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ABSTRACT

This paper studies a voluntary disclosure model in which the manager can choose to disclose across two different disclosure channels: one processed by both informed and uninformed investors, and one processed only by informed investors. Firm value is established in a competitive equilibrium setting with risk averse investors and noisy information, based on participants' expectations of firm value given the manager's disclosure choice. Long-run firm value is established through a rational expectations equilibrium. This model demonstrates a situation in which a manager will disclose more information to informed investors than to uninformed investors in equilibrium. Compared against a disclosure regime in which the manager only discloses publicly, having both disclosure channels leads the manager to increase disclosure overall while decreasing disclosure to uninformed investors. If the manager only discloses to informed investors, the overall level of disclosure is identical, but expected stock price is maximized by having both channels.

Keywords: voluntary disclosure, disclosure channels, limited attention

Data availability: Simulation data are available from the author by request.

I. INTRODUCTION

The model detailed in this paper directly addresses a concern of Francis, Nanda, and Olsson (2008): determining when managers use different channels (venues) of disclosure. This is an important issue, as empirically managers disclose information across many different disclosure channels, such as press releases, government filings, conference calls, websites, etc. By developing a model where the manager can choose between two channels for disclosure or not releasing the information at all, this study can build a theory for when a manager will choose one channel over another for voluntary disclosure. The model is developed under a structure where some investors are informed and others are not, as in Dye (1998).

To demonstrate the impact of including multiple disclosure channels, equilibria using each of the two channels in isolation are discussed. The channels considered here differ based on the observability of the channels by different investor types where one channel available for disclosure is quickly[1] processed by all investors, while the second channel only quickly processed by informed investors. This underlying structure is most closely related to the disclosure regime prior to Reg FD (Regulation Fair Disclosure). Disclosing through multiple channels was a common practice prior to Reg FD (Regulation Fair Disclosure), where companies could disclose publicly as well as privately to certain individuals. Disclosing through multiple channels is also common today, with many companies making disclosures through traditional channels such as SEC filings and conference calls, as well as through newer channels such as websites and social media. As early as 1995, the U.S. Securities and Exchange Commission (SEC) maintained a policy on electronic disclosure (SEC, 1995). In 2000, the use of firm websites for disclosure was specifically discussed (SEC, 2000), and in 2013 the SEC approved of the use of social media networks, such as Twitter and Facebook, as a channel for firm disclosure (SEC, 2013). Thus, the SEC's own policies acknowledge that firms disclose through a variety of disclosure channels.

The model takes a basic structure under which the firm is either good or bad, i.e., a market for lemons (Akerlof, 1970). No one in the model knows whether the firm is good or bad, but the manager probabilistically receives a signal from the firm that follows the distribution of the firm's type. This signal could be thought of as a forecast about the potential value of a follow-on project, or another indicator that is correlated with firm type but not correlated with firm value conditional on knowing the firm type. This signal does not tell the manager the actual value of the firm, but it is useful in determining whether the firm is good or bad. The manager then decides whether to truthfully disclose or to not disclose the information at all. In the main model, if the manager chooses to disclose the information, the manager can then choose to disclose it through an easy-to-process disclosure channel (easy channel) or a hard-to-process disclosure channel (hard channel)[2]. In the

[1]Quickly, in this context, is taken to mean costless processing within the time-frame between disclosure and trading.

[2]The naming of these channels reflects the perspective of the uninformed investors. For instance, consider SEC filings and firm websites. SEC filings are, on average, written in a difficult to read manner, with a Gunning Fog index

short run, informed investors see all information that was disclosed, while uninformed investors only see the information that was released via the easy channel. This is a strong manipulation of disclosure channel understandability, akin to a pre-Reg FD world or to disclosure via a press release or webpage (easy channel) or burying the disclosure in a 10-K footnote (hard channel). Both investor types are tasked with allocating their initial capital across the firm's stock and an inelastically priced riskless asset. As in Dye (1998), if the manager receives information but chooses not to disclose it through either channel, then the informed investors are aware that the information exists and was withheld, but they do not know what the information is, whereas the uninformed investors have no information. If the manager receives no information, the informed investors likewise know this, whereas the uninformed investors again have no information. A diagram of the flow of information in this system is presented in Figure 1.

Under this framework, the manager does take advantage of this second channel when the manager's signal is in an intermediate range. Above this intermediate range the manager chooses to disclose in the easy channel, and below this intermediate range the manager will choose to withhold information even though the informed investors are aware of the withholding. When the second channel is not present, the manager will withhold information whenever the signal falls below a certain cutoff, which happens to fall in the intermediate range. This result agrees with the primary result of Dye (1998). After deriving the expected management disclosure pattern, general expressions for the competitive equilibrium price are obtained.

In the long-run, the manager's disclosure pattern is unchanged. The uninformed investors, however, are able to completely discern the informed investors' information from the initial price, and thus all investors have the same information in the long run. Overall, the long-run results imply that managers can use multiple channels of disclosure to increase firm value in the short run without affecting firm value in the long run.

above 18 (Li, 2008). Firm websites, on the other hand, are more likely to employ plain language. Thus, firm websites should be easier for uninformed investors to process. From an informed investor's perspective, SEC filings may be easier to process, since the documents should have more consistent structures from firm to firm. However, this is not relevant to the model, as it is assumed that informed investors can costlessly process any disclosure made by the manager, regardless of channel.

Through simulation, certain model characteristics are verified (to augment proofs), unconditional expected firm value is examined, and the dynamics of the equilibrium price and management actions are observed and discussed. The simulation finds that the two channel model leads to the highest unconditional expected value of the firm, on average, and does so more than 95 percent of the time when compared against models with only an easy-to-process channel or a hard-to-process channel. This demonstrates one reason why, ex ante, managers would use multiple disclosure channels.

Related literature on informed investors in voluntary disclosure is discussed in the next section. In Section III, the general structure of the model is described. Section IV derives the main model under a competitive equilibrium and a rational expectations equilibrium and compares the main model against models containing each disclosure channel in isolation. Section V uses simulation to examine implications of the model as well as the dynamics of the equilibrium. Section VI concludes. Proofs not included inline are contained in the appendix.

II. REVIEW OF THEORY

Most of the research on voluntary disclosure focuses on the unraveling result (Grossman and Hart, 1980; Grossman, 1981; Milgrom, 1981; Milgrom and Roberts, 1986), altering one or more of the six conditions for full disclosure. The most pertinent element of the unraveling result to this study is the uniformity of investor response. Suijs (2007) found that violating this condition while holding all others constant is sufficient to induce less than full disclosure. A specific variant of violating uniform investor response that has been considered is varying investor sophistication. Dye (1998) uses rational investors with probabilistic information acquisition, finding that managers disclose more when investors have a higher probability of acquiring information. Fishman and Hagerty (2003) demonstrates a model with two levels of sophistication, where one level is capable of interpreting disclosures and the other only knows the disclosure happened.

The endogenous model in this study builds upon these models, particularly Dye (1998), by having two separate channels of information: an easy-to-process disclosure channel (easy channel) which can be quickly processed by all investors, and a hard-to-process disclosure channel (hard channel) which can only be quickly processed by informed investors. The hard channel can be processed by all investors in the long run, however. Furthermore, the information that can be quickly processed by the uninformed investors is not determined probabilistically, but is determined endogenously by the manager.

This paper is also related to the stream of literature on limited investor attention. Limited attention has been modeled in the disclosure literature to examine how investors might react to different disclosure formats and rules (Hirshleifer and Teoh, 2003), as well as earnings news (Hirshleifer, Lim, and Teoh, 2011). Empirically, some extent of limited investor attention has been documented by Dellavigna and Pollet (2009), showing that stock prices react more slowly to information on Fridays. Likewise, Barber and Odean (2008) empirically demonstrate that investors are more likely to purchase than sell stocks due to attention grabbing news, as investors holding the stock are likely to be paying closer attention to firm news. The implementation of informed and uninformed investors in this study can be interpreted as a strong use of limited attention, since the uninformed investors do not observe any information in the hard channel. However, such a strong use is needed to maintain tractability in the model.

III. MODEL STRUCTURE

The model takes place in a one period setting in which there are 3 types of players, one manager working for one firm and two investor types. The main model considers a case with two disclosure channels. The number and types of investors are taken to be exogenous, with N_U uninformed investors and N_I informed investors, N investors in total. At the start of the first period, the firm type is chosen to be either good or bad; the firm is good with probability p_G. If the firm is good (bad), its expected value at period 1 will follow a normal distribution with mean μ_G (μ_B) and variance σ^2. Variance is kept constant across firm types for parsimony in the model results. A signal $\tilde{y}$ exists, such that the signal follows the true distribution of the firm; thus, if the firm is good (bad), $\tilde{y} \sim N(\mu_G, \sigma^2)$ ($\tilde{y} \sim N(\mu_B, \sigma^2)$). It is assumed that the manager receives the signal with some probability, $\hat{p}$, as in Dye (1985), Jung and Kwon (1988), and Dye (1998). If the manager receives the signal, then the manager can choose whether or not to release the signal. If the manager chooses to release the signal, the manager must do so truthfully, and the manager will then choose which channel to release the signal through. Investors then receive information based on their type and the channel chosen by the manager, determine their desired amount of shares at each possible share price, and participate in one round of silent trading in order to establish the firm's stock price. Throughout this model it is assumed that the manager does not and cannot manipulate the content of any disclosure, as in Dye (1985), Jung and Kwon (1988), and Dye (1998). Consequently, this model assumes that the information contained in a disclosure must be the same regardless of the manager's disclosure channel choice. While this is an idealized case, disclosure regulation, which could account for some variation in information content ex ante, should be constant across channels—per SEC Release No. 33-7856 (US SEC, 2000), the disclosing party is responsible for the accuracy of the disclosures regardless of the channel or medium through which the statement is made.

Model setup

Disclosure channels

By allowing for multiple channels, voluntary disclosures can, at the manager's discretion, have different costs of access for investors. The main model in this paper implements this by having two channels for disclosure, an easy channel and a hard channel. If the two investor groups have a different ability to process disclosures among these two channels, then the manager could potentially use this structure for personal gain when compared to a single disclosure channel model. In practice, given that firm disclosures naturally vary in readability based on the channel of the disclosures, it is expected that disclosure channels will vary in the ease of processing across investor types. For instance, annual reports tend to have low readability (Li, 2008), whereas disclosures via social media are likely to be more readable. Based on the information that the investors receive, a price per share of the firm, P_0, will be determined. There are $\bar{x}$ shares available.

Investors

In the models, there are two types of investors. The first type of investor, informed investors, has sufficient processing capability to quickly process all information voluntarily released by the manager. Consequently, informed investors see all information in both channels before trading. Furthermore, informed investors are aware of whether the manager received a signal or not (as in Dye (1998)).[3] The second type of investor, uninformed investors, is able to quickly process only the information disclosed through the easy channel. Uninformed investors, however, are not capable of detecting or quickly processing the information in the hard channel, and the cost to process the hard channel is assumed to be too high to attempt processing. All investors' expected utility is based on their expectation of the underlying value of the firm, the amount of the stock they purchase, and the amount of the risk-free asset that they hold. The risk-free asset provides a return plus 1 of R_f and is perfectly elastic in terms of quantity. Furthermore, it is assumed that the investors are risk averse, with a utility function following constant absolute risk aversion (CARA), and in particular the investors have a utility function $U(x) = -e^{-ax}$, where $a > 0$ is the coefficient of risk aversion.[4] Lastly, it is assumed that there are $N_I > 0$ informed investors and $N_U > 0$ uninformed investors.

Manager

The model assumes the presence of just one firm with one manager. Inside the firm there is private information in the form of a signal, $\tilde{y}$, related to the expected value of the firm at time 1. The manager then receives this signal with some probability $\hat{p}$. The signal $\tilde{y}$ is noisy and follows the distribution of the firm's value for the actual firm type, providing information about the firm type in a Bayesian sense. If the firm is good, firm value at time 1 ($\tilde{P}_1$) and $\tilde{y}$ will follow $N(\mu_G, \sigma^2)$; if the firm is bad, $\tilde{P}_1$ and $\tilde{y}$ will follow $N(\mu_B, \sigma^2)$. When the manager receives the signal, the manager will decide whether or not to disclose the signal based on the signal's value, the manager's utility function, and the expected interpretation and actions by other market participants. If the manager chooses to disclose information, the manager must pick between the easy and hard channels to disclose through. While the manager could disclose through both channels simultaneously, doing

[3]This assumption is required in order to induce partial disclosure, as otherwise the manager has no incentive to treat the two types of investors differently. Without this assumption, informed investors would have no extra information compared to uninformed investors when the managers does not disclose. Thus, in equilibrium, the manager would ignore the hard channel.

