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# Assessment and student motivations through the lens of agency theory

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**Abstract:** Assessment is used not only to evaluate students' learning but also to motivate them to learn. Common types of assessment include individual and group assessments. Using the principal-agent model, this study examines how each assessment type affects students' incentives to learn. Group assessment imposes more uncertainties on students than individual assessment, but it also encourages students to cooperate and collaborate. Therefore, instructors should consider these effects of group assessment when designing optimal assessments. This study also examines the optimal weights for each assessment component and the directional effects of changes in these weights on student motivation. The results can inform instructors on the design of teaching pedagogy and educational administrators on the design of educational assessment policies.

**Keywords**: assessment, group assessment, individual assessment, motivation, agency theory

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# 1 Introduction

Studies have shown that assessment is important for evaluating students' learning and for motivating them to learn (Boud, Cohen, and Sampson 1999; Fernandes, Flores, and Lima 2012; Gardner 2012; Webber 2012). Traditionally, assessments have mainly been conducted at the individual level, as the measurement of individual ability has typically been emphasized in higher education (Boud, Cohen, and Sampson 1999; Sharp 2006; Meijer et al. 2020). However, in past decades, group assessments have been increasingly incorporated into universities' curricula, as group work has been shown to improve various social skills required by employers (Norman, Rose, and Lehmann 2004; Jackling and De Lange 2009; Bailey, Barber, and Ferguson 2015). Given the prevalence of both individual and group assessments, this study provides a better understanding of how each assessment type affects students' incentives to study and of how the component weights of different assessment types affect these incentives.

Through the lens of agency theory, this study examines the effect of individual and group assessments on students' motivation to learn in the presence of cooperative opportunities for students with heterogeneous abilities. It shows that group assessments can encourage cooperation and collaboration between students so that students help each other learn. This finding is consistent with the notion of peer learning (Topping 2005; Boud and Cohen 2014). However, group assessments also impose uncertainties on students and thus disincentivize them from exerting study effort. According to the principal-agent theory, it is easier to motivate an agent to exert effort if the agent has greater control over the output (Holmstrom and Milgrom 1991; Feltham and Xie 1994). In the context of educational assessment, individual assessments can better motivate students to study because the results of individual assessments largely depend on the effort of each student. In contrast, group assessment results depend on the joint efforts of all group members. As such, each member has less control over the group's overall performance and may have less incentive to exert effort, to begin with.

Building on these findings, this study shows that the optimal component weights given to group and individual assessments depend on the strength of these factors. When it is challenging to encourage weak students to study, it is optimal to use a combination of group and individual assessments. In contrast, when encouraging weak students to study is not a major concern, assigning little to no weight to group assessments is optimal. This study also shows that the relationship between the weights applied to each assessment component and students' incentives to exert study effort is not monotonic. When the weight applied to the group assessment component is low or high, increasing this weight imposes more uncertainties on students and thus makes it more difficult to incentivize them to exert effort. However, when the weight applied to the group assessment component is moderate, increasing this weight incentivizes students to cooperate, which makes it easier to incentivize them to learn.

Agency theory, or principal-agent theory, has been applied in various fields (Eisenhardt 1989). Agency theory considers the interactions between principals and agents, information asymmetry, and conflicts of interests (Noreen 1988; Douglas 1989). In educational research, the application of agency theory has been limited, and most studies using agency theory have focused on policies or institutional management (Yallew et al. 2018). This study uses agency theory to model the relationship between instructors (principals) and students (agents). The model assumes that students are effort-averse, and therefore, instructors have to use assessments to motivate students to exert study effort. It is important to note that this study makes no attempt to model all aspects of students' behavior, nor does it argue that assessment is the only tool that instructors can use to motivate students to study. Instead, this study focuses on the direct effects of different assessment methods on students' economic incentives to exert study effort. In practice, instructors should also consider the effects of each assessment type on other dimensions of student learning to determine their optimal assessment system.

Studies have shown that each assessment type has its own advantages and disadvantages. In terms of construct validity, which refers to the extent to which a measure evaluates what it is intended to, individual assessments have been shown to be better than group assessments (Nordberg 2008; Meijer et al. 2020). The results of individual assessments are more likely to be valid because they reflect individual students' academic performance. In contrast, group assessments may not have strong construct validity because of the difficulty of measuring the learning or contribution of each individual member of a group. In this study, I also argue that students consider individual assessment to be less risky because it gives them more control over the assessment results. Thus, individual assessment can better motivate students to exert study effort.

Although group assessments may not provide good construct validity, they have been widely used in higher education, as they have numerous benefits.[1] For example, group work helps students improve their social skills by requiring them to work with other students with different characteristics and opinions (Healy, Doran, and McCutcheon 2018; Opdecam and Everaert 2018). In addition, group learning enhances

[1] This study does not distinguish between different group learning techniques as long as group assessments are used. Examples of these group learning techniques include cooperative learning, collaborative learning, and team-based learning.

students' active participation in the learning process (Peek, Winking, and Peek 1995; Clerici-Arias 2021; Ruder, Maier, and Simkins 2021), increases students' interest in learning (Caldwell, Weishar, and William 1996; Espey 2018), improves students' exam performance (Caldwell, Weishar, and William 1996; Hwang, Lui, and Tong 2005), improves students' teamwork abilities (Christensen et al. 2019), and creates a positive learning experience (Cadiz Dyball et al. 2007; Simkins, Maier, and Ruder 2021). This study contributes to the growing literature on group work and group assessments by suggesting an additional benefit of group assessments, an economic incentive, as group assessments can incentivize students to exert effort. Accordingly, it may be optimal to incorporate group assessments into the overall course assessment framework even without considering the benefits of improving students' soft skills.

This study contributes to the literature in several ways. First, the results can inform instructors in the design of teaching pedagogy. Although it is well documented that group work is beneficial to students, instructors should not put too much component weight on group assessments, as they may discourage students from exerting study effort. This study can also inform educational administrators in the design of educational assessment policies. Specifically, the results suggest that educational administrators should not impose the same requirements on assessment component weights across all courses, as some courses may benefit more from group assessments than others.

The remainder of the paper is organized as follows. Section 2 describes the model and assumptions used in the analysis. I analyze benchmark cases in which an instructor uses only individual assessment and only group assessment in Section 3 and Section **??**, respectively. In Section 4, I consider the case in which an instructor uses both individual and group assessments and show how the component weights affect students' incentives to exert effort. Section 5 describes the optimal assessment environment. Section 6 shows numerical examples and graphical illustration of the main results. Section 7 concludes the paper. All proofs are shown in the Appendix.

## 2 Model and Assumptions

To analyze the relationship between assessment and students' motivation, I consider a model with an instructor and two risk-neutral students—a strong student (she) and a weak student (he).[2] The objective of the instructor is to ensure that the students master the material, which can occur only when they exert high study effort. As the instructor cannot directly observe the students' effort level, the instructor has to use assessment to incentivize them to exert high study effort. There are two types of assessments available—individual assessments (e.g., an individual quiz, exam, etc.) and group assessments (e.g., a group project, group exam, etc.). If the instructor uses both types of assessments, the component weight of each assessment component must also be specified (e.g., 70% individual assessment and 30% group assessment).

