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Behavioural incentivization in healthcare operations

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Behavioural Incentivization in Healthcare Operations

by

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I hereby declare that this dissertation is my original work
and it has been written by me in its entirety.
I have duly acknowledged all the sources of information
which have been used in this dissertation.

This dissertation has also not been submitted for any degree
in any university previously.

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Abstract

Common across current research in healthcare operations is the conclusion that there exist many inefficiencies in today's healthcare systems. Governments and healthcare organisations have sought to address these inefficiencies through the introduction of new policies and operational procedures or by relying on incentives to encourage specific behaviour. However, despite these attempts to reduce inefficiencies in the healthcare systems, the problem persists and is further exacerbated by growing medical complexities coupled with a rapidly ageing population. Against this backdrop, this dissertation investigates two issues within healthcare operations: (i) colorectal cancer (CRC) screening adherence, and (ii) blood donor management. A distinguishing feature of this dissertation looks at the incentivization of participants' behaviours within the two main operations in healthcare operations management.

The first chapter of the dissertation empirically examines the determinants and barriers of CRC screening adherence. Using responses drawn from a nationwide survey, the data highlights that CRC screening adherence levels continue to remain low despite the government's implementation of nationwide screening programs. To study the reasons behind the low adherence rate, I conduct a stepwise logistic regression model and identify several key predictors of the screening adherence. I found that age and individual perceived risk of developing CRC have significant quadratic trends towards screening participation. The results further show that participant's proficiency in probability literacy has an impact on perceiving an individual's risk of developing

CRC towards screening adherence. Linear predictors consisting of CRC knowledge and factor of trust in government are also significant predictors towards screening participation. Motivated by the significant quadratic trend of age, I further investigate the nonmonotonic relationship between age and the adherence rate and provide policymakers with insights on possible interventions to CRC screening policies via a mediation model. I found that policy mediation factors in the form of financial means - CPF account balance and ownership of private insurance were statistically significant mediators that drives the nonmonotonic relationship.

The second chapter studies the strategic management of blood inventory through donor incentivization policies where under-incentivization may lead to shortage of critical blood supply while over-incentivization potentially causes excessive wastage. Incorporating key features of the blood donation such as perishable inventory, observation queue and stochastic demand and supply, I propose an optimization model to solve the donor incentivization decisions in the blood donor management problem by modelling both the blood inventory and donor flow process. Building on the techniques of the Pipeline Queues framework, the optimization model can be reformulated into a convex problem and be efficiently solved. Numerical experiments were further conducted to study how the structure of the optimal policies can change with respect to donors' responsiveness, inventory levels, changes in demand for blood, new donor recruitment rate and distribution of donors in the observation window. Based on the results, the study also puts forward important practical implications relevant in supply chains with social impact.

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Chapter 1

Introduction

The study of healthcare remains a pervasive and pertinent area of research. The growing body of research suggests that there are many challenges policymakers face in the current healthcare landscape, such as promoting healthy ageing in the face of a rapidly ageing population, reducing obesity as well as encouraging participation in cancer screening and vaccination programs. For many of these problems, a fundamental change in individual behaviour is needed. Governments, non-profit healthcare organizations and researchers have sought to influence individual behaviour through conventional strategies such as enacting legislation and regulations as well as providing evidence-based information in the public sphere. However, not all of these efforts have been successful. This is a cause for concern as unlike other sectors, suboptimal decision-making by individuals in health-related matters invariably results in hefty costs for the society. Some of these costs include the direct costs associated with providing the medical resources to diagnose and treat an illness, loss of productivity due to illness or mortality and non-medical costs. Encouraging individuals to make optimal decisions is therefore critical in healthcare operations.

A common approach studied in the literature is the use of incentivization schemes to modify behaviours. By influencing (increasing or decreasing) participation levels

in public healthcare programs through incentivization, governments and healthcare organizations could potentially raise overall standards in healthcare operations. For instance, increasing participation levels in the case of cancer screening can lead to an overall reduction in social costs and a lower burden of care while decreasing participation in a blood donation drive when the blood bank has excess inventory could prevent wastage.

My dissertation, titled 'behavioural incentivization in healthcare operations' seeks to explore incentivization schemes in guiding more effective healthcare operations and policy guidelines. The first essay examines the determinants and barriers of colorectal cancer (CRC) screening adherence in Singapore. The city-state reports CRC as the most commonly diagnosed cancer. Despite the fact that early detection of CRC can significantly reduce an individual's risk of developing the disease, adherence levels towards the nation's cancer screening programs continue to fall below expectations. The preventable nature of CRC and the inconsistencies within the CRC screening literature described below highlights the importance of this research and triggers the need for this project. To begin to address the low adherent behaviour, this essay uses econometric models to highlight the predictors of screening adherence. Although earlier studies have derived useful insights in understanding CRC screening participation, each study is confined to its unique sample and consequently, results across the literature are inconsistent. Our model attempts to address this by incorporating quadratic and interaction terms. In my review of the literature, I further found that several studies measure how participants perceive their individual risk of developing CRC differently. In our study, I provided a narrative describing cancer risk statistics to the participant before asking the participant to assess their individual risk. Given that most statistics rely on probability theory, I further consider the impact of how an individual's probability literacy impairs or strengthens her ability to assess her own risk. Lastly, in the later section, I introduced a mediation model to explore

the impacts of two financial mediators, namely the retirement account (CPF) and ownership of private insurance, towards screening adherence.

In the second essay, I consider the incentivization policies in the blood donor management problem. Specifically, the management of blood inventory faces two critical challenges. First, supply is extremely variable and blood donation depends on the time of the year. There is a need to intervene when supply is inadequate as there can be severe implications on healthcare systems when blood demand is not met. Therefore, these interventions critically depend on the state of the blood donation system. Second, current regulations mandate that donors are required to observe a 3-month observation period after a donation. However, the lifespan of blood is 42 days, which consequently exposes blood banks to shortages. At the same time, excessive blood inventory leads to unnecessary wastage. As a result, the flow of the donor's donation process requires the blood bank to control two networks simultaneously – one for the donor and the other for the blood inventory. I propose an optimization model that accounts for the dynamics in the blood inventory and the donor's donation process, as a coupled queueing network. In my model, I adopt the Pipeline Queue (Bandi & Loke, 2018) paradigm, which leads us to a tractable convex reformulation. In the numerical study, I further examine donor heterogeneity, variability in donor arrivals and drop-outs under different levels of inventory.

My dissertation has several contributions to healthcare and supply chain management research. In the first essay, the study contributes to the literature in two ways. First, it is one of the few studies to feature a large-scale cross-sectional data set in Singapore which comprehensively assesses and studies the key determinants towards screening adherence with a view to raising screening participation as an outcome. Second, I explore policy interventions towards CRC screening adherence that adds to the healthcare and operations literature. Specifically, the study found that there exists nonmonotonic predictors of age and individual perceived risk of developing CRC

towards screening participation. Furthermore, the study found that linear predictors such as CRC knowledge and awareness as well as factor of trust in government are also statistically significant in predicting screening adherence. Departing from the majority of the studies within the literature, we further studied mediators in the form of financial factors (CPF account balance and ownership of private insurance) towards screening participation. These results emphasize the importance of designing multifaceted interventions to improve CRC knowledge and screening compliance, and a concerted effort to develop further financial nets to address the needs and concerns of eligible screening participants is necessary. Although our study is unique to the context of Singapore, our results have highlighted several key determinants of screening adherence which can potentially inspire fellow researchers and healthcare practitioners to develop future strategies in improving uptake of CRC screening and foster better public health outcomes.

The second essay contributes by extending the theoretical techniques employed in the Pipeline Queue paradigm. I model the blood donation and inventory management problem as a novel tandem-queue network and further obtain the tractable formulations to solve the donor incentivization strategy without relying on the independency assumption in the traditional Pipeline Queue framework. Apart from technical contributions, my second essay also contributes to the literature in being the first to operationalize donor incentivization schemes by determining the optimal number of donors within each sub-population to receive each type of incentive.

In the numerical study, I present the optimal structure of the call up policy under varying levels of existing blood inventory and responsiveness of donors towards incentivization, which captures the key parameters which decision makers are most interested in. Through the simulations, I observed three main trends. First, for a fixed inventory level, the higher the responsiveness of donors, the fewer donors are incentivized, which is unsurprising, given that the model seeks to incentivize enough

donors to meet demand and prevent wastage from excessive call ups. Second, for a fixed responsiveness level of donors, the number of donors incentivized first increases, then decreases with existing inventory levels. The latter is intuitive, as this is done to minimize wastage. The initial increase is counter intuitive and might be due to the fact that the model minimizes both the probability of shortages and the extent of shortages for each time period and control the risk under some threshold violation. By balancing shortages while fulfilling demand under this threshold, the model results in a policy that avoids incentivizing too many donors at critically low inventory levels, especially at the start. Lastly, there is a small triangular feature within the optimal structure that preserves the donor pool by calling up donors who would already donate without incentivization, given that doing so would not increase the instantaneous wastage by too much. As an extension, I vary the contextual parameters such as the average demand for blood, new donors' arrival rate, and initial level of donors within the observation queue to understand how the structure of the optimal policy changes. Lastly, I consider segmenting eligible donors into different classes of high and low responsive donors to allow decision makers to manage these different groups of donors depending on current inventory levels. Notably, the results suggest that in times of excess, calling low-responsive donors as a risk-pooling measure would help alleviate downstream shortages, which runs counter to current practices. Collectively, the numerical experiments indicate a dynamic situation of how the optimal policies can change with various factors which can effectively direct policy makers in their efforts towards a more strategic management of blood donors.

To better explore the role of behavioural dimensions of health policy, the dissertation is structured in the following manner: The second chapter of my thesis sets out to explore the underlying factors towards CRC screening adherence. The third chapter explores incentivization policies for blood donors' behaviours in response to blood inventory management. The final chapter briefly concludes the dissertation.

Chapter 2

On the Adherence of Colorectal Cancer (CRC) Screening Policies – A Survey Study in Singapore

2.1 Introduction

Colorectal cancer (CRC) remains one of the leading causes of cancer mortality and morbidity globally. This is surprising given that the survival rate of CRC can significantly increase when it is detected early (Dodd, Carey, Mansfield, Oldmeadow, & Evans, 2019; SEER, 2020). The high mortality can be attributed to the fact that CRC is asymptomatic in early stages, resulting in fewer cases being detected before it progresses to late stages.

To achieve early CRC detection, many countries have introduced public screening programs, e.g. Japan, US, UK and Singapore. Yet, despite stringent efforts, these nationwide cancer screening programs have been ineffective due to low screening adherence levels. This poses a problem given that the need for regular CRC screenings becomes more important against the backdrop of an ageing population, particularly

as there is a higher risk of contracting CRC as age increases (Haggar & Boushey, 2009). Consequently, this has spurred many studies to investigate factors affecting the adherence behavior in order to aid policy makers in designing policies that could potentially raise adherence levels. Various studies have been conducted in the United States (Perencevich, Ojha, Steyerberg, & Syngal, 2013; Sy et al., 2018), Australia (Duncan et al., 2014; Gregory et al., 2013), Korea (Suh et al., 2015) and Europe (Kroupa et al., 2019; Miles, Rainbow, & von Wagner, 2011) to identify predictors that may have an effect on CRC screening adherence. Although these studies have derived useful insights in understanding CRC screening participation, the results of each study are unique and confined to its sample. For instance, residents in the United States are required to collect their Fecal Occult Blood Test (FOBT) kits from their General Practitioner (GP) or local hospital (American Cancer Society, 2020), while Australia's National Bowel Cancer Screening Program (Parkin, Bell, & Mirbagheri, 2018) and UK's NHS Bowel Cancer Screening Program mail the test kits to the homes of eligible residents (National Health Service, 2020a), which may influence participation in screening adherence. There is no consistent solution amongst these studies and the strategies to promote cancer screening uptake varies widely. As such, we have chosen to focus primarily on the landscape of Singapore to study these adherence factors for our study.

Singapore: Screen for Life Cancer Screening Programme

In 2017, the Singapore Ministry of Health (MOH) introduced a public screening program, 'Screen for Life' (SFL) to raise awareness and promote cancer screening participation. The SFL programme strongly encourages residents to regularly screen for cancer by heavily subsidizing screening fees and ensuring that screening kits are widely available in various towns nationwide (Health Hub, 2020). For CRC screening, eligible participants above the age of 50 are recommended to annually screen for CRC

with Faecal Immunochemical Test (FIT) kits (an alternative to FOBT) and to visit a physician if they receive a positive FIT result. A nominal fee is imposed for the screening under the SFL program, which includes the test kit, test examination and, if necessary, the first post screening consultation with a physician. By introducing the public screening program, the Singapore government aims to achieve a national indicator of 70% adherence for cancer screening (Ministry of Health, 2010). However, according to our nationwide survey to Singaporean aged 50 and above, 54% (n = 2115) of the respondents have never done a CRC screening and only 12% regularly screened for CRC. Of those who received a positive result after FIT testing, 26% (n = 294) did not follow up with a physician, as per CRC screening guidelines. Despite the Singapore government's efforts to ensure accessibility and repeatedly promoted the need for early and regular screening, local take up rates in CRC screening have been lacklustre and remain low. This leads us to our research question: What are the determinants of adherence and barriers towards CRC screening in Singapore?

2.2 Hypothesis Development

In selecting the candidates of attributes to include in our study, we reviewed the literature and conducted discussions with healthcare professionals. This led us to assess demographic attributes, knowledge and awareness of CRC, participant's preferences, as well as individual perceived risk patterns. Furthermore, the objective of the study aims to look understanding key determinants of cancer screening adherence that could potentially shape policy decisions. In doing so, we look at the following factors: age, individual perceived risk of developing CRC, CRC knowledge and awareness, and factor of trust in government. By examining these factors, the study can explain the behavioural trend towards screening decisions. The detailed hypotheses are presented below.

2.2.1 Age Factor

Our first hypothesis explores the impact of age factors on screening adherence given that medical research has long acknowledged the increase of an individual's risk of CRC as age increases. Following this trend, policymakers have acted on these medical research studies by tailoring the national screening programs based on age, sending invitations for CRC screening once a participant reaches a predetermined age. Not surprisingly, this has resulted in age being heavily and consistently featured as a main variable in assessing the determinants towards CRC screening adherence within the literature. Studies have shown that older participants tend to be more health conscious and as a result, are more likely to explore basic health screening as well as be more aware of screening tests options, such as recommended testing guidelines and available test options (Shapiro et al., 2012). In another study conducted by Reynolds, Bissett, and Consedine (2018), the authors have also found that older participants were more likely to discuss cancer symptoms amongst their peers, effectively sharing information about CRC screening adherence. Collectively, these factors promote adherent behaviours towards CRC screening.

While the majority of the research within the literature suggests that participants are more likely to adhere to screening guidelines as age increases, there have been extended studies conducted that highlights that participants are in fact less likely to adhere to screening guidelines (Taskila et al., 2009). In a sample study conducted in United States, Denberg et al. (2005) found that older participants regarded themselves too old for testing and considered the drawback of treatment too large, resulting in non-adherent behaviours. In particular, these studies have cited factors relating to old age such as lack of financial means (cost of testing and treatments) as well as preferences and inconvenience factors such as discomfort in testing and inconvenience in testing and recevieing subsequent treatment. These factors suggest that elderly participants intentionally choose not to screen for cancer due to age-related reasons

and may potentially be more adherent towards screening guidelines when they are younger. Hence, this apparent conflict within the literature calls for further investigation to arrive at a consistent conclusion and we hypothesize that participants are more likely to screening for CRC as they age, up until a certain threshold, before being less adherent towards screening decisions.

Hypothesis 2.1. *The age of a participant exhibits an inverse U-shaped relationship with CRC screening adherence.*

2.2.2 Perceived Risk of Developing CRC

The next hypothesis relates to a participant's perception of her individual risk of developing CRC. Based on the review of the literature and discussions with healthcare professionals, one of the key motivation towards screening decisions lies in how an individual perceived her individual risk of developing CRC - where a participant's perceived odds of developing CRC increases, her adherence towards screening likewise increases (Hughes, Watanabe-Galloway, Schnell, & Soliman, 2015). However, the literature has been inconsistent in the approaches to assess an individual's perceived risk. In a study conducted by Hay, Coups, and Ford (2006), the authors sought to determine participants' perceived risk of developing CRC without providing any prior information. Our study has instead chosen to provide the national statistics on an average individual's risk of developing CRC before formally asking participants to assess their own level of risk. By doing so, we provide participants with the opportunity to make an informed assessment of their risk of developing CRC. From the response that they provide, we are also able to infer the participant's risk attitude by comparing the response to the statistic that we had earlier provided to them.

Although results from prior literature have collectively shown increased screening adherence as an individual's perceived risk of CRC increases, a recent stream of

literature suggests that participants may intentionally defer from learning their health status. In particular, Golman, Hagmann, and Loewenstein (2017) introduced the phenomenon of information avoidance and proposed that under certain conditions, individuals will choose not to obtain knowledge that is freely accessible. Specifically in the case of cancer diseases, the authors highlighted that individuals may choose not to obtain information on their health status if they believe themselves highly likely to have developed the disease, even if such information was freely accessible. According to the authors, individuals behave in this manner in order to avoid the feelings of worry and anxiety that usually accompany a diagnosis of disease. This suggests that while an individual's screening adherence will increase as her perceived risk of developing CRC increases, up to a threshold, before intentionally choosing not to adhere to screening guidelines. This sets up the following hypothesis.

Hypothesis 2.2. *An individual's perceived risk of CRC exhibits an inverse U-shaped relationship with CRC screening adherence.*

With respect to risk measures, information describing the statistics of CRC can also play a part in how an individual perceive her risk towards CRC. At present, most medical campaigns often rely on probability statistics to highlight the severity of the illness or disease. For example, conventional medical collaterals tend to feature the following: '1 in 20 will develop CRC in their lifetimes' or '6% of CRC patients detected their cancers at later stages'. However, an individual's proficiency in probability theory can affect his interpretation of statistical information and impact screening decisions. When presented with the same set of statistics, different individuals may interpret the information and perceive their risk levels differently. This interaction between an individual's probability literacy and his/her perceived level of risk can impact screening adherence. This leads us to the following hypothesis:

Hypothesis 2.3. *The interaction between probability literacy and an individual's perceived risk of CRC has a positive association towards CRC screening adherence.*

Simply, the hypothesis implies that as probability literacy increase, the marginal effect of perceived risk of CRC on the adherence towards cancer screening increases, and vice-versa.

2.2.3 Knowledge of CRC

The impacts of participants' knowledge of CRC on screening adherence have been discussed widely within the literature (Arnold et al., 2012; Greiner, Born, Nollen, & Ahluwalia, 2005). The majority of the studies argued that participants with deeper CRC knowledge can better understand the risks and treatments available (Bardach, Schoenberg, Fleming, & Hatcher, 2012; Christou & Thompson, 2012; Katz et al., 2004) and as a result, are more likely to screen for CRC. On the other hand, using multivariate models, Weinberg et al. (2009) found that higher basic knowledge of CRC screening and guidelines were not significant predictors towards screening intent and suggests that implementing education interventions alone will not be adequate in raising screening compliance. However, these studies are unique to the sample population of the study. Our study aims to examine these factors and investigate the impacts of CRC knowledge and awareness can have towards participation towards CRC screening and introduce the following hypothesis:

Hypothesis 2.4. *A participant's knowledge of CRC has a positive association towards CRC screening adherence.*

2.2.4 Trust in Government

Finally, we discuss the relationship between participants' trust in government and CRC screening adherence. At present, most national screening programs are spearheaded by the government and commonly associated with the nation's health ministry. This association may cause individuals with a strong distrust towards the government to reject participation in the screening programs (Ward, Coffey, Javanparast, Wilson, & Meyer, 2015). Douma, Uiters, and Timmermans (2018) found that trust in the government regarding national screening programmes is associated with positive attitude towards CRC screening, leading to higher screening adherence. By studying the factors of trust in government, our study can influence policy decisions relating to government efforts in encouraging screening behaviours. We hypothesize that participants who trust the government would participate in cancer screening and adhere to the policy's guidelines. Thus, we propose the following:

Hypothesis 2.5. *A participant's level of trust in the government has a positive association towards CRC screening adherence.*

2.3 Research Methodology

To test the proposed hypothesis, we conducted a nationwide survey in Singapore. In this section, we provide detailed information on the data collection process. In particular, there are two main phases: data collection and data pre-processing.

2.3.1 Data Collection - Survey

Survey participants are recruited by the Centre of Research on Successful Ageing (ROSA), a research institute in Singapore Management University which seeks to better understand the economic and social well-being of the older population in Sin-

gapore. The participants recruited are either a citizen or Permanent Resident between the ages of 50-70 and the members collectively form the Singapore Life Panel (SLP). The profile of ROSA's SLP aligns with the CRC screening eligible population that our study aims to target. Under ROSA, SLP members will be invited to participate in a monthly wave, which consists of many survey modules besides our CRC screening module, including employment status and outlook, financial literacy, health and life satisfaction, as well as views on government policies. The survey is administered via a digital platform or tele-conversation. Through our access to SLP's data bank, we extracted various responses from past modules to include in our study, such as participants' trust in government, risk preference, and other demographic variables.

Colorectal Cancer Screening Module

The CRC screening module was conducted from February 2019 to May 2019 and our study was approved by the Institutional Review Board (IRB) at Singapore Management University. Our survey consists of questions regarding individuals' perception on colorectal cancer (CRC) and its screening guidelines. In particular, the survey was divided into 3 parts: (1) participants' demographic information and barriers towards CRC screening; (2) awareness on CRC symptoms and perception towards developing CRC and polyps; and (3) knowledge towards CRC risks, test options and screening guidelines. Given that the nature of our topic is considered sensitive, participants were given the choice to skip questions they find uncomfortable. Lastly, participants who complete our survey module will receive a SGD5 voucher. For a detailed sample of the survey, the reader is referred to Appendix B.

Under the first section of our module, the survey assessed patient demographic information, such as previous history of cancer, family history and history of chronic illnesses. We assessed potential barriers towards Faecal Immunology Test (FIT) which include lack of awareness, uncomfortable with the test procedures and unreliable test

results for FIT testing. We also considered the potential barriers towards colonoscopy which include high cost, embarrassment, age, discomfort with testing procedures, wait time for an appointment, amongst others. A follow up question was administered for each of the chosen barrier to assess how strongly they feel about that particular barrier.

In the second section, we tested the individual's level of awareness towards CRC of our participants. To this end, participants were quizzed on their knowledge on CRC symptoms through a variety of yes-no questions relating to potential symptoms such as 'Bleeding from back passage' and 'Persistent pain in your abdomen'. We further assessed participant's awareness towards CRC by their perception towards developing CRC through a 5-point range scale on behavioural patterns such as 'Drinking more than 1 unit of alcohol a day', 'Having a diet low in fibre' among others.

Our final section of the survey features a questionnaire relating to CRC screening, symptoms, and testing guidelines in three broad categories: knowledge in FIT testing guidelines, CRC symptoms and risks, and colonoscopy testing guidelines. Each question allocates 1 point and the aggregate score represents the individual's knowledge towards CRC.

2.3.2 Data Pre-processing

Our survey module invited a total of 7,540 eligible participants from the panel and collected a total of 3,920 responses (52%) at the end of our survey, with 3,914 respondents consenting the use of their responses for the purpose of research. From the consented responses, we crossmatch with other waves to obtain their responses in our modules, which include risk-appetite, probability literacy, income level, BMI, medical conditions, financial goals, among others. We subsequently compared the demographics of our sample respondents to the population Census data and found that the respondents were consistent with the population profile. The median age of

the respondents is 62 and 53% of the respondents are female. The racial mix are as follows: 89% Chinese, 4% Malay, 5% Indian and 2% others. Median income is $880 and 79% of the respondents are married. In terms of screening rates, 48% of the participants have ever participated in FIT screening, with only 12% of the participants currently up-to-date for regular FIT screening based on the SFL guidelines. Of those who participated in FIT screening, only 74% of the participants who received positive FIT results followed up with a doctor consultation. A detailed list of variables is presented in Appendix (5.1).

All data were analyzed using STATA 16.1. At the end of our survey module, we found that missing values accounted for less than 5% for every question which qualifies us to treat these missing values through multiple imputation using the multivariate imputation by chained equations (MICE). According to Schafer and Graham (2002), multiple imputation accounts for the statistical uncertainty compared with the single imputation and these missing values are imputed based on the observed data for a single individual and the relations observed in other participants. Hence, we performed a MICE on the continuous and categorical variables in our model and applied a predictive model (logistic regression) for the binary variables. For a more detailed explanation on MICE, we refer the reader to the work from Azur, Stuart, Frangakis, and Leaf (2011).