[4]This form of utility is used as it allows for tractability when the information processes described in the next section are normally distributed.

so is equivalent to disclosing through only the easy channel in this setting, given the investors' information acquisition process and that disclosing information is costless. The manager's utility is assumed to be risk neutral and linearly increasing in the initial stock price, P_0, of the firm.[5]

IV. MAIN MODEL

Under the framework outlined above, a short run competitive equilibrium can be determined.

Definition 1 (Competitive equilibrium)**.** *A competitive equilibrium for the model is the set of manager actions and investor actions covering all possible states of nature; whether the manager receives a signal and, if so, the value of the signal. The manager's actions must lead to the highest price P_0 given the optimal actions of the investors, and the investors must maximize their expected utility with respect to their own information sets. The uninformed investors' information set includes whether or not a disclosure was made through the easy channel, and if so, what the signal was. The informed investors' information set includes whether or not the manager received a signal, whether or not the manager disclosed a signal through either channel, and if so, what the value of the signal was. Information on all probabilities, distributions, risk aversion, number of each investor type, and number of shares available is common knowledge to all participants.*

This competitive equilibrium can be thought of as follows. Suppose that the manager understands the makeup of the market, and strategically discloses to maximize the outcome of a one period auction in which all shares must be sold. Then, investors use the information they receive (either the disclosure or the lack thereof), and participate in a silent auction in which the highest price that clears all market shares is chosen. As such, the investors cannot obtain information from the auction itself, as the moment any usable information is generated, i.e., the market clearing price, the auction ends. A rational expectations equilibrium following the same structure is considered in Section IV.

Initially, there are four states to consider: a signal is obtained by the manager and disclosed

[5]Risk neutral utility is chosen for tractability. Imposing a CARA utility function for the manager is not tractable in this model.

to all investors (Full Disclosure, FD), a signal is obtained but is disclosed only to informed investors (Partial Disclosure, PD), a signal is obtained and is not disclosed (Withholding, W), and no signal is obtained (No Information, NI). As in Dye (1998), the manager may have an incentive not to disclose if the signal $\tilde{y}$ is too low. This causes the manager's non-disclosure signal to not be a credible sign of having no information, and investors will behave accordingly. If the manager could credibly contract to disclose the signal at any value of $\tilde{y}$, non-disclosure could be a credible signal that the manager did not receive a signal. Without such a contract, there may instead exist a stable point c_W at which the manager is indifferent between full disclosure and withholding when $\tilde{y} = c_W$. Existence of c_W is equivalent to the manager having an incentive to not always fully disclose under a model in which only the easy channel exists. Furthermore, there may be two additional stable points: one where the manager is indifferent between partial disclosure and full disclosure (when $\tilde{y} = c_{FD}$) and one where the manager is indifferent between partial disclosure and withholding (when $\tilde{y} = c_{PD}$). These two points, c_{FD} and c_{PD}, will define the manager's incentives under equilibrium in this model.

To simplify the discussion of the equilibrium going forward, the parameter space considered will be restricted. The analysis will focus on the the parameter spaces such that under the on-path competitive equilibrium, a high value of $\tilde{y}$ is indicative of a good firm and a low value of $\tilde{y}$ is indicative of a bad firm. These equilibriums will be termed "non-degenerate." While it is possible for a low $\tilde{y}$ to be indicative of a bad firm if $\sigma_G > \sigma_B$, the results for such equilibriums will mirror the results for the non-degenerate equilibriums.

Definition 2. *A non-degenerate competitive equilibrium for the competitive equilibrium as described in Definition 1 is one such that there exists some y_1 and y_2 such that $P_0(\tilde{y}) > P_0(y_1) \forall \tilde{y} > y_1$ and $P_0(\tilde{y}) < P_0(y_2) \forall \tilde{y} < y_2$.*

Essentially, a non-degenerate equilibrium requires investors to infer that a high value of the signal is good and a low value of the signal is bad, though this need not be true for intermediate values of $\tilde{y}$. Before deriving the equilibrium, an existence condition and uniqueness are discussed.

Theorem 1 (Condition for existence of a non-degenerate competitive equilibrium)**.** *A non-degenerate competitive equilibrium as described in Definitions 1 and 2 exists whenever the following condition holds:*

$$\mu_G - \mu_B > 2ap_G \frac{\bar{x}}{N}\sigma^2.$$

Proof. See Appendix B. ∎

This existence criterion is intuitive. Under full disclosure, while the expected value of the firm is greater for high values of $\tilde{y}$, the variance of the outcome will also be higher. As the investors are risk averse, they do not only care about the difference in the means, but also in the variance of the outcome. The price is penalized by a function that is increasing in the risk aversion of investors and the number of shares available per investor. As risk aversion increases, investors will decrease their willingness to pay for a share of the firm as they will be less willing to take on the risk of the asset. Likewise, as the variance of the unconditional firm type distribution increases, investors will assess a greater penalty due to their risk aversion. As p_G increases, the difference between the expectation of $\tilde{P}_1$ conditional on $\tilde{y}$ and the unconditional expectation of $\tilde{P}_1$ decreases. Lastly, as the number of shares in the market relative to the number of investors increases, the price of the shares decrease as all shares must be traded in order to complete the market.

Under the existence criterion, any disclosure pattern by the manager is likely to be unique. However, proving uniqueness under the structure of this model is elusive due to the lack of monotonicity in the price under partial disclosure, $P_{0,PD}$. Instead, uniqueness is tested in the simulation in Section V. Based on the simulation, any competitive equilibrium under the condition of Theorem 1 appears to be unique.

Investors' actions

Before solving for the manager or investor actions, the perceived probabilities of ending up in a state must be defined.

Lemma 1 (Conditional probabilities)**.** *When c_{FD} and c_{PD} exist and $c_{FD} > c_{PD}$, the conditional probability that each investor type $i \in \{I, U\}$ perceives the firm is good in state $s \in \{FD, PD, W, NI\}$ are given by:*

$$p_{I,FD} = p_{I,PD} = p_{U,FD} = \frac{p_G \phi(\beta_{y,G})}{p_G \phi(\beta_{y,G}) + (1 - p_G)\phi(\beta_{y,B})},$$

$$p_{I,W} = \frac{p_G \Phi(\beta_{c_{PD},G})}{p_G \Phi(\beta_{c_{PD},G}) + (1 - p_G)\Phi(\beta_{c_{PD},B})},$$

$$p_{I,NI} = p_G,$$

$$p_{U,PD} = p_{U,W} = p_{U,NI} = \frac{p_G \left(\hat{p}\Phi(\beta_{c_{FD},G}) + (1 - \hat{p})\right)}{\hat{p}\left(p_G \Phi(\beta_{c_{FD},G}) + (1 - p_G)\Phi(\beta_{c_{FD},B})\right) + (1 - \hat{p})},$$

Where:

$$\beta_{x,T} = \frac{x - \mu_T}{\sigma},$$

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2},$$

$$\Phi(x) = \frac{1}{2}\left(1 + \mathit{Erf}\left(x/\sqrt{2}\right)\right),$$

Erf is the error function, given by Erf$(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$

Each of these probabilities follows directly from Bayes' theorem using the underlying probability distributions of the two firm types as well as the information set known to each investor type on each disclosure state. Given the above probabilities and that the firm value distributions under each firm type (good and bad) are normal, the conditional distribution the firm follows under a given investor-state pair (i, s) is given by

$$N\left(p_{i,s}\mu_G + (1 - p_{i,s})\mu_B, (2{p_{i,s}}^2 - 2p_{i,s} + 1)\sigma^2\right). \tag{1}$$

For simplicity, assume that all investors have the same starting wealth W_0 and the same coefficient of risk aversion, a. The investors' problem will be to determine the number of shares to purchase, x_i, that maximizes their expected utility based on the information set they receive, I_i, where i denotes the investor:

$$\begin{aligned} & \max_{x_i} \mathbb{E}\left[U(\tilde{W}_{1,i})\right], \\ = \quad & \max_{x_i} \mathbb{E}\left[U(R_f W_0 + (\tilde{P}_1 - R_f P_0)x_i)|I_i\right]. \end{aligned}$$

As U is exponential and $\tilde{P}_1$ is conditionally normal, this is equivalent to:

$$\begin{aligned} & \max_{x_i} a\mathbb{E}\left[\tilde{W}_{1,i}|I_i\right] + \frac{1}{2}a^2\mathbb{V}\left[\tilde{W}_{1,i}|I_i\right], \\ = \quad & \max_{x_i} R_f W_0 - R_f P_0 x_i + \mathbb{E}\left[\tilde{P}_1|I_i\right]x_i + \frac{1}{2}a^2 {x_i}^2\mathbb{V}\left[\tilde{P}_1|I_i\right], \\ \Rightarrow \quad & x_i = \frac{\mathbb{E}\left[\tilde{P}_1|I_i\right] - R_f P_0}{a\mathbb{V}\left[\tilde{P}_1|I_i\right]}. \end{aligned} \tag{2}$$

Once investors' behavior under a certain state is derived, the initial price can be determined by aggregating the each x_i up to the number of shares available, $\bar{x}$. This allows for solving the price, P_0, that clears the market for the $\bar{x}$ shares.

Equilibrium

Now an expression for the basic behavior under each state can be derived. This follows from summing equation (2) for each investor and setting the sum equal to $\bar{x}$, the total number of shares available. The resulting equation can then be solved to determine $P_{0,s}$. Likewise, given the optimal strategy for investors to follow, the manager's strategy can also be determined. Thus, the equilibrium can be defined.

Theorem 2 (Competitive equilibrium)**.** *If the manager receives a signal, $\tilde{y}$, then the manager's optimal action is to withhold if $\tilde{y} < c_{PD}$, partially disclosure if $c_{PD} \leq \tilde{y} < c_{FD}$, and fully disclose if $c_{FD} \leq \tilde{y}$. The point c_{FD} is the value of the signal at which an uninformed investors' value of the firm is identical between full disclosure and partial disclosure, while the point c_{PD} is the value of the signal at which an informed investors' value of the firm is identical between partial disclosure and withholding. If the manager has no information, then the manager's only action is to choose* *not to disclose.*

Aggregate investor actions under each state $s \in \{FD, PD, W, NI\}$ *are summarized by the attained price* P_0 *and and the number of shares purchased by each investor,* $x_{i,s}$*:*

$$P_{0,s} = \frac{N_I \frac{p_{I,s}\mu_G + (1-p_{I,s})\mu_B}{(2p_{I,s}{}^2 - 2p_{I,s}+1)\sigma^2} + N_U \frac{2p_{U,s}\mu_G + (1-p_{U,s})\mu_B}{(2p_{U,s}{}^2 - 2p_{U,s}+1)\sigma^2} - a\bar{x}}{R_f \left[\frac{N_I}{(2p_{I,s}{}^2 - 2p_{I,s}+1)\sigma^2} + \frac{N_U}{(2p_{U,s}{}^2 - 2p_{U,s}+1)\sigma^2} \right]},$$

$$x_{i,s} = \frac{\mathbb{E}\left[\tilde{P}_1 | I_i\right] - R_f P_{0,s}}{a\mathbb{V}\left[\tilde{P}_1 | I_i\right]},$$

where $p_{i,s}$ *is defined as in Lemma 1 above.*

Proof. See Appendix C for the derivation of investors' optimal actions and Appendix D for the derivation of the optimal manager actions. ∎

The manager will choose to take advantage of both channels that are available. For high enough signals, the manager will disclose through the easy channel, informing all investors of the signal's value. For low values of the signal, the manager will withhold the signal. For intermediate values of the signal, the manager will disclose through the hard channel, only informing the informed investors of the signal's value. The manager discloses differently due to being able to pool with the no information state for uninformed investors, but not being able to do so for informed investors. Instead, the manager provides extra information to informed investors to raise their perception of the firm's value. This result implies that the availability of multiple disclosure channels affects the information environment of the market, as the two disclosure channels are used to disclose different information. Consequently, the existence of multiple disclosure channels may have an impact on firm stock price formation.

