sizes.

turnover rate is larger at firms with initially lower worker turnover rates. At firms with initially large worker turnover rates, the hiring subsidy actually reduces their worker turnover rate. In a cross-country comparison, Haltiwanger, Scarpetta and Schweiger (2010) show that firms in the industries and size classes that require more often employment changes are affected proportionally more from hiring and firing restrictions. This evidence supports the results obtained from the worker selection model.

7 Conclusion

The hiring-vacancy ratio in the U.S. shows variation in the cross-section: it rises steeply with employment growth rate and worker turnover rate, but declines with firm size. The standard DMP model with multi-worker firms implies no variation in the cross-section, because all the firms face the same job filling rate determined via an aggregate matching function. I extended the standard model to allow firms to selectively hire among a pool of applicants. Selection of workers is motivated by introducing unobserved match-specific quality shocks to the model, which determine the productivity of a worker at the hiring firm. Firms can fill vacancies at different rates by adjusting their hiring standards and this mechanism generates a cross sectional variation in the hiring-vacancy ratio. In the calibrated model, the worker selection accounts for about 30% of the variation in the hiring-vacancy ratio across different growth rates. The worker turnover and size relationships are also qualitatively matched. This is the first paper to account for all the three properties at the same time.

In the quantitive section, I analyzed the effects of a hiring subsidy on labor market outcomes. The results from the worker selection and the standard models are both qualitatively and quantitively different. The standard model predicts that a hiring subsidy would reduce unemployment only slightly. The decline in the worker selection model is substantial: the unemployment rate would go down by half of a percentage point if firms are subsidized for half of the wages of a newly hired worker. The analysis also gives new insights about how different firm groups are affected by the subsidy. The standard model implies that worker turnover rate increases more in more stable firms. In the worker selection model, firms with initially larger worker turnover rate also experience a larger increase in the worker turnover rate after the hiring subsidy. Empirical evidence support the predictions of the worker selection model.

The hiring subsidy also increases the vacancy yield in the worker selection model. This suggests that the hiring subsidy could help speed up the transition to the new equilibrium. Analysis of the transition to the new steady state is an interesting avenue for future research.

8 Appendix

A Derivation of Wage Functions

The derivation of the wage functions exploits the fact that the continuation values for the firm and the workers cancel each other from the first order condition for n' and the envelope condition. Let $J(n', \varepsilon) = E_{\varepsilon'|\varepsilon}\Pi_{n'}(n', \varepsilon')$. From (13),the marginal value of an existing and potential worker are:

$$D_{\tilde{n}}(.) = \quad \alpha A\varepsilon {n'}^{\alpha-1} - (w^n)'(.)\tilde{n} - w^n(.) - (w^p)'(.)r + J(.) \tag{27}$$

$$D_r(.) = \quad g(p)\alpha A\varepsilon {n'}^{\alpha-1} - g(p)(w^n)'(.)\tilde{n} - g(p)(w^p)'(.)r - w^p(.) + g(p)J(.) \tag{28}$$

where $(w^n)'(.)$ and $(w^p)'(.)$ are the derivatives of the wage functions with respect to n'. Rearranging the bargaining solution in (14), I get:

$$\begin{aligned}(1-\phi)(w^n(.) - \Omega + \beta(1-\delta)(1-\lambda)[E_{\varepsilon'|\varepsilon}V^n(n', \varepsilon') - V^u])\\ = \phi(\alpha A\varepsilon {n'}^{\alpha-1} - (w^n)'(.)\tilde{n} - w^n(.) - (w^p)'(.)r + J(.))\end{aligned} \tag{29}$$

where Ω is defined in (18). First, I show that $J(n', \varepsilon) = \beta(1-\delta)(1-\lambda)[E_{\varepsilon'|\varepsilon}V^n(n', \varepsilon') - V^u])$. To see that, re-write the dynamic problem of a hiring firm before inserting the wage functions and after replacing the constraint in 4:

$$\begin{aligned}\Pi^h(n, \varepsilon) = \max\nolimits_{n',p} \quad & -\frac{c_v}{q}\frac{\gamma(n' - (1-\lambda)n)}{1-p^\gamma} - \frac{c_s}{z}\exp\{p^2\}\left(\frac{\gamma(n' - (1-\lambda)n)}{1-p^\gamma}\right)^z\\ & + A\varepsilon {n'}^{\alpha} - (1-\lambda)nw^n(n', \varepsilon) - \frac{(1-p)\gamma(n' - (1-\lambda)n)}{1-p^\gamma}w^p(n', \varepsilon)\\ & + \beta(1-\delta)E_{\varepsilon'|\varepsilon}\Pi(n', \varepsilon')\end{aligned} \tag{30}$$