We further considered selection bias within our population sample. Specifically, we accounted for incomplete coverage of the target population, either by lack of access or refusal nonresponse for our survey module. To this end, we applied a response propensity weight adjustment introduced by Brick (2013) by taking the inverse of the estimated propensities of the respondents before performing a multi-variate propensity-weighted stepwise logistic regression to investigate the determinants of cancer FIT screening adherence. Lastly, we standardized the variables to efficiently compare the results between different types of variables.

Table 2.1: List of Main Variables

Variable	Description
Age	Age of participant
Age^2	Quadratic term of Age
P_riskcrc	Individual perceived risk of developing CRC
$P\_riskcrc^2$	Quadratic term of P_riskcrc
P_riskcrc × PScr	Interaction between P_riskcrc and probability literacy
FIT_Knowledge	Knowledge on FIT testing
Screening_Knowledge	Knowledge on Screening
Colo_Knowledge	Knowledge on Colonoscopy testing
Trust_gov	Participant's factor of trust in Government

2.4 Stepwise Logistic Regression and Results

2.4.1 Model

Using regression analysis, we test our hypotheses which propose relationships between the variables of interest and FIT participation. Specifically, we test Hypothesis 2.1 to 2.5 using a stepwise logit model illustrated in equation (2.1).

$$\begin{aligned} \textbf{Base: In(FIT)} =& \alpha_1 Age + \alpha_2 Age^2 + \alpha_3 P\_riskcrc + \alpha_4 P\_riskcrc^2 \\ &+ \alpha_5 P\_riskcrc \times PScr + \alpha_6 FIT\_Knowledge \\ &+ \alpha_7 Screening\_Knowledge + \alpha_8 Colo\_Knowledge \\ &+ \alpha_9 Trust\_gov + \nu \textbf{Control\_Variables} \end{aligned} \tag{2.1}$$

In order to test H2.1, we introduced the squared term of age to study if a quadratic impact of age on screening adherence is significant. Similarly, the squared term of perceived risk of developing CRC represented in fourth term of model 2.1 with the same intention to test H2.2. Testing H2.3 requires the interaction term of perceived risk of CRC and probability literacy which is represented by the fifth term. In test-

ing the impact of participant's knowledge of CRC towards screening adherence, we capture the variables of FIT knowledge, screening knowledge and colorectal testing knowledge through the sixth, seventh and eighth term respectively. Lastly, H2.5 is examined through the factor of trust in government, represented by the ninth term. In the analysis, we control for demographics varaibles which includes gender, citizenship, highest academic qualification, presence of chronic disease, weekly physical activity time, emotional health, perceived health condition, as reported in Appendix (5.1).

2.4.2 Results and Discussion

Under the results of the base model illustrated in table (2.2), the positive coefficient on age ($\alpha_1 = 0.1711$) and negative coefficient on age^2 ($\alpha_2 = -0.1445$) are significant predictors on screening adherence, thereby supporting H2.1. Earlier works described in section 2.2.1 have shown competing linear (negative and positive) relationships without a conclusive result. By including the quadratic term Age^2, our result is not only consistent with current literature, but also provides a coherent explanation for the competing linear relationships found in the literature. In the succeeding section, we extend the analysis to provide managerial insights for policy makers by developing a mediation model to study drivers of the nonmonotonic trend.

H2.2 is supported by the results on P_riskcrc ($\alpha_3 = 0.2583$) and P_riskcrc^2 ($\alpha_4 = -0.0417$). Another key observation reflected in the result is the inverse U-shaped relationship between an individual's perceived risk of CRC and FIT screening. Specifically, the 'information avoidance' phenomenon exists in this sample group, especially in high perceived risk individuals where these participants intentionally choose not to screen for cancer despite the screening being freely and easily accessible as they do not wish to learn of their cancer status. Our results also suggest that participants past a perceived risk threshold will more likely opt out of screening in order to avoid

learning of their cancer status.

In our analysis, CRC knowledge variables comprising of knowledge in FIT testing ($\alpha_6 = 0.2893$), CRC screening knowledge ($\alpha_7 = 0.1059$), and knowledge of colonoscopy testing ($\alpha_8 = 0.1162$) are statistically significant towards FIT screening adherence in the analysis which support H2.4. Within the base model (2.1), we evaluate the linear predictors CRC knowledge through survey questionnaire. Consistent with the literature, we find that higher knowledge on CRC screening and the tests available are positively associated with CRC screening adherence. Given that most screening programs are spearheaded by the governments, policy makers can raise awareness and CRC knowledge through publicity campaigns to spread public awareness and promote cancer screening adherence (Gimeno Garcia, Hernandez Alvarez Buylla, Nicolas-Perez, & Quintero, 2014). Among various interventions, Cameron, Persell, Brown, Thompson, and Baker (2011) and Kramish Campbell et al. (2004) conducted RCTs and reported higher screening rates through the distribution of informational or educational small media, such as pamphlets, brochures, newsletters, and videos on cancer screening information. Similarly, other research has shown that one-to-one education by health professionals (Menon et al., 2011; Tu et al., 2006) coupled with the delivery of information about benefits, how to overcome barriers to screening and providing motivation in a group setting (Powe, Ntekop, & Barron, 2004) are effective interventions.

Furthermore, participants' factor of trust in the government ($\alpha_9 = 0.1378$) is significant in predicting screening compliance. Consistent with the literature, our results reflects how the government has a substantial influence on screening adherence.

In testing H2.3, the results of the analysis reports a negative coefficient of the interaction term between probability literacy and individual perceived risk of developing CRC ($\alpha_5 = -1.1097$), failing to support H2.3. As mentioned at the outset, aside from the hypotheses examined in this study, the literature does not address how an

individual's proficiency in probability theory might interact with his perceived risk of CRC. Counter to conventional intuition, the negative coefficient asserts that as participant's probability increases, the marginal effect of individual perceived risk of developing CRC on screening adherence decreases, suggesting that participants who can better understand their risk are more likely to choose not to screen for cancer. The detailed results of the stepwise logistic regressions of the base are presented in table (2.2).

Lastly, in our final step of the statistical analysis, we tested the performance of our model using a receiver operating characteristics (ROC) curve. We segment 70% of the data to train the model and relied on the remaining 30% of the data to test our base modelmodel. Our final results report training and testing AUC performance of 0.7028 and 0.6856 respectively.

2.5 Investigation of the Non-Monotone Effect of Age

Motivated by the analysis on the basic model which provides support for Hypothesis 2.1, we were keen to investigate policy driven factors that drives the nonmonotone trend. Drawing from the literature, we find that noncompliance towards screening guidelines largely involves older participants, citing reasons of discomfort in testing and considering the drawbacks of treatments too large. These observations prompts us to consider if the quadratic trend of age is driven by older participants lacking of financial means and subsequently dominating the results.

Upon closer inspection, we observed the critical point of the quadratic trend of age exist at 66.37 years of age, which incidentally coincides with the reemployment age[1]in Singapore. This observation reaffirms our decision to consider financial factors

	Basic Model	95% CI
Age	0.1711***	(0.0933, 0.2490)
Age^2	-0.1445***	(-0.1968, -0.0921)
P_riskcrc	0.2583***	(0.0879, 0.4287)
$P\_riskcrc^2$	-0.0417**	(-0.0813, -0.0020)
P_riskcrc × PScr	-0.1097**	(-0.1973, -0.0212)
FIT_Knowledge	0.2869***	(0.2123, 0.3614)
Screening_Knowledge	0.1059**	(0.0347, 0.1771)
Colo_Knowledge	0.1162**	(0.0470, 0.1853)
Trust_gov	0.1378***	(0.0657, 0.2100)
Gender		
Female	1	
Male	-0.1675**	(-0.3172, -0.0178)
Non citizenship	-0.4144**	(-0.7236, -0.1052)
AcadQual		
No Schooling	1	
Secondary	0.6619***	(0.4606, 0.8631)
University	1.180***	(0.9724, 1.389)
Chronic_Illness	0.3547***	(0.1456, 0.5638)
BMI	-0.1009**	(-0.1747,-0.0270)
Phy_Act	0.1226**	(0.0506, 0.1946)
Alcohol	0.3016***	(0.1473, 0.4559)
Emotion	-0.0849**	(-0.1645, -0.0053)
Symp_awareness	0.1432***	(0.0732, 0.2133)
Perceived_health_con	0.1082***	(0.0285, 0.1879)
Observations	3914	

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2.2: Logit Results

in the mediation analysis.

We turn to the literature and found that there is a large body of literature exploring the link between financial means and the age of participants towards screening

[1]In Singapore, employers must offer reemployment to eligible employees who turn 62, up to the re-employment age of 67, thereby providing older workers with more opportunities to work longer

adherence. Namely, these studies explored income levels and financial savings towards screening decisions. However, for the CRC screening eligible individuals, a considerable proportion have retired and income may not be a suitable indicator of the participants' financial position. Instead, the ability to afford the cost of testing and treatments heavily depends on their (i) retirement accounts, and (ii) ownership insurance that offsets these associated costs for the older participants. These factors motivates the study to provide explanations to H2.1 and investigate possible policy interventions.

In Singapore, the Central Provident Fund (CPF) is a compulsory savings and pension plan for working Singaporeans primarily to fund their retirement, healthcare, and housing needs in Singapore. Typically, medical needs constitute the largest expense for retired citizens and we predict that CPF balance will impact screening decisions for older participants. Another financial lever commonly used to substitute for CPF to offset medical expense is private insurance. However, private insurance increasingly becomes less affordable for older participants due their increased risk of illnesses. Often, this expense to own private insurance become even less affordable as the participant retires, calling for further investigations on these variables. Therefore, based on the link between financial means and age of the participant, we postulate the following:

Hypothesis 2.6. *CPF balance mediates the relationship between age and CRC screening adherence.*

Hypothesis 2.7. *Private insurance mediates the relationship between age and CRC screening adherence.*

2.5.1 Methodology - Mediation Model

We develop a mediation model to test the mediating relationships outlined in H2.6 and H2.7. Specifically, the mediation analysis tests if the relationship between two variables can be explained by an intermediate variable (Baron & Kenny, 1986). By performing a mediation analysis, researchers can determine how certain relationships occur and in theory, explain statistical interventions. The mediation analysis can be expanded by incorporating multiple mediators (Lange, Rasmussen, & Thygesen, 2014).

The test mediation relationship requires three steps. In the first step, the variables of age must predict the mediating variables - CPF account balance and ownership of private insurance. The second step of the mediation analysis tests the impact of the dependent variable (DV) on both the independent variables (IV) and the mediators. The final step involves testing the indirect effect of the IV on the DV through the mediators - where the subsequent result of the DV on the IV will determine the impact of mediation. Under a fully mediated model, the factors of age must become nonsignificant when controlling for the mediating variables, while a mediated model is considered partial if the confidence interval of the indirect effect does not include zero. Models (2.2) and (2.3) below conducts Steps 1 while model (2.4) performs Step 2 respectively. We conduct step 3 by testing for the indirect effect and state the results in the succeeding subsection.

$$\begin{aligned}\textbf{CPF\_Balance} =& \zeta_1 Age + \zeta_2 Age^2 + \zeta_3 P\_riskcrc + \zeta_4 P\_riskcrc^2 + \zeta_5 P\_riskcrc \times PScr \\ &+ \zeta_6 FIT\_Knowledge + \zeta_7 Screening\_Knowledge \\ &+ \zeta_8 Colo\_Knowledge + \zeta_9 trust\_gov + \eta \textbf{Control\_Variable}\end{aligned} \tag{2.2}$$

$$\begin{aligned}\textbf{Private\_Ins} =&\gamma_1 Age + \gamma_2 Age^2 + \gamma_3 P\_riskcrc + \gamma_4 P\_riskcrc^2 + \gamma_5 P\_riskcrc \times PScr \\ &+ \gamma_6 FIT\_Knowledge + \gamma_7 Screening\_Knowledge \\ &+ \gamma_8 Colo\_Knowledge + \gamma_9 trust\_gov + \delta \textbf{Control\_Variables}\end{aligned} \tag{2.3}$$

$$\begin{aligned}\textbf{Mediation: In(FIT)} =&\beta_1 Age + \beta_2 Age^2 + \beta_3 P\_riskcrc + \beta_4 P\_riskcrc^2 \\ &+ \beta_5 P\_riskcrc \times PScr + \beta_6 FIT\_Knowledge \\ &+ \beta_7 Screening\_Knowledge + \beta_8 Colo\_Knowledge \\ &+ \beta_9 trust\_gov + \beta_{10} CPF\_Balance + \beta_{11} Private\_Ins \\ &+ \xi \textbf{Control\_Variables}\end{aligned} \tag{2.4}$$

2.5.2 Mediation Result and Discussion

The results of the mediation analysis illustrated in table(2.3) verified the statistical significance of the impacts of the mediators. Both CPF Account Balance ($\beta_{10} = 0.2522$) and ownership of private insurance ($\beta_{11} = 0.1765$) are significant predictors in model (2.4). In addition, the difference in the coefficients on the factors of age are statistically significant. The test of indirect effects of the mediators of CPF account balance (-0.02, $p < 0.01$) and private insurance (-0.10, $p < 0.05$) are statistically significant. Collectively, these results suggests that CPF account balance and ownership of private insurance partially mediates the effect of the factors of age on FIT screening adherence. An illustration of the mediation network is presented in figure (2.1).

Our mediation analysis determines the extent to which the effect of age is me-

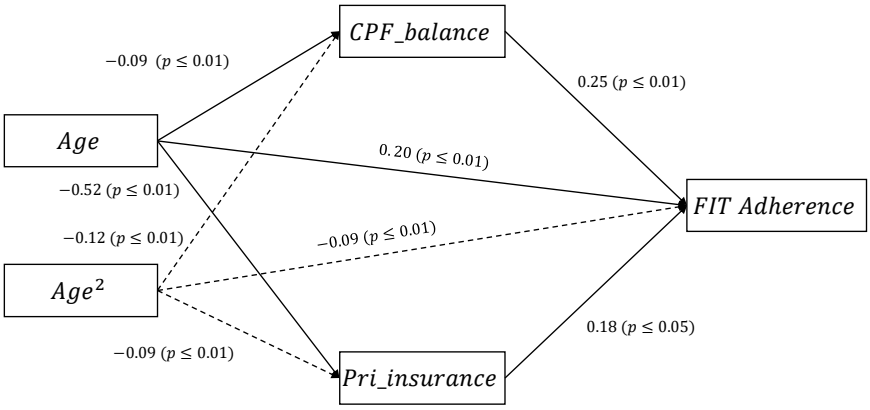


Figure 2.1: Mediation Network

diated by higher CPF account balance (H2.6) and ownership of private insurance (H2.7). We find evidence for partial mediation, which suggests that there could be other mechanisms aside from higher CPF balances and ownership of private insurance underlying the effect of age and age^2 on FIT screening. Nevertheless, the partial mediating impact of CPF balance and private insurance supports our hypothesis that participants are more likely to screen for cancer when in possession of financial safety nets in the form of an adequate CPF balance and/or a private insurance plan as they age.

Our finding on the mediating variables offers policymakers several levers to effect policy changes that could potentially raise screening adherence levels. As a first lever, policymakers can choose to raise the minimum balance requirements on retirement accounts so that more funds would be set aside to offset medical expenses for older citizens. This has in fact been implemented by the Singapore government, where CPF minimum balance requirements were previously raised in order to better fund retirement plans (Central Provident Fund, 2021). The second lever which policymakers can utilize is to mandate a national health insurance program with a coverage that is sufficiently comparable to private health insurance plans. For countries with existing national health insurance programs, policymakers should look into closing the gaps

in the scope of coverage between its national health insurance plan and a private insurance plan. While the government might incur costs in doing so, our position is that society is better off as net social cost will be lowered. Research has shown that the social cost of CRC is far lower when it can be identified and treated at earlier stages (Lansdorp-Vogelaar, Van Ballegooijen, Zauber, Habbema, & Kuipers, 2009).

We also tested the performance of the mediation model and report a training AUC of 0.7155 and a testing AUC of 0.6926. Finally, as an addition layer of robustness check, we compared the results of the mediation model without imputation and the results of the mediators still hold.

2.6 Conclusion and Future Work

The findings from this study helped to elucidate the non-linear impacts of age, linear determinants, and the interactions of probability literacy on perceived risk towards CRC screening participation and adherence in Singapore. Specifically, our results show that (i) knowledge and awareness of CRC, and individual's trust in the government were significantly associated with intentions to screen for CRC, (ii) balance in CPF accounts and ownership of private insurance are significant mediators towards age, and (iii) proficiency in probability literacy significantly impacts how an individual perceives her risk and consequently the adherence towards cancer screening.

Returning to the study's specific motivation of raising screening adherence, the rich data set collected from the nationwide survey presents multiple future works. First, the current study investigates the average effect within the population. However, based on our findings, we observe that particular subgroups of the participants have varied preferences and habits. This sets up further opportunities to investigate mulitple subgroups within the population to evaluate salient barriers for each group separately and proposed tailored interventions. Second, the survey features an RCT

	Mediation Model	95% CI
CPF_Balance	0.2522***	(0.1756, 0.3288)
Private_Ins	0.1765**	(0.0015, 0.3514)
Age	0.2042***	(0.1243, 0.2841)
Age^2	-0.0899***	(-0.1389, -0.0408)
P_riskcrc	0.2411***	(0.0693, 0.4130)
$P\_riskcrc^2$	-0.0379*	(-0.0779, 0.0021)
P_riskcrc × PScr	-0.1070**	(-0.1966, -0.0174)
FIT_Knowledge	0.2899***	(0.2151, 0.3646)
Screening_Knowledge	0.0998***	(0.0285, 0.1712)
Colo_Knowledge	0.1190***	(0.0495, 0.1886)
Trust_gov	0.1269***	(0.0542, 0.1997)
Gender		
Female	1	
Male	-0.1929**	(-0.3439, -0.0419)
Non citizenship	-0.3478**	(-0.6550, -0.0406)
AcadQual		
No Schooling	1	
Secondary	0.5702***	(0.3665, 0.7740)
University	0.9635***	(0.7460, 1.1811)
Chronic_Illness	0.3362***	(0.1263, 0.5461)
BMI	-0.0981**	(-0.1730, -0.0233)
Phy_Act	0.1141***	(0.0418, 0.1864)
Alcohol	0.2386***	(0.0822, 0.3949)
Emotion	-0.0724*	(-0.1529, 0.0082)
Symp_awareness	0.1349***	(0.0644, 0.2055)
Perceived_health_con	0.1166***	(0.0362, 0.1969)
Observations	3914	

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2.3: Logit Results

component to study the effects of education in relation to screening intent. Participants were divided into two groups where participants in the control group were not shown the results of a questionnaire consisting of CRC knowledge and testing. On the other hand, participants in the treatment group were provided the correct answers to the questionnaire, and both control and treatment groups were subsequently asked to indicate their intent towards CRC screening.

In conclusion, results from the current study also emphasize the importance of designing multifaceted interventions to improve CRC knowledge and screening compliance. To achieve national screening goals, a concerted effort to develop financial nets to needs and concerns of eligible screening participants is needed. The results of this study point to potentially useful directions in this important effort.

Chapter 3

Inventory-Responsive Donor Management Policy: A Tandem Queueing Network Model

3.1 Introduction

Blood transfusion is an essential aspect of many medical treatments. In the United States alone, approximately 21 million blood components are transfused every year, saving over four million lives annually in the process (Satyavarapu & Wagle, 2020). The global demand for blood is also on an upward trajectory, fuelled primarily by increased complexities in medical procedures and ageing populations. Consequently, blood donation and its management remain an integral part of the medical infrastructure. In addition to traditional difficulties in the management of blood, like its short shelf life and variability in its demand and supply, recent trends also exacerbate the challenges, such as shrinking eligible donor populations observed in many countries (Greinacher, Fendrich, & Hoffmann, 2010; Müller-Steinhardt, Weidmann, & Klüter, 2017). These pressures on both the demand and supply side call for more effective

blood supply management to meet growing demand while minimizing wastage.

In managing the supply of blood, the decision-maker possesses two main control levers: recruiting new donors and ensuring the regularity of donation amongst existing donors (*i.e.*, donor retention) (Mugion, Pasca, Di Di Pietro, & Renzi, 2021). Our study is motivated by Singapore's blood donation landscape, where there are significant challenges to the recruitment of new donors, rendering the first lever less effective (Health Sciences Authority, 2020). This is also observed in other developed states (World Health Organization, 2017). Consequently, the central blood bank will need to focus on the management of the existing donor pool. This is often conducted through donor incentivization, with the goal of increasing the regularity of donation and preventing drop-outs. While there is ample discussion in both literature and practice on the adoption of economic rewards to motivate blood donations (Lacetera, Macis, & Slonim, 2013, 2014; Sun, Gao, & Jin, 2019), many nations adopt the World Health Organization's (WHO) stance against monetary incentives (World Health Organization, 2020). This reasoning is based on the fact that economic rewards may spur donation for the wrong reasons and, in some cases, the withholding of important health information that can compromise blood inventory safety. Singapore adopts a similar stance. Hence, our paper primarily focuses on incentivization mechanisms that are altruistic in nature. One of such mechanism is proposed in Heger, Slonim, Garbarino, Wang, and Waller (2020), where donors are invited to join a "Registry" to be called up when there is a shortage. The empirical study found that registered donors have significantly higher likelihood of donations. Nonetheless, the question remains if doing so is optimal. In our paper, we consider a general incentivization mechanism that changes the *likelihood of donation* and aim to optimizing such a mechanism, so as to minimize blood shortage and wastage.

Formulating such an incentivization scheme can be difficult. In particular, it faces three main challenges. Firstly, the demand and supply of blood can vary over time.

For example, in Singapore, donation rates are observed to fall during the Lunar New Year period due to cultural beliefs. This leads to periodic shortages of supply during festivities and cultural celebrations. Planning for such variations is crucial, as seen in the scale-up of incentivization weeks before such periods (Shi et al., 2014). Secondly, there are critical hard time-based constraints that must be observed. Specifically, donors must observe a fixed duration of time between consecutive donations for health and safety reasons. This is termed the 'observation window' and is commonly three months in most countries (Health Sciences Authority, 2019; National Health Service, 2020b). However, blood units have an expiration window of 42 days, which is significantly shorter than the three-month observation period. The longer timescale in the supply than the demand indicates intrinsic bottlenecks in mustering enough supply to meet demand at short notice. As such, forward planning becomes critical. The combination of this and the previous challenge on time-variability dictates the need for a multi-period model that is anticipatory and reacts dynamically to the existing and expected inventory levels. Consequently, in our paper, we shall specifically study the donor management problem in the transient, multi-period setting that incorporates blood expiry windows and fixed observation windows between donations. Lastly, while the ultimate goal of incentivization optimization is to change the supply and inventory to the desired level, it can be challenging to explicitly characterize the impact of incentivization on the eventual blood supply and inventory. This is because incentivization alters donors' donation patterns and their likelihood of donation. This uncertainty in how donors react to the incentivization, in turn, influences future incentivization decisions. The complex *endogeneous* interaction between the decision to incentivize and the uncertainty in the donations needs to be carefully modeled to arrive at an accurate description of the blood inventory.

Key Approaches in the Literature

Blood supply chain management has been studied widely and extensively (Beliën & Forcé, 2012; Gunpinar & Centeno, 2015; Karaesmen, Scheller-Wolf, & Deniz, 2011). Here, we review the most related literature on both donor management and blood inventory management, which will be relevant to our work as we attempt to model both aspects. Broadly, we shall discuss the literature based on the adopted methodologies in three streams, namely dynamic programming, queueing and the work on Bayesian Persuasion (Lingenbrink & Iyer, 2019).

Dynamic Programming Approach: Dynamic programming is frequently seen in the literature as it can be applied to model the perishable inventory process. Nahmias and Pierskalla (1973) studied the setting of a perishable product with a two-period product life and characterized optimal ordering policy facing random demand. This was extended to m periods (Nahmias, 1976). Prastacos (1981) developed an optimal policy to allocate perishable inventory to demand from n locations, minimizing both expected shortages and outdates under random supply and demand. Chen, Li, and Zhou (2019) considered a blood center facing two different demand streams with different freshness requirements and characterized the structure of the optimal blood collection quantities and inventory policies. Ayer, Zhang, Zeng, White III, and Joseph (2019) studied blood collection operations and obtained near-optimal blood collection schedules through a two-interval Markov decision process (MDP) formulation.

Moving to our setting of donor incentivization, it would be difficult to implement a dynamic programming approach. The presence of finite time windows necessitate the tracking of the age of blood and donor observation times in the state space, drastically increasing its complexity. Together with the complex dynamics of donor eligibility, it would be difficult to avoid tractability challenges.