To better understand the effect of having a second disclosure channel, a competitive equilibrium under the same framework is derived when only one channel, the easy channel, is available to the manager. Such a model is analogous to Dye (1998) in that it examines one channel of disclosure that is visible to all investors along with investor sophistication. However, the model in Dye

(1998) is under different assumptions: 1) this paper uses a trading model, and consequently includes a risk-free asset along with the risky asset of the firm; 2) the signal in Dye (1998) is the firm value, rather than a random signal from the firm value's distribution[6]; 3) the makeup of investors is pre-determined, with N_I informed investors and N_U uninformed investors[7]; 4) firm value follows a normal distribution as opposed to a general distribution with a weakly decreasing probability density function (PDF) and investors are risk averse as opposed to risk neutral. These assumptions flow from the standard setup of a competitive equilibrium market pricing model (Grossman, 1976). Still, the base results of Dye (1998) continue to hold under this new framework. Corollary 2.1 presents the equilibrium of this easy channel only model.

Corollary 2.1 (Manager action under 1 channel: easy channel)**.** *If the manager is restricted to have only the easy channel to disclose through, and the condition specified in Theorem 1 holds, the manager's optimal disclosure pattern still depends on if the manager receives a signal and the value of the signal. If the manager receives a signal, there exists some point c'_{FD} such that the manager will fully disclose when $\tilde{y} > c'_{FD}$. The manager will withhold the signal when $\tilde{y} < c'_{FD}$. If the manager does not receive a signal, the manager will not disclose. Furthermore, $c_{PD} < c'_{FD} < c_{FD}$.*

Proof. That $c_{PD} < c'_{FD} < c_{FD}$ is a consequence of the ordering in Theorem 2. From Theorem 2 it is known that $P_{0,FD}(\tilde{y}) > P_{0,W}(\tilde{y})$ at $\tilde{y} = c_{FD}$ and $P_{0,FD}(\tilde{y}) < P_{0,W}(\tilde{y})$ at $\tilde{y} c_{PD}$. Since $P_{0,FD}(\tilde{y})$ and $P_{0,W}(\tilde{y})$ are continuous, then $f(\tilde{y}) = P_{0,FD}(\tilde{y}) - P_{0,W}(\tilde{y})$ is continuous and maps (c_{PD}, c_{FD}) to (v_1, v_2) where $v_1 < 0$ and $v_2 > 0$. If f evaluates to 0 for some point, then the manager would be indifferent between full disclosure and withholding. By the Intermediate Value Theorem, there exists some point $c'_{FD} \in \{c_{PD}, c_{FD}\}$ such that $f(c'_{FD}) = 0$, proving existence. ∎

This result is identical in spirit to the equilibrium in Dye (1998), and this model allows the

[6]This difference is needed in order to maintain tractability. Under the Dye (1998) framework, introducing a second disclosure channel leads to an intractable distribution of bids for uninformed investors. Likewise, under a standard competitive equilibrium in which the signal is the firm value, the expected utility under any withholding case becomes intractable as the distribution of expected firm values follow a distribution akin to a product of a normal PDF and normal CDF weighted to integrate to 1.

[7]This assumption improves tractability. In particular, this assumption removes the need to sum across a binomial distribution to get the expected investor reactions.

management actions underlying the two channel model can be compared against a one channel model that is consistent with prior literature. In particular, notice that since $c'_{FD} < c_{FD}$, the manager will choose to decrease disclosure through the easy channel when the manager has the option of using a hard-to-process disclosure channel. Furthermore, the manager *increases* the overall level of disclosure when having access to both channels, as $c_{PD} < c'_{FD}$. This implies the existence of potential market efficiencies and inefficiencies from managers having multiple disclosure channels.

To further explore the consequences of having multiple disclosure channels, the case in which only the hard channel exists is also examined. While this model is farther from reality, comparing it against the two channel model shows the impact that allowing for public disclosure through a channel similar to the easy channel. The manager's disclosure under this setup is as follows.

Corollary 2.2 (Manger action under 1 channel: hard channel)**.** *If the manager is restricted to have only the hard channel to disclose through, and the condition specified in Theorem 1 holds, the manager's optimal disclosure pattern still depends on if the manager receives a signal and the value of the signal. If the manager receives a signal, there exists some point c'_{PD} such that the manager will fully disclose when $\tilde{y} > c'_{PD}$. The manager will withhold the signal when $\tilde{y} < c'_{PD}$. If the manager does not receive a signal, the manager will not disclose. Furthermore, $c'_{PD} = c_{PD}$.*

Proof. First, note that the informed investors' information set under this setup is identical to that of the main model – partial disclosure and full disclosure lead to the same inference about firm type given the same $\tilde{y}$ for these investors. Furthermore, note that the uninformed investors cannot receive any information in this setting. As such, disclosure will only impact the informed investors. The manager will be indifferent between disclosure and non-disclosure at $c'_{PD} = c_{PD}$, as the managers problem is the same as in determining c_{PD} in Theorem 2 (solution detailed in Appendix D). ∎

Interestingly, the absence of the easy channel does not affect the manager's use of the hard channel, unlike in the case with only the easy channel. This is because the hard channel is sufficient to mimic the same strategy for setting the information set of the informed investors by disclosing through the hard channel over both the full disclosure and partial disclosure regions of the two channel model. This disclosure strategy stemming from the lack of an easy channel does impact the attained stock price though, as the uninformed investors will unconditionally discount the value of the firm since they have no possibility to observe the signal of the ex post value of the firm. As the price impact is not solvable, the impact of removing the easy channel will be discussed in greater detail in the simulation in Section V. The primary result of the simulation is that removing the easy channel almost always decreases unconditional expected firm value.

Long run implications

To further understand the dynamics of the investor and manager actions in the underlying environment, this section considers a multi-period steady-state style equilibrium. In particular, it considers an equilibrium in which investors not only consider the disclosure information they receive, but also the stock price as determined in the first period, i.e., in the competitive equilibrium. Such an equilibrium is a rational expectations equilibrium.

Definition 3 (Rational expectations equilibrium)**.** *A rational expectations equilibrium for the model is the set of manager actions and investor actions covering all possible states of nature, namely whether the manager receives a signal and, if so, the value of the signal. Furthermore, the manager's actions must lead to the highest price* P_0 *given the optimal actions of the investors, and the investors must maximize their expected utility with respect to their own information sets. The uninformed investors' information set includes whether or not a disclosure was made through the easy channel, and if so, what the signal was. The informed investors' information set includes whether or not the manager received a signal, whether or not the manager disclosed a signal through either channel, and if so, what the value of the signal was. Information on all probabilities, distributions, risk aversion, number of each investor type, and number of shares available is common knowledge to all participants. Furthermore, the current price as determined by the investors' trade is visible to all parties and is used to construct a law of motion* f_I *and* f_U *mapping the beliefs of each* *investor type from one state to the next.*

Because the price is visible to all parties, it is possible that the investors may be able to infer the other party's information from the price. Because the uninformed investors' information is a subset of the informed investors' information, the informed investors cannot gather any additional information, but the uninformed investors can. For uninformed investors, if the price is a sufficient statistic for the informed investors' information, then it would be possible for the them to perfectly determine the informed investors' information. In such a case, the equilibrium is said to be fully-revealing.

Theorem 3 (Fully revealing equilibrium)**.** *The equilibrium discussed in Theorem 2 is fully revealing in a rational expectations context whenever*

$$\mu_G - \mu_B > 2a\frac{\bar{x}}{N}\sigma^2. \tag{3}$$

Proof. See Appendix E for the derivation showing that the rational expectations equilibrium will be fully revealing. ∎

Given the above theorem, a rational expectations equilibrium will exist and will be fully-revealing, so long as the condition in equation (3) holds. Note that this is the same condition as for existence of a non-degenerate competitive equilibrium from Theorem 1. This means that after observing the price that attains in the competitive equilibrium, all investors will act as though they are informed.

Theorem 4 (Rational expectations equilibrium)**.** *When $\mu_G - \mu_B > 2a\frac{\bar{x}}{N}\sigma^2$, the law of motion f_I is the function $f(x) = x$, as informed investors' beliefs are unchanged, while the law of motion f_U maps $P_{0,s}$ to $p_{I,s}$. Initially, investor actions under each state s are defined by*

$$P_{0,s} = \frac{N_I \frac{p_{I,s}\mu_G + (1-p_{I,s})\mu_B}{(2{p_{I,s}}^2 - 2p_{I,s}+1)\sigma^2} + N_U \frac{2p_{U,s}\mu_G + (1-p_{U,s})\mu_B}{(2{p_{U,s}}^2 - 2p_{U,s}+1)\sigma^2} - a\bar{x}}{R_f\left[\frac{N_I}{(2{p_{I,s}}^2 - 2p_{I,s}+1)\sigma^2} + \frac{N_U}{(2{p_{U,s}}^2 - 2p_{U,s}+1)\sigma^2}\right]},$$

where $p_{I,s}$ is defined as in Lemma 1. Factoring in the law of motion above and that the initial equilibrium is fully revealing, investor actions under each state s for any subsequent period k are given by:

$$P_{k,s} = \frac{1}{R_f}\left(p_{I,s}\mu_G + (1 - p_{I,s})\mu_B - a\frac{\bar{x}}{N}(2{p_{I,s}}^2 - 2p_{I,s} + 1)\sigma^2\right),$$

The manager's optimal action set when receiving a signal is to disclose when $\tilde{y} > c_{PD}$, and to withhold when $\tilde{y} < c_{PD}$. The manager is indifferent between disclosure and withholding when $\tilde{y} = c_{PD}$.

Proof. The investors' action is identical to that of Theorem 2, except with $N_I = N$ and $N_U = 0$. This is a direct consequence of all investors acting as though they are informed.

The manager's action is simply to disclose only when it leads to a higher expected value than withholding for informed investors. As the cutoff, c_{PD}, is invariant to the number of informed investors, it is the same as c_{PD} in the competitive equilibrium case. Since no investors will behave as anything other than an informed investor, the manager need not consider other investor types. ∎

This shows that the manager's competitive equilibrium strategy and rational expectations equilibrium strategy are compatible. This result allows the manager to obtain a maximum stock price in both the first and latter periods. Furthermore, this result implies that the manager can achieve a higher stock price in the short run without harming the stock price in the long run. Thus, there is no direct agency issue in terms of harming firm value, but there is a potential expropriation of wealth if the manager has incentives based on the period 1 (competitive equilibrium) stock price rather than the long-run (rational expectations equilibrium) stock price.

V. MODEL CHARACTERISTICS

This section further examines the model through a numerical example and simulation. The simulation is used to provide supplemental analysis for some of the proofs in the preceding section and to examine comparative statics of the model.

Numerical example

Parameter values for the numerical example are detailed in Table 1. Values for the numerical example are chosen to be representative of an easy to illustrate case – the numbers abide by the condition of Theorem 1, and the numbers lead to slopes that are neither extremely close to 0 nor extremely large around the cutoff points. The behavior of slopes of P_0 on $\tilde{y}$ at cutoff points is particularly important in being able to see how the equilibrium is determined, though the equilibrium holds equally well without it.

The method to obtain the manager's optimal action is illustrated in two steps in Figure 2. Panel A shows the first part of the manager's optimization problem: Given that investors know that the manager has an incentive to withhold information and that the manager has the option to use a second channel which uninformed investors cannot quickly process, the manager must determine the information to disclose through the easy channel. In Panel A, the crossing of the full disclosure and partial disclosure lines indicates the value of the signal at which the manager is indifferent between partial and full disclosure. In this example, that cutoff is at 0.8825 – between the mean unconditional expected values of the good and bad firm types. Once this cutoff is determined, the cutoff between partial disclosure and withholding can be determined, which is illustrated in Panel B of Figure 2. In Panel B, the intersection of the two lines again indicates the point at which the manager would be indifferent between partial disclosure and withholding. This occurs at 0.3678. Note that while the graphs appear to converge on the left side, they are not quite equal – withholding leads to a higher price for all cutoffs less than the optimal cutoff, though the two lines limit to the same value.

In this numerical example, if the manager only had one channel available, a cutoff of 0.7535 would be optimal. Consistent with Corollary 2.1, the inclusion of a second channel leads to a decrease in the information available through the easy channel. Likewise, the inclusion of a second channel leads the manager to release *more* information to the market overall.