The first order condition for n' is:

$$\begin{aligned}& -\frac{c_v}{q}\frac{\gamma}{1-p^\gamma} - c_s\exp\{p^2\}\left(\frac{\gamma}{1-p^\gamma}\right)^z (n' - (1-\lambda)n)^{z-1}\\ & + \alpha A\varepsilon {n'}^{\alpha-1} - (1-\lambda)n(w^n)' - \frac{(1-p)\gamma}{1-p^\gamma}w^p(.) - \frac{(1-p)\gamma(n' - (1-\lambda)n)}{1-p^\gamma}(w^p)'(.)\\ & + \beta(1-\delta)E_{\varepsilon'|\varepsilon}\Pi_{n'}(n', \varepsilon') \leq 0\end{aligned} \tag{31}$$

Next, conditional on hiring, the envelope condition implies:

$$\begin{aligned}\Pi_{n'}(n',\varepsilon) \;&= (1-\lambda)\left(\frac{c_v}{q}\frac{\gamma}{1-p^\gamma} + c_s\exp\{p^2\}\left(\frac{\gamma}{1-p^\gamma}\right)^z (n'-(1-\lambda)n)^{z-1}\right) \\ &+(1-\lambda)\left(-w_n(.) + \frac{(1-p)\gamma}{1-p^\gamma}w^p(.)\right)\end{aligned} \tag{32}$$

Replacing the first line in (32) from (31) and substituting for $\tilde{n}$ and r, one obtains:

$$\Pi_{n'}(n',\varepsilon) \;= (1-\lambda)D_{\tilde{n}}(.) \tag{33}$$

If the firm is neither hiring nor firing, (33) holds by definition. Finally, if the firm is firing workers, then the marginal surplus of an existing worker is equal to 0. Equivalently, $\Pi_{n'}(n',\varepsilon) = 0$. Hence, (33) is trivially satisfied. From the definition of $J(.)$ and using (33), one gets $J(n) = \beta(1-\delta)(1-\lambda)D_{\tilde{n}}(.)$, which further implies $J(n',\varepsilon) = \beta(1-\delta)(1-\lambda)[E_{\varepsilon'|\varepsilon}V^n(n',\varepsilon') - V^u])$ by (14).

The bargaining equations now can be written as follows:

$$\phi\left(\alpha A\varepsilon {n'}^{\alpha-1} - (w^n)'(.)\tilde{n} - (w^p)'(.)r\right) \;= (1-\phi)(w^n(.) - \Omega) \tag{34}$$

$$g(p)\phi\left(\alpha A\varepsilon {n'}^{\alpha-1} - (w^n)'(.)\tilde{n} - (w^p)'(.)r\right) \;= (1-\phi)(w^n(.) - \Omega) \tag{35}$$

Multiplying (34) by $g(p)$ and subtracting from (35) implies:

$$(w^p)(.) = g(p)(w^n)(.) + (1-g(p))\Omega \tag{36}$$

After taking the derivative with respective to n' and plugging this back in (27), I obtain the following first order differential equation in n':

$$w^n(.) + \phi n'(w^n)'(.) = \phi\alpha A\varepsilon {n'}^{\alpha-1} + (1-\phi)\Omega \tag{37}$$

The solution to this differential equation is given by (26). The constant of integration is set to zero so that $n'w(.) \to 0$ as $n' \to 0$. The wage equation for newly hired workers can be obtained from (36).

B Recursive Stationary Equilibrium

Definition 1. (Recursive Stationary Equilibrium) The recursive stationary equilibrium consists of value function for firms, $\Pi(n,\varepsilon)$; a set of decision rules for vacancies, hiring standard, firings and employment, $v(n,\varepsilon)$, $p(n,\varepsilon)$, $f(n,\varepsilon)$ and $n'(n,\varepsilon)$;value functions for workers, $V^n(n,\varepsilon)$ and $V^p(n,\varepsilon)$; wage functions, $w^n(n',\varepsilon)$ and $w^p(n',\varepsilon,p)$; market tightness and aggregate matching probability, θ and q; value of unemployment at the beginning and bargaining stages, $\tilde{V}^u$ and V^u; and a stationary distribution firms across productivities and employment, $G(n,\varepsilon)$, such that:

1. θ and q are related according to (2).
2. Firm's Optimization: Given q, $w^n(n',\varepsilon)$ and $w^p(n',\varepsilon,p)$, the set of decision rules, $v(n,\varepsilon)$, $p(n,\varepsilon)$, $f(n,\varepsilon)$ and $n'(n,\varepsilon)$, solve firms' problem described by equations (4), (5), (6) and (7).
3. Worker Value Functions: Given θq, $w^n(n',\varepsilon)$, $w^p(n',\varepsilon,p)$ and firms' decision rules, $v(n,\varepsilon)$, $p(n,\varepsilon)$, and $n'(n,\varepsilon)$, value functions for workers, $V^n(n,\varepsilon)$, $V^p(n,\varepsilon)$, $\tilde{V}^u$ and V^u, satisfy equations (8), (9), (10) and (11).
4. Wage Bargaining: The wage equations, $w^n(n',\varepsilon)$ and $w^p(n',\varepsilon,p)$, satisfy (13), (14) and (15).
5. Free-entry condition in (19) holds.
6. Consistency: The stationary distribution $G(n,\varepsilon)$ is consistent with the firm's decision rules and satisfies (12).

C Properties of the Adjustment Cost Function

As in the text, let $\Delta = \gamma(n' - (1-\lambda)n)$. We are seeking the optimal choice for p given Δ to minimize total cost of adjustment. For brevity, let $f(p) = \frac{1}{1-p^\gamma}$ and $h(p) = c_p \exp\{p^2\}\{f(p)\}^z$. Note that the natural logarithm of $h(p)$, $\{f(p)\}^z$ and $c_p \exp\{(p^2)\}$ are all convex. I use this observation to show the convexity of $C(\Delta)$. The minimization problem of the firm is:

$$C(\Delta) = \min_p \Delta\left((\Omega + c_v/q)f(p) - \Omega f(p)p + h(p)\Delta^{z-1}\right) \tag{38}$$

Let $\Upsilon(p,\Delta)$ denote the objective function of the problem above. I have already established in the text that the solution is interior and unique. Then, the first order condition (FOC) and the second order condition (SOC) are:

$$\begin{aligned} \Upsilon_p(p,\Delta) &= 0 \\ \Upsilon_{pp}(p,\Delta) &> 0 \end{aligned}$$

Totally differentiating the FOC, one obtains:

$$p'(\Delta) = \frac{dp}{d\Delta} = -\frac{\Upsilon_{p\Delta}}{\Upsilon_{pp}} = -\frac{(z-1)h'(p)\Delta^{z-1}}{\Upsilon_{pp}} \leq 0$$

The last inequality follows from the SOC. Further, the cost function satisfies:

$$C(\Delta) = \Upsilon(p(\Delta),\Delta)$$

Taking the derivative with respect to Δ:

$$C'(\Delta) = \Upsilon_p p'(\Delta) + \Upsilon_\Delta$$

By the FOC, the first term is zero. Hence:

$$C'(\Delta) = (\Omega(1-p) + c_v/q)f(p) + zh(p)\Delta^{z-1} > 0$$

Finally, differentiating the expression for $C'(\Delta)$ yields:

$$C''(\Delta) = \Upsilon_p p'(\Delta) + \Upsilon_{p\Delta p'(\Delta)} + \Upsilon_{\Delta\Delta}$$

By the FOC, the first term is zero. Rearranging the terms and replacing for $p'(\Delta)$ yields:

$$C''(\Delta) = \Upsilon_{\Delta\Delta} - \frac{\Upsilon^2_{p\Delta}}{\Upsilon_{pp}}$$

By the SOC, the adjustment cost function is convex iff $\Upsilon_{\Delta\Delta}\Upsilon pp - \Upsilon^2_{p\Delta} \geq 0$. This requires:

$$\begin{gathered} \Delta(\Omega(1-p) + c_v/q)f''(p) - 2\Omega f'(p)))z(z-1)h(p)\Delta^{z-1} \\ +\Delta z(z-1)(\Delta)^{2z-2}h''(p)h(p) - (z-1)^2\Delta^{2z-1}(h'(p))^2 \geq 0 \end{gathered}$$

The first term is positive from the definition of $f(p)$. A sufficient condition for the second term to be positive is that $h(p)h''(p) - (h'(p))^2 \geq 0$. This condition is also satisfied by log-convexity of $h(p)$.

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