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Stochastic capacity management in the presence of production resource disruption

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I hereby declare that this PhD dissertation is my original work
and it has been written by me in its entirely.
I have duly acknowledged all the sources of information
which have been used in this dissertation.

This PhD dissertation has also not been submitted for any degree
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Abstract

This dissertation studies the capacity investment decision of a manufacturing firm facing demand uncertainty in the presence of shortage possibility in production resources, as often ignored in the literature. These production resources can be physical resources (component / raw material) or financial resources (working capital / budget). The shortage in these resources can be caused by a variety of supply chain disruptions; examples include global disruptions like COVID-19 and financial crisis in 2008 and local disruptions like shortage of components/workforce. The dissertation analyses two important issues related to capacity management: (i) the effect of production resource disruption on the capacity investment strategy and the profitability of the firm (including the significance of profitability loss incurred when the resource shortage possibility is ignored, and (ii) the role of production resource disruption management strategies, i.e. using pre-shipment financing to mitigate the effect of financial resource disruption and using hedging to mitigate physical resource disruption.

The first part (Chapter 3) examines a two stage capacity-production framework that capacity investment decision is in anticipation of demand and production resource uncertainties and production quantity is decided after the revelation of uncertainties. I characterize the optimal decisions and investigate how the uncertainties (demand and production resource variability and the correlation between the two) affect the optimal capacity investment level and the profitability. My results provide rule of thumb for the managers in capacity management. I also study the significance of profitability loss incurred when the resource uncertainty is ignored in choosing capacity level. Through both analytical and extensive numerical analysis, I show that the profitability loss is high when 1) correlation is high; 2) either production resource variability is sufficiently high or sufficiently

low; and 3) either demand variability is sufficiently high or sufficiently low.

The second part (Chapter 4) examines the role of pre-shipment finance in managing financial production resource (working capital/budget) disruption. Pre-shipment finance allows the firm to transfer the purchase orders (which will be paid after production) to an external party that provides immediate cash flow (at a cost) that can be used for financing the production process. To this end, I characterize the optimal pre-shipment finance level (proportion of sales revenues transferred) and the production volume in the production stage and the optimal capacity investment level in the capacity stage. I make comparisons with the results in the first chapter to understand how pre-shipment financing alters the effects of demand and production resource uncertainties on the optimal capacity investment level, expected profit and profitability loss due to ignoring resource uncertainty. I identify that applying pre-shipment finance makes the capacity investment and profits more resilient to changes in spot price uncertainty.

The third part (Chapter 5) studies the role of procurement hedging contract in managing physical production resource (e.g., component/raw material) disruption. With the hedging contract, the firm can engineer the production resource uncertainty at the capacity investment stage—for example, with full hedging this uncertainty can be completely removed. I provide the joint characterization of the optimal hedging level and capacity investment decisions. I find that these decisions critically depend on the covariance between demand and production resource uncertainties and the unit capacity investment cost. For example, I find that fully hedging is always optimal when the correlation is non-positive. I highlight conditions under which the firm optimally does not hedge at all or use partial hedging strategy. I then investigate the significance of the profitability loss due to i) misspecification of capacity level by ignoring production resource uncertainty and ii) misspecification of hedging strategy (using full hedging which is easy to implement), and provide conditions under which these profitability losses are significant.

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Chapter 1

Introduction

It is well established that capacity investment decision, which is an important operational decision of manufacturing firms in a variety of industries, is subject to demand uncertainty; the capacity investment level for a product needs to be decided long before the actual product demand is realized. In practice, aside from the demand uncertainty, these manufacturing firms may also experience uncertainty in the production resources. When the production resources are variable, a shortage in these resources can limit the actual production; in particular, when the realized production resources volume is less than the planned production quantity. A key feature of this paper is to consider production resources uncertainty together with demand uncertainty.

These production resources can be financial (e.g., budget) or physical (e.g., labor, raw material, component) resources. On the financial production resources, the shortage in these resources can be attributed to the worsened external financing conditions (e.g., credit crunch, liquidity shocks); or to a decrease in internal financing (where the financial resources are allocated to another subsidiary). For example, in 2018, it is reported that majority of the small and medium enterprises (SMEs) in UK have experienced unavailability of financial resources (Financial Times, Bounds (2018)). Such liquidity problem due to lack of capital resources is challenging not only to SMEs but also to capital intensive companies. For example, China's National Electric Vehicle Sweden (NEVS) temporarily halted output of its Saab car due to shortage of funds in 2014 (The Wall Street Journal, Stoll (2014)). In Diamond mining industry, Indian diamond manufacturers released the excess inventory due to struggling to get credit, even though there is a growth of the middle class in China and India should boost demand (Financial Times, Dempsey and Parkin (2019)). On the physical resources, the shortage in these resources can be attributed to a variety of factors, e.g. reduction in raw material yield, less supply of components and tightening immigration policy. In recent breakout of Covid-19, the wreaking havoc of production resources disruption has caught attention in variety of industries (Harvard Business Review, Haren and Simchi-Levi (2020)). For example, drug-makers are facing significant disruption to global production due to Chinese lock-down and cutting off supplies of Chinese-made essential ingredients (Financial Times, Findlay et al. (2020)). Coincidentally, the world's fifth-biggest carmaker, Hyundai shut down all its car factories in South Korea after running out of components from China and searches new sources of engine wire-harness (Financial Times, White et al. (2020)). These examples demonstrate that it is important to consider the uncertainty in the availability of these (financial or physical) production resources in choosing the right capacity to invest.

The stochastic capacity investment problem has received wide attention in the operations management literature; see Van Mieghem (2003) for a comprehensive review. The majority of this literature examine how demand uncertainty affect capacity investment and profitability in the absence of production resources uncertainty. Among the few papers that consider production resource and demand uncertainties, such as Ciarallo et al. (1994) for physical resources, Boyabatlı et al. (2016) for financial resources, there is no paper that examines how production resource uncertainty affects the optimal capacity investment and profitability of the firm. Intuitively, ignoring production resource uncertainty leads to over-investment in capacity and, thus profitability loss, however, it is still an open question under what conditions this profitability loss is significant. In this paper, we attempt to fill this void by studying the optimal capacity investment problem with the presence of demand and production resource uncertainties.

To this end, we formulate a two-stage profit maximization problem. In the first stage, the firm chooses the capacity level to invest with the presence of demand and production resource uncertainties. In the second stage, after both the demand and production resource is realized, the firm then decides on the optimal production quantity. We note that the demand and resource shocks might be independent or positively/negatively correlated, as discussed in Babich (2010). With the model, we characterize the optimal capacity investment policy and answer the following research questions:

1) How would the optimal capacity level and profitability be impacted by the demand and production resource variability and the correlation between the two?

2) If the possibility of production resource shortage is ignored in capacity planning, as often done in practice and the academic literature, would the resulting profitability loss be significant and how do the demand and production resource uncertainties affect this profitability loss?

The Optimal Capacity Investment Policy. In answering the first question, we provide analytical results assuming that the demand and production resource follow a bivariate normal distribution. In addressing the second question, we conduct extensive numerical experiments when analytical results are not attainable. To delineate the impact of production resource uncertainty on our results, whenever applicable, we make comparisons with a benchmark scenario in which the possibility of production resource shortage is ignored in capacity planning. We summarize our main findings as below:

Impacts of Demand and production resource Uncertainties. We conduct sensitivity analysis, both analytically and numerically, to investigate the impact of demand and production resource variabilities and their correlation on both optimal capacity investment level and corresponding expected profit. When the possibility of production resource shortage is ignored, conventional understanding suggests that a higher demand variability increases optimal capacity level and profitability, as established in the traditional stochastic capacity investment literature. Interestingly, we find that a higher demand variability *decreases* both optimal capacity level and profitability when the demand variability is lower than certain threshold and the correlation is negative. Otherwise, the sensitivity results are consistent with conventional understanding. In terms of the sensitivity to production resource variability, we find that a higher production resource variability is beneficial (i.e., increases profitability) only when this variability is lower than certain threshold, the correlation is positive and unit capacity investment cost is low; otherwise, a lower production resource variability is beneficial. However, the optimal capacity level is monotonically increasing (decreasing) in the production resource variability when unit capacity cost is low (high) and the correlation is high (low). We also find that a higher correlation between the demand and production resource leads to higher optimal capacity level and the profitability. These results provide rules of thumb to manufacturing firms in managing their capacity investment decisions with respect to changing environmental conditions once the possibility of production resource shortage is taken into consideration.

Profitability Loss Incurred by Ignoring the Possibility of production resource Shortage. We calculate the percentage profit loss when the firm, instead using the optimal capacity investment policy, heuristically uses the benchmark policy in which the possibility of production resource shortage is ignored. We also analytically and numerically perform sensitivity analysis of profitability-loss to understand the effects of uncertainties. We prove that the profitability-loss is decreasing in the correlation, which implies that the profitability loss would be most significant when the demand and production resource is negatively correlated. We also prove that profitability-loss strictly increases in the production resource variability when the unit capacity cost is higher than certain threshold. We run extensive numerical experiments to obtain a comprehensive understanding on effects of demand and production resource variabilities on this profitability loss. There are patterns consistently observed throughout numerical analyses regarding to the sensitivity of profitability loss. For the impact of production resource variability under a low unit capacity cost scenario, profitability loss decreases in production resource variability when the variability is smaller than a threshold; profitability loss increases in production resource variability otherwise. For the sensitivity on demand variability, when the correlation between demand and production resource is positive, profitability loss first decreases then increases in demand variability; otherwise, as demand variability increases, profitability loss increases.

Pre-shipment Finance in Managing Financial Production Resource Disruption. Pre-shipment financing creates necessary liquidity for the firm when the budget is constraining. The firm chooses a loan that is fully secured within the product selling revenue. Then, we characterize the optimal pre-shipment finance level (proportion of sales revenues transferred) and the production volume in the production stage and the optimal capacity investment level in the capacity stage. We make comparisons with the results in Chapter 3 to understand how pre-shipment financing alters the effects of demand and production budget uncertainties on the optimal capacity investment level, expected profit and profitability-loss due to ignoring budget uncertainty. We identify that applying pre-shipment finance makes the capacity investment and profits more resilient to changes in both demand and production budget uncertainties. The profitability-loss from miss-specifying capacity is significantly reduced by using pre-shipment financing.

Procurement Hedging Contract in Managing Physical Production Resource Disruption. When production resource is physical such as raw material or component, pricing of material costs is directly linked to fluctuations in firm's production capability. Therefore, one of the way to manage the potential disruption of production resource would be to lock in prices for a production resource at a pre-determined fixed price through arriving at a fixed price procurement contract. In managing production resource disruption in the capacity investment stage, we apply procurement hedging contract, which is beneficial if it can allow the firm to avoid unnecessary fluctuations of physical production resource, e.g. raw material or component, in capacity investment spending. In this procurement hedging contract, the firm alters the distribution of production resource to manage the risk of production resource disruption. We find that the partial hedging dominates full hedging and no-hedging when demand and production resource is positively correlated and the unit cost of investing capacity is low; no-hedging dominates when the positive correlation between demand and production resource is high and the unit capacity investment cost is even lower. We identify that optimal partial hedging decreases in both demand variability and the correlation, whereas it increases in production resource variability. We show that the profitability increases in demand variability, production resource variability and the correlation. Our numerical analysis shows that the profitability loss due to ignoring production resource shortage possibility in choosing capacity investment level is significant when demand variability is large, production resource variability is large and the correlation is low. And the profitability loss due to heuristically choosing always full hedging increases in both demand and production resource variability.

1.1 Organization of the Dissertation

The structure of the rest of dissertation is organized as follows. In Chapter 2, I provide an extensive overview of the literature and discuss the contribution of our work by comparison with existing papers. The work contributes to following two streams of literature: 1) the stream of literature that studies inventory-production systems under uncertain capacity, 2) the stream of literature in OM-finance that studies the impact of financial constraints on operational decisions. In addition, I discuss the relevance of our work to the financial economic literature about the relation between productive investment and uncertainties (demand or/and production resource).

In Chapter 3, we investigate the impact of production resource and demand uncertainties on capacity investment of a manufacturing firm. The stochastic capacity management problem of the firm is formulated in Section 3.1, its optimal strategy is defined in Section 3.2. Then Section 3.3 is the sensitivity analyses of this basic model and, lastly, Section 3.4 examine the sensitivity analyses profitability-loss with numerical experiments.

In Chapter 4, we conduct an extension that the firm can finance its production after the purchase of the products (after demand is resolved) has been committed, that is, the basic model with a financing instrument called pre-shipment finance where the formulation is and the corresponding optimal policy is introduced in Section 4.2. Through sensitivity analyses in Section 4.3, we show that the financing makes the capacity investment decision and profitability more resilient to uncertainties. In Section 4.4, we conduct the numerical analysis to show that the profitability loss due to miss-specify capacity level is negligible.

In Chapter 5, we conduct an extension that the firm can hedge away the production resource uncertainty at the capacity investment stage, where the model formulation and the optimal strategy are introduced in Section 5.2 and 5.3, respectively. Through sensitivity analyses with respect to optimal capacity level, optimal hedging strategy and the corresponding profitability in Section 5.4, we show that the unfavourable uncertainties are removed by hedging strategy, in other words, the demand variability, production resource variability and the correlation between the two are either not affect the optimal decisions or increases the optimal capacity and profitability. Also, we perform profitability loss in Section 5.5. In the last, Chapter 6 concludes the thesis with a summary on the research results, managerial insights and potential research directions.

Chapter 2

Literature Review

Our work contribute to two streams of literature that study the production resources uncertainty. The first stream studies the inventory-production systems with unreliable physical production resources; the second stream is related to OM-finance interface that studies operational capacity and production decisions under financial constraints, by reason that financial constraints behave the same as production resource. I will highlight papers that investigate the impact of demand uncertainty or/and financial constraints uncertainty on the investment and point out the research gap that our work try to fill. Throughout the review, I summarize their research questions, main results and contributions of these relevant papers, more importantly, based on this review, our contribution to the literature is proposed.

The stream of literature that studies the inventory-production systems with supply uncertainty is discussed in Section 2.1. I review researches that study optimal inventory and production decisions under physical production resources in different business environments. The physical production resources uncertainty in the manufacturing industry was earliest identified by Lee and Billington (1993) who state, in the production process of HP printers, both process and supply activities may incur uncertainty. Process uncertainty comes from workforce level uncertainty and production rework; and supply uncertainty comes from quantity and quality of raw materials, components and delivery. Therefore, from formulation perspective, there are four types of uncertainty: random capacity, random yield, random supply lead time, and lastly, probabilistic on/off of production capacity which is called supply disruption and can be considered as special case of the rest types of uncertainties. Physical production resources uncertainty refers to random capacity category. Our work contributes to the literature that relaxing the assumption that random demand and random capacity are independent and increasing the understanding of random capacity in operations management, therefore, this part of review would focus on papers study operational models with random capacity.

The extensive review of operational capacity and production models under financial constraints is in Section 2.2. Financial constraints come from capital market imperfection indicating that firms are not always able to secure enough funds for production. This constraints also service similar effect as what random capacity does. Specifically, financial constraints may lead to cash crunch, that further cause the manufacturers unable to proceed the production up to the planned volume due to lack of working capital. Therefore, financial constraints can also be regarded as limited production resources. Financial constraints have been widely studied in OM-finance field, however, only few researches about the impact of its uncertainty on the operational decisions and corresponding profitability (excepting Babich (2010) and Boyabatlı et al. (2016)). I will provide an extensive review of the development of the literature.

2.1 Physical Production Resources Uncertainty

Physical production resources as the maximum productive capacity usually varies stochastically because of uncertainties in production processes. It may truncate planned production volume to the materialized volume of production resources. There exists a stream of literature that studies the impact of the uncertain capacity in the content of inventory-production problem.

Ciarallo et al. (1994) is the first to introduce uncertain capacity to the newsvonder framework. The decision of the framework is the production target (also called planned production quantity) in the presence of uncertain capacity and uncertain demand. After the uncertain capacity and demand are realized, the actual production quantity is the minimum between capacity and production target. The objective is to minimize the total expected costs that is composed by production, holding and penalty costs. Since then, researches that study multi-period newsvonder frameworks with uncertain capacity primarily focus on adding more operational features, characterizing the structure of the optimal policy, and understanding the impact of uncertain capacity on the optimal policy structure comparing with the one without uncertain capacity, so as to serve as a building block for dynamic inventory models. I will review these researches in the following paragraph.

The seminal work of Ciarallo et al. (1994) show that for a periodic-review finite-horizon newsvendor model with uncertain capacity, an order-up-to policy is optimal, and the optimal planned production quantity is identical to the results of the model without uncertainty capacity in the single period scenario. Wang and Gerchak (1996) add another uncertain feature on top of Ciarallo et al. (1994)'s work, that is random yield in the model describing the fact that only a random proportion of the actual production quantity could be qualified to satisfy demand. They also show that the random capacity is irrelevant to the optimal policy, because the optimal policy possesses the same structure as the optimal policy obtained by Henig and Gerchak (1990) for the case in which the random capacity is not considered. Inspired by frozen seafood industry operations, Khang and Fujiwara (2000) formulate a multi-period newsvendor model with raw material supply uncertainty in each period and constant demand, different from Ciarallo et al. (1994), the raw material supply volume is known and constraints production volume (one unit of raw material turns to one unit of finished product). They show that optimal order-up-to level is capped by the realized raw material supply volume and myopic ordering policy (buy all raw materials or satisfy all demand, whichever is possible) can be optimal under certain condition. Yang (2004) also study a multi-period newsvendor model with demand uncertainty and raw material supply uncertainty in each period. Similar as Khang and Fujiwara (2000), the raw material is materialized as the beginning of each period, but differently, raw material is storable and the firm accepts all raw material; also raw material is tradeable to its spot market. They contribute to the literature by providing a combination of two base-stock policy: one for raw material inventory and one for finished product inventory. Yang et al. (2005) extend the multi-period newsvendor problem with Markovian in-house production capacity and outsourcing option with setup cost and variable cost. The production level is decided after the realization of capacity. They obtain that the optimal outsourcing policy is (s, S) policy due to setup cost and optimal inventory policy is modified base-stock policy. For the sensitivity results: a higher current capacity level (stochastically) leaves the firm better off and both base-stock level and outsourcing level decrease. Feng (2010) extend Ciarallo et al. (1994)'s newsvendor framework with price-dependent random demand. She obtains that basestock policy is not optimal and the capacity uncertainty induces the optimal policy depending on the inventory level. She shows that optimal policy is monotone with respect the average capacity level but not with respect to the variability of the capacity. Feng and Shi (2012) extend Feng (2010)'s framework by adding multiple supply resources with uncertain capacity. They reveal that both supply diversification (due to multiple supply resources) and dynamic pricing are effective in raise profit when newsvendor's cost parameters decreases. Tan et al. (2016) also extend Ciarallo et al. (1994)'s newsvendor framework with one more 'slow' supply resource with random capacity, therein, 'slow' means one more period delivery lead time than 'normal' supply resource. They show that the slower supplier plays a crucial role in mitigating stockout risk when demand surge and the fast supplier is restricted by either capacity limitation or capacity uncertainty.

Besides papers that study multi-period newsvendor framework focus on the optimal policy development, there are papers investigate single-period newsvendor framework, in concern with other issues. Hu et al. (2008) address the optimal transshipment for a firm that produces in two manufacturing facilities each of which serves its individual uncertain market demand and faces capacity uncertainty. They analytically discuss how optimal policy with transshipment strategy is affected by stochastically higher facilities capacities; and numerically discuss under which condition the benefit of transshipment is high. The focus of this work are the effect of random capacity on optimal policy and profitability of transshipment strategy. Different from their work, firstly, we don't consider transshipment and directly address how capacity and demand variability affecting optimal policy; secondly, we discuss the significance of profitability-loss due to ignoring random capacity that is missing in their work. Wang et al. (2010) study a newsvendor model that can source from two unreliable suppliers, where the unreliability comes from the random loss of design capacity of suppliers. They examine two process improvement strategy: dual sourcing and exerting effort to increase supplier's reliability (process improvement). And they identify the conditions, under which dual sourcing or process improvement is more favourable. The effect of uncertainties is not discussed.