Figure 3 depicts how these disclosure choices impact the firm's stock price. In Panel A, the stock price under each of the three disclosure strategies (along with the no information case) are presented for each level of the signal $\tilde{y}$. The points of indifference discussed above, c_{FD} and c_{PD}, are included as solid vertical lines. As the price determined by withholding is formed purely based on expectations derived from the endogenously determined cutoffs, the price is constant with respect to the value of the signal. On the far left, this constant value dominates the price that would be obtained through any type of disclosure. For middle values of the signal, the price from partial and full disclosure increases at a rapid rate. As the uninformed investors' risk-adjusted expectation of firm value is higher than that of the informed investors, partial disclosure leads to a higher overall price in this region. On the right portion of the graph, the price from full disclosure dominates, as the signal is sufficiently high enough to raise the uninformed investor's risk-adjusted expected value of the firm compared to the no disclosure case for them. In particular, when no disclosure is visible to the uninformed investors, they endogenously determine a 38.5 percent chance that the firm is good. Likewise, the rightmost cutoff occurs when full disclosure leads to a 38.5 percent probability that the firm is good. This is lower than the unconditional probability that the firm is good of 50 percent.

Simulation

In order to understand how changes in the model parameters affect stock price and disclosure behavior in the model, 10,000 iterations of a simulation of the model were run. Distributions for model parameters were chosen such that they always satisfy Theorem 1; the distributions are presented in Table 1. Parameters were chosen to represent a wide variety of values and for tractability. The probability of the manager acquiring the signal is allowed to vary from 0 to 1, encompassing all possible values. The number of investors, N, is fixed at 100 for simplicity in applying a distribution to the number of shares available, $\bar{x}$, as the ratio of these quantities drives the effect of $\bar{x}$. $\bar{x}$ is allowed to vary between 50 and 100 shares, at integer values only. The number of uninformed investors is allowed to vary across the entire range of integers from 1 to $N-1$, inclusive, guaranteeing that there is at least one of each investor type. The coefficient of risk aversion, a, is allowed to vary between 0 and 1, as that level covers a large amount of empirically identified risk aversion levels (see Table 1 of Babcock, Choi, and Feinerman (1993)). The percentage chance that the firm is good is chosen to be between 0 and 0.5 for tractability in the simulation. The probability that the firm is good heavily influences the slope of the stock price, P_0, with respect to the signal $\tilde{y}$, making the slopes at the indifference points between disclosure choices more similar as p_G increases. As such, the numerical solver employed converges more efficiently when $p_G < 0.50$. The standard deviation, σ is bounded between 0.5 and 1, while μ_G is bounded between 1 and 3, and μ_B is set to 0. These choices assure that the condition of Theorem 1 is fulfilled and allow μ_G to vary between 1 and 6 standard deviations above μ_B.

Proof verification

The first verification focuses on existence and uniqueness. As expected given the proof of existence, a solution is found for the main model simulation in all 10,000 iterations, as the parameters conform to the condition in Theorem 1. For each of these 10,000 iterations, one unique utility maximizing solution is found for the manager in terms of the disclosure cutoffs c_{FD} and c_{PD}. This also holds true for both one channel models. This provides evidence that the equilibrium described in Theorem 2 is unique.

The second verification, for Theorem 3, regards the monotonicity of P_0 in $\tilde{y}$ over the partial disclosure region. In the simulation, P_0 is monotonic in $\tilde{y}$ in all 10,000 iterations, as the minimum derivative of p_0 with respect to $\tilde{y}$ over the partial disclosure region across all iterations is 2.39×10^{-6}. As such, this provides evidence that the value of P_0 is always increasing in the value of the signal throughout the partial disclosure range, so long as the existence condition is met. The proved portion of Theorem 3, that P_0 is monotonic in $\tilde{y}$ over the full disclosure region, is also verified by the simulation. In all 10,000 iterations, the derivative of P_0 with respect to $\tilde{y}$ is positive, with a minimum derivative of 5.32×10^{-6}.

Results and statics

This section examines the outcomes of the simulation to draw conclusions on unconditional expected stock prices and the amount of disclosure by managers.

When examining the levels of the unconditional expected stock price, it is observed that the average price is highest with two channels, at 0.8093. With just the easy channel available, the average unconditional expected price is just 0.16 percent lower, while with just the hard channel available the average unconditional expected price is 31.5 percent lower. Without any disclosure possible, the average unconditional expected price is 56.2 percent lower than with both channels available. As such, it appears that the easy channel is more valuable than the hard channel, as it influences a greater number of investors beliefs about the firm's value. Even so, the hard channel, in isolation, does have a large impact on price by itself, but has a much lower impact when combined with the easy channel.

Despite the relatively close unconditional expected stock prices from the two channel and the easy channel only models, the two channel model leads to a higher unconditional expected stock price over 96 percent of the time in the simulation (9,609 of 10,000 iterations). Compared to the hard channel only model, both the two channel and easy channel only models lead to a higher unconditional expected stock price in all but 16 iterations.

Regarding the amount of information disclosed, the two channel case led to less disclosure in the easy channel in each iteration, compared to the easy channel only model, though more information was disclosed overall in each iteration. This is consistent with Corollary 2.1. Consistent with Corollary 2.2, the amount of information disclosed under the hard channel only model was the same as the amount of information disclosed under the two channel model.

In order to understand how the unconditional stock price and the disclosure pattern change with respect to the various parameters of the model, the simulation is necessary, as statics for this model are not analytically attainable.[8] The simulation varies each parameter of the model to understand the overall effects of each parameter.

Figure 4 illustrates how the unconditional expected stock price varies with each of 7 parameters. As expected, increasing the mean of the distribution of good firm value increases the expected price, since that directly increases the investors expectation of firm value. Likewise, increasing the standard deviation leads to a decrease in stock price, as the investors are risk averse.

[8]In particular, maximizing over the disclosure options leads to a maximization problem in which the main determinant, $\tilde{y}$ is contained in both the PDF and CDF of the normal distribution, leading to a seemingly intractable maximization problem.

Increasing the probability the firm is good has two main effects. First, the firm is more likely to be good, and thus the expected value of the firm will be higher. Second, increasing p_G will, over certain ranges, lead to an increase in the perceived volatility of the firm. On average, it appears that the volatility effects are overwhelmed by the mean effects (for $p_G < 0.5$). However, when splitting the unconditional expectation into two conditional components, this dichotomy is revealed. When the manager does not receive a signal, the average conditional expected stock price is *increasing* in p_G; when the manager receives a signal, the average conditional expected stock price is $decreasing$ in p_G. Similarly, increasing the probability that the manager receives the signal decreases the expected stock price, due to a worse perception of non-disclosure. Increasing the number of uninformed investors leads to a small positive effect, driven by the larger information asymmetry between uninformed investors and the manager when compared to informed investors and the manager. Unsurprisingly, increasing the number of shares has a negative effect on the stock price, as the number of shares only enters into the stock price in a negative manner (see Theorem 2). Lastly, the firm's stock price decreases as the level of risk aversion among the investors rises, since investors will need to be compensated more for the risk they take on by investing.

Regarding the effects of the model parameters on the manager's disclosure choices, Figure 5 illustrates the effects on the cutoff between full disclosure and partial disclosure (c_{FD}), while Figure 6 illustrates the effects on the cutoff between partial disclosure and withholding (c_{PD}). Increasing the mean value of the good firm leads to higher levels for both cutoffs.[9] This is due to the wider spread between the good and bad firms. Increasing the standard deviation has the opposite effect, leading the manager to disclose more fully and to lower both disclosure cutoffs. While the remaining factors do not exhibit a clear pattern for the lower cutoff, the probability that the manager receives a signal does appear to affect the cutoff between full and partial disclosure. As the probability the manager receives a signal increases, the manager appears to disclose more through the easy channel. The manager has an incentive to do this, as disclosing more makes uninformed investors believe that a lack of disclosure will be more likely caused by the manager not

[9]This pattern also holds when examining the pattern between the cutoffs and a z-score, μ_G/σ.

receiving the signal. The cutoff between partial and full disclosure is unaffected, as the informed investors will know whether or not the manager received a signal, regardless of the probability.

VI. EMPIRICAL IMPLICATIONS AND CONCLUSION

The primary model of this paper demonstrates a manager's disclosure incentives when the manager has two voluntary disclosure channels in the presence of both uninformed and informed investors. The model follows a market for lemons structure in which the firm may be good or bad, and the manager probabilistically receives a signal that is potentially useful in determining the firm's type, good or bad. If the manager receives a signal, the manager can choose to disclose through an easy-to-process channel or a hard-to-process channel, or the manager can choose to withhold the signal from investors. When there is sufficient difference between the firm types, and when both informed and uninformed investors are present, the manager will adopt a three part disclosure strategy. For high values of the signal, the manager will disclose the signal through the easy-to-process channel. For low values of the signal, the manager will choose to not disclose, withholding the information from the market. If the value of the signal is in the middle, the manager will disclose through the hard-to-process channel.

This disclosure pattern is also consistent with maximizing the long-run stock price of the firm. The derived rational expectations equilibrium shows that the manager's optimal actions are identical when maximizing the short-run and long-run stock prices. Furthermore, when the manager has two disclosure channels available, the stock price will be higher initially after the manager's optimal action and will drop in the subsequent trading round, except for when the manager disclosures in the easy-to-process channel. If the manager discloses in the easy-to-process channel, then this information will be fully impounded in the stock price in the first round of trading. Taken together, this indicates that, given any short run incentive to increase the stock price, the manager in this setting can costlessly capture a gain through the use of multiple disclosure channels.

The model further demonstrates that, compared to when having only an easy-to-process channel available, the manager will decrease disclosure to uninformed investors when given the option of disclosing via an additional hard-to-process channel, though the manager will disclose more to the market overall. Such a model structure is akin to a pre-Reg FD setting where managers could disclose privately, or to the present day setting where managers could intentionally place certain voluntary disclosures in 10-K footnotes or other hard-to-process channels. The results of this comparison could indicate a possible short-run inefficiency in the market while the uninformed investors learn the information disclosed through the hard-to-process disclosure channel. From the simulation, it appears that this short run effect could be priced, and would be driven by uninformed investors valuing the firm at a higher price than they would if they had processed all available information. However, the model also indicates that multiple channels of disclosure may serve to make the market more efficient over longer periods of time, as the market should fully impound more information than under a one disclosure channel system. As managers have the option to disclose through multiple sources, it is an open empirical issue 1) if and how managers use these channels in different ways and 2) if firms disclosing different information across disclosure channels leads to an impact on the market.

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APPENDIX A

Variable definitions

a	The coefficient of risk aversion for investors
c_{FD}	The signal cutoff between full disclosure and partial disclosure
c'_{FD}	The signal cutoff between full disclosure and withholding, 1 channel
c_{PB}	The signal cutoff between partial disclosure and withholding
FD	The full disclosure state
I	Informed investors
i	A subscript indicating an individual investor
I_i	The information set of investor i
N	The number of investors
N_I	The number of informed investors
N_U	The number of uninformed investors
NI	The no information state
$\hat{p}$	The probability the manager receives a signal
$P_{0,s}$	The expected price of the stock under disclosure choice s
$\tilde{P}_1$	The price of the firm in period 1
p_G	The unconditional probability the firm is good
$p_{i,s}$	The probability the firm is good, conditional on I_i and s
PD	The partial disclosure state
R_f	The risk-free rate
s	A subscript indicating a state: FD, PD, W, or NI
U	Uninformed investors
W	The withholding state
W_0	Investor wealth at time 0
$\tilde{W}_{1,i}$	The investor wealth process at time 1 for investor i
$\bar{x}$	The number of shares available
$x_{i,s}$	The number of shares per investor demanded by investor i in state s
$\tilde{y}$	The random signal of firm value received by the manager
$\beta_{x,k}$	$\frac{x-\mu_k}{\sigma_k}$
μ_B	The unconditional expected value of a bad firm
μ_G	The unconditional expected value of a good firm
σ	The shared standard deviation of the good and bad firm's payoffs
σ_B	The standard deviation of the bad firm's payoff
σ_G	The standard deviation of the good firm's payoff

APPENDIX B

Existence of a non-degenerate equilibrium

Proof. For a non-degenerate equilibrium to exist, a higher value of the signal $\tilde{y}$ must indicate a better firm while a lower value must indicate a worse firm. For such an equilibrium to exist, then, there must exist some point y_1 such that $P_{0,FD} > P_{0,PD}$ and $P_{0,FD} > P_{0,W}$ for all $\tilde{y} > y_1$, and some point y_2 such that $P_{0,FD} < P_{0,PD}$ and $P_{0,FD} < P_{0,W}$ for all $\tilde{y} < y_2$. These conditions imply that investors would see the signal as good for values above y_1 and bad for values below y_2. These conditions can be approached through a limit argument. As is shown below, the conditions on the $P_{0,PD}$ do not constrain the solution space after constraining based on $P_{0,FD}$ and $P_{0,W}$.

