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Export-Learning and FDI with Heterogeneous Firms

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Abstract

I present a dynamic general equilibrium model with heterogeneous firms that can innovate, learn how to export and then go on to become multinational firms. Entering the foreign market is a dynamic process where firms first learn how to export and then can learn how to adapt production to a low-wage location (multinational production). I solve the model numerically and, starting from a 1990 benchmark of US and Mexico, study how policy changes such as stronger patent protection and trade liberalization affect innovation, technology transfer and consumer welfare. In particular, I disentangle how labor resources are reallocated within regions in response to policy changes: across sectors (production, innovation, export-learning and adaption to multinational production), across high-productivity and low-productivity firms, and within firms as they produce more (less) for the home market visavi the export market. I obtain higher rates of export-learning and FDI for high-productivity firms than for low-productivity firms. As a result, exporters are on average more productive than non-exporters, and multinational firms are on average more productive than exporters. In equilibrium, there are still some low-productivity exporters, some low-productivity multinational firms and some high-productivity non-exporters. Low-productivity firms export and engage in FDI but they are just not as successful in these activities as high-productivity firms.

Keywords: Multinational Firms, Heterogeneous Firms, North-South Trade, Intellectual Property Rights, Foreign Direct Investment, Product Cycles, Economic Growth.

JEL Classification: F12, F23, F43, O31, O34.

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1 Introduction

From the firm-level datasets that became available in the 1990s, it was evident that only a small share of firms export and an even smaller share of firms are multinationals. The data also showed that exporters and multinational firms are different from non-exporting firms. In particular, exporting firms tend to be more productive than firms that do not export, and multinational firms tend to be even more productive than exporting firms. Existing trade theory could not explain these interesting facts, and consequently, the last decade has witnessed an explosion in research on firm entry into foreign markets.

In Helpman, Melitz and Yeaple (2004), building on the influential paper by Melitz (2003), monopolistically competitive firms are heterogeneous in productivity and face fixed costs for selling domestically, for entering a foreign market via exports, and for entering a foreign market via foreign direct investment (FDI). In their model, the fixed cost of FDI is higher than the fixed cost for exporting, but a firm that serves the foreign market through a foreign affiliate does not need to pay the variable trade cost for shipping its product from one port to another. All firms that are above a particular productivity threshold decide to engage in FDI and become multinational firms. Firms that are not productive enough to cover the fixed costs of FDI, but have a productivity above another lower threshold level decide to not just serve the domestic market but also to export.

In Helpman et al (2004), the decision to enter the foreign market through either exporting or FDI is a one-time decision. However, Conconi, Sapir and Zanardi (2013) find that, looking at all Belgian manufacturing firms that started to engage in FDI during 1998-2008, these firms were already serving the foreign market via exports in almost 90 per cent of the cases. This suggests that learning how to serve foreign markets (via exports and then via FDI) is a gradual process that takes time. The static model in Helpman et al (2004) and the many extensions that followed Melitz (2003) cannot capture a gradual learning process where learning how to export is a stepping stone to doing FDI and becoming a multinational firm. Instead, a dynamic model is needed to capture this process for firms' international activities.

Another striking fact about FDI is that the recent wave of globalization has been associated with a huge increase in FDI going to developing countries. For example, from 1990 to 2005, there was a ten-fold increase in FDI going to developing countries. The only way this can be explained using the Helpman et al (2004) model is through a reduction in the fixed cost of FDI. In that model, a decrease in the fixed FDI cost would lead to more multinationals, but it would also lead to fewer exporters since the productivity cutoff for becoming a multinational is lowered while the productivity cutoff for exporting is unchanged. Since 1990, while there has been a large increase in FDI, there has been no corresponding decrease in exporting by firms.

In this paper, I present an explanation for the large increase in FDI inflow that does not involve changing the cost of FDI. I show that a more favorable environment for firms in the host-country (after the cost of FDI is incurred) can lead to dramatically more FDI. During the time period 1990-2005, many developing countries were strengthening their intellectual property rights (IPR) protection to comply with the Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPS). I calibrate the model to fit a benchmark in 1990 which represents the world prior to the signing of the TRIPS agreement and as an exercise, also a 2005 benchmark, when southern IPR protection is stronger due to implementation of the TRIPS agreement. By simply imposing stronger IPR protection, I am able to replicate the large observed increase in FDI inflow going to developing countries during the time period 1990-2005.

A common feature in Helpman et al (2004) and the many extensions that followed Melitz (2003) is the sharp productivity cutoffs in the productivity sorting across exporters and non-exporters. Firm-level evidence show that there are still some non-exporters that have higher productivity than exporters (as seen for the US in Bernard, Eaton, Jensen and Kortum (2003) and for Belgium in Mayer and Ottaviano (2008)), even though exporters on average are more productive than non-exporting firms (Bernard and Jensen (2004) and Lileeva and Trefler (2010)). Castro, Li, Maskus and Xie (2013) attribute this pattern to varying fixed costs of exporting. Using Chilean firm-level data, they examine how firms' export decisions vary with both firm productivity and the fixed export costs the firm faces in a particular year, industry and region. In the dynamic model presented in this paper, I find that the export-learning rate is higher for high-productivity firms than for low-productivity firms and that the FDI rate is higher for high-productivity firms than for low-productivity firms. The resulting pattern is that, on average, exporting firms are more productive than non-exporting firms, and multinational firms are more productive than exporting firms. However, there will still be some low-productivity multinational firms, some low-productivity exporting firms, and some high-productivity non-exporting firms in equilibrium.

In the model, firms in the North (developed countries) engage in innovative research and development (R&D) to develop new product varieties. Upon successful innovation, a northern firm starts to produce in the North (serving the domestic market) and learns if it is a high-cost (low-productivity) firm or a low-cost (high-productivity) firm. Firms in the North can engage in export-learning R&D to access the southern market and earn higher profits from selling to both markets. The export-learning R&D costs are of a similar nature to the fixed export costs in Arkolakis (2010), where firms need to pay a fixed cost for marketing (or setting up a distribution network) to enter into each export market. Exporting northern firms can then choose to engage in adaptive R&D (FDI) to learn how to produce their product variety through a foreign affiliate in the South where wages (and hence, production costs) are lower. Multinational firms that produce in foreign affiliates in the South face the risk of imitation from southern firms. If imitation occurs, the multinational firm is pushed out of the market and southern firms immediately serve both the southern market and export to the North.

I find that stronger IPR protection in the South induces both high-cost and low-cost foreign affiliates of northern firms to increase their R&D expenditures and also results in faster rates of technology transfer within these multinational firms, consistent with the empirical evidence in Branstetter, Fisman and Foley (2006).[1] Low-cost firms respond more to FDI-related policies than high-cost firms by transferring more production to the South than high-cost firms. As a result of stronger IPR protection, more product varieties end up being produced in the South and exports of new products from the South to the North increase, consistent with the empirical evidence in Branstetter, Fisman, Foley and Saggi (2011).[2] I also find that stronger IPR protection stimulates innovative R&D spending by northern firms and results in faster economic growth, consistent with the empirical evidence in Gould and Gruben (1996).[3] Consumers in both regions benefit from increased product variety and lower prices as more production takes place in the South, resulting in higher long-run consumer welfare.

This paper also relates to the theoretical literature on FDI and IPR protection in developing

[1]Branstetter, Fisman and Foley (2006) study how international technology transfer within US-based multinational firms changes in response to IPR reforms in developing countries. They find that due to IPR reform, royalty payments for technology that has been transferred to foreign affiliates increase and the R&D expenditures of these foreign affiliates increase.

[2]Branstetter, Fisman, Foley and Saggi (2011) find that following patent reform aimed at strengthening IPR protection, US-based multinational firms expand the scale of their activities in reforming countries and exports of new goods increase in reforming countries.

[3]Gould and Gruben (1996) use cross-country data on patent protection, trade regime and country-specific characteristics and find evidence that IPR protection is a significant determinant of economic growth. Countries with stronger IPR protection tend to have higher average yearly per capita GDP growth from 1960 to 1988. Furthermore, the effects are slightly stronger in relatively open economies. They attribute this to the linkage between innovation and IPR protection playing a stronger role in more competitive (open) markets.

countries, for example Glass and Saggi (2002), Glass and Wu (2007), Branstetter and Saggi (2011) and Jakobsson and Segerstrom (2012). The standard assumption in these dynamic general equilibrium models of North-South trade is that all firms sell to their domestic market and also immediately export to the foreign market. This assumption is not consistent with the empirical evidence from firm-level data that the majority of firms only sell to the domestic market (for example, Bernard et al (2003) find in their sample of US firms that about 80 per cent of plants do not export any of their output). With a simple heterogeneity structure and by introducing R&D costs for export-learning, I am able to generate results that are consistent with such a large share of non-exporting firms.

By using a dynamic modeling framework where firms engage in R&D, I can study how innovation, international technology transfer and ultimately consumer welfare are affected by heterogeneous firms' export-learning and FDI activities. In addition, I can study how labor resources are allocated across sectors (production, innovation, export-learning and adaption to multinational production), across high-productivity and low-productivity firms, and within firms as they choose to produce more (less) for the home market visavi the export market in response to policy changes. In the time period 1999-2009, innovative R&D expenditures in the U.S. relative to local manufacturing value-added grew from 8.7 percent to 12.7 percent and the share of employment of U.S. firms located in their foreign affiliates grew from 22 percent to 31 percent (OECD STAN, US Bureau of Economic Analysis, cited in Arkolakis et al (2013)). The paper that relates closest to mine is Arkolakis, Ramondo, Rodríguez-Claire and Yeaple (2013). In their static monopolistic competition model of trade and FDI, location of innovation and production is determined by comparative advantage and home market effects arising from variable trade costs and variable costs for multinational production with increasing returns to scale. In contrast, in the dynamic model that I present, firms' internationalization is a gradual process, where the amount of innovation and the rates of export-learning and adaption for multinational production are endogenously determined.

The paper proceeds as follows. Section 2 describes the model and derives the steady-state equations. The model is solved numerically in Section 3. I present two benchmark scenarios and solve the model for several counterfactuals to study the effect of policy changes related to exporting and FDI. In particular, I study the effects of trade liberalization, strengthened IPR protection, lower fixed costs of exporting (export market entry) and lower fixed costs of FDI (adaption for multinational production). Section 5 concludes. Calculations done to solve the model in more detail can be found in the Appendix.

2 The Model

2.1 Overview

Consider a global economy with two regions, the North and the South. Labor is the only factor of production. It is used to manufacture product varieties for final consumption, develop new product varieties (innovation), adapt exisiting product varieties for entry into the foreign market (export-learning) and adapt exported varieties for production in the South (FDI). Labor is perfectly mobile across activities within a region, but cannot move across regions. Since labor markets are perfectly competitive, there is one single wage rate paid to all northern workers w_N and one single wage rate paid to all southern workers w_S. Although labor cannot move across regions, goods can. International trade between the North and the South is subject to iceberg trade costs: $\tau > 1$ units of a good must be shipped for one unit to arrive at its destination.

Only firms in the North, *northern* firms, have the capacity to innovate. A northern firm can hire workers to engage in innovative R&D with the purpose of developing the blueprint for a new product variety. After successful innovation, the firm earns monopoly profits from selling to the domestic market (the North). When the northern firm makes the decision of how much labor to hire for innovation, the firm does not know its own productivity in manufacturing, and there is therefore uncertainty about the expected profit flow. With probability $q^H = q$, the northern firm will be a high-cost firm with unit labor requirement c^H for manufacturing output. With probability $q^L = 1 - q$, the northern firm will be a low-cost firm with unit labor requirement c^L, where $c^L < c^H$. The northern firm is fully informed about the probabilities for the marginal cost draw. Even though firms are heterogeneous in their productivities, high- and low-cost firms face the same labor requirement for R&D.

When the northern firm starts producing and selling to the domestic market, the firm learns its own productivity. The northern firm can subsequently choose to hire southern workers to engage in export-learning R&D with the purpose of introducing its product variety to the southern market. Such R&D costs can be thought of as marketing, setting up distribution networks and learning how to comply with regulations in the foreign market. Upon successful export-learning, the firm earns higher monopoly profits since it earns profits from two markets instead of one. Such a firm is called an *exporter*. A firm that has learned to export can then choose to hire southern workers to engage in adaptive R&D with the purpose of transferring its manufacturing operations to the

South where the wage rate is lower.[4] When successful in adaptive R&D, a firm earns higher global monopoly profits compared to when the firm was a northern exporter because of the lower wage rate in the South. Such a firm is called a *foreign affiliate* since, even though all production takes place in the South, a fraction of its profits is repatriated back to its stockholders in the North in the form of royalty payments for the right to use the blueprint of the particular product variety. Adaptive R&D is the cost that firms incur when transferring their technology to foreign affiliates, and can therefore be interpreted as an index of FDI. Southern R&D (northern firms' export learning R&D and exporters' adaptive R&D) is financed by southern savings, but northern domestic firms and northern exporters, respectively, control the amount of R&D in order to maximize their global expected discounted profits. Upon successfully adapting production to the South, the foreign affiliate sells to the southern market and also exports back to the North without incurring any additional export learning costs.

Foreign affiliates are exposed to a positive rate of imitation from *southern* firms. Once a product variety has been imitated, the blueprint becomes available to all southern firms. No southern firm can set its price higher than marginal cost and southern firms earn zero profits. Foreign affiliates need to set the price higher than marginal cost to recover the cost of adaptive R&D. Therefore, a foreign affiliate cannot compete with southern firms. Southern imitators sell to the southern market and also export the product variety to the North without having to incur any export-learning costs to sell to the northern market.

A product variety experiences a one-way product cycle à la Vernon (1966), illustrated in Figure 1. Each product variety is initially developed by a northern firm. The number of varieties in the economy grows at the rate g as a result of northern firms' innovative R&D activities. Each northern firm starts to produce the product variety it invented in the first stage and sells to the domestic market. It is at this point that the northern firm learns its own productivity. With probability q^H the firm draws a high marginal cost and with probability q^L it draws a low marginal cost. The firm can then engage in export-learning R&D with the aim of exporting to the southern market. Export-learning as a result of firms' R&D activities occurs at the endogenous rate χ^H for high-cost firms and at the endogenous rate χ^L for low-cost firms. After the firm has become an exporter, it can engage in adaptive R&D. If the firm is successful, the product variety is transferred to the

[4] I will only solve for equilibria where $w_N > w_S$, since lower production costs in the South creates the incentive for FDI in the model.

South where it is produced by a foreign affiliate of the northern firm. Such international technology transfer occurs at the endogenous rate ϕ^H for high-cost northern firms and at the endogenous rate ϕ^L for low-cost northern exporters. Imitation of both types of foreign affiliates occurs at the common exogenous rate ι_S, resulting in southern firms producing the product variety for the entire world market.

2.2 Households

In both the North and the South, there is a fixed measure of households that provide labor services in exchange for wage payments. Each individual member of a household lives forever and is endowed with one unit of labor, which is inelastically supplied. The size of each household, measured by the number of its members, grows exponentially at a fixed rate g_L, the population growth rate. Let $L_{Nt} = L_{N0}e^{g_L t}$ denote the supply of labor in the North at time t, let $L_{St} = L_{S0}e^{g_L t}$ denote the corresponding supply of labor in the South, and let $L_t = L_{Nt} + L_{St}$ denote the world supply of labor. In addition to wage income, households also receive asset income from their ownership of firms.

Households in both the North and the South share identical preferences. Each household is modeled as a dynastic family that maximizes discounted lifetime utility

$$U = \int_0^\infty e^{-(\rho - g_L)t} \ln(u_t) dt \qquad (1)$$

where $\rho > g_L$ is the subjective discount rate and u_t is the static utility of an individual at time t. The static constant elasticity of substitution (CES) utility function is given by

$$u_t = \left[\int_0^{n_t} x_t(\omega)^\alpha d\omega \right]^{\frac{1}{\alpha}}, \qquad 0 < \alpha < 1. \qquad (2)$$

In the CES utility function (2), $x_t(\omega)$ is the per capita quantity demanded of the product variety ω at time t and n_t is the total number of invented varieties at time t. The parameter α measures the degree of product differentiation across varieties. Varieties are assumed to be gross substitutes and the elasticity of substitution between different product varieties is $\sigma \equiv 1/(1-\alpha) > 1$.

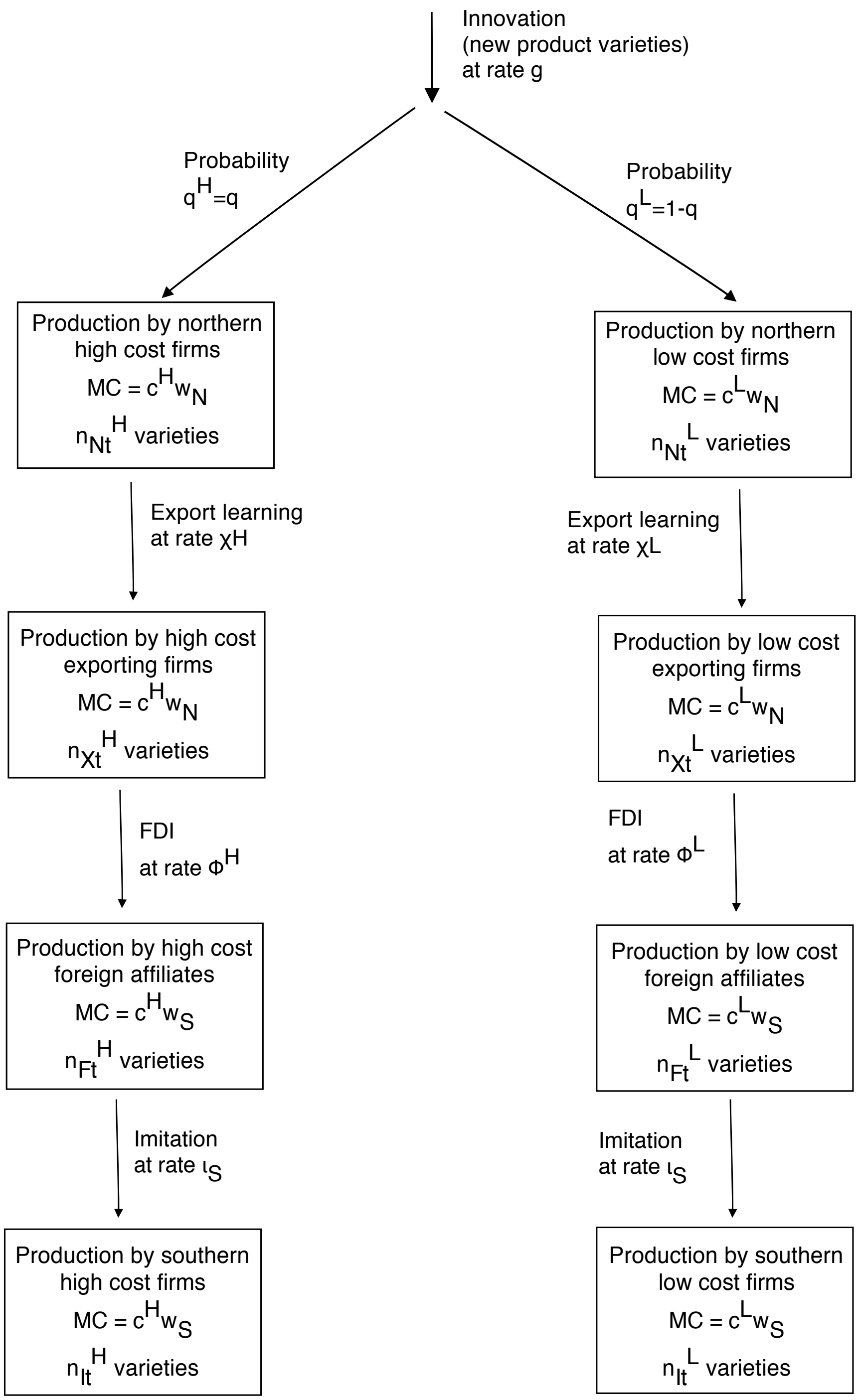

Figure 1: Product cycle

Solving the static consumer optimization problem yields the familiar demand function

$$x_t(\omega) = \frac{p_t(\omega)^{-\sigma} c_t}{P_t^{1-\sigma}} \tag{3}$$

where c_t is individual consumer expenditure at time t, $p_t(\omega)$ is the price of variety ω at time t, and $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ is an index of consumer prices. I will shortly define one such price index for each of the two regions. By substituting the demand function (3) into (2) and using the definition of the price index P_t, it can be shown that $u_t = c_t/P_t$. The consumer takes all prices as given when maximizing intertemporal utility. Maximizing (1) subject to the relevant intertemporal budget constraint yields the familiar intertemporal optimization condition

$$\frac{\dot{c}_t}{c_t} = r_t - \rho \tag{4}$$

implying that individual consumer expenditure only grows over time if the market interest rate r_t is larger than the subjective discount rate ρ.

The representative consumer in each region has different wage income ($w_N > w_S$) and hence different consumer expenditure. Let c_N and c_S denote the representative consumer's expenditure in the North and the South, respectively. I treat the southern wage rate as the numeraire price ($w_S = 1$) so all prices are measured relative to the price of southern labor. I solve the model for a steady-state equilibrium where wages w_N and w_S and consumer expenditures c_N and c_S are all constant over time. Therefore, in steady-state equilibrium, $\dot{c}_t/c_t = 0$ in (4) and $r_t = \rho$. The market interest rate is constant over time and equal in the two regions in steady-state equilibrium, $r_N = r_S = \rho$. The interest rate in one region can be different from that in the other region along the transition path to the new steady-state equilbrium since there is no international capital mobility.

