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Control-Theoretic, Mission-Driven, Optimization Techniques for Wireless Sensor Networks

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***Abstract*—Network Utility Maximization (NUM) techniques, which cast resource sharing problems as one of distributed utility maximization, have been investigated for a variety of optimization problems in wireless and wired networks. Our recent work has extended the NUM framework to consider the case of resource sharing by multiple competing missions in a military-centric wireless sensor network (WSN) environment. Our enhanced NUM-based protocols provide rapid and dynamic mission-based adaptation of tactical wireless networks to support the transport of sensor data streams with very small control overhead. In particular, we focus specifically on mechanisms that capture the joint nature of mission utilities and the presence of prioritized mission demands. We then introduce a new problem, of joint utility and network lifetime maximization, as a representative of a new class of multi-metric optimization problems, and provide early evidence that techniques from optimal control theory can be used to derive distributed adaptation protocols conforming to the basic NUM paradigm. We also enumerate and motivate a list of open cross-layer dynamic adaptation problems of direct relevance to military C4I operations.**

I. Introduction

Data feeds from various sophisticated sensors (e.g., video, acoustic, shortrange radar, infra-red and magnetic) are expected to provide critical situational intelligence for a variety of missions in future military battlefield operations. In contrast to conventional civilian wireless sensor network (WSN) applications that focus on long-term, low data-rate environmental monitoring and are principally *energy-constrained*, many military missions (e.g., search and rescue, gunfire localization, insurgent tracking) are tactical in nature, lasting for relatively shorter and often well-defined durations. Given the high sampling rates and stream-oriented nature of the data from the underlying sensors, bandwidth (link capacities) is also a critical shared resource in such wireless networks–accordingly, the military WSN environment is both *energy and bandwidth constrained*.

In this paper, we investigate the use of Network Utility Maximization (NUM) based techniques for dynamic adaptation of various network parameters in such mission-centric WSN environments. The NUM problem and its distributed implementation have been extensively studied as a resource allocation mechanism for unicast flows in wireline [1], [2], wireless networks [3], and, more recently, for multicast flows in adhoc wireless scenarios [9]. The NUM paradigm, pioneered in [1] for Internet congestion control, models resource sharing among competing applications as a form of cooperative utility maximization and uses the theory of decomposition [7] to develop distributed algorithms that achieve close-to optimal performance without any global coordination. In this paper, we derive several extensions to the NUM framework to capture the following unique features of mission-oriented operations:

- *Joint Utility Functions and Multiple Mission Subscriptions*: An individual missions utility is often derived from multiple sensor sources, implying that it is not possible to articulate a missions benefit from a specific sensor independently of the data rates that it is receiving from other sensors. Conversely, multiple missions often consume the data stream from a single sensor, implying that a change in a sensor's data rate potentially alters the 'goodness' of different missions in different ways.
- *Differentiated Mission Priorities and Inelastic Demands*: Different missions may have different priorities, with some missions deemed more critical than others (e.g., gunfire localization vs. perimeter monitoring). In general, prioritized missions can be modeled by a combination of both a priority order and a lower bound on the acceptable utility. These requirements differ significantly from the basic best-effort oriented NUM framework that focuses purely on cumulative utility, and thus permits the traffic rates for some missions to become arbitrarily low, if it contributes to the collective good.
- *Utility vs. Network Lifetime Tradeoff:* In many environments, different missions have well-defined durations; in many instances, the entire WSN infrastructure may be required to be operational for a predetermined time period. Given the energy constraints on individual nodes, the instantaneous utilities of missions must be maximized in a manner that does not compromise the required operational lifetime.

Figure 1 illustrates some of these distinct features of mission-oriented WSNs. We see a high priority surveillance application that consumes streams from one video sensor and a short range radar, while a medium priority 'asset verification' mission uses data from two video sensors and a lower priority 'individual identification' mission utilizes the feed from one video and

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one audio sensor.

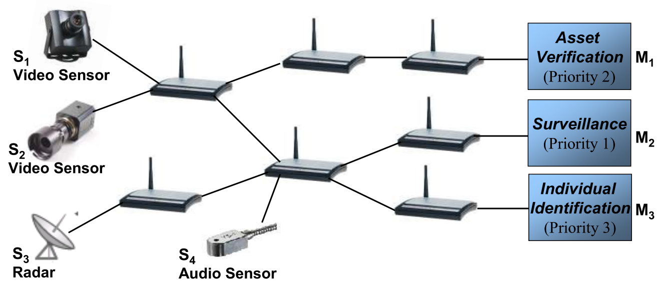

Fig. 1: An example WSN network with missions of different priorities, consuming data from multiple shared sensors.