First, let $\tilde{y}, c_{FD}, c_{PD} \to \infty$. Then $p_{I,FD} = p_{U,FD} = p_{I,PD} \to 1$, as, for all $\delta > 0$, $\lim_{x\to\infty} \frac{p_G\phi(x)}{p_G\phi(x)+(1-p_G)\phi(x-\delta)} = 1$. Furthermore, $p_{I,W}$, $p_{U,PD}$, and $p_{U,W}$ will all approach p_G, the unconditional probability that the firm is good, as, for all δ finite, $\lim_{x\to\infty} \frac{p_G\Phi(x)}{p_G\Phi(x)+(1-p_G)\Phi(x-\delta)} = p_G$. Let $p' = 2{p_G}^2 - 2p_G + 1$. Substituting these probabilities into the time 0 price expressions derived in Appendix C yields:

$$
\begin{aligned}
& P_{0,FD} > P_{0,PD} \text{ as } \tilde{y}, c_{FD}, c_{PD} \to \infty, \\
\Rightarrow \quad & \frac{1}{R_f}\left[\mu_G - a\frac{\bar{x}}{N}\sigma^2\right] > \frac{N_I\mu_G + N_U\frac{p_G\mu_G+(1-p_G)\mu_B}{2{p_G}^2-2p_G+1} - a\bar{x}\sigma^2}{R_f\left[N_I + \frac{N_U}{2{p_G}^2-2p_G+1}\right]}, \\
\Rightarrow \quad & \mu_G\left(1 - \frac{N_I + \frac{p_G N_U}{p'}}{N_I + \frac{N_U}{p'}}\right) > \frac{N_U\frac{(1-p_G)\mu_B}{p'}}{N_I + \frac{N_U}{p'}} + a\bar{x}\sigma^2\left(\frac{1}{N} - \frac{1}{N_I + \frac{N_U}{p'}}\right), \\
\Rightarrow \quad & \frac{N_U(1-p_G)}{N_I p' + N_U}\mu_G > \frac{N_U(1-p_G)}{N_I p' + N_U}\mu_B + a\bar{x}\sigma^2\frac{N_U(1-p')}{N(N_I p' + N_U)}, \\
\Rightarrow \quad & \frac{N_U(1-p_G)}{N_I p' + N_U}(\mu_G - \mu_B) > a\bar{x}\sigma^2\frac{2N_U p_G(1-p_G)}{N(N_I p' + N_U)}, \\
\Rightarrow \quad & \mu_G - \mu_B > 2p_G a\frac{\bar{x}}{N}\sigma^2.
\end{aligned}
$$

$$
P_{0,FD} > P_{0,W} \text{ as } \tilde{y}, c_{FD}, c_{PD} \to \infty,
$$

$$\Rightarrow \quad \frac{1}{R_f}\left[\mu_G - a\frac{\bar{x}}{N}\sigma^2\right] > \frac{N_I \frac{p_G\mu_G+(1-p_G)\mu_B}{2p_G{}^2-2p_G+1} + N_U \frac{p_G\mu_G+(1-p_G)\mu_B}{2p_G{}^2-2p_G+1} - a\bar{x}\sigma^2}{R_f\left[\frac{N_I}{2p_G{}^2-2p_G+1} + \frac{N_U}{2p_G{}^2-2p_G+1}\right]},$$

$$\Rightarrow \quad \mu_G - a\sigma^2\frac{\bar{x}}{N} > p_G\mu_G + (1-p_G)\mu_B - ap'\sigma^2\frac{\bar{x}}{N},$$

$$\Rightarrow \quad (1-p_G)(\mu_G-\mu_B) > a\sigma^2\frac{\bar{x}}{N}(2p_G(1-p_G)),$$

$$\Rightarrow \quad \mu_G - \mu_B > 2p_G a\frac{\bar{x}}{N}\sigma^2.$$

Both conditions are identical, requiring that the difference between the mean of the distribution for P_1 for the good firm and bad firm be above a threshold of $2p_G a\frac{\bar{x}}{N}\sigma^2$. This condition assures that a sufficiently high signal, $\tilde{y}$, is interpretted as an indicator of the company being good.

Next, let $\tilde{y}, c_{FD}, c_{PD} \rightarrow -\infty$. Under this limit, $p_{I,FD} = p_{U,FD} = p_{I,PD} = p_{I,W} \rightarrow 0$. On the other hand, as the conditional probability of the manager having information given that no disclosure took place approaches 0 in this case, uninformed investors will assume the manager has no information if they do not observe a disclosure, and thus $p_{U,PD} = p_{U,W} \rightarrow p_G$. Let $p' = 2p_G{}^2 - 2p_G + 1$. Substituting these probabilities into the time 0 price expressions derived in Appendix C yields:

$$\lim_{c\to-\infty} P_{0,FD} = \frac{1}{R_f}\left[\mu_B - a\frac{\bar{x}}{N}\sigma^2\right],$$

$$\lim_{c\to-\infty} P_{0,PD} = \frac{N_I\frac{\mu_B}{\sigma^2} + N_U\frac{p_G\mu_G+(1-p_G)\mu_B}{p'\sigma^2} + a\bar{x}}{R_f\left(\frac{N_I}{\sigma^2} + \frac{N_U}{p'\sigma^2}\right)},$$

$$\lim_{c\to-\infty} P_{0,W} = \frac{N_I\frac{\mu_B}{\sigma^2} + N_U\frac{p_G\mu_G+(1-p_G)\mu_B}{p'\sigma^2} + a\bar{x}}{R_f\left(\frac{N_I}{\sigma^2} + \frac{N_U}{p'\sigma^2}\right)}.$$

Note that the limits for $P_{0,PD}$ and $P_{0,W}$ are the same. Thus, if, in the limit, $P_{0,FD} < P_{0,W}$, then $P_{0,FD} < P_{0,PD}$.

$$P_{0,FD} < P_{0,W} \text{ as } \tilde{y}, c_{FD}, c_{PD} \rightarrow -\infty,$$

$$\frac{1}{R_f}\left[\mu_B - a\frac{\bar{x}}{N}\sigma^2\right] < \frac{N_I\frac{\mu_B}{\sigma^2} + N_U\frac{p_G\mu_G+(1-p_G)\mu_B}{p'\sigma^2} + a\bar{x}}{R_f\left(\frac{N_I}{\sigma^2}+\frac{N_U}{p'\sigma^2}\right)},$$

$$\Rightarrow \quad \frac{N_U}{p'\sigma^2}\mu_B - a\bar{x}\left(\frac{N_I}{N}+\frac{N_U}{Np'}\right) < \frac{N_U p_G}{p'\sigma^2}\mu_G + \frac{N_U(1-p_G)}{p'\sigma^2}\mu_B - a\bar{x},$$

$$\Rightarrow \quad -\frac{N_U p_G}{p'\sigma^2}(\mu_G - \mu_B) < a\bar{x}\frac{N_U}{Np'}(1-p').$$

Since $\mu_G > \mu_B$ by definition and since the support of p' is $(0.5, 1)$ (as $p \in (0,1)$), the statement holds for all parameter values.

Taking the two sets of limits together, so long as $\mu_G - \mu_B > 2p_G a\frac{\bar{x}}{N}\sigma^2$, a non-degenerate solution exists.

∎

APPENDIX C

Derivation

This section derives P_0 under each of the four states: full disclosure, partial disclosure, withholding, and no information.

First, the probabilities in Lemma 1 must be derived. Per Bayes' theorem, $\mathbb{P}(A|B) = \frac{\mathbb{P}(A)\mathbb{P}(B|A)}{\mathbb{P}(B)}$, and $\mathbb{P}(B) = \mathbb{P}(A)\mathbb{P}(B|A) + \mathbb{P}\left(A^{\complement}\right)\mathbb{P}\left(B|A^{\complement}\right)$.

Furthermore, note that under full disclosure, both investor types know $\tilde{y}$. Under partial disclosure, the informed investors will likewise know $\tilde{y}$, but uninformed investors will not. Under withholding, informed investors know that $\tilde{y} < c_{PD}$, where c_{PD} is an endogenously determined cutoff. Under no information, informed investors are aware that the manager did not receive a signal, and thus have no information to condition on. For uninformed investors, they cannot distinguish between the partial disclosure, withholding, and no information states. As such, they must endogenously determine probabilities based on a cutoff c_{FD} below which full disclosure will not occur. Thus, for investor i and state s, the conditional probability that the firm is good conditioned on the given information can be defined as $p_{i,s}$. Let $\beta_{x,T} = \frac{x-\mu_T}{\sigma}$, $\phi(x)$ be the normal distribution PDF, and $\Phi(x)$ be the normal distribution CDF.

1. Since informed investors know the manager didn't receive the signal under the no information state, it is easy to see that $p_{I,NI} = p_G$, as this is the unconditional probability that the firm is good.

2. Let firm type be T, good or bad. $\mathbb{P}\left(\tilde{y} < c|T\right) = \Phi\left(\frac{c-\mu_T}{\sigma}\right) = \Phi\left(\beta_{c,T}\right)$. Thus, $p_{I,W} = \frac{p_G\Phi(\beta_{c_{PD},G})}{p_G\Phi(\beta_{c_{PD},G})+(1-p_G)\Phi(\beta_{c_{PD},B})}$.

3. Likewise, $\mathbb{P}\left(\tilde{y} < c \text{ or } \nexists\tilde{y}|T\right) = \hat{p}\Phi(\beta_{c_{FD},G}) + (1-\hat{p})$. Thus, $p_{U,PD} = p_{U,W} = p_{U,NI} = \frac{p_G\left(\hat{p}\Phi(\beta_{c_{FD},G})+(1-\hat{p})\right)}{\hat{p}\left(p_G\Phi(\beta_{c_{FD},G})+(1-p_G)\Phi(\beta_{c_{FD},B})\right)+(1-\hat{p})}$.

4. When a specific $\tilde{y}$ is known, the functional form of the probability is similar, but based on the PDF instead of the CDF, since an individual value is known. Thus, $p_{I,FD} = p_{I,PD} = p_{U,FD} = \frac{p_G\phi(\beta_{y,G})}{p_G\phi(\beta_{y,G})+(1-p_G)\phi(\beta_{y,B})}$.

Thus, the derivation of Lemma 1 is complete. Next, using equation (2) the price can be derived under a general state. From (2),

$$x_{i,s} = \frac{\mathbb{E}\left[\tilde{P}_1|I_{i,s}\right] - R_f P_0}{a\mathbb{V}\left[\tilde{P}_1|I_{i,s}\right]}.$$

Aggregating $x_{i,s}$ up to $\bar{x}$:

$$\bar{x} = N_I x_{I,s} + N_U x_{U,s}.$$

Solving the above for $P_{0,s}$ yields:

$$P_{0,s} = \frac{N_I\frac{\mathbb{E}\left[\tilde{P}_1|I_{I,s}\right]}{\mathbb{V}\left[\tilde{P}_1|I_{I,s}\right]} + N_U\frac{\mathbb{E}\left[\tilde{P}_1|I_{U,s}\right]}{\mathbb{V}\left[\tilde{P}_1|I_{U,s}\right]} - a\bar{x}}{R_f\left[\frac{N_I}{\mathbb{V}\left[\tilde{P}_1|I_{I,s}\right]} + \frac{N_U}{\mathbb{V}\left[\tilde{P}_1|I_{U,s}\right]}\right]}.$$

