Key-Aggregate Cryptosystem for Scalable Data Sharing in Cloud Storage

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Abstract—Data sharing is an important functionality in cloud storage. In this article, we show how to securely, efficiently, and flexibly share data with others in cloud storage. We describe new public-key cryptosystems which produce constant-size ciphertexts such that efficient delegation of decryption rights for any set of ciphertexts are possible. The novelty is that one can aggregate any set of secret keys and make them as compact as a single key, but encompassing the power of all the keys being aggregated. In other words, the secret key holder can release a constant-size aggregate key for flexible choices of ciphertext set in cloud storage, but the other encrypted files outside the set remain confidential. This compact aggregate key can be conveniently sent to others or be stored in a smart card with very limited secure storage. We provide formal security analysis of our schemes in the standard model. We also describe other application of our schemes. In particular, our schemes give the first public-key patient-controlled encryption for flexible hierarchy, which was yet to be known.

Index Terms—Cloud storage, data sharing, key-aggregate encryption, patient-controlled encryption

1 INTRODUCTION

Cloud storage is gaining popularity recently. In enterprise settings, we see the rise in demand for data outsourcing, which assists in the strategic management of corporate data. It is also used as a core technology behind many online services for personal applications. Nowadays, it is easy to apply for free accounts for email, photo album, file sharing and/or remote access, with storage size more than 25GB (or a few dollars for more than 1TB). Together with the current wireless technology, users can access almost all of their files and emails by a mobile phone in any corner of the world.

Considering data privacy, a traditional way to ensure it is to rely on the server to enforce the access control after authentication (e.g., [1]), which means any unexpected privilege escalation will expose all data. In a shared-tenancy cloud computing environment, things become even worse. Data from different clients can be hosted on separate virtual machines (VMs) but reside on a single physical machine. Data in a target VM could be stolen by instantiating another VM co-resident with the target one [2]. Regarding availability of files, there are a series of cryptographic schemes which go as far as allowing a third-party auditor to check the availability of files on behalf of the data owner without leaking anything about the data [3], or without compromising the data owners anonymity [4]. Likewise, cloud users probably will not hold the strong belief that the cloud server is doing a good job in terms of confidentiality. A cryptographic solution, e.g., [5], with proven security relied on number-theoretic assumptions is more desirable, whenever the user is not perfectly happy with trusting the security of the VM or the honesty of the technical staff. These users are motivated to encrypt their data with their own keys before uploading them to the server.

Data sharing is an important functionality in cloud storage. For example, bloggers can let their friends view a subset of their private pictures; an enterprise may grant her employees access to a portion of sensitive data. The challenging problem is how to effectively share encrypted data. Of course users can download the encrypted data from the storage, decrypt them, then send them to others for sharing, but it loses the value of cloud storage. Users should be able to delegate the access rights of the sharing data to others so that they can access these data from the server directly. However, finding an efficient and secure way to share partial data in cloud storage is not trivial. Below we will take Dropbox\(^1\) as an example for illustration.

Assume that Alice puts all her private photos on Dropbox, and she does not want to expose her photos to everyone. Due to various data leakage possibility Alice cannot feel relieved by just relying on the privacy protection mechanisms provided by Dropbox, so she encrypts all the photos using her own keys before uploading. One day, Alice’s friend, Bob, asks her to share the photos

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\(^1\) http://www.dropbox.com
taken over all these years which Bob appeared in. Alice can then use the share function of Dropbox, but the problem now is how to delegate the decryption rights for these photos to Bob. A possible option Alice can choose is to securely send Bob the secret keys involved. Naturally, there are two extreme ways for her under the traditional encryption paradigm:

- Alice encrypts all files with a single encryption key and gives Bob the corresponding secret key directly.
- Alice encrypts files with distinct keys and sends Bob the corresponding secret keys.

Obviously, the first method is inadequate since all unchosen data may be also leaked to Bob. For the second method, there are practical concerns on efficiency. The number of such keys is as many as the number of the shared photos, say, a thousand. Transferring these secret keys inherently requires a secure channel, and storing these keys requires rather expensive secure storage. The costs and complexities involved generally increase with the number of the decryption keys to be shared. In short, it is very heavy and costly to do that.

Encryption keys also come with two flavors — symmetric key or asymmetric (public) key. Using symmetric encryption, when Alice wants the data to be originated from a third party, she has to give the encryptor her secret key; obviously, this is not always desirable. By contrast, the encryption key and decryption key are different in public-key encryption. The use of public-key encryption gives more flexibility for our applications. For example, in enterprise settings, every employee can upload encrypted data on the cloud storage server without the knowledge of the company’s master-secret key.

Therefore, the best solution for the above problem is that Alice encrypts files with distinct public-keys, but only sends Bob a single (constant-size) decryption key. Since the decryption key should be sent via a secure channel and kept secret, small key size is always desirable. For example, we can not expect large storage for decryption keys in the resource-constraint devices like smart phones, smart cards or wireless sensor nodes. Especially, these secret keys are usually stored in the tamper-proof memory, which is relatively expensive. The present research efforts mainly focus on minimizing the communication requirements (such as bandwidth, rounds of communication) like aggregate signature [6]. However, not much has been done about the key itself (see Section 3 for more details).