The prices of goods will differ between the two regions because of trade costs. Product prices are denoted by p_N^L and p_N^H for low- and high-cost northern varieties sold in the domestic market and p_N^{L*} and p_N^{H*} for exported northern varieties in the South, p_F^L and p_F^H for foreign affiliate varieties sold in the South, and p_F^{L*} and p_F^{H*} for foreign affiliate varieties sold in the North. Varieties produced by southern firms are sold at the prices p_S^L and p_S^H in the South, and p_S^{L*} and p_S^{H*} in the North. In steady-state equilibrium, all product prices are constant over time.

2.3 Some Steady-State Dynamics

There are four types of varieties produced by high-cost firms: n_{Nt}^{H} varieties produced by high-cost non-exporting northern firms, n_{Xt}^{H} varieties produced by high-cost exporting northern firms (”X” for ”export”), n_{Ft}^{H} varieties produced by high-cost foreign affiliates (”F” for ”FDI”) and n_{It}^{H} varieties produced by high-cost southern firms that have imitated high-cost foreign affiliates (”I” for ”imitation”). Likewise, there are four types of varieties produced by low-cost firms: n_{Nt}^{L} varieties produced by low-cost non-exporting northern firms, n_{Xt}^{L} varieties produced by low-cost exporting northern firms, n_{Ft}^{L} varieties produced by low-cost foreign affiliates and n_{It}^{L} varieties produced by low-cost southern firms that have imitated low-cost foreign affiliates.

Let $g \equiv \dot{n}_t/n_t$ denote the steady-state growth rate of the number of varieties. From the variety condition $n_t = n_{Nt}^{H} + n_{Nt}^{L} + n_{Xt}^{H} + n_{Xt}^{L} + n_{Ft}^{H} + n_{Ft}^{L} + n_{It}^{H} + n_{It}^{L}$, it follows that the number of varieties produced by each type of firm must grow at the same rate g. Therefore the variety shares $\gamma_N^H \equiv n_{Nt}^H/n_t$, $\gamma_N^L \equiv n_{Nt}^L/n_t$, $\gamma_X^H \equiv n_{Xt}^H/n_t$, $\gamma_X^L \equiv n_{Xt}^L/n_t$, $\gamma_F^H \equiv n_{Ft}^H/n_t$, $\gamma_F^L \equiv n_{Ft}^L/n_t$, $\gamma_I^H \equiv n_{It}^H/n_t$ and $\gamma_I^L \equiv n_{It}^L/n_t$ are necessarily constant over time in any steady-state equilibrium and satisfy $\gamma_N^H + \gamma_N^L + \gamma_X^H + \gamma_X^L + \gamma_F^H + \gamma_F^L + \gamma_I^H + \gamma_I^L = 1$.

The price index in the North will be different than the price index in the South for two reasons. First, products prices differ across regions because of trade costs τ. Second, the set of product varieties available in the northern market is larger than the set of product varieties available in the southern market, since some northern product varieties are only sold domestically. Let P_{Nt} denote the price index for the North and P_{St} denote the price index for the South. From the definition of the price index $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ it follows that the northern price index satisfies $P_{Nt}^{1-\sigma} = \sum_{i=H,L}\left[n_{Nt}^i \left(p_N^i\right)^{1-\sigma} + n_{Xt}^i \left(p_N^i\right)^{1-\sigma} + n_{Ft}^i \left(p_F^{i*}\right)^{1-\sigma} + n_{It}^i \left(p_I^{i*}\right)^{1-\sigma}\right]$ and the southern price index satisfies $P_{St}^{1-\sigma} = \sum_{i=H,L}\left[n_{Xt}^i \left(p_N^{i*}\right)^{1-\sigma} + n_{Ft}^i \left(p_F^{i}\right)^{1-\sigma} + n_{It}^i \left(p_I^{i}\right)^{1-\sigma}\right]$. Using the variety shares defined earlier, I can rewrite these expressions as

$$P_{Nt}^{1-\sigma} = \sum_{i=H,L}\left[\gamma_N^i \left(p_N^i\right)^{1-\sigma} + \gamma_X^i \left(p_N^i\right)^{1-\sigma} + \gamma_F^i \left(p_F^{i*}\right)^{1-\sigma} + \gamma_I^i \left(p_I^{i*}\right)^{1-\sigma}\right] n_t \tag{5}$$

$$P_{St}^{1-\sigma} = \sum_{i=H,L}\left[\gamma_X^i \left(p_N^{i*}\right)^{1-\sigma} + \gamma_F^i \left(p_F^{i}\right)^{1-\sigma} + \gamma_I^i \left(p_I^{i}\right)^{1-\sigma}\right] n_t \tag{6}$$

where the terms in brackets are constant over time. Thus, $P_{Nt}^{1-\sigma}$ and $P_{St}^{1-\sigma}$ both grow over time at the rate g in any steady-state equilibrium.

Let $\chi^i \equiv (\dot{n}^i_{Xt} + \dot{n}^i_{Ft} + \dot{n}^i_{It})/n^i_{Nt}$ denote the steady-state rate at which non-exporting northern firms of marginal cost type i (where $i = H, L$) learn how to export to the South (the rate at which i-cost product varieties become available to southern consumers as a result of export-learning activities of the northern firms). The export-learning rate is constant over time in any steady-state equilibrium because $\chi^i \equiv \frac{\dot{n}^i_{Xt}+\dot{n}^i_{Ft}+\dot{n}^i_{It}}{n^i_{Nt}} = \frac{\dot{n}^i_{Xt}}{n^i_{Xt}}\frac{n^i_{Xt}/n_t}{n^i_{Nt}/n_t} + \frac{\dot{n}^i_{Ft}}{n^i_{Ft}}\frac{n^i_{Ft}/n_t}{n^i_{Nt}/n_t} + \frac{\dot{n}^i_{It}}{n^i_{It}}\frac{n^i_{It}/n_t}{n^i_{Nt}/n_t} = g\frac{\gamma^i_X}{\gamma^i_N} + g\frac{\gamma^i_F}{\gamma^i_N} + g\frac{\gamma^i_I}{\gamma^i_N}$. In the definition of export-learning rate, I take into account that some of the exported varieties are adapted for production by foreign affiliates, and in turn, some of these foreign affiliate varieties are imitated by southern firms.

Let $\phi^i \equiv (\dot{n}^i_{Ft} + \dot{n}^i_{It})/n^i_{Xt}$ denote the steady-state rate of international transfer of i-cost technology from the North to the South as a result of FDI. This FDI rate is constant over time in any steady-state equilibrium since $\phi^i \equiv \frac{\dot{n}^i_{Ft}+\dot{n}^i_{It}}{n^i_{Xt}} = \frac{\dot{n}^i_{Ft}}{n^i_{Ft}}\frac{n^i_{Ft}/n_t}{n^i_{Xt}/n_t} + \frac{\dot{n}^i_{It}}{n^i_{It}}\frac{n^i_{It}/n_t}{n^i_{Xt}/n_t} = g\frac{\gamma^i_F}{\gamma^i_X} + g\frac{\gamma^i_I}{\gamma^i_X}$. In the definition of the FDI rate, I take into account that adaption to production by foreign affiliates in the South involves exposure to a positive rate of imitation from southern firms. The flow $\dot{n}^i_{It}$ represents the flow of foreign affiliate varieties that are imitated by southern firms.

Let $\iota_S \equiv \dot{n}^i_{It}/n^i_{Ft}$ denote the imitation rate of foreign affiliate-produced varieties. It is constant over time in steady-state equilibrium since $\iota_S \equiv \frac{\dot{n}^i_{It}}{n^i_{Ft}} = \frac{\dot{n}^i_{It}}{n^i_{It}}\frac{n^i_{It}/n_t}{n^i_{Ft}/n_t} = g\frac{\gamma^i_I}{\gamma^i_F}$.

By the law of large numbers, $\gamma^i_N + \gamma^i_X + \gamma^i_F + \gamma^i_I = q^i$. From the variety condition $n_t = \sum_{i=H,L} n^i_{Nt} + n^i_{Xt} + n^i_{Ft} + n^i_{It}$, it follows that a share $q^H = q$ of total varieties consists of the high-cost varieties and the remaining share $q^L = 1 - q$ consists of low-cost varieties. By taking the derivative of $q^i n_t = n^i_{Nt} + n^i_{Xt} + n^i_{Ft} + n^i_{It}$ with respect to time, dividing by n_t and using the definitions of the northern i-cost variety share γ^i_N, the growth rate g and the FDI rate ϕ^i, I obtain a steady-state expression for γ^i_N:

$$\gamma^i_N = q^i \frac{g}{g + \chi^i}, \qquad (i = H, L). \tag{7}$$

A faster innovation rate corresponds to larger shares of varieties on the world market produced by non-exporting northern firms ($g \uparrow \Longrightarrow \gamma^i_N \uparrow$). Also, i-cost firms becoming exporters at a faster rate corresponds to a smaller share of world production being done by i-cost non-exporting firms in the North ($\chi^i \uparrow \Longrightarrow \gamma^i_N \downarrow$).

From the definition of the export-learning rate, it follows that $\chi^i = \left(g + \phi^i\right) \gamma^i_X/\gamma^i_N$. Inserting the steady-state expressions for γ_N^i from (7) yields

$$\gamma_X^i = q^i \frac{\chi^i}{g + \chi^H} \frac{g}{g + \phi^i}, \qquad (i = H, L). \tag{8}$$

Faster export-learning rates for northern firms correspond to larger shares of world production being done by northern exporters ($\chi^i \uparrow \Longrightarrow \gamma_X^i \uparrow$). Northern exporters becoming multinational firms and transferring production to the South at a faster rate corresponds to smaller shares of world production being done by northern exporters ($\phi^i \uparrow \Longrightarrow \gamma_X^i \downarrow$).

From the definition of the FDI rate ϕ^i and using the definition of the imitation rate $\iota_S \equiv \dot{n}_{It}^i / n_{Ft}^i$, it follows that $\gamma_F^i = \gamma_X^i \phi^i / (g + \iota_S)$. Inserting the steady-state expressions for γ_X^i yields

$$\gamma_F^i = q^i \frac{\chi^i}{g + \chi^i} \frac{\phi^i}{g + \phi^i} \frac{g}{g + \iota_S}, \qquad (i = H, L). \tag{9}$$

A higher FDI rate for i-cost technology translates into a larger share of world production taking place in foreign affiliates of marginal cost type i ($\phi^i \uparrow \Longrightarrow \gamma_F^i \uparrow$). Since only exporting firms can become multinationals, a higher export-learning rate also corresponds to a higher share of foreign affiliate production ($\chi^i \uparrow \Longrightarrow \gamma_F^i \uparrow$). On the other hand, a faster imitation rate corresponds to a smaller share of world production being done by foreign affiliates ($\iota_S \uparrow \Longrightarrow \gamma_F^i \downarrow$).

From the definition of the imitation rate $\iota_S \equiv \frac{\dot{n}_{It}^i}{n_{Ft}^i}$ it follows that $\iota_S = g \frac{\gamma_I^i}{\gamma_F^i}$. Using this expression along with the expressions for γ_F^i from (9), I obtain

$$\gamma_I^i = q^i \frac{\chi^i}{g + \chi^i} \frac{\phi^i}{g + \phi^i} \frac{\iota_S}{g + \iota_S}, \qquad (i = H, L). \tag{10}$$

As expected, a faster imitation rate corresponds to larger shares of world varieties being produced by southern firms ($\iota_S \uparrow \Longrightarrow \gamma_I^i \uparrow$). Imitation targets foreign affiliates so more foreign affiliate varieties in the South means that more product varieties can be imitated and produced by southern firms ($\phi^i \uparrow \Longrightarrow \gamma_I^i \uparrow$). Since only exporters engage in adaptive R&D, a faster rate of export-learning corresponds to more production taking place in southern firms ($\chi^i \uparrow \Longrightarrow \gamma_I^i \uparrow$) .

2.4 Product Markets

Firms compete in prices and maximize profits. There are constant returns to scale in production. Production of one unit of output requires c^H units of labor for a high-cost firm, and c^L units of labor for a low-cost firm. There are iceberg trade costs, such that $\tau > 1$ units of a good must be shipped for one unit of the good to arrive at its destination. A northern high-cost non-exporting firm has the marginal cost $c^H w_N$ and a northern low-cost non-exporting firm has the marginal cost $c^L w_N$. A northern high-cost exporting firm has the marginal cost $c^H w_N$ when selling to the domestic market, but the marginal cost $\tau c^H w_N$ when selling to the southern market. The corresponding marginal cost for a low-cost northern exporter is $c^L w_N$ when selling to the domestic market (the North), and $\tau c^L w_N$ when selling to the export market (the South). A high-cost foreign affiliate and a high-cost southern firm has the marginal cost $c^H w_S$ when selling to the domestic market (the South) and $\tau c^H w_S$ when selling to their export market (the North). The corresponding marginal costs for low-cost foreign affiliates and low-cost southern firms are $c^L w_S$ and $\tau c^L w_S$, respectively.

All northern firms earn domestic profits. The domestic profit flow for a northern firm is given by $\pi^i_{Nt} = \left(p^i_N - c^i w_N\right) x^i_{Nt} L_{Nt}$, $(i = H, L)$, where p^i_N is the price of a northern variety of marginal cost type i in the domestic market, c^i is the unit labor requirement in production and x^i_{Nt} is the quantity of the northern firm's product demanded by the typical northern consumer. (Note that c_N is consumer expenditure of the typical northern consumer.) From (3), a northern firm faces domestic consumer demand $x^i_{Nt} = (p^i_N)^{-\sigma} c_N / P^{1-\sigma}_{Nt}$. A northern firm chooses its price to maximize profits, and it is straightforward to verify that the profit-maximizing price is the monopoly price $p^i_N = \frac{c^i w_N}{\alpha}$. A high-cost northern firm has a higher marginal cost than a low-cost northern firm so the price of a high-cost firm's product variety will be higher. Using the obtained monopoly prices and defining population size-adjusted aggregate domestic demand for non-exported northern i-cost varieties $X^i_N \equiv \frac{\left(p^i_N\right)^{-\sigma} c_N L_{Nt} n^i_{it}}{P^{1-\sigma}_{Nt} L_t}$, the northern domestic profit flow can be written as

$$\pi^i_{Nt} = \frac{c^i w_N X^i_N}{(\sigma - 1)\gamma^i_N} \frac{L_t}{n_t} \qquad (i = H, L). \tag{11}$$

The aggregate demand terms are constant over time in steady-state equilibrium since prices, consumer expenditure and varitety shares are constant, L_t and L_{Nt} grow at the same rate g_L and n^i_{Nt} and $P^{1-\sigma}_{Nt}$ grow at the same rate g. In the profit expressions, the marginal cost terms c^i and the elasticity of substitution σ are parameters, while the wage rate w_N and the variety share γ^i_N are constant over time in steady-state equilibrium. Therefore, profits only change over time in steady-state equilibrium as the market size term L_t/n_t changes over time. Population growth increases the size of the market, while variety growth decreases the relevant market size for an individual firm.

A northern firm that has learned how to export to the South earns the global profit flow $\pi^i_{Xt} = \left(p^i_N - c^i w_N\right) x^i_{Xt} L_{Nt} + \left(p^{i*}_N - \tau c^i w_N\right) x^{i*}_{Xt} L_{St}$, $(i = H, L)$ where p^i_N is the price of the exporter's product variety in the domestic market, p^{i*}_N the price of the same variety in the export market, $x^i_{Xt} = \left(p^i_N\right)^{-\sigma} c_N / P^{1-\sigma}_{Nt}$ is the quantity of the northern exporter's product demanded by the typical northern consumer and $x^{i*}_{Xt} = \left(p^{i*}_N\right)^{-\sigma} c_S / P^{1-\sigma}_{St}$ is the quantity of the same product variety demanded by the typical southern consumer. A northern exporter faces the same domestic consumer demand and profit-maximizing considerations in the domestic market as the northern firms that do not export. The exporter's profit-maximizing price in the domestic market is $p^i_N = \frac{c^i w_N}{\alpha}$, same as for non-exporting northern firms. In the export market, a northern exporting firm sets the profit-maximizing price $p^{i*}_N = \frac{\tau c^i w_N}{\alpha}$. Using the obtained prices and defining population size-adjusted aggregate demand for northern exporters' varieties in the domestic and export market, respectively, as $X^i_X \equiv \frac{\left(p^i_N\right)^{-\sigma} c_N L_{Nt} n^i_{Xt}}{P^{1-\sigma}_{Nt} L_t}$ and $X^{i*}_X \equiv \frac{\left(p^{i*}_N\right)^{-\sigma} c_S L_{St} n^i_{Xt}}{P^{1-\sigma}_{St} L_t}$, the global profit flow of a northern exporter can be written as

$$\pi^i_{Xt} = \frac{c^i w_N}{(\sigma - 1)} \frac{\left(X^i_X + \tau X^{i*}_X\right)}{\gamma^i_X} \frac{L_t}{n_t}, \qquad (i = H, L). \tag{12}$$

Since X^i_X and X^{i*}_X are constant over time, the profits of a northern exporter only change over time because the market size L_t/n_t changes over time.

The global profit flow for a foreign affiliate of marginal cost type i is $\pi^i_{Ft} = \left(p^i_F - c^i w_S\right) x^i_{Ft} L_{St} + \left(p^{i*}_F - \tau c^i w_S\right) x^{i*}_{Ft} L_{Nt}$, where p^i_F and p^{i*}_F are the prices of a foreign affiliate variety in the South and the North, respectively, $x^i_{Ft} = \left(p^i_F\right)^{-\sigma} c_S / P^{1-\sigma}_{St}$ is the quantity of the foreign affiliate variety demanded by the typical southern consumer and $x^{i*}_{Ft} = \left(p^{i*}_F\right)^{-\sigma} c_N / P^{1-\sigma}_{Nt}$ the quantity of that variety demanded by the typical northern consumer. Profit-maximizing monopoly prices can be shown to be $p^i_F = \frac{c^i w_S}{\alpha}$ in the domestic market (the South) and $p^{i*}_F = \frac{\tau c^i w_S}{\alpha}$ in the export market (the North). The incentive for an exporter to become a multinational firm and move production to the South is not market access, but to earn higher profits by lowering production cost. Therefore I will solve for equilibria where the inequality condition $w_N > \tau w_S$ holds so foreign affiliates export back to the North and the parent firm in the North ceases to produce there. In Helpman et al (2004), firms choose to enter into the foreign market either through exporting or through FDI. Market access is driving (horizontal) FDI in their model since a multinational firm continues to serve the parent firm's market via production at home.[5] Using the profit-maximizing prices and defining population size-adjusted aggregate demand for foreign affiliate varieties in the South and the North, respectively, as $X_F^i \equiv \frac{\left(p_F^i\right)^{-\sigma} c_S L_{St} n_{Ft}^i}{P_{St}^{1-\sigma} L_t}$ and $X_F^{i*} \equiv \frac{\left(p_F^{i*}\right)^{-\sigma} c_N L_{Nt} n_{Ft}^i}{P_{Nt}^{1-\sigma} L_t}$, the global profit flow for a foreign affiliate can be written as

$$\pi_{Ft}^i = \frac{c^i w_S}{(\sigma-1)} \frac{\left(X_F^i + \tau X_F^{i*}\right)}{\gamma_F^i} \frac{L_t}{n_t}, \qquad (i = H, L). \tag{13}$$

Once imitation has occurred, the blueprint is freely available to all southern firms. Southern firms do not incur any imitation costs. A southern firm that uses high marginal cost technology becomes a high marginal cost southern firm, and a southern firm that uses low marginal cost technology becomes a low marginal cost southern firm. Southern firms do not need to engage in export-learning. Instead, an imitated product variety can be immediately sold to the entire world market. If only a few southern firms would export to the North, the risk of imitation for foreign affiliates and exporters would be less severe, which is a weaker case than the one considered here. No southern firm can set its price higher than marginal cost, and all southern firms earn zero profits. The resulting prices are $p_S^i = c^i w_S$ and $p_S^{i*} = \tau c^i w_S$. Population size-adjusted aggregate demand for the imitated varieties produced by southern firms are defined as $X_I^i \equiv \frac{\left(p_S^i\right)^{-\sigma} c_S L_{St} n_{It}^i}{P_{St}^{1-\sigma} L_t}$ and $X_I^{i*} \equiv \frac{\left(p_S^{i*}\right)^{-\sigma} c_N L_{Nt} n_{It}^i}{P_{Nt}^{1-\sigma} L_t}$.

2.5 Technology for Innovation, FDI and Export-Learning

There is free entry into innovative R&D activities in the North, with every northern firm having access to the same R&D technology. To innovate and develop a new product variety, a representative northern firm j must devote $a_N g^\beta / n_t^\theta$ units of labor to innovative R&D, where a_N is an innovative R&D productivity parameter, n_t is the disembodied stock of knowledge at time t and θ is an intertemporal knowledge spillover parameter.[6] The parameter $\beta > 0$ captures decreasing returns to R&D at the industry level. When there is more innovation in the economy (the growth rate of

[5]The assumption that exporters always keep serving the domestic market in my model is the same as in Helpman et al (2004). However, they assume that firms that engage in FDI serve the foreign market through the foreign affiliate but do not export back to the host country. This assumption is relaxed in the working paper version of their paper where they allow for export platform FDI. I assume that once a firm has successfully adapted production to a foreign affiliate, the parent firm no longer produces the variety in the domestic market. Instead it is exported from the foreign affiliate in the South to the northern market.