Queueing Approach: The other common approach in the literature is to model the dynamics as a queueing system. Graves (1982) studied a single queue and server

inventory system with constant replenishment and exponential demand stream. The authors postulate that the lifetime on the oldest unit in stock is equivalent to a virtual waiting time of an M/M/1 queue under a first-in-first-out (FIFO) policy. Goh, Greenberg, and Matsuo (1993) considered a model with two classes of demands, where each of the classes represents separate demand streams. Sarhangian, Abouee-Mehrizi, Baron, and Berman (2018) extended the literature by incorporating inventory freshness within the system and evaluating a threshold-based blood allocation policy that considered the age of the blood units.

These studies mainly focus on characterizing and evaluating the queueing process. Hence, they are not readily amenable to the question of policy design. The assumption of the steady state, while useful in the analysis of the queueing system, does not gel with the fundamentally transient and state-dependent nature of the problem we intend to consider.

Bayesian Persuasion Approach: The Bayesian persuasion approach has also been studied as means for modelling behavioural responses to incentives and signals in the setting of queues and inventory networks. Specifically, the most related work, Lingenbrink and Iyer (2019), considered revealing information to participants within an unobservable system to encourage or discourage participation. Relying on steady-state assumptions, the authors designed a state-dependent policy while considering behavioural patterns. However, this approach will be difficult to accommodate the blood donation features, particularly the safety observation window and perishable nature of blood inventory. Furthermore, similar to the queueing literature, the reliance on steady-state assumption leaves little room for optimization in the transient setting.

3.1.1 Main Approach and Contributions

The literature is particular scant on how donor dynamics affect the supply of blood. Blood supply is often either modeled as stochastic with a known distribution, or as a decision variable. The omission of donor dynamics fundamentally renders it challenging to examine the impact of donor incentivization, and hence extending the existing models in the literature is untenable.

To this end, we instead adopt the paradigm recently introduced in Pipeline Queues (P-Queues, for short, Bandi & Loke, 2018). In this paper, the authors introduced a new framework specifically targeting problems that involve the optimization of flows within a queueing network. The framework is introduced in the transient setting, leads to state-dependent policies and is polynomial-time solvable. We believe this fits the needs of our intended problem. This is illustrated by the two works in the literature that apply the technique, namely Tang, Zhang, and Zhou (2020), who studied the vehicle re-positioning problem as a transient queueing network, and Zhou, Loke, Bandi, Liau, and Wang (2021), who examined the patient scheduling problem under patient re-entry. In both cases, they involve feedback loops in the network, which is especially relevant to our work, where similarly donors return for multiple donations. Despite this, both works arrived at tractable models that also perform strongly against benchmarks.

To this end, in our paper, we present a model for optimizing donor incentivization using a P-Queue model that aims to reduce the risk of blood shortages and wastage. Specifically, we make the following contributions:

a) **Model for donor incentivization**: To the best of our knowledge, our paper is the first to consider the question of optimizing donor incentivization and present a polynomial-solvable tool to generate the optimal policy. Moreover, in the numerical experiments, we present multiple insights into how the state of the inventory, the effectiveness of the incentivization, relative rate of new donor

recruitment, the demand for blood and the number of donors in observation affect the optimal policy. Finally, our model can operationalize a donor management system to optimize incentivization of two donor classes depending on their responsiveness to incentivization; and,

b) **Theoretical extensions**: Our paper advances the technique of P-Queues in three aspects. First, we propose a modification that allows relaxing some independence assumptions in the original framework (Proposition 4 and subsequent discussion). Second, we introduce the novel concept of reducing the one-period delay that exists between the queues and servers (discussion after Equation (3.2)). Finally, our model involves a coupled queueing system, which is not immediately clear could be executed from the original framework.

We also believe that our work has wider relevance to supply incentivization and perishability, such as in two-sided markets (e.g., home-sharing), and supply chains with social impacts (e.g., charity organizations and food donations). In particular, the coupled network model that we consider in our paper is able to handle both demand and supply dynamics separately, and the P-Queue technique easily handles hard temporal cut-offs, such as perishability.

Organization of the paper

The remainder of this paper is organized as follows. In §2, we introduce the basic model setup without the donor incentivizing intervention and detail the model reformulation. In §3, we present the main model that extends the basic model to incorporate the incentivization mechanism and present tractable reformulations. In §4, we design a numerical simulation study and discuss managerial implications of our model. We conclude the paper in §5 and discuss extensions and possible future

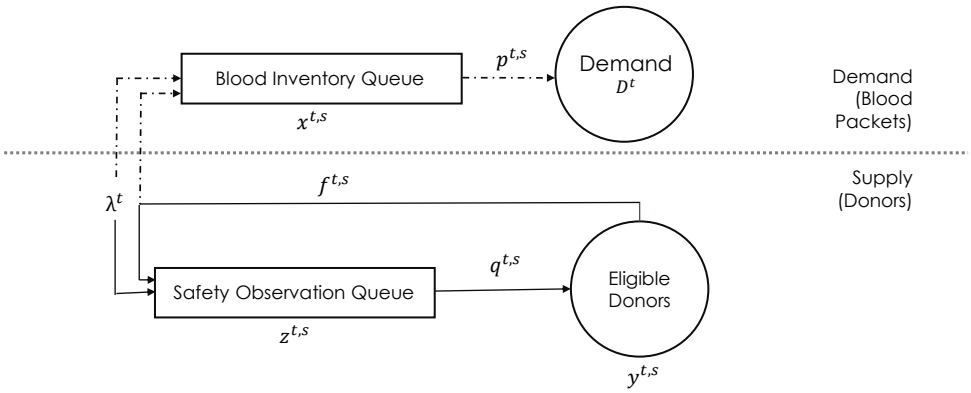


Figure 3.1: Blood Donation Network Flow

research. To keep the discourse succinct, we relegate all the proofs to Appendix 5.3.

Notation. Given $N \in \mathbb{N}$, let $[N]$ represent $\{1, ..., N\}$ and denote $[N]_0 := \{0\} \cup [N]$. We adopt the convention that $\min \emptyset = \infty$, where $\emptyset$ is the empty set and $\log 0 = \max \emptyset = -\infty$. For brevity, for a given indexed variable $p^{t,s}$, we shall abuse the notation $(1-p)^{t,s}$ to mean $1 - p^{t,s}$.

3.2 Basic Blood Donation Model

In this section, we present an integrated model of blood supply and demand with particular focus on donation behaviour and dynamics. We first ease the reader by omitting the incentivization, which we term the *basic model*, and return to the full model in §3.3. A brief flow of events is illustrated as a schematic in Figure 3.1. Although knowledge of P-Queue is not required, readers may find it helpful to refer to Zhou et al. (2021) and Tang et al. (2020).

Consider a finite time horizon $t \in [T]_0$, where $t = T$ is the last modelling time period, and $t = 0$ represents the initial state. Our model comprises two queuing networks, one modelling the eligibility of donors for donation and the other, the blood inventory. The two networks are related via the process of blood donation —for each donor who successfully completes a donation, that corresponding blood packet will

trigger an inflow for the inventory network. We term this setting a tandem network. Before proceeding, it is worth emphasizing that critical to the P-Queue framework is the introduction of two time indices to track system states. In addition to the current time period, indexed by $t \in [T]_0$, we also track "present delay" of each job in a server or a queue denoted by superscript $s \in [S]_0$, where it is assumed that $S > T$.

We first describe the network for donors. In the "eligible donors" server, each job represents a donor who has completed their safety observation window (*i.e.*, is at present eligible for donation). Specifically, let $y^{t,s}$, $t \in [T]_0$, $s \in [S]_0$ be the random variable representing the number of eligible donors in the server, who have yet to donate by time t after s periods of completing their observation window. Donation is modelled as service completion in the server. This is captured by $f^{t,s}$, the number of donors who donated at time t after s periods of completing their observation window. After donation, the donor returns to the observation queue, where they must observe a $S_o < S$ safety observation period before they are next eligible for donation. We denote the state of the observation queue with the random variable $z^{t,s}$, $t \in [T]_0$, $s \in [S]_0$, representing the number of donors who have observed exactly s periods of the S_o observation window at time t. Once the observation window is completed, the donor may rejoin the pool of eligible donors in the server. The movement of donors who have observed s periods by time t into the server is modelled by the auxiliary decision variable $q^{t,s}$, $t \in [T]$, $s \in [S]_0$. Here, by definition, $q^{t,s} = 0$, $\forall s < S_o$. Note that the dynamics is not defined for $s > S$. Donors who stay past $s = S$ are assumed to have left the donor pool as *drop-outs*. We also model new donors to the system via an inflow into the observation queue, where the assumption is that they have just donated and hence need to wait till the end of the observation window. This inflow is denoted by random variable λ^t at time t.

In the other network, we track the donated blood packets until they have expired or are used to fulfill the time non-homogeneous stochastic demand d^t. Here, each

job in the network represents one blood packet. Denote the random variable $x^{t,s}$, $t \in [T]_0$, $s \in [S]_0$ as the number of blood packets at time t that has spent s period in the blood inventory queue. Amongst these blood packets, the decision variable $p^{t,s}$, $t \in [T], s \in [S]_0$ of them will be used to fulfill demand d^t. Let $S_e (< S)$ represent the shelf life of blood. Hence, $x^{t,s}$ for any $s \geq S_e$ would contribute to wastage as a result of expired blood. The parameters and notations used in the basic model are detailed in Table 3.1.

Table 3.1: List of Parameters and Variables

		Dimensions
T	:	Last modelling time
S_e	:	Shelf life of blood packets
S_o	:	Safety observation period of donors
S	:	Index upper bound of the present delay in each server or queue
		Parameters
λ^t	:	Random arrivals of first-time donors at time t
$\omega^{t,s}$	:	Likelihood (probability) of donation of donors who waited for s periods after completing safety observation period
d^t	:	Demand of blood packets
W	:	Upper bound on total wastage by the last time period
		State and Decision variables
$y^{t,s}$	:	Random variable of eligible donors at time t, completed observation period and yet to donate for s periods
$z^{t,s}$	:	Random variable of donors observing safety observation for s periods at time t
$x^{t,s}$	:	Random variable of blood units in inventory queue for s periods at time t
$q^{t,s}$	:	Push variable of donors dispatched to donor pool after spending s periods in the observation queue at time t
$p^{t,s}$	:	Push variable of blood units dispatched to fulfill demand after spending s periods in blood inventory queue at time t

Assumption 1. *Inflow $\lambda^t \sim \Lambda^t$ and demand $d^t \sim D^t$ distributions can be time non-homogeneous, but are independent across t. We assume that their moment generating functions exist.*

These assumptions are relatively loose. Time non-homogeneity can capture seasonal patterns, and if there is a need to observe non-independent distributions across time, the model may still be solved on a rolling horizon manner, where the information of the new distribution is updated when new data is realized. We also consider the

setting where inflow and demand are bounded, which guarantees that their moment generating function exists.

3.2.1 Dynamics

We begin by describing the dynamics of the donors and start with the server consisting of eligible donors. Specifically, at time t, inflow into the server, which is equivalently all donors who have spent 0 time in the server so far, is represented by $y^{t,0}$. This is fully made up of donors who have completed their safety observation period: $y^{t,0} = \sum_{s=0}^{S} q^{t,s}$, $\forall t \in [T]$.

Before describing the rest of the dynamics of the server, which entails service completion, we need to make some assumptions about the service distribution.

Assumption 2. *The decision of each donor to donate is independent of any other donor. Furthermore, we assume that at any time $t \geq 0$, the probability of whether the donor will donate in the next time period, condition on the fact that they had not donated $s \geq 0$ periods since they became eligible, is the same for all donors; and this is denoted by $\omega^{t+1,s+1}$.*

Independence is a reasonable assumption in our setting of blood donation. At first glance, requiring the conditional probabilities of donation to be the same appears restrictive. However, note that this representation is general —any discrete-time service time distribution can be represented in the form of $\omega^{t,s}$ (Dai & Shi, 2017). Moreover, as we shall later see, due to the structure of our model, its tractability is not influenced by adding an index i to the conditional probabilities; in other words, we can define different conditional probabilities $\omega_i^{t,s}$ on sub-populations of donors.

Assumption 2 justifies the definition for all $t \in [T]$, $s \in [S]$,

$$y^{t,s} = \text{Bin}\left(y^{t-1,s-1}, 1 - \omega^{t,s}\right).$$

Proposition 1. *(a) Let* $\Omega^{t,s} \coloneqq \prod_{\tau=0}^{\min\{s,t\}-1}(1-\omega^{t-\tau,s-\tau})$*, then* $\forall t \in [T]$*,*

$$y^{t,s} = \begin{cases} \text{Bin}(y^{t-s,0}, \Omega^{t,s}) & \text{for } 0 < s < t, \\ \text{Bin}(y^{0,s-t}, \Omega^{t,s}) & \text{for } t \leq s \leq S. \end{cases}$$

(b) ***Independence of Pipelines****: For a fixed time* $t \in [T]$*, for any* $s, s' \in [S]$ *such that* $s \neq s'$*,* $y^{t,s}$ *and* $y^{t,s'}$ *are independent.*

Under Proposition 1, $y^{t,s}$ can be represented in terms of the inflows where $s = 0$ or start states where $t = 0$. Here, $\Omega^{t,s}$ represents the cumulative survival probability of not donating after s periods. This concludes the dynamics for the server.

We move on to the dynamics of the observation queue. Its inflows consist of all donors who just donated blood, both registered donors as well as new donors. Therefore, $\forall t \in [T]$,

$$z^{t,0} = \lambda^t + \sum_{s=1}^{S} f^{t,s} = \lambda^t + \sum_{s=1}^{S} \text{Bin}\left(y^{t-1,s-1}, \omega^{t,s}\right). \tag{3.1}$$

Here, the first term pertains to new donors. The second term, as a consequence of Assumption 2, can be written in terms of the Binomial distribution, via $f^{t,s} = \text{Bin}\left(y^{t-1,s-1}, \omega^{t,s}\right)$.

The rest of the dynamics for z involves the outflow from the queue as donors complete their observation window: $\forall t \in [T]$,

$$z^{t,s} = z^{t-1,s-1} - q^{t,s} = \begin{cases} z^{t-s,0} - \sum_{\tau=0}^{s-1} q^{t-\tau,s-\tau} & \text{for } 0 < s < t, \\ z^{0,s-t} - \sum_{\tau=0}^{t-1} q^{t-\tau,s-\tau} & \text{for } t \leq s \leq S. \end{cases}$$

Finally, we describe the dynamics of the blood packets in the blood inventory. Note that every inflow into the observation queue is marked by a donation. Hence, the inflow of blood packets into the blood inventory queue is exactly equal to the

inflow of donors into the observation queue. As such, the inflow dynamics into the blood inventory x would be the same as that for the blood donors z. Indeed, for all $t \in [T]$,

$$x^{t,0} = \lambda^t + \sum_{s=1}^{S} f^{t,s} - p^{t,0} = \lambda^t + \sum_{s=1}^{S} \mathrm{Bin}\left(y^{t-1,s-1}, \omega^{t,s}\right) - p^{t,0}. \tag{3.2}$$

Here, unlike for z, there is the additional third term $p^{t,0}$. This term ensures that newly donated blood packets can be dispatched to fulfill demand. Without it, all newly donated blood packets must minimally wait for one period, which is undesirable in a high-demand setting. This formulation is a novelty that is absent in the original P-Queue framework. Notice that we could have added the same term into $z^{t,0}$. However, this would not be consequential as $q^{t,0} = 0$ given the observation period requirement.

Finally, the outflow from the blood inventory corresponds to the blood packets utilized to fulfill the demand: For all $t \in [T]$,

$$x^{t,s} = \begin{cases} x^{t-s,0} - \sum_{\tau=0}^{s-1} p^{t-\tau,s-\tau} & \text{for } 0 < s < t, \\ x^{0,s-t} - \sum_{\tau=0}^{t-1} p^{t-\tau,s-\tau} & \text{for } t \leq s \leq S. \end{cases}$$

3.2.2 Constraints

The main objective of a blood bank is to fulfill the demand for blood while minimizing wastage. To handle this multi-objective problem, while it may be possible to write a combined objective that quantifies the trade-off for blood shortages against the cost of wastage, we avoid this, as it is generally difficult to prescribe such a trade-off. Instead, in the P-Queue framework, both objectives are modelled as constraints with the aim of finding a feasible policy that runs a high likelihood of meeting the demand and keeping wastage below a certain level.

At time t, demand is fulfilled by blood allocated from the inventory $p^{t,s}$ of different age s:

$$\sum_{s=0}^{S_e-1} p^{t,s} \geq d^t \quad \forall t \in [T].$$

Here, the limits run till S_e-1 as expired blood cannot be used to fulfill the demand. Hence, we also requires $p^{t,s} = 0, \forall s \geq S_e$. Notice that we modelled demand fulfillment at every time period separately, as opposed to summing over all time periods t. This is because the latter can potentially result in large shortages at a particular time point, in exchange for low or no shortages at other times.

The blood bank also has to limit the total wastage of blood over the planning horizon under some level W:

$$\sum_{s=S_e}^{S} x^{T,s} + \sum_{t=1}^{T} \left(\sum_{s=0}^{S_e-1} p^{t,s} - d^t \right) \leq W.$$

The first term represents the total expired blood packets while the second term captures the total number of excess blood packets that were over-committed to fulfilling the demand, summed over all time periods $t \in [T]$.

On servers, we can impose capacity constraints, whereas, for queues, constraints can be imposed to ensure that the queue does not build up when there is spare capacity in the server. In both cases, such constraints can be written in linear forms (3.3). This is done by $a_n^s = 1, n \in \{x, y, z\}$, where the LHS of (3.3) represent the total number of donors or blood packets in the server or queue at any given time t. The corresponding linear forms give rise to the capacity constraints and queue-clearing constraints. Note that those linear forms in (3.3) can also represent other types of constraints required to control the system, such as waiting time in the queue and a blood demand fulfillment policy. Specifically, when $a_z^s = \max\{0, s - S_o\}$, the LHS gives rise to the total waiting time after completing the observation window and before

being dispatched to the eligible donors server, which are situations we seek to reduce. Commonly adopted blood demand fulfillment policies include the FIFO policy where demand is fulfilled by the least fresh blood packet to ensure little wastage or the last-in-first-out (LIFO) policy which allocates the freshest blood packet to the demand (Sarhangian et al. (2018)). The linear forms can specify both types of policies, and there may be many ways to model them. In our paper, we adopt the FIFO demand fulfillment policy as a demonstration which is enforced by constraint $a_x^s = s^2$. LIFO can be similarly modeled (*e.g.*, $a_x^s = 1/(s+1)^2$).

$$\sum_{s=0}^{S} a_y^s y^{t,s} \leq b_y^t, \qquad \sum_{s=0}^{S} a_z^s z^{t,s} \leq b_z^t, \qquad \sum_{s=0}^{S} a_x^s x^{t,s} \leq b_x^t. \tag{3.3}$$

We also have the feasibility constraints on push variables to ensure that there will not be more donors made eligible than there are donors already in the observation queue: $z^{t,s}, x^{t,s} \geq 0, \forall t \in [T], s \in [S]_0$, which are equivalent to $q^{t,s} \leq z^{t-1,s-1}, p^{t,s} \leq x^{t-1,s-1}$. This is a special case of (3.3) where $a^s = -1$ for some s, and 0 otherwise. As we shall imminently see in the next section, this does not hinder our attempts to arrive at a tractable formulation.

Lastly, after donation, donors must observe the safety period, and cannot be pushed into the eligible donors server: $q^{t,s} = 0, \forall t \in [T], s < S_o$. For demand fulfillment, expired blood packets cannot be used: $p^{t,s} = 0, \forall t \in [T], s \geq S_e$.

Most critically, note that the constraints introduced above are all linear in the state variables $x^{t,s}, z^{t,s}, y^{t,s}$, decision variables $p^{t,s}, q^{t,s}$, and exogenous uncertainties λ^t, d^t. This will help us achieve tractable reformulations.

3.2.3 Model and Reformulation

In the P-Queue framework, stochastic constraints $\tilde{\zeta} \leq 0$ are modelled via their corresponding surrogates defined under the Aumann and Serrano (2008)'s riskiness index,

$C_{k,\theta}[\tilde{\zeta}]\left(:= k\log\mathbb{E}\left[\exp\left(\tilde{\zeta}/k\theta\right)\right]\right) \leq 0$. These surrogate constraints control the probability of violation of the original constraint, as Proposition 2 details.

Proposition 2. *Let $k, \theta > 0$. For a random variable $\tilde{\zeta}$, define:* $C_{k,\theta}[\tilde{\zeta}] = k\log\mathbb{E}\Big[\exp\left(\tilde{\zeta}/k\theta\right)\Big]$[1]. *If $C_{k,\theta}[\tilde{\zeta}] \leq 0$, then,*

$$\mathbb{P}\left[\tilde{\zeta} \geq \Delta\right] \leq \exp\left(-\Delta/k\theta\right) \quad \forall \Delta > 0.$$

From Proposition 2 we can see that both k and θ control the probability of constraint violation —the smaller k or θ, the sharper the guarantees. In the literature, k is treated as the global risk level, which we shall attempt to minimize, while θ is viewed as an idiosyncratic parameter that controls the tightness of each constraint. Here, we label each θ according to the constraint type "n" and time t, denoted as $\theta_{n,t}$. Indexing by t allows the decision-maker to flexibly vary the tightness of the constraint in time t. For example, if demand shortages at early times is more critical than later times, the decision-maker may make $\theta_{d,t}$ smaller for small t. In this paper, we consider two levels of tightness, θ_{hard}, for hard constraints that should never be violated (*e.g.* capacity constraints) and θ_{soft} for soft constraints that can be violated but should only be done so infrequently (*e.g.* waiting time). The value of θ_{hard} is much smaller than that of θ_{soft} (*e.g.* $\theta_{\text{hard}} = 0.01$ versus $\theta_{\text{soft}} = 1$). In practice, the decision-maker would calibrate θ based on the probability $\exp\left(-\Delta/k\theta\right)$ of incurring violation Δ.

Proposition 2 motivates the following optimization problem:

$$\min \quad k$$

Demand:

$$C_{k,\theta_{d,t}}\left[d^t - \sum_{s=0}^{S_e-1} p^{t,s}\right] \leq 0, \qquad \forall t \in [T]$$

[1]We adopt the convention in the literature: $C_{k,\theta}[\tilde{\zeta}] = \text{ess sup}\{\tilde{\zeta}/\theta\}$ as $k \to 0$ and $\mathbb{E}[\tilde{\zeta}/\theta]$ as $k \to \infty$.

Wastage:

$$C_{k,\theta_{W,t}}\left[\sum_{s=S_e}^{S} x^{T,s} + \sum_{t=1}^{T}\left(\sum_{s=0}^{S_e-1} p^{t,s} - d^t\right) - W\right] \leq 0,$$

Capacity, Queue-clearing, Waiting-time, FIFO:

$$C_{k,\theta_{n,t}}\left[\sum_{s=0}^{S} a_n^s n^{t,s} - b_n^t\right] \leq 0, \qquad n \in \{x, y, z\}, \qquad \forall t \in [T]$$

Push Constraints:

$$C_{k,\theta_{n,t}}\left[-n^{t,s}\right] \leq 0, n \in \{x, z\}, \qquad \forall t \in [T], s \in [S]_0$$

Logic Constraints:

$$q^{t,s} = 0, \qquad \forall t \in [T], \forall s \leq S_o$$

$$p^{t,s} = 0, \qquad \forall t \in [T], \forall s \geq S_e$$

In the rest of this section, we will show that the above surrogate constraints can be reformulated into computationally tractable forms that are jointly convex in the decision variables. Define the function $\rho_p^{t,s}(x) := \log\left(1 - p^{t,s} + p^{t,s}\exp(x)\right)$.

Proposition 3. *For a given $t \in [T]$,*

$$C_{k,\theta_{y,t}}\left[\sum_{s=0}^{S} a_y^s y^{t,s}\right] = k\sum_{s=0}^{t-1}\sum_{s'=0}^{S} q^{t-s,s'}\rho_\Omega^{t,s}(a_y^s/k\theta_{y,t}) + k\sum_{s=t}^{S} y^{0,s-t}\rho_\Omega^{t,s}(a_y^s/k\theta_{y,t}). \quad (3.4)$$

Proposition 3 is a standard example of the reformulation of the surrogate constraints. In all of the cases, due to the properties of the operator $C_{k,\theta}[\cdot]$ and the dynamics defined, any linear expression in the state variables, $y^{t,s}$, can be evaluated to the initial conditions. The double index of t and s induces two types of initial conditions, that where $s = 0$, which is the inflow, leading to the first term in (3.4), and $t = 0$, which is the start state, corresponding to the second term.