### 1.1 Our Contributions

In modern cryptography, a fundamental problem we often study is about leveraging the secrecy of a small piece of knowledge into the ability to perform cryptographic functions (e.g., encryption, authentication) multiple times. In this paper, we study how to make a decryption key more powerful in the sense that it allows decryption of multiple ciphertexts, without increasing its size. Specifically, our problem statement is –

Fig. 1. Alice shares files with identifiers 2, 3, 6 and 8 with Bob by sending him a single aggregate key.

“To design an efficient public-key encryption scheme which supports flexible delegation in the sense that any subset of the ciphertexts (produced by the encryption scheme) is decryptable by a constant-size decryption key (generated by the owner of the master-secret key).”

We solve this problem by introducing a special type of public-key encryption which we call key-aggregate cryptosystem (KAC). In KAC, users encrypt a message not only under a public-key, but also under an identifier of ciphertext called class. That means the ciphertexts are further categorized into different classes. The key owner holds a master-secret called master-secret key, which can be used to extract secret keys for different classes. More importantly, the extracted key have can be an aggregate key which is as compact as a secret key for a single class, but aggregates the power of many such keys, i.e., the decryption power for any subset of ciphertext classes.

With our solution, Alice can simply send Bob a single aggregate key via a secure e-mail. Bob can download the encrypted photos from Alice’s Dropbox space and then use this aggregate key to decrypt these encrypted photos. The scenario is depicted in Figure 1.

The sizes of ciphertext, public-key, master-secret key and aggregate key in our KAC schemes are all of constant size. The public system parameter has size linear in the number of ciphertext classes, but only a small part of it is needed each time and it can be fetched on demand from large (but non-confidential) cloud storage.

Previous results may achieve a similar property featuring a constant-size decryption key, but the classes need to conform to some pre-defined hierarchical relationship. Our work is flexible in the sense that this constraint is eliminated, that is, no special relation is required between the classes. The detail and other related works can be found in Section 3.

We propose several concrete KAC schemes with different security levels and extensions in this article. All
constructions can be proven secure in the standard model. To the best of our knowledge, our aggregation mechanism\(^2\) in KAC has not been investigated.

2 Key-Aggregate Encryption

We first give the framework and definition for key-aggregate encryption. Then we describe how to use KAC in a scenario of its application in cloud storage.

2.1 Framework

A key-aggregate encryption scheme consists of five polynomial-time algorithms as follows.

The data owner establishes the public system parameter via Setup and generates a public/master-secret\(^3\) key pair via KeyGen. Messages can be encrypted via Encrypt by anyone who also decides what ciphertext class is associated with the plaintext message to be encrypted. The data owner can use the master-secret to generate an aggregate decryption key for a set of ciphertext classes via Extract. The generated keys can be passed to delegates securely (via secure e-mails or secure devices). Finally, any user with an aggregate key can decrypt any ciphertext provided that the ciphertext’s class is contained in the aggregate key via Decrypt.

- Setup\((1^\lambda, n)\): executed by the data owner to setup an account on an untrusted server. On input a security level parameter \(1^\lambda\) and the number of ciphertext classes \(n\) (i.e., class index should be an integer bounded by 1 and \(n\)), it outputs the public system parameter \(\text{param}\), which is omitted from the input of the other algorithms for brevity.
- KeyGen: executed by the data owner to randomly generate a public/master-secret key pair \((\text{pk}, \text{msk})\).
- Encrypt\((\text{pk}, i, m)\): executed by anyone who wants to encrypt data. On input a public-key \(\text{pk}\), an index \(i\) denoting the ciphertext class, and a message \(m\), it outputs a ciphertext \(\text{c}\).
- Extract\((\text{msk}, S)\): executed by the data owner for delegating the decrypting power for a certain set of ciphertext classes to a delegatee. On input the master-secret key \(\text{msk}\) and a set \(S\) of indices corresponding to different classes, it outputs the aggregate key for set \(S\) denoted by \(K_S\).
- Decrypt\((K_S, S, i, \text{c})\): executed by a delegatee who received an aggregate key \(K_S\) generated by Extract. On input \(K_S\), the set \(S\), an index \(i\) denoting the ciphertext class the ciphertext \(\text{c}\) belongs to, and \(\text{c}\), it outputs the decrypted result \(m\) if \(i \in S\).

There are two functional requirements:

- Correctness For any integers \(\lambda\) and \(n\), any set \(S \subseteq \{1, \ldots, n\}\), any index \(i \in S\) and any message \(m\),
  \[
  \Pr[\text{Decrypt}(K_S, S, i, \text{c}) = m : \text{param} \leftarrow \text{Setup}(1^\lambda, n),
  (\text{pk}, \text{msk}) \leftarrow \text{KeyGen}(),
  \text{c} \leftarrow \text{Encrypt}(\text{pk}, i, m),
  K_S \leftarrow \text{Extract}(\text{msk}, S)] = 1.
  \]

- Compactness For any integers \(\lambda\), \(n\), any set \(S\), any index \(i \in S\) and any message \(m\), \(\text{param} \leftarrow \text{Setup}(1^\lambda, n)\), \((\text{pk}, \text{msk}) \leftarrow \text{KeyGen}()\), \(K_S \leftarrow \text{Extract}(\text{msk}, S)\) and \(\text{c} \leftarrow \text{Encrypt}(\text{pk}, i, m)\); \(|K_S|\) and \(|\text{c}|\) only depend on the security parameter \(\lambda\) but independent of the number of classes \(n\).