Next, this expression can be explicitly derived for each of the four states. Note that $\mathbb{E}\left[\tilde{P}_1|I_{i,s}\right] = p_{i,s}\mu_G + (1-p_{i,s})\mu_B$ and $\mathbb{V}\left[\tilde{P}_1|I_{i,s}\right] = {p_{i,s}}^2\sigma^2 + (1-p_{i,s})^2\sigma^2 = (2{p_{i,s}}^2 - 2p_{i,s} + 1)\sigma^2$. Thus, the price at time 0 can explicitly be written, under each of the four states, as:

$$P_{0,FD} = \frac{1}{R_f}\left[p_{I,FD}\mu_G + (1-p_{I,FD})\mu_B - a\frac{\bar{x}}{N}(2p_{I,FD}{}^2 - 2p_{I,FD} + 1)\sigma^2\right],$$

$$P_{0,PD} = \frac{N_I \frac{p_{I,FD}\mu_G + (1-p_{I,FD})\mu_B}{2p_{I,FD}{}^2 - 2p_{I,FD}+1} + N_U \frac{p_{U,W}\mu_G + (1-p_{U,W})\mu_B}{2p_{U,W}{}^2 - 2p_{U,W}+1} - a\bar{x}\sigma^2}{R_f\left[\frac{N_I}{2p_{I,FD}{}^2 - 2p_{I,FD}+1} + \frac{N_U}{2p_{U,W}{}^2 - 2p_{U,W}+1}\right]},$$

$$P_{0,W} = \frac{N_I \frac{p_{I,W}\mu_G + (1-p_{I,W})\mu_B}{2p_{I,W}{}^2 - 2p_{I,W}+1} + N_U \frac{p_{U,W}\mu_G + (1-p_{U,W})\mu_B}{2p_{U,W}{}^2 - 2p_{U,W}+1} - a\bar{x}\sigma^2}{R_f\left[\frac{N_I}{2p_{I,W}{}^2 - 2p_{I,W}+1} + \frac{N_U}{2p_{U,W}{}^2 - 2p_{U,W}+1}\right]},$$

$$P_{0,NI} = \frac{N_I \frac{p_G\mu_G + (1-p_G)\mu_B}{2p_G{}^2 - 2p_G+1} + N_U \frac{p_{U,W}\mu_G + (1-p_{U,W})\mu_B}{2p_{U,W}{}^2 - 2p_{U,W}+1} - a\bar{x}\sigma^2}{R_f\left[\frac{N_I}{2p_G{}^2 - 2p_G+1} + \frac{N_U}{2p_{U,W}{}^2 - 2p_{U,W}+1}\right]}.$$

APPENDIX D

Disclosure patterns

Proof. From the proof of Theorem 1, as the two disclosure cutoffs and $\tilde{y}$ approach infinity, full disclosure will have a higher price than partial disclosure and withholding when the existence condition is met. Thus, full disclosure will be used for at least all $\tilde{y}$ above some point $\bar{y}$. Furthermore, from the proof of Theorem 1, as the two disclosure cutoffs and $\tilde{y}$ approach negative infinity, full disclosure will have a lower price than partial disclosure and withholding. Thus, full disclosure will not be the only strategy employed.

By the above argument, it is known that c_{FD} exists. At $\tilde{y} = c_{FD}$, it must be that $p_{I,PD} = p_{I,FD}$, as the informed investors have the exact same information sets under full and partial disclosure. However, as everything but the probabilities is constant with respect to $\tilde{y}$ and c_{FD} in the equation for the price P_0, this implies that $p_{U,PD} = p_{U,FD}$ at $\tilde{y} = c_{FD}$ as well.

To prove existence of c_{PD}, note that since $p_{I,W}$ and $p_{U,W} = p_{U,PD}$ are fixed and positive given any specified c_{FD} and c_{PD}, then P_0 under withholding is fixed. Also note that P_0, as a function, is always positive. Since $p_{U,PD} = p_{U,W}$, then for $c_P D$ to exist, there must be some value of c_{PD} such that at $\tilde{y} = c_{PD}$, P_0 is equal under both withholding and partial disclosure.

First, note that the value of $p_{I,PD}$ does not depend on c_{PD} or c_{FD}. Further note that, from

Appendix B, $p_{I,PD}$ limits to 1 as $\tilde{y}$ limits to infinity, and $p_{I,PD}$ limits to 0 as $\tilde{y}$ limits to negative infinity. Also, note that $p_{I,PD}$, as fa function, is comprised of three terms, each term relating to $\phi(\cdot)$, the PDF of the standard normal distribution. This function is continuous, mapping ϕ : $(-\infty, \infty) \to (0, 1/\sqrt{2\pi})$. Since the numerator of $p_{I,PD}$ is ϕ times a constant, and the denominator of $p_{I,PD}$ is a weighted sum of phi functions (with positive weights), then both the numerator and denominator are continuous functions mapping to positive real numbers, and thus the function $p_{I,PD}$ itself is continuous.

Since $p_{I,PD}$ is continuous and approaches both 0 and 1, and since p_W is a constant value between 0 and 1, then by the Intermediate Value Theorem, these two functions must cross for some value of $\tilde{y} = c_{PD}$. Thus, c_{PD} must exist.

Furthermore, note that at c_{FD}, $P_{0,PD} = P_{0,FD} > P_{0,W}$. Consequently, the manager will prefer partial disclosure when $c_{PD} < \tilde{y} < c_{FD}$.

To show that below c_{PD} the manager will prefer withholding relies on showing that the derivative of $P_{0,PD}$ at $\tilde{y} = c_{PD}$ is positive. However, determining the sign of the derivative analytically is seemingly intractible. Simulating the derivative (following the methodology in Section V) finds that the derivative is positive in every single iteration. A positive derivative here indicates that for $\tilde{y}$ below c_{PD} the manager would prefer withholding, and for $\tilde{y}$ above c_{PD} the manager would prefer partial disclosure (up to c_{FD}).

Thus, all three disclosure options will be used in the manager's optimal disclosure strategy: withholding for low values of $\tilde{y}$, partial disclosure for middle values, and full disclosure for high values.[10] ∎

APPENDIX E

Fully revealing equilibrium

Proof. Let $\mu_G - \mu_B > 2a\frac{\bar{x}}{N}\sigma^2$ and consider the sign of $\frac{\partial P_0}{\partial p_{I,FD}}$ under state FD:

[10] Note that while this proof shows that the manager will have 2 cutoff points, it does not preclude the manager from having cutoff points other than the 2 mentioned. Thus, it does not constitute a proof of uniqueness. Such a proof would need to show that no cutoff point can exist when the probability that the firm is good from the investors' perspective changes based on the manager's disclosure choice. This is discussed in the simulation in Section V.

$$\left.\frac{\partial P_0}{\partial p_{I,FD}}\right|_{FD} = \frac{1}{R_f}\left((\mu_G - \mu_B) - a\frac{\bar{x}}{N}(4p_{I,FD} - 2)\sigma^2\right) > 0$$
$$\Rightarrow p_{I,FD} < \frac{1}{2}\left(1 + \frac{\mu_G - \mu_B}{2a\frac{\bar{x}}{N}\sigma^2}\right) > 1$$

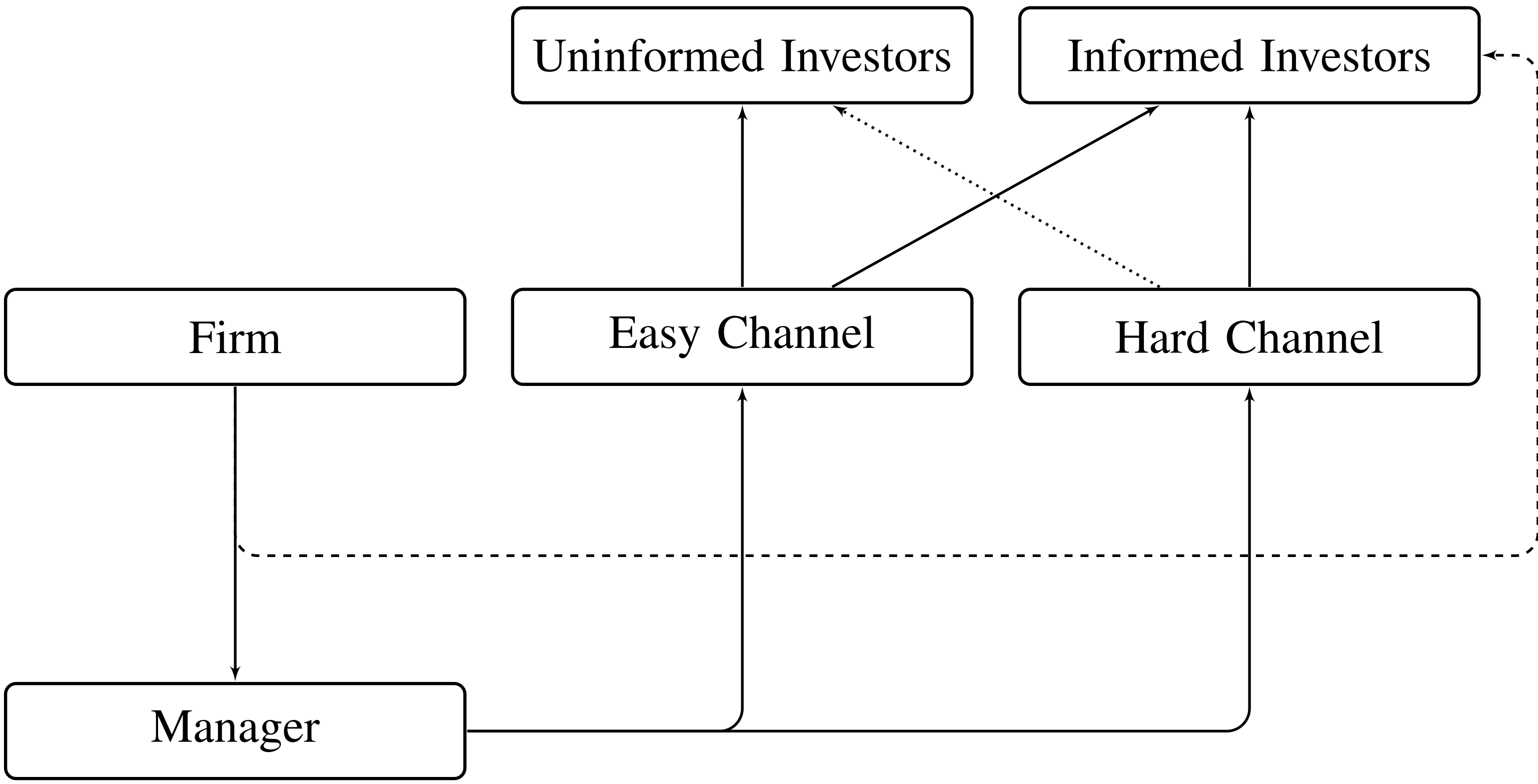
Since $p_{I,FD} < 1$ by definition, P_0 under state FD is monotonic in $p_{I,FD}$. Since $p_{I,FD}$ is monotonic in $\tilde{y}$, P_0 is monotonic in $\tilde{y}$ when $\mu_G - \mu_B > 2a\frac{\bar{x}}{N}\sigma^2$.

Showing that P_0 under state PD is monotonic in $\tilde{y}$ is significantly more difficult analytically. As such, this is instead demonstrated in the simulation in Section V. In all iterations when $\mu_G - \mu_B > 2a\frac{\bar{x}}{N}\sigma^2$, $P_0|_{PD}$ is monotonic in $\tilde{y}$.