After observing the assessment types and the component weight, the students will exert their study effort to improve their learning and their likelihood of scoring in the assessment. The strong student exerts effort $a^s \in \{a_l^s, a_h^s, a_c^s\}$ where $a_l^s$ refers to low effort, $a_h^s$ refers to high effort, and $a_c^s$ refers to high effort with cooperation. By exerting each type of effort, the strong student incurs a personal cost of $K(a_l^s)$, $K(a_h^s)$, and $K(a_c^s)$, respectively.[3] The effort choices of the weak student depend on

[2]This study focuses on students with various abilities because previous studies have shown that heterogeneity of the skill level of team members is essential in cooperative learning (Ravenscroft et al. 1995).

[3]For notational convenience, I use the subscript to denote the effort level (low, high, and high with cooperation) and the superscript to denote the type of student (weak and strong).

the effort chosen by the strong student. If the strong student exerts effort $a_l^s$ or $a_h^s$ (i.e., she does not cooperate), the weak student can exert effort $a^w \in \{a_l^w, a_h^w\}$. However, if the strong student cooperates, $a^s = a_c^s$, the weak student can exert effort $a^w \in \{a_l^w, a_c^w\}$. Similar to the effort of the strong student, $a_l^w$ refers to low effort, $a_h^w$ refers to high effort, and $a_c^w$ refers to high effort when the strong student cooperates. The cost of each effort choice is denoted as $K(a_l^w)$, $K(a_h^w)$, and $K(a_c^w)$, respectively. Each student's effort choice affects the likelihood that the student will score in the assessment.[4] In the individual assessment, by exerting effort $a$, the student will score with probability $a$ and will not score with probability $(1-a)$. In the group assessment, however, the likelihood of scoring depends on the joint effort of both students. Specifically, in the group assessment, the group will score with probability $a^s a^w$ and will not score with probability $(1 - a^s a^w)$.

I make the following assumptions:

$$\text{Assumption 1} : \quad K(a_l^s) = K(a_l^w) = 0$$

$$\text{Assumption 2} : \quad K(a_h^w) > K(a_c^w) > K(a_c^s) > K(a_h^s) > 0$$

$$\text{Assumption 3} : \quad a_l^s = a_l^w = 0$$

$$\text{Assumption 4} : \quad 1 \geq a_c^s = a_h^s > a_c^w = a_h^w > 0.$$

Assumption 1 states that the students do not incur any personal costs if they exert low effort.

Assumption 2 states that the cost of exerting high effort is higher than the cost

[4] The effort in this model refers to the effort to study the course material in general, not the effort towards each specific assessment. As a consequence, if the student exerts high effort, the likelihood of scoring in both the team assessment component and the individual assessment component will increase.

of exerting low effort, and this cost is higher for the weak student than for the strong student. In addition, if the strong student chooses to cooperate, she will have to incur a higher cost, $K\left(a_c^s\right) > K\left(a_h^s\right)$, while the weak student will benefit by incurring a lower cost of effort, $K\left(a_h^w\right) > K\left(a_c^w\right)$. The cost of effort can be interpreted as the time required by students to study to understand the materials. Therefore, this assumption reflects the notion that, without cooperation, the weak student will have to spend more time studying than the strong student to understand the same materials. Also, relative to studying by her/himself, cooperation requires the strong student to spend more time while the weak student can spend less time understanding the same materials.

Assumption 3 states that the students have no chance of scoring if they exert low effort.

Assumption 4 states that, for both students, the likelihood of scoring in the individual assessment is positive if and only if they exert high effort, and this likelihood is higher for the strong student than for the weak student.

Finally, I assume that both students have a strictly positive utility of $x$ if they score in either the individual assessment or the group assessment. The parameter $x$ can be interpreted as the grades obtained by the students, the students' satisfaction, or the students' learning. Throughout the analysis, I focus on deriving the minimum value of $x$ required to induce both students to exert high effort. This minimum value of $x$ can be interpreted as the level of difficulty to motivate the students to exert high effort. The higher the minimum value of $x$ required to induce high effort, the more difficult it is to induce the students to exert high effort.

## 3 Individual vs. Group Assessment

In this section, I show how each assessment type affects the students' motivation to study. If the instructor uses only individual assessment, the students will never have an incentive to cooperate because each student's grade is determined by one's own effort. In this case, the minimum value of $x$ that can induce both students to exert high effort is

$$x \geq \frac{K\left(a_h^w\right)}{a_h^w}. \tag{1}$$

Condition (1) ensures that the weak student has an incentive to exert high effort. When this condition holds, the strong student will also have an incentive to exert high effort. Intuitively, because the weak student has to incur a higher cost to exert high effort while having a lower probability of scoring on the assessment, relative to the strong student, it is more difficult to motivate the weak student to exert high effort than to motivate the strong student to do so. Put differently, if the instructor can successfully incentivize the weak student to exert high effort, the strong student will also exert high effort. The following proposition summarizes this result.

**Proposition 1** *When the assessment consists of individual assessment only, there exists a threshold* $x_1^* \equiv \frac{K\left(a_h^w\right)}{a_h^w}$ *such that when* $x \geq x_1^*$*, both the strong and the weak students will exert high effort.*

Let us now consider group assessment. Unlike the individual assessment, the group assessment can encourage the students to cooperate and collaborate. Specifically, the strong student will have an incentive to help the weak student learn because she is aware that she has a chance of scoring only when her weaker peer also exerts high effort. If she chooses to study on her own, although she can save time, it is likely that her weaker peer will not exert high effort and she will have no chance of scoring in the group assessment. This argument points out a benefit of the group assessment as it incentivizes stronger students to collaborate and cooperate with weaker students to help them learn.

Although group assessment encourages collaboration, it can discourage students from exerting high effort as it imposes additional uncertainties on them. Agency theory suggests that agents are more willing to exert high effort if they have greater control over their results. In group assessment, the assessment result not only depends on one's own effort but also on the effort of one's group members. As a consequence, students can be less willing to exert high effort in a group assessment than in an individual assessment due to additional uncertainties imposed on them. The following proposition shows the minimum value of $x$ required to induce both students to exert high effort when the assessment consists of group assessment only.

**Proposition 2** *When the assessment consists of the group assessment only, there exists a threshold* $x_2^* \equiv \frac{K(a_c^w)}{a_c^s a_c^w}$ *such that when* $x \geq x_2^*$*, both the strong and the weak students will exert high effort.*

The results in Proposition 1 and Proposition 2 show a trade-off of using group assessment. On the one hand, it is more difficult to induce the students to exert high effort in the group assessment because it imposes unnecessary uncertainties on them. On the other hand, group assessment provides incentives for the students to cooperate, which can help motivate weaker students to exert high effort.