In this stream of literature, Babich (2010) is the closest paper to ours. He investigates a manufacturer's capacity reservation and subsidy decisions to a supplier who has risky financial state to generate the capacity. The goal of his paper is to model the relationship between the supplier's financial state and the supplier's capabilities to fulfill the manufacturer's order. Different from his model, we consider a manufacturer internalizes the capacity establishment and the financial trouble may happen in the production stage. In addition, his model is dynamic periodic-review model and ours is two-stage stochastic programming model. In his work, the effect of financial constraints from the other party in supply chain on capacity investment decision has been numerically discussed.

Remark: In this inventory-production system with uncertain capacity stream of literature, the potential correlation between uncertain capacity and demand are neglected, therefore they obtain either the relation between optimal inventory/production level and capacity variability is monotone. Our research fill this gap by assuming uncertain capacity and demand are correlated and analytically provide the sensitivity results of how capacity and demand uncertainties and their correlation shape the optimal inventory level.

2.2 Financial Constraints in Capacity Management

Our paper also related to the literature that considers financial market frictions in capacity investment management, because the production resources uncertainty can also be variable financial constraints. Our contribution to the literature is to understand when production demand and financial constraint are correlated how financial constraint variability shapes capacity investment level. Stochastic models for capacity management has been well studied from operations literature, see Van Mieghem (2003) for an extensive review, in which all of the models assume that one is always able to secure funds to adopt 'optimal' capacity investment and production plans. As the value of the interplay between capacity management and financial risk is illustrated (Birge, 2015), increasingly number of papers consider stochastic capacity investment models accommodating the risk due to financial market imperfections, the constraints could be in different forms, e.g. transaction costs, exchange rate, bankruptcy costs, taxes and regulations, costly or slow information diffusion, agency problems, moral hazards and so on. Particularly, the contemporary researches that demonstrate the value of financial constraints and its uncertainty in capacity management are listed below.

There are papers that involve the financial constraints as constant so that the variability feature of it is missing. Babich and Sobel (2004) study a multi-period model of capacity expansion and production where the IPO event is treated as a stopping time for entrepreneurs to cash-out. Each period the entrepreneurs make operational and financial decisions: capacity increment, production, bank loan amount and whether or not to IPO. They provide monotone threshold rule to yield an optimal IPO decisions. In the paper, they mentioned that operational capacity may deteriorate over time in a random rate. The impact of this capacity deteriorate rate uncertainty and demand uncertainty are not discussed in the paper. Xu and Birge (2004) extend the newsvendor model to include financing constraint, whom is called capital-constrained-newsvendor , to elevate the financial constraint, the newsvendor can issue both debt and dividend. They demonstrate that facing the bankruptcy risk due to demand uncertainty, the firm will reduce inventory investment facing financial constraint. Dada and Hu (2008) also consider a capital-constrained-newsvendor, therein, he borrows from endogenous bank that determines interest rate. Both of the seminal works prove the negative impact of financial constraint on the profitability that showing the importance of taking the financial constraint into account. Ning and Sobel (2018) study a price-taking firm using only internal financing and lives in a stochastic market environment to make multi-period capacity investment/divestment, production and dividend issue decisions. The effects of financial frictions in the form of internal financing and the goal of their paper is to study how this internal financing affect the optimal policy. In terms of financing constraint, in their model, all operations are financed by the cash reserve, whereas in our paper only the production is constrained by a random budget. In addition, capacity investment is irreversible in our model.

Boyabatlı and Toktay (2011), Chod and Zhou (2014) and Boyabatlı et al. (2016)

analyze flexible capacity investment in financially constrained environments. They all study a budget constraint that applies to the capacity investment stage. Boyabatlı and Toktay (2011) endogenize the cost of borrowing and examine the effect of capital market imperfections on the firm's technology choice. Chod and Zhou (2014) consider the optimal mix of flexible and dedicated capacity showing that flexibility reduces the risk of costly default and the agency cost. Also as flexible capacity investment cost decreases and the optimal capacity mix becomes more flexible, the optimal amount of debt increases monotonically.

In addressing financial market fictions, optimal financial hedging strategy with capacity investment decision are studied as follows: Chod et al. (2010) examine a value-maximizing firm that produces two products and show that the firm can use operational flexibility to mitigate the demand risk and financial hedging to cope with profit risk. The firm value is a concave function of pre-tax profit due to market imperfections such as taxes, the cost of financial distress, and costly external financing (Smith and Stulz, 1985). They show that product flexibility (flexible capacity for producing two different products) and financial hedging tend to be complements (substitutes) when demands are positively (negatively) correlated. In their paper, the financial market imperfections twist the firm being risk-averse. We model different type of financial market imperfection that is the risk that production decision is truncated by limited financial resource. Chen et al. (2014) develop a mean-variance model to investigate manufacturer's the optimal financial hedging strategy and capacity investment decision, so as to mitigate the manufacturer's risk of multi-country foreign currency exposures due to overseas suppliers. They numerically study the impact of correlation between production demand and currency exchange rates on the optimal utility and capacity of the firm, obtaining that when the exchange rates and demands are perfectly correlated, the optimal capacities and utilities between a risk-averse and a risk-neutral firm are identical. They take financial market fictions in a risk preference angle that is different from our work.

Other financial issues incurred by financial market friction are also studied: Natarajan and Swaminathan (2014) study multi-period stochastic inventory problem in the presence of funding constraints over a finite planning period in the content of humanitarian operations. The funding constraints in humanitarian operations have special property, that is, given the total fund fixed, there are scenarios with uncertain funding and funding timing among the time horizon, each of the scenario is called a stochastic funding schedule. They identify the optimal replenishment policy for any given funding schedule and analyze the impact of uncertainty in funding timing on the total operating costs. Iancu et al. (2017) study the risk of liquidation for a capital constrained newsvendor with operating flexibility provided by two-period selling seasons. For leverage, the newsvendor contracts borrowing base covenant with a bank and thus in the risk of liquidation. They examine the value of operating flexibility in the presence of capital market frictions and debt covenants. They find that by providing risk-shifting incentives in the debt covenant, operating flexibility can substantially increase borrowing costs. Alan and Gaur (2018) examine the effect of bankruptcy costs and information asymmetry on the firm's operating plans under asset-based lending, where commercial bank screens newsvendor-type firms. They show that asset-based lending enables the bank to mitigate information asymmetry by screening firms and control each firm type. de Véricourt and Gromb (2019) investigate the behavioural feature of capacity investment when investor finance the activity. They study firm's capacity choice given that it must be financed by investor, as a result, sharing profits with investors causes governance problems (two moral hazards), i.e. the firm may "steal" capital which reduces effective capacity, and "shirk" on market development which reduces demand, in the sequel, affecting both capacity and demand.

Start-up firms as few trade records and low loan credit corporates have different operating target and financial concerns from established firms. Swinney et al. (2011) analyze the competitive capacity investment timing decisions of both established firms and start-ups entering new markets in face with demand uncertainty. Therein, start-up firm aims to minimize the possibility of bankruptcy due to unable to repay debt whereas the established firm has no financial constraint and focuses on maximizing the expected profit. They demonstrate the threat of start-up bankruptcy significantly impacts the dynamics of the competition. Tanrısever et al. (2012) address debt-financed start-ups' concern of production cost reducing R&D investment, that is, financial distress of unable to meet bank's financial requirement in short term resulting in liquidation versus better business growth in long term. They incorporate important start-ups' concerns of uncertain R&D performance, uncertain demand and uncertain production cost of competitor. Tanrisever et al. (2019) also study a production cost-reduction investment model. Different from Tanrısever et al. (2012)'s work, the bank is endogenous and loan is in the present of capital market frictions, specifically, there is a cost of bankruptcy when firm's revenue is unable to recover the face value of the loan. Both of the papers demonstrate that the production-cost-reduction investment affects firm's operational and financial capabilities. It is worth noting that Tanrisever et al. (2019) numerically perform the impact of demand variability and the aforementioned capital market frictions on the optimal investment.

The closest paper to ours is Boyabatlı et al. (2016). They also consider production stage budget uncertainty in the capacity investment content. The differences are 1) we don't focus on the technology choice of the capacity investment; 2) we make allowance for the influence of variability of the production resources on the capacity investment and the profitability.

Chapter 3

Stochastic Capacity Investment in the Presence of Physical and Financial Production Resource Disruptions

In this chapter, we consider a manufacturing firm which produces and sells a single type of products on the market. The firm first makes the capacity investment for the production of the single product, with the presence of random market demand and random production resources. We define production resource broadly so that it can represent either a financial budget or the availability of limiting physical resource, e.g. raw material, components and workforce level. We adopt stylized stochastic programming approach to characterize the firm's optimal capacity investment strategy. We conduct sensitivity analysis analytically provide the answers of how production resource and demand uncertainties jointly affect optimal capacity level and the profitability of the manufacturing firm. In addition, extensive numerical experiments are conducted to verify the above sensitivity analyses and provide the condition under which the profitability-loss is significant.

The organization of this chapter is as follows. In Section 3.1, we introduce the formulation and assumptions of the model. In Section 3.2, we establish the optimal capacity investment policy and compare it with the benchmark policy which ignores the possibility of production resource shortage. Then we derive analytical results for the sensitivity of optimal capacity investment policy as well as the profitability in Section 3.3. Specifically, we answer the research question: How would the optimal capacity level and profitability be impacted by the demand and production resource variability and the correlation between the two? Finally, in Section 3.4, we use analytical analysis as well as complementary numerical study to answer second research question, if the possibility of production resource shortage is ignored in capacity planning, as often done in practice and the academic literature, would the resulting profitability loss be significant and how do the demand and production resource uncertainties affect this profitability loss?

3.1 Notations and Assumptions of Basic Model

In the basic model, the capacity investment decision is made in the presence of uncertainties. Other than consumer demand uncertainty that is widely studied in the field, we take into account production resource uncertainty. After the realization of uncertainties, the firm makes decision on the optimal production quantity, subject to its earlier capacity investment and available resources. Finally, the firm sells the products in the market and collects revenue. The objective function is to maximize the firm's expected profit. In the following section, the notations and assumptions are introduced.

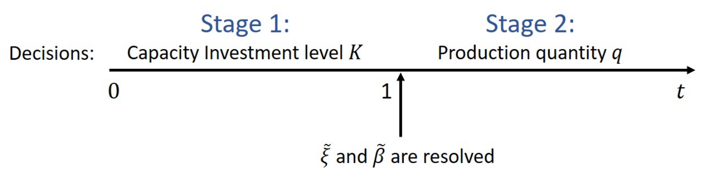
We formulate the problem as a two-stage stochastic programming model, *capacity investment* stage and *product manufacturing* stage, in time sequence. At the start of capacity investment stage $t = 0$, an amount equal to ωK is invested in capacity, where firm decides K units of productive capacity to purchase at the net price of ω per unit. This decision, that maximizes expected operating profit less the total capacity investment cost, is made in anticipation of the demand and production resource uncertainties denoted by $\tilde{\xi}$ and $\tilde{\beta}$ respectively.

Specifically, the uncertainty of demand is denoted by $\tilde{\xi}$ which comes from a demand-dependent price function denoted by $p(q) = \tilde{\xi} - bq$, that is the price of the product is a linear inverse demand function. $\tilde{\xi}$ indicates the maximal price of this product, above which there would be no buyer willing to purchase. b is a positive constant parameter that represents the price sensitivity of the the buyers. This demand-price relation provides that if the firm sells q unit of the product to the market, it has to charge price at $(\xi - bq)$. This demand-price relationship has been widely adopted in literature, e.g. Van Mieghem and Dada (1999), Caldentey and Haugh (2009), Swinney et al. (2011) and Tanrısever et al. (2012). Also this demand-price relationship is justifiable because in practice, unsold units are generally liquidated through other channels, e.g. secondary markets, at a discount price. Therefore, on average, higher quantity of products sold leads to lower price.

Production resource uncertainty $\tilde{\beta}$ (in dollar) represents the production resource availability for providing the product. $\tilde{\beta}$ can be the physical production resource uncertainty proposed by Ciarallo et al. (1994), motivated by the facts that the productivity level may fluctuate because of variations in worker skills or operating conditions, in the case of raw materials or components souring, there may be a lack of information about the production capability. $\tilde{\beta}$ can also denote random production budget proposed by Boyabatlı et al. (2016), motivated by the facts that firm may not have access to sufficient external financing due to credit crunch, liquidity shocks and financial crisis; also its internal financing may not enough if the parent firm reallocates the fund to other divisions. It has a continuous distribution with positive support $[\underline{\beta}, \overline{\beta}]$ and bounded expectation μ_β.

After making investment decision with the presence of these two uncertainties, these uncertainties are realized, and the firm then makes decisions on the production $t = 1$, which is the beginning of production stage. For clarity of exposition, (ξ, β) is the realization of the uncertainties $(\tilde{\xi}, \tilde{\beta})$. Having demand function and total production resource materialized, the firm decides the production quantity q, noting that $q \in [0, \xi]$. Producing one unit of product requires one unit of capacity and y (in dollar \$) unit of production resource, which means the product quantity is constrained either by capacity level K decided in the capacity investment stage or the realized production resource, whichever is lower. The objective of the firm is to maximize the expected profit at the beginning of capacity investment stage. To summarize, we illustrate the sequence of events for the problem in Figure 3.1.

Figure 3.1: Timeline of events



Let Π denote the expected profit for the firm at a capacity level K at capacity investment stage and $\pi^*(K, \xi, \beta)$ denote the firm's the optimal profit at production stage given a capacity level K and realization of uncertainties, the formulation of the problem is as follows. The capacity investment stage problem is

$$\max_{K \geq 0} \Pi(K) = \max_{K \geq 0} \quad - \omega K + \mathop{\mathbf{E}}_{(\xi, \beta)} \left[\pi^*(K, \tilde{\xi}, \tilde{\beta})\right], \tag{3.1}$$

and the production stage problem is

$$\begin{aligned} \pi^*(K, \xi, \beta) = \max_{q} \quad & (\xi - bq)q - yq \\ \text{s.t.} \quad & 0 \leq q \leq \min\left\{K, \tfrac{\beta}{y}\right\}. \end{aligned}$$

According to Theory of Constraints proposed by Goldratt (1990), among all other constraints that may be binding the production quantity, we assume that either capacity investment level or production resource is the weakest link that may get in the way of the optimal production plan, besides the product demand.

Since on the one hand, capacity investment level can be managed in the first stage, on the other hand, the production resource is uncertain at the first stage and its possible shortage leads to production resource crunch. As a result, in line with the theory of constraints, understand the optimal capacity investment strategy is equivalent to identify the most important limiting factor.

Without loss of generality, throughout the paper, we assume that the capacity investment is an unconstrained expenditure and is irreversible. In practice, capacity investment could also be constrained by some resource availability, i.e. capital budget or/and supply reliability, practically, capacity investment is usually financed by equity far earlier than production process. Therefore, having a capital budget constraint in capacity investment stage wouldn't change the optimal production strategy. To isolate the role of the uncertainties in changing capacity investment level, we abstract away the constraint in capacity investment stage. Analytically, the analyses on how production resource uncertainty and demand uncertainty influencing capacity investment are easier without capacity investment capital constraint, ruling out financial constraint in capacity investment stage is beneficial in concern with both research focus and analytical convenience.

Assumption 1 *The mean value of demand function intercept denoted by μ_ξ is larger than unit production cost, specifically, $\mu_\xi > y$.*

This condition ensures that investing in capacity to carry out the production is admissible. Moreover, this assumption implies that the firm engages in a reasonable production stage profitability. In particular, the expected production stage profit with no consideration of production resource uncertainty, that is $\mathbf{E}_{\xi}\left[(\tilde{\xi} - bq)q - yq\right] = (\mu_\xi - y)q - bq^2$, takes positive value for some $q \in [0, \infty)$.

We complete this section with a summary of mathematical notations and conventions throughout the remainder of the paper. The probability density function of probability distribution $(\tilde{\xi}, \tilde{\beta})$ is denoted by $f(\xi, \beta)$. The conditional distributions $\tilde{\xi}|\beta$ and $\tilde{\beta}|\xi$ have probability density function denoted by $f_{\xi|\beta}(\xi)$ and $f_{\beta|\xi}(\beta)$ respectively. The correlation between $\tilde{\xi}$ and $\tilde{\beta}$ is denoted by ρ.

The marginal distribution $\tilde{\xi}$ is characterized by $(0, \infty)$, which has mean $\mu_\xi > y$, standard deviation $\sigma_\xi > 0$. In a similar, the marginal distribution $\tilde{\beta}$ is characterized by $[\underline{\beta}, \overline{\beta}]$, which has mean $\mu_\beta > 0$, standard deviation $\sigma_\beta > 0$. All notations for the model are summarized at Table A.1 in Appendix. In addition, some standard mathematical representation are summarized: $\mathbf{E}$ denotes the expectation operator; Pr denotes probability; $(\cdot)^+$ denotes the maximum between 0 and the value $\cdot$, that is to say $(\cdot)^+ := \max\{0, \cdot\}$. Any other notation will be introduced as necessary. Monotonic relations are in the weak sense unless otherwise stated.

3.2 Characterization of the Optimal Strategy

In this section, we characterize the firm's optimal capacity investment and production decisions. The problem is solved using backward induction. In particular, we first analyse the optimal production decision given capacity K. We partition the state space $(\xi, \beta) \in \{(\xi, \beta) : \xi > 0, \beta \in [\underline{\beta}, \overline{\beta}]\}$ into four regions, Ω_i, $i = 0, 1, 2, 3$ to denote different optimal production quantity scenarios that we will show in Theorem 1. The formal definitions of these regions are

$$
\begin{aligned}
\Omega_0 &:= \left\{(\xi, \beta) : 0 \le \xi \le y,\ \beta \in \left(\underline{\beta}, \overline{\beta}\right]\right\}, \\
\Omega_1(K) &:= \left\{(\xi, \beta) : y < \xi < y + 2b \min\left\{\frac{\beta}{y}, K\right\},\ \underline{\beta} < \beta \le \overline{\beta}\right\}, \\
\Omega_2(K) &:= \left\{(\xi, \beta) : \xi \ge y + 2bK; \max\left\{\underline{\beta}, \min\{yK, \overline{\beta}\}\right\} < \beta \le \overline{\beta}\right\}, \\
\Omega_3(K) &:= \left\{(\xi, \beta) : \underline{\beta} \le \beta < \max\left\{\underline{\beta}, \min\{yK, \overline{\beta}\}\right\},\ \xi \ge y + \frac{2b\beta}{y}\right\}.
\end{aligned}
$$

With the analysis of production constraints, the optimal production strategy is characterized in Theorem 1 and the expected profit $\Pi(K)$ under optimal allocation is obtained. In Theorem 2, the optimal resource capacity investment level is obtained from $\Pi(K)$ and characterized by unit capacity cost ω.

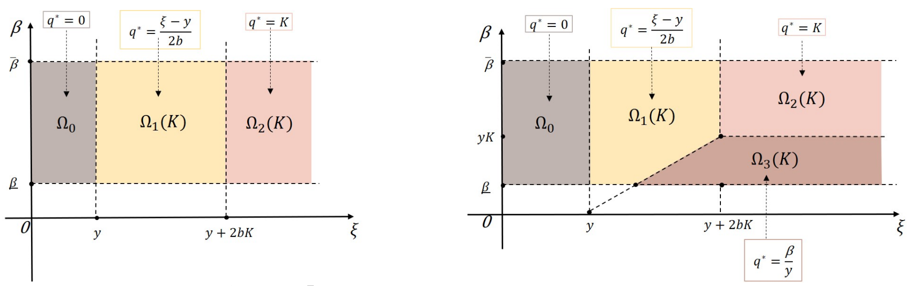
Theorem 1 (Optimal Production Strategy of Basic Model) *At given capacity K and realizations of random variables ($\tilde{\xi} = \xi, \tilde{\beta} = \beta$), the optimal production* *level $q^*(K, \xi, \beta)$ satisfies that*

$$q^*(K, \xi, \beta) = \begin{cases} 0, & \text{if } (\xi, \beta) \in \Omega_0, \\ \dfrac{\xi - y}{2b}, & \text{if } (\xi, \beta) \in \Omega_1(K), \\ K, & \text{if } (\xi, \beta) \in \Omega_2(K), \\ \dfrac{\beta}{y}, & \text{if } (\xi, \beta) \in \Omega_3(K), \end{cases}$$

and the optimal profit is $\pi^*(K, \xi, \beta) = \begin{cases} 0, & \text{if } (\xi, \beta) \in \Omega_0, \\ \frac{(\xi - y)^2}{4b}, & \text{if } (\xi, \beta) \in \Omega_1(K), \\ (\xi - y)K - bK^2, & \text{if } (\xi, \beta) \in \Omega_2(K), \\ \frac{\beta(\xi - y)}{y} - b\left(\frac{\beta}{y}\right)^2, & \text{if } (\xi, \beta) \in \Omega_3(K). \end{cases}$

General understanding of this theorem is that, given that the internal optimal production quantity is $q^* = \frac{(\xi - y)^+}{2b}$, this quantity can be either binding by capacity K or financial capability β/y. More specifically, the optimal production quantity decision depend on the values of (ξ, β). We show this result in Figure 3.2 where the demand intercept realization is on the horizontal axis and the production resource realization is on the vertical axis.

