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Essays on new business models in operations

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Essays on New Business Models in Operations

by

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I hereby declare that this dissertation is my original work
and it has been written by me in its entirety.
I have duly acknowledged all the sources of information
which have been used in this dissertation.

This dissertation has also not been submitted for any degree
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LU, LI LING

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Abstract

This dissertation consists of three essays about problems of managing operations with emerging new business models that are broadly related to anti-counterfeiting, car subscription programs, and on-demand ride-hailing services. In the following three chapters, each studies one type of new business model with opportunities and challenges, and builds analytical models to explore the implications on firms' operational decisions.

Chapter 2 studies the emergence of "super fakes," and investigates the effectiveness of the new anti-counterfeiting measure — converting counterfeiters to authorized suppliers. We employ a game-theoretic model to examine interactions between a brand-name firm with its home supplier, and a counterfeiter who produces high-quality counterfeits and can be potentially converted to an authorized overseas supplier. We demonstrate that it is easier for the brand-name firm to combat counterfeiting through conversion than by driving the counterfeiter out of the market. We examine the impact of this new measure on consumer and social surplus, and find that it may hurt consumer surplus and does not always improve social surplus.

Chapter 3 studies flexible versus dedicated technology choice and capacity investment decision of a two-product manufacturing firm under demand uncertainty in the presence of subscription programs. The key feature of subscription programs is that a proportion of customers that are allocated a particular product later switches to using the other product (if available). We

build a two-stage stochastic program to study the optimal technology choice and capacity investment decision, and the subsequent product allocation and reservation for each product. We investigate how the demand correlation and the switching proportion affect the profitability with each technology, and shape the optimal technology choice decision.

Chapter 4 studies an on-demand ride-hailing platform partnering with traditional taxi companies for expanding the supply of drivers, and the government's regulation problem of access control of taxi drivers to on-demand ride-hailing requests under such emerging partnership. We examine the conditions under which taxi drivers participate in providing both street-hailing and on-demand ride-hailing services. We investigate whether and how the government should make regulatory decisions to maximize social welfare. We find that advocating their partnership by allowing taxi drivers to get "full access" to the platform may not be optimal and the regulation is needed.

Contents

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To my grandpa.

Chapter 1

Introduction

The rapid development of manufacturing/service capabilities and the trend of applying digital platforms not only greatly affect traditional business models, but also provide lots of opportunities and challenges to firms with new business models in different industries. In this dissertation, I study the problems of managing the operations with emerging new business models that are broadly related to anti-counterfeiting, car subscription programs, on-demand ride-hailing services, with a focus on the supply side.

Chapter 2 is about high-quality counterfeits in emerging markets. With more products being infringed by counterfeits, anti-counterfeiting measures, such as law enforcement and consumer education, have been commonly adopted in emerging markets. In recent years, "super fakes," *i.e.,* counterfeits with high quality, have become popular. Super fake manufacturers' capability to produce high-quality products inspires a new anti-counterfeiting measure, *converting counterfeiters to authorized suppliers.* To study the effectiveness of this anti-counterfeiting measure, our paper employs a game-theoretic model to examine the interactions between a brand-name firm with its home supplier, and a counterfeiter who produces high-quality counterfeits and can be potentially converted to an authorized overseas supplier. Our results show that when the difference in production costs between the two suppliers and the discount factor of using the overseas supplier are low, the brand-name firm may not have the incentive to convert the counterfeiter due to the limited cost saving and significant brand value loss. Otherwise, the brand-name firm has the incentive to convert the counterfeiter through either dual sourcing or single sourcing. However, the brand-name firm may still fail to do so when the wholesale price required for the conversion is too high because of a large overseas market size, a low penalty from law enforcement, or a high perceived quality of the counterfeit. We demonstrate that it is easier for the brand-name firm to combat counterfeiting through conversion than by driving the counterfeiter out of the market. We also examine the impact of this anti-counterfeiting measure on consumer and social surplus, and find that it may hurt consumer surplus and does not always improve social surplus.

Chapter 3 is about subscription programs in automotive markets. We studies flexible versus dedicated technology choice and capacity investment decision of a two-product manufacturing firm under demand uncertainty in the presence of subscription programs. The firm offers two subscription programs, each one starts with allocation of one of the products. The key feature of the subscription program is that a proportion of customers that are allocated a particular product later switches to using the other product (if available). We build a two-stage stochastic program to study the optimal flexible vs. dedicated technology choice and the capacity investment decision, and the subsequent product allocation and reservation decisions for each product. We investigate how the correlation between the two subscription demands affect the profitability with each technology, and shape the optimal technology choice decision. Our result shows that with dedicated technology, a higher demand correlation increases the profitability, which is novel and different from the traditional ownership model as it is well known that the demand correlation does not affect the profitability. However, with flexible technology, the profitability could be no-monotone with the demand correlation, and only under some conditions, the demand correlation positively affects the profitability. Interestingly, we find that even when the demand correlation is not perfectly positively correlated, it is possible that there is no value of flexible technology, which is not possible under a traditional ownership model where flexible technology always has a capacity-pooling benefit. We calibrate our model with the public available data and based on the calibration, we find that paralleling with the literature, in the presence of subscription programs, a higher demand correlation always favours the dedicated technology adoption. Managerially, our results underline that in the presence of subscription programs, firms should manage technology adoption together with future switching requests, which shapes capacity flexibility.

Chapter 4 is about the ride service market with labor shortage, and investigate the emerging partnership between on-demand ride-hailing platforms and traditional taxi companies. We develop a game-theoretical model of an on-demand ride-hailing platform who may expand its supply pool with private car drivers, by forming a partnership with traditional taxi company to connect taxi drivers into its platform, and examine whether and how the government regulates their partnership by controlling taxi drivers' access level to the ride-hailing platform to maximize social welfare. We find that when the allowed maximum access level of taxi drivers is high enough, it is optimal for the ride-hailing platform to offer a high wage compensation to attract both taxi drivers and private car drivers to provide on-demand ride-hailing services. In particular, as the number of private car drivers increases, taxi drivers are less likely to serve ride-hailing requests with "mixed" service mode. Interestingly, we also find that advocating the partnership by enabling taxi drivers to get "full access" to the online platform may not be optimal for the government. Without a restriction on street-hailing availability, "partial access" is optimal if the labor welfare becomes more important, and the number of private car drivers is moderate. With a restriction on street-hailing availability, "partial access" is optimal when the number of private car drivers is not high, and it becomes less possible as the restriction level increases. Our results provide guidance to platform managers and regulators who are seeking to integrate traditional taxi services with ride-hailing platforms. Under certain conditions, forming a partnership with traditional taxi companies is effective in which ride-hailing platforms are beneficial to attract the participation of taxi drivers to serve ride-hailing requests. Regulators should be cautious about encouraging this partnership by enabling taxi drivers to get "full access" to ride-hailing platforms.

Chapter 2

Converting Counterfeiters in Emerging Markets to Authorized Suppliers: A New Anti-counterfeiting Measure

2.1 Introduction

Counterfeits are illegal products that imitate and infringe on brands of genuine items. Globalization has contributed to an alarming expansion of the types of products being infringed, from luxury goods (e.g., fashion apparel and watches) to other consumer products (e.g., electronics and stationery). The Organisation for Economic Cooperation and Development (OECD) reports that counterfeit and pirated goods made up 3.3% of global trade volumes in 2016, totalling $509 billion, compared with an estimate of up to 2.5% in 2013 (OECD (2019)). Counterfeiting is estimated to drain $4.2 trillion from the global economy and put 5.4 million legitimate jobs at risk by 2022 (Economics (2017)).

Although industries and governments have been actively devoting their efforts to combating counterfeiting, companies still suffer serious trademark infringement from counterfeiters. For years, the luxury goods industry has invested heavily in fighting against counterfeiters, such as encouraging governments to strengthen regulations and law enforcement to seize counterfeits, and running public awareness campaigns on the risks of purchasing counterfeits (Fontana et al. (2019)). In the United States, the government has been pushing for stronger global law enforcement on trademarks and intellectual property (IP) rights. These measures can be used to drive counterfeiters out of markets. However, in some developing economies, although anti-counterfeiting laws have been passed, their enforcement is still weak. For example, the Turkish parliament passed regulations against counterfeits in 2016, which involve prison terms and steep fines. But these laws have not stopped counterfeiting, and their effectiveness remains unclear (Smith (2018)). In China, local governments established laws to crack down on fake products, but in some cases, the incentive to enforce the laws is not strong enough. For example, the "fake shoe market" has become an invisible pillar of the local economy in China Putian (Hirtenstein (2017)).

With technological developments in manufacturing, there is a trend that the quality of counterfeits is improving (Yao (2014)). As quoted from Alibaba's Jack Ma, "Fake products today are of better quality and better price than the real names" (Dou (2016)). In recent years, there have been many cases in which counterfeiters indeed have the capability to produce high-quality products. Turkish fake Louis Vuitton (LV) bags are notorious for being high-end "genuine fake" because these fake bags are made from leather sourced from the same channel as genuine bags and are made by experienced craftsmen. It makes high-quality fake bags hard to be distinguished from originals (Letsch (2011)). Chinese imitations used to be of low quality, but some manufacturers are now producing so-called "super fakes": the quality of products is so good that even experts cannot tell the difference from genuine products (Mau (2018b)).

Economist (2022) points out that the quality of counterfeit consumer goods has never been greater.

Due to the improved manufacturing capability of counterfeiters, we observe a new anti-counterfeiting measure, that is, some brand-name firms outsource their production to counterfeiters *converting them to authorized suppliers*. For example, in the luxury goods industry, Balenciaga's Triple S was initially made in Italy but is now made in China's Putian factories. A Balenciaga official explained that the key reason for outsourcing to China was because Chinese factories "have the savoir-faire and capacity to produce a lighter shoe" (Silbert (2018)). In the consumer goods industry, Japanese stationery maker Kokuyo tied up with the Chinese "shanzhai" stationery brand Gambol, who once imitated Kokuyo's famous brand Campus and sold the knockoff at a much lower price in more than 5,000 retail stores in China. The strategy that Kokuyo has adopted to join hands with its counterfeiter helps it gain a bigger market share in China (Sugawara (2015)). Similarly, Honda set up a joint venture to make and sell motorcycles in China with Hainan Sundiro Motorcycle CO., which used to produce Honda knockoffs (Zaun and Leggett (2016)). The reason behind this collaboration is that the Chinese company can produce parts at one-fourth the price of Honda.

The collaboration between brand-name firms and counterfeiters has various implications. For brand-name firms, outsourcing to overseas suppliers who are converted from counterfeiters may have two benefits. First, if counterfeiters can produce high-quality products at low costs, it will save production costs for brand-name firms. Second, converting counterfeiters to authorized suppliers can help brand-name firms combat counterfeiting in overseas markets and potentially obtain larger market shares. However, not all brand-name firms accept this type of collaboration due to potential harm to their brand values.

For counterfeiters, becoming brand-name firms' authorized suppliers can help them not only secure profits but also avoid lawsuits and penalties from law enforcement. However, counterfeiters may not always be willing to become authorized suppliers for brand-name firms. For example, according to our conversations with counterfeit producers in the Grand Bazaar of Istanbul in Turkey, some counterfeiters of high-quality leather goods, such as fake LV bags, are not willing to become authorized suppliers for genuine brands due to a significant cut of their profit margin by the contract offered by the brand-name firm. Therefore, it remains unclear when such a collaboration can be established and how effective it is in terms of combating counterfeiting.

Motivated by the above observations, in this paper, we consider this new anti-counterfeiting measure: converting counterfeiters who are capable of producing high-quality products to authorized overseas suppliers. We study the following three research questions.

First, with the option to convert a counterfeiter, what is the equilibrium sourcing strategy for a brand-name firm? Under what conditions is the counterfeiter willing to be authorized as an overseas supplier?

Second, how does this new anti-counterfeiting measure interact with the conventional measures such as consumer education and law enforcement?

Third, does converting the counterfeiter to an authorized supplier benefit consumers or society?

To examine these questions, we develop a game-theoretic model to capture interactions between a brand-name firm with a home supplier and a counterfeiter who produces "super fakes" in the overseas market. The brand-name firm potentially outsources the production to the home supplier and/or the counterfeiter via wholesale-price contracts, and sells the brand-name product in both home and overseas markets. The counterfeiter may enter the overseas market to sell the counterfeit if she rejects the brand-name firm's contract. In this case, the counterfeiter faces an expected penalty from law enforcement. In our paper, we assume that the counterfeiter sells a non-deceptive counterfeit; *i.e.*, consumers know that it is a counterfeit at the time of purchase (e.g., Grossman and Shapiro (1988), Zhang et al. (2012), Cho et al. (2015), Gao et al. (2016), Yi et al. (2022)). Observed from our motivating examples, the counterfeiters who have the capability to produce high-quality products are generally non-deceptive, and thus brand-name firms consider them as potential suppliers, rather than deceptive counterfeiters who lack such capability.

The brand-name firm's possible sourcing strategies can be classified into four types: single sourcing from the home supplier (H), dual sourcing (D), single sourcing from the overseas supplier (O) and no sourcing (N). We develop the conditions that lead to each of these four sourcing strategies as game equilibrium. Our analysis examines the impact of the following factors on the equilibrium sourcing strategy: the penalty from law enforcement, the "perceived quality of the counterfeit" that captures how overseas market consumers value the counterfeit and can be altered by consumer education, the overseas market size, the difference in production costs between two suppliers, and the "discount factor of using the overseas supplier" that reflects the brand value loss from converting the counterfeiter. We next summarize our main findings as follows:

(1) Equilibrium sourcing strategies: When the difference in production costs between the two suppliers and the discount factor of using the overseas supplier are low, the brand-name firm may not have the incentive to convert the counterfeiter due to the limited cost saving and significant brand value loss. Otherwise, the brand-name firm has the incentive to do so, and Table 2.1 provides a summary of three possible equilibrium sourcing strategies. In-

terestingly, in this case, the brand-name firm may still adopt single sourcing from the home supplier, in spite of the benefits of converting the counterfeiter, when the overseas market size is high, the penalty from law enforcement is low, or the perceived quality of the counterfeit is high. This is because the counterfeiter can obtain a high profit by selling the counterfeit in such a market so that a high wholesale price is needed for the conversion, which may wipe out the benefits. Therefore, the conventional anti-counterfeiting measures, such as lawsuits to increase the penalty from law enforcement for counterfeiting and education campaigns to reduce consumers' perceived value of the counterfeit, may be adopted to facilitate the conversion so that the brand-name firm can convert the counterfeiter to the overseas supplier through either dual sourcing or single sourcing.

Sourcing strategy	Overseas market size	Penalty from law enforcement	Perceived quality of the counterfeit
Single sourcing from the home supplier (H)	High	Low	High
Single sourcing from the overseas supplier (O)	Low	High	Moderate
Dual sourcing (D)	High	High	Low

Table 2.1: Equilibrium Sourcing Strategies with the Option to Convert the Counterfeiter

(2) Interaction with the conventional measures: Our result shows that the new measure of converting the counterfeiter to an authorized overseas supplier also complements the conventional anti-counterfeiting measures, such as consumer education and law enforcement. Specifically, without the option to convert the counterfeiter, stronger efforts in consumer education and law enforcement are required in order to mitigate the counterfeiting threat. Therefore, brand-name firms should consider converting counterfeiters especially when driving counterfeiters out of markets is challenging. In addition,

we find that converting the counterfeiter can benefit both the brand-name firm and the counterfeiter. Interestingly, increasing penalty from law enforcement enhances these benefits whereas education campaigns to reduce consumers' perceived value of counterfeits may reduce these benefits.

(3) Impacts on consumer and social surplus: We find that converting the counterfeiter to an overseas supplier may hurt consumer surplus and does not always improve social surplus. This is because consumers may prefer the product produced by the home supplier to that produced by the overseas supplier, and more importantly, converting the counterfeiter mitigates the competition in the overseas market resulting in a surplus loss. When the penalty from law enforcement is high or the perceived quality of the counterfeit is low, converting the counterfeiter benefits society. On the contrary, when law enforcement is weak or consumers find the counterfeit attractive, caution should be taken about converting the counterfeiter.

The rest of this paper is organized as follows. Section 2.2 reviews the related literature. In Sections 2.3 and 2.4, we present the model and the game equilibrium. In Section 2.5, we discuss the impacts of this new anti-counterfeiting measure on the profits of firms, consumer and social surplus, focusing on its interaction with the conventional measures. Section 2.6 concludes the paper. To make the presentation concise, we present an extension of the base model in Appendix A.1. We relegate all the proofs to Appendix A.2.

2.2 Literature Review

In this section, we review related literature mainly on product counterfeiting and strategies to combat counterfeiting. There is a stream of literature that examines product counterfeiting. Grossman and Shapiro (1988) consider the status and quality attributes of brand-name products and analyze the positive and normative effects of counterfeiting. By collecting panel data from Chinese shoe companies, Qian (2008) finds that the original producer tends to offer a higher quality product with a higher price in the presence of counterfeit entry. Qian (2014) shows that counterfeits have both advertising effects and substitution effects for brand-name products with various quality levels. Qian et al. (2015) study strategies that an authentic firm responds to the entry of counterfeiters by improving the experiential quality (e.g., functionality) and searchable quality (e.g., appearance). Gao et al. (2016) analyze entry decisions of copycats by incorporating both the physical resemblance and product quality features. Pun and DeYong (2017) study the competition between an authentic manufacturer and a copycat firm in the presence of impatient consumers with strategic behavior. Chen and Papanastasiou (2021) show that the deceptive seller may seed consumers' observational learning process with a fake purchase in order to convince consumers to purchase the deceptive product. Wu et al. (2021) study a retailer's strategic choice of channel structures in the presence of supplier encroachment and counterfeiting. Chen et al. (2022) find that the benefit brought by consumer wealth status signaling to the firm is neutralized by the presence of the counterfeit. Yuan et al. (2023) discuss how the manufacturer determines information disclosure strategy about product fit with the potential risk of supplier copycatting. Gao et al. (2023) consider direct channel and indirect online channel of an authentic luxury brand and show that the authentic luxury brand may share the same online channel with the counterfeiters to improve consumer surplus. In our paper, we focus on the emerging "supper fakes" produced by non-deceptive counterfeiters, who have the capability to produce brand-name products of high quality. Apart from the competition between the brand-name firm and the counterfeiter analyzed in the literature, we investigate the collaboration between the brand-name firm and the counterfeiter; *i.e.*, the brand-name firm converts the counterfeiter to an authorized overseas supplier by adopting different sourcing strategies.

There are several strategies to combat counterfeiting discussed in the literature. Grossman and Shapiro (1988) examine the effect of enforcement policy to combat foreign counterfeits with low quality for international trade. Zhang et al. (2012) investigate strategies for brand-name companies to fight non-deceptive counterfeiting by raising consumers' awareness of intellectual properties and the potential harm of counterfeits, or pushing the government for enforcement. Cho et al. (2015) study the effectiveness of different approaches for a brand-name firm competing with deceptive and non-deceptive counterfeiters, including law enforcement effort and consumer education. Gao et al. (2016) show that higher quality and enhancement of status image through advertising can prevent the copycat from entering the market. Yi et al. (2022) discuss the supply chain members' anti-counterfeiting efforts such as enforcing closure of factories supplying counterfeits and educating consumers. Wu et al. (2022) consider an online counterfeiter who can strategically choose the type of counterfeits to sell and suggest that the brand-name firm can implement stricter intellectual property protections to reduce the probability of consumers being deceived. Li et al. (2023) examine the customer-to-customer platform's inspection service to detect counterfeits. Gao and Wu (2023) consider the retailers who sell both authentic products and counterfeits, and the manufacturer's strategic response to regulation, such as a penalty for counterfeit sales imposed by regulators. There are also papers considering anti-counterfeiting technologies that help consumers distinguish genuine products from the fake. Gao (2018) examines how pharmaceutical firms adopt overt anti-counterfeiting technologies to increase the fixed entry cost to combat deceptive counterfeiters. Pun et al. (2021) discuss the value of blockchain technology adoption to combat deceptive counterfeits. Yao et al. (2023) analyze the effect of an authentic company's anti-counterfeit technology and regulatory authorities' law enforcement on combating deceptive counterfeit products. Unlike the existing literature, we study the innovative anti-counterfeiting measure that the brand-name firm can use to combat the counterfeiter by converting her to an authorized overseas supplier. Furthermore, we examine the interaction between this new anti-counterfeiting measure and the conventional measures in the literature, such as law enforcement and consumer education.

Related to counterfeiting, gray markets (or parallel importing) are unauthorized channels in which retailers sell brand-name products, see, for example, Ahmadi and Yang (2000), Hu et al. (2013), Ahmadi et al. (2015, 2017), Autrey et al. (2015), Shao et al. (2016). Unlike counterfeits that are produced or sold by unauthorized imitators, products at gray markets are genuine and sourced from authorized sellers. Recent work by Wang et al. (2020) provides an overview of this topic.

Our paper is also related to the literature that studies firms' global sourcing decisions. Feng and Lu (2012) consider production cost and contract negotiations between manufacturers and suppliers. Wu and Zhang (2014) consider supply lead time to capture the trade-off between cost and responsiveness. Sun et al. (2010) study the firm's technology outsourcing strategy to a foreign firm with imitation risk. Berry and Kaul (2015) empirically examine how foreign knowledge-seeking impacts the firm's global sourcing choices between offshore integration and offshore outsourcing. Guo et al. (2016) consider consumers being socially conscious and analyze a buyer's sourcing decision between a responsible supplier and a supplier with violation risk. Orsdemir et al. (2019) study how the firm decides between vertical integration and horizontal sourcing under corporate social and environmental responsibility violation risk of suppliers and demand externalities. Hu et al. (2020) study when an innovator may source from competitor-supplier under technical and non-technical innovation spillover risks. Pun and Hou (2022) consider a manufacturer's outsourcing decision of production tasks to a supplier with imitation risk. Skowronski and Benton (2017) empirically evaluate how brand-name firms protect IP from poaching when outsourcing to suppliers in countries with weak IP rights. A stream of literature considers sourcing decisions with suppliers who potentially sell products through a direct channel to consumers (e.g., Arya et al. (2007), Li et al. (2014), and Ha et al. (2016)). Different from the above literature, we focus on the sourcing decisions in the setting where a counterfeiter exists and may be converted to an authorized overseas supplier. In our paper, the counterfeiter decides on whether to accept the brand-name firm's contract and be converted to a supplier, or to reject the contract and sell the counterfeit in the overseas market. When selling the counterfeit, the counterfeiter faces law enforcement penalty from the local government in the overseas market.

In summary, our model incorporates the recent trend of high-quality counterfeits to examine interactions between the brand-name firm and two types of potential suppliers, one of which is converted from the counterfeiter. To the best of our knowledge, our paper is the first to study combating counterfeiting through conversion. Based on our model, we derive novel insights into anti-counterfeiting and global sourcing.

2.3 Model

In this section, we describe the model setting and sequence of events before we formulate the consumer utility and expected profit of each firm. A summary of the model notations is presented in Table 2.2.

Decision Variables	
w_i	Wholesale price for supplier $i \in \{1, 2\}$
d_i	Whether supplier i accepts the contract, $d_i \in \{0, 1\}$, $i \in \{1, 2\}$
s	Whether the counterfeiter enters the overseas market to sell the counterfeit, $s \in \{0, 1\}$
p_2	Retail price of the counterfeit in the overseas market
Parameters	
θ	Taste of consumers, $\theta \sim U[0, 1]$
α	Overseas market size, $\alpha > 0$
β	Perceived quality discount factor of the counterfeit, $\beta \in (0, 1)$
γ	Perceived quality discount factor of the brand-name product produced by the overseas supplier, $\gamma \in (\beta, 1]$
q_j	Perceived quality of product $j \in \{B, 2\}$
p_B	Retail price of the brand-name product
k_i	Unit production cost of supplier $i \in \{1, 2\}$, $k_1 \geq k_2 > 0$
Δ	Difference in production costs between two suppliers, $\Delta = k_1 - k_2$
e	Expected penalty from law enforcement for counterfeiting, $e \geq 0$
Profits and Demands	
π_B	Expected profit of the brand-name firm
π_i	Expected profit of supplier $i \in \{1, 2\}$
m_{Bi}	Demand of the brand-name product in market $i \in \{1, 2\}$
m_2	Demand of the counterfeit in the overseas market

Table 2.2: Model Notations

2.3.1 Model Setting and Sequence of Events

We consider a setting in which a brand-name firm ('he') potentially outsources his production to a home supplier and/or an overseas supplier who is converted from a counterfeiter, and sells the brand-name product to consumers in both home and overseas markets. The size of the home market is normalized to 1, and the size of the overseas market is α, where $\alpha > 0$. In each market, there is a potential supplier ('she'). We use subscript $i = 1$ to indicate the home supplier and $i = 2$ to indicate the counterfeiter in the overseas market who potentially can be converted to an authorized overseas supplier. The marginal production cost of supplier i is k_i, and we assume $k_1 \geq k_2 > 0$ to capture the lower production cost of the the counterfeiter in the overseas market. We define $\Delta = k_1 - k_2$ as the difference in production costs between the two.

The brand-name firm determines which suppliers to source from via wholesale-price contracts. We consider the following two-stage sequence of events: contract stage and selling stage, as shown in Figure 2.1. At the beginning of the first stage, the brand-name firm offers wholesale price w_1 to the home supplier and wholesale price w_2 to the counterfeiter. Then, the home supplier and the counterfeiter simultaneously decide on whether to accept the contract. Let $d_i \in \{0, 1\}$ denote potential supplier i's decision: $d_i = 0$ means that supplier i rejects the contract, and $d_i = 1$ means that supplier i accepts the contract. We assume that if a potential supplier is indifferent between accepting and rejecting, she would accept it.

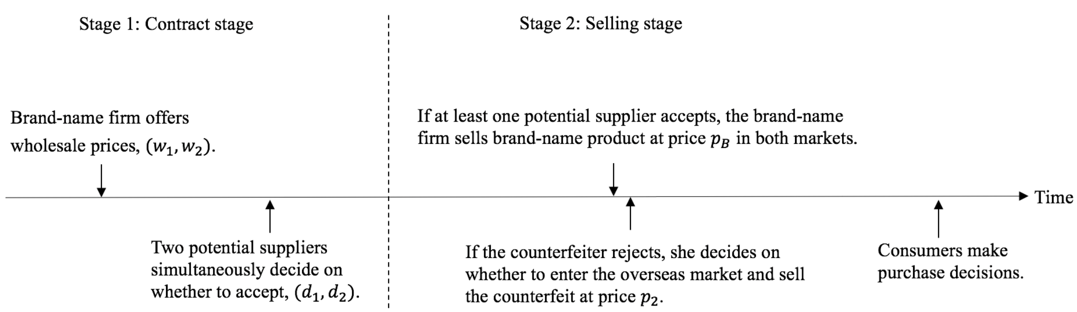
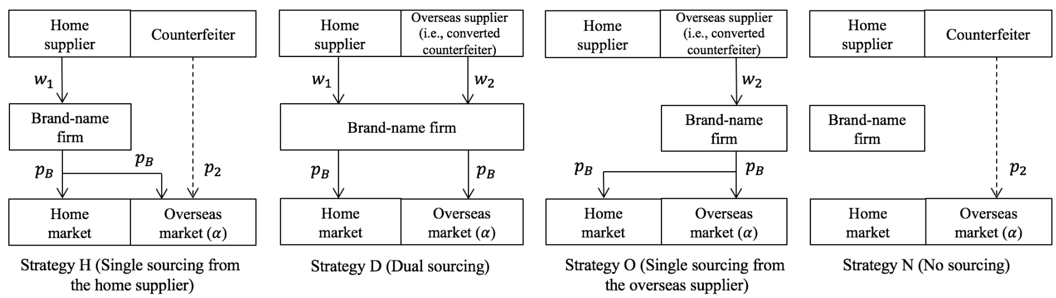


Figure 2.1: Sequence of Decisions and Events

In the second stage, if the home supplier and/or the counterfeiter accepts the contract, the brand-name firm sells the brand-name product at a retail price p_B in two markets. In order to focus on the analysis of converting the counterfeiter, the retail price p_B in our model is exogenous. Such an assumption has been adopted in the counterfeiting literature (e.g., Gao et al. 2016 and Gao et al. 2023) as it is rarely seen that a brand-name firm would launch a price war against an infamous counterfeiter. We also assume that the retail prices of brand-name products in the two markets are the same because of the anti-dumping law (Macrory 2005), and this assumption can be easily relaxed. Note that $p_B \geq k_i$ for $i \in \{1, 2\}$ ensures that the brand-name firm enters the market with non-negative marginal profit when any potential supplier accepts the contract. If supplier i rejects the contract, she may become a counterfeiter by entering the market to sell the counterfeit. In this case, she faces the risk of getting caught and paying the penalty from law enforcement. We assume that the home market's expected penalty from law enforcement is high enough so that the home supplier would not produce or sell the counterfeit if rejecting the contract. On the contrary, the expected penalty from law enforcement in the overseas market, denoted as e (≥ 0), is not very high so that if the counterfeiter rejects the contract, she may obtain a non-negative expected profit by selling the counterfeit. In the base model, the law enforcement penalty for counterfeiters selling counterfeits is fixed at e. In Appendix A.1, we demonstrate the robustness of our results by considering a scenario in which the law enforcement penalty depends on the counterfeiter's revenue. In the case of the counterfeiter rejecting the contract, we use $s \in \{0, 1\}$ to denote the counterfeiter's entry decision: $s = 1$ means that she enters the market to sell the counterfeit, and $s = 0$ means that she does not enter the market. If the counterfeiter rejects the contract and decides to enter the overseas market to sell the counterfeit, she endogenously determines the retail price, p_2, to compete with the brand-name product. We use $j = B$ and $j = 2$ to denote the brand-name product and the counterfeit, respectively. Due to the strict law enforcement in the home market, the counterfeiter in the overseas market would not sell the counterfeit in the home market. At the end of the second stage, upon observing the price(s) of the product(s), consumers in each market make their purchase decisions.

There are four types of supply chain structures based on the brand-name firm's possible sourcing strategies: single sourcing from the home supplier (H), dual sourcing (D), single sourcing from the overseas supplier (O), and no sourcing (N), as illustrated in Figure 2.2.



Note. Solid lines represent the product flow of the authentic brand-name product and dash lines represent the product flow of the counterfeit.

Figure 2.2: Supply Chain Structures

Strategy H (Single sourcing from the home supplier): When the home supplier accepts her contract and the counterfeiter rejects her contract, the brand-name firm uses brand-name products sourced from the home supplier to satisfy demands in both home and overseas markets. He has to compete with the counterfeiter if she enters the overseas market to sell the counterfeit. We normalize the transportation cost from the home supplier to the overseas market to zero, and our main results can be easily extended to a case with a positive transportation cost.

Strategy D (Dual sourcing): When both the home supplier and the counterfeiter accept their contracts, the brand-name firm uses brand-name products sourced from the home (overseas) supplier to satisfy the home (overseas) market demand. Consistent with our motivating examples, we assume if the counterfeiter accepts the contract, she will not sell the counterfeit due to close monitoring by the brand-name firm.

Strategy O (Single sourcing from the overseas supplier): When the home supplier rejects her contract and the counterfeiter accepts her contract, the brand-name firm uses brand-name products produced by the overseas supplier who is converted from the counterfeiter to satisfy demands in both home and overseas markets. The home supplier does not enter the home market as a counterfeiter because of the high penalty from law enforcement in the home market.

Strategy N (No sourcing): When both the home supplier and the counterfeiter reject their contracts, the brand-name firm does not enter markets. The counterfeiter sells the counterfeit if she enters the overseas market. This strategy is unlikely in practice, and later we will show that it is not an equilibrium strategy.

2.3.2 Consumer Utility

In our model, each consumer demands one unit of the product at most and does not purchase across markets. They make their purchase decisions to maximize their utilities. A consumer's utility of purchasing product $j \in \{B, 2\}$ is given by $u_j = \theta q_j - p_j$, where θ denotes consumer's taste and is uniformly distributed between 0 and 1, *i.e.*, $\theta \sim U[0, 1]$, q_j denotes the "perceived quality" of product j, and p_j is the retail price of product j. Note that the perceived quality can be different from the actual quality of a product. For example, consumers with a social conscience will be reluctant to purchase a product from a brand with an infamous reputation. Although the quality is not inferior, consumers' perceived value of the product can be low. In our context, as we study the non-deceptive counterfeit, consumers know whether the product is purchased from a counterfeiter or from a brand-name firm. Therefore, the perceived quality can be directly and knowingly derived.

Let q (> 0) denote the actual product quality of the brand-name product. If purchasing a product from the counterfeiter, despite that the actual product quality may be the same as the authentic one, consumers perceive it as of low value and discount the quality by a factor $\beta \in (0, 1)$, *i.e.*, $q_2 = \beta q$. That is, although these high-quality counterfeits imitate the design of brand-name products and are produced with premium materials, they only get a part of the brand value from the brand-name products, as consumers knowingly purchase the counterfeits.

The perceived quality of the brand-name product can also be different from its actual quality, q, if it is produced by the overseas supplier converted from a counterfeiter. For example, some consumers in China were upset when Balenciaga outsourced the triple S production in 2018 (Mau 2018a). In order to capture the possible brand value loss due to converting the counterfeiter, we use γ, where $\gamma \in (\beta, 1]$, to represent consumers' perceived quality discount factor of the brand-name products produced by the overseas supplier, *i.e.*, $q_B = \gamma q$. We assume that there is no such discount if the product is produced by the home supplier.

For simplicity, we normalize the actual quality of the brand-name product q to be one, *i.e.*, $q = 1$. Then, the perceived quality of the counterfeit is $q_2 = \beta$. We assume $\beta > \underline{\beta} = \frac{k_2}{p_B}$ to eliminate an uninteresting case in which the perceived quality of the counterfeit is so low that there is no demand for the counterfeit if the counterfeiter rejects the contract. The perceived qualities of the brand-name product produced by the home supplier and the overseas supplier are $q_B = 1$ and $q_B = \gamma$, respectively. In the following, we derive the demand of each product based on consumer utility.

Strategy H: In the home market, only brand-name product with price p_B is available. Consumers in the home market decide on whether to purchase the brand-name product or not. A consumer with taste $\hat{\theta}_B = \frac{p_B}{q_B}$ is indifferent between buying and not buying as it satisfies $\hat{\theta}_B q_B - p_B = 0$. We assume that p_B is not very high such that the brand-name product in the home market has a positive demand, *i.e.*, $p_B < q_B$, which implies $\hat{\theta}_B < 1$. Thus, consumers with $\theta \in \left[\hat{\theta}_B, 1\right]$ purchase the brand-name product, and consumers with $\theta \in \left[0, \hat{\theta}_B\right)$ purchase nothing, as illustrated in Figure 2.3(a).

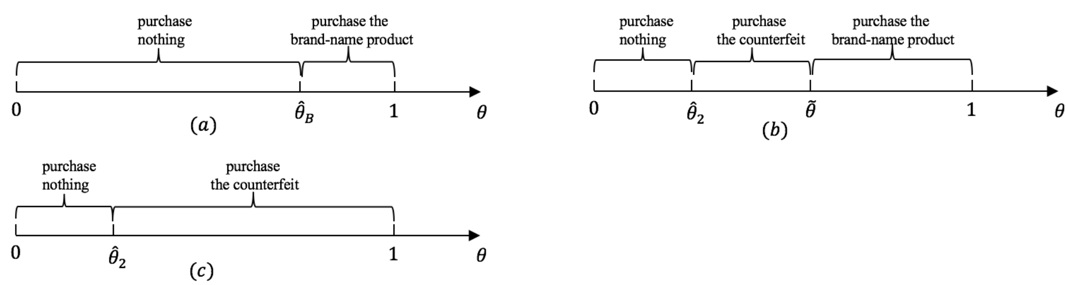


Figure 2.3: Consumer Utility Thresholds. Note: (a) The home or overseas market only has the brand-name product; (b) the overseas market has both the brand-name product and the counterfeit; (c) the overseas market only has the counterfeit.

In the overseas market, there are two products if the counterfeiter enters the market, the brand-name product with price p_B as well as the counterfeit with price p_2. Consumers in the overseas market decide whether to purchase the brand-name product, the counterfeit, or nothing. A consumer who is indifferent between purchasing the counterfeit and not purchasing at all has the taste $\hat{\theta}_2 = \frac{p_2}{q_2}$, which satisfies $\hat{\theta}_2 q_2 - p_2 = 0$. A consumer who is indifferent between purchasing the brand-name product and the counterfeit has taste $\tilde{\theta} = \frac{p_B - p_2}{1-\beta}$, which satisfies $\tilde{\theta} q_B - p_B = \tilde{\theta} q_2 - p_2$. Under the assumption $\beta > \underline{\beta}$, we obtain $\hat{\theta}_2 < \min\{\tilde{\theta}, 1\}$. If $\tilde{\theta} < 1$, consumers with $\theta \in \left[\tilde{\theta}, 1\right]$ purchase the brand-name product, consumers with $\theta \in \left[\hat{\theta}_2, \tilde{\theta}\right)$ purchase the counterfeit, and consumers with $\theta \in \left[0, \hat{\theta}_2\right)$ purchase nothing, as illustrated in Figure 2.3(b).

If $\tilde{\theta} \geq 1$, consumers with $\theta \in \left[\hat{\theta}_2, 1\right]$ purchase the counterfeit, and consumers with $\theta \in \left[0, \hat{\theta}_2\right)$ purchase nothing, as illustrated in Figure 2.3(c).

In the overseas market, only brand-name product with price p_B is available if the counterfeiter does not enter the market. In this case, similar to the home market, consumers with $\theta \in \left[\hat{\theta}_B, 1\right]$ purchase the brand-name product, and consumers with $\theta \in \left[0, \hat{\theta}_B\right)$ purchase nothing, as illustrated in Figure 2.3(a).

Strategy D and Strategy O: The brand-name firm sells the brand-name product at p_B in both markets. A consumer with taste $\hat{\theta}_B = \frac{p_B}{q_B}$ is indifferent between buying and not buying. Thus, in each market, consumers with $\theta \in \left[\hat{\theta}_B, 1\right]$ purchase the brand-name product, and consumers with $\theta \in \left[0, \hat{\theta}_B\right)$ do not purchase any products, as illustrated in Figure 2.3(a).

Strategy N: In the home market, there is no product for purchasing. In the overseas market, only the counterfeit with price p_2 is available if the counterfeiter enters the market. A consumer with taste $\hat{\theta}_2 = \frac{p_2}{q_2}$ is indifferent between buying and not buying. Under the assumption $\beta > \underline{\beta}$, we know $\hat{\theta}_2 < 1$. Thus, in the overseas market, consumers with $\theta \in \left[\hat{\theta}_2, 1\right]$ purchase the counterfeit, and consumers with $\theta \in \left[0, \hat{\theta}_2\right)$ purchase nothing, as illustrated in Figure 2.3(c). If the counterfeiter does not enter the market, there is also no product for purchasing in the overseas market.

To summarize, given the home supplier's decision on whether to accept the contract, *i.e.*, d_1, and the counterfeiter's decisions on whether to accept the contract and whether to enter the overseas market to sell the counterfeit if rejecting the contract, *i.e.*, d_2 and s, we obtain the demands of the brand-name product and the counterfeit. Let $(x)^+$ denote $\max(x, 0)$. The demand of the brand-name product in the home market, m_{B1}, is represented as below,

$$m_{B1} = \begin{cases} 1 - p_B, & \text{if } d_1 = 1, \\ 1 - \frac{p_B}{\gamma}, & \text{if } d_1 = 0 \text{ and } d_2 = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{2.1}$$

The demand of the brand-name product in the overseas market, m_{B2}, is represented as below,

$$m_{B2} = \begin{cases} \alpha \left(1 - \frac{p_B - p_2}{1-\beta}\right)^+, & \text{if } d_1 = 1,\ d_2 = 0 \text{ and } s = 1, \\ \alpha \left(1 - p_B\right), & \text{if } d_1 = 1,\ d_2 = 0 \text{ and } s = 0, \\ \alpha \left(1 - \frac{p_B}{\gamma}\right), & \text{if } d_2 = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{2.2}$$

Finally, the demand of the counterfeit in the overseas market, m_2, is represented as below,

$$m_2 = \begin{cases} \alpha \left(\min\{\frac{p_B - p_2}{1-\beta}, 1\} - \frac{p_2}{\beta}\right), & \text{if } d_1 = 1,\ d_2 = 0 \text{ and } s = 1, \\ \alpha \left(1 - \frac{p_2}{\beta}\right), & \text{if } d_1 = 0,\ d_2 = 0 \text{ and } s = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{2.3}$$

2.3.3 Expected Profits of Firms

In this subsection, we derive the expected profits of the brand-name firm, the home supplier and the counterfeiter. Given the wholesale prices and players' decisions (w_1, w_2, d_1, d_2, s), the brand-name firm's expected profit π_B is given as

$$\begin{aligned} \pi_B \left(w_1, w_2, d_1, d_2, s\right) = & \left(p_B - d_1 w_1 - \left(1 - d_1\right) d_2 w_2\right) m_{B1} \\ & + \left(\left(1 - s\right)\left(p_B - \left(1 - d_2\right) d_1 w_1 - d_2 w_2\right) \right. \\ & \left. + s \left(1 - d_2\right)\left(p_B - d_1 w_1\right)\right) m_{B2}, \end{aligned}$$

where m_{B1} and m_{B2} are given in Equations (2.1) and (2.2), respectively. The first and second terms represent the expected profits of the brand-name firm from selling products in the home market and the overseas market, respectively.

The home supplier's expected profit π_1 is given as

$$\pi_1 (w_1, d_1, d_2, s) = d_1 (w_1 - k_1) m_{B1} + (1 - d_2) d_1 (w_1 - k_1) m_{B2}.$$

The first and second terms represent the expected profits of the home supplier from producing products for the brand-name firm to sell in the home market and the overseas market, respectively, if she accepts the contract. If she rejects the contract, we normalize her profit to zero. This assumption can be easily relaxed by considering an outside option with a positive profit for the supplier.

The counterfeiter's expected profit π_2 is given as

$$\begin{aligned}\pi_2 (w_2, d_1, d_2, s, p_2) = &(1 - s) d_2 ((1 - d_1) (w_2 - k_2) m_{B1} + (w_2 - k_2) m_{B2}) \\ &+ s (1 - d_2) ((p_2 - k_2) m_2 - e),\end{aligned}$$

where m_2 is given in Equation (2.3). The first term represents the expected profit of the authorized overseas supplier from producing brand-name products for the home market and the overseas market, if the counterfeiter accepts the contract; the second term represents the expected profit of the counterfeiter from selling the counterfeit in the overseas market if she rejects the contract and enters this market.

2.4 Equilibrium Analysis

In this section, we use backward induction to analyze the sequential game between the brand-name firm and the two potential suppliers, as depicted in

Figure 2.1. In Section 2.4.1, we analyze the contract acceptance decisions of the home supplier and the counterfeiter as well as the market entry decision of the counterfeiter if she rejects the contract. In Section 2.4.2, under each possible sourcing strategy, we derive the optimal profits of the brand-name firm and the two potential suppliers in the second stage, and determine the optimal wholesale prices. In Section 2.4.3, we obtain conditions under which a particular sourcing strategy arises in equilibrium in the first stage, and analyze the impact of different factors on the equilibrium.

2.4.1 Suppliers' Best Responses

Under each possible sourcing strategy, we first obtain the profit expression of each firm given the optimal retail price of the counterfeit. Then we investigate the optimal market entry decision of the counterfeiter as well as the optimal contract acceptance decisions of the home supplier and the counterfeiter.

Strategy H: Given wholesale prices w_1 and w_2, the home supplier accepts the contract and the counterfeiter rejects the contract, *i.e.*, $d_1 = 1$ and $d_2 = 0$. Thus, the brand-name firm only sources from the home supplier.

(1) If the counterfeiter enters the overseas market to sell the counterfeit, *i.e.*, $s = 1$, she determines retail price p_2^H of the counterfeit to compete with the brand-name firm and obtain the below profit:

$$\pi_2^H\left(p_2^H\right) = \alpha\left(p_2^H - k_2\right)\left(\min\{\frac{p_B - p_2^H}{1-\beta}, 1\} - \frac{p_2^H}{\beta}\right)^+ - e.$$

We assume $\frac{1+k_2}{2} < p_B < 1$ to avoid a trivial case in which Strategy H will never be an equilibrium. By taking the first order derivative of $\pi_2^H(p_2^H)$ with respect to p_2^H, the optimal retail price of the counterfeit is

$$p_2^{H*} = \begin{cases} \frac{\beta p_B + k_2}{2}, & \text{if } \underline{\beta} < \beta < \beta_1, \\ \min\{p_B - (1-\beta), \frac{\beta + k_2}{2}\}, & \text{if } \beta_1 \le \beta < 1, \end{cases}$$

where $\beta_1 = \frac{k_2 + 2(1-p_B)}{2-p_B}$.

Substituting the expression of p_2^{H*} into Equations (2.2) and (2.3), we obtain $m_{B2} = \alpha \left(1 - \frac{(2-\beta)p_B - k_2}{2(1-\beta)}\right)^+$ and $m_2 = \min\left\{\frac{\alpha(\beta p_B - k_2)}{2\beta(1-\beta)}, \alpha\left(1 - \frac{\min\{p_B-(1-\beta), \frac{\beta+k_2}{2}\}}{\beta}\right)\right\}$.

(i) If $\underline{\beta} < \beta < \beta_1$, both the brand-name firm and the counterfeiter have positive market shares in the overseas market. Thus, their profits are:

$$\pi_B^H(w_1) = (p_B - w_1)(1 - p_B) + \alpha(p_B - w_1)\left(1 - \frac{(2-\beta)p_B - k_2}{2(1-\beta)}\right),$$
$$\pi_1^H(w_1) = (w_1 - k_1)\left((1 - p_B) + \alpha\left(1 - \frac{(2-\beta)p_B - k_2}{2(1-\beta)}\right)\right), \quad \pi_2^H = \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} - e.$$

(ii) If $\beta_1 \le \beta < 1$, only the counterfeiter has a positive market share in the overseas market. Thus, their profits are:

$$\pi_B^H(w_1) = (p_B - w_1)(1 - p_B), \quad \pi_1^H(w_1) = (w_1 - k_1)(1 - p_B),$$
$$\pi_2^H = \frac{\alpha \min\{\beta - k_2 - (1-p_B), \frac{\beta - k_2}{2}\} \max\{1 - p_B, \frac{\beta - k_2}{2}\}}{\beta} - e.$$

(2) If the counterfeiter does not enter the overseas market to sell the counterfeit, *i.e.*, $s = 0$, the brand-name firm is the monopoly in the overseas market. Thus, their profits expressions are:

$$\pi_B^H(w_1) = (1+\alpha)(p_B - w_1)(1 - p_B),$$
$$\pi_1^H(w_1) = (1+\alpha)(w_1 - k_1)(1 - p_B), \quad \pi_2^H = 0.$$

Strategy D: Given wholesale prices w_1 and w_2, the home supplier and the counterfeiter accept their contracts, respectively, *i.e.*, $d_1 = 1$ and $d_2 = 1$. Thus, their profits expressions are:

$$\pi_B^D(w_1, w_2) = (p_B - w_1)(1 - p_B) + \alpha(p_B - w_2)\left(1 - \frac{p_B}{\gamma}\right),$$
$$\pi_1^D(w_1) = (w_1 - k_1)(1 - p_B), \quad \pi_2^D(w_2) = \alpha(w_2 - k_2)\left(1 - \frac{p_B}{\gamma}\right).$$

Strategy O: Given wholesale prices w_1 and w_2, the home supplier rejects the contract and the counterfeiter accepts the contract, *i.e.*, $d_1 = 0$ and $d_2 = 1$. Thus, their profits are:

$$\pi_B^O(w_1, w_2) = (1 + \alpha)(p_B - w_2)\left(1 - \frac{p_B}{\gamma}\right), \quad \pi_1^O(w_1) = 0,$$
$$\pi_2^O(w_2) = (1 + \alpha)(w_2 - k_2)\left(1 - \frac{p_B}{\gamma}\right).$$

Strategy N: Given wholesale prices w_1 and w_2, the home supplier and the counterfeiter reject their contracts, respectively, *i.e.*, $d_1 = 0$ and $d_2 = 0$. Under this strategy, the brand-name firm does not have suppliers, and later we will show that it is not an equilibrium strategy.

(1) If the counterfeiter enters the overseas market to sell the counterfeit, *i.e.*, $s = 1$, she is the monopoly in the overseas market and determines retail price p_2^N of the counterfeit and obtains the below profit:

$$\pi_2^N\left(p_2^N\right) = \alpha\left(p_2^N - k_2\right)\left(1 - \frac{p_2^N}{\beta}\right) - e.$$

By taking the first order derivative of $\pi_2^N(p_2^N)$ with respect to p_2^N, the optimal retail price of the counterfeit is $p_2^{N*} = \frac{\beta + k_2}{2}$. Substituting the expression of p_2^{N*}

into Equation (2.3), we obtain $m_2 = \alpha\left(1 - \frac{\beta+k_2}{2\beta}\right)$. Thus, their profits are:

$$\pi_B^N(w_1, w_2) = 0, \quad \pi_1^N(w_1) = 0, \quad \pi_2^N = \frac{\alpha(\beta-k_2)^2}{4\beta} - e.$$

(2) If the counterfeiter does not enter the overseas market to sell the counterfeit, *i.e.*, $s = 0$, then, their profits are:

$$\pi_B^N(w_1, w_2) = 0, \quad \pi_1^N(w_1) = 0, \quad \pi_2^N = 0.$$

Under Strategy H and Strategy N, the counterfeiter rejects the contract. The counterfeiter enters the overseas market if she can obtain non-negative profit by selling the counterfeit. Thus, the optimal entry decision of the counterfeiter is

$$s^*(w_1, w_2) = \begin{cases} 0, & \text{if } \pi_2^H(s=1, p_2^{H*}) < \pi_2^H(s=0), \\ & \text{or, if } \pi_2^N(s=1, p_2^{N*}) < \pi_2^N(s=0), \\ 1, & \text{otherwise.} \end{cases}$$

Next, given w_1 and w_2, we derive the home supplier's and the counterfeiter's optimal decisions about whether to accept their contracts. Define

$$\pi_0 = \begin{cases} \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}, & \text{if } \underline{\beta} < \beta < \beta_1, \\ \frac{\alpha \min\{\beta-k_2-(1-p_B), \frac{\beta-k_2}{2}\} \max\{1-p_B, \frac{\beta-k_2}{2}\}}{\beta}, & \text{if } \beta_1 \le \beta < 1, \end{cases} \tag{2.4}$$

$M = (\pi_0 - e)^+$ and $K = \left(\frac{\alpha(\beta-k_2)^2}{4\beta} - e\right)^+$. By evaluating the difference in each potential supplier's expected profit between accepting and rejecting the contract, we obtain their optimal decisions as follows:

$$(d_1^*, d_2^*) = \begin{cases} (1,1), & \text{if } w_1 \geq k_1 \text{ and } w_2 \geq \frac{M}{\alpha\left(1-\frac{p_B}{\gamma}\right)} + k_2, \\ (1,0), & \text{if } w_1 \geq k_1 \text{ and } w_2 < \frac{M}{\alpha(1-\frac{p_B}{\gamma})} + k_2, \\ (0,1), & \text{if } w_1 < k_1 \text{ and } w_2 \geq \frac{K}{(1+\alpha)\left(1-\frac{p_B}{\gamma}\right)} + k_2, \\ (0,0), & \text{if } w_1 < k_1 \text{ and } w_2 < \frac{K}{(1+\alpha)\left(1-\frac{p_B}{\gamma}\right)} + k_2. \end{cases} \tag{2.5}$$

2.4.2 Brand-Name Firm's Optimal Wholesale Prices

In this subsection, we determine the optimal wholesale prices. The brand-name firm chooses wholesale prices w_1 and w_2 to maximize his expected profit by solving the following program:

$$\max_{w_1, w_2} \pi_B(w_1, w_2);\ \text{s.t. } (2.5)\,.$$

Lemma 2.1 presents the optimal wholesale prices of the brand-name firm under each possible sourcing strategy.

Lemma 2.1. *The optimal wholesale price(s) of the brand-name firm, which will be accepted by the home supplier and the counterfeiter, satisfies the following:*
(a) under Strategy H, $w_1^H = k_1$;
(b) under Strategy D, $w_1^D = k_1$, $w_2^D = \frac{M}{\alpha(1-\frac{p_B}{\gamma})} + k_2$;
(c) under Strategy O, $w_2^O = \frac{K}{(1+\alpha)(1-\frac{p_B}{\gamma})} + k_2$.

In leader-follower games with wholesale-price contracts, generally, the leader can extra all the benefits and set a wholesale price equal to the marginal production cost (Cachon 2003). In our model, under Strategy H and Strategy D, the brand-name firm sources from the home supplier by providing the wholesale price equal to the supplier's marginal cost, *i.e.*, $w_1^H = w_1^D = k_1$. For the counterfeiter, however, since she has the option to reject the contract and sell the counterfeit to compete with the brand-name firm in the overseas market, the brand-name firm might need to offer a wholesale price w_2 higher than k_2 in order to convert her under Strategy D and Strategy O. The optimal wholesale price for the counterfeiter depends on how much profit she obtains if rejecting the contract and selling the counterfeit. Specifically, when e is high, β is low or α is low, M and K in the lemma take the value of zero. This implies that the counterfeiter selling the counterfeit earns zero profit, so the brand-name firm only needs to offer the marginal production cost as the wholesale price, *i.e.*, $w_2^D = k_2$ and $w_2^O = k_2$, to convert the counterfeiter. When e is low, β is high or α is high, which leads to $M > 0$ and $K > 0$, the counterfeiter has a strictly positive profit by entering the overseas market to sell the counterfeit. The brand-name firm has to offer a premium wholesale price to convert the counterfeiter, *i.e.*, $w_2^D > k_2$ and $w_2^O > k_2$.

2.4.3 Equilibrium Sourcing Strategies

When the brand-name firm decides on his sourcing strategy facing the counterfeiter who can potentially be converted to an overseas supplier, he considers the following tradeoff. On the one hand, the brand-name firm can enjoy the benefit of production cost savings by sourcing from a low-cost authorized overseas supplier and mitigating the threat from the counterfeiter who will compete with him in the overseas market. On the other hand, the brand-name firm suffers a brand value loss if sourcing from the overseas supplier, and he may have to offer a premium wholesale price to attract the counterfeiter to accept the contract. If the overall benefit exceeds the overall loss, the brand-name firm prefers to convert the counterfeiter to the overseas supplier, *i.e.*, Strategy D or Strategy O. Otherwise, the brand-name firm is better off sourcing only from the home supplier, *i.e.*, Strategy H.

By comparing the optimal profits of the brand-name firm under the four possible sourcing strategies, we obtain the equilibrium strategy as stated in the following proposition. We denote the optimal profits of the brand-name firm, the home supplier and the counterfeiter as π_B^*, π_1^* and π_2^*, respectively. For ease of exposition, in the following, we focus on the case in which the counterfeiter obtains a positive profit by entering the overseas market to sell the counterfeit, *i.e.*, $M > 0$ and $K > 0$, which is equivalent to $e < \pi_0$ (defined in Equation (2.4)). For the analysis below, it is convenient to define

$$\begin{aligned}
e_1 = &\quad \pi_0 + \alpha(p_B - k_2 - \Delta)\left(1 - \tfrac{(2-\beta)p_B - k_2}{2(1-\beta)}\right)^+ - \alpha(p_B - k_2)\left(1 - \tfrac{p_B}{\gamma}\right), \\
e_2 = &\quad \tfrac{\alpha(\beta-k_2)^2}{4\beta} + \alpha(p_B - k_2 - \Delta)\left(1 - \tfrac{(2-\beta)p_B - k_2}{2(1-\beta)}\right)^+ - \alpha(p_B - k_2)\left(1 - \tfrac{p_B}{\gamma}\right) \\
&\quad -\Delta(1 - p_B) - (p_B - k_2)\left(p_B - \tfrac{p_B}{\gamma}\right), \\
\Delta_1 = &\quad \frac{\frac{\alpha(\beta-k_2)^2}{4\beta} - \pi_0 - (p_B - k_2)\left(p_B - \frac{p_B}{\gamma}\right)}{1 - p_B}.
\end{aligned}$$

Proposition 2.1. *The equilibrium sourcing strategy of the brand-name firm is as follows:*

(a) Strategy H with $w_1^* = k_1$ *if* $e < \min\{(e_1)^+, (e_2)^+\}$*;*

(b) Strategy D with $w_1^* = k_1$ *and* $w_2^* = w_2^D$ *if* $e \geq (e_1)^+$ *and* $\Delta < (\Delta_1)^+$*;*

(c) Strategy O with $w_2^* = w_2^O$ *if* $e \geq (e_2)^+$ *and* $\Delta \geq (\Delta_1)^+$*.*

Proposition 2.1 presents three possible equilibrium outcomes: Strategy H, Strategy D and Strategy O. Strategy N is not an equilibrium strategy because it is dominated by Strategy H, under which the brand-name firm is able to earn a non-negative profit from the home market.

Figure 2.4 shows the equilibrium sourcing strategy from the perspective of the interplay between the penalty from law enforcement, e, and the difference in production costs between two suppliers, Δ. We also highlight how the region boundaries change as the discount factor of using the overseas supplier,

γ, increases.

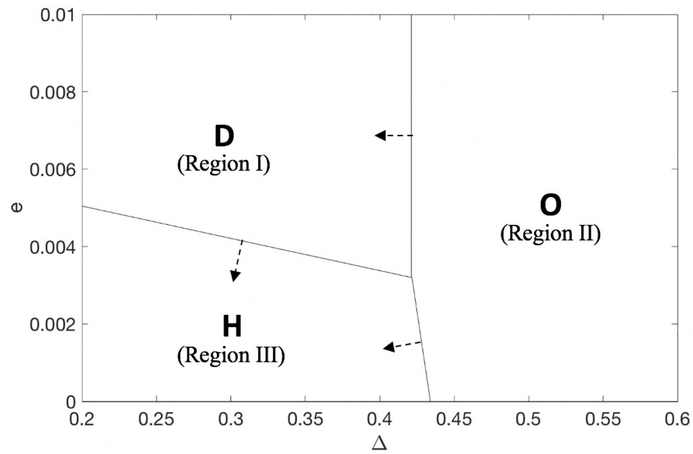


Figure 2.4: Equilibrium Sourcing Strategy as a Function of the Difference in Production Costs Between Two Suppliers (Δ) and the Penalty from Law Enforcement in the Overseas Market (e). (Dashed arrows indicate how threshold lines change as γ increases. In this example, $p_B = 0.75$, $k_2 = 0.01$, $\beta = 0.4$, $\gamma = 0.85$, $\alpha = 1$.)

When the penalty from law enforcement e is relatively high, *i.*e., $e \geq \min\{(e_1)^+, (e_2)^+\}$ (Regions I and II), the brand-name firm can offer wholesale price w_2^D or w_2^O to incentivize the counterfeiter to accept the contract (*i.e.*, $d_2^* = 1$). Such a wholesale contract with a price premium to the counterfeiter can successfully convert her into an overseas supplier. The difference in production costs between two suppliers Δ represents the cost advantage of the overseas supplier. If the cost advantage is not very prominent (Region I), the brand-name firm sources from both suppliers, leading to Strategy D. This strategy helps the brand-name firm mitigate the competition in the overseas market without depending too much on the production of the overseas supplier. For example, Kokuyo teams up with Chinese Gambol to produce and sell their notebooks in China and still keeps its home supplier (Sugawara 2015). If the cost advantage is large (Region II), the brand-name firm chooses to source only from the overseas supplier, leading to Strategy O. This result is consistent with industrial examples, in which brand-name firms source only from overseas suppliers due to lower production costs. For example, the luxury brand Balenciaga switches the Triple S product line from Italy to China Putian factories due to lower labor costs.

When e is low, the counterfeiter obtains a high profit if rejecting the contract and entering the overseas market to sell the counterfeit, and hence becomes less willing to be converted. Thus, the wholesale price w_2^D or w_2^O has to be high enough to attract the counterfeiter, which squeezes the brand-name firm's profit. At the same time, if the cost advantage Δ is low, the benefit from sourcing from the overseas supplier is less prominent. Therefore, the brand-name firm prefers to source only from the home supplier, leading to Strategy H, as shown in Figure 2.4 in Region III. This result helps us understand why some counterfeiters in Turkey are not willing to be authorized suppliers for brand-name firms. Firstly, the production cost of high-quality counterfeits is not too far from authentic products. Secondly, the penalty from law enforcement in Turkey is not high and counterfeiting is penalized only if it is reported. Some brand-name firms even tolerate counterfeiters and treat them as a means to raise brand awareness (Letsch 2011).

Dashed arrows in Figure 2.4 illustrate that as γ increases, the region of Strategy H shrinks and the region of Strategy O expands. To explain this, recall that consumers value the brand-name product produced by the overseas supplier more when there is a higher γ, which leads to two effects. On the one hand, the brand-name firm becomes more willing to source from the overseas supplier due to a higher demand for her product. On the other hand, a lower wholesale price is needed to convert the counterfeiter to the overseas supplier; *i.e.*, w_2^D or w_2^O decreases as γ increases (which can be shown from Lemma 2.1). Therefore, when γ increases, the brand-name firm becomes less likely to source only from the home supplier under Strategy H and more likely to adopt Strategy O.

Figure 2.5 further illustrates the equilibrium with respect to the perceived quality of the counterfeit, β, and the overseas market size, α. The impact of β under a high γ is different from that under a low γ. Specifically, when γ is high (*i.e.*, $\gamma > \frac{p_B(p_B-k_2)}{p_B(p_B-k_2)+\Delta(1-p_B)}$), consumers value the brand-name product produced by the overseas supplier similarly to the one produced by the home supplier. In Figure 2.5(a), as β increases, the equilibrium sourcing strategy first changes from Strategy D to Strategy O, then to Strategy H. The reasons are as follows. Recall that when β is high, consumers enjoy a high value from purchasing the counterfeit. This leads to a high profit for the counterfeiter if she rejects the contract and sells the counterfeit in the overseas market. As a result, in order to convert the counterfeiter, the brand-name firm has to offer a high enough wholesale price. In the case when β is high, it is not beneficial for the brand-name firm to do so. Therefore, he chooses not to convert the counterfeiter and adopts Strategy H. For example, consumers in Turkey generally have a high acceptance of fake products. According to the International Chamber of Commerce (ICC), 58% of them admit to "regularly" purchasing illicit products (Review 2016). Therefore, it is not easy to convert counterfeiters in Turkey to authorized suppliers. When β is moderate or low, the brand-name firm can use a relatively low wholesale price to convert the counterfeiter. Therefore, we have Strategy O and Strategy D as the equilibrium. Specifically, when β is moderate, Strategy O is adopted. This is because compared with Strategy D under which the counterfeiter only supplies a single market (the overseas market) if being converted, Strategy O under which the counterfeiter supplies both markets is more attractive for the counterfeiter. Since selling the counterfeit is still relatively profitable for the counterfeiter with moderate

β, the brand-name firm chooses to adopt Strategy O in exchange for a lower wholesale price to convert the counterfeiter. This is in contrast to the case with low β, when Strategy D is adopted to convert the counterfeiter.