# 4 The Effect of Component Weights on Students' Motivation

In Section 3, I assume that the instructor uses only one type of assessment, either individual or group assessment, to motivate the students to study. In practice, instructors can employ multiple assessment tools and assign weights to different assessment components. As an example, instructors can use both group projects and individual tests and assign a weight to each component (e.g. 30% for the group projects and 70% for the individual tests). In this section, I analyze how the component weight of each assessment component affects the students' incentives to exert effort. I assume that the instructor assigns the weight of $\lambda \in [0,1]$ to the group assessment component and the weight of $(1-\lambda)$ to the individual assessment component. The following proposition shows the minimum value of $x$ required to induce both students to exert high effort when the instructor assigns the weight $\lambda$ and $(1-\lambda)$ to group and individual assessments, respectively.

**Proposition 3** *For a given level of $\lambda$, when the assessment consists of both group assessment and individual assessment, there exists a threshold $x_3^*$,*

$$x_3^* \equiv \min\left\{\frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w + (1-\lambda)\, a_h^w}, \max\left\{\frac{K\left(a_c^w\right)}{\lambda a_c^s a_c^w + (1-\lambda)\, a_c^w}, \frac{K\left(a_c^s\right) - K\left(a_h^s\right)}{\lambda a_c^s a_h^w}\right\}\right\}$$

*such that if $x \geq x_3^*$, both the strong and the weak student will exert high effort.*

Proposition 3 shows the minimum value of $x$ required for both students to exert high effort. The result suggests that both students may or may not cooperate in equilibrium. Specifically, when $\lambda$ is high, both students will cooperate to exert high effort. Intuitively, when the percentage of the group assessment component ($\lambda$) is very high, it is easy to encourage the strong student to cooperate with the weak student as a high percentage of score depends on the joint effort of both students. As a result, students will choose to cooperate. In contrast, when the weight applied to the group assessment ($\lambda$) is low, both students will exert high effort but they will not cooperate. This result follows because the strong student will have low incentive to cooperate when the individual assessment accounts for a high percentage of score, as cooperation has no impact on individual assessment.

I next analyze how a change in the weight applied to each assessment component affects the difficulty of inducing both students to exert high effort, $x_3^*$. Understanding this relationship is important to instructors as it can help them adjust this weight to increase student motivation. In addition, this relationship provides a testable empirical prediction that can be validated in future studies. Interestingly, I show that the relationship between the weight applied to each assessment component and the difficulty in motivating both students to exert high effort, $x_3^*$, is not monotonic.

When the weight applied to the group assessment is low ($\lambda$ is low), increasing the weight applied to the group assessment makes it more difficult to induce students to exert high effort, $\frac{dx_3^*}{d\lambda} > 0$. Intuitively, when little weight is applied to the group assessment component, students will not have incentives to collaborate and cooperate. By marginally increasing this weight, students will still not cooperate but the level of uncertainty imposed on the students has increased. As a consequence, applying more weight to the group assessment component will impose unnecessary uncertainties on the students and discourage them from exerting high effort.

When the weight applied to the group assessment is moderate ($\lambda$ is moderate), increasing this weight makes it easier to induce students to exert high effort, $\frac{dx_3^*}{d\lambda} < 0$.

In this case, the strong student needs to be incentivized to cooperate, rather than to exert high effort without cooperation. By increasing the group assessment percentage, the result from the group assessment becomes more important to the students. As a consequence, the strong student will have a greater incentive to cooperate because she will rationally anticipate that if she does not cooperate, the weak student may shirk, resulting in a poor score on the group assessment. In other words, the instructor can increase the group assessment percentage to motivate the strong student to cooperate with the weak student so that both of them exert high effort.

When the level of $\lambda$ is high, increasing this weight makes it more difficult to induce students to exert high effort, $\frac{dx_3^*}{d\lambda} > 0$. When the group assessment weight is high, students will already have incentives to cooperate because the score from the group assessment is very important. Therefore, an increase in the weight applied to the group assessment component will not provide any additional benefits (because students are already willing to cooperate) but it will increase the uncertainty imposed on the students, similar to the case of the low weight. As such, it becomes harder to incentivize the students to exert high effort.

The following proposition summarizes the non-monotonic effect of $\lambda$ on $x_3^*$.

**Proposition 4** *There exist thresholds $\lambda_1$ and $\lambda_2$ such that*

*(i) when $\lambda < \lambda_1$, an increase in $\lambda$ makes it more difficult to encourage high effort from the students, $\frac{dx_3^*}{d\lambda} > 0$,*

*(ii) when $\lambda \in [\lambda_1, \lambda_2]$, an increase in $\lambda$ makes it easier to encourage high effort from the students, $\frac{dx_3^*}{d\lambda} < 0$, and*

*(iii) when $\lambda > \lambda_2$, an increase in $\lambda$ makes it more difficult to encourage high effort from the students, $\frac{dx_3^*}{d\lambda} > 0$.*

$\lambda_1$ and $\lambda_2$ are defined in the Appendix. Proposition 4 shows that the effect of the weight applied to the group assessment component on the difficulty of inducing high effort from the students is not monotonic. The graphical illustration of this non-monotonic relationship is shown in Figure 1 and Figure 2 in Section 6. A related study to this result is Bacon, Stewart, and Silver (1999) who showed that the percentage associated with the group assessment does not affect group project experiences among MBA students. This result is not inconsistent with the non-monotonic relationship suggested in this study. Instead, this study provides an alternative explanation to the result shown in Bacon, Stewart, and Silver (1999) that the percentage associated with group assessment can affect group project experiences but this result is not monotonic. Therefore, to examine this relationship with data, it is important to consider it at various levels of the group assessment percentage.

# 5 Optimal Assessment Weights

Having analyzed different assessment environments in Sections 3 and 4, in this section, I show the optimal weight applied to each assessment component.

I first show that it is not optimal to use group assessment as the only assessment component, $\lambda = 1$, because this imposes too much uncertainty on the students. Therefore, the instructor will always find it easier to induce both students to exert high effort by reducing the weight applied to the group assessment component. This result is consistent with the practice observed in almost all courses across various disciplines that typically include a large percentage of individual assessments.

When it is relatively easy to motivate the weak student to exert high effort without cooperation ($K(a_h^w)$ is low), it is optimal to use the individual assessment as the only assessment component, $\lambda = 0$. In this case, the students will study independently without cooperation. This result is consistent with the result shown in Section 3 that when the instructor uses only the individual assessment, the students will not have an incentive to cooperate. Intuitively, if it is relatively easy to motivate the weak student to exert high effort without cooperation, using individual assessment as the only assessment type is sufficient to motivate both students to exert effort and this assessment environment does not impose too much uncertainty on the students.