Figure 3.2: The Optimal Production Strategy of Basic Model



(a) When $K \in \left(0, \underline{\beta}/y\right]$ (b) When $K \in \left(\underline{\beta}/y, \overline{\beta}/y\right)$

Noting that panel (a) of Figure 3.2 describing a case that production resource is ample. By ample, we mean that the minimal amount of resource $\underline{\beta}$ is sufficient for manufacturing the products up to the capacity level, because of $\underline{\beta} \geq yK$. In this case, the value of production resource doesn't affect the production decision so that we call it as *resource-unconstrained case*. When $(\xi, \beta) \in \Omega_0$, $q^* = 0$

because the demand intercept ξ is so low that the selling price is even lower than the production cost y. When $(\xi, \beta) \in \Omega_1(K)$, optimal production quantity $q^* = \frac{\xi - y}{2b}$ is consistent with the internal optimal production quantity. It indicates no constraint is binding production, because the realization of demand intercept is not large enough such that both capacity and production resource is larger than this optimal quantity. When $(\xi, \beta) \in \Omega_2(K)$, the demand realization is very large such that the internal optimal product quantity is not attainable due to capacity constraint. At the same time, the production resource is large such that the production resource constraint becomes immaterial comparing with capacity constraint, as a result, optimal production strategy is utilizing all available capacity for the production in order to close to the unconstrained optimal quantity.

In contrast, panel (b) of Figure 3.2 as *resource-constrained case* illustrates a case that production resource could be constraining. The reason is the given capacity level is in between of minimal and maximal production quantity that supported by the minimum and maximum value of production resource realization, namely $K^* \in (\underline{\beta}/y, \overline{\beta}/y)$. Consequently, in a large demand realization scenario when internal optimal production quantity is not attainable, $\Omega_2(K)$ and $\Omega_3(K)$ indicate that either the capacity constraint is tighter or the resource constraint is, respectively. Particularly, when $(\xi, \beta) \in \Omega_3(K)$, production resource is tighter than capacity level such that the optimal production strategy is to use up all production resource. It motivates the firm to alarm the potential uncertainty of production resource in the first stage to alleviate the shortage and spillover of production capacity.

In the following, we characterize the optimal capacity investment level in capacity investment stage. The optimization problem in this stage follows from Equation 3.1 by substituting $\underset{(\xi,\beta)}{\mathbf{E}}\left[\pi^*(K, \tilde{\xi}, \tilde{\beta})\right]$ with the characterization provided in Theorem 1. To be specific, the expected profit as a function of capacity level under optimal allocation is in following form, $\Pi(K) =$

$$
\begin{aligned}
&-\omega K+\int_{\underline{\beta}}^{\overline{\beta}}\int_{y}^{y+2b\min\left\{K,\frac{\beta}{y}\right\}}\frac{(\xi-y)^2}{4b}f(\xi,\beta)d\xi d\beta\\
&+\int_{\underline{\beta}}^{\overline{\beta}}\int_{y+2b\min\left\{K,\frac{\beta}{y}\right\}}^{\infty}\left((\xi-y)\min\left\{K,\frac{\beta}{y}\right\}-b\left(\min\left\{K,\frac{\beta}{y}\right\}\right)^2\right)f(\xi,\beta)d\xi d\beta.
\end{aligned}
\tag{3.2}
$$

Theorem 2 (Optimal Capacity Investment Strategy of Basic Model) *The optimal capacity investment level K^* is characterized as follows:*

$$
K^*(\omega)=\begin{cases}
0, & \text{if } \mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi}-y)^+\right]\le\omega;\\
K^U(\omega), & \text{if } \mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi}-y-2b\underline{\beta}/y)^+\right]\le\omega<\mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi}-y)^+\right];\\
K^B(\omega), & \text{if } 0<\omega<\mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi}-y-2b\underline{\beta}/y)^+\right].
\end{cases}
$$

$K^U(\omega)$ uniquely satisfies optimality condition:

$$
\omega=\int_{y+2bK^U}^{\infty}(\xi-y-2bK^U)f_{\xi}(\xi)d\xi \tag{3.3}
$$

and $K^B(\omega)$ uniquely satisfies optimality condition:

$$
\omega=\int_{y+2bK^B}^{\infty}(\xi-y-2bK^B)\Pr\left\{\tilde{\beta}>yK^B\middle|\xi\right\}f_{\xi}(\xi)d\xi. \tag{3.4}
$$

Theorem 2 states that the optimal capacity investment level is characterized by unit capacity cost. When the unit capacity investment cost is high, it is optimal for the firm to not engage in the investment. When the cost is moderate, the firm invests in moderate amount of capacity $K^U(\omega)\in\left(0,\underline{\beta}/y\right]$ such that the lowest resource realization is sufficient to finance the production at a level that the capacity is fully utilized. We define $K^U(\omega)$ as *resource-unconstrained capacity level* in view of the corresponding optimality condition (3.3), the right side of which representing the expected marginal revenue of investing one more unit of capacity doesn't depend on production resource. Also, this capacity is corresponding to the panel (a) in Figure 3.2. Lastly, when the unit capacity cost is low, optimal capacity investment level $K^B(\omega) \in (\underline{\beta}/y, \overline{\beta}/y)$ is defined as *resource-constrained capacity level*, because its marginal revenue (right side of the optimality condition (3.4)) is affected by uncertain production resource and this capacity level is corresponding to resource-constrained case in production stage, that is depicted in the panel (b) of Figure 3.2.

An alternative explanation of this optimal capacity investment strategy through unified marginal profit of investing one more unit of capacity, that is, both optimality condition (3.3) and (3.4) can be written as $\omega = \iint_{\Omega_2(K^*)}(\xi - y - 2bK^*)f(\xi, \beta)d\xi d\beta$. Since $\Pi(K)$ is concave in K (referring to the proof of Theorem 2), optimal capacity investment level K^* decreases in unit capacity cost ω is easily derived. As a result, the optimal structure of optimal capacity investment level is derived by critical values of ω, by reason that $K^* = 0$ when $\omega = \mathop{\mathbf{E}}_{\tilde{\xi}}\left[(\tilde{\xi} - y)^+\right]$ and $K^* = \underline{\beta}/y$ when $\omega = \mathop{\mathbf{E}}_{\tilde{\xi}}\left[(\tilde{\xi} - y - 2b\underline{\beta}/y)^+\right]$. Another observation from above unified optimality condition is that an additional unit of the capacity only has positive effect on revenue in $(\xi, \beta) \in \Omega_2(K^*)$. The reason is that the capacity level is binding the production when $(\xi, \beta) \in \Omega_2(K^*)$ so that the product quantity is less than the internal optimal value, adding capacity relaxes the constrain and therefore leads to a profit nearer to the unconstrained optimal profit.

The influence of production resource shortfall is captured by the term $\Pr\left\{\tilde{\beta} > yK^B \middle| \tilde{\xi}\right\}$ of optimality condition (3.4) capturing the probability that the firm has ample production resource to support production up to capacity level given a demand intercept $\tilde{\xi}$. The structure of the optimality condition (3.4) and optimality condition (3.3) are constructed by a unit capacity investment cost at the left side of the equal sign and expected marginal revenue of investing one additional unit of capacity at the right side of the equal sign. The only difference between these two marginal revenues is that there is an additional term in the integrand of basic model marginal revenue, that is production resource flexibility level $\Pr\left\{\tilde{\beta} > yK \middle| \xi\right\}$. Specifically, the probability of production resource shortfall is $1 - \Pr\left\{\tilde{\beta} > yK^B \middle| \tilde{\xi}\right\}$. In the case of optimality condition (3.3) that derives resource-unconstrained capacity level, $\Pr\left\{\tilde{\beta} > yK^U \middle| \tilde{\xi}\right\} \equiv 1$. The Similar concept of this probability was once introduced by Boyabatlı et al. (2016) called financial flexibility level in which it is a fixed probability, or equivalently it is exogenous. Whereas in our model, we define $\Pr\left\{\tilde{\beta} > yK \middle| \tilde{\xi}\right\}$ as *production resource flexibility level* which is endogenous in a way that is a function of capacity level such that the firm optimally determines production resource flexibility level to maximize the optimal expected profit in consideration of production resource crunch. A key observation from optimality condition (3.4) is that the correlation between demand and production resource uncertainties, demand volatility and production resource volatility affect resource-constrained capacity level through their impacts on the production resource flexibility level. We use this observation to explain corresponding sensitivity results of resource-constrained capacity in the following section.

In addition, the impact of price sensitivity to product quantity on the optimal capacity investment level and corresponding expected profit is presented in the following corollary.

Corollary 1 (Impact of price sensitivity to product quantity b**)** *Given the optimal capacity investment strategy in Theorem 2, we have*

1. $K^U(\omega)$ *decreases in b;*
2. $K^B(\omega)$ *decreases in b;*
3. $\Pi\big(K^*(\omega)\big)$ *decreases in b.*

This corollary shows that a higher price sensitivity to product quantity, not only shrinks the optimal capacity investment level but also hurts the profitability. Intuitively, as buyers care more about the price, the responsive pricing is less powerful, because the price of the product they are welling to pay drops quickly as the firm manufactures more products.

3.3 Sensitivity of Optimal Capacity Level and Profitability for Basic Model

In this section, we answer the second primary objective of this research, that is to understand the impact of demand variability, production resource variability and the correlation between them on the firm's optimal capacity investment level and profitability of the firm. Having found the optimal capacity investment strategy, we analytically prove the properties of the firm's optimal capacity and profitability with respect to above mentioned uncertainty parameters. To this end, we impose distribution assumption for demand intercept and production resource uncertainties. They follow a bivariate normal distribution throughout all sensitivity analyses. Due to the tractability of the analysis, bivariate normal distribution is widely used in the literature to represent two random variables with correlation, e.g. demand uncertainties (Chod and Rudi, 2005) and revenue uncertainties (Boyabatlı et al., 2019), since its correlation structure is amenable to analysis. More specifically, we make the following assumption:

Assumption 2 $(\tilde{\xi}, \tilde{\beta})$ *follows a bivariate normal distribution with mean vector* $(\mu_\xi, \mu_\beta)'$ *and variance-covariance matrix* $\begin{pmatrix} \sigma_\xi^2 & \rho\sigma_\xi\sigma_\beta \\ \rho\sigma_\xi\sigma_\beta & \sigma_\beta^2 \end{pmatrix}$, *where the correlation coefficient* $\rho \in (-1, 1)$.

To deal with the contradiction between non-negativity of demand intercept and the production resource realizations and real-valued normal distribution outcomes in Assumption 2, we assume that their variability are not extremely large, hence, the effect of the negative values are negligible.

Based on Assumption 2, we find the rotational symmetry of production quantity in section. Define bivariate normal distribution $(\tilde{D}, \tilde{B}) := \left(\frac{\tilde{\xi}-y}{2b}, \frac{\tilde{\beta}}{y}\right)$ and it has mean vector $\left(\frac{\mu_\xi - y}{2b}, \frac{\mu_\beta}{y}\right)'$ and variance-covariance matrix $\begin{pmatrix} \left(\frac{\sigma_\xi}{2b}\right)^2 & \frac{\rho\sigma_\xi\sigma_\beta}{2by} \\ \frac{\rho\sigma_\xi\sigma_\beta}{2by} & \left(\frac{\sigma_\beta}{y}\right)^2 \end{pmatrix}$ where the correlation is also ρ. In the new distribution, we find the rotational symmetry of optimal production decision such that

$$q^* = \begin{cases} 0 \text{ if } (D, B) \in \{(D, B) : D \leq 0\} \\ D \text{ if } (D, B) \in \{(D, B) : 0 < D \leq \min\{B, K\}\} \\ B \text{ if } (D, B) \in \{(D, B) : B \leq \min\{D, K\}\} \\ K \text{ if } (D, B) \in \{(D, B) : D \geq K \text{ and } B \geq K\} \end{cases}$$

Even if the demand and production resource uncertainties in the production stage affecting the decision in a symmetrical way, however, the impact of their uncertainty parameters such as variabilities on optimal capacity investment decision aren't not symmetrical. The optimal capacity investment decision of the basic model is given by

$$K^*(\omega) = \begin{cases} 0, & \text{if } \omega \geq \omega_{\max}; \\ K^B(\omega), & \text{if } 0 < \omega < \omega_{\max} \end{cases}$$

where $\omega_{\max} := \mathbf{E}_{\xi}\left[(\tilde{\xi} - y)^+ \Pr\left\{\tilde{\beta} > 0 \middle| \tilde{\xi}\right\}\right]$. Comparing with the optimal capacity investment structure in Theorem 2, there is no interval of unit capacity cost ω under which the optimal capacity investment level equals to K^U, equivalently K^* is always solved by optimal condition (3.4). It is because the domain of the production resource becomes $\beta \in (-\infty, \infty)$ so that production resource flexibility level $\Pr\left\{\tilde{\beta} > yK \middle| \tilde{\xi}\right\}$ is strictly less than 1 for all finite $K \geq 0$.

In the following, we conduct sensitivity analyses to study the effects of the correlation between demand and production resource uncertainties, demand variability σ_ξ and production resource variability σ_β on resource-constrained capacity K^B and corresponding optimal expected profit $\Pi(K^B)$. Noting that both optimal capacity level K^B and expected profit $\Pi(K^B)$ are functions of parameters ω, ρ, σ_ξ and σ_β, for the ease of exposition, we compress the functional relationship in the following analysis if it is not necessary to point out a particular functional relationship, e.g. the functional relationship in terms of ρ are $K^B(\rho)$ and

$\Pi\big(K^B(\rho), \rho\big)$, we would omit writing these functional relationships and just use notation K^B and $\Pi(K^B)$, if this is not likely to lead to confusion.

The organization of the sensitivity analyses in this section is as follows, we investigate the effects on optimal capacity investment level K^B and corresponding expected profit $\Pi(K^B)$ of correlation between production resource and demand uncertainties ρ in Section 3.3.1, of demand variability σ_ξ in Section 3.3.2 and of production resource variability σ_β in Section 3.3.3.

3.3.1 Sensitivity to the Correlation between Production Resource and Demand Uncertainties

We start with the effect of the correlation between production resource and demand uncertainties. Note that the correlation could be positive, negative or zero, we illustrate the value of correlation when production resource uncertainty is either under financial constraints scenario or physical resource scenario, respectively. 1) When the production resource referring to financial constraints, the correlation depends on the characteristics of the product being produced. It is likely that the production resource is positively correlated with the demand of discretionary purchase products, or luxury goods, while negatively correlated with the staple products. To understand the relationship, consider that both the financial resource availability and the product demand are closely related to the economic condition. Under condition that the economy is good (bad), the firm would be able to raise large (small) amount of finance; and at the same time, the market demand for discretionary purchase goods is high (low), and the demand for staple products is low (high) as consumers afford higher-end substitutes. 2) When the production resource referring to physical resource, e.g. raw materials, components and workforce level. If the production resource uncertainty comes from raw materials or components, it can happen that raw materials or components are provided by other factories of the manufacturing firm so that production resource and demand could be positively correlated due to abundantly exchange of information; it can also happen that raw materials or components are provided by external firms that are competing with the manufacturing firm in the same final product market, which may result in the demand for the manufacturing firm negatively correlated with production resource. If the production resource uncertainty comes from variations in worker skills, it is likely that production resource uncertainty is independent to demand uncertainty.

As shown in Proposition 1, resource-constrained capacity level increases in the correlation and the firm benefits from a higher correlation.

Proposition 1 (Impact of correlation between demand and production resource)

1. *The resource-constrained capacity level K^B is strictly increasing in ρ;*

2. *The optimal optimal expected profit $\Pi(K^B)$ are strictly increasing in ρ.*

To understand the effect of ρ on resource-constrained capacity level K^B, recall that for the production resource flexibility level $\Pr\left\{\tilde{\beta} > yK \middle| \tilde{\xi}\right\}$ it increases with the correlation between demand and production resource uncertainties. For a any fixed K, higher the correlation between $\tilde{\beta}$ and $\tilde{\xi}$, this production resource flexibility level is higher, which resulting in the increase of expected marginal revenue from optimality condition (3.4). Therefore, the capacity level K^B increases in ρ.

For the effect of ρ on optimal expected profit $\Pi(K^B)$, intuitively, as the correlation increases, high (low) demand is more likely to be associated with high (low) production resource, and thus the production resource is less constraining for the high demand scenario. On average, this higher correlation brings production stage optimal structure closer to the resource-unconstrained case (referring to panel (a) of Figure 3.2), thus the higher correlation is more beneficial for the firm.

3.3.2 Sensitivity to Demand Variability - Comparison with the Resource-unconstrained Benchmark

In this section, we investigate how demand variability σ_ξ affects the optimal capacity investment level and corresponding expected profit. In examining the demand variability, we make a comparison with the sensitivity result of a *benchmark model* where the production resource is always ample for the firm to proceed the production. We will provide the effect of demand variability on optimal capacity level and corresponding expected profit of benchmark model, so as to see the changes in insight by introducing the production resource uncertainty comparing with the literature.

To show the sensitivity results of benchmark model, we first identify the optimal strategy. In this case, the production decisions is made with no constraint on the amount of production resource. Tracing the literature, Van Mieghem and Dada (1999) introduce price and production postponement strategy to capacity-production framework that is identical to the benchmark model. Specifically, the benchmark model is formulated as two-stage problem, the first stage problem is $\max_{K\geq 0} \Pi_u(K) = \max_{K\geq 0} \left\{ -\omega K + \mathbf{E}_{\tilde{\xi}}\left[\pi_u^*\left(K,\tilde{\xi}\right)\right]\right\}$, where '$u$' denotes that the production process is unconstrained; and the second stage problem is $\pi_u^*\left(K,\xi\right) = \max_{q\in[0,K]} \left\{(\xi - bq)q - yq\right\}$ showing that the production resource constraint β/y no longer exists. The optimal strategy of this benchmark model is presented in Lemma 1.

Lemma 1 (Optimal Strategy of Benchmark Model)

1. *Optimal production quantity is* $q^*\left(K,\xi\right) = \begin{cases} 0, & \textit{if } \xi \in [0,\ y] \\ \frac{\xi - y}{2b}, & \textit{if } \xi \in (y,\ y+2bK] \\ K, & \textit{if } \xi \in (y+2bK,\ \infty)\,. \end{cases}$

   The corresponding expected profit in capacity-investment stage is

$$\Pi_u(K) = -\,\omega K + \int_y^{y+2bK} \frac{(\xi - y)^2}{4b} f_\xi(\xi) d\xi + \int_{y+2bK}^{\infty} \left((\xi - y)K - bK^2\right) f_\xi(\xi) d\xi;$$

2. *Optimal capacity investment level is* $\begin{cases} 0, & \text{if } \omega \geq \mathbf{E}\big[(\tilde{\xi} - y)^+\big]; \\ K^U(\omega), & \text{otherwise}. \end{cases}$

The proof of this Lemma 1 is skipped due to its similarity to the proof of Theorem 1 and 2. Not surprisingly, the optimal production strategy allocation of the state space is the same as what shows in the panel (a) of the Figure 3.2, where the strategy only depends on demand realization. For the optimal capacity investment level, the no investment strategy has the same unit capacity cost condition as what in Theorem 2. Also, under condition $\mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y - 2b\underline{\beta}/y)^+\big] \leq \omega < \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y)^+\big]$, the optimal capacity level for both basic model and benchmark model is K^U.

According to above Lemma 1, the effect of demand variability of benchmark model is introduced in Proposition 2. In line with Assumption 2 on $(\tilde{\xi}, \tilde{\beta})$, the distribution of $\tilde{\xi}$ used for proving Proposition 2 is normally distribution with mean μ_ξ and variance σ_ξ^2.