Proposition 4. *For a given $t \in [T]$, $C_{k,\theta_{z,t}}\left[\sum_{s=0}^{S} a_z^s z^{t,s} - b_z^t\right] \leq 0$ is equivalent to*

$$C_{k,\theta_{z,t}}\left[\sum_{s=0}^{t-1} a_z^s \lambda^{t-s}\right] - \frac{1}{\theta_{z,t}} \sum_{s=0}^{t-1}\sum_{\tau=0}^{s-1} a_z^s q^{t-\tau,s-\tau} + \frac{1}{\theta_{z,t}} \sum_{s=t}^{S} a_z^s \left(z^{0,s-t} - \sum_{\tau=0}^{t-1} q^{t-\tau,s-\tau}\right) + Z_t(k) \leq b_z^t/\theta_{z,t}, \tag{3.5}$$

where

$$Z_t(k) := k\sum_{j=1}^{t-1}\sum_{s'=0}^{S} q^{t-j,s'}\Upsilon_{1,j}^{t,j} + k\sum_{j=t}^{S-1} y^{0,j-t}\Upsilon_{j-t+1,j}^{t,j} + k\sum_{j=S}^{t+S-1} y^{0,j-t}\Upsilon_{j-t+1,S}^{t,j}, \tag{3.6}$$

$\Upsilon_{l,h}^{t,j} := \log\left(1 + \sum_{s'=l}^{h}\left(\exp(a_z^{j-s'}/k\theta_{z,t}) - 1\right)\bar{\Omega}^{t-j+s',s'}\right)$, *and*
$\bar{\Omega}^{t,s} := \omega^{t,s}\prod_{\tau=1}^{\min\{s,t\}-1}(1-\omega^{t-\tau,s-\tau})$.

In (3.5), the first term corresponds to the first-time donors while the second and third terms represent the outflows via pushes into $y^{t,s}$ and the initial conditions, respectively. The reformulation is significantly complicated by the last term $Z_t(k)$, which represents the contribution from the eligible donors (*i.e.*, the second term in (3.1)). Here, we introduce a novelty that deviates from the original P-Queue paradigm of Bandi and Loke (2018). In the original framework, an independence assumption, additional upon Assumption 2, is required to evaluate the stochastic term corresponding to $Z_t(k)$. Such an assumption could be considered strong in our context. Thus, we introduce new techniques to generalize the framework to one that does not depend on this assumption to conduct the reformulation while maintaining tractability. This eventually gives rise to the three terms in $Z_t(k)$ as opposed to one term if adopting the original paradigm. The reader may refer to the proof in Appendix 5.3 for more details.

As mentioned previously in §3.2.1, the dynamics of $x^{t,s}$ is analogous to $z^{t,s}$; consequently, its reformulation is also similar. Notice that if the right-hand side terms

b_n^t for $n = \{x, y, z\}$ are decision variables, the expressions in Propositions 3 and 4 remain exactly as they are. Additionally, for the demand and wastage constraints, the additive nature of the $C_{k,\theta}[\cdot]$ operator and the fact that demand is conditionally independent with the rest of the dynamics means that the demand terms d^t can be taken out, *e.g.*, $C_{k,\theta_{d,t}}\left[d^t - \sum_{s=0}^{S_e-1} p^{t,s}\right] = C_{k,\theta_{d,t}}\left[d^t\right] + C_{k,\theta_{d,t}}\left[-\sum_{s=0}^{S_e-1} p^{t,s}\right]$, which leaves demand terms that are separately evaluated and a linear component in the state in the form of Propositions 3 and 4. As such, we omit their reformulations for brevity.

As a last observation, note that in all of the above reformulations, we obtain constraints that are jointly convex in all decision variables (auxiliary or otherwise). This preserves model tractability. More will be discussed later in §3.4.2, where we describe methods for solving the convex constraints.

3.3 Blood Donation with Incentivization

In this section, we build upon the basic model by including the decision lever of donor incentivization. We shall do this by adding a parallel series of queue and server to represent the donors who have been called up by the blood bank, as illustrated in Figure 3.2.

In the incentivization model, we preserve all of the original definitions in the basic model, including the observation queue $z^{t,s}$, the eligible donor server $y^{t,s}$, and the blood inventory $x^{t,s}$. At this point, however, we consider an incentivization mechanism that acts upon the eligible donors amongst $y^{t,s}$. We assume the following properties on the incentivization mechanism:

Assumption 3. *a) Incentivization can occur at any time point t or period since eligibility s;*

b) For any donor, upon incentivization, the likelihood of donation is independent

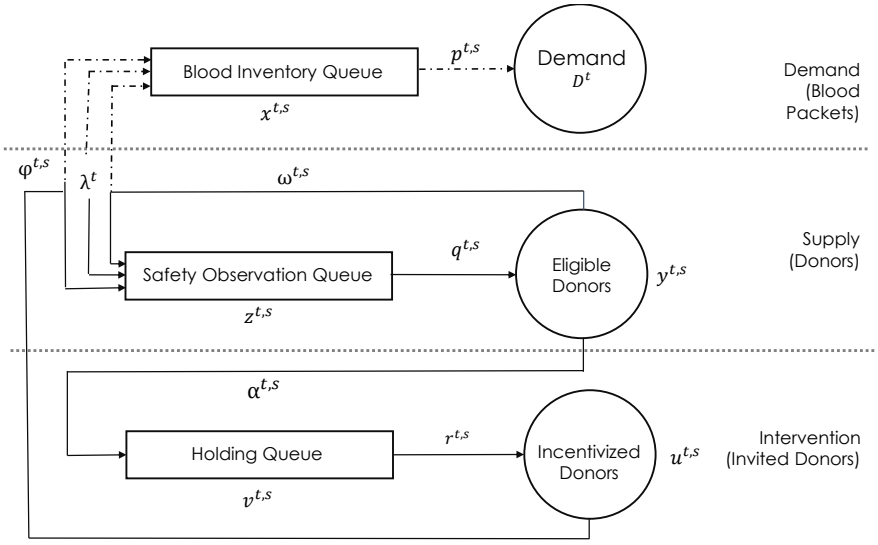


Figure 3.2: Blood Donation Network Flow

of their prior likelihood of donation;

c) *The likelihood of donation in the next time period, given incentivization, is independent across donors. This probability, at any time* $t(\geq 0)$*, is the same for all donors conditional on the time since they were incentivized at* $s(\geq 0)$*; and we denote this probability by* $\phi^{t+1,s+1}$*; and,*

d) *The act of incentivization occurs prior to the act of donation at each time period.*

Inspired by the literature on Bayesian Persuasion, we wish to consider incentivization mechanisms that influence donors' likelihood of donation, as opposed to modelling incentivization as an inclusion or exclusion policy. This allows us to cover a large range of incentivization mechanisms, such as (i) incentivizing donors to return to donate or to hasten their donations, (ii) incentivizing donors to delay their donations, and (iii) dispensing different incentives to different subpopulations of donors.

We shall discuss this more in §3.3.3 and also in the numerical section. Now, Assumption 3a) allows the mechanism to be as general as possible in time. Assumption 3b) sets up a setting similar to Bayesian Persuasion, where donors act differently post-incentivization. Assumption 3c) is simply analogous to Assumption 2. Finally,

Assumption 3d) just describes the staging of the events and does not have any real implications on the model.

The set-up of Assumption 3 motivates our approach to model incentivization as an outflow decision variable from the eligible donor server $y^{t,s}$. Keeping with the primitives of a P-Queue, we define this outflow as an adaptive decision: Let the *decision* variables $\alpha^{t,s} \in [0,1]$ represent the proportion of donors who have been eligible since s periods ago at time t, who are to receive the incentivization.

Most precisely, this outflow would go into a new server, which we term the "incentivized donors" server, with state variables $u^{t,s}$, representing the number of donors who have yet at time t to donate, s periods since incentivization. Adhering to the structure of P-Queues, we model an intermediate and auxiliary queue, termed the "holding queue", with state variables $v^{t,s}$, which receives the incentivized donors and dispatches them to invited donors server via auxiliary decision variables, $r^{t,s}$. As before, upon successful donation, incentivized donors will return to the safety observation queue. A list of additional parameters is given in Table 3.2.

Table 3.2: List of Additional Parameters and Variables

		Parameters
$\phi^{t,s}$	:	Conditional probability of donor donating after being called to donate
		State and Decision variables
$\alpha^{t,s}$	:	Decision variable of proportion of donors to approach
$u^{t,s}$	:	Random variable of invited donors who has yet to donate for s periods at time t
$v^{t,s}$	:	Random variable registered and invited donors in holding queue for s periods at time t
$r^{t,s}$	:	Recourse variable of invited donors dispatched to server after waiting for s periods at time t

3.3.1 Dynamics and Constraints

The incentivization model preserves all of the dynamics from the basic model, with minor amendments.We start with the dynamics of the eligible donor server. As before,

inflow is governed by

$$y^{t,0} = \sum_{s=0}^{S} q^{t,s}.$$

In the basic model, we defined the rest of the dynamics on $y^{t,s}$ as $y^{t,s} \sim \text{Bin}\left(y^{t-1,s-1}, 1-\omega^{t,s}\right)$, with $\text{Bin}\left(y^{t-1,s-1}, \omega^{t,s}\right)$ representing the number of donors who have donated and left the eligible pool. With the inclusion of incentivization, Assumption 3d) motivates the definition:

$$y^{t,s} = \text{Bin}\left((1-\alpha^{t,s})y^{t-1,s-1}, 1-\omega^{t,s}\right), \quad t \in [T], s \in [S].$$

Likewise, it is understood that a $\text{Bin}\left((1-\alpha^{t,s})y^{t-1,s-1}, \omega^{t,s}\right)$ number of donors would complete donation and return to the observation queue. The remaining $\alpha^{t,s}y^{t-1,s-1}$ are targets for incentivization and will proceed to the holding queue, forming its inflow dynamics:

$$v^{t,0} = \sum_{s=1}^{S} \alpha^{t,s} y^{t-1,s-1} - r^{t,0}.$$

The rest of the dynamics of the holding queue comprises the outflows. For $t \in [T]$,

$$v^{t,s} = \begin{cases} v^{t-s,0} - \sum_{\tau=0}^{s-1} r^{t-\tau,s-\tau} & \text{for } 0 < s < t, \\ v^{0,s-t} - \sum_{\tau=0}^{t-1} r^{t-\tau,s-\tau} & \text{for } t \leq s \leq S. \end{cases}$$

The dynamics in the invited donors server is akin to the eligible donors server, where inflow is given by the dispatches from the holding queue: $\forall t \in [T]$,

$$u^{t,0} = \sum_{s=0}^{S} r^{t,s}.$$

Assumption 3c) being identical to Assumption 2 allows us to define a Binomial distribution on the state $u^{t,s}$ with probability $\phi^{t,s}$. This similarly leads to the following

Proposition:

Proposition 5. *(a) Let* $\Phi^{t,s} \coloneqq \prod_{\tau=0}^{\min\{s,t\}-1}(1-\phi^{t-\tau,s-\tau})$*, then* $\forall t \in [T]$*,*

$$u^{t,s} = \begin{cases} \text{Bin}(u^{t-s,0}, \Phi^{t,s}) & \text{for } 0 < s < t, \\ \text{Bin}(u^{0,s-t}, \Phi^{t,s}) & \text{for } t \leq s \leq S. \end{cases}$$

(b) ***Independence of Pipelines****: For a fixed time* $t \in [T]$*, for any* $s, s' \in [S]$ *such that* $s \neq s'$*,* $u^{t,s}$ *and* $u^{t,s'}$ *are independent.*

Finally, in the observation queue, the only difference is the addition of the term resulting from the donations from the incentivized donors, constituting the second term in (3.7) below. The dynamics of its outflow remains as before.

$$z^{t,0} = \lambda^t + \sum_{s=1}^{S} \text{Bin}\left(u^{t-1,s-1}, \phi^{t,s}\right) + \sum_{s=1}^{S} \text{Bin}\left((1-\alpha^{t,s})y^{t-1,s-1}, \omega^{t,s}\right). \tag{3.7}$$

Like in the basic model, the dynamics in the blood inventory queue traces that of the observation queue for the same logic. Its only difference is the novelty we introduced by adding the $-p^{t,0}$ term as in the basic model. The outflow dynamics is unchanged from the basic model.

$$x^{t,0} = \lambda^t + \sum_{s=1}^{S} \text{Bin}\left(u^{t-1,s-1}, \phi^{t,s}\right) + \sum_{s=1}^{S} \text{Bin}\left((1-\alpha^{t,s})y^{t-1,s-1}, \omega^{t,s}\right) - p^{t,0}.$$

In the incentivization model, we extend the constraints previously considered in the basic model where appropriate – extending capacity constraints to all servers, queue-clearing constraints to all queues and push constraints to all auxiliary push decision variables. FIFO demand fulfillment policy, logical constraints, wastage and demand target constraints remain. The only new constraint we introduce is a budget on the maximum number of donors that can be demarcated as targets for incentivization. Here, we can consider the constraint $\sum_{s=0}^{S} c^{t,s}y^{t,s} \leq B^t$, where $c^{t,s}$ is the per

donor cost of dispensing the incentive to donors who have yet to donate for s periods by time t, and B^t is the budget at time t. Most often, this would simply take the form of $c^{t,s} \equiv 1$ and B^t being the maximum number of donors that is administratively possible to contact within the planning window. Note that this is once again linear, hence reformulating it would be no different from a general linear constraint in state variable $y^{t,s}$.

3.3.2 Model and Reformulation

We are now able to present the full incentivization model:

$$\min \quad k \tag{3.8}$$

Demand:

$$C_{k,\theta_{d,t}} \left[d^t - \sum_{s=0}^{S_e-1} p^{t,s} \right] \leq 0, \qquad \forall t \in [T]$$

Wastage:

$$C_{k,\theta_{W,t}} \left[\sum_{s=S_e}^{S} x^{T,s} + \sum_{t=1}^{T} \left(\sum_{s=0}^{S_e-1} p^{t,s} - d^t \right) - W \right] \leq 0,$$

Capacity, Budget, Queue-clearing, Waiting-time, FIFO:

$$C_{k,\theta_{n,t}} \left[\sum_{s=0}^{S} a_n^s n^{t,s} - b_n^t \right] \leq 0, \qquad n \in \{x, y, z, u, v\}, \qquad \forall t \in [T]$$

Push Constraints:

$$C_{k,\theta_{n,t}} \left[-n^{t,s} \right] \leq 0, n \in \{x, z, v\}, \qquad \forall t \in [T], s \in [S]_0$$

Logic Constraints:

$$q^{t,s} = 0, \qquad \forall t \in [T], \forall s \leq S_o$$

$$p^{t,s} = 0, \qquad \forall t \in [T], \forall s \geq S_e$$

As before, we attempt to reformulate the surrogate constraints into a convex form.

We only introduce below the results for the new server and queue introduced or if they involve new techniques for brevity. Given the complexity of the expressions, detailed interpretations of each term in the reformation are omitted. Also note that in the following propositions, if index sets are empty, the corresponding terms or constraints are understood to be omitted.

Proposition 6. *For a given $t \in [T]$,*

$$C_{k,\theta_{u,t}}\left[\sum_{s=0}^{S} a_u^s u^{t,s}\right] = k\sum_{s=0}^{t-1}\sum_{s'=0}^{S} r^{t-s,s'}\rho_\Phi^{t,s}(a_u^s/k\theta_{u,t}) + k\sum_{s=t}^{S} u^{0,s-t}\rho_\Phi^{t,s}(a_u^s/k\theta_{u,t}). \quad (3.9)$$

Proposition 7. *For any $t \in [T]$, $C_{k,\theta_{y,t}}\left[\sum_{s=0}^{S} y^{t,s}a_y^s - b_y^t\right] \leq 0$ is equivalent to the collection of the following constraints:*

$$\begin{aligned}
&y^{t,0}a_y^0/\theta_{y,t} + k\sum_{j=0}^{t-1}\xi^{t-j,1} + k\sum_{j=t+1}^{S}\xi^{1,j-t+1} \leq b_y^t/\theta_{y,t} && \\
&\xi^{t,s} \geq \beta^{t,s}\rho_{1-\omega}^{t,s}(a_y^s/k\theta_{y,t}) && \forall s \in [S] \\
&\xi^{t-\tau,s-\tau} \geq \beta^{t-\tau,s-\tau}\rho_{1-\omega}^{t-\tau,s-\tau}(\xi^{t-\tau+1,s-\tau+1}/\beta^{t-\tau,s-\tau}) && \forall \tau \in [t-1], s \in [S]\setminus[\tau].
\end{aligned}$$

Importantly, decision variables have been reformulated as $1-\alpha^{t,s} = \beta^{t,s}/\beta^{t-1,s-1}$ to arrive at the perspective function $\beta\rho_{1-\omega}^{t,s}(\xi/\beta)$, which is *jointly* convex in β and ξ. This reformulation uses more decision variables than there originally are. This grants us the degree of freedom to determine the boundary values as $\beta^{t,0} = y^{t,0} := \sum_{s=0}^{S} q^{t,s}$ and initial conditions $\beta^{0,s} := y^{0,s}$.

Proposition 8. *For any $t \in [T]$, $C_{k,\theta_{v,t}}\left[\sum_{s=0}^{S} a_v^s v^{t,s} - b_v^t\right] \leq 0$ is equivalent to the*

collection of the following constraints.

$$\sum_{j=1}^{t-1}\frac{\beta^{j,0}-\beta^{j+1,1}}{\theta_{v,t}/a_v^{t-j-1}}+\sum_{j=t}^{S-1}\frac{\beta^{0,j-t}-\beta^{1,j-t+1}}{\theta_{v,t}/a_v^{t-1}}+\sum_{j=S-t}^{S-1}\frac{\beta^{0,j}-\beta^{1,j+1}}{\theta_{v,t}/a_v^{t-1}}$$
$$+k\Bigg(\sum_{j=2}^{t-1}\eta^{t-j+1,1}+\sum_{j=t}^{S-1}\eta^{1,j-t+1}+\sum_{j=S}^{t+S-2}\eta^{1,j-t+1}\Bigg)-\frac{1}{\theta_{v,t}}\sum_{s=0}^{t-1}\sum_{\tau=0}^{s}a_v^s r^{t-\tau,s-\tau}$$
$$+\frac{1}{\theta_{v,t}}\sum_{s=t}^{S}a_v^s\Bigg(v^{0,s-t}-\sum_{\tau=0}^{t-1}r^{t-\tau,s-\tau}\Bigg)\le b_v^t/\theta_{v,t}$$

$$\eta^{t-1,j-1}\ge\beta^{t-1,j-1}\rho_{1-\omega}^{t-1,j-1}\left(\frac{a_v^0(\beta^{t-1,j-1}-\beta^{t,j})}{k\theta_{v,t}\beta^{t-1,j-1}}\right)\qquad\forall j\in[S-1]\setminus\{1\}$$
$$\eta^{t-\tau,j-\tau}\ge\beta^{t-\tau,j-\tau}\rho_{1-\omega}^{t-\tau,j-\tau}\left(\frac{\beta^{t-\tau,j-\tau}-\beta^{t-\tau+1,j-\tau+1}}{k\theta_{v,t}\beta^{t-\tau,j-\tau}/a_v^{\tau-1}}+\frac{\eta^{t-\tau+1,j-\tau+1}}{\beta^{t-\tau,j-\tau}}\right)$$
$$\forall\tau\in[t-2]\setminus\{1\},j\in[t-1]\setminus[\tau]$$
$$\eta^{t-\tau,j-\tau}\ge\beta^{t-\tau,j-\tau}\rho_{1-\omega}^{t-\tau,j-\tau}\left(\frac{\beta^{t-\tau,j-\tau}-\beta^{t-\tau+1,j-\tau+1}}{k\theta_{v,t}\beta^{t-\tau,j-\tau}/a_v^{\tau-1}}+\frac{\eta^{t-\tau+1,j-\tau+1}}{\beta^{t-\tau,j-\tau}}\right)$$
$$\forall\tau\in[t-1]\setminus\{1\},j\in[S-1]\setminus[t-1]$$
$$\eta^{t-j,S-1}\ge\beta^{t-j,S-1}\rho_{1-\omega}^{t-j,S-1}\left(\frac{\beta^{t-j,S-1}-\beta^{t-j+1,S}}{k\theta_{v,t}\beta^{t-j,S-1}/a_v^{j-1}}\right)\quad\forall j\in[t-1]$$
$$\eta^{t-j-\tau,S-\tau}\ge\beta^{t-j-\tau,S-\tau}\rho_{1-\omega}^{t-j-\tau,S-\tau}\left(\frac{\beta^{t-j-\tau,S-\tau}-\beta^{t-j-\tau+1,S-\tau+1}}{k\theta_{v,t}\beta^{t-j-\tau,S-\tau}/a_v^{j+\tau-1}}+\frac{\eta^{t-j-\tau+1,S-\tau+1}}{\beta^{t-j-\tau,S-\tau}}\right)$$
$$\forall\tau\in[t-1]\setminus\{1\},j\in[t-\tau-1]_0.$$

Proposition 9. *For any $t\in[T]$, $C_{k,\theta_{z,t}}\left[\sum_{s=0}^{S}a_z^s z^{t,s}-b_z^t\right]\le 0$ is equivalent to*

$$C_{k,\theta_{z,t}}\left(\sum_{s=0}^{t-1}a_z^s\lambda^{t-s}\right)-\frac{1}{\theta_{z,t}}\sum_{s=0}^{t-1}\sum_{\tau=0}^{s-1}a_z^s q^{t-\tau,s-\tau}+\frac{1}{\theta_{z,t}}\sum_{s=t}^{S}a_z^s\Bigg(z^{0,s-t}-\sum_{\tau=0}^{t-1}q^{t-\tau,s-\tau}\Bigg)+\bar{Z}_t(k)$$
$$+k\Bigg(\sum_{j=1}^{t-1}\psi^{t-j+1,1}+\sum_{j=t}^{t+S-1}\psi^{1,j-t+1}\Bigg)\le b_z^t/\theta_{z,t}$$

$$\psi^{t,j}\ge\beta^{t,j}\rho_\omega^{t,j}(a_z^0/k\theta_{z,t})\qquad\forall j\in[S-1]$$
$$\psi^{t-j,S}\ge\beta^{t-j,S}\rho_\omega^{t-j,S}(a_z^j/k\theta_{z,t})\qquad\forall j\in[t-1]_0$$

$$\psi^{t-\tau,j-\tau} \geq \beta^{t-\tau,j-\tau}\sigma_\omega^{t-\tau,j-\tau}(a_z^\tau/k\theta_{z,t}, \psi^{t-\tau+1,j-\tau+1}/\beta^{t-\tau,j-\tau}) \quad \forall \tau \in [t-2], j \in [t-1] \setminus [\tau]$$

$$\psi^{t-\tau,j-\tau} \geq \beta^{t-\tau,j-\tau}\sigma_\omega^{t-\tau,j-\tau}(a_z^\tau/k\theta_{z,t}, \psi^{t-\tau+1,j-\tau+1}/\beta^{t-\tau,j-\tau}) \quad \forall \tau \in [t-1], j \in [S-1] \setminus [t-1]$$

$$\psi^{t-j-\tau,S-\tau} \geq \beta^{t-j-\tau,S-\tau}\sigma_\omega^{t-j-\tau,S-\tau}\left(\frac{a_z^{j+\tau}}{k\theta_{z,t}}, \frac{\psi^{t-j-\tau+1,S-\tau+1}}{\beta^{t-j-\tau,S-\tau}}\right) \quad \forall \tau \in [t-1], j \in [t-\tau-1]_0,$$

where

$$\bar{Z}_t(k) := k\sum_{j=1}^{t-1}\sum_{s'=0}^{S} r^{t-j,s'}\bar{\Upsilon}_{1,j}^{t,j} + k\sum_{j=t}^{S-1} u^{0,j-t}\bar{\Upsilon}_{j-t+1,j}^{t,j} + k\sum_{j=S}^{t+S-1} u^{0,j-t}\bar{\Upsilon}_{j-t+1,S}^{t,j},$$

$\bar{\Upsilon}_{l,h}^{t,j} := \log\left(1 + \sum_{s'=l}^{h}\left(\exp(a_z^{j-s'}/k\theta_{z,t}) - 1\right)\bar{\Phi}^{t-j+s',s'}\right)$, $\bar{\Phi}^{t,s} := \phi^{t,s}\prod_{\tau=1}^{\min\{s,t\}-1}(1 - \phi^{t-\tau,s-\tau})$,

and

$$\sigma_p^{t,s}(x,y) := \log\left(p^{t,s}\exp(x) + (1-p^{t,s})\exp(y)\right).$$

The reformulation for the constraints involving x is analogous and is omitted for brevity.