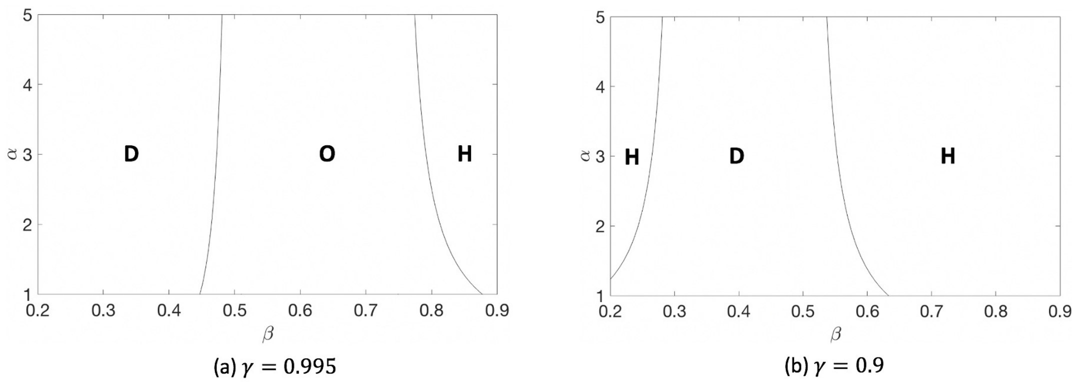


Figure 2.5: Equilibrium Sourcing Strategy as a Function of the Perceived Quality of the Counterfeit (β) and the Overseas Market Size (α). (In these examples, $p_B = 0.75$, $k_2 = 0.01$, $\Delta = 0.02$, $e = 0.03$.)

When γ is low, consumers do not value the brand-name product produced by the overseas supplier. In Figure 2.5(b), as β increases, the equilibrium sourcing strategy first changes from Strategy H to Strategy D, then back to Strategy H. Note that we have two regions with Strategy H being the equilibrium sourcing strategy. Interestingly, these two equilibria stem from different rationales. When β is low, consumers have a low valuation of the counterfeit, so the brand-name firm does not face fierce competition if the counterfeiter enters the overseas market. Given that consumers do not value the product produced by the overseas supplier, the brand-name firm has no incentive to convert the counterfeiter. As β increases, the benefit of mitigating the competition in the overseas market becomes more prominent. Therefore, the equilibrium sourcing strategy switches to Strategy D for a moderate β. When β is high, similar to the discussion of the high γ case above, converting the counterfeiter requires a high wholesale price. It is not beneficial for the brand-name firm to do so even though he has an incentive to mitigate the competition. Therefore, he will solely rely on the home supplier.

There may be a positive correlation between β and γ in practice since both parameters reflect consumers' quality perception of the products produced by the same entity, *i.e.,* the counterfeit produced by the counterfeiter or the brand-name product produced by the overseas supplier. In this case, our results can still be applied. For example, if the perceived quality of both the counterfeit and the brand-name product produced by the overseas supplier is high, the equilibrium strategy would be Strategy O or Strategy H, as illustrated in Figure 2.5(a).

In Figure 2.5, as the overseas market size α increases, interestingly, the brand-name firm may become less dependent on the overseas supplier, switching from Strategy O to Strategy D or Strategy H. This is because if the counterfeiter rejects the contract and sells the counterfeit, as α increases, the demand of the counterfeit increases. As a result, a higher wholesale price is needed to convert the counterfeiter; *i.e.*, w_2^D or w_2^O increases as α increases. This squeezes the brand-name firm's profit from converting the counterfeiter. Therefore, as α increases, the brand-name firm is more likely to adopt Strategy D or Strategy H.

To summarize, we observe that when the difference in production costs between the two suppliers (Δ) and the discount factor of using the overseas supplier (γ) are low, the brand-name firm may not have the incentive to convert the counterfeiter due to the limited cost saving and significant brand value loss. When Δ or γ is high, the brand-name firm has the incentive to convert the counterfeiter, but he may not be able to do so when the wholesale price needed to convert the counterfeiter is too high due to: a large overseas market size (α), a low penalty from law enforcement (e), or a high perceived quality of the counterfeit (β). In this case, the conventional anti-counterfeiting measures, such as raising lawsuits against counterfeiters (*i*.e., increasing e) and educating consumers about the risk of purchasing counterfeits (*i*.e., reducing β), can facilitate the conversion.

2.5 Impact of Converting the Counterfeiter

In this section, we compare our model with the new anti-counterfeiting measure that converts the counterfeiter to an authorized overseas supplier (referred to as the base model) and a benchmark in which only conventional anti-counterfeiting measures (*i.e.*, law enforcement and consumer education) can be used. In Section 2.5.1, we introduce the benchmark and compare converting the counterfeiter with driving her out of the market by conventional measures. Sections 2.5.2 and 2.5.3 further compare the base model with the benchmark in terms of profits, consumer and social surplus. In particular, we examine how the penalty from law enforcement (e) and the perceived quality of the counterfeit (β) affect these comparisons.

2.5.1 Benchmark

We first present a benchmark that the brand-name firm does not have the option to convert the counterfeiter and only sources from the home supplier. We use the accent "$-$" to denote the benchmark.

Strategy H: given wholesale price $\bar{w}_1$, the home supplier accepts the contract, *i.e.*, $\bar{d}_1 = 1$. The brand-name firm competes with the counterfeiter if she enters the overseas market to sell the counterfeit with the optimal retail price $\bar{p}_2^{H*}$, where $\bar{p}_2^{H*} = p_2^{H*}$. The expected profit of each firm is the same as that under the base model, respectively:

$$\bar{\pi}_B^H\left(\bar{w}_1\right)=\pi_B^H\left(w_1,w_2\right),\quad \bar{\pi}_1^H\left(\bar{w}_1\right)=\pi_1^H\left(w_1\right),\quad \bar{\pi}_2^H=\pi_2^H.$$

Strategy N: given wholesale price $\bar{w}_1$, the home supplier rejects the contract, *i.e.*, $\bar{d}_1=0$. The counterfeiter determines the optimal retail price $\bar{p}_2^{N*}$ of the counterfeit to maximize the profit if she enters. Note that $\bar{p}_2^{N*}=p_2^{N*}$. The expected profit of each firm is the same as that under the base model, respectively:

$$\bar{\pi}_B^N\left(\bar{w}_1\right)=\pi_B^N\left(w_1,w_2\right),\quad \bar{\pi}_1^N\left(\bar{w}_1\right)=\pi_1^N\left(w_1\right),\quad \bar{\pi}_2^N=\pi_2^N.$$

In the benchmark, if the counterfeiter cannot obtain a non-negative profit by selling the counterfeit in the overseas market, she will not enter. Thus, the optimal entry decision of the counterfeiter to sell the counterfeit is

$$\bar{s}^*(\bar{w}_1)=\begin{cases}0, & \text{if } \bar{\pi}_2^H(\bar{s}=1,\bar{p}_2^{H*})<\bar{\pi}_2^H(\bar{s}=0),\\ & \text{or, if } \bar{\pi}_2^N(\bar{s}=1,\bar{p}_2^{N*})<\bar{\pi}_2^N(\bar{s}=0),\\ 1, & \text{otherwise.}\end{cases}$$

From $\bar{\pi}_B\left(\bar{w}_1\right)$ and $\bar{\pi}_1\left(\bar{w}_1\right)$, it is easy to obtain that the brand-name firm's optimal wholesale price for the home supplier is $\bar{w}_1^*=k_1$, and the home supplier would accept the contract, *i.e.*, $\bar{d}_1^*=1$. That is to say, in the benchmark, the brand-name firm adopts Strategy H, in which the counterfeiter enters the market and competes with the brand-name firm by setting the optimal retail price $p_2^*=\bar{p}_2^{H*}$ for the counterfeit. We denote the optimal profits of the brand-name firm, the home supplier and the counterfeiter as $\bar{\pi}_B^*$, $\bar{\pi}_1^*$ and $\bar{\pi}_2^*$, respectively, in this benchmark.

In the following, we compare converting the counterfeiter with driving her out of the market. Specifically, we compare the thresholds of e and β, at which the counterfeiter does not enter the overseas market to sell the counterfeit, in the base model (*i.e.*, $s^* = 0$) and those in the benchmark (*i.e.*, $\bar{s}^* = 0$). In the base model, the counterfeit is not in the market when the counterfeiter accepts the contract to be converted to an overseas supplier under either Strategy D or Strategy O. Thus, we use e_0 to denote the minimum level of the penalty from law enforcement and use β_0 to denote the maximum level of the perceived quality of the counterfeit, at which the counterfeiter would be converted. For ease of exposition, in the following, we focus on the case in which the brand-name firm has the incentive to convert the counterfeiter, *i.e.*, $\gamma \geq \frac{p_B(p_B-k_2)}{p_B(p_B-k_2)+\Delta(1-p_B)}$. In the benchmark, the counterfeit is not in the market when e is high enough or β is low enough such that the profit of the counterfeiter becomes zero if she enters the market. We use $\bar{e}_0$ to denote the minimum level of the penalty from law enforcement and use $\bar{\beta}_0$ to denote the maximum level of the perceived quality of the counterfeit, at which the counterfeiter cannot enter the market. We have the following proposition.

Proposition 2.2. $e_0 < \bar{e}_0$, $\beta_0 > \bar{\beta}_0$.

Proposition 2.2 shows that, compared with the benchmark in which the brand-name firm combats counterfeiting by driving the counterfeiter out of the market through a high penalty from law enforcement or a low perceived quality of the counterfeit, the brand-name firm can mitigate the risk of counterfeiting through converting her to an authorized overseas supplier at a lower penalty or a higher perceived quality. Alternatively said, it is easier for the brand-name firm to combat counterfeiting through conversion than by driving the counterfeiter out of the market. Proposition 2.2 implies that the brand-name firm should consider converting the counterfeiter to an authorized supplier when driving the counterfeiter out of the market is difficult. This result is consistent with observations from practice in which more brand-name firms choose to source from imitators with high manufacturing capabilities in those countries with weaker law enforcement and lower consumer awareness.

2.5.2 Comparison of Profits

By comparing the equilibrium profits of firms in the base model with those in the benchmark, we obtain Corollary 2.1.

Corollary 2.1. *(a) For the brand-name firm,* $\pi_B^H = \bar{\pi}_B^*$, $\pi_B^D \geq \bar{\pi}_B^*, \pi_B^O \geq \bar{\pi}_B^*$.
(b) For the counterfeiter, $\pi_2^H = \pi_2^D = \bar{\pi}_2^*$, $\pi_2^O \geq \bar{\pi}_2^*$.
(c) For the home supplier, $\pi_1^H = \pi_1^D = \pi_1^O = \bar{\pi}_1^*$.

Corollary 2.1 shows that converting the counterfeiter brings a win-win outcome for the brand-name firm and the counterfeiter. It is straightforward that when Strategy H is the equilibrium strategy, the profit of each firm in the base model is the same as that in the benchmark. In the following, we focus on the cases in which Strategy D or Strategy O is the equilibrium strategy in the base model. Corollary 2.1(a) shows that, compared with the benchmark, Strategy D and Strategy O benefit the brand-name firm. By converting the counterfeiter to an overseas supplier, the brand-name firm not only takes advantage of the low production cost of the overseas supplier but also expands his market share in the overseas market by mitigating the competition with the counterfeiter. This is consistent with practical examples of Japanese stationery makers Kokuyo (Sugawara 2015) and Honda (Zaun and Leggett 2016). Corollary 2.1(b) shows that compared with the benchmark, converting the counterfeiter by Strategy D does not affect the counterfeiter's profit. This is because, with the home supplier being available, the brand-name firm sets the wholesale price w_2^D so that the counterfeiter is indifferent between accepting and rejecting the contract for entering the overseas market to sell the counterfeit, under which the counterfeiter obtains the same profit as in the benchmark. However, converting the counterfeiter by Strategy O can bring extra benefits to the counterfeiter over the benchmark. This is because the profit of the overseas supplier obtained under Strategy O equals the profit that the counterfeiter would have obtained if she sells the counterfeit in the overseas market as a monopoly, which is larger than her profit in the benchmark. Corollary 2.1(c) is intuitive because, in equilibrium, the brand-name firm can always source from the home supplier at the marginal cost, k_1, in both the benchmark and the base model with either Strategy H or Strategy D.

Next, we examine how the penalty from law enforcement (e) and the perceived quality of the counterfeit (β) affect the comparison of the profits between the base model and the benchmark. Let $\tilde{\beta}$ be the unique boundary between the equilibrium with Strategy O and that with Strategy D, as shown in Figure 2.5(a).

Proposition 2.3. *(a) When the penalty from law enforcement, $e(\in [0, \pi_0))$, increases,*

(i) $(\pi_B^* - \bar{\pi}_B^*)$ increases;

(ii) $(\pi_2^* - \bar{\pi}_2^*)$ increases.

(b) When the perceived quality of the counterfeit, $\beta(\in (\underline{\beta}, 1])$, increases,

(i) $(\pi_B^* - \bar{\pi}_B^*)$ increases if $\beta \leq \beta_1$, and decreases if $\beta > \beta_1$;

(ii) $(\pi_2^* - \bar{\pi}_2^*)$ increases if $\beta \leq \tilde{\beta}$, and decreases if $\beta > \tilde{\beta}$.

Proposition 2.3(a) shows that the benefits of converting the counterfeiter (*i.e.,* the profit differences between the base model and the benchmark) for both the brand-name firm and the counterfeiter increase in e. For the brand-name firm, by Corollary 2.1(a), there is a benefit (*i.e.,* $(\pi_B^* - \bar{\pi}_B^*) > 0$) only when Strategy O or Strategy D is adopted. Under these two strategies, the brand-name firm's profit π_B^* increases in e. This is because as e increases, the counterfeiter is less profitable if rejecting the contract and selling the counterfeit, so the brand-name firm is able to offer a lower wholesale price w_2^D or w_2^O to convert the counterfeiter. In the benchmark, the brand-name firm's profit $\bar{\pi}_B^*$ is not affected by e. Therefore, the benefit of converting the counterfeiter, $(\pi_B^* - \bar{\pi}_B^*)$, becomes larger for the brand-name firm with a higher e.

For the counterfeiter, it is easy to see that in the benchmark, the counterfeiter's profit, $\bar{\pi}_2^*$, decreases in e. Interestingly, in the base model, the profit of the counterfeiter π_2^* can be non-monotone in e. Specifically, as e increases, the equilibrium may change either from Strategy H to Strategy D, or from Strategy H to Strategy O (see Figure 2.4). For the former case, according to Corollary 2.1(b), $(\pi_2^* - \bar{\pi}_2^*)$ is zero, meaning that there is no benefit of converting the counterfeiter. For the latter case, as e increases, $(\pi_2^* - \bar{\pi}_2^*)$ increases from zero to a positive value. This is because as e increases, being converted to an overseas supplier becomes more attractive and under Strategy O, the counterfeiter can obtain a higher profit due to the wholesale price premium and the opportunity to produce brand-name products for both markets.

Proposition 2.3(b) shows that the benefits of converting the counterfeiter are non-monotone in β for both the brand-name firm and the counterfeiter. Specifically, as β increases, the benefit of the brand-name firm increases when β is low (*i.e.*, $\beta \leq \beta_1$), and decreases when β is high (*i.e.*, $\beta > \beta_1$), as illustrated in Figure 2.6(a). Note that β_1 is the critical value of the perceived quality of the counterfeit, above which the brand-name firm loses the entire overseas market to the counterfeiter in the benchmark.Thus, when $\beta > \beta_1$, in the benchmark, the brand-name firm does not earn any profit in the overseas market, and his profit does not change with β. In the base model, the brand-name firm can adopt Strategy O and make a profit in the overseas market. So converting the counterfeiter grants the brand-name firm a profit benefit (*i.e.*, $(\pi_B^* - \bar{\pi}_B^*) > 0$). However, as β increases, the counterfeit is perceived of higher quality, so a higher wholesale price is needed to convert the counterfeiter. Therefore, the profit benefit for the brand-name firm decreases. When $\beta \leq \beta_1$, the profits of the brand-name firm decrease in both the benchmark and the base model as β increases. However, as the brand-name firm is able to convert the counterfeiter, the negative impact from a higher β on the profit in the base model is less than that in the benchmark where the brand-name firm directly competes with the counterfeiter. Therefore, as β increases, $(\pi_B^* - \bar{\pi}_B^*)$ increases.

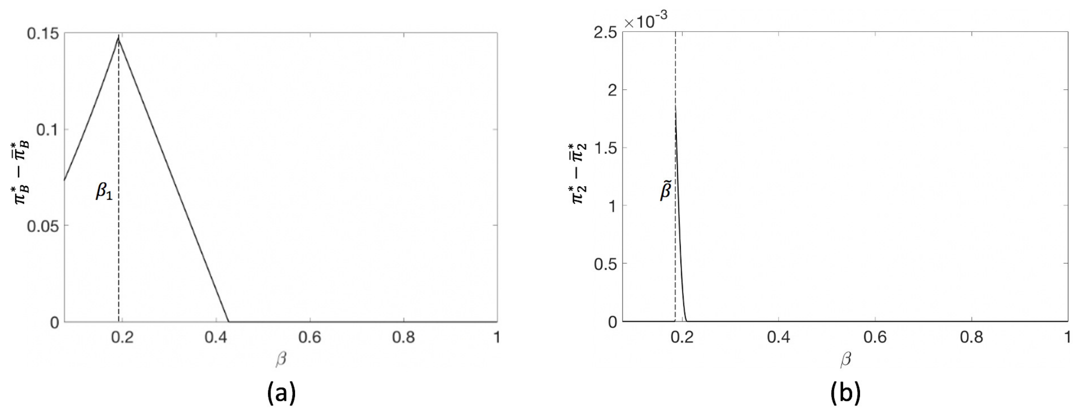


Figure 2.6: Benefits of Converting the Counterfeiter as a Function of the Perceived Quality of the Counterfeit (β). (In this example, $p_B = 0.9$, $k_2 = 0.01$, $\Delta = 0.02$, $e = 0.03$, $\gamma = 1$, $\alpha = 2.5$.)

For the counterfeiter, as β increases, her benefit of being converted increases when $\beta \leq \tilde{\beta}$, and decreases when $\beta > \tilde{\beta}$, as illustrated in Figure 2.6(b). Note that $\tilde{\beta}$ is the critical value of the perceived quality of the counterfeit, above which the brand-name firm adopts Strategy O to convert the counterfeiter. When $\beta < \tilde{\beta}$, Strategy D is adopted, and the benefit for the counterfeiter being converted, $(\pi_2^* - \bar{\pi}_2^*)$, is zero (Corollary 2.1(b)). When $\beta = \tilde{\beta}$, Strategy O is adopted, and the benefit becomes positive due to the wholesale price premium and the opportunity to supply both markets. When $\beta > \tilde{\beta}$, as β increases, the counterfeiter's profits increase in both the benchmark and the base model. However, the positive impact from a higher β on the profit is more profound in the benchmark where the counterfeiter directly competes with the brand-name firm than that in the base model where the counterfeiter is converted to the overseas supplier. Therefore, $(\pi_2^* - \bar{\pi}_2^*)$ decreases in β.

2.5.3 Comparison of Consumer and Social Surplus

In the benchmark without the option to convert the counterfeiter, in equilibrium, the consumer surplus in the home market, $\overline{CS}_1$, and that in the overseas market, $\overline{CS}_2$, are given as follows:

$$\overline{CS}_1 = \int_{p_B}^{1} (\theta - p_B)d\theta,$$

$$\overline{CS}_2 = \begin{cases} \alpha \int_{\hat{\theta}_2}^{\widetilde{\theta}} (\theta\beta - \bar{p}_2^*)d\theta + \alpha \int_{\widetilde{\theta}}^{1} (\theta - p_B)d\theta, & \text{if } \underline{\beta} < \beta < \beta_1, \\ \alpha \int_{\hat{\theta}_2}^{1} (\theta\beta - \bar{p}_2^*)d\theta, & \text{if } \beta_1 \leq \beta < 1, \end{cases}$$

where $\widetilde{\theta} = \frac{(2-\beta)p_B - k_2}{2(1-\beta)}$, $\hat{\theta}_2 = \frac{\bar{p}_2^*}{\beta}$. In particular, when $\underline{\beta} < \beta < \beta_1$, the first term in $\overline{CS}_2$ represents the surplus of consumers who purchase the counterfeit, and the second term is the surplus of consumers who purchase the brand-name product. Let $\overline{CS}$ and $\overline{SS}$ denote the total consumer surplus and the social surplus in the benchmark, that is, $\overline{CS} = \overline{CS}_1 + \overline{CS}_2$, and $\overline{SS} = \overline{CS} + \bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^*$.

In the base model with the option to convert the counterfeiter, in equilibrium, under Strategy H, the consumer surplus in each market, CS_1^H and CS_2^H, the total consumer surplus in the two markets, CS^H, and the social surplus, SS^H, are the same as those in the benchmark. In equilibrium, under either Strategy D or Strategy O, the brand-name firm is the only firm in the two markets. Let CS_1^D and CS_2^D denote the consumer surplus in each market under Strategy D in equilibrium:

$$CS_1^D = \int_{p_B}^{1} (\theta - p_B)d\theta, \quad CS_2^D = \alpha \int_{\frac{p_B}{\gamma}}^{1} (\theta\gamma - p_B)d\theta.$$

Let CS_1^O and CS_2^O denote the consumer surplus in each market under Strategy O in equilibrium:

$$CS_1^O = \int_{\frac{p_B}{\gamma}}^{1} (\theta\gamma - p_B)d\theta, \quad CS_2^O = \alpha \int_{\frac{p_B}{\gamma}}^{1} (\theta\gamma - p_B)d\theta.$$

$CS^D(CS^O)$ denotes the total consumer surplus under Strategy D(O) in equilibrium, that is, $CS^D = CS_1^D + CS_2^D$, and $CS^O = CS_1^O + CS_2^O$. Similarly, $SS^D(SS^O)$ denotes the social surplus under Strategy D(O) in equilibrium: $SS^D = CS^D + \pi_B^D + \pi_1^D + \pi_2^D$ and $SS^O = CS^O + \pi_B^O + \pi_1^O + \pi_2^O$.

Proposition 2.4. *(a) In the home market,* $CS_1^H = CS_1^D = \overline{CS}_1$, $CS_1^O \leq \overline{CS}_1$.
(b) In the overseas market, $CS_2^H = \overline{CS}_2, CS_2^D = CS_2^O < \overline{CS}_2$.
(c) In the two markets, $CS^H = \overline{CS}, CS^D < \overline{CS}$, $CS^O < \overline{CS}$.

Proposition 2.4(a) shows that compared with the benchmark, in equilibrium, Strategy D has no impact on the consumer surplus in the home market whereas Strategy O reduces consumer surplus in the home market. This happens because of the perceived quality discount factor γ; *i.e.*, consumer surplus is reduced in the home market when consumers purchase the products produced by the overseas supplier under Strategy O. Proposition 2.4(b) shows that both Strategy D and Strategy O cut down the consumer surplus in the overseas market. In addition to the effect of γ, in the benchmark, the brand-name firm competes with the counterfeiter in the overseas market. This competition benefits consumers through lower retail prices. However, under either Strategy D or Strategy O, there is a lack of competition in the overseas market. Proposition 2.4(c) shows that the total consumer surplus of the two markets is lower in the base model than in the benchmark.

Moreover, we find that this loss in consumer surplus does not change with the penalty from law enforcement e, but can be non-monotone in the perceived quality of the counterfeit β. For e, since the retail price of either the brand-name product p_B or the counterfeit p_2^* is not affected by e, consumer surplus does not change with e. For β, in the base model with either Strategy D or Strategy O, consumer surplus is not affected by β as the brand-name firm does not compete with the counterfeiter. However, in the benchmark, an increase of β makes the competition in the overseas market more fierce, which may increase the consumer surplus. As a result, when β is high enough, the difference in consumer surplus between the base model and the benchmark decreases as β decreases.

Next, we analyze the impact of converting the counterfeiter on the social surplus. According to Corollary 2.1 and Proposition 2.4, compared with the benchmark, converting the counterfeiter benefits the profits of firms, but leads to consumer surplus loss. Thus, for the comparison of social surplus with the benchmark, it depends on whether the gain in profits or the loss in consumer surplus dominates.

Proposition 2.5. $SS^H = \overline{SS}$, $SS^D > \overline{SS}$ *if* $e > (e_1')^+$, $SS^O > \overline{SS}$ *if* $e > (e_2')^+$.

The thresholds e_1' and e_2' are provided in the proof of the proposition. Proposition 2.5 shows that neither Strategy D nor Strategy O necessarily improves the social surplus compared with the benchmark. Specifically, Strategy D and Strategy O improve the social surplus when e is high. The reason is as follows. Compared with the benchmark, under Strategy D or Strategy O, as e

increases, the loss in consumer surplus remains the same, whereas the gain in profits increases as the counterfeiter's profit in the benchmark gets lower with higher e. When e is sufficiently high, the gain in profits dominates the loss in consumer surplus. Thus, Strategy D and Strategy O benefit society with a high e, and this benefit increases in e as well.

We can further show that the thresholds e'_1 and e'_2 first decrease and then increase in β when $\beta < \beta_0$, where β_0 is the critical value above which Strategy H is adopted. Hence, Strategy D and Strategy O lead to a higher social surplus when β is low. This is because, under Strategy D or Strategy O, the gain in profits do not change with β. As β decreases, the loss in consumer surplus becomes smaller and is more likely to be dominated by the gain in profits, which makes the social surplus under Strategy D or Strategy O higher than that in the benchmark. Therefore, the social surplus benefits from converting the counterfeiter with a low β, but this benefit may be non-monotone in β.

The above analysis of the consumer surplus and social surplus provides insights for governments on whether they should encourage converting counterfeiters to authorized overseas suppliers. On the one hand, when consumers in emerging markets have a high brand awareness and a low perceived value of counterfeits, converting counterfeiters should be encouraged because it would benefit brand-name firms, overseas suppliers and society. On the other hand, when consumers in emerging markets find counterfeits attractive and have a high perceived value of counterfeits, caution should be taken about converting counterfeiters.

2.6 Summary

High-quality counterfeits are becoming prevalent in emerging markets. With the option to convert these counterfeiters to authorized overseas suppliers, we seek to better understand the equilibrium sourcing strategy of brand-name firms and the effectiveness of this new anti-counterfeiting measure. We develop a game-theoretic model based on a two-tier supply chain to capture the interactions between a brand-name firm with a licit home supplier and a counterfeiter producing "super fakes," who may be converted to an authorized overseas supplier. We find that each of three possible sourcing strategies —single sourcing from the home supplier, dual sourcing and single sourcing from the overseas supplier —can be optimal under different circumstances. We examine how factors, such as the penalty from law enforcement, the perceived quality of the counterfeit, the overseas market size, the difference in production costs between two suppliers and the discount factor of using the overseas supplier, affect the equilibrium sourcing strategy. In addition, we discuss the interaction between this new anti-counterfeiting measure of converting the counterfeiter and the conventional measures, such as law enforcement and consumer education. Lastly, we investigate the impact of converting the counterfeiter to an overseas supplier on consumers and society.

Our results provide guidance to brand-name firms on the adoption of the new anti-counterfeiting measure of converting counterfeiters, which is captured by brand-name firms' equilibrium sourcing strategy in the presence of counterfeiters. When the difference in production costs between the two suppliers and the discount factor of using the overseas supplier are low, the brand-name firm may not have the incentive to convert the counterfeiter due to the limited cost saving and significant brand value loss. Otherwise, the brand-name firm has the incentive to convert the counterfeiter through either dual sourcing or single sourcing. However, the brand-name firm may still fail to convert the counterfeiter to an authorized overseas supplier and adopt single sourcing from the home supplier, when the overseas market size is large, the penalty from law enforcement is low, or the perceived quality of the counterfeit is high. This result provides an explanation for why some counterfeiters producing "super fakes" refuse to be authorized to become licit suppliers. In such cases, the conventional anti-counterfeiting measures, such as lawsuits to increase the penalty from law enforcement for counterfeiting and education campaigns to reduce consumers' perceived value of counterfeits, may be adopted to facilitate the conversion.

In addition, we find that without the option to convert the counterfeiter, stronger efforts in consumer education and law enforcement are required in order to mitigate the counterfeiting threat. Therefore, brand-name firms should consider adopting this new anti-counterfeiting measure of converting counterfeiters especially when driving counterfeiters out of markets is challenging. Interestingly, increasing penalty from law enforcement enhances the benefits of converting counterfeiters whereas education campaigns to reduce consumers' perceived value of counterfeits may reduce the benefits.

Lastly, we find that converting the counterfeiter to an overseas supplier may hurt consumer surplus and does not always improve social surplus. This is because consumers may prefer the product produced by the home supplier to that produced by the overseas supplier, and more importantly, converting the counterfeiter mitigates the competition in the overseas market resulting in a surplus loss. When the penalty from law enforcement is high or the perceived quality of the counterfeit is low, converting the counterfeiter benefits society. On the contrary, when law enforcement is weak or consumers find the counterfeit attractive, caution should be taken about converting the counterfeiter.

Our study can be extended in various directions for future research. First, we assume in our model that if the counterfeiter is converted to an overseas supplier, she will not sell the counterfeit and there is no other counterfeiter in the overseas market. However, in some cases, the authorized overseas supplier may still sell the counterfeit. Then the brand-name firm may have to closely monitor the overseas supplier and choose a high wholesale price to increase her opportunity cost. It is also possible that there exist multiple counterfeiters in the overseas market. Converting one counterfeiter then will not eliminate counterfeiting completely, which can be interesting to explore. Second, we assume that consumers would not purchase across markets, but some consumers in the home market might be attracted by low-price counterfeits in the overseas market. These switching consumers increase the market shares of counterfeiters, which makes it more difficult for brand-name firms to convert them to overseas suppliers. Hence, an exploration of strategic consumers might yield further insights. Third, one may consider the risk aversion of counterfeiters facing the penalty of law enforcement. In this case, they are more likely to be converted to authorized suppliers. Lastly, we assume that the home supplier obtains zero profit when she rejects the brand-name firm's contract. It would be insightful to study the case in which the home supplier may have the option to produce products for other companies when she rejects the contract.

Chapter 3

Stochastic Capacity Investment and Flexible vs. Dedicated Technology Choice in the Presence of Subscription Programs

3.1 Introduction

For classic technology choice problem, dedicated technology provides product-dedicated capacity to produce single product, flexible technology provides product-flexible capacity to produce multiples products. Under traditional business model where a manufacturer sells the ownership of products, although flexible technology has a higher unit capacity investment cost than dedicated technology, it brings capacity-pooling benefit by reallocating capacity between products in response to demand realizations. In recent years, a new emerging business model, car subscription, is prevalent in automotive industry. Several leading automotive manufacturers have started offering their car subscription programs as a part of new mobility-based business models, such as, "Care by Volvo," "Porsche Passport," and "Access by BMW" program (Campbell and Waldmeir 2017). Different from selling the ownership of cars, car subscription programs sell the usage of cars and allowing consumers to switch in a fleet of cars. In 2021, the Boston Consulting Group (BCG) predicted: “by 2030, car subscriptions may become a 30 billion to 40 billion market opportunity in Europe and the United States” (Schellong et al. 2021). In this paper, we aim to develop a theoretical basis for understanding the impact of subscription programs on a manufacturer in stochastic capacity investment and flexible versus dedicated technology choice.

There are two key features of these car subscription programs. Firstly, customers have the ability to switch vehicles. In these car subscription programs, instead of purchasing cars, customers pay an all-inclusive monthly fee (including the cost of insurance, maintenance, and road assistance) to use a fleet of vehicles from different models during the subscription period. For example, in ICON subscription program of BMW, if you choose one car model, and after usage, you can switch to another car model in the subscription period. BCG report also shows that one out of four customers list switching as a major criterion for subscribing to this service (Schellong et al. 2021). Secondly, the manufacturer is responsible for managing the inventory of each car model. It means that the manufacturer needs to decide the fleet size for each car model and manage switching customers for their desired car model. However, until start of year 2021, the leading car manufacturers, BMW, Audi and Mercedes-Benz shut down their subscription programs (Elliott 2021). For Audi, in order to allow customers swapping in and out of vehicles, they need to hold enough inventory, which is too costly (Baldwin 2021). For BMW, it is also difficult to figure out subscription fleet size and product mix (Bell 2021). Cadillac also expressed that let customers drive multiple cars each month was too costly. Thus, the fleet size for each car model and the volume of cars reserved for satisfying the requests from switching customers are two challenging decisions for the manufacturer (Porter 2018). Since the manufacturers are responsible for producing cars, they need to decide on the production technology and capacity investment levels for car models with uncertain subscription demands and incorporating these two key features in these programs.

Thus, it is challenging to make capacity investment and technology choice decisions in providing subscription programs. In this paper, we aim to develop a theoretical basis for understanding the challenges associated with offering subscription programs and its implications on the flexible versus dedicated technology choice and the subsequent capacity investment decision.

We focus on answering following research questions.

(1) What are the optimal capacity investment decisions and subsequent product allocation and reservation decisions with each technology in the presence of subscription programs?

(2) What are the impacts of proportion of switching requests and correlation between two types of subscription demands on the profitability with each technology?

(3) How do subscription programs shape the flexible versus dedicated technology choice? In particular, we are interested in addressing the following three aspects.

(i) How does the introduction of subscription programs affect the technology choice?

(ii) How does the proportion of switching requests affect the technology choice?

(iii) How does the subscription demand correlation affect the technology choice?

To answer our research questions, we consider a firm that produces two products to be offered under subscription programs with demand uncertainty to maximize its expected profit in a single planning period. The firm offers two types of subscription programs. In particular, in type i subscription program, customers are first allocated to product i, and after usage, a fixed proportion (which we denote as switch proportion) of these customers make an attempt to switch to the other product $-i$. We model the firm's decisions as a two-stage stochastic program. In the first stage, the firm makes the technology (flexible versus dedicated) and capacity investment (with the chosen technology) decisions under subscription demand uncertainties. In the second stage, after these uncertainties are realized, the firm, constrained by the capacity invested, decides on product i's volume for (i) allocation to satisfy the type i subscription demand and (ii) reservation to satisfy future switching requests under type $-i$ subscription program. The firm satisfies the switching requests for product i under type $-i$ subscription program with the reserved units as well as the returned units under type i subscription program. A penalty cost is incurred to the firm due to the unsatisfied switching requests. At the end of the second stage (i.e., when the subscription program is over), the firm sells all the products used in the second-hand market. For tractability, we assume that the two products are symmetric. We solve for the optimal product allocation and reallocation decisions in closed form. We find that with dedicated technology, the switching requests can be fully satisfied; whereas the switching requests can not be fully satisfied with flexible technology. We further characterize the optimality conditions for the capacity investment decision with each technology and examine how the demand correlation and the switch-proportion affect the profitability of each technology, respectively. To characterize the optimal technology choice decision, we establish a unique flexible capacity investment cost threshold that determines the optimal technology choice; that is, flexible (dedicated) technology is chosen if flexible capacity cost is lower (higher) than this threshold.

We summarize our main findings below.

(1) With dedicated technology, we find that a higher demand correlation

increases the profitability by decreasing the expected cost associated with satisfying switching requests. This is a novel insight as it is well known that demand correlation does not affect profitability with the dedicated technology in a traditional ownership model. Because in the subscription model, each product's profit depends not only on the uncertainty in subscription demand that this product is allocated to but also uncertainties in other subscription demands that this product is used later on for satisfying future switching requests. Specifically, in the expected profit, as demand correlation increases, the expected total optimal reservation volume, and thus, the expected loss associated with switching requests for both products decreases. As a result, the expected profit with dedicated technology increases. In addition, we show that a higher switch proportion decreases the profitability with dedicated technology, which is consistent with the practice about the difficulty of managing switching requests.

(2) With flexible technology, in a traditional ownership model, the firm enjoys capacity-pooling benefit due to the ability of reallocating the flexible capacity based on demand realizations. We find that while the firm enjoys the same benefit in a subscription model, this benefit is curbed as the re-allocation of capacity between the two products creates a costly imbalance between the future switching requests of each subscription program. Interestingly, we find that under some conditions (which is relevant for any demand correlation) the expected cost associated with satisfying switching requests completely nullifies the capacity-pooling benefit of flexible technology. In this case, demand correlation does not affect profitability with flexible technology. This is contrary to the common intuition based on a traditional model that postulates a higher demand correlation decreases profitability due to a reduction in capacity-pooling benefit. Outside of these conditions, we find that demand correlation matters

for profitability with flexible technology. In particular, we find that a higher demand correlation decreases the capacity-pooling benefit in the subscription model, but at the same time decreases the expected cost associated with satisfying switching requests. We find that for sufficiently high switch-proportion, the latter effect outweighs the former and contrary to common intuition, a higher demand correlation increases the profitability with flexible technology. Moreover, we also show that a higher switch-proportion decreases the profitability with flexible technology.

(3) To examine how the optimal technology choice is impacted by demand correlation and switch proportion, we conduct numerical experiments based on realistic instances. To this end, we calibrate our model to represent the Care by Volvo subscription program using publicly available data, complemented by data obtained from other academic studies. We already establish that a higher demand correlation increases profitability with dedicated technology and decreases (increases) profitability with flexible technology when switch proportion is low (high). From our numerical study, we observe that a higher demand correlation always favors dedicated technology adoption (even when switch proportion is high). This result is consistent with the traditional ownership model. While a higher switch proportion has a negative impact on profitability with each technology, we numerically observe that the impact is less significant with dedicated technology; that is, a higher switch-proportion favors dedicated technology adoption. To complement these numerical observations, we introduce a reformulation in our model to analytically quantify the flexibility premium which captures the additional expected profit generated by optimally reallocating flexible capacity based on the subscription demand realizations. We prove that, consistent with our numerical observations, flexibility premium decreases in demand correlation and switch-proportion.

The remainder of this paper is organized as follows. Section 3.2 surveys the related literature and discusses the contribution of our work. Section 3.3 describes the model and discusses the basis for our assumptions. Section 3.4 and Section 3.5 derive the optimal strategy for a given technology, respectively. Section 3.6 analyzes the optimal technology choice and investigates the impact of proportion of switching requests and demand correlation between two types of subscription programs on this choice. Section 3.7 concludes with the main insights and future research directions.

3.2 Literature Review

We draw upon two related literature streams: 1) stochastic capacity investment and flexible versus dedicated technology choice; 2) servicizing business models.

Our paper is closely related to the literature on stochastic capacity investment and flexible versus dedicated technology choice, in which an extensive number of papers have studied capacity investment problem and the benefit of resource flexibility. Fine and Freund (1990) demonstrate numerically that the value of flexible capacity can disappear in the presence of perfect correlation. Van Mieghem (1998) shows analytically that, given perfect correlation, it may yet be optimal to invest in flexible capacity if there are differences in the profit margins of the products. Netessine et al. (2002) consider the firm providing multiple services using both specialized and flexible capacity in a single period, and study the value of a single level upgrading for lack of capacity of the desired product, in which customers may be upgraded to a higher level of service at no cost to the customer. They find that the correlation structure of demand has a significant impact on the value of resource flexibility: increasing correlation induces a shift from flexible to dedicated capacity. Van Mieghem and Rudi

(2021) introduce a class of models, called newsvendor networks, in which any leftover stock at the end of one period carries over as input to the next period, to study the link to the classic newsvendor model. In this setting, the optimal capacity and inventory decisions balance overages with underages. Bish and Wang (2004) incorporate the effect of the firm's ex-post pricing strategy on the resource investment decision, considering resource flexibility and the impact of demand correlation. Chod and Rudi (2005) consider investment in a flexible capacity with endogenous pricing decision, and demonstrate that the optimal capacity investment level increases with a lower demand correlation assuming a bivariate normal demand uncertainty. Goyal and Netessine (2007) studies the impact of competition on a firm's choice of technology, and find that a firm that invests in flexibility benefits from a low correlation between demands for two products, but the extent of this benefit differs depending on the competitor's technology choice. Goyal and Netessine (2011) find that volume flexibility is indeed a potent tool to harness high (positive) demand correlation between products. Boyabatlı (2015) studies the supply management of a primary input, where this input gives rise to multiple products in fixed proportions. He identifies the critical role that demand correlation plays with the fixed proportions technology: in contrast to the capacity-pooling value, which decreases in demand correlation, the cost-pooling value increases in demand correlation. A number of papers in this literature investigate the interplay between flexible capacity investment and financing frictions (Van Mieghem 2003, Chod et al. 2010, Boyabatlı and Toktay 2011, Boyabatlı et al. 2016). Dong et al. (2022) discuss 3D printing as a production technology with the production flexibility and operational efficiency, and analyze the firm's choice between 3D printing and two conventional technologies (dedicated and traditional flexible). A number of contributions have extended the Netessine et al. (2002) model, including those of Shumsky and Zhang (2009), Yu et al. (2015) and Notz and Pibernik (2022).

For the literature in this first stream, the common findings are as follows: (i) manufacturer's profitability with dedicated technology is independent of the demand correlation; (ii) manufacturer's profitability with flexible technology decreases with the demand correlation; (iii) if the demands are not perfectly positive corrected, flexible technology would always have value over dedicated technology. Different from this stream of literature, we focus on the subscription programs with the key feature of allowing future switching requests. We show that with subscription programs, (i) manufacturer's profitability with dedicated technology increases with the demand correlation; (ii) manufacturer's profitability with flexible technology may increase with the demand correlation.

Our paper is also related to an emerging stream of operations management literature that study servicizing business models, under which the firm selling the functionality of a product instead of the product itself. The papers in this stream examine the manufacturer's servicizing business model design for a particular product and its economic and environmental implications. Agrawal et al. (2012) consider three life-cycle phases, including production, use and disposal, by comparing leasing and selling models, they find that leasing can be environmentally worse despite remarketing all off-lease products and greener than selling despite the mid-life removal of off-lease products. Agrawal and Bellos (2017) endogenize the firm's choice between a pure sales model, a pure servicizing model, and a hybrid model with both sales and servicizing options; and study the environmental impact of servicization with and without resource pooling. Bellos et al. (2017) consider a manufacturer that contemplates car sharing and designs its product line by accounting for the trade-off between driving performance and fuel efficiency. Örsdemir et al. (2019) allow the servicizing firm to tailor the service to consumers' needs, endogenize product durability choice, and use an environmental impact metric that captures the low discretionary use nature of products. Tian et al. (2021) consider the emerging of consumer-to-consumer (C2C) sharing, and studies a manufacturer's optimal entry strategy in the product sharing market to provide business-to-consumer (B2C) rental services in addition to outright sales to consumers, and the economic implications of its entry. Niu et al. (2021) provides empirical evidence disclosing the impact of manufacturers' service offerings on two bullwhip issues, namely "felt" demand variability and intra-firm demand distortion. Agrawal and Toktay (2021) consider a solar power company installs solar panels for a customer, who purchases the electricity generated from the panels. They discuss the solar power company's optimal business model decisions under the context that the adoption of solar panels is promoted by investment and generation subsidies. Shi and Hu (2022) study flexible electric vehicle (EV) battery leasing enabled by a swappable battery design, and investigate whether and when profit-maximizing flexible battery leasing reduces total battery capacity. Different from these papers, our paper focuses on decisions associated with the manufacturing of this product by investigating stochastic capacity investment and technology choice under the setting of exogenous subscription programs.

In summary, there is no work in the literature that characterizes the capacity investment decision and technology choice in the presence of subscription programs. In this paper, we attempt to fill this void.

3.3 Model Description and Assumptions

In this section, we introduce our model setting (§3.3.1) and key assumptions (§3.3.2).

3.3.1 Model Description

The following mathematical representation is used throughout the text: the random variable $\tilde{\xi}$ has a realization ξ. Boldface letters represent column vectors of the required size and the prime symbol ($'$) denotes the transpose operator; Pr denotes probability and $\mathbb{E}$ denotes the expectation operator; $(x)^+ \doteq \max(x, 0)$; and $\Omega_{12}^T = \Omega_1^T \cup \Omega_2^T$. Monotonic relations (increasing, decreasing) are used in the weak sense unless otherwise stated.

A firm produces and sells two products in two types of subscription programs. Under type 1 subscription program, customers firstly request product 1 and might switch to product 2 after the usage of product 1, whereas under type 2 subscription program, customers firstly request product 2 and might switch to product 1 after the usage of product 2.

The firm faces a stochastic demand for each type of subscription program, represented by $\tilde{\boldsymbol{\xi}}' = \left(\tilde{\xi}_1, \tilde{\xi}_2\right)$. We assume $(\tilde{\xi}_1, \tilde{\xi}_2)$ follows a bivariate distribution with mean μ_1, μ_2, and covariance matrix $\boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma}_{jj} = \sigma_j^2$ and $\boldsymbol{\Sigma}_{ij} = \rho\sigma_i\sigma_j$ for $i \neq j$ and ρ denotes the correlation coefficient. Without loss of generality, we assume $\mu_1 = \mu_2 = \mu$, and $\sigma_1 = \sigma_2 = \sigma$. Flexible technology has a single resource with capacity level K^F that is capable of producing two products. Dedicated technology has two resources that can each produce a single product. Since we assume symmetric two products with $\mu_1 = \mu_2 = \mu$, the firm optimally invests in identical capacity levels for each product with dedicated technology. Therefore, a single capacity level K^D is sufficient to characterize the capacity investment decision.

We model the firm's problem as a two-stage stochastic problem. Figure 3.1 illustrates the sequence of decisions and events.

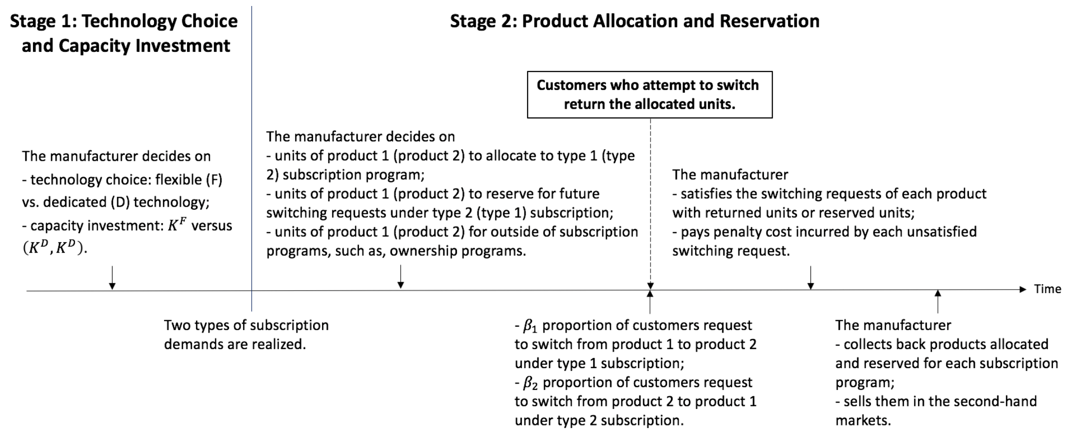


Figure 3.1: Timeline of Events

In stage 1, the firm chooses between dedicated (D) and flexible (F) technologies that incur investment costs, denoted as c^T, where $T \in \{D, F\}$, and determines the capacity levels with the chosen technology, (K^D, K^D) for dedicated technology and K^F for flexible technology. In stage 2, after observing the subscription demand realizations, (ξ_1, ξ_2), two products are produced by incurring unit production costs w_1, w_2, respectively. The firm decides on x_i^T, the units of product i to allocate for the demand for type i subscription program, and y_i^T, the units of product i to reserve for future switching requests under type $-i$ subscription program, $i = 1, 2$. Let r_i^s denote the subscription fee of type i subscription program at the beginning of stage 2. After one period of usage, β_i proportion of customers under type i subscription program return the allocated units of product i to the firm, and attempt to *switch* from product i to product $-i$, where $\beta_i \in [0, 1]$. We assume there is no additional fee for customers to switch between the products. The firm satisfies the switching requests of each product as much as possible with reserved units and *returned units* by switching customers. Any unsatisfied switching requests of product

i incurs penalty cost to the firm, which is denoted as p_i. At the end of stage 2, the firm sells products allocated for subscription programs in second-hand markets with their residual values. Let r_i^r denote the resale price of product i in the beginning of stage 2, d_i^u denote the depreciation cost of one period when product i is under use, d_i^n denote the depreciation cost of one period when product i is not in use. To provide subscription programs, the firms incurs additional operation cost. Let m_i^u denote the unit maintenance cost of one period when product i is under use and m_i^n denote the unit maintenance cost of one period when product i is not in use.

3.3.2 Assumptions

In this paper, we focus on symmetric products in the sense that the cost and revenue parameters are the same for both products. Specifically, the proportions of switching requests are the same as well; that is, we have $\beta \doteq \beta_i$, $i = 1, 2$.

Without switching requests (i.e., $\beta = 0$), we use this case as the benchmark model with traditional business; with switching requests (i.e., $\beta > 0$), we use this case as the subscription model.

We define $\Delta m = m^u - m^n$, $\Delta d = d^u - d^n$, and

$$
\begin{aligned}
R &= p - \Delta m - \Delta d, \\
P &= r^s - 2\Delta m - 2\Delta d, \qquad (3.1) \\
S &= r^r - 2m^n - 2d^n,
\end{aligned}
$$

where $R(> 0)$ denotes the marginal penalty cost for unsatisfied switching requests as these customers return back the allocated product but do not have the other product when request to switch, $P(> 0)$ denotes the marginal revenue for subscription sales as the firm provides each product for usage and with maintenance for two periods in subscription programs, $S(>0)$ denotes the residual value for the products that are unused but with maintenance for two periods in subscription programs.

To sustain that the subscription program is profitable to be introduced with reservations for future switching requests, we make the following assumptions to rule out some uninteresting and trivial cases.

Assumption 3.1. $P \geq 2R$. *This assumption implies that the unit penalty cost of unsatisfied switching requests is no more than half of the subscription fee in stage 2, i.e.,* $p \leq \frac{r^s}{2}$.

Assumption 3.2. $S > w$. *This assumption represents that the marginal profit from the products that are unused in subscription programs is positive, which implies that the firm is profitable for leftover capacity that is unused in subscription programs.*

Assumption 3.3. $S < O < R + S$, *i.e.,* $0 < O - S < R$. *This assumption represents that for unused units in each subscription program, their marginal revenue* O *in outside of subscription programs, such as ownership programs, is no less than the reselling value* S *in the end of stage 2, and is lower than the value of reservation* $(R + S)$ *which is captured by the saved penalty cost and the reselling value in the end of stage 2.*

Based on above discussions, products can be used for three different ways: (1) allocated products for subscription demands, which bring the marginal profit $P + S - w$; (2) reserved products for future switching requests, which bring the marginal profit $R+S-w$; (3) leftover products for out of subscription programs, such as, ownership programs, which bring the marginal profit $O-w$. With Assumption 3.1-3.3, the marginal profits for these products have the following relationships:

$$P + S - w > R + S - w > O - w > S - w > 0. \tag{3.2}$$

Thus, to obtain the maximal profit, for given capacity K^T, where $T \in \{D, F\}$, the firm firstly allocates product capacity for satisfying both subscription demands, then reserves product capacity (if any) for satisfying future switching requests, then leaves the leftover capacity (if any) for outside of subscription programs.

Assumption 3.4. *$c^T > O - w$, where $T \in \{D, F\}$. This assumption represents that the unit capacity investment cost for each technology is not low.*

Assumption 3.4 implies that the optimal capacity investment level with each technology is finite. For the technology choice (§3.6), we focus on $c^F > c^D$.

3.4 Dedicated Technology

In this section, we first characterize the firm's optimal strategy with dedicated technology (§3.4.1). We then examine the impacts of correlation between subscription demands (ρ) and switch-proportion (β) on the optimal expected profit with dedicated technology (§3.4.2). To highlight the new insights resulting from the subscription model, whenever applicable, we make a comparison with the benchmark case (the special case of $\beta = 0$) which represents the traditional ownership model.

3.4.1 The Optimal Strategy

We describe the optimal solution for the firm's capacity investment and subsequent production decisions with dedicated technology. We solve the firm's problem using backward induction starting from stage 2. All the proofs are relegated to the appendix.

In stage 1, the firm invested in K^D amount of capacity for each product. In stage 2, the firm observes the subscription demand realizations $\boldsymbol{\xi'} = (\xi_1, \xi_2)$ and makes the production decisions constrained by the capacity $\mathbf{K}^{\boldsymbol{D}\prime} = (K^D, K^D)$. In particular, for each product $i \in \{1, 2\}$, the firm determines the allocation volume x_i^D to satisfy the demand ξ_i under type i subscription program and reservation volume y_i^D to satisfy the future switching requests under type $-i$ subscription program to maximize the profit. The firm's stage-2 profit maximization problem is given by

$$\begin{aligned} \pi^D\left(\mathbf{K}^{\boldsymbol{D}}, \boldsymbol{\xi}\right) \quad = \max_{\mathbf{x}^{\boldsymbol{D}} \geq \mathbf{0}, \mathbf{y}^{\boldsymbol{D}} \geq \mathbf{0}} \quad & \sum_{i=1}^{2} \Big[-w\left(x_i^D + y_i^D\right) + P x_i^D - R\left(\beta x_{-i}^D - \left(\beta x_i^D + y_i^D\right)\right)^+ \\ & \quad + S\left(x_i^D + y_i^D\right) + (O - w)\left(K^D - x_i^D - y_i^D\right)\Big] \\ \text{s.t.} \quad & \mathbf{x}^{\boldsymbol{D}} \leq \boldsymbol{\xi}, \\ & \mathbf{x}^{\boldsymbol{D}} + \mathbf{y}^{\boldsymbol{D}} \leq \mathbf{K}^{\boldsymbol{D}}, \end{aligned} \tag{3.3}$$

where $\pi^D\left(\mathbf{K}^{\boldsymbol{D}}, \boldsymbol{\xi}\right)$ denotes the optimal stage-2 profit with dedicated technology.

In the objective function of (B.1), for each product $i \in \{1, 2\}$, the first term represents the total production cost for the allocation volume x_i^D and the reservation volume y_i^D. The second term represents the revenue from type i subscription program which is given by the product of marginal subscription revenue P and the allocation volume x_i^D. The third term represents the penalty cost associated with unsatisfied switching requests for product i. This cost is given by the product of marginal penalty cost R and the amount of unsatisfied switching requests (if any); that is, the difference between the switching requests under type $-i$ subscription program (βx_{-i}^D) and the product i's total available volume including the reservation volume y_i^D and the returned units βx_i^D under type i subscription program. The fourth term represents the salvage value for product i's total production volume $(x_i^D + y_i^D)$ at the end of the subscription program. The last term represents the profit associated with the leftover capacity $K^D - x_i^D - y_i^D$ outside of the subscription program. The first constraint in (B.1) ensures that the allocation volume for each product cannot exceed its demand. The second constraint ensures that the total production volume for each product is constrained by its production capacity.

Proposition 3.1 characterizes the optimal product allocation and reservation volumes with dedicated technology.

Proposition 3.1. *The optimal allocation volume for product* $i \in \{1, 2\}$ *is given by* $x_i^{D*} = \min\left(\xi_i, K^D\right)$. *Let* $[i]$ *denote the product with the minimum realized subscription demand, i.e.,* $\xi_{[i]} = \min\left(\xi_1, \xi_2\right)$, *and* $-[i]$ *denote the other product. The optimal reservation volumes for product* $[i]$ *and product* $-[i]$ *are given by* $y_{[i]}^{D*} = \beta x_{-[i]}^{D}{}^* - \beta x_{[i]}^{D*}$ *and* $y_{-[i]}^{D}{}^* = 0$, *respectively.*

Intuitively, the optimal allocation volume x_i^{D*} for product i is given by the minimum of its demand ξ_i and the production capacity K^D. The optimal reservation volume for each product crucially depends on which product's optimal allocation volume is larger than the other. In particular, the firm optimally does not reserve product $-[i]$, which has a larger allocation volume by definition, because the returned units $\beta x_{-[i]}^{D}{}^*$ are sufficient to satisfy the switching requests $\beta x_{[i]}^{D*}$ for this product under type $[i]$ subscription program. For product $[i]$, the firm optimally reserves the balance $\beta x_{-[i]}^{D}{}^* - \beta x_{[i]}^{D*}$ between the switching requests and the returned units unless constrained by the remaining production capacity $K^D - x_{[i]}^{D*}$ after allocation. It can be proven that the remaining production capacity is always sufficient to cover the balance at any demand realization. Therefore, the optimal reservation volume for product $[i]$ is given by $\beta x_{-[i]}^{D}{}^* - \beta x_{[i]}^{D*}$.

Proposition 3.1 characterizes the optimal reservation volumes based on product indexes $[i]$ and $-[i]$ that specify the product with minimum and maximum realized subscription demand, respectively. Moreover, these optimal decisions are characterized based on the optimal allocation decisions x_i^{D*} and x_{-i}^{D*}. To solve for the optimal capacity investment decision at stage 1, there is a need to explicitly characterize these optimal reservation volumes based on the demand realizations (ξ_1, ξ_2). To this end, we define the following 5-region partitioning of the (ξ_1, ξ_2) space:

$$\begin{aligned}
\Omega_1^D &\doteq \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, \xi_2 \leq \xi_1 \leq K^D\right\}, \\
\Omega_2^D &\doteq \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, \xi_1 < \xi_2 \leq K^D\right\}, \\
\Omega_3^D &\doteq \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, \xi_2 \leq K^D < \xi_1\right\}, \\
\Omega_4^D &\doteq \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, \xi_1 \leq K^D < \xi_2\right\}, \\
\Omega_5^D &\doteq \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, \xi_1 > K^D, \xi_2 > K^D\right\}.
\end{aligned} \tag{3.4}$$

Using (3.4), panel b of Figure 3.2 illustrates the optimal reservation volumes (y_1^{D*}, y_2^{D*}) based on the demand realizations (ξ_1, ξ_2) (for completeness, we also illustrate the optimal allocation volumes (x_1^{D*}, x_2^{D*}) in panel a).

When $\boldsymbol{\xi} \in \Omega_{13}^D$, we have $\xi_2 \leq \min\left(\xi_1, K^D\right)$ and as depicted in panel a of Figure 3.2, the optimal allocation decisions are $x_1^{D*} = \min\left(\xi_1, K^D\right)$ and $x_2^{D*} = \xi_2$. In this region, because product 1's optimal allocation volume is larger than product 2's optimal allocation volume, the firm optimally does not reserve product 1 (i.e., $y_1^{D*} = 0$). Moreover, switching requests $\beta \min\left(\xi_1, K^D\right)$ for product 2 is larger than the returned units $\beta\xi_2$ of this product. Because ξ_2 is low, the remaining product 2 capacity $K^D - \xi_2$ after allocation is sufficient to cover the balance between the switching requests and the returned units, and thus, the firm optimally reserves this balance for product 2 (i.e., $y_2^{D*} = \beta \min\left(\xi_1, K^D\right) - \beta\xi_2$) as depicted in panel b. When $\boldsymbol{\xi} \in \Omega_{24}^D$, we have $\xi_1 \leq \min\left(\xi_2, K^D\right)$ and as depicted in panel a, the optimal allocation decisions are $x_1^{D*} = \xi_1$ and $x_2^{D*} = \min\left(\xi_2, K^D\right)$. The characterizations of the optimal

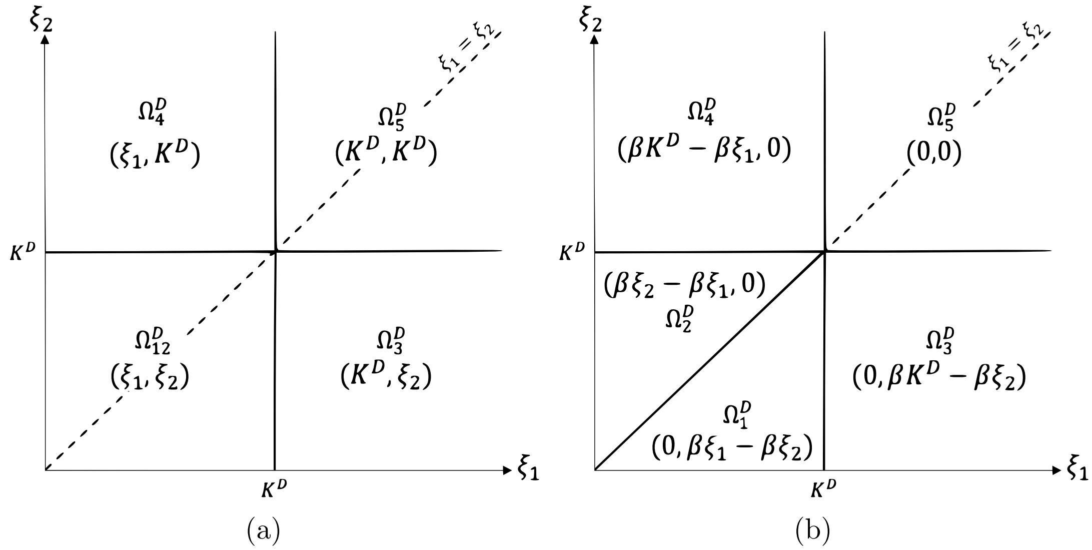


Figure 3.2: Effects of rainfall and recycling cost variabilities and correlation on the differences between the unit costs of satisfying household demand (Panel a) and non-household demand (Panel b) under the optimal framework and the benchmark case. In the two panels, a negative value implies that the unit cost under the optimal framework is smaller than that under the benchmark case. The dashed-dotted lines (represent the effects of rainfall variability) refer to the y-axis on the right.

reservation volumes follow from the same intuition with Ω_{13}^D region except for product 2's optimal allocation volume is larger than product 1's optimal allocation volume. Therefore, we have $y_1^{D*} = \beta \min\left(\xi_2, K^D\right) - \beta\xi_1$ and $y_2^{D*} = 0$ as depicted in panel b. When $\boldsymbol{\xi} \in \Omega_5^D$, the optimal allocation volume for each product is given by K^D. In this region, because there is no capacity left for reservation the firm optimally does not reserve for each product (i.e., $y_1^{D*} = y_2^{D*} = 0$). Even though there is no reservation at these demand realizations, the firm does not experience an unsatisfied switching request for each product because the number of switching requests (βK^D) is identical to the number of returned units (βK^D).

In stage 1, the firm chooses the optimal capacity level $K^{D*} \geq 0$ for each product with respect to the subscription demand uncertainty $\tilde{\boldsymbol{\xi}}$ so as to maximize the expected profit $\mathbb{E}\left[\pi^D\left(\mathbf{K}^{\boldsymbol{D}}, \tilde{\boldsymbol{\xi}}\right)\right] - 2c^D K^D$. Let $\Pi^D\left(\mathbf{K}^{\boldsymbol{D}}\right)$ denote the expected profit for a given capacity $\mathbf{K}^{\boldsymbol{D}\prime} = (K^D, K^D)$ where

$$\begin{aligned} \Pi^D\left(\mathbf{K}^{\boldsymbol{D}}\right) = \quad & -2c^D K^D + \sum_{i=1}^{2}\left[(P+S-w)\,\mathbb{E}\left[\min(\tilde{\xi}_i, K^D)\right] + (O-w)\,\mathbb{E}\left[\left(K^D - \tilde{\xi}_i\right)^+\right]\right] \\ & -(O-S)\,\mathbb{E}\left[\beta\min\left(\max\left(\tilde{\xi}_1,\tilde{\xi}_2\right), K^D\right) - \beta\min\left(\min\left(\tilde{\xi}_1,\tilde{\xi}_2\right), K^D\right)\right]. \end{aligned} \tag{3.5}$$

In (3.5), the first term denotes the capacity investment cost whereas the remaining terms denote the expected stage-2 profit $\mathbb{E}\left[\pi^D\left(\mathbf{K}^{\boldsymbol{D}}, \tilde{\boldsymbol{\xi}}\right)\right]$ with dedicated technology. In this expected profit term, for each product $i \in \{1,2\}$, the first term represents the profit from subscription sales which is given by the product of expected type i subscription sales $\mathbb{E}\left[\min(\tilde{\xi}_i, K^D)\right]$ and the unit subscription sales margin; that is, the sum of the marginal subscription revenue P and the salvage value at the end of the subscription program S minus the unit production cost w. The second term represents the profit generated by product i's leftover capacity after allocation $\mathbb{E}\left[\left(K^D - \tilde{\xi}_i\right)^+\right]$ outside of the subscription program *in the absence of reservation.* The last term denotes the expected total loss associated with switching requests for both products which also captures the effect of optimal reservation decisions. Recall from (B.1) that the firm incurs a marginal penalty cost R per unit of unsatisfied switching request. As discussed above, at stage 2, the firm experiences a larger amount of switching requests than the returned units for the low-demand product. As follows from Proposition 3.1, because the firm optimally reserves the balance between the switching requests $\beta\min\left(\max\left(\xi_1,\xi_2\right), K^D\right)$ and the returned units $\beta\min\left(\min\left(\xi_1,\xi_2\right), K^D\right)$ of this product, the firm does not incur the penalty cost R. However, for each reserved unit, the firm incurs a marginal cost $O-S$ because the firm takes this unit away from the leftover capacity after allocation, and hence, loses the unit revenue O; but salvages this unit at the end of the subscription program, and hence, reaps the unit revenue S. Therefore, the expected loss associated with switching requests is given by the product of marginal cost $O - S$ and the expected total optimal reservation volume $\mathbb{E}\left[\beta \min\left(\max\left(\tilde{\xi}_1, \tilde{\xi}_2\right), K^D\right) - \beta \min\left(\min\left(\tilde{\xi}_1, \tilde{\xi}_2\right), K^D\right)\right]$ for both products. In the traditional ownership model, as captured by the benchmark case of $\beta = 0$, this expected loss term does not exist and the expected profit for a given capacity is given by the terms in the first line of (3.5).