Thus, P_0 appears to be monotonic in all situations when there is a signal and it is disclosed. Furthermore, when there is a signal and it is not disclosed, the expected value is identical to the price when a signal equal to c_{PD} is disclosed. Consequently, P_0 is invertible over the set $(c_{PD}, \infty)\backslash\{{P_0}^{-1}(P_0|NI)\}$. Thus, P_0 is invertible for all but two prices: the price when no information exists and the highest price at which information is withheld. When information is withheld, however, the price is identical to $P_0(c_{PD})$. As withholding occurs with finite probability, $\mathbb{P}\left(\tilde{y} < c_{PD}\right) \cdot \hat{p}$, whereas $\tilde{y} = c_{PD}$ occurs with probability approximately 0, investors can infer that if the price is initially $P_0(c_{PD})$, the manager chose to withhold information. When no information exists, the price will initially be at some value P_0^*. As no signal existing occurs with finite probability $\hat{p}$, whereas, $\tilde{y} = P_0^{-1}(P_0^*)$ occurs with probability approximately 0, investors can infer that if P_0 equals P_0^*, no signal exists. Thus, investors can determine the available information at every possible price, and consequently a fully-revealing rational expectations equilibrium will attain. Furthermore, since the price is invertible, the solution must be unique. ∎

Figures

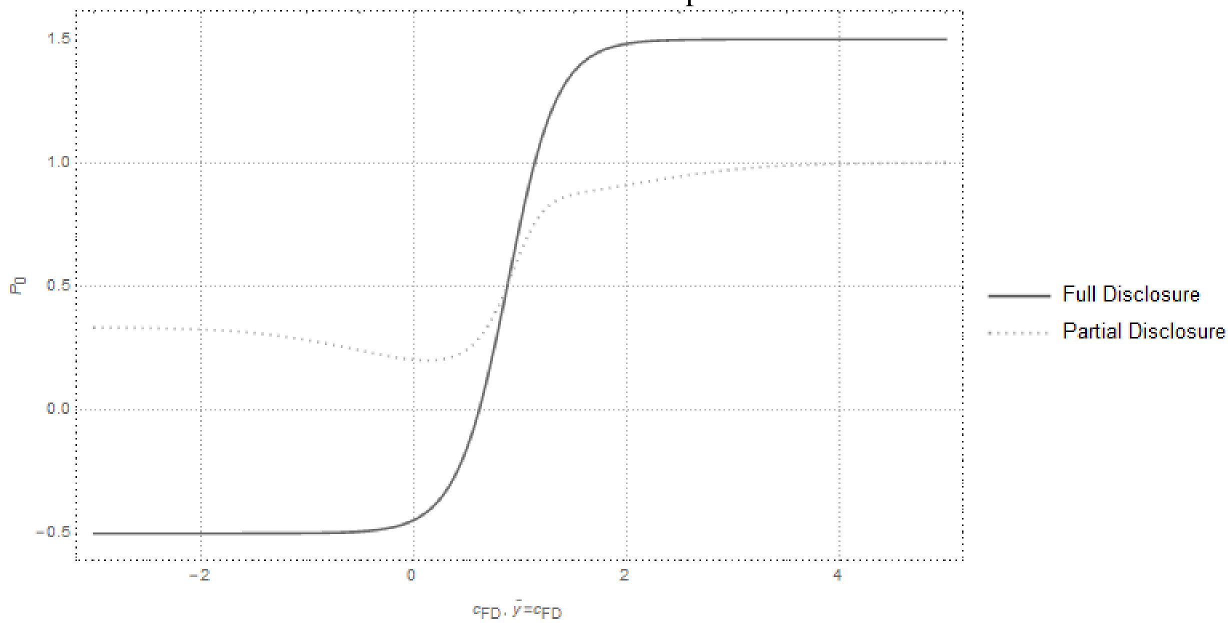
Figure 1: Diagram of the full model



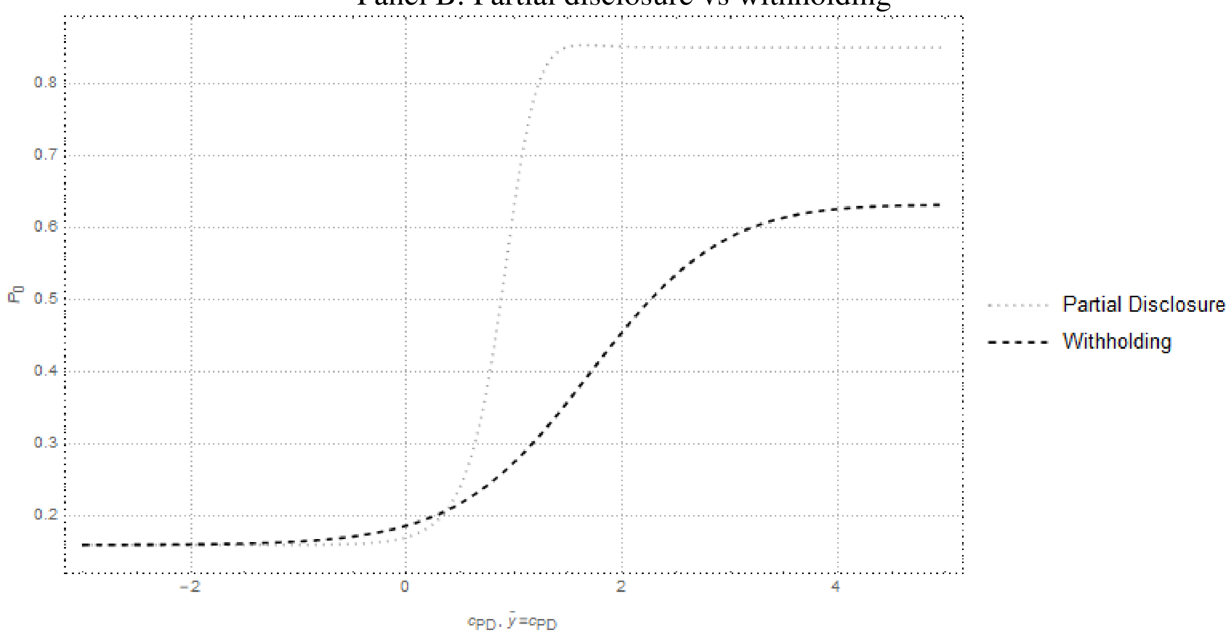
A diagram of the flow of information in the market. Solid lines represent information transfers, dashed lines represent partial information transfers, and dotted lines represent information transfers that only occur in the long run.

Figure 2: Optimizing disclosure

Panel A: Full disclosure vs partial disclosure



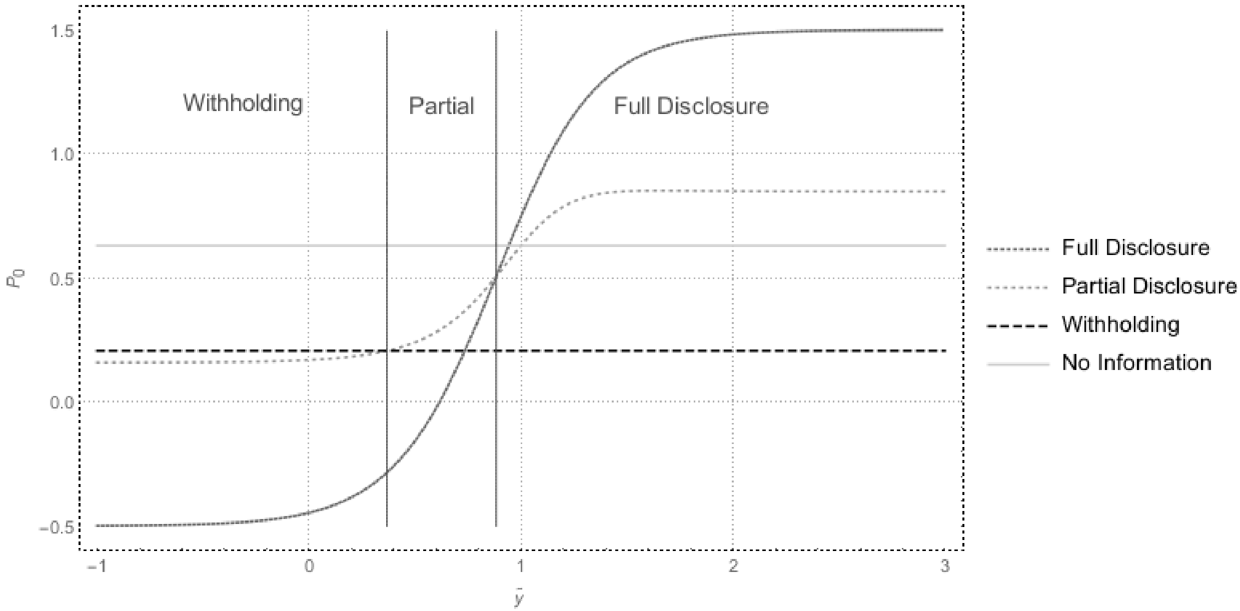
Panel B: Partial disclosure vs withholding



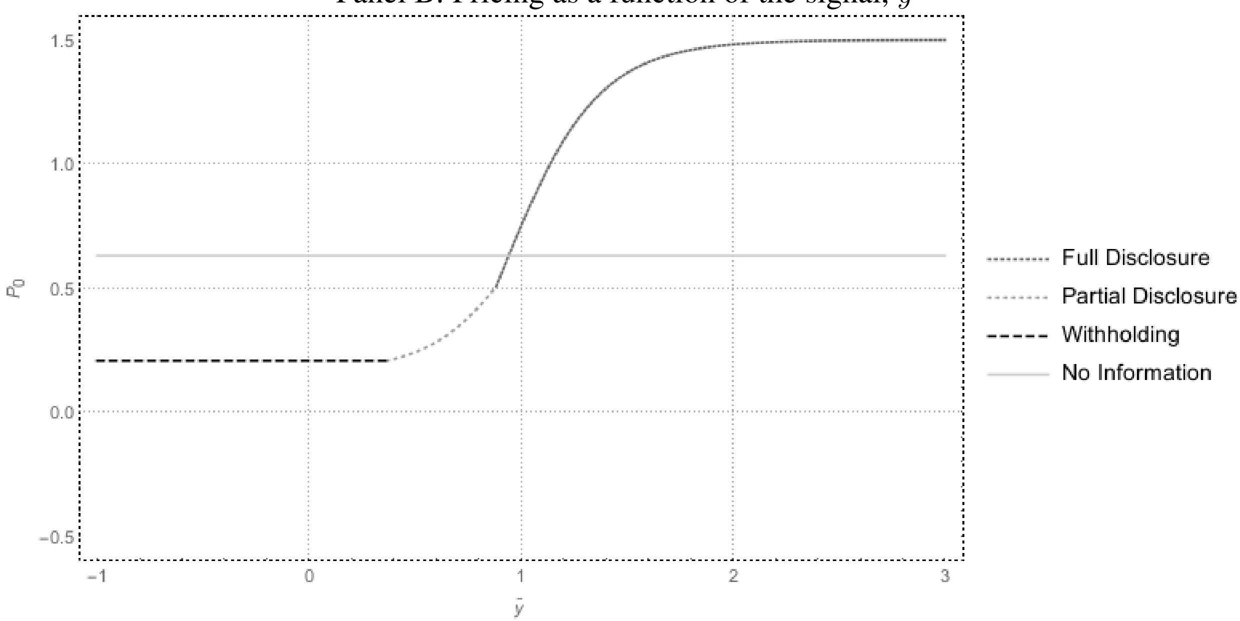
These two graphs illustrate the process for choosing the optimal disclosure cutoff points from the manager's perspective. The graphs are based on the numerical example parameters listed in Table 1. Parameter definitions are included in Appendix A.