2.2 Sharing Encrypted Data

A canonical application of KAC is data sharing. The key aggregation property is especially useful when we expect the delegation to be efficient and flexible. The schemes enable a content provider to share her data in a confidential and selective way, with a fixed and small ciphertext expansion, by distributing to each authorized user a single and small aggregate key.

Here we describe the main idea of data sharing in cloud storage using KAC, illustrated in Figure 2. Suppose Alice wants to share her data \(m_1, m_2, \ldots, m_n\) on the server. She first performs Setup\((1^\lambda, n)\) to get \(\text{param}\) and execute KeyGen to get the public/master-secret key pair \((\text{pk}, \text{msk})\). The system parameter \(\text{param}\) and public-key \(\text{pk}\) can be made public and master-secret key \(\text{msk}\) should be kept secret by Alice. Anyone (including Alice herself) can then encrypt each \(m_i\) by \(\text{c}_i = \text{Encrypt}(\text{pk}, i, m_i)\). The encrypted data are uploaded to the server.

With \(\text{param}\) and \(\text{pk}\), people who cooperate with Alice can update Alice’s data on the server. Once Alice is willing to share a set \(S\) of her data with a friend Bob, she can compute the aggregate key \(K_S\) for Bob by performing Extract\((\text{msk}, S)\). Since \(K_S\) is just a constant-size key, it is easy to be sent to Bob via a secure e-mail.

After obtaining the aggregate key, Bob can download the data he is authorized to access. That is, for each \(i \in S\), Bob downloads \(\text{c}_i\) (and some needed values in \(\text{param}\)) from the server. With the aggregate key \(K_S\), Bob can decrypt each \(\text{c}_i\) by Decrypt\((K_S, S, i, \text{c}_i)\) for each \(i \in S\).

3 Related Work

This section we compare our basic KAC scheme with other possible solutions on sharing in secure cloud storage. We summarize our comparisons in Table 1.

3.1 Cryptographic Keys for a Predefined Hierarchy

We start by discussing the most relevant study in the literature of cryptography/security. Cryptographic key assignment schemes (e.g., [11], [12], [13], [14]) aim to

\(^2\)It is obvious that we are not proposing an algorithm to compress the decryption key. On one hand, cryptographic keys come from a high-entropy source and are hardly compressible. On the other hand, decryption keys for all possible combinations of ciphertext classes are all in constant-size — information theoretically speaking such compression scheme cannot exist.

\(^3\)We call this as master-secret key to avoid confusion with the delegated key we will explain later.

\(^4\)For simplicity, we omit the inclusion of a decryption algorithm for the original data owner using the master-secret key. In our specific constructions, we will show how the knowledge of the master-secret key allows a faster decryption than using Extract followed by Decrypt.
<table>
<thead>
<tr>
<th>Key assignment schemes</th>
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<td>most likely non-constant</td>
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</tr>
<tr>
<td>Symmetric-key encryption with Compact Key (e.g., [8])</td>
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<td>constant</td>
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<td>IBE with Compact Key (e.g., [9])</td>
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<td>Attribute-Based Encryption (e.g., [10])</td>
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<tr>
<td>KAC</td>
<td>constant</td>
<td>constant</td>
<td>public-key</td>
</tr>
</tbody>
</table>

TABLE 1
Comparisons between our basic KAC scheme and other related schemes

minimize the expense in storing and managing secret keys for general cryptographic use. Utilizing a tree structure, a key for a given branch can be used to derive the keys of its descendant nodes (but not the other way round). Just granting the parent key implicitly grants all the keys of its descendant nodes. Sandhu [15] proposed a method to generate a tree hierarchy of symmetric-keys by using repeated evaluations of pseudorandom function/block-cipher on a fixed secret. The concept can be generalized from a tree to a graph. More advanced cryptographic key assignment schemes support access policy that can be modeled by an acyclic graph or a cyclic graph [16], [17], [7]. Most of these schemes produce keys for symmetric-key cryptosystems, even though the key derivations may require modular arithmetic as used in public-key cryptosystems, which are generally more expensive than "symmetric-key operations" such as pseudorandom function.

We take the tree structure as an example. Alice can first classify the ciphertext classes according to their subjects like Figure 3. Each node in the tree represents a secret key, while the leaf nodes represent the keys for individual ciphertext classes. Filled circles represent the keys for the classes to be delegated and circles circumvented by dotted lines represent the keys to be granted. Note that every key of the non-leaf node can derive the keys of its descendant nodes.