We next characterize the optimal capacity investment decision with dedicated technology.

Proposition 3.2. *When $c^D \geq P + S - w$, $K^{D^*} = 0$. Otherwise, $K^{D^*} > 0$ is the unique solution to*

$$\begin{aligned}(P + S - w) \sum_{i=1}^{2} \Pr\left(\tilde{\xi}_i > K^{D^*}\right) \quad &+ \quad (O - w) \sum_{i=1}^{2} \Pr\left(\tilde{\xi}_i \leq K^{D^*}\right) \\ &- \quad \beta (O - S) \Pr\left(\min\left(\tilde{\xi}_1, \tilde{\xi}_2\right) \leq K^{D^*} < \max\left(\tilde{\xi}_1, \tilde{\xi}_2\right)\right) = 2c^D.\end{aligned} \tag{3.6}$$

When the unit capacity investment cost c^D is larger than the unit subscription sales margin $P + S - w$, the firm optimally does not invest in any capacity for each product. Otherwise, the firm optimally invests in identical capacity levels for both products and the optimal capacity investment level K^{D^*} is characterized by (3.6). Here, the right-hand-side ($2c^D$) denotes the marginal investment cost whereas the left-hand-side corresponds to the expected marginal revenue of capacity investment for both products. In particular, at stage 2, an additional unit of capacity investment for each product generates the marginal sales margin $P + S - w$ when there is unsatisfied subscription demand; otherwise, it generates the marginal profit $O - w$ outside of the subscription program *in the absence of reservation.* This is captured by the first two terms on the left-hand-side of (3.6). The last term on the left-hand-side captures the additional unit of capacity investment's effect on the expected loss associated with switching requests for both products as characterized by the last term in (3.5). In particular, at stage 2, an additional unit of capacity investment for each product affects the loss associated with switching requests when this investment creates a subscription sale only with the high-demand product; that is, when $\min(\xi_1, \xi_2) \leq K^D < \max(\xi_1, \xi_2)$. In this case, the firm uses the additional unit of capacity for the high-demand product to create a subscription sale and the additional unit of capacity for the low-demand product to use outside of the subscription program *in the absence of reservation*. Because the subscription sale for the high-demand product also creates an additional β unit of switching request for the low-demand product, the firm optimally uses β unit of the additional capacity for the low-demand product to reserve and satisfy this request. Therefore, the firm incurs a marginal cost of $(O - S)$ for this β unit. In the benchmark case; that is, when $\beta = 0$, this term does not exist and using symmetric demand distribution $\left(\tilde{\xi}, \tilde{\xi}\right)$ it is easy to establish from (3.6) that the optimal capacity level is given by the well-known newsvendor solution with dedicated technology: $\Pr(\tilde{\xi} \leq K^{D*}) = \frac{c_u^D}{c_u^D + c_o^D} = \frac{P+S-w-c^D}{P+S-O}$ where $c_u^D = P + S - w - c^D$ is the unit under-investment cost and $c_o^D = c^D - (O - w)$ is the unit over-investment cost.

3.4.2 Impacts of Demand Correlation and Switch-proportion on Profitability

We now conduct sensitivity analyses to study the effects of two key parameters; specifically, correlation between subscription demands (ρ) and switch-proportion (β), on the optimal expected profit with dedicated technology. To avoid uninteresting cases, we assume $c^D < P + S - w$ throughout this section so that, as follows from Proposition 3.2, the firm optimally invests in a positive amount of capacity.

We first examine the effect of demand correlation ρ. It is well-known from the extant literature that in a traditional ownership model the firm's profitability with the dedicated technology is independent of demand correlation. This is because each product's profit depends only on that product's demand uncertainty and not on the other products' demand uncertainties. This can also be observed in our setting from the expected total profit expression for a given capacity investment as given by (3.5). In particular, in the benchmark case as captured by $\beta = 0$, the expected total profit from both products depends on the expected subscription sales $\mathbb{E}\left[\min(\tilde{\xi}_i, K^D)\right]$ and the expected leftover capacity $\mathbb{E}\left[\left(K^D - \tilde{\xi}_i\right)^+\right]$ with each product $i \in \{1, 2\}$, and hence, it is independent of ρ.

In the subscription model, demand correlation matters with dedicated technology. This is because each product's profit depends not only on the uncertainty in subscription demand that this product is allocated to but also uncertainties in other subscription demands that this product is used later on for satisfying future switching requests. In the context of our model, this can be observed from the expected total profit expression as given by (3.5). In particular, in the subscription model as captured by $\beta > 0$, the expected total profit from both products depends on demand correlation ρ through the expected loss associated with switching requests for both products (i.e., the last term in (3.5)). Proposition 3.3 proves that with a higher demand correlation ρ the optimal expected profit increases with dedicated technology.

Proposition 3.3. *Assume* $\left(\tilde{\xi}_1, \tilde{\xi}_2\right)$ *to follow a bivariate Normal distribution. When* $\beta > 0$*, we have* $\frac{\partial \Pi^{D*}}{\partial \rho} \geq 0$.

To delineate the intuition, let us focus on the expected profit for a given capacity investment level K^D as characterized by (3.5): if the expected profit for any given K^D increases in demand correlation ρ, then the optimal expected profit also increases in ρ. It is sufficient to show that the expected total optimal reservation volume $\mathbb{E}\left[\beta \min\left(\max\left(\tilde{\xi}_1, \tilde{\xi}_2\right), K^D\right) - \beta \min\left(\min\left(\tilde{\xi}_1, \tilde{\xi}_2\right), K^D\right)\right]$ for both products decreases in ρ. With a higher ρ, there will be a higher likelihood that when the type i subscription demand ξ_i is low (high), the type $-i$ subscription demand ξ_{-i} will be low (high). Therefore, there will be a higher likelihood that the difference between $\max(\xi_1, \xi_2)$ and $\min(\xi_1, \xi_2)$ realizations will be low. In summary, as ρ increases, the expected total optimal reservation volume, and thus, the expected loss associated with switching requests for both products decreases. As a result, the expected profit with dedicated technology increases.

We next examine the effect of switch-proportion β on the profitability.

Proposition 3.4. *Assume $\beta > 0$. We have $\frac{\partial \Pi^{D*}}{\partial \beta} \leq 0$.*

Paralleling the impact of ρ, the expected profit from both products depends on the switch-proportion β only through the last term in (3.5); that is, the expected loss associated with switching requests for both products. With a higher β, there will be a higher imbalance between the switching requests $\beta \min\left(\max(\xi_1, \xi_2), K^D\right)$ and the returned units $\beta \min\left(\min(\xi_1, \xi_2), K^D\right)$ for the low-demand product at stage 2 for any given demand realization. Therefore, the expected loss associated with switching requests for both products increases, and thus, the optimal expected profit with dedicated technology decreases.

3.5 Flexible Technology

In this section, we characterize the firm's optimal strategy with flexible technology (§3.5.1) and examine the impacts of correlation between subscription demands (ρ) and switch-proportion (β) on the optimal expected profit with flexible technology (§3.5.2).

3.5.1 The Optimal Strategy

In stage 1, the firm invested in K^F amount of capacity that can be used to produce both products. In stage 2, the firm observes the subscription demand realizations $\boldsymbol{\xi}' = (\xi_1, \xi_2)$ and makes the production decisions constrained by the capacity K^F. In particular, for product $i \in \{1, 2\}$, the firm determines the allocation volume x_i^F to satisfy the demand ξ_i under type i subscription program and reservation volume y_i^F to satisfy the future switching requests under type $-i$ subscription program to maximize the profit. The firm's stage-2 profit maximization problem is given by

$$\begin{aligned} \pi^F\left(K^F, \boldsymbol{\xi}\right) \quad = \max_{\mathbf{x}^F \geq \mathbf{0}, \mathbf{y}^F \geq \mathbf{0}} \quad & \sum_{i=1}^{2} \Big[-w\left(x_i^F + y_i^F\right) + P x_i^F - R\left(\beta x_{-i}^F - \left(\beta x_i^F + y_i^F\right)\right)^+ \\ & \quad + S\left(x_i^F + y_i^F\right)\Big] + (O - w)\left(K^F - \sum_{i=1}^{2}(x_i^F + y_i^F)\right) \\ \text{s.t.} \quad & \mathbf{x}^F \leq \boldsymbol{\xi}, \\ & \sum_{i=1}^{2}(x_i^F + y_i^F) \leq K^F, \end{aligned} \tag{3.7}$$

where $\pi^F\left(K^F, \boldsymbol{\xi}\right)$ denotes the optimal stage-2 profit with flexible technology. The intuition behind the formulation in (B.4) is similar to the formulation with dedicated technology in (B.1) with two main differences. First, the profit associated with the leftover capacity outside of the subscription program, the last term in the objective function of (B.4), is jointly characterized for both products as a single capacity K^F is used for producing both products. Second, there is a single constraint (second constraint in (B.4)) which ensures that total production volume for both products is limited by the capacity K^F where there is such capacity constraint for each product with dedicated technology.

We next characterize the optimal allocation and reservation volumes with flexible technology.

Proposition 3.5. *Let* $[i]$ *denote the product with the minimum realized sub-*

scription demand, i.e., $\xi_{[i]} = \min(\xi_1, \xi_2)$*, and* $-[i]$ *denote the other product. The optimal allocation volumes for product* $[i]$ *and product* $-[i]$ *are given by* ${x^F_{[i]}}^* = \min\left(\xi_{[i]}, \frac{K^F}{2}\right)$ *and* ${x^F_{-[i]}}^* = \min\left(\xi_{-[i]}, \left(K^F - {x^F_{[i]}}^*\right)^+\right)$*, respectively. The optimal reservation volumes for product* $[i]$ *and product* $-[i]$ *are given by* ${y^F_{[i]}}^* = \min\left(\left(K^F - {x^F_{[i]}}^* - {x^F_{-[i]}}^*\right)^+, \beta {x^F_{-[i]}}^* - \beta {x^F_{[i]}}^*\right)$ *and* ${y^F_{-[i]}}^* = 0$*, respectively.*

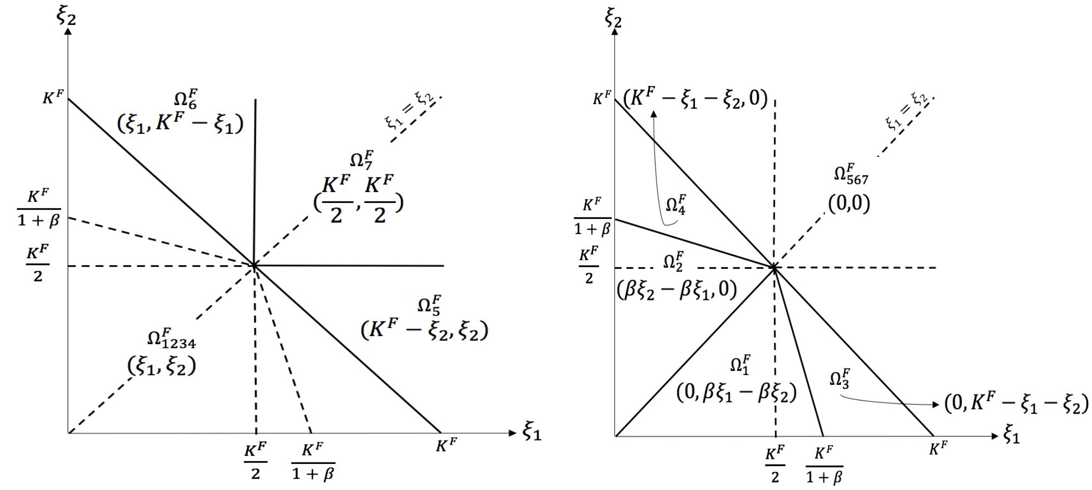
The optimal allocation and reservation volumes critically depend on whether the flexible capacity K^F is sufficient to fully satisfy the total subscription demand $\xi_1 + \xi_2$. When the flexible capacity is sufficient (i.e., $K^F \geq \xi_1 + \xi_2$), the optimal allocation volume for each product is given by its demand: ${x^F_1}^* = \xi_1$ and ${x^F_2}^* = \xi_2$. In this case, the firm optimally does not reserve product $-[i]$, which has a larger allocation volume by definition, because the returned units $\beta {x^F_{-[i]}}^*$ are sufficient to satisfy the switching requests $\beta {x^F_{[i]}}^*$ for this product. For product $[i]$, the firm optimally reserves the balance $\left(\beta {x^F_{-[i]}}^* - \beta {x^F_{[i]}}^*\right)$ between the switching requests and the returned units unless constrained by the remaining production capacity $\left(K^F - {x^F_{[i]}}^* - {x^F_{-[i]}}^*\right)$ after allocation. When the flexible capacity is not sufficient to fully satisfy the total subscription demand (i.e., $K^F < \xi_1 + \xi_2$), intuitively, all capacity is used as allocation volume for both products and because there is no capacity left for reservation, the firm optimally does not reserve for each product: ${y^F_1}^* = {y^F_2}^* = 0$. In this case, the firm optimally allocates the capacity K^F to each product in a manner to avoid significant imbalance between the switching requests and returned units to reduce the penalty cost. To this end, the optimal allocation volumes are characterized based on the following two strategies: first, identical volume is allocated to each product up to the minimum demand $\xi_{[i]}$ (so that the firm does not experience an unsatisfied switching request for each product because the number of switching requests is identical to the number of returned units) and second, any leftover capacity after implementing the first strategy is also allocated to product $-[i]$ which has a larger demand by definition. In particular, when $K^F < 2\xi_{[i]}$, the capacity is not sufficient to allocate $\xi_{[i]}$ to each product, and thus, the firm optimally allocates $K^F/2$ to each product (i.e., $x_1^{F^*} = x_2^{F^*} = K^F/2$). When $2\xi_{[i]} \leq K^F < \xi_1 + \xi_2$, the capacity is sufficient to allocate $\xi_{[i]}$ to each product; therefore, the firm optimally allocates $\xi_{[i]}$ to product $[i]$ and the remaining capacity $K^F - \xi_{[i]}$ to product $-[i]$.

Proposition 3.5 characterizes the optimal allocation and reservation volumes based on product indexes $[i]$ and $-[i]$ that specify the product with minimum and maximum realized subscription demand, respectively. Moreover, ${x^F_{-[i]}}^*$ and $y_{[i]}^{F^*}$ are characterized based on other optimal decision variables. To solve for the optimal capacity investment decision at stage 1, we explicitly characterize the optimal allocation and reservation volumes based on the demand realizations (ξ_1, ξ_2). To this end, we define the following 7-region partitioning of the (ξ_1, ξ_2) space:

$$
\begin{aligned}
\Omega_1^F &\doteq \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, \xi_1 + \xi_2 + \beta(\xi_1 - \xi_2) \leq K^F, \xi_1 \geq \xi_2\right\}, \\
\Omega_2^F &\doteq \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, \xi_1 + \xi_2 + \beta(\xi_2 - \xi_1) \leq K^F, \xi_2 > \xi_1\right\}, \\
\Omega_3^F &\doteq \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, \xi_1 + \xi_2 \leq K^F < \xi_1 + \xi_2 + \beta(\xi_1 - \xi_2), \xi_1 \geq \xi_2\right\}, \\
\Omega_4^F &\doteq \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, \xi_1 + \xi_2 \leq K^F < \xi_1 + \xi_2 + \beta(\xi_2 - \xi_1), \xi_2 > \xi_1\right\}, \\
\Omega_5^F &\doteq \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, 2\xi_2 \leq K^F < \xi_1 + \xi_2, \xi_1 \geq \xi_2\right\}, \\
\Omega_6^F &\doteq \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, 2\xi_1 \leq K^F < \xi_1 + \xi_2, \xi_2 > \xi_1\right\}, \\
\Omega_7^F &\doteq \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, 2\xi_1 > K^F, 2\xi_2 > K^F\right\}.
\end{aligned}
\tag{3.8}
$$

Using (3.8), Figure 3.3 illustrates the optimal allocation volumes $(x_1^{F^*}, x_2^{F^*})$ in panel a and the optimal reservation volumes $(y_1^{F^*}, y_2^{F^*})$ in panel b based on the demand realizations (ξ_1, ξ_2).

Based on the characterizations in Figure 3.3 we make the following two



(a) (b)

Figure 3.3: Effects of rainfall and recycling cost variabilities and correlation on the differences between the unit costs of satisfying household demand (Panel a) and non-household demand (Panel b) under the optimal framework and the benchmark case. In the two panels, a negative value implies that the unit cost under the optimal framework is smaller than that under the benchmark case. The dashed-dotted lines (represent the effects of rainfall variability) refer to the y-axis on the right.

important observations:

1) In the traditional ownership model, as captured by the benchmark case of $\beta = 0$, it is well-known that when the flexible capacity is insufficient to fully satisfy the total demand, any allocation of the capacity to each product is optimal as long as the allocation does not exceed the product demand; otherwise, the optimal allocation for each product is given by its demand. In the subscription model, as depicted in a panel a, while the latter observation continues to hold (specifically, when $\boldsymbol{\xi} \in \Omega^F_{1234}$), interestingly, the former observation does not hold. In particular, to reduce the penalty cost associated with unsatisfied switching requests, the firm either allocates $K^F/2$ to each product (when $\boldsymbol{\xi} \in \Omega^F_7$) or allocates the demand to the low-demand product and the remaining capacity to the high-demand product (when $\boldsymbol{\xi} \in \Omega^F_{56}$). In the benchmark case, these two allocations are both optimal when flexible capacity

is insufficient to fully satisfy the demand (specifically, when $\boldsymbol{\xi} \in \Omega^F_{567}$).

2) When $\boldsymbol{\xi} \in \Omega^F_{123456}$, the firm experiences a larger amount of switching requests than the returned units for the low-demand product due to asymmetric optimal allocation volumes for both products. To counteract against this, as depicted in panel b, the firm optimally reserves the low-demand product when there is capacity left after allocation (i.e., when $\boldsymbol{\xi} \in \Omega^F_{1234}$). However, unlike the case with dedicated technology, the remaining capacity after allocation is not always sufficient to cover the balance between switching requests and returned units. In particular, when $\boldsymbol{\xi} \in \Omega^F_{34}$, the remaining capacity $K^F - \xi_1 - \xi_2$ is not sufficient to cover the balance $\beta \max(\xi_1, \xi_2) - \beta \min(\xi_1, \xi_2)$ (hence, the firm optimally reserves the remaining capacity) and when $\boldsymbol{\xi} \in \Omega^F_{12}$, the remaining capacity is sufficient to cover the balance (hence, the firm optimally reserves this balance).

In stage 1, the firm chooses the optimal flexible capacity level $K^{F^*} \geq 0$ with respect to the subscription demand uncertainty $\tilde{\boldsymbol{\xi}}$ so as to maximize the expected profit $\mathbb{E}\left[\pi^F(K^F, \tilde{\boldsymbol{\xi}})\right] - c^F K^F$. Let $\Pi^F\left(K^F\right)$ denote the expected profit for a given capacity K^F, where

$$\begin{aligned}
\Pi^F(K^F) = \quad & -c^F K^F + (P + S - w)\,\mathbb{E}\left[\min\left(\tilde{\xi}_1 + \tilde{\xi}_2, K^F\right)\right] + (O - w)\,\mathbb{E}\left[\left(K^F - \left(\tilde{\xi}_1 + \tilde{\xi}_2\right)\right)^+\right] \\
& -R\,\mathbb{E}\left[\left(\beta \min\left(K^F - \min\left(\tilde{\xi}_1, \tilde{\xi}_2\right), \max\left(\tilde{\xi}_1, \tilde{\xi}_2\right)\right) - \beta \min\left(\tilde{\xi}_1, \tilde{\xi}_2\right)\right)^+\right] \\
& +(R - (O - S))\,\mathbb{E}\left[\min\left(\left(K^F - \left(\tilde{\xi}_1 + \tilde{\xi}_2\right)\right)^+, \beta \max\left(\tilde{\xi}_1, \tilde{\xi}_2\right) - \beta \min\left(\tilde{\xi}_1, \tilde{\xi}_2\right)\right)\right].
\end{aligned} \tag{3.9}$$

In (3.9), the first term denotes the capacity investment cost whereas the remaining terms denote the expected stage-2 profit $\mathbb{E}\left[\pi^F(K^F, \tilde{\boldsymbol{\xi}})\right]$ with flexible technology. In particular, the second term in (3.9) represents the profit from the expected total subscription sales $\mathbb{E}\left[\min(\tilde{\xi}_1 + \tilde{\xi}_2, K^F)\right]$ whereas the third term represents the profit generated by the expected leftover capacity after allocation $\mathbb{E}\left[\left(K^F - (\tilde{\xi}_1 + \tilde{\xi}_2)\right)^+\right]$ outside of the subscription program *in the absence of reservation*. The last two terms represent the total expected loss associated with *unsatisfied* switching requests for both products *in the absence of reservation* and the reduction in this expected total loss due to optimal reservation decisions, respectively. Recall from (B.4) that the firm incurs the marginal penalty cost R per unit of unsatisfied switching request. As discussed above, unless $K^F < 2\min(\xi_1, \xi_2)$ at stage 2 (where the firm optimally allocates $K^F/2$ to both products), the firm experiences a larger amount of switching requests than the returned units $\beta\min(\xi_1, \xi_2)$ for the low-demand product. In those cases, the number of switching requests for the low-demand product is characterized by the product of β and the optimal allocation volume for the high-demand product which, as follows from Proposition 3.5, is given by $\min\left(K^F - \min(\xi_1, \xi_2), \max(\xi_1, \xi_2)\right)$. This explains the term in the second line of (3.9). The last term in (3.9) represents the reduction in the expected total loss associated with unsatisfied switching requests due to the optimal reservation decisions. Recall from our discussion with dedicated technology that by reserving a unit, the firm saves the marginal penalty cost R but incurs a marginal cost of $O - S$ because this reserved unit is taken away from the leftover capacity after allocation and it is salvaged at the end of the subscription program. Therefore, the last term in (3.9) is given by the product of the marginal cost reduction $R - (O - S) > 0$ and the total expected optimal reservation volume for both products. As discussed above, when there is leftover capacity after allocation (i.e., $K^F > \xi_1 + \xi_2$) at stage 2, as follows from Proposition 3.5, the firm optimally reserves the balance $\beta\max(\xi_1, \xi_2) - \beta\min(\xi_1, \xi_2)$ between the switching requests and the returned units for the low-demand product unless constrained by the remaining production capacity $K^F - (\xi_1 + \xi_2)$ after allocation. Therefore, total expected optimal reservation volume for both products is given by $\mathbb{E}\left[\min\left(\left(K^F - \left(\tilde{\xi}_1 + \tilde{\xi}_2\right)\right)^+, \beta \max\left(\tilde{\xi}_1, \tilde{\xi}_2\right) - \beta \min\left(\tilde{\xi}_1, \tilde{\xi}_2\right)\right)\right]$. It is easy to establish that the sum of the last two terms in (3.9), which represents the expected loss associated with switching requests for both products, is negative. In particular, at stage 2, the firm incurs the penalty cost R for the unsatisfied switching requests and the opportunity cost $O - S$ for the satisfied switching requests by the optimal reservation decisions. In the traditional ownership model, as captured by the benchmark case of $\beta = 0$, the last two terms in (3.9) do not exist.

We next characterize the optimal capacity investment decision with flexible technology.

Proposition 3.6. *When* $c^F \geq P + S - w$, $K^{F^*} = 0$. *Otherwise,* $K^{F^*} > 0$ *is the unique solution to*

$$\begin{aligned}
&(P + S - w) \Pr\left(\tilde{\xi}_1 + \tilde{\xi}_2 > K^{F^*}\right) + (O - w) \Pr\left(\tilde{\xi}_1 + \tilde{\xi}_2 \leq K^{F^*}\right) \\
&-\beta R \Pr\left(2 \min\left(\tilde{\xi}_1, \tilde{\xi}_2\right) \leq K^{F^*} < \tilde{\xi}_1 + \tilde{\xi}_2\right) \\
&+ (R - (O - S)) \Pr\left(\tilde{\xi}_1 + \tilde{\xi}_2 \leq K^{F^*} < \tilde{\xi}_1 + \tilde{\xi}_2 + \left(\beta \max\left(\tilde{\xi}_1, \tilde{\xi}_2\right) - \beta \min\left(\tilde{\xi}_1, \tilde{\xi}_2\right)\right)\right) = c^F.
\end{aligned} \tag{3.10}$$

When the unit capacity investment cost c^F is larger than the unit subscription sales margin $P + S - w$, the firm optimally does not invest in any capacity with flexible technology. Otherwise, the optimal capacity investment level $K^{F^*} > 0$ is characterized by (3.10). Here, the right-hand-side (c^F) denotes the marginal cost whereas the left-hand-side corresponds to the expected marginal revenue of capacity investment. In particular, at stage 2, an additional unit of capacity generates the sales margin $P + S - w$ when there is unsatisfied total subscription demand; otherwise, it generates the marginal profit $O - w$ outside of the subscription program *in the absence of reservation.* This is captured by the first two terms on the left-hand-side of (3.10). The third term on the left-hand-side captures the additional unit of capacity investment's effect on the expected total loss associated with unsatisfied switching requests for both products in the absence of reservation as characterized by the second line in (3.9). In particular, an additional unit of capacity investment leads to a marginal cost increase βR when this investment creates a subscription sale only with the high-demand product at stage 2 which, in turn, leads to β unit of unsatisfied switching request in the absence of reservation. This happens when the capacity is not sufficient to fully satisfy the total subscription demand (i.e., $K^F < \xi_1 + \xi_2$) so that the capacity affects the optimal allocation decisions but at the same time it is sufficient for allocating the minimum demand to each product (i.e., $K^F \geq 2\min(\xi_1, \xi_2)$) so that it does not affect the optimal allocation decisions in both markets; that is, it is not optimal to allocate $K^F/2$ to each product. The last term on the left-hand-side of (3.10) captures the additional unit of capacity investment's effect on the reduction in the expected total loss associated with unsatisfied switching requests due to optimal reservation decisions as characterized by the third line in (3.9). In particular, an additional unit of capacity investment leads to a marginal cost reduction $R - (O - S)$ when this investment is used for reservation (for the low-demand product) at stage 2. This happens when there is capacity left for reservation after allocation to both products; that is, when $K^F \geq \xi_1 + \xi_2$, but at the same time the leftover capacity $K^F - (\xi_1 + \xi_2)$ is not sufficient to fully cover the balance $\beta\max(\xi_1, \xi_2) - \beta\min(\xi_1, \xi_2)$ between the switching requests and the returned units for the low-demand product. In the traditional ownership model, as captured by the benchmark case of $\beta = 0$, the last two terms on the left-hand-side of (3.10) do not exist and the optimal capacity investment level is given by the well-known newsvendor solution with flexible technology: $\Pr(\tilde{\xi}_1 + \tilde{\xi}_2 \leq K^{F*}) = \frac{c_u^F}{c_u^F + c_o^F} = \frac{P+S-w-c^F}{P+S-O}$ where $c_u^F = P + S - w - c^F$ is the unit under-investment cost and $c_o^F = c^F - (O - w)$ is the unit over-investment cost with flexible technology.

3.5.2 Impacts of Demand Correlation and Switch-proportion on Profitability

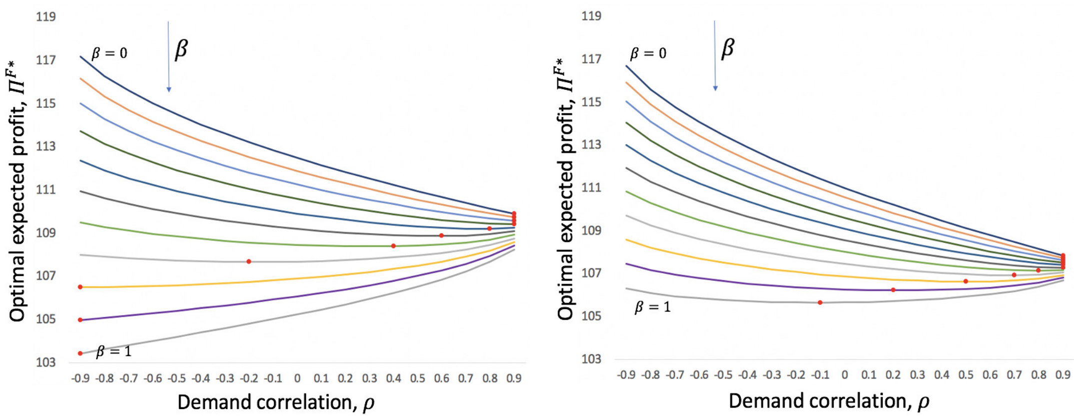
In this section, paralleling Section 3.4.2, we conduct sensitivity analyses to study the effects of correlation between subscription demands (ρ) and switch-proportion (β) on the optimal expected profit with flexible technology. Throughout this section, we assume $c^F < P+S-w$ so that, as follows from Proposition 3.6, the firm optimally invests in a positive amount of capacity.

We first examine the effect of demand correlation ρ. It is well-known from the extant literature that in a traditional ownership model an increase in ρ decreases the profitability with flexible technology. This is because the capacity-pooling benefit; that is, the benefit from firm's ability to reallocate the flexible capacity among multiple products based on their demand realizations, decreases. This can be observed in our setting from the expected profit expression for a given capacity investment as characterized by (3.9). In particular, in the benchmark case as captured by $\beta = 0$, the expected stage-2 profit is given by the sum of the profit $(P+S-w)\,\mathbb{E}\left[\min\left(\tilde{\xi}_1+\tilde{\xi}_2, K^F\right)\right]$ from the expected total subscription sales and the profit $(O-w)\,\mathbb{E}\left[\left(K^F-\left(\tilde{\xi}_1+\tilde{\xi}_2\right)\right)^+\right]$ generated by the leftover capacity. Using the identity $\min\left(K^F, \xi_1+\xi_2\right) = K^F - \left(K^F-(\xi_1+\xi_2)\right)^+$, the expected profit for a given capacity K^F can be rewritten as

$$-c^F K^F + (P+S-w)\,K^F - (P+S-O)\,\mathbb{E}\left[\left(K^F-\left(\tilde{\xi}_1+\tilde{\xi}_2\right)\right)^+\right], \quad (3.11)$$

where $P+S-O>0$ by assumption. When $\left(\tilde{\xi}_1, \tilde{\xi}_2\right)$ follows a bivariate Normal distribution, $\tilde{\xi}_1+\tilde{\xi}_2$ follows a Normal distribution with standard deviation $\sigma\sqrt{2(1+\rho)}$ and it is well-known that $\mathbb{E}\left[\left(K^F-\left(\tilde{\xi}_1+\tilde{\xi}_2\right)\right)^+\right]$ increases in this standard deviation. Therefore, as ρ increases, the expected profit for a given capacity K^F in (3.11) decreases, and thus, the optimal expected profit with flexible technology in the benchmark model decreases.

In the subscription model (i.e., $\beta > 0$), the effect of demand correlation on profitability with flexible technology is more nuanced. To illustrate this we assume $\left(\tilde{\xi}_1, \tilde{\xi}_2\right)$ to follow a bivariate Normal distribution and examine how demand correlation ρ affects the expected profit for a given capacity level as characterized by (3.9). Here, paralleling the benchmark case, the sum of the profit from expected total subscription sales and profit generated by the expected leftover capacity outside of the subscription program in the absence of reservation decreases in ρ due to decreasing capacity-pooling benefit. It can be proven that the total expected loss associated with unsatisfied switching requests for both products in the absence of reservation (i.e., the second line in (3.9)) decreases in ρ. This is because with a higher ρ there will be a higher likelihood that (i) the difference between $\max(\xi_1, \xi_2)$ and $\min(\xi_1, \xi_2)$ will be low, and thus, (ii) there are fewer unsatisfied switching requests in the absence of reservation leading to a lower expected loss. The impact of ρ on the reduction in the expected total loss due to optimal reservation decisions (i.e., the third line in (3.9)) cannot be proven analytically. With a higher ρ on one hand the expected leftover capacity after allocation $\mathbb{E}\left[\left(K^F - \left(\tilde{\xi}_1 + \tilde{\xi}_2\right)\right)^+\right]$; that is, the supply available for reservation, increases. On the other hand the expected volume for unsatisfied switching request $\mathbb{E}\left[\max(\tilde{\xi}_1, \tilde{\xi}_2) - \min(\tilde{\xi}_1, \tilde{\xi}_2)\right]$; that is, demand for reservation, decreases. We leverage numerical analysis to investigate the impact of ρ on profitability. We plot the optimal expected profits against the value of ρ ($\in [-0.9, 0.9]$ with a step size of 0.1) in Figure 3.4. Each curve is for a given value of β, and we plot it for all $\beta \in [0, 1]$ with a step size of 0.1. We consider two cost-parameter cases, i.e., $P = 2R$ (panel (a)) and $P > 2R$ (panel (b)). We present the key observation for all instances as follows.



(a) (b)

Figure 3.4: (Color online) Impact of demand correlation ρ on the optimal expected profit with flexible technology. Note: In each panel, $\rho \in [-0.9, 0.9]$ with a step size of 0.1 and $\beta \in [0, 1]$ with a stepsize of 0.1. In these instances, we have $P = 14$, $S = 32$. In panel (a): $R = 7$, $O = 35.5$; in panel (b): $R = 5.2$, $O = 34.6$. The red dots indicate the threshold $\overline{\rho}(\beta)$ in Observation 3.1.

Observation 3.1. *For a given switch-proportion β, we observe a unique demand correlation threshold $\overline{\rho}(\beta) \in [-1, 1]$ such that when $\rho \leq \overline{\rho}(\beta)$, the optimal expected profit with flexible technology decreases in ρ; otherwise, it increases in ρ. The threshold $\overline{\rho}(\beta)$ decreases in β.*

In both panels, we observe a decreasing trend in the optimal expected profit, Π^{F^*}, as ρ increases when β is small. The reason for this is that with a small β, the profit is less impacted by switching requests. In other words, the impact of ρ on profit terms involving β is less pronounced, including the total expected loss associated with unsatisfied switching requests in the absence of reservation (i.e., the second line in (3.9)) and the expected total loss due to optimal reservation decisions (i.e., the third line in (3.9)). In this case, the sum of profits from expected total subscription sales and profit generated by the expected leftover capacity outside of the subscription program in the absence of reservation has a dominating impact on the profit. Since this sum decreases with increasing ρ, the optimal expected profit also decreases. On the contrary, when β is large, the impact of ρ on the last two terms becomes stronger, leading to a non-monotonic relationship. As a result, we observe that the optimal expected profit, Π^{F^*}, first decreases and then increases as ρ increases. In particular, the optimal expected profit monotonically increases in ρ when β is close to 1, for the cost parameter case $P = 2R$ (panel (a)). We can analytically prove this special case when $\beta = 1$ in the following proposition.

Proposition 3.7. *Assume* $\left(\tilde{\xi}_1, \tilde{\xi}_2\right)$ *to follow a bivariate Normal distribution. When* $\beta = 1$ *and* $P = 2R$*, we have* $\frac{\partial \Pi^{F*}}{\partial \rho} \geq 0$.

The result for the special case is particularly noteworthy as it presents one possibility that a high demand correlation favors profitability with flexible technology, which is contrary to the conventional wisdom in the traditional ownership model that a high demand correlation typically diminishes capacity-pooling benefits and adversely affects profitability. In the subscription model (i.e., $\beta > 0$), a high demand correlation still has an adverse effect on capacity-pooling benefit. However, it can also reduce the imbalance between switching requests for both products, thereby decreasing the number of unsatisfied switching requests and increasing profitability. Proposition 3.7 describes a scenario (when $\beta = 1$ and $P = 2R$) in which the increase in profitability due to reduced unsatisfied switching requests compensates for the decrease in capacity-pooling benefits, leading to higher overall profitability.

We next examine the effect of switch-proportion β on the profitability with flexible technology.

Proposition 3.8. *Assume* $\beta > 0$*. We have* $\frac{\partial \Pi^{F*}}{\partial \beta} \leq 0$.

The expected profit for a given capacity depends on the switch-proportion β through the last two terms in (3.9); that is, the expected loss associated with unsatisfied switching requests for both products in the absence of reservation and the reduction in this expected loss due to optimal reservation decisions, respectively. With a higher β, there will be a higher imbalance between the switching requests and the returned units for the low-demand product at stage 2 for any given demand realization. As a result, the expected loss associated with unsatisfied switching requests for both products in the absence of reservation will increase. A higher β also increases the reduction in this expected loss by increasing the optimal reservation volume at stage 2. In particular, this happens at stage 2 when the firm optimally reserves $\beta \max(\xi_1, \xi_2) - \beta \min(\xi_1, \xi_2)$ amount for the low-demand product. However, in those cases, as follows from the last term in (3.9), while a higher β leads to a marginal cost reduction of $R - (O - S)$ per unit of the reserved volume, it also leads to an increase in its marginal cost by R because of an increase in unsatisfied switching requests in the absence of reservation. Therefore, the net effect is an increase in the marginal cost by $O - S$ per unit of the reserved volume. In summary, with a higher β, the sum of the last two terms in (3.9); that is, the total expected loss associated with switching requests for both products, increases, and thus, the optimal expected profit with flexible technology decreases.

3.6 Technology Choice

In §3.4 and §3.5, we investigated the impact of subscription demand correlation (ρ) and switch-proportion (β) for a given (dedicated and flexible, respectively) technology. In this section, we characterize the optimal technology choice (§3.6.1) and examine how this choice is affected by ρ and β, respectively (§3.6.2). To highlight the new insights resulting from the subscription model, we make a comparison with the benchmark case (the special case of $\beta = 0$) which represents the traditional ownership model. Throughout this section, paralleling the industry practice and the extant academic literature, we assume $c^F > c^D$; that is, flexible technology investment is more expensive than dedicated technology investment.

3.6.1 Optimal Technology Choice

The firm decides the optimal technology choice (dedicated or flexible technology) at stage 1 before determining the optimal capacity level with the chosen technology. It is easy to establish that the optimal expected profit strictly decreases in the unit capacity investment cost with each technology. Therefore, for a given unit dedicated capacity investment cost c^D, there exists a unique unit flexible capacity investment cost threshold such that it is optimal to invest in flexible technology when c^F is no more than this threshold (and it is optimal to invest in dedicated technology otherwise). Let $\bar{c}^F_\beta(c^D;\rho)$ denote this unique unit flexible capacity investment cost threshold for a given switch-proportion (β) and demand correlation (ρ).

We first discuss the optimal technology choice in the benchmark case of $\beta = 0$ where $\bar{c}^F_0(c^D;\rho)$ denotes the unit flexible capacity investment cost threshold for a given ρ. It is well-known from the extant literature that in a traditional ownership model the optimal technology choice is determined based on the trade-off between the capacity-pooling benefit of flexible technology and the lower unit capacity investment cost of dedicated technology. This can also be observed in our setting:

Remark 1. *We have $\bar{c}^F_0(c^D;1) = c^D$ and $\bar{c}^F_0(c^D;\rho) > c^D$ for $\rho < 1$.*

When the demands are perfectly positive correlated (i.e., $\rho = 1$), because two demands are always identical to each other, the firm does not benefit from the ability to reallocate the flexible capacity between the two products based on the demand realizations. Therefore, there is no capacity-pooling benefit of flexible technology and $\bar{c}_0^F(c^D;1) = c^D$. In this case, because $c^F > c^D$ the firm optimally does not invest in flexible technology. When the demands are not perfectly positive correlated (i.e., $\rho < 1$), there is capacity-pooling benefit of flexible technology and hence, $\bar{c}_0^F(c^D;\rho) > c^D$. In other words, there is a feasible range of $c^F > c^D$ in which the firm optimally invests in flexible technology. In this case, establishing the specific form of $\bar{c}_0^F(c^D;\rho)$ is not analytically tractable because the optimal expected profit with each technology cannot be characterized explicitly.

In the subscription model (i.e., $\beta > 0$) the optimal technology choice decision is more nuanced. While the trade-off between the capacity-pooling benefit of flexible technology and the lower unit capacity investment cost of dedicated technology continues to be relevant, there are two additional drivers that affect the optimal technology choice: (i) the total expected loss associated with unsatisfied switching requests for both products in the absence of reservation with each technology and (ii) the reduction in this expected total loss due to optimal reservation decisions with each technology. It is not a priori clear which technology's optimal expected profit is more significantly impacted by these two additional drivers. As we formally illustrate next, unless the subscription demands are perfectly positive correlated these two additional drivers are relevant and they may affect the optimal technology choice in such a way that, unlike the traditional ownership model, the firm optimally does not invest in flexible technology even when there is capacity-pooling benefit.

Proposition 3.9. *Assume $\beta > 0$. We have $\bar{c}_\beta^F(c^D;1) = c^D$. When $\rho < 1$, we have*

(i) $\bar{c}_\beta^F(c^D;\rho) > c^D$ for $\beta < 1$;

(ii) $\bar{c}_1^F(c^D;\rho) = c^D$ if $P = 2R$ and $\bar{c}_1^F(c^D;\rho) > c^D$ otherwise.

When the demands are perfectly positive correlated (i.e., $\rho = 1$), because two subscription demands are always identical to each other (i.e., $\min(\xi_1, \xi_2) = \max(\xi_1, \xi_2) = \xi$) there is no imbalance between the allocation volumes of both products with each technology at stage 2. In particular, as follows from Proposition 3.1, the optimal allocation volume for each product is given by $\min(\xi, K^D)$ with dedicated technology whereas, as follows from Proposition 3.5, the optimal allocation volume for each product is given by $\min(\xi, \frac{K^F}{2})$ with flexible technology. As a result, there is no loss associated with unsatisfied switching requests in the absence of reservation (and hence, there is no reservation) with each technology. Therefore, the optimal technology choice is identical to the benchmark case of $\beta = 0$. In particular, because there is no capacity-pooling benefit of flexible technology we have $\bar{c}^F_\beta(c^D; 1) = c^D$ and the firm optimally does not invest in flexible technology.

When the demands are not perfectly positive correlated (i.e., $\rho < 1$), because two subscription demands are not always identical to each other, there is imbalance between the allocation volumes of both products with each technology at some demand realizations. As a result, two additional drivers—that is, (i) the total expected loss associated with unsatisfied switching requests for both products in the absence of reservation with each technology and (ii) the reduction in this expected total loss due to optimal reservation decisions with each technology—matter for the optimal technology choice decision. The relevance of these two additional drivers for each technology can be observed from the second line of expected profit expression with dedicated technology in (3.5) and from the last two lines of expected profit expression with flexible technology in (3.9). Interestingly, Proposition 3.9 demonstrates that these two additional drivers may nullify the capacity-pooling benefit of flexible technology under some conditions (specifically, when $\beta = 1$ and $P = 2R$) so that, in contrast to the traditional ownership model, the firm does not invest in flexible technology. In other cases, there is a feasible range of $c^F > c^D$ in which the firm optimally invests in flexible technology. In these cases, paralleling the traditional ownership model, establishing the specific form of $\bar{c}^F_\beta(c^D;\rho)$ is not analytically tractable.

To delineate the intuition behind the characterizations in Proposition 3.9 for the $\rho < 1$ case, we introduce a reformulation to the expected profit expression with flexible technology for a given unit capacity investment cost c^F and capacity investment level K^F in (3.9):

$$\begin{aligned}\Pi^F(c^F,K^F) = \quad & -c^F K^F + (P+S-w)\mathbb{E}\left[\sum_{i=1}^{2}\min\left(\tfrac{K^F}{2},\tilde{\xi}_i\right)\right] + (O-w)\mathbb{E}\left[K^F - \sum_{i=1}^{2}\min\left(\tfrac{K^F}{2},\tilde{\xi}_i\right)\right] \\ & -(O-S)\mathbb{E}\left[\beta\min\left(\tfrac{K^F}{2},\max\left(\tilde{\xi}_1,\tilde{\xi}_2\right)\right) - \beta\min\left(\tfrac{K^F}{2},\min\left(\tilde{\xi}_1,\tilde{\xi}_2\right)\right)\right] + \Delta(\rho,\beta),\end{aligned} \tag{3.12}$$

where the sum of the first four terms represents the expected profit generated by allocating $K^F/2$ to each product in a dedicated manner (which resembles the expected profit expression with dedicated technology in (3.5)) and $\Delta(\rho,\beta)$, which we will explicitly characterize shortly, denotes the *flexibility premium* that captures the additional expected profit generated by optimally reallocating the flexible capacity based on the subscription demand realizations at stage 2. The reformulation in (3.12) is useful because when we set $c^F = c^D$ and $K^F = 2K^{D*}(c^D)$ (where $K^{D*}(c^D)$ denotes the optimal dedicated capacity investment level for a given c^D), the first four terms in (3.12) become identical to the optimal expected profit with dedicated technology. In this case, if $\Delta(\rho,\beta) > 0$, then the expected profit with flexible technology at this capacity investment level, and thus, the optimal expected profit with flexible technology is larger than the optimal expected profit with dedicated technology. As a result, we can conclude that $\bar{c}^F_\beta(c^D;\rho) > c^D$. If $\Delta(\rho,\beta) = 0$, then there is a need to check if the optimal flexible capacity investment level is different from $2K^{D*}(c^D)$. If the optimal flexible capacity level is different, then we can conclude that $\bar{c}^F_\beta(c^D;\rho) > c^D$; otherwise, we can conclude that $\bar{c}^F_\beta(c^D;\rho) = c^D$.

We next characterize the flexibility premium $\Delta(\rho,\beta)$ in (3.13) for a given ρ and β:

$$\begin{aligned}\Delta(\rho,\beta) &= (P-(O-S)-\beta R)\mathbb{E}\left[\left(\tfrac{K^F}{2}-\min\left(\tilde{\xi}_1,\tilde{\xi}_2\right)\right)^+\mathbb{I}\{\tilde{\xi}_1+\tilde{\xi}_2 > K^F\} + \left(\max\left(\tilde{\xi}_1,\tilde{\xi}_2\right)-\tfrac{K^F}{2}\right)^+\mathbb{I}\{\tilde{\xi}_1+\tilde{\xi}_2 \le K^F\}\right] \\ &\quad +(R-(O-S))\mathbb{E}\left[-\beta\left(\tfrac{K^F}{2}-\min\left(\tilde{\xi}_1,\tilde{\xi}_2\right)\right)^+\mathbb{I}\{\tilde{\xi}_1+\tilde{\xi}_2 > K^F\}\right. \\ &\qquad + \left(K^F-(\tilde{\xi}_1+\tilde{\xi}_2)-\beta\left(\tfrac{K^F}{2}-\min\left(\tilde{\xi}_1,\tilde{\xi}_2\right)\right)\right)\mathbb{I}\left\{\tilde{\xi}_1+\tilde{\xi}_2 \le K^F < (1+\beta)\max\left(\tilde{\xi}_1,\tilde{\xi}_2\right)-(1-\beta)\min\left(\tilde{\xi}_1,\tilde{\xi}_2\right)\right\} \\ &\qquad \left. +\beta\left(\max\left(\tilde{\xi}_1,\tilde{\xi}_2\right)-\tfrac{K^F}{2}\right)^+\mathbb{I}\{(1+\beta)\max\left(\tilde{\xi}_1,\tilde{\xi}_2\right)-(1-\beta)\min\left(\tilde{\xi}_1,\tilde{\xi}_2\right) \le K^F\}\right].\end{aligned} \tag{3.13}$$

The flexibility premium is particularly written as the sum of two terms. The first term represents the additional expected profit (over the profit generated by allocating $K^F/2$ to each product in a dedicated manner) for capacity reallocation. This is given by the product of the unit value of capacity reallocation, $P-(O-S)-\beta R$, and the expected capacity allocation volume,

$$\mathbb{E}\left[\left(\frac{K^F}{2}-\min\left(\tilde{\xi}_1,\tilde{\xi}_2\right)\right)^+\mathbb{I}\{\tilde{\xi}_1+\tilde{\xi}_2 > K^F\} + \left(\max\left(\tilde{\xi}_1,\tilde{\xi}_2\right)-\frac{K^F}{2}\right)^+\mathbb{I}\{\tilde{\xi}_1+\tilde{\xi}_2 \le K^F\}\right].$$

Note that one unit of reallocation creates β unit of imbalance between the switching requests and returned units. The unit value of capacity reallocation, therefore, is the difference between the unit margin $P-(O-S)$ and the penalty of the imbalance, βR. The second term is the net value of the reservation volume difference between allocating K^F capacity to two products in a flexible manner and allocating $K^F/2$ capacity to each product in a dedicated manner. It is the product of the marginal value for an additional reservation, $R-(O-S)$, and the expected reservation volume difference. Specifically, the reservation volume difference takes different forms under different ranges of K^F, i.e., $-\beta(\frac{K^F}{2}-\min(\tilde{\xi}_1,\tilde{\xi}_2))^+$ when K^F is small (i.e., $K^F < \tilde{\xi}_1+\tilde{\xi}_2$), $K^F-(\tilde{\xi}_1+\tilde{\xi}_2)-\beta(\frac{K^F}{2}-\min(\tilde{\xi}_1,\tilde{\xi}_2))$ when K^F is medium (i.e, $\tilde{\xi}_1+\tilde{\xi}_2 \le K^F < (1+\beta)\max(\tilde{\xi}_1,\tilde{\xi}_2)-(1-\beta)\min(\tilde{\xi}_1,\tilde{\xi}_2)$), and $\beta(\max(\tilde{\xi}_1,\tilde{\xi}_2)-\frac{K^F}{2})^+$ when K^F is large (i.e., $K^F \ge (1+\beta)\max(\tilde{\xi}_1,\tilde{\xi}_2)-(1-\beta)\min(\tilde{\xi}_1,\tilde{\xi}_2)$).

To delineate the intuition behind (3.13), let us first focus on the special case of $\beta = 0$ that represents the traditional ownership model. In this case, the flexibility premium, $\Delta(\rho, 0)$, is derived purely from the additional expected profit of capacity reallocation. Note that the additional expected profit of capacity reallocation is positive and thus $\Delta(\rho, 0) > 0$ for $\rho < 1$. This implies $\bar{c}_0^F(c^D; \rho) > c^D$ for $\rho < 1$, which aligns with Remark 1 for the traditional ownership model.

For the subscription model (i.e., $\beta > 0$), we can establish a lower bound of $\Delta(\rho, \beta)$ in (3.14) by replacing the reservation volume differences when $K^F \geq \tilde{\xi}_1 + \tilde{\xi}_2$ with a lower value of $-\beta\left(\max\left(\tilde{\xi}_1, \tilde{\xi}_2\right) - \frac{K^F}{2}\right)^+$.

$$\begin{aligned}
&\Delta(\rho, \beta) \\
&\geq (P - (O - S) - \beta R)\mathbb{E}\left[\left(\frac{K^F}{2} - \min\left(\tilde{\xi}_1, \tilde{\xi}_2\right)\right)^+ \mathbb{I}\{\tilde{\xi}_1 + \tilde{\xi}_2 > K^F\} + \left(\max\left(\tilde{\xi}_1, \tilde{\xi}_2\right) - \frac{K^F}{2}\right)^+ \mathbb{I}\{\tilde{\xi}_1 + \tilde{\xi}_2 \leq K^F\}\right] \\
&\quad + (R - (O - S))\mathbb{E}\left[-\beta\left(\frac{K^F}{2} - \min\left(\tilde{\xi}_1, \tilde{\xi}_2\right)\right)^+ \mathbb{I}\{\tilde{\xi}_1 + \tilde{\xi}_2 > K^F\} - \beta\left(\max\left(\tilde{\xi}_1, \tilde{\xi}_2\right) - \frac{K^F}{2}\right)^+ \mathbb{I}\{\tilde{\xi}_1 + \tilde{\xi}_2 \leq K^F\}\right] \\
&= (P - (1+\beta)R + (1-\beta)(R - (O - S)))\mathbb{E}\left[\left(\frac{K^F}{2} - \min\left(\tilde{\xi}_1, \tilde{\xi}_2\right)\right)^+ \mathbb{I}\{\tilde{\xi}_1 + \tilde{\xi}_2 > K^F\} + \left(\max\left(\tilde{\xi}_1, \tilde{\xi}_2\right) - \frac{K^F}{2}\right)^+ \mathbb{I}\{\tilde{\xi}_1 + \tilde{\xi}_2 \leq K^F\}\right].
\end{aligned} \tag{3.14}$$

Note that the first inequality in (3.14) is strict when $\beta < 1$. With $P \geq 2R$ from Assumption 3.1 and $R > O - S$ from Assumption 3.3, the lower bound is strictly larger than 0 for $\beta < 1$, and therefore, $\Delta(\rho, \beta) > 0$ for $\beta < 1$. In this case, the benefit from capacity reallocation can always compensate for the loss from reservation volume difference. It implies $\bar{c}_\beta^F(c^D; \rho) > c^D$ for $\beta < 1$ as shown in Proposition 3.9(i). In other words, there is a feasible range of $c^F > c^D$ in which the firm optimally invests in flexible technology in the subscription model when $\beta < 1$.

For the case that $\beta = 1$, $\Delta(\rho, 1)$ equals the lower bound, which can be simplified as below.

$$\Delta(\rho, 1) = (P - 2R)\mathbb{E}\left[\left(\frac{K^F}{2} - \min\left(\tilde{\xi}_1, \tilde{\xi}_2\right)\right)^+ \mathbb{I}\{\tilde{\xi}_1 + \tilde{\xi}_2 > K^F\} + \left(\max\left(\tilde{\xi}_1, \tilde{\xi}_2\right) - \frac{K^F}{2}\right)^+ \mathbb{I}\{\tilde{\xi}_1 + \tilde{\xi}_2 \leq K^F\}\right].$$

When $P > 2R$, we have $\Delta(\rho, 1) > 0$. This implies that $\bar{c}_1^F(c^D; \rho) > c^D$ as in the second part of Proposition 3.9(ii). When $P = 2R$, $\Delta(\rho, 1) = 0$.

This implies that under the condition $P = 2R$ and $\beta = 1$, the expected revenue generated from additional subscription sales via capacity reallocation is nullified by the expected penalty from the additional imbalance between switching requests and returned units, and the loss from reservation volume difference if we invest the flexible capacity $K^F = 2K^{D^*}(c^D)$. Interestingly, we can show that when $\beta = 1$ and $P = 2R$, the optimal flexible capacity investment satisfies $K^{F^*}(c^D) = 2K^{D^*}(c^D)$ in Lemma 3.1.

Lemma 3.1. *When $c^F = c^D$, we have $K^{F^*} = 2K^{D^*}$, if $\beta = 1$ and $P = 2R$.*

This implies $\bar{c}_1^F(c^D; \rho) = c^D$ when $P = 2R$, as in the first part of Proposition 3.9(ii). Therefore, in this case, the firm optimally does not invest in flexible technology contrary to the traditional ownership model.

3.6.2 Impacts of Demand Correlation and Switch-proportion on Technology Choice

In this session, we will perform a sensitivity analysis to investigate how the correlation between subscription demand (ρ) and the switch-proportion (β) affects technology choice. Recall from Proposition 3.3 and the sensitivity analysis of demand correlation ρ in Section 3.5.2, the optimal expected profit with dedicated technology increases in ρ, and the optimal expected profit with flexible technology could be non-monotone with ρ. It remains unclear how the demand correlation ρ affects the technology choice. The closed-form expression of $\bar{c}_\beta^F(c^D; \rho)$ is not analytically tractable, so we will resort to numerical experiments to examine how the threshold $\bar{c}_\beta^F(c^D; \rho)$ is affected by ρ, for various values of β, in cases where $\bar{c}_\beta^F(c^D; \rho) > c^D$. In terms of the impact of switching-proportion, Proposition 3.4 and Proposition 3.8 show that the optimal expected profits under both dedicated technology and flexible technology decrease in β, then, we are interested in under which technology the negative impact of β is stronger. Will the impact β on technology choice be monotone?

In this numerical study, we use the real case of the XC60 car model from Volvo's car subscription program to calibrate the model. The XC60 is a popular compact SUV that was released in 2013 and became part of the subscription program in 2019. The subscription contract duration for each Volvo car was two years. The baseline parameter values used in our study are presented in Table 3.1, with further calibration details provided in Appendix B.1. We consider ρ values ranging from -1 to 1, with increments of 0.1, and β values ranging from 0 to 1, with increments of 0.05, resulting in a total of 21×21 (ρ, β) combinations for both dedicated and flexible technology. We assume subscription demand $\left(\tilde{\xi}_1, \tilde{\xi}_2\right)$ follow a bivariate Normal distribution with mean μ and variance σ specified in Table 3.1. For each β (or β), we calculate the threshold $\bar{c}^F_\beta(c^D; \rho)$, and examine how this threshold change in ρ (or β). Thus, in this baseline instance, we investigate the 21×21 (ρ, β) combinations for both of the technologies. We consistently have the following observation.

Notation	Description	Value
P	Marginal revenue for subscription sales in the beginning of stage 2	14
R	Marginal penalty cost for unsatisfied switching requests in one period	7
S	Residual value for subscription product per unit in the end of stage 2	32
O	Marginal revenue for non-subscription product in the beginning of stage 2	35.5
c^D	Unit capacity investment cost of dedicated technology	26
μ	Average subscription demand for each product in the beginning of stage 2	7.46
σ	Variance of each subscription demand in the beginning of stage 2	1.12

Table 3.1: Description of the Baseline Parameters Used in Our Numerical Experiments. Note: In this baseline instance, we assume $P = 2R$, $O = 0.5R + S$, and $\sigma = 0.15\mu$.

Observation 3.2. *Given a $\beta \in (0, 1]$, for any ρ_0 and ρ_1, such that $-1 \leq \rho_0 < \rho_1 \leq 1$, we observe $\bar{c}^F_\beta(c^D; \rho_0) \geq \bar{c}^F_\beta(c^D; \rho_1)$, where the equality holds when*

$\beta = 1$ and $P = 2R$.

Observation 3.2 suggest that paralleling with traditional ownership model, a higher demand correlation (ρ) favors the dedicated technology adoption.

Observation 3.3. *Given a $\rho \in [-1, 1]$, for any β_0 and β_1, such that $0 \leq \beta_0 < \beta_1 \leq 1$, we observe $\bar{c}^F_{\beta_0}(c^D; \rho) \geq \bar{c}^F_{\beta_1}(c^D; \rho)$, where the equality holds when $\rho = 1$.*

Observation 3.3 implies that a higher switch-proportion (β) less significantly affects the optimal expected profit with dedicated technology, and does favor the dedicated technology adoption. Observation 3.3 also indicates that $\bar{c}^F_0(c^D; \rho) > \bar{c}^F_\beta(c^D; \rho)$ for $\beta > 0$. It provides the insight that in comparison with the traditional ownership model, the introduction of subscription programs favors the dedicated technology adoption.

To ensure the robustness of the key observations, we vary the baseline model parameters by considering combinations of the following values, $R \in \{3.4, 5.2, 7\}$, $O \in \{0.25R + S, 0.5R + S, 0.75R + S\}$, $c^D \in \{25, 26, 27\}$, $\mu \in \{3.73, 7.46, 11.19\}$, $\sigma \in \{10\%, 15\%, 20\%\}$ of μ. In summary, we conduct the same numerical studies on 243 instances: each with 21×21 (ρ, β) combinations to characterize $\bar{c}^F_\beta(c^D; \rho)$. Among all these instances, for any given β, we constantly observe the same pattern, as reported in Observation 3.2; for any given ρ, we constantly observe the same pattern, as reported in Observation 3.3.

Parallel to the analysis in Section 3.6.1, to delineate the intuition behind Observation 3.2, we investigate how the flexibility premium, i.e., $\Delta(\rho, \beta)$, is affected by the demand correlation, ρ, for different values of switch-proportion, β. As we formally present in Proposition 3.10, the analysis shows consistent results with our numerical observations for the threshold $\bar{c}^F_\beta(c^D; \rho)$.

Proposition 3.10. *Assume* $\left(\tilde{\xi}_1, \tilde{\xi}_2\right)$ *to follow a bivariate Normal distribution. When* $\beta > 0$*, we have* $\frac{\partial \Delta(\rho,\beta)}{\partial \rho} \leq 0$.

Our analytical result suggests that a high demand correlation reduces the flexibility premium and favors dedicated technology adoption. To examine the intuition behind our result, let us focus on the flexibility premium for a given capacity investment level K^F as characterized by (3.13). Proposition 3.10 demonstrates that the flexibility premium $\Delta(\rho, \beta)$ decreases as demand correlation ρ increases. Recall that the first term of $\Delta(\rho, \beta)$ represents the additional expected profit obtained from capacity reallocation, which decreases as ρ increases. The second term represents the expected profit from the reservation volume difference between the two technologies, which is non-monotonic in ρ. However, the negative impact of ρ on the first term dominates, resulting in a decrease in $\Delta(\rho, \beta)$ as ρ increases. When $\rho = 1$, the flexibility premium decreases to zero.

Below, we investigate how the flexibility premium, i.e., $\Delta(\rho, \beta)$, is affected by the switch-proportion, β, for different demand correlation, ρ.

Proposition 3.11. *Assume* $\left(\tilde{\xi}_1, \tilde{\xi}_2\right)$ *to follow a bivariate Normal distribution. When* $\beta > 0$*, we have* $\frac{\partial \Delta(\rho,\beta)}{\partial \beta} \leq 0$.

Proposition 3.11 shows that the flexibility premium $\Delta(\rho, \beta)$ decrease in β. It is intuitive because as β increases, the unbalanced additional allocation volume increases, which implies that the marginal profit from capacity reallocation $(P - (O - S) - \beta R)$ decreases. At the same time, it could lead to the additional loss for reservation in the second term of $\Delta(\rho, \beta)$ increase, and bring additional benefit for reservation in the second term of $\Delta(\rho, \beta)$ increase. Since the additional marginal penalty R is larger than the marginal reservation benefit $R - (O - S)$, then, the negative effect of β dominates, and thus $\Delta(\rho, \beta)$

decreases in β. In particular, when β increases to 1, the flexibility premium decreases to zero if $P = 2R$.

3.7 Summary

This paper contributes to the operations management literature on stochastic capacity and technology investment in multiproduct firms by analyzing the impact of subscription programs on the flexible versus dedicated technology choice, which have significant different features than the traditional ownership model. In comparison with the traditional ownership model, the subscription programs brings challenges for the manufacturer in choosing its production technology (flexible versus dedicated technology) and capacity investment level with the chosen technology under demand uncertainty. This is because the manufacturer needs to make the production decision for each car model while considering the management of switching customers during the subscription period. Specifically, in the presence of subscription programs, the correlation between subscription demands and the proportion of switching requests brings new challenges to the firm's flexible versus dedicated technology choice and the optimal capacity investment with each technology. This is the first paper that studies how subscription programs shape the firm's decisions.

With dedicated technology, we find that a higher demand correlation increases the profitability by decreasing the expected cost associated with satisfying switching requests. With flexible technology, the impact of demand correlation would be more complication. Because customers' switching requests create a link between allocation and reservation of two products, and results in the trade-off between allocation and reservation. Interestingly, we theoretically show that under some conditions, the expected cost associated with satisfying switching requests completely nullifies the capacity-pooling benefit of flexible technology, which leads that the demand correlation does not affect profitability with flexible technology. Moreover, we find that in the presence of subscription programs, a higher switch proportion decreases the profitability with both dedicated and flexible technology. This is consistent with the practical observation that it is difficult to manage customers' switching requests.