[6]For $\theta > 0$, R&D labor becomes more productive as time passes and a northern firm needs to devote less labor to develop a new variety as the stock of knoweldge increases. For $\theta < 0$, R&D becomes more difficult over time.

the number of varieties g is higher), each individual northern firm must devote more resources to innovation in order to successfully develop one new product variety. In the following description of the model, I set $\beta = 1$ which is within the range estimated by Kortum (1993).[7] When firms decide to invest (employ labor) in innovation, they do not know their own productivity for producing output, only the probabilities for high and low marginal cost draws. Given this technology, the flow of new products developed by northern firm j is

$$\dot{n}_{jt} = \frac{l_{Rjt}}{a_N g / n_t^{\theta}} = \frac{n_t^{\theta} l_{Rjt}}{a_N g}, \tag{14}$$

where l_{Rjt} is the northern labor employed by firm j in innovative R&D. Aggregating over all northern firms, the aggregate flow of new products developed in the North is

$$\dot{n}_t = \frac{n_t^{\theta} L_{Rt}}{a_N g}, \tag{15}$$

where $L_{Rt} \equiv \sum_j l_{Rjt}$ is the total amount of northern labor employed in innovative activities.

In any steady-state equilibrium, the share of labor employed in innovative R&D must be constant over time. Given that the northern supply of labor grows at the population growth rate g_L, northern R&D employment L_{Rt} must grow at this rate as well. Dividing both sides of (15) by n_t yields

$$g \equiv \frac{\dot{n}_t}{n_t} = \frac{n_t^{\theta-1} L_{Rt}}{a_N g}.$$

Since g is constant over time in any steady-state equilibrium, $n_t^{\theta-1}$ and L_{Rt} must grow at offsetting rates, that is, $(\theta - 1) \frac{\dot{n}_t}{n_t} + \frac{\dot{L}_{Rt}}{L_{Rt}} = (\theta - 1) g + g_L = 0$. It immediately follows that

$$g \equiv \frac{\dot{n}_t}{n_t} = \frac{g_L}{1 - \theta}. \tag{16}$$

Thus, the steady-state rate of innovation g is pinned down by parameter values and is proportional to the population growth rate g_L. As in Jones (1995), when there is positive population growth, the parameter restriction $\theta < 1$ is needed to guarantee that the steady-state rate of innovation is positive and finite.

I can now solve for the steady-state rate of economic growth. The representative northern

[7] Kortum (1993) estimates that $1/(1+\beta)$ is between 0.1 and 0.6. $\beta = 1$ yields $1/(1+\beta) = 0.5$.

consumer has utility $u_{Nt} = c_N/P_{Nt}$ and the representative southern consumer has utility $u_{St} = c_S/P_{St}$. In steady-state equilibrium, individual consumer expenditure is constant over time but consumer utility nevertheless grows because the price indexes fall over time. Since $P_{Nt}^{1-\sigma}$ and $P_{St}^{1-\sigma}$ both grow over time at the rate g, it follows that consumer utility growth is

$$g_u \equiv \frac{\dot{u}_{Nt}}{u_{Nt}} = \frac{\dot{u}_{St}}{u_{St}} = \frac{g}{\sigma - 1} = \frac{g_L}{(1-\theta)(\sigma - 1)}. \tag{17}$$

With consumer utility in both regions being proportional to consumer expenditure holding prices fixed, consumer utility growth equals real wage growth, which I use as a measure of economic growth. The economic growth rate is completely pinned down by parameters of the model (the population growth rate g_L, the elasticity of substitution σ, and the knowledge spillover parameter θ). This means that public policy changes like trade liberalization (a decrease in τ) or export subsidies (a decrease in a_X, as will be discussed later) have no effect on the steady-state rate of economic growth, so growth is "semi-endogenous".

In the unit labor requirement for innovation $a_N g/n_t^{\theta}$, the term $1/n_t^{\theta}$ is a measure of absolute R&D difficulty. It increases over time if $\theta < 0$ and decreases over time if $\theta \in (0, 1)$. Following Jakobsson and Segerstrom (2012), I define relative R&D difficulty as absolute R&D difficulty divided by the market size term L_t/n_t:

$$\delta \equiv \frac{n_t^{-\theta}}{L_t/n_t} = \frac{n_t^{1-\theta}}{L_t}.$$

From (16), it follows that relative R&D difficuly is constant over time in steady-state equilibrium. As discussed earlier, the innovation rate g is constant in steady-state equilibrium, but a larger δ in one steady-state compared to an earlier steady-state means that there has been more innovation in the transition to the new steady-state, and that the stock of knowledge (number of varieties) has increased permanently. In the short run, the rate of innovation increases, but in the long run, the rate of innovation returns to its steady-state rate.

Export-learning R&D is undertaken after the firm knows its own productivity. To learn how to export one product variety to the South, a northern firm of type i ($i = H, L$) must employ $a_X \left(\chi^i\right)^{\beta} / n_t^{\theta}$ units of southern labor to export-learning R&D.[8] This kind of R&D can be thought of as marketing and setting up distribution networks. The parameter a_X is an exporting R&D

[8] I assume that southern labor is employed for northern firms' export-learning activities. This facilitates comparison between FDI activities and export-learning activities in the model.

productivity parameter. As with innovation, $\beta > 0$ captures the decreasing returns to export-learning R&D. When more firms learn how to become exporters (the rate of export-learning χ^i is higher), each individual northern firm must devote more resourcers to successfully become an exporter. To simplify calculations, I set $\beta = 1$. The flow of new products that southern consumers can buy due to exporter j's activities is given by

$$\dot{n}^i_{Xjt} + \dot{n}^i_{Fjt} + \dot{n}^i_{Ijt} = \frac{l^i_{Xjt}}{a_X \chi^i / n^\theta_t} = \frac{n^\theta_t l^i_{Xjt}}{a_X \chi^i}, \qquad (i = H, L) \tag{18}$$

where l^i_{Xjt} is the southern labor employed by firm j of marginal cost type i $(i = H, L)$ in export-learning R&D. Aggregating over all northern firms, the flow of new products sold in the South as a consequence of export-learning activities is

$$\dot{n}^i_{Xt} + \dot{n}^i_{Ft} + \dot{n}^i_{It} = \frac{n^\theta_t L^i_{Xt}}{a_X \chi^i}, \qquad (i = H, L) \tag{19}$$

where $L^i_{Xt} \equiv \sum_j l^i_{Xjt}$ is the total amount of southern labor employed in export-learning activities by firms of marginal cost type i. Some exporters then go on to become multinational firms after engaging in adaptive R&D, and some of these foreign affiliate-produced varieties become imitated by southern firms. Therefore, the flows $\dot{n}^i_{Ft}$ and $\dot{n}^i_{It}$ must be taken into account in the exported product flow.

Adaptive R&D (or FDI) is undertaken by northern exporters. To learn how to produce an exported variety in the South, the foreign affiliate of a northern exporting firm of marginal cost type i must devote $a_F \left(\phi^i\right)^\beta / n^\theta_t$ units of southern labor to adaptive R&D. The parameter a_F is an adaptive R&D productivity parameter that is common to all firms and can be thought of as measuring the ease of doing FDI in the South. There are decreasing returns also to adaptive R&D. When northern exporters are doing more FDI (ϕ^i is higher), each individual exporting firm must devote more resources to adaptive R&D in order to be successful in transferring production to a foreign affiliate in the South. Again, I set $\beta = 1$. The flow of products that are transferred to the South due to the adaptive R&D activities of firm j of marginal cost type i is

$$\dot{n}^i_{Fjt} + \dot{n}^i_{Ijt} = \frac{l^i_{Fjt}}{a_F \phi^i / n^\theta_t} = \frac{n^\theta_t l^i_{Fjt}}{a_F \phi^i}, \qquad (i = H, L) \tag{20}$$

where l^i_{Fjt} is the southern labor employed by firm j of marginal cost type i in adaptive R&D.

Aggregating over all foreign affiliates generates the product flow

$$\dot{n}_{Ft}^{i}+\dot{n}_{It}^{i}=\frac{n_t^{\theta}L_{Ft}^{i}}{a_F\phi^{i}},\qquad (i=H,L) \tag{21}$$

where $L_{Ft}^{i}\equiv\sum_j l_{Fjt}^{i}$ is the aggregate amount of southern labor employed in adaptive R&D by firms of marginal cost type i.

Imitation targets foreign affiliates in the South. Let $\iota_S\equiv 1/a_I$ where a_I is a measure of the strength of southern IPR protection. With stronger southern IPR protection, the rate of imitation is lower ($a_I\uparrow\Longrightarrow\iota_S\downarrow$).

2.6 R&D Incentives

At the time when firms decide how much innovation to engage in, they do not yet know their own productivity but the firms know the probability of becoming a high-cost or a low-cost firm. Denote the expected discounted profits associated with innovating in the North at time t for a firm of marginal cost type i ($i=H,L$) by v_{Nt}^{i}. The R&D labor used to develop one new variety is a_Ng/n_t^{θ} and the cost of developing this variety is w_Na_Ng/n_t^{θ}. Taking into account the probability of becoming a high- and low-cost producer, free entry into innovative R&D activities in the North implies that the cost of innovating must be exactly balanced by the expected benefit from innovating in equilibrium:

$$qv_{Nt}^{H}+(1-q)\,v_{Nt}^{L}=\frac{w_Na_Ng}{n_t^{\theta}}. \tag{22}$$

After successful innovation, a northern firm learns its productivity in manufacturing, produces in the North and serves the northern market. The firm can then choose to do export-learning R&D with the purpose of exporting to the southern market. Let v_{Xt}^{i} be the expected discounted profits that an exporter of marginal cost type i ($i=H,L$) earns. The export-learning R&D needed for a firm of marginal cost type i to learn how to export one product variety to the South is $a_X\chi^{i}/n_t^{\theta}$ and the cost of this export-learning is $w_Sa_X\chi^{i}/n_t^{\theta}$. The benefit from becoming an exporter is given by $v_{Xt}^{i}-v_{Nt}^{i}$. Note that v_{Nt}^{i} must be subtracted since the expected discounted profits earned in the domestic market are already included in v_{Xt}^{i}.[9] A firm of marginal cost type i will decide to become an exporter if $v_{Xt}^{i}-v_{Nt}^{i}\geq\frac{w_Sa_X\chi^{i}}{n_t^{\theta}}$. If this holds with strict inequality, there will be infinite export

[9]There are no "pure exporters" in the model. All exporting firms also serve their domestic market.

learning and if $v_{Xt}^i - v_{Nt}^i < \frac{w_S a_X \chi^i}{n_t^\theta}$, no northern firm will choose to become an exporter. Since the cost of learning how to export must be exactly balanced by the benefit of exporting in steady-state equilibrium, I obtain

$$v_{Xt}^i - v_{Nt}^i = \frac{w_S a_X \chi^i}{n_t^\theta}, \qquad (i = H, L). \tag{23}$$

Let v_{Ft}^i be the expected discounted profits that a foreign affiliate of marginal cost type i earns from producing a product variety in the South. A northern exporter uses $a_F \phi^i / n_t^\theta$ units of southern labor to adapt production of a product variety to the South and the cost of this transfer is $w_S a_F \phi^i / n_t^\theta$. A northern exporter will choose to become a multinational firm if $v_{Ft}^i - v_{Xt}^i \geq \frac{w_S a_F \phi^i}{n_t^\theta}$. For finite levels of adaptive R&D, this must hold with equality. The expected benefit from becoming a multinational is the gain in expected profits, since the exporting firm is already earning profits from producing in the North and serving both markets. The foreign affiliate pays its parent firm a royalty payment v_{Xt}^i for using the technology that the parent firm has transferred to the South. In steady-state equilibrium the cost of transferring production to the South must be exactly balanced by the benefit, and therefore

$$v_{Ft}^i - v_{Xt}^i = \frac{w_S a_F \phi^i}{n_t^\theta}, \qquad (i = H, L). \tag{24}$$

There is a stock market in each region that channels household savings to firms that engage in different kinds of R&D. There is no international capital mobility so northern savings finance R&D in the North (innovation) and southern savings finance R&D in the South (export entry and adaption). Households earn a safe return from holding the market portfolio in each region since there is no aggregate risk. Ruling out any arbitrage opportunities, the total return on equity equals the opportunity cost of invested capital, which is the risk-free market interest rate ρ.

The relevant no-arbitrage condition for a northern firm j of marginal cost type i $(i = H, L)$ is

$$\left(\pi_{Nt}^i - w_S l_{Xjt}^i\right) dt + \dot{v}_{Nt}^i dt + \left(\dot{n}_{Xjt}^i + \dot{n}_{Fjt}^i + \dot{n}_{Ijt}^i\right) dt \left(v_{Xt}^i - v_{Nt}^i\right) = \rho v_{Nt}^i dt.$$

During the time interval dt, the northern firm earns the profit flow $\pi_{Nt}^i dt$, but also incurs the export-learning cost $w_S l_{Xjt}^i dt$ and experiences a gradual capital gain $\dot{v}_{Nt}^i dt$. In the time interval dt, the firm is responsible for introducing $\left(\dot{n}_{Xjt}^i + \dot{n}_{Fjt}^i + \dot{n}_{Ijt}^i\right) dt$ varieties to the southern market as a result of its export-learning activities. When the firm is successful in becoming an exporter, its market value jumps up by $v_{Xt}^i - v_{Nt}^i$. To rule out any arbitrage opportunities for investors, the rate of return for a northern firm must be the same as the return on an equal sized investment in a risk-free bond $\rho v_{Nt}^i dt$. From (18) and (23), it follows that

$$\left(\dot{n}_{Xjt}^i + \dot{n}_{Fjt}^i + \dot{n}_{Ijt}^i\right)\left(v_{Xt}^i - v_{Nt}^i\right) = w_S l_{Xjt}^i.$$

Equation (22) implies that v_{Nt}^i must grow at the rate $-\theta g$. Using this and dividing by $v_{Nt}^i dt$, the no-arbitrage condition for the i-cost northern firm becomes $\frac{\pi_{Nt}^i}{v_{Nt}^i} - \theta g = \rho$, or $v_{Nt}^i = \frac{\pi_{Nt}^i}{\rho + \theta g}$. Combining this expression with (22), the northern no-arbitrage condition can be written as

$$\frac{q\pi_{Nt}^H + (1-q)\,\pi_{Nt}^L}{\rho + \theta g} = \frac{w_N a_N g}{n_t^\theta}. \tag{25}$$

The left-hand side is the expected discounted profit from innovating, taking into account the probability of becoming a high-cost firm, appropriately discounted by the market interest rate ρ and the capital loss term θg. The right-hand side is the cost of innovating. Inserting the earlier derived profit expressions into (25), dividing both sides by w_N and then by the market size term L_t/n_t yields the northern steady-state no-arbitrage condition

$$\frac{\frac{1}{\sigma-1}\left(\frac{qc^H X_N^H}{\gamma_N^H} + \frac{(1-q)c^L X_N^L}{\gamma_N^L}\right)}{\rho + \theta g} = a_N g \delta. \tag{26}$$

The left-hand side of (26) is the market size-adjusted expected benefit from innovating (taking into account the probability of becoming a high-cost producer) and the right-hand side is the market size-adjusted cost of innovating. As long as the population growth rate differs from the growth rate of the number of varieties ($g_L \neq g$) and there are knowledge spillovers ($\theta \neq 0$), the market size L_t/n_t changes over time and I need to adjust for that in the steady-state calculations. The market size-adjusted benefit from innovating is higher when the average consumer buys more of non-exported northern varieties ($X_N^i \uparrow$ where $i = H, L$), future profits are less heavily discounted ($\rho \downarrow$), and northern firms experience smaller capital losses over time ($\theta g \downarrow$). The market size-adjusted cost of innovating is higher when northern researchers employed in innovative R&D are less productive ($a_N \uparrow$), innovating is relatively more difficult ($\delta \uparrow$) and the innovation rate is higher ($g \uparrow$), because then each northern firm must hire more researchers to be successful in developing a new product variety.

For a northern exporter j of marginal cost type i, the relevant no-arbitrage condition is

$$\left(\pi^i_{Xt} - w_S l^i_{Fjt}\right) dt + \dot{v}^i_{Xt} dt + \left(\dot{n}^i_{Fjt} + \dot{n}^i_{Ijt}\right) dt \left(v^i_{Ft} - v^i_{Xt}\right) = \rho v^i_{Xt} dt.$$

During the time interval dt, the exporter earns the profit flow $\pi^i_{Xt} dt$, incurs the FDI cost $w_S l^i_{Fjt} dt$ and experiences the gradual capital gain $\dot{v}^i_{Xt} dt$. In the time interval dt, the exporter is successful in transferring prodution of $\left(\dot{n}^i_{Fjt} + \dot{n}^i_{Ijt}\right) dt$ varieties to the South. The firm's market value jumps up by $v^i_{Ft} - v^i_{Xt}$ when it is successful in moving production to the South. As seen for non-exporting firms, the rate of return for a northern exporting firm must be the same as the return on an equal sized investment in a risk-free bond $\rho v^i_{Xt} dt$ to rule out any arbitrage opportunities for investors. From (20) and (24), it follows that

$$\left(\dot{n}^i_{Fjt} + \dot{n}^i_{Ijt}\right) \left(v^i_{Ft} - v^i_{Xt}\right) = w_S l^i_{Fjt}.$$

Also, from (23), it follows that $\frac{\dot{v}^i_{Xt}}{v^i_{Xt}} = -\theta g$. Thus, after dividing by $v^i_{Xt} dt$, the no-arbitrage condition simplifies to $v^i_{Xt} = \frac{\pi^i_{Xt}}{\rho + \theta g}$. Combining this with (23) yields the northern exporter no-arbitrage conditions

$$\frac{\pi^i_{Xt}}{\rho + \theta g} - \frac{\pi^i_{Nt}}{\rho + \theta g} = \frac{w_S a_X \chi^i}{n^\theta_t}, \qquad (i = H, L) \tag{27}$$

where the left-hand side is the increase in expected discounted profits from becoming an exporter and the right-hand side is the export-learning cost. Substituting for π^H_{Xt} and π^L_{Xt} using (12) and dividing both sides by w_S and the market size term L_t/n_t yields the steady-state northern exporter no-arbitrage conditions

$$\frac{c^i w}{\sigma - 1} \left[\frac{\frac{X^i_X + \tau X^{i*}_X}{\gamma^i_X} - \frac{X^i_N}{\gamma^i_N}}{\rho + \theta g} \right] = a_X \chi^i \delta, \qquad (i = H, L) \tag{28}$$

where $w = w_N/w_S$ is the northern relative wage. The left-hand side of (28) is the market size-adjusted benefit from becoming an exporter, and the right-hand side is the market size-adjusted cost of learning how to export to the South (for a high-cost and a low-cost firm). The market size-adjusted benefit from becoming an exporter is higher when the average consumer buys more of exported varieties ($X^i_X + \tau X^{i*}_X \uparrow$), future profits are less heavily discounted ($\rho \downarrow$) and northern exporters experience smaller capital losses over time ($\theta g \downarrow$). The market size-adjusted cost of learning how to export is higher when workers employed in export entry-related activites in the South are less productive ($a_X \uparrow$) and export-learning is relatively more difficult ($\delta \uparrow$). Also, when export-learning is occuring at a faster rate ($\chi^i \uparrow$), each individual firm needs to hire more labor in order to be successful in exporting to the southern market.

A foreign affiliate j of marginal cost type i faces the no-arbitrage condition

$$\pi^i_{Ft}dt + \dot{v}^i_{Ft}dt - (\iota_S dt)\, v^i_{Ft} = \rho v^i_{Ft}dt.$$

In the time interval dt the foreign affiliate earns the profit flow $\pi^i_{Ft}dt$ and experiences a gradual capital gain $\dot{v}^i_{Ft}dt$. However, it is exposed to a positive rate of imitation by southern firms and experiences a total capital loss if it is imitated, which occurs with the probability $\iota_S dt$ during the time interval dt. From (24), it follows that $\frac{\dot{v}^i_{Ft}}{v^i_{Ft}} = -\theta g$, so after dividing the no-arbitrage condition by $v^i_{Ft}dt$, I obtain $v^i_{Ft} = \frac{\pi^i_{Ft}}{\rho+\theta g+\iota_S}$. Combining this with (24) yields

$$\frac{\pi^i_{Ft}}{\rho + \theta g + \iota_S} - \frac{\pi^i_{Xt}}{\rho + \theta g} = \frac{w_S a_F \phi^i}{n^\theta_t}, \qquad (i = H, L) \tag{29}$$

where the left-hand side of (29) is the increase in expected discounted profits from moving production to the South and the right-hand side is the adaptive R&D cost. The expected profits of the foreign affiliate are discounted by the market interest rate ρ, the capital loss term θg and the imitation rate ι_S. Substituting for π^i_{Xt} using (12) and for π^i_{Ft} using (13), and then dividing both sides by w_S and the market size term L_t/n_t, yields the foreign affiliate steady-state no-arbitrage conditions

$$\frac{c^i}{\sigma - 1}\left[\frac{\frac{X^i_F+\tau X^{i*}_F}{\gamma^i_F}}{\rho + \theta g + \iota_S} - \frac{\frac{w\left(X^i_X+\tau X^{i*}_X\right)}{\gamma^i_X}}{\rho + \theta g}\right] = a_F \phi^i \delta, \qquad (i = H, L). \tag{30}$$

The left-hand side is the market size-adjusted benefit from becoming a multinational firm, and the right-hand side is the corresponding market size-adjusted cost of FDI. The market size-adjusted benefit is higher when the average consumer buys more of foreign affiliate varieties ($X^i_F + \tau X^{i*}_F \uparrow$), future profits are less heavily discounted ($\rho \downarrow$), foreign affiliates experience smaller capital losses over time ($\theta g \downarrow$) and foreign affiliates are exposed to a lower imitation rate ($\iota_S \downarrow$). The market size-adjusted cost is higher when workers employed in adaptive R&D are less productive ($a_F \uparrow$), adaption is relatively more difficult ($\delta \uparrow$), and when more FDI occurs ($\phi^i \uparrow$).