Theorem 1 (Reformulation)**.** *Problem (3.8) has a reformulation into a convex optimization problem with $O(ST^3)$ constraints. Moreover, it can be solved via a sequence of convex sub-problems.*

Here, we summarize in Table 3.3 how each of the reformulations contributes to the eventual total number of constraints.

3.3.3 Practical Settings in Applications

Our model readily extends to situations commonly seen in different applications. For example, we may consider a multi-tiered incentivization setting, where the planner

Table 3.3: Summary of number of constraints per reformulation

Constraint	# to reformulate		# constraints per reformulation		Total constraints
$\sum_s a^s u^{t,s} \leq b^t$	$O(T)$	$\times$	1	$=$	$O(T)$
$\sum_s a^s y^{t,s} \leq b^t$	$O(T)$	$\times$	$O(ST)$	$=$	$O(ST^2)$
$\sum_s a^s v^{t,s} \leq b^t$	$O(T)$	$\times$	$O(ST)$	$=$	$O(ST^2)$
$\sum_s a^s z^{t,s} \leq b^t$	$O(T)$	$\times$	$O(ST)$	$=$	$O(ST^2)$
$\sum_s a^s x^{t,s} \leq b^t$	$O(T)$	$\times$	$O(ST)$	$=$	$O(ST^2)$
$p^{t,s} \leq x^{t-1,s-1}$	$O(T^2)$	$\times$	$O(ST)$	$=$	$O(ST^3)$
$q^{t,s} \leq z^{t-1,s-1}$	$O(T^2)$	$\times$	$O(ST)$	$=$	$O(ST^3)$
$r^{t,s} \leq v^{t-1,s-1}$	$O(T^2)$	$\times$	$O(ST)$	$=$	$O(ST^3)$

has a series of different incentives to execute. For example, a two-tiered system might include one option to incentivize donors to hasten their donation and the other to delay the donation. Our model can be altered trivially to handle such a situation by adding more layers, one for each type of incentive, just like how we have added a queue-server layer in moving from the basic model in §3.2 to the incentivization model in §3.3. As the dynamics of the model is defined additively, doing so has no implications on the tractability of the model.

Another situation is the setting of different donor classes, for example, two distinct groups of high and low responsive donors, where the decision-maker might be interested in incentivizing them differently. This can be done by labelling all state variables with an additional index i to represent the class. Due to the additive nature of the dynamics and the constraints, and with the further addition of assumptions to ensure independence across the classes, we can recover analogous reformulations to the surrogate constraints and the tractability guaranteed in Theorem 1.

We shall explore both of these settings later in the numerical studies, specifically in §3.4.5.

3.4 Numerical Studies

In this section, we present our numerical study. In particular, we test our model on synthetic data that allows us to test different scenarios to understand the behaviour of our model as the baseline parameters vary.

In our numerical study, we shall consider the setting of a blood bank adaptively making donor incentivization decisions under a finite planning horizon. The blood bank possesses data about their donors, including the time (and thus frequency) of their donations and how they responded to incentives in the past. Specifically, the latter refers to both the likelihood that they donated after being incentivized and the speed at which they did so. From this data, the blood bank obtains estimates of $\omega^{t,s}$ and $\phi^{t,s}$. The blood bank also has data about the arrival of new donors to the system and the consumption of blood, which allows estimates of the inflow distribution Λ^t and demand distribution D^t. Finally, the blood bank knows the state of its inventory and the state of its donors, be it whether they are in observation, for how long they have been in observation, and if they are eligible, how long since have they been eligible. This enables the blood bank to determine the initial state of the system, $x^{0,s}, y^{0,s}$ and $z^{0,s}$. The goal of the blood bank is to determine the number of donors to be incentivized as a *decision rule* $\alpha^{t,s}$. It is expected that the blood bank would not execute the whole decision rule across all times t from 1 to the horizon T, but rather, to implement the decision at $t = 1$, allowing the new uncertainty to materialize, and then re-solve the model thereafter, in such a rolling horizon manner. Naturally, this would mean that the decision rule is always a function of the last observed data as the initial state.

In this numerical study, the decision-maker obtains the optimal incentivization decision rule via the proposed model, Problem (3.8). We shall not impose any capacity constraints on the servers, except only keeping the total number of incentivized donors under some budget. This might arise out of a financial budget or capacity constraints.

As in (3.8), the blood bank aims to keep the wastage low, and the demand met as far as possible.

In what ensues, we conduct our numerical study as follows: First, we shall analyze the optimal number of donors to incentivize, as proposed by our model, under different configurations of the current inventory level and the "effectiveness of incentivization" defined as the ratio between $\phi^{t,s}$ and $\omega^{t,s}$. We term this the *structure* of the optimal policy, and these two factors are the most critical in influencing the structure. Next, we will vary other variables, including the average demand for blood, the rate at which new donors enter the donor pool, and the distribution of donors between those in observation and those eligible. Finally, we extend the numerical study to the two settings of multiple interventions and multiple donor classes with different responsiveness to incentivization.

Here, we make a quick remark that, in this chapter, we shall not embark on any comparisons against other theoretical benchmarks for a few reasons. First, to the best of our knowledge, we do not find any models in the literature to optimize donor incentivization policies in a comparable setting. Second, even a methodology like Sample Average Approximations (SAA) would be difficult to execute. As explained in Zhou et al. (2021), due to the sequential nature of the decisions and the uncertainty unfolding, and their interdependence, the number of data points required to conduct the SAA accurately would be exponentially large. Instead, our model handles the interdependence through the dynamics, as opposed to the data. Moreover, as Zhou et al. (2021) illustrated, SAA would unlikely lead to good performance once this dynamics is included.

3.4.1 The Parameter Setup

We first describe the basic parameter setup of our model, which we will employ for the first part of the analysis. This is followed by varying some of these parameters to

understand their impact on the structure of the optimal policy.

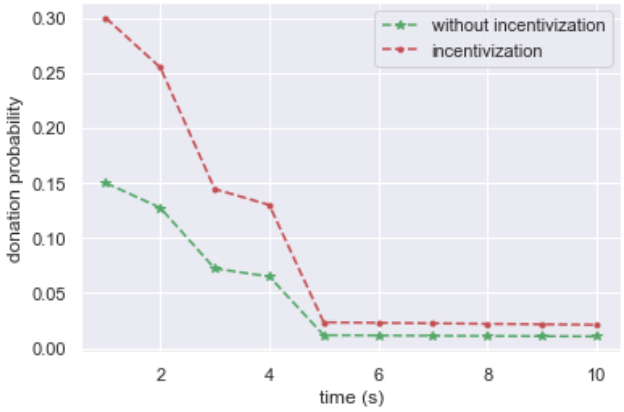


Figure 3.3: Figure 6(a) Increased responsiveness of donors

We model the problem over a $T = 5$ period window, and $S = 10$, where the shelf life of blood inventory is $S_e = 3$ and the observation period is $S_o = 4$. We also fix the probability of donation without intervention, $\omega^{t,s}$, as specified in Figure 3.7a.

The values of $\omega^{t,s}$ are designed to represent the phenomenon that the longer an eligible donor delays their donation, the less likely they would do so. Thus, the effective likelihood of donation after a certain point ($s \geq 5$) is insignificant. The conditional probability of donation post-intervention s periods ago shall be given as a multiple $\phi^{t,s} = I \cdot \omega^{t,s}$, where I represents the effectiveness of the incentivization and for brevity, we call this the *intensity*. We initialize the system in the blood inventory with all the blood packets being fully fresh ($s = 0$) so as to let the model handle the wastage itself. The demand of the blood d^t is time-homogeneous and distributed over the support of $\{51, \cdots, 58\}$ with a mean of 54. The wastage threshold is set at $W = 60$. We also initialize the eligible donor pool with 360 eligible donors, and they are front-loaded in their distribution over s over the range of non-negligible donation probabilities. The number 360 was chosen so that the expected number of donations without intervention would work out to be about 40, which can never satisfy the

demand in the long run. Hence, some incentivization is always necessary. Donors in the observation window totalled 240, evenly distributed over $s \leq S_o$. Finally, we model a saturated market, where the rate of new donors is very low, averaging at 2.55.

In the simulations, we shall compute the statistic of the average number of donors incentivized. Because we have proposed a decision rule as the optimal policy, this number is a random variable. Hence, we compute the statistic as an average, under simulation of the dynamics, given the optimal policy, for $1,000$ instances.

3.4.2 Solution Methodology

We solve Problem (3.8) via interval bisection on k. For each given k, we have to solve the feasibility problem consisting of all the reformulations of the surrogate constraints in (3.8). These reformulations are convex. However, due to their nature of having the form akin to the function $\log(1 - p + pe^{(\cdot)})$, which is asymptotically linear, we can efficiently solve this via sequential cutting plane approach. Specifically, we first declare a tolerance for the accuracy of the linear estimates for these constraints, solve the feasibility problem with the linear constraints, and then generate a cutting plane as a new linear constraint if the accuracy is not met. If at any point, the feasibility problem becomes infeasible, then there is also no need to proceed further, and we can declare the problem infeasible for that given k. Otherwise, if the problem remains feasible and the tolerance for the surrogate constraints is met, then the problem is declared feasible for that given k. Once the optimal k^* is determined by interval bisection, we re-solve the model, given that k^*, to obtain a set of solutions $\alpha^{t,s}$, which we use as the optimal policy. For more details of the solution methodology, please refer to the pseudo-code provided in Appendix 5.4.

The above methodology is relatively rapid. Solving each instance of the problem to optimality takes less than 90 seconds on a Mac-OS computer with 16 GB of RAM

and a 2.0-GHz CPU using Gurobi on Python. As a quick note, as $k \to \infty$, the surrogate constraints tend to the expectation value. It is also possible that the model is not even feasible in expectation, which will also return a value of unbounded k. As such, we do not reflect any results in subsequent discussions if the value of optimal k^* does not come down below $k^* < 1$. They will be reflected as grey cells in the figure.

3.4.3 Structure of the Optimal Policy

We first study the structure of the optimal policy under varying levels of existing blood inventory and the intensity, which represents the responsiveness of the donors to incentivization.

In Figure 3.4, we plot this structure, where on both axes, we vary these factors and in each grid, we plot the total number of donors that were incentivized under the optimal policy, summed over the first 4 time periods. We omit the fifth and last time period due to potential end-of-horizon effects. For ease of interpretation, we suppressed the numbers in each grid, and instead reflect it as a colour in the heat map.

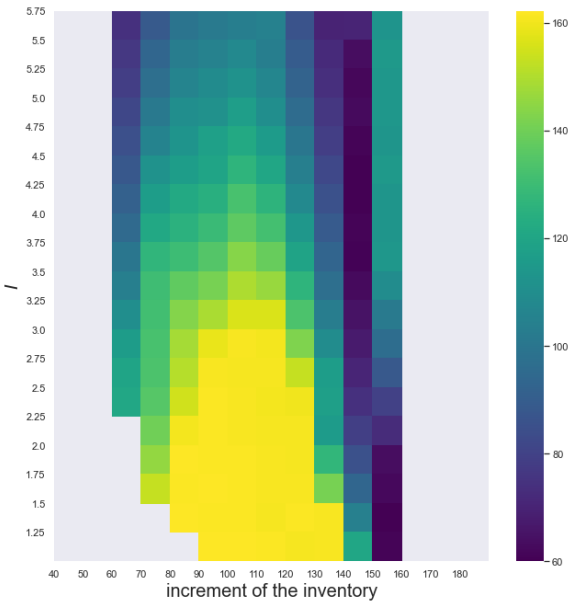


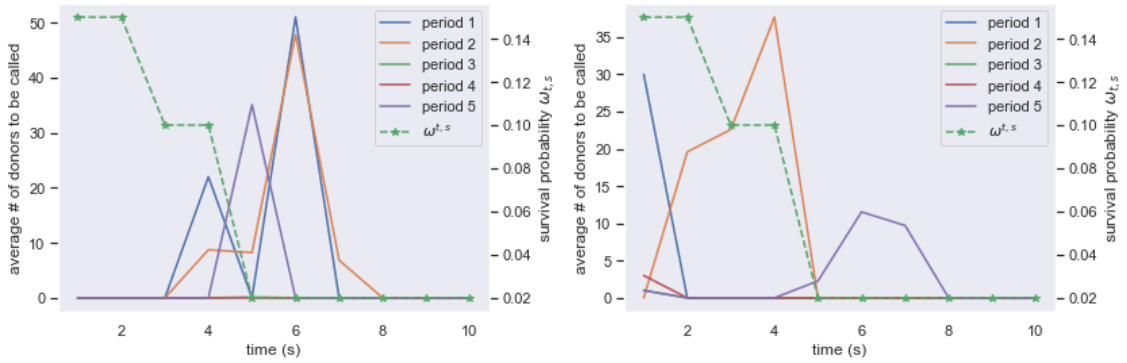
Figure 3.4: Optimal Incentivization Policy with Average Demand of 54

From Figure 3.4, we observe three main trends. First, for a fixed inventory level, the higher the intensity, the fewer donors are incentivized. This is unsurprising as the model only seeks to incentivize enough donors to meet demand, rather than over-incentivizing donors and resulting in wastage. Second, for a fixed intensity level, the number of incentivized donors first increase, then decrease with existing inventory levels. The latter decrease is intuitive, as this is done to minimize wastage. The initial increase is counter-intuitive. This might be due to the fact that our model minimizes both the probability of shortages and the extent of shortages for every time period, as opposed to the total shortages over the entire horizon. Specifically, the model aims to minimize the risk of violation at each time period and control under some threshold violations. As such, at critically low inventory levels in earlier time periods, the model chooses not to aggressively incentivize donors to restrict the probability of violations and thereby effectively spread the risk over all time periods instead. By balancing shortages while fulfilling demand and ensure shortages remain as small as possible over all time periods, the model results in a policy that avoids incentivizing too many donors at the start. Lastly, there is a small triangular feature at the top right corner when inventory level is high where both trends discussed above reverse. This is counter-intuitive, and we shall imminently discuss it.

Faster Donations versus Lower Drop-outs

To understand the reason for the triangular feature in the upper right corner of the structure of the optimal policy in Figure 3.4, we drill down to the behaviour of the policy at each time period for two grid points, one in the intuitive large central spur-like feature of the Figure ($I := \phi^{t,s}/\omega^{t,s} = 3$ with inventory level 100), and one in the triangular corner ($I = 5$ with inventory level 150). Figure 3.5 below plots the decomposition of the number of donors to each time period, given by each line in the chart, across how long they have been eligible for in the horizontal axis. For reference,

we also plotted in broken line, the likelihood of donation without incentivization, $\omega^{t,s}$.



(a) I = 3 and Inventory = 100

(b) I = 5 and Inventory = 150

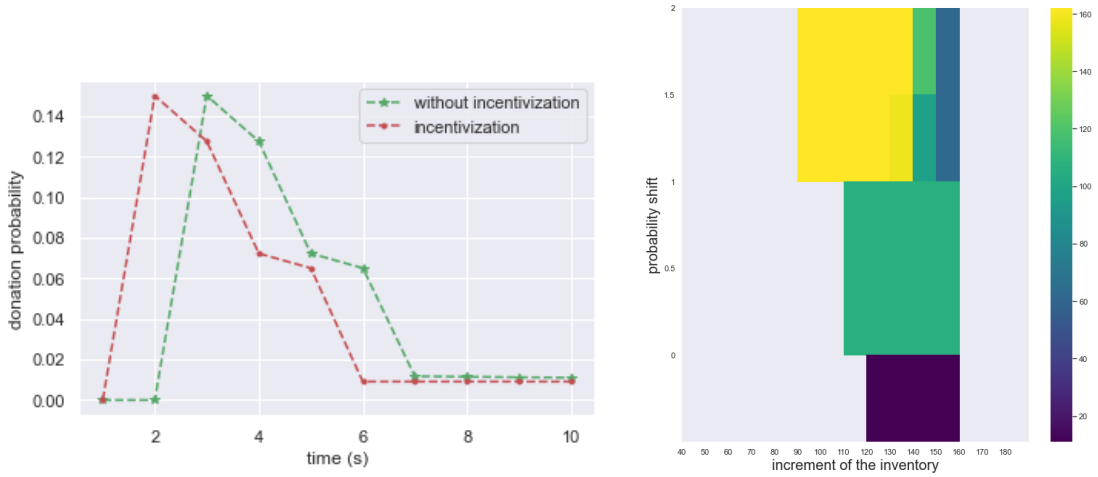
Figure 3.5: Comparison of Instances within the Spur Feature and the Triangular Feature in the Structure

We observe that in the intuitive spur-like region, the model prioritizes incentivizing donors who have negligible likelihood of donation without incentivization, in other words, have likely dropped out from the donor pool. Incentivizing them would then provide them a second opportunity to return to the active donor population. In the triangular region, however, the model, especially in the second time period, incentivizes donors who have significant likelihood of donating. This does two things. First, given those donors' already high donation likelihood without incentivization, calling them up would not increase the instantaneous wastage by too much. On the contrary, it may reduce the expected number of donations downstream, by letting those high responsive donors donate early to leave the eligible donors server and reduce the overall expected number of donations. Second, it preserves the donor pool and prevents drop-outs by returning these donors, post-donation, to the observation window.

The above analysis allows us to draw two important lessons and insights. First, even in times of excess, it may be sensible to continue incentivizing donors in order to even out the supply over time. This is especially critical as most blood banks

today only operate on the basis of incentivization only at times of shortages. We will revisit this idea later in §3.4.5. Second, incentivization essentially has two types of effects: (i) it lowers the likelihood of drop-outs from the donor pool, in other words, the probability of donation at some $s < S$ is increased, and (ii) the actual time-to-donation, *i.e.* s, has been reduced. In this analysis, what we see is that the model is intricately balancing between these two effects, especially the former, in controlling the availability of the supply.

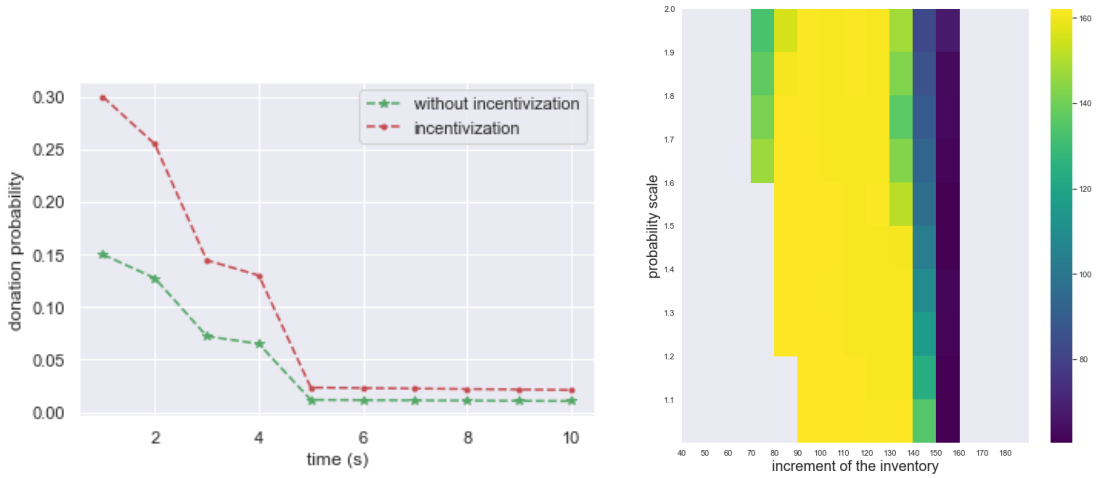
To this effect, we dive deeper to analyze the differences. Here, we consider two kinds of mechanism. In the first instance, we consider a situation where incentivization only shifts forward the time-to-donation of the donors (see Figure 3.6a). We then plot the structure of the policy according to the amount of shifting forward in Figure 3.6b. In the second instance, we consider a situation where incentivization decreases the probability of drop-outs, but does not fundamentally change the pattern of the time-to-donation (as represented in Figure 3.7a). Similarly, we plot the structure of the policy according to the amount of scaling in Figure 3.7b.



(a) Increased Responsiveness of Donors

(b) Optimal Structure

Figure 3.6: Increased Responsiveness of Donors



(a) Increased Probability of Donations (b) Optimal Structure

Figure 3.7: Increased Probability of Donations

By comparing Figures 3.6b and 3.7b, what we noticed is a vertical pattern in the former and a horizontal pattern in the latter. Specifically, in the former case, we observe that when the scale of the shift is small, the incentivization policy does not respond to the changes in inventory levels. When the shift is significant, a slight horizontal pattern emerges, where more donors are incentivized when inventory is low. Therefore, in order for a mechanism that only shifts the time-to-donation to be an effective lever on the blood inventory, the scale of the shift needs to be high. In contrast, for the mechanism that decreases drop-out probability and hence directly alters the expected number of donations, the optimal policy only depends on the present state of inventory, not the scale of altering.

The culmination of these analyses explains potential insights about how blood banks can operate. Specifically, it is important to be cognizant that there are two kinds of incentivization. Incentivizing for faster time-to-donation may be useful only if the shift is significantly large, whereas incentivizing fewer drop-outs has a consistent effect, and primarily solves the longer-term issue since it alters the number of donations over time.

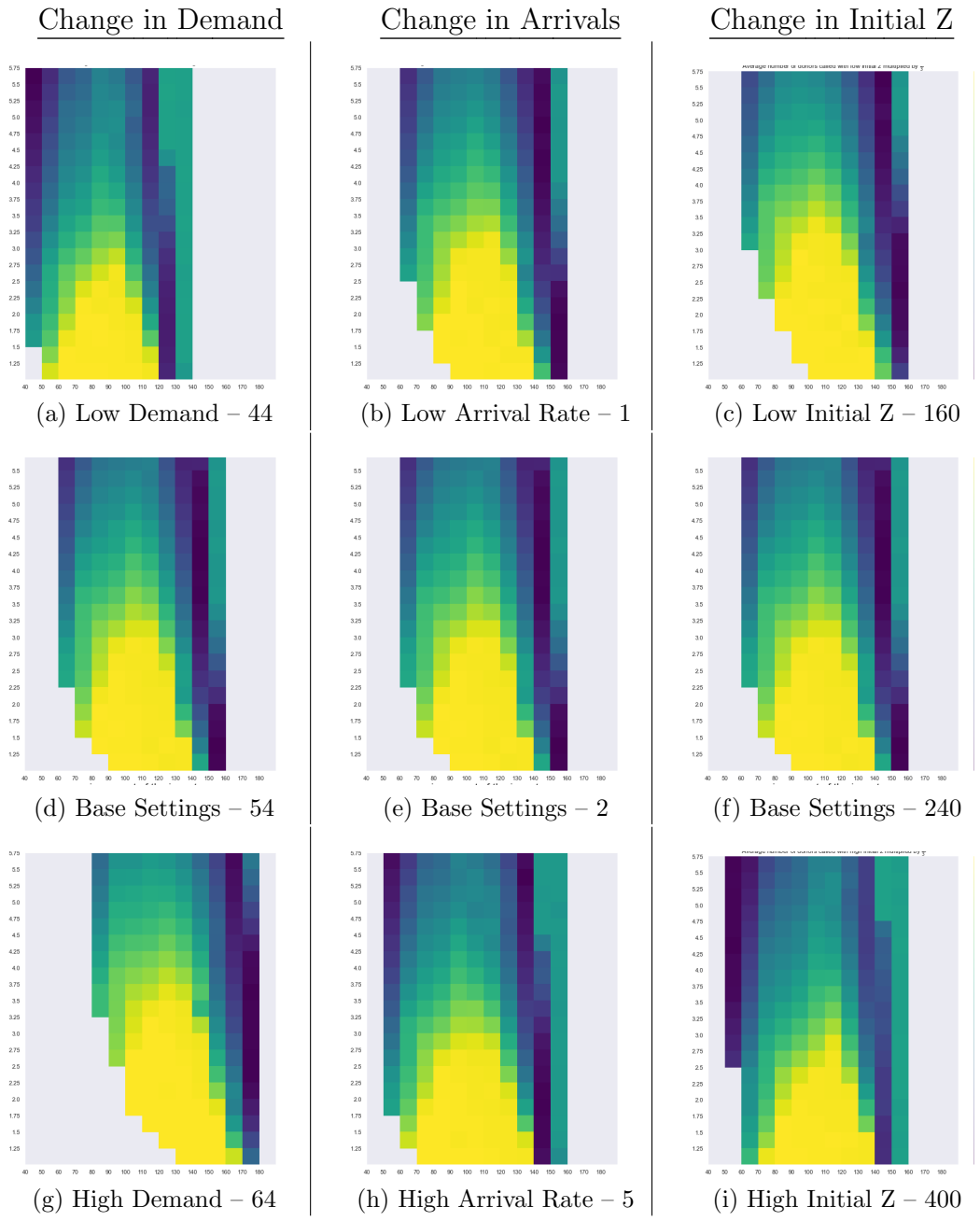
3.4.4 Altering the Context

In this section, we shall let the the contextual parameters vary, in order to understand how the structure of the optimal policy changes. In particular, we vary: (i) the average demand for blood, (ii) new donors arrival rate, and (iii) initial levels of donors within the observation queue. We present the results in Figure 3.8, where the columns show how the structure of the optimal policy changes with each of the factors.