Our primary goal in this paper is to address progressively more sophisticated and realistic resource optimization problems for mission-oriented WSN environments, and demonstrate the broad applicability of "decomposable optimization techniques" [7], embodied by NUM-based protocols, to these scenarios. More broadly, we shall also show how the generic resource optimization problems may be captured within the broader, and more general framework of "optimal control" theory. The use of optimal-control based tools allows us to address not just the much-studied problem of instantaneous 'rate' or 'congestion' control, but also a more generic class of multi-objective problems that relate to the optimal allocation of resources over a temporal horizon. Besides presenting the optimal control framework, we also discuss a set of open and relevant dynamic cross-layer optimization problems in resource-constrained mission-oriented wireless networks. We shall also discuss why suitable modifications of existing NUM protocols are likely to provide close-to-optimal solutions to these problems. From a broader perspective, our goal in this paper is to demonstrate the broad applicability, and motivate additional investigation, of NUM-based "decomposable optimization" techniques for a variety of intelligent cross-layer adaptation challenges in future WSN-based environments.

The rest of this paper is organized as follows. Section II details the modification of the basic NUM algorithms to capture the many-to-many relationships between sensor data streams and subscribing missions. Section III then considers the additional challenges that arise from the introduction of differentiated mission priorities and the specification of quasi-elastic demands. Section IV then focuses on the problem of joint utility and network lifetime optimization and demonstrates the use of an optimal control-theoretic framework for a specific, well-defined problem. Section V describes some of the open and relevant cross-layer problems that we are addressing in our ongoing work; finally, Section VI concludes the paper.

II. The Generalized NUM Framework for Joint Mission Utilities and Sensor Sharing

To mathematically abstract the mission-oriented WSN environment, consider a set of M missions operating over a set of S sensors. Let the utility function of the m^{th} mission be denoted as $U_m(X_m)$, where X_m represents the S-dimensional vector of rates associated with the set of sensors S (i.e., let $X_m[i]$ be the transmission rate of sensor s_i and $X_m[i] = 0$ if sensor s_i is not a source for mission m). Furthermore, for any mission m, let $set(m)$ be the set of sensors to which mission m subscribes; conversely, let $Miss(s)$ be the set of missions subscribing to the data stream from sensor s. Also, let K denote the set of all (source, sink and forwarding) nodes in the network.

The capacity constraint in wireless environments needs to reflect the fact that the wireless channel is shared and that a node can transmit its data only when there are no *interfering* transmissions. To express these interference constraints, we characterize the link-layer broadcast transmission by a node k to its child nodes by the tuple (k, s), where s is the sensor source node for this flow; a sensor flow is thus a series of transmissions by the nodes in the corresponding multicast forwarding tree. The forwarding tree is itself assumed to be created by some external routing protocol. Given this model, the capacity constraint is shown in [10] to apply to each maximal clique (denoted by l) in the conflict graph (CG) formed from the underlying set of transmissions, and can be expressed as:

$$\sum_{\forall (k,s) \in l} \frac{x_s}{c_{k,s}} \leq 1, \tag{1}$$

where x_s is the flow rate, $c_{k,s}$ is the transmission rate for transmission (k, s) and L is the set of all maximal cliques in the CG corresponding to the WSN.

The problem of mission-oriented adaptation of the sensor data rates in the WSN may then be expressed by the optimization problem $SENSOR$ as:
$SENSOR(U; L)$:

$$\text{maximize} \sum_{m \in M} U_m(X_m) \tag{2}$$
$$\text{subject to} \sum_{\forall (k,s) \in l} \frac{x_s}{c_{k,s}} \leq 1, \;\; \forall l \in L.$$

By taking the Lagrangian of this constrained problem and setting the partial derivatives to 0, we can then show that if

- the *sensor* node s_i adjusts it rate according to the equation

$$\frac{d}{dt} x_{s_i}(t) = \kappa\Big(\sum_{m \in Miss(s_i)} w_{ms_i}(t) - x_{s_i}(t) * \sum_{\forall l \in flow(s_i)} \mu_l(t) * \sum_{\forall (k,s_i) \in l} \frac{1}{c_{k,s_i}} \Big) \tag{3}$$

  where μ_l is the 'cost' charged per bit by each forwarding clique, and is given by $\mu_l(t) =$

$$p_l\Big(\sum_{\forall (k,s) \in l} \frac{x_s(t)}{c_{k,s}}\Big) = \Big(\sum_{\forall (k,s) \in l} \frac{x_s(t)}{c_{k,s}} - 1 + \epsilon\Big)^+ / \epsilon^2 \tag{4}$$

- each *mission* adjust's its so-called 'willingness to pay' value for each subscribed sensor as:

$$w_{ms}(t) = x_s(t) \frac{\partial U_m}{\partial x_s} \tag{5}$$

then the distributed and iterative rate adjustment scheme will converge to rates that optimize a relaxation of the problem $SENSOR(U; L)$.

A. Protocol Version of the NUM Algorithm

The above NUM algorithm can be adapted into a distributed dynamic rate adaptation protocol, called *WSN-NUM* that requires minor modifications at the sensor and sink (mission) nodes and somewhat more sophisticated functionality at intermediate (forwarding) nodes. Each mission has to periodically compute the partial derivatives of its utility with respect to the individual sensor rates and transmits these w_{ms} terms back as feedback signals to the corresponding sensors. Each sensor, in turn, collects the feedback terms from each of its subscribed sensors, computes the cumulative 'willingness to pay' across all its missions and then adapts its transmission rate according to Equation 3. To enable individual missions to compute the w_{ms} terms, the sensor must piggyback its instantaneous data rate on a fraction of its data packets. While Equation 3 specifies a continuous adaptation process, the practical protocol operates by having the sensor s_i collect the w_{ms} terms over a relatively small 'time window' and then perform a 'step adjustment' of its transmission rate; this transmission rate is then held constant until the end of the next time window. In particular, it has been demonstrated that this discrete version of the NUM algorithm will converge to the global optimum even in the presence of different round-trip delays in the feedback provided by individual missions.