In contrast, when it is relatively difficult to motivate the weak student to exert high effort without cooperation ($K(a_h^w)$ is high), it is optimal to use a combination of both the group and individual assessments. The optimal weight applied to the group assessment component is $\lambda_2$ and the optimal weight applied to the individual assessment component is $(1 - \lambda_2)$, where $\lambda_2$ is defined in the Appendix. In this assessment environment, both students will cooperate to exert high effort.

The following proposition shows the optimal assessment weights.

**Proposition 5** *There exists a threshold* $\widehat{K(a_h^w)}$ *such that*

*(i) if* $K(a_h^w) < \widehat{K(a_h^w)}$*, it is optimal to use the individual assessment only,* $\lambda = 0$*, and*

*(ii) if* $K(a_h^w) \geq \widehat{K(a_h^w)}$*, it is optimal to use both the individual assessment and the group assessment with the weight* $\lambda_2$ *applied to the group assessment component and the weight* $(1 - \lambda_2)$ *applied to the individual assessment component.*

The threshold $\widehat{K(a_h^w)}$ is defined in the Appendix. Proposition 5 shows the optimal assessment component weights. When it is relatively easy to motivate the weak student to exert high effort without cooperation, using individual assessment only is optimal because it does not impose unnecessary uncertainties on the students.

However, when it is difficult to do so, it is optimal to use both individual assessment and group assessment instead to motivate both students to cooperate.

# 6 Numerical Examples

To better understand the results shown in the previous sections, I illustrate the main results using numerical examples and graphs. I use the following set of parameters:

| $a_c^s = a_h^s$ | $K(a_c^s)$ | $K(a_h^s)$ | $a_c^w = a_h^w$ | $K(a_c^w)$ | $K(a_h^w)$ |
|---|---|---|---|---|---|
| 0.8 | 3 | 1 | 0.3 | 4 | 6 |

This set of parameters is consistent with the assumptions outlined in Section 2. With these parameters, the values of $\lambda_1$, $\lambda_2$, and $\widehat{K(a_h^w)}$ can be calculated as follows.

| $\lambda_1$ | $\lambda_2$ | $\widehat{K(a_h^w)}$ |
|---|---|---|
| 0.3846 | 0.5556 | 4.5 |

Using these results, I show the relationship between the weight applied to group assessment ($\lambda$) and the minimum value of $x$ required to induce students to exert high effort ($x_3^*$) in Figure 1.

In Figure 1, the X-axis represents the weight applied to group assessment ($\lambda$) while the Y-axis represents the difficulty in inducing students to exert high effort ($x_3^*$). This figure illustrates the result shown in Proposition 4. Specifically, when $\lambda$ is lower than $\lambda_1$ or greater than $\lambda_2$, an increase in the percentage applied to the group assessment makes it more difficult to motivate students to exert effort. However, when $\lambda$ is between $\lambda_1$ and $\lambda_2$, increasing the group assessment percentage makes it easier to induce effort from students.

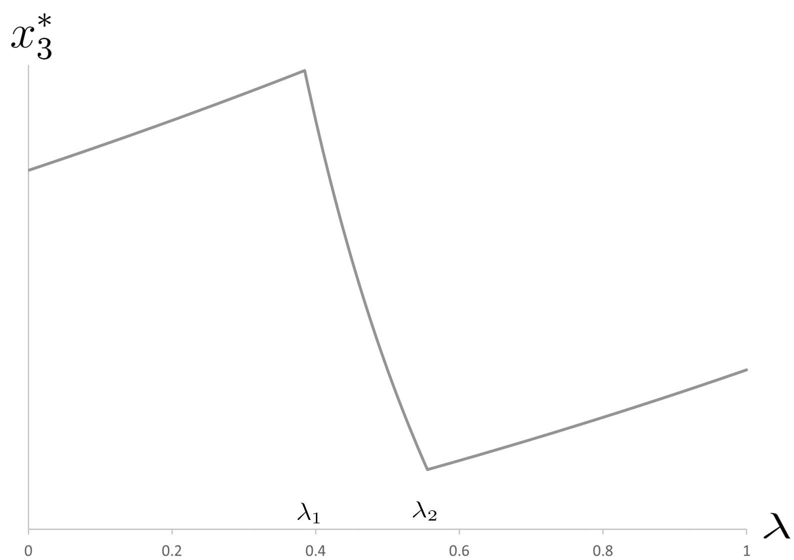


Figure 1: The relationship between the percentage applied to the team assessment ($\lambda$) and the difficulty in inducing students to exert high effort ($x_3^*$) when $K\left(a_h^w\right) \geq \widehat{K\left(a_h^w\right)}$.

Figure 1 also illustrates the result shown in Proposition 5. Given that the Y-axis represents the difficulty in inducing students to exert high effort, the optimal level of $\lambda$ is the point where the corresponding value in the Y-axis is minimal. In Figure 1, the optimal level of $\lambda$ is $\lambda = \lambda_2$. This is because when $K\left(a_h^w\right) \geq \widehat{K\left(a_h^w\right)}$ (in this case, $6 > 4.5$), it is optimal to use both the individual assessment and the group assessment with the weight of $\lambda_2$ applied to the group assessment component, as described in Part (ii) of Proposition 5.

In contrast, if $K\left(a_h^w\right) < \widehat{K\left(a_h^w\right)}$, it is optimal to use the individual assessment only, $\lambda = 0$. To illustrate this result, I use a similar set of parameters except that I change the value of $K\left(a_h^w\right)$ from 6 to 4.2. This new set of parameters is shown below.

| $a_c^s = a_h^s$ | $K\left(a_c^s\right)$ | $K\left(a_h^s\right)$ | $a_c^w = a_h^w$ | $K\left(a_c^w\right)$ | $K\left(a_h^w\right)$ |
|---|---|---|---|---|---|
| 0.8 | 3 | 1 | 0.3 | 4 | 4.2 |

With this new set of parameters, the values of $\lambda_1$, $\lambda_2$, and $\widehat{K\left(a_h^w\right)}$ are

| $\lambda_1$ | $\lambda_2$ | $\widehat{K(a_h^w)}$ |
|---|---|---|
| 0.5319 | 0.5556 | 4.5 |

and the graph is shown in Figure 2.

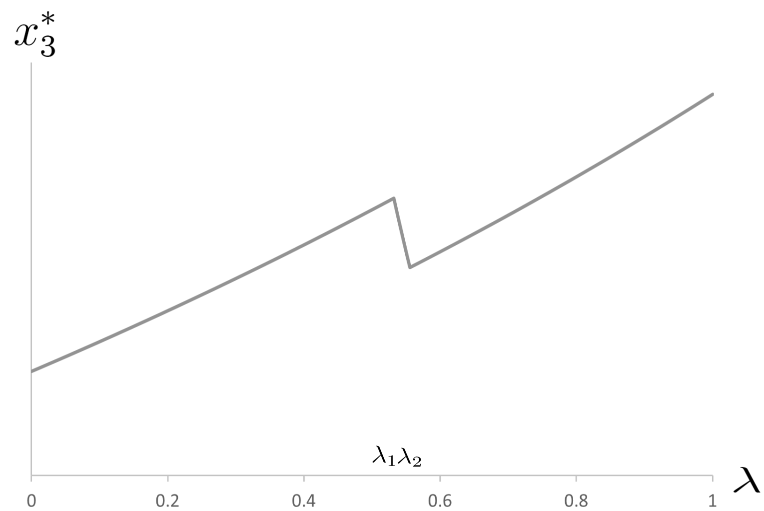


Figure 2: The relationship between the percentage applied to the team assessment ($\lambda$) and the difficulty in inducing students to exert high effort ($x_3^*$) when $K(a_h^w) < \widehat{K(a_h^w)}$.