Fig. 2. Using KAC for data sharing in cloud storage

In Figure 3(a), if Alice wants to share all the files in the "personal" category, she only needs to grant the key for the node "personal", which automatically grants the delegatee the keys of all the descendant nodes ("photo", "music"). This is the ideal case, where most classes to be shared belong to the same branch and thus a parent key of them is sufficient.

However, it is still difficult for general cases. As shown in Figure 3(b), if Alice shares her demo music at work ("work"→"casual"→"demo" and "work"→"confidential"→"demo") with a colleague who also has the rights to see some of her personal data, what she can do is to give more keys, which leads to an increase in the total key size. One can see that this approach is not flexible when the classifications are more complex and she wants to share different sets of files to different people. For this delegatee in our example, the number of granted secret keys becomes the same as the number of classes.

In general, hierarchical approaches can solve the problem partially if one intends to share all files under a certain branch in the hierarchy. On average, the number of keys increases with the number of branches. It is unlikely to come up with a hierarchy that can save the number of total keys to be granted for all individuals (which can access a different set of leaf-nodes) simultaneously.

3.2 Compact Key in Symmetric-Key Encryption
Motivated by the same problem of supporting flexible hierarchy in decryption power delegation (but in symmetric-key setting), Benaloh et al. [8] presented an encryption scheme which is originally proposed for concisely transmitting large number of keys in broadcast scenario [18]. The construction is simple and we briefly
review its key derivation process here for a concrete description of what are the desirable properties we want to achieve. The derivation of the key for a set of classes (which is a subset of all possible ciphertext classes) is as follows. A composite modulus \( N = p \cdot q \) is chosen where \( p \) and \( q \) are two large random primes. A master-secret key \( Y \) is chosen at random from \( \mathbb{Z}_N^* \). Each class is associated with a distinct prime \( e_i \). All these prime numbers can be put in the public system parameter \( Y \).

A constant-size key for set \( S \) can be generated (with the knowledge of \( \phi(N) \)) as \( k_{S'} = Y^{1/\prod_{i \in S}(e_i)} \mod N \). For those who have been delegated the access rights for \( S \) where \( S' \subset S \), \( k_{S'} \) can be computed by \( k_{S}^{\prod_{i \in S \setminus S'}(e_i)} \). As a concrete example, a key for classes represented by \( e_1, e_2, e_3 \) can be generated as \( Y^{1/(e_1 \cdot e_2 \cdot e_3)} \), from which each of \( Y^{1/e_1}, Y^{1/e_2}, Y^{1/e_3} \) can easily be derived (while providing no information about keys for any other class, say, \( e_4 \)). This approach achieves similar properties and performances as our schemes. However, it is designed for the symmetric-key setting instead. The encryptor needs to get the corresponding secret keys to encrypt data, which is not suitable for many applications. Since their method is used to generate a secret value rather than a pair of public/secret keys, it is unclear how to apply this idea for public-key encryption scheme.

Finally, we note that there are schemes which try to reduce the key size for achieving authentication in symmetric-key encryption, e.g., [19]. However, sharing of decryption power is not a concern in these schemes.

### 3.3 Compact Key in Identity-Based Encryption

Identity-based encryption (IBE) (e.g., [20], [21], [22]) is a type of public-key encryption in which the public-key of a user can be set as an identity-string of the user (e.g., an email address). There is a trusted party called private key generator (PKG) in IBE which holds a master-secret key and issues a secret key to each user with respect to the user identity. The encryptor can take the public parameter and a user identity to encrypt a message. The recipient can decrypt this ciphertext by his secret key.

Guo et al. [23], [9] tried to build IBE with key aggregation. One of their schemes [23] assumes random oracles but another [9] does not. In their schemes, key aggregation is constrained in the sense that all keys to be aggregated must come from different “identity divisions”. While there are an exponential number of identities and thus secret keys, only a polynomial number of them can be aggregated. Most importantly, their key-aggregation [23], [9] comes at the expense of \( O(n) \) sizes for both ciphertexts and the public parameter, where \( n \) is the number of secret keys which can be aggregated into a constant size one. This greatly increases the costs of storing and transmitting ciphertexts, which is impractical in many situations such as shared cloud storage. As we mentioned, our schemes feature constant ciphertext size, and their security holds in the standard model.

In fuzzy IBE [21], one single compact secret key can decrypt ciphertexts encrypted under many identities which are close in a certain metric space, but not for an arbitrary set of identities and therefore it does not match with our idea of key aggregation.

### 3.4 Other Encryption Schemes

Attribute-based encryption (ABE) [10], [24] allows each ciphertext to be associated with an attribute, and the master-secret key holder can extract a secret key for a policy of these attributes so that a ciphertext can be decrypted by this key if its associated attribute conforms to the policy. For example, with the secret key for the policy \( (2 \lor 3 \lor 6 \lor 8) \), one can decrypt ciphertext tagged with class \( 2, 3, 6 \) or \( 8 \). However, the major concern in ABE is collusion-resistance but not the compactness of secret keys. Indeed, the size of the key often increases linearly with the number of attributes it encompasses, or the ciphertext-size is not constant (e.g., [25]).