Intermediate forwarding nodes must perform several additional functions. First, each node has to compute the maximal cliques to which it belongs-this is computed using the corrected Bierstone algorithm applied to a local conflict graph, which itself is computed by exchanging information about the set of all distinct (k, s) transmissions within its $r + 1$ neighborhood (where the interference range is assumed to be r times the transmission range). Second, to compute the shadow prices[1] associated with each clique, the nodes that have transmissions belonging to the same clique must exchange their current airtime fractions, i.e., $\frac{x_s}{c_{k,s}}$ at a time scale faster than that employed for rate adaptation by each sensor. Finally, to avoid duplicate and incorrect counting of the shadow price, each forwarding node must first divide the total cumulative shadow prices (from the upstream nodes) by the number of downstream child nodes on the forwarding tree, and also attach a unique clique ID (to ensure that downstream nodes belonging to the same clique do not repeat the same price). Table I summarizes the principal modifications required in our WSN-NUM protocol, compared to the unicast oriented base NUM protocol. Note that the WSN-NUM protocol described here assumes a completely *cooperative* environment, where all nodes accurately exchange and compute their shadow prices and individual sensors use Equation 3 to adjust their data rates. The general problem of NUM-oriented adaptation in the face of inaccurate or partially accurate information or rate adaptation is an open problem, beyond the scope of this paper.

[1]The term 'shadow price' refers to the price per bit computed in Equation 4 and implicitly reflects a per-bit resource consumption charge that typically increases rapidly as the consumption rate (in this case, the bandwidth utilization) approaches the capacity constraint.

Node	Prior Unicast Adaptation	Modified WSN-NUM Protocol
Source Node	performs rate and w adaptation	performs only rate adaptation based on sum of w_{ms} values
Sink Node (Mission)	echoes back costs	computes w_{ms}; echoes back cost and w_{ms}
Intermediate Forwarding Node	adds link cost(s) before relaying	adds and splits clique's cost before broadcasting to multiple downstream neighbors; computes maximal cliques

TABLE I: **Key Differences in Distributed Adaptation Procedure**

B. WSN-NUM Performance and Convergence

Simulation experiments performed using an implementation of the WSN-NUM protocol on the Qualnet discrete-event simulator, and using an 802.11b MAC protocol, demonstrate that the algorithm can converge close to the theoretical optimum, with only a very modest signaling overhead. Figure 2 shows the evolution of the cumulative system utility for a randomly distributed topology consisting of 50 nodes; Figures 3 and 4 show the corresponding signaling packet overhead and the mean end-to-end flow latency and packet delivery ratios achieved by our WSN-NUM protocol. It is easy to see that the WSN-NUM protocol can dynamically adjust the data rates of individual sensor streams and ensure that the packet delivery rate (even without a reliable MAC layer) stays above $80-90\%$.

III. Dynamic Adaptation with Prioritized Missions

The basic NUM adaptation strategy focuses only on maximizing the cumulative utility, i.e., there is no notion of priorities across the missions and it is acceptable to provide arbitrarily small rates (and low utility) to one or more missions, if that contributes to the "collective good". In many military scenarios, there is often a clear notion of relative priority assigned to missions, with the priority values used for arbitrating across missions in the face of unplanned or unexpected resource shortages. To incorporate the notion of priorities, we must also introduce the notion of a minimum-acceptable demand; accordingly, we now model each prioritized mission through a *priority* order and a quasi-elastic demand function (as the mission will always benefit from sensor data rates that exceed its bare minimum requirements). Thus, a prioritized mission m is associated with a priority value PO_m and a minimal demand function that is represented by the function $f_m(set(m)) \geq D_m$, where D_m is a scalar constant. This formulation effectively specifies an $|set(m)|$-dimensional surface (where $|set(m)|$ is the number of sensors used by the mission) in the utility space, demarcating the unsatisfied region, corresponding to the various combinations of rates from the $set(m)$ sensors that satisfy the minimal utility; see Figure 5 for an example. The modified NUM problem can then be defined as: $SENSOR - P(U; L)$:

$$\begin{aligned} &\textbf{maximize} && \textstyle\sum_{m \in M} U_m(X_m) \textbf{ over } x_s \geq 0 \\ &\textbf{subject to } \text{i)} && \textstyle\sum_{\forall (k,s) \in l} \frac{x_s}{c_{k,s}} \leq 1, \textbf{ and} \\ &\text{ii)} && f_m(X) \geq D_m, \ \forall m \in \{1, \ldots, H\} \end{aligned} \quad (6)$$