In Figure 2, the optimal level of $\lambda$ is $\lambda = 0$ because it is the point where the corresponding value of the Y-axis is minimum. This result is consistent with the result shown in Part (i) of Proposition 5 that when $K(a_h^w) < \widehat{K(a_h^w)}$, it is optimal to use the individual assessment only.

# 7 Conclusion

Individual and group assessments are commonly used to evaluate student learning. Through the lens of agency theory, this study shows the impact of these assessment schemes on students' incentives to study. Relative to group assessments, individual assessments give students more control over their performance, thereby reducing their uncertainty and increasing their motivation to study. However, individual assessments do not incentivize students to cooperate. Therefore, it can be challenging to motivate certain groups of students (i.e., weaker students) to study when only individual assessments are used. The results of this study suggest that instructors should take these two effects into account when designing assessments. If it is not difficult to incentivize weak students to study in the absence of cooperation, using only individual assessments can be appropriate. However, if it is challenging to incentivize weaker students to exert study effort without any help from their stronger peers, then it is optimal to use both individual and group assessments.

Another important issue is how the weight given to each assessment component affects students' motivation to study. This study shows that as the weight of the group assessment component increases, it may become easier or more difficult to motivate students to study, as this relationship is non-monotonic. When the weight assigned to the group assessment component is sufficiently low or high, increasing this weight makes it more difficult to incentivize students to study because of the increased uncertainty imposed on them. However, when the weight assigned to the group assessment component is moderate, increasing it motivates students to cooperate, which makes it easier to motivate them to study. This result suggests that if researchers wish to examine how the weights applied to each assessment component affect students' incentives to study, it is important to consider this effect at various levels, as the relationship can be positive or negative.

This study is not without limitations. As is common in analytical research, this study does not consider all of the benefits and costs of each assessment method.

Instead, this study focuses on the impact of different assessment types on students' economic incentives to study. Another limitation, which is typically associated with analytical studies, is that this study does not use empirical data. Therefore, this study does not suggest the specific weight of each component in an optimal assessment. Instead, this study provides empirical predictions of the directional effects of the weight assigned to each assessment component on student motivation.

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# Appendix

### Proof of Proposition 1

If the strong student exerts high effort, her expected utility is

$$a_h^s \cdot x - K\left(a_h^s\right),$$

and if she exerts low effort, her expected utility is

$$a_l^s \cdot x - K\left(a_l^s\right).$$

Therefore, to ensure that she exerts high effort, the following incentive compatibility constraint must hold:

$$a_h^s \cdot x - K\left(a_h^s\right) \geq a_l^s \cdot x - K\left(a_l^s\right). \tag{2}$$

Using $a_l^s = 0$ and $K\left(a_l^s\right) = 0$, Condition (2) can be re-written as

$$x \geq \frac{K\left(a_h^s\right)}{a_h^s}. \tag{3}$$

Similarly, to ensure that the weak exerts high effort, the following incentive compatibility constraint must hold:

$$a_h^w \cdot x - K\left(a_h^w\right) \geq a_l^w \cdot x - K\left(a_l^w\right),$$

which can be re-written as

$$x \geq \frac{K\left(a_h^w\right)}{a_h^w}. \tag{4}$$

Given that $K(a_h^w) > K(a_h^s)$ and $a_h^w < a_h^s$, by assumption, it follows that (4) is the binding constraint. Therefore, when $x > x_1^* \equiv \frac{K(a_h^w)}{a_h^w}$, both students will exert high effort. ■

**Proof of Proposition 2**

When group assessment is used, two possible equilibria can occur. In the first equilibrium, the strong student exerts $a_h^s$ and the weak student exerts $a_h^w$. In the second equilibrium, the strong student exerts $a_c^s$ and the weak student exerts $a_c^w$.

Suppose the strong student exerts $a_h^s$. The weak student will exert $a_h^w$ when

$$a_h^s a_h^w \cdot x - K(a_h^w) \geq a_h^s a_l^w \cdot x - K(a_l^w). \tag{5}$$

Using $a_l^w = 0$ and $K(a_l^w) = 0$, Condition (5) can be re-written as

$$x \geq \frac{K(a_h^w)}{a_h^s a_h^w}. \tag{6}$$

Suppose $x \geq \frac{K(a_h^w)}{a_h^s a_h^w}$. If the strong student exerts $a_h^s$, the weak student will exert $a_h^w$. However, if the strong student exerts $a_l^s$, the weak student will exert $a_l^w$. The strong student will exert $a_h^s$ when

$$a_h^s a_h^w \cdot x - K(a_h^s) \geq a_l^s a_l^w \cdot x - K(a_l^s),$$

which can be re-written as

$$x \geq \frac{K(a_h^s)}{a_h^s a_h^w}. \tag{7}$$

The first equilibrium ($a_h^s$ and $a_h^w$) occurs when both Conditions (6) and (7) hold. Given that $K(a_h^w) > K(a_h^s)$, by assumption, it follows that (6) is the binding constraint. Therefore, the first equilibrium occurs when $x > \frac{K\left(a_h^w\right)}{a_h^s a_h^w}$.

Suppose the strong student exerts $a_c^s$. The weak student will exert $a_c^w$ when

$$a_c^s a_c^w \cdot x - K\left(a_c^w\right) \geq a_c^s a_l^w \cdot x - K\left(a_l^w\right). \tag{8}$$

Using $a_l^w = 0$ and $K\left(a_l^w\right) = 0$, Condition (8) can be re-written as

$$x \geq \frac{K\left(a_c^w\right)}{a_c^s a_c^w}. \tag{9}$$

Conditions (6) and (9) suggest that there are three ranges that should be considered.

When $x < \frac{K\left(a_c^w\right)}{a_c^s a_c^w}$, the weak student will exert low effort regardless of the effort of the strong student. Since the instructor cannot induce both students to exert high effort, this range can be ignored.

When $x \in \left[\frac{K\left(a_c^w\right)}{a_c^s a_c^w}, \frac{K\left(a_h^w\right)}{a_h^s a_h^w}\right)$, the weak student will exert high effort if and only if the strong student exerts high effort with cooperation.

When $x \geq \frac{K\left(a_h^w\right)}{a_h^s a_h^w}$, if the strong student exerts $a_c^s$, the weak student will exert $a_c^w$. However, if the strong student exerts $a_h^s$, the weak student will exert $a_h^w$. Therefore, the strong student will never exert $a_c^s$ because she can achieve the same expected outcome at a lower cost by exerting $a_h^s$ instead.