To delegate the decryption power of some ciphertexts without sending the secret key to the delegatee, a useful primitive is proxy re-encryption (PRE) (e.g., [26], [27], [28], [29]). A PRE scheme allows Alice to delegate to the server (proxy) the ability to convert the ciphertexts encrypted under her public-key into ones for Bob. PRE is well known to have numerous applications including cryptographic file system [30]. Nevertheless, Alice has to trust the proxy that it only converts ciphertexts according to her instruction, which is what we want to avoid at the first place. Even worse, if the proxy colludes with Bob, some form of Alice’s secret key can be recovered which can decrypt Alice’s (convertible) ciphertexts without Bob’s further help. That also means that the transformation key of proxy should be well-protected. Using PRE just moves the secure key storage requirement from the delegatee to the proxy. It is thus undesirable to let the proxy reside in the storage server. That will also be inconvenient since every decryption requires separate interaction with the proxy.

### 4 Concrete Constructions of KAC

Let \( G \) and \( G_T \) be two cyclic groups of prime order \( p \) and \( \tilde{\epsilon} : G \times \mathbb{G} \to \mathbb{G}_T \) be a map with the following properties:

- **Bilinear**: \( \forall g_1, g_2 \in G, a, b \in \mathbb{Z}_p, \tilde{\epsilon}(g_1^a, g_2^b) = (g_1g_2)^{ab} \).
- **Non-degenerate**: for some \( g \in G \), \( \tilde{\epsilon}(g,g) \neq 1 \).

\( G \) is a bilinear group if all the operations involved above are efficiently computable. Many classes of elliptic curves feature bilinear groups.

#### 4.1 A Basic Construction

The design of our basic scheme is inspired from the collusion-resistant broadcast encryption scheme proposed by Boneh et al. [31]. Although their scheme supports constant-size secret keys, every key only has
the power for decrypting ciphertexts associated to a particular index. We thus need to devise a new Extract algorithm and the corresponding Decrypt algorithm.

- **Setup(1^λ, n):** Randomly pick a bilinear group \( G \) of prime order \( p \) where \( 2^λ \leq p \leq 2^{λ+1} \), a generator \( g \in G \) and \( α \in R \mathbb{Z}_p \). Compute \( g_i = g^{αi} \in G \) for \( i = 1, \ldots, n, n+2, \ldots, 2n \). Output the system parameter as \( \text{param} = (g, g_1, \ldots, g_n, g_{n+2}, \ldots, g_{2n}) \) (α can be safely deleted after Setup).

Note that each ciphertext class is represented by an index in the integer set \( \{1, 2, \ldots, n\} \), where \( n \) is the maximum number of ciphertext classes.

- **KeyGen(γ):** Pick \( γ \in R \mathbb{Z}_p \), output the public and master-secret key pair: \( (pk = v = g^γ, msk = γ) \).

- **Encrypt(pk, i, m):** For a message \( m \in \mathbb{G}_T \) and an index \( i \in \{1, 2, \ldots, n\} \), randomly pick \( t \in R \mathbb{Z}_p \) and compute the ciphertext as \( \mathcal{C} = \langle g^i, (vg^i)^t, m \cdot \hat{e}(g_1, g_n)^t \rangle \).

- **Extract(msk = γ, S):** For the set \( S \) of indices \( j \)'s, the aggregate key is computed as \( K_S = \prod_{j \in S} g_{n+1-j} \).

Since \( S \) does not include 0, \( g_{n+1-j} = g^{αn+1-j} \) can always be retrieved from param.

- **Decrypt(K_S, S, i, \mathcal{C} = \langle c_1, c_2, c_3 \rangle):** If \( i \notin S \), output ⊥. Otherwise, return the message: \( m = c_3 \cdot \hat{e}(K_S, \prod_{j \in S \setminus \{i\}} g_{n+1-j+i}, c_1) / \hat{e}(\prod_{j \in S \setminus \{i\}} g_{n+1-j}, c_2) \).

For the data owner, with the knowledge of \( γ \), the term \( \hat{e}(g_1, g_n) \) can be easily recovered by \( \hat{e}(c_1, g_n) = \hat{e}(g^i, g_n) = \hat{e}(g_1, g_n)^i \).

For correctness, we can see that
\[
\begin{align*}
\hat{e}(K_S, \prod_{j \in S \setminus \{i\}} g_{n+1-j+i}, c_1) &= \hat{e}(\prod_{j \in S \setminus \{i\}} g_{n+1-j}, c_2) \\
&= c_3 \cdot \hat{e}(\prod_{j \in S \setminus \{i\}} g_{n+1-j+i}, (vg^i)^t) / \hat{e}(\prod_{j \in S \setminus \{i\}} g_{n+1-j}, g^j) \\
&= c_3 \cdot \hat{e}(\prod_{j \in S \setminus \{i\}} g_{n+1-j+i}, g^j) / \hat{e}(\prod_{j \in S \setminus \{i\}} g_{n+1-j}, g^j) \\
&= m \cdot \hat{e}(g_1, g_n)^i / \hat{e}(g_{n+1}, g^j) = m.
\end{align*}
\]

### 4.2 Public-Key Extension

If a user needs to classify his ciphertexts into more than \( n \) classes, he can register for additional key pairs \( (pk_j, msk_j), \ldots, (pk_l, msk_l) \). Each class now is indexed by a 2-level index in \( \{(i, j)|1 \leq i \leq ℓ, 1 \leq j \leq n\} \) and the number of classes is increased by \( n \) for each added key.