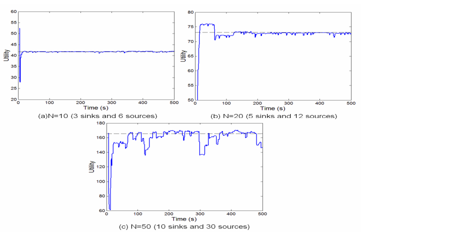

Fig. 2: Evolution of total network utility ($N = 50$)

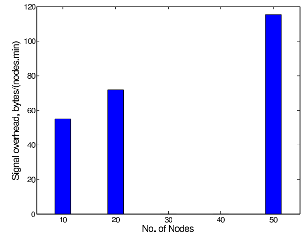

Fig. 3: Total packet overhead/node/minute (bytes) vs. network size

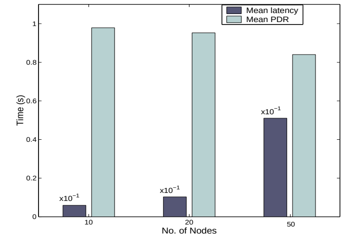

Fig. 4: Average packet delivery ratio (PDR) and latency vs. network size

where $f_m(X)$ denotes the minimum demand of a prioritized mission in terms of its utility and H is the total number of such 'prioritized' missions (let HP denote the set of such prioritized missions).

A. Solving the Prioritized NUM Problem For Feasible Scenarios

Using the basic principle of decomposable optimization, the SENSOR-P(U;L) problem can be decomposed into the following independent $SINK$ and $NETWORK$ problems. $SINK_m(U_m; \lambda_m)$:

$$\textbf{maximize } U_m(\frac{\bar{w}_m}{\bar{\lambda}_m}) - (\sum_{s \in set(m)} w_{ms}) \textbf{ over } w_{ms} > 0 \quad (7)$$

where $\bar{w}_m$ is a vector of terms w_{ms}, $\bar{\lambda}_m$ is a vector of λ_{ms}, and element-wise division of $\bar{w}_m$ by $\bar{\lambda}_m$ is assumed. $NETWORK(L; w)$:

$$\begin{aligned} \textbf{maximize} \quad & \textstyle\sum_{s \in S} \sum_{m \in M} w_{ms} log(x_s) \textbf{ over } x_s \geq 0 \\ \textbf{subject to i)} \quad & \textstyle\sum_{\forall (k,s) \in l} \frac{x_s}{c_{k,s}} \leq 1, \text{ for each clique } l \in L, \\ \textbf{and ii)} \quad & f_m(X) \geq D_m, \ \forall m \in \{1, \ldots, H\} \quad (8) \end{aligned}$$

Note the absence of the priority values PO_i in this specification–this arises from the assumption that all mission demands are collectively feasible; the PO_i values become relevant in determining the set of missions only in cases where the network can satisfy only a partial set of the mission demands.

By taking the Lagrangian of this joint problem and setting the first-order derivatives to 0, we can now show that (under the assumption that the set of demands is collectively feasible) if

- An individual sensor s continues to adapt its rate according to Equation 3
- Each mission computes its 'willigness to pay' according to the modified 'η-corrected' equation:

$$w_{ms}(t) = x_s(t) \frac{\partial U_m}{\partial x_s} + x_s(t) * \eta_{ms} * \left(\frac{\partial f_m(X)}{\partial x_s} \right), \quad (9)$$

where η_{ms} is an additional'payment term' computed according to:

$$\begin{aligned} \eta_{ms} &= (1 - f_m(X(t))/D_m)^+ / \epsilon^p \text{ if } f_m(X) < D_m \\ &= 0 \text{ otherwise,} \quad (10) \end{aligned}$$

then this iterative rate adjustment scheme will converge to rates that optimize a relaxation of the problem $SENSOR - P(U; L)$. (Here p is a scaling coefficient that determines the sensitivity of the gradient ascent algorithm.) The 'additional payment term' η_{ms} is non-zero only when the mission's current utility is lower than its minimal specified demand D_m; a non-zero value of η_{ms} effectively causes the sensor rate adjustment (via Equation 3) to have an additional 'positive bias' towards a higher rate (to drive the subscribed mission's utility to a higher, minimally acceptable, value). The important point to note here is that this relatively more-complicated optimization objective has been met through a simple modification (localized only to each receiving mission) of the distributed WSN-NUM protocol.

However, this simple adjustment in the adaptation scheme does not, by itself, guarantee that all prioritized missions will have their minimum demands satisfied. However, if we set an upper bound N_{max} on the number of missions that the system can admit, and restrict the maximum slope of a mission's utility function (which occurs at the origin, due to the concavity of utility functions) to an arbitrary value S_{max}, we can ensure that all the prioritized missions will have their minimal demand met (to within ϵ) by ensuring that:

$$\frac{1}{\epsilon^{p-1}} > N_{max} * S_{max}. \quad (11)$$

Intuitively, this condition ensures that the marginal cost for a priority flow receiving utility that is more than ϵ lower than its minimum demand is always greater than the marginal utility of all other missions. In practice, given N_{max} and S_{max}, we set $\epsilon = \frac{1}{N_{max} * S_{max}}^{\frac{1}{p-1}}$.