2.7 Labor Markets

Northern labor is employed in innovative R&D, production in high-cost and low-cost firms serving the domestic market and in high-cost and low-cost exporting firms that serve both the domestic and the foreign market. Each innovation requires $a_N g/n_t^\theta$ units of northern labor for innovative R&D. There are $\dot{n}_t$ varieties developed at time t, so total employment in innovative R&D at time t is $\frac{a_N g}{n_t^\theta}\dot{n}_t = a_N g \frac{n_t^{1-\theta}}{L_t}\frac{\dot{n}_t}{n_t}L_t = a_N g^2 \delta L_t$. Each consumer in the North demands x_{Nt}^i units of output of a northern variety of type i, $(i = H, L)$. There are L_{Nt} northern consumers. For a high-cost firm, c^H units of labor are required to produce 1 unit of output. For a low-cost firm, c^L units of labor are required to produce 1 unit of output. Given consumer demand, each variety produced for the northern market requires $c^i x_{Nt}^i L_{Nt} = \frac{c^i\left(p_N^i\right)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}}$ units of labor and there are n_{Nt}^i such varieties. Aggregate demand for northern labor from production of i-type varieties is therefore $c^i x_{Nt}^i L_{Nt} n_{Nt}^i = \frac{c^i\left(p_N^i\right)^{-\sigma} c_N L_{Nt} n_{Nt}^i}{P_{Nt}^{1-\sigma}} = c^i X_N^i L_t$. For exporting firms, aggregate demand for northern labor is $c^i x_{Xt}^i L_{Nt} n_{Xt}^i + \tau c^i x_{Xt}^{i*} L_{St} n_{Xt}^i = \frac{c^i\left(p_N^i\right)^{-\sigma} c_N L_{Nt} n_{Xt}^i}{P_{Nt}^{1-\sigma}} + \frac{\tau c^i\left(p_N^{i*}\right)^{-\sigma} c_S L_{St} n_{Xt}^i}{P_{St}^{1-\sigma}} = \left(X_X^i + \tau X_X^{i*}\right) c^i L_t$. As L_{Nt} denotes the supply of labor in the North, full employment of labor requires that $L_{Nt} = L_t\left[a_N g^2\delta + \sum_{i=H,L} c^i X_N^i + c^i\left(X_X^i + \tau X_X^{i*}\right)\right]$. Evaluating at time $t = 0$ yields the steady-state full employment of labor condition for the North:

$$L_{N0} = L_0\left[a_N g^2\delta + \sum_{i=H,L} c^i X_N^i + c^i\left(X_X^i + \tau X_X^{i*}\right)\right]. \tag{31}$$

Southern labor is employed in adaptive R&D, export-learning R&D, production by foreign affiliates and production by southern firms that have imitated foreign affiliates. For each exported northern product variety, $a_X\chi^i/n_t^\theta$ units of southern labor are employed in export-learning R&D. There are $\dot{n}_{Xt}^i + \dot{n}_{Ft}^i + \dot{n}_{It}^i$ i-type varieties that start to become exported at time t, so total employment in export-learning R&D by i-type firms is $\frac{a_X\chi^i}{n_t^\theta}\frac{\left(\dot{n}_{Xt}^i + \dot{n}_{Ft}^i + \dot{n}_{It}^i\right)}{n_{Nt}^i}\frac{n_{Nt}^i}{n_t}\frac{n_t}{L_t}L_t = a_X\delta\left(\chi^i\right)^2\gamma_N^i L_t$. For each product variety that is adapted to the production conditions of the South, $a_F\phi^i/n_t^\theta$ units of southern labor are employed in adaptive R&D. There are $\dot{n}_{Ft}^i + \dot{n}_{It}^i$ varieties adapted at time t, so total southern employment in adaptive R&D by i-type firms is $\frac{a_F\phi^i}{n_t^\theta}\frac{\left(\dot{n}_{Ft}^i + \dot{n}_{It}^i\right)}{n_{Xt}^i}\frac{n_{Xt}^i}{n_t}\frac{n_t}{L_t}L_t = a_F\delta\left(\phi^i\right)^2\gamma_X^i L_t$. Each foreign affiliate-produced variety of type i requires $\frac{c^i\left(p_F^i\right)^{-\sigma} c_S L_{St} n_{Ft}^i}{P_{St}^{1-\sigma}} + \frac{\tau c^i\left(p_F^{i*}\right)^{-\sigma} c_N L_{Nt} n_{Ft}^i}{P_{Nt}^{1-\sigma}} =$

$\left[X_F^i + \tau X_F^{i*}\right] c^i L_t$ units of labor. Similarly, labor demand from production in southern imitating firms of type i is $\left[X_I^i + \tau X_I^{i*}\right] c^i L_t$. As L_{St} denotes the supply of labor in the South, full employment requires that $L_{St} = L_t \left[\sum_{i=H,L} a_X \delta \left(\chi^i\right)^2 \gamma_N^i + a_F \delta \left(\phi^i\right)^2 \gamma_X^i + \left(X_F^i + \tau X_F^{i*} + X_I^i + \tau X_I^{i*}\right) c^i\right]$. Evaluating at time $t = 0$, I obtain the steady-state full employment of labor condition for the South:

$$L_{S0} = L_0 \left[\sum_{i=H,L} a_X \delta \left(\chi^i\right)^2 \gamma_N^i + a_F \delta \left(\phi^i\right)^2 \gamma_X^i + \left(X_F^i + \tau X_F^{i*} + X_I^i + \tau X_I^{i*}\right) c^i\right]. \tag{32}$$

2.8 Aggregate Demand

To solve the model, I need steady-state values for the aggregate demand terms X_N^i, X_X^i, X_X^{i*}, X_F^i, X_F^{i*}, X_I^i and X_I^{i*}. I start by expressing aggregate demand for i-type varieties in terms of X_F^i and X_F^{i*}.

Solving for the ratio X_N^i / X_F^{i*} yields

$$\frac{X_N^i}{X_F^{i*}} = \frac{\frac{\left(p_N^i\right)^{-\sigma} c_N L_{Nt} n_{Nt}^i}{P_{Nt}^{1-\sigma} L_t}}{\frac{\left(p_F^{i*}\right)^{-\sigma} c_N L_{Nt} n_{Ft}^i}{P_{Nt}^{1-\sigma} L_t}} = \left(\frac{p_N^i}{p_F^{i*}}\right)^{-\sigma} \frac{n_{Nt}^i / n_t}{n_{Ft}^i / n_t} = \left(\frac{\frac{c^i w_N}{\alpha}}{\frac{\tau c^i w_S}{\alpha}}\right)^{-\sigma} \left[\frac{q^i \frac{g}{g+\chi^i}}{q^i \frac{\chi^i}{g+\chi^H} \frac{\phi^i}{g+\phi^i} \frac{g}{g+\iota_S}}\right],$$

from which it follows that

$$X_N^i = X_F^{i*} \left(\frac{w}{\tau}\right)^{-\sigma} \frac{\left(g + \phi^i\right)\left(g + \iota_S\right)}{\chi^i \phi^i}, \qquad (i = H, L). \tag{33}$$

By doing similar calculations looking at other ratios, I obtain that

$$X_X^i = X_F^{i*} \left(\frac{w}{\tau}\right)^{-\sigma} \frac{g + \iota_S}{\phi^i}, \qquad (i = H, L) \tag{34}$$

$$X_X^{i*} = X_F^i \left(w\tau\right)^{-\sigma} \frac{g + \iota_S}{\phi^i}, \qquad (i = H, L) \tag{35}$$

$$X_I^i = X_F^i \left(\frac{1}{\alpha}\right)^{\sigma} \frac{\iota_S}{g}, \qquad (i = H, L) \tag{36}$$

and

$$X_I^{i*} = X_F^{i*} \left(\frac{1}{\alpha}\right)^{\sigma} \frac{\iota_S}{g}, \qquad (i = H, L). \tag{37}$$

Finally, I can express X_F^H and X_F^{H*} in terms of X_F^L and X_F^{L*} by solving for the ratios

$$\frac{X_F^H}{X_F^L} = \frac{\frac{\left(p_F^H\right)^{-\sigma} c_S L_{St} n_{Ft}^H}{P_{St}^{1-\sigma} L_t}}{\frac{\left(p_F^L\right)^{-\sigma} c_S L_{St} n_{Ft}^L}{P_{St}^{1-\sigma} L_t}} = \left(\frac{p_F^H}{p_F^L}\right)^{-\sigma} \frac{n_{Ft}^H/n_t}{n_{Ft}^L/n_t} = \left(\frac{\frac{c^H w_S}{\alpha}}{\frac{c^L w_S}{\alpha}}\right)^{-\sigma} \frac{\gamma_F^H}{\gamma_F^L} = \left(\frac{c^H}{c^L}\right)^{-\sigma} \frac{\gamma_F^H}{\gamma_F^L}$$

and

$$\frac{X_F^{H*}}{X_F^{L*}} = \frac{\frac{\left(p_F^{H*}\right)^{-\sigma} c_N L_{Nt} n_{Ft}^H}{P_{Nt}^{1-\sigma} L_t}}{\frac{\left(p_F^{L*}\right)^{-\sigma} c_N L_{Nt} n_{Ft}^L}{P_{Nt}^{1-\sigma} L_t}} = \left(\frac{p_F^{H*}}{p_F^{L*}}\right)^{-\sigma} \frac{n_{Ft}^H/n_t}{n_{Ft}^L/n_t} = \left(\frac{\frac{\tau c^H w_S}{\alpha}}{\frac{\tau c^L w_S}{\alpha}}\right)^{-\sigma} \frac{\gamma_F^H}{\gamma_F^L} = \left(\frac{c^H}{c^L}\right)^{-\sigma} \frac{\gamma_F^H}{\gamma_F^L}.$$

Using steady-state variety share expressions, I obtain

$$X_F^H = X_F^L \left(\frac{c^H}{c^L}\right)^{-\sigma} \left(\frac{q}{1-q}\right) \left(\frac{g+\chi^L}{g+\chi^H}\right) \left(\frac{\chi^H}{\chi^L}\right) \left(\frac{g+\phi^L}{g+\phi^H}\right) \left(\frac{\phi^H}{\phi^L}\right), \tag{38}$$

$$X_F^{H*} = X_F^{L*} \left(\frac{c^H}{c^L}\right)^{-\sigma} \left(\frac{q}{1-q}\right) \left(\frac{g+\chi^L}{g+\chi^H}\right) \left(\frac{\chi^H}{\chi^L}\right) \left(\frac{g+\phi^L}{g+\phi^H}\right) \left(\frac{\phi^H}{\phi^L}\right). \tag{39}$$

2.9 Asset Ownership and Consumer Expenditure

Long-run consumer welfare is determined by consumer expenditures c_N and c_S and the price indexes P_{Nt} and P_{St}. Consumers earn income from working and from earning a return on asset holdings in firms that engage in R&D. I assume that R&D done in the North is financed by northern household savings and R&D done in the South is financed by southern household savings. In equilibrium, northern firms that are only active in the domestic market will be fully owned by northern consumers while exporting firms and foreign affiliates will be owned jointly by northern and southern consumers.

Denote aggregate northern assets by A_{Nt} and aggregate southern assets by A_{St}. The aggregate value of all financial assets is $A_t = A_{Nt} + A_{St} = \sum_{i=H,L} n_{Nt}^i v_{Nt}^i + n_{Xt}^i v_{Xt}^i + n_{Ft}^i v_{Ft}^i$. Aggregate southern assets are $A_{St} = \sum_{i=H,L} \left(n_{Xt}^i + n_{Ft}^i\right)\left(v_{Xt}^i - v_{Nt}^i\right) + n_{Ft}^i \left(v_{Ft}^i - v_{Xt}^i\right) = \sum_{i=H,L} n_{Xt}^i \left(v_{Xt}^i - v_{Nt}^i\right) + n_{Ft}^i \left(v_{Ft}^i - v_{Nt}^i\right)$. From (23) and (24), it follows that

$$v_{Ft}^i - v_{Nt}^i = \frac{w_S a_F \phi^i}{n_t^\theta} + \frac{w_S a_X \chi^i}{n_t^\theta}, \qquad (i = H, L).$$

Substituting into the expression for A_{St} using the obtained expressions for $v_{Ft}^H - v_{Nt}^H$ and $v_{Ft}^L - v_{Nt}^L$, the values of exporting firms (23), steady-state variety shares (8) and (9) along with the definition of relative R&D difficulty δ yields

$$A_{St} = w_S L_t \delta \left[\sum_{i=H,L} \gamma_X^i a_X \chi^i + \gamma_F^i \left(a_F \phi^i + a_X \chi^i \right) \right]. \tag{40}$$

Aggregate northern assets are $A_{Nt} = \sum_{i=H,L} \left(n_{Nt}^i + n_{Xt}^i + n_{Ft}^i \right) v_{Nt}^i$. Substituting into this expression using northern firm values $v_{Nt}^i = \pi_{Nt}^i / \left(\rho + \theta g \right)$ and profit expressions from (11) yields

$$A_{Nt} = \frac{w_N L_t}{\left(\sigma - 1 \right) \left(\rho + \theta g \right)} \left[\sum_{i=H,L} c^i X_N^i \frac{\gamma_N^i + \gamma_X^i + \gamma_F^i}{\gamma_N^i} \right]. \tag{41}$$

The intertemporal budget constraint of the typical consumer in region k ($k = N, S$) is $\dot{a}_{kt} = w_k + \rho a_{kt} - c_k - g_L a_{kt}$, where individual assets are denoted by $a_{kt} = A_{kt}/L_{kt}$. In any steady-state equilibrium, $\dot{a}_{kt}/a_{kt} = 0$ since wages w_k and consumer expenditures c_k are constant over time. Individual steady-state consumer expenditure for the typical consumer is therefore $c_k = w_k + \left(\rho - g_L \right) a_{kt}$. Combining the intertemporal budget constraint of the typical consumer in steady-state equilibrium with the derived aggregate assets in each region (40) and (41) and evaluating at time $t = 0$ yields steady-state consumer expenditures

$$c_S = w_S + \left(\rho - g_L \right) w_S \delta \frac{L_0}{L_{S0}} \left[\sum_{i=H,L} \gamma_X^i a_X \chi^i + \gamma_F^i \left(a_F \phi^i + a_X \chi^i \right) \right] \tag{42}$$

and

$$c_N = w_N + \frac{\left(\rho - g_L \right) w_N}{\left(\sigma - 1 \right) \left(\rho + \theta g \right)} \frac{L_0}{L_{N0}} \left[\sum_{i=H,L} c^i X_N^i \frac{\gamma_N^i + \gamma_X^i + \gamma_F^i}{\gamma_N^i} \right]. \tag{43}$$

Having specified the ownership of firms and derived steady-state consumer expenditures c_N and c_S, I can derive the final steady-state condition. This steady-state asset condition is found by taking the ratio X_F^{L*}/X_F^L, substituting equilibrium prices $p_F^L = c^L w_S / \alpha$ and $p_F^{L*} = \tau c^L w_S / \alpha$ and evaluating at time $t = 0$, which yields

$$\frac{X_F^{L*}}{X_F^L} = \left(\frac{1}{\tau} \right)^\sigma \frac{c_N L_{N0}}{c_S L_{S0}} \frac{P_{St}^{1-\sigma}}{P_{Nt}^{1-\sigma}}. \tag{44}$$

Thus, solving the model for a steady-state equilibrium reduces to solving a system of 8 equations

((26), (28) for $i = H$ and $i = L$, (30) for $i = H$ and $i = L$, (31), (32) and (44)) in 8 unknowns (w, δ, χ^L, χ^H, ϕ^L, ϕ^H, X_F^L and X_F^{L*}) , where the 8 equations are: five R&D conditions (innovation, two export-learning, two FDI), two labor market conditions (North and South) and one asset condition.

3 Numerical Results - PRELIMINARY

3.1 Parameters

The system of 8 equations in 8 unknowns is solved numerically. I calibrate the model to fit the world prior to the signing of the TRIPS agreement, in 1990, and after its implementation, in 2005. The following benchmark parameter values are used in the calibration: $\rho = 0.07$, $\alpha = 0.714$, $g_L = 0.014$, $\theta = 0.72$, $L_{N0} = 1$, $L_{S0} = 2$, $\tau = 1.54$ for 1990 and $\tau = 1.33$ for 2005, $q = 0.7$, $c^H = 1$, $c^L = .612$, $a_N = 1$, $a_X = 4.8$, $a_F = 23.1$, $a_I = 4$ for 1990 and $a_I = 38.5$ for 2005.

The subjective discount rate ρ is set at 0.07 to reflect a real interest rate of 7 percent, consistent with the average real return on the US stock market over the 20th century (Mehra and Prescott, 1985). The measure of product differentiation α determines the markup of price over marginal cost $1/\alpha$. It is set at 0.714 to generate a northern markup of 40 percent, which is within the range of estimates from Basu (1996) and Norrbin (1993). The parameter g_L is set at 0.014 to reflect a 1.4 percent population growth rate. This was the average annual world population growth rate during the 1990s according to the World Development Indicators (World Bank, 2011). The steady-state economic growth rate is calculated from $g_u = g_L / ((\sigma - 1)(1 - \theta))$. In order to generate a steady-state economic growth rate of 2 percent, consistent with the average US GDP per capita growth rate from 1950 to 1994 (Jones, 1995), the R&D spillover parameter θ is set at 0.72. When $0 < \theta < 1$ knowledge spillovers are positive but weak. Since only the ratio L_{N0}/L_{S0} matters, I set $L_{N0} = 1$ and $L_{S0} = 2$ so L_{N0}/L_{S0} equals the ratio of working-age population in high-income countries to that in middle-income countries (World Bank, 2003).

During the time period 1990-2005 when the TRIPS agreement was being implemented, North-South trade costs were falling. I use the micro-founded measure of bilateral trade costs developed by Novy (2013) that indirectly infers trade frictions from observable trade data. By linear extrapolation of the bilateral trade cost estimates between the US and Mexico in 1970 and 2000, I obtain a tariff-equivalent of 54 percent for 1990 ($\tau = 1.54$) and 33 percent in 2005 ($\tau = 1.33$).

It is only the relative productivity advantage of low-cost firms over high-cost firms that matter, so I normalize $c^H = 1$. Helpman et al (2004) find that, for US firms, the productivity advantage of exporters over domestic firms is 0.388 (and the productivity advantage of multinationals over domestic firms is 0.537). Consistent with this evidence, I set $c^L = 1 - 0.388 = 0.612$.

Remaining parameters are the R&D productivity parameters a_N (innovation), a_X (export-learning) and a_F (adaption). I also need to set the parameter a_I that is the measure of southern IPR protection (the imitation rate of foreign affiliate varieties is given by $\iota_S = 1/a_I$) and the probability q for drawing a high marginal cost. First, since only the relative difference between the R&D productivity parameters matters, I can normalize $a_N = 1$. I set $a_I = 4$ in the 1990 benchmark so that one in four foreign affiliate varieties are imitated in 1990. I set $a_I = 38.5$ in 2005 to ensure that the model is consistent with the evidence of a ten-fold increase in the FDI inflow to developing countries between 1990 and 2005 (UNCTAD, 2011). FDI inflow is the total amount of researchers employed in adaptive R&D ($L^H_{F0} + L^L_{F0}$) multiplied by the wage rate paid to these workers (w_S). From (21), I derive steady-state FDI expenditure by firms of marginal cost type i

$$L^i_{F0} = \left(\phi^i\right)^2 \gamma^i_X \delta a_F L_0, \qquad (i = H, L),$$

where the southern wage rate has been normalized to 1 ($w_S = 1$). Total FDI spending is $L_{F0} = L^H_{F0} + L^L_{F0} = \left(\phi^H\right)^2 \gamma^H_X \delta a_F L_0 + \left(\phi^L\right)^2 \gamma^L_X \delta a_F L_0$. In 1990, the FDI inflow to developing countries (including transition economies) was 34.9 billion US dollars and in 2005 that FDI inflow was 363.4 billion US dollars (UNCTAD, 2011). This represents a roughly ten-fold increase in the FDI inflow to developing countries measured in current prices. Adjusting the FDI inflow in 1990 for population growth and inflation from 1990 to 2005 generates an expected FDI inflow of 59.7 billion US dollars for 2005.[10] The ratio of the observed FDI inflow to this expected FDI inflow yields a six-fold increase in my measure of FDI inflow during the time period 1990-2005 that can be attributed to policy changes (lower τ and higher a_I). Consistent with this evidence, I set $a_I = 38.5$ in 2005 to ensure that the model generates a six-fold increase in $L^H_{F0} + L^L_{F0}$ from 1990 to 2005. (In the 1990 benchmark, $L^H_{F0} + L^L_{F0} = .01745$. Setting $a_I = 38.5$ in the 2005 benchmark generates $L^H_{F0} + L^L_{F0} = .10470$.)

When calibrating the model, I need to match the stylized fact that a majority of firms do not

[10]From 1990 to 2005, the US GDP implicit price deflator increased by 38.4 percent (Federal Reserve Bank of St Louis, 2011). During the same time period, the world population grew by 23.4 percent using the 1.4 percent annual population growth rate. Multiplying the observed FDI inflow in 1990 by the population growth and inflation over the period generates the expected FDI inflow in 2005 in the absence of any policy changes.

export. In particular, Bernard et al (2003) find that 79 per cent of US plants do not export any of their output. I set the probability of a high marginal cost draw $q = 0.7$ and the export-learning productivity parameter $a_X = 4.8$ to ensure that the model generates a variety share of 79 per cent for non-exporting northern firms while northern consumer expenditure is twice as large as southern consumer expenditure in the 1990 benchmark ($c_N/c_S = 2$). This is consistent with the observed average US-Mexico consumption share adjusted GDP per worker ratio during the time period 1990-2005 (Heston, Summers and Aten, 2011). Finally, I set the FDI productivity parameter $a_F = 23.1$ to ensure that the model generates a foreign affiliate share in world GDP near 5 per cent in 1990.[11] This is consistent with that, in 1990, foreign affiliate value added (product) as share of world GDP was 4.6 percent[12] (UNCTAD, 2012).