Suppose it is the case that $x \in \left[\frac{K\left(a_c^w\right)}{a_c^s a_c^w}, \frac{K\left(a_h^w\right)}{a_h^s a_h^w}\right)$. The strong student will exert $a_c^s$, instead of $a_l^s$, when

$$a_c^s a_c^w \cdot x - K\left(a_c^s\right) \geq a_l^s a_l^w \cdot x - K\left(a_l^s\right),$$

which is equivalent to

$$x \geq \frac{K\left(a_c^s\right)}{a_c^s a_c^w}. \tag{10}$$

In addition, the strong student will exert $a_c^s$, instead of $a_h^s$, when

$$a_c^s a_c^w \cdot x - K\left(a_c^s\right) \geq a_h^s a_l^w \cdot x - K\left(a_h^s\right),$$

which is equivalent to

$$x \geq \frac{K\left(a_c^s\right) - K\left(a_h^s\right)}{a_c^s a_c^w}. \tag{11}$$

The second equilibrium ($a_c^s$ and $a_c^w$) occurs when Condition (10), Condition (11), and $x \in \left[\frac{K(a_c^w)}{a_c^s a_c^w}, \frac{K\left(a_h^w\right)}{a_h^s a_h^w}\right)$ hold. Given that $K\left(a_c^w\right) > K\left(a_c^s\right)$ and $K\left(a_h^s\right) > 0$, it follows that the minimum value of $x$ that can satisfy all three conditions is $x_2^* \equiv \frac{K(a_c^w)}{a_c^s a_c^w}$.■

**Proof of Proposition 3**

Suppose the strong student exerts $a_h^s$. The weak student will exert $a_h^w$ when

$$\lambda\left(a_h^s a_h^w \cdot x\right) + (1-\lambda)\left(a_h^w \cdot x\right) - K\left(a_h^w\right) \geq \lambda\left(a_h^s a_l^w \cdot x\right) + (1-\lambda)\left(a_l^w \cdot x\right) - K\left(a_l^w\right),$$

which can be expressed as

$$x \geq \frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w + (1-\lambda) a_h^w}. \tag{12}$$

Now suppose the strong student exerts $a_c^s$. The weak student will exert $a_c^w$ when

$$\lambda\left(a_c^s a_c^w \cdot x\right) + (1-\lambda)\left(a_c^w \cdot x\right) - K\left(a_c^w\right) \geq \lambda\left(a_c^s a_l^w \cdot x\right) + (1-\lambda)\left(a_l^w \cdot x\right) - K\left(a_l^w\right),$$

which can be expressed as

$$x \geq \frac{K\left(a_c^w\right)}{\lambda a_c^s a_c^w + (1-\lambda)\, a_c^w}. \tag{13}$$

(12) and (13) together imply that there are three ranges that we need to consider.

When $x < \frac{K\left(a_c^w\right)}{\lambda a_c^s a_c^w + (1-\lambda) a_c^w}$, the weak student will exert low effort regardless of the effort of the strong student. Since the instructor cannot induce both students to exert high effort, this range can be ignored.

When $x \in \left[\frac{K\left(a_c^w\right)}{\lambda a_c^s a_c^w + (1-\lambda) a_c^w}, \frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w + (1-\lambda) a_h^w}\right)$, the weak student will exert high effort if and only if the strong student exerts high effort with cooperation.

When $x \geq \frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w + (1-\lambda) a_h^w}$, if the strong student exerts $a_c^s$, the weak student will exert $a_c^w$. However, if the strong student exerts $a_h^s$, the weak student will exert $a_h^w$. Therefore, the strong student will never exert $a_c^s$ because she can achieve the same expected outcome at a lower cost by exerting $a_h^s$ instead.

Suppose it is the case that $x \geq \frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w + (1-\lambda) a_h^w}$. The strong student will exert $a_h^s$, instead of $a_l^s$, when

$$\lambda\left(a_h^s a_h^w \cdot x\right) + (1-\lambda)\left(a_h^s \cdot x\right) - K\left(a_h^s\right) \geq \lambda\left(a_l^s a_l^w \cdot x\right) + (1-\lambda)\left(a_l^w \cdot x\right) - K\left(a_l^w\right),$$

which is equivalent to

$$x \geq \frac{K\left(a_h^s\right)}{\lambda a_h^s a_h^w + (1-\lambda)\, a_h^s}. \tag{14}$$

For both students to exert high effort (without cooperation), Conditions (12), and (14) must hold. Given that $K\left(a_h^w\right) > K\left(a_h^s\right)$ and $a_h^s > a_h^w$, by assumption, it follows that (12) is the binding constraint. Therefore, the minimum value of $x$ that can incentivize both students to exert high effort without cooperation is

$$x \geq \frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w + (1-\lambda)\, a_h^w}. \tag{15}$$

Now, suppose it is the case that $x \in \left[\frac{K(a_c^w)}{\lambda a_c^s a_c^w + (1-\lambda) a_c^w}, \frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w + (1-\lambda) a_h^w}\right)$. The strong student will exert $a_c^s$, instead of $a_l^s$, when

$$\lambda\left(a_c^s a_c^w \cdot x\right) + (1-\lambda)\left(a_c^s \cdot x\right) - K\left(a_c^s\right) \geq \lambda\left(a_l^s a_l^w \cdot x\right) + (1-\lambda)\left(a_l^s \cdot x\right) - K\left(a_l^s\right),$$

which is equivalent to

$$x \geq \frac{K\left(a_c^s\right)}{\lambda a_c^s a_c^w + (1-\lambda)\, a_c^s}. \tag{16}$$

In addition, the strong student will exert $a_c^s$, instead of $a_h^s$, when

$$\lambda\left(a_c^s a_c^w \cdot x\right) + (1-\lambda)\left(a_c^s \cdot x\right) - K\left(a_c^s\right) \geq \lambda\left(a_h^s a_l^w \cdot x\right) + (1-\lambda)\left(a_h^s \cdot x\right) - K\left(a_h^s\right),$$

which, using $a_c^s = a_h^s$, is equivalent to

$$x \geq \frac{K\left(a_c^s\right) - K\left(a_h^s\right)}{\lambda a_c^s a_c^w}. \tag{17}$$

For both students to exert high effort with cooperation ($a_c^s$ and $a_c^w$), Condition (16), Condition (17), and $x \in \left[\frac{K(a_c^w)}{\lambda a_c^s a_c^w + (1-\lambda) a_c^w}, \frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w + (1-\lambda) a_h^w}\right)$ must hold. Given that $K\left(a_c^w\right) > K\left(a_c^s\right)$ and $a_c^s > a_c^w$, Condition (16) is not binding. Therefore, the minimum value of $x$ that can incentivize both students to exert high effort with cooperation is

$$x \geq \max\left\{x \geq \frac{K\left(a_c^s\right) - K\left(a_h^s\right)}{\lambda a_c^s a_c^w}, \frac{K\left(a_c^w\right)}{\lambda a_c^s a_c^w + (1-\lambda)\, a_c^w}\right\}. \tag{18}$$