Since the new public-key can be essentially treated as a new user, one may have the concern that key aggregation across two independent users is not possible. It seems that we face the problem of hierarchical solution as reviewed in Section 1, but indeed, we still achieve shorter key size and gain flexibility as illustrated in Figure 4. Figure 4 shows the flexibility of our approach. We achieve “local aggregation”, which means the secret keys under the same branch can always be aggregated. We use a quaternary tree for the last level just for better illustration of our distinctive feature. Our advantage is still preserved when compared with quaternary trees in hierarchical approach, in which the latter either delegates the decryption power for all 4 classes (if the key for their parent class is delegated) or the number of keys will be the same as the number of classes. For our approach, at most 2 aggregate keys are needed in our example.

Below we give the details on how encryption and decryption work when the public-key is extended, which is similar to the “√n-approach” [31].

- **Setup and KeyGen:** Same as the basic construction.
- **Extend(pk_l, msk_l):** Execute KeyGen() to get \( (v_{l+1}, γ_{l+1}) \in G \times \mathbb{Z}_p \), output the extended public and master-secret keys as \( pk_{l+1} = (pk_l, v_{l+1}), msk_{l+1} = (msk_l, γ_{l+1}) \).
• Encrypt(pk_r, (a, b), m): Let pk_r = \{v_1, \ldots, v_l, m\}. For an index (a, b), 1 \leq a \leq l, 1 \leq b \leq n, pick t \in \mathbb{Z}_p, output the ciphertext as \( C = (g^t, (v_a g_b)^t, m \cdot (g_1, g_n)^t) \).

• Extract(msk_r, S_r): Let msk_r = \{\gamma_1, \gamma_2, \ldots, \gamma_l\}. For a set S_r of indices \((i, j), 1 \leq i \leq l, 1 \leq j \leq n\), get \( g_{n+1-j} = g^{n+1-j} \) from params, output:

\[
K_{S_r} = \prod_{(1,j) \in S_r} g_{n+1-j}^{\gamma_1} \cdot \prod_{(2,j) \in S_r} g_{n+1-j}^{\gamma_2} \cdot \ldots \cdot \prod_{(l,j) \in S_r} g_{n+1-j}^{\gamma_l}.
\]

• Decrypt(K_{S_r}, S_r, (a, b), C): If \((a, b) \notin S_r\), output \( \perp \). Otherwise, let \( K_{S_r} = (d_1, \ldots, d_l) \) and \( C = \langle c_1, c_2, c_3 \rangle \). Output the message:

\[
m = \frac{c_3 \cdot \hat{e}(d_a, \prod_{(a,j) \in S_r, j \neq b} g_{n+1-j+b}, c_1)}{\hat{e}(\prod_{(a,j) \in S_r} g_{n+1-j}, c_2)}.
\]

Just like the basic construction, the decryption can be done more efficiently with the knowledge of \( \gamma_i \)'s. Correctness is not much more difficult to see:

\[
c_3 \cdot \hat{e}(d_a, \prod_{(a,j) \in S_r, j \neq b} g_{n+1-j+b}, c_1) / \hat{e}(\prod_{(a,j) \in S_r} g_{n+1-j}, c_2)
\]
\[
= c_3 \cdot \hat{e}(\prod_{(a,j) \in S_r} g_{n+1-j}^{\gamma_a}, \prod_{(a,j) \in S_r, j \neq b} g_{n+1-j+b}^{\gamma_j}, (v_a g_b)^t) / \hat{e}(\prod_{(a,j) \in S_r} g_{n+1-j}, g_1^t)
\]
\[
= c_3 \cdot \hat{e}(\prod_{(a,j) \in S_r, j \neq b} g_{n+1-j+b}^{\gamma_a}, (v_a g_b)^t) / \hat{e}(\prod_{(a,j) \in S_r} g_{n+1-j}^{\gamma_a}, g_1^t)
\]
\[
= m \cdot \hat{e}(g_1, g_n)^t / \hat{e}(g_1, g_n)^t = m.
\]

We can also prove the semantic security of this extended scheme. The proof is very similar to that for the basic scheme and therefore is omitted. The public-key of the CCA construction to be presented below can also be extended using the same Extend algorithm.

### 4.2.1 Discussions

To make the best out of our extended scheme (i.e., to make the key size as small as possible), we suggest that the ciphertext classes for different purposes should be corresponded to different public-keys. This is reasonable in practice and does not contradict our criticism on hierarchical methods that an efficient assignment of hierarchy requires a priori knowledge on what to be shared. Using our example, pk_1 and pk_2 correspond to “personal” and “work”. It is likely to have many subcategories under either of them but it may not be equally likely to share both of them (if the user does not gossip about office drama with friends and does not expose party photos to colleagues). Another example, say a user’s categorization include “music” and “game”. One day she becomes a graduate student and needs to publish, and therefore find the new need to add a category “paper”, which is probably independent of “music” and “game”.