3.2 Results

3.2.1 Average Productivity for Non-Exporters, Exporters and Multinational Firms

I solve the model numerically using the parameter values discussed in Section 3.1. The pre-TRIPS 1990 benchmark and the post-TRIPS 2005 benchmark are presented in Columns 1 and 2 of Table 1. The stylized facts that emerge from Bernard et al (2003) and Bernard et al (2007), among others, are that multinationals are on average more productive than exporters and that exporters are on average more productive than non-exporters. The model generates a pattern that is consistent with this. The export-learning rate of northern firms is higher for low-cost firms than for high-cost firms ($\chi^L > \chi^H$ in both 1990 and 2005). Also, the rate of FDI is higher for low-cost firms than for high-cost firms ($\phi^L > \phi^H$ in both 1990 and 2005). Therefore, the share of high-productivity firms is higher for exporting northern firms than for non-exporting northern firms, and the share of high-productivity firms is higher for multinational firms than for northern exporters. In particular, in 1990, $\gamma_N^L/(\gamma_N^H + \gamma_N^L) = .241$, $\gamma_X^L/(\gamma_X^H + \gamma_X^L) = .484$ and $\gamma_F^L/(\gamma_F^H + \gamma_F^L) = .762$.

[11]Foreign affiliate share in world GDP is measured by $\left(X_F^L + X_F^H\right) / \left[\sum_{i=H,L} X_N^i + X_X^i + X_F^i + X_I^i\right]$.

[12]Value added (product) of foreign affiliates in 1990 was 1,018 billion US dollars and world GDP was 22,206 billion US dollar, measured in 2012 US dollar (UNCTAD, 2012). As mentioned in Antras and Helpman (2008) using UNCTAD data, in 2000, about 10 percent of world GDP was accounted for by foreign affiliates, leaving out the value added generated by parent firms. The pre-crisis average in 2005-2007 for this ratio was also 10 per cent (UNCTAD 2012), indicating a slow-down in FDI spending.

3.2.2 TRIPS Agreement and Trade Liberalization

Going from the 1990 to the 2005 benchmark (with trade liberalization and stronger southern IPR protection to comply with TRIPS), exporting increases ($\chi^L \uparrow$ and $\chi^H \uparrow$) and FDI increases ($\phi^L \uparrow$ and $\phi^H \uparrow$). Overall, there is a geographical redistribution of production from the North to the South ($\gamma_N^H + \gamma_N^L + \gamma_X^H + \gamma_X^L$ decreases from .973 to .932 and $\gamma_F^H + \gamma_F^L + \gamma_I^H + \gamma_I^L$ increases from .027 to .068). In the post-TRIPS scenario, the share of non-exporting firms in the North is smaller (the share of non-exporters decrease from .790 to .743). More northern firms have learned to export, but among northern exporters, the share of world production done by low-cost firms is lower (γ_X^L decreases from .089 to .081) and the corresponding share of high-cost (less productive) firms is higher (γ_X^H increases from .094 to .109). The low-cost firms to a larger extent went on to become multinational firms, whereas high-cost firms remain as exporters (keep producing in the North and serve the southern market via exports). In the post-TRIPS 2005 benchmark, foreign affiliates are more important in the world economy as seen by the increase in the share of varieties that are produced in foreign affiliates ($\gamma_F^L + \gamma_F^H$ increases from .004 to .045) and the large increase in foreign affiliate value-added in world GDP from .042 to .264.

Going from the 1990 to the 2005 benchmark, southern consumer welfare is improved (u_{S0} increases from 123.3 to 156.4) but northern consumer welfare is worsened (u_{N0} decreases from 343.9 to 335.0). To understand these welfare changes, I solve the model for two counterfactual scenarios. In the first counterfactual, presented in Column 3 of Table 1, trade costs are assumed to be at their 1990 level, but IPR protection is set at its post-TRIPS 2005 level. This would have been the case if TRIPS had not been accompanied by any trade liberalization. Stronger IPR protection leads to a faster rate of FDI for both high-cost and low-cost firms in the North (ϕ^L increases from .0117 to .0335 and ϕ^H increases from .0034 to .0098). Stronger IPR protection does not encourage export-learning (the share of non-exporters in the North remains at .790 and the export learning rates remain the same), but it encourages the northern firms that are already exporting to go on to become multinational firms. Low-cost firms respond more to stronger southern IPR protection by transferring production to the South (γ_X^L decreases from .089 to .065 and γ_F^L increases from .003 to .029 while γ_X^H decreases from .094 to .084 and γ_F^H increases from .001 to .011).

Consumer welfare is measured by $u_{N0} = c_N/P_{N0}$ and $u_{S0} = c_S/P_{S0}$, respectively. With stronger southern IPR protection, consumer welfare is improved in both regions (u_{N0} increases from 343.9 to

367.3 and u_{S0} increases from 123.3 to 160.1). Southern consumer expenditure is higher (c_S increases from 1.028 to 1.071) and the southern price index is lower (P_{S0} decreases from .008 to .007), which contributes to higher long-run consumer welfare. For northern consumers, consumer expenditure is actually lower (c_N decreases from 2.056 to 1.783) but this is dominated by the effect of the lower price index (P_{N0} decreases from .006 to .005). There is a geographical redistribution of production from the North to the South as less production is done by northern exporters, and more production is done by foreign affiliates in the South. This has two effects on consumer welfare. First, more production taking place in the lower-wage South translates to lower product prices. Second, labor resources are freed up from production by exporting firms, which puts downward pressure on the northern wage rate (w_N/w_S decreases from 1.782 to 1.533) and this lowers the cost of innovation. Therefore, there is more innovation (δ increases from 19.08 to 20.12) and the resulting increase in invented varieties benefits consumers in both regions.

In the second counterfactual presented in Column 4 of Table 1, trade costs are set at their 2005 level but IPR protection is the same as in the 1990 benchmark. This would have been the case if trade liberalization had occurred between 1990 and 2005 without being accompanied by any stronger southern IPR protection. Overall, the share of non-exporters in the North is lower when trade costs are lower (from .790 to .744). Lower trade costs leads to higher exporting rates (χ^L increases from .0286 to .0376 and χ^H increases from .0084 to .0111). Consequently, there is a redistribution of variety shares from northern non-exporters to northern exporters (γ_N^L decreases from .191 to .171 and γ_N^H decreases from .599 to .573 while γ_X^L increases from .089 to .107 and γ_X^H increases from .094 to .120). As a result of trade liberalization, the rate of FDI is lower for both types of firms (ϕ^L decreases from .0117 to .0101 and ϕ^H decreases from .0034 to .0030). However, since there are more exporters that have the option to become multinationals, the variety shares of foreign affiliates increase slightly. Surprisingly, consumer welfare in both regions is worsened by trade liberalization (u_{N0} decreases from 343.9 to 313.7 and u_{S0} decreases from 123.3 to 118.9). Consumer expenditure is higher in both regions (c_N increases from 2.056 to 2.062 and c_S increases from 1.028 to 1.041). This is because the market value of asset holdings in exporting firms increase, benefitting consumers in both regions that jointly own exporting northern firms. Northern consumers also benefit from a higher relative wage (w_N/w_S increases from 1.782 to 1.805). Trade liberalization draws resources into production by exporting firms in the North, which puts upward pressure on the northern wage rate, thus making innovation more costly. This results in less innovation (δ decreases from 19.08 to

	(1) 1990	(2) 2005	(3) $a_I \uparrow$	(4) $\tau \downarrow$	(5) $a_X \downarrow$	(6) $a_F \downarrow$
	$\tau = 1.54$	$\tau = 1.33$	$\tau = 1.54$	$\tau = 1.33$	$\tau = 1.54$	$\tau = 1.54$
	$a_I = 4$	$a_I = 38.5$	$a_I = 38.5$	$a_I = 4$	$a_I = 4$	$a_I = 4$
w_N/w_S	1.782	1.524	1.533	1.805	1.805	1.631
δ	19.08	18.67	20.12	17.69	18.61	19.24
ϕ^L	.0117	.0299	.0335	.0101	.0092	.0160
ϕ^H	.0034	.0088	.0098	.0030	.0027	.0047
χ^L	.0286	.0380	.0287	.0376	.0551	.0282
χ^H	.0084	.0112	.0084	.0111	.0162	.0083
γ_N^L	.191	.170	.191	.171	.143	.192
γ_N^H	.599	.572	.599	.573	.529	.601
γ_X^L	.089	.081	.065	.107	.133	.082
γ_X^H	.094	.109	.084	.120	.163	.091
γ_F^L	.003	.032	.029	.004	.004	.004
γ_F^H	.001	.013	.011	.001	.001	.001
γ_I^L	.017	.017	.015	.018	.020	.022
γ_I^H	.005	.007	.006	.006	.007	.007
ι_S	.25	.026	.026	.25	.25	.25
L_{F0}^L	.016	.094	.103	.013	.014	.012
L_{F0}^H	.001	.011	.011	.001	.002	.001
Non-exp.	.790	.743	.790	.744	.671	.792
FA in VA	.042	.264	.266	.042	.042	.042
c_N	2.056	1.754	1.783	2.062	2.074	1.882
c_S	1.028	1.081	1.071	1.041	1.033	1.026
c_N/c_S	2.000	1.622	1.665	1.981	2.007	1.834
P_{N0}	.006	.005	.005	.007	.006	.005
P_{S0}	.008	.007	.007	.009	.008	.008
P_{N0}/P_{S0}	.717	.757	.726	.751	.786	.721
u_{N0}	343.9	335.0	367.3	313.7	333.3	346.8
u_{S0}	123.3	156.4	160.1	118.9	130.5	136.4

Table 1: Pre- and post-TRIPS benchmarks and four counterfactual scenarios. (1) 1990 benchmark, (2) 2005 benchmark (trade liberalization and stronger IPR protection), (3) Counterfactual with stronger IPR protection without trade liberalization, (4) Counterfactual with trade liberalization without stronger IPR protection, (5) Counterfactual with same parameter values as 1990 benchmark except lower export learning cost, $a_X = 2$, (6) Counterfactual with same parameter values as 1990 benchmark except lower cost of FDI, $a_F = 10$. Non-exp. is the share of non-exporting firms. FA in VA is foreign affiliate share of world value added (GDP).

17.69). Consumers are worse off since northern product varieties are more expensive and because there is less product variety in the long run (P_{N0} increases from .006 to .007 and P_{S0} increases from .008 to .009).

3.2.3 Lower Costs of Export-Learning and FDI

I can also study the response to a decrease in the cost of export-learning for northern firms (a decrease in the export-learning productivity parameter a_X). The results from setting $a_X = 2$ (instead of the benchmark $a_X = 4.8$) are presented in Column 5 of Table 1. In response to the activity being less costly, the rates of export-learning are higher (χ^L increases from .0286 to .0551 and χ^L increases from .0084 to .0162). The share of northern non-exporters decrease from .790 to .671 and the variety shares of northern exporters increase (γ_X^L increases from .089 to .133 and γ_X^H increases from .094 to .163). With a lower export-learning cost, the incentives for exporting are stronger, but the incentives to engage in FDI once the firm has become an exporter remains the same, and hence the FDI rates are slightly lower in equilibrium (ϕ^L decreases from .0117 to .0092 and ϕ^H decreases from .0034 to .0027). Interestingly, as a result of lower export-learning costs in the North, northern consumers are worse off (u_{N0} decreases from 343.9 to 333.3) but southern consumers are better off (u_{S0} increases from 123.3 to 130.5). The redistribution of production towards northern exporters leads to higher long-run consumer expenditure in both regions by increasing the market value of northern exporting firms (c_N and c_S increases) but also less innovation (a lower δ). Less product variety affects consumers in both regions negatively, but in this case the benefit for southern consumers from being able to purchase more product varieties as more northern varieties are exported from the North to the South outweighs the negative effect of less innovation. When more northern firms export, it benefits the South since a larger share of invented products are made available to southern consumers.[13]

Finally, I can study the response to a decrease in the cost of FDI (a decrease in the FDI productivity parameter a_F). The results from setting $a_F = 10$ (instead of the benchmark $a_F = 23.1$) can be found in Column 6 of Table 1. With less costly FDI, there is a decrease in total FDI spending (for the parameter values chosen, $L_{F0}^H + L_{F0}^L$ decreases from .017 to .013). Lowering the cost of FDI leads to an increase in the rates of FDI, but a slight decrease in the rates of export-learning. The

[13] In the model, southern product varieties are imitations of products invented in the North. There are no export-learning costs for southern firms. All southern firms immediately export to the North, so there are no variety gains from trade in the North from gaining access to "new goods".

share of non-exporters remains roughly the same, but a larger share of exporters choose to become multinationals. Low-cost firms respond more than high-cost firms to the decrease in the cost of FDI (in this example, γ_X^L decrease from .089 to .082 and γ_X^H decrease from .094 to .091 while γ_F^L increases from .003 to .004 and γ_F^H remains at .001). In response to a lower FDI cost, there is a geographical redistribution of production from the North to the South as less production is done by northern exporting firms and more production is done by foreign affiliates and the southern firms that imitates them. As discussed earlier, this benefits consumers in both regions through lower prices and increased product variety from innovation.

3.2.4 Aggregate Labor Demand

To understand the effects of the different policy changes, it is useful to look at labor demand by activity and across high-productivity and low-productivity firms. From expanding the left-hand side of (15) by $n_t L_t / n_t L_t$ and evaluating at time $t = 0$, it follows that $L_{R0} = g^2 \delta a_N L_0$. It was seen earlier that aggregate labor demand from adaptive R&D by firms of marginal cost type i is $L_{F0}^i = \left(\phi^i\right)^2 \gamma_X^i \delta a_F L_0$, $(i = H, L)$. Similarly, using (19), I can derive the aggregate demand from export-learning R&D activities by firms of marginal cost type i, $L_{X0}^i = \left(\chi^i\right)^2 \gamma_N^i \delta a_X L_0$. Aggregate labor demand from production in northern non-exporting firms of marginal cost type i is $c^i X_N^i L_0$. For northern exporters, aggregate labor demand from production for the home market is $c^i X_X^i L_0$ and from production for the export market $\tau c^i X_X^{i*} L_0$. Foreign affiliates in the South have aggregate labor demand $c^i X_F^i L_0$ for production for the domestic market and $\tau c^i X_X^{i*} L_0$ for production for the export market. Similarly, southern imitating firms have labor demand $c^i X_I^i L_0$ for production for the domestic market and $\tau c^i X_I^{i*} L_0$ for production for the export market.

I calculate labor demand by activity and productivity type for the two benchmarks and for each of the counterfactual scenarios. The results are presented in Table 2. The top panel represents labor demand by activitity and by productivity type for firms in the North, and the lower panel represents labor demand by activity and productivity type for firms in the South.[14] With stronger IPR protection (Column 3 of Table 2), northern labor moves from production in low-cost exporting firms to innovation activities ($c^L X_X^L L_0$ decreases from .132 to .103 and $\tau c^L X_X^{L*} L_0$ decreases from .103 to .094 while L_{R0} increases from .143 to .150.) Interestingly, less labor is employed in high-cost exporting firms for production for the domestic market ($c^H X_X^H L_0$ decreases from .041 to .039) but

[14]When calibrating the model, I set $L_{N0} = 1$ and $L_{S0} = 2$.

	(1) 1990	(2) 2005	(3) $a_I \uparrow$	(4) $\tau \downarrow$	(5) $a_X \downarrow$	(6) $a_F \downarrow$
	$\tau = 1.54$ $a_I = 4$	$\tau = 1.33$ $a_I = 38.5$	$\tau = 1.54$ $a_I = 38.5$	$\tau = 1.33$ $a_I = 4$	$\tau = 1.54$ $a_I = 4$	$\tau = 1.54$ $a_I = 4$
North						
Inn. R&D L_{R0}	.143	.140	.150	.132	.139	.144
Dom. prod. $c^L X_N^L L_0$	.285	.249	.300	.237	.208	.289
Dom. prod. $c^H X_N^H L_0$	.263	.246	.277	.233	.226	.266
Dom. prod. $c^L X_X^L L_0$	.132	.119	.103	.149	.194	.124
Exp. prod. $\tau c^L X_X^{L*} L_0$	.103	.144	.094	.151	.120	.104
Dom. prod. $c^H X_X^H L_0$	.041	.047	.039	.049	.070	.040
Exp. prod. $\tau c^H X_X^{H*} L_0$	.032	.057	.036	.049	.043	.034
South						
Exp. R&D L_{X0}^L	.043	.066	.045	.062	.048	.042
Exp. R&D L_{X0}^H	.012	.019	.012	.018	.015	.011
FDI R&D L_{F0}^L	.016	.094	.103	.013	.014	.012
FDI R&D L_{F0}^H	.001	.011	.011	.001	.002	.001
Dom. prod. $c^L X_F^L L_0$	.089	.503	.543	.081	.085	.090
Exp. prod. $\tau c^L X_F^{L*} L_0$	.013	.100	.069	.019	.016	.012
Dom. prod. $c^H X_F^H L_0$	.008	.058	.060	.008	.009	.009
Exp. prod. $\tau c^H X_F^{H*} L_0$	.001	.012	.008	.002	.002	.001
Dom. prod. $c^L X_I^L L_0$	1.449	.850	.917	1.323	1.379	1.461
Exp. prod. $\tau X_I^{L*} L_0$	.215	.169	.117	.314	.258	.202
Dom. prod. $c^H X_I^H L_0$	.133	.098	.102	.127	.145	.140
Exp. prod. $\tau c^H X_I^{H*} L_0$	.020	.020	.013	.030	.027	.019

Table 2: Pre- and post-TRIPS benchmarks and four counterfactual scenarios. (Inn. R&D - innovative R&D, Exp. R&D - export-learning R&D, FDI R&D - adaptive R&D in foreign affiliates, Exp. Prod. - Production for export market, Dom. Prod - Production for domestic market.) (1) 1990 benchmark, (2) 2005 benchmark (trade liberalization and stronger IPR protection), (3) Counterfactual with stronger IPR protection but without trade liberalization, (4) Counterfactual with trade liberalization but without stronger IPR protection, (5) Counterfactual with same parameter values as 1990 benchmark except lower export learning cost, $a_X = 2$, (6) Counterfactual with same parameter values as 1990 benchmark except lower cost of FDI, $a_F = 10$.

more labor is employed in these firms for production for the export market ($\tau c^H X_X^{H*}$ increases from .032 to .036.) Because there are more new varieties in the North as a result of more innovation, more labor resources are demanded for production in northern non-exporting firms ($c^L X_N^L L_0$ increases from .285 to .300 and $c^H X_N^H L_0$ increases from .263 to .277). In the South, stronger IPR protection leads to a large increase in labor employed in adaptive R&D (L_{F0}^L increases from .016 to .103 and L_{F0}^H increases from .001 to .011). As expected, there is a redistribution of production labor from imitating firms to foreign affiliates. For low-cost firms, $c^L X_I^L L_0$ decreases from 1.449 to .917 and $\tau c^L X_I^{L*} L_0$ decreases from .215 to .117 whereas $c^L X_F^L L_0$ increases from .089 to .543 and $\tau c^L X_F^{L*} L_0$ increases from .013 to .069, and there is a similar pattern for high-cost firms in the South.

As a result of trade liberalization (Column 4 of Table 2), production labor in the North moves from non-exporting firms to exporting firms ($c^L X_N^L L_0$ decreases from .285 to .237 and $c^H X_N^H L_0$ decreases from .263 to .233 while $c^L X_X^L L_0$ increases from .132 to .149, $\tau c^L X_X^{L*} L_0$ increases from .103 to .151, $c^H X_X^H L_0$ increases from .041 to .049 and $\tau c^H X_X^{H*} L_0$ increases from .032 to .049). There is a redistribution of resources within exporting firms in the North. For northern exporters, trade liberalization makes the export market more attractive, as seen by the proportionally larger increase in labor demand from production for the export market. Production in exporting firms increases by so much that it also draws labor from innovative R&D activities (L_{R0} decreases from .143 to .132). In the South, there is a redistribution of researchers from adaptive R&D to R&D done by exporting northern firms (L_{F0}^i decreases and L_{X0}^i increases). In addition to this redistribution of labor resources *across* firms in the South, lower trade costs results in a relative redistribution of labor resources *within* firms in the South. All southern firms export to the North and as a result of lower barriers to trade, firms redistribute resources from production for the domestic market to production for the export market. For foreign affiliates and southern firms, labor demand from production for the domestic market decreases and labor demand from production for the export market increases (for example, $c^L X_F^L L_0$ decreases from .089 to .081 while $\tau c^L X_F^{L*} L_0$ increases from .013 to .019 and $c^L X_I^L L_0$ decreases from 1.449 to 1.323 while $\tau c^L X_I^{L*} L_0$ increases from .215 to .314).

Lowering the export-learning cost in the North (Column 5 of Table 2) have similar effects on employment in different activities and productivity type of firms as lower variable trade costs. In the North, labor is redistributed from production by firms that only serve the domestic market towards production by exporting firms ($c^i X_N^i L_0$ decrease while $c^i X_X^i L_0$ and $\tau c^i X_X^{i*} L_0$ increase).

Some labor is drawn from innovation (L_{R0} decreases) towards production by exporting firms. This is straightforward since lower costs of export-learning create stronger incentives for all northern non-exporting firms to learn how to export, similar to the effect of lower trade costs. A lower northern export-learning cost has no effect on the actual costs of exporting for southern firms but it still leads to a redistribution of labor within southern firms from production for the domestic market towards production for the export market. Lower export-learning costs (as well as lower trade costs) lead to an increase in the North-South relative wage. This increases the purchasing power of northern consumers relative to southern consumers, making the export market relatively more attractive for firms in the South. Therefore, there is a redistribution of labor within southern firms from production for the domestic market to production for the export market.