Most noticeably, the general shape of the optimal policy has not changed —the spur-like and the upper corner triangular features have remained. It is expected as the reasons for their shape remain. What has changed is the position of these features and the covering of the grey boxes, representing the configurations that are infeasible in expectation. Here, we discuss these changes in relation to the factors that vary.

Change in demand: With higher demand for blood, the optimal policy has migrated upwards and to the right. The upward drift is due to the need for more blood packets, and hence even at higher intensity, there needs to be more donors incentivized. The rightward shift also occurs for the same reason —the higher demand offsets the higher level of initial inventory.

Change in new donor arrivals: For the new donor arrival rate, the change in the structure of the optimal policy is minimal. This is likely due to the fact that the arrivals were initiated at a low level, and the arrival rate is an order of magnitude away from the actual demand. However, if the arrival rate is drastically increased, a behaviour quite akin to a lowering of the demand should be observed, as evidenced by the movement of the infeasible regions.



(a) Low Demand – 44 (b) Low Arrival Rate – 1 (c) Low Initial Z – 160

(d) Base Settings – 54 (e) Base Settings – 2 (f) Base Settings – 240

(g) High Demand – 64 (h) High Arrival Rate – 5 (i) High Initial Z – 400

Figure 3.8: Variation in Contextual Parameters

Change in initial observation queue length: Most prominently, having a higher initial number of donors in the observation queue tilts the shape of the spur-like feature to the right. With more donors in the observation queue, the future supply of donations is expected to increase. As such, the point that requires the same level of incentivization as before would correspond to the point with higher inventory, which

would offset the expected higher future supply of blood. Note that there are no variations vertically in the chart as the expected number of donations does not change with only changes in the distribution of the donor pool.

Summary: We see that the optimal policy reacts well to the varying conditions in this series of experiments. Knowledge of these insights is important. For example, for the case of arrival rates, recognizing that increasing new donors, especially for states that face challenges in growing the donor pool like Singapore, would not have visible benefits over interventions that directly influence the repeat donors. Changes in demand for blood are especially significant as this impacts both the required safety inventory levels and the intensity of the incentivization. As the demand for blood climbs, blood banks will face ever more challenging operating circumstances.

3.4.5 Applying to Multi-class Donors

Segmenting eligible donors into different classes of high and low-responsive donors allows for managing them differently depending on inventory levels. With increasing data tools today, performing such segmentation through Machine Learning is already a reality. It may be possible to segment donors into more than 2 classes or even determine a responsiveness score for every donor. Our goal is not to explore the latter, and for brevity, we consider the two-class context as an indication of how such a donor management mechanism can be operationalized and how the incentivization policy would be. To do so, we add a subscript $i(= 1, 2)$ on all of the state variables, which correspond to high and low responsive donors respectively.

In the first instance, we consider the situation as described in Figure 3.9, where one class of donors, named high-responsive donors, experiences higher donation probabilities after incentivization (*i.e.*, $I_1 > 1$) and another class of donors, low-responsive donors, displays lower donation probabilities after incentivization (*i.e.*, $I_2 < 1$). This

can be used to model an incentive mechanism with two levers, one incentivizing donors to expedite their donation, the other incentivizing donors to *delay* their donation. In Figure 3.10, we plot the structure of the optimal policy for each of the classes of donors, where on the vertical axis, scale refers to the amount of scaling up of the donation probability of the high-responsive donors, *i.e.*, I_1. I_2 is kept constant throughout the simulation.

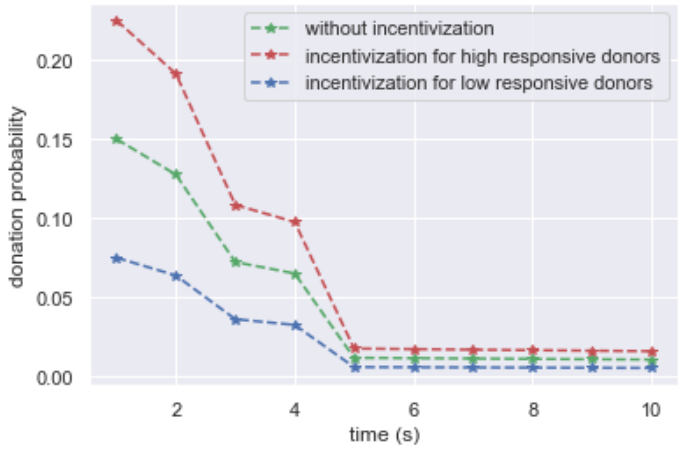
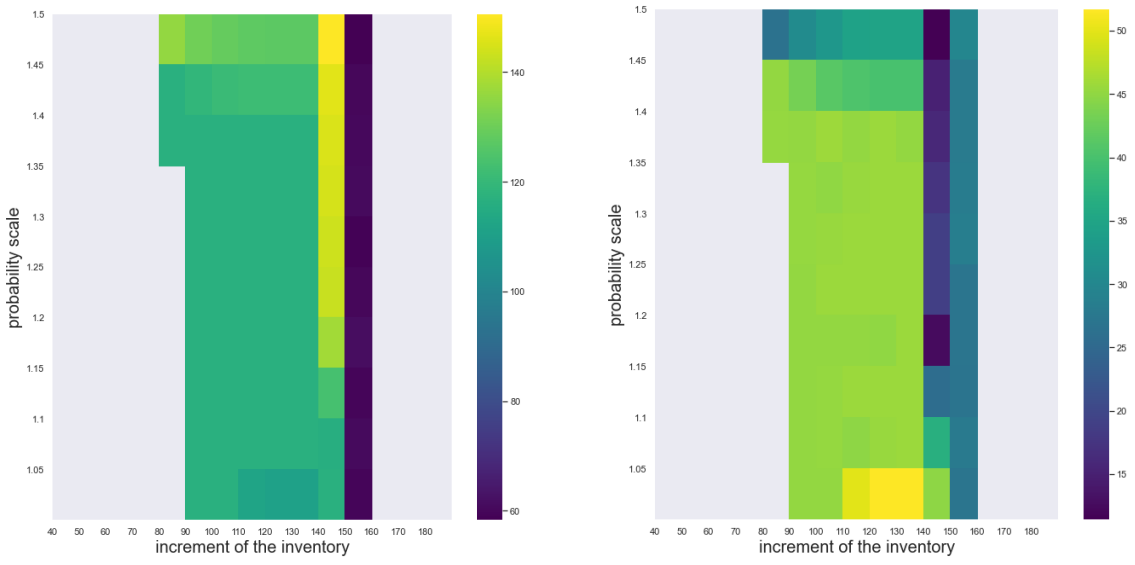


Figure 3.9: Two classes of donors



(a) High-responsive donors

(b) Low-responsive donors

Figure 3.10: Two classes with scaled probabilities of donation

From Figure 3.10, we draw two main observations. First, the structure of the optimal policy appears to be reversed for high and low responsive donors. This is intuitive as in situations where many donors are incentivized to expedite their donations, we would not expect many donors to be incentivized to delay their donations simultaneously. Second, the policy appears to be more sensitive to present inventory levels. Again, this makes sense as the inventory level should dictate whether or not to incentivize donors to expedite or delay their donations.

In the second instance, we study the two-class donor setting. Here, the effect of incentivization on donation likelihood is shown in Figure 3.11. Low responsive donors are assumed to not react to incentivization. The shifting of their donation likelihood forward is a simple time-updating due to the one-period delay in the incentivization process. In contrast, high responsive donors will show an increase in their donation probabilities, which will entail a lower rate of drop-out. The structures of the optimal policies are plotted in Figure 3.12.

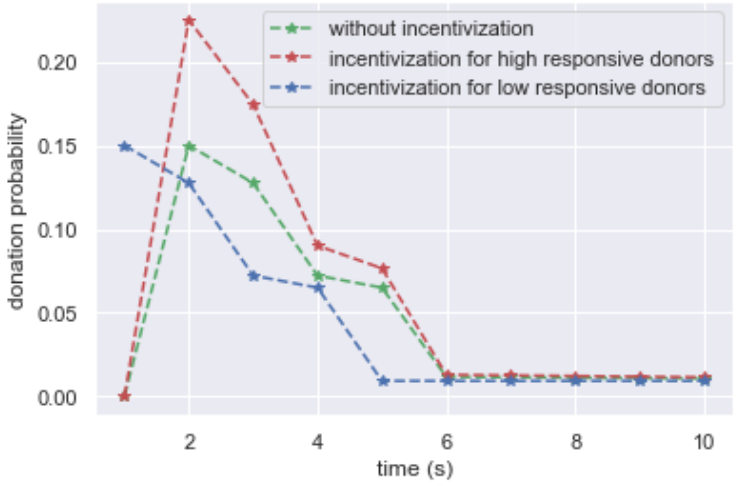
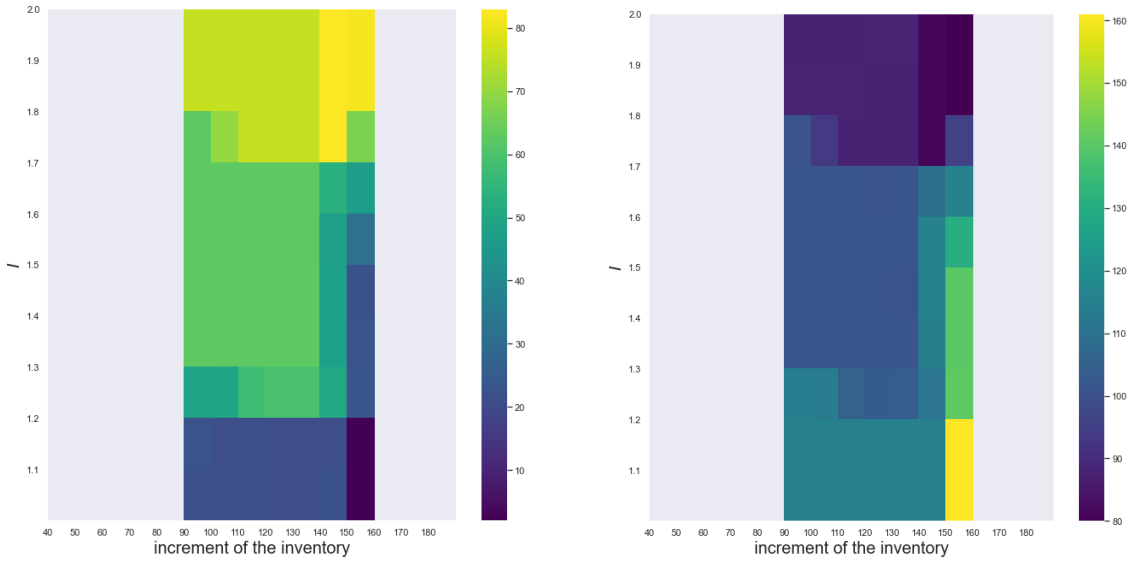


Figure 3.11: Two classes of donors



(a) High-responsive donors (b) Low-responsive donors

Figure 3.12: Two classes with shifting and scaling

There are two features of note. First, as before, the structure of the optimal policy is reversed for high and low responsive donors for the same reasons. Second, there is a small lower-right corner where the inventory is high and the intensity I_1 is low, that the model suggests incentivizing low-responsive donors. While this might seem strange at first glance, the policy essentially suggests that in times of excess, calling low responsive donors as a risk-pooling measure would help alleviate downstream shortages. There is also the added element of helping to preserve the donor pool by delaying donor drop-outs. This insight may be critical for blood banks, as they often only begin to incentivize donors when the inventory is low, which complements the earlier discussion in §3.4.3.

3.5 Conclusion

In this paper, we proposed an optimization framework to solve the donor incentivization policy in the blood donor management problem. Our framework is novel in which it simultaneously models the dynamics of both the blood inventory and the

donor flow process in a manner that can tractably solve the incentivization decisions. The numerical experiments indicate a dynamic situation of how the optimal policies can change with various factors from the responsiveness of donors and present inventory levels, to changes in demand for blood, new donor recruitment rate and distribution of donors in the observation window. Moreover, our framework can be easily extended to practical situations, particularly in operationalizing an optimization decision-making support system for the proposed strategy of managing high and low-responsive donors.

Work has begun in Singapore to classify blood donors into high and low-responsive classes. It has opened the opportunity for blood donor management, and most excitingly, is the basis for the application of our proposed framework. With this, I hope to collaborate with the local authorities as part of our future work in this area.

Chapter 4

Conclusion

The global demographic shifts (ageing population) and growing medical complexities continue to raise multiple challenges for academic researchers, practitioners and policymakers in the field of healthcare operations. Motivated by the problems outlined in the above chapters, my dissertation seeks to identify incentivization schemes to strengthen the delivery of healthcare operations by modifying individual behaviours. Specifically, this dissertation comprises two essays involving (1) the econometric framework studying colorectal cancer screening adherence, and (2) an optimization model that seeks to solve optimal donor incentivization policies. These two essays broadly covers various perspectives at the strategic, operational, and policy levels of healthcare systems.

The empirical frameworks developed in chapter 2 describes the predictors of screening adherence. In studying the potential interventions which policymakers can consider, the essay further investigates the impact of financial mediators in relation to screening decisions and propose strategic interventions to raise adherence levels. Given the rich data set that the nationwide survey had collected, I intend to examine the predictors between different subgroups of adherence in participants as a future work. Furthermore, the survey also features an RCT component and I intend to study

the effects of education in relation to screening adherence. These analyses can further provide managerial insights for policymakers in designing effective interventions to promote adherence on a nationwide level.

In chapter 3, my personal experience as a consistent blood donor for more than ten years was the motivation for me to look into improving the inventory-responsive blood donor management system. My work develops optimal donor incentivization policy and incorporates the unique properties relating to blood donation, including donor heterogeneity, rate of donor arrival, present inventory levels, varying demand patterns, mandatory observation period and perishable property of blood inventory. As discussed, classification of blood donors by their respective responsiveness has already been in the works. I hope to collaborate with local authorities to operationalize the proposed framework.

As an extension of my study in behavioural incentivization policies, I would like to explore future works in the domain of humanitarian operations and in particular, disaster relief supply chain management.

In every disaster or humanitarian crisis, international aid from foreign governments and non-profit international humanitarian organizations can mean the difference between life and death. Typically, after a disaster or crisis has occurred, international humanitarian organizations will send out calls for donations across the various countries which it operates in. During these heightened calls for aid, these humanitarian organizations tend to experience a spike in the donations received. However, a significant proportion of these donations by well-meaning individuals tend to be goods in kind which are unsuitable and do not meet the needs of the receiving populations. For instance, many of the donations received when the Tropical Cyclone Pam struck Vanuatu in 2016 were unsolicited food items that had expired by the time they were accessed or apparel that were inappropriate for Vanuatu's weather and culture (Australian Red Cross, 2020). For local governments and disaster relief workers on the

ground, instead of aiding relief efforts, these donations lead to wastage and results in additional costs being incurred in the processing, organizing and disposing of the goods. This puts additional strain on the humanitarian supply chain, which is already operating at its maximum capacity in ensuring that relief is properly administered to the needy populations. I would like to explore the strategic design and use of incentivization schemes to influence altruistic donation patterns, particularly for goods in kind.

The dissertation features multiple research methods, including empirical and modelling frameworks, and advances the literature in both data and methodological techniques. As described in the dissertation, the challenges and complexities in healthcare operations management continues to afflict both researchers and practitioners. I hope my research findings shed light on healthcare process understanding pertaining to behavioural incentivization and non-profit operations affecting the healthcare OM decisions.

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Chapter 5

Appendix

5.1 Variable Description

Participant Demographics

Gender	Gender of participant
Race	Race of participant
Age	Age of participant
Age^2	Quadratic term of age
AcadQual	Academic qualification
BMI	Body Mass Index
Chronic_Illness	Presence of chronic illnesses
Citizenship	Citizenship

Participant's Habits and Views

Phy_act	Physical activity in hours per week
Alcohol	Binary variable of alcohol consumption
Trust_gov	Factor of trust in government
Emotion	Emotionally sad over the past 30 days

Participant's Individual Perceived Beliefs

Perceived_health_con	Perceived health condition
P_riskcrc	Perceived risk of CRC
P_riskcrc^2	Quadratic term of p_riskcrc

Probability Score and Interaction terms

Prob_score	Probability proficiency
P_riskcrc $\times$_PScr	Interaction term between p_riskcrc and prob_score

Participants CRC Knowledge and Health Literacy

FIT_Knowledge	FIT awareness questions
Screening_Knowledge	CRC screening questions
Colo_Knowledge	Colonoscopy questions
Symp_awareness	Questionnaire on CRC symptoms

Financial Factors

CPF_Balance	CPF account balance
Private_Ins	Private insurance

5.2 Survey

(1) Background for Colorectal Cancer and its screening

Colorectal cancer, also known as bowel cancer, colon cancer, or rectal cancer, is any cancer that affects the colon and the rectum.

Colorectal cancer can develop from a "polyp", a nonspecific term to describe a growth on the inner surface of the colon. This survey aims to understand your awareness and opinions towards colorectal cancer and its screening process.

Which of the following apply to you? Please tick all that apply.

1) I have medical history of colitis (inflammation of the inner lining of the colon)
2) I have medical history of polyps
3) I have medical history of colorectal cancer
4) I have a close relative with colorectal cancer (parents, siblings, or children)
5) I have 2 or more close relatives with colorectal cancer (parents, siblings, or children)
6) I have a close relative with other cancers (parents, siblings, or children)
0) None of the above

If question (1) option 3 is empty:

(2) Lifetime chance of developing CRC

On average, 3-5% of Singaporeans will develop colorectal cancer in their lifetime.

What do you think are the chances that you will develop colorectal cancer in your lifetime?

1) Much less likely than average (Less than 1%)
2) Less likely than average (1–3%)
3) Average (3–5%)
4) More likely than average (5–10%)
5) Much more likely than average (More than 10%)

If question (1) option 2 AND 3 is empty:

(3) Lifetime chance of developing polyps

On average, 10-15% of Singaporeans will develop polyps in their lifetime.

What do you think are the chances that you will develop polyps in your lifetime?

1 Much less likely than average (Less than 5%)
2 Less likely than average (5–10%)
3 Average (10–15%)
4 More likely than average (15–20%)
5 Much more likely than average (More than 20%)

(4) False negative on FIT Testing

A fecal occult blood test (FOBT) looks at a sample of your stool (feces) to check for hidden (occult) blood that you can't see with the naked eye. The Faecal Immunochemical Test (FIT) is an advanced version of FOBT. These tests are preliminary tests that are used to estimate the chance of having polyps or bowel cancer through detecting occult blood in the stool sample.

We are interested in your views of the accuracy of these tests.

If 100 people who have colorectal cancer take a FOBT/FIT test, how many do you think will incorrectly be classified as not having colorectal cancer?

0) None, 0

1) 1 - 5

2) 6 - 10

3) 11 - 20

4) 21 - 30

5) 31 - 50

6) 51 - 70

7) 71 - 99

8) All 100

(5) False positive on FIT testing

Now consider the opposite scenario.

If 100 people who do not have colorectal cancer take a FOBT/FIT test, how many do you think will incorrectly be classified as having colorectal cancer?

0) None, 0

1) 1 - 5

2) 6 - 10

3) 11 - 20

4) 21 - 30

5) 31 - 50

6) 51 - 70

7) 71 - 99

8) All 100

(6) Perform a FOBT/FIT before?

Have you ever done a FOBT/FIT before? If yes, how many times?

If 100 people who have colorectal cancer take a FOBT/FIT test, how many do you think will incorrectly be classified as not having colorectal cancer?

1) No, never

2) Yes, once in the past

3) Yes, more than once, but not regularly

4) Yes, regularly, but less than once a year

5) Yes, regularly, once a year

If question (6) option 2 OR 3 OR 4 OR 5 is selected:

(7) Retrieval of FIT kit

Where did you get the test kit for your most recent FOBT/FIT??

1) Hospital
2) General Practitioner (GP)
3) Polyclinics
4) Pharmacy or Health & Beauty Store
5) Singapore Cancer Society (SCS)
6) Private Clinic

If question (6) option 2 OR 3 OR 4 OR 5 is selected:

(8) Expense of the FIT kit

How much did you pay out-of-pocket for your most recent FOBT/FIT?

If 100 people who have colorectal cancer take a FOBT/FIT test, how many do you think will incorrectly be classified as not having colorectal cancer?

0) $0, it was free
1) $1 - $4.90
2) $5 - $9.90
3) $10 - $29.90
4) $30 - $49.90
5) More than $50

(9) Reasons to take the most recent FIT

Why did you decide to take that most recent FOBT/FIT?

Please check all that apply.

1) I know the FOBT/FIT is recommended for people over 50 years of age
2) It was cheap or free
3) I was familiar with the procedure of the test
4) I felt uncomfortable/there were some symptoms
5) I had no symptoms, I just wanted to know my health condition
6) I felt that the test was safe and convenient
7) I thought that the FOBT/FIT was very accurate
8) I have a medical history of colon diseases like colitis, polyps, etc.
9) My doctor/friends/relatives recommended that I should take FOBT/FIT
10) Other, please specify

(10) Number of times receive most positive results on FIT

How many times have you received a positive result (indicating the presence of blood in your sample) from an FOBT/FIT test?

0) None
1) One
2) Two or more

If question (6) option 1 is selected:

(11) Reason for not taking up FIT testing

What are the reasons why you have never done an FOBT/FIT test?

Please check all that apply

1) I do not know much about the test or where to take it
2) I do not want to pay the costs to do the test
3) I feel the test is inconvenient and complicated
4) I think I am healthy enough and do not need to take the test
5) I am afraid to know any bad news from the test
6) I am too busy to do the test
7) I do not think that the test is accurate enough
8) I think I am too old for test
9) None or very few of my friends or relatives have done the test

(12) Chance of screening with FIT next year

In fact, FOBT/FIT screening is free for Singaporeans and Permanent Residents over 50 years old from the Singapore Cancer Society. Now that you know that you can get the FOBT/FIT kit for free, what are the chances that you will do an FOBT/FIT test in the next year?

(0% means no chance and 100% means absolutely certain)

(13) View on health condition

Which of the following statements best describes how you felt about your health condition?

1) I was extremely confident that I was healthy
2) I was somewhat confident that I was healthy
3) I was a bit worried
4) I was very pessimistic

(14) Knowledge on actual health condition

How much did you want to know your **actual health condition**?

1) I was very eager to know my actual health condition
2) I wanted to know my actual health condition
3) I was afraid to know my actual health condition

If question (10) option 0 is selected

(15) Hypothetical factors to follow doctor's recommendation

Imagine you received a positive result from an FOBT/FIT test and the doctor recommended that you have a colonoscopy, which of the following factors would you consider when deciding whether to follow the doctor's recommendations?

Please check all that apply

1) My medical history
2) My age
3) How confident I am about my health condition
4) How much I trust my doctor
5) Accuracy of FOBT/FIT
6) How much I want to know my actual health condition
7) Embarrassment of doing a colonoscopy
8) Comfort of a colonoscopy
9) Price of a colonoscopy
10) How supportive my family/friends were
99) None of the above

(16) Random Control Trial

We are interested in your understanding of the facts relating to CRC.

Please indicate whether you think that each of the following statements are True or False.

1) Colorectal cancer is the most diagnosed cancer in Singapore and second most common cause of cancer-related deaths. Every day, around five Singaporeans are diagnosed with colorectal cancer and two die of it
2) Even if colorectal cancer is detected at an early stage, it can only be controlled and is unlikely to be cured
3) If colorectal cancer is detected at stage 1 the survival rates can be as high as 92%, compared to 11% for stage 4
4) Few instances of colorectal cancer begin as polyps. So even if polyps are removed in a timely manner, patients can still develop colorectal cancer from other sources.
5) Early stage polyps or colorectal cancer cannot be detected by regular screening.
6) If polyps are detected during a colonoscopy, the patient needs to make another appointment for a procedure to remove the polyps.
7) Colonoscopy is very accurate (almost 100%), but it is very costly and has potential serious side effects.
8) The nationwide Screen for Life (SFL) programme offers affordable and convenient screening for colorectal cancer.
9) Singaporeans and PRs aged 50 and above can obtain FOBT/FIT kits for free from the Singapore Cancer Society (SCS), once a year.
10) FOBT/FIT should be done once every year. If the result is positive, colonoscopy is strongly recommended.