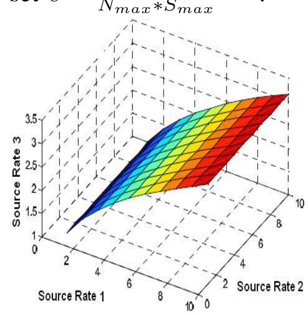

Fig. 5: Surface demarcating feasible and infeasible regions for the minimum utility demand of a prioritized mission.

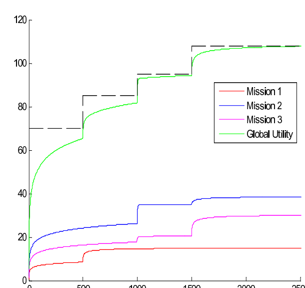

Fig. 6: Utility of three missions in a network, with demands at different times

B. Protocol Performance and More Complex Situations

To demonstrate our modified NUM framework, we simulated the operation of the modified WSN-NUM-P protocol over a 20-node wireless sensor network with random topology. There are 6 sources and 10 missions of which m_1,

m_2, m_3 are prioritized. The utility functions are given by $U_{m1}(x1, x2) = log(1+x1)+log(1+x2)$, $U_{m2}(x1, x3, x5) = log(1 + x1) + log(1 + x2) + log(1 + x3)$ and $U_{m3}(x3, 4) = log(1 + x3) + log(1 + x4)$. At time t=500, m_1 demands a minimum utility of 15; at t=1000, m_2 demands 35; at t=1500, m_3 demands 30. The utilities of the three missions over time is shown in Fig. 6. We see that when the demands arrive, the utilities are adjusted to meet the minimal demands. Moreover, we can see that for any network configuration, the global utility always approaches the optimal utility (shown in dotted lines).

The above formulation assumes that the set of demands made by the set of prioritized missions are inherently feasible. However, there may be cases when impairments in the underlying network cause a violation of this assumption–in such cases, the NUM protocols must be modified to find the 'optimal' set of satisfied flows SF ($SF \subset HP : m_i \in SF$ iff $f_i(X) > D_i$). In this case, optimality is defined by the following properties: (i) *Priority Property*: A mission m_i with priority PO_i cannot be in the set SF if removing this mission from SF (i.e., reducing its utility below its minimum demand) enables a higher priority mission, $P_j : P_j > P_i$, not in set SF, to become a member of SF, and (ii) *Utility Property*: Given the set SF, the set of sensor rates chosen, $\{x_s\}$, maximizes system utility (subject to capacity constraints) and meets the minimum demand for all $m_i \in SF$. This is a considerably more complex problem, requiring more elaborate modification to the base WSN-NUM protocols. However, as detailed in [11], our initial results suggest that heuristic modifications to the distributed protocols enable us to achieve close-to-optimal performance, with only modestly higher signaling overhead.

IV. Balancing Instantaneous Utility and Lifetime Objectives

We now turn our attention to a new problem–that of jointly maximizing the cumulative system utility and the operational lifetime of the WSN. This is a departure from the prior body of NUM work, in that our objective is now *multi-dimensional*. Indeed, the conventional NUM protocols focus on the adaptation of the current source data rates (and optionally, the link capacities) to maximize the *instantaneous* system utility; such *greedy* optimization might result in rapid depletion of the residual energy on the intermediate network nodes and lead to unsatisfactorily small operational lifetime. In contrast, a multi-dimensional objective helps capture the presence of both bandwidth and lifetime (or energy) constraints more accurately. We show that such constraints can be captured by the general "optimal control" framework, where the problem of determining the instantaneous rates is equivalent to selecting, not just instantaneous sensor data rates, but rather a *functional* (or time-indexed function) of the sensor data rates. This type of multi-dimensional optimization is applicable to a variety of military problems, e.g., determining the maximum amount of video feeds that can be transmitted from land-based sensors, given that a mission has to last for 48 hours.

A. Review of Optimal Control

The basic theory for optimizing the mission utility under a lifetime objective comes from Optimal Control, a mathematical framework used for determining adaptive behavior of a system over time. It has wide applications in modeling dynamic engineering systems and also in economics [15]. The general form of the objective function of an inter-temporal optimization problem can be written as:

$$\text{maximize} \int_{t_0}^{T} f(s(t), c(t), t)dt \tag{12}$$

$$\text{subject to } \frac{ds}{dt} = g(s(t), c(t), t) \tag{13}$$

$$\text{with } s(t_0) = s_0 \text{ and } s(T) = s_T$$

Here $s(t)$ is a state variable and $c(t)$ is a control variable. The constraint in Eq.(13) denotes the rate of change of the state of the system. By taking Lagrangian and simplifying using integration by parts, we get

$$J = \int_{t_0}^{T} [f(s, c, t)+\psi g(s, c, t)+s\frac{d\psi}{dt}]dt-\psi(T)s(T)+\psi(t_0)s(t_0) \tag{14}$$

By using the first-order necessary condition for optimality ($\frac{dJ}{dt} = 0$), we can show that the optimal solution satisfies the relationships:

$$H_c = f_c + \psi g_c = 0 \tag{15}$$

$$\frac{d\psi}{dt} = -H_s = -(f_s + \psi g_s) \tag{16}$$

where $H = f(s, c, t) + \psi(t)g(s, c, t)$ and is referred to as the Hamiltonian function. Equations (15) and (16) are both necessary and sufficient for optimization over time [16].