The effects on employment from lowering a_F, such that adaptive R&D is more productive (essentially making FDI less costly), are presented in Column 6 of Table 2. The results suggest that, in the North, labor is redistributed towards innovation activities and towards production by firms that have not yet learned how to export (L_{R0} and $c^i X_N^i L_0$ increase). In the South, labor is freed up from adaptive R&D (L_{F0}^L decreases). Low-cost firms are doing more FDI than high-cost firms, and as adaptive R&D is more productive ($a_F \downarrow$), less researchers are demanded by low-cost firms. With lower FDI cost, the rates of FDI is higher ($\phi^i, (i = H, L)$ increase as seen in Column 6 of Table 1). There is a relative redistribution of labor within imitating firms, from production for the export market towards production for the domestic market ($c^i X_I^i L_0$ increase while $\tau c^i X_I^{i*} L_0$ decrease). A similar pattern can be seen also for labor demand from production by foreign affiliates in the South. A lower FDI cost decreases consumer expenditure in the North proportionally more than consumer expenditure in the South (as seen in Column 6 of Table 1, in this example, c_N decreases from 2.056 to 1.882 while c_S decreases from 1.028 to 1.026). For southern firms, this makes the domestic market relatively more attractive than the export market, causing a redistribution of production labor within firms with production in the South.

4 Concluding Comments

I present a dynamic general equilibrium model of North-South trade where high-cost and low-cost firms can engage in innovation, learn how export and then do FDI to become multinational firms and engage in multinational production. I find that export-learning rates and FDI rates are higher for high-productivity firms than for low-productivity firms. As a result, exporters are on average more productive than non-exporters and multinational firms are on average more productive than exporters. In equilibrium, there are still some low-productivity exporters, some low-productivity multinationals and some-high productivity non-exporters. This is consistent with empirical evidence from Bernard et al (2007) and Mayer and Ottaviano (2008), but cannot be explained in traditional Melitz (2003)-style models where all firms above a certain productivity threshold engage in either exporting or FDI, or only produce for the domestic market. In my model, low-productivity firms invest in learning how to export and do FDI, but they are just not as successful in these activites as high-productivity firms.

The model allows me to study the long-run implications of trade liberalization, lower fixed costs of exporting, lower fixed costs of FDI and stronger intellectual property rights protection on innovation, international technology transfer and consumer welfare. I disentangle how labor resources are reallocated within regions in response to these changes: across sectors (production, innovation, export-learning and adaption to multinational production), across high-productivity and low-productivity firms, and within firms as they produce more (less) for the home market visavi the export market. Stronger IPR protection and lower costs of FDI lead to more technology transfer within multinational firms and more innovation. Consumers in both regions benefit from lower prices and incresed product variety. Low-cost firms respond more to FDI-related policies than high-cost firms by transferring more production to the South than high-cost firms. However, lower costs of FDI cannot explain the large increase in FDI inflow going to developing countries during 1990-2005, but stronger IPR protection can. As a result of lower fixed costs of exporting, increased labor demand from production activities in exporting firms in the North puts upward pressure on the northern wage rate. Higher production costs are passed on to consumers in both regions through higher prices for exporting northern firms' varieties. Lower fixed cost of exporting in the North makes northern consumers worse off in the long run but can make southern consumers better off if the benefit from access to more northern product varieties outweighs the effect of higher prices of imports.

References

[1] Antras, Pol and Elhanan Helpman (2008), "Contractual Frictions and Global Sourcing", In *The Organization of Firms in a Global Economy* eds. Elhanan Helpman, Dalia Marin and Thierry Verdier, 9-54. Cambridge, MA: Harvard University Press.

[2] Arkolakis, Costas (2010), "Market Penetration Costs and New Consumers Margin in International Trade", *Journal of Political Economy*, 118(6), 1151-1199.

[3] Arkolakis, Costas, Natalia Ramondo, Andrés Rodríguez-Claire and Stephen Yeaple (2013), "Innovation and Production in the Global Economy", NBER Working Paper No. 18972.

[4] Basu, Susanto (1996), "Procyclical Productivity: Increasing Returns or Cyclical Utilization," *Quarterly Journal of Economics*, 111(3), 709-751.

[5] Bernard, Andrew B., Jonathan Eaton, Bradford Jensen and Samuel S. Kortum (2003), "Plants and Productivity in International Trade", *American Economic Review*, 93(4), 1268-1290.

[6] Benard, Andrew B., J. Bradford Jensen, Stephen J Redding and Peter K. Schott (2007), "Firms in International Trade", *Journal of Economic Perspectives*, 21(3), 105-130.

[7] Bernard, Andrew B. and J. Bradford Jensen (2004), "Why Some Firms Export", *Review of Economics and Statistics*, 86(2), 561-569.

[8] Branstetter, Lee and Kamal Saggi (2011), "Intellectual Property Rights, Foreign Direct Investment and Industrial Development," *Economic Journal*, 121(555), 1161-1191.

[9] Branstetter, Lee, Raymond Fisman, Fritz Foley and Kamal Saggi (2011), "Does Intellectual Property Rights Reform Spur Industrial Development?", *Journal of International Economics*, 83(1), 27-36.

[10] Branstetter, Lee, Raymond Fisman and Fritz Foley (2006), "Do Stronger Intellectual Property Rights Increase International Technology Transfer? Empirical Evidence from U.S. Firm-Level Panel Data," *Quarterly Journal of Economics*, 121(1), 321-349.

[11] Castro, Luis, Ben Li, Keith E. Maskus and Yiqing Xie (2013), "Fixed Export Costs and Firm-Level Export Behavior", University of Colorado at Boulder, Mimeo.

[12] Conconi, Paola, André Zapir and Maurizio Zanardi (2013), "The Internationalization Process of Firms: from Exports to FDI", *Working Papers ECARES*, ECARES 2013-09, Université Libre de Bruxelles.

[13] Federal Reserve Bank of St. Louis (2011), *Federal Reserve Economic Data*.

[14] Glass, Amy and Kamal Saggi (2002), "Intellectual Property Rights and Foreign Direct Investment", *Journal of International Economics*, 56(2), 387-410.

[15] Glass, Amy and Xiaodong Wu (2007), "Intellectual Property Rights and Quality Improvement," *Journal of Development Economics*, 82(2), 393-415.

[16] Gould, David M. and William C. Gruben (1996), "The Role of Intellectual Property Rights in Economic Growth", *Journal of Development Economics*, 48(2), 323-350.

[17] Helpman, Elhanan, Marc J. Melitz and Stephen R. Yeaple (2004), "Export Versus FDI with Heterogeneous Firms", *American Economic Review*, 94(1), 300-316.

[18] Heston, Alan, Robert Summers and Bettina Aten (2011), "Penn World Table Version 7.0," Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, June 2011.

[19] Jakobsson, Amanda (2013), "A Simple Model of TRIPS", Stockholm School of Economics, mimeo.

[20] Jakobsson, Amanda and Paul S. Segerstrom (2012), "In Support of the TRIPs Agreement," Stockholm School of Economics, mimeo.

[21] Jones, Charles I. (1995), "R&D-based Models of Economic Growth," *Journal of Political Economy*, 103(4), 759-784.

[22] Kortum, Samuel (1993), "Equilibrium R&D and the Patent-R&D Ratio: U.S. Evidence," *American Economic Review*, 83(2), 450-457.

[23] Lileeva, Alla and Daniel Trefler (2010), "Improved Access to Foreign Markets Raises Plant-Level Productivity... for Some Plants," *Quarterly Journal of Economics*, 125(3), 1051-1099.

[24] Mayer, Thierry and Gianmarco Ottaviano (2008), “The Happy Few: The Internationalisation of European Firms,” *Intereconomics: Review of European Economic Policy*, 43(3), 135-148.

[25] McCaig, Brian and Pavcnik, Nina (2013), “Export Markets and labor reallocation,” NBER Working Paper No. 19616.

[26] McMillan, Margaret S. and Dani Rodrik (2011), “Globalization, Structural Change and Productivity Growth,” NBER Working Paper No. 17143.

[27] Mehra, Rajnish and Edward Prescott (1985), “The Equity Premium: A Puzzle,” *Journal of Monetary Economics*, 15(2), 145-161.

[28] Melitz, Marc J. (2003), “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity,” *Econometrica*, 71(6), 1695-1725.

[29] Norrbin, S.C. (1993), “The Relationship between Price and Marginal Cost in US Industry: A Contradiction,” *Journal of Political Economy*, 101(6), 1149-1164.

[30] Novy, Dennis (2013), “Gravity Redux: Measuring International Trade Costs with Panel Data,” *Economic Inquiry*, 51(1), 101-121.

[31] UNCTAD (2011), *FDI Statistics*, United Nations Conference on Trade and Development, at http://www.unctad.org/fdistatistics.

[32] UNCTAD (2012), *World Investment Report 2012*, United Nations Conference on Trade and Development.

[33] World Bank (2011), *World Development Indicators*, Washington, D.C.

[34] World Bank (2003), *World Development Indicators*, Washington, D.C.

Appendix: Solving The Model

In this appendix, calculations done to solve the model are spelled out in more detail.

Households

The static consumer optimization problem is

$$\max_{x_t(\cdot)} \int_0^{n_t} x_t(\omega)^\alpha d\omega \qquad \text{s.t.} \qquad \dot{y}(\omega) = p_t(\omega)x_t(\omega),\ y(0) = 0,\ y(n_t) = c_t.$$

where $y(\omega)$ is a new state variable and $\dot{y}(\omega)$ is the derivative of y with respect to ω. The Hamiltonian function for this optimal control problem is

$$H = x_t(\omega)^\alpha + \gamma(\omega)p_t(\omega)x_t(\omega)$$

where $\gamma(\omega)$ is the costate variable. The costate equation $\frac{\partial H}{\partial y} = 0 = -\dot{\gamma}(\omega)$ implies that $\gamma(\omega)$ is constant across ω. $\frac{\partial H}{\partial x} = \alpha x_t(\omega)^{\alpha-1} + \gamma \cdot p_t(\omega) = 0$ implies that

$$x_t(\omega) = \left(\frac{\alpha}{-\gamma \cdot p_t(\omega)}\right)^{1/(1-\alpha)}.$$

Substituting this back into the budget constraint yields

$$c_t = \int_0^{n_t} p_t(\omega)x_t(\omega)d\omega = \int_0^{n_t} p_t(\omega)\left(\frac{\alpha}{-\gamma \cdot p_t(\omega)}\right)^{1/(1-\alpha)} d\omega = \left(\frac{\alpha}{-\gamma}\right)^{1/(1-\alpha)} \int_0^{n_t} p_t(\omega)^{\frac{1-\alpha-1}{1-\alpha}} d\omega.$$

Now $\sigma \equiv \frac{1}{1-\alpha}$ implies that $1 - \sigma = \frac{1-\alpha-1}{1-\alpha} = \frac{-\alpha}{1-\alpha}$, so

$$\frac{c_t}{\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega} = \left(\frac{\alpha}{-\gamma}\right)^{1/(1-\alpha)}.$$

It immediately follows that the consumer demand function is

$$x_t(\omega) = \frac{p_t(\omega)^{-\sigma} c_t}{P_t^{1-\sigma}} \tag{3}$$

where $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ is an index of consumer prices.

Substituting this consumer demand function back into the consumer utility function yields

$$u_t = \left[\int_0^{n_t} x_t(\omega)^\alpha d\omega\right]^{\frac{1}{\alpha}} = \left[\int_0^{n_t} \frac{p_t(\omega)^{-\sigma\alpha} c_t^\alpha}{P_t^{(1-\sigma)\alpha}} d\omega\right]^{\frac{1}{\alpha}} = c_t \left[\int_0^{n_t} \frac{p_t(\omega)^{-\sigma\alpha}}{P_t^{(1-\sigma)\alpha}} d\omega\right]^{\frac{1}{\alpha}}.$$

Taking into account that $-\sigma\alpha = \frac{-\alpha}{1-\alpha} = 1 - \sigma$, consumer utility can be simplified further to

$$u_t = \frac{c_t}{P_t^{1-\sigma}} \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} \, d\omega \right]^{\frac{1}{\alpha}} = \frac{c_t}{P_t^{1-\sigma}} \left[P_t^{1-\sigma} \right]^{\frac{1}{\alpha}} = \frac{c_t}{P_t^{1-\sigma}} P_t^{-\sigma} = \frac{c_t}{P_t}$$

or

$$\ln u_t = \ln c_t - \ln P_t.$$

The individual household takes the prices of all products as given, as well as how prices change over time, so the $\ln P_t$ term can be ignored in solving the household's dynamic optimization problem. This problem simplifies to:

$$\max_{c_t} \int_0^{\infty} e^{-(\rho - g_L)t} \ln c_t \, dt \qquad \text{s.t.} \qquad \dot{a}_t = w_t + r_t a_t - g_L a_t - c_t,$$

where a_t represents the asset holding of the representative consumer, w_t is the wage rate and r_t is the interest rate.

The Hamiltonian function for this optimal control problem is

$$H = e^{-(\rho - g_L)t} \ln c_t + \lambda_t \left[w_t + r_t a_t - g_L a_t - c_t \right]$$

where λ_t is the relevant costate variable. The costate equation $-\dot{\lambda}_t = \frac{\partial H}{\partial \tilde{a}} = \lambda_t \left[r_t - g_L \right]$ implies that

$$\frac{\dot{\lambda}_t}{\lambda_t} = g_L - r_t.$$

$\partial H / \partial c_t = e^{-(\rho - g_L)t} \frac{1}{c_t} - \lambda_t = 0$ implies that $e^{-(\rho - g_L)t} \frac{1}{c_t} = \lambda_t$. Taking logs of both sides yields $-(\rho - g_L)t - \ln c_t = \ln \lambda_t$ and then differentiating with respect to time yields

$$-(\rho - g_L) - \frac{\dot{c}_t}{c_t} = \frac{\dot{\lambda}_t}{\lambda_t} = g_L - r_t.$$

It immediately follows that

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{4}$$

Steady-State Dynamics

In this section, I derive some steady-state equilibrium implications of the model.

Because prices differ between the North and the South due to trade costs, and because the set of varieties available to consumers in the South is a subset of the set of varieties available to consumers in the North, I need to define a different price index for each region. Let P_{Nt} be the price index for the North and P_{St} be the price index for the South. Given the definition of the price index

$P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$, it follows that the northern price index satisfies

$$\begin{aligned}
P_{Nt}^{1-\sigma} &= \int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega \\
&= \sum_{i=H,L} \left[n_{Nt}^i \left(p_N^i\right)^{1-\sigma} + n_{Xt}^i \left(p_N^i\right)^{1-\sigma} + n_{Ft}^i \left(p_F^{i*}\right)^{1-\sigma} + n_{It}^i \left(p_I^{i*}\right)^{1-\sigma} \right] \\
&= \sum_{i=H,L} \left[\gamma_N^i n_t \left(p_N^i\right)^{1-\sigma} + \gamma_X^i n_t \left(p_N^i\right)^{1-\sigma} + \gamma_F^i n_t \left(p_F^{i*}\right)^{1-\sigma} + \gamma_I^i n_t \left(p_I^{i*}\right)^{1-\sigma} \right] \\
&= \sum_{i=H,L} \left[\gamma_N^i \left(p_N^i\right)^{1-\sigma} + \gamma_X^i \left(p_N^i\right)^{1-\sigma} + \gamma_F^i \left(p_F^{i*}\right)^{1-\sigma} + \gamma_I^i \left(p_I^{i*}\right)^{1-\sigma} \right] n_t
\end{aligned}$$

where the term in brackets is constant over time. Likewise, the southern price index satisfies

$$\begin{aligned}
P_{St}^{1-\sigma} &= \int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega \\
&= \sum_{i=H,L} \left[n_{Xt}^i \left(p_N^{i*}\right)^{1-\sigma} + n_{Ft}^i \left(p_F^i\right)^{1-\sigma} + n_{It}^i \left(p_I^i\right)^{1-\sigma} \right] \\
&= \sum_{i=H,L} \left[\gamma_X^i n_t \left(p_N^{i*}\right)^{1-\sigma} + \gamma_F^i n_t \left(p_F^i\right)^{1-\sigma} + \gamma_I^i n_t \left(p_I^i\right)^{1-\sigma} \right] \\
&= \sum_{i=H,L} \left[\gamma_X^i \left(p_N^{i*}\right)^{1-\sigma} + \gamma_F^i \left(p_F^i\right)^{1-\sigma} + \gamma_I^i \left(p_I^i\right)^{1-\sigma} \right] n_t
\end{aligned}$$

where the term in brackets is constant over time.

The representative northern consumer's static utility is $u_{Nt} = c_{Nt}/P_{Nt}$ and the representative southern consumer's static utility is $u_{St} = c_{St}/P_{St}$. In any steady-state equilibrium, consumer expenditure is constant but the price indexes P_{Nt} and P_{St} fall over time, and therefore consumer utility grows over time in steady-state equilibrium. Define $g_u \equiv \dot{u}_{Nt}/u_{Nt} = \dot{u}_{St}/u_{St}$. It is straightforward to see that $\dot{u}_{Nt}/u_{Nt} = -\dot{P}_{Nt}/P_{Nt} = g/(\sigma-1)$.

I also derive steady-state expressions for the variety shares. First, I solve for γ_N^i. By differentiating the variety condition for i-cost firms $q^i n_t = n_{Nt}^i + n_{Xt}^i + n_{Ft}^i + n_{It}^i$, I obtain

$$\begin{aligned}
q^i \dot{n}_t &= \dot{n}_{Nt}^i + \dot{n}_{Xt}^i + \dot{n}_{Ft}^i + \dot{n}_{It}^i \\
q^i \frac{\dot{n}_t}{n_t} &= \frac{\dot{n}_{Nt}^i + \dot{n}_{Xt}^i + \dot{n}_{Ft}^i + \dot{n}_{It}^i}{n_t} \\
q^i g &= \frac{\dot{n}_{Nt}^i}{n_{Nt}^i}\frac{n_{Nt}^i}{n_t} + \frac{\dot{n}_{Xt}^i + \dot{n}_{Ft}^i + \dot{n}_{It}^i}{n_{Nt}^i}\frac{n_{Nt}^i}{n_t} \\
q^i g &= g\gamma_N^i + \chi^i \gamma_N^i
\end{aligned}$$

and solving for γ_N^i yields

$$\gamma_N^i = q^i \frac{g}{g+\chi^i}, \qquad (i = H, L). \tag{7}$$

From the definition of the export-learning rate for northern firms of marginal cost type i, I obtain that

$$\begin{aligned}\chi^i &= \frac{\dot{n}^i_{Xt}+\dot{n}^i_{Ft}+\dot{n}^i_{It}}{n^i_{Nt}} \\ &= \frac{\dot{n}^i_{Xt}}{n^i_{Xt}}\frac{n^i_{Xt}/n_t}{n^i_{Nt}/n_t}+\frac{\dot{n}^i_{Ft}+\dot{n}^i_{It}}{n^i_{Xt}}\frac{n^i_{Xt}/n_t}{n^i_{Nt}/n_t} \\ &= \left(g+\phi^i\right)\frac{\gamma^i_X}{\gamma^i_N}\end{aligned}$$

and it follows that $\gamma^i_X=\gamma^i_N\left(\frac{\chi^i}{g+\phi^i}\right)$. Inserting the steady-state expression for γ^i_N from (7) yields

$$\gamma^i_X = q^i\frac{\chi^i}{g+\chi^i}\frac{g}{g+\phi^i}, \qquad (i=H,L). \tag{8}$$

From the definition of the FDI rate for exporting firms of marginal cost type i, I obtain that

$$\begin{aligned}\phi^i &= \frac{\dot{n}^i_{Ft}+\dot{n}^i_{It}}{n^i_{Xt}} \\ &= \frac{\dot{n}^i_{Ft}}{n^i_{Ft}}\frac{n^i_{Ft}/n_t}{n^i_{Xt}/n_t}+\frac{\dot{n}^i_{It}}{n^i_{Ft}}\frac{n^i_{Ft}/n_t}{n^i_{Xt}/n_t} \\ &= \left(g+\iota_S\right)\frac{\gamma^i_F}{\gamma^i_X}\end{aligned}$$

and it follows that $\gamma^i_F=\gamma^i_X\phi^i/\left(g+\iota_S\right)$. Inserting the steady-state expressions for γ^i_X from (8) yields

$$\gamma^i_F = q^i\frac{\chi^i}{g+\chi^i}\frac{\phi^i}{g+\phi^i}\frac{g}{g+\iota_S}, \qquad (i=H,L). \tag{9}$$

From the definition of the imitation rate, I obtain that

$$\begin{aligned}\iota_S &\equiv \frac{\dot{n}^i_{It}}{n^i_{Ft}} \\ &= \frac{\dot{n}^i_{It}}{n^i_{It}}\frac{n^i_{It}/n_t}{n^i_{Ft}/n_t} \\ &= g\frac{\gamma^i_I}{\gamma^i_F}.\end{aligned}$$

and it follows that $\gamma^i_I=\left(\iota_S/g\right)\gamma^i_F$. Inserting the steady-state expressions for γ^i_F from (9) yields

$$\gamma^i_I = q^i\frac{\chi^i}{g+\chi^i}\frac{\phi^i}{g+\phi^i}\frac{\iota_S}{g+\iota_S}, \qquad (i=H,L). \tag{10}$$