Using (15) and (18), it follows that the minimum value of $x$ that can incentivize both students to exert high effort (with or without cooperation) is

$$x_3^* \equiv \min\left\{\frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w + (1-\lambda)\, a_h^w}, \max\left\{\frac{K\left(a_c^s\right) - K\left(a_h^s\right)}{\lambda a_c^s a_c^w}, \frac{K\left(a_c^w\right)}{\lambda a_c^s a_c^w + (1-\lambda)\, a_c^w}\right\}\right\}. \blacksquare \tag{19}$$

**Proof of Proposition 4**

$\lambda_1$ and $\lambda_2$ are defined as follows:

$$\lambda_1 \equiv \frac{\left(K\left(a_c^s\right) - K\left(a_h^s\right)\right)}{a_c^s K\left(a_h^w\right) + \left(K\left(a_c^s\right) - K\left(a_h^s\right)\right)\left(1 - a_c^s\right)}, \text{ and} \tag{20}$$

$$\lambda_2 \equiv \frac{\left(K\left(a_c^s\right) - K\left(a_h^s\right)\right)}{a_c^s K\left(a_c^w\right) + \left(K\left(a_c^s\right) - K\left(a_h^s\right)\right)\left(1 - a_c^s\right)}. \tag{21}$$

*Step 1: Prove that* $0 < \lambda_1 < \lambda_2 < 1$.

Using $K\left(a_h^w\right) > K\left(a_c^w\right)$, it follows that $0 < \lambda_1 < \lambda_2$. In addition,

$$\begin{aligned}
\lambda_1 &= \frac{\left(K\left(a_c^s\right) - K\left(a_h^s\right)\right)}{a_c^s K\left(a_h^w\right) + \left(K\left(a_c^s\right) - K\left(a_h^s\right)\right)\left(1 - a_c^s\right)} \\
&< \frac{\left(K\left(a_c^s\right) - K\left(a_h^s\right)\right)}{a_c^s\left(K\left(a_c^s\right) - K\left(a_h^s\right)\right) + \left(K\left(a_c^s\right) - K\left(a_h^s\right)\right)\left(1 - a_c^s\right)} \\
&= 1,
\end{aligned}$$

because $K\left(a_h^w\right) > K\left(a_c^s\right) > K\left(a_c^s\right) - K\left(a_h^s\right)$. Similarly,

$$\begin{aligned}
\lambda_2 &= \frac{\left(K\left(a_c^s\right) - K\left(a_h^s\right)\right)}{a_c^s K\left(a_c^w\right) + \left(K\left(a_c^s\right) - K\left(a_h^s\right)\right)\left(1 - a_c^s\right)} \\
&< \frac{\left(K\left(a_c^s\right) - K\left(a_h^s\right)\right)}{a_c^s\left(K\left(a_c^s\right) - K\left(a_h^s\right)\right) + \left(K\left(a_c^s\right) - K\left(a_h^s\right)\right)\left(1 - a_c^s\right)} \\
&= 1,
\end{aligned}$$

because $K\left(a_c^w\right) > K\left(a_c^s\right) > K\left(a_c^s\right) - K\left(a_h^s\right)$.

*Step 2: Prove that when* $\lambda < \lambda_1$, $x_3^* = \frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w + (1-\lambda) a_h^w}$ *and* $\frac{dx_3^*}{d\lambda} > 0$.

When $\lambda < \lambda_1$, using (20), (21), and $0 < \lambda_1 < \lambda_2 < 1$, we obtain

$$\begin{aligned}
\frac{K\left(a_c^s\right) - K\left(a_h^s\right)}{\lambda a_c^s a_c^w} &> \frac{K\left(a_c^s\right) - K\left(a_h^s\right)}{\lambda_1 a_c^s a_c^w} \\
&> \frac{K\left(a_c^s\right) - K\left(a_h^s\right)}{\lambda_2 a_c^s a_c^w} \\
&= \frac{K\left(a_c^w\right)}{\lambda_2 a_c^s a_c^w + \left(1 - \lambda_2\right) a_c^w} \\
&> \frac{K\left(a_c^w\right)}{\lambda_1 a_c^s a_c^w + \left(1 - \lambda_1\right) a_c^w} \\
&> \frac{K\left(a_c^w\right)}{\lambda a_c^s a_c^w + \left(1 - \lambda\right) a_c^w}.
\end{aligned}$$

Therefore, $\max\left\{\frac{K(a_c^s)-K\left(a_h^s\right)}{\lambda a_c^s a_c^w}, \frac{K(a_c^w)}{\lambda a_c^s a_c^w+(1-\lambda)a_c^w}\right\} = \frac{K(a_c^s)-K\left(a_h^s\right)}{\lambda a_c^s a_c^w}$. In addition,

$$\begin{aligned}
\frac{K\left(a_c^s\right) - K\left(a_h^s\right)}{\lambda a_c^s a_c^w} &> \frac{K\left(a_c^s\right) - K\left(a_h^s\right)}{\lambda_1 a_c^s a_c^w} \\
&= \frac{K\left(a_h^w\right)}{\lambda_1 a_h^s a_h^w + \left(1 - \lambda_1\right) a_h^w} \\
&> \frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w + \left(1 - \lambda\right) a_h^w},
\end{aligned}$$

suggesting that $\min\left\{\frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w+(1-\lambda)a_h^w}, \frac{K(a_c^s)-K\left(a_h^s\right)}{\lambda a_c^s a_c^w}\right\} = \frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w+(1-\lambda)a_h^w}$. Thus, $x_3^* = \frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w+(1-\lambda)a_h^w}$.

$$\begin{aligned}
\frac{dx_3^*}{d\lambda} &= \frac{d}{d\lambda}\left(\frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w + \left(1 - \lambda\right) a_h^w}\right) \\
&= \frac{a_h^w\left(1 - a_h^s\right) K\left(a_h^w\right)}{\left(\lambda a_h^s a_h^w + \left(1 - \lambda\right) a_h^w\right)^2} > 0.
\end{aligned}$$

*Step 3: Prove that when* $\lambda \in [\lambda_1, \lambda_2]$, $x_3^* = \frac{K(a_c^s)-K\left(a_h^s\right)}{\lambda a_c^s a_c^w}$ *and* $\frac{dx_3^*}{d\lambda} < 0$.