### 4.2.2 Other Implication

This key extension approach can also be seen as a key update process. In case a secret value is compromised, we can replace the compromised pk_r with a new key pk_r'. The small aggregate key size minimizes the communication overhead for transferring the new key.

### 5 Performance Analysis

#### 5.1 Compression Factors

For a concrete comparison, we investigate the space requirements of the tree-based key assignment approach we described in Section 3.1. This is used in the Complete Subtree scheme, which is a representative solution to the broadcast encryption problem following the well-known Subset-Cover framework [33]. It employs a static logical key hierarchy, which is materialized with a full binary key tree of height h (equals to 3 in Figure 3), and thus can support up to \( 2^h \) ciphertext classes, a selected part of which is intended for an authorized delegate.

In an ideal case as depicted in Figure 3(a), the delegate can be granted the access to \( 2^n \) classes with the possession of only one key, where \( h_s \) is the height of a certain subtree (e.g., \( h_s = 2 \) in Figure 3(a)). On the other hand, to decrypt ciphertexts of a set of classes, sometimes the delegate may have to hold a large number of keys, as depicted in Figure 3(b). Therefore, we are interested in \( n_a \), the number of symmetric-keys to be assigned in this hierarchical key approach, in an average sense.

We assume that there are exactly \( 2^h \) ciphertext classes, and the delegate of concern is entitled to a portion \( r \) of them. That is, \( r \) is the delegation ratio, the ratio of the delegated ciphertext classes to the total classes. Obviously, if \( r = 0 \), \( n_a \) should also be 0, which means no access to any of the classes; if \( r = 100\% \), \( n_a \) should be as low as 1, which means that the possession of only the root key in the hierarchy can grant the access to all the \( 2^h \) classes. Consequently, one may expect that \( n_a \) may first increase with \( r \), and may decrease later. We set \( r = 10\%, 20\%, \ldots, 90\% \), and choose the portion in a random manner to model an arbitrary “delegation pattern” for different delegates. For each combination of \( r \) and \( h_s \), we randomly generate \( 10^4 \) different combinations of classes to be delegated, and the output key set size \( n_a \) is the average over random delegations.

We tabulate the results in Table 2, where \( h = 16, 18, 20 \) respectively. For a given \( h \), \( n_a \) increases with the delegation ratio \( r \) until \( r \) reaches \( \sim 70\% \). An amazing fact is that, the ratio of \( n_a \) to \( N = 2^{h+1} - 1 \), the total number of keys in the hierarchy (e.g., \( N = 15 \) in Figure 3), appears to be only determined by \( r \) but irrelevant of \( h \). This is because when the number of ciphertext classes \( (2^h) \) is large and the delegation ratio \( r \) is fixed, this kind of random delegation achieves roughly the same key

6. Legend: \( h \): The height of the binary tree; there are total \( 2^h \) ciphertext classes, \( n_a \): The number of keys to be assigned, \( N \): The total number of keys in the hierarchy, \( r \): The delegation ratio: the ratio of the delegated ciphertext classes to the total classes.


## Table 2

<table>
<thead>
<tr>
<th>$h$</th>
<th>$r$</th>
<th>$n_a$</th>
<th>$n_a/N$</th>
<th>$r2^h$</th>
<th>$n_a/g$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>0.1</td>
<td>6224.8</td>
<td>11772.5</td>
<td>16579.3</td>
<td>20545.8</td>
<td>23520.7</td>
</tr>
<tr>
<td>18</td>
<td>0.5</td>
<td>25263.8</td>
<td>25400.1</td>
<td>23252.6</td>
<td>17334.6</td>
<td>11670.2</td>
</tr>
<tr>
<td>20</td>
<td>0.9</td>
<td>25400.1</td>
<td>23252.6</td>
<td>17334.6</td>
<td>11670.2</td>
<td>6224.8</td>
</tr>
</tbody>
</table>

**TABLE 2**

Compression ratios for different delegation ratios and tree heights

---

**5.2 Performance of Our Proposed Schemes**

Our approaches allow the compression factor $F$ ($F = n$ in our schemes) to be a tunable parameter, at the cost of $O(n)$-sized system parameter. Encryption can be done in constant time, while decryption can be done in $O(|S|)$ group multiplications (or point addition on elliptic curves) with 2 pairing operations, where $S$ is the set of ciphertext classes decryptable by the granted aggregate key and $|S| \leq n$. As expected, key extraction requires $O(|S|)$ group multiplications as well, which seems unavoidable. However, as demonstrated by the experiment results, we do not need to set a very high $n$ to have better compression than the tree-based approach. Note that group multiplication is a very fast operation.