Further, we identify that based on the demand realizations, the flexibility in how to allocate the leftover capacity for outside of subscription programs in the absence of reservation plays a key role on the optimal technology choice. We use flexibility premium to capture the capacity pooling benefit generated in the subscription program. Consistent with our numerical observations, we theoretically find that the flexibility premium decreases in the demand correlation and switch-proportion. These results implies that a higher demand correlation or a higher switch-proportion favors the dedicated technology adoption. Interestingly, when the demand correlation or switch-proportion becomes high enough under some conditions, the flexibility premium becomes zero. Managerially, these results are important because they imply that the benefit of flexible technology may not exist in the presence of subscription program.

Relaxing the assumptions we made on the production environment gives rise to a number of interesting possibilities, both in the theory of capacity management and technology choice. For example, first, we assume the usage periods of customers before requesting switching are the same; that is, we assume the penalty costs for unsatisfied switching requests are the same, and the value of reservation products are the same. The timing of request of switching can be random, which leads to the availability of freed-up units complex. We may predict that in order to reduce the penalty cost of unsatisfied switching requests, the manufacturer may need to increase the production volume for reservation. Thus, considering more details of customers switching requests would be a promising direction. Second, we assume that ,the subscription fee is exogenous and all customers will accept upgrades and there is no cost for the customer to upgrade to a higher level of service if there is a lack for desired product. With a positive switching cost, one the one hand, the switching proportion could decrease, which reduces the flexibility of customers with subscription programs. On the other hand, the demand of subscription program could decrease, as the convenience of subscription programs becomes smaller. It would be fruitful to add the impact of subscription price or switching cost on subscription demand, and discuss the capacity investment and technology choice decisions. Thirdly, we assume each subscription program contains two products, which have the same demand average. The subscription programs may contain more than two products, in which the products may be different in demand and retailing price. It would be interesting to analyze the subscription product portfolio management and flexible versus dedicated technology choice.

Chapter 4

Partnership Between Taxis and On-Demand Ride-Hailing Platforms: How Should It Be Regulated?

4.1 Introduction

The rise of on-demand ride-hailing platforms, such as Uber, Grab, and Didi Chuxing, creates digital disruption across the taxi industry (Schiller 2017). One of the key differences between these on-demand ride-hailing platforms and traditional taxi companies is independent service providers (i.e., private car drivers) in these platforms. On-demand ride-hailing platforms set ride fares for riders who request on-demand ride-hailing services and offer wage compensations to attract private car drivers to serve these requests. However, the rapid development of on-demand ride-hailing platforms boosts the driver shortage of their ride-hailing services, especially due to Covid, which increases riders' waiting time (Fanusie 2021). Private car drivers are reluctant to return despite relaxed Covid restrictions (Lean and Teo 2022). What's more, luring back drivers and on-boarding new ones is taking time and money, revealing the fragility of these platforms' supply (Davalos 2022).

Interestingly, there is a trend that with the approval of authorities, on-demand ride-hailing platforms and traditional taxi companies reach agreements to collaboratively provide on-demand services. For example, in 2016, Didi Chuxing, China's largest ride-hailing company, announced a partnership with taxi companies across ten Chinese cities to improve matching efficiency and service quality (He 2016). In 2017, Grab, southeast Asia's leading ride-hailing company, formed a partnership with the Singapore taxi operators to provide JustGrab service (Grab 2017). In 2022, Uber formed an partnership with traditional taxi companies in New York City and San Francisco by listing taxicabs on its app (Hu et al. 2022).

With the partnership, the listed taxi drivers are able to access riders' on-demand ride-hailing requests from on-demand platforms. Thus, for ride service like "Justgrab", one key feature is that on-demand ride-hailing requests are served by a mixed pool of traditional taxi drivers and private car drivers. For example, by using Justgrab to hail a cab, a rider is assigned the nearest vehicle, *regardless of whether it is a private car or a standard taxi*. That is to say, the rider could be picked up by either a taxi driver or a private car driver and pay the flat fare, which is determined by the on-demand ride-hailing platform.

The partnership generates several critical impacts on the ride-hailing service system. (i) On-demand ride-hailing platforms could have more drivers to serve riders' requests with less waiting time. (ii) Taxi drivers gain a new service option by getting access to on-demand ride-hailing requests, and paid by wage compensations set by ride-hailing platforms, which are different from the traditional metered fare set by the government. (iii) For private car drivers, although the platforms can serve more riders, since a fraction of these riders are served by traditional taxis, people are skeptical whether they can gain higher earnings (Linebaugh and Knutson 2022). Besides these impacts, the partnership is faced with challenges and brings some concerns. For the supply side of drivers, there is a challenge of taxi drivers' accessibility: (i) taxi drivers acceptance level is low due to the entry barrier of digital technology; (ii) taxi drivers may be reluctant to join this alliance because they are sceptical about whether they can earn more. For example, in Singapore, lots of older experienced taxi drivers state that adapting to the new model is difficult (Ong 2022). It mean that not all traditional taxi drivers can have the benefit of getting full access to both street-hailing and ride-hailing services.

However, even if all taxi drivers get full access to both street-hailing and ride-hailing services, it also could be problematic for the demand side of riders. As more taxi drivers serve on-demand ride-hailing requests, it becomes harder for riders to find taxis on streets and afford the ride fee of on-demand ride-hailing, especially for elderly who are unfamiliar with using smartphones. For example, in New York city, during the Covid period, the ride fares of Uber and Lyft are very high, and the traditional cab fare based on meters could be more affordable for them. However, the number of cabs on the street becomes fewer, making it harder for riders to access this service. This intensifies the "empty seats, busy street" phenomenon.

In practice, governments in different cities or countries take different measures to address these challenges and concerns, as summarized in Table 4.1. In China, in 2015, to make taxis more available to roadside riders, the Shanghai government restricted the ride information of taxi drivers for ride-hailing app and require them to serve riders in traditional manner (Hewitt 2015). What's more, Chinese governments regulate ride-hailing platforms to retain street-hailing service channels and urge them to develop their digital services friendly to the elderly (Chen 2020). In Singapore, to improve taxi drivers' accessibility, the government launched several initiatives to help drivers, especially senior ones, adapt to digital platforms (Goh 2019). To tackle the issue of riders' street-hailing availability, in 2017, Singapore government uses percentage of taxis on the roads to improve the supply (Lim 2016). In 2020, due to the decrease of street-hailing requests and the increase of on-demand ride-hailing requests, the government lift the taxi availability standard (Abdullah 2020) and pass rules to both street-hailing and ride-hailing services to ensure that commuters benefit from more transportation options. Differently, the Malaysian government has undertaken efforts to encourage ride-hailing operators to "adopt" taxi drivers, and incentivise taxi drivers to migrate to ride-hailing platform by relaxing taxi requirements (Mohamad Izham 2018). In US, even people has concern about the availability of street-hailing as the partnership between Uber and taxi expands, currently regulators are still open and does not have any actions (Daus 2022). Thus, it is unclear whether and how the government should intervene into their partnership by controlling taxi drivers' access level.

City or Country	Year	Platform	Government Measure
China	2015	Didi Chuxing	Restrict the access (e.g., restrict time of taxi drivers' access to ride app)
Singapore	2017	Grab	Mixed measures (e.g., set standards for drivers providing a public service)
Malaysia	2017	Grab	Encourage the access (e.g., encourage taxi drivers to migrate to use ride app)
New York City, San Francisco	2022	Uber	No measures taken

Table 4.1: Measures in Different Regions

This paper aims to shed light on the ongoing debate concerning government regulation to the partnership between traditional taxi companies and on-demand ride-hailing platforms. The novelty of our analysis lies in incorporating the government's control measures regarding access policies for drivers in the on-demand ride-hailing framework. Specifically, with their partnership, we focus on the following research questions.

1.With the allowed access of traditional taxis to on-demand ride-hailing requests, what are the optimal decisions for agents (i.e., the on-demand ride-hailing platform, taxi drivers, private car drivers, and riders)? Specifically, is it possible for taxi drivers to provide both street-hailing and ride-hailing services?

2.Should the government intervene by promoting or restricting taxis' access to on-demand ride-hailing requests? If so, what are the government's optimal regulatory decisions to maximize the social welfare?

3.What if a certain level of street-hailing availability from taxi drivers is desired?

To address these questions, we develop a game-theoretical model to capture an on-demand ride-hailing platform who may expand its supply pool with private car drivers, by forming a partnership with a traditional taxi company to connect taxi drivers into the online platform. In our model, riders are sensitive to service delays. Taxi drivers and private car drivers are different in participation costs and service qualities. The platform sets a ride fee for riders who request ride-hailing services and provides a wage compensation for participating drivers who serve ride-hailing requests. The government decides on the maximum access level of taxi drivers to serve the platform-based ride-hailing requests and the taxi fee for street-hailing services to maximize the social welfare. Firstly, we derive the optimal decisions for agents, i.e., the on-demand ride-hailing platform, taxi drivers, private car drivers, and riders, for a given regulatory framework. We further examine the optimal regulatory decisions of the government without and with a restriction on street-hailing availability, respectively.

Our main findings are as follows.

1.Only when the allowed maximum access level of taxi drivers is high enough, which means the available service time of taxi drivers to on-demand ride-hailing requests is sufficient, the online platform offers a high enough wage compensation to attract the participation of both taxi drivers and private car drivers (Proposition 4.3).

As the number of private car drivers increases, taxi drivers are less likely to serve ride-hailing requests, particularly in "mixed" service mode. Because the pooling effect from private car drivers becomes stronger, the online platform becomes less willing to provide a high wage compensation to attract taxi drivers.

2.If there is no restriction on street-hailing availability, the government could optimally choose to allow taxi drivers to get "partial access" or "full access." In particular, "partial access" is optimal if the labor welfare is more important, and the number of private car drivers is moderate (Proposition 4.4). This result suggests that the partnership should be encouraged. However, advocating the partnership by enabling taxi drivers to get "full access" to the online platform is not always optimal for the government.

3.If there is a restriction on street-hailing availability, the government could optimally choose to allow taxi drivers to get "partial access" or "no access". In particular, "partial access" is optimal when the number of private car drivers is not high (Proposition 4.5), and it becomes less possible as the restriction level increases. This result suggests that promoting the partnership is not necessary when the number of private car drivers is very high. Otherwise, the partnership should be encouraged by allowing "partial access," especially as the riders' street-hailing requests decreases.

The remainder of this study is organized as follows. In Section 4.2, we review the related literature. Section 4.3 introduces the model setting and decision sequence and events. Section 4.4 provides the parameter estimation of our base model. Section 4.5 analyzes riders' and drivers' participation decisions and the online platform's optimal decisions on wage compensation and ride fee. Section 4.6 investigates the optimal regulation decisions of the government to optimize social welfare. Section 4.7 summarizes our results and outlines future research directions. Proofs of the technical results are relegated to the Appendix C.

4.2 Literature Review

This paper is related to three streams of literature: (i) monopoly on-demand service platform; (ii) competition and coopetition of on-demand platforms, and (iii) government regulation on ride services.

4.2.1 Single On-Demand Service Platform

Our work is related to the growing operations management literature on on-demand service platforms. The focus of this literature has been on settings with a single platform that operates as a monopolist. Taylor (2018) examines how the delay sensitivity and agent independence features impact an on-demand service platform's optimal per-service price and wage. Cachon et al. (2017) study several pricing schemes of a service platform, and show the benefit from the use of surge pricing on a platform with self-scheduling capacity. Bai et al. (2019) present an analytical queueing model to capture both heterogeneous customers and independent providers, and analyze a single on-demand service platform's optimal price and wage rates that maximize the profit of the platform. Feng et al. (2021) investigate how various matching mechanisms of on-demand platforms affect the efficiency of the transportation system—in particular, whether it will help reduce passengers' average waiting time compared with traditional street-hailing systems. They find that the on-demand matching mechanism could result in lower efficiency than the traditional street-hailing mechanism. He et al. (2023) focus on the phenomenon of leakage, where customer-provider pairs may decide to transact "off-platform" to avoid the platform's commission, study the key characteristics of a service vulnerable to leakage, and propose and evaluate a potential approach that a platform may employ to curb leakage. Tang et al. (2021) study the safety concern of female users (riders and drivers), and examine how female users' safety concerns affect the system configuration of ride-hailing platforms. Cachon et al. (2022) study the price control of the platform service, by the platform or the servers. They find that when the platform uses a simple commission contract to earn revenue, the price delegation decision depends on the importance of regulating competition among the large population of servers relative to the value of allowing servers to tailor their prices to their privately known costs. Bimpikis et al. (2022) investigate the information provision policy of suppliers' service quality, with respect to managing supply-side decisions, including supplier entry/exit and pricing. Liu et al. (2023a) show that the jobs assigned by the better matching technology can unintentionally reveal more information about uncertain labor demand to workers, and thus unfavorably change workers' participation decisions, resulting in a revenue loss for the platform.

Differently, in our paper, we consider the partnership between on-demand ride-hailing platform and the traditional taxi company, and investigate the access level of taxi drivers to on-demand ride-hailing requests.

4.2.2 Competition and Co-opetition of On-Demand Service Platforms

A number of researchers have recently studied the impact of competition between on-demand service platforms. Bernstein et al. (2021) study competition between on-demand service platforms under two settings: one in which there is a dedicated pool of workers for each platform (single homing) and one in which workers work for both platforms (multi-homing). Benjaafar et al. (2020) discuss the impact of competition between two on-demand service platforms on worker welfare and consumer surplus. Bai and Tang (2022) examine a base case when both firms operate under seven operational assumptions and find that only one platform can sustain a payoff dominant stable equilibrium. Siddiq and Taylor (2022) explore the case that a platform's access to supply-side (namely, AV) technology changes prescriptions for its demand-side (namely, pricing) decisions, and the implications of competition and access to AVs for the management of ride-hailing platforms. Wu et al. (2020) examine how the characteristics of the embedded workers–customers subgame affect outcomes in equilibrium. Nikzad (2020) investigates competition by comparing the monopoly and duopoly equilibria, and finds that competition benefits workers: their wage and average welfare are always higher in the duopoly equilibrium. Daniels and Turcic (2023) consider two strategies of taxi serve to improve their competitiveness with ride-hailing services. Zhang et al. (2022) examine the impacts of the self-scheduled nature of the supply on competing platforms and the role of the wage scheme in the platform competition.

There are papers considering the co-opetition between ride service platforms. Cohen and Zhang (2022) study the impact of co-opetition between two-sided platforms such as ride-sharing firms or on-demand service platforms and design a profit-sharing contract that will benefit both the users and the firms. Zhang et al. (2021) examine the cooperation between the platform and the traditional rental firm, under which they allow a (secondary) driver to rent from the rental firm and drive for the platform. However, this paper does not consider the time-sensitivity feature of riders and assumes the rental firm set the price charged to riders.

There are papers considering the integration of multiple on-demand service platforms. Lin et al. (2022) study the impact of mergers between on-demand service platforms on consumer surplus and labor welfare. They show that customers may benefit from a merger due to the risk-pooling effect and reduced waiting times. Zhou et al. (2022b) study the impact of third-party platform-integration on the ride-sourcing market with multiple competing platforms.

In our paper, we consider the partnership between an on-demand service platform and a traditional service company that is regulated by the government, that is, the integration of online and offline services, which is different from these papers considering collaboration or integration of multiple on-demand service platforms. Moreover, under our model, the partnership between the on-demand service platform and the traditional service company makes the driver pool become a mix of two types of drivers, i.e., taxi drivers, and private car drivers who are independent service providers.

4.2.3 Government Regulations to Ride Services

To regulate ride-hailing platforms, some policies have already taken effect, mainly including a minimum wage for drivers, a cap on the number of drivers, entry restrictions for drivers, the congestion fee, and the utilization rate of vehicles. Ke et al. (2021) provide a review of these regulatory policies. Several papers investigate the effectiveness of these regulation policies to ride service platforms.

Yu et al. (2020) focus on the regulatory policy that regulates the "maximum" number of registered Uber/DiDi drivers, and examine the impact of these policies on the welfare of different stakeholders. They show that lowering the taxi fare instead of imposing a strict policy toward on-demand ride services can improve the total social welfare. Benjaafar et al. (2022) consider the welfare of independent agents, and investigate the effectiveness of two regulation measures that reduce the labor pool size or that impose a floor on the nominal wage or effective wage (i.e., the product of the nominal wage and agent utilization). Huang et al. (2022) compare three quality regulation strategies: the platform excludes access to low-quality complementors; it provides a fixed amount of subsidy to high-quality complementors; it develops its own high-quality applications in addition to those from third-party complementors. Rhee et al. (2022) empirically investigate the policy to restrict taxi drivers' access to and acceptance of ride requests via ride-hailing apps during certain hours, and reveal the underlying mechanism that information sharing via ride-hailing apps reduces drivers' search cost and thus enables them to match not only with more orders but also with those of higher marginal profit. Hu et al. (2023b) study the policy question about workers being classified as independent contractors or employees for providing on-demand services, and empirically investigate the implication of 2019 Assembly Bill No. 5 (AB5) in California.

There are papers studying the government's regulations on private and public services. Hu et al. (2023a) study a service system where the job of providing a public service is delegated to a private firm subject to government regulation, and discuss the two practically motivated regulation instruments, i.e., price and wait time. Siddiq et al. (2022) study the strategic partnership between a public transit agency and a ride-hailing platform to improve public transit adoption by solving the last mile problem, and investigate two incentive mechanisms: provide a subsidy to commuters who adopt a "mixed mode" of taking public transit and hailing rides to/from a transit station; provide funding subsidies through a private sector partner. Zhou et al. (2022a) consider a mixed duopoly service system with two service providers (SPs), one private and the other public. They demonstrate that the maximum social welfare is achieved by partially privatizing the public SP, that is, by including both welfare and profit maximization as arguments in its objective function.

In the economics literature, a number of papers study the efficiency of governmental regulations of taxi industries. Douglas (1972) studies the optimal regulated taxi price and service quality. Cetin and Deakin (2019) provide a review of pricing and entry regulation of taxi markets.

The most related paper to ours is Yu et al. (2020). They discuss the competition between taxi drivers and Didi drivers under the "maximum" number of registered Uber/DiDi drivers regulation and a complete ban policy, respectively. Differently, in our paper, we focus on the government's regulations about the partnership between the taxi company and the on-demand ride-hailing platform. Specifically, we examine the government's regulatory decision on the taxi drivers' service capacity allocation by controlling their access level to on-demand ride-hailing requests.

4.3 Modeling a Mixed Private Transportation Ecosystem

In this section, we start by describing the main entities comprising the ecosystem of private urban transportation that mixes traditional and online-based modes of providing riding services and then follow up by presenting the time sequence of events in our model.

4.3.1 Participating Entities

We consider an on-demand ride-hailing platform and a traditional taxi company operating within the same urban market. Our model involves six distinct entities: the government, the online platform, the taxi company, taxi drivers and private car drivers, and riders, as illustrated in Figure 4.1. We use index $i = 1$ to denote taxi drivers (i.e., type-1 drivers), and index $i = 2$ to denote private car drivers (i.e., type-2 drivers). We also use $j = s$ to denote the traditional street-hailing service, and $j = p$ to denote the ride-hailing service requested via the online platform. Before moving into details of the model, we describe each of the six entities.

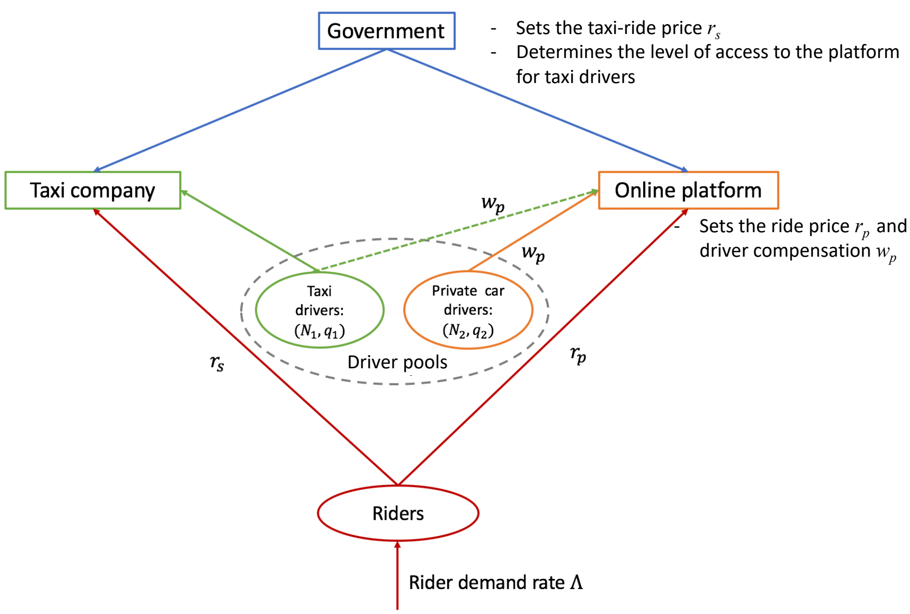


Figure 4.1: (Color online) Modelling urban private transportation services: participating entities and their decisions

The government: The government controls the price (per ride), r_s, that the taxi company charges to its customers, as well as the level of access that the taxi drivers have to the ride requests arriving via the online platform. In our analysis, we model this level of access using the parameter $\bar{\alpha}$, $\bar{\alpha} \in [0, 1]$ which describes the maximum fraction of taxi drivers' working time that can spent serving customers on the platform. In particular, $\bar{\alpha} = 0$ describes the "no access" setting where taxi drivers are blocked from using the platform, and must rely exclusively on street-hailing requests, while $\bar{\alpha} = 1$ describes the "complete access" setting where taxi drivers are free to allocate their working time between the street-hailing and on-platform modes of serving customers. Thus, by setting the value of $\bar{\alpha}$, the government can control the conditions of co-existence of the "traditional" and "new" components of the transportation services ecosystem, and, in particular, a degree to which the traditional taxi company can benefit from the innovative platform technology while maintaining its own way of providing high-quality transportation services. The government uses these two levers - r_s and $\bar{\alpha}$ - to create conditions that are maximally beneficial from the point of view of total social welfare.

Under any choice of $\bar{\alpha}$, riders who request street-hailing services will be served by taxi drivers with high service quality. Under the "no access" mode ($\bar{\alpha} = 0$), riders who request transportation services via the online platform will only be served by private car drivers with lower service quality, while in any other setting those riders will face a mixed pool of drivers, and enjoy the benefits of pooled service capacity.

The taxi company: The taxi company owns taxicabs and rents them to taxi drivers for a fixed rental fee h (per unit of time), which we assume to be an exogenous parameter.

The online platform: The online platform does not own any vehicles, and its job is to match drivers who join the online platform with riders who request ride-hailing services. The online platform sets the ride-hailing price r_p (per ride) to be charged to riders and the wage compensation w_p (per ride) for drivers working on the platform.

Drivers: In our model, there are two types of drivers providing ride services: type-1 drivers with service quality q_1 (i.e., taxi drivers), and type-2 drivers with service quality q_2 (i.e., private car drivers). They come from two separate driver pools and cannot switch from one type to the other. In many countries, the requirements for being a taxi driver are more strict than those for a private car driver joining the online platform. Consequently, in our model we assume that $q_1 > q_2$. The number of potential drivers of each type is N_i, $i \in \{1, 2\}$, and the total number of potential drivers is $N = N_1 + N_2$. Taxi drivers rent taxis at a fixed rental fee h (per unit of time) from the taxi company. In contrast, private car drivers drive their own cars. Without loss of generality, we normalize vehicle maintenance costs of taxicabs and private cars to zero. Each type i driver, $i \in \{1, 2\}$, has an outside opportunity cost k_i per unit of time, $k_1 > k_2$. We assume that the rides have an expected duration of $1/\mu$ that does not depend on the driver's type. Let n_i denote the realized number of type i drivers who choose to participate in the ride services market, $n_i \in \{0, 1, ..., N_i\}$. Private car drivers who join the online platform serve rider requests exclusively via the platform, while taxi drivers who join the taxi company serve street-hailing requests, and, for $\bar{\alpha} > 0$, may decide to also serve riders via the online platform. When taxi drivers decide to allocate α fraction of their time to providing services via the on-demand platform, we use $n^s = (1 - \alpha)\, n_1$ and $n^p = \alpha n_1 + n_2$ to denote the supply levels of street-hailing and platform-based riding capacity, respectively, i.e., the "effective" numbers of drivers providing street-hailing and platform-based rides.

Riders: Riders are both price- and delay-sensitive, and request the ride services according to a homogeneous Poisson process with the rate Λ. We assume that the ride services market is supply-constrained,

$$\Lambda > N\mu, \tag{4.1}$$

so that some of the demand for transportation services is satisfied by public transportation options. Following Lian et al. (2022) and Liu et al. (2023b), we model the expected time in the system, W^j, for riders using service $j = s, p$, via the $M/M/1$ queueing dynamics. In particular, let λ^j be the arrival rate for the riders who request service $j \in \{s, p\}$, and n^j be the number of drivers who participate in service $j \in \{s, p\}$. Then,

$$W^j\left(n^j, \lambda^j\right) = \begin{cases} \frac{1}{n^j\mu - \lambda^j}, & \text{if } \lambda^j < n^j\mu, \\ +\infty, & \text{otherwise}, \end{cases} \tag{4.2}$$

where μ denotes the average service rate corresponding to transportation services. Note that the time-in-the-system expression (4.2) includes the "pickup time," i.e., the time required for the driver to arrive at rider's pick-up point (Castillo et al. 2022, Feng et al. 2021).

If riders choose to request street-hailing services, they will be served by higher-quality type-1 drivers, while the riding requests placed via the online platform may, under $\bar{\alpha} > 0$, be served by a mixed driver pool. We assume that, for a typical rider it is easier to place a riding request via the platform, and model the difference for riders between the street-hailing and platform-based service using the extra per-ride "inconvenience" cost $I > 0$ incurred by riders who use street-hailing.

Under the assumptions described above, the utility for the riders who choose street-hailing can be expressed as

$$U_r^s(r_s, n^s, \lambda^s) = q_1 - I - r_s - cW(n^s, \lambda^s), \tag{4.3}$$

where, for simplicity, we equate the utility of service to its quality, c denotes the rider's waiting cost per unit of time, and $n^s = (1-\alpha)\, n_1$. The corresponding utility for the riders who choose on-demand ride hailing is

$$U_r^p(r_p, n^p, \lambda^p) = q^p - r_p - cW(n^p, \lambda^p), \tag{4.4}$$

where $n^p = \alpha n_1 + n_2$ and $q^p = \frac{\alpha n_1 q_1 + n_2 q_2}{\alpha n_1 + n_2}$.

Let λ_1^p and λ_2^p denote the participating number of riders who request ride-hailing services and are served by type-1 and type-2 drivers, respectively.

The process of matching rider requests with driver capacity is illustrated in Figure 4.2.

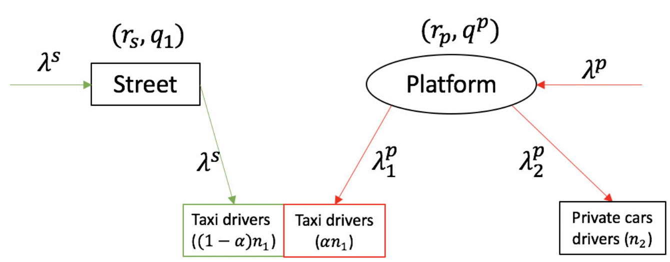


Figure 4.2: Matching demand for rides with supply of driver capacity

4.3.2 Sequence of Decisions and Events

Our modeling framework is based on a four-stage sequential game, as shown in Figure 4.3. Below we describe each stage in detail.

1. At the first stage, the government sets the fee r_s that the traditional taxi firm is allowed to charge for street-hailing services, and the upper

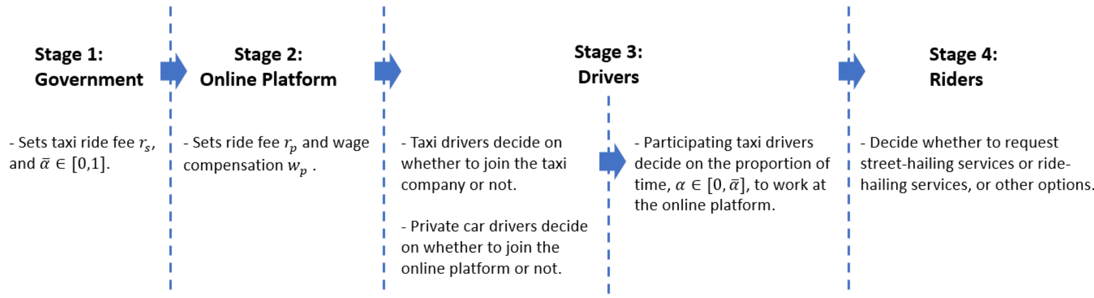


Figure 4.3: Timeline and Decision Sequence

bound $\bar{\alpha}$ on the proportion of time that the taxi drivers can spend serving ride-hailing requests on the platform.

2. At the second stage, given the values of r_s and $\bar{\alpha}$, the online platform sets the per-ride fee r_p it will charge its customers, and the wage compensation w_p it will pay the drivers who will serve ride-hailing requests on the platform.

3. At the third stage, drivers make their decisions on how to use their service capacity. Taxi drivers decide on whether to join the taxi company or not, and private car drivers decide on whether to join the online platform or not. Next, after observing the number of participating drivers (n_1, n_2), taxi drivers who joined the taxi company set the proportion of their service capacity, $0 \leq \alpha \leq \bar{\alpha}$, to be allocated to serve the platform-based ride-hailing requests.

4. Finally, at the fourth stage, riders, upon observing all the ride fees and participating decisions of drivers, decide on using street-hailing services, ride-hailing services via the platform, or other transportation options, such as public transportation.

For convenience, the summary of modeling notation we use is presented in Table 4.2.

Notation	Explanation
r_j	Fee (per ride) for service j, $j \in \{s, p\}$
$\bar{\alpha}$	The maximum proportion of service time that participating taxi drivers can spend serving ride-hailing requests via the platform, $\bar{\alpha} \in [0, 1]$
w_p	Wage compensation (per ride) for participating drivers on the platform
α	Proportion of service time that participating taxi drivers allocate to serving ride-hailing requests via the platform, $0 \leq \alpha \leq \bar{\alpha}$, where $\bar{\alpha} \in [0, 1]$
h	Taxi rental fee (per unit of time)
μ	Expected service rate ($1/\mu$ represents the expected duration of each ride)
q_i	Service quality for type-i drivers, $i \in \{1, 2\}$, $q_1 > q_2$
N_i	Potential number of type-i drivers, $i \in \{1, 2\}$, $N = N_1 + N_2$
n_i	Participating number of type-i drivers, $i \in \{1, 2\}$
ρ_i	Utilization level of type-i drivers, $i \in \{1, 2\}$
k_i	Opportunity cost for type-i drivers (per unit of time), $i \in \{1, 2\}$, $k_1 > k_2$
n^j	Participating number of drivers providing service j, $j \in \{s, p\}$
W^j	Riders' expected time in the system associated with service j, $j \in \{s, p\}$
Λ	Arrival rate of potential riders
λ^j	Arrival rate of riders choosing service j, $j \in \{s, p\}$
c	Riders' waiting cost (per unit of time)
I	Inconvenience cost for riders requesting street-hailing services
$\mathcal{N}_1^s(r_p, w_p)$	Equilibrium number of taxi drivers who serve street-hailing requests
$\mathcal{N}_1^p(r_p, w_p)$	Equilibrium number of taxi drivers who serve ride-hailing requests on the platform
$\mathcal{N}_1(r_p, w_p)$	Equilibrium number of taxi drivers who join the taxi company
$\mathcal{N}_2(r_p, w_p)$	Equilibrium number of private car drivers who join the online platform

Table 4.2: Model Notation

4.4 Parameter Estimation: Base Case

In this section, we calibrate our model parameters based on the Singapore Taxi Market in the year 2019.

- **Estimating service quality, q_1 and q_2, and the inconvenient cost, I.**

  In Singapore, the taxi fee for street-hailing service is metered price, which depends on factors such as location and time of the day. For a taxi trip with 10km, the range of actual taxi fee for non-peak hours and non-late time is $[\text{S\$}8.15, \text{S\$}8.75]$ for standard taxi, and $[\text{S\$}10.65, \text{S\$}11.25]$ for fancy taxi (MoneySmart 2019). In our model, based on (4.16) in Assumption 4.1, we establish the range of the street-hailing fee, $r_s \in [\underline{r}_s, \overline{r}_s]$, ensuring a positive number of riders for street-hailing. Thus, we choose $q_1 = 30$ and $I = 18$ to derive the range of r_s as $[7.69, 11.99]$, which cover the aforementioned actual taxi fee ranges.

  We choose q_2 to satisfy Assumption 4.3.

- **Estimating waiting cost per unit of time, c.**

  In Singapore, in year 2019, the average monthly salary for full-time workers was S\$4563, and full-time workers typically worked an average of 42.9 hours per week (of Manpower 2020). According to Gomez-Ibanez et al. (1999), the waiting cost for a working class passenger in San Francisco is approximately 195% of the passenger's after-tax wages. Then, the average hourly wage of workers after-tax is around S\$23.39, and the waiting cost for an average passenger is around S\$45.61 per hour, i.e., S\$1094.64 per day. Thus, we choose $c = 1095$.

- **Estimating expected service rate per unit of time, μ.**

In Singapore, the average length of each taxi trip is 10km, and the average drive speed of each driver is 30kmh. Then, the duration per trip with 10km is 20 minutes. Assuming an average waiting time of riders for picking up is 3mins. then, the average service time (including pickup time) for each trip is 23 minutes. Thus, we choose $\mu = \frac{1}{23}$ trips per minute, i.e., $\mu = 62.6$ trips per day.

- **Estimating the effective number of taxi and private car drivers, N_1 and N_2.**

  In 2019, the number of taxi fleet was 18542 (Authority 2021), which could be operated in either one-shift or two shift. For example, for two-shift fleet, the daily-daytime shift is from 6a.m to 6p.m, and the daily-night time shift is from 6p.m to 6a.m. In 2019, the two-shifts fleet was 58.9% (hailing Committee 2020). For each shift, we assume the average usual hours for them is the same as that of full time workers, which was 42.9 hours per week (of Manpower 2020). Thus, we estimate the number of effective taxi fleet per day as follows: $18542 \times (58.9\% + (31.1\%/2)) \times 2 \times \frac{42.9}{24 \times 7} = 18542 \times 79.45\% \times 51.07\% = 14731 \times 51.07\% = 7523$.

  In 2019, the number of private hired cars was 77141 (Authority 2021). We assume all private-hired (self-drive) cars were doing the part-time jobs at the online platform. In Singapore, the average usual hours worked of part-timers was 21.1 hours per week in year 2019 (of Manpower 2020). Thus, we estimate the effective number of private cars per day as follows: $77141 \times \frac{21.1}{24 \times 7} = 77141 \times 12.56\% = 9688$.

- **Estimating the opportunity cost per unit of time of two type of drivers, k_1 and k_2.**

  We choose k_1 to satisfy Assumptions 4.1 and 4.3, and choose k_2 to satisfy

Assumption 4.2.

- **Estimating the rental fee per unit of time, h.**

  In Singapore, since ComfortDelGro is the largest cab operator, we use the rental fee of ComfortDelGro taxi for our estimation. In 2020, ComfortDelGro paid half the usual rental rates, which means the rental waivers ranged from S$45 to S$86 a day, depending on the make, model and age of the taxi (Tan 2020). We can know the range of average rental fee is [S$90, S$172]. Thus, the average rental fee can be estimated as follows: $h = \frac{90+172}{2}$ =S$131 per day.

Table 4.3 summarizes the values of our base case.

Parameter related to Riders	Notation	Value
Service quality of taxi drivers per trip	q_1^b	S$30
Service quality of private car drivers per trip	q_2^b	S$29.85
Inconvenient cost of riders for street-hailing per trip	I^b	S$18
Waiting time cost per day per trip	c^b	S$1095
Parameter related to Drivers	**Notation**	**Value**
Service rate of drivers, i.e., number of trips per day per driver	μ^b	62.6
Number of (effective) taxi fleet	N_1^b	7523
Number of (effective) private cars	N_2^b	9688
Outside opportunity cost of taxi drivers per day	k_1^b	S$350
Outside opportunity cost of private car drivers per day	k_2^b	S$300
Rental fee of a taxicab per day	h^b	S$131

Table 4.3: Description of the Baseline Parameters Used in Our Numerical Experiments. Note: The super script ^{b} represents the base case.

4.5 Assessing the Impact of a Regulatory Framing: Analysis of the Optimal Financial and Demand-Supply Matching Decisions

In this section, we investigate the optimal decisions of riders, drivers, and the online platform under a fixed regulatory regime. In particular, we assume that the mixed transportation ecosystem operates under a fixed combination of the parameters r_s and $\bar{\alpha}$ that regulate the street-hailing revenues and the level of access that taxi drivers have to platform-generated demand for services, respectively. We use backward induction to analyze the sequential game depicted in Figure 4.3. In Section 4.5.1, we derive the demand rates for transportation services for a given set of financial parameters and driver capacity supply decisions. In Section 4.5.2, we obtain the expressions for the optimal proportion of service time that the participating taxi drivers allocate to the online platform. In Section 4.5.3, we analyze the participating decisions for all drivers. In Section 4.5.4, we provide the analytical characterization of the optimal service fees and wage compensation that the platform sets.

In our model, given the regulator's decision $\bar{\alpha} \in [0, 1]$, the taxi drivers set the proportion of their working time $\alpha \in [0, \bar{\alpha}]$ to serve customer requests generated via the online platform. As a result, three distinct modes of service provision for taxi drivers can emerge. In the first mode, $\alpha = 0$, i.e., the taxi drivers choose to serve only street-hailing requests (we refer to this mode as "street-only"). In the second mode, $0 < \alpha < 1$, and the taxi drivers serve both street-hailing and platform-based ride-hailing requests ("mixed" mode). Finally, in the third, "platform-only" mode, the taxi drivers choose to exclusively serve platform-based ride-hailing requests.

4.5.1 Riders' Problem: Demand Expressions for Taxi and Platform-Based Services

In the sequence of decisions depicted in Figure 4.3, riders choose among three options: street-hailing services, ride-hailing services via the online platform, and the outside option. This choice is made after observing the government's regulation decisions $(\bar{\alpha}, r_s)$, the online platform's ride fee and wage compensation decisions (r_p, w_p), and the drivers' participation decisions (n_1, n_2, α).

Under the supply-constrained setting, the participating numbers of riders in equilibrium satisfy

$$U_r^s(r_s, n^s, \lambda^s) = U_r^p(r_p, n^p, \lambda^p) = 0, \tag{4.5}$$

where $n^s = (1-\alpha)n_1$, $n^p = \alpha n_1 + n_2$, and where we normalized the utility associated with the outside option to zero. Using the utility expressions for the riders (4.3) and (4.4), we have

$$q_1 - I - r_s - \frac{c}{n^s\mu - \lambda^s} = q^p - r_p - \frac{c}{n^p\mu - \lambda^p} = 0, \tag{4.6}$$

where $q^p = \frac{\alpha n_1 q_1 + n_2 q_2}{\alpha n_1 + n_2}$. We report the resulting demand rates for street-hailing and platform-based riding services in the form of a lemma.

Lemma 4.1. (Equilibrium Demand Rates) *For given $(\bar{\alpha}, r_s, r_p, n_1, n_2, \alpha)$, the equilibrium demand rate for the street-hailing rides is*

$$\lambda^s(r_s, n_1, \alpha) = n_1 \left((1-\alpha)\mu - \frac{c}{n_1(q_1 - I - r_s)^+} \right)^+, \tag{4.7}$$

and the equilibrium demand rate for the platform-based rides is

$$\lambda^p(r_p, n_1, n_2, \alpha) = (\alpha n_1 + n_2)\left(\mu - \frac{c}{(\alpha n_1(q_1 - r_p) + n_2(q_2 - r_p))^+}\right)^+, \quad (4.8)$$

where $x^+ = \max(x, 0)$.

Note that in order to produce positive demand rates, the ride fees r_s and r_p must not be too high, i.e., $r_s < q_1 - I - \frac{c}{(1-\alpha)n_1\mu}, r_p < \frac{\alpha n_1 q_1 + n_2 q_2}{\alpha n_1 + n_2} - \frac{c}{(\alpha n_1 + n_2)\mu}$.

4.5.2 Drivers' Problem: Supply of Riding Capacity

As illustrated in Figure 4.3, potential drivers make their participation decisions after observing government's regulation decisions $(\bar{\alpha}, r_s)$, the online platform's ride fee and wage compensation decisions (r_p, w_p), and anticipating the demand responses from the riders described by (4.7) and (4.8). Among the rides satisfied by the online platform, the rate of rides serviced by taxi drivers is

$$\lambda_1^p(r_p, n_1, n_2, \alpha) = \frac{\alpha n_1}{\alpha n_1 + n_2}\lambda^p(r_p, n_1, n_2, \alpha), \quad (4.9)$$

and the rate of rides serviced by private car drivers is

$$\lambda_2^p(r_p, n_1, n_2, \alpha) = \frac{n_2}{\alpha n_1 + n_2}\lambda^p(r_p, n_1, n_2, \alpha), \quad (4.10)$$

where $\lambda^p(r_p, n_1, n_2, \alpha)$ is given by (4.8).

Consider the setting where n_1 taxi drivers join the taxi company and allocate the fraction α of their working time to serving clients via the platform and n_2 private care drivers join the platform. Then, the utility rate for the taxi drivers is

$$\begin{aligned} U_{d,1}\left(n_1, n_2, \alpha\right) &= \frac{r_s \lambda^s\left(r_s, n_1, \alpha\right) + w_p \lambda_1^p\left(r_p, n_1, n_2, \alpha\right)}{n_1} - k_1 - h \\ &= \frac{r_s \lambda^s\left(r_s, n_1, \alpha\right)}{n_1} + \frac{\alpha w_p \lambda^p\left(r_p, n_1, n_2, \alpha\right)}{\alpha n_1 + n_2} - k_1 - h, \end{aligned} \tag{4.11}$$

where $\lambda^s(r_s, n_1, \alpha)$ is given by (4.7). In (4.11), the first and second terms represent the average earning (per unit of time) of taxi drivers from providing street-hailing and platform-based ride-hailing services, respectively; the third and fourth terms represent their outside opportunity cost and rental fee, respectively.

The private car drivers' utility rate is

$$U_{d,2}\left(n_1, n_2, \alpha\right) = \frac{w_p \lambda_2^p\left(r_p, n_1, n_2, \alpha\right)}{n_2} - k_2 = \frac{w_p \lambda^p\left(r_p, n_1, n_2, \alpha\right)}{\alpha n_1 + n_2} - k_2. \tag{4.12}$$

In (4.12), the first term represents the average earning (per unit of time) of private car drivers from providing platform-based ride-hailing services, the second term represents their outside opportunity cost.

Below, we first discuss the optimal percentage of time taxi drivers allocate to platform-based ride hailing after observing (n_1, n_2).

$$\max_{\alpha \in [0, \bar{\alpha}]} U_{d,1}\left(n_1, n_2, \alpha\right). \tag{4.13}$$

The optimal time allocation for taxi drivers is described in the following result.

Proposition 4.1. (Optimal Time Allocated for the Online Platform by Taxi Drivers) *For given* $(\bar{\alpha}, r_s, r_p, w_p)$, *define*

$$L\left(\bar{\alpha}, r_s, r_p, w_p, n_1\right) = \frac{1}{q_2 - r_p}\left(\frac{c w_p}{\mu\left(w_p - \frac{r_s}{\bar{\alpha}} \min\left\{\bar{\alpha}, \left(1 - \frac{c}{n_1 \mu (q_1 - I - r_s)^+}\right)\right\}\right)^+} - \bar{\alpha} n_1\left(q_1 - r_p\right)\right). \tag{4.14}$$

Then, the optimal fraction of time taxi drivers spend on serving ride-hailing requests is given by

$$\alpha^*(\bar{\alpha}, r_s, r_p, w_p, n_1, n_2) = \begin{cases} \bar{\alpha}, & \textit{if } q_2 < r_p \textit{ and } n_2 \leq (L(\bar{\alpha}, r_s, r_p, w_p, n_1))^+, \\ & \textit{or if } q_2 \geq r_p \textit{ and } n_2 \geq (L(\bar{\alpha}, r_s, r_p, w_p, n_1))^+, \\ 0, & \textit{otherwise.} \end{cases} \tag{4.15}$$

Proposition 4.1 indicates that the taxi drivers either forgo the online platform option of serving customers or go "all in" allocating the maximum allowed fraction of their time to online service delivery. For a given number of participating taxi drivers, n_1, the switch between these two regimes is governed by the service fee the platform charges, r_p, and the number of participating private car drivers, n_2. In particular, when the platform fee r_p is set above the utility delivered by private car drivers, q_2, the taxi drivers participate in online service delivery if and only if the number of participating private car drivers is below a critical threshold, $(L(\bar{\alpha}, r_s, r_p, w_p, n_1))^+$. However, once the platform fee r_p drops below q_2, the taxi drivers limit their service provision to the street hailing mode unless the number of participating private car drivers is above $(L(\bar{\alpha}, r_s, r_p, w_p, n_1))^+$. In summary, taxi drivers use the online platform if the platform fee is high and there are few private car drivers, or if the platform fee is low, and the private car drivers are numerous.

The intuition behind the results of Proposition 4.1 can be described as follows. As the number of private car drivers n_2 increases, two competing effects are at work. On the one hand, the expected quality $q^p = \frac{\alpha n_1 q_1 + n_2 q_2}{\alpha n_1 + n_2}$ for platform-based ride-hailing services decreases, which means that riders' willingness to pay decreases as well. We denote this is a "quality" effect. On the other hand, having more drivers on the platofrm reduces the expected waiting time of riders - the "network" effect.

When the quality of ride-hailing services from private car drivers is lower than the fee charged to riders, i.e., when $q_2 < r_p$, the higher n_2, the more the loss from the quality effect dominates the benefit from the network effect for riders requesting platform-based ride-hailing services,. Thus, for each driver on the platform, the average number of riders requesting transportaiton services decreases. Then, for given w_p, the average earning of each taxi driver from providing platform-based ride-hailing services decreases, which undermines the incentive for a taxi driver to join the platform. Therefore, in this case, taxi drivers will be more willing to join the online platform if the number of private car drivers n_2 is lower. However, when $q_2 > r_p$, the higher n_2, the more the benefit from network effect dominates the loss from quality effect for riders using the platform. As a result, the average number of riders per driver on the platform increases, and, for given w_p, taxi drivers will be more willing to join the online platform if the number of private car drivers n_2 is higher.

4.5.3 Participation Decisions of Taxi Drivers and Private Car Drivers

Given $(\bar{\alpha}, r_s, r_p, w_p)$, potential taxi drivers and private car drivers simultaneously make participation decisions to maximize their own utilities. Taxi drivers decide on whether to join the traditional taxi company with a rental fee h (per unit of time). Private car drivers decide on whether to join the online platform. To avoid trivial settings, we make the following two assumptions on the allowable parameter combinations.

Assumption 4.1. *When taxi drivers only provide street-hailing services, all of them are willing to join the traditional taxi company:* $r_s\left(\mu - \frac{c}{N_1(q_1 - I - r_s)^+}\right) - k_1 - h \geq 0$*, where* $x^+ = \max(x, 0)$*.*

We present this and other assumptions in our analysis in both the verbal and algebraic forms, with the verbal forms reflecting the intuitive meaning of the assumptions and with the details and explanations of the algebraic forms relegated to the proofs of our analytical results. Note that Assumption 4.1 implies that r_s must satisfy

$$\underline{r}_s \leq r_s \leq \overline{r}_s, \tag{4.16}$$

where

$$\begin{aligned} \overline{r}_s &= \frac{N_1(\mu(q_1-I)+k_1+h)-c+\sqrt{(N_1(\mu(q_1-I)+k_1+h)-c)^2-4\mu N_1^2(q_1-I)(k_1+h)}}{2\mu N_1}, \\ \underline{r}_s &= \frac{N_1(\mu(q_1-I)+k_1+h)-c-\sqrt{(N_1(\mu(q_1-I)+k_1+h)-c)^2-4\mu N_1^2(q_1-I)(k_1+h)}}{2\mu N_1}. \end{aligned} \tag{4.17}$$

In other words, r_s cannot not be too low or too high, since for $r_s < \underline{r}_s$ the taxi fee cannot bring enough earnings for taxi drivers to participate, and if $r_s > \overline{r}_s$, then the taxi fee is too high to generate sufficient street-hailing demand. Note that in order for the interval $[\underline{r}_s, \overline{r}_s]$ to be non-empty, we must also have

$$N_1\left(\sqrt{\mu\,(q_1 - I)} - \sqrt{k_1 + h}\right)^2 > c. \tag{4.18}$$

In summary, Assumption 4.1 reflects the setting where the traditional taxi service constituted a viable incumbent business before the arrival of online ride-hailing. The following assumption ensures that the online platform also is a viable stand-alone business.

Assumption 4.2. *The online platform can run profitably with the participation of all potential private car drivers and no taxi car drivers:* $\sqrt{\mu N_2 q_2} - \sqrt{c} \geq \sqrt{k_2 N_2}$.

Assumption 4.2 ensures that there is a demand for online ride-hailing services when all potential private car drivers join the platform ($\mu N_2 q_2 > c$), and also that this demand is sufficient to make it profitable for all potential private car drivers to join the platform rather than to choose an outside option ($k_2 \leq \frac{\left(\sqrt{\mu N_2 q_2}-\sqrt{c}\right)^2}{N_2}$).

Let $\mathcal{N}_1\left(\bar{\alpha}, r_s, r_p, w_p\right)$ and $\mathcal{N}_2\left(\bar{\alpha}, r_s, r_p, w_p\right)$ denote the equilibrium numbers of drivers who join the taxi company and the online platform, respectively. In order to clearly represent the service capacity allocation of taxi drivers at each service system, we use $\mathcal{N}_1^s\left(\bar{\alpha}, r_s, r_p, w_p\right)$ to denote the participating number of taxi drivers serving street-hailing requests, i.e., $\mathcal{N}_1^s = \left(1-\alpha^*\right)\mathcal{N}_1$, and $\mathcal{N}_1^p\left(\bar{\alpha}, r_s, r_p, w_p\right)$ to denote the participating number of taxi drivers serving platform-based ride-hailing requests, i.e., $\mathcal{N}_1^p = \alpha^* \mathcal{N}_1$.

For the analysis below it is convenient to define the following quantities:

$$
\begin{aligned}
f\left(\alpha n_1, n_2, r_p\right) &:= \left(\mu - \frac{c}{\left(\alpha n_1 q_1 + n_2 q_2 - \left(\alpha n_1 + n_2\right) r_p\right)^+}\right)^+, && (4.19)\\
W_1\left(\bar{\alpha}, r_s, r_p\right) &:= \frac{r_s \mu \min\{\bar{\alpha}, 1 - \frac{c}{N_1 \mu (q_1 - I - r_s)^+}\}}{\bar{\alpha} f\left(\bar{\alpha} N_1, N_2, r_p\right)}, && (4.20)\\
W_2\left(\bar{\alpha}, r_s, r_p\right) &:= \frac{r_s \mu \min\{\bar{\alpha}, 1 - \frac{c}{N_1 \mu (q_1 - I - r_s)^+}\}}{\bar{\alpha} f\left(\bar{\alpha} N_1, 0, r_p\right)}, && (4.21)\\
W_3\left(r_p\right) &:= \frac{k_2}{f\left(0, N_2, r_p\right)}. && (4.22)
\end{aligned}
$$

Proposition 4.2. (Participation of Drivers) *Suppose that Assumptions 4.1 and 4.2 hold. Then, For given $(\bar{\alpha}, r_s, r_p, w_p)$, the equilibrium numbers of participating taxi drivers and private car drivers, and the corresponding fraction of time taxi drivers allocate to the platform-based service are given by*

$$
\left(\mathcal{N}_1, \mathcal{N}_2, \alpha^*\right) = \begin{cases}
\left(N_1, N_2, \bar{\alpha}\right), & \text{if } w_p \geq W_1\left(\bar{\alpha}, r_s, r_p\right),\\
\left(N_1, \left(L\left(\bar{\alpha}, r_s, r_p, w_p, N_1\right)\right)^+, \bar{\alpha}\right), & \text{if } r_p > q_2 \text{ and } W_2\left(\bar{\alpha}, r_s, r_p\right) \leq w_p < W_1\left(\bar{\alpha}, r_s, r_p\right),\\
\left(N_1, N_2, 0\right), & \text{if } r_p \leq q_2 \text{ and } W_3\left(r_p\right) \leq w_p < W_1\left(\bar{\alpha}, r_s, r_p\right),\\
\left(N_1, 0, 0\right), & \text{otherwise,}
\end{cases} \quad (4.23)
$$

where $L\left(\bar{\alpha}, r_s, r_p, w_p, N_1\right)$ is defined in (4.14).

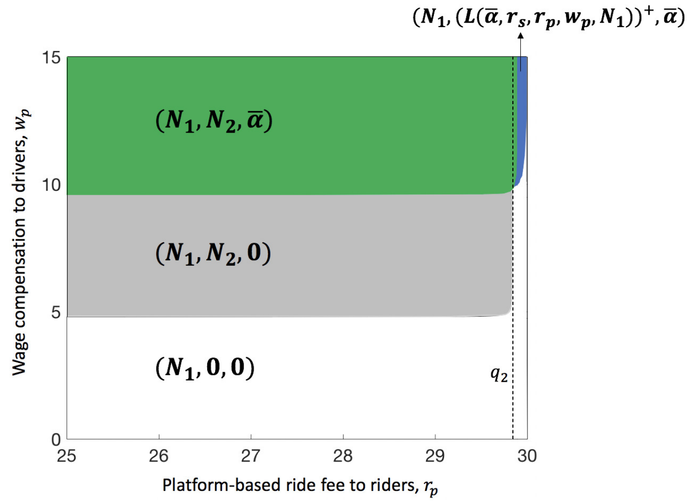


Figure 4.4: (Color online) Drivers' Participation Decisions as Functions of Platform-based Ride Fee r_p and Wage Compensation w_p. Note: $r_s = 9.6$, $\bar{\alpha} = 0.5$ and the rest of parameter values are given in Table 4.3.

Figure 4.4 illustrates four possible participation outcomes described in Proposition 4.2. Note that in this example all taxi drivers join the system, switching from street-only ($\alpha^* = 0$) to mixed or platform-only($\alpha^* = \bar{\alpha}$) mode of service delivery as the wage compensation on the platform grows. In contrast, for the private car drivers three distinct outcomes are possible: all of them join the platform, or only some of them, or none. In particular, for $r_p > q_2$, only some private car drivers join the platform to "make space" for taxi drivers who will allocate the maximum allowed fraction of their time to platform-based service.

4.5.4 Online Platform's Problem

Given $(\bar{\alpha}, r_s)$, the online platform sets the ride fee r_p and wage compensation w_p to maximize its profit. The online platform's profit is

$$\pi(r_p, w_p) = (r_p - w_p)\,\lambda^p\left(r_p, \alpha^* \mathcal{N}_1, \mathcal{N}_2\right), \tag{4.24}$$

where $\alpha^* \mathcal{N}_1$, $\mathcal{N}_2$ are the equilibrium numbers of taxi drivers and private car drivers who serve for the online platform in (4.23), respectively.

Then, the optimization problem of the online platform can be formulated as

$$\max_{(r_p, w_p) \in \mathbb{R}_+^2} \pi\left(r_p, w_p\right). \tag{4.25}$$

By attracting the participation of private car drivers, the online platform enjoys the reduction of expected waiting time of riders requesting platform-based ride-hailing services and suffers from the low service quality of private car drivers. The online platform provides a low wage compensation to attract the participation of private car drivers. On the one hand, by attracting the participation of taxi drivers, the online platform can enjoy both the reduction of expected waiting time of riders requesting platform-based ride-hailing services and the benefit from the high service quality of taxi drivers. On the other hand, the online platform needs to provide a higher enough wage compensation to attract taxi drivers to serve platform-based ride-hailing requests, compared with that of private car drivers. If the benefit of attracting taxi drivers exceeds the cost of attracting them, the online platform prefers to provide a high wage compensation to engage both taxi drivers and private car drivers.

Similar to the analysis of the driver participation decisions, we will use the following assumptions to eliminate trivial problem settings.

Assumption 4.3. *Private car drivers deliver service of competitive quality:* $N_1 \mu \left(q_1 - q_2\right)^2 < c q_1$.

Note that Assumption 4.3 implies: (1) the service quality difference is not too large; (2) the unit waiting cost is not low, i.e., $c > \frac{N_1 \mu (q_1 - q_2)^2}{q_1}$; (3) the service quality of private car drivers is not low, i.e., $q_1 - \sqrt{\frac{c q_1}{N_1 \mu}} < q_2 < q_1$; (4) $r_p^* < q_2$, which means that the service quality of private car drivers is good enough.

Assumption 4.4. *It may be optimal for the platform to allow taxi drivers to operate in a "mixed" mode, serving both street-hailing and platform-based ride-hailing requests:* $N_2 < N_2^{max}$, *where* N_2^{max} *satisfies*

$$\begin{aligned}(k_1 + h)N_1 + (\underline{r}_s\mu - k_2)N_2^{max} = \ & \left(\sqrt{\mu\left(N_1 q_1 + N_2^{max} q_2\right) - \frac{cq_1}{q_1 - I - \underline{r}_s}} - \sqrt{c}\right)^2 \\ & - \left(\sqrt{\mu N_2^{max} q_2} - \sqrt{c}\right)^2,\end{aligned} \tag{4.26}$$

and $\underline{r}_s$ *is defined in (4.17).*

Assumptions 4.3 ensures the taxi service does not completely dominate private care mode of transportation, while Assumptions 4.4 eliminates settings where it is never possible for taxi drivers, under optimal incentives, to share their service capacity between the traditional and platform-based modes of operation.

Proposition 4.3. (Optimal Ride Fee and Wage Compensation for Online Platform) *Suppose that Assumptions 4.1- 4.4 hold. Define*

$$s_3(\bar{\alpha}) := \frac{\left(\sqrt{\mu(\bar{\alpha}N_1 q_1 + N_2 q_2)} - \sqrt{c}\right)^2 - (\sqrt{\mu N_2 q_2} - \sqrt{c})^2 + k_2 N_2}{\bar{\alpha}N_1 + N_2}. \tag{4.27}$$

Then, for given $(\bar{\alpha}, r_s)$, *the optimal platform-based ride fee and wage compensation are given by*

$$(r_p^*, w_p^*) = \begin{cases} \left(r_p^m, w_p^m\right), & \textit{if } \bar{\alpha} < 1 - \frac{c}{N_1\mu(q_1 - I - r_s)^+} \textit{ and } r_s\mu \leq s_3(\bar{\alpha}), \text{ [Mixed]} \\ \left(r_p^m, \frac{w_p^m\left(1 - \frac{c}{N_1\mu(q_1 - I - r_s)^+}\right)}{\bar{\alpha}}\right), & \textit{if } \bar{\alpha} \geq 1 - \frac{c}{N_1\mu(q_1 - I - r_s)^+} \textit{ and } \frac{r_s\mu\left(1 - \frac{c}{N_1\mu(q_1 - I - r_s)^+}\right)}{\bar{\alpha}} \leq s_3(\bar{\alpha}), \\ & \text{[Platform-only]} \\ \left(q_2 - \sqrt{\frac{cq_2}{N_2\mu}}, \frac{k_2}{\mu - \sqrt{\frac{c\mu}{N_2 q_2}}}\right), & \textit{otherwise. } \text{[Street-only]} \end{cases} \tag{4.28}$$

where

$$r_p^m = \frac{\bar{\alpha}N_1q_1 + N_2q_2 - \sqrt{\frac{c(\bar{\alpha}N_1q_1+N_2q_2)}{\mu}}}{\bar{\alpha}N_1 + N_2}, \tag{4.29}$$

$$w_p^m = \frac{r_s\mu}{\mu - \sqrt{\frac{c\mu}{\bar{\alpha}N_1q_1+N_2q_2}}}, \tag{4.30}$$

and the boundary $s_3(\bar{\alpha})$ is increasing function of $\bar{\alpha}$ characterized in the proof of the proposition.

Proposition 4.3 shows that all N_2 number of private car drivers participate in serving platform-based ride-hailing requests. This is intuitive. Recall that private car drivers have a lower service quality and lower opportunity cost. It implies that when using the high wage compensation to attract the participation of taxi drivers, private car drivers can also be induced to participate. However, there are three possible service modes of taxi drivers in equilibrium, as illustrated in Figure 4.5: in Region I, no taxi drivers serves platform-based ride-hailing requests, which leads to taxi drivers in "street-only" service mode; in Region II, $\bar{\alpha}N_1$ number of participating taxi drivers serve platform-based ride-hailing requests, and their left available service time $(1-\bar{\alpha})N_1$ still serve street-hailing requests with positive rider demand rate $\lambda^s > 0$, which leads to taxi drivers in "mixed" service mode; similarly, in Region III, $\bar{\alpha}N_1$ number of participating taxi drivers serve platform-based ride-hailing requests, but their left available service time $(1-\bar{\alpha})N_1$ do not serve street-hailing requests due to zero rider demand rate $\lambda^s = 0$, which is because rides would experience long waiting time requesting street-hailing, and thus, leading to taxi drivers in "platform-only" service mode.

From Figure 4.5, we observe that when $\bar{\alpha}$ is low (i.e., Region I), which means the available service time of taxi drivers to the platform is low, the online plat-

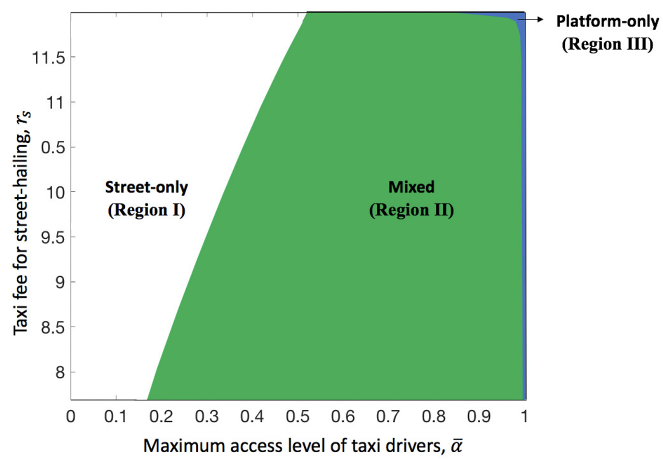


Figure 4.5: (Color online) Online Platform: Equilibrium as Functions of the Maximum Access Level ($\bar{\alpha}$) and Taxi Fee of Street-hailing (r_s). Note: the rest of the parameter values are given in Table 4.3.

form is only willing to use a low wage compensation (i.e., $w_p^* = \frac{k_2}{\mu - \sqrt{\frac{c\mu}{N_2 q_2}}}$) to attract the participation of private car drivers, and thus taxi drivers work in "street-only" service mode. Only when $\bar{\alpha}$ is not very low, it is optimal for the online platform to attract the participation of taxi drivers to serve platform-based ride-hailing requests in "mixed" or "platform-only" service mode. Specifically, when $\bar{\alpha}$ is moderate (i.e., Region II), the online platform is willing to attract taxi drivers to serve platform-based ride-hailing requests with a high wage compensation (i.e., $w_p^* = w_p^m$). In this case, taxi drivers work in "mixed" service mode by serving both street-hailing and platform-based ride-hailing requests. As $\bar{\alpha}$ increases, the allocated service time of taxi drivers to the platform increases, while their left available service time for street-hailing services decreases. When $\bar{\alpha}$ is very high (i.e., Region IIII), the remaining $(1 - \bar{\alpha})$ proportion of service time for taxi drivers to street-hailing services is very low such that there is no street-hailing requests served, which means they cannot obtain any earnings from street-hailing services. Thus, taxi drivers will switch to "platform-only" service mode. In this case, with a higher $\bar{\alpha}$, the online platform can attract the participation of taxi drivers by a relatively lower wage compensation (i.e., $w_p^* = \frac{w_p^m\left(1-\frac{c}{N_1\mu(q_1-I-r_s)^+}\right)}{\bar{\alpha}}$) than that in Region II.