B. Application of Optimal Control-based NUM Adaptation in Mission-oriented WSNs

As an easy-to-understand but representative example of the application of this technique to mission-oriented WSN environments, we consider the problem of utility optimization of a set of missions over a pre-defined network operational lifetime. We consider the scenario where the number of missions M remains fixed throughout a time interval T, and the goal is to continuously regulate the individual sensor rates to maximize the global utility over this *entire* network lifetime. We assume that a node k in the network has an initial, non-renewable battery capacity E_k; furthermore, as communication costs typically dominate computing costs, we assume that the transmission and reception energy for one bit of data by node k is α_t^k and α_r^k units, respectively. Our objective function can then be framed as follows: $SENSOR - LIFE(U; L)$:

$$\text{maximize} \int_{0}^{T} \sum_{\forall m \in M} U_m(x_{s:s \in set(m)}(t))dt \tag{17}$$

subject to

$$\textbf{Energy} \text{ (i)} \quad \frac{dp_k}{dt} = -(\sum_{\forall i \in InFlows(k)} \alpha_r^k x_i(t) + \sum_{\forall i \in OutFlows(k)} \alpha_t^k x_i(t) = -\alpha_k \sum_{\forall i \in Flows(k)} x_i(t) \forall\, k \in K, \tag{18}$$

$$\textbf{Capacity (ii)} \sum_{\forall(k,s)\in l} \frac{x_s(t)}{C_{ks}} \leq 1 \quad \forall t \;\; \forall l \in L \tag{19}$$

$$\text{(iii) } p_k(T) \geq 0 \;\; \forall \text{nodes k}, \tag{20}$$

where $p_k(t)$ is the residual battery in node k at time t, $p_k(0) = E_k$ and $\alpha_k = \alpha_r^k + \alpha_t^k$. The constraint in Eq.(18) models the consumption of battery power at each node, while Equation (19) reflects the bandwidth constraints of the network.

The Hamiltonian for the problem in Eq.(17)-(20) is given as:

$$H = \sum_{\forall m\in M} U_m(x_{s:s\in set(m)}) - \sum_{\forall l\in L} \mu_l \sum_{\forall(k,s)\in l} (\frac{x_s}{C_{ks}} - 1) -$$
$$\sum_{\forall k\in K} \psi_k \alpha_k \sum_{\forall i\in Flows(k)} x_i$$

where μ_q and ψ_k are now (time-dependent) Lagrangian multipliers.

The necessary and sufficient conditions for optimization then reduce to:

$$(i) H_{x_s} = \sum_{\forall m\in M} \frac{\partial U_m}{\partial x_s} - \sum_{\forall l\in L} \mu_l \sum_{(k,i)\in l} \frac{1}{C_{ki}}$$
$$- \sum_{\forall k\in Path(s)} \psi_k \alpha_k = 0 \text{ and} \tag{21}$$
$$(ii) \frac{d\psi_k}{dt} = -H_{p_k} = 0 \tag{22}$$

Differentiating Eq.(21) and using Eq.(22), we get $\frac{dx_s}{dt} = 0$. Similarly, we can also derive that $\frac{d\psi_k}{dt} = 0$. This implies that the marginal cost associated with the depletion of a node's battery resources stays constant over time. Taken together, these relationships indicate that the optimal strategy is *for the sensors to transmit data at a constant rate over the entire duration T*, subject to the constraints that the rates do not collectively violate either the capacity or power depletion constraints. Accordingly, for this specific problem, we can show that if:

- each individual *forwarding clique l* and *forwarding node k* computes its capacity shadow cost μ_l and 'power' shadow cost ν_k according to (with the time index for both of these costs being implicitly dropped):

$$\mu_l = (\sum_{\forall(k,s)\in l} \frac{x_s(t)}{c_{k,s}} - 1 + \epsilon_1)^+/\epsilon^2$$
$$\nu_k = (\alpha_k * \sum_{\forall s\in Path(k)} x_s(t) - \frac{E_k}{T} + \epsilon_2)^+/\epsilon_2^2$$

  where ϵ_1 and ϵ_2 are arbitrarily small constants,
- each *sensor* node s_i adjusts it rate according to the equation

$$\frac{d}{dt} x_{s_i}(t) = \kappa(\sum_{m\in Miss(s_i)} w_{ms_i}(t) - x_{s_i}(t) * \{$$
$$\sum_{\forall l\in flow(s_i)} \mu_l(t) * \sum_{\forall(k,s_i)\in l} \frac{1}{c_{k,s_i}} + \sum_{\forall k\in Path(s_i)} \nu_k(t)\}) \tag{23}$$

- each *mission* adjust's its so-called 'willingness to pay' value for each subscribed sensor as:

$$w_{ms}(t) = x_s(t) \frac{\partial U_m}{\partial x_s} \tag{24}$$

then the distributed and iterative rate adjustment scheme converges to a relaxation of the problem $SENSOR-LIFE(U; L)$.