Again, we confirm empirically that our analysis is true. We implemented the basic KAC system in C with the Pairing-Based Cryptography (PBC) Library\(^8\) version 0.4.18 for the underlying elliptic-curve group and pairing operations. Since the granted key can be as small as one $G$ element, and the ciphertext only contains two

---

7. As discussed, we are not proposing a compression mechanism, yet we effectively save the costly secure storage requirement.

and one \( G_T \) elements, we used (symmetric) pairings over Type-A (supersingular) curves as defined in the PBC library which offers the highest efficiency among all types of curves, even though Type-A curves do not provide the shortest representation for group elements. In our implementation, \( p \) is a 160-bit Solinas prime, which offers 1024-bit of discrete-logarithm security. With this Type-A curves setting in PBC, elements of groups \( G \) and \( G_T \) take 512 and 1024 bits to represent, respectively.

The test machine is a Sun UltraSparc IIIi system with dual CPU (1002 MHz) running Solaris, each with 2GB RAM. The timings reported below are averaged over 100 randomized runs. In our experiment, we take the number of ciphertext classes \( n = 2^{16} = 65536 \). The Setup algorithm, while outputting \((2n + 1)\) elements by doing \((2n - 2)\) exponentiations, can be made efficient by preprocessing function offered by PBC, which saves time for exponentiating the same element \( g \) in the long run. This is the only “low-level” optimization trick we have used. All other operations are implemented in a straightforward manner. In particular, we did not exploit the fact that \( \phi(g_1, g_n) \) will be exponentiated many times across different encryptions. However, we pre-computed its value in the setup stage, such that the encryption can be done without computing any pairing.

Our experiment results are shown in Table 3. The execution times of Setup, KeyGen, Encrypt are independent of the delegation ratio \( r \). In our experiments, KeyGen takes 3.3 milliseconds and Encrypt takes 0.8 milliseconds. As expected, the running time complexities of Extract and Decrypt increase linearly with the delegation ratio \( r \) (which determines the size of the delegated set \( S \)). Our timing results also conform to what can be seen from the equation in Extract and Decrypt — two pairing operations take negligible time, the running time of Decrypt is roughly a double of Extract. Note that our experiments dealt with up to 65536 number of classes (which is also the compression factor), and should be large enough for fine-grained data sharing in most situations.

Finally, we remark that for applications where the number of ciphertext classes is large but the non-confidential storage is limited, one should deploy our schemes using the Type-D pairing bundled with the PBC, which only requires 170-bit to represent an element in \( G \). For \( n = 2^{16} \), the system parameter requires approximately 2.6 megabytes, which is as large as a lower-quality MP3 file or a higher-resolution JPEG file that a typical cellphone can store more than a dozen of them. But we saved expensive secure storage without the hassle of managing a hierarchy of delegation classes.

### 6 New Patient-Controlled Encryption

Motivated by the nationwide effort to computerize America’s medical records, the concept of patient-controlled encryption (PCE) has been studied [8]. In PCE, the health record is decomposed into a hierarchical representation based on the use of different ontologies, and patients are the parties who generate and store secret keys. When there is a need for a healthcare personnel to access part of the record, a patient will release the secret key for the concerned part of the record. In the work of Benaloh et al. [8], three solutions have been provided, which are symmetric-key PCE for fixed hierarchy (the “folklore” tree-based method in Section 3.1), public-key PCE for fixed hierarchy (the IBE analog of the folklore method, as mentioned in Section 3.1), and RSA-based symmetric-key PCE for “flexible hierarchy” (which is the “set membership” access policy as we explained).

Our work provides a candidate solution for the missing piece, public-key PCE for flexible hierarchy, which the existence of an efficient construction was an open question. Any patient can either define her own hierarchy according to her need, or follow the set of categories suggested by the electronic medical record system she is using, such as “clinic visits”, “x-rays”, “allergies”, “medications” and so on. When the patient wishes to give access rights to her doctor, she can choose any subset of these categories and issue a single key, from which keys for all these categories can be computed. Thus, we can essentially use any hierarchy we choose, which is especially useful when the hierarchy can be complex. Finally, one healthcare personnel deals with many patients and the patient record is possible stored in cloud storage due to its huge size (e.g., high resolution medical imaging employing x-ray), compact key size and easy key management are of paramount importance.

### 7 Conclusion and Future Work

How to protect users’ data privacy is a central question of cloud storage. With more mathematical tools, cryptographic schemes are getting more versatile and often involve multiple keys for a single application. In this article, we consider how to “compress” secret keys in public-key cryptosystems which support delegation of secret keys for different ciphertext classes in cloud storage. No matter which one among the power set of classes, the delegatee can always get an aggregate key of constant size. Our approach is more flexible than hierarchical key assignment which can only save spaces if all key-holders share a similar set of privileges.
A limitation in our work is the predefined bound of the number of maximum ciphertext classes. In cloud storage, the number of ciphertexts usually grows rapidly. So we have to reserve enough ciphertext classes for the future extension. Otherwise, we need to expand the public-key as we described in Section 4.2.

Although the parameter can be downloaded with ciphertexts, it would be better if its size is independent of the maximum number of ciphertext classes. On the other hand, when one carries the delegated keys around in a mobile device without using special trusted hardware, the key is prompt to leakage, designing a leakage-resilient cryptosystem [22], [34] yet allows efficient and flexible key delegation is also an interesting direction.

REFERENCES


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