(17) Chance to test with FIT next year after learning CRC screening facts

IF Testing Group: "After learning these facts about colorectal cancer and its screening tests, what"

IF Control Group: "What"

... are the chances that you will do an FOBT/FIT test in the next year?

(0% means no chance and 100% means absolutely certain)

(18) Warning signs of colorectal cancer

We are interested in the kinds of symptoms that you think could be a warning sign of cancer. For each of the following symptoms, please tick 'Yes' if you think it could be a warning sign for colorectal cancer, or 'No' if you think it is not a warning sign for colorectal cancer.

1) Bleeding from back passage
2) Persistent pain in your abdomen (tummy)
3) A change in colorectal habits (diarrhoea, constipation or both) over a period of weeks
4) A feeling that your colorectal does not completely empty after using the toilet
5) Blood in your stools
6) Pain in your back passage
7) A lump in your abdomen (tummy)
8) Tiredness/anaemia
9) Unexplained weight loss

(19) How soon will participant contact doctor if notice signs of cancer

If you had a symptom that you thought might be a sign of cancer how soon would you contact your doctor to make an appointment to discuss it?

1) Immediately
2) Within three days
3) Within one week
4) Within one month
5) I would wait and see

(20) Average physical activity

On average, about how many hours per week do you do physical activities? Some examples may include walking for exercise, working out in neighbourhood parks, jogging, swimming, cycling, and so on.

1) Less than 1 hour per week
2) 1 – 2 hours per week
3) 3 – 4 hours per week
4) 5 – 6 hours per week
5) More than 7 hours per week

5.3 Proof of Results

Proof of Proposition 1. The first statement can be easily shown by using the law of iterated expectations. The first statement implies that for a fixed t, the state variables $y^{t,s}$ are independent across $s \in [S]$, which completes the proof. □

Proof of Proposition 2. For $k, \theta > 0$, we have

$$\begin{aligned}\mathbb{P}\left[\tilde{\zeta} \geq \Delta\right] =& \mathbb{P}\left[\exp\left(\tilde{\zeta}/k\theta\right) \geq \exp\left(\Delta/k\theta\right)\right] \\ \leq& \mathbb{E}\left[\exp\left(\tilde{\zeta}/k\theta\right)\right] / \exp\left(\Delta/k\theta\right) \\ \leq& \exp\left(-\Delta/k\theta\right).\end{aligned}$$

Here the first inequality is due to the Markov's inequality and the second inequality follows from the condition that $C_{k,\theta}[\tilde{\zeta}] \leq 0$, which implies $\mathbb{E}\left[\exp\left(\tilde{\zeta}/k\theta\right)\right] \leq 1$. □

Proof of Proposition 3. For a specific $t \in [T]$,

$$\begin{aligned}& k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{S} y^{t,s} a_y^s / k\theta_{y,t}\right)\right] \\ =& k \sum_{s=0}^{S} \log \mathbb{E}\left[\exp\left(y^{t,s} a_y^s / k\theta_{y,t}\right)\right] \\ =& k \sum_{s=0}^{t-1} y^{t-s,0} \log\left(1 - \Omega^{t,s} + \Omega^{t,s} \exp(a_y^s / k\theta_{y,t})\right) \\ & \quad + k \sum_{s=t}^{S} y^{0,s-t} \log\left(1 - \Omega^{t,s} + \Omega^{t,s} \exp(a_y^s / k\theta_{y,t})\right) \\ =& k \sum_{s=0}^{t-1} \sum_{s'=0}^{S} q^{t-s,s'} \log\left(1 - \Omega^{t,s} + \Omega^{t,s} \exp(a_y^s / k\theta_{y,t})\right) \\ & \quad + k \sum_{s=t}^{S} y^{0,s-t} \log\left(1 - \Omega^{t,s} + \Omega^{t,s} \exp(a_y^s / k\theta_{y,t})\right) \\ =& k \sum_{s=0}^{t-1} \sum_{s'=0}^{S} q^{t-s,s'} \rho_{\Omega}^{t,s}(a_y^s / k\theta_{y,t}) + k \sum_{s=t}^{S} y^{0,s-t} \rho_{\Omega}^{t,s}(a_y^s / k\theta_{y,t}),\end{aligned}$$

where the first equality is due to Proposition 1(b), the second equality is due to

Proposition 1(a), and the third equality is due to the fact that $y^{t,0} = \sum_{s'=0}^{S} q^{t,s'}$, for all $t \in [T]$. □

Proof of Proposition 4. First, we have

$$
\begin{aligned}
& C_{k,\theta_{z,t}}\left[\sum_{s=0}^{S} a_z^s z^{t,s}\right] \\
=& k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1} z^{t,s} a_z^s / k\theta_{z,t} + \sum_{s=t}^{S} z^{t,s} a_z^s / k\theta_{z,t}\right)\right] \\
=& k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1}\left(z^{t-s,0} - \sum_{\tau=0}^{s-1} q^{t-\tau,s-\tau}\right) a_z^s / k\theta_{z,t}\right.\right. \qquad (5.1) \\
& \qquad \left.\left. + \sum_{s=t}^{S}\left(z^{0,s-t} - \sum_{\tau=0}^{t-1} q^{t-\tau,s-\tau}\right) a_z^s / k\theta_{z,t}\right)\right] \\
=& \underbrace{k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1} z^{t-s,0} a_z^s / k\theta_{z,t}\right)\right]}_{\text{first term}} \underbrace{- \frac{1}{\theta_{z,t}} \sum_{s=0}^{t-1}\sum_{\tau=0}^{s-1} a_z^s q^{t-\tau,s-\tau}}_{\text{second term}} \qquad (5.2) \\
& \qquad \underbrace{+ \frac{1}{\theta_{z,t}} \sum_{s=t}^{S} a_z^s \left(z^{0,s-t} - \sum_{\tau=0}^{t-1} q^{t-\tau,s-\tau}\right)}_{\text{third term}}. \qquad (5.3)
\end{aligned}
$$

Now we consider the first term in the above result,

$$
\begin{aligned}
& k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1} z^{t-s,0} a_z^s / k\theta_{z,t}\right)\right] \\
=& k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1}\left(\lambda^{t-s} + \sum_{s'=1}^{S}\left[\mathrm{Bin}\left(y^{t-s-1,s'-1}, \omega^{t-s,s'}\right)\right]\right) a_z^s / k\theta_{z,t}\right)\right] \\
=& \underbrace{k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1}\sum_{s'=1}^{S}\left[\mathrm{Bin}\left(y^{t-s-1,s'-1}, \omega^{t-s,s'}\right)\right] a_z^s / k\theta_{z,t}\right)\right]}_{Z_t(k)} + C_{k,\theta_{z,t}}\left[\sum_{s=0}^{t-1} a_z^s \lambda^{t-s}\right],
\end{aligned}
$$

where the first equality is due to Equation (3.1).

We use $\mathbb{1}$ to represent the indicator function to simplify our exposition in the following calculations; thus $\mathbb{1}\{\mathcal{C}\} = 1$ if set $\mathcal{C}$ is nonempty and $\mathbb{1}\{\mathcal{C}\} = 0$ if $\mathcal{C}$ is

empty. Then, we have

$$
\begin{aligned}
& k \log \mathbb{E}\Bigg[\exp\Bigg(\sum_{s=0}^{t-1} \sum_{s'=1}^{S} (a_z^s / k\theta_{z,t}) \mathrm{Bin}(y^{t-s-1,s'-1}, \omega^{t-s,s'}) \Bigg) \Bigg] \\
=& k \log \mathbb{E}\Bigg[\exp\Bigg(\sum_{j=1}^{t-1} \sum_{s'=1}^{j} (a_z^{j-s'} / k\theta_{z,t}) \mathrm{Bin}\big(y^{t-j+s'-1,s'-1}, \omega^{t-j+s',s'}\big) \\
& + \sum_{j=t}^{S-1} \sum_{s'=j-t+1}^{j} (a_z^{j-s'} / k\theta_{z,t}) \mathrm{Bin}\big(y^{t-j+s'-1,s'-1}, \omega^{t-j+s',s'}\big) \\
& + \sum_{j=S}^{t+S-1} \sum_{s'=j-t+1}^{S} (a_z^{j-s'} / k\theta_{z,t}) \mathrm{Bin}\big(y^{t-j+s'-1,s'-1}, \omega^{t-j+s',s'}\big) \Bigg) \Bigg] \\
=& k \sum_{j=1}^{t-1} \log \mathbb{E}\Bigg[\exp\Bigg(\sum_{s'=1}^{j} (a_z^{j-s'} / k\theta_{z,t}) \mathrm{Bin}\big(y^{t-j+s'-1,s'-1}, \omega^{t-j+s',s'}\big) \Bigg) \Bigg] \\
& + k \sum_{j=t}^{S-1} \log \mathbb{E}\Bigg[\exp\Bigg(\sum_{s'=j-t+1}^{j} (a_z^{j-s'} / k\theta_{z,t}) \mathrm{Bin}\big(y^{t-j+s'-1,s'-1}, \omega^{t-j+s',s'}\big) \Bigg) \Bigg] \\
& + k \sum_{j=S}^{t+S-1} \log \mathbb{E}\Bigg[\exp\Bigg(\sum_{s'=j-t+1}^{S} (a_z^{j-s'} / k\theta_{z,t}) \mathrm{Bin}\big(y^{t-j+s'-1,s'-1}, \omega^{t-j+s',s'}\big) \Bigg) \Bigg] \\
=& k \sum_{j=1}^{t-1} \log \mathbb{E}\Bigg[\exp\Bigg(\sum_{\ell=1}^{y^{t-j,0}} \sum_{s'=1}^{j} (a_z^{j-s'} / k\theta_{z,t}) \\
& \qquad \mathbb{1}\{\text{Individual } \ell \text{ will donate at time } t-j+s'\} \Bigg) \Bigg] \\
& + k \sum_{j=t}^{S-1} \log \mathbb{E}\Bigg[\exp\Bigg(\sum_{\ell=1}^{y^{0,j-t}} \sum_{s'=j-t+1}^{j} (a_z^{j-s'} / k\theta_{z,t}) \\
& \qquad \mathbb{1}\{\text{Individual } \ell \text{ will donate at time } t-j+s'\} \Bigg) \Bigg] \\
& + k \sum_{j=S}^{t+S-1} \log \mathbb{E}\Bigg[\exp\Bigg(\sum_{\ell=1}^{y^{0,j-t}} \sum_{s'=j-t+1}^{S} (a_z^{j-s'} / k\theta_{z,t}) \\
& \qquad \mathbb{1}\{\text{Individual } \ell \text{ will donate at time } t-j+s'\} \Bigg) \Bigg] \\
=& k \sum_{j=1}^{t-1} \sum_{\ell=1}^{y^{t-j,0}} \Upsilon_{1,j}^{t,j} + k \sum_{j=t}^{S-1} \sum_{\ell=1}^{y^{0,j-t}} \Upsilon_{j-t+1,j}^{t,j} + k \sum_{j=S}^{t+S-1} \sum_{\ell=1}^{y^{0,j-t}} \Upsilon_{j-t+1,S}^{t,j} \\
=& k \sum_{j=1}^{t-1} \sum_{s'=0}^{S} q^{t-j,s'} \Upsilon_{1,j}^{t,j} + k \sum_{j=t}^{S-1} y^{0,j-t} \Upsilon_{j-t+1,j}^{t,j} + k \sum_{j=S}^{t+S-1} y^{0,j-t} \Upsilon_{j-t+1,S}^{t,j}.
\end{aligned}
$$

Here, the first equality follows from a regrouping of terms, and the second equality comes from Proposition 1, which implies the terms under the outer sum in the first equality are independent across $j \in [t+S-1]$. The third equality follows from the property of binomial random variables and the fact that the terms under the inner sum in the second equality are fully correlated, the fourth equality is because of the property of disjoint events and Assumption 2, and the last equality is due to $y^{t,0} = \sum_{s'=0}^{S} q^{t,s'}$, for all $t \in [T]$. The assertion now follows if we apply the above results to $C_{k,\theta_{z,t}} \left[\sum_{s=0}^{S} a_z^s z^{t,s} - b_z^t\right] \leq 0$. □

Proof of Proposition 5. The proof is analogous to the proof of Proposition 1. □

Proof of Proposition 6. The proof is analogous to the proof of Proposition 3. □

Proof of Proposition 7. Note that we use $\beta^{t',0} = y^{t',0}$ for all $t' \in [t-1]$ and $\beta^{0,t'} = y^{0,t'}$ for all $t' \in [S] \setminus [t-1]$ to simplify the reformulation. First, we have

$$k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{S} y^{t,s} a_y^s / k\theta_{y,t}\right)\right] = y^{t,0} a_y^0/\theta_{y,t} + k \sum_{s=1}^{S} \log \mathbb{E}\left[\exp\left(y^{t,s} a_y^s / k\theta_{y,t}\right)\right],$$

where the equality is due to Proposition 1(b). Then, using the law of iterated expectation and the fact that $\exp(\cdot)$ is convex increasing, we have

$$\begin{aligned}
& k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{S} y^{t,s} a_y^s / k\theta_{y,t}\right)\right] \leq b_y^t/\theta_{y,t} \\
\iff & y^{t,0} a_y^0/\theta_{y,t} + k \sum_{s=1}^{S} \log \mathbb{E}\left[\exp\left(y^{t,s} a_y^s / k\theta_{y,t}\right)\right] \leq b_y^t/\theta_{y,t} \\
\iff & y^{t,0} a_y^0/\theta_{y,t} + k \sum_{s=1}^{S} \log \mathbb{E}_{<t}\left[\exp\left(y^{t-1,s-1} \frac{\beta^{t,s}}{\beta^{t-1,s-1}} \rho_{1-\omega}^{t,s}\left(a^s{}_y / k\theta_{y,t}\right)\right)\right] \leq b_y^t/\theta_{y,t} \qquad (5.4) \\
\iff & \begin{cases} y^{t,0} a_y^0/\theta_{y,t} + k \sum_{s=1}^{S} \log \mathbb{E}_{<t}\left[\exp\left(y^{t-1,s-1} \frac{\xi^{t,s}}{\beta^{t-1,s-1}}\right)\right] \leq b_y^t/\theta_{y,t} \\ \xi^{t,s} \geq \beta^{t,s} \rho_{1-\omega}^{t,s}(a_y^s/k\theta_{y,t}) \qquad \forall s \in [S] \end{cases} \qquad (5.5)
\end{aligned}$$

$$
\iff \begin{cases} y^{t,0}a_y^0/\theta_{y,t} + k\xi^{t,1} + k\sum_{s=2}^{S} \log \mathbb{E}_{<t}\left[\exp\left(y^{t-1,s-1}\frac{\xi^{t,s}}{\beta^{t-1,s-1}}\right)\right] \le b_y^t/\theta_{y,t} \\ \xi^{t,s} \ge \beta^{t,s}\rho_{1-\omega}^{t,s}(a_y^s/k\theta_{y,t}) \qquad \forall s \in [S] \end{cases} \tag{5.6}
$$

$$
\iff \begin{cases} y^{t,0}a_y^0/\theta_{y,t} + k\xi^{t,1} \\ \quad +k\sum_{s=2}^{S} \log \mathbb{E}_{<t-1}\left[\exp\left(y^{t-2,s-2}\frac{\beta^{t-1,s-1}}{\beta^{t-2,s-2}}\rho_{1-\omega}^{t-1,s-1}\left(\xi^{t,s}/\beta^{t-1,s-1}\right)\right)\right] \le b_y^t/\theta_{y,t} \\ \xi^{t,s} \ge \beta^{t,s}\rho_{1-\omega}^{t,s}(a_y^s/k\theta_{y,t}) \qquad \forall s \in [S] \end{cases}
$$

$$
\iff \begin{cases} y^{t,0}a_y^0/\theta_{y,t} + k\sum_{\tau=0}^{1} \xi^{t-\tau,1} + k\sum_{s=3}^{S} \log \mathbb{E}_{<t-1}\left[\exp\left(y^{t-2,s-2}\frac{\xi^{t-1,s-1}}{\beta^{t-2,s-2}}\right)\right] \le b_y^t/\theta_{y,t} \\ \xi^{t,s} \ge \beta^{t,s}\rho_{1-\omega}^{t,s}(a_y^s/k\theta_{y,t}) \qquad \forall s \in [S] \\ \xi^{t-1,s-1} \ge \beta^{t-1,s-1}\rho_{1-\omega}^{t-1,s-1}(\xi^{t,s}/\beta^{t-1,s-1}) \qquad \forall s \in [S]\setminus\{1\} \end{cases}
$$

$$
\iff \cdots
$$

$$
\iff \begin{cases} y^{t,0}a_y^0/\theta_{y,t} + k\sum_{\tau=0}^{t-2} \xi^{t-\tau,1} + k\sum_{s=t}^{S} \log \mathbb{E}\left[\exp\left(y^{0,s-t}\frac{\xi^{1,s-t+1}}{\beta^{0,s-t}}\right)\right] \le b_y^t/\theta_{y,t} \\ \xi^{t,s} \ge \beta^{t,s}\rho_{1-\omega}^{t,s}(a_y^s/k\theta_{y,t}) \qquad \forall s \in [S] \\ \xi^{t-\tau,s-\tau} \ge \beta^{t-\tau,s-\tau}\rho_{1-\omega}^{t-\tau,s-\tau}(\xi^{t+1-\tau,s+1-\tau}/\beta^{t-\tau,s-\tau}) \qquad \forall \tau \in [t-1], s \in [S]\setminus[\tau] \end{cases}
$$

$$
\iff \begin{cases} y^{t,0}a_y^0/\theta_{y,t} + k\sum_{\tau=0}^{t-2} \xi^{t-\tau,1} + k\sum_{\tau=t}^{S} \xi^{1,\tau-t+1} \le b_y^t/\theta_{y,t} \\ \xi^{t,s} \ge \beta^{t,s}\rho_{1-\omega}^{t,s}(a_y^s/k\theta_{y,t}) \qquad \forall s \in [S] \\ \xi^{t-\tau,s-\tau} \ge \beta^{t-\tau,s-\tau}\rho_{1-\omega}^{t-\tau,s-\tau}(\xi^{t+1-\tau,s+1-\tau}/\beta^{t-\tau,s-\tau}) \quad \forall \tau \in [t-1], s \in [S]\setminus[\tau]. \end{cases}
$$

Here (5.4) is using the law of iterated expectation, (5.5) is due to the convexity of $\rho_{1-\omega}^{t,s}(\cdot)$ and the fact that $\exp(\cdot)$ is convex increasing, and (5.6) is using $\beta^{t-1,0} = y^{t-1,0}$. Recursively, we will obtain the final reformulation. □

Proof of Proposition 8. Observe that

$$
\begin{aligned}
& k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{S} v^{t,s} a_v^s / k\theta_{v,t}\right)\right] \\
=& k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1} v^{t,s} a_v^s / k\theta_{v,t} + \sum_{s=t}^{S} v^{t,s} a_v^s / k\theta_{v,t}\right)\right] \\
=& k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1}\left(v^{t-s,0} - \sum_{\tau=0}^{s-1} r^{t-\tau,s-\tau}\right)(a_v^s / k\theta_{v,t})\right.\right. && (5.7) \\
& \qquad \left.\left. + \sum_{s=t}^{S}\left(v^{0,s-t} - \sum_{\tau=0}^{t-1} r^{t-\tau,s-\tau}\right)(a_v^s / k\theta_{v,t})\right)\right] \\
=& k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1} v^{t-s,0} a_v^s / k\theta_{v,t}\right)\right] - \frac{1}{\theta_{v,t}} \sum_{s=0}^{t-1}\sum_{\tau=0}^{s-1} a_v^s r^{t-\tau,s-\tau} && (5.8) \\
& \qquad + \frac{1}{\theta_{v,t}} \sum_{s=t}^{S} a_v^s \left(v^{0,s-t} - \sum_{\tau=0}^{t-1} r^{t-\tau,s-\tau}\right) && (5.9)
\end{aligned}
$$

Hence, it suffices to reformulate $k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1} v^{t-s,0} a_v^s / k\theta_{v,t}\right)\right] \leq V$. Recall that $v^{t,0} = \sum_{s=1}^{S} \alpha^{t,s} y^{t-1,s-1} - r^{t,0}$, for all $t \in [T]$. We have

$$
\begin{aligned}
& k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1} v^{t-s,0} a_v^s / k\theta_{v,t}\right)\right] \\
=& k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1}\sum_{s'=1}^{S} \alpha^{t-s,s'} y^{t-s-1,s'-1}\left(a_v^s / k\theta_{v,t}\right)\right)\right] - \frac{1}{\theta_{v,t}} \sum_{s=0}^{t-1} a_v^s r^{t-s,0} \\
=& k \log \mathbb{E}\Bigg[\exp\Bigg(\sum_{j=1}^{t-1}\sum_{s'=1}^{j} \alpha^{t-j+s',s'} y^{t-j+s'-1,s'-1}\left(a_v^{j-s'} / k\theta_{v,t}\right) \\
& \quad + \sum_{j=t}^{S-1}\sum_{s'=j-t+1}^{j} \alpha^{t-j+s',s'} y^{t-j+s'-1,s'-1}\left(a_v^{j-s'} / k\theta_{v,t}\right) \\
& \quad + \sum_{j=S}^{t+S-1}\sum_{s'=j-t+1}^{S} \alpha^{t-j+s',s'} y^{t-j+s'-1,s'-1}\left(a_v^{j-s'} / k\theta_{v,t}\right)\Bigg)\Bigg] - \frac{1}{\theta_{v,t}} \sum_{s=0}^{t-1} a_v^s r^{t-s,0}
\end{aligned}
$$

$$= \underbrace{k \sum_{j=1}^{t-1} \log \mathbb{E}\Big[\exp\Big(\sum_{s'=1}^{j} \alpha^{t-j+s',s'} y^{t-j+s'-1,s'-1} \big(a_v^{j-s'}/k\theta_{v,t}\big) \Big) \Big]}_{\text{first term}}$$
$$+ \underbrace{k \sum_{j=t}^{S-1} \log \mathbb{E}\Big[\exp\Big(\sum_{s'=j-t+1}^{j} \alpha^{t-j+s',s'} y^{t-j+s'-1,s'-1} \big(a_v^{j-s'}/k\theta_{v,t}\big) \Big) \Big]}_{\text{second term}}$$
$$+ \underbrace{k \sum_{j=S}^{t+S-1} \log \mathbb{E}\Big[\exp\Big(\sum_{s'=j-t+1}^{S} \alpha^{t-j+s',s'} y^{t-j+s'-1,s'-1} \big(a_v^{j-s'}/k\theta_{v,t}\big) \Big) \Big]}_{\text{third term}} - \frac{1}{\theta_{v,t}} \sum_{s=0}^{t-1} a_v^s r^{t-s,0} \tag{5.10}$$

where the first equality is due to inflow dynamics of the holding queue, i.e., $v^{t,0} = \sum_{s=1}^{S} \alpha^{t,s} y^{t-1,s-1} - r^{t,0}$, for all $t \in [T]$, and the third equality stems from Proposition 1, which implies the terms under the outer sum in the second equality are independent across $j \in [t+S-1]$. Therefore, reformulating $k \log \mathbb{E}\left[\exp\left(\sum_{s=0}^{t-1} v^{t-s,0} a_v^s / k\theta_{v,t}\right)\right] \leq V$ would be equivalent to reformulating

$$\text{1st term} \leq V_1,\ \text{2nd term} \leq V_2,\ \text{3rd term} \leq V_3,\ V_1 + V_2 + V_3 - \sum_{s=0}^{t-1} a_v^s r^{t-s,0}/\theta_{v,t} \leq V, \tag{5.11}$$

where V_1, V_2 and V_3 are auxiliary variables.