The important point here is that the sensor data rates are controlled by the more-restrictive of the capacity or power consumption constraints at each node. While the above adaptation is fairly simple, the basic dual-layer NUM adaptation strategy described here is applicable to a much broader set of problems. For example, optimal-control based NUM adaptation can be used to optimize other, more complex scenarios, such as when the network has replenishable battery or when missions arrive and depart at unpredictable instants (but can be described through statistical arrival and departure processes).

V. Open and Ongoing Problems

The NUM-related modifications described in the previous sections demonstrate the ability to solve many complex resource optimization problems via the principle of decomposable optimization. However, each of the three problems described before relate largely to the control of 'source data' rates and thus correspond to various adaptations performed purely at the *network layer*. We now describe a set of ongoing research activities related to more sophisticated cross-layer optimization problems of relevance to mission-centric military operations, especially in resource-constrained battlefield environments.

- *Joint Allocation of Sensor Assets and Rate Control:* In the problem formulations described here, the set of sensors assigned to each mission (i.e., $set(m)$) and the underlying routes for the data flows are assumed to be provided *a priori*. In practice, the adaptive rate control techniques described here are preceded by a separate *sensor assignment* phase, where the sensor assets are assigned or *matched* to both static and dynamic mission demands. When sensor data are consumed by multiple missions, the assignment process should, in general, arbitrate both the competition among contending missions for common sensor resources *and* the (bandwidth,lifetime) constraints associated with the wireless transport network. So far, the practice, however, has largely been to solve these two aspects of mission-based network adaptation independently–in particular, the sensor-mission matching activity typically does not consider the bandwidth constraints in the transport network.
  Given the desire to avoid selecting sensors whose traffic flows over congested paths, significant improvements in overall system utility can be expected if we can jointly optimize both the set of sensors $set(m)$ assigned to a mission and the data rates of the corresponding sensor data streams. In a formal way, the problem may be

expressed as $SENSOR - JOINT(U; L)$:

$$\textbf{maximize} \sum_{m \in M} U_m(x_s \in A(S)_m) \textbf{ over } x_s \geq 0$$

$$\textbf{subject to i)} \sum_{\forall(k,s) \in l} \frac{x_s}{c_{k,s}} \leq 1 \quad \forall l \in L$$

$$\text{ii) } A(S)_m \subset Req(M) \ \ \forall m \in \{1, \ldots, M\} \tag{25}$$

where $A(X)$ is viewed as a vector of projections fromt the set of sensors S, such that $A(S)_m$ (the m^{th} row of the vector) corresponds to the set of sensors $set(m)$ to which mission m subscribes, and $Req(m)$ denotes the specification of the number (and type) of sensors required by mission m. Note that this problem is, in general, a mixed-integer problem (as the decision of including a sensor in $set(m)$ is an integral one), and is NP-Hard. However, it is very likely that smart, iterative heuristics can be formulated that preserve the decentralized operation of the NUM protocols and that significantly improve the overall utility of the network. An early and limited example of this is available in [14], which considers the more limited problem of jointly assigning each sensor to a unique single mission and in maximizing the rate-based utility of each mission. Results in [14] show that a distributed but coupled optimization heuristic can result in $\approx 15\%$ improvement in overall utility, and provide early evidence that this form of joint optimization can result in significantly higher mission utilities.

- *In-Network Sensor Fusion and Computational Energy Constraints:* In-network processing has been widely proposed (e.g., [4]) as a fundamental technique for mitigating the communication load incurred in transporting sensor data over energy-constrained wireless networks. In-network processing or fusion typically reduces the volume of data transmitted to the sink at the expense of a potential loss of information that occurs during the aggregation process. For many *stream-oriented* military mission scenarios, where the data comes from a variety of relatively high data-rate streaming sources (e.g., video or radar feeds), in-network processing comprises relatively sophisticated operations (e.g., MPEG compression or wavelet transformations). Due to the non-negligble energy footprint of in-network computation for such applications, an interesting problem is to determine both the *degree* and *location* of such in-network operations that help to maximize the cumulative utility, subject to both wireless bandwidth and energy constraints. While limited past work (e.g., [6], [8]) has considered the presence of simultaneous energy and communication constraints, they do not consider the possibility of *varying the level or quality of the in-network processing* performed to adapt to these constraints. In many applications, however, opportunities exist for either variable *compression* (of a single stream; e.g. variable compression of MPEG streams) or *fusion* (of multiple streams; e.g. mixing of multiple video streams), resulting in a non-linear tradeoff between computation and communication energy, with corresponding effect on the overall mission utility.
For the relatively simple case where the location of the fusion operators is *fixed*, this problem can be expressed as $SENSOR - INP(U; C; P)$:

$$\textbf{maximize} \sum_{m \in M} U_m(\{x_s^{rec}\}_{s \in set(m)}) - \delta \sum_{\forall nodes, k} P_{tot}^k. \tag{26}$$

subject to

i) **Capacity Constraint:**

$$\sum_{\forall(k,i) \in l} \frac{x_{out}(i,k)}{c_{ki}} \leq 1, \forall l \in L \tag{27}$$

ii) **Energy Constraint:** $P_{tot}^k \leq P_{max}^k, \forall k \in K$ (28)

where $P_{tot}^k = P_{rec}^k + P_{trans}^k + P_{comp}^k$,

$0 \leq \delta \leq 1$ **and** $x_i, x_{out} \geq 0 \forall i$

where x_s^{rec} is the data rate from sensor s received by mission m and P_{tot}^k is a threshold on the maximum power that can be consumed by node k. Clearly, the NUM protocol must now be enhanced to capture the fact that the rate received by a mission may differ from the sensor's source rate, due to variable compression or fusion performed by nodes on the forwarding path.