Product Markets

A northern firm of marginal cost type i where $i = H, L$ earns the flow of domestic profits

$$\pi^i_{Nt} = \left(p^i_N - c^i w_N\right) x^i_{Nt} L_{Nt}$$

where x^i_{Nt} is the quantity demanded by the typical northern consumer of the northern firm's product. From the earlier demand function, it follows that $x^i_{Nt} = \left(p^i_N\right)^{-\sigma} c_N / P^{1-\sigma}_{Nt}$. Hence, I can write a northern firm's profit flow as:

$$\pi^i_{Nt} = \left(p^i_N - c^i w_N\right) \frac{\left(p^i_N\right)^{-\sigma} c_N L_{Nt}}{P^{1-\sigma}_{Nt}}.$$

Maximizing π^i_{Nt} with respect to p^i_N yields the first-order condition

$$\frac{\partial \pi^i_{Nt}}{\partial p^i_N} = \left[(1-\sigma)\left(p^i_N\right)^{-\sigma} + \sigma c^i w_N \left(p^i_N\right)^{-\sigma-1}\right] \frac{c_N L_{Nt}}{P^{1-\sigma}_{Nt}} = 0,$$

which implies that $(1-\sigma)\left(p^i_N\right)^{-\sigma} + \sigma c^i w_N \left(p^i_N\right)^{-\sigma-1} = 0$ since $\frac{c_N L_{Nt}}{P^{1-\sigma}_{Nt}} \neq 0$. Dividing by $\left(p^i_N\right)^{-\sigma}$ yields $\frac{\sigma c^i w_N}{p^i_N} = \sigma - 1$ or

$$p^i_N = \frac{\sigma c^i w_N}{\sigma - 1} = \frac{c^i w_N}{\alpha}.$$

To demonstrate the second equality, first note that $\sigma \equiv \frac{1}{1-\alpha}$ implies that $\sigma - 1 = \frac{1-(1-\alpha)}{1-\alpha} = \frac{\alpha}{1-\alpha}$. It follows that $\frac{\sigma}{\sigma-1} = (\frac{1}{1-\alpha})/(\frac{\alpha}{1-\alpha}) = \frac{1}{\alpha}$. Plugging the prices back into the profit expression, I obtain

$$\begin{aligned}
\pi^i_{Nt} &= \left(p^i_N - c^i w_N\right) \frac{\left(p^i_N\right)^{-\sigma} c_N L_{Nt}}{P^{1-\sigma}_{Nt}} \\
&= \left(\frac{c^i w_N}{\alpha} - c^i w_N\right) \frac{\left(p^i_N\right)^{-\sigma} c_N L_{Nt}}{P^{1-\sigma}_{Nt}} \\
&= \frac{c^i w_N}{\sigma - 1} \left[\frac{\left(p^i_N\right)^{-\sigma} c_N L_{Nt}}{P^{1-\sigma}_{Nt}}\right]
\end{aligned}$$

where I have used that $\frac{1}{\alpha} - 1 = \frac{\sigma}{\sigma-1} - \frac{\sigma-1}{\sigma-1} = \frac{1}{\sigma-1}$. It turns out to be convenient to reexpress profits by multiplying the RHS by $\frac{L_t}{L_t}\frac{n^i_{Nt}}{n^i_{Nt}}\frac{n_t}{n_t}$:

$$\pi^i_{Nt} = \frac{c^i w_N}{\sigma - 1} \left[\frac{\left(p^i_N\right)^{-\sigma} c_N L_{Nt} n^i_{Nt}}{P^{1-\sigma}_{Nt} L_t}\right] \frac{L_t}{n_t \frac{n^i_{Nt}}{n_t}}.$$

Now $\gamma_N^i \equiv \frac{n_{Nt}^i}{n_t}$ is constant over time, $X_N^i \equiv \frac{\left(p_N^i\right)^{-\sigma} c_N L_{Nt} n_{Nt}^i}{P_{Nt}^{1-\sigma} L_t}$ is constant over time since $P_{Nt}^{1-\sigma}$ grows at the same rate g as n_{Nt}^i. Thus I can write π_{Nt}^i more simply as:

$$\pi_{Nt}^i = \frac{c^i w_N X_N^i}{(\sigma-1)\,\gamma_N^i} \frac{L_t}{n_t}. \tag{11}$$

A northern exporting firm earns the flow of global profits

$$\pi_{Xt}^i = \left(p_N^i - c^i w_N\right) x_{Xt}^i L_{Nt} + \left(p_N^{i*} - \tau c^i w_N\right) x_{Xt}^{i*} L_{St}$$

where $x_{Xt}^i = \left(p_N^i\right)^{-\sigma} c_N / P_{Nt}^{1-\sigma}$ is the quantity demanded by the typical northern consumer of the northern exporting firm's product and $x_{Xt}^{i*} = \left(p_N^{i*}\right)^{-\sigma} c_S / P_{St}^{1-\sigma}$ is the quantity demanded by the typical southern consumer of the northern exporting firm's product. Hence, I can write a northern exporting firm's global profit flow as:

$$\pi_{Xt}^i = \left(p_N^i - c^i w_N\right) \frac{\left(p_N^i\right)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}} + \left(p_N^{i*} - \tau c^i w_N\right) \frac{\left(p_N^{i*}\right)^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}}.$$

Maximizing π_{Xt}^i with respect to p_N^i yields the first-order condition

$$\frac{\partial \pi_{Xt}^i}{\partial p_N^i} = \left[(1-\sigma)\left(p_N^i\right)^{-\sigma} + \sigma c^i w_N \left(p_N^i\right)^{-\sigma-1}\right] \frac{c_N L_{Nt}}{P_{Nt}^{1-\sigma}} = 0,$$

which implies that $(1-\sigma)\left(p_N^i\right)^{-\sigma} + \sigma c^i w_N \left(p_N^i\right)^{-\sigma-1} = 0$ since $\frac{c_N L_{Nt}}{P_{Nt}^{1-\sigma}} \neq 0$. Dividing by $\left(p_N^i\right)^{-\sigma}$ yields $\frac{\sigma c^i w_N}{p_N^i} = \sigma - 1$ or

$$p_N^i = \frac{\sigma c^i w_N}{\sigma - 1} = \frac{c^i w_N}{\alpha}.$$

Similarly, maximizing π_{Xt}^i with respect to p_N^{i*} yields the first-order condition

$$\frac{\partial \pi_{Xt}^i}{\partial p_N^{i*}} = \left[(1-\sigma)\left(p_N^{i*}\right)^{-\sigma} + \sigma\tau c^i w_N \left(p_N^{i*}\right)^{-\sigma-1}\right] \frac{c_S L_{St}}{P_{St}^{1-\sigma}} = 0,$$

which implies that $(1-\sigma)\left(p_N^{i*}\right)^{-\sigma} + \sigma\tau c^i w_N \left(p_N^{i*}\right)^{-\sigma-1} = 0$. Dividing by $\left(p_N^{i*}\right)^{-\sigma}$ yields $\frac{\sigma\tau c^i w_N}{p_N^{i*}} = \sigma - 1$ or

$$p_N^{i*} = \frac{\sigma\tau c^i w_N}{\sigma - 1} = \frac{\tau c^i w_N}{\alpha}.$$

Plugging the prices back into the profit expression, I obtain

$$\begin{aligned}
\pi_{Xt}^{i} &= \left(p_{N}^{i}-c^{i}w_{N}\right)\frac{\left(p_{N}^{i}\right)^{-\sigma}c_{N}L_{Nt}}{P_{Nt}^{1-\sigma}}+\left(p_{N}^{i*}-\tau c^{i}w_{N}\right)\frac{\left(p_{N}^{i*}\right)^{-\sigma}c_{S}L_{St}}{P_{St}^{1-\sigma}}\\
&= \left(\frac{c^{i}w_{N}}{\alpha}-c^{i}w_{N}\right)\frac{\left(p_{N}^{i}\right)^{-\sigma}c_{N}L_{Nt}}{P_{Nt}^{1-\sigma}}+\left(\frac{\tau c^{i}w_{N}}{\alpha}-\tau c^{i}w_{N}\right)\frac{\left(p_{N}^{i*}\right)^{-\sigma}c_{S}L_{St}}{P_{St}^{1-\sigma}}\\
&= \frac{c^{i}w_{N}}{\sigma-1}\left[\frac{\left(p_{N}^{i}\right)^{-\sigma}c_{N}L_{Nt}}{P_{Nt}^{1-\sigma}}+\tau\frac{\left(p_{N}^{i*}\right)^{-\sigma}c_{S}L_{St}}{P_{St}^{1-\sigma}}\right]
\end{aligned}$$

where I have used that $\frac{1}{\alpha}-1=\frac{\sigma}{\sigma-1}-\frac{\sigma-1}{\sigma-1}=\frac{1}{\sigma-1}$. It turns out to be convenient to reexpress profits by multiplying the RHS by $\frac{L_t}{L_t}\frac{n_{Xt}^{i}}{n_{Xt}^{i}}\frac{n_t}{n_t}$:

$$\pi_{Xt}^{i}=\frac{c^{i}w_{N}}{\sigma-1}\left[\frac{\left(p_{N}^{i}\right)^{-\sigma}c_{N}L_{Nt}n_{Xt}^{i}}{P_{Nt}^{1-\sigma}L_t}+\tau\frac{\left(p_{N}^{i*}\right)^{-\sigma}c_{S}L_{St}n_{Xt}^{i}}{P_{St}^{1-\sigma}L_t}\right]\frac{L_t}{n_t\frac{n_{Xt}^{i}}{n_t}}.$$

Now $\gamma_{X}^{i}\equiv\frac{n_{Xt}^{i}}{n_t}$ is constant over time, $X_{X}^{i}\equiv\frac{\left(p_{N}^{i}\right)^{-\sigma}c_{N}L_{Nt}n_{Xt}^{i}}{P_{Nt}^{1-\sigma}L_t}$ is constant over time since $P_{Nt}^{1-\sigma}$ grows at the same rate g as n_{Xt}^{i}, and $X_{X}^{i*}\equiv\frac{\left(p_{N}^{i*}\right)^{-\sigma}c_{S}L_{St}n_{Xt}^{i}}{P_{St}^{1-\sigma}L_t}$ is constant over time since $P_{St}^{1-\sigma}$ grows at the same rate g as n_{Xt}^{i}. Thus I can write π_{Xt}^{i} more simply as:

$$\pi_{Xt}^{i}=\left[\frac{c^{i}w_{N}\left(X_{X}^{i}+\tau X_{X}^{i*}\right)}{\left(\sigma-1\right)\gamma_{X}^{i}}\right]\frac{L_t}{n_t}. \tag{12}$$

A foreign affiliate earns the flow of global profits:

$$\pi_{Ft}^{i}=\left(p_{F}^{i}-c^{i}w_{S}\right)x_{Ft}^{i}L_{St}+\left(p_{F}^{i*}-\tau c^{i}w_{S}\right)x_{Ft}^{i*}L_{Nt}$$

where $x_{Ft}^{i}=\left(p_{F}^{i}\right)^{-\sigma}c_{S}/P_{St}^{1-\sigma}$ is the quantity demanded by the typical southern consumer of the foreign affiliate's product and $x_{Ft}^{i*}=\left(p_{F}^{i*}\right)^{-\sigma}c_{N}/P_{Nt}^{1-\sigma}$ is the quantity demanded by the typical northern consumer of the foreign affiliate's product. Hence, I can write a foreign affiliate's profit flow as

$$\pi_{Ft}^{i}=\left(p_{F}^{i}-c^{i}w_{S}\right)\frac{\left(p_{F}^{i}\right)^{-\sigma}c_{S}L_{St}}{P_{St}^{1-\sigma}}+\left(p_{F}^{i*}-\tau c^{i}w_{S}\right)\frac{\left(p_{F}^{i*}\right)^{-\sigma}c_{N}L_{Nt}}{P_{Nt}^{1-\sigma}}.$$

Maximizing π_{Ft}^{i} with respect to p_{F}^{i} yields the first-order condition

$$\frac{\partial\pi_{Ft}^{i}}{\partial p_{F}^{i}}=\left[\left(1-\sigma\right)\left(p_{F}^{i}\right)^{-\sigma}+\sigma c^{i}w_{S}\left(p_{F}^{i}\right)^{-\sigma-1}\right]\frac{c_{S}L_{St}}{P_{St}^{1-\sigma}}=0$$

which implies that $\left(1-\sigma\right)\left(p_{F}^{i}\right)^{-\sigma}+\sigma c^{i}w_{S}\left(p_{F}^{i}\right)^{-\sigma-1}=0$. Dividing by $\left(p_{F}^{i}\right)^{-\sigma}$ yields $\frac{\sigma c^{i}w_{S}}{p_{F}^{i}}=\sigma-1$ or

$$p_{F}^{i}=\frac{\sigma c^{i}w_{S}}{\sigma-1}=\frac{c^{i}w_{S}}{\alpha}.$$

Similarly, maximizing π^i_{Ft} with respect to p^{i*}_F yields the first-order condition

$$\frac{\partial \pi^i_{Ft}}{\partial p^{i*}_F} = \left[(1-\sigma)\left(p^{i*}_F\right)^{-\sigma} + \sigma\tau c^i w_S \left(p^{i*}_F\right)^{-\sigma-1}\right] \frac{c_N L_{Nt}}{P^{1-\sigma}_{Nt}} = 0,$$

which implies that $(1-\sigma)\left(p^{i*}_F\right)^{-\sigma} + \sigma\tau c^i w_S \left(p^{i*}_F\right)^{-\sigma-1} = 0$. Dividing by $\left(p^{i*}_F\right)^{-\sigma}$ yields $\frac{\sigma\tau c^i w_S}{p^{i*}_F} = \sigma - 1$ or

$$p^{i*}_F = \frac{\sigma\tau c^i w_S}{\sigma - 1} = \frac{\tau c^i w_S}{\alpha}.$$

When the inequality $\tau w_S < w_N$ holds, each foreign affiliate exports to the northern market. The trade costs parameter τ cannot be too high. Plugging the prices back into the profit expression, I obtain

$$\begin{aligned}
\pi^i_{Ft} &= \left(\frac{c^i w_S}{\alpha} - c^i w_S\right) \frac{\left(p^i_F\right)^{-\sigma} c_S L_{St}}{P^{1-\sigma}_{St}} + \left(\frac{\tau c^i w_S}{\alpha} - \tau c^i w_S\right) \frac{(p^{i*}_F)^{-\sigma} c_N L_{Nt}}{P^{1-\sigma}_{Nt}} \\
&= \frac{c^i w_S}{\sigma - 1} \left[\frac{\left(p^i_F\right)^{-\sigma} c_S L_{St}}{P^{1-\sigma}_{St}} + \tau \frac{\left(p^{i*}_F\right)^{-\sigma} c_N L_{Nt}}{P^{1-\sigma}_{Nt}}\right].
\end{aligned}$$

I reexpress profits by multiplying the RHS by $\frac{L_t}{L_t}\frac{n^i_{Ft}}{n^i_{Ft}}\frac{n_t}{n_t}$:

$$\pi^i_{Ft} = \frac{c^i w_S}{\sigma - 1}\left[\frac{\left(p^i_F\right)^{-\sigma} c_S L_{St} n^i_{Ft}}{P^{1-\sigma}_{St} L_t} + \tau \frac{\left(p^{i*}_F\right)^{-\sigma} c_N L_{Nt} n^i_{Ft}}{P^{1-\sigma}_{Nt} L_t}\right] \frac{L_t}{n_t \frac{n^i_{Ft}}{n_t}}.$$

Now $\gamma^i_F \equiv \frac{n^i_{Ft}}{n_t}$ is constant over time, $X^i_F \equiv \frac{\left(p^i_F\right)^{-\sigma} c_S L_{St} n^i_{Ft}}{P^{1-\sigma}_{St} L_t}$ is constant over time since $P^{1-\sigma}_{St}$ grows at the same rate g as n^i_{Ft}, and $X^{i*}_F \equiv \frac{\left(p^{i*}_F\right)^{-\sigma} c_N L_{Nt} n^i_{Ft}}{P^{1-\sigma}_{Nt} L_t}$ is constant over time since $P^{1-\sigma}_{Nt}$ grows at the same rate g as n^i_{Ft}. Thus I can write π^i_{Ft} more simply as:

$$\pi^i_{Ft} = \left[\frac{c^i w_S \left(X^i_F + \tau X^{i*}_F\right)}{(\sigma - 1)\gamma^i_F}\right] \frac{L_t}{n_t}. \tag{13}$$

A foreign affiliate's variety is imitated by southern firms at the exogenously given rate ι_S. Once the imitation technology is available to southern firms, competition drives down price to marginal cost and southern firms therefore earn zero profits. The quantity demanded by the typical southern consumer of southern firm products is $x^i_{It} = p^{-\sigma}_S c_S / P^{1-\sigma}_{St}$ and $x^{i*}_{It} = \left(p^{i*}_S\right)^{-\sigma} c_N / P^{1-\sigma}_{Nt}$ is the quantity demanded by the typical northern consumer of southern firm products. Since southern firms set price equal to marginal cost, I must have $p^i_S = c^i w_S$ and $p^{i*}_S = \tau c^i w_S$.

R&D Incentives

For a non-exporting northern firm, the no-arbitrage condition is

$$v_{Nt} = \frac{q\pi^H_{Nt} + (1-q)\pi^L_{Nt}}{\rho + \theta g} = \frac{w_N a_N g}{n^\theta_t}.$$

Substituting for π_{Nt}^H and π_{Nt}^L yields

$$\begin{aligned}
\frac{\frac{qc^H w_N X_N^H}{(\sigma-1)\gamma_N^H}\frac{L_t}{n_t} + \frac{(1-q)c^L w_N X_N^L}{(\sigma-1)\gamma_N^L}\frac{L_t}{n_t}}{\rho+\theta g} &= \frac{w_N a_N g}{n_t^\theta} \\
\frac{\frac{qc^H X_N^H}{(\sigma-1)\gamma_N^H} + \frac{(1-q)c^L X_N^L}{(\sigma-1)\gamma_N^L}}{\rho+\theta g} &= a_N g \frac{n_t^{1-\theta}}{L_t}.
\end{aligned}$$

Using the definition of relative R&D difficulty the steady-state northern no-arbitrage condition becomes

$$\frac{\frac{1}{\sigma-1}\left(\frac{qc^H X_N^H}{\gamma_N^H} + \frac{(1-q)c^L X_N^L}{\gamma_N^L}\right)}{\rho+\theta g} = a_N g\delta. \tag{26}$$

For a northern exporting firm, the no-arbitrage condition is

$$v_{Xt}^i - v_{Nt}^i = \frac{\pi_{Xt}^i}{\rho+\theta g} - \frac{\pi_{Nt}^i}{\rho+\theta g} = \frac{w_S a_X \chi^i}{n_t^\theta}.$$

Using the profits for northern exporters and non-exporters from earlier, I can write this as:

$$\begin{aligned}
\frac{\frac{c^i w_N}{(\sigma-1)}\frac{X_X^i+\tau X_X^{i*}}{\gamma_X^i}\frac{L_t}{n_t}}{\rho+\theta g} - \frac{\frac{c^i w_N X_N^i}{(\sigma-1)\gamma_N^i}\frac{L_t}{n_t}}{\rho+\theta g} &= \frac{w_S a_X \chi^i}{n_t^\theta} \\
\frac{\frac{c^i w}{(\sigma-1)}\frac{X_X^i+\tau X_X^{i*}}{\gamma_X^i}}{\rho+\theta g} - \frac{\frac{c^i w X_N^i}{(\sigma-1)\gamma_N^i}}{\rho+\theta g} &= a_X \chi^i \frac{n_t^{1-\theta}}{L_t}.
\end{aligned}$$

It follows that the steady-state exporter no-arbitrage condition is

$$\frac{c^i w}{\sigma-1}\left[\frac{\frac{X_X^i+\tau X_X^{i*}}{\gamma_X^i} - \frac{X_N^i}{\gamma_N^i}}{\rho+\theta g}\right] = a_X \chi^i \delta \tag{28}$$

where $w \equiv w_N/w_S$ is the northern relative wage.