Proposition 2 (Impact of σ_ξ on K^U and $\Pi_u(K^U)$) *When the production resource of the firm is large enough, the influence of demand variability on optimal capacity investment level and optimal expected profit are summarized as follows:*

1. *Resource-unconstrained capacity level K^U is strictly increasing in σ_ξ;*

2. *Optimal profit $\Pi_u(K^U)$ is strictly increasing in σ_ξ.*

From Proposition 2, we obtain that higher demand variability is, higher resource-unconstrained capacity level and corresponding optimal expected profit are. As increasing demand variability means more low/high demand intercept realizations, the intuition of the results are developed by understanding how these low or high realizations of demand affect the optimal capacity investment level and the profitability.

Firstly, we discuss the intuition underlying part 1, Proposition 2, that is the impact on the resource-unconstrained capacity level K^U. We define the right-hand unit normal linear loss function as $L(t) := \int_t^\infty (z - t)\phi(z)dz$, which is a function monotonically decreasing in t. As for demand intercept $\tilde{\xi}$ is normally distributed in accordance with Assumption 2, we have close form of resource-unconstrained capacity level, that is $K^U = \frac{\mu_\xi - y + \sigma_\xi L^{-1}(\omega/\sigma_\xi)}{2b}$, where L^{-1} is the inverse function of $L(t)$. Given a fixed unit cost of capacity investment, the optimal capacity investment level changes the same way as the marginal revenue of capacity investment changes in demand variability, as shown by Equation (3.3). Since marginal revenue is not sensitive to low demand realization and larger in high demand realization, it increases in demand variability.

For the intuition of Proposition 2 part 2 result, more demand variability is beneficial due to the price-demand relationship $p(q) = \xi - bq$ where price is responsive to product quantity demanded, particularly when the production quantity is decided after the resolution of uncertainties. The reason behind is developed by Chod and Rudi (2005): on the one hand, the firm can charge a high price when capacity is constraining and demand realization is high; on the other hand, the firm can adjust a low price to have more products sold in the case of low demand realization. To summarize, demand variability is beneficial to both marginal revenue of investing capacity and optimal expected profitability of the firm.

In contract to the effect of demand variability σ_ξ on resource-unconstrained capacity K^U, the resource-constrained capacity level K^B is not always monotonically increasing in σ_ξ. Proposition 3 and Conjecture 1 together present that the resource-constrained capacity K^B increases in demand variability σ_ξ when the correlation satisfies $\rho \in [0, 1)$; the resource-constrained capacity K^B first decreases then increases in σ_ξ when $\rho \in (-1, 0)$.

Proposition 3 (Impact of σ_ξ on K^B) *Define*

$$\sigma_\xi^i := \left\{ \sigma_\xi \middle| \sigma_\xi < \frac{2b\sigma_\beta}{y}, \lim_{\rho \to -1} K^B(\sigma_\xi) = \frac{\frac{\mu_\xi - y}{\sigma_\xi} - \frac{\mu_\beta}{\sigma_\beta}}{\frac{2b}{\sigma_\xi} - \frac{y}{\sigma_\beta}} \right\} \textit{ and}$$

$$\sigma_\xi^{ii} := \left\{ \sigma_\xi | \sigma_\xi > \frac{2b\sigma_\beta}{y}, \lim_{\rho \to -1} K^B(\sigma_\xi) = \frac{\frac{\mu_\xi - y}{\sigma_\xi} - \frac{\mu_\beta}{\sigma_\beta}}{\frac{2b}{\sigma_\xi} - \frac{y}{\sigma_\beta}} \right\} \textit{ if exist.}$$

Further define $\sigma_\xi^K := \begin{cases} \min\{\sigma_\xi^i, \frac{2b\sigma_\beta}{y}\}, & \text{if } \frac{\mu_\xi - y}{2b} < \frac{\mu_\beta}{y}; \\ \max\{\frac{2b\sigma_\beta}{y}, \sigma_\xi^{ii}\}, & \text{if } \frac{\mu_\xi - y}{2b} > \frac{\mu_\beta}{y}. \end{cases}$ *The impact of the demand variability* σ_ξ *on the resource-constrained capacity level is:*

1. *If* $\rho \geq 0$*, then* K^B *increases in* σ_ξ*;*

2. *If* $\rho \to -1$*, then* K^B *decreases in* σ_ξ *if* $\sigma_\xi < \sigma_\xi^K$ *and* K^B *increases in* σ_ξ *if* $\sigma_\xi > \sigma_\xi^K$*.*

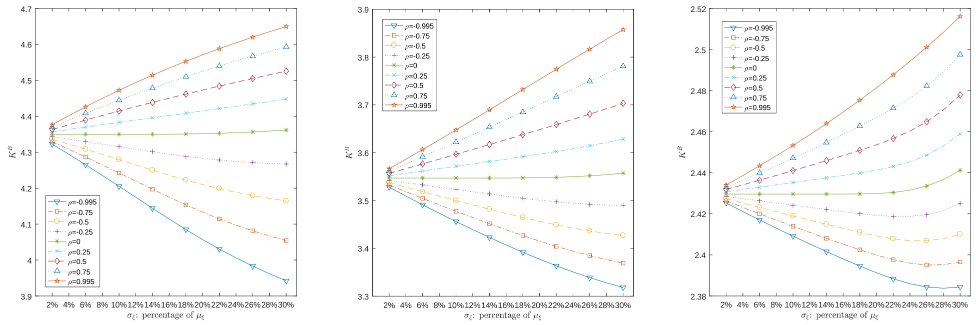
Additional to analytical sensitivity results in Proposition 3, it is not analytically tractable to prove the effect of σ_ξ on K^B when $\rho \in (-1, 0)$. Under condition $\rho \in (-1, 0)$, we can prove the sign of $\frac{dK^B}{d\sigma_\xi}$ for some particular value of σ_ξ, specifically, $\lim_{\sigma_\xi \to 0} \frac{dK^B}{d\sigma_\xi} < 0$ and $\frac{dK^B}{d\sigma_\xi} > 0$ when $\sigma_\xi \geq \frac{\omega}{\sqrt{1-\rho^2}\int_{\frac{\frac{y(\mu_\xi - y)}{2b} - \mu_\beta}{\sigma_\beta}}^{\infty} \mathbf{E}\left[\left(\tilde{z} + \frac{\rho z_0}{\sqrt{1-\rho^2}}\right)^+\right]\phi(z_0)dz_0}$. Furthermore, we observe the following pattern described in Conjecture 1 through 18,225 numerical instances introduced in Section 3.4.1.

Conjecture 1 (Impact of σ_ξ on K^B when $\rho < 0$) *When* $\rho \in (-1, 0)$*, there exists a unique* σ_ξ^K *such that* K^B *decreases in* σ_ξ *if* $\sigma_\xi < \sigma_\xi^K$ *and increases in* σ_ξ *if* $\sigma_\xi > \sigma_\xi^K$*.*

Noting that we purposely define the σ_ξ threshold in Conjecture 1 as σ_ξ^K which is the same notation as what's in Proposition 3 part 2 in order to show the continuity of the sensitivity results. A graphical representation of the effect of σ_ξ on K^B in Proposition 3 as well as in Conjecture 1 is shown in Figure 3.3.

The effect of σ_ξ on K^B is illustrated by comparing with the effect of demand variability on resource-unconstrained capacity level K^U in benchmark model. The result is presented in part 1 of Proposition 2 showing that resource-unconstrained capacity level always increases in the demand variability. In contrast, in basic model, Proposition 3 and Conjecture 1 show that demand variability would lead to a negative effect on resource-constrained capacity level under certain condition and this deviation of the impact is driven by the correlation between demand and production resource. Specifically, when the correlation is negative, the result

Figure 3.3: Effect of demand variability σ_ξ on resource-constrained capacity level K^B.



(a) $\omega = 1$ (b) $\omega = 5$ (c) $\omega = 10$

Therein, the baseline scenario is $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\beta = 16\%\mu_\beta$ and $\rho \in \{-0.995, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.995\}$, with σ_ξ as percentage of μ_ξ from 2% to 30%.

deviates from the traditional understanding, that is, higher demand variability would lead to **lower** optimal capacity investment level K^B when $\sigma_\xi < \sigma_\xi^K$. Otherwise, the resource-constrained capacity K^B increases in demand variability, which is in line with the sensitivity result of benchmark case in Proposition 2.

The intuition behind Proposition 3 and Conjecture 1 is developed by understanding why the correlation is the key driver for the impact of demand variability on resource-constrained capacity level. To undertake the fact that the correlation between demand and production resource uncertainties is vital, the intuition of the result is discussed with classification of no correlation, positive correlation and negative correlation, since changing the sign of correlation changes the behaviour of marginal revenue in optimality condition (3.4). When there is no correlation i.e. $\rho = 0$, the financial flexibility level is not influenced by demand uncertainty, as a result, demand variability has the same effect for optimal capacity level in both models, to put it in another way, we can simply write the optimality condition (3.4) as $\omega = \Pr\left\{\tilde{\beta} > yK^B\right\} \int_{y+2bK^B}^{\infty} (\xi - y - 2bK^B) f_\xi(\xi) d\xi$. When the correlation is positive i.e. $\rho > 0$, high demand realization is associated with high production resource, or equivalently, when additional capacity is available because of the high demand, the production resource is less constraining comparing with no correlation case. As a result, financial flexibility level is high and thus marginal revenue is high so that the the optimal capacity level is increasing in demand variability. When the demand and production resource is negatively correlated i.e. $\rho < 0$, as demand variability increasing, more and more high (low) demand associated with low (high) production resource, the financial flexibility level is therefore getting lower gradually. Even though basic model inherits higher demand variability higher marginal revenue trend from basic model marginal revenue similar part, it only dominates the financial flexibility level decreasing trend when demand variability is larger than a threshold. Our result provides a new result by illustrating that higher demand variability does not necessarily result in higher capacity investment level when the uncertain demand is correlated with the production resource. It depends on the sign of the correlation as well as the magnitude of the demand variability.

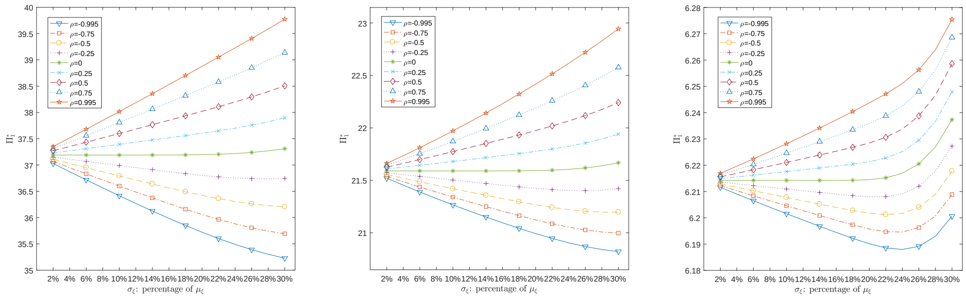
Next to that, the impact of demand variability on the optimal expected profit is summarized in Proposition 4 in comparison with part 2 of Proposition 2. Also, we find that the impact of demand variability on the optimal expected profit shares a similar pattern as that on the resource-constrained capacity level.

Proposition 4 (Impact of σ_ξ on $\Pi(K^B)$) *In terms of the impact of the demand intercept variability σ_ξ on the optimal expected profit, we have the following results:*

1. *If $\rho \geq 0$, $\Pi(K^B)$ increases in σ_ξ;*
2. *If $\rho < 0$, there exists a unique σ_ξ^Π such that $\Pi(K^B)$ decreases in σ_ξ when $\sigma_\xi \leq \sigma_\xi^\Pi$; and $\Pi(K^B)$ increases in σ_ξ when $\sigma_\xi > \sigma_\xi^\Pi$.*

The sensitivity of $\Pi(K^B)$ on demand variability is represented graphically in Figure 3.4. The impact of demand variability on the optimal expected profit in Proposition 4 is illustrated by comparing with Proposition 2 part 2. When the correlation is negative, different from the benchmark model, the result shows that there exists a threshold of demand variability denoted by σ_ξ^Π, below which

Figure 3.4: Effect of demand variability σ_ξ on optimal expected profit $\Pi(K^B)$.



(a) $\omega = 1$ (b) $\omega = 5$ (c) $\omega = 10$

The baseline scenario is $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\beta = 16\%\mu_\beta$, σ_ξ is picked as the percentage of μ_ξ ranging from 2% to 30% and $\rho \in \{-0.995, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.995\}$.

the profit **decreases** in demand variability, above which the profit increases in demand variability; when the correlation is non-negative, the profit increasing in demand variability holds in both basic model and benchmark model.

The intuition of the result is discussed with classification of no correlation, positive correlation and negative correlation. We first discuss the result when the correlation is non-negative (part 1 of Proposition 4). If that is the case, it is not hard to see that the optimal expected profit in the second stage is convex in the demand intercept, consequently, the optimal profit increases in the demand variability. Secondly, when the demand and production resource is negatively correlated (part 2 of Proposition 4), high (low) demand realization is associated with low (high) production resource realization, indicating that the production resource is more significantly constraining the production quantity when the market demand is higher.

Pursuing the intuition for negative correlation case (part 2 of Proposition 4) further, as demand variability increases from zero, we would observe more high and low demand realizations. Regardless of the effect of production resource, the resulting higher demand realization would continue to contribute more revenue by responsive pricing, and the resulting lower demand would lead to revenue loss because of the demand shrinkage. However, when the variability is smaller than certain threshold, comparing with the revenue loss brought by the lower demand realizations, the extra revenue brought by higher demand realizations is less significant due to the association with tightening production resource realizations. Note that the revenue loss by the the lower demand realizations is diminishing as the variability increases while the revenue gain by the higher demand realizations keeps increasing in the demand variability. Therefore, when the demand variability is large enough, the gain would dominate the loss and thus the optimal expected revenue starts to increase in the demand variability.

3.3.3 Sensitivity to Production Resource Variability

In this section, we conduct sensitivity analyses to study how manufacturing firms should adjust their capacity investment level as a response to changing production resource variability. Also how production resource variability affects the profitability of the firm is analysed. Starting from the impact of production resource variability σ_β on resource-constrained capacity level K^B, Proposition 5 characterizes this effect through unit capacity investment cost threshold and correlation threshold.

Proposition 5 (Impact of σ_β on K^B) *Define*

$$\omega_\beta^K(\rho) := \sigma_\xi\sqrt{(1-\rho^2)}\int_0^\infty \mathbf{E}\left[\left(\tilde{z}_2 - \left(\frac{y+2b\mu_\beta/y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\right)\right)^+\right]\phi(z_0)dz_0 \text{ and}$$

further define $\omega_\beta^K(-1) := \lim_{\rho\to-1}\omega_\beta^K(\rho)$ *and* $\omega_\beta^K(1) := \lim_{\rho\to1}\omega_\beta^K(\rho)$*. The impact of* σ_β *on* K^B *is:*

1. *If* $\omega \in \left(0,\ \omega_\beta^K(-1)\right]$*, then* K^B *strictly increases in* σ_β*. Note that* $\omega_\beta^K(-1) = 0$ *when* $\frac{\mu_\xi - y}{2} \le \frac{\mu_\beta}{y}$*.*

2. *For any* $\omega \in \left(\omega_\beta^K(-1), \omega_\beta^K(1)\right)$*, there exists a unique* $\rho_\beta^K(\omega)$ *satisfying* $\omega_\beta^K(\rho_\beta^K(\omega)) = \omega$ *such that*

   (a) *if* $\rho > \rho_\beta^K(\omega)$*, then* K^B *strictly increases in* σ_β*;*

(b) if $\rho = \rho_\beta^K(\omega)$, then $K^B = \mu_\beta/y$, which is constant to σ_β;

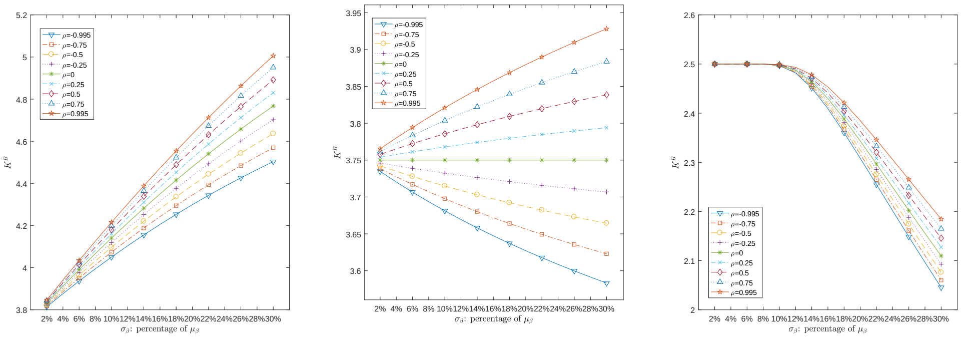
(c) if $\rho < \rho_\beta^K(\omega)$, then K^B strictly decreases in σ_β;

Noting that $\rho_\beta^K(\omega)$ increases in ω.

3. *If $\omega \in \left[\omega_\beta^K(1),\ \omega_{\max}\right)$, K^B strictly decreases in σ_β.*

Proposition 5 demonstrates that resource-constrained capacity level K^B is monotone in production resource variability σ_β, and whether it is increasing or decreasing in σ_β critically depends on the unit capacity cost and the correlation between demand and production resource uncertainties. Specifically, when the unit capacity investment cost ω is low enough (part 1 of Proposition 5), higher production resource variability results in higher resource-constrained capacity level; when the capacity investment cost is sufficiently high (part 3 of Proposition 5), as production resource variability increases, the resource-constrained capacity level decreases. When the capacity investment cost is intermediate (part 2 of Proposition 5), the impact of σ_β on K^B crucially depends on value of the correlation ρ. In particular, there exists a threshold value of ρ denoted by $\rho_\beta^K(\omega)$, above which the resource-constrained capacity level increases in σ_β; and below which the capacity level decreases in σ_β. For visualizing Proposition 5, Figure 3.5 graphically shows the impact of production resource variability on resource-constrained capacity level, where three panels have different scenarios of unit capacity cost.

Figure 3.5: Effect of production resource variability σ_β on resource-constrained capacity level K^B.



(a) $\omega = 1 < \omega_\beta^K(-1)$

(b) $\omega = 0.5\big(\omega_\beta^K(-1) + \omega_\beta^K(1)\big)$

(c) $\omega = 10 > \omega_\beta^K(1)$

The figures are depicted using baseline data $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\xi = 16\%\mu_\xi$, σ_β is picked as the percentage of μ_β ranging from 2% to 30% and and $\rho \in \{-0.995, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.995\}$. We calculated that $\omega_\beta^K(-1) = 2.73$ and $\omega_\beta^K(1) = 4.7713$.

For the effect of production resource variability σ_β on resource-constrained capacity level K^B presented in Proposition 5, we illustrate intuition behind in sequence of its part 1, part 3 and part 2. Since the effect critically depends on unit capacity cost ω and correlation between demand and production resource uncertainties ρ, we first summarize the effects of ω and ρ on K^B so as to understand how ω and ρ jointly shape the impact of σ_β on K^B. It is easy to verify that K^B decreases in the unit investment cost ω (because of $\frac{dK^B}{d\omega} < 0$). It is consistent with the intuition that when the unit capacity investment cost is high (low), the firm invests small (large) amount of capacity to control over-investment (under-investment) cost. For the effect of ρ on K^B, the monotone increasing relationship is illustrated in Proposition 1 part 1. Observing from part 1 (part 3) of Proposition 5, the unit capacity investment cost is sufficiently low (high) respectively, the resulting under-investment (over-investment) effect is so significant that the effect of the correlation is negligible. We first discuss part 1 and part 3.

In part 1 of Proposition 5, the unit capacity investment is sufficiently low so that controlling under-investment cost of capacity becomes the most important consideration. The reason is that investing large amount of capacity so as to prevent production process being constrained by capacity is less costly more profitable in terms of capturing potential demand. Under this condition, we discuss the impact of more high and low production resource realizations on resource-constrained capacity level respectively as the production resource variability increases. More high production resource realizations service the same role as high capacity level in terms of relaxing production process constraint so as to satisfy potential high demand. Even through more low production resource realizations constrain the production resulting more leftover capacity, it is not costly due to low unit capacity investment cost. To control under-investment cost of capacity, the firm would invest in more capacity when production resource variability is higher.