In Figure 4.5, as N_2 increases, the regions of "mixed" and "platform-only" shrink. Interestingly, when N_2 is very high, "mixed" service mode does not exist. The intuition is as follows. With a high N_2, the pooling effect of private car drivers is strong enough such that the online platform only attracts the participation of private car drivers with a low wage compensation. Only when the service time of taxi drivers to the platform, $\bar{\alpha}$, is high enough, and the cost associated with wage compensation needed, which is the earning of taxi drivers from street-hailing controlled by r_s, is low enough, the online platform has the incentive to attract the participation of taxi drivers. This is consistent with the practical observation in US that as the driver shortage problem becomes more prominent, Uber has more incentives to form a partnership with traditional taxi companies.

In C.2, we present a summary of the subsequent payoff for each agent. Note that the access level of taxi drivers $\bar{\alpha}$ is one key control measure for the government to the labor pool size on the online platform. To what extent are the interests of the drivers and the online platform aligned? Below we characterize the impact of the access level of taxi drivers $\bar{\alpha}$, which controls the online platform's labor pool size, on the platform's profit and labor welfare.

By substituting (r_p^*, w_p^*) in (4.28) to (4.24), we have the online platform's platform $\pi^*(\bar{\alpha}, r_s)$. We use LW to denote the labor welfare, which the utility sum of all participating taxi drivers and private car drivers, and can be represented as

$$LW(\bar{\alpha}, r_s) = \mathcal{N}_1(\bar{\alpha}, r_s)\, U_{d,1}(\bar{\alpha}, r_s) + \mathcal{N}_2(\bar{\alpha}, r_s)\, U_{d,2}(\bar{\alpha}, r_s)\,. \tag{4.31}$$

Lemma 4.2. (Agents' Perofrmance) *As the access level allowed for taxi drivers, $\bar{\alpha}$, increases,*

(a) the online platform's profit, $\pi^*(\bar{\alpha}, r_s)$, increases;

(b) the labor welfare, $LW(\bar{\alpha}, r_s)$, firstly increases then decreases.

With optimal decisions $(r_p^*(\bar{\alpha}, r_s), w_p^*(\bar{\alpha}, r_s))$ for the online platform in Proposition 4.3, it is easy to know that as $\bar{\alpha}$ increases, the optimal profit $\pi^*(\bar{\alpha}, r_s)$ increases in $\bar{\alpha}$. By offering a high enough wage compensation $w_p^*(\bar{\alpha}, r_s)$ to induce the participation of taxi drivers, the online platform is able to set a high ride fee $r_p^*(\bar{\alpha}, r_s)$ to serve more riders, which leads to the increase of platform's revenue. Since the revenue with a high ride fee dominates the cost associated with the high wage compensation offered to attract taxi drivers, the optimal profit of online platform $\pi^*(\bar{\alpha}, r_s)$ increases in $\bar{\alpha}$. It implies that the online platform is always beneficial to induce the participation of taxi drivers with a high enough wage compensation.

Similarly, from Proposition 4.3, we can easily know that taxi drivers' utility $U_{d,1}(\bar{\alpha}, r_s)$ is not affected by $\bar{\alpha}$, no matter whether there exists the partnership; private car drivers' utility $U_{d,2}(\bar{\alpha}, r_s)$ gets improved due to the partnership, and decreases in $\bar{\alpha}$ under the partnership. Recall from (4.11), taxi drivers have two possible earning sources: one part from serving street-hailing, i.e., $\frac{r_s \lambda^s}{N_2}$, another part from serving platform-based ride-hailing requests, i.e., $\frac{w_p^* \lambda_1^p}{N_1}$. Under either "mixed" and "platform-only" service modes, the wage compensation offered by the online platform is high enough so that taxi drivers are indifferent between serving street-hailing and ride-hailing requests. Thus, in Proposition 4.3, under the partnership, even though the wage compensations between "mixed" and "platform-only" service modes are different, taxi drivers have the same total earnings with that under "street-only" service mode, which is independent of $\bar{\alpha}$. However, under the partnership, the different wage compensations between these two scenarios are critical to private car drivers. Because private car drivers only serve ride-hailing requests, and from (4.12), their earning, i.e., $\frac{w_p^* \lambda_2^p}{N_2}$, highly depends on the wage compensation offered by the platform, which is affected by $\bar{\alpha}$.

(i) Under "mixed" service mode, with the wage compensation $w_p^* = w_p^m$, it takes the online platform a total cost of $r_s \mu \bar{\alpha} N_1$ to attract $\bar{\alpha} N_1$ number of taxi drivers, where $\bar{\alpha} < 1 - \frac{c}{N_1 \mu (q_1 - I - r_s)^+}$. It means that the value of one unit of service capacity is $r_s \mu$.

(ii) Under "platform-only" service mode, with the wage compensation $w_p^* = \frac{w_p^m \left(1 - \frac{c}{N_1 \mu (q_1 - I - r_s)^+}\right)}{\bar{\alpha}}$, it takes the online platform a total cost of $r_s \mu (1 - \frac{c}{N_1 \mu (q_1 - I - r_s)^+}) N_1$ to attract $\bar{\alpha} N_1$ number of taxi drivers, where $\bar{\alpha} \geq 1 - \frac{c}{N_1 \mu (q_1 - I - r_s)^+}$. It means that the value of one unit of service capacity is $\frac{r_s \mu (1 - \frac{c}{N_1 \mu (q_1 - I - r_s)^+})}{\bar{\alpha}}$, which is lower than $r_s \mu$ and decreases in $\bar{\alpha}$.

The different values of per unit of service capacity between "mixed" and "platform-only" implies that with the partnership, there exists a *supply congestion effect*, that is, *the "congestion" among taxi drivers over the earning "pie"* which is the total cost associated with wage compensation paid by the online platform. As $\bar{\alpha}$ increases, which means taxi drivers allocate more service time to the platform, it may become easier for the online platform to attract the participation of taxi drivers due to a relatively lower wage compensation needed. However, a higher $\bar{\alpha}$ may decrease private car drivers' earning due to the congestion effect on supply side. This reveals a situation of private car drivers that their utilization level gets improved, however, their earning gets reduced. That is, they work longer, but earn less.

An implication of Lemma 4.2 is that when $\bar{\alpha}$ is low or moderate, which means the online platform's drivers pool size under either "street-only" or "mixed" is not that high, the interests of drivers and the online platform are aligned: both drivers' earning and the online platform's profit increase in $\bar{\alpha}$. However, when $\bar{\alpha}$ is very high, which means the driver pool size (i.e., $\bar{\alpha}N_1+N_2$) under "platform-only" is very high, the online platform's profit continuously increases in $\bar{\alpha}$, whereas the drivers' earning decreases in $\bar{\alpha}$. Thus, in this case, the government coordination may be needed.

4.6 Government's Regulation Problem: Optimal Regulatory Decisions

In this section, we examine how the government decides on the maximum access level of taxi drivers allowed to on-demand ride-hailing requests, $\bar{\alpha}$, and the taxi fee for street-hailing services, r_s, to maximize the social welfare. Figure 4.5 illustrates whether taxi drivers participate in providing on-demand ride-hailing requests as we vary $\bar{\alpha}$ and r_s.

Following Yu et al. (2020) and Zhou et al. (2022a), we define social welfare as the weighted sum of utilities of all riders requesting street-hailing or platform-based ride-hailing services, utilities of all participating taxi and private car drivers, profits of the online platform and traditional taxi company, represented by SW. Note that we assume there are sufficient amount of riders in (4.1), it implies that in equilibrium, riders' utilities are $U_r^p(\bar{\alpha}, r_s) = U_r^s(\bar{\alpha}, r_s) = 0$. Then, the social welfare SW can be expressed as

$$\begin{aligned} SW(\bar{\alpha}, r_s) = \; & \gamma\left(\mathcal{N}_1(\bar{\alpha}, r_s)\, U_{d,1}(\bar{\alpha}, r_s) + \mathcal{N}_2(\bar{\alpha}, r_s)\, U_{d,2}(\bar{\alpha}, r_s)\right) \\ & + (1-\gamma)\left(\pi^*(\bar{\alpha}, r_s) + \mathcal{N}_1(\bar{\alpha}, r_s)\, h\right), \end{aligned} \tag{4.32}$$

where $\mathcal{N}_i(\bar{\alpha}, r_s)$, $U_{d,i}(\bar{\alpha}, r_s)$ for $i \in \{1, 2\}$ and $\pi^*(\bar{\alpha}, r_s)$ are from the subsequent equilibrium of Proposition 4.3; h is the fix rental fee for each participating taxi drivers; and $0 \leq \gamma \leq 1$ is the weight parameter for the labor welfare including the utility of all participating taxi and private car drivers, and $(1-\gamma)$ is the weight for the total profit of the online platform and traditional taxi company.

We first present a benchmark case that the partnership between the online platform and traditional taxi company is not an option, i.e., $\bar{\alpha}^* = 0$. That is to say, under "no access" with $\bar{\alpha}^* = 0$, the government optimally decides on the taxi fee r_s^* to maximize the social welfare. In this benchmark, there has the largest number of riders being served for requesting street-hailing services.

Lemma 4.3. (Benchmark: Demand Rate for Street-hailing Sevices) *Under "no access", the demand rate for street-hailing rides is*

$$\lambda_0^s = N_1\mu - \sqrt{\frac{N_1\mu c}{q_1 - I}}. \tag{4.33}$$

Next, when the partnership between the the online platform and traditional taxi company is an option, the government jointly decides on the maximal access level of taxi drivers to the online platform, $\bar{\alpha}$, and the taxi fee for street-hailing services, r_s, to maximize the social welfare. Then, we formulate the government's optimization problem as follows:

$$\begin{aligned} \max_{\bar{\alpha}\in[0,1], r_s\in[\underline{r}_s, \overline{r}_s]} \quad & SW(\bar{\alpha}, r_s) \\ \text{s.t.} \quad & \lambda^s(\bar{\alpha}, r_s) \geq \beta\lambda_0^s, \end{aligned} \tag{4.34}$$

where $0 \leq \beta \leq 1$ represents the proportion of riders sustained to be served for requesting street-hailing services, and the region of $r_s \in [\underline{r}_s, \overline{r}_s]$ is given in Assumption 4.1.

In the following, we consider two sub-cases: (1) the government imposes no restriction on the demand for street-hailing services, i.e., $\beta = 0$; (2) there is a finite number of demands for street-hailing services that the government wants to reserve, i.e., $0 < \beta \leq 1$.

Before analyzing the optimization problem in (4.34), we calibrate parameter values of β and γ for the base case, which is based on the Singapore government.

Estimating the weight of labor welfare, γ^b. Since the entry of on-demand ride-hailing platforms helps Singapore build a Smart Nation, in Singapore, as of December 2019, there has been minimal action on regulating platforms (Chia and Chan 2021), because the government needs to have a balance between economic, social priorities to achieve long-term, sustainable development. In this context, we take the weight of labor welfare as $\gamma^b = 0.5$ to capture that the government cares equally between the welfare of workers and the economic impact for the development of innovation technology.

Estimating the proportion of street-hailing requests sustained, β^b. In 2019, after the entry of ride-hailing apps with 6 years, up to 70% of taxi giant ComfortDelGro's rides are still street hails (Tan 2019). Therefore, we take the realized value of $\bar{\alpha}^{b*} = 0.3$ as the optimal allowed access level of taxi drivers to on-demand ride-hailing requests. Thus, we choose the corresponding level of $\beta^b = 0.71$, under which we can obtain $\bar{\alpha}^{b*} = 0.3$ by optimizing the social welfare. In this setting, the optimal social welfare achieved is S\$$9,796,312.6$.

4.6.1 Without a Restriction from Street-hailing Availability

The government optimally decides on the taxi fee r_s and the maximal access level $\bar{\alpha}$ to balance between labor welfare and the total profit of firms. In (4.32), we note that the decisions of the taxi fee r_s and the maximal access level $\bar{\alpha}$ may have different impacts on the social welfare, which depends on the weight parameter γ.

To remove unnecessary technical complexities from the possibility of multiple equilibria, we assume that the government takes the smallest value of the allowed access level $\bar{\alpha}^*$ to get ride of the degeneracy of solutions, and thus $\alpha^* = \bar{\alpha}^*$.

Proposition 4.4. (Optimal Decision $\bar{\alpha}^*$ under $\beta = 0$)

(a) When N_2 is high, the government chooses "full access," i.e., $\bar{\alpha}^* = 1$. In this case, the subsequent service mode of taxi drivers is "platform-only."

(b) When N_2 is low, there exists a threshold such that if γ is lower than the threshold, the government chooses "full access," i.e., $\bar{\alpha}^* = 1$; if γ is higher than the threshold, the government chooses "partial access," i.e., $0 < \bar{\alpha}^* < 1$. In both cases, the subsequent service mode of taxi drivers is "platform-only."

In C.3, we present complete formulations in the Proposition 4.4 with all technical details.

Proposition 4.4 shows that given $\beta = 0$, the government's optimal regulatory decisions could lead to two types of taxi service systems: (1)"full access, platform-only," i.e., $\bar{\alpha}^* = 1$; (2) "partial access, platform-only," i.e., $0 < \bar{\alpha}^* < 1$, as illustrated in Figure 4.6.

In Figure 4.6, when the number of private car drivers N_2 is very high, the government always chooses to promote the partnership by "full access" policy. Because a higher N_2 indicates a stronger pooling effect from private car drivers. In this case, the online platform is less willing to attract the taxi drivers by offering a high wage compensation. Only when taxi drivers contribute all of their service time to platform-based ride-hailing services, the online platform has the incentive to provide a high wage compensation, which can also improve the earnings of drivers.

When N_2 is not very high, the government would choose to promote the partnership by using "partial access" or "full access" policy. Because in this case, the pooling effect from private car drivers is not very strong, the online

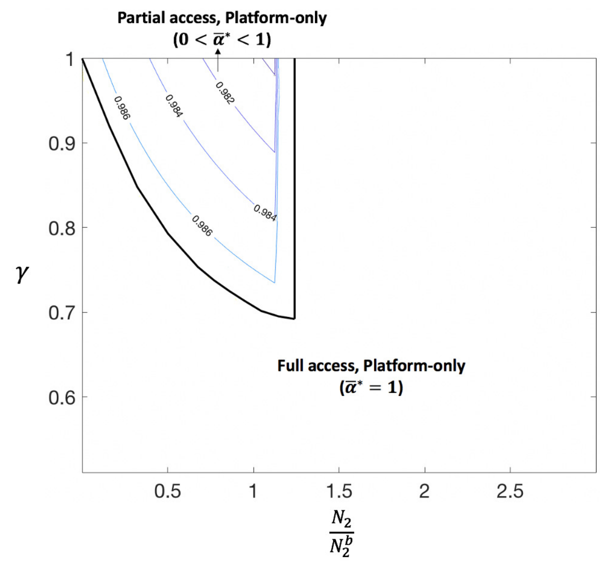


Figure 4.6: (Color online) Given $\beta = 0$, Government's Optimal Decision $\bar{\alpha}^*$. Note: the lines with colours are contour lines of $\bar{\alpha}^*$; the rest of the parameter values are given in Table 4.3.

platform has the incentive to attract taxi drivers by offering a high wage compensation, which can also improve the total earnings of drivers. The weight of labor welfare γ represents how much the government cares about the total earning of drivers, comparing to the profit of firms. (i) If the labor welfare is not much important, i.e., γ is not high, the government optimally chooses "full access" policy to maximize the social welfare. Because the profit benefit for the online platform from $\bar{\alpha}^* = 1$ is very large, which mainly contributes to the maximization of social welfare. (ii) As γ increases, the government would change from "full access" to "partial access" policy. The reasons are as follows. Note that under "full access" policy, as γ increases, the social welfare decreases due to the less importance of the platform's profit. In order to mitigate the decreasing of social welfare with γ, the government can switch to "partial access" policy by decreasing the access level $\bar{\alpha}^*$ and increasing r_s^* which leads the total earning of two types of drivers to increase. In that way, for a given γ, the labor welfare gets improved, which is because the decreasing of the allowed access level $\bar{\alpha}^*$ mitigates the *supply congestion effect*, and the profit of the online platform gets reduced, which is because the revenue of the online platform becomes lower and the cost associated with wage compensation becomes larger (as shown in Lemma 4.2). (iii) Thus, with a very high γ, which means the labor welfare becomes very important, the decreasing social welfare get improved by switching to "partial access" policy. In particular, under this policy, the optimal access level $\bar{\alpha}^*$ decreases in γ.

Interestingly, from Figure 4.6, we observe that for a high γ, as N_2 increases, the government would firstly change from "full access" to "partial access," then change to "full access." This result may appear counterintuitive. Conventional wisdom would suggest that all high-quality taxi drivers should be allowed to join the online platform. Because including high-quality taxi drivers as much as possible not only raises the average quality of services on the platform to q^p, but also expands the driver pool to a higher number of drivers $N^p(= \bar{\alpha}^* N_1 + N_2)$, which could increase the attractiveness of the platform to riders by decreasing their waiting time. Our result highlights an opposite effect associated with this "full access" policy to the social welfare: as N_2 increases, the online platform is less willing to induce the participation of taxi drivers with a high wage compensation, and thus in order to increase the attractiveness of taxi drivers to the platform, the government needs to adjust the taxi fee r_s to reduce taxi drivers' earning from street-hailing, so that the total cost associated with wage compensation needed to offer for the online platform decreases. Furthermore, the government reduces $\bar{\alpha}^*$ to avoid the *supply congestion effect* on the platform.

In particular, under "partial access" policy, i.e., $0 < \bar{\alpha}^* < 1$, the value of $\bar{\alpha}^*$ firstly decreases then increases in N_2. This is because, as N_2 increases, the *supply congestion effect* increases. In order to improve the labor welfare, the government would choose a lower $\bar{\alpha}^*$ to reduce the proportion of service time for taxi drivers to the online platform. In this case, even though the pooling effect from private car drivers is not high, the online platform is still beneficial to provide a high wage compensation to attract taxi drivers with a slightly low $\bar{\alpha}^*$. When N_2 is relatively high, the pooling effect from private car drivers is high enough so that the online platform becomes less willing to attract taxi drivers. In this case, in order to incentivise the platform to provide a high wage compensation, taxi drivers need to allocate more service time to the online platform. Thus, in this case, the optimal decision $\bar{\alpha}^*$ increase in N_2.

Figure 4.7 illustrates the optimal social welfare SW^* obtained under optimal decisions in Proposition 4.4. We observe that SW^* increases in N_2, and decreases in γ. (i) A higher number of private car drivers N_2 can directly improve the welfare of private car drivers. What's more, as N_2 increases, more riders requesting on-demand ride-hailing services will be served, and thus it brings a higher profit to the online platform, which improves the social welfare. (ii) For the impact of γ, as γ increases, on one hand, the total profit from firms is less counted into the social welfare. On the other hand, the optimal decision $\bar{\alpha}^*$ slightly decreases, which decreases the optimal profit of the online platform. Even though the decreasing of $\bar{\alpha}^*$ could benefit the labor welfare, the loss in the online platform's profit dominates the benefit in the labor welfare, and thus, the optimal social welfare decreases in γ.

4.6.2 With a Restriction from Street-hailing Availability

In practice, riders may request ride services through either ride-hailing or street-hailing. With the heterogeneity of riders' needs, the governments have an incentive to sustain the street-hailing services by allocating enough service

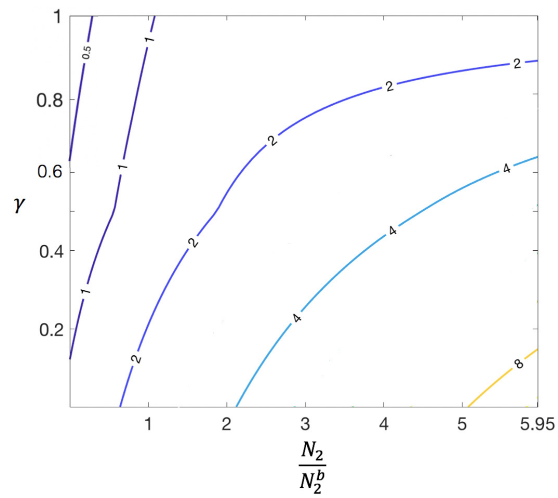


Figure 4.7: (Color online) Given $\beta = 0$, Optimal Social Welfare Ratio Between SW^* and SW^{b*}, i.e., $\frac{SW^*}{SW^{b*}}$. Note: the lines with colours are contour lines of $\frac{SW^*}{SW^{b*}}$; SW^{b*} is the optimal social welfare under the base case, where the rest of the parameter values are given in Table 4.3.

capacity to serve the demand. That is, the participant number of riders for street-hailing services could be positive, i.e.,

$$0 < \beta \leq 1. \tag{4.35}$$

Different from the problem without a restriction to street-hailing demand in Section 4.6.1, the constraint for β in (4.35) rules out the service mode of "platform-only", and leads the government to balance between the service modes of "mixed" or "street-only."

Recall that the demand of street-hailing $\lambda^s(\bar{\alpha}, r_s)$ decreases in $\bar{\alpha}$. Then, the constraint for $\lambda^s(\bar{\alpha}, r_s)$ in (4.34) indicates that there is an upper bound of the proportion of service time ($\bar{\alpha}$) that taxi drivers can allocate for the platform-based ride-hailing requests. However, from Proposition 4.3, only when $\bar{\alpha}$ is high enough, the online platform is willing to offer a high wage compensation to attract taxi drivers.

Thus, there exists a threshold of β, only when β is lower than this threshold, which means the demand for street-hailing is low enough, the "mixed" service mode of taxi drivers is possible. Let this threshold as

$$\beta^{max} := \frac{N_1\mu\,(1-\bar{\alpha}_1) - \frac{c}{(q_1-I-\underline{r}_s)^+}}{N_1\mu - \sqrt{\frac{N_1\mu c}{q_1-I}}}, \tag{4.36}$$

where $\underline{r}_s$ is defined in (4.17), $\bar{\alpha}_1$ satisfies $\underline{r}_s\mu = s_3(\bar{\alpha}_1)$ and $s_3(\cdot)$ is defined in (4.27). Note that β^{max} decreases in N_2, it indicates that as N_2 increases, it becomes harder to work in "mixed" service mode by forming the partnership.

Proposition 4.5. (Sufficient Conditions for Optimal Decision $\bar{\alpha}^*$ under $\beta > 0$)

(a) When $\min\{1, \beta^{max}\} \leq \beta \leq 1$*, the government chooses "no access," i.e.,* $\bar{\alpha}^* = 0$*. The subsequent service mode of taxi drivers is "street-only."*

(b) When $0 < \beta < \min\{1, \beta^{max}\}$*, the sufficient conditions for the case that the government chooses "partial access," i.e.,* $0 < \bar{\alpha}^* < 1$*, under which the subsequent service mode of taxi drivers is "mixed," are: both* β *and* γ *are low if* N_2 *is low.*

In C.3, we present complete formulations in the Proposition 4.5 with all technical details.

Figure 4.8 illustrates the result of Proposition 4.5 as we vary β, the proportion of street-hailing demand sustained, and N_2, the number of private car drivers. Note that when the number of private car drivers N_2 is high, β^{max} is small with $\beta^{max} < 1$.

Proposition 4.5(a) shows that if street-hailing demand that needs to be sustained is very high, i.e., $\min\{1, \beta^{max}\} \leq \beta \leq 1$, it is optimal for the government to choose $\bar{\alpha}^* = 0$, which leads the ride service system to be "no access, street-only," see the right upper side of Figure 4.8. Because after allocating enough time to serve the street-hailing demand sustained, taxi drivers have a very low service time left, so that the online platform is not willing to offer a high wage compensation to attract them. It implies that in this case, the partnership should not be encouraged.

Proposition 4.5(b) shows that if β is not very high, i.e., $0 < \beta < \min\{\beta^{max}, 1\}$, the government's optimal regulatory decisions could lead to two types of taxi service system: (1)"no access, street-only," i.e., $\bar{\alpha}^* = 0$; (2) "partial access, mixed," i.e., $0 < \bar{\alpha}^* < 1$. In particular, when β is high enough, i.e., the left upper side region of Figure 4.8, as the profit benefit for the online platform to attract taxi drivers with "mixed" service mode is very limited, it is optimal for the government to use "no access" policy. When β is low, i.e., the left down side region of Figure 4.8, "partial access" policy is optimal. Furthermore, under "partial access,", we observe, when N_2 is low, the optimal decision $\bar{\alpha}^*$ is high if β is low, or low if β is high. This is because as β increases, which means the left service time of taxi drivers to ride-hailing requests decreases, taxi drivers becomes less attractive for the online platform to provide a high wage compensation. This also implies that it becomes more easily for the online platform to not incentivize the taxi drivers with a high wage compensation.

Figure 4.9 shows the optimal social welfare SW^* with respect to β and γ given different levels of N_2. We have several key observations, compared with SW^{b*} under the base case.

With the number of private car drivers N_2 in Figure 4.9(b), i.e., $N_2 = N_2^b$, the government is able to obtain a high optimal social welfare if both β and γ are low, or a low social welfare if both β and γ are high. For the former result, a lower β represents a softer restriction to $\bar{\alpha}^*$, and thus, taxi drivers can allocate more service time to platform-based ride-hailing requests, which makes taxi drivers more attractive for the online platform. A lower γ represents

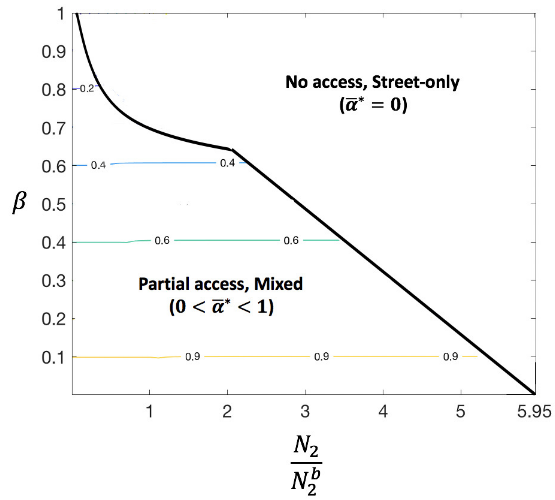
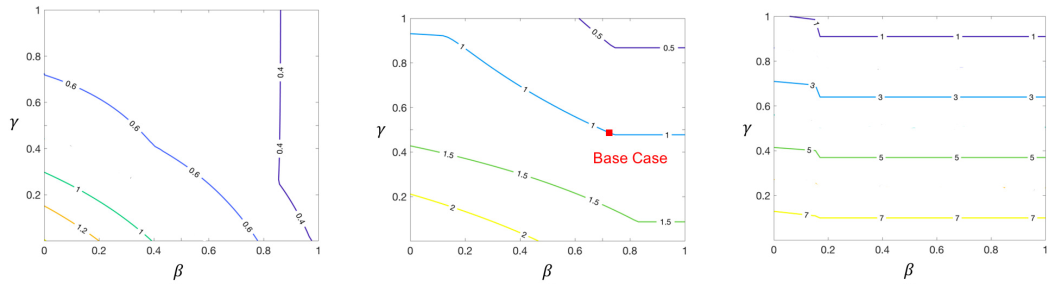


Figure 4.8: (Color online) Given $\gamma = 0.5$, Government's Optimal Decision $\bar{\alpha}^*$. Note: the lines with colours are contour lines of $\bar{\alpha}^*$; the rest of parameter values are given in Table 4.3.

a higher weight for the total profit of firms, which indicates that a lower γ can amplify the profit benefit of the online platform from attracting taxi drivers. As a result, low values of both β and γ can greatly improve the optimal social welfare.

If N_2 is very low, the government can still obtain the comparable optimal social welfare (i.e., SW^{b*}) by decreasing both β and γ, see the bottom left corner in Figure 4.9(a); if N_2 is very high, the government is able to achieve the comparable optimal social welfare (i.e., SW^{b*}) even with very high values of both β and γ, see the top right corner in Figure 4.9(c).

In Figure 4.9, we have a further observation that, when N_2 increases, the optimal social welfare SW^* becomes less sensitive to the increase of β. This is because with a higher N_2, the online platform is less willing to incentive taxi drivers to serve platform-based ride-hailing, which makes "street-only" service mode of taxi drivers easily occur.



(a) $\frac{N_2}{N_2^b} = 0.2$ (b) $\frac{N_2}{N_2^b} = 1$ (c) $\frac{N_2}{N_2^b} = 5$

Figure 4.9: (Color online) Optimal Social Welfare Ratio Between SW^* and SW^{b*}, i.e., $\frac{SW^*}{SW^{b*}}$, as Functions of β and γ, where $\beta \in (0, 1]$, $\gamma \in [0, 1]$. Note: the lines with colours are contour lines of $\frac{SW^*}{SW^{b*}}$; SW^{b*} is the optimal social welfare under the base case, and the rest of parameter values are given in Table 4.3.

4.7 Summary

Faced with the challenge of a labor shortage, traditional taxi drivers are a largely untapped resource for on-demand ride-hailing platforms. The great potential service capacity of taxi drivers motivates the expanding of the partnerships between traditional taxi companies and on-demand ride-hailing platforms. However, there are concerns that this partnership may result in harm to drivers or riders, and the intervention of governments is required.

We develop a game-theoretical model of an on-demand ride-hailing platform who may expand its supply pool with private car drivers, by forming a partnership with traditional taxi company to connect taxi drivers into its platform. Our analysis provides practical and intuitive guidelines to regulators about how the regulatory measure of controlling the access level allowed of taxi drivers to on-demand ride-hailing requests is useful in improving the social welfare, which includes both the labor welfare and the profits of firms. These guidelines can also be insightful to platform managers in gaining a better understanding of different service modes of taxi drivers to on-demand ride-hailing requests.

Our results show that when there is no restriction for street-hailing demand served, taxi drivers always work on “platform-only” service mode. In particular, the specific access level depends on both the weight of labor welfare and the number of private car drivers. If there is a restriction for street-hailing demand needed to be served, taxi drivers change to either “mixed” or “street-only” service mode. Specifically, only when the number of private car drivers is not very high, there is a possible of “mixed” service mode for taxi drivers, and further, the specific access level allowed highly depends on the restriction level for street-hailing demand.

Figure 4.10 depicts the evolution of the ride service system, which includes taxi drivers and private car drivers, as the number of private car drivers (N_2) and street-hailing demand sustained (β) change. When there is street-hailing demand needed to be sustained, there are several different cases. (i) When N_2 and β are both high, with “no access” policy, taxi drivers will not be attracted to join the online platform and work in “street-only” service mode. In this case, two types of drivers work in two separate ride service systems. (ii) When N_2 is high and β is low, with “partial access” policy, taxi drivers will join the online platform and work in “mixed” service mode. In particular, taxi drivers get a high access level to on-demand ride-hailing requests, and the specific access level $0 < \bar{\alpha}^* < 1$ is not that sensitive to the level of N_2. (iii) When N_2 is low and β is high, with “partial access” policy, taxi drivers are in “mixed” service mode. In particular, taxi drivers get a low access level to on-demand ride-hailing requests. (iv) When N_2 and β are both low, with “partial access” policy, taxi drivers work in “mixed” service mode. In particular, taxi drivers get a high access level to on-demand ride-hailing requests. When there is no restriction from street-hailing demand, no matter whether using “full access” or “partial access” policy, taxi drivers are always attracted to join the online platform and work in "platform-only" service mode. Consequently, the street-hailing service diminishes.

	Low N_2		High N_2
High β	Partial access, Mixed (low $\bar{\alpha}^*$)		No access, Street-only ($\bar{\alpha}^* = 0$)
Low β	Partial access, Mixed (high $\bar{\alpha}^*$)		Partial access, Mixed (high $\bar{\alpha}^*$)
$\beta = 0$ (and high γ)	Full access, Platform-only ($\bar{\alpha}^* = 1$)	Partial access, Platform-only (high $\bar{\alpha}^*$)	Full access, Platform-only ($\bar{\alpha}^* = 1$)

Figure 4.10: Regulatory Policies and the Subsequent Service Mode of Taxi Drivers.

Our results suggest several managerial implications that governments can take into consideration. First, with a high β, which represents aging or traditional conservation cities with a low usage of ride app, then, if N_2 is high, which implies private car drivers are sufficient, there is no need to encourage the partnership because it is ineffective for the online platform. However, if education efforts on improving riders' usage of ride app to stay a low β are devoted, the partnership should be promoted by allowing "partial access." Secondly, if N_2 is low, which implies the driver shortage is severe, the partnership should be encouraged; at the same time, "partial access" is needed. As β decreases, the allowed access level of taxi drivers increases. Thirdly, when β decreases to zero, that is, there is no restriction from demand side, which represents young or modern mature cities with a high usage of ride app: the partnership should always be encouraged. However, cautious should be taken by allowing "partial access," especially when γ is high and N_2 is moderate, because of the supply congestion effect of drivers.

We acknowledge a number of limitations of our work. First, we assume that when receiving the same wage compensation level, taxi drivers would choose to work instead to keep idle. However, in practice, taxi drivers may be lazy and less motivated to work more if without extra monetary incentives. Second, we assume the number of taxi drivers and private car drivers keep unchanged in a period of time. However, in practice, the taxi drivers or private car drivers may provide other services, such as, food/parcel delivery services. Thirdly, practically, there also exists the competition between online platforms, which may affect the online requests allocation of the online platform.

Chapter 5

Conclusion

This dissertation studies problems of managing the operations with new business models that are broadly related to anti-counterfeiting, car subscription programs, on-demand ride-hailing services. We study these new business models with unique challenges: (i) With the rising of high-quality counterfeits in emerging markets, is it effective for the brand-name firm to adopt the new measure of anti-counterfeiting, i.e., converting them into authorized overseas suppliers? (ii) To provide car subscription programs, how should the manufacturer decide on flexible vs. dedicated technology choice and make capacity investments by considering both uncertain subscription demands and customers' future switching requests? (iii) For ride service systems, with the issue of labor shortage for on-demand ride-hailing, how should the government regulate the partnership between on-demand ride-hailing platforms and traditional taxi companies to maximize the social welfare? We use mathematical modeling to tackle these problems, and provide new managerial insights to deal with the challenges on the management of different types of new business models.

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Appendix A

Appendix of Chapter 2

A.1 Revenue Dependent Penalty for Counterfeiting

Denote the probability of a counterfeiter getting caught as ϕ, where $\phi \in (0, 1)$, we examine the effect of revenue related penalty for counterfeiting: after getting caught, the counterfeiter pays the penalty from law enforcement e and get her investment of counterfeiting confiscated, which means she cannot sell and produces the counterfeit in the market.

With the revenue related penalty for counterfeiting, if the overseas supplier rejects the offer and sells the counterfeit in the overseas market, her profit under Strategy H and Strategy N can be represented as follows:

$$\pi_2(p_2) = (1 - \phi)(p_2 - k_2)m_2 - \phi e, \tag{A.1}$$

where m_2 is given in Equation (2.3). In Equation (A.1), the first term represents the expected profit of selling the counterfeit, the second term represents the expected penalty from law enforcement.

When the overseas supplier rejects the contract, she decides on retail price p_2 of the counterfeit to maximize her profit $\pi_2(p_2)$ in Equation (A.1). Since the second term in Equation (A.1) does not depend on p_2, the optimal p_2 is only related to the first term. That is, at this extension, optimal prices p_2 of the counterfeit under Strategy H and Strategy N are the same as those in the model of Section 2.4.1, respectively. However, the optimal profit of the overseas supplier selling the counterfeit will decrease, since her investment of production will be confiscated. This means, the brand-name firm could set a lower wholesale price w_2 to source from the overseas supplier.

Similar with the analysis in Section 2.4, by evaluating the difference in each potential suppliers's expected profit between accepting and rejecting the contract, we obtain the best response function of two potential suppliers. Recall

$$\pi_0 = \begin{cases} \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}, & \text{if } \underline{\beta} < \beta < \beta_1, \\ \frac{\alpha \min\{\beta - k_2 - (1-p_B), \frac{\beta - k_2}{2}\} \max\{1-p_B, \frac{\beta-k_2}{2}\}}{\beta}, & \text{if } \beta_1 \le \beta < 1, \end{cases}$$

and we define $M = ((1-\phi)\pi_0 - \phi e)^+$, $K = (\frac{\alpha(\beta-k_2)^2(1-\phi)}{4\beta} - \phi e)^+$. Then,

(i) under Strategy D, the wholesale price that will be accepted by the counterfeiter is

$$w_2^D = \frac{M}{\alpha(1-\frac{p_B}{\gamma})} + k_2;$$

(ii) under Strategy O, the wholesale price that will be accepted by the counterfeiter is

$$w_2^O = \frac{K}{(1+\alpha)(1-\frac{p_B}{\gamma})} + k_2.$$

Define

$$\begin{aligned} e_1 &= \frac{\pi_0(1-\phi)}{\phi} + \frac{1}{\phi}(\alpha(p_B - k_2 - \Delta)(1 - \frac{(2-\beta)p_B - k_2}{2(1-\beta)})^+ - \alpha(p_B - k_2)(1 - \frac{p_B}{\gamma})), \\ e_2 &= \frac{\alpha(\beta-k_2)^2(1-\phi)}{4\beta\phi} + \frac{1}{\phi}(-\Delta(1-p_B) - \alpha(p_B-k_2)(1-\frac{p_B}{\gamma}) - (p_B-k_2)(p_B - \frac{p_B}{\gamma}) \\ &\quad + \alpha(p_B - k_2 - \Delta)\left(1 - \frac{(2-\beta)p_B - k_2}{2(1-\beta)}\right)^+), \\ \Delta_1 &= \frac{\frac{\alpha(\beta-k_2)^2(1-\phi)}{4\beta} - \pi_0(1-\phi) - (p_B-k_2)(p_B - \frac{p_B}{\gamma})}{1-p_B}. \end{aligned}$$

The equilibrium sourcing strategy of the brand-name firm satisfies the following:

(i) Strategy H is optimal with $w_1^* = k_1$ and $w_2^* < w_2^D$ when $e < \min\{(e_1)^+, (e_2)^+\}$;

(ii) Strategy D is optimal with $w_1^* = k_1$ and $w_2^* = w_2^D$ when $e \ge (e_1)^+$ and $\Delta \le (\Delta_1)^+$;

(iii) Strategy O is optimal with $w_1^* < k_1$ and $w_2^* = w_2^O$ when $e \ge (e_2)^+$ and $\Delta > (\Delta_1)^+$.

In this extension, the equilibrium is similar to that in the base model. We find that the consumer surplus under each optimal strategy is the same as that in the base model, while the social surplus can be lower or higher than that in the base model.

A.2 Proofs for Analytical Results.

Analysis: Best Responses of The Home Supplier and The Counterfeiter.

Given supplier i's decision, $d_i \in \{0,1\}$, $i \in \{1,2\}$, we derive the indifference condition for supplier $-i$ by comparing supplier $-i$'s profits between accepting and rejecting the contract.

(1) For the overseas supplier, if $\widetilde{d}_1 = 1$, then, we compare the overseas supplier's profits between Strategy D and Strategy H, *i.e.*, π_2^D and π_2^H. If the

overseas supplier decides to accept, then it should satisfy

$$\pi_2^D(w_2) \geq \pi_2^H \Rightarrow \alpha(w_2 - k_2)(1 - \tfrac{p_B}{\gamma}) \geq M \Rightarrow w_2 \geq \tfrac{M}{\alpha(1-\frac{p_B}{\gamma})} + k_2.$$

If $\widetilde{d_1} = 0$, then, we compare the overseas supplier's profits between Strategy O and Strategy N, *i.e.*, π_2^O and π_2^N. If the overseas supplier decides to accept, then it should satisfy

$$\pi_2^O(w_2) \geq \pi_2^N \Rightarrow (1+\alpha)(w_2 - k_2)(1 - \tfrac{p_B}{\gamma}) \geq K \Rightarrow w_2 \geq \tfrac{K}{(1+\alpha)(1-\frac{p_B}{\gamma})} + k_2.$$

Thus, we obtain the best response function of the overseas supplier $d_2(\widetilde{d_1})$ to the home supplier's each possible decision $\widetilde{d_1} \in \{0, 1\}$, that is,

$$d_2(\widetilde{d_1}) = \begin{cases} d_2\left(\widetilde{d_1} = 1\right) = 1, & \text{if } w_2 \geq \frac{M}{\alpha\left(1-\frac{p_B}{\gamma}\right)} + k_2, \\ d_2\left(\widetilde{d_1} = 1\right) = 0, & \text{if } w_2 < \frac{M}{\alpha(1-\frac{p_B}{\gamma})} + k_2, \\ d_2\left(\widetilde{d_1} = 0\right) = 1, & \text{if } w_2 \geq \frac{K}{(1+\alpha)\left(1-\frac{p_B}{\gamma}\right)} + k_2, \\ d_2\left(\widetilde{d_1} = 0\right) = 0, & \text{if } w_2 < \frac{K}{(1+\alpha)\left(1-\frac{p_B}{\gamma}\right)} + k_2. \end{cases}$$

(2) For the home supplier, if $\widetilde{d_2} = 1$, then, we compare the home supplier's profits between Strategy D and Strategy O, *i.e.*, π_1^D and π_1^O. If the home supplier decides to accept, then it should satisfy

$$\pi_1^D(w_1) \geq \pi_1^O \Rightarrow w_1 \geq k_1.$$

If $\widetilde{d_2} = 0$, then, we compare the home supplier's profits between Strategy H and Strategy N, *i.e.*, π_1^H and π_1^N. If the home supplier decides to accept, then it should satisfy

$$\pi_1^H(w_1) \geq \pi_1^N \Rightarrow w_1 \geq k_1.$$

Thus, we obtain the best response function of the home supplier $d_1(\widetilde{d_2})$ to the overseas supplier's each possible decision $\widetilde{d_2} \in \{0, 1\}$, that is,

$$d_1(\widetilde{d_2}) = \begin{cases} d_1\left(\widetilde{d_2} = 1\right) = 1, & \text{if } w_1 \geq k_1, \\ d_1\left(\widetilde{d_2} = 0\right) = 1, & \text{if } w_1 \geq k_1, \\ d_1\left(\widetilde{d_2} = 1\right) = 0, & \text{if } w_1 < k_1, \\ d_1\left(\widetilde{d_2} = 0\right) = 0, & \text{if } w_1 < k_1. \end{cases}$$

Given best response functions $d_1(\widetilde{d_2})$ and $d_2(\widetilde{d_1})$, we obtain the following fixed point (d_1^*, d_2^*) that satisfies $(d_1(\widetilde{d_2}), \widetilde{d_2}) = (\widetilde{d_1}, d_2(\widetilde{d_1}))$:

$$(d_1^*, d_2^*) = \begin{cases} (1,1), & \text{if } w_1 \geq k_1 \text{ and } w_2 \geq \frac{M}{\alpha\left(1-\frac{p_B}{\gamma}\right)} + k_2, \\ (1,0), & \text{if } w_1 \geq k_1 \text{ and } w_2 < \frac{M}{\alpha(1-\frac{p_B}{\gamma})} + k_2, \\ (0,1), & \text{if } w_1 < k_1 \text{ and } w_2 \geq \frac{K}{(1+\alpha)\left(1-\frac{p_B}{\gamma}\right)} + k_2, \\ (0,0), & \text{if } w_1 < k_1 \text{ and } w_2 < \frac{K}{(1+\alpha)\left(1-\frac{p_B}{\gamma}\right)} + k_2. \end{cases}$$

□

Proof of Lemma 2.1.

First, we simplify the brand-name firm's problem by substituting (d_1^*, d_2^*) in Equation (2.5), and obtain

$$\max_{w_1, w_2} \pi_B = \begin{cases} \pi_B^H(w_1, w_2) = (p_B - w_1)\left((1-p_B) + \alpha\left(1 - \frac{(2-\beta)p_B - k_2}{2(1-\beta)}\right)^+\right), & \text{if } w_1 \geq k_1 \text{ and } w_2 < \frac{M}{\alpha(1-\frac{p_B}{\gamma})} + k_2, \\ \pi_B^D(w_1, w_2) = (p_B - w_1)(1-p_B) + \alpha(p_B - w_2)\left(1 - \frac{p_B}{\gamma}\right), & \text{if } w_1 \geq k_1 \text{ and } w_2 \geq \frac{M}{\alpha\left(1-\frac{p_B}{\gamma}\right)} + k_2, \\ \pi_B^O(w_1, w_2) = (p_B - w_2)\left(\left(1 - \frac{p_B}{\gamma}\right) + \alpha\left(1 - \frac{p_B}{\gamma}\right)\right), & \text{if } w_1 < k_1 \text{ and } w_2 \geq \frac{K}{(1+\alpha)\left(1-\frac{p_B}{\gamma}\right)} + k_2, \\ \pi_B^N(w_1, w_2) = 0, & \text{if } w_1 < k_1,\ w_2 < \frac{K}{(1+\alpha)\left(1-\frac{p_B}{\gamma}\right)} + k_2, \end{cases} \tag{A.2}$$

where $M = (\pi_0 - e)^+$, $K = (\frac{\alpha(\beta-k_2)^2}{4\beta} - e)^+$.

Next, we discuss optimal wholesale prices that the brand-name firm would like to offer under each strategy. Note that $\pi_B(w_1, w_2)$ decreases in w_1 and w_2, respectively.

(1) Strategy H. The brand-name firm's profit is π_B^H in the first line of Equation (A.2). In order to get maximal profit, the optimal wholesale prices are $w_1^H = k_1$ and $w_2^H < \frac{M}{\alpha(1-\frac{p_B}{\gamma})} + k_2$.

(2) Strategy D. The brand-name firm's profit is π_B^D in the second line of Equation (A.2). In order to get maximal profit, the optimal wholesale price for the home supplier is $w_1^D = k_1$. The optimal wholesale price for the overseas supplier is $w_2^D = \frac{M}{\alpha(1-\frac{p_B}{\gamma})} + k_2$, where $M = (\pi_0 - e)^+$. Note that when $e \geq \pi_0$, $w_2^D = k_2$. When $e < \pi_0$, we know, $w_2^D > k_2$ and w_2^D increases in β and α, respectively, decreases in e.

(3) Strategy O. The brand-name firm's profit is π_B^O in the third line of Equation (A.2). In order to get maximal profit, the optimal wholesale price for the home supplier is $w_1^O < k_1$. The optimal wholesale price for the overseas supplier is $w_2^O = \frac{K}{(1+\alpha)(1-\frac{p_B}{\gamma})} + k_2$, where $K = (\frac{\alpha(\beta-k_2)^2}{4\beta} - e)^+$. Note that when $e \geq \frac{\alpha(\beta-k_2)^2}{4\beta}$, $w_2^O = k_2$. When $e < \frac{\alpha(\beta-k_2)^2}{4\beta}$, $w_2^O > k_2$ and w_2^O increases in β and α, respectively, decreases in e.

(4) Strategy N. The brand-name firm's profit is π_B^N in the fourth line of Equation (A.2). The optimal wholesale prices are $w_1^N < k_1$ and $w_2^N < \frac{K}{(1+\alpha)(1-\frac{p_B}{\gamma})} + k_2$. The brand-name firm does not have suppliers to product any products, and his profit π_B^N is always zero. □

Proof of Proposition 2.1.

Firstly, substituting the optimal wholesale prices at Lemma 2.1 into the brand-name firm's profit expressions, respectively, we obtain the brand-name firm's optimal profits under each strategy.

Next, we compare the brand-name firm's optimal profit among four strategies to determine the equilibrium sourcing strategy. The optimal profit is $\pi_B^* = \max\{\pi_B^H, (\pi_B^D)^+, (\pi_B^O)^+\}$, that is,

$$\pi_B^* = \begin{cases} \pi_B^H, & \text{if } \pi_B^H \geq \max\{(\pi_B^D)^+, (\pi_B^D)^+\}, \\ \pi_B^D, & \text{if } \pi_B^D \geq \max\{\pi_B^H, (\pi_B^O)^+\}, \\ \pi_B^O, & \text{if } \pi_B^O \geq \max\{\pi_B^H, (\pi_B^D)^+\}. \end{cases}$$

Recall $\underline{\beta} = \frac{k_2}{p_B}$ and $\beta_1 = \frac{k_2+2(1-p_B)}{2-p_B}$. We define $\beta_2 = k_2 + 2(1-p_B)$.

1. With $\underline{\beta} < \beta < \beta_1$, we know, $\pi_B^H \geq 0$, $\pi_B^D \geq 0$ and $\pi_B^O \geq 0$, then, we compare π_B^D and π_B^H, π_B^O and π_B^D, respectively.

$$\pi_B^H = \begin{cases} (p_B - k_1)(1-p_B) + \alpha(p_B - k_1)(1-p_B), & \text{if } e \geq \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}, \\ (p_B - k_1)\left((1-p_B) + \alpha\left(1 - \frac{(2-\beta)p_B - k_2}{2(1-\beta)}\right)\right), & \text{if } e < \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}, \end{cases}$$

$$\pi_B^D = (p_B - k_1)(1-p_B) + \alpha(p_B - k_2)(1 - \frac{p_B}{\gamma}) - M,$$

$$\pi_B^O = (p_B - k_2)(1 - \frac{p_B}{\gamma}) + \alpha(p_B - k_2)(1 - \frac{p_B}{\gamma}) - K,$$

$$\pi_B^N = 0,$$

where $M = \left(\frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} - e\right)^+$, $K = (\frac{\alpha(\beta - k_2)^2}{4\beta} - e)^+$.

(1) By comparing profit expressions of π_B^D and π_B^H, if $e < \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}$, then,

$$\begin{aligned}\pi_B^D - \pi_B^H = \; & \alpha\Delta(1 - \tfrac{(2-\beta)p_B - k_2}{2(1-\beta)}) + \tfrac{\alpha(p_B - k_2)(\beta p_B - k_2)}{2(1-\beta)} - \tfrac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} \\ & + e + \alpha(p_B - k_2)(p_B - \tfrac{p_B}{\gamma}).\end{aligned} \tag{A.3}$$

From $\pi_B^D \geq \pi_B^H$ in Equation (A.3), we obtain,

$$\begin{aligned} e \geq \; & \tfrac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} - \alpha\Delta(1 - \tfrac{(2-\beta)p_B - k_2}{2(1-\beta)}) - \tfrac{\alpha(p_B - k_2)(\beta p_B - k_2)}{2(1-\beta)} \\ & - \alpha(p_B - k_2)(p_B - \tfrac{p_B}{\gamma}).\end{aligned} \tag{A.4}$$

(2) By comparing profit expressions of π_B^O and π_B^D, if $e < \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}$, then,

$$\pi_B^O - \pi_B^D = \Delta(1-p_B) - \tfrac{\alpha(\beta - k_2)^2}{4\beta} + \tfrac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} + (p_B - k_2)(p_B - \tfrac{p_B}{\gamma}). \tag{A.5}$$

From $\pi_B^O \geq \pi_B^D$ in Equation (A.5), we obtain,

$$\Delta \geq \frac{\frac{\alpha(\beta-k_2)^2}{4\beta} - \frac{\alpha(\beta p_B-k_2)^2}{4\beta(1-\beta)} - (p_B-k_2)(p_B-\frac{p_B}{\gamma})}{(1-p_B)}.$$

(3) By comparing profit expressions of π_B^O and π_B^H, if $e < \frac{\alpha(\beta p_B-k_2)^2}{4\beta(1-\beta)}$, then,

$$\begin{aligned}\pi_B^O - \pi_B^H = \; & \Delta(1-p_B) + \Delta\alpha\left(1-\frac{(2-\beta)p_B-k_2}{2(1-\beta)}\right) + \frac{\alpha(p_B-k_2)(\beta p_B-k_2)}{2(1-\beta)} - \frac{\alpha(\beta-k_2)^2}{4\beta} \\ & + e + (1+\alpha)(p_B-k_2)(p_B-\frac{p_B}{\gamma}).\end{aligned} \tag{A.6}$$

From $\pi_B^O \geq \pi_B^H$ in Equation (A.6), we obtain,

$$\begin{aligned}e \geq \; & \frac{\alpha(\beta-k_2)^2}{4\beta} - \Delta(1-p_B) - \Delta\alpha(1-\frac{(2-\beta)p_B-k_2}{2(1-\beta)}) \\ & -\frac{\alpha(p_B-k_2)(\beta p_B-k_2)}{2(1-\beta)} - (1+\alpha)(p_B-k_2)(p_B-\frac{p_B}{\gamma}).\end{aligned} \tag{A.7}$$

2. With $\beta_1 \leq \beta < \beta_2$, we compare π_B^D and π_B^H, π_B^O and π_B^H, π_B^O and $(\pi_B^D)^+$, respectively.

$$\pi_B^H = \begin{cases} (p_B-k_1)(1-p_B) + \alpha(p_B-k_1)(1-p_B), & \text{if } e \geq \frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta}, \\ (p_B-k_1)(1-p_B), & \text{if } e < \frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta}, \end{cases}$$

$$\pi_B^D = (p_B-k_1)(1-p_B) + \alpha(p_B-k_2)(1-\frac{p_B}{\gamma}) - M,$$

$$\pi_B^O = (p_B-k_2)(1-\frac{p_B}{\gamma}) + \alpha(p_B-k_2)(1-\frac{p_B}{\gamma}) - K,$$

$$\pi_B^N = 0,$$

where $M = \left(\frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta} - e\right)^+$, $K = (\frac{\alpha(\beta-k_2)^2}{4\beta} - e)^+$.

(1) By comparing profit expressions of π_B^D and π_B^H, if $e < \frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta}$, then,

$$\pi_B^D - \pi_B^H = \alpha(p_B-k_2)(1-p_B) - \frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta} + e + \alpha(p_B-k_2)(p_B-\frac{p_B}{\gamma}). \tag{A.8}$$

From $\pi_B^D \geq \pi_B^H$ in Equation (A.8), we obtain,

$$e \geq \frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta} - \alpha(p_B-k_2)(1-\frac{p_B}{\gamma}).$$

(2) By comparing profit expressions of π_B^O and π_B^D, if $e < \frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta}$, then,

$$\pi_B^O - \pi_B^D = \Delta(1-p_B) - \frac{\alpha(\beta-k_2)^2}{4\beta} + \frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta} + (p_B-k_2)(p_B-\frac{p_B}{\gamma}). \tag{A.9}$$

From $\pi_B^O \geq \pi_B^D$ in Equation (A.9), we obtain,

$$\Delta \geq \frac{\frac{\alpha(\beta-k_2)^2}{4\beta} - \frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta} - (p_B - k_2)(p_B - \frac{p_B}{\gamma})}{(1-p_B)}.$$

(3) By comparing profit expressions of π_B^O and π_B^H, if $e < \frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta}$, then,

$$\begin{aligned}\pi_B^O - \pi_B^H = \;& \Delta(1-p_B) + \alpha(p_B-k_2)(1-p_B) - \tfrac{\alpha(\beta-k_2)^2}{4\beta} + e \\ & +(1+\alpha)(p_B-k_2)(p_B - \tfrac{p_B}{\gamma}).\end{aligned} \tag{A.10}$$

From $\pi_B^O \geq \pi_B^H$ in Equation (A.10), we obtain,

$$e \geq \frac{\alpha\left(\beta-k_2\right)^2}{4\beta} - \Delta(1-p_B) - \alpha(p_B-k_2)(1-p_B) - (1+\alpha)(p_B-k_2)(p_B - \frac{p_B}{\gamma}).$$

3. With $\beta_2 \leq \beta < 1$, we compare π_B^D and π_B^H, π_B^O and π_B^H, π_B^O and $(\pi_B^D)^+$, respectively.

$$\begin{aligned}\pi_B^H &= \left(p_B-k_1\right)\left(1-p_B\right), \\ \pi_B^D &= (p_B-k_1)(1-p_B) + \alpha(p_B-k_2)(1-\frac{p_B}{\gamma}) - M, \\ \pi_B^O &= (p_B-k_2)(1-\frac{p_B}{\gamma}) + \alpha(p_B-k_2)(1-\frac{p_B}{\gamma}) - K, \\ \pi_B^N &= 0,\end{aligned}$$

where $M = K = (\frac{\alpha(\beta-k_2)^2}{4\beta} - e)^+$.

(1) By comparing profit expressions of π_B^D and π_B^H, if $e < \frac{\alpha(\beta-k_2)^2}{4\beta}$, then,

$$\pi_B^D - \pi_B^H = \alpha(p_B-k_2)(1-\tfrac{p_B}{\gamma}) - (\tfrac{\alpha(\beta-k_2)^2}{4\beta} - e). \tag{A.11}$$

From $\pi_B^D \geq \pi_B^H$ in Equation (A.11), we obtain,

$$e \geq \frac{\alpha\left(\beta-k_2\right)^2}{4\beta} - \alpha(p_B-k_2)(1-\frac{p_B}{\gamma}).$$

(2) By comparing profit expressions of π_B^D and π_B^O, if $e < \frac{\alpha(\beta-k_2)^2}{4\beta}$, then,

$$\pi_B^O - \pi_B^D = \Delta(1-p_B) + (p_B-k_2)(p_B-\tfrac{p_B}{\gamma}). \tag{A.12}$$

From $\pi_B^O \geq \pi_B^D$ in Equation (A.12), we obtain,

$$\Delta \geq -\frac{(p_B-k_2)(p_B-\frac{p_B}{\gamma})}{(1-p_B)} = \frac{(p_B-k_2)(\frac{p_B}{\gamma}-p_B)}{(1-p_B)}.$$

(2) By comparing profit expressions of π_B^O and π_B^H, if $e < \frac{\alpha(\beta-k_2)^2}{4\beta}$, then,

$$\begin{aligned}\pi_B^O - \pi_B^H = \; & \Delta(1-p_B) + (p_B - k_2)(p_B - \tfrac{p_B}{\gamma}) + \alpha(p_B - k_2)(1 - \tfrac{p_B}{\gamma}) \\ & -(\tfrac{\alpha(\beta-k_2)^2}{4\beta} - e).\end{aligned} \tag{A.13}$$

From $\pi_B^O \geq \pi_B^H$ in Equation (A.13), we obtain,

$$e \geq \frac{\alpha\,(\beta-k_2)^2}{4\beta} - [\Delta(1-p_B) + (p_B - k_2)(p_B - \frac{p_B}{\gamma}) + \alpha(p_B - k_2)(1 - \frac{p_B}{\gamma})].$$

Recall

$$\begin{aligned}e_1 &= \pi_0 + \alpha(p_B - k_2 - \Delta)(1 - \tfrac{(2-\beta)p_B - k_2}{2(1-\beta)})^+ - \alpha(p_B - k_2)(1 - \tfrac{p_B}{\gamma}),\\ e_2 &= \tfrac{\alpha(\beta-k_2)^2}{4\beta} - \Delta(1-p_B) - \alpha(p_B - k_2)(1 - \tfrac{p_B}{\gamma}) - (p_B - k_2)(p_B - \tfrac{p_B}{\gamma}) \\ &\quad + \alpha(p_B - k_2 - \Delta)\left(1 - \tfrac{(2-\beta)p_B - k_2}{2(1-\beta)}\right)^+,\\ \Delta_1 &= \frac{\frac{\alpha(\beta-k_2)^2}{4\beta} - \pi_0 - (p_B - k_2)(p_B - \frac{p_B}{\gamma})}{1-p_B}.\end{aligned}$$

Then, to summarize, the equilibrium sourcing strategies are as follows.

(i) Strategy H is optimal with $w_1^* = k_1$ and $w_2^* < w_2^D$ when $e < \min\{(e_1)^+, (e_2)^+\}$;

(ii) Strategy D is optimal with $w_1^* = k_1$ and $w_2^* = w_2^D$ when $e \geq (e_1)^+$ and $\Delta \leq (\Delta_1)^+$;

(iii) Strategy O is optimal with $w_1^* < k_1$ and $w_2^* = w_2^O$ when $e \geq (e_2)^+$ and $\Delta > (\Delta_1)^+$. □

Analysis: Equilibrium Sourcing Strategy with respect to β and α.

In order to further analyze the factors that affect the equilibrium outcomes in Proposition 2.1, we rewrite the conditions of equilibrium outcomes as a function of α and β. Below, we derive boundary lines between each two sourcing strategies in Figure 2.5. Define $\pi_0' = \frac{\pi_0}{\alpha}$, that is,

$$\pi_0' = \begin{cases} \frac{(\beta p_B - k_2)^2}{4\beta(1-\beta)}, & \text{if } \underline{\beta} \leq \beta < \beta_1, \\ \frac{\min\{\beta - k_2 - (1-p_B), \frac{\beta - k_2}{2}\}\max\{1-p_B, \frac{\beta-k_2}{2}\}}{\beta}, & \text{if } \beta_1 < \beta < 1. \end{cases}$$

From $e \leq \pi_0$, we have, $\alpha \geq \alpha_0(\beta)$, where $\alpha_0(\beta) = \frac{e}{\pi_0'}$.

From the boundary line between Strategy D and Strategy H, *i.e.*, $e > e_1$, we have: if $\pi_0' + (p_B - k_2 - \Delta)(1 - \frac{(2-\beta)p_B - k_2}{2(1-\beta)})^+ - (p_B - k_2)(1 - \frac{p_B}{\gamma}) > 0$, then,

$$\begin{aligned} e > e_1 \Rightarrow \;& e > \alpha[\pi_0' + (p_B - k_2 - \Delta)(1 - \tfrac{(2-\beta)p_B - k_2}{2(1-\beta)})^+ - (p_B - k_2)(1 - \tfrac{p_B}{\gamma})];\\ \Rightarrow \;& \alpha < \alpha_1(\beta), \text{ where } \alpha_1(\beta) = \frac{e}{\pi_0' + (p_B - k_2 - \Delta)(1 - \frac{(2-\beta)p_B - k_2}{2(1-\beta)})^+ - (p_B - k_2)(1 - \frac{p_B}{\gamma})}.\end{aligned}$$

From the boundary line between Strategy O and Strategy D, *i.e.*, $\Delta > \Delta_1$, we have: if $\beta < \beta_3$, then,

$$\begin{aligned}\Delta > \Delta_1 \Rightarrow \quad & \Delta > \frac{\alpha[\frac{(\beta-k_2)^2}{4\beta}-\pi_0']-(p_B-k_2)(p_B-\frac{p_B}{\gamma})}{1-p_B};\\ \Rightarrow \quad & \alpha < \alpha_\Delta(\beta), \text{ where } \alpha_\Delta(\beta) = \frac{\Delta(1-p_B)+(p_B-k_2)(p_B-\frac{p_B}{\gamma})}{\frac{(\beta-k_2)^2}{4\beta}-\pi_0'}.\end{aligned}$$

From the boundary line between Strategy O and Strategy H, *i.e.*, $e > e_2$, we have: if $\frac{(\beta-k_2)^2}{4\beta} + (p_B - k_2 - \Delta)(1 - \frac{(2-\beta)p_B-k_2}{2(1-\beta)})^+ - (p_B - k_2)(1 - \frac{p_B}{\gamma}) > 0$, then,

$$\begin{aligned}e > e_2 \Rightarrow \quad & e > \alpha[\frac{(\beta-k_2)^2}{4\beta} + (p_B - k_2 - \Delta)(1 - \frac{(2-\beta)p_B-k_2}{2(1-\beta)})^+ - (p_B - k_2)(1 - \frac{p_B}{\gamma})]\\ & -\Delta(1-p_B) - (p_B-k_2)(p_B - \frac{p_B}{\gamma});\\ \Rightarrow \quad & \alpha < \alpha_2(\beta), \text{ where } \alpha_2(\beta) = \frac{e+\Delta(1-p_B)+(p_B-k_2)(p_B-\frac{p_B}{\gamma})}{\frac{(\beta-k_2)^2}{4\beta}+(p_B-k_2-\Delta)(1-\frac{(2-\beta)p_B-k_2}{2(1-\beta)})^+-(p_B-k_2)(1-\frac{p_B}{\gamma})}.\end{aligned}$$

Moreover, we obtain additional boundary conditions of α from $\pi_B^D(\alpha) \geq 0$, $\pi_B^O(\alpha) \geq 0$ in equilibrium. □

Analysis: Equilibrium Sourcing Strategy with respect to γ.