Figure 3: Optimal disclosure

Panel A: Stock price as a function of disclosure choice

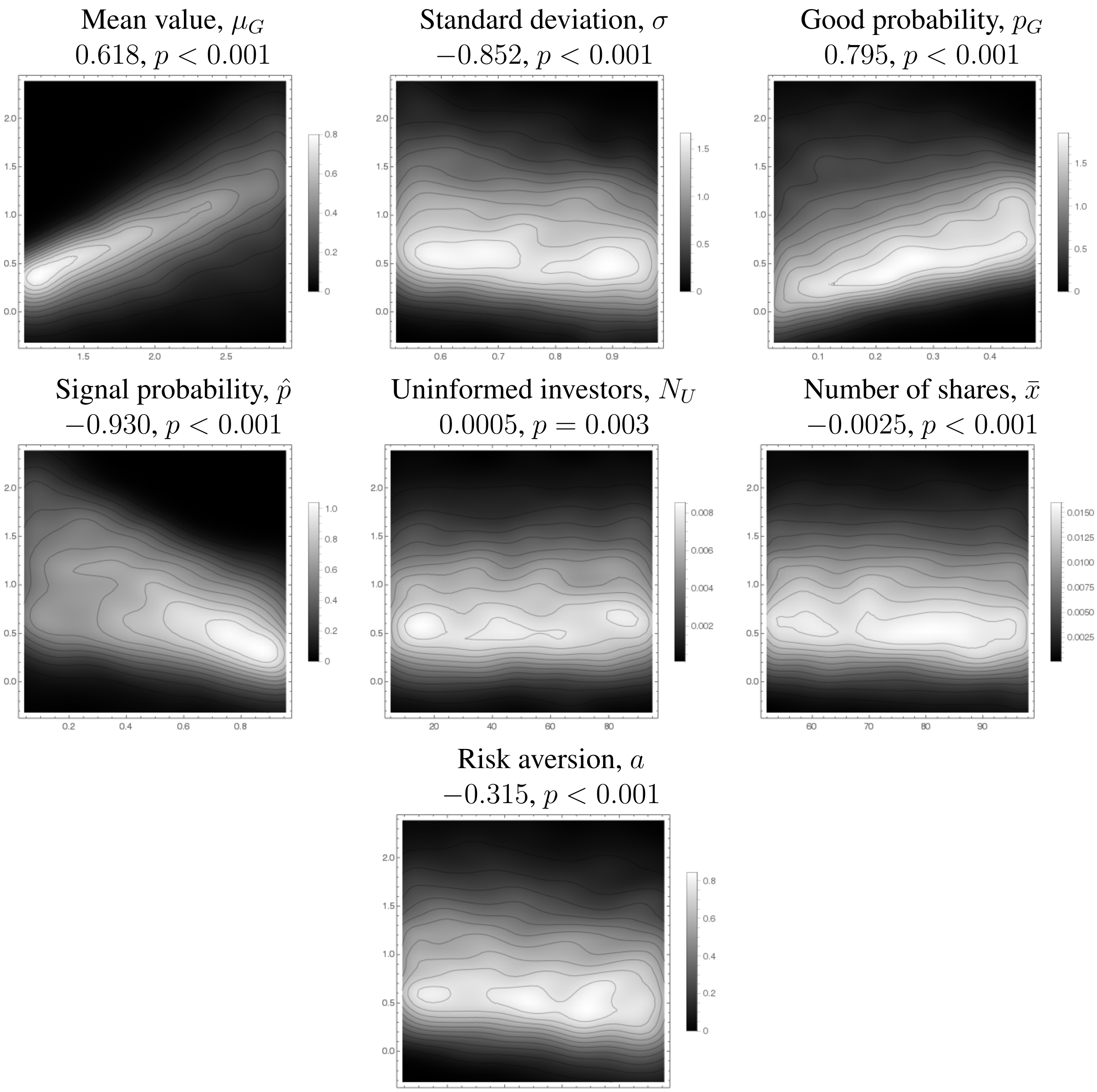


Panel B: Pricing as a function of the signal, $\tilde{y}$



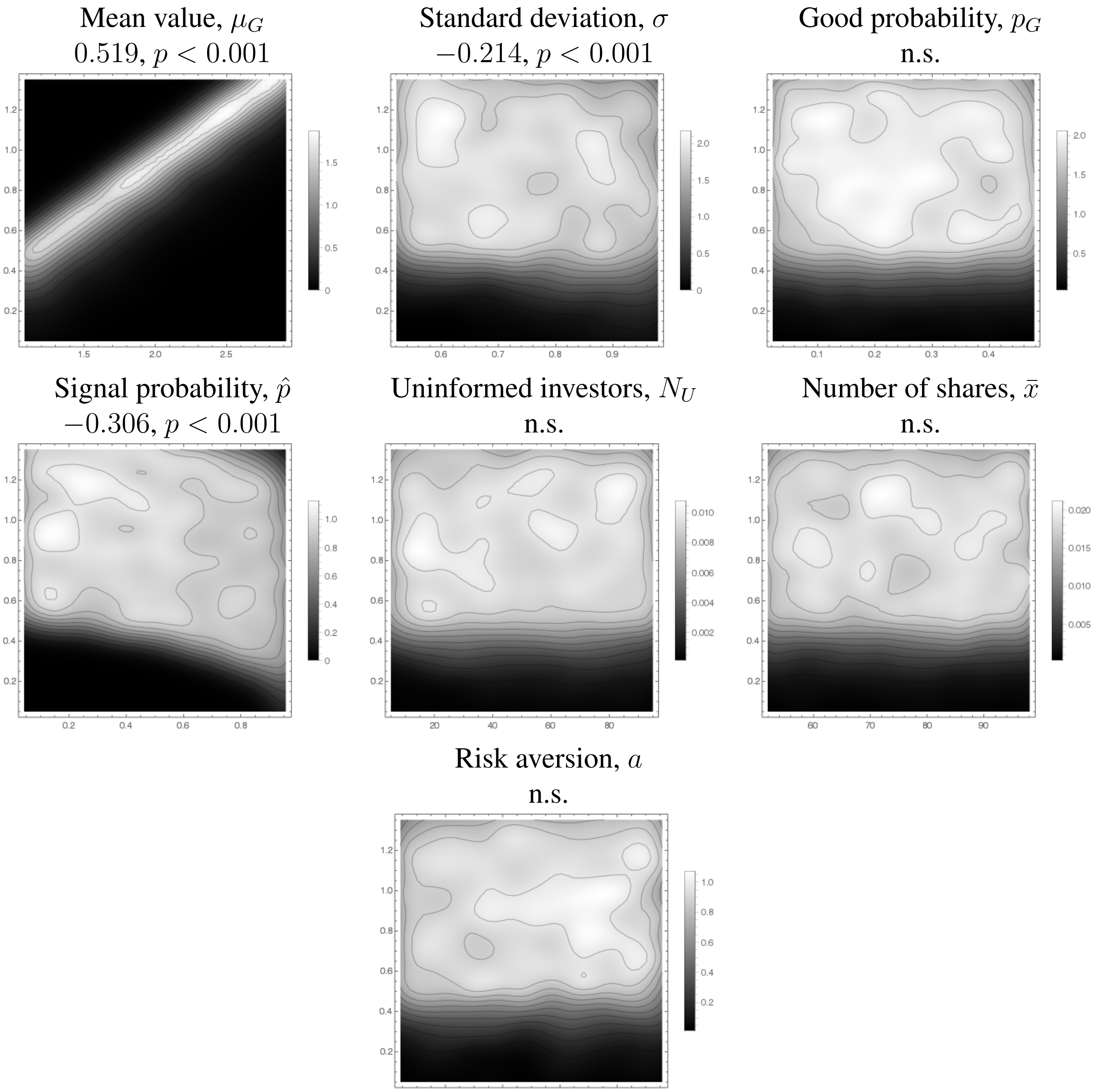
These graphs show the stock price at time 0, P_0, as a function of the signal the manager receives (if received) and the manager's disclosure choice. Panel A shows the payoffs for all disclosure choices, whereas Panel B shows the price as a function of the manager's optimal choices. These graphs are based on the numerical example parameters listed in Table 1. Parameter definitions are included in Appendix A.

Figure 4: Unconditional expected price statics



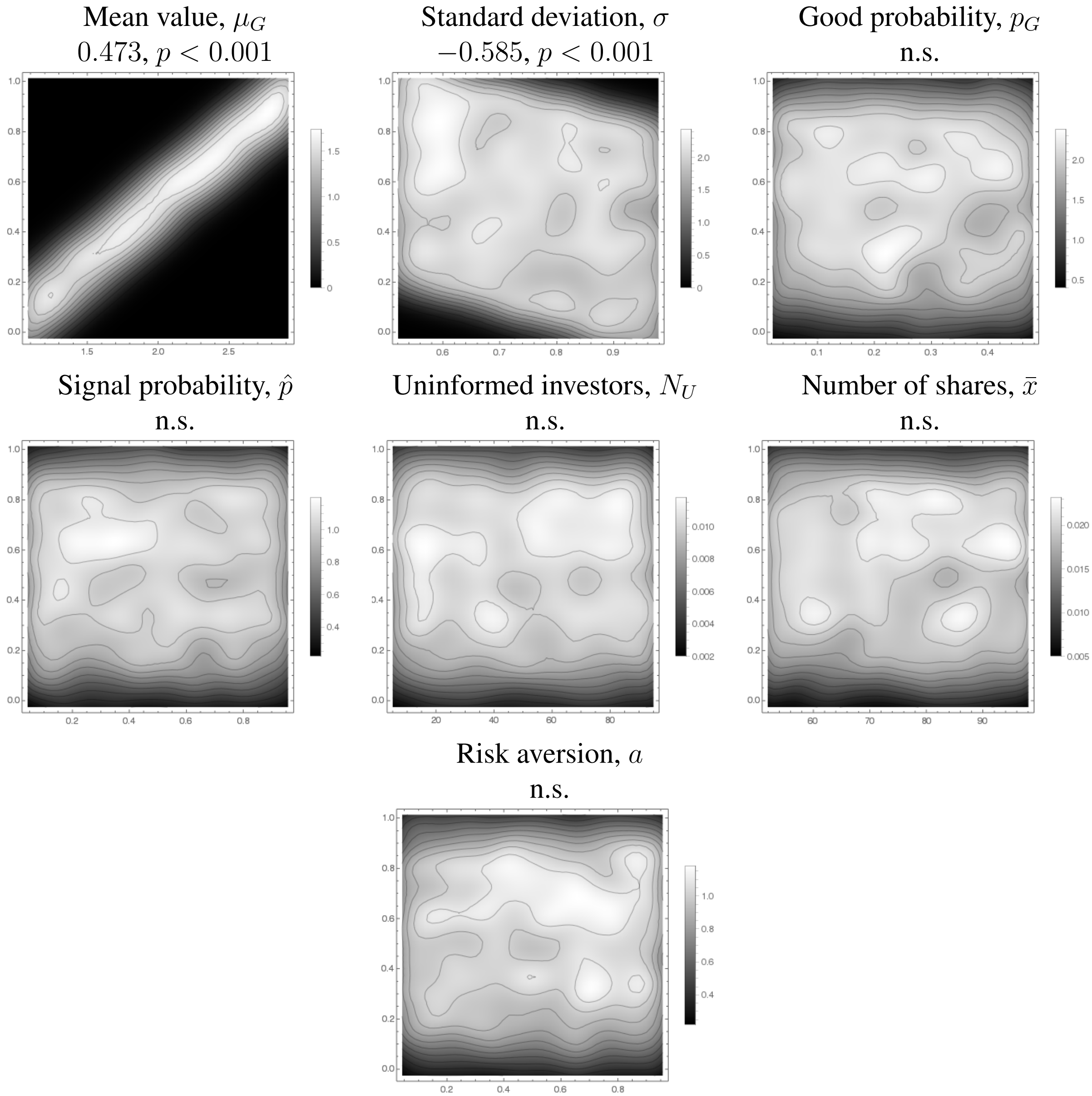
These smoothed density histograms show the relationship between the unconditional expected stock price at time 0 (P_0) on the vertical axes and various parameters of the model on the horizontal axes. These plots are based on a simulation of 10,000 iterations of the model (see Table 1 for specific details). Lighter colors represent a greater concentration of values near the specific parameter-price pairing, and the enclosed regions represent level curves of the density at 10% intervals. Significant trends based on OLS regressions are noted above the graphs; graphs with no significant trends at the $p < 0.05$ level are instead marked as n.s. Parameter definitions are included in Appendix A.

Figure 5: Disclosure statics, cutoff between full disclosure and partial disclosure (c_{FD})



These smoothed density histograms show the relationship between the disclosure cutoff between full disclosure and partial disclosure on the vertical axes and various parameters of the model on the horizontal axes. These plots are based on a simulation of 10,000 iterations of the model (see Table 1 for specific details). Lighter colors represent a greater concentration of values near the specific parameter-disclosure cutoff, and the enclosed regions represent level curves of the density at 10% intervals. Significant trends based on OLS regressions are noted above the graphs; graphs with no significant trends at the $p < 0.05$ level are instead marked as n.s. Parameter definitions are included in Appendix A.

Figure 6: Disclosure statics, cutoff between partial disclosure and withholding (c_{PD})



These smoothed density histograms show the relationship between the disclosure cutoff between partial disclosure and withholding on the vertical axes and various parameters of the model on the horizontal axes. These plots are based on a simulation of 10,000 iterations of the model (see Table 1 for specific details). Lighter colors represent a greater concentration of values near the specific parameter-disclosure cutoff, and the enclosed regions represent level curves of the density at 10% intervals. Significant trends based on OLS regressions are noted above the graphs; graphs with no significant trends at the $p < 0.05$ level are instead marked as n.s. Parameter definitions are included in Appendix A.

Tables

Table 1: Parameters for numerical example and simulation

Parameter	Example value	Simulation distribution
μ_G	2	Uniform$[1,3]$
μ_B	0	0
σ	1	Uniform$[0.5,1]$
p_G	0.5	Uniform$[0,0.5]$
$\hat{p}$	0.5	Uniform$[0,1]$
R_f	1	1
N	100	100
N_U	50	$\lfloor$Uniform$[1,100]\rfloor$
N_I	50	$N - N_U$
$\bar{x}$	100	$\lfloor$Uniform$[50,101]\rfloor$
a	1	Uniform$[0,1]$

This table shows the distributions for each parameter in the simulation. The parameter choices are chosen such that $2p_G\sigma^2\frac{\bar{x}}{N} < 1$ and $\mu_G - \mu_B > 1$, thus fulfilling the condition of Theorem 1, ensuring existence of a non-degenerate competitive equilibrium for any choices of parameter values. Parameter definitions are included in Appendix A.