When $\lambda \in [\lambda_1, \lambda_2]$, using (20), (21), and $0 < \lambda_1 < \lambda_2 < 1$, we obtain

$$\begin{aligned}
\frac{K(a_c^s) - K(a_h^s)}{\lambda a_c^s a_c^w} &\geq \frac{K(a_c^s) - K(a_h^s)}{\lambda_2 a_c^s a_c^w} \\
&= \frac{K(a_c^w)}{\lambda_2 a_c^s a_c^w + (1-\lambda_2) a_c^w} \\
&\geq \frac{K(a_c^w)}{\lambda a_c^s a_c^w + (1-\lambda) a_c^w}.
\end{aligned}$$

Therefore, $\max\left\{\frac{K(a_c^s)-K(a_h^s)}{\lambda a_c^s a_c^w}, \frac{K(a_c^w)}{\lambda a_c^s a_c^w + (1-\lambda) a_c^w}\right\} = \frac{K(a_c^s)-K(a_h^s)}{\lambda a_c^s a_c^w}$. In addition,

$$\begin{aligned}
\frac{K(a_c^s) - K(a_h^s)}{\lambda a_c^s a_c^w} &\leq \frac{K(a_c^s) - K(a_h^s)}{\lambda_1 a_c^s a_c^w} \\
&= \frac{K(a_h^w)}{\lambda_1 a_h^s a_h^w + (1-\lambda_1) a_h^w} \\
&\leq \frac{K(a_h^w)}{\lambda a_h^s a_h^w + (1-\lambda) a_h^w},
\end{aligned}$$

suggesting that $\min\left\{\frac{K(a_h^w)}{\lambda a_h^s a_h^w + (1-\lambda) a_h^w}, \frac{K(a_c^s)-K(a_h^s)}{\lambda a_c^s a_c^w}\right\} = \frac{K(a_c^s)-K(a_h^s)}{\lambda a_c^s a_c^w}$. Thus, $x_3^* = \frac{K(a_c^s)-K(a_h^s)}{\lambda a_c^s a_c^w}$.

$$\begin{aligned}
\frac{dx_3^*}{d\lambda} &= \frac{d}{d\lambda}\left(\frac{K(a_c^s) - K(a_h^s)}{\lambda a_c^s a_c^w}\right) \\
&= -\left(\frac{K(a_c^s) - K(a_h^s)}{\lambda^2 a_c^s a_c^w}\right) < 0.
\end{aligned}$$

*Step 4: Prove that when* $\lambda > \lambda_2$, $x_3^* = \frac{K(a_c^w)}{\lambda a_c^s a_c^w + (1-\lambda) a_c^w}$ *and* $\frac{dx_3^*}{d\lambda} > 0$.

When $\lambda > \lambda_2$, using (20), (21), and $0 < \lambda_1 < \lambda_2 < 1$, we obtain

$$\begin{aligned}
\frac{K(a_c^s) - K(a_h^s)}{\lambda a_c^s a_c^w} &< \frac{K(a_c^s) - K(a_h^s)}{\lambda_2 a_c^s a_c^w} \\
&= \frac{K(a_c^w)}{\lambda_2 a_c^s a_c^w + (1-\lambda_2) a_c^w} \\
&\leq \frac{K(a_c^w)}{\lambda a_c^s a_c^w + (1-\lambda) a_c^w}.
\end{aligned}$$

Therefore, $\max\left\{\frac{K(a_c^s)-K\left(a_h^s\right)}{\lambda a_c^s a_c^w}, \frac{K(a_c^w)}{\lambda a_c^s a_c^w+(1-\lambda)a_c^w}\right\} = \frac{K(a_c^w)}{\lambda a_c^s a_c^w+(1-\lambda)a_c^w}$. In addition, since $K(a_h^w) > K(a_c^w)$, $a_h^s = a_c^s$, and $a_h^w = a_c^w$, it follows that

$$\frac{K(a_c^w)}{\lambda a_c^s a_c^w + (1-\lambda)\, a_c^w} < \frac{K(a_h^w)}{\lambda a_h^s a_h^w + (1-\lambda)\, a_h^w}$$

for all $\lambda \in [0,1]$, suggesting that $\min\left\{\frac{K\left(a_h^w\right)}{\lambda a_h^s a_h^w+(1-\lambda)a_h^w}, \frac{K(a_c^w)}{\lambda a_c^s a_c^w+(1-\lambda)a_c^w}\right\} = \frac{K(a_c^w)}{\lambda a_c^s a_c^w+(1-\lambda)a_c^w}$. Thus, $x_3^* = \frac{K(a_c^w)}{\lambda a_c^s a_c^w+(1-\lambda)a_c^w}$.

$$\begin{aligned} \frac{dx_3^*}{d\lambda} &= \frac{d}{d\lambda}\left(\frac{K(a_c^w)}{\lambda a_c^s a_c^w + (1-\lambda)\, a_c^w}\right) \\ &= \frac{a_c^w\,(1-a_c^s)\, K(a_c^w)}{\left(\lambda a_c^s a_c^w + (1-\lambda)\, a_c^w\right)^2} > 0. \blacksquare \end{aligned}$$

**Proof of Proposition 5**

$\widehat{K(a_h^w)}$ is defined as follows:

$$\widehat{K(a_h^w)} \equiv K(a_c^w) + \frac{\left(K(a_c^s) - K(a_h^s)\right)(1-a_c^s)}{a_c^s}. \tag{22}$$

It suffices to show that the optimal assessment can be determined by solving for the value of $\lambda$ that minimizes $x_3^*$ shown in (19).

Using the results in Proposition 4, the minimum value of $x_3^*$ can occur only when $\lambda = 0$ or $\lambda = \lambda_2$.

Suppose $K(a_h^w) < \widehat{K(a_h^w)}$. Using (19),

$$\begin{aligned} x_3^*|_{\lambda=\lambda_2} &= \frac{K(a_c^s) - K(a_h^s)}{\lambda_2 a_c^s a_c^w} \\ &= \frac{a_c^s K(a_c^w) + (K(a_c^s) - K(a_h^s))(1 - a_c^s)}{a_c^s a_c^w} \\ &= \frac{\widehat{K(a_h^w)}}{a_c^w} > \frac{K(a_h^w)}{a_c^w} = \frac{K(a_h^w)}{a_h^w} = x_3^*|_{\lambda=0}, \end{aligned}$$

suggesting that it is optimal to set $\lambda = 0$ (individual assessment only).

In contrast, suppose $K(a_h^w) \geq \widehat{K(a_h^w)}$. Using (19),

$$\begin{aligned} x_3^*|_{\lambda=\lambda_2} &= \frac{K(a_c^s) - K(a_h^s)}{\lambda_2 a_c^s a_c^w} \\ &= \frac{a_c^s K(a_c^w) + (K(a_c^s) - K(a_h^s))(1 - a_c^s)}{a_c^s a_c^w} \\ &= \frac{\widehat{K(a_h^w)}}{a_c^w} \leq \frac{K(a_h^w)}{a_c^w} = \frac{K(a_h^w)}{a_h^w} = x_3^*|_{\lambda=0}, \end{aligned}$$

suggesting that it is optimal to set $\lambda = \lambda_2$ (both group and individual assessment with the weight of $\lambda_2$ applied to the group assessment component).■