We derive the condition of γ that the brand-name firm is willing to consider converting the counterfeiter. From Figure 2.5(b), when Strategy H exists under a low β, in this case, the brand-name firm is not willing to consider converting the counterfeiter. Recall that the boundary line between Strategy H and Strategy D is $\alpha_1(\beta) = \frac{e}{\pi_0'+(p_B-k_2-\Delta)(1-\frac{(2-\beta)p_B-k_2}{2(1-\beta)})^+-(p_B-k_2)(1-\frac{p_B}{\gamma})}$. Thus, if $\alpha_1(\beta = \underline{\beta}) \leq 0$, Strategy H would not exist under a low β in panel(b). That is, under $\beta = \underline{\beta} = \frac{k_2}{p_B}$, if $\frac{(\beta p_B-k_2)^2}{4\beta(1-\beta)} + (p_B-k_2-\Delta)(1-\frac{(2-\beta)p_B-k_2}{2(1-\beta)})^+ - (p_B-k_2)(1-\frac{p_B}{\gamma}) \leq 0$, which implies $\gamma \geq \frac{p_B(p_B-k_2)}{p_B(p_B-k_2)+\Delta(1-p_B)}$, then, there is no Strategy H under a low β. That is, in this condition of $\gamma \geq \frac{p_B(p_B-k_2)}{p_B(p_B-k_2)+\Delta(1-p_B)}$, the brand-name firm is willing to consider converting the counterfeiter. □

Analysis: Properties of π_B^* and π_2^*.

To prove the statement of the Proposition 2.3, we examine the properties of π_B^* and π_2^* in below Lemma A.1.

Lemma A.1. *(a) The brand-name firm's optimal profit π_B^* increases as e increases, or as β decreases;*
(b) The counterfeiter's optimal profit π_2^* is non-monotone as e increases, and decreases as β decreases.

Proof of Lemma A.1.

Recall from the equilibrium in Proposition 2.1, as e increases, the optimal strategy could change from Strategy H to Strategy D if $\Delta < (\Delta_1)^+$, or change from Strategy H to Strategy O if $\Delta \geq (\Delta_1)^+$, as illustrated in Figure 2.4.

(i) When $\Delta < (\Delta_1)^+$, we have,

$$\pi_B^* = \begin{cases} \pi_B^H & \text{if } e < (e_1)^+, \\ \pi_B^D & \text{if } e \geq (e_1)^+. \end{cases}$$

(ii) When $\Delta \geq (\Delta_1)^+$, we have,

$$\pi_B^* = \begin{cases} \pi_B^H & \text{if } e < (e_2)^+, \\ \pi_B^O & \text{if } e \geq (e_2)^+. \end{cases}$$

In above two subcases, we easily know that as e increases, π_B^* firstly keeps unchanged then increases.

Recall from Proposition 2.1, with the condition of $\gamma \geq \frac{p_B(p_B-k_2)}{p_B(p_B-k_2)+\Delta(1-p_B)}$: as β increases, the optimal strategy will firstly change from Strategy D to Strategy O, then to Strategy H, as illustrated in Figure 2.5(a).

From the boundary line between Strategy H and Strategy O, *i.e.*, $e = e_2(\beta)$, where $\beta > \beta_2$, we get the critical point of β_0: define

$$y_0 = \frac{e + (\Delta + \alpha(p_B - k_2))(1 - p_B) + (1 + \alpha)(p_B - k_2)(p_B - \frac{p_B}{\gamma})}{\alpha},$$

then, by solving $\frac{(\beta-k_2)^2}{4\beta} = \frac{e+\Delta(1-p_B)+(p_B-k_2)(p_B-\frac{p_B}{\gamma})}{\alpha} + (p_B - k_2)(1 - \frac{p_B}{\gamma})$, then,

$$\beta_0 = k_2 + 2y_0 - 2\sqrt{y_0(k_2 + y_0)}. \tag{A.14}$$

From the boundary line between Strategy O and Strategy D, *i.e.*, $\Delta = \Delta_1(\beta)$, where $\beta < \beta_2$, we get the critical point of $\tilde{\beta}$: given $\pi_0' = \frac{(\beta p_B-k_2)^2}{4\beta(1-\beta)}$, define $y_1 = k_2 + \frac{(1-p_B)^2}{2} + \frac{2\Delta(1-p_B)+2(p_B-k_2)(p_B-\frac{p_B}{\gamma})}{\alpha}$, then, by solving $\frac{(\beta-k_2)^2}{4\beta} - \frac{(\beta p_B-k_2)^2}{4\beta(1-\beta)} = \frac{\Delta(1-p_B)+(p_B-k_2)(p_B-\frac{p_B}{\gamma})}{\alpha}$, then,

$$\tilde{\beta} = y_1 + \sqrt{y_1^2 - (2k_2(1 - p_B) - k_2^2 + \frac{4\Delta(1 - p_B) + 4(p_B - k_2)(p_B - \frac{p_B}{\gamma})}{\alpha})}. \tag{A.15}$$

From the above discussions of threshold boundaries between strategies, we have:

$$\pi_B^* = \begin{cases} \pi_B^D & \text{if } \beta < \tilde{\beta}, \\ \pi_B^O & \text{if } \tilde{\beta} \leq \beta < \beta_0, \\ \pi_B^H & \text{if } \beta \geq \beta_0. \end{cases}$$

We can easily show that π_B^* is continuous and increases as β decreases. Figure A.1(a)-(b) and Figure A.1(c) show the impact of e and β, respectively, on the profit of the brand-name firm.

Similarly, we get the optimal expression of π_2^* in equilibrium. If $\Delta < (\Delta_1)^+$, as e increases, the optimal strategy could change from Strategy H to Strategy D. Thus, the optimal profit π_2^* decreases in e. If $\Delta \geq (\Delta_1)^+$, as e increases, the

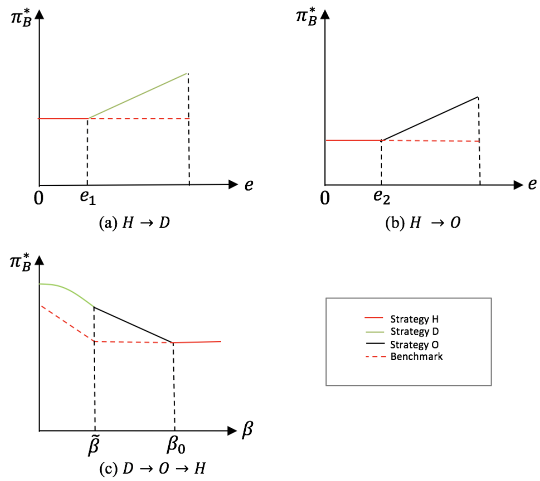


Figure A.1: (Color online) Brand-name Firm's Profit as a Function of the Penalty from Law Enforcement (e) and the Perceived Quality of the Counterfeit (β), respectively. Note: Solid arrows at the panel name indicate the changes of equilibrium sourcing strategy as e increases in panels (a)-(b), or as β increases in panel (c).

optimal strategy could change from Strategy H to Strategy O. Under the strategy D or Strategy O, the optimal profit π_2^D or π_2^O decreases in e. Differently, in this change from Strategy H to Strategy O, there is a discontinuous point that the optimal profit jumps to a larger value. Thus, the optimal profit π_2^* is non-monotone with e, as illustrated in Figure A.2(a)-(b). It is also easy for us to show that π_2^* decreases as β decreases, as illustrated in Figure A.2(c). □

Proof of Proposition 2.2.

Recall that under $\gamma \geq \frac{p_B(p_B-k_2)}{p_B(p_B-k_2)+\Delta(1-p_B)}$, the brand-name firm is willing to convert the counterfeiter. In this condition, the brand-name firm optimally choose Strategy H when the wholesale price w_2^D or w_2^O is so high that the brand-name firm can not afford to convert the counterfeiter.

Below, we show that when the brand-name firm is willing to consider converting, the thresholds of e_0 and β_0 satisfy the relationships in this proposition. The proof proceeds as follows: we firstly obtain $(\bar{e}_0, \bar{\beta}_0)$ by analyzing the counterfeiter's profit $\bar{\pi}_2^* < 0$ under the benchmark; then, obtain (e_0, β_0) by analyzing the brand-name firm's profit relationship from $\pi_B^H \leq \pi_B^D$, $\pi_B^H \leq \pi_B^O$ under the base model; lastly, make comparisons between $\bar{e}_0$ and e_0, $\bar{\beta}_0$ and β_0.

Under the benchmark, the counterfeiter is driven out of market when $\bar{\pi}_2(\beta) < 0$, where

$$\bar{\pi}_2(\beta) = \begin{cases} \frac{\alpha(\beta p_B-k_2)^2}{4\beta(1-\beta)} - e, & \text{if } \underline{\beta} < \beta < \beta_1, \\ \frac{\alpha \min\{\beta-k_2-(1-p_B), \frac{\beta-k_2}{2}\} \max\{1-p_B, \frac{\beta-k_2}{2}\}}{\beta} - e, & \text{if } \beta_1 < \beta < 1. \end{cases}$$

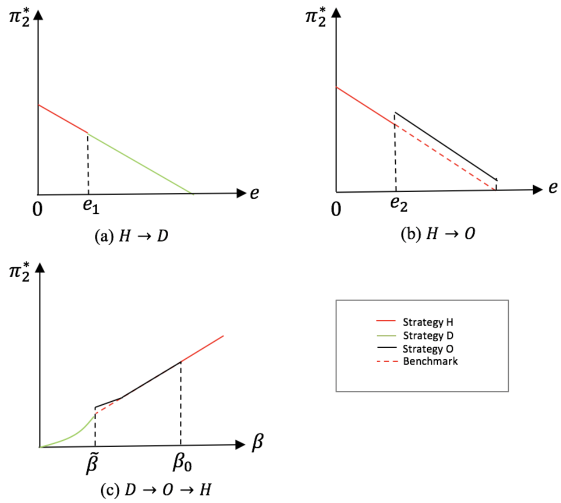


Figure A.2: (Color online) Counterfeiter's Profit with respect to the Penalty from Law Enforcement (e) and the Perceived Quality of the Counterfeit (β), respectively. Note: Solid arrows at the panel name indicate the changes of equilibrium sourcing strategy as e increases in panels (a)-(b), or as β increases in panel (c).

Under the benchmark, we know, if $e > \pi_0(\beta)$, the counterfeiter does not enter to sell counterfeits, *i.e.*, $\bar{\pi}_2(\beta) < 0$. Thus, $\bar{e}_0 = \pi_0(\beta)$, and $\bar{\beta}_0$ satisfies $\pi_0(\beta = \bar{\beta}_0) - e = 0$.

With the option of converting, when Strategy D or Strategy O is optimal, it means that the counterfeiting is combated. Thus, we need to compare the thresholds between the base model, *i.e.*, $\min\{(e_1(\beta))^+, (e_2(\beta))^+\}$ and the benchmark, *i.e.*, $\pi_0(\beta)$, and derive conditions that satisfy

$$\min\{(e_1(\beta))^+, (e_2(\beta))^+\} < \pi_0(\beta).$$

1. For the threshold of e: $e_0 = \min\{(e_1)^+, (e_2)^+\}$, $\bar{e} = \pi_0$. From Figure 2.4, with the condition $\gamma \geq \frac{p_B(p_B-k_2)}{p_B(p_B-k_2)+\Delta(1-p_B)}$, we know, $\min\{(e_1)^+, (e_2)^+\} < \pi_0(\beta)$ always holds. Thus, $e_0 < \bar{e}_0$.

2. For the threshold of β: $\beta_0 = \tilde{\beta}$, $\bar{\beta}_0 = \pi_0$. From Figure 2.5(a), when the counterfeiter is converted by either Strategy D or Strategy O, the maximal value of β lines in the boundary of $\max\{\alpha_2(\beta), \alpha_0(\beta)\}$, where $\alpha > \alpha_0(\beta)$ guarantees $\pi_2^H > 0$. Because, with $\gamma \geq \frac{p_B(p_B-k_2)}{p_B(p_B-k_2)+\Delta(1-p_B)}$, $\alpha_2(\beta) > \alpha_0(\beta)$, we know, $\beta_0 > \bar{\beta}_0$. □

Proof of Corollary 2.1.

Note that in equilibrium of the base model, under Strategy H, the profit of each firm is the same as that under the benchmark, *i.e.*, $\pi_1^H = \bar{\pi}_1^*$, $\pi_2^H = \bar{\pi}_2^*$, $\pi_B^H = \bar{\pi}_B^*$. In equilibrium, under Strategy D or Strategy O, the home supplier obtains zero profit, that is, $\pi_1^D = \pi_1^O = \bar{\pi}_1^* = 0$. Thus, in the following, we focus on comparing profits of the brand-name firm, the overseas supplier, between the benchmark and Strategy D as well as Strategy O in equilibrium from Proposition 2.1, respectively. Note that we assume $e < \pi_0$, and hence we have $M > 0$ and $K > 0$.

1. If $\underline{\beta} < \beta < \beta_1$, we have the comparison of profits as follows. In this case, $M = \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} - e$, $K = \frac{\alpha(\beta - k_2)^2}{4\beta} - e$.

(1) When Strategy D is optimal,

$$\text{brand-name firm: } \pi_B^D - \bar{\pi}_B^* = \frac{\alpha(p_B - k_1)(\beta p_B - k_2)}{2(1-\beta)} + \alpha(1-p_B)\Delta + \alpha(p_B - k_2)(p_B - \frac{p_B}{\gamma}) - M; \quad (A.16)$$

$$\text{overseas supplier: } \pi_2^D - \bar{\pi}_2^* = M - (\frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} - e) = 0.$$

(2) When Strategy O is optimal,

$$\text{brand-name firm: } \pi_B^O - \bar{\pi}_B^* = \frac{\alpha(p_B - k_1)(\beta p_B - k_2)}{2(1-\beta)} + (1+\alpha)(1-p_B)\Delta + (1+\alpha)(p_B - k_2)(p_B - \frac{p_B}{\gamma}) - K; \quad (A.17)$$

$$\text{overseas supplier: } \pi_2^O - \bar{\pi}_2^* = K - (\frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} - e) > 0. \quad (A.18)$$

2. If $\beta_1 \le \beta < \beta_2$, we have the comparison of profits as follows. In this case, $M = \frac{\alpha(\beta p_B - k_2)((2-p_B)\beta - k_2)}{4\beta} - e$.

(1) When Strategy D is optimal,

$$\text{brand-name firm: } \pi_B^D - \bar{\pi}_B^* = \alpha(p_B - k_2)(1-p_B) + \alpha(p_B - k_2)(p_B - \frac{p_B}{\gamma}) - M; \quad (A.19)$$

$$\text{overseas supplier: } \pi_2^D - \bar{\pi}_2^* = M - (\frac{\alpha(\beta - k_2 - (1-p_B))(1-p_B)}{\beta} - e) = 0.$$

(2) When Strategy O is optimal,

$$\text{brand-name firm: } \pi_B^O - \bar{\pi}_B^* = (\alpha(p_B - k_2) + \Delta)(1-p_B) + (1+\alpha)(p_B - k_2)(p_B - \frac{p_B}{\gamma}) - K; \quad (A.20)$$

$$\text{overseas supplier: } \pi_2^O - \bar{\pi}_2^* = K - (\frac{\alpha(\beta - k_2 - (1-p_B))(1-p_B)}{\beta} - e) > 0. \quad (A.21)$$

3. If $\beta_2 \le \beta < 1$, we have the comparison of profits as follows. In this case, $M = K = \frac{\alpha(\beta - k_2)^2}{4\beta} - e$.

(1) When Strategy D is optimal,

$$\text{for brand-name firm, } \pi_B^D - \bar{\pi}_B^* = \alpha(p_B - k_2)(1 - p_B) + \alpha(p_B - k_2)(p_B - \frac{p_B}{\gamma}) - M; \tag{A.22}$$

$$\text{for overseas supplier, } \pi_2^D - \bar{\pi}_2^* = M - (\frac{\alpha\,(\beta - k_2)^2}{4\beta} - e) = 0.$$

(2) When Strategy O is optimal,

$$\text{brand-name firm: } \pi_B^O - \bar{\pi}_B^* = (\alpha(p_B - k_2) + \Delta)(1 - p_B) + (1 + \alpha)(p_B - k_2)(p_B - \frac{p_B}{\gamma}) - K; \tag{A.23}$$

$$\text{overseas supplier: } \pi_2^O - \bar{\pi}_2^* = K - (\frac{\alpha\,(\beta - k_2)^2}{4\beta} - e) = 0.$$

Thus, for the brand-name firm, based on the equilibrium in Proposition 2.1, under Strategy D and Strategy O, $\pi_B^D \geq \bar{\pi}_B^*$, $\pi_B^O \geq \bar{\pi}_B^*$, respectively.

For the overseas supplier, for $e < \pi_0$, under the benchmark, the counterfeiter enters the overseas market to sell counterfeits. Under the base model, when Strategy D is optimal, we know, $\pi_2^D = \bar{\pi}_2^*$. When Strategy O is optimal, if $\beta < \beta_2$, we have $\pi_2^O > \bar{\pi}_2^*$; if $\beta > \beta_2$, we have $\pi_2^O = \bar{\pi}_2^*$. □

Proof of Proposition 2.3.

The proof of Proposition 2.3 proceeds as follows: we firstly derive the profit difference between each possible strategy in the base model and in the benchmark, then discuss how e and β affect the difference between the optimal strategy in equilibrium and the benchmark.

1. For the brand-name firm, in equilibrium, when Strategy H is optimal, $\pi_B^H = \bar{\pi}_B^*$. Thus, below we focus on deriving the profit difference between π_B^D and $\bar{\pi}_B^*$, π_B^O and $\bar{\pi}_B^*$, respectively.

(1) The impact of e on $(\pi_B^* - \bar{\pi}_B^*)$. Note that as e increases, the equilibrium sourcing strategy could change from Strategy H to either Strategy D or Strategy O. Then, from Equations (A.16), (A.19) and (A.22), we know,
if $\Delta < (\Delta_1)^+$, then,

$$\pi_B^* - \bar{\pi}_B^* = \begin{cases} \pi_B^H - \bar{\pi}_B^* = 0, & \text{if } e < (e_1)^+, \\ \pi_B^D - \bar{\pi}_B^*, & \text{if } e \geq (e_1)^+; \end{cases}$$

if $\Delta \geq (\Delta_1)^+$, then,

$$\pi_B^* - \bar{\pi}_B^* = \begin{cases} \pi_B^H - \bar{\pi}_B^* = 0, & \text{if } e < (e_2)^+, \\ \pi_B^O - \bar{\pi}_B^*, & \text{if } e \geq (e_2)^+. \end{cases}$$

Since $(\pi_B^D - \bar{\pi}_B^*)$ and $(\pi_B^O - \bar{\pi}_B^*)$ increases with e. Thus, as e increases, $(\pi_B^* - \bar{\pi}_B^*)$ firstly keeps unchanged with zero and then increases with e.

(2) The impact of β on $(\pi_B^* - \bar{\pi}_B^*)$. Note that as β increases, the optimal

strategy firstly changes from Strategy D to Strategy O, then to Strategy H. Then, from Equations (A.16), (A.19) and (A.22), we have,

$$\pi_B^* - \bar{\pi}_B^* = \begin{cases} \pi_B^D - \bar{\pi}_B^*, & \text{if } \beta \le \tilde{\beta}, \\ \pi_B^O - \bar{\pi}_B^*, & \text{if } \tilde{\beta} < \beta \le \beta_0, \\ \pi_B^H - \bar{\pi}_B^* = 0, & \text{if } \beta > \beta_0. \end{cases}$$

Define $f_1^D(\beta) = \frac{\alpha(p_B-k_1)(\beta p_B-k_2)}{2(1-\beta)} - \frac{\alpha(\beta p_B-k_2)^2}{4\beta(1-\beta)}$, $f_2^D(\beta) = -\frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta}$; $f_1^O(\beta) = \frac{\alpha(p_B-k_1)(\beta p_B-k_2)}{2(1-\beta)} - \frac{\alpha(\beta-k_2)^2}{4\beta}$, $f_2^O(\beta) = -\frac{\alpha(\beta-k_2)^2}{4\beta}$. By taking first order derivative with respect to β, we can easily show that as β decreases, (i) when Strategy D is optimal, $f_1^D(\beta)$ decreases with $\beta \in (\beta_1, \beta_2)$; $f_2^D(\beta)$ increases with $\beta \in (\beta_2, 1)$; (ii) when Strategy O is optimal, $f_1^O(\beta)$ decreases with $\beta \in (\beta_1, \beta_2)$, $f_2^O(\beta)$ increases with $\beta \in (\beta_2, 1)$.

Thus, as β decreases, (i) with π_B^H in $\beta > \beta_0$, $(\pi_B^H - \bar{\pi}_B^*)$ keeps unchanged with zero; (ii) with π_B^O in $\tilde{\beta} < \beta < \beta_0$, $(\pi_B^O - \bar{\pi}_B^*)$ increases for $\beta > \beta_1$, decreases for $\beta < \beta_1$; (iii) with π_B^D in $\beta < \tilde{\beta}$, $(\pi_B^D - \bar{\pi}_B^*)$ decreases. Thus, in equilibrium, $(\pi_B^* - \bar{\pi}_B^*)$ is non-monotone with β, as illustrated in Figure 2.6(a).

2. For the counterfeiter, since $\pi_2^H = \bar{\pi}_2^*$ when Strategy H is optimal, $\pi_2^D = \bar{\pi}_2^*$ when Strategy D is optimal, then, below we focus on the profit difference between π_2^O and $\bar{\pi}_2^*$ when Strategy O is optimal.

(1) The impact of e on $(\pi_2^* - \bar{\pi}_2^*)$. Note that as e decreases, the optimal strategy could change from Strategy H to Strategy D, or change from Strategy H to Strategy O. Thus, (i) for the change from Strategy H to Strategy D, $(\pi_2^D - \bar{\pi}_2^*) = 0$; (ii) for the change from Strategy H to Strategy O, from Equations (A.18) and (A.21), $(\pi_2^O - \bar{\pi}_2^*) \ge 0$. Thus, the profit difference could firstly keep unchanged with zero, and then jump to a positive value.

(2) The impact of β on $(\pi_2^* - \bar{\pi}_2^*)$. Note that as β increases, the optimal strategy firstly changes from Strategy D to Strategy O, then to Strategy H. Then, from Equations (A.18) and (A.21), we have,

$$\pi_2^* - \bar{\pi}_2^* = \begin{cases} \pi_2^D - \bar{\pi}_2^* = 0, & \text{if } \beta \le \tilde{\beta}, \\ \pi_2^O - \bar{\pi}_2^*, & \text{if } \tilde{\beta} < \beta \le \beta_0, \\ \pi_2^H - \bar{\pi}_2^* = 0, & \text{if } \beta > \beta_0, \end{cases}$$

where $\pi_2^O - \bar{\pi}_2^* = 0$ if $\beta \ge \beta_2$.

Define $f_{1\prime}^O(\beta) = \frac{\alpha(\beta-k_2)^2}{4\beta} - \frac{\alpha(\beta p_B-k_2)^2}{4\beta(1-\beta)}$, $f_{2\prime}^O(\beta) = \frac{\alpha(\beta-k_2)^2}{4\beta} - \frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta}$. By taking first order derivative with respect to β, we can easily show that as β increases, $f_{2\prime}^O(\beta)$ decreases; and for $f_{1\prime}^O(\beta)$, we get the critical point $\hat{\beta} = 1 + k_2 - p_B$, where $f_{1\prime}^O(\beta)$ increases if $\beta < \hat{\beta}$ and decreases if $\beta > \hat{\beta}$. Recall that $\tilde{\beta}$ is the threshold that the optimal strategy changes from Strategy D to Strategy O, *i.e.*, $\pi_B^O > \pi_B^D$ for $\beta > \tilde{\beta}$. Since $\hat{\beta} < \tilde{\beta}$, then, it means that in equilibrium, $f_{1\prime}^O(\beta)$ decreases for $\beta > \tilde{\beta}$.

Thus, as β decreases, (i) with the Strategy H for $\beta > \beta_0$, $(\pi_2^H - \bar{\pi}_2^*)$ keeps unchanged with zero; (ii) with the Strategy O for $\tilde{\beta} < \beta < \beta_0$, $(\pi_2^O - \bar{\pi}_2^*)$ keeps unchanged with zero if $\beta > \beta_2$, and increases if $\beta < \beta_2$; (ii) with the Strategy D for $\beta < \tilde{\beta}$, $(\pi_2^D - \bar{\pi}_2^*)$ keeps unchanged with zero. Thus, in equilibrium, $(\pi_2^* - \bar{\pi}_2^*)$ is non-monotone with β, as illustrated in Figure 2.6(b). □

Proof of Proposition 2.4.

Firstly, under the benchmark: in the equilibrium,
(1) consumer surplus in the home market is

$$\overline{CS}_1 = \frac{1-(p_B)^2}{2} - p_B(1-p_B) = \frac{(1-p_B)^2}{2};$$

(2) consumer surplus in the overseas market is

$$\overline{CS}_2 = \begin{cases} \alpha(\frac{\beta(\widetilde{\theta}^2-(\hat{\theta}_2)^2)}{2} - \frac{(\beta p_B+k_2)(\widetilde{\theta}-\hat{\theta}_2)}{2} + \frac{1-\widetilde{\theta}^2}{2} - p_B(1-\widetilde{\theta})), & \text{if } \underline{\beta} < \beta < \beta_1, \\ \alpha(\frac{\beta(1-(\hat{\theta}_2)^2)}{2} - \bar{p}_2^*(1-\hat{\theta}_2)), & \text{if } \beta_1 \le \beta < 1, \end{cases}$$

where $\widetilde{\theta} = \frac{(2-\beta)p_B-k_2}{2(1-\beta)}$, $\hat{\theta}_2 = \frac{\bar{p}_2^*}{\beta}$ and

$$\bar{p}_2^* = \begin{cases} \frac{\beta p_B+k_2}{2}, & \text{if } \underline{\beta} < \beta < \beta_1, \\ \min\{p_B-(1-\beta), \frac{\beta+k_2}{2}\}, & \text{if } \beta_1 \le \beta < 1, \end{cases}$$

Thus, we obtain,

$$\overline{CS}_2 = \begin{cases} \alpha(\frac{\beta(\widetilde{\theta}-\hat{\theta}_2)^2}{2} + \frac{1-\widetilde{\theta}^2}{2} - p_B(1-\widetilde{\theta})), & \text{if } \underline{\beta} < \beta < \beta_1, \\ \alpha(\frac{\beta(1-\hat{\theta}_2)^2}{2}), & \text{if } \beta_1 \le \beta < 1. \end{cases}$$

Secondly, under the base model: recall $\hat{\theta}_B = \frac{p_B}{\gamma}$,
when Strategy H is optimal, consumer surplus in the home and overseas markets are as follows, respectively:

$$CS_1^H = \overline{CS}_1, CS_2^H = \overline{CS}_2;$$

when Strategy D is optimal, consumer surplus in the home and overseas markets are as follows, respectively:

$$CS_1^D = \frac{1-(p_B)^2}{2} - p_B(1-p_B) = \frac{(1-p_B)^2}{2},$$
$$CS_2^D = \alpha(\gamma(\frac{1-(\hat{\theta}_B)^2}{2}) - p_B(1-\hat{\theta}_B)) = \alpha\gamma(\frac{(1-\hat{\theta}_B)^2}{2});$$

when Strategy O is optimal, consumer surplus in the home and overseas markets are as follows, respectively:

$$CS_1^O = \gamma(\frac{1-(\hat{\theta}_B)^2}{2}) - p_B(1-\hat{\theta}_B) = \gamma(\frac{(1-\hat{\theta}_B)^2}{2}),$$
$$CS_2^O = \alpha(\gamma(\frac{1-(\hat{\theta}_B)^2}{2}) - p_B(1-\hat{\theta}_B)) = \alpha\gamma(\frac{(1-\hat{\theta}_B)^2}{2}).$$

Lastly, by comparing consumer surplus between the benchmark and Strategy D as well as Strategy O in equilibrium, respectively, we have the following relationships.

(1) In the home market, $CS_1^O < CS_1^D = \overline{CS}_1$. Because

$$\begin{aligned} CS_1^O - \overline{CS}_1 &= \gamma(\frac{1-(\hat{\theta}_B)^2}{2}) - p_B(1-\hat{\theta}_B) - (\frac{1-(p_B)^2}{2} - p_B(1-p_B)) \\ &= \frac{\gamma(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2} < 0. \end{aligned}$$

(2) In the overseas market, $CS_2^D = CS_2^O$. By comparing CS_2^D with $\overline{CS}_2$, we get the following results.

(i) If $\underline{\beta} < \beta < \beta_1$, where $\underline{\beta} = \frac{k_2}{p_B}$ and $\beta_1 = \frac{k_2+2(1-p_B)}{2-p_B}$, then, in the overseas market, we have

$$\begin{aligned} CS_2^D - \overline{CS}_2 &= \alpha(\frac{\gamma(1-\hat{\theta}_B)^2}{2} - (\frac{\beta(\widetilde{\theta}-\hat{\theta}_2)^2}{2} + \frac{1-\widetilde{\theta}^2}{2} - p_B(1-\widetilde{\theta})) \\ &= -\frac{\alpha\left(\beta p_B - k_2\right)^2}{8\beta\left(1-\beta\right)} + \alpha(\frac{\gamma(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2}) < 0. \end{aligned}$$

(ii) If $\beta_1 \le \beta < \beta_2$, where $\beta_1 = \frac{k_2+2(1-p_B)}{2-p_B}$ and $\beta_2 = k_2 + 2\left(1-p_B\right)$, then, in the overseas market, we have

$$\begin{aligned} CS_2^D - \overline{CS}_2 &= \alpha(\frac{\gamma(1-\hat{\theta}_B)^2}{2} - \frac{\beta(1-\frac{p_B-(1-\beta)}{\beta})^2}{2}) \\ &= \frac{\alpha((1-p_B)^2 - \beta(\frac{1-p_B}{\beta})^2)}{2} + \alpha(\frac{\gamma(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2}) < 0. \end{aligned}$$

Because $(CS_2^D - \overline{CS}_2)$ increases in $\beta \in [\beta_1, \beta_2)$, and $(CS_2^D - \overline{CS}_2)|_{\beta=1} = 0$, then, $CS_2^D - \overline{CS}_2 < 0$ for $\beta \in [\beta_1, \beta_2)$.

(iii) If $\beta_2 \le \beta < 1$, where $\beta_2 = k_2 + 2\left(1-p_B\right)$, then, in the overseas market, we have

$$\begin{aligned} CS_2^D - \overline{CS}_2 &= \alpha(\frac{\gamma(1-\hat{\theta}_B)^2}{2} - \frac{\beta(1-\frac{\beta+k_2}{2\beta})^2}{2}) \\ &= \frac{\alpha((1-p_B)^2 - \beta(1-\frac{\beta+k_2}{2\beta})^2)}{2} + \alpha(\frac{\gamma(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2}) < 0. \end{aligned}$$

Because $(CS_2^D - \overline{CS}_2)$ decreases in $\beta \in [\beta_2, 1)$, and $(CS_2^D - \overline{CS}_2)|_{\beta=\beta_2} = (1-p_B)^2 - \frac{(1-p_B)^2}{k_2+2(1-p_B)} < 0$, then, $CS_2^D - \overline{CS}_2 < 0$ for $\beta \in [\beta_2, 1)$.

Thus, for $\beta \in (\underline{\beta}, 1)$, in the overseas market, $CS_2^D = CS_2^O < \overline{CS}_2$.

For the total consumer surplus, $CS = CS_1 + CS_2$: $CS^O < CS^D < \overline{CS}$. $\square$

Proof of Proposition 2.5.

Firstly, under the benchmark: in the equilibrium,

(1) if $\underline{\beta} < \beta < \beta_1$, the social surplus is
$\overline{SS} = \overline{CS}_1 + \overline{CS}_2 + \bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^* = \frac{(1-p_B)^2}{2} + \alpha(\frac{\beta(\widetilde{\theta}^2-(\hat{\theta}_2)^2)}{2} - \frac{(\beta p_B+k_2)(\widetilde{\theta}-\hat{\theta}_2)}{2} + \frac{1-\widetilde{\theta}^2}{2} - p_B(1-\widetilde{\theta})) + (p_B-k_1)(1-p_B) + \alpha(p_B-k_1)\left(\frac{2(1-\beta)(1-p_B)-\beta p_B+k_2}{2(1-\beta)}\right) + (\frac{\alpha(\beta p_B-k_2)^2}{4\beta(1-\beta)} - e)^+$;

(2) if $\beta_1 \le \beta < \beta_2$, the social surplus is
$\overline{SS} = \overline{CS}_1 + \overline{CS}_2 + \bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^* = \frac{(1-p_B)^2}{2} + \alpha(\frac{\beta(1-(\hat{\theta}_2)^2)}{2} - \bar{p}_2^*(1-\hat{\theta}_2)) + (p_B-k_1)(1-p_B) + (\frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta} - e)^+$.

(3) if $\beta_2 \le \beta < 1$, the social surplus is
$\overline{SS} = \overline{CS}_1 + \overline{CS}_2 + \bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^* = \frac{(1-p_B)^2}{2} + \alpha(\frac{\beta(1-(\hat{\theta}_2)^2)}{2} - \bar{p}_2^*(1-\hat{\theta}_2)) + (p_B-k_1)(1-p_B) + (\frac{\alpha(\beta-k_2)^2}{4\beta} - e)^+$.

Secondly, under the base model:

when Strategy D is optimal, the social surplus is
$SS^D = CS_1^D + CS_2^D + \pi_B^D + \pi_1^D + \pi_2^D = \frac{(1-p_B)^2}{2} + \alpha\gamma(\frac{(1-\hat{\theta}_B)^2}{2}) + (p_B-k_1)(1-p_B) + \alpha(p_B-k_2)(1-p_B)$;

when Strategy O is optimal, the social surplus is
$SS^O = CS_1^O + CS_2^O + \pi_B^O + \pi_1^O + \pi_2^O = (1+\alpha)\gamma(\frac{(1-\hat{\theta}_B)^2}{2}) + (p_B-k_2)(1-p_B) + \alpha(p_B-k_2)(1-p_B)$.

Lastly, by comparing the social surplus between the benchmark and Strategy D as well as Strategy O in equilibrium, respectively, we have the following relationships. Recall that

$$\begin{aligned} e_1 &= \pi_0 + \alpha(p_B-k_2-\Delta)(1-\tfrac{(2-\beta)p_B-k_2}{2(1-\beta)})^+ - \alpha(p_B-k_2)(1-\tfrac{p_B}{\gamma}), \\ e_2 &= \tfrac{\alpha(\beta-k_2)^2}{4\beta} - \Delta(1-p_B) - \alpha(p_B-k_2)(1-\tfrac{p_B}{\gamma}) - (p_B-k_2)(p_B-\tfrac{p_B}{\gamma}) \\ &\quad + \alpha(p_B-k_2-\Delta)\left(1-\tfrac{(2-\beta)p_B-k_2}{2(1-\beta)}\right)^+. \end{aligned}$$

We define

$$e_1' = e_1 - g_1, \tag{A.24}$$

$$e_2' = e_2 - g_2 - \frac{\alpha(\beta-k_2)^2}{4\beta} + \pi_0, \tag{A.25}$$

where

$$g_1 = \begin{cases} -\frac{\alpha(\beta p_B - k_2)^2}{8\beta(1-\beta)} + \alpha(\frac{\gamma(1-\frac{p_B}{\gamma})^2}{2} - \frac{(1-p_B)^2}{2}), & \text{if } \underline{\beta} < \beta < \beta_1, \\ \frac{\alpha((1-p_B)^2 - \beta(1-\min\{\frac{p_B-(1-\beta)}{\beta}, \frac{\beta+k_2}{2\beta}\})^2)}{2} + \alpha(\frac{\gamma(1-\frac{p_B}{\gamma})^2}{2} - \frac{(1-p_B)^2}{2}), & \text{if } \beta_1 \leq \beta < 1, \end{cases}$$

$g_2 = g_1 + (\frac{\gamma(1-\frac{p_B}{\gamma})^2}{2} - \frac{(1-p_B)^2}{2})$. Note that $g_1 < 0$ and $g_2 < 0$ represent the loss of consumer surplus under Strategy D and Strategy O, respectively. Then, we have the following comparisons about social welfare.

(1) If $\underline{\beta} < \beta < \beta_1$,
when Strategy D is optimal,

$$\begin{aligned} &SS^D - \overline{SS} = (CS_1^D - \overline{CS}_1) + (CS_2^D - \overline{CS}_2) + (\pi_B^D + \pi_1^D + \pi_2^D) - (\bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^*) \\ &= -\frac{\alpha\,(\beta p_B - k_2)^2}{8\beta\,(1-\beta)} + \alpha(\frac{\gamma(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2}) \\ &+ \frac{\alpha(p_B - k_2 - \Delta)(\beta p_B - k_2)}{2\,(1-\beta)} + \alpha(1-p_B)\Delta + \alpha(p_B - k_2)(p_B - \frac{p_B}{\gamma}) - (\frac{\alpha(\beta p_B - k_2)^2}{4\beta\,(1-\beta)} - e)^+; \end{aligned}$$

when Strategy O is optimal,

$$\begin{aligned} &SS^O - \overline{SS} = (CS_1^O - \overline{CS}_1) + (CS_2^O - \overline{CS}_2) + (\pi_B^O + \pi_1^O + \pi_2^O) - (\bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^*) \\ &= -\frac{\alpha\,(\beta p_B - k_2)^2}{8\beta\,(1-\beta)} + (1+\alpha)(\frac{\gamma(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2}) \\ &+ \frac{\alpha(p_B - k_2 - \Delta)(\beta p_B - k_2)}{2\,(1-\beta)} + (1+\alpha)(1-p_B)\Delta + (1+\alpha)(p_B - k_2)(p_B - \frac{p_B}{\gamma}) - (\frac{\alpha(\beta p_B - k_2)^2}{4\beta\,(1-\beta)} - e)^+. \end{aligned}$$

Since $\frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} \geq e$, then, $SS^D > \overline{SS}$ when $e > e_1 - g_1$; $SS^O > \overline{SS}$ when $e > e_2 - g_2 - \frac{\alpha(\beta-k_2)^2}{4\beta} + \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}$.

(2) If $\beta_1 \leq \beta < \beta_2$,
when Strategy D is optimal,

$$\begin{aligned} &SS^D - \overline{SS} = (CS_1^D - \overline{CS}_1) + (CS_2^D - \overline{CS}_2) + (\pi_B^D + \pi_1^D + \pi_2^D) - (\bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^*) \\ &= \frac{\alpha((1-p_B)^2 - \beta(\frac{1-p_B}{\beta})^2)}{2} + \alpha(\frac{\gamma(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2}) \\ &+ \alpha(p_B - k_2)(1-p_B) + \alpha(p_B - k_2)(p_B - \frac{p_B}{\gamma}) - (\frac{\alpha(\beta - k_2 - (1-p_B))(1-p_B)}{\beta} - e)^+; \end{aligned}$$

when Strategy O is optimal,

$$\begin{aligned} &SS^O - \overline{SS} = (CS_1^O - \overline{CS}_1) + (CS_2^O - \overline{CS}_2) + (\pi_B^O + \pi_1^O + \pi_2^O) - (\bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^*) \\ &= \frac{\alpha((1-p_B)^2 - \beta(\frac{1-p_B}{\beta})^2)}{2} + (1+\alpha)(\frac{\gamma(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2}) \\ &+ (\alpha(p_B - k_2) + \Delta)(1-p_B) + (1+\alpha)(p_B - k_2)(p_B - \frac{p_B}{\gamma}) - (\frac{\alpha(\beta - k_2 - (1-p_B))(1-p_B)}{\beta} - e)^+. \end{aligned}$$

Since $\frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta} - e > 0$, then, $SS^D > \overline{SS}$ if $e > e_1 - g_1$; $SS^O > \overline{SS}$

if $e > e_2 - g_2 - \frac{\alpha(\beta-k_2)^2}{4\beta} + \frac{\alpha(\beta-k_2-(1-p_B))(1-p_B)}{\beta}$.

(3) If $\beta_2 \le \beta < 1$,
when Strategy D is optimal,

$$SS^D - \overline{SS} = (CS_1^D - \overline{CS}_1) + (CS_2^D - \overline{CS}_2) + (\pi_B^D + \pi_1^D + \pi_2^D) - (\bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^*)$$
$$= \frac{\alpha((1-p_B)^2 - \beta(1-\frac{\beta+k_2}{2\beta})^2)}{2} + \alpha(\frac{\gamma(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2})$$
$$+ \alpha(p_B - k_2)(1-p_B) + \alpha(p_B - k_2)(p_B - \frac{p_B}{\gamma}) - (\frac{\alpha\left(\beta-k_2\right)^2}{4\beta} - e);$$

when Strategy O is optimal,

$$SS^O - \overline{SS} = (CS_1^O - \overline{CS}_1) + (CS_2^O - \overline{CS}_2) + (\pi_B^O + \pi_1^O + \pi_2^O) - (\bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^*)$$
$$= \frac{\alpha((1-p_B)^2 - \beta(1-\frac{\beta+k_2}{2\beta})^2)}{2} + (1+\alpha)(\frac{\gamma(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2})$$
$$+ (\alpha(p_B - k_2) + \Delta)(1-p_B) + (1+\alpha)(p_B - k_2)(p_B - \frac{p_B}{\gamma}) - (\frac{\alpha\left(\beta-k_2\right)^2}{4\beta} - e)^+.$$

Since $\frac{\alpha(\beta-k_2)^2}{4\beta} - e > 0$, then, $SS^D > \overline{SS}$ if $e > e_1 - g_1$; $SS^O > \overline{SS}$ if $e > e_2 - g_2$.

Note that $e_1'(\beta) = e_1(\beta) - g_1(\beta)$, $e_2'(\beta) = e_2(\beta) - g_2(\beta) - \frac{\alpha(\beta-k_2)^2}{4\beta} + \pi_0(\beta)$, then, the results follow. □

Appendix B

Appendix of Chapter 3

Section B.1 describes the data and calibration used for numerical experiments in Section 3.6.2 in the paper. Section B.2 contains the proofs for our technical statements in the paper.

A summary of model parameters and notations is presented in Table B.1.

B.1 Model Calibration for Numerical Experiments

Below are details for our calibration of baseline parameters.

- Prices and costs information:

$r^s = 18$, $r = 40$, $p = 9$, $w = 12$, $c^D = 26$; $m^n = 0.8$, $m^u = 1.2$; $r^r = 40$, $d^n = 3.2$, $d^u = 4.8$.

(1) Under traditional ownership programs, the profit ratio is $\frac{r-w-c^D}{r} = \frac{40-12-26}{40} = 5\%$. Note, c^D should satisfy $0 < S - w < c^D < P + S - w$, that is, $20 < c^D < 34$.

(2) $p = \frac{r^s}{2}$ implies the penalty cost of unsatisfied the switching requests of the rest 1 year is the half of subscription fee of two years.

(3) The marginal profits: $R = 7, P = 14, S = 32$ and $O = 0.5R + S = 35.5$. Note that with $c^F = 26$, the critical ratio is $\frac{P+S-w-c^F}{(P+S-w-c^F)+(c^F-(O-w))} = \frac{14+32-12-26}{14+32-12-26+(26-(35.5-12))} = \frac{8}{10.5} = 0.76$.

- Subscription demand information: parameter estimation μ_2 by using public sales data. Estimation procedures of μ_2 are as below.

Step 1: We assume 10% of sale is from subscription program. Based on XC60's total sales in US market during year 2020-2021, we can get the ownership sale by applying that ownership sale is $(1 - 10\%)$ of total sale.
-The total sale of XC60 during year 2020-2021 in US is: 73,616 units.
-Estimated ownership sale is: $73616 * 90\% = 66,254.4$ units.
-Estimated subscription sale is: $73616 * 10\% = 7361.6$ units.
-Ownership price of XC60 is $\$43,000$ in Oct of 2020.

Decision Variables	
K_i^D	Capacity investment decision for product i under dedicated technology
K^F	Capacity investment decision under flexible technology
x_i^T	Units of product i allocated to satisfy the demand for Type i subscription program, $T \in \{D, F\}$
y_i^T	Units of product i reserved for customers who switch from Type $-i$ subscription program to Type i subscription program, $T \in \{D, F\}$
Demand Variables	
ξ_i	Demand for Type i subscription program in the beginning of stage 2
Demand Parameters	
β_i	Proportion of customers for product i switches to product $-i$ under Type i subscription
μ_i	Average subscription demand for Type j subscription program
σ_i	Variance of subscription demand for Type j subscription program
ρ	Correlation between the two types of subscription demands
Cost Parameters	
c^T	Unit capacity investment cost of technology, $T \in \{D, F\}$
w_i	Unit production cost for product i
m_i^u	Unit maintenance cost for product i when it is under use of one period
m_i^n	Unit maintenance cost for product i when it is not in use of one period
Δm_i	Unit maintenance cost difference for product i between usage and not usage, $\Delta m_i = m_i^u - m_i^n$
r_i^s	Subscription fee of Type i subscription program
p_i	Unit penalty cost for unsatisfied switching requests for product i in the second period
r_i^r	Resale price for product i allocated to Type i subscription program in the beginning of stage 2
d_i^u	Unit depreciation cost for product i when it is under use of one period
d_i^n	Unit depreciation cost for product i when it is not in use of one period
Δd_i	Unit depreciation cost difference for product i between usage and not usage, $\Delta d_i = d_i^u - d_i^n$

Table B.1: Summary of Notations

Step 2: We assume the ownership demand D_{2o} for product 2 follows normal distribution, i.e., $D_{2o} \sim \mathcal{N}(\mu_{2o}, \sigma_{2o})$. By applying traditional newsvendor model to estimated ownership sale information, we estimate average ownership demand μ_{2o} for product 2, with $\sigma_{2o} = 0.1\mu_{2o}$. We want to choose the average demand of ownership μ_{2o} that can bring around 66254.4 unit of sales in two years. The relationship of ownership sale and average demand of Product 2 under dedicated technology: for given μ_{2o}, we get the optimal capacity investment level K_{2o}^{D*}. Then, we know, the sales volume of Product 2 is

$$E[\min\{(K_{2o}^{D*}(\mu_{2o}), \tilde{D}_{2o}(\mu_{2o}))\}] = \int_{-\infty}^{K_{2o}^D} D_{2o} f(D_{2o}) dD_{2o} + K_{2o}^D \int_{K_{2o}^D}^{\infty} f(D_{2o}) dD_{2o}$$
$$= \mu_{2o}[\tfrac{B-c^D}{B-R_L}) - \sigma_{2o}\phi(\Phi^{-1}(\tfrac{B-c^D}{B-R_L})] + K_{2o}^{D*}(1 - \tfrac{B-c^D}{B-R_L}),$$

where $B = r-w$ represents the profit margin of traditional ownership program, $R_L = (S-w)^+$ represents the salvage value.

By setting $r = 43,000$, choosing other economic parameters based on the relative percentage with respect to r, and calculating under newsvendor model, we know, the average ownership demand is around 72.17 thousand in 2020-2021.

Step 3: With the calculated average ownership demand μ_{2o} in Step 2, and $\frac{subscription\ demand}{ownership\ demand} = \frac{10\%}{90\%}$, we get the estimation average value of subscription demand:

$$\frac{10\%}{90\%} = \frac{estimated\ subscription\ demand}{72170}.$$

Thus, with $r = 43,000$, the estimated average subscription demand is $\mu_{2o} = 8019$ units.

We assume price and demand mean has linear relationship. By applying the same scale with ownership price, we have, $\frac{40}{43000} = \frac{\mu_2}{8019}$. Thus, we obtain, $\mu_2 = 7.46$.

B.2 Proofs for Analytical Results.

Proof of Proposition 3.1.

In order to show the results in Proposition 3.1, we have below lemma about switching requests.

Lemma B.1. (Three Cases based on whether switching requests is fully served)

Only the following cases can occur for switching demand under each subscription program. Note $T \in \{F, D\}$.

Case 1: *Switching demand for Product 1 under Type 2 subscription program is fully served, but switching demand for Product 2 under Type 1 subscription program is underserved, i.e.,* $\beta_2 x_2^T \leq \beta_1 x_1^T + y_1^T$, *and* $\beta_1 x_1^T \geq \beta_2 x_2^T + y_2^T$.

Case 2: *Switching demand for Product 1 under Type 2 subscription program is underserved, but switching demand for Product 1 under Type 1 subscription program is fully served, i.e.,* $\beta_2 x_2^T \geq \beta_1 x_1^T + y_1^T$, *and* $\beta_1 x_1^T \leq \beta_2 x_2^T + y_2^T$.

Case 3: *Switching demand for Product 1 under Type 2 subscription program is fully served, and switching demand for Product 1 under Type 1 subscription program is fully served, i.e.,* $\beta_2 x_2^T \leq \beta_1 x_1^T + y_1^T$, *and* $\beta_1 x_1^T \leq \beta_2 x_2^T + y_2^T$.

Proof of Lemma B.1.

Case 1: switching demand for product 1 is fully served, but switching demand for product 2 is underserved.

$\Rightarrow \beta_2 x_2^T \leq \beta_1 x_1^T + y_1^T, \beta_1 x_1^T \geq \beta_2 x_2^T + y_2^T \Rightarrow \beta_2 x_2^T \leq \beta_1 x_1^T - y_2^T$.

Case 2: switching demand for product 1 is underserved, switching demand for product 2 is fully served.

$\Rightarrow \beta_2 x_2^T \geq \beta_1 x_1^T + y_1^T, \beta_1 x_1^T \leq \beta_2 x_2^T + y_2^T \Rightarrow \beta_2 x_2^T \geq \beta_1 x_1^T + y_1^T$.

Case 3: switching demand for product 1 is fully served, demand for product 2 is fully served.

$\Rightarrow \beta_2 x_2^T \leq \beta_1 x_1^T + y_1^T, \beta_1 x_1^T \leq \beta_2 x_2^T + y_2^T \Rightarrow \beta_1 x_1^T - y_2^T \leq \beta_2 x_2^T \leq \beta_1 x_1^T + y_2^T$.

Case 4: switching demand for product 1 is under served, switching demand for product 2 is under served.

$\Rightarrow \beta_2 x_2^T \geq \beta_1 x_1^T + y_1^T, \beta_1 x_1^T \geq \beta_2 x_2^T + y_2^T$.

It is easy to know: Case 4 is an infeasible case.

From Lemma B.1, we know,

- Case 1: capacity of product 2 is scarce, $\beta_2 x_2^T \leq \beta_1 x_1^T - y_2^T$;
- Case 2: capacity of product 1 is scarce, $\beta_2 x_2^T \geq \beta_1 x_1^T + y_1^T$;
- Case 3: capacity of product 1 and capacity of product 2 are ample, $\beta_1 x_1^T - y_2^T \leq \beta_2 x_2^T \leq \beta_1 x_1^T + y_2^T$. $\square$

Define z_i^T for product i as the leftover capacity after allocation and reservation. With the assumption of symmetric parameters of $\beta_1 = \beta_2 = \beta$, and by rearranging the profit function, we know, under Case 1, the profit function is:

$$\pi^D = (P - \beta R + S - w)\, x_1^T + (S - w) y_1^T + (O - w) z_1^T + (P + \beta R + S - w)\, y_2^T$$
$$+ (R + (S - w)) y_2^T + (O - w) z_2^T.$$

Define $R_L = S - w$, $B = O - w$, and

$$\begin{aligned} S_H &= R_L + P + \beta R; \\ S &= R_L + P; \\ S_L &= R_L + P - \beta R; \\ R_H &= R_L + R. \end{aligned}$$

With our assumptions, we know: $0 < R_L < B < R_H < S_L < S_H$ holds.

With these redefinitions, we can rewrite the profit in stage-2 as follows:

$$\pi^T = S_L x_1^T + R_L y_1^T + S_H y_2^T + R_H y_2^T + B(z_1^T + z_2^T).$$

Similarly, we can rearrange the profit under other two cases, and obtain the following three case about profits from subscription programs in stage 2.

Case 1: $\pi^T = S_L x_1^T + R_L y_1^T + S_H x_2^T + R_H y_2^T + B(z_1^T + z_2^T)$;
Case 2: $\pi^T = S_L x_2^T + R_L y_2^T + S_H x_1^T + R_H y_1^T + B(z_1^T + z_2^T)$;
Case 3:

$$\pi^T = \begin{cases} S_L x_1^T + R_L y_1^T + S_H x_2^T + R_H \beta \left(x_1^T - x_2^T\right) + R_L \left(y_2^T - \beta \left(x_1^T - x_2^T\right)\right) + B(z_1^T + z_2^T), & \text{if } x_1^T \geq x_2^T, \\ S_L x_2^T + R_L y_2^T + S_H x_1^T + R_H \beta \left(x_2^T - x_1^T\right) + R_L \left(y_1^T - \beta \left(x_2^T - x_1^T\right)\right) + B(z_1^T + z_2^T), & \text{if } x_1^T < x_2^T, \end{cases}.$$

Part 1: Model Formulation With Dedicated Technilogy

At Stage 1, the objective function of the optimization problem under dedicated technology is

$$\Pi^D(\mathbf{K}^{\boldsymbol{D}}) = \mathbb{E}\left[\pi^D(\mathbf{K}^{\boldsymbol{D}}, \tilde{\boldsymbol{\xi}})\right] - 2c^D K^D.$$

At Stage 2, based on whether the switching demand under each type of subscription program are fully served at Lemma B.1, we have following discussions.

(1) If product i is fully served and product $-i$ is under served, then, the production allocation problem is as follows.

$$\begin{aligned}
& \pi^D(x_1^D, y_1^D, x_2^D, y_2^D; K^D, \xi_1, \xi_2) \\
= \quad & \max \quad S_L x_i^D + R_L y_i^D + S_H x_{-i}^D + R_H y_{-i}^D + B(z_1^T + z_2^T) \\
& s.t. \quad x_j^D + y_j^D \leq K^D,\ \forall j = 1, 2 \\
& \qquad\ \ x_j^D \leq \xi_j,\ \forall j = 1, 2 \\
& \qquad\ \ y_j^D \geq 0,\ \forall j = 1, 2 \\
& \qquad\ \ x_j^D \geq 0,\ \forall j = 1, 2 \\
& \qquad\ \ \beta x_i^D \geq \beta x_{-i}^a + y_{-i}^D
\end{aligned}$$

(2) If both products are fully served and $x_i^a \geq x_{-i}^a$, then, the production allocation problem is as follows.

$$\begin{aligned}
& \pi^D(x_1^D, y_1^D, x_2^D, y_2^D; K^D, \xi_1, \xi_2) \\
= \quad & \max \quad S_L x_i^D + R_L y_i^D + S_H x_{-i}^D + R_H \beta \left(x_i^D \ - y_{-i}^D\right) + R_L \left(y_{-i}^D - \beta \left(x_i^D - x_{-i}^D\right)\right) + B(z_1^T + z_2^T) \\
& s.t. \quad x_j^D + y_j^D \leq K^D,\ \forall j = 1, 2 \\
& \qquad\ \ x_j^D \leq \xi_j,\ \forall j = 1, 2 \\
& \qquad\ \ x_j^D \geq 0\, \forall j = 1, 2 \\
& \qquad\ \ y_i^D \geq 0,\ \forall j = 1, 2 \\
& \qquad\ \ \beta x_j^D \leq \beta x_{-j}^D + y_{-j}^D,\ \forall j = 1, 2 \\
& \qquad\ \ x_i^D \geq x_{-i}^D
\end{aligned}$$

We first show that the profit function is concave in the first-stage capacity investment decisions through the following Proposition B.1.

Proposition B.1. *The objective function* $\Pi^D(K^D)$ *is concave in capacity variable* K^D*.*

Part 2: in stage-2, product allocation and reservation with dedicated technology.

- **Product 1 or Product 2 is fully served**

As shown in Lemma 1, when the switching requests is fully satisfied for only one product, only the following two scenarios will occur:
S1. $\beta_1 x_1^D \geq \beta_2 x_2^D + y_2^D$, or
S2. $\beta_2 x_2^D \geq \beta_1 x_1^D + y_1^D$.

If we consider S1, in stage 2, we solve the following problem.

$$
\begin{array}{llllc}
\pi^D(K^D,\xi_1,\xi_2) = & \max & B(z_1^D + z_2^D) + S_L x_1^D + S_H x_2^D + R_L y_1^D + R_H y_2^D & & \text{Dual Variables} \\
 & s.t. & x_i^D + y_i^D + z_i^D - K^D \le 0 & i = 1,2 & \mu_i \\
 & & x_i^D - \xi_i \le 0 & i = 1,2 & \lambda_i \\
 & & \beta x_2^D + y_2^D - \beta x_1^D \le 0 & & \eta \\
 & & -z_j^D \le 0, & j = 1,2 & \kappa_{z_j^D} \\
 & & -x_i^D \le 0, & i = 1,2 & \kappa_{x_i^D} \\
 & & -y_i^D \le 0 & i = 1,2 & \kappa_{y_i^D}
\end{array}
$$

KKT conditions are given as follows.

$$
\begin{array}{llcll}
S_L = \mu_1 + \lambda_1 - \beta\eta - \kappa_{x_1^D} & (x_i^D + y_i^D + z_i^D - K^D)\mu_i & = & 0 & i = 1,2 \\
S_H = \mu_2 + \lambda_2 + \beta\eta - \kappa_{x_2^D} & (x_i^D - \xi_i)\lambda_i & = & 0 & i = 1,2 \\
R_L = \mu_1 - \kappa_{y_1^D} & (\beta x_2^D + y_2^D - \beta x_1^D)\eta & = & 0 & \\
R_H = \mu_2 - \kappa_{y_2^D} + \eta & z_j^D \kappa_{z_j^D} & = & 0 & j = 1,2 \\
B = \mu_1 - \kappa_{z_1^D} & x_i^D \kappa_{x_i^D} & = & 0 & i = 1,2 \\
B = \mu_2 - \kappa_{z_2^D} & y_i^D \kappa_{y_i^D} & = & 0 & i = 1,2 \\
\text{Dual variables are nonnegative and prime feasibility} & & & &
\end{array}
$$

From $0 < R_L < B < R_H < S_L < S_H$, we can get the following relationships of dual variables.

$R_L > 0$, implies $\mu_1 > 0$, thus, $x_1^D + y_1^D + z_1^D = K^D$.
$S_L > R_L$, that is, $\mu_1 + \lambda_1 - k_{x_1^D} - \beta\eta > \mu_1 - k_{y_1^D}$, then, $\lambda_1 + k_{y_1^D} > k_{x_1^D} + \beta\eta$, at least one of $\lambda_1, k_{y_1^D}$ is positive.
$S_L > B$, that is, $\mu_1 + \lambda_1 - k_{x_1^D} - \beta\eta > \mu_1 - k_{z_1^D}$, then, $\lambda_1 + k_{z_1^D} > k_{x_1^D} + \beta\eta$, at least one of $\lambda_1, k_{z_1^D}$ is positive.
$B > 0$, that is, $\mu_2 > k_{z_2^D}$, it implies $\mu_2 > 0$. Thus, $x_2^D + y_2^D + z_2^D = K^D$.
$B > R_L$, that is, $\mu_1 - k_{z_1^D} \ge \mu_1 - k_{y_1^D}$, then, $k_{y_1^D} > k_{z_1^D}$. It implies, $k_{y_1^D}$ is positive. Thus, $y_1^D = 0$.
$R_H > B$, that is, $\mu_2 - k_{y_2^D} + \eta > \mu_2 - k_{z_2^D}$, then, $k_{z_2^D} + \eta > k_{y_2^D}$, at least one of $\eta, k_{z_2^D}$ is positive.
$S_H > B$, that is, $\mu_2 + \lambda_2 - k_{x_2^D} + \beta\eta > \mu_2 - k_{z_2^D}$, then, $\lambda_2 + k_{z_2^D} + \beta\eta > k_{x_2^D}$, at least one of $\lambda_2, k_{z_2^D}, \eta$ is positive.
$S_H > R_H$, that is, $\mu_2 + \lambda_2 - k_{x_2^D} + \beta\eta > \mu_2 - k_{y_2^D} + \eta$, then $\lambda_2 + k_{y_2^D} > (1-\beta)\eta + k_{x_2^D}$, at least one of $\lambda_2, k_{y_2^D}$ is positive.

Based on the KKT conditions, we have the following discussions.

(1) When $K^D < \xi_1$, then, $x_1^D < \xi_1$. Thus, $\lambda_1 = 0$, and $k_{y_1^D} > 0, k_{z_1^D} > 0$. Then, $y_1^D = 0, z_1^D = 0, x_1^D = K^D$. Solution: $x_1^D = K^D, y_1^D = 0,\ z_1^D = 0$.

(2) When $\xi_1 < K^D$, then,
If $\lambda_1 = 0$, then, $k_{y_1^D} > 0,\ k_{z_1^D} > 0$. It implies, $y_1^D = 0, z_1^D = 0, x_1^D = K^D > \xi_1$. This is infeasible.
If $\lambda_1 > 0$, then, $x_1^D = \xi_1 < K^D, y_1^D + z_1^D = K^D - \xi_1 > 0$. Then, we know, $z_1^D > 0$. Thus, $z_1 = K^D - \xi_1$. Solution: $x_1^D = \xi_1, y_1^D = 0,\ z_1^D = K^D - \xi_1$.

(3) When $K^D < \xi_2$, and $x_1^{D*} < K^D < \xi_2$, then: since $x_2^D < \xi_2$, we know, $\lambda_2 = 0$. It implies, $k_{y_2^D} > 0$. Thus, $y_2^D = 0$. It implies $x_2^D + z_2^D = K^D$. There is at least one of $k_{z_2^D}, \eta$ is positive.
If $k_{z_2^D} > 0$, then, $z_2^D = 0$. Then, $x_2^D = K^D$. Solution: $x_2^D = K^D, y_2^D = 0, z_2^D = 0$.
If $\eta > 0$, then, $y_2^D = \beta\left(x_1^{D*} - x_2^D\right) = 0$. Then, $x_2^D = x_1^{D*}$. Solution: $x_2^D = x_1^{D*}, y_2^D = 0, z_2^D = K^D - x_1^{D*}$.

(4) When $K^D < \xi_2$, and $K^D < x_1^{D*} < \xi_2$, then: since $x_2^D < \xi_2$, we know, $\lambda_2 = 0$. It implies, $k_{y_2^D} > 0$. Thus, $y_2^D = 0$. We know, $x_1^{D*} > x_2^D$,

thus, $\eta = 0$. It implies that, $k_{z_2^D} > 0, \mu_2 > 0$. That is, $z_2^D = 0$. Solution: $x_2^D = K^D, y_2^D = 0, z_2 = 0$.

(5) When $K^D < \xi_2$, and $K^D < \xi_2 < x_1^{D*}$, then: since $x_2^D < \xi_2$, we know, $\lambda_2 = 0$. It implies, $k_{y_2^D} > 0$. Thus, $y_2^D = 0$. We know, $x_1^{D*} > x_2^D$, thus, $\eta = 0$. It implies that, $k_{z_2} > 0, \mu_2 > 0$. That is, $z_2^D = 0, x_2^D = K^D$. Solution: $x_2^D = K^D, y_2^D = 0, z_2 = 0$.