For a foreign affiliate, the no-arbitrage condition is

$$\frac{\pi_{Ft}^i}{\rho+\theta g+\iota_S} - \frac{\pi_{Xt}^i}{\rho+\theta g} = \frac{w_S a_F \phi^i}{n_t^\theta}.$$

Substituting for π_{Ft}^i and π_{Xt}^i yields

$$\begin{aligned}
\frac{\frac{c^i w_S}{\sigma-1}\frac{X_F^i+\tau X_F^{i*}}{\gamma_F^i}\frac{L_t}{n_t}}{\rho+\theta g+\iota_S} - \frac{\frac{c^i w_N}{\sigma-1}\frac{X_X^i+\tau X_X^i}{\gamma_X^i}\frac{L_t}{n_t}}{\rho+\theta g} &= \frac{w_S a_F \phi^i}{n_t^\theta} \\
\frac{\frac{c^i}{\sigma-1}\frac{X_F^i+\tau X_F^{i*}}{\gamma_F^i}}{\rho+\theta g+\iota_S} - \frac{\frac{c^i w}{\sigma-1}\frac{X_X^i+\tau X_X^{i*}}{\gamma_X^i}}{\rho+\theta g} &= a_F \phi^i \frac{n_t^{1-\theta}}{L_t}.
\end{aligned}$$

It follows that the steady-state foreign affiliate no-arbitrage condition is

$$\frac{c^i}{\sigma-1}\left[\frac{\frac{X_F^i+\tau X_F^{i*}}{\gamma_F^i}}{\rho+\theta g+\iota_S}-\frac{\frac{w\left(X_X^i+\tau X_X^{i*}\right)}{\gamma_X^i}}{\rho+\theta g}\right]=a_F\phi^i\delta. \tag{30}$$

Aggregate Demand

To solve the model, I need steady-state values for the aggregate demand terms X_N^i, X_X^i, X_X^{i*}, X_F^i, X_F^{i*}, X_I^i and X_I^*. The calculations

$$\begin{aligned}
\frac{X_N^i}{X_F^{i*}} &= \frac{\frac{\left(p_N^i\right)^{-\sigma}c_N L_{Nt} n_{Nt}^i}{P_{Nt}^{1-\sigma}L_t}}{\frac{\left(p_F^{i*}\right)^{-\sigma}c_N L_{Nt} n_{Ft}^i}{P_{Nt}^{1-\sigma}L_t}} \\
&= \left(\frac{p_N^i}{p_F^{i*}}\right)^{-\sigma}\frac{n_{Nt}^i/n_t}{n_{Ft}^i/n_t} \\
&= \left(\frac{\frac{c^i w_N}{\alpha}}{\frac{\tau c^i w_S}{\alpha}}\right)^{-\sigma}\frac{\gamma_N^i}{\gamma_F^i} \\
&= \left(\frac{w}{\tau}\right)^{-\sigma}\frac{q^i\frac{g}{g+\chi^i}}{q^i\frac{\chi^i}{g+\chi^i}\frac{\phi^i}{g+\phi^i}\frac{g}{g+\iota_S}} \\
&= \left(\frac{w}{\tau}\right)^{-\sigma}\frac{\left(g+\phi^i\right)\left(g+\iota_S\right)}{\chi^i\phi^i},
\end{aligned}$$

$$\begin{aligned}
\frac{X_X^i}{X_F^{i*}} &= \frac{\frac{\left(p_N^i\right)^{-\sigma}c_N L_{Nt} n_{Xt}^i}{P_{Nt}^{1-\sigma}L_t}}{\frac{\left(p_F^{i*}\right)^{-\sigma}c_N L_{Nt} n_{Ft}^i}{P_{Nt}^{1-\sigma}L_t}} \\
&= \left(\frac{p_N^i}{p_F^{i*}}\right)^{-\sigma}\frac{n_{Xt}^i/n_t}{n_{Ft}^i/n_t} \\
&= \left(\frac{\frac{c^i w_N}{\alpha}}{\frac{\tau c^i w_S}{\alpha}}\right)^{-\sigma}\frac{\gamma_X^i}{\gamma_F^i} \\
&= \left(\frac{w}{\tau}\right)^{-\sigma}\frac{q^i\frac{\chi^i}{g+\chi^i}\frac{g}{g+\phi^i}}{q^i\frac{\chi^i}{g+\chi^i}\frac{\phi^i}{g+\phi^i}\frac{g}{g+\iota_S}} \\
&= \left(\frac{w}{\tau}\right)^{-\sigma}\frac{g+\iota_S}{\phi^i},
\end{aligned}$$

$$
\begin{aligned}
\frac{X_X^{i*}}{X_F^i} &= \frac{\frac{\left(p_N^{i*}\right)^{-\sigma} c_S L_{St} n_{Xt}^i}{P_{St}^{1-\sigma} L_t}}{\frac{\left(p_F^{i}\right)^{-\sigma} c_S L_{St} n_{Ft}^i}{P_{St}^{1-\sigma} L_t}} \\
&= \left(\frac{p_N^{i*}}{p_F^i}\right)^{-\sigma} \frac{n_{Xt}^i/n_t}{n_{Ft}^i/n_t} \\
&= \left(\frac{\frac{\tau c^i w_N}{\alpha}}{\frac{c^i w_S}{\alpha}}\right)^{-\sigma} \frac{\gamma_X^i}{\gamma_F^i} \\
&= (\tau w)^{-\sigma} \frac{q^i \frac{\chi^i}{g+\chi^i} \frac{g}{g+\phi^i}}{q^i \frac{\chi^i}{g+\chi^i} \frac{\phi^i}{g+\phi^i} \frac{g}{g+\iota_S}} \\
&= (\tau w)^{-\sigma} \frac{g+\iota_S}{\phi^i},
\end{aligned}
$$

$$
\begin{aligned}
\frac{X_I^{i}}{X_F^i} &= \frac{\frac{\left(p_S^{i}\right)^{-\sigma} c_S L_{St} n_{It}^i}{P_{St}^{1-\sigma} L_t}}{\frac{\left(p_F^{i}\right)^{-\sigma} c_S L_{St} n_{Ft}^i}{P_{St}^{1-\sigma} L_t}} \\
&= \left(\frac{p_S^{i}}{p_F^i}\right)^{-\sigma} \frac{n_{It}^i/n_t}{n_{Ft}^i/n_t} \\
&= \left(\frac{c^i w_S}{\frac{c^i w_S}{\alpha}}\right)^{-\sigma} \frac{\gamma_I^i}{\gamma_F^H} \\
&= \left(\frac{1}{\alpha}\right)^{\sigma} \frac{q^i \frac{\chi^i}{g+\chi^i} \frac{\phi^i}{g+\phi^i} \frac{\iota_S}{g+\iota_S}}{q^i \frac{\chi^i}{g+\chi^i} \frac{\phi^i}{g+\phi^i} \frac{g}{g+\iota_S}} \\
&= \left(\frac{1}{\alpha}\right)^{\sigma} \frac{\iota_S}{g},
\end{aligned}
$$

$$
\begin{aligned}
\frac{X_I^{i*}}{X_F^{i*}} &= \frac{\frac{\left(p_S^{i*}\right)^{-\sigma} c_N L_{Nt} n_{It}^i}{P_{Nt}^{1-\sigma} L_t}}{\frac{\left(p_F^{i*}\right)^{-\sigma} c_N L_{Nt} n_{Ft}^i}{P_{Nt}^{1-\sigma} L_t}} \\
&= \left(\frac{p_S^{i*}}{p_F^{i*}}\right)^{-\sigma} \frac{n_{It}^i/n_t}{n_{Ft}^i/n_t} \\
&= \left(\frac{\tau c^i w_S}{\frac{\tau c^i w_S}{\alpha}}\right)^{-\sigma} \frac{\gamma_I^i}{\gamma_F^i} \\
&= \left(\frac{1}{\alpha}\right)^{\sigma} \frac{q^i \frac{\chi^i}{g+\chi^i} \frac{\phi^i}{g+\phi^i} \frac{\iota_S}{g+\iota_S}}{q^i \frac{\chi^i}{g+\chi^i} \frac{\phi^i}{g+\phi^i} \frac{g}{g+\iota_S}} \\
&= \left(\frac{1}{\alpha}\right)^{\sigma} \frac{\iota_S}{g}
\end{aligned}
$$

imply that

$$X_N^i = X_F^{i*}\left(\frac{w}{\tau}\right)^{-\sigma}\frac{\left(g+\phi^i\right)\left(g+\iota_S\right)}{\chi^i\phi^i}, \tag{33}$$

$$X_X^i = X_F^{i*}\left(\frac{w}{\tau}\right)^{-\sigma}\frac{g+\iota_S}{\phi^i}, \tag{34}$$

$$X_X^{i*} = X_F^i\left(w\tau\right)^{-\sigma}\frac{g+\iota_S}{\phi^i}, \tag{35}$$

$$X_I^i = X_F^i\left(\frac{1}{\alpha}\right)^{\sigma}\frac{\iota_S}{g}, \tag{36}$$

and

$$X_I^{i*} = X_F^{i*}\left(\frac{1}{\alpha}\right)^{\sigma}\frac{\iota_S}{g}. \tag{37}$$

Finally, I need to express X_F^H in terms of X_F^L and X_F^{H*} in terms of X_F^{L*}. The calculations

$$\begin{aligned}
\frac{X_F^H}{X_F^L} &= \frac{\frac{\left(p_F^H\right)^{-\sigma}c_S L_{St} n_{Ft}^H}{P_{St}^{1-\sigma}L_t}}{\frac{\left(p_F^L\right)^{-\sigma}c_S L_{St} n_{Ft}^L}{P_{St}^{1-\sigma}L_t}} \\
&= \left(\frac{p_F^H}{p_F^L}\right)^{-\sigma}\frac{n_{Ft}^H/n_t}{n_{Ft}^L/n_t} \\
&= \left(\frac{\frac{c^H w_S}{\alpha}}{\frac{c^L w_S}{\alpha}}\right)^{-\sigma}\frac{\gamma_F^H}{\gamma_F^L} \\
&= \left(\frac{c^H}{c^L}\right)^{-\sigma}\frac{q\left(\frac{\chi^H}{g+\chi^H}\frac{\phi^H}{g+\phi^H}\frac{g}{g+\iota_S}\right)}{(1-q)\left(\frac{\chi^L}{g+\chi^L}\frac{\phi^L}{g+\phi^L}\frac{g}{g+\iota_S}\right)}
\end{aligned}$$

yields

$$X_F^H = X_F^L\left(\frac{c^H}{c^L}\right)^{-\sigma}\left(\frac{q}{1-q}\right)\left(\frac{g+\chi^L}{g+\chi^H}\right)\left(\frac{\chi^H}{\chi^L}\right)\left(\frac{g+\phi^L}{g+\phi^H}\right)\left(\frac{\phi^H}{\phi^L}\right) \tag{38}$$

where I have used that $q^H = q$ and $q^L = 1-q$.

Similarly, the calculations

$$
\begin{aligned}
\frac{X_F^{H*}}{X_F^{L*}} &= \frac{\frac{\left(p_F^{H*}\right)^{-\sigma} c_N L_{Nt} n_{Ft}^H}{P_{Nt}^{1-\sigma} L_t}}{\frac{\left(p_F^{L*}\right)^{-\sigma} c_N L_{Nt} n_{Ft}^L}{P_{Nt}^{1-\sigma} L_t}} \\
&= \left(\frac{p_F^{H*}}{p_F^{L*}}\right)^{-\sigma} \frac{n_{Ft}^H/n_t}{n_{Ft}^L/n_t} \\
&= \left(\frac{\frac{\tau c^H w_S}{\alpha}}{\frac{\tau c^L w_S}{\alpha}}\right)^{-\sigma} \frac{\gamma_F^H}{\gamma_F^L} \\
&= \left(\frac{c^H}{c^L}\right)^{-\sigma} \frac{q\left(\frac{\chi^H}{g+\chi^H}\frac{\phi^H}{g+\phi^H}\frac{g}{g+\iota_S}\right)}{(1-q)\left(\frac{\chi^L}{g+\chi^L}\frac{\phi^L}{g+\phi^L}\frac{g}{g+\iota_S}\right)}
\end{aligned}
$$

yields

$$X_F^{H*} = X_F^{L*}\left(\frac{c^H}{c^L}\right)^{-\sigma}\left(\frac{q}{1-q}\right)\left(\frac{g+\chi^L}{g+\chi^H}\right)\left(\frac{\chi^H}{\chi^L}\right)\left(\frac{g+\phi^L}{g+\phi^H}\right)\left(\frac{\phi^H}{\phi^L}\right). \tag{39}$$

Asset Ownership and Consumer Expenditure

Northern household savings finance northern R&D (innovation) and southern household savings finance southern R&D (export learning and FDI). Denote aggregate northern assets by A_{Nt} and aggregate southern assets by A_{St}. Total assets are $A_t = A_{Nt}+A_{St} = \sum_{i=H,L} n_{Nt}^i v_{Nt}^i + n_{Xt}^i v_{Xt}^i + n_{Ft}^i v_{Ft}^i$. Aggregate northern assets are $A_{Nt} = \sum_{i=H,L}\left(n_{Nt}^i + n_{Xt}^i + n_{Ft}^i\right) v_{Nt}^i$ while aggregate southern assets are $A_{St} = A_t - A_{Nt} = \sum_{i=H,L} n_{xt}^i\left(v_{Xt}^i - v_{Nt}^i\right) + n_{Ft}^i\left(v_{Ft}^i - v_{Nt}^i\right)$. From

$$v_{Xt}^i - v_{Nt}^i = \frac{w_S a_X \chi^i}{n_t^\theta}, \qquad (i = H, L)$$

and

$$v_{Ft}^i - v_{Xt}^i = \frac{w_S a_F \phi^i}{n_t^\theta}, \qquad (i = H, L)$$

it follows that

$$
\begin{aligned}
v_{Ft}^i - v_{Xt}^i &= \frac{w_S a_F \phi^i}{n_t^\theta} \\
v_{Ft}^i - \left(\frac{w_S a_X \chi^i}{n_t^\theta} + v_{Nt}^i\right) &= \frac{w_S a_F \phi^i}{n_t^\theta} \\
v_{Ft}^i - v_{Nt}^i &= \frac{w_S a_F \phi^i}{n_t^\theta} + \frac{w_S a_X \chi^i}{n_t^\theta}.
\end{aligned}
$$

Substituting for $v_{Ft}^i - v_{Nt}^i$ in the expression for aggregate southern assets

$$\begin{aligned} A_{St} &= \sum_{i=H,L} n_{xt}^i \left(v_{Xt}^i - v_{Nt}^i\right) + n_{Ft}^i \left(v_{Ft}^i - v_{Nt}^i\right) \\ &= \sum_{i=H,L} n_{xt}^i \frac{w_S a_X \chi^i}{n_t^\theta} + n_{Ft}^i \left(\frac{w_S a_F \phi^i}{n_t^\theta} + \frac{w_S a_X \chi^i}{n_t^\theta}\right) \\ &= w_S L_t \frac{n_t^{1-\theta}}{L_t} \left[\sum_{i=H,L} \left(\frac{n_{Xt}^i}{n_t}\right) a_X \chi^i + \left(\frac{n_{Ft}^i}{n_t}\right) \left(a_F \phi^i + a_X \chi^i\right)\right] \end{aligned}$$

yields

$$A_{St} = w_S L_t \delta \left[\sum_{i=H,L} \gamma_X^i a_X \chi^i + \gamma_F^i \left(a_F \phi^i + a_X \chi^i\right)\right]. \tag{40}$$

Using $v_{Nt}^i = \frac{\pi_{Nt}^i}{\rho+\theta g}$ and the steady-state profit expressions $\pi_{Nt}^i = \frac{c^i w_N X_N^i}{(\sigma-1)\gamma_N^i} \frac{L_t}{n_t}, (i = H, L)$, northern aggregate assets can be written as

$$\begin{aligned} A_{Nt} &= \sum_{i=H,L} \left(n_{Nt}^i + n_{Xt}^i + n_{Ft}^i\right) v_{Nt}^i \\ &= \sum_{i=H,L} \left(\frac{n_{Nt}^i}{n_t} + \frac{n_{Xt}^i}{n_t} + \frac{n_{Ft}^i}{n_t}\right) \frac{\pi_{Nt}^i n_t}{\rho + \theta g} \\ &= \sum_{i=H,L} \left(\gamma_N^i + \gamma_X^i + \gamma_F^i\right) \frac{c^i w_N X_N^i n_t}{(\sigma - 1)\, \gamma_N^i \,(\rho + \theta g)} \frac{L_t}{n_t} \end{aligned}$$

which yields

$$A_{Nt} = \frac{w_N L_t}{(\sigma - 1)(\rho + \theta g)} \left[\sum_{i=H,L} c^i X_N^i \frac{\gamma_N^i + \gamma_X^i + \gamma_F^i}{\gamma_N^i}\right]. \tag{41}$$

The intertemporal budget constraint for the typical consumer in region k $(k = N, S)$ is given by $\dot{a}_{kt} = w_k + \rho a_{kt} - c_k - g_L a_{kt}$, where individual assets are $a_{kt} = A_{kt}/L_{kt}$. In any steady-state equilibrium $\dot{a}_{kt}/a_{kt} = 0$. Individual consumer expenditure for the typical consumer is therefore $c_k = w_k + (\rho - g_L)\, a_{kt}$. Consumer expenditure for the typical southern consumer is

$$\begin{aligned} c_S &= w_S + (\rho - g_L)\, a_{St} \\ &= w_S + (\rho - g_L) \frac{A_{St}}{L_{St}} \\ &= w_S + (\rho - g_L)\, w_S \frac{L_t}{L_{St}} \delta \left[\sum_{i=H,L} \gamma_X^i a_X \chi^i + \gamma_F^i \left(a_F \phi^i + a_X \chi^i\right)\right]. \end{aligned}$$

Evaluating at time 0 yields steady-state southern consumer expenditure

$$c_S = w_S + (\rho - g_L) w_S \frac{L_0}{L_{S0}} \delta \left[\sum_{i=H,L} \gamma_X^i a_X \chi^i + \gamma_F^i \left(a_F \phi^i + a_X \chi^i \right) \right]. \tag{42}$$

Consumer expenditure for the typical northern consumer is

$$\begin{aligned} c_N &= w_N + (\rho - g_L) a_{Nt} \\ &= w_N + (\rho - g_L) \frac{A_{Nt}}{L_{Nt}} \\ &= w_N + (\rho - g_L) \frac{w_N}{(\sigma - 1)(\rho + \theta g)} \frac{L_t}{L_{Nt}} \left[\sum_{i=H,L} c^i X_N^i \frac{\gamma_N^i + \gamma_X^i + \gamma_F^i}{\gamma_N^i} \right]. \end{aligned}$$

Evaluating at time 0 yields steady-state northern consumer expenditure

$$c_N = w_N + \frac{(\rho - g_L) w_N}{(\sigma - 1)(\rho + \theta g)} \frac{L_0}{L_{N0}} \left[\sum_{i=H,L} c^i X_N^i \frac{\gamma_N^i + \gamma_X^i + \gamma_F^i}{\gamma_N^i} \right]. \tag{43}$$

Having solved for steady-state consumer expenditure c_N and c_S, I can take the ratio

$$\frac{X_F^{L*}}{X_F^L} = \frac{\frac{\left(p_F^{L*}\right)^{-\sigma} c_N L_{Nt} n_{Ft}^L}{P_{Nt}^{1-\sigma} L_t}}{\frac{\left(p_F^L\right)^{-\sigma} c_S L_{St} n_{Ft}^L}{P_{St}^{1-\sigma} L_t}} = \left(\frac{p_F^{L*}}{p_F^L} \right)^{-\sigma} \frac{c_N L_{Nt}}{c_S L_{St}} \frac{P_{St}^{1-\sigma}}{P_{Nt}^{1-\sigma}} = \left(\frac{\frac{\tau c^L w_S}{\alpha}}{\frac{c^L w_S}{\alpha}} \right)^{-\sigma} \frac{c_N L_{Nt}}{c_S L_{St}} \frac{P_{St}^{1-\sigma}}{P_{Nt}^{1-\sigma}} = \left(\frac{1}{\tau} \right)^{\sigma} \frac{c_N L_{Nt}}{c_S L_{St}} \frac{P_{St}^{1-\sigma}}{P_{Nt}^{1-\sigma}}.$$

Evaluating at time 0 yields the steady-state asset condition

$$\frac{X_F^{L*}}{X_F^L} = \left(\frac{1}{\tau} \right)^{\sigma} \frac{c_N L_{N0}}{c_S L_{S0}} \frac{P_{St}^{1-\sigma}}{P_{Nt}^{1-\sigma}}. \tag{44}$$

Aggregate Labor Demand

Total employment in innovative R&D L_{Rt} is derived from the flow of new products developed in the North. From (15) it follows that

$$\begin{aligned} \dot{n}_t &= \frac{n_t^\theta L_{Rt}}{a_N g} \\ \frac{\dot{n}_t}{n_t} n_t \frac{L_t}{L_t} &= \frac{n_t^\theta L_{Rt}}{a_N g} \\ g^2 a_N \frac{n_t^{1-\theta}}{L_t} L_t &= L_{Rt}. \end{aligned}$$

Evaluating at time $t = 0$ yields steady-state employment in innovative R&D

$$L_{R0} = g^2 a_N \delta L_0.$$

Total employment in adaptive R&D by firms of marginal cost type i is denoted by L^i_{Ft}. It is derived from the flow of products that are adapted for production in the South as a result of firms' FDI activities. From (21), I obtain

$$\begin{aligned}
\dot{n}^i_{Ft} + \dot{n}^i_{It} &= \frac{n^\theta_t L^i_{Ft}}{a_F \phi^i} \\
\frac{\dot{n}^i_{Ft} + \dot{n}^i_{It}}{n^i_X} \frac{n^i_{Xt}}{n_t} n_t \frac{L_t}{L_t} &= \frac{n^\theta_t L^i_{Ft}}{a_F \phi^i} \\
\left(\phi^i\right)^2 a_F \gamma^i_X \frac{n^{1-\theta}_t}{L_t} L_t &= L^i_{Ft}.
\end{aligned}$$

Evaluating at time $t = 0$ yields steady-state employment in adaptive R&D by firms of marginal cost type i

$$L^i_{F0} = \left(\phi^i\right)^2 \gamma^i_X \delta a_F L_0, \qquad (i = H, L).$$

Total employment in export-learning R&D by firms of marginal cost type i is denoted by L^i_{Xt}. It is derived from the flow of new products sold in the South as a consequence of export-learning activities. From (19) it follows that

$$\begin{aligned}
\dot{n}^i_{Xt} + \dot{n}^i_{Ft} + \dot{n}^i_{It} &= \frac{n^\theta_t L^i_{Xt}}{a_X \chi^i} \\
\frac{\dot{n}^i_{Xt} + \dot{n}^i_{Ft} + \dot{n}^i_{It}}{n^i_{Nt}} \frac{n^i_{Nt}}{n_t} n_t \frac{L_t}{L_t} &= \frac{n^\theta_t L^i_{Xt}}{a_X \chi^i} \\
\left(\chi^i\right)^2 a_X \gamma^i_N \frac{n^{1-\theta}_t}{L_t} L_t &= L^i_{Xt}
\end{aligned}$$

Evaluating at time $t = 0$ yields steady-state employment in export-learning R&D by firms of marginal cost type i

$$L^i_{X0} = \left(\chi^i\right)^2 a_X \gamma^i_N \delta L_0, \qquad (i = H, L).$$