In part 3 of Proposition 5, the unit capacity investment is sufficiently high such that the over-investment cost is unbearable, the firm invests small amount of capacity and refer the production resource to be less volatile. As the production resource variability increases, there are more high and low production resource realizations. High resource do not affect the capacity investment decision, yet low resource constrain production process and therefore increases over-investment cost of capacity. To control the over-investment cost, resource-constrained capacity investment level should be decreasing in production resource variability.

To understand the result in part 2 of Proposition 5, note that the effect of intermediate unit capacity cost ω on resource-constrained capacity K^B does not dominate the effect of the correlation between demand and production resource uncertainties ρ any more. The value of ρ determines how K^B changes in σ_β. Therefore, we recall the impact of ρ on K^B: as the correlation increases, the revenue margin of investing capacity increases due to the increase of production resource flexibility level $\Pr\left\{\tilde{\beta} > yK \middle| \tilde{\xi}\right\}$, as a result, K^B increases. The trade-off between over- and under- investment is observed from production resource flexibility. When production resource flexibility level is high (in the case of high correlation $\rho > \rho_\beta^K(\omega)$), due to relevantly ample production resource, it is more likely to capture high demand. Therefore the firm should pay attention to control under-investment cost by invest in moderately large amount of capacity. As production resource variability increases, more high resource realizations resulting in a higher production resource flexibility level so that the firm should invest in more capacity. When production resource flexibility level is low (in the case $\rho < \rho_\beta^K(\omega)$), on expectation, the production resource is not enough, the firm should invest moderately small amount of capacity and prevent further over-investment cost as more low production resource realizations caused by production resource variability increasing.

In summary, the impact of production resource variability on resource-constrained capacity level crucially depends on the unit capacity investment cost and the correlation between demand and production resource uncertainties. Balancing over-investment and under-investment of capacity so as to control cost is the key factor that explains the impact of σ_β on K^B. Then we introduce the impact of σ_β on the optimal expected profit $\Pi(K^B)$ of the basic model.

Proposition 6 (Impact of σ_β on $\Pi(K^B)$) *Define $\omega_\beta^\Pi(\rho)$ as the unique solution of*

$$\lim_{\sigma_\beta \to 0} \left\{ \frac{\partial \Pi(K)}{\partial \sigma_\beta} \Big|_{K^B(\omega_\beta^\Pi)} \right\} = 0 \text{ when } \rho > 0$$

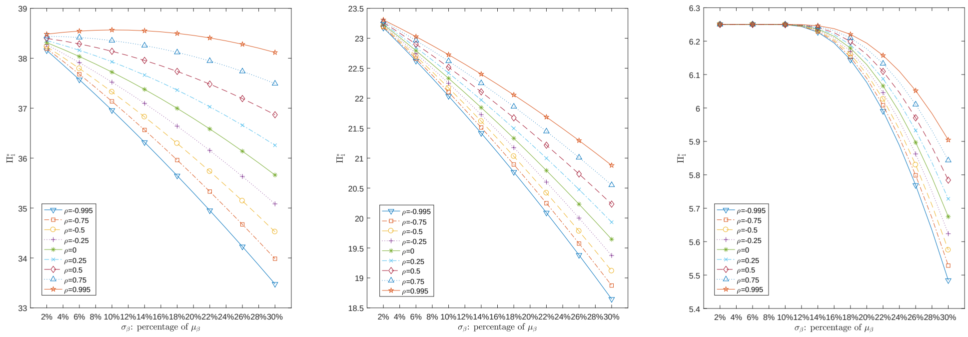
and $\omega_\beta^\Pi(1) := \lim_{\rho \to 1} \omega_\beta^\Pi(\rho)$. Define $\rho_\beta^\Pi(\omega)$ that uniquely solves $\lim_{\sigma_\beta \to 0} \left\{ \frac{\partial \Pi(K)}{\partial \sigma_\beta} \Big|_{K=K^B(\omega), \rho=\rho_\beta^\Pi(\omega)} \right\} = 0$. For any given correlation and unit capacity investment cost (ρ, ω),

1. *if $(\rho, \omega) \in \left\{(\rho, \omega) \middle| \rho \leq \rho_\beta^\Pi(\omega) \text{ or } \omega \geq \omega_\beta^\Pi(1)\right\}$, then $\Pi(K^B)$ decreases in σ_β.*

2. *if $(\rho, \omega) \in \left\{(\rho, \omega) \middle| \rho > \rho_\beta^\Pi(\omega) \text{ and } 0 < \omega < \omega_\beta^\Pi(1)\right\}$, then there exists a threshold $\sigma_\beta^\Pi(\omega, \rho) \in (0, \frac{\rho \sigma_\xi y}{2b}]$ such that $\Pi(K^B)$ increases in σ_β when $\sigma_\beta < \sigma_\beta^\Pi(\omega, \rho)$; and $\Pi(K^B)$ decreases in σ_β when $\sigma_\beta > \sigma_\beta^\Pi(\omega, \rho)$.*

Proposition 6 presents the impact of production resource variability on optimal expected profit and this result also crucially depends on unit capacity investment

Figure 3.6: Effect of production resource σ_β on optimal expected profit $\Pi(K^B)$.



(a) $\omega = 1$ (b) $\omega = 5$ (c) $\omega = 10$

The baseline data applied are $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\xi = 16\%\mu_\xi$, σ_β is picked as the percentage of μ_β ranging from 2% to 30% and $\rho \in \{-0.995, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.995\}$.

cost ω and the correlation ρ. There exist threshold value of ω and ρ denoted by $\omega_\beta^\Pi(1)$ and $\rho_\beta^\Pi(\omega)$ respectively, such that only when $\omega < \omega_\beta^\Pi(1)$ and $\rho > \rho_\beta^\Pi(\omega)$ (condition in part 2 of Proposition 6) firm's optimal expected profit first increases then decreases in production resource variability; otherwise (condition in part 1 of the Proposition 6), the profit monotonically decreases in production resource variability. A graphic representation of this proposition is shown in Figure 3.6.

Intuitively, variability of production resource may harm the firm's profit as result showed in part 1 of Proposition 6. One supporting example is when there is no correlation between demand and production resource uncertainties, i.e. $\rho = 0$, the optimal profit in the production stage $\pi^*(K, \xi, \beta)$ is concave in the realization of production resource and thus higher resource variability would result in lower expected profit in the capacity investment stage according to Jensen's inequality. Also, high volatility of production resource is more harmful for the firm's profitability fits business insight since on the one hand, more high production resource realizations may not be beneficial to the firm due to tightening capacity in the production process, on the other hand, more low production resource realizations may severely hurt firm's profit because the not only capacity is over-invested but also product demand is not satisfied.

However, in part 2 of Proposition 6, we obtain different pattern, indicating that the optimal expected profit increases in production resource variability when the production resource variability is less than a threshold value $\sigma_\beta^\Pi(\omega, \rho)$, the correlation is sufficiently high and the unit capacity investment cost is sufficiently low. To understand the result we first show how the low value of unit capacity cost ω impacts the optimal expected profit. For one thing, a a very low ω results in a very high resource-constrained capacity level so that the production process is almost always constrained by production resource, for another, optimal profit $\Pi(K^B)$ decreases in ω meaning that over-investment cost is negligible. Then we recall that when the demand and production resource correlation is very high, high production resource realizations are more likely to associated with high demand realizations, which gives more room to satisfy high demand so as to reduce under-investment cost, or equivalently to capture higher profit. But as production resource variability increases to a value larger than $\sigma_\beta^\Pi(\omega, \rho)$, more production resource realizations tend to take more extreme values, i.e. the value is either too high such that the realizations fill into Ω_2 or too low such that the realizations fill into Ω_0. Since high production resource doesn't increases production quantity when uncertainties are realized in Ω_2 and the firm doesn't produce when uncertainties are realized in Ω_0, higher production resource variability starts to hurt profitability of the firm.

So far we have discussed the impact of production resource variability on resource-constrained capacity level and the corresponding expected profit in Proposition 5 and Proposition 6 respectively. We find that the critical unit capacity investment cost thresholds as a function of the correlation between demand and production resource uncertainties in both propositions are analytically comparable:

Corollary 2 $\omega_\beta^\Pi(\rho) < \omega_\beta^K(\rho) < \omega_{\max}, \forall \rho > 0.$

This corollary presents the comparison between the unit capacity cost threshold $\omega_\beta^K(\rho)$ in Proposition 5 and $\omega_\beta^\Pi(\rho)$ in Proposition 6 given the correlation ρ.

Recalling that in Proposition 5, K^B increases in σ_β when unit capacity cost satisfies $\omega < \omega_\beta^K(\rho)$; K^B decreases in σ_β otherwise; in Proposition 6, optimal profit $\Pi(K^B)$ first increases then decreases in σ_β when unit capacity cost satisfies $\omega < \omega_\beta^\Pi(\rho)$; $\Pi(K^B)$ decreases in σ_β otherwise. The intuition is that even through there is a wide range of $(\omega, \rho) \in \left\{(\omega, \rho) | \omega < \omega_\beta^K(\rho), \rho \in (-1, 1)\right\}$ under which a higher production resource variability increases optimal capacity level, only a proper subset of above (ω, ρ) set, denoted by $\left\{(\omega, \rho) | \omega < \omega_\beta^\Pi(\rho), \rho \in (0, 1)\right\}$, is the range under which higher production resource variability is profitable when production resource variability is less than a certain threshold.

3.4 Profitability-loss

In this section, we address the third research question by extending our analyses of the impact of the demand and production resource uncertainties on the profitability-loss incurred once the production resource uncertainty is ignored in choosing the capacity investment level. Since the expected profit taking into consideration of the production resource is $\Pi(\cdot)$, we define the rate of the profitability-loss due to miss-specifying capacity level as $\Delta\Pi := \frac{\Pi(K^B)-\Pi(K^U)}{\Pi(K^B)}$. Recalling that K^U is resource-unconstrained capacity level that the firm would choose if the production resource is ignored. Therefore, the expected profit with miss-specified capacity level is $\Pi(K^U)$. K^B is resource-constrained capacity level that is the optimal capacity investment level of the basic model. For the ease of the analysis, we bring the bivariate normal distribution assumption back and limit unit capacity cost in the range $\omega \in (0,\ \omega_{\max})$ where $\omega_{\max} = \mathop{\mathbf{E}}_{\tilde{\xi}}\left[(\tilde{\xi} - y)^+ \Pr\left\{\tilde{\beta} > 0 \middle| \tilde{\xi}\right\}\right]$. The following lemma provides the basic theoretical support of the existence of profitability-loss.

Lemma 2 $K^U(\omega) > K^B(\omega)$ *and* $\Pi_u(K^U) > \Pi(K^B) > \Pi(K^U)$ *for all* $\omega \in (0,\ \omega_{\max})$.

This result fits the standard folklore that 1) if the firm has a sufficiently large production resource, the firm would make higher capacity investment and 2) the investment project is not proceed as profitable as planned if the firm has insufficient production resource for production. Briefly speaking, the firm would incur an optimality gap, that is called profitability-loss, when the firm mistakenly chooses higher capacity investment level by ignoring uncertain production resource. Due to the optimality of K^B we have $\Delta\Pi > 0$, in addition, $\Delta\Pi$ may greater than 1 in the case of $\Pi(K^U) < 0$. Noting that above inequalities in Lemma 2 also hold without Distribution Assumption 2 when the range of unit capacity cost is $0 < \omega < \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y - 2b\underline{\beta}/y)^+\big]$.

In analysing the sensitivity, results are analytically and numerically provided, specifically, we show that under what conditions the profitability-loss is significant by examining the effects of uncertainties on this profitability-loss. At the same time, we use extensive numerical experiments to show the results that are not analytically proven. In the following, we investigate the impact of the uncertainty parameters (ρ, σ_ξ and σ_β) on the profitability-loss. We first present a proposition showing how profitability-loss changes in the correlation between demand and production resource uncertainties.

Proposition 7 (Impact of ρ on Profitability-loss)

1. *Profitability-loss* $\Delta\Pi$ *is decreasing in* ρ*;*

2. *The lower bound of profitability-loss is* $\lim\limits_{\rho\to 1}\Delta\Pi$.

Proposition 7 demonstrates that a high the correlation between demand and production resource uncertainties results in a low profitability-loss. First of all, K^U is not a function of the correlation. Base on this, we discussion the general insight. On the one hand, a higher correlation decreases the difference between product quantity provided by realized production resource $\frac{\beta}{y}$ and the internal optimal production quantity $\frac{\xi-y}{2b}$, thus, K^B could be closer to K^U as the correlation increases. On the other hand, from the proof of Proposition 1, we obtain that $\frac{\partial\Pi(K)}{\partial K}$ increases in ρ, meaning that as ρ increasing, the decreasing trend of $\Pi(K)$ on K when $K > K^B(\rho)$ tends to be more flat. As a result, the rate of profitability-loss between optimal expected profit and the profit without considering production-loss is shrink when ρ increases.

In analysing the impact of production resource variability on the profitability-loss, we provide following analytical result.

Proposition 8 (Impact of σ_β on Profitability-loss) *Define* $\omega_U := \mathbf{E}\left[\left(\tilde{\xi} - \left(y + 2b\mu_\beta/y\right)\right)^+\right] > \omega_\beta^K(\rho)$*. The sensitivity result is calibrated as follows:*

1. *when* $\omega \geq \omega_U$*,* $\Delta\Pi$ *strictly increases in* σ_β*;*

2. *when* $\omega < \omega_U$ *and* $\rho > 0$*,* $\frac{d\Delta\Pi}{d\sigma_\beta} < 0$ *for all* $\sigma_\beta \leq \max\left\{0,\ \sigma_{\beta 0}\left(K^U\right)\right\}$*, where* $\sigma_{\beta 0}(K^U)$ *denotes the unique* σ_β *that solves implicit equation* $\left.\frac{\partial\Pi(K)}{\partial\sigma_\beta}\right|_{K=K^U} = 0$.

The first part of Proposition 8 shows that when unit capacity cost is higher than ω_U, then the profitability-loss increases in production resource variability. The part 2 of Proposition 8 is for theoretically supporting Conjecture 2. Specifically, second part indicates when the unit capacity cost is lower than this threshold ω_U and the correlation between demand and production resource is positive, we only know that the profitability-loss decreases in production resource variability when the variability is no more than threshold $\max\left\{0,\ \sigma_{\beta 0}\left(K^U\right)\right\}$.

Analytical explanation of part 1 of Proposition 8 is as follows. First of all, K^U is not sensitive to production resource variability σ_β by definition. For the effect of σ_β on K^B, when unit capacity cost is sufficiently large, $K^B(\sigma_\beta)$ decreases in σ_β as discussed in part 3 of Proposition 5. As a result, the difference between two capacity levels $K^U - K^B(\sigma_\beta)$ increases in σ_β. Also, $\frac{\partial\Pi(K)}{\partial K}$ decreases in σ_β when $\omega < \omega_U$, which implies that as $K^U - K^B(\sigma_\beta)$ increases $\Pi\left(K^U, \sigma_\beta\right) - \Pi\left(K^B(\sigma_\beta), \sigma_\beta\right)$ also increases due to sharper decreasing trend of $\frac{\partial\Pi(K)}{\partial K}$ caused by increasing σ_β. The general insight is twofold. For one thing, similar as what we discussed in Section 3.3.3, that is, the need for control over-investment cost outweighs the need for under-investment cost when unit capacity cost is sufficiently large. For another, higher the production resource variability more unprofitable the optimal expected profit in basic model is. The part 2 of Proposition is the analytical support of Conjecture 2 introduced in Section 4.4.2. In obtaining conjectures of sensitivity of profitability-loss, we conduct numerical experiment and the numerical study design is introduced in the following section.

3.4.1 Numerical Study Design

The numerical study is conducted with $18,225$ numerical instances. A wide range of parameter values extended around the baseline scenario: the correlation between demand and production resource uncertainties takes value $\rho \in \{-0.995, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.995\}$; unit production cost is standardized as $b = 1$, $y = 1$; the unit capacity investment cost $\omega \in \{1, 5, 10\}$; mean value of demand uncertainty has range $\mu_\xi \in \{14y, 16y, 18y\}$; mean value of production resource uncertainty takes $\frac{\mu_\beta}{y} = \frac{1}{2}\frac{(\mu_\xi - y)}{2b}$ for each μ_ξ; the demand variability $\sigma_\xi \in [2\%, 30\%]$ is the percentage of μ_ξ with 2%-unit increments, and similarly, production resource variability $\sigma_\beta \in [2\%, 30\%]$ is also picked as the percentage of μ_β, varying with 2%-unit increments. For this numerical study design, we have several specifications in what follows.

Firstly, the one to one correspondence between μ_ξ and μ_β, specifically $\frac{\mu_\beta}{y} = \frac{1}{2}\frac{(\mu_\xi - y)}{2b}$, indicates the quantity of products processed by mean production resource value equals to half of the product quantity that denotes the internal optimal production quantity given demand intercept materialized as its average level. In other words, this equation means, on expectation, the production resource is short to satisfy the product demand. On count of widely studied less- or non-resource constrained firms' capacity management in the OM literature, we would not focus on the case where the production resource is high on expectation.

Secondly, we choose the standard deviations of the distribution carefully so that the probability of having negative realizations is negligible, particularly, the coefficient of variation no more than 30%. This is because, as a matter of fact, neither demand intercept nor production resource should be realized in negative values even if we follow Assumption 2 that their joint distribution is bivariate normally distributed. In this way, the non-negativity of the $(\tilde{\xi}, \tilde{\beta})$ distribution is unproblematic.

Lastly, we carefully choose three values of unit capacity investment cost $\omega \in \{1, 5, 10\}$. Mathematically, ω should be less than $\omega_{\max}$ in order to have positive K^*. Defined in Section 3.3, $\omega_{\max}$ is an unit capacity cost threshold such that $K^* = K^B$ when $\omega < \omega_{\max}$ and $K^* = 0$ otherwise. As $\omega_{\max}$ is a function of $b, y, \rho, \mu_\xi, \sigma_\xi, \mu_\beta$ and σ_β, ω should be less than the lowest $\omega_{\max}$ for all instances of $\{b, y, \rho, \mu_\xi, \sigma_\xi, \mu_\beta, \sigma_\beta\}$. Denoting $\overline{\omega}_{\max}$ as the lowest value mentioned above, we can safely pick ω less than $\overline{\omega}_{\max} = 12.9888$. In addition, the early stage unit capacity cost is usually more expensive than unit production cost, that is $\omega > y$, as we fix $y = 1$, the capacity cost ω should no less than 1.

We numerically compute the percentage profitability-loss $\Delta\Pi \times 100\%$ after obtaining both resource-unconstrained capacity level K^U and resource-constrained capacity level K^B and optimal expected profit in basic model (Equation (3.2)) using standard MatLab optimization procedures. The baseline scenario for sensitivity results described in Section 3.3 is based on the numerical study design by calculating optimal capacity and corresponding expected profit for all ρ.

3.4.2 Profitability-loss under Different Production Resource Variability

Continuing on the discussion regarding to the impact of σ_β on Profitability-loss in Proposition 8, we perform complementary computational experiments. We numerically calculate the average profitability-loss $\Delta\Pi \times 100\%$ across all scenarios fixing σ_β/μ_β, ρ and ω and report results in Table 3.1. Similarly, we calculate the average profitability-loss $\Delta\Pi \times 100\%$ fixing only σ_β/μ_β and ρ in Table 3.2. We draw a conjecture according to Proposition 8 together with observations in Table 3.1 and Table 3.2. Specifically, we consistently observe following pattern in each scenario:

Conjecture 2 *Observing from Table 3.1 and Table 3.2 and according to Proposition 8, there exists a threshold $\overline{\sigma}_\beta^{PL}(\omega, \rho)$ decreasing in ω and increasing in ρ, such that Profitability-loss $\Delta\Pi$ decreases in σ_β when $\sigma_\beta < \overline{\sigma}_\beta^{PL}(\omega, \rho)$ and increases in σ_β when $\sigma_\beta > \overline{\sigma}_\beta^{PL}(\omega, \rho)$.*

This conjecture shows that the impact of production resource variability on profitability-loss is not always monotone. There exists a production resource variability threshold $\overline{\sigma}_\beta^{PL}(\omega, \rho)$ which critically depends on unit capacity cost and correlation. Specifically, when production resource variability is below this threshold, profitability-loss decreases as production resource variability increases; when production resource variability is above this threshold, profitability-loss increases in production resource variability. In a special case that unit capacity cost is no less than ω_U, the threshold $\overline{\sigma}_\beta^{PL}(\omega, \rho)$ equals to 0. Then, we discuss observations from Table 3.1 and Table 3.2 for supporting the conjecture.