(6) When $\xi_2 < K^D$, and $x_1^{D*} < \xi_2 < K^D$, then: $x_2^D = \xi_2$.
(i) If $\lambda_2 > 0$, then, $x_2^D = \xi_2 < K^D$, and $y_2^D + z_2^D = K^D - \xi_2$.
–if $\eta > 0$, then, $y_2^D = \beta(x_1^{D*} - x_2^D) < 0$. Thus, it is infeasible.
–if $\eta = 0$, then, $k_{z_2^D} > 0$. Thus, $z_2^D = 0$, and $y_2^D = K^D - \xi_2$. However, since $\beta x_2^D + y_2^D - \beta x_1^D \leq 0$ does not hold, it is infeasible.
(ii) If $\lambda_2 = 0$, then, $k_{y_2^D} > 0$. Thus, $y_2^D = 0$, and $x_2^D + z_2^D = K^D$. There is at least one of $k_{z_2^D}, \eta$ is positive. –if $k_{z_2^D} > 0$, then, $z_2^D = 0$. Then, $x_2^D = K^D$. This breaks the condition of $x_2^D \leq \xi_2$, then, it is infeasible. –if $\eta > 0$, then, $y_2^D = \beta\left(x_1^{D*} - x_2^D\right) = 0$. Then, $x_2^D = x_1^{D*}$. This breaks the condition of $x_1^{D*} < \xi_2$, then, it is infeasible.

(7) When $\xi_2 < K^D$, and $\xi_2 < x_1^{D*} < K^D$, then: $x_2^D = \xi_2$.
(i) If $\lambda_2 > 0$, then, $x_2^D = \xi_2 < K^D$, and $y_2^D + z_2^D = K^D - \xi_2$.
–if $\eta > 0$, then, $y_2^D = \beta(x_1^{D*} - x_2^D) = \beta(x_1^{D*} - \xi_2)$. Thus, $z_2^D = K^D - \xi_2 - \beta(x_1^{D*} - \xi_2)$. Solution: $x_2^D = \xi_2, y_2^D = \beta(x_1^{D*} - \xi_2), z_2^D = K^D - \xi_2 - \beta(x_1^{D*} - \xi_2)$.
–if $\eta = 0$, then, $k_{z_2^D} > 0$. Thus, $z_2^D = 0$, and $y_2^D = K^D - \xi_2$. However, since $\beta x_2^D + y_2^D - \beta x_1^D \leq 0$ does not hold, it is infeasible.
(ii) If $\lambda_2 = 0$, then, $k_{y_2^D} > 0$. Thus, $y_2^D = 0$, and $x_2^D + z_2^D = K^D$. There is at least one of $k_{z_2^D}, \eta$ is positive. –if $k_{z_2^D} > 0$, then, $z_2^D = 0$. Then, $x_2^D = K^D$. This breaks the condition of $x_2^D \leq \xi_2$, then, it is infeasible. –if $\eta > 0$, then, $y_2^D = \beta\left(x_1^{D*} - x_2^D\right) = 0$. Then, $x_2^D = x_1^{D*}$. This breaks the condition of $x_1^{D*} < \xi_2$, then, it is infeasible.

(8) When $\xi_2 < K^D$, and $\xi_2 < K^D < x_1^{D*}$, then: $x_2^D = \xi_2$.
(i) If $\lambda_2 > 0$, then, $x_2^D = \xi_2 < K^D$, and $y_2^D + z_2^D = K^D - \xi_2$.
–if $\eta > 0$, then, $y_2^D = \beta(x_1^{D*} - x_2^D) = \beta(x_1^{D*} - \xi_2)$. Thus, $z_2^D = K^D - \xi_2 - \beta(x_1^{D*} - \xi_2)$. Solution: $x_2^D = \xi_2, y_2^D = \beta(x_1^{D*} - \xi_2), z_2^D = K^D - \xi_2 - \beta(x_1^{D*} - \xi_2)$.
–if $\eta = 0$, then, $k_{z_2^D} > 0$. Thus, $z_2^D = 0$, and $y_2^D = K^D - \xi_2$. However, since $\beta x_2^D + y_2^D - \beta x_1^D \leq 0$ does not hold, it is infeasible.
(ii) If $\lambda_2 = 0$, then, $k_{y_2^D} > 0$. Thus, $y_2^D = 0$, and $x_2^D + z_2^D = K^D$. There is at least one of $k_{z_2^D}, \eta$ is positive. –if $k_{z_2^D} > 0$, then, $z_2^D = 0$. Then, $x_2^D = K^D$. This breaks the condition of $x_2^D \leq \xi_2$, then, it is infeasible. –if $\eta > 0$, then, $y_2^D = \beta\left(x_1^{D*} - x_2^D\right) = 0$. Then, $x_2^D = x_1^{D*}$. This breaks the condition of $x_1^{D*} < \xi_2$, then, it is infeasible.

Therefore,

$$\begin{array}{rclrcl} x_1^{D*} & = & \min\{K^D, \xi_1\}, & x_2^{D*} & = & \min\{K^D, \xi_2, x_1^{D*}\}, \\ y_1^{D*} & = & 0, & y_2^{D*} & = & \min\{K^D - x_2^{D*}, \beta(x_1^{D*} - x_2^{D*})^+\}, \\ z_1^{D*} & = & K^D - x_1^{D*} - y_1^{D*}, & z_2^{D*} & = & K^D - x_2^{D*} - y_2^{D*}. \end{array}$$

S2 is the symmetric case to S1.

$$\begin{array}{rclrcl} x_1^{D*} &=& \min\{K^D, \xi_1, y_2^{D*}\}, & y_2^{D*} &=& \min\{K^D, \epsilon_2, \}, \\ y_1^{D*} &=& \min\{K^D - y_1^{D*}, \beta(y_2^{D*} - y_1^{D*})^+\}, & y_2^{D*} &=& 0, \\ z_1^{D*} &=& K^D - x_1^{D*} - y_1^{D*}, & z_2^{D*} &=& K^D - x_2^{D*} - y_2^{D*}. \end{array}$$

We can easily show that if $\min\{K^D, \xi_1\} > \min\{K^D, \xi_2\}$, S1 gives higher profit and otherwise, S2 gives higher profit. The optimal solutions are

$$\begin{array}{rclrcl} x_1^{D*} &=& \min\{K^D, \xi_1\}, & x_2^{D*} &=& \min\{K^D, \xi_2\}, \\ y_1^{D*} &=& \min\{(K^D - x_1^{D*}), \beta(x_2^{D*} - x_1^{D*})^+\}, & y_2^{D*} &=& \min\{(K^D - x_2^{D*}), \beta(x_1^{D*} - x_2^{D*})^+\}, \\ z_1^{D*} &=& K^D - x_1^{D*} - y_1^{D*}, & z_2^{D*} &=& K^D - x_2^{D*} - y_2^{D*}. \end{array}$$

Thus, the optimal gross profit from subscription program is $\pi^D = S_L x_i^{D^*} + R_L y_i^{D*} + S_H x_{-i}^{D*} + R_H y_{-i}^{D*} + B(z_1^{D*} + z_2^{D*})$. It can be rewritten as

$$\pi^D = S_L x_i^{D*} + R_L y_i^{D*} + S_H x_{-i}^{D*} + R_H \min\{(K^D - x_{-i}^{D*}), \beta\left(x_i^{D*} - x_{-i}^{D*}\right)^+\}$$
$$+B(2K^D - x_1^{D*} - y_1^{D*} - x_2^{D*} - y_2^{D*}).$$

- **Both Product 1 and Product 2 are fully served**

As shown in Lemma B.1, when the switching requests is fully satisfied for for both two products, it should satisfy: $\beta_1 x_1^D \leq \beta_2 x_2^D + y_2^D$, and $\beta_2 x_2^D \leq \beta_1 x_1^D + y_1^D$.

Similarly, from the KKT conditions, we can get the corresponding optimal solutions.

- **Comparisons of these three cases under dedicated technology**

Based on the above discussions of three Cases under dedicated technology, we compare maximal profits of these three Cases under Lemma B.1. Thus, we get, under $\min\left\{K^D, \xi_i\right\} > \min\left\{K^D, \xi_{-i}\right\}$,
(1) when $K^D - \xi_{-i} \leq \beta(\min\left\{K^D, \xi_i\right\} - \min\left\{K^D, \xi_{-i}\right\})$, the switching requests of only one product being fully served is optimal;
(2) when $K^D - \xi_{-i} \geq \beta(\min\left\{K^D, \xi_i\right\} - \min\left\{K^D, \xi_{-i}\right\})$, the switching requests of both products being fully served is optimal.

For dedicated technology, the optimal product allocation for subscription programs are as follows.

$$\begin{array}{rclrcl} x_1^{D*} &=& \min\{K^D, \xi_1\}, & x_2^{D*} &=& \min\{K^D, \xi_2\}, \\ y_1^{D*} &=& \min\{(K^D - x_1^{D*}), \beta(x_2^{D*} - x_1^{D*})^+\}, & y_2^{D*} &=& \min\{(K^D - x_2^{D*}), \beta(x_1^{D*} - x_2^{D*})^+\}. \end{array}$$

Under dedicated technology, the optimal profit at the second stage is

$$\begin{array}{rcl} \pi^D &=& S_L x_i^{D*} + R_L y_i^{D*} + S_H x_{-i}^{D*} + R_H \min\{y_{-i}^{D*}, \beta\left(x_i^{D*} - x_{-i}^{D*}\right)^+\} + B(z_1^{D*} + z_2^{D*}) \\ &=& S_L x_i^{D*} + R_L y_i^{D*} + S_H x_{-i}^{D*} + R_H \min\{y_{-i}^{D*}, \beta\left(x_i^{D*} - x_{-i}^{D*}\right)^+\} + B(2K^D - (x_1^{D*} + y_1^{D*}) - (x_2^{D*} + y_2^{D*})). \end{array}$$

To summarize, given demand realizations $\boldsymbol{\xi}' = (\xi_1, \xi_2)$, the optimal allocation and reservation volumes of product i, for $i = 1, 2$, are $x_i^{D*} = \min\left(\xi_i, K_i^D\right)$ and $y_i^{D*} = \min\left(K_i^D - x_i^{D*}, \left(\beta x_{-i}^{D*} - \beta x_i^{D*}\right)^+\right)$, respectively. The optimal stage 2 profit is given by

$$\begin{aligned}\pi^D\left(\mathbf{K}^{\boldsymbol{D}}, \boldsymbol{\xi}\right) &= \sum_{i=1}^{2}\Big[(S-w)K_i^D + Px_i^{D*} - R\left(\beta x_{-i}^{D*} - \beta x_i^{D*}\right)^+ \\ &+R\left(\min\left(K_i^D - x_i^{D*}, \left(\beta x_{-i}^{D*} - \beta x_i^{D*}\right)^+\right)\right) \\ &+(O-S)\left(K_i^D - x_i^{D*} - \min\left(K_i^D - x_i^{D*}, \left(\beta x_{-i}^{D*} - \beta x_i^{D*}\right)^+\right)\right)\Big].\end{aligned}$$

□

Proof of Proposition 3.2.

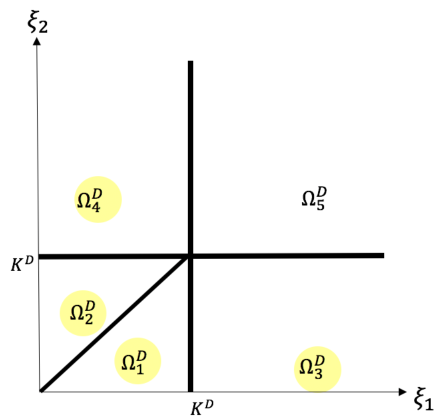


Figure B.1: (Color online) Demand Realization Space of $\boldsymbol{\xi}$. Note: The circle in yellow colour represents the regions that there has positive reservation volume of product 1 or product 2, and the future switching requests under each subscription program are fully satisfied.

The expected profit can be represented as below:

$$\begin{aligned}&\Pi^D(K^D) = -2c^D K^D \\ &+\mathbb{E}\left[P(\tilde{\xi}_1+\tilde{\xi}_2) + 2(S-w)K^D + (O-S)\left((K^D-\xi_1) + (K^D - \xi_2 - \beta(\xi_1-\xi_2))\right) | \tilde{\boldsymbol{\xi}} \in \Omega_1^D\right] \Pr\left(\tilde{\boldsymbol{\xi}} \in \Omega_1^D\right) \\ &+\mathbb{E}\left[P(\tilde{\xi}_1+\tilde{\xi}_2) + 2(S-w)K^D + (O-S)\left((K^D-\xi_2) + (K^D - \xi_1 - \beta(\xi_2-\xi_1))\right) | \tilde{\boldsymbol{\xi}} \in \Omega_2^D\right] \Pr\left(\tilde{\boldsymbol{\xi}} \in \Omega_2^D\right) \\ &+\mathbb{E}\left[P(K^D+\xi_2) + 2(S-w)K^D + (O-S)\left(K^D - \xi_2 - \beta\left(K^D-\xi_2\right)\right) | \tilde{\boldsymbol{\xi}} \in \Omega_3^D\right] \Pr\left(\tilde{\boldsymbol{\xi}} \in \Omega_3^D\right) \\ &+\mathbb{E}\left[P(\tilde{\xi}_1+K^D) + 2(S-w)K^D + (O-S)\left(K^D - \xi_1 - \beta\left(K^D-\xi_1\right)\right) | \tilde{\boldsymbol{\xi}} \in \Omega_4^D\right] \Pr\left(\tilde{\boldsymbol{\xi}} \in \Omega_4^D\right) \\ &+2(P+S-w)K^D \Pr\left(\tilde{\boldsymbol{\xi}} \in \Omega_5^D\right).\end{aligned} \tag{B.1}$$

Then, by taking the first-order derivative of $\Pi^D(K^D)$ with respect to K^D, we have,

$$\begin{aligned}\frac{\partial \Pi^D(K^D)}{\partial K^D} &= -2c^D + 2(S-w) + (O-S)\iint_{\Omega_{12}^D:\xi_1\le K^D,\xi_2\le K^D} f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2 \\ &+(P+(1-\beta)(O-S))\iint_{\Omega_3^D:\xi_2\le K^D<\xi_1} f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2 \\ &+(P+(1-\beta)(O-S))\iint_{\Omega_4^D:\xi_1\le K^D<\xi_2} f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2 \\ &+2P\iint_{\Omega_5^D:\xi_1>K^D,\xi_2>K^D} f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2.\end{aligned}$$

Since the optimal capacity level should satisfy $\frac{\partial \Pi^D(K^D)}{\partial K^D}|_{K^D=K^{D*}}=0$, then, K^{D*} is obtained by solving

$$\begin{aligned}&((P+S-w)-(O-w))\sum_{i=1}^{2}\Pr\left(\tilde{\xi}_i\le K^{D*}\right)\\ &+\beta(O-S)\Pr\left(\min\left(\tilde{\xi}_1,\tilde{\xi}_2\right)\le K^{D*}<\max\left(\tilde{\xi}_1,\tilde{\xi}_2\right)\right)=2\left((P+S-w)-c^D\right).\end{aligned} \tag{B.2}$$

When $c^D \ge P+S-w$, optimal decision is $K^{D*}=0$.
When $O-w<c^D<P+S-w$, optimal decision satisfies $K^{D*}>0$. □

Proof of Proposition 3.3.
In order to analyze the impact of demand correlation ρ on profitability with dedicated technology, by referring to the methods used in in Boyabatlı and Toktay (2011) and Boyabatlı (2015), we conduct proofs as follows.

Since Π^{D*} is obtained at $K^D=K^{D*}$, it is sufficient to show that the property of $\Pi^D(K^D)$ with respect to ρ. As defined $\Pi^D(K^D)=\mathbb{E}\left[\pi^D\left(\mathbf{K^D},\tilde{\boldsymbol{\xi}}\right)\right]-2c^DK^D$, it is sufficient to show that $\pi^D(\boldsymbol{\xi})$ is supermodular in $\boldsymbol{\xi}$. To prove supermodularity, it is sufficient to show that $\frac{\partial(\pi^D(\boldsymbol{\xi}))}{\partial\xi_1}$ increases with ξ_2. From $\pi^D(\boldsymbol{\xi})$ in (B.1), we obtain,

$$\frac{\partial \pi^D(\boldsymbol{\xi})}{\partial \xi_1}=\begin{cases} P-(1+\beta)(O-S), & \text{if } \boldsymbol{\xi}\in\Omega_1^D,\\ P-(1-\beta)(O-S), & \text{if } \boldsymbol{\xi}\in\Omega_{24}^D,\\ 0, & \text{otherwise.}\end{cases} \tag{B.3}$$

Then, for a given ξ_1 and $\beta>0$, since we know $P-(1-\beta)(O-S)>P-(1+\beta)(O-S)\ge 0$, then, $\frac{\partial(\pi^D(\boldsymbol{\xi}))}{\partial\xi_1}$ increases as ξ_2 increases. This concludes the proof. □

Proof of Proposition 3.4.
Since Π^{D*} is obtained at $K^D=K^{D*}$, it is sufficient to show that the property of $\Pi^D(K^D)$ with respect to β. Recall the expected profit is represented

as below:

$$
\begin{aligned}
&\Pi^D(K^D) = -2c^D K^D \\
&+\Pr\left\{\tilde{\boldsymbol{\xi}} \in \Omega_1^D\right\}\mathbb{E}\left\{P(\tilde{\xi}_1+\tilde{\xi}_2)+2(S-w)K^D+(O-S)\left((K^D-\xi_1)+(K^D-\xi_2-\beta(\xi_1-\xi_2))\right)|\tilde{\boldsymbol{\xi}} \in \Omega_1^D\right\} \\
&+\Pr\left\{\tilde{\boldsymbol{\xi}} \in \Omega_2^D\right\}\mathbb{E}\left\{P(\tilde{\xi}_1+\tilde{\xi}_2)+2(S-w)K^D+(O-S)\left((K^D-\xi_2)+(K^D-\xi_1-\beta(\xi_2-\xi_1))\right)|\tilde{\boldsymbol{\xi}} \in \Omega_2^D\right\} \\
&+\Pr\left\{\tilde{\boldsymbol{\xi}} \in \Omega_3^D\right\}\mathbb{E}\left\{P(K^D+\xi_2)+2(S-w)K^D+(O-S)\left(K^D-\xi_2-\beta\left(K^D-\xi_2\right)\right)|\tilde{\boldsymbol{\xi}} \in \Omega_3^D\right\} \\
&+\Pr\left\{\tilde{\boldsymbol{\xi}} \in \Omega_4^D\right\}\mathbb{E}\left\{P(\tilde{\xi}_1+K^D)+2(S-w)K^D+(O-S)\left(K^D-\xi_1-\beta\left(K^D-\xi_1\right)\right)|\tilde{\boldsymbol{\xi}} \in \Omega_4^D\right\} \\
&+\Pr\left\{\tilde{\boldsymbol{\xi}} \in \Omega_5^D\right\}2(P+S-w)K^D.
\end{aligned}
$$

Then, by taking the first-order derivative of $\Pi^D(K^D)$ with respect to β, we have,

$$
\begin{aligned}
\frac{\partial \Pi^D(K^D)}{\partial \beta} \quad = \quad & -(O-S)\iint_{\Omega_1^D:\xi_2\le\xi_1\le K^D}(\tilde{\xi}_1-\tilde{\xi}_2)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2 \\
& +(O-S)\iint_{\Omega_2^D:\xi_1<\xi_2\le K^D}(\tilde{\xi}_2-\tilde{\xi}_1)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2 \\
& -(O-S)\iint_{\Omega_3^D:\xi_2\le K^D<\xi_1}(K^D-\tilde{\xi}_2)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2 \\
& -(O-S)\iint_{\Omega_4^D:\xi_1\le K^D<\xi_2}(K^D-\tilde{\xi}_1)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2 \\
& \le 0.
\end{aligned}
$$

Thus, we know, $\frac{\partial \Pi^D(K^D)}{\partial \beta} \le 0$. $\square$

Proof of Proposition 3.5.

Similar with the discussion in the proof of Proposition 3.1, we first build our stage-2 product allocation problem.

Part 1: Model Formulation With Flexible Technology

At Stage 1, the objective function of the optimization problem under flexible technology is

$$\Pi^F(K^F) = \mathbb{E}\left[\pi^F(K^F,\tilde{\boldsymbol{\xi}})\right] - c^F K^F.$$

At Stage 2, based on whether switching demands of two products are fully served, we have following discussions.

(1) If product i is fully served and product $-i$ is under served, then, the product allocation problem is as follows.

$$
\begin{aligned}
& \pi^F(x_1^F, y_1^F, x_2^F, y_2^F; K^F, \xi_1, \xi_2) \\
= \quad & \max \quad S_L x_i^F + R_L y_i^F + S_H x_{-i}^F + R_H y_{-i}^F + B(z_1^T + z_2^T) \\
& s.t. \quad \sum_{j=1}^{2}\left(x_j^F + y_j^F\right) \le K^F \\
& \qquad\;\; x_j^F \le \xi_j,\ \forall j = 1,2 \\
& \qquad\;\; x_j^F \ge 0,\ \forall j = 1,2 \\
& \qquad\;\; y_j^F \ge 0,\ \forall j = 1,2 \\
& \qquad\;\; \beta x_i^F \ge \beta x_{-i}^F + y_{-i}^F
\end{aligned}
$$

(2) If both products are fully served and $x_i^F \ge x_{-i}^F$, then, the product

allocation problem is as follows.

$$
\begin{aligned}
& \pi^F(x_1^F, y_1^F, x_2^F, y_2^F; K^F, \xi_1, \xi_2) \\
= \; & \max \quad S_L x_i^F + R_L y_i^F + S_H x_{-i}^F + R_H \beta\left(x_i^F - x_{-i}^F\right) + R_L\left(y_{-i}^F - \beta\left(x_i^F - x_{-i}^F\right)\right) + B(z_1^F + z_2^F) \\
& \quad s.t. \quad \sum_{j=1}^{2}\left(x_j^F + y_j^F\right) \le K^F \\
& \qquad\qquad x_j^F \le \xi_j, \forall j = 1,2 \\
& \qquad\qquad x_j^F \ge 0, \forall j = 1,2 \\
& \qquad\qquad y_j^F \ge 0, \forall j = 1,2 \\
& \qquad\qquad \beta x_j^F \le \beta x_{-j}^F + y_{-j}^F, \forall j = 1,2 \\
& \qquad\qquad x_i^F \ge x_{-i}^F
\end{aligned}
$$

We first show that the profit function is concave in the first-stage capacity investment decisions through the following Proposition B.2.

Proposition B.2. *The objective function $\Pi^F(K^F)$ is concave in K^F.*

Part 2: in stage-2, product allocation and reservation with flexible technology.

- **Both Product 1 and Product 2 are fully served**

From $0 < R_L < B < R_H < S_L < S_H$, KKT conditions implies the following optimal solutions. Given $\xi_1 > \xi_2$, we have:

If $K^F \ge \sum_{i=1}^{2}\xi_i$, then

$$
\begin{aligned}
x_1^{F*} &= \min\{\xi_1, \xi_2\}, \qquad x_2^{F*} = \min\{\xi_1, \xi_2\}; \\
y_1^{F*} + y_2^{F*} &= \min\left\{\beta\left(x_1^{F*} - x_2^{F*}\right), (K^F - \textstyle\sum_{i=1}^{2} x_i^{F*})^+\right\}; \\
z_1^{F*} + z_2^{F*} &= K^F - (x_1^{F*} + x_2^{F*}) - (y_1^{F*} + y_2^{F*}).
\end{aligned}
$$

If $2\xi_2 \le K^F \le \sum_{i=1}^{2}\xi_i$, then, $x_1^{F*} = K^F - \xi_2$, $x_2^{F*} = \xi_2$; $y_1^{F*} = y_2^{F*} = 0$; $z_1^{F*} = z_2^{F*} = 0$.

If $K^F \le 2\xi_2$, then, $x_1^{F*} = x_2^{F*} = \frac{K^F}{2}$; $y_1^{F*} = y_2^{F*} = 0$; $z_1^{F*} = z_2^{F*} = 0$.

- **Comparisons of three cases under flexible technology**

Based on the discussion of three Cases under flexible technology, we compare maximal profits of these three Cases under Lemma B.1. Thus, we get,

if $K^F \le 2\min\{\xi_1, \xi_2\}$, the switching requests of both products being fully satisfied is optimal;

if $2\min\{\xi_1, \xi_2\} \le K^F \le \xi_1 + \xi_2 + \beta(\max\{\xi_1, \xi_2\} - \min\{\xi_1, \xi_2\})$, the switching requests of only one product being fully satisfied is optimal;

if $K^F \ge \xi_1 + \xi_2 + \beta(\max\{\xi_1, \xi_2\} - \min\{\xi_1, \xi_2\})$, the switching requests of both products being fully satisfied is optimal.

For flexible technology, the optimal product allocation for subscription programs are as follows.

1. if $K^F \geq \xi_1 + \xi_2$, then,

$$\begin{array}{lcl} x_1^{F*} & = & \xi_1, \qquad x_2^{F*} = \xi_2; \\ y_1^{F*} = 0, y_2^{F*} & = & \min\{K^F - (\xi_1+\xi_2), \beta(\max\{\xi_1,\xi_2\} - \min\{\xi_1,\xi_2\})\}; \\ z_1^{F*} + z_2^{F*} & = & K^F - (x_1^{F*} + x_2^{F*}) - (y_1^{F*} + y_2^{F*}); \end{array}$$

2. if $K^F < \xi_1 + \xi_2$, then,

$$\begin{array}{lcl} x_1^{F*} & = & \min\{\xi_1, \max\{\frac{K^F}{2}, K^F - \xi_2\}\}, \quad x_2^{F*} = \min\{\xi_2, \max\{\frac{K^F}{2}, K^F - \xi_1\}\}; \\ y_1^{F*} = y_2^{F*} & = & 0; \\ z_1^{F*} = z_2^{F*} & = & 0. \end{array}$$

The optimal gross profit in the second stage is

$$\begin{aligned} \pi^F &= S_L x_i^{F*} + R_L y_i^{F*} + S_H x_{-i}^{F*} + R_H \min\{y_{-i}^{F*}, \beta\left(x_i^{F*} - x_{-i}^{F*}\right)^+\} + B(z_1^{F*} + z_2^{F*}) \\ &= S_L x_i^{F*} + R_L y_i^{F*} + S_H x_{-i}^{F*} + R_H \min\{y_{-i}^{F*}, \beta\left(x_i^{F*} - x_{-i}^{F*}\right)^+\} + B(K^F - x_1^{F*} - y_1^{F*} - x_2^{F*} - y_2^{F*}). \end{aligned}$$

To summarize, given demand realizations $\boldsymbol{\xi}' = (\xi_1, \xi_2)$, with flexible technology, the optimal allocation and reservation volumes of product i, for $i = 1, 2$, are as follows:

1. if $K^F \geq \xi_1 + \xi_2$, then,

$$x_i^{F*} = \xi_i \text{ and } y_i^{F*} = \min\left(K^F - x_1^{F*} - x_2^{F*}, \left(\beta x_{-i}^{F*} - \beta x_i^{F*}\right)^+\right);$$

2. if $K^F < \xi_1 + \xi_2$, then,

$$x_i^{F*} = \min\left(\xi_i, \max\left(\frac{K^F}{2}, K^F - \xi_{-i}\right)\right) \text{ and } y_i^{F*} = 0.$$

Under flexible technology, the optimal profit at the second stage is

$$\begin{aligned} &\pi^F = S_L x_i^{F*} + R_L y_i^{F*} + S_H x_{-i}^{F*} + R_H \min\{y_{-i}^{F*}, \beta\left(x_i^{F*} - x_{-i}^{F*}\right)^+\} \\ &+B(K^F - x_1^{F*} - y_1^{F*} - x_2^{F*} - y_2^{F*}). \end{aligned}$$

Thus, the optimal stage 2 profit is expressed as

$$\begin{aligned} \pi^F(K^F, \boldsymbol{\xi}) &= (S-w)K^F + P\min\left(K^F, \xi_1+\xi_2\right) \\ &-R\left(\beta\min\left(K^F - \min(\xi_1,\xi_2), \max(\xi_1,\xi_2)\right) - \beta\min(\xi_1,\xi_2)\right)^+ \\ &+R\min\left(\left(K^F - (\xi_1+\xi_2)\right)^+, \beta\max(\xi_1,\xi_2) - \beta\min(\xi_1,\xi_2)\right) \\ &+(O-S)\left(K^F - (\xi_1+\xi_2) - \min\left(\left(K^F-(\xi_1+\xi_2)\right)^+, \left(\beta\max(\xi_1,\xi_2) - \beta\min(\xi_1,\xi_2)\right)\right)\right)^+. \end{aligned}$$

□

Proof of Proposition 3.6.

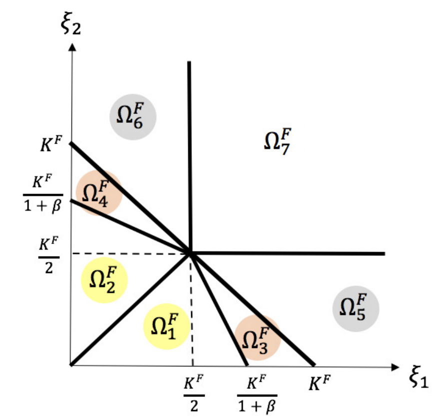


Figure B.2: (Color online) Demand Realization Space of $\boldsymbol{\xi}$. Note: The circle in yellow colour represents the regions that there has positive reservation volume of product 1 or product 2, and all switching requests are fully satisfied. The circle in light pink colour represents the regions that there has positive reservation volume of product 2 (or product 1), and switching requests for product 2 (or product 1) are not fully satisfied. The circle in grey colour represents the regions that there has no reservation, and switching requests for product 1 or product 2 are not fully satisfied.

Recall the expected profit is represented as below:

$$
\begin{aligned}
&\Pi^F(K^F) = -c^F K^F \\
&+\mathbb{E}\left[P(\tilde{\xi}_1+\tilde{\xi}_2)+(S-w)K^F+(O-S)((K^F-\tilde{\xi}_1-\tilde{\xi}_2)-\beta(\tilde{\xi}_1-\tilde{\xi}_2))|\tilde{\boldsymbol{\xi}}\in\Omega_1^F\right]\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega_1^F\right) \\
&+\mathbb{E}\left[P(\tilde{\xi}_1+\tilde{\xi}_2)+(S-w)K^F+(O-S)((K^F-\tilde{\xi}_1-\tilde{\xi}_2)-\beta(\tilde{\xi}_2-\tilde{\xi}_1))|\tilde{\boldsymbol{\xi}}\in\Omega_2^F\right]\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega_2^F\right) \\
&+\mathbb{E}\left[P(\tilde{\xi}_1+\tilde{\xi}_2)+(S-w)K^F+R((K^F-\tilde{\xi}_1-\tilde{\xi}_2)-\beta(\tilde{\xi}_1-\tilde{\xi}_2))|\tilde{\boldsymbol{\xi}}\in\Omega_3^F\right]\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega_3^F\right) \\
&+\mathbb{E}\left[P(\tilde{\xi}_1+\tilde{\xi}_2)+(S-w)K^F+R((K^F-\tilde{\xi}_1-\tilde{\xi}_2)-\beta(\tilde{\xi}_2-\tilde{\xi}_1))|\tilde{\boldsymbol{\xi}}\in\Omega_4^F\right]\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega_4^F\right) \\
&+\mathbb{E}\left[PK^F+(S-w)K^F-\beta R(K^F-2\tilde{\xi}_2)|\tilde{\boldsymbol{\xi}}\in\Omega_5^F\right]\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega_5^F\right) \\
&+\mathbb{E}\left[PK^F+(S-w)K^F-\beta R(K^F-2\tilde{\xi}_1)|\tilde{\boldsymbol{\xi}}\in\Omega_6^F\right]\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega_6^F\right) \\
&+(PK^F+(S-w)K^F)\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega_7^F\right).
\end{aligned}
$$

Then, by taking the first-order derivative of $\Pi^F(K^F)$ with respect to K^F, we can obtain the optimally conditions for K^{F^*}. □

Proof of Proposition 3.7.

Under the special case of $\beta = 1$: with $P = 2R$, the expected profit is as follows.

$$
\begin{aligned}
\Pi^F(K^F) \quad &= -c^F K^F + (P+S-w)K^F - (P-R)\iint_{(\Omega^F_{1234}:\tilde{\xi}_1+\tilde{\xi}_2<K^F}\left(K^F-\tilde{\xi}_1-\tilde{\xi}_2\right)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2\\
&-R\int_{\Omega^F_{1246}:2\tilde{\xi}_1<K^F}\left(K^F-2\tilde{\xi}_1\right)f(\tilde{\xi}_1)d\tilde{\xi}_1 - R\int_{\Omega^F_{1235}:2\tilde{\xi}_2<K^F}\left(K^F-2\tilde{\xi}_2\right)f(\tilde{\xi}_2)d\tilde{\xi}_2\\
&+R\int_{\Omega^F_{1246}:2\tilde{\xi}_1<K^F}\left(K^F-2\tilde{\xi}_1\right)f(\tilde{\xi}_1)d\tilde{\xi}_1 + R\int_{\Omega^F_{1235}:2\tilde{\xi}_2<K^F}\left(K^F-2\tilde{\xi}_2\right)f(\tilde{\xi}_2)d\tilde{\xi}_2\\
&-R\iint_{\Omega^F_{13}:\tilde{\xi}_1+\tilde{\xi}_2<K^F,\tilde{\xi}_2<\tilde{\xi}_1}\left(\tilde{\xi}_1-\tilde{\xi}_2\right)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2\\
&-R\iint_{\Omega^F_{24}:\tilde{\xi}_1+\tilde{\xi}_2<K^F,\tilde{\xi}_1<\tilde{\xi}_2}\left(\tilde{\xi}_2-\tilde{\xi}_1\right)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2\\
&+\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega^F_1\right)\mathbb{E}\left[(O-S)(K^F-2\xi_1)|\tilde{\boldsymbol{\xi}}\in\Omega^F_1\right]\\
&+\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega^F_2\right)\mathbb{E}\left[(O-S)(K^F-2\xi_2)|\tilde{\boldsymbol{\xi}}\in\Omega^F_2\right].
\end{aligned}
$$

Then, by rearrangement with $P = 2R$, we obtain

$$
\begin{aligned}
\Pi^F(K^F) \quad &= -c^F K^F + (P+S-w)K^F\\
&-R\int_{\Omega^F_{1246}:2\tilde{\xi}_1<K^F}\left(K^F-2\tilde{\xi}_1\right)f(\tilde{\xi}_1)d\tilde{\xi}_1 - R\int_{\Omega^F_{1235}:2\tilde{\xi}_2<K^F}\left(K^F-2\tilde{\xi}_2\right)f(\tilde{\xi}_2)d\tilde{\xi}_2\\
&+\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega^F_1\right)\mathbb{E}\left[(O-S)(K^F-2\xi_1)|\tilde{\boldsymbol{\xi}}\in\Omega^F_1\right]\\
&+\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega^F_2\right)\mathbb{E}\left[(O-S)(K^F-2\xi_2)|\tilde{\boldsymbol{\xi}}\in\Omega^F_2\right].
\end{aligned}
$$

Thus, with $\beta = 1$ and $P = 2R$, $\frac{\partial \Pi^F(K^F)}{\partial \rho} > 0$. □

Proof of Proposition 3.8.

Since Π^{F^*} is obtained at $K^F = K^{F^*}$, it is sufficient to show that the property of $\Pi^F(K^F)$ with respect to β. Then, by taking the first-order derivative of $\Pi^F(K^F)$ with respect to β, we have,

$$
\begin{aligned}
\frac{\partial \Pi^F(K^F)}{\partial \beta} = \quad & -(O-S)\iint_{\Omega^F_1}(\tilde{\xi}_1-\tilde{\xi}_2)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2 - (O-S)\iint_{\Omega^F_2}(\tilde{\xi}_2-\tilde{\xi}_1)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2\\
& -R\iint_{\Omega^F_3}(\tilde{\xi}_1-\tilde{\xi}_2)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2 - R\iint_{\Omega^F_4}(\tilde{\xi}_2-\tilde{\xi}_1)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2\\
& -R\iint_{\Omega^F_5}(K^F-2\tilde{\xi}_2)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2 - R\iint_{\Omega^F_6}(K^F-2\tilde{\xi}_1)f(\tilde{\xi}_1,\tilde{\xi}_2)d\tilde{\xi}_1 d\tilde{\xi}_2\\
\leq \quad & 0.
\end{aligned}
$$

Thus, we know, $\frac{\partial \Pi^F(K^F)}{\partial \beta} \leq 0$. □

Proof of Proposition 3.9.

In order to show the results in Proposition 3.9, we have below lemma about the flexibility premium $\Delta(\rho, \beta)$.

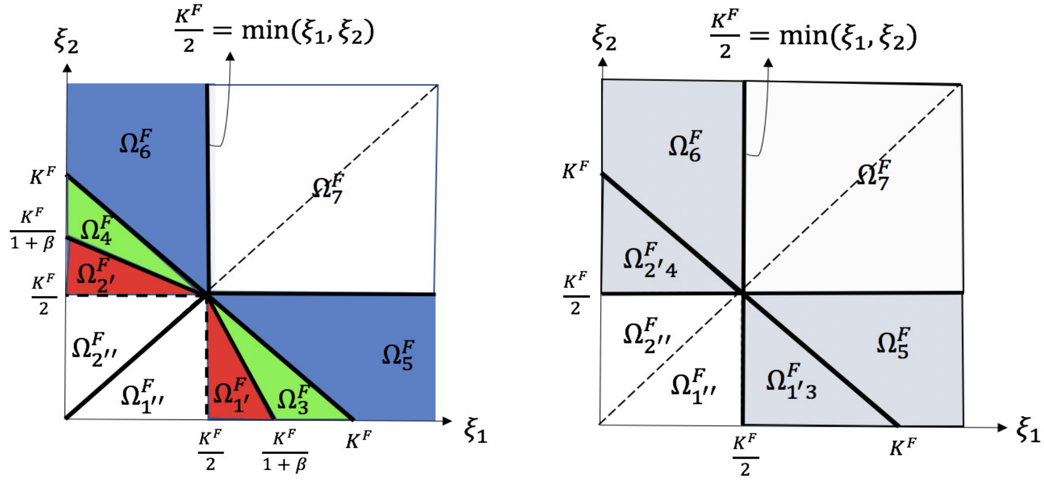
Lemma B.2. *Assume $\beta > 0$. We have $\Delta(1, \beta) = 0$. When $\rho < 1$, we have*
(i) $\Delta(\rho, 1) = 0$ if $P = 2R$ and $\Delta(\rho, 1) > 0$ otherwise;
(ii) $\Delta(\rho, \beta) > 0$ for $\beta < 1$.

Proof of Lemma B.2:

In order to illustrate the regions with reallocation quantity, we define below two sub-regions:

$$
\begin{aligned}
\Omega^F_{1'} \quad &\doteq \quad \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, \xi_1+\xi_2+\beta(\xi_1-\xi_2) > K^F, \xi_1 > \xi_2, \xi_1 > \tfrac{K^F}{2}\right\},\\
\Omega^F_{2'} \quad &\doteq \quad \left\{\boldsymbol{\xi} : \boldsymbol{\xi} \geq \mathbf{0}, \xi_1+\xi_2+\beta(\xi_2-\xi_1) > K^F, \xi_2 > \xi_1, \xi_2 > \tfrac{K^F}{2}\right\}.
\end{aligned}
$$

See from Figure B.3 (1) and (2), for $\boldsymbol{\xi} \in \Omega^F_{1'2'3456}$, there are positive reallocation volumes.



(a) (b)

Figure B.3: (Color online) Demand Realization Space of $\boldsymbol{\xi}$. Note: In panel(a), the red regions of $\boldsymbol{\xi} \in \Omega^F_{1'2'}$ represent that there are reallocations, positive reservation volume, no unsatisfied switching requests; the green regions of $\boldsymbol{\xi} \in \Omega^F_{34}$ represent that there are reallocations, positive reservation volume, positive unsatisfied switching requests; the blue regions of $\boldsymbol{\xi} \in \Omega^F_{56}$ represent that there are reallocations, no reservation volume, positive unsatisfied switching requests. In panel(b), the grey regions of $\boldsymbol{\xi} \in \Omega^F_{1'2'3456}$ represent that there are reallocations, no reservation volume, no unsatisfied switching requests

The flexibility premium $\Delta(\rho, \beta)$ can be represented as follows:

$$
\begin{aligned}
&\Delta(\rho,\beta)\\
&= \Pr\left(\tilde{\boldsymbol{\xi}} \in \Omega^F_{1'}\right)\mathbb{E}\left[P(\tilde{\xi}_1 - \tfrac{K^F}{2}) - (O-S)\,(\tilde{\xi}_1 - \tfrac{K^F}{2}) - (O-S)\,\beta(\tilde{\xi}_1 - \tfrac{K^F}{2})|\tilde{\boldsymbol{\xi}} \in \Omega^F_{1'}\right]\\
&+ \Pr\left(\tilde{\boldsymbol{\xi}} \in \Omega^F_{2'}\right)\mathbb{E}\left[P(\tilde{\xi}_2 - \tfrac{K^F}{2}) - (O-S)\,(\tilde{\xi}_2 - \tfrac{K^F}{2}) - (O-S)\,\beta(\tilde{\xi}_2 - \tfrac{K^F}{2})|\tilde{\boldsymbol{\xi}} \in \Omega^F_{2'}\right]\\
&+ \Pr\left(\tilde{\boldsymbol{\xi}} \in \Omega^F_{3}\right)\mathbb{E}\left[P(\tilde{\xi}_1 - \tfrac{K^F}{2}) - (O-S)\,(\tilde{\xi}_1 - \tfrac{K^F}{2}) - \beta R(\tilde{\xi}_1 - \tilde{\xi}_2) + R(K^F - \tilde{\xi}_1 - \tilde{\xi}_2) - (O-S)\,((1-\beta)\,(\tfrac{K^F}{2} - \tilde{\xi}_2) - (\tilde{\xi}_1 - \tfrac{K^F}{2}))|\tilde{\boldsymbol{\xi}} \in \Omega^F_{3}\right]\\
&+ \Pr\left(\tilde{\boldsymbol{\xi}} \in \Omega^F_{4}\right)\mathbb{E}\left[P(\tilde{\xi}_2 - \tfrac{K^F}{2}) - (O-S)\,(\tilde{\xi}_2 - \tfrac{K^F}{2}) - \beta R(\tilde{\xi}_2 - \tilde{\xi}_1) + R(K^F - \tilde{\xi}_1 - \tilde{\xi}_2) - (O-S)\,((1-\beta)\,(\tfrac{K^F}{2} - \tilde{\xi}_1) - (\tilde{\xi}_2 - \tfrac{K^F}{2}))|\tilde{\boldsymbol{\xi}} \in \Omega^F_{4}\right]\\
&+ \Pr\left(\tilde{\boldsymbol{\xi}} \in \Omega^F_{5}\right)\mathbb{E}\left[P(\tfrac{K^F}{2} - \tilde{\xi}_2) - (O-S)\,(\tfrac{K^F}{2} - \tilde{\xi}_2) - 2\beta R(\tfrac{K^F}{2} - \tilde{\xi}_2) + (O-S)\,\beta(\tfrac{K^F}{2} - \tilde{\xi}_2)|\tilde{\boldsymbol{\xi}} \in \Omega^F_{5}\right]\\
&+ \Pr\left(\tilde{\boldsymbol{\xi}} \in \Omega^F_{6}\right)\mathbb{E}\left[P(\tfrac{K^F}{2} - \tilde{\xi}_1) - (O-S)\,(\tfrac{K^F}{2} - \tilde{\xi}_1) - 2\beta R(\tfrac{K^F}{2} - \tilde{\xi}_1) + (O-S)\,\beta(\tfrac{K^F}{2} - \tilde{\xi}_1)|\tilde{\boldsymbol{\xi}} \in \Omega^F_{6}\right].
\end{aligned}
$$

By rearranging the expression of $\Delta(\rho, \beta)$ and further using our assumptions that $P \geq 2R$ and $R > O - S > 0$, we have

$$
\begin{aligned}
&\Delta(\rho,\beta)\\
&=\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega^F_{1'}\right)\mathbb{E}\left[(P-(1+\beta)(O-S))\left(\tilde{\xi}_1-\tfrac{K^F}{2}\right)|\tilde{\boldsymbol{\xi}}\in\Omega^F_{1'}\right]\\
&+\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega^F_{2'}\right)\mathbb{E}\left[(P-(1+\beta)(O-S))\left(\tilde{\xi}_2-\tfrac{K^F}{2}\right)|\tilde{\boldsymbol{\xi}}\in\Omega^F_{2'}\right]\\
&+\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega^F_{3}\right)\mathbb{E}\left[(P-(1+\beta)R)\left(\tilde{\xi}_1-\tfrac{K^F}{2}\right)+(1-\beta)(R-(O-S))\left(\tfrac{K^F}{2}-\tilde{\xi}_2\right)|\tilde{\boldsymbol{\xi}}\in\Omega^F_{3}\right]\\
&+\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega^F_{4}\right)\mathbb{E}\left[(P-(1+\beta)R)\left(\tilde{\xi}_2-\tfrac{K^F}{2}\right)+(1-\beta)(R-(O-S))\left(\tfrac{K^F}{2}-\tilde{\xi}_1\right)|\tilde{\boldsymbol{\xi}}\in\Omega^F_{4}\right]\\
&+\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega^F_{5}\right)\mathbb{E}\left[(P-(1-\beta)(O-S)-2\beta R)(\tfrac{K^F}{2}-\tilde{\xi}_2)|\tilde{\boldsymbol{\xi}}\in\Omega^F_{5}\right]\\
&+\Pr\left(\tilde{\boldsymbol{\xi}}\in\Omega^F_{6}\right)\mathbb{E}\left[(P-(1-\beta)(O-S)-2\beta R)(\tfrac{K^F}{2}-\tilde{\xi}_1)|\tilde{\boldsymbol{\xi}}\in\Omega^F_{6}\right]\\
&>0.
\end{aligned}
$$

□

Proof of Proposition 3.10.

- The impact of β: omitted.
- The impact of ρ:

Define $\Delta(\rho,\beta)=\mathbb{E}\left[\Delta\pi^F\left(K^F,\tilde{\boldsymbol{\xi}}\right)\right]$, it is sufficient to show that $\Delta\pi^F(\boldsymbol{\xi})$ is supermodular in $\boldsymbol{\xi}$. To prove supermodularity, it is sufficient to show that $\frac{\partial(\Delta\pi^F(\boldsymbol{\xi}))}{\partial\xi_1}$ decreases in ξ_2. Applying the supermodularity method to $\Delta(\rho,\beta)$, we have,

$$
\frac{\partial\Delta\pi^F(\boldsymbol{\xi})}{\partial\xi_1}=\begin{cases}P-(1+\beta)(O-S), & \text{if } \boldsymbol{\xi}\in\Omega^F_{1'},\\ P-(1+\beta)R, & \text{if } \boldsymbol{\xi}\in\Omega^F_{3},\\ (1-\beta)(O-S-R), & \text{if } \boldsymbol{\xi}\in\Omega^F_{4},\\ -P+(1-\beta)(O-S)+2\beta R, & \text{if } \boldsymbol{\xi}\in\Omega^F_{6},\\ 0, & \text{otherwise.}\end{cases}
$$

Then, for a given ξ_1, since we know $-P+(1-\beta)(O-S)+2\beta R<(1-\beta)(O-S-R)<0$ and $0<P-(1+\beta)R<P-(1+\beta)(O-S)$, then, $\frac{\partial\Delta\pi^F(\boldsymbol{\xi})}{\partial\xi_1}$ decreases as ξ_2 increases. Thus, $\Delta(\rho,\beta)$ decreases in ρ. □

Appendix C

Appendix of Chapter 4

C.1 Proofs for Analytical Results.

Proof of Proposition 4.1.

It is convenient to define the following quantities

$$\alpha_1(r_s, n_1) = 1 - \frac{c}{n_1\mu\left(q_1 - I - r_s\right)^+}, \tag{C.1}$$

$$\alpha_2(r_p, n_1, n_2) = \frac{c - n_2\mu\left(q_2 - r_p\right)}{n_1\mu\left(q_1 - r_p\right)}. \tag{C.2}$$

To eliminate trivial settings, we restrict our attention to the case of $\alpha_1\left(r_s, n_1\right) > 0$, i.e., the case of $n_1 > \frac{c}{\mu(q_1 - I - r_s)^+}$.

Note that with the approximation formula of service time in (4.2), the demand rate for either street-hailing and platform-based ride-hailing services could be zero. Thus, $\lambda^s\left(r_s, n_1, \alpha\right) > 0$ if and only if $0 \le \alpha < \alpha_1\left(r_s, n_1\right)$, and $\lambda^p\left(r_p, n_1, n_2, \alpha\right) > 0$ if and only if $\alpha_2^+\left(r_p, n_1, n_2\right) < \alpha \le 1$. For ease of exposition, we use $\alpha_1(n_1)$ for $\alpha_1(r_s, n_1)$ and $\alpha_2(n_1, n_2)$ for $\alpha_2(r_p, n_1, n_2)$.

We obtain,

$$U_{d,1}(\alpha) = \begin{cases} r_s\left((1-\alpha)\mu - \frac{c}{n_1(q_1 - I - r_s)^+}\right) - k_1 - h & \text{if } 0 \le \alpha \le \min\{\alpha_1^+\left(n_1\right), \alpha_2^+\left(n_1, n_2\right)\}, \\ r_s\left((1-\alpha)\mu - \frac{c}{n_1(q_1 - I - r_s)^+}\right) + \alpha w_p\left(\mu - \frac{c}{(\alpha n_1(q_1 - r_p) + n_2(q_2 - r_p))^+}\right) - k_1 - h & \text{if } \alpha_2^+\left(n_1, n_2\right) < \alpha < \alpha_1^+\left(n_1\right), \\ -k_1 - h & \text{if } \alpha_1^+\left(n_1\right) < \alpha < \alpha_2^+\left(n_1, n_2\right), \\ \alpha w_p[\mu - \frac{c}{(\alpha n_1(q_1 - r_p) + n_2(q_2 - r_p))^+}] - k_1 - h & \text{if } \max\{\alpha_1^+\left(n_1\right), \alpha_2^+\left(n_1, n_2\right)\} \le \alpha \le 1. \end{cases}$$

In the following, we analyze the optimal α to maximize $U_{d,1}(\alpha)$ in the range of $\alpha \le \bar{\alpha}$.

1. If $0 \le \bar{\alpha} \le \min\{\alpha_1^+\left(n_1\right), \alpha_2^+\left(n_1, n_2\right)\}$, then for $\alpha \in [0, \bar{\alpha}]$,

$$U_{d,1}(\alpha) = r_s\left((1-\alpha)\mu - \frac{c}{n_1(q_1 - I - r_s)^+}\right) - k_1 - h,$$

which decreases in α. Therefore in this case, $\alpha^* = 0$.

2. If $\alpha_2^+(n_1, n_2) < \bar{\alpha} < \alpha_1^+(n_1)$, then

$$U_{d,1}(\alpha) = r_s\left((1-\alpha)\mu - \frac{c}{n_1(q_1 - I - r_s)^+}\right) + \alpha w_p\left(\mu - \frac{c}{(\alpha n_1(q_1 - r_p) + n_2(q_2 - r_p))^+}\right) - k_1 - h.$$

By taking the first-order derivative of $U_{d,1}(\alpha)$ with respect to α, we get,

$$\frac{\partial U_{d,1}(\alpha)}{\partial \alpha} = (w_p - r_s)\mu - \frac{c w_p n_2(q_2 - r_p)}{(\alpha n_1(q_1 - r_p) + n_2(q_2 - r_p))^2}$$

and

$$\frac{\partial^2 U_{d,1}(\alpha)}{\partial^2 \alpha} = \frac{2cn_2(q_2 - r_p)n_1(q_1 - r_p)}{(\alpha n_1(q_1 - r_p) + n_2(q_2 - r_p))^3}.$$

We discuss in the following three subcases:

(a) If $q_2 > r_p$, then $\frac{\partial^2 U_{d,1}(\alpha)}{\partial^2 \alpha} > 0$ and thus $U_{d,1}$ is convex in α when $\alpha \in [\alpha_2^+, \bar{\alpha}]$. In this case, α^* is either 0 or $\bar{\alpha}$, that is, $\alpha^* = \underset{\alpha \in \{0, \bar{\alpha}\}}{\text{argmin}} U_{d,1}(\alpha)$.

(b) If $q_2 \leq r_p$ and $r_s < w_p$, then $\frac{\partial^2 U_{d,1}(\alpha)}{\partial^2 \alpha} < 0$ and $\frac{\partial U_{d,1}}{\partial \alpha} > 0$. As $U_{d,1}(\alpha)$ increases with $\alpha \in [\alpha_2^+, \bar{\alpha}]$, $\alpha^* = \underset{\alpha \in \{0, \bar{\alpha}\}}{\text{argmin}} U_{d,1}(\alpha)$..

(c) If $q_2 \leq r_p$ and $r_s \geq w_p$, then, $\frac{\partial^2 U_{d,1}(\alpha)}{\partial^2 \alpha} < 0$. We know, $U_{d,1}(\alpha)$ is concave in $\alpha \in [\alpha_2^+, \bar{\alpha}]$. In this case, define $\tilde{\alpha}$ satisfying $\frac{\partial U_{d,1}(\alpha)}{\partial \alpha}|_{\alpha=\tilde{\alpha}} = 0$. The optimal α would be one of the following three values: $\{0, \bar{\alpha}, \tilde{\alpha}\}$. Solving $\frac{\partial U_{d,1}(\alpha)}{\partial \alpha}|_{\alpha=\tilde{\alpha}} = 0$, we obtain that $\tilde{\alpha} = \frac{\sqrt{\frac{cn_2(r_p - q_2)w_p}{\mu(r_s - w_p)}} + n_2(r_p - q_2)}{n_1(q_1 - r_p)}$. As a result,

$$U_{d,1}(\tilde{\alpha}) = r_s(\mu - \frac{c}{n_1(q_1 - I - r_s)}) - \frac{(\sqrt{\mu n_2(r_p - q_2)(r_s - w_p)} + \sqrt{cw_p})^2}{n_1(q_1 - r_p)} - k_1 - h.$$

It can be easily verified that $U_{d,1}(0) > U_{d,1}(\tilde{\alpha})$. Because $U_{d,1}(\tilde{\alpha}) > U_{d,1}(\bar{\alpha})$ at the same time. Therefore, $\alpha^* = 0$.

3. If $\alpha_1^+(n_1, n_2) < \bar{\alpha} < \alpha_2^+(n_1)$, then for $\alpha \in [\alpha_1^+, \bar{\alpha}]$, we have $U_{d,1}(\alpha) = -k_1 - h < U_{d,1}|_{\alpha=0}$. As a result, $\alpha^* = 0$.

4. If $\max\{\alpha_1^+(n_1), \alpha_2^+(n_1, n_2)\} \leq \bar{\alpha} \leq 1$, then, for $\alpha \in [\max\{\alpha_1^+(n_1), \alpha_2^+(n_1, n_2)\}, \bar{\alpha}]$,

$$U_{d,1}(\alpha) = \alpha w_p(\mu - \frac{c}{\alpha n_1(q_1 - r_p) + n_2(q_2 - r_p)}) - k_1 - h,$$

By taking the first-order derivative of $U_{d,1}(\alpha)$ with respect to α, we get

$$\frac{\partial U_{d,1}(\alpha)}{\partial \alpha} = w_p\left(\mu - \frac{cn_2(q_2 - r_p)}{(\alpha n_1(q_1 - r_p) + n_2(q_2 - r_p))^2}\right),$$

and
$$\frac{\partial^2 U_{d,1}(\alpha)}{\partial^2 \alpha} = \frac{2cn_2(q_2 - r_p) * n_1(q_1 - r_p)}{(\alpha n_1(q_1 - r_p) + n_2(q_2 - r_p))^3}.$$

(a) If $q_2 > r_p$, then, $\frac{\partial^2 U_{d,1}(\alpha)}{\partial^2 \alpha} > 0$. We know, $U_{d,1}$ is convex in this region. Thus, for $\alpha \in [0, \bar{\alpha}]$, the optimal decision α^* is either 0 or $\bar{\alpha}$. that is, $\alpha^* = \underset{\alpha \in \{0,\bar{\alpha}\}}{\operatorname{argmin}}(U_{d,1}(\alpha))$.

(b) If $q_2 \leq r_p$, then, $\frac{\partial^2 U_{d,1}(\alpha)}{\partial^2 \alpha} < 0$ and $\frac{\partial U_{d,1}(\alpha)}{\partial \alpha} > 0$. We know, $U_{d,1}(\alpha)$ increases in α for $\alpha \in (\max\{\alpha_1^+(n_1), \alpha_2^+(n_1, n_2)\}, \bar{\alpha})$. Thus, for $\alpha \in [0, \bar{\alpha}]$, the optimal decision α^* is 0 or $\bar{\alpha}$.

Based on the discussion so far, we know that α^* is either 0 or $\bar{\alpha}$. Therefore $\alpha^* = \bar{\alpha}$ if and only if $U_{d,1}(\bar{\alpha}) > U_{d,1}(0)$, which is possible in the following two cases.

(a) If $\alpha_2^+(n_1, n_2) < \bar{\alpha} < \alpha_1(n_1)$,

$$U_{d,1}(\bar{\alpha}) = r_s((1-\bar{\alpha})\mu - \frac{c}{n_1(q_1 - I - r_s)}) + w_p\bar{\alpha}(\mu - \frac{c}{(\bar{\alpha}n_1(q_1 - r_p) + n_2(q_2 - r_p))^+}) - k_1 - h.$$

$$U_{d,1}(0) = r_s(\mu - \frac{c}{n_1(q_1 - I - r_s)}) - k_1 - h = r_s\mu\alpha_1(r_s, n_1) - k_1 - h.$$

From $U_{d,1}(\bar{\alpha}) \geq U_{d,1}(0)$, we obtain, $w_p f(\bar{\alpha}n_1, n_2, r_p) \geq r_s\mu$, where $f(\bar{\alpha}n_1, n_2, r_p) = \left(\mu - \frac{c}{(\bar{\alpha}n_1q_1 + n_2q_2 - (\bar{\alpha}n_1 + n_2)r_p)^+}\right)^+$.

(b) If $\max\{\alpha_2^+(n_1, n_2), \alpha_1(n_1)\} \leq \bar{\alpha} \leq 1$,

$$U_{d,1}(\bar{\alpha}) = w_p\bar{\alpha}(\mu - \frac{c}{(\bar{\alpha}n_1(q_1 - r_p) + n_2(q_2 - r_p))^+}) - k_1 - h.$$

From $U_{d,1}(\bar{\alpha}) \geq U_{d,1}(0)$, we obtain that $w_p f(\bar{\alpha}n_1, n_2, r_p) \geq \frac{r_s\mu\alpha_1(n_1)}{\bar{\alpha}}$.

To summarize, $\alpha^* = \bar{\alpha}$ if and only if
$\{\alpha_2^+(n_1, n_2) < \bar{\alpha} < \alpha_1(n_1), w_p f(\bar{\alpha}n_1, n_2, r_p) \geq r_s\mu\} \cup \{\max\{\alpha_2^+(n_1, n_2), \alpha_1(n_1)\} \leq \bar{\alpha} \leq 1, w_p f(\bar{\alpha}n_1, n_2, r_p) \geq \frac{r_s\mu\alpha_1(n_1)}{\bar{\alpha}}\}$, which is equivalent to

$$\{\alpha_2^+(n_1, n_2) < \bar{\alpha} < 1, w_p f(\bar{\alpha}n_1, n_2, r_p) \geq \frac{r_s\mu \min\{\bar{\alpha}, \alpha_1(n_1)\}}{\bar{\alpha}}\}. \qquad \text{(C.3)}$$

Below we rearrange the conditions for $\alpha^* = \bar{\alpha}$ in (C.3) to know the impact of n_1 and n_2.

From $\alpha_2^+(n_1, n_2) < \bar{\alpha} < 1$ in (C.3), we obtain, $\bar{\alpha}n_1(q_1 - r_p) + n_2(q_2 - r_p) > \frac{c}{\mu}$, that is, $f(\bar{\alpha}n_1, n_2, r_p) > 0$.

From $w_p f(\bar{\alpha}n_1, n_2, r_p) \geq \frac{r_s\mu \min\{\bar{\alpha}, \alpha_1(n_1)\}}{\bar{\alpha}}$ in (C.3), we know, $f(\bar{\alpha}n_1, n_2, r_p) >$

0, and

$$\begin{aligned} & w_p(\mu - \frac{c}{(\bar{\alpha}n_1(q_1-r_p)+n_2(q_2-r_p))^+}) \geq \frac{r_s\mu\min\{\bar{\alpha},\alpha_1(n_1)\}}{\bar{\alpha}} \\ \Rightarrow\ & \bar{\alpha}n_1(q_1-r_p)+n_2(q_2-r_p) \geq \frac{cw_p}{\mu(w_p-\frac{r_s}{\bar{\alpha}}\min\{\bar{\alpha},(1-\frac{c}{n_1\mu(q_1-I-r_s)^+})\})^+}. \end{aligned} \tag{C.4}$$

Define $L(n_1) = \frac{1}{q_2-r_p}\left(\frac{cw_p}{\mu\left(w_p-\frac{r_s}{\bar{\alpha}}\min\left\{\bar{\alpha},\left(1-\frac{c}{n_1\mu(q_1-I-r_s)^+}\right)\right\}\right)^+} - \bar{\alpha}n_1(q_1-r_p)\right)$.

Based on inequality (C.4), we discuss conditions of n_2 as below.
(1) If $q_2 < r_p$, then, $\bar{\alpha}n_1(q_1-r_p) > \frac{cw_p}{\mu(w_p-\frac{r_s}{\bar{\alpha}}\min\{\bar{\alpha},(1-\frac{c}{n_1\mu(q_1-I-r_s)^+})\})^+}$. That is to say, the denominator of $L(n_1)$ should be negative; otherwise, the feasible region of n_2 is empty. Thus, we obtain, $n_2 \leq (L(n_1))^+$.
(2) If $q_2 \geq r_p$, then, $n_2 \geq L(n_1)$. As $n_2 \geq 0$, thus, we have $n_2 \geq (L(n_1))^+$. □

Proof of Proposition 4.2.

For ease of exposition, we use $\alpha_1(n_1)$ for $\alpha_1(r_s, n_1)$.

Define $\Omega_1 = \{(n_1,n_2) : \alpha_1(n_1) > \bar{\alpha}, w_p f(\bar{\alpha}n_1, n_2, r_p) \geq r_s\mu\}$, $\Omega_2 = \{(n_1,n_2) : \alpha_1(n_1) \leq \bar{\alpha}, w_p f(\bar{\alpha}n_1, n_2, r_p) \geq \frac{r_s\mu\alpha_1(n_1)}{\bar{\alpha}}\}$, and then $\Omega = \Omega_1 \cup \Omega_2 = \{(n_1,n_2) : w_p f(\bar{\alpha}n_1, n_2, r_p) \geq \frac{r_s\mu\min\{\bar{\alpha},\alpha_1(n_1)\}}{\bar{\alpha}}\}$.

$$U_{d,1}(n_1,n_2,\alpha^*(n_1,n_2)) = \begin{cases} r_s\left((1-\bar{\alpha})\mu - \frac{c}{n_1(q_1-I-r_s)^+}\right) + \bar{\alpha}w_p\left(\mu - \frac{c}{(\alpha n_1(q_1-r_p)+n_2(q_2-r_p))^+}\right) - k_1 - h & \text{if } (n_1,n_2)\in\Omega_1, \\ \bar{\alpha}w_p(\mu - \frac{c}{(\bar{\alpha}n_1(q_1-r_p)+n_2(q_2-r_p))^+}) - k_1 - h & \text{if } (n_1,n_2)\in\Omega_2, \\ r_s\left(\mu - \frac{c}{n_1(q_1-I-r_s)^+}\right) - k_1 - h & \text{otherwise.} \end{cases}$$

For $n_2 = \mathcal{N}_2$, we first analyze taxi drivers best response, that is, to get n_1 which optimises $U_{d,1}(n_1,\mathcal{N}_2,\alpha^*(n_1,\mathcal{N}_2))$.