- *Mission-Interdependence and Adaptive Selection:* The existing NUM models assume that each mission is independent of all others; accordingly, the cumulative utility of the set of missions is simply the sum of the utilities of each individual mission. While this abstraction holds for most conventional applications, it may not be strictly true for military mission-oriented environments, where missions may exhibit several different types of interdependence. For example, it is possible that missions may be deployed in redundant configurations (e.g. it may be OK for two out of 3 gunfire monitoring applications to receive their sensor feeds) or be viable only in groups (e.g. a mission monitoring the adversary's movement may not be useful unless an alternate mission monitoring the location of friendly artillery assets receives adequate data). For such scenarios, the basic NUM formulation needs to be modified to capture the fact that the utilities are not additive, but exhibit more complex relationships. It is likely that the non-linear nature of the relationships will make exact computation of the optimal sensor rates an NP-hard problem. Accordingly, we feel that the most promising way to view such scenarios is to model them as cross-layer optimization problems, with one layer focusing on the best selection of sensor data rates for a set of 'active' missions and another higher layer dynamically controlling the activation and deactivation of missions. The goal in these cases is not to achieve the global optimum, but use a set of coupled iterative loops to achieve significantly better mission utilities than those achieved without considering these inter-dependencies. Such cross-layer NUM-based optimizations have been explored for a limited set of optimization problems–e.g. for the joint optimization [12] of source rates and node transmission powers (for modulating link capacity) for unicast flows. In general, most cross-layer techniques have, however, been limited to the joint optimization of variables within a single 'layer' (e.g., within the communications network.) We believe that the principle of decomposable optimization can be used for a much richer set of cross-layer optimization problems, for example, when one control parameter lies in the communications layer (e.g., sensor transmission rate) and the other control parameter lies in the information layer (e.g., the choice of multiple locations where sensor data will be cached for efficient search.)

- *NUM Sensitivity to Incorrect Information:* Besides application-specific adaptations of NUM, an important open question relates to the fundamental stability and performance of the NUM-based framework in the face of incorrect, missing or inaccurate information. In military environments, it is likely that one or more individual nodes may be compromised, causing the overall NUM protocol to deviate from the optimal. For example, an individual forwarding node may report abnormally high shadow costs, thereby throttling the data rates of sensors whose data dissemination paths traverse this node. Accordingly, practical deployment of NUM-based protocols in tactical military networks requires the development of both adequate *distributed monitoring mechanisms* (so that nodes can detect potential incorrect behavior by neighboring nodes) and corresponding *modifications to the adaptation protocols* (so that the nodes can counteract such faulty behavior). In addition to deliberately malicious behavior, NUM-based protocols may also exhibit performance degradation simply due to erroneous or missing feedback (e.g., if clique congestion messages are lost). An early study of the sensitivity of the basic NUM protocol in [17] shows how noisy feedback can slow the rate of NUM convergence and how the use of multiple NUM-loops can be more susceptible to noisy feedback. Further research is required to understand and characterize the sensitivity of WSN-NUM protocols, where sensor data streams are typically multicast to and consumed by multiple sinks (missions) and where different adaptation loops often operate over widely differing time scales.

VI. Conclusions

This paper attempts to establish the wide applicability of the Network Utility Maximization (NUM) optimization technique as a tool for decentralized resource sharing in mission-based wireless sensor networks. By framing the requirements of individual missions as, potentially quasi-elastic, utility functions, the problem of resource sharing can be viewed as a decentralized optimization problem and solved through relatively low-overhead localized signaling among the wireless network nodes. In this paper, we have first shown the modifications to the basic NUM strategy required to tackle two unique features of missions that utilize a common set of sensor data streams: i) the use by a mission of data from multiple sensor data streams, and ii) the specification of strict priorities across missions with quasi-elastic demand constraints. Our simulation studies show that the modified NUM protocols can achieve close-to-optimal utility with only modest signaling overheads of $\approx$ 100s of bytes/min at each node. In addition, we have provided initial evidence of how more complex objectives of joint utility and lifetime maximization can be formulated as an optimal control problem, and how the optimal behavior can be achieved through NUM-based adaptive feedback.

Looking forward, we believe that this form of decoupled optimization may be used to develop robust and quick-reacting resource sharing protocols for a variety of problems in mission-based operating environments. A particularly interesting application of this technique may be in optimization of resources 'across' different networks–for example, for finding a good combination of active missions, sensor-mission assignments and sensor data rates that satisfy the constraints in both the communications and information networks.

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