For the observation of Table 3.1, we start from high unit capacity cost scenario Table 3.1 (c) $\omega = 10$. In line with part 1 of the Proposition 8 where the maximal ω_U among all instances equals to 8.6334 that is less than given value of unit capacity cost 10, the profitability-loss increases in production resource variability σ_β. And the value of profitability-loss is in a very low range 0% − 8.8%. In Table 3.1 (b) where unit capacity cost is moderate, we observe profitability-loss decreases in σ_β when σ_β is smaller than a threshold, which is in accordance with part 2 of Proposition 8. The value of profitability-loss is relevantly high 14% − 26.9%. For low unit capacity cost scenario $\omega = 1$ in Table 3.1 (a), the profitability-loss span the range of 5.2% − 10%. In this case, as σ_β increases profitability-loss is lower, which implies that the production resource variability threshold $\overline{\sigma}_\beta^{PL}(\omega, \rho)$ may go to ∞.

An additional observation from comparing three sub-tables of Table 3.1 is

Table 3.1: Effect of production resource variability σ_β on profitability loss $\Delta\Pi$ for each ω

$\Delta\Pi$ \ ρ / σ_β	−0.995	−0.75	−0.5	−0.25	0	0.25	0.5	0.75	0.995
$2\%\mu_\beta$	10.026%	9.987%	9.951%	9.919%	9.89%	9.863%	9.838%	9.815%	9.793%
$4\%\mu_\beta$	9.887%	9.809%	9.738%	9.674%	9.615%	9.561%	9.511%	9.465%	9.422%
$6\%\mu_\beta$	9.757%	9.639%	9.532%	9.435%	9.346%	9.265%	9.19%	9.121%	9.058%
$8\%\mu_\beta$	9.635%	9.477%	9.333%	9.203%	9.084%	8.976%	8.876%	8.784%	8.701%
$10\%\mu_\beta$	9.521%	9.322%	9.141%	8.977%	8.829%	8.693%	8.568%	8.454%	8.351%
$12\%\mu_\beta$	9.416%	9.175%	8.957%	8.759%	8.58%	8.416%	8.267%	8.131%	8.007%
$14\%\mu_\beta$	9.318%	9.036%	8.78%	8.548%	8.338%	8.146%	7.972%	7.814%	7.671%
$16\%\mu_\beta$	9.228%	8.905%	8.61%	8.344%	8.103%	7.883%	7.684%	7.503%	7.341%
$18\%\mu_\beta$	9.147%	8.781%	8.448%	8.148%	7.874%	7.626%	7.401%	7.198%	7.018%
$20\%\mu_\beta$	9.073%	8.665%	8.294%	7.958%	7.653%	7.376%	7.126%	6.9%	6.701%
$22\%\mu_\beta$	9.008%	8.557%	8.147%	7.776%	7.439%	7.132%	6.856%	6.609%	6.39%
$24\%\mu_\beta$	8.95%	8.457%	8.007%	7.601%	7.231%	6.896%	6.593%	6.323%	6.086%
$26\%\mu_\beta$	8.901%	8.364%	7.875%	7.433%	7.031%	6.666%	6.337%	6.044%	5.789%
$28\%\mu_\beta$	8.859%	8.278%	7.75%	7.272%	6.838%	6.443%	6.088%	5.772%	5.498%
$30\%\mu_\beta$	8.824%	8.2%	7.632%	7.119%	6.652%	6.227%	5.845%	5.507%	5.214%

(a) $\omega = 1$

$\Delta\Pi$ \ ρ / σ_β	−0.995	−0.75	−0.5	−0.25	0	0.25	0.5	0.75	0.995
$2\%\mu_\beta$	26.954%	26.786%	26.618%	26.455%	26.296%	26.142%	25.994%	25.851%	25.718%
$4\%\mu_\beta$	27.007%	26.671%	26.335%	26.008%	25.691%	25.383%	25.086%	24.801%	24.535%
$6\%\mu_\beta$	27.077%	26.572%	26.068%	25.577%	25.099%	24.637%	24.192%	23.766%	23.368%
$8\%\mu_\beta$	27.166%	26.491%	25.818%	25.161%	24.524%	23.908%	23.314%	22.747%	22.219%
$10\%\mu_\beta$	27.278%	26.432%	25.589%	24.768%	23.972%	23.203%	22.463%	21.757%	21.102%
$12\%\mu_\beta$	27.417%	26.4%	25.389%	24.407%	23.455%	22.537%	21.656%	20.816%	20.039%
$14\%\mu_\beta$	27.587%	26.402%	25.226%	24.086%	22.984%	21.924%	20.907%	19.941%	19.05%
$16\%\mu_\beta$	27.793%	26.441%	25.104%	23.812%	22.567%	21.37%	20.227%	19.142%	18.145%
$18\%\mu_\beta$	28.038%	26.521%	25.027%	23.589%	22.206%	20.882%	19.619%	18.425%	17.331%
$20\%\mu_\beta$	28.323%	26.644%	24.998%	23.417%	21.904%	20.458%	19.084%	17.788%	16.605%
$22\%\mu_\beta$	28.651%	26.812%	25.015%	23.297%	21.658%	20.098%	18.619%	17.229%	15.966%
$24\%\mu_\beta$	29.023%	27.024%	25.079%	23.228%	21.467%	19.797%	18.22%	16.743%	15.405%
$26\%\mu_\beta$	29.439%	27.281%	25.189%	23.206%	21.327%	19.552%	17.881%	16.323%	14.916%
$28\%\mu_\beta$	29.902%	27.582%	25.344%	23.23%	21.236%	19.358%	17.599%	15.962%	14.492%
$30\%\mu_\beta$	30.411%	27.928%	25.542%	23.298%	21.189%	19.212%	17.366%	15.656%	14.126%

(b) $\omega = 5$

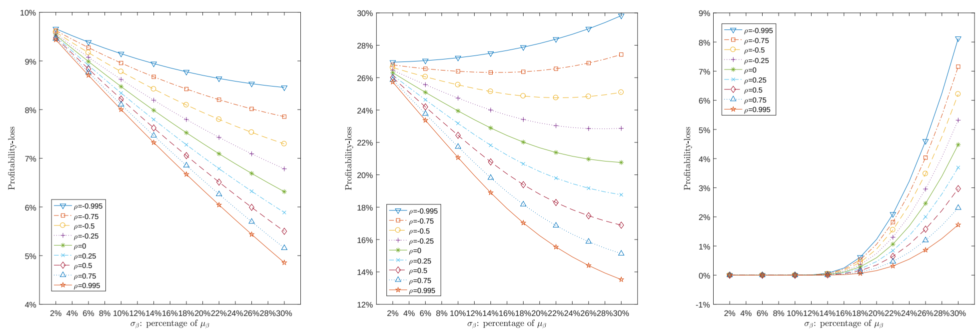
$\Delta\Pi$ \ ρ / σ_β	−0.995	−0.75	−0.5	−0.25	0	0.25	0.5	0.75	0.995
$2\%\mu_\beta$	0%	0%	0%	0%	0%	0%	0%	0%	0%
$4\%\mu_\beta$	0%	0%	0%	0%	0%	0%	0%	0%	0%
$6\%\mu_\beta$	0.001%	0.001%	0%	0%	0%	0%	0%	0%	0%
$8\%\mu_\beta$	0.025%	0.021%	0.017%	0.013%	0.01%	0.007%	0.004%	0.003%	0.002%
$10\%\mu_\beta$	0.132%	0.112%	0.094%	0.076%	0.059%	0.045%	0.033%	0.023%	0.016%
$12\%\mu_\beta$	0.349%	0.303%	0.257%	0.213%	0.173%	0.136%	0.104%	0.076%	0.056%
$14\%\mu_\beta$	0.684%	0.597%	0.512%	0.431%	0.355%	0.286%	0.224%	0.171%	0.128%
$16\%\mu_\beta$	1.142%	1.002%	0.864%	0.734%	0.611%	0.499%	0.398%	0.309%	0.237%
$18\%\mu_\beta$	1.732%	1.524%	1.32%	1.127%	0.945%	0.778%	0.627%	0.495%	0.385%
$20\%\mu_\beta$	2.463%	2.171%	1.886%	1.615%	1.362%	1.128%	0.917%	0.73%	0.574%
$22\%\mu_\beta$	3.347%	2.953%	2.57%	2.206%	1.866%	1.553%	1.27%	1.018%	0.808%
$24\%\mu_\beta$	4.398%	3.884%	3.383%	2.909%	2.467%	2.06%	1.691%	1.364%	1.09%
$26\%\mu_\beta$	5.638%	4.98%	4.34%	3.736%	3.173%	2.656%	2.188%	1.773%	1.424%
$28\%\mu_\beta$	7.088%	6.259%	5.455%	4.699%	3.996%	3.351%	2.767%	2.251%	1.815%
$30\%\mu_\beta$	8.769%	7.74%	6.745%	5.812%	4.946%	4.152%	3.437%	2.803%	2.269%

(c) $\omega = 10$

Table 3.2: Effect of production resource variability σ_β on profitability loss $\Delta\Pi$

$\Delta\Pi$ \ ρ / σ_β	−0.995	−0.75	−0.5	−0.25	0	0.25	0.5	0.75	0.995
$2\%\mu_\beta$	12.327%	12.258%	12.19%	12.125%	12.062%	12.002%	11.944%	11.889%	11.837%
$4\%\mu_\beta$	12.298%	12.16%	12.024%	11.894%	11.769%	11.648%	11.532%	11.422%	11.319%
$6\%\mu_\beta$	12.278%	12.071%	11.867%	11.671%	11.482%	11.301%	11.127%	10.962%	10.809%
$8\%\mu_\beta$	12.276%	11.996%	11.722%	11.459%	11.206%	10.963%	10.732%	10.511%	10.307%
$10\%\mu_\beta$	12.31%	11.955%	11.608%	11.274%	10.953%	10.647%	10.355%	10.078%	9.823%
$12\%\mu_\beta$	12.394%	11.959%	11.534%	11.126%	10.736%	10.363%	10.009%	9.674%	9.368%
$14\%\mu_\beta$	12.53%	12.012%	11.506%	11.022%	10.559%	10.119%	9.701%	9.309%	8.95%
$16\%\mu_\beta$	12.721%	12.116%	11.526%	10.963%	10.427%	9.917%	9.436%	8.985%	8.574%
$18\%\mu_\beta$	12.972%	12.275%	11.599%	10.954%	10.342%	9.762%	9.216%	8.706%	8.244%
$20\%\mu_\beta$	13.287%	12.494%	11.726%	10.997%	10.306%	9.654%	9.042%	8.473%	7.96%
$22\%\mu_\beta$	13.668%	12.774%	11.911%	11.093%	10.321%	9.594%	8.915%	8.285%	7.721%
$24\%\mu_\beta$	14.124%	13.121%	12.156%	11.246%	10.388%	9.584%	8.835%	8.143%	7.527%
$26\%\mu_\beta$	14.659%	13.541%	12.468%	11.458%	10.51%	9.625%	8.802%	8.047%	7.376%
$28\%\mu_\beta$	15.283%	14.04%	12.85%	11.734%	10.69%	9.717%	8.818%	7.995%	7.268%
$30\%\mu_\beta$	16.001%	14.623%	13.307%	12.076%	10.929%	9.864%	8.883%	7.989%	7.203%

Figure 3.7: Effect of production resource variability σ_β on profitability loss $\Delta\Pi$.



(a) $\omega = 1$ (b) $\omega = 5$ (c) $\omega = 10$

Figures are depicted using baseline scenario $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\xi = 16\%\mu_\xi$ and σ_β is the percentage of μ_β which are $\{2\%, 4\%, 6\%, 8\%, 10\%, 12\%, 14\%, 16\%, 18\%, 20\%, 22\%, 24\%, 26\%, 28\%, 30\%\}$.

that, as unit capacity cost increases, the production resource variability threshold $\overline{\sigma}_\beta^{PL}(\omega, \rho)$ decreases. To further understand how correlation shapes the impact of production resource variability on the profitability-loss, we calculate the average profitability-loss by only fixing σ_β/μ_β and ρ. The result is summarized in Table 3.2, in which we observe that as correlation increasing, production resource variability threshold $\overline{\sigma}_\beta^{PL}(\omega, \rho)$ increases.

So far we have explained how we draw the Conjecture 2 from both analytical Proposition and numerical experiments. Now, in complementing intuition of the impact of profitability-loss on production resource variability, we consider this impact in a low unit capacity cost scenario. We know that in this case, there exists a production resource variability threshold $\overline{\sigma}_\beta^{PL}(\omega, \rho)$ critically depends on unit capacity cost and correlation. This is because the impact of

Table 3.3: Effect of demand variability σ_ξ on profitability loss $\Delta\Pi$.

$\Delta\Pi$ \ ρ / σ_ξ	-0.995	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	0.995
$2\%\mu_\xi$	10.198%	10.13%	10.06%	9.991%	9.923%	9.855%	9.787%	9.72%	9.655%
$4\%\mu_\xi$	10.489%	10.349%	10.207%	10.068%	9.93%	9.795%	9.661%	9.53%	9.403%
$6\%\mu_\xi$	10.812%	10.596%	10.38%	10.169%	9.962%	9.759%	9.561%	9.367%	9.182%
$8\%\mu_\xi$	11.166%	10.871%	10.578%	10.293%	10.016%	9.747%	9.486%	9.232%	8.992%
$10\%\mu_\xi$	11.547%	11.171%	10.799%	10.439%	10.091%	9.755%	9.432%	9.121%	8.828%
$12\%\mu_\xi$	11.958%	11.498%	11.044%	10.608%	10.189%	9.787%	9.403%	9.036%	8.695%
$14\%\mu_\xi$	12.404%	11.857%	11.321%	10.807%	10.317%	9.849%	9.405%	8.985%	8.597%
$16\%\mu_\xi$	12.89%	12.254%	11.635%	11.044%	10.482%	9.949%	9.446%	8.974%	8.542%
$18\%\mu_\xi$	13.421%	12.695%	11.992%	11.323%	10.69%	10.092%	9.531%	9.009%	8.536%
$20\%\mu_\xi$	13.998%	13.182%	12.394%	11.648%	10.944%	10.283%	9.664%	9.092%	8.579%
$22\%\mu_\xi$	14.621%	13.714%	12.842%	12.019%	11.245%	10.52%	9.846%	9.225%	8.674%
$24\%\mu_\xi$	15.291%	14.292%	13.335%	12.436%	11.592%	10.805%	10.075%	9.407%	8.818%
$26\%\mu_\xi$	16.005%	14.915%	13.873%	12.897%	11.984%	11.135%	10.35%	9.635%	9.01%
$28\%\mu_\xi$	16.763%	15.581%	14.455%	13.402%	12.419%	11.508%	10.669%	9.909%	9.248%
$30\%\mu_\xi$	17.564%	16.289%	15.077%	13.947%	12.895%	11.923%	11.031%	10.225%	9.528%

production resource variability on both resource-constrained capacity K^B and optimal expected profit in basic model $\Pi(K^B)$ are characterized by unit capacity cost and correlation, referring to Proposition 5 and Proposition 6 for details. In this small unit capacity cost scenario, profitability-loss decreases in production resource variability when production resource variability is less than the threshold $\overline{\sigma}_\beta^{PL}(\omega, \rho)$. This is because not only resource-constrained capacity level getting closer to resource-unconstrained capacity level, but the expected profit in basic model is less sensitivity to capacity investment level. As a result, we conclude that either too low or too high production resource variability leads to a large profitability-loss.

3.4.3 Profitability-loss under Different Demand Variability

Then, we investigate the impact of demand variability σ_ξ on the profitability-loss $\Delta\Pi \times 100\%$. Since the analytical result is intractable, we conduct numerical experiment across all scenarios fixing σ_ξ/μ_ξ, ρ and report results in Table 3.3. All entries denote the average profitability-loss with all numerical instances. In addition, we underline the minimal value of each column, since we observe the non-monotone trend of how profitability-loss changes in demand variability and this demand variability threshold changes in the correlation ρ.

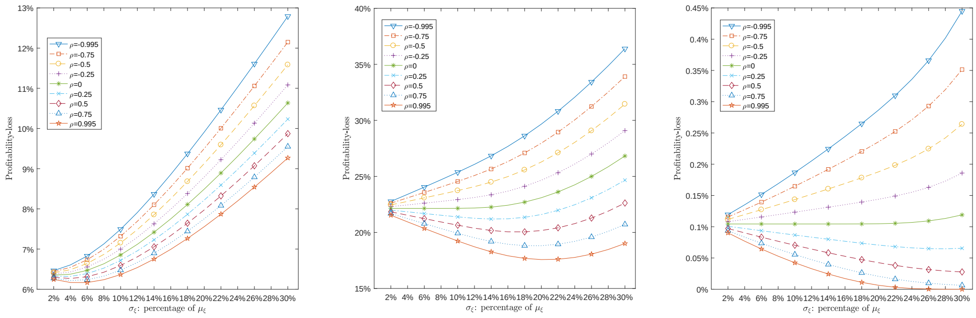
From this table, we first observe that given fixed demand variability σ_ξ, profitability-loss is decreasing in correlation ρ, which verify the Proposition 7 again. When ρ is low, profitability-loss is increasing in demand variability and when ρ is high, this monotonicity does not hold anymore. In conclusion, the observation from Table 3.3 about the impact of σ_ξ on profitability-loss is summarized in following conjecture.

Conjecture 3 *There exists a threshold $\overline{\sigma}_\xi^{PL}(\rho)$ increasing in ρ, such that*

1. *when $\rho > 0$, $\Delta\Pi$ decreases in σ_ξ when $\sigma_\xi < \overline{\sigma}_\xi^{PL}(\rho)$, otherwise, it increases in σ_ξ;*
2. *when $\rho \leq 0$, $\Delta\Pi$ increases in σ_ξ, or equivalently, $\overline{\sigma}_\xi^{PL}(\rho) = 0$.*

The graphical exposition of the conjecture using baseline scenario is as follows: Intuitively, how profitability-loss being affected by demand variability critically depends on the impact of demand variability on resource-unconstrained capacity level K^U (part 1 of Proposition 2), resource-constrained capacity level K^B and the expected profit function $\Pi(K)$. As we obtain from Proposition 3 and 4, the sign of correlation shapes the sensitivity results. We explain the intuitions in two scenarios, negative correlation and positive correlation. Firstly, when the demand and production resource is negatively correlated, both resource-constrained capacity level and corresponding optimal expected profit in basic model first decrease then increase in demand variability. Since resource-unconstrained

Figure 3.8: Effect of demand variability σ_ξ on profitability loss $\Delta\Pi$.



(a) $\omega = 1$ (b) $\omega = 5$ (c) $\omega = 10$

Therein, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\beta = 16\%\mu_\beta$ and σ_β is percentage of μ_β, where the percentage set is $\{2\%, 4\%, 6\%, 8\%, 10\%, 12\%, 14\%, 16\%, 18\%, 20\%, 22\%, 24\%, 26\%, 28\%, 30\%\}$.

capacity level always increases in demand variability, the difference between K^U and K^B is larger as demand variability increases. Therefore, when correlation is negative, profitability-loss higher as demand variability increases. When the correlation is positive, higher production resource flexibility level relaxes the pressure of under-investment, so that miss-specify a higher capacity level can be less hurtful when demand variability increases.

To summarize, we discuss the conditions under which the profitability-loss is significant. When the correlation between demand and production resource is non-positively correlated, high demand variability leads to large profitability-loss; When the correlation is positive, both too low and too high demand variability results in large profitability-loss.