We first consider the constraint given by first term $\leq V_1$, which gives

$$
\begin{aligned}
& k\sum_{j=1}^{t-1}\log\mathbb{E}\Big[\exp\Big(\sum_{s'=1}^{j}\alpha^{t-j+s',s'}y^{t-j+s'-1,s'-1}\big(a_v^{j-s'}/k\theta_{v,t}\big)\Big)\Big]\leq V_1 \\
\iff & \begin{cases} k\sum_{j=2}^{t-1}\log\mathbb{E}_{<t-1}\Big[\exp\Big(y^{t-2,j-2}\dfrac{\eta^{t-1,j-1}}{\beta^{t-2,j-2}}+\sum_{s'=1}^{j-1}\dfrac{\alpha^{t-j+s',s'}y^{t-j+s'-1,s'-1}}{k\theta_{v,t}/a_v^{j-s'}}\Big)\Big] \\ \qquad\qquad +\left(a_v^0\left(\beta^{t-1,0}-\beta^{t,1}\right)/theta_{v,t}\right)\leq V_1 \\ \eta^{t-1,j-1}\geq\beta^{t-1,j-1}\rho_{1-\omega}^{t-1,j-1}\Big(\dfrac{a_v^0(\beta^{t-1,j-1}-\beta^{t,j})}{k\theta_{v,t}\beta^{t-1,j-1}}\Big) \qquad \forall j\in[t-1]\setminus\{1\} \end{cases} \\
\iff & \begin{cases} k\sum_{j=3}^{t-1}\log\mathbb{E}_{<t-2}\Big[\exp\Big(y^{t-3,j-3}\dfrac{\eta^{t-2,j-2}}{\beta^{t-3,j-3}}+\sum_{s'=1}^{j-2}\dfrac{\alpha^{t-j+s',s'}y^{t-j+s'-1,s'-1}}{k\theta_{v,t}/a_v^{j-s'}}\Big)\Big] \\ \qquad\qquad +\dfrac{\beta^{t-1,0}-\beta^{t,1}}{\theta_{v,t}/a_v^0}+\dfrac{\beta^{t-2,0}-\beta^{t-1,1}}{\theta_{v,t}/a_v^1}+k\eta^{t-1,1}\end{cases}\leq V_1 \\
\iff & \begin{cases} \eta^{t-1,j-1}\geq\beta^{t-1,j-1}\rho_{1-\omega}^{t-1,j-1}\Big(\dfrac{a_v^0(\beta^{t-1,j-1}-\beta^{t,j})}{k\theta_{v,t}\beta^{t-1,j-1}}\Big) \qquad \forall j\in[t-1]\setminus\{1\} \\ \eta^{t-2,j-2}\geq\beta^{t-2,j-2}\rho_{1-\omega}^{t-2,j-2}\Big(\dfrac{a_v^1(\beta^{t-2,j-2}-\beta^{t-1,j-1})+k\theta_{v,t}\eta^{t-1,j-1}}{k\theta_{v,t}\beta^{t-2,j-2}}\Big) \\ \qquad\qquad \forall j\in[t-1]\setminus\{1,2\} \end{cases} \\
\iff & \cdots \\
\iff & \begin{cases} \sum_{j=1}^{t-1}(\beta^{j,0}-\beta^{j+1,1})a_v^{t-j-1}/\theta_{v,t}+k\sum_{j=2}^{t-1}\eta^{t-j+1,1}\leq V_1 \\ \eta^{t-1,j-1}\geq\beta^{t-1,j-1}\rho_{1-\omega}^{t-1,j-1}\Big(\dfrac{a_v^0(\beta^{t-1,j-1}-\beta^{t,j})}{k\theta_{v,t}\beta^{t-1,j-1}}\Big) \quad \forall j\in[t-1]\setminus\{1\} \\ \eta^{t-\tau,j-\tau}\geq\beta^{t-\tau,j-\tau}\rho_{1-\omega}^{t-\tau,j-\tau}\Big(\dfrac{a_v^{\tau-1}(\beta^{t-\tau,j-\tau}-\beta^{t-\tau+1,j-\tau+1})+k\theta_{v,t}\eta^{t-\tau+1,j-\tau+1}}{k\theta_{v,t}\beta^{t-\tau,j-\tau}}\Big) \\ \qquad\qquad \forall\tau\in[t-2]\setminus\{1\},j\in[t-1]\setminus[\tau]. \end{cases}
\end{aligned}
\tag{5.12}
$$

Here we use the law of iterated expectation, the convexity of $\rho_{1-\omega}^{t,s}(\cdot)$, and the fact that $\exp(\cdot)$ is convex increasing. Moreover, we use $\beta^{j,0}=y^{j,0}$, for all $j\in[t-1]$, to simplify the reformulation.

Similarly, for the constraint given by second term $\leq V_2$, we have

$$
\begin{aligned}
& k\sum_{j=t}^{S-1}\log\mathbb{E}\Big[\exp\Big(\sum_{s'=j-t+1}^{j}\alpha^{t-j+s',s'}y^{t-j+s'-1,s'-1}\big(a_v^{j-s'}/k\theta_{v,t}\big)\Big)\Big]\leq V_2\\
\Longleftrightarrow\ & \begin{cases}
\displaystyle\sum_{j=t}^{S-1}(\beta^{0,j-t}-\beta^{1,j-t+1})a_v^{t-1}/\theta_{v,t}+k\sum_{j=t}^{S-1}\eta^{1,j-t+1}\leq V_2\\
\eta^{t-1,j-1}\geq\beta^{t-1,j-1}\rho_{1-\omega}^{t-1,j-1}\left(\dfrac{a_v^0(\beta^{t-1,j-1}-\beta^{t,j})}{k\theta_{v.t}\beta^{t-1,j-1}}\right)\quad\forall j\in[S-1]\setminus[t-1]\\
\eta^{t-\tau,j-\tau}\geq\beta^{t-\tau,j-\tau}\rho_{1-\omega}^{t-\tau,j-\tau}\left(\dfrac{a_v^{\tau-1}(\beta^{t-\tau,j-\tau}-\beta^{t-\tau+1,j-\tau+1})+k\theta_{v,t}\eta^{t-\tau+1,j-\tau+1}}{k\theta_{v,t}\beta^{t-\tau,j-\tau}}\right)\\
\qquad\qquad\qquad\qquad\qquad\qquad\forall\tau\in[t-1]\setminus\{1\},j\in[S-1]\setminus[t-1];
\end{cases}
\end{aligned}
\tag{5.13}
$$

for the constraint given by third term $\leq V_3$, we have

$$
\begin{aligned}
& k\sum_{j=S}^{t+S-1}\log\mathbb{E}\Big[\exp\Big(\sum_{s'=j-t+1}^{S}\alpha^{t-j+s',s'}y^{t-j+s'-1,s'-1}\big(a_v^{j-s'}/k\theta_{v,t}\big)\Big)\Big]\leq V_3\\
\Longleftrightarrow\ & \begin{cases}
\displaystyle\sum_{j=S-t}^{S-1}(\beta^{0,j}-\beta^{1,j+1})a_v^{t-1}/\theta_{v,t}+k\sum_{j=S}^{t+S-2}\eta^{1,j-t+1}\leq V_3\\
\eta^{t-j,S-1}\geq\beta^{t-j,S-1}\rho_{1-\omega}^{t-j,S-1}\left(\dfrac{a_v^{j-1}(\beta^{t-j,S-1}-\beta^{t-j+1,S})}{k\theta_{v,t}\beta^{t-j,S-1}}\right)\quad\forall j\in[t-1]\\
\eta^{t-j-\tau,S-\tau}\geq\beta^{t-j-\tau,S-\tau}*\\
\qquad\rho_{1-\omega}^{t-j-\tau,S-\tau}\left(\frac{a_v^{j+\tau-1}(\beta^{t-j-\tau,S-\tau}-\beta^{t-j-\tau+1,S-\tau+1})+k\theta_{v,t}\eta^{t-j-\tau+1,S-\tau+1}}{k\theta_{v,t}\beta^{t-j-\tau,S-\tau}}\right)\\
\qquad\qquad\qquad\qquad\forall\tau\in[t-1]\setminus\{1\},j\in[t-\tau-1]_0.
\end{cases}
\end{aligned}
\tag{5.14}
$$

Applying (5.9)–(5.14) to $C_{k,\theta_{v,t}}\left[\sum_{s=0}^{S}v^{t,s}a_v^s-b_v^t\right]\leq 0$ and eliminating the redundant auxiliary variables V_1,V_2,V_3,V, we will obtain the final reformulation provided in Proposition 8. $\square$

Proof of Proposition 9. By Equation (5.3), we have

$$\begin{aligned}
&C_{k,\theta_{z,t}}\left[\sum_{s=0}^{S} a_z^s z^{t,s}\right] \\
=&k\log\mathbb{E}\Big[\exp\Big(\sum_{s=0}^{t-1} z^{t-s,0}a_z^s/k\theta_{z,t}\Big)\Big] - \frac{1}{\theta_{z,t}}\sum_{s=0}^{t-1}\sum_{\tau=0}^{s-1} a_z^s q^{t-\tau,s-\tau} \\
&+\frac{1}{\theta_{z,t}}\sum_{s=t}^{S} a_z^s\Big(z^{0,s-t} - \sum_{\tau=0}^{t-1} q^{t-\tau,s-\tau}\Big).
\end{aligned}$$

Next, we consider the first term in the above result,

$$\begin{aligned}
&k\log\mathbb{E}\Big[\exp\Big(\sum_{s=0}^{t-1} z^{t-s,0}a_z^s/k\theta_{z,t}\Big)\Big] \\
=&k\log\mathbb{E}\Big[\exp\Big(\sum_{s=0}^{t-1}(a_z^s/k\theta_{z,t})\Big[\lambda^{t-s} + \sum_{s'=1}^{S}\mathrm{Bin}\Big(u^{t-s-1,s'-1},\phi^{t-s,s'}\Big) \\
&+\sum_{s'=1}^{S}\mathrm{Bin}\Big((1-\alpha^{t-s,s'})y^{t-s-1,s'-1},\omega^{t-s,s'}\Big)\Big]\Big)\Big] \\
=&C_{k,\theta_{z,t}}\Big[\sum_{s=0}^{t-1} a_z^s\lambda^{t-s}\Big] + \underbrace{k\log\mathbb{E}\Big[\exp\Big(\sum_{s=0}^{t-1}\sum_{s'=1}^{S}\mathrm{Bin}\Big(u^{t-s-1,s'-1},\phi^{t-s,s'}\Big)a_z^s/k\theta_{z,t}\Big)\Big]}_{\bar{Z}_t(k)} \\
&+k\log\mathbb{E}\Big[\exp\Big(\sum_{s=0}^{t-1}\sum_{s'=1}^{S}\mathrm{Bin}\Big((1-\alpha^{t-s,s'})y^{t-s-1,s'-1},\omega^{t-s,s'}\Big)a_z^s/k\theta_{z,t}\Big)\Big],
\end{aligned}$$

where the first equality is due to Equation (3.7), the second equality is guaranteed by Assumptions 1–3, and the evaluation of $\bar{Z}_t(k)$ is analogous to $Z_t(k)$ in the proof of Proposition 4. Hence, it suffices to reformulate $k\log\mathbb{E}\Big[\exp\Big(\sum_{s=0}^{t-1}\sum_{s'=1}^{S}\mathrm{Bin}\Big((1-\alpha^{t-s,s'})y^{t-s-1,s'-1},\omega^{t-s,s'}\Big)a_z^s/k\theta_{z,t}\Big)\Big] \le Z$.

Note that

$$
\begin{aligned}
& k \log \mathbb{E}\Big[\exp\Big(\sum_{s=0}^{t-1} \sum_{s'=1}^{S} a_z^s \mathrm{Bin}\Big((1-\alpha^{t-s,s'}) y^{t-s-1,s'-1}, \omega^{t-s,s'} \Big) / k\theta_{z,t} \Big) \Big] \\
=& k \log \mathbb{E}\Big[\exp\Big(\sum_{s=0}^{t-1} \sum_{s'=1}^{S} a_z^s \mathrm{Bin}\Big((\frac{\beta^{t-s,s'}}{\beta^{t-s-1,s'-1}}) y^{t-s-1,s'-1}, \omega^{t-s,s'} \Big) / k\theta_{z,t} \Big) \Big] \\
=& \underbrace{k \sum_{j=1}^{t-1} \log \mathbb{E}\Big[\exp\Big(\sum_{s'=1}^{j} a_z^{j-s'} \mathrm{Bin}\Big((\frac{\beta^{t-j+s',s'}}{\beta^{t-j+s'-1,s'-1}}) y^{t-j+s'-1,s'-1}, \omega^{t-j+s',s'} \Big) / k\theta_{z,t} \Big) \Big]}_{\text{first term}} \\
& \underbrace{+ k \sum_{j=t}^{S-1} \log \mathbb{E}\Big[\exp\Big(\frac{\sum_{s'=j-t+1}^{j} a_z^{j-s'} \mathrm{Bin}\Big((\frac{\beta^{t-j+s',s'}}{\beta^{t-j+s'-1,s'-1}}) y^{t-j+s'-1,s'-1}, \omega^{t-j+s',s'} \Big)}{k\theta_{z,t}} \Big) \Big]}_{\text{second term}} \\
& \underbrace{+ k \sum_{j=S}^{t+S-1} \log \mathbb{E}\Big[\exp\Big(\frac{\sum_{s'=j-t+1}^{S} a_z^{j-s'} \mathrm{Bin}\Big((\frac{\beta^{t-j+s',s'}}{\beta^{t-j+s'-1,s'-1}}) y^{t-j+s'-1,s'-1}, \omega^{t-j+s',s'} \Big)}{k\theta_{z,t}} \Big) \Big]}_{\text{third term}}
\end{aligned}
$$

Here, the first equality is due to $1-\alpha^{t,s} = \beta^{t,s}/\beta^{t-1,s-1}$, and the second equality stems from Proposition 1, which implies the terms under the outer sum in the second equality are independent across $j \in [t+S-1]$. Therefore, reformulating $k \log \mathbb{E}\Big[\exp\Big(\sum_{s=0}^{t-1} \sum_{s'=1}^{S} \mathrm{Bin}\Big((1-\alpha^{t-s,s'}) y^{t-s-1,s'-1}, \omega^{t-s,s'} \Big) a_z^s / k\theta_{z,t} \Big) \Big] \leq Z$ would be equivalent to reformulating first term $\leq Z_1$, second term $\leq Z_2$, third term $\leq Z_3$, $Z_1+Z_2+Z_3 \leq Z$, where Z_1, Z_2 and Z_3 are auxiliary variables.

We first reformulate the constraint given by first term $\leq Z_1$, which gives

$$
\begin{aligned}
& k \sum_{j=1}^{t-1} \log \mathbb{E}\Big[\exp\Big(\sum_{s'=1}^{j} a_z^{j-s'} \mathrm{Bin}\Big((\frac{\beta^{t-j+s',s'}}{\beta^{t-j+s'-1,s'-1}}) y^{t-j+s'-1,s'-1}, \omega^{t-j+s',s'} \Big) / k\theta_{z,t} \Big) \Big] \leq Z_1 \\
\iff & k \sum_{j=1}^{t-1} \log \mathbb{E}_{<t}\Big[\exp\Big(\sum_{s'=1}^{j-1} \mathrm{Bin}\Big((\frac{\beta^{t-j+s',s'}}{\beta^{t-j+s'-1,s'-1}}) y^{t-j+s'-1,s'-1}, \omega^{t-j+s',s'} \Big) (\frac{a_z^{j-s'}}{k\theta_{z,t}}) \\
& \quad + (\frac{y^{t-1,j-1}}{\beta^{t-1,j-1}}) \beta^{t,j} \rho_\omega^{t,j} (\frac{a_z^0}{k\theta_{z,t}}) \Big) \Big] \leq Z_1
\end{aligned}
$$

$$
\iff \begin{cases} k\sum_{j=2}^{t-1}\log\mathbb{E}_{<t}\Big[\exp\Big(\sum_{s'=1}^{j-1}\mathrm{Bin}\Big((\frac{\beta^{t-j+s',s'}}{\beta^{t-j+s'-1,s'-1}})y^{t-j+s'-1,s'-1},\omega^{t-j+s',s'}\Big)(\frac{a_z^{j-s'}}{k\theta_{z,t}}) \\ \qquad +(\frac{y^{t-1,j-1}}{\beta^{t-1,j-1}})\psi^{t,j}\Big)\Big]+k\psi^{t,1}\le Z_1 \\ \psi^{t,j}\ge\beta^{t,j}\rho_\omega^{t,j}(a_z^0/k\theta_{z,t}) \qquad\qquad \forall j\in[t-1] \end{cases}
$$

$$
\iff \begin{cases} k\sum_{j=2}^{t-1}\log\mathbb{E}_{<t-1}\Big[\exp\Big(\sum_{s'=1}^{j-2}\mathrm{Bin}\Big((\frac{\beta^{t-j+s',s'}}{\beta^{t-j+s'-1,s'-1}})y^{t-j+s'-1,s'-1},\omega^{t-j+s',s'}\Big)(\frac{a_z^{j-s'}}{k\theta_{z,t}})\Big) \\ \qquad \times\mathbb{E}_{t-1}\Big[\exp\Big(\frac{a_z^1}{k\theta_{z,t}}\mathrm{Bin}\Big((\frac{\beta^{t-1,j-1}}{\beta^{t-2,j-2}})y^{t-2,j-2},\omega^{t-1,j-1}\Big) \\ \qquad +\frac{\psi^{t,j}}{\beta^{t-1,j-1}}\mathrm{Bin}\Big((\frac{\beta^{t-1,j-1}}{\beta^{t-2,j-2}})y^{t-2,j-2},1-\omega^{t-1,j-1}\Big)\Big)\Big]\Big]+k\psi^{t,1}\le Z_1 \\ \psi^{t,j}\ge\beta^{t,j}\rho_\omega^{t,j}(a_z^0/k\theta_{z,t}) \qquad\qquad \forall j\in[t-1] \end{cases}
$$

$$
\iff \begin{cases} k\sum_{j=2}^{t-1}\log\mathbb{E}_{<t-1}\Big[\exp\Big(\sum_{s'=1}^{j-2}\mathrm{Bin}\Big((\frac{\beta^{t-j+s',s'}}{\beta^{t-j+s'-1,s'-1}})y^{t-j+s'-1,s'-1},\omega^{t-j+s',s'}\Big)(\frac{a_z^{j-s'}}{k\theta_{z,t}})\Big) \\ \underbrace{\quad\times\prod_{\ell=1}^{\frac{\beta^{t-1,j-1}}{\beta^{t-2,j-2}}y^{t-2,j-2}}\Big(\omega^{t-1,j-1}\exp\Big(\frac{a_z^1}{k\theta_{z,t}}\Big)+(1-\omega^{t-1,j-1})\exp\Big(\frac{\psi^{t,j}}{\beta^{t-1,j-1}}\Big)\Big)\Big]+k\psi^{t,1}}_{\le Z_1} \\ \psi^{t,j}\ge\beta^{t,j}\rho_\omega^{t,j}(a_z^0/k\theta_{z,t}) \qquad\qquad \forall j\in[t-1] \end{cases}
$$

(5.15)

$$
\iff \begin{cases} k\sum_{j=3}^{t-1}\log\mathbb{E}_{<t-1}\Big[\exp\Big(\sum_{s'=1}^{j-2}\mathrm{Bin}\Big((\frac{\beta^{t-j+s',s'}}{\beta^{t-j+s'-1,s'-1}})y^{t-j+s'-1,s'-1},\omega^{t-j+s',s'}\Big)(\frac{a_z^{j-s'}}{k\theta_{z,t}}) \\ \qquad +\frac{\psi^{t-1,j-1}}{\beta^{t-2,j-2}}y^{t-2,j-2}\Big)\Big]+k(\psi^{t,1}+\psi^{t-1,1})\le Z_1 \\ \psi^{t,j}\ge\beta^{t,j}\rho_\omega^{t,j}(a_z^0/k\theta_{z,t}) \qquad\qquad \forall j\in[t-1] \\ \psi^{t-1,j-1}\ge\beta^{t-1,j-1}\sigma_\omega^{t-1,j-1}(a_z^1/k\theta_{z,t},\psi^{t,j}/\beta^{t-1,j-1}) \qquad \forall j\in[t-1]\setminus\{1\} \end{cases}
$$

$$
\iff \cdots
$$

$$
\iff \begin{cases} k\sum_{j=1}^{t-1}\psi^{t-j+1,1}\le Z_1 \\ \psi^{t,j}\ge\beta^{t,j}\rho_\omega^{t,j}(a_z^0/k\theta_{z,t}) \qquad\qquad \forall j\in[t-1] \\ \psi^{t-\tau,j-\tau}\ge\beta^{t-\tau,j-\tau}\sigma_\omega^{t-\tau,j-\tau}(a_z^\tau/k\theta_{z,t},\psi^{t-\tau+1,j-\tau+1}/\beta^{t-\tau,j-\tau}) \\ \qquad\qquad\qquad\qquad \forall\tau\in[t-2],j\in[t-1]\setminus[\tau] \end{cases}
$$

Here we use the law of iterated expectation, and the fact that $\exp(\cdot)$ is convex increasing. Moreover, we use $\beta^{j,0} = y^{j,0}$, for all $j \in [t-1]$, to simplify the reformulation. It should be noted that (5.15) is due to the property of binomial random variables and disjoint events.

Similarly, the constraint given by second term $\leq Z_2$ is equivalent to

$$\begin{cases} k \sum_{j=t}^{S-1} \psi^{1,j-t+1} \leq Z_2 \\ \psi^{t,j} \geq \beta^{t,j} \rho_\omega^{t,j}(a_z^0 / k\theta_{z,t}) & \forall j \in [S-1] \setminus [t-1] \\ \psi^{t-\tau,j-\tau} \geq \beta^{t-\tau,j-\tau} \sigma_\omega^{t-\tau,j-\tau}(a_z^\tau / k\theta_{z,t}, \psi^{t-\tau+1,j-\tau+1} / \beta^{t-\tau,j-\tau}) \\ & \forall \tau \in [t-1], j \in [S-1] \setminus [t-1]; \end{cases}$$

the constraint given by third term $\leq Z_3$ is equivalent to

$$\begin{cases} k \sum_{j=S}^{t+S-1} \psi^{1,j-t+1} \leq Z_3 \\ \psi^{t-j,S} \geq \beta^{t-j,S} \rho_\omega^{t-j,S}(a_z^j / k\theta_{z,t}) & \forall j \in [t-1]_0 \\ \psi^{t-j-\tau,S-\tau} \geq \beta^{t-j-\tau,S-\tau} \sigma_\omega^{t-j-\tau,S-\tau}\left(a_z^{j+\tau} / k\theta_{z,t}, \frac{\psi^{t-j-\tau+1,S-\tau+1}}{\beta^{t-j-\tau,S-\tau}}\right) \\ & \forall \tau \in [t-1], j \in [t-\tau-1]_0. \end{cases}$$

Applying the above results to $C_{k,\theta_{z,t}} \left[\sum_{s=0}^{S} a_z^s z^{t,s} - b_z^t \right] \leq 0$ and eliminating the redundant auxiliary variables Z_1, Z_2, Z_3, Z, we will obtain the final form provided in Proposition 9. □

Proof of Theorem 1. This result follows directly from Propositions 3–9. □

5.4 Pseudo Code for the Solution Approach

Algorithm 1 Cutting-Plane Algorithm for the Inventory-Responsive Donor Management Policy

Require: Tolerances ϵ and $\epsilon_{\mathcal{K}}$. Set as objective to minimize any constraint for $t = T$ (say capacity for the eligible registered donors pool $y^{T,0}a_y^0/\theta_{y,T} + k\sum_{j=0}^{T-2}\xi^{T-j,1} + k\sum_{j=T}^{S}\xi^{1,j-T+1}$). Let $b_y^T/\theta_{y,T}$ be its target.

Initialization: $k_- \leftarrow 0$, sufficiently large k_+.

Let $\mathcal{Q}(k)$ be the model consisting of all exponential cone constraints in (3.8) having the form

$$f(\beta^-, \beta^+, \gamma^+; \omega) := \beta^- \log\left(\omega\exp(a) + (1-\omega)\exp\left(\frac{b\beta^- + c\beta^+ + d\gamma^+}{\beta^-}\right)\right) \leq \gamma^-,$$

are replaced with asymptotic linear estimates,

$$\begin{cases} \beta^-(a + \log\omega) \leq \gamma^- \\ \beta^-\log(1-\omega) + b\beta^- + c\beta^+ + d\gamma^+ \leq \gamma^-. \end{cases}$$

while $k_+ - k_- > \epsilon$ **do**
  $k := \bar{k} \leftarrow (k_- + k_+)/2$
  Solve $\mathcal{Q}(k)$. If feasible, obtain optimal value b^* and optimal policy $(\beta^{-*}, \beta^{+*}, \gamma^{-*}, \gamma^{+*})$.

  if $\mathcal{Q}(k)$ infeasible **or** $b^* > b_y^T/\theta_{y,T}$ **then**
    $k_- \leftarrow \bar{k}$

  else
    $\mathcal{F} \leftarrow \{f | f(\beta^{-*}, \beta^{+*}, \gamma^{+*}; \omega) - \gamma^{-*} > \epsilon_{\mathcal{K}}\}$

    if $\mathcal{F} \neq \emptyset$ **then**
      Add hyperplane $\forall f \in \mathcal{F}$ at $(\beta^{-*}, \beta^{+*}, \gamma^{-*}, \gamma^{+*})$ into $\mathcal{Q}(k)$.

    else
      $k_+ \leftarrow \bar{k}$

    end if
  end if
end while

Output: Optimal $k^* = k_+$ and optimal policy $(\beta^{-*}, \beta^{+*}, \gamma^{-*}, \gamma^{+*})$.