Note that $U_{d,1}(n_1,\mathcal{N}_2,\alpha^*(n_1,\mathcal{N}_2))$ increases in n_1, therefore, $n_1^*(\mathcal{N}_2) = N_1$ if and only if $U_{d,1}(N_1,\mathcal{N}_2,\alpha^*(N_1,\mathcal{N}_2)) \geq 0$; otherwise, $n_1^*(\mathcal{N}_2) = 0$. With Assumption 4.1, we restrict our attention to the scenario that all type 1 drivers can participate in when without collaboration, that is, $r_s\left(\mu - \frac{c}{N_1(q_1-I-r_s)^+}\right) - k_1 - h \geq 0$, $\alpha_1(N_1) > 0$. Then, we have that $U_{d,1}(N_1,\mathcal{N}_2,\alpha^*(N_1,\mathcal{N}_2)) \geq r_s\left(\mu - \frac{c}{N_1(q_1-I-r_s)^+}\right) - k_1 - h \geq 0$. Therefore, $n_1^*(\mathcal{N}_2) = N_1$. Furthermore, $\alpha^*(N_1,\mathcal{N}_2) = \bar{\alpha}$ if $(N_1,\mathcal{N}_2)\in\Omega$; otherwise, $\alpha^*(N_1,\mathcal{N}_2) = 0$.

Now we analyse private car drivers' best response for given $n_1 = \mathcal{N}_1$. By joining the platform,

$$U_{d,2}(\mathcal{N}_1,n_2,\alpha^*(\mathcal{N}_1,n_2)) = \begin{cases} w_p(\mu - \frac{c}{(\bar{\alpha}\mathcal{N}_1(q_1-r_p)+n_2(q_2-r_p))^+})^+ - k_2 & \text{if } (\mathcal{N}_1,n_2)\in\Omega, \\ w_p(\mu - \frac{c}{(n_2(q_2-r_p))^+})^+ - k_2 & \text{otherwise.} \end{cases}$$

1. If $q_2 \geq r_p$, then $U_{d,2}(\mathcal{N}_1,n_2,\alpha^*(\mathcal{N}_1,n_2))$ is non-decreasing in n_2. We also notice that if $(\mathcal{N}_1,N_2)\notin\Omega$, then $(\mathcal{N}_1,n_2)\notin\Omega$ for any $n_2 < N_2$. The optimal participation number of private car drivers satisfies $n_2^*(\mathcal{N}_1) = N_2$

if and only if $U_{d,2}(\mathcal{N}_1, N_2, \alpha^*(\mathcal{N}_1, N_2)) \geq 0$; $n_2^*(\mathcal{N}_1) = 0$, otherwise. From Assumption 4.2, $\mu N_2 q_2 > c$ holds. Then, in this case, with $q_2 > r_p$, $f(0, N_2, r_p) > 0$ holds. Note that $U_{d,2}(\mathcal{N}_1, N_2, \alpha^*(\mathcal{N}_1, N_2)) \geq 0$ if and only if $\{(\mathcal{N}_1, N_2) \in \Omega, w_p f(\bar{\alpha}\mathcal{N}_1, N_2, r_p) \geq k_2\} \cup \{(\mathcal{N}_1, N_2) \notin \Omega, w_p f(0, N_2, r_p) \geq k_2\}$. In particular, when $\{(\mathcal{N}_1, N_2) \in \Omega, w_p f(\bar{\alpha}\mathcal{N}_1, N_2, r_p) \geq k_2\}$, that is, $w_p f(\bar{\alpha}\mathcal{N}_1, N_2, r_p) \geq \max\{\frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(\mathcal{N}_1)\}}{\bar{\alpha}}, k_2\}$, we have $\alpha^*(\mathcal{N}_1, N_2) = \bar{\alpha}$; if $\{(\mathcal{N}_1, N_2) \notin \Omega, w_p f(0, N_2, r_p) \geq k_2\}$, that is, $\{w_p f(0, N_2, r_p) \geq k_2, w_p f(\bar{\alpha}\mathcal{N}_1, N_2, r_p) < \frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(\mathcal{N}_1)\}}{\bar{\alpha}}\}$, we have $\alpha^*(\mathcal{N}_1, N_2) = 0$. Using $(\mathcal{N}_1, n_2^*(\mathcal{N}_1)) = (n_1^*(\mathcal{N}_2), \mathcal{N}_2)$, we have the following results: if $q_2 \geq r_p$, then in equilibrium,

$$
\begin{aligned}
&(\mathcal{N}_1, \mathcal{N}_2, \alpha^*(\mathcal{N}_1, \mathcal{N}_2)) \\
&= \begin{cases} (N_1, N_2, \bar{\alpha}) & \text{if } w_p f(\bar{\alpha} N_1, N_2, r_p) \geq \max\{\frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(N_1)\}}{\bar{\alpha}}, k_2\}; \\ (N_1, N_2, 0) & \text{if } w_p f(0, N_2, r_p) \geq k_2, w_p f(\bar{\alpha} N_1, N_2, r_p) < \frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(N_1)\}}{\bar{\alpha}}; \\ (N_1, 0, 0) & \text{otherwise.} \end{cases}
\end{aligned}
$$

2. If $q_2 < r_p$, define $\bar{n}_2(\mathcal{N}_1) = \max\{n_2 : (\mathcal{N}_1, n_2) \in \Omega, w_p f(\bar{\alpha}\mathcal{N}_1, n_2, r_p) \geq k_2\}$. Note that $f(\bar{\alpha}\mathcal{N}_1, n_2, r_p)$ is non-increasing in n_2, as we can see,

   - $\bar{n}_2(\mathcal{N}_1) < 0$ if and only if $w_p f(\bar{\alpha}\mathcal{N}_1, 0, r_p) < \max\{\frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(\mathcal{N}_1)\}}{\bar{\alpha}}, k_2\}$. In this case, $n_2^* = 0$ and $\alpha^*(\mathcal{N}_1, 0) = 0$.
   - $\bar{n}_2(\mathcal{N}_1) > N_2$ if and only if $w_p f(\bar{\alpha}\mathcal{N}_1, N_2, r_p) > \max\{\frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(\mathcal{N}_1)\}}{\bar{\alpha}}, k_2\}$. In this case, $n_2^* = N_2$ and $\alpha^*(\mathcal{N}_1, N_2) = \bar{\alpha}$.
   - $0 \leq \bar{n}_2(\mathcal{N}_1) \leq N_2$ if and only if $w_p f(\bar{\alpha}\mathcal{N}_1, 0, r_p) \geq \max\{\frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(\mathcal{N}_1)\}}{\bar{\alpha}}, k_2\} \geq w_p f(\bar{\alpha}\mathcal{N}_1, N_2, r_p)$. In this case, $n_2^* = \bar{n}_2(\mathcal{N}_1)$ satisfies that $w_p f(\bar{\alpha}\mathcal{N}_1, \bar{n}_2(\mathcal{N}_1), r_p) = \max\{\frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(\mathcal{N}_1)\}}{\bar{\alpha}}, k_2\}$, and $\alpha^*(\mathcal{N}_1, \bar{n}_2(\mathcal{N}_1)) = \bar{\alpha}$.

   Using $(\mathcal{N}_1, n_2^*(\mathcal{N}_1)) = (n_1^*(\mathcal{N}_2), \mathcal{N}_2)$, we have the following results: If $q_2 < r_p$, then in equilibrium,

$$
\begin{aligned}
&(\mathcal{N}_1, \mathcal{N}_2, \alpha^*(\mathcal{N}_1, \mathcal{N}_2)) \\
&= \begin{cases} (N_1, N_2, \bar{\alpha}) & \text{if } w_p f(\bar{\alpha} N_1, N_2, r_p) \geq \max\{\frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(N_1)\}}{\bar{\alpha}}, k_2\}; \\ (N_1, \bar{n}_2(N_1), \bar{\alpha}) & \text{if } w_p f(\bar{\alpha} N_1, 0, r_p) \geq \max\{\frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(N_1)\}}{\bar{\alpha}}, k_2\} \geq w_p f(\bar{\alpha} N_1, N_2, r_p); \\ (N_1, 0, 0) & \text{otherwise.} \end{cases}
\end{aligned}
$$

To summarize, in equilibrium the participation of taxi drivers and private car drivers satisfy:

$$
\begin{aligned}
&(\mathcal{N}_1, \mathcal{N}_2, \alpha^*(\mathcal{N}_1, \mathcal{N}_2)) \\
&= \begin{cases} (N_1, N_2, \bar{\alpha}) & \text{if } w_p f(\bar{\alpha} N_1, N_2) \geq \max\{\frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(N_1)\}}{\bar{\alpha}}, k_2\}; \\ (N_1, \bar{n}_2(N_1), \bar{\alpha}) & \text{if } q_2 < r_p,\ w_p f(\bar{\alpha} N_1, 0, r_p) \geq \max\{\frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(N_1)\}}{\bar{\alpha}}, k_2\} \geq w_p f(\bar{\alpha} N_1, N_2, r_p); \\ (N_1, N_2, 0) & \text{if } q_2 \geq r_p,\ w_p f(0, N_2, r_p) \geq k_2, w_p f(\bar{\alpha} N_1, N_2, r_p) < \frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(N_1)\}}{\bar{\alpha}}; \\ (N_1, 0, 0) & \text{otherwise.} \end{cases}
\end{aligned}
$$

□

Proof of Proposition 4.3.

In order to show the optimal decisions of the online platform, there are two following two parts: firstly, we discuss the possible optimal profit under each case; secondly, we make comparisons among these cases to gain the maximal profit value.

$$\lambda^p(r_p, n^p) = \left(n^p\mu - \frac{c}{(q^p - r_p)^+}\right)^+ = (\alpha n_1 + n_2) f(\alpha n_1, n_2, r_p). \tag{C.5}$$

where $n^p = \alpha n_1 + n_2$ and $q^p = \frac{\alpha n_1 q_1 + n_2 q_2}{\alpha n_1 + n_2}$.

The optimization problem for the online platform is

$$\max_{(r_p, w_p)} \quad \pi(r_p, w_p) = (r_p - w_p)\lambda^p(r_p, n^p).$$

In the following discussion, for ease of notations, we define $s(r_s, \bar{\alpha}) = \max\{\frac{r_s\mu\min\{\bar{\alpha}, \alpha_1(r_s, N_1)\}}{\bar{\alpha}}, k_2\}$. With Assumption 4.1, we have,

$$s(r_s, \bar{\alpha}) = \frac{r_s\mu\min\{\bar{\alpha}, \alpha_1(r_s, N_1)\}}{\bar{\alpha}}.$$

1. $(\mathcal{N}_1, \mathcal{N}_2, \alpha^*(\mathcal{N}_1, \mathcal{N}_2)) = (N_1, N_2, \bar{\alpha})$

   In this case, the optimization problem for the online platform is

$$\begin{aligned} \max_{(r_p, w_p)} \quad & \pi(r_p, w_p) = (r_p - w_p)\lambda^p(r_p, \bar{\alpha}N_1 + N_2) = (r_p - w_p)(\alpha N_1 + N_2) f(\alpha N_1, N_2, r_p) \\ \text{s.t.} \quad & w_p f(\bar{\alpha}N_1, N_2, r_p) \geq s(r_s, \bar{\alpha}), \end{aligned}$$

   where $f(\bar{\alpha}N_1, N_2, r_p) = \left(\mu - \frac{c}{(\bar{\alpha}N_1(q_1 - r_p) + N_2(q_2 - r_p))^+}\right)^+$. We firstly optimize over w_p for any given r_p. It is easy to see $w_p^*(r_p)$ satisfies

$$w_p^* f(\bar{\alpha}N_1, N_2, r_p) = s(r_s, \bar{\alpha}).$$

   The remaining problem for the online platform is

$$\max_{r_p} \quad \pi(r_p, w_p^*(r_p)) = (r_p f(\alpha N_1, N_2, r_p) - s(r_s, \bar{\alpha}))(\alpha N_1 + N_2).$$

   It is easy to see that $\pi(r_p, w_p^*(r_p))$ is concave in r_p. By solving the first order condition $\frac{\partial \pi(r_p, w_p^*(r_p))}{\partial r_p} = 0$, we get the optimal ride price

$$r_p^* = \frac{\bar{\alpha}N_1 q_1 + N_2 q_2 - \sqrt{\frac{c(\bar{\alpha}N_1 q_1 + N_2 q_2)}{\mu}}}{\bar{\alpha}N_1 + N_2}.$$

   As a result, $w_p^* = \frac{s(r_s, \bar{\alpha})}{\mu - \sqrt{\frac{c\mu}{\bar{\alpha}N_1 q_1 + N_2 q_2}}}$, $f(\bar{\alpha}N_1, N_2, r_p) = \mu - \sqrt{\frac{c\mu}{\bar{\alpha}N_1 q_1 + N_2 q_2}}$, $\lambda^p = (\bar{\alpha}N_1 + N_2)\mu - (\bar{\alpha}N_1 + N_2)\sqrt{\frac{c\mu}{\bar{\alpha}N_1 q_1 + N_2 q_2}}$. The optimal profit of the online platform is

$$\pi^{(1)} = (\sqrt{\mu(\bar{\alpha}N_1q_1 + N_2q_2)} - \sqrt{c})^2 - (\bar{\alpha}N_1 + N_2)s(r_s, \bar{\alpha}). \qquad \text{(C.6)}$$

In this case, with Assumption 4.3, which states that $N_1\mu\left(q_1 - q_2\right)^2 < cq_1$, we know, $r_p^* < q_2$ holds for $\bar{\alpha} \in (0, 1]$.

2. $(\mathcal{N}_1, \mathcal{N}_2, \alpha^*(\mathcal{N}_1, \mathcal{N}_2)) = (N_1, N_2, 0)$

   In this case, the optimization problem for the online platform is

$$\begin{aligned} \max_{(r_p, w_p)} \quad & \pi(r_p, w_p) = (r_p - w_p)\lambda^p(r_p, N_2) = (r_p - w_p)N_2 f(0, N_2, r_p) \\ \text{s.t.} \quad & w_p f(\bar{\alpha}N_1, N_2, r_p) < s(r_s, \bar{\alpha}), \\ & w_p f(0, N_2, r_p) \geq k_2, \end{aligned}$$

   where $f(0, N_2, r_p) = (\mu - \frac{c}{N_2(q_2 - r_p)^+})^+$. Note that $q_2 \geq r_p$ is implied in the feasible region.

$$\begin{aligned} \max_{(r_p, w_p)} \quad & \pi(r_p, w_p) = (r_p - w_p)\lambda^p(r_p, N_2) = (r_p - w_p)N_2 f(0, N_2, r_p) \\ \text{s.t.} \quad & \tfrac{k_2}{f(0, N_2, r_p)} \leq w_p < \tfrac{s(r_s, \bar{\alpha})}{f(\bar{\alpha}N_1, N_2, r_p)}. \end{aligned}$$

   For given r_p, $w_p^*(r_p) = \frac{k_2}{f(0, N_2, r_p)}$. Define

$$g(r_p) = s(r_s, \bar{\alpha})f(0, N_2, r_p) - k_2 f(\bar{\alpha}N_1, N_2, r_p).$$

   The remaining problem is to optimize

$$\begin{aligned} \max_{r_p} \quad & \pi(r_p, w_p^*(r_p)) = N_2(r_p f(0, N_2, r_p) - k_2) = N_2(r_p \left(\mu - \tfrac{c}{N_2(q_2 - r_p)}\right)^+ - k_2) \\ \text{s.t.} \quad & g(r_p) \geq 0. \end{aligned}$$

   Note that

$$\frac{\partial g\left(r_p\right)}{\partial r_p} = -\frac{s(r_s, \bar{\alpha})c}{N_2\left(q_2 - r_p\right)^2} + \frac{k_2 c}{(\bar{\alpha}N_1 + N_2)(\frac{\bar{\alpha}N_1q_1 + N_2q_2}{\bar{\alpha}N_1 + N_2} - r_p)^2} < 0.$$

   Thus, $g(r_p)$ is decreasing with r_p. The feasible region $g(r_p) \geq 0$ is non-empty as long as

$$g(0) = s(r_s, \bar{\alpha})\left(\mu - \frac{c}{N_2q_2}\right) - k_2\left(\mu - \frac{c}{\bar{\alpha}N_1q_1 + N_2q_2}\right) \geq 0.$$

   Now we discuss the optimisation problem when $g(0) \geq 0$. Note that $\pi(r_p, w_p^*(r_p))$ is concave in r_p. By taking the first-order derivative of $\pi(r_p, w_p^*(r_p))$ with respect to r_p, we get $\frac{\partial \pi(r_p, w_p^*(r_p))}{\partial r_p} = N_2\mu - \frac{cq_2}{(q_2 - r_p)^2}$, and $\pi(r_p)$ is concave in r_p. From $\frac{\partial \pi(r_p)}{\partial r_p} = 0$, we get the critical point

$\widetilde{r}_p = q_2 - \sqrt{\frac{cq_2}{N_2\mu}}$. Next, we need to check whether this critical point satisfies $g(\widetilde{r}_p) \geq 0$.

- If $g(\widetilde{r}_p) = s(r_s, \bar{\alpha})(\mu - \sqrt{\frac{c\mu}{N_2 q_2}}) - k_2 \left(\mu - \frac{c}{\bar{\alpha} N_1 (q_1 - q_2 + \sqrt{\frac{cq_2}{N_2\mu}}) + \sqrt{\frac{cq_2 N_2}{\mu}}} \right) \geq 0$, then, the optimal ride fee is

$$r_p^* = \widetilde{r}_p = q_2 - \sqrt{\frac{cq_2}{N_2\mu}}.$$

  $f(0, N_2, r_p) = \mu - \sqrt{\frac{c\mu}{N_2 q_2}}$; $f(\bar{\alpha} N_1, N_2, r_p) = \mu - \frac{c}{\bar{\alpha} N_1 (q_1 - q_2 + \sqrt{\frac{cq_2}{N_2\mu}}) + \sqrt{\frac{cq_2 N_2}{\mu}}}$. The optimal wage and demand rate are

$$w_p^* = \frac{k_2}{\mu - \sqrt{\frac{c\mu}{N_2 q_2}}}, \lambda^p = N_2\mu - \sqrt{\frac{cN_2\mu}{q_2}}.$$

  The optimal profit of the online platform is

$$\pi^* = (\sqrt{\mu N_2 q_2} - \sqrt{c})^2 - k_2 N_2.$$

  With Assumption 4.2, we know, in this case, $\pi^* \geq 0$.

- If $g(\widetilde{r}_p) = s(r_s, \bar{\alpha})(\mu - \sqrt{\frac{c\mu}{N_2 q_2}}) - k_2 \left(\mu - \frac{c}{\bar{\alpha} N_1 (q_1 - q_2 + \sqrt{\frac{cq_2}{N_2\mu}}) + \sqrt{\frac{cq_2 N_2}{\mu}}} \right) < 0$, then, there exists a unique point $\widehat{r}_p$ which satisfies

$$g(\widehat{r}_p) = 0.$$

  Then, $r_p^* = \widehat{r}_p$. The optimal wage and demand rate are

$$w_p^* = \frac{k_2}{\mu - \frac{c}{N_2(q_2 - \widehat{r}_p)}}, \lambda^p = N_2\mu - \frac{c}{q_2 - \widehat{r}_p}.$$

  The optimal profit of online platform is

$$\pi^* = \widehat{r}_p(N_2\mu - \frac{c}{q_2 - \widehat{r}_p}) - k_2 N_2.$$

To summarize, when $g(0) < 0$, $(\mathcal{N}_1, \mathcal{N}_2, \alpha^*(\mathcal{N}_1, \mathcal{N}_2)) = (N_1, N_2, 0)$ does not exist. Otherwise, if $g(0) \geq 0$, the optimal profit achieved under $(\mathcal{N}_1, \mathcal{N}_2, \alpha^*(\mathcal{N}_1, \mathcal{N}_2)) = (N_1, N_2, 0)$ is

$$\pi^{(2)} = \begin{cases} \pi^{(2a)} = (\sqrt{\mu N_2 q_2} - \sqrt{c})^2 - k_2 N_2 & \text{if } g(\widetilde{r}_p) \geq 0; \\ \pi^{(2b)} = \widehat{r}_p(N_2\mu - \frac{c}{q_2 - \widehat{r}_p}) - k_2 N_2 & \text{if } g(0) > 0 \text{ and } g(\widetilde{r}_p) < 0; \end{cases} \tag{C.7}$$

where $\widehat{r}_p$ satisfies $g(\widehat{r}_p) = 0$, $g(0) = s(r_s, \bar{\alpha}) * \left(\mu - \frac{c}{N_2 q_2}\right) - k_2 \left(\mu - \frac{c}{\bar{\alpha} N_1 q_1 + N_2 q_2}\right)$,
$g(\widetilde{r}_p) = s(r_s, \bar{\alpha}) * (\mu - \sqrt{\frac{c\mu}{N_2 q_2}}) - k_2 \left(\mu - \frac{c}{\bar{\alpha} N_1 (q_1 - q_2 + \sqrt{\frac{c q_2}{N_2 \mu}}) + \sqrt{\frac{c q_2 N_2}{\mu}}}\right)$.

3. $(\mathcal{N}_1, \mathcal{N}_2, \alpha^*(\mathcal{N}_1, \mathcal{N}_2)) = (N_1, \bar{n}_2(N_1), \bar{\alpha})$

$$\begin{aligned} \max_{(r_p, w_p, \bar{n}_2)} \quad & \pi(r_p, w_p, \bar{n}_2) = (r_p - w_p)\lambda^p(r_p, \bar{\alpha} N_1 + \bar{n}_2) = (r_p - w_p)(\alpha N_1 + \bar{n}_2) f(\alpha N_1, \bar{n}_2, r_p) \\ \text{s.t.} \quad & w_p f(\bar{\alpha} N_1, 0, r_p) \geq s(r_s, \bar{\alpha}), \\ & w_p f(\bar{\alpha} N_1, N_2, r_p) \leq s(r_s, \bar{\alpha}), \\ & w_p f(\bar{\alpha} N_1, \bar{n}_2, r_p) = s(r_s, \bar{\alpha}). \end{aligned}$$

For given r_p and $\bar{n}_2$, we have $w_p^*(r_p, \bar{n}_2)$ satisfies that $w_p^* f(\bar{\alpha} N_1, \bar{n}_2, r_p) = s(r_s, \bar{\alpha})$.

$$\begin{aligned} \max_{(r_p, \bar{n}_2)} \quad & \pi(r_p, \bar{n}_2) = (r_p f(\alpha N_1, \bar{n}_2, r_p) - s(r_s, \bar{\alpha}))(\alpha N_1 + \bar{n}_2) \\ \text{s.t.} \quad & q_2 \leq r_p, \\ & 0 \leq \bar{n}_2 \leq N_2. \end{aligned}$$

For given $\bar{n}_2$, we first optimize over r_p. Note that $\pi(r_p, \bar{n}_2)$ is concave in r_p, from $\frac{\partial \pi(r_p, \bar{n}_2)}{\partial r_p} = 0$, we get

$$\bar{r}_p(\bar{n}_2) = \frac{\bar{\alpha} N_1 q_1 + \bar{n}_2 q_2 - \sqrt{\frac{c(\bar{\alpha} N_1 q_1 + \bar{n}_2 q_2)}{\mu}}}{\bar{\alpha} N_1 + \bar{n}_2}.$$

We discuss the problem in two subcases.

- In the region $\{\bar{n}_2 : \bar{r}_p \geq q_2\}$, $r_p^*(\bar{n}_2) = \bar{r}_p$. Define $h(\bar{n}_2) = \sqrt{\frac{c(\bar{\alpha} N_1 q_1 + \bar{n}_2 q_2)}{\mu}} - \bar{\alpha} N_1 (q_1 - q_2)$, which increases in $\bar{n}_2$. Then $\bar{r}_p \geq q_2$ is equivalent to $h(\bar{n}_2) \leq 0$. In this region, the optimal profit is achieved by solving the following problem:

$$\begin{aligned} \max_{\bar{n}_2} \quad & \pi(\bar{n}_2) = (\sqrt{\mu(\bar{\alpha} N_1 q_1 + \bar{n}_2 q_2)} - \sqrt{c})^2 - s(r_s, \bar{\alpha}) * (\bar{\alpha} N_1 + \bar{n}_2) \\ \text{s.t.} \quad & h(\bar{n}_2) \leq 0, \\ & 0 \leq \bar{n}_2 \leq N_2. \end{aligned}$$

  Note that if $h(0) > 0$, that is, $\bar{\alpha} N_1 \mu (q_1 - q_2)^2 < c q_1$, then this region is empty. From Assumption 4.3, we know that $\bar{\alpha} N_1 \mu (q_1 - q_2)^2 < c q_1$ always holds, as a result, this region is empty.

- In the region $\{\bar{n}_2 : \bar{r}_p \leq q_2\}$, $r_p^*(\bar{n}_2) = q_2$. In this region, the optimal profit is achieved by solving the following problem:

$$\begin{array}{ll} \max\limits_{\bar{n}_2} & \pi(\bar{n}_2) = (q_2(\mu - \frac{c}{\bar{\alpha}N_1(q_1-q_2)})^+ - s(r_s,\bar{\alpha}))(\bar{\alpha}N_1 + \bar{n}_2) \\ \text{s.t.} & h(\bar{n}_2) \geq 0, \\ & 0 \leq \bar{n}_2 \leq N_2. \end{array}$$

(a) If $h(N_2) < 0$, then the feasible region is empty.

(b) If $h(N_2) \geq 0$ and $(q_2(\mu - \frac{c}{\bar{\alpha}N_1(q_1-q_2)})^+ - s(r_s,\bar{\alpha})) > 0$, then $\bar{n}_2 = N_2$ and $\pi^* = (q_2(\mu - \frac{c}{\bar{\alpha}N_1(q_1-q_2)})^+ - s(r_s,\bar{\alpha}))(\bar{\alpha}N_1 + N_2)$. Note that $\pi^* < \pi^{(1)}$, therefore, it would not be the global optimal profit.

(c) If $h(N_2) \geq 0$ and $(r_p(\mu - \frac{c}{\bar{\alpha}N_1(q_1-q_2)})^+ - s(r_s,\bar{\alpha})) \leq 0$, then $\pi^* \leq 0$. Therefore, it would not be the global optimal profit.

4. $(\mathcal{N}_1, \mathcal{N}_2, \alpha^*(\mathcal{N}_1, \mathcal{N}_2)) = (N_1, 0, 0)$, $\pi^* = 0$.

With all the discussions above, the optimal profit is $\pi^* = \max\{\pi^{(1)}, \pi^{(2)}, 0\}$, where

$$\pi^{(1)} = (\sqrt{\mu(\bar{\alpha}N_1q_1 + N_2q_2)} - \sqrt{c})^2 - (\bar{\alpha}N_1 + N_2) * s(r_s,\bar{\alpha});$$

$$\pi^{(2)} = \begin{cases} \pi^{(2a)} = (\sqrt{\mu N_2 q_2} - \sqrt{c})^2 - k_2N_2 & \text{if } g(\widetilde{r}_p) \geq 0; \\ \pi^{(2b)} = \widehat{r}_p(N_2\mu - \frac{c}{q_2 - \widehat{r}_p}) - k_2N_2 & \text{if } g(0) > 0 \text{ and } g(\widetilde{r}_p) < 0. \end{cases}$$

That is, π^* can be represented as

$$\pi^* = \begin{cases} \pi^{(1)} & \text{if } \{g(0) > 0,\ g(\widetilde{r}_p) \geq 0 \text{ and } \pi^{(1)} \geq \pi^{(2a)}\} \\ & \cup \{g(0) > 0,\ g(\widetilde{r}_p) < 0 \text{ and } \pi^{(1)} \geq (\pi^{(2b)})^+\} \\ & \cup \{g(0) \leq 0 \text{ and } \pi^{(1)} \geq 0\}; \\ \pi^{(2a)} & \text{if } \{g(0) > 0,\ g(\widetilde{r}_p) \geq 0 \text{ and } \pi^{(2a)} > \pi^{(1)}\}; \\ \pi^{(2b)} & \text{if } \{g(0) > 0,\ g(\widetilde{r}_p) < 0 \text{ and } \pi^{(2b)} > (\pi^{(1)})^+\}; \\ 0 & \text{otherwise.} \end{cases} \tag{C.8}$$

Recall that

$$g(0) = s(r_s,\bar{\alpha})\left(\mu - \frac{c}{N_2q_2}\right) - k_2\left(\mu - \frac{c}{\bar{\alpha}N_1q_1 + N_2q_2}\right)$$

and

$$g(\widetilde{r}_p) = s(r_s,\bar{\alpha})(\mu - \sqrt{\frac{c\mu}{N_2q_2}}) - k_2\left(\mu - \frac{c}{\bar{\alpha}N_1(q_1 - q_2 + \sqrt{\frac{cq_2}{N_2\mu}}) + \sqrt{\frac{cq_2N_2}{\mu}}}\right).$$

For conditions of $\pi^* = \pi^{(1)}$, we know,

$$\begin{array}{rll} g(0) > 0 & \text{is equivalent to} & s(r_s, \bar{\alpha}) > s_1(\bar{\alpha}), \\ g(\widetilde{r}_p) \geq 0 & \text{is equivalent to} & s(r_s, \bar{\alpha}) \geq s_2(\bar{\alpha}), \\ \pi^{(1)} \leq \pi^{(2a)} & \text{is equivalent to} & s(r_s, \bar{\alpha}) \geq s_3(\bar{\alpha}), \end{array} \tag{C.9}$$

where $s_1(\bar{\alpha}), s_2(\bar{\alpha}), s_3(\bar{\alpha})$ are defined as

$$\begin{aligned} s_1(\bar{\alpha}) &= \frac{k_2\left(\mu - \frac{c}{\bar{\alpha}N_1q_1 + N_2q_2}\right)}{\mu - \frac{c}{N_2q_2}}, \\ s_2(\bar{\alpha}) &= \frac{k_2(\mu - \frac{c}{\bar{\alpha}N_1(q_1-q_2)+(\bar{\alpha}N_1+N_2)\sqrt{\frac{cq_2}{N_2\mu}}})}{\mu - \sqrt{\frac{c\mu}{N_2q_2}}}, \\ s_3(\bar{\alpha}) &= \frac{(\sqrt{\mu(\bar{\alpha}N_1q_1 + N_2q_2)} - \sqrt{c})^2 - (\sqrt{\mu N_2q_2} - \sqrt{c})^2 + k_2N_2}{\bar{\alpha}N_1 + N_2}. \end{aligned}$$

From $g(0) > g(\widetilde{r}_p)$, we have $s_1(\bar{\alpha}) < s_2(\bar{\alpha})$. Next we prove that $s_2(\bar{\alpha}) \leq s_3(\bar{\alpha})$ for any $0 \leq \bar{\alpha} \leq 1$.

We define

$$\begin{aligned} F_1(\bar{\alpha}) = {} & (\mu\bar{\alpha}N_1q_1 - 2\sqrt{c\mu\,(\bar{\alpha}N_1q_1 + N_2q_2)} + 2\sqrt{c\mu N_2q_2} + k_2N_2)(\mu - \sqrt{\frac{c\mu}{N_2q_2}}) \\ & -k_2(\bar{\alpha}N_1 + N_2)(\mu - \frac{c}{\bar{\alpha}N_1(q_1 - q_2) + (\bar{\alpha}N_1 + N_2)\sqrt{\frac{cq_2}{\mu N_2}}}). \end{aligned}$$

It is easy to see that $s_2(\bar{\alpha}) \leq s_3(\bar{\alpha})$ if and only if $F_1(\bar{\alpha}) \geq 0$ for any $0 \leq \bar{\alpha} \leq 1$.

Note that $F_1(0) = 0$, and by taking derivative of $F_1(\bar{\alpha})$ with respect to $\bar{\alpha} \geq 0$, we have

$$\frac{\partial F_1(\bar{\alpha})}{\partial \bar{\alpha}} = \mu N_1q_1 - \frac{c\mu N_1q_1}{\sqrt{c\mu\,(\bar{\alpha}N_1q_1 + N_2q_2)}}(\mu - \sqrt{\frac{c\mu}{N_2q_2}}) - k_2\mu N_1 - \frac{k_2cN_1N_2(q_1 - q_2)}{\bar{\alpha}N_1(q_1 - q_2) + (\bar{\alpha}N_1 + N_2)\sqrt{\frac{cq_2}{\mu N_2}}}.$$

It can be verified that $\frac{\partial F_1(\bar{\alpha})}{\partial \bar{\alpha}}$ is increasing in $\bar{\alpha}$, and

$$\frac{\partial F_1(\bar{\alpha})}{\partial \bar{\alpha}}|_{\bar{\alpha}=0} = \frac{N_1q_1\mu}{q_2}((\left(\sqrt{\mu N_2q_2} - \sqrt{c}\right)^2 \frac{1}{N_2} - k_2) > 0.$$

As a result, $s_2(\bar{\alpha}) \leq s_3(\bar{\alpha})$ for any $0 \leq \bar{\alpha} \leq 1$. So far we have proved that $s_1(\bar{\alpha}) \leq s_2(\bar{\alpha}) \leq s_3(\bar{\alpha})$.

- $\pi^* = \pi^{(2a)}$ if and only if $\{g(0) > 0, g(\widetilde{r}_p) \geq 0, \pi^{(2a)} > \pi^{(1)}\}$. As $\{g(0) > 0, g(\widetilde{r}_p) \geq 0, \pi^{(2a)} > \pi^{(1)}\} = \{s(r_s, \bar{\alpha}) \geq \max\{s_1(\bar{\alpha}), s_2(\bar{\alpha}), s_3(\bar{\alpha})\}\} = \{s(r_s, \bar{\alpha}) \geq s_3(\bar{\alpha})\} = \{\pi^{(1)} \leq \pi^{(2a)}\}$. Therefore, $\pi^* = \pi^{(2a)}$ if and only if $\pi^{(1)} \leq \pi^{(2a)}$.

- $\pi^* = \pi^{(2b)}$ if and only if $\{g(0) > 0, g(\widetilde{r}_p) < 0, \pi^{(2b)} > \pi^{(1)}, \pi^{(2b)} > 0\}$. As

$\pi^{(2b)} < \pi^{(2a)}$, $\{g(0) > 0, g(\widetilde{r}_p) < 0, \pi^{(2b)} > \pi^{(1)}, \pi^{(2b)} > 0\} \subseteq \{g(\widetilde{r}_p) < 0, \pi^{(2a)} > \pi^{(1)}\} = \{s_3(\bar{\alpha}) \le s(r_s, \bar{\alpha}) \le s_2(\bar{\alpha})\}$, which is empty set. Therefore, $\pi^* = \pi^{(2b)}$ does not hold.

- So far we have obtained that $\pi^* = \pi^{(2a)}$ if and only if $\pi^{(1)} \le \pi^{(2a)}$. As $\pi^{(2b)}$ would not be optimal, when $\pi^{(1)} > \pi^{(2a)}$, $\pi^* = \max\{\pi^{(1)}, 0\} = \pi^{(1)}$, which follows because $\pi^{(2a)} > 0$.

$$\pi^* = \begin{cases} \pi^{(1)} = (\sqrt{\mu(\bar{\alpha}N_1 q_1 + N_2 q_2)} - \sqrt{c})^2 - (\bar{\alpha}N_1 + N_2) * s(r_s, \bar{\alpha}) & \text{if } \pi^{(1)} \ge \pi^{(2a)}; \\ \pi^{(2a)} = (\sqrt{\mu N_2 q_2} - \sqrt{c})^2 - k_2 N_2 & \text{if } \pi^{(1)} < \pi^{(2a)}. \end{cases}$$

Note that $\pi^{(1)} \ge \pi^{(2a)}$ is equivalent to $s(r_s, \bar{\alpha}) \le s_3(\bar{\alpha})$, where $s(r_s, \bar{\alpha}) = \frac{r_s \mu \min\{\bar{\alpha}, \alpha_1(r_s, N_1)\}}{\bar{\alpha}}$ and $\alpha_1(r_s) = 1 - \frac{c}{N_1 \mu (q_1 - I - r_s)^+}$. □

Proof of Proposition 4.3. (Further Check Conditions for Non-empty.)

In order to get the explicit condition in $(\bar{\alpha}, r_s)$ space from solving $s(r_s, \bar{\alpha}) \le s_3(\bar{\alpha})$, where,

$$s(\bar{\alpha}, r_s) = \frac{r_s \mu \min\{\bar{\alpha}, 1 - \frac{c}{N_1 \mu (q_1 - I - r_s)^+}\}}{\bar{\alpha}},$$
$$s_3(\bar{\alpha}) = \frac{\left(\sqrt{\mu(\bar{\alpha}N_1 q_1 + N_2 q_2)} - \sqrt{c}\right)^2 - (\sqrt{\mu N_2 q_2} - \sqrt{c})^2 + k_2 N_2}{\bar{\alpha}N_1 + N_2},$$

we have below discussions. Recall

$$R_1(\bar{\alpha}) = q_1 - I - \frac{c}{N_1 \mu (1 - \bar{\alpha})},$$
$$R_2(\bar{\alpha}) = \frac{\left(\sqrt{\mu(\bar{\alpha}N_1 q_1 + N_2 q_2)} - \sqrt{c}\right)^2 - \left(\sqrt{\mu N_2 q_2} - \sqrt{c}\right)^2 + k_2 N_2}{\mu(\bar{\alpha}N_1 + N_2)}.$$

$s(r_s, \bar{\alpha}) \le s_3(\bar{\alpha})$ can be divided into two sub-regions.
(1) Region 1, which is feasible region of "mixed" service mode:

$$\{\bar{\alpha} < \alpha_1(r_s, N_1), r_s \mu \le s_3(\bar{\alpha})\}.$$

Then, the feasible region can be represented as

$$r_s < \min\{R_1(\bar{\alpha}), R_2(\bar{\alpha})\}. \tag{C.10}$$

With $\underline{r}_s \le r_s \le \overline{r}_s$ in Assumption 4.1, we need to further check whether the inequality in (C.10) is non-empty. Recall

$$\bar{\alpha}_1 = \frac{x_0^2 - \mu N_2 q_2}{\mu N_1 q_1} \text{ where } x_0 = \frac{q_1 \sqrt{c}}{q_1 - \underline{r}_s} + \sqrt{\frac{\underline{r}_s \mu N_2 (q_1 - q_2) + \left(\left(\sqrt{\mu N_2 q_2} - \sqrt{c}\right)^2 - k_2 N_2 - c\right) q_1}{q_1 - \underline{r}_s} + \left(\frac{q_1 \sqrt{c}}{q_1 - \underline{r}_s}\right)^2},$$
$$\bar{\alpha}_2 = 1 - \frac{\frac{c}{\mu}}{N_1 (q_1 - I - \underline{r}_s)}.$$

where $\bar{\alpha}_1$ satisfies $\underline{r}_s = R_2(\bar{\alpha}_1)$, and $\bar{\alpha}_2$ satisfies $\underline{r}_s = R_1(\bar{\alpha}_2)$.

Note that $R_1(\bar{\alpha})$ decreases in $\bar{\alpha}$, $R_2(\bar{\alpha})$ increases in $\bar{\alpha}$. In order to guarantee this feasible region is nonempty, $\bar{\alpha}_1 < \alpha < \bar{\alpha}_2$ should hold. Below we further discuss about the parameter combinations that ensures that the feasible region of $(\bar{\alpha}, r_s)$ for "mixed" case is non-empty, i.e., $\bar{\alpha}_1 < \alpha < \bar{\alpha}_2$ is non-empty.

Since $\bar{\alpha}_1$ satisfies $\underline{r}_s = R_2(\bar{\alpha} = \bar{\alpha}_1)$, and $R_2(\bar{\alpha})$ increases in $\bar{\alpha}$, then, $\bar{\alpha}_1 < \bar{\alpha}_2$ is equivalent to $\underline{r}_s < R_2(\bar{\alpha} = \bar{\alpha}_2)$, which implies

$$\underline{r}_s < \frac{\left(\sqrt{\mu\left(\bar{\alpha}_2 N_1 q_1 + N_2 q_2\right)} - \sqrt{c}\right)^2 - \left(\sqrt{\mu N_2 q_2} - \sqrt{c}\right)^2 + k_2 N_2}{\mu\left(\bar{\alpha}_2 N_1 + N_2\right)}. \tag{C.11}$$

By solving (C.11), we obtain a threshold of N_2, which is defined as N_2^{max} in (4.26). That is to say, if $N_2 < N_2^{max}$, Assumption 4.4 ensures this feasible region is non-empty.

(2) Region 2, which is feasible region of "platform-only" service mode:

$$\{\bar{\alpha} \geq \alpha_1(r_s, N_1), \tfrac{r_s \mu \alpha_1(r_s, N_1)}{\bar{\alpha}} \leq s_3(\bar{\alpha})\}. \tag{C.12}$$

Then, we need to obtain the explicit conditions of $(\bar{\alpha}, r_s)$ by solving

$$\begin{aligned} & r_s\left(1 - \tfrac{c}{N_1\mu(q_1 - I - r_s)^+}\right) \leq \bar{\alpha} R_2(\bar{\alpha}), \\ \Leftrightarrow\ & \left(\tfrac{N_1\mu(q_1 - I + \bar{\alpha}R_2(\bar{\alpha})) - c}{2N_1\mu}\right)^2 - (q_1 - I)\,\bar{\alpha}R_2(\bar{\alpha}) \leq \left(r_s - \tfrac{N_1\mu(q_1 - I + \bar{\alpha}R_2(\bar{\alpha})) - c}{2N_1\mu}\right)^2. \end{aligned} \tag{C.13}$$

To prove the statement of the Proposition, we first establish the following Lemma C.1.

Lemma C.1. *Given $r_s \in [\underline{r}_s, \overline{r}_s]$ and $\bar{\alpha} \in [0,1]$, with Assumption 4.4, by solving $r_s\left(1 - \frac{c}{N_1\mu(q_1 - I - r_s)^+}\right) \leq \bar{\alpha} R_2(\bar{\alpha})$, we obtain:*

$$\begin{aligned} & \{(\bar{\alpha}, r_s) : r_s \in [\underline{r}_s, R_3(\bar{\alpha})] \cup [R_4(\bar{\alpha}), \overline{r}_s] \text{ for } \bar{\alpha} < \min\{\bar{\alpha}_4, 1\}\} \\ \cup\ & \{(\bar{\alpha}, r_s) : r_s \in [\underline{r}_s, \overline{r}_s] \text{ for } \bar{\alpha} > \min\{\bar{\alpha}_4, 1\}\}. \end{aligned} \tag{C.14}$$

Proof. To solve $r_s\left(1 - \frac{c}{N_1\mu(q_1 - I - r_s)^+}\right) \leq \bar{\alpha} R_2(\bar{\alpha})$ in (C.13), we derive the result in two steps: (1) first, we analyze the concavity of the left hand part, $r_s\left(1 - \frac{c}{N_1\mu(q_1 - I - r_s)^+}\right)$, and obtain the maximal and minimal values of the left hand part; (2) second, since the right hand part, $\bar{\alpha}R_2(\bar{\alpha})$, increases in $\bar{\alpha}$, we discuss the solution of this inequality (C.13) based on whether the maximal value of $\bar{\alpha}R_2(\bar{\alpha})$ is larger than the maximal and minimal values of the left hand part.

Step 1: Define $F_L(r_s) = r_s\left(1 - \frac{c}{N_1\mu(q_1 - I - r_s)^+}\right)$. Then, $F_L(r_s) \geq \frac{k_1 + h}{\mu}$ due to Assumption 4.1. Since $F_L(r_s)$ is concave in r_s for $r_s \in [\underline{r}_s, \overline{r}_s]$, then, from the first-order condition, the maximal value of $F_L(r_s)$ is $F_L^{max} = F_L(r_s = r_{s4}) = \frac{(\sqrt{(q_1 - I)N_1\mu} - \sqrt{c})^2}{N_1\mu}$, where $r_{s4} = q_1 - I - \sqrt{\frac{c(q_1 - I)}{N_1\mu}}$. Then, we know,

$\frac{k_1+h}{\mu} \leq F_L(r_s) \leq \frac{(\sqrt{(q_1-I)N_1\mu}-\sqrt{c})^2}{N_1\mu}$.

Step 2: Define $F_R(\bar{\alpha}) = \bar{\alpha}R_2(\bar{\alpha})$. Since $F_R(\bar{\alpha}) = \bar{\alpha}R_2(\bar{\alpha})$ increases in $\bar{\alpha} \in [0,1]$, then, the maximal value of the right hand part is $F_R(\bar{\alpha}=1)$.

From Assumption 4.4, we know, $F_R(\bar{\alpha}=1) > \frac{k_1+h}{\mu}$ always holds, which implies,

$$\left(\sqrt{\mu\left(N_1q_1+N_2q_2\right)}-\sqrt{c}\right)^2-\left(\sqrt{\mu N_2q_2}-\sqrt{c}\right)^2 > (k_1+h)(N_1+N_2)-k_2N_2. \tag{C.15}$$

This means that the feasible region from (C.13) is non-empty.

Next, we focus on discussing whether $F_R(\bar{\alpha}=1)$ is larger than F_L^{max}.

By solving $F_L^{max} = F_R(\bar{\alpha})$, where $F_L^{max} = \frac{(\sqrt{(q_1-I)N_1\mu}-\sqrt{c})^2}{N_1\mu}$, which means,

$$\frac{(\sqrt{(q_1-I)\,N_1\mu}-\sqrt{c})^2}{N_1\mu} = \frac{\bar{\alpha}\left(\left(\sqrt{\mu\left(\bar{\alpha}N_1q_1+N_2q_2\right)}-\sqrt{c}\right)^2-\left(\sqrt{\mu N_2q_2}-\sqrt{c}\right)^2+k_2N_2\right)}{\mu\left(\bar{\alpha}N_1+N_2\right)}, \tag{C.16}$$

we obtain the unique solution $\bar{\alpha} = \bar{\alpha}_4$ satisfying (C.16).

When $\bar{\alpha}_4 > 1$, which means,

$$\frac{(\sqrt{(q_1-I)\,N_1\mu}-\sqrt{c})^2}{N_1\mu} > \frac{\left(\sqrt{\mu\left(N_1q_1+N_2q_2\right)}-\sqrt{c}\right)^2-\left(\sqrt{\mu N_2q_2}-\sqrt{c}\right)^2+k_2N_2}{\mu\left(N_1+N_2\right)},$$

then, for $\bar{\alpha} \in [0,1]$, there always exists valid solutions of r_s for solving inequality in (C.13). Recall

$$\begin{aligned}
R_3\left(\bar{\alpha}\right) &= \frac{N_1\mu\left(q_1-I+\bar{\alpha}R_2\left(\bar{\alpha}\right)\right)-c}{2N_1\mu}-\sqrt{\left(\frac{N_1\mu\left(q_1-I+\bar{\alpha}R_2\left(\bar{\alpha}\right)\right)-c}{2N_1\mu}\right)^2-\bar{\alpha}R_2\left(\bar{\alpha}\right)\left(q_1-I\right)},\\
R_4\left(\bar{\alpha}\right) &= \frac{N_1\mu\left(q_1-I+\bar{\alpha}R_2\left(\bar{\alpha}\right)\right)-c}{2N_1\mu}+\sqrt{\left(\frac{N_1\mu\left(q_1-I+\bar{\alpha}R_2\left(\bar{\alpha}\right)\right)-c}{2N_1\mu}\right)^2-\bar{\alpha}R_2\left(\bar{\alpha}\right)\left(q_1-I\right)}.
\end{aligned}$$

In this case, for $\bar{\alpha} \in [0,1]$, by solving (C.13), we obtain, $r_s \leq R_3(\bar{\alpha})$ or $R_4(\bar{\alpha}) \leq r_s$.

When $\bar{\alpha}_4 \leq 1$, then, for $\bar{\alpha} \in [0, \bar{\alpha}_4]$, there exists valid solutions of r_s solving inequality in (C.13); for $\bar{\alpha} \in [\bar{\alpha}_4, 1]$, the inequality in (C.13) always holds for r_s.

Therefore, With $\underline{r}_s \leq r_s \leq \overline{r}_s$ in Assumption 4.1, we have:

if $\bar{\alpha} < \min\{\bar{\alpha}_4, 1\}$, then, $\underline{r}_s \leq r_s \leq R_3(\bar{\alpha})$ or $R_4(\bar{\alpha}) \leq r_s \leq \overline{r}_s$;

if $\min\{\bar{\alpha}_4, 1\} \leq \bar{\alpha} \leq 1$, then, $r_s \in [\underline{r}_s, \overline{r}_s]$. □

Since $\bar{\alpha} \geq \alpha_1(r_s, N_1)$ in (C.12) implies that $r_s \geq R_1(\bar{\alpha})$, then, the statement of Proposition follows directly by taking the intersection between $r_s \geq R_1(\bar{\alpha})$ and the results in Lemma C.1. □

Proof of Lemma 4.2: Given r_s, where $\underline{r}_s \leq r_s \leq \min\{r_{s0}, \overline{r}_s\}$, the profit of the online platform is

$$\pi^*(\bar{\alpha}) = \begin{cases} (\sqrt{\mu N_2 q_2} - \sqrt{c})^2 - k_2 N_2, & \text{if } \bar{\alpha} < \alpha_1(r_s, N_1) \text{ and } r_s\mu > s_3(\bar{\alpha}), \\ (\sqrt{\mu\,(\bar{\alpha}N_1 q_1 + N_2 q_2)} - \sqrt{c})^2 - r_s\mu\,(\bar{\alpha}N_1 + N_2)\,, & \text{if } \bar{\alpha} < \alpha_1(r_s, N_1) \text{ and } r_s\mu \leq s_3(\bar{\alpha}), \\ (\sqrt{\mu(\bar{\alpha}N_1 q_1 + N_2 q_2)} - \sqrt{c})^2 - \frac{r_s\mu\alpha_1(r_s,N_1)(\bar{\alpha}N_1+N_2)}{\bar{\alpha}}, & \text{if } \bar{\alpha} \geq \alpha_1(r_s, N_1) \text{ and } \frac{r_s\mu\alpha_1(r_s,N_1)}{\bar{\alpha}} \leq s_3(\bar{\alpha}). \end{cases} \tag{C.17}$$

Note that in (C.17), for the first line, $\pi^*(\bar{\alpha})$ keeps unchanged with $\bar{\alpha}$; for the third line, $\pi^*(\bar{\alpha})$ increases with $\bar{\alpha}$. Below, we focus on discuss the property of the expression in the second line, i.e., $\pi^*(\bar{\alpha}) = (\sqrt{\mu(\bar{\alpha}N_1 q_1 + N_2 q_2)} - \sqrt{c})^2 - r_s\mu(\bar{\alpha}N_1 + N_2)$. In this case, by taking the first-order derivative of $\pi^*(\bar{\alpha})$ with respect to $\bar{\alpha}$, we obtain,

$$\frac{\partial(\pi^*(\bar{\alpha}))}{\partial(\bar{\alpha})} = \mu N_1 q_1 - \frac{\mu N_1 q_1 \sqrt{c}}{\sqrt{\mu(\bar{\alpha}N_1 q_1 + N_2 q_2)}} - r_s\mu N_1.$$

From $\frac{\partial(\pi^*(\bar{\alpha}))}{\partial(\bar{\alpha})} = 0$, we obtain the critical value of $\bar{\alpha} = \frac{(\frac{q_1\sqrt{c}}{q_1-r_s})^2 - \mu N_2 q_2}{\mu N_1 q_1}$. Since $\frac{\partial^2(\pi^*(\bar{\alpha}))}{\partial^2(\bar{\alpha})} > 0$, $\frac{\partial(\pi^*(\bar{\alpha}))}{\partial(\bar{\alpha})}$ is increasing in $\bar{\alpha}$.

Then, $\frac{\partial(\pi^*(\bar{\alpha}))}{\partial(\bar{\alpha})} > 0$ for $\bar{\alpha} > \frac{(\frac{q_1\sqrt{c}}{q_1-r_s})^2 - \mu N_2 q_2}{\mu N_1 q_1}$, and $\frac{\partial(\pi^*(\bar{\alpha}))}{\partial(\bar{\alpha})} < 0$ for $\bar{\alpha} < \frac{(\frac{q_1\sqrt{c}}{q_1-r_s})^2 - \mu N_2 q_2}{\mu N_1 q_1}$. Since for a given r_s, where $\underline{r}_s \leq r_s \leq \min\{r_{s0}, \overline{r}_s\}$, the value of $\bar{\alpha}$ in the feasible region is higher than the critical value $\frac{(\frac{q_1\sqrt{c}}{q_1-r_s})^2 - \mu N_2 q_2}{\mu N_1 q_1}$, i.e., $s_3(\frac{(\frac{q_1\sqrt{c}}{q_1-r_s})^2 - \mu N_2 q_2}{\mu N_1 q_1}) < r_s\mu$, we know, $\frac{\partial(\pi^*(\bar{\alpha}))}{\partial(\bar{\alpha})} > 0$ for $\{\bar{\alpha} < \alpha_1(r_s, N_1)$ and $r_s\mu \leq s_3(\bar{\alpha})\}$. Thus, for the second line of (C.17), $\pi^*(\bar{\alpha})$ is increasing in $\bar{\alpha}$. □

Proof of Proposition 4.4: Omitted. □

Proof of Proposition 4.5: Omitted. □

C.2 Subsequent Results for the Equilibrium in Proposition 4.3

In order to clearly know the optimal profit and decisions in each case, Table C.1 summarizes expressions of the optimal profit π^*, optimal ride fee r_p^* and optimal wage compensation w_p^* of the online platform.

Note that in Table C.1, the optimal profit of online platform $\pi^*(\bar{\alpha}, r_s)$ increases in $\bar{\alpha}$, the optimal ride fee $r_p^*(\bar{\alpha}, r_s)$ increases in $\bar{\alpha}$, and the optimal wage compensation $w_p^*(\bar{\alpha}, r_s)$ decreases in $\bar{\alpha}$.

In order to clearly know the performances of drivers and riders in each case, Table C.2 summarizes the utilities of taxi drivers and private car drivers, and the participant number of riders for street-hailing services and platform-based ride-hailing services.

Note that in Table C.2, taxi drivers' utility $U_{d,1}(\bar{\alpha}, r_s)$ keeps unchanged in

How do taxis serve customers?	Optimal profit of online platform π^*	Optimal ride fee of online platform r_p^*	Optimal wage compensation per ride of online platform w_p^*
Street-only	$(\sqrt{\mu N_2 q_2} - \sqrt{c})^2 - k_2 N_2$	$q_2 - \sqrt{\frac{cq_2}{N_2\mu}}$	$\frac{k_2}{\mu - \sqrt{\frac{c\mu}{N_2 q_2}}}$
Mixed	$(\sqrt{\mu(\bar{\alpha}N_1 q_1 + N_2 q_2)} - \sqrt{c})^2 - r_s\mu(\bar{\alpha}N_1 + N_2)$	$\frac{\bar{\alpha}N_1 q_1 + N_2 q_2 - \sqrt{\frac{c(\bar{\alpha}N_1 q_1 + N_2 q_2)}{\mu}}}{\bar{\alpha}N_1 + N_2}$	$\frac{r_s\mu}{\mu - \sqrt{\frac{c\mu}{\bar{\alpha}N_1 q_1 + N_2 q_2}}}$
Platform-only	$(\sqrt{\mu(\bar{\alpha}N_1 q_1 + N_2 q_2)} - \sqrt{c})^2 - r_s\left(\mu - \frac{c}{N_1(q_1 - I - r_s)^+}\right)\frac{(\bar{\alpha}N_1 + N_2)}{\bar{\alpha}}$	$\frac{\bar{\alpha}N_1 q_1 + N_2 q_2 - \sqrt{\frac{c(\bar{\alpha}N_1 q_1 + N_2 q_2)}{\mu}}}{\bar{\alpha}N_1 + N_2}$	$\frac{r_s\left(\mu - \frac{c}{N_1(q_1 - I - r_s)^+}\right)}{\bar{\alpha}(\mu - \sqrt{\frac{c\mu}{\bar{\alpha}N_1 q_1 + N_2 q_2}})}$

Table C.1: Optimal Profit, Ride Fee and Wage Compensation per ride

How do taxis serve customers?	Taxi drivers' utility $U_{d,1}$	Private car drivers utility $U_{d,2}$	Participant number of riders for street-hailing services λ^s	Participant number of riders for platform-based ride-hailing services λ^p
Street-only	$r_s\left(\mu - \frac{c}{N_1(q_1 - I - r_s)}\right) - k_1 - h$	0	$N_1\mu - \frac{c}{(q_1 - I - r_s)^+}$	$N_2\mu - \sqrt{\frac{cN_2\mu}{q_2}}$
Mixed	$r_s\left(\mu - \frac{c}{N_1(q_1 - I - r_s)}\right) - k_1 - h$	$r_s\mu - k_2$	$(1 - \bar{\alpha})N_1\mu - \frac{c}{(q_1 - I - r_s)^+}$	$(\bar{\alpha}N_1 + N_2)(\mu - \sqrt{\frac{c\mu}{\bar{\alpha}N_1 q_1 + N_2 q_2}})$
Platform-only	$r_s\left(\mu - \frac{c}{N_1(q_1 - I - r_s)}\right) - k_1 - h$	$\frac{r_s\left(\mu - \frac{c}{N_1(q_1 - I - r_s)}\right)}{\bar{\alpha}} - k_2$	0	$(\bar{\alpha}N_1 + N_2)(\mu - \sqrt{\frac{c\mu}{\bar{\alpha}N_1 q_1 + N_2 q_2}})$

Table C.2: Drivers' utility and participant number of riders

$\bar{\alpha}$, private car drivers' utility $U_{d,2}(\bar{\alpha}, r_s)$ decreases in $\bar{\alpha}$; $\lambda^s(\bar{\alpha}, r_s)$ decreases in α. □

C.3 Additional Results for the Optimal Decision of r_s^*

Optimal decision of r_s^* in Proposition 4.4.

Recall that $r_{s5} \in \{\underline{r}_s, \overline{r}_s\}$, and $\widehat{r}_{s5} \in C_1$, where

$$C_1 = \begin{cases} \{r_s : r_s \in \mathbb{R}, \underline{r}_s \leq r_s \leq \overline{r}_s\} & \text{if } \bar{\alpha}_4 < 1, \\ \{r_s : r_s \in \mathbb{R}, \underline{r}_s \leq r_s \leq r_{s1} \text{ or } r_{s2} \leq r_s \leq \overline{r}_s\} & \text{if } \bar{\alpha}_4 \geq 1. \end{cases}$$

Before applying the selection rule with multiple equilibria, we have the below results.

Proposition C.1. (Optimal decisions $(\bar{\alpha}^*, r_s^*)$: without a restriction to λ^s.) *Assume $c > \frac{N_1\mu(q_1 - q_2)^2}{q_1}$ and $\bar{\alpha}_1 < \bar{\alpha}_2$. To maximize the social welfare, the government's optimal decisions $(\bar{\alpha}^*, r_s^*)$ are as follows:*
(a) when $\gamma < \frac{1}{2}$, then, $(\bar{\alpha}^*, r_s^*) = (1, r_{s5})$;
(b) when $\gamma = \frac{1}{2}$, then, $(\bar{\alpha}^*, r_s^*) = (1, \widehat{r}_{s5})$;
(c) when $\gamma > \frac{1}{2}$, and
1. if $\bar{\alpha}_4(N_2) > 1$, then, $(\bar{\alpha}^*, r_s^*) = (1, \widehat{r}_{s4})$;
2. if $\bar{\alpha}_4(N_2) < 1$ and $\gamma < \widehat{\gamma}_0$, then, $(\bar{\alpha}^*, r_s^*) = (1, r_{s4})$;
3. if $\bar{\alpha}_4(N_2) < 1$ and $\widehat{\gamma}_0 < \gamma < 1$, then, $(\bar{\alpha}^*, r_s^*) = (\max\{\bar{\alpha}_3, \bar{\alpha}_4\}, r_{s4})$;
4. if $\bar{\alpha}_4(N_2) < 1$, $\gamma = 1$, and
(i) when $r_{s3} < \min\{r_{s0}, \overline{r}_s\}$, then,

$$(\bar{\alpha}^*, r_s^*) = (\bar{\alpha}^{(1)}, r_{s3}),$$

where $\bar{\alpha}^{(1)} \in [\bar{\alpha}_1', \bar{\alpha}_2']$ *and* $\bar{\alpha}_1'$ *satisfies* $r_{s3} = \frac{s_3(\bar{\alpha}=\bar{\alpha}_1')}{\mu}$, $\bar{\alpha}_2' = 1 - \frac{1}{N_1}\sqrt{\frac{c(N_1+N_2)}{\mu(q_1-I)}}$;

(ii) when $r_{s3} \geq \min\{r_{s0}, \overline{r}_s\}$, *then,*

$$(\bar{\alpha}^*, r_s^*) = \begin{cases} (\bar{\alpha}_4, r_{s4}) & \text{if } r_{s0} < r_{s4} < \min\{r_{s3}, \overline{r}_s\}, \\ (\bar{\alpha}_0, r_{s0}) & \text{if } r_{s4} < r_{s0} < \min\{r_{s3}, \overline{r}_s\}, \\ (\alpha_1(\overline{r}_s), \overline{r}_s) & \text{if } r_{s4} < \min\{r_{s3}, \overline{r}_s\} < r_{s0}. \end{cases}$$

□

Optimal decision of r_s^* in Proposition 4.5.

Recall $\beta^{max} = \frac{N_1\mu(1-\bar{\alpha}_1) - \frac{c}{(q_1-I-\underline{r}_s)^+}}{N_1\mu - \sqrt{\frac{N_1\mu c}{q_1-I}}}$, $\bar{\alpha}_0''(\beta) = 1 - \frac{c}{(q_1-I-\underline{r}_s)N_1\mu} - \frac{\beta\lambda_b^s}{N_1\mu}$ and

$$f_1(\underline{r}_s) = \underline{r}_s(N_1\mu - \frac{c}{q_1-I-\underline{r}_s}) + \underline{r}_s\mu N_2 - k_2N_2, f_2 = (\sqrt{\mu N_1(q_1-I)} - \sqrt{c})^2,$$
$$g_1(\bar{\alpha}_0''(\beta)) = (\sqrt{\mu(\bar{\alpha}_0''(\beta)N_1q_1 + N_2q_2)} - \sqrt{c})^2 - r_s\mu(\bar{\alpha}_0''(\beta)N_1 + N_2),$$
$$g_2 = (\sqrt{\mu N_2 q_2} - \sqrt{c})^2 - k_2N_2;$$

and β_0 satisfies $g_1(\bar{\alpha}_0''(\beta_0)) = f_1(\underline{r}_s) - f_2 + g_2$; β_1 satisfies $g_1(\bar{\alpha}_0''(\beta_1)) = g_2$.

Proposition C.2. (Sufficient Conditions for optimal decisions $(\bar{\alpha}^*, r_s^*)$: with a restriction to λ^s.) *Assume* $c > \frac{N_1\mu(q_1-q_2)^2}{q_1}$ *and* $\bar{\alpha}_1 < \bar{\alpha}_2$. *The government's optimal decisions* $(\bar{\alpha}^*, r_s^*)$ *are as follows:*

(a) when $\min\{1, \beta^{max}\} < \beta < 1$, *then,* $(\bar{\alpha}^*, r_s^*) = (\bar{\alpha}^{(0)}, r_{s4})$, *where* $\bar{\alpha}^{(0)} \in [0, 1]$ *satisfies* $s(\bar{\alpha}^{(0)}, r_{s4}) > s_3(\bar{\alpha}^{(0)})$;

(b) when $0 < \beta < \min\{1, \beta^{max}\}$, *the sufficient condition for the optimal decisions* $(\bar{\alpha}^*, r_s^*) = (\bar{\alpha}_0''(\beta), \underline{r}_s)$ *are*

1. $\beta < \beta_1$ *and* $0 < \gamma < \min\{1, \frac{(g_2 - g_1(\bar{\alpha}_0''))}{f_1(\underline{r}_s) - f_2 + (g_2 - g_1(\bar{\alpha}_0''(\beta)))}\}$;
2. *or,* $\beta > \beta_1$ *and* $\min\{1, \frac{(g_2 - g_1(\bar{\alpha}_0''(\beta)))}{f_1(\underline{r}_s) - f_2 + (g_2 - g_1(\bar{\alpha}_0''(\beta)))}\} < \gamma < 1$.

□