Chapter 4

Managing Financial Production Resource Disruption: Role of Pre-shipment Financing

This chapter examines the role of pre-shipment finance in managing financial production resource (working capital/budget) disruption. Hereafter, we use 'budget' to denote the financial production resource. Pre-shipment finance allows the firm to transfer the purchase orders (which will be paid after production) to an external party that provides immediate cash flow (at a cost) that can be used for financing the production process. Pre-shipment financing creates necessary liquidity for the firm when the budget is constraining. In comparing with basic model, we name the model with pre-shipment finance as *pre-shipment finance model* (abbreviated as PSF model). And we assume the partial equilibrium, which means 1) the buyer(s) exogenously accepts any product quantity and its unit price on the realized inverse demand curve; 2) the finance is risk free, as the firm would always chooses a loan that can be fully secured within the product selling revenue. Then, we characterize the optimal pre-shipment finance level (proportion of sales revenues transferred) and the production volume in the production stage and the optimal capacity investment level in the capacity stage. We make comparisons with the results in Chapter 3 to understand how pre-shipment financing alters the effects of demand and production budget uncertainties on the optimal capacity investment level, expected profit and profitability-loss due to ignoring budget uncertainty. We identify that applying pre-shipment finance makes the capacity investment and profits more resilient to changes in both demand and production budget uncertainties. The profitability-loss by miss-choosing capacity level due to ignore the constraint of production budget is significantly reduced compared with that of basic model.

The organization of this chapter is as follows. We review the literature in Section 4.1. In Section 4.2, we introduce the formulation and assumptions of the model and establish the optimal capacity investment policy and compare it with the basic model. Then we derive analytical results for the sensitivity of optimal capacity investment policy as well as the profitability in Section 4.3. Specifically, we answer the research question: How would the optimal capacity level and profitability be impacted by the demand and production resource variability and the correlation between the two? Finally, in Section 4.4, we use analytical analysis as well as complementary numerical study to answer second research question, if the possibility of production resource shortage is ignored in capacity planning, as often done in practice and the academic literature, would the resulting profitability loss be significant and how do the demand and production resource uncertainties affect this profitability loss?

4.1 Literature Review

Pre-shipment finance is a category of financing instruments issued when the manufacturing firm wants payment (by its buyers directly or financial institutions) of product selling revenue *before* the product shipment. In operations management, the main objective behind is to release the financial distress of production process. There are several types of pre-shipment finance including purchase order financing

(Reindorp et al., 2018; Tang et al., 2018; Zhao and Huchzermeier, 2019), advance payment discount (Boyacı and Özer, 2010; Zhao and Huchzermeier, 2019) and buyer intermediated financing (Tunca and Zhu, 2018).

The only paper that consider capacity planning with pre-shipment finance is Boyacı and Özer (2010), they investigate a capacity planning strategy that collects commitments to purchase before the capacity decision and uses the acquired advance sales information to decide on the capacity. In their model, a finite number of price adjustments are made prior to the capacity decisions, the demand uncertainty becomes endogenous and demand parameters get updated overtime when applying the advance sell. Tang et al. (2018) address buyer direct financing and purchase order financing without the buyer's guarantee in a signalling game, and focus on the effect of information asymmetry between buyer and bank on the firm's cost. Reindorp et al. (2018) study a two-stage supply chain where a retailer buys from a supplier who faces financial constraints, also and the retailer commits purchase order finance to supplier. They address the potential of purchase commitments for mitigating capital market frictions. Zhao and Huchzermeier (2019) investigate the interaction between firm's operational and financing choice between advance payment discount and buyer-backed purchase order financing. Tunca and Zhu (2018) compare commercial loan and buyer intermediated financing in the same supply chain and find that buyer intermediated financing will improve channel profit.

4.2 Optimal Strategy of Pre-shipment Finance Model

The timeline of events for pre-shipment finance starts from $(\tilde{\xi}, \tilde{\beta})$ being resolved. The firm is aware of the demand pattern of buyer, so does the selling revenue providing any quantity of product $(\xi - bq_p)q_p$. Then if necessary, together with making production quantity decision q_p, the firm finances $(1 + r_p)L_p$ amount against the product selling revenue to support production. Therein, only an advance value L_p is used for relaxing production budget constraint, a constant fraction $r_p \in [0, 1]$ is repaid to financial institution and spent as transaction cost due to capital market friction. The pre-shipment financing L_p is decided almost at the same time as product quantity but a little later, because pre-shipment financing amount is decided after product selling revenue $(\xi - bq_p)q_p$ is settled with buyer. For the ease of exposition, we write down the cash flow (in dollar) of the firm regarding to the model in chronological order: at the beginning of the production stage $t = 1$, firm has production budget worth β; and shortly after receiving pre-shipment finance the total amount becomes $(\beta + L_p)$; then firm produces and ships the products so that leaves $(\beta + L_p - yq_p)$ on hand; finally, at the end of the production stage $t = 2$, firm receives selling revenue minus the pre-shipment finance, therefore, end up has $(\xi - bq_p)q_p - (1 + r_p)L_p + (\beta + L_p - yq_p) = (\xi - bq_p)q_p - yq - r_pL_p + \beta$ on hand.

In practice, besides firm's decisions mentioned above, there are inter-plays among buyer(s), finance provider and the firm to get the deal done, for example, buyer(s) commitment on purchase quantity and finance provider's approval of pre-shipment finance and interest rate (referring to Reindorp et al. (2018) for endogenous buyer and exogenous finance provider problem; referring to Tang et al. (2018) for the model having all three parties endogenized). We assume away the supply chain effect and the bankruptcy risk management of the bank in order to answer our core research question: how much does the financing release the impact of production budget uncertainty on the capacity investment level and the profitability. As the firm is in the profit maximizing scenario, we define optimal profit the production stage as $\Pi_p^*(K, \xi, \beta)$ in which the firm decides production quantity q^* and pre-shipment finance principal L_p. The expected profit function in capacity investment stage is defined as $\Pi_p(K)$. Noting that 'p' denotes pre-shipment finance. The pre-shipment finance model production stage formulation is as follows:

$$\begin{aligned}
\pi_p^*\big(K,\xi,\beta\big) = \max_{q,L_p} \quad & (\xi - bq)q - yq - r_pL_p \\
\text{s.t.} \quad & 0 \le q \le \min\left\{K, \tfrac{\beta+L_p}{y}\right\} \\
& 0 \le (1+r_p)L_p \le (\xi - bq)q.
\end{aligned}$$

The objective function quantifies the firm's profit at the end of the time horizon. The first constraint represents both capacity constraint: the production volume cannot exceed the capacity level decided in the first stage; and financial (production budget) constraint: the production cost is less than the financing available which is released by pre-shipment financing. Therein, the pre-shipment finance adds flexibility on the budget constraint. The second constraint represents the financing limits that the firm should always be able to repay the loan. In capacity investment stage, the profit maximizing objective function is denoted by $\max_{K\ge 0} \Pi_p(K) = \max_{K\ge 0} \left\{ -\omega K + \mathbf{E}_{\tilde{\xi}}\big[\Pi_p^*\big(K,\tilde{\xi},\tilde{\beta}\big)\big]\right\}$, where the decision in concern is the capacity investment level K in anticipation of the production stage optimal profit. We presents the result in reverse chronological order since it is solved by backward induction. Therein, Theorem 3 describes the unique optimal solution to the production stage of PSF model and Theorem 4 provides unique optimal capacity investment level. In order to present the optimal production stage decisions, we define state space Ω_i^p $i = 3,4,5$ as follows:

$$\Omega_3^p(K,r_p) := \left\{ (\xi,\beta) : \begin{array}{l} \dfrac{\big(\xi-(1+r_p)y\big)y}{2b} \le \beta \le \dfrac{(\xi-y)y}{2b}, \\ \underline{\beta} < \beta \le \max\left\{\underline{\beta}, \min\{yK, \overline{\beta}\}\right\} \end{array} \right\},$$

$$\Omega_4^p(K,r_p) := \left\{ (\xi,\beta) : \begin{array}{l} (1+r_p)y < \xi < (1+r_p)y + 2bK, \\ \underline{\beta} < \beta \le \max\left\{\underline{\beta}, \min\left\{\dfrac{\big(\xi-(1+r_p)y\big)y}{2b}, \overline{\beta}\right\}\right\} \end{array} \right\} \text{ and}$$

$$\Omega_5^p(K,r_p) := \left\{ (\xi,\beta) : \begin{array}{l} \xi \ge (1+r_p)y + 2bK, \\ \underline{\beta} < \beta \le \max\left\{\underline{\beta}, \min\{yK, \overline{\beta}\}\right\} \end{array} \right\}.$$

Theorem 3 (Optimal Production Stage Strategy of Pre-shipment Finance Model)

Given the interest rate of financing $r_p \in (0,1)$, the firm's optimal production quantity q_p^* and optimal loan principal L_p are characterized by

$$\left(q_p^*(K,\xi,\beta),\ L_p^*(K,\xi,\beta)\right) = \begin{cases} (0,\ 0), & \text{if } (\xi,\beta) \in \Omega_0 \\ \left(\frac{\xi-y}{2b},\ 0\right), & \text{if } (\xi,\beta) \in \Omega_1(K) \\ (K,\ 0) & \text{if } (\xi,\beta) \in \Omega_2(K) \\ \left(\frac{\beta}{y},\ 0\right), & \text{if } (\xi,\beta) \in \Omega_3^p(K,r_p) \\ \left(\frac{\xi-(1+r_p)y}{2b},\ \frac{\left(\xi-(1+r_p)y\right)y}{2b} - \beta\right), & \text{if } (\xi,\beta) \in \Omega_4^p(K,r_p) \\ \left(K,\ yK-\beta\right), & \text{if } (\xi,\beta) \in \Omega_5^p(K,r_p) \end{cases}$$

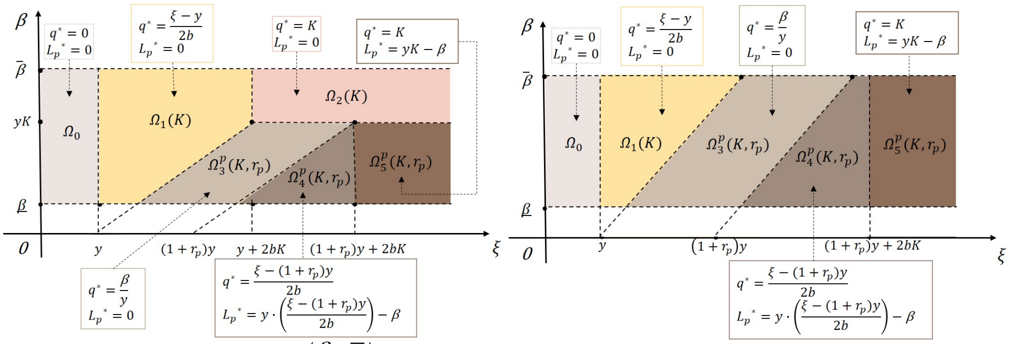
The optimal sales profit in the product market $\Pi_p^*(K,\xi,\beta)$ is characterized by

$$\pi_p^*(K,\xi,\beta) = \begin{cases} 0, & \text{if } (\xi,\beta) \in \Omega_0, \\ \dfrac{(\xi-y)^2}{4b}, & \text{if } (\xi,\beta) \in \Omega_1(K), \\ (\xi-y)K - bK^2, & \text{if } (\xi,\beta) \in \Omega_2(K), \\ (\xi-y)\beta/y - b\left(\beta/y\right)^2, & \text{if } (\xi,\beta) \in \Omega_3^p(K,r_p), \\ \dfrac{\left(\xi-(1+r_p)y\right)^2}{4b} + r_p\beta, & \text{if } (\xi,\beta) \in \Omega_4^p(K,r_p), \\ \left(\xi-(1+r_p)y\right)K - bK^2 + r_p\beta, & \text{if } (\xi,\beta) \in \Omega_5^p(K,r_p). \end{cases}$$

Recalling the optimal production quantity for basic model, the value takes the minimal among internal optimal $\frac{(\xi-y)^+}{2b}$, capacity constraint K and production budget constraint β/y. Now in Theorem 3, the only difference is that as production quantity takes value β/y according to above comparison, it could be optimal to use pre-shipment finance to weaken the negative effect of production budget shortage.

Specifically, comparing with the optimal production strategy for basic model in Theorem 1, this theorem indicates that, given same capacity level K, the production budget tightening region Ω_3 in basic model is replaced by union of Ω_i^p $i = 3,4,5$, where $\Omega_4^p \cup \Omega_5^p$ is the region with positive L_p^* and optimal production quantity are the same in region Ω_3 and Ω_3^p (see Figure 4.1 Panel (a)). This indicates that the pre-shipment finance is only valuable when the

Figure 4.1: State space (ξ, β) of optimal production strategy of pre-shipment finance model.



(a) When $K \in \left(\frac{\underline{\beta}}{y}, \frac{\bar{\beta}}{y}\right)$ (b) When $K \geq \frac{\bar{\beta}}{y}$

Note: from both Panel (a) and (b), state space becomes identical to resource-unconstrained benchmark case when $r_p \to 0$; and Panel (a) becomes identical to the Basic model when $r_p \to \infty$.

production budget is realized low. And comparing with Ω_i^p $i = 3, 4, 5$ themselves, it shows that higher the demand realization more valuable the pre-shipment financing. Figure 4.1 graphically represents how optimal production quantity and pre-shipment principal are allocated in state space as stated in Theorem 3. In the ideal case, higher the realization of demand, higher the optimal production quantity q_p^*, however, there are capacity constraint K and production budget constraint β/y get in the way of increasing the production volume. As a result, the intuition of applying pre-shipment finance is when demand is realized high and production budget is realized low, as what in $\Omega_4^p(K, r_p)$ and $\Omega_5^p(K, r_p)$. The intuitions behind production and finance decisions in terms of stage space partition are as follows.

$\Omega_3^p(K, r_p)$ is the region having relatively larger market size versus relatively not enough production budget, quantitatively $\beta \leq \frac{(\xi - y)y}{2b}$, and the production decision is bounded by available budget at the beginning of production-stage. What's interesting is, even though on hand budget are not enough, it is optimal to not apply pre-shipment finance. This plausible counter-intuitive phenomenon happens because the cost of this financing is also identified as following statement: using pre-shipment finance enhances unit production cost to $(1 + r_p)y$, but firm gets $r_p\beta$ amount of compensation after production. According to this statement, if carrying out pre-shipment finance, the optimal production quantity would be $\frac{\xi-(1+r_p)y}{2b}$, in addition, under state space $\Omega_3^p(K, r_p)$, inequality $\frac{\xi-(1+r_p)y}{2b} < \frac{\beta}{y}$ holds, current available budget are enough to produce $\frac{\xi-(1+r_p)y}{2b}$, it shows the needless of pre-shipment finance. So the firm optimally chooses to produce up to financial limit.

$\Omega_4^p(K, r_p)$, by definition, the optimal strategy is producing $\frac{\xi-(1+r_p)y}{2b}$ amount of products against on hand production budget plus requisite pre-shipment finance, because cost of production is larger than production budget, $\left(\frac{\xi-(1+r_p)y}{2b}\right)y > \beta$. Also $K > \frac{\xi-(1+r_p)y}{2b}$ indicates that the optimal quantity is attainable rather than bounded by K. In $\Omega_5^p(K, r_p)$, the optimal strategy is in line with which for $\Omega_4^p(K, r_p)$, except that as demand increases further, the internal optimal production amount is not attainable because the production is bounded by capacity K.

Adopting above optimal strategy in production stage, the corresponding first stage expected profit $\Pi_p(K)$ can be expended as $\Pi_p(K) =$

$$
\begin{aligned}
&-\omega K + \int_{\underline{\beta}}^{\overline{\beta}} \int_{y}^{y+2b\min\left\{K,\frac{\beta}{y}\right\}} \frac{(\xi-y)^2}{4b} f(\xi,\beta)d\xi d\beta \\
&+ \int_{\max\left\{\underline{\beta},\min\{yK,\overline{\beta}\}\right\}}^{\overline{\beta}} \int_{y+2bK}^{\infty} \left((\xi-y)K - bK^2\right) f(\xi,\beta)d\xi d\beta \\
&+ \int_{\underline{\beta}}^{\max\left\{\underline{\beta},\min\{yK,\overline{\beta}\}\right\}} \int_{y+\frac{2b\beta}{y}}^{(1+r_p)y+\frac{2b\beta}{y}} \left(\frac{(\xi-y)\beta}{y} - b\left(\frac{\beta}{y}\right)^2\right) f(\xi,\beta)d\xi d\beta \\
&+ \int_{\underline{\beta}}^{\max\left\{\underline{\beta},\min\{yK,\overline{\beta}\}\right\}} \int_{(1+r_p)y+\frac{2b\beta}{y}}^{(1+r_p)y+2bK} \left(\frac{\left(\xi-(1+r_p)y\right)^2}{4b} + r_p\beta\right) f(\xi,\beta)d\xi d\beta \\
&+ \int_{\underline{\beta}}^{\max\left\{\underline{\beta},\min\{yK,\overline{\beta}\}\right\}} \int_{(1+r_p)y+2bK}^{\infty} \left(\left(\xi-(1+r_p)y\right)K - bK^2 + r_p\beta\right) f(\xi,\beta)d\xi d\beta.
\end{aligned}
$$

Now we are in a position to characterize the firm's optimal investment level for PSF model.

Theorem 4 (Optimal Capacity Investment Level of PSF Model)

The optimal capacity investment level $K_p^*(\omega)$ is characterized as follows:

$$K_p^*(\omega) = \begin{cases} 0, & \text{if } \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y)^+\big] \le \omega; \\ K^U(\omega), & \text{if } \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y - 2b\underline{\beta}/y)^+\big] \le \omega < \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y)^+\big]; \\ K_p^B(\omega), & \text{if } 0 \le \omega < \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y - 2b\underline{\beta}/y)^+\big]; \end{cases}$$

where $K_p^B(\omega)$ is the unique solution of

$$\begin{aligned} \omega = &\int_{yK}^{\overline{\beta}} \int_{y+2bK}^{\infty} (\xi - y - 2bK) f(\xi,\beta) d\xi d\beta \\ &+ \int_{\underline{\beta}}^{\min\{yK,\overline{\beta}\}} \int_{(1+r_p)y+2bK}^{\infty} \big(\xi - (1+r_p)y - 2bK\big) f(\xi,\beta) d\xi d\beta. \end{aligned} \tag{4.1}$$

The above characterization of optimal capacity investment level by unit capacity cost is the same as the one for basic model. The only difference is that when $0 \le \omega < \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y - 2b\underline{\beta}/y)^+\big]$, the capacity level in PSF model should be larger than the one in basic model because of the leverage of financing. Following corollary shows the relation.

Corollary 3 *Define $\omega_p := \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - (1+r_p)y - 2b\overline{\beta}/y)^+\big]$, the capacity investment level of basic model and PSF model have following relation:*

1. $\frac{\overline{\beta}}{y} \ge K_p^B(\omega) > K^B(\omega) > \frac{\underline{\beta}}{y}$ *and* $\Pi_p(K_p^B) > \Pi_p(K^B)$*, when* $\omega_p \le \omega < \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y - \frac{2b\underline{\beta}}{y})^+\big]$*;*

2. $K_p^B(\omega) = K^U(\omega) - \frac{r_p y}{2b} > \frac{\overline{\beta}}{y} > K^B(\omega)$*, when* $0 < \omega < \omega_p$*.*

Noting that K^U is resource-unconstrained capacity level and K^B is resource-constrained capacity level.

This corollary shows that with pre-shipment finance, both the optimal capacity investment level and expected profit are higher than the case without the finance. In addition, In second part of Corollary 3, when the unit capacity investment level is very low, the firm optimally chooses a capacity level larger than the maximal possible level the production budget β can provide. The optimal capacity investment level K_p^B doesn't not depend on production budget uncertainty and is only less than resource-unconstrained capacity level a value $\frac{r_p y}{2b}$. This value is additional marginal production cost when applying pre-shipment finance.

4.3 Sensitivity Analyses for Pre-shipment Finance Model - Comparison with Basic Model

In this section, we investigate the role of pre-shipment finance by preforming sensitivity analysis in comparison with results in basic model. Similarly, we establish optimal capacity investment based on bivariate normal distribution assumption on $(\tilde{\xi}, \tilde{\beta})$ (referring to Assumption 2). Define $\omega_{\max}^p := \mathbf{E}_{\xi}\left[(\tilde{\xi} - y)^+ \Pr\left\{\tilde{\beta} > 0 \middle| \tilde{\xi}\right\}\right] + \mathbf{E}_{\xi}\left[(\tilde{\xi} - (1+r_p)y)^+ \Pr\left\{\tilde{\beta} \le 0 \middle| \tilde{\xi}\right\}\right]$, the optimal capacity investment strategy is simplified as $K_p^*(\omega) = \begin{cases} 0, & \text{if } \omega \ge \omega_{\max}^p; \\ K_p^B(\omega), & \text{if } 0 < \omega < \omega_{\max}^p \end{cases}$. An immediate comparison with basic model is that the unit capacity cost threshold $\omega_{\max}^p$ is larger than that of basic model $\omega_{\max}$. The unit capacity cost threshold for both models indicates that when unit capacity cost below this value the optimal capacity investment level is positive. This shows that having the pre-shipment finance, investing capacity is valuable in a wider range of capacity intensity. This result fits the intuition that larger assume $0 < \omega < \omega_{\max}^p$ in order to focus on the analysis of K_p^B. To this end, we first start with the effect of correlation between demand and production budget uncertainties on the capacity investment decision and the profitability of the firm.

Proposition 9 (Impact of correlation ρ - PSF Model)

1. *The optimal capacity investment K_p^B increases in ρ;*

2. *the corresponding optimal expected profit $\Pi_p(K_p^B)$ increases in ρ.*

With a higher ρ, there will be a higher possibility for high demand associated with high production budget, so that there will be a lower chance for the production constrained by production budget. Therefore, same as the basic model case in Proposition 1, both optimal capacity investment level and optimal profit are increasing in the correlation.

4.3.1 Sensitivity to Demand Variability

In contrast to the correlation effect where both pre-shipment finance model and basic model have same monotone sensitivity result, the effect of demand variability is more complicate. Define a unit capacity cost threshold $\omega_\xi := \left(1-\Phi\left(\frac{\frac{y(\mu_\xi-(1+r_p)y)}{2b}-\mu_\beta}{\sigma_\beta}\right)\right)r_p y$, we briefly summarize the effect of demand variability. When unit capacity cost is larger than ω_ξ, the effect of demand variability on optimal capacity level and profit for pre-shipment model have the same pattern as those for budget unconstrained benchmark model; otherwise, the effect of demand variability on both optimal capacity level and profit for pre-shipment model have same pattern as those for basic model.

We first introduce the effect on the optimal expected that is fully characterized.

Proposition 10 (Impact of σ_ξ on $\Pi_p(K_p^B)$ - PSF Model)

1. *When either $\rho \geq 0$ or $\omega > \omega_\xi$, $\Pi_p(K_p^B)$ increases in σ_ξ;*

2. *When $\rho < 0$ and $\omega \leq \omega_\xi$, there exists a threshold $\sigma_\xi^{\Pi_p}(\omega,\rho)$ such that $\Pi_p(K_p^B)$ decreases in σ_ξ if $\sigma_\xi < \sigma_\xi^{\Pi_p}(\omega,\rho)$ and $\Pi_p(K_p^B)$ increases in σ_ξ if $\sigma_\xi > \sigma_\xi^{\Pi_p}(\omega,\rho)$.*

As what shown in Proposition 10, when unit capacity cost is larger than ω_ξ, the optimal profit always increases in demand variability regardless of the value of the correlation; when the unit capacity cost is no more than ω_ξ, the result critically depends on the sign of the correlation, specifically, if the correlation is non-negative the expected profit increases in demand variability; otherwise, there exists a demand variability threshold, below which a higher demand variability decreases the profit and above which a higher demand variability increases the profit.

As benchmark model indicates the first best result that the firm can get if there is no production budget constraint, above result shows that a larger unit capacity cost leads to production budget less affecting optimal profit in pre-shipment finance model. The intuition is that, on the one hand, a higher unit capacity cost is weaken the impact of production budget uncertainty. As we look back the state space allocation in Figure 4.1 panel (a), the region production budget binding the optimal decisions $\Omega_3^p(K, r_p) \cup \Omega_4^p(K, r_p) \cup \Omega_5^p(K, r_p)$ is shrinking due to the decrease of K and a higher unit capacity cost decreases optimal capacity investment level in general, it explains why the impact of production budget uncertainty is weaken. On the other hand, with pre-shipment finance, the optimal production quantity can be as high as capacity level when demand is high even if the production budget is realized low. For this reason, the optimal expected profit in pre-shipment model behaves similar to that in the benchmark model.

Then we move on to the effect of demand variability on optimal capacity level in Proposition 11 and Conjecture 4, where the conjecture is supported by partial analytical result and extensive numerical analysis with $54,675$ numerical instances that will introduce in Section 4.4.1.

Proposition 11 (Impact of σ_ξ on K_p^B if $\rho \geq 0$ - PSF Model) *If $\rho \geq 0$, K_p^B increases in σ_ξ.*

When the correlation between demand and production budget is non-negative, the increasing trend of capacity investment level in demand variability. However, when the correlation is negative pre-shipment finance reshapes the sensitivity result.

Conjecture 4 (Impact of σ_ξ on K_p^B when $\rho < 0$ - PSF Model) *When $\rho < 0$,*

1. *if $\omega > \omega_\xi$, K_p^B increases in σ_ξ;*

2. *if $\omega \le \omega_\xi$, there exists a threshold $\sigma_\xi^{K_p}$ such that K_p^B decreases in σ_ξ if $\sigma_\xi < \sigma_\xi^{K_p}$ and K_p^B increases in σ_ξ if $\sigma_\xi > \sigma_\xi^{K_p}$.*

The conjecture is from partial analytical result we proved. Specifically, under condition $\rho < 0$, we are able to prove that $\lim_{\sigma_\xi \to 0} \frac{dK_p^B}{d\sigma_\xi} = 0$ when $\omega > \omega_\xi$ and $\lim_{\sigma_\xi \to 0} \frac{dK_p^B}{d\sigma_\xi} < 0$ when $\omega \le \omega_\xi$. In addition, $\frac{dK_p^B}{d\sigma_\xi} > 0$ when $\sigma_\xi \ge \overline{\sigma}_\xi^{K_p}$ where $\overline{\sigma}_\xi^{K_p}$ is the unique solution of $\frac{\partial \Pi_p(K)}{\partial K}\Big|_{K=\frac{\mu_\xi - y}{2b}} = 0$.

From Conjecture 4 and Proposition 11, we can also find that when unit capacity cost ω is larger than threshold ω_ξ, the impact of demand variability on optimal capacity level in PSF model has the same pattern as what in benchmark model; whereas when the unit capacity cost is no larger than the threshold, the impact of demand variability on capacity level in PSF model has the same pattern as what in basic model. The reason is the same as the one for Proposition 10.

In summary, with pre-shipment finance, both optimal capacity investment level and expected profit are closer to the first best case in benchmark model.

4.3.2 Sensitivity to Production Budget Variability

In this section, we investigate the impact of production budget variability on both capacity investment level K_p^B and profitability $\Pi_p(K_p^B)$. Firstly, we find that the impact of production budget variability on capacity investment level doesn't change pattern in comparison with that for basic model.

Proposition 12 (Impact of σ_β on K_p^B - PSF Model) *Define*

$$\begin{aligned}\omega_\beta^{K_p}(\rho) :=& \sigma_\xi \sqrt{(1-\rho^2)} \int_0^\infty \mathbf{E}\left[\left(\tilde{z}_2 - \left(\frac{y + 2b\mu_\beta/y - \mu_\xi - \rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\right)\right)^+\right]\phi(z_0)dz_0 \\ &+ \sigma_\xi \sqrt{(1-\rho^2)} \int_{-\infty}^0 \mathbf{E}\left[\left(\tilde{z}_2 - \left(\frac{(1+r_p)y + 2b\mu_\beta/y - \mu_\xi - \rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\right)\right)^+\right]\phi(z_0)dz_0,\end{aligned}$$

we have that it is not related with σ_β and $\omega_\beta^{K_p} > \omega_\beta$. Further define $\omega_\beta^{K_p}(-1) := \lim_{\rho \to -1} \omega_\beta^{K_p}$ and $\omega_\beta^{K_p}(1) := \lim_{\rho \to 1} \omega_\beta^{K_p}$. The sensitivity result is characterized by

1. *When* $\omega \in \left(0,\ \omega_{\beta}^{K_p}(-1)\right]$, K_p^B *increases in* σ_β*;*

2. *When* $\omega \in \left(\omega_{\beta}^{K_p}(-1), \omega_{\beta}^{K_p}(1)\right)$, *there exists a unique* $\rho_{\beta}^{K_p}(\omega)$ *that satisfies* $\omega_{\beta}^{K_p}(\rho_{\beta}^{K_p}(\omega)) = \omega$ *where* $\rho_{\beta}^{K_p}(\omega)$ *increases in* ω*. The sensitivity result is*

   (a) *under condition* $\rho > \rho_{\beta}^{K_p}(\omega)$, K_p^B *increases in* σ_β*;*

   (b) *under condition* $\rho = \rho_{\beta}^{K_p}(\omega)$, $K_p^B = \mu_\beta/y$ *which is not sensitive to* σ_β*;*

   (c) *under condition* $\rho < \rho_{\beta}^{K_p}(\omega)$, K_p^B *decreases in* σ_β*;*

3. *When* $\omega \in \left[\omega_{\beta}^{K_p}(1),\ \omega_{\max}^{p}\right)$, K_p^B *decreases in* σ_β*.*

We find the exact same result structure as what in basic model (Proposition 5), that is larger ω leads to optimal capacity investment level monotonically increasing in production budget variability, low ω leads to optimal capacity monotonically decreasing in budget variability; when ω is intermediate, the sensitivity result critically depends on ρ, specifically, capacity level increases in budget variability when ρ is high and decreases in budget variability when ρ is low. What different between the result for pre-shipment finance model and basic model is that they have different ω threshold to separate the increasing trend and the decreasing trend. The comparison between these ω thresholds is provided in following corollary.

Corollary 4 $\omega_{\beta}^{K_p}(\rho) > \omega_{\beta}^{K}(\rho)$.

Noting that $\omega_{\beta}^{K}(\rho)$ is the unit capacity cost threshold for the effect of budget variability on capacity level in basic model. This indicate that when pre-shipment finance is applied, under a larger range of unit capacity cost, the optimal capacity increasing in production budget variability. This suggests that the firm can invest in capacity more aggressively when there are external financing options available.

Then we introduce the impact of σ_β on the optimal expected profit of the PSF model, where it also has the same pattern as that of the basic model.

Proposition 13 (Impact of σ_β on $\Pi_p(K_p^B)$ - PSF Model) *Define* $\omega_\beta^{\Pi_p} := \mathbf{E}\Big[\Big(\tilde{\xi} - \big((1+r_p)y + 2b\mu_\beta/y\big)\Big)^+\Big]$,

1. *Under condition* $\rho \le 0$ *or* $\omega \ge \omega_\beta^{\Pi_p}$*, we have* $\Pi_p(K_p^B)$ *decreases in* σ_β*;*

2. *Under condition* $\rho > 0$ *and* $\omega < \omega_\beta^{\Pi_p}$*, there exists a unique* $\sigma_\beta^{\Pi_p}(\omega, \rho) \in (0, \frac{\rho\sigma_\xi y}{2b})$ *such that* $\Pi_p(K_p^B)$ *increases in* σ_β *when* $\sigma_\beta < \sigma_\beta^{\Pi_p}(\omega, \rho)$ *and* $\Pi_p(K_p^B)$ *decreases in* σ_β *when* $\sigma_\beta > \sigma_\beta^{\Pi_p}(\omega, \rho)$*.*

Proposition 13 shows that firm's optimal expected profit first increases then decreases in production budget variability only when unit capacity cost is low and the correlation is high positive; otherwise, the profit monotonically decreases in production budget variability.

Corollary 5 $\omega_\beta^{\Pi_p} < \omega_\beta^{K_p}(\rho) < \omega_{\max}^p$, $\forall \rho \in (-1, 1)$.

Above corollary is in parallel to Corollary 2 presenting the comparison between the unit capacity cost threshold $\omega_\beta^{K_p}(\rho)$ in Proposition 12 and $\omega_\beta^{\Pi_p}$ in Proposition 13 given the correlation ρ. Recalling that in Proposition 12, K_p^B increases in σ_β when unit capacity cost satisfies $\omega < \omega_\beta^{K_p}(\rho)$; K_p^B decreases in σ_β otherwise; in Proposition 13, optimal profit $\Pi_p(K_p^B)$ first increases then decreases in σ_β when unit capacity cost satisfies $\omega < \omega_\beta^{\Pi_p}$; $\Pi_p(K_p^B)$ decreases in σ_β otherwise. The intuition is that even though there is a wide range of $(\omega, \rho) \in \{(\omega, \rho) | \omega < \omega_\beta^{K_p}(\rho), \rho \in (-1, 1)\}$ under which a higher production budget variability increases optimal capacity level, only a proper subset of above (ω, ρ) set, denoted by $\{(\omega, \rho) | \omega < \omega_\beta^{\Pi_p}, \rho \in (0, 1)\}$, is the range under which higher production budget variability is profitable when production budget variability is less than a certain threshold.

4.4 Profitability-loss

In the previous section, we have characterized the role of pre-shipment finance in shaping the effects of demand and production budget uncertainties on capacity investment level and profitability. In this section, we further discuss the role of pre-shipment finance in reducing profitability loss occurred when the firm mistakenly chooses higher capacity investment level by ignoring uncertain production budget. Also, we discuss the role of pre-shipment finance in how it shapes the impact of the demand and production budget uncertainties on the profitability-loss.

We first introduce the function of profitability-loss. When the production budget uncertainty is ignored, the firm believes that the problem faced by himself is corresponding to the benchmark model, therefore, capacity level is miss-chosen as K^U, that is resource-unconstrained capacity level that the firm would choose if there is no production budget constraint. In fact, the pre-shipment model is corresponding to firm's problem and the expected profit is $\Pi_p(\cdot)$ and the optimal capacity level should be K_p^B. Briefly speaking, the firm would incur an optimality gap, since $K^U > K_p^B$ showed in Corollary 3. We define the rate of the profitability-loss due to miss choosing capacity level as $\Delta_r\Pi_p := \frac{\Pi_p(K_p^B)-\Pi_p(K^U)}{\Pi_p(K_p^B)}$. As both resource-unconstrained capacity level K^U and K_p^B critically depend on unit capacity investment cost ω and both capacity levels are positive only when limit unit capacity cost in the range $\omega \in \left(0,\ \omega_{\max}^p\right)$ where $\omega_{\max}^p = \mathop{\mathbf{E}}_{\tilde{\xi}}\left[(\tilde{\xi}-y)^+ \Pr\left\{\tilde{\beta} > 0 \middle| \tilde{\xi}\right\}\right] + \mathop{\mathbf{E}}_{\tilde{\xi}}\left[(\tilde{\xi}-(1+r_p)y)^+ \Pr\left\{\tilde{\beta} \le 0 \middle| \tilde{\xi}\right\}\right]$, we restrict our analysis in this range of unit capacity cost.

In analyzing the sensitivity, we investigate the impact of the uncertainty parameters (ρ, σ_ξ and σ_β) on the profitability-loss. We first present a proposition showing how profitability-loss changes in the correlation between demand and production budget uncertainties.

Proposition 14 (Impact of ρ on Profitability-loss - PSF Model) *Profitability-loss $\Delta_r\Pi_p$ is decreasing in ρ.*

In line with the effect of correlation on profitability-loss in basic model, Proposition 14 demonstrates that a higher the correlation between demand and production budget uncertainties results in a lower profitability-loss. The reason is that, K^U is not a function of the correlation. Base on this, we discussion the general insight. On the one hand, a higher correlation decreases the difference between product quantity provided by realized production budget $\frac{\beta}{y}$ and the internal optimal production quantity $\frac{\xi-y}{2b}$, thus, K_p^B could be closer to K^U as the correlation increases. On the other hand, we have that $\frac{\partial \Pi_p(K)}{\partial K}$ increases in ρ, meaning that as ρ increasing, the decreasing trend of $\Pi_p(K)$ on K when $K > K_p^B(\rho)$ tends to be more flat. As a result, the rate of profitability-loss between optimal expected profit and the profit without considering production-loss is shrink when ρ increases.

4.4.1 Numerical Study Design

The numerical study is conducted relying on $54,675$ numerical instances with wide range of parameter values extended around the baseline scenario. The only difference of numerical study design from which for basic model is that we add the pre-shipment finance interest rate where it takes value $r_p \in \{6\%, 12\%, 18\%\}$. The value of financing rates are picked based on offers quoted by finance provider, e.g. Paragon Financial Group charges interest rate '3% to 4% for the first 30 days; 1.25% every 10 days after that' and payment terms that buyers can postpone the payment date to the product manufacturing firm even after receiving the products, where the duration of delay payment depends on negotiation result between trade parties and the global or domestic trade regulation, ranging from 60 days to 180 days.

Recalling that the numerical design for the rest of parameters are: unit production cost is standardized as $b = 1$, $y = 1$; mean value of demand uncertainty has range $\mu_\xi \in \{14y, 16y, 18y\}$; mean value of production budget uncertainty takes $\frac{\mu_\beta}{y} = \frac{1}{2}\frac{(\mu_\xi - y)}{2b}$ for each μ_ξ; the demand variability $\sigma_\xi \in [2\%, 30\%]$ is the percentage of μ_ξ with 2%-unit increments, and similarly, production budget variability $\sigma_\beta \in [2\%, 30\%]$ is also picked as the percentage of μ_β, varying with 2%-unit increments; the correlation between demand and production budget uncertainties takes value $\rho \in \{-0.995, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.995\}$; lastly,

Table 4.1: Effect of demand variability σ_ξ on profitability loss $\Delta_r \Pi_p$.

$\Delta_r \Pi_p$ \ ρ / σ_ξ	-0.995	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	0.995
$2\%\mu_\xi$	0.009%	0.009%	0.009%	0.009%	0.009%	0.009%	0.009%	0.009%	0.009%
$4\%\mu_\xi$	0.008%	0.008%	0.008%	0.008%	0.008%	0.008%	0.008%	0.008%	0.008%
$6\%\mu_\xi$	0.008%	0.008%	0.008%	0.008%	0.008%	0.008%	0.008%	0.008%	0.008%
$8\%\mu_\xi$	0.008%	0.008%	0.008%	0.008%	0.008%	0.008%	0.008%	0.008%	0.008%
$10\%\mu_\xi$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$12\%\mu_\xi$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$14\%\mu_\xi$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$16\%\mu_\xi$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$18\%\mu_\xi$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$20\%\mu_\xi$	0.007%	0.007%	0.007%	0.007%	0.007%	0.006%	0.006%	0.006%	0.006%
$22\%\mu_\xi$	0.007%	0.007%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%
$24\%\mu_\xi$	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%
$26\%\mu_\xi$	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%
$28\%\mu_\xi$	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%
$30\%\mu_\xi$	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%

the unit capacity investment cost $\omega \in \{1, 5, 10\}$.

Similarly as the range of unit capacity cost in basic model picked, ω takes value $\{1, 5, 10\}$ because $\omega_{\max}^p$ is an unit capacity cost threshold such that $K^* = K_p^B$ when $\omega < \omega_{\max}^p$ and $K^* = 0$ otherwise. Moreover, $\omega_{\max}^p$ is a function of $r_p, b, y, \rho, \mu_\xi, \sigma_\xi, \mu_\beta$ and σ_β such that ω should be less than the lowest $\omega_{\max}^p$ for all instances of $\{r_p, b, y, \rho, \mu_\xi, \sigma_\xi, \mu_\beta, \sigma_\beta\}$. Denoting $\overline{\omega}_{\max}^p$ as the lowest value mentioned above, we can safely pick ω less than $\overline{\omega}_{\max}^p = 12.9999$.

4.4.2 Sensitivity to Demand Variability and Production Budget Variability

In this section, we investigate the impact of demand variability σ_ξ and the impact of production budget variability σ_β on the profitability loss. These impacts depend on the correlation between demand and production budget uncertainties and we conduct numerical analysis to uncover the effects.

Firstly, we discuss the impact of demand variability σ_ξ on the profitability-loss $\Delta_r \Pi_p \times 100\%$. The numerical experiment is across all scenarios fixing σ_ξ / μ_ξ and ρ and corresponding results are reported in Table 4.1.

From Table 4.1, we first observe that given fixed demand variability σ_ξ, profitability-loss is decreasing in correlation ρ, which verify the Proposition 7. However, the decreasing trend is flat, which indicates that the firm is more resilient to the correlation between uncertainties. Secondly, all entries are in the range of

Table 4.2: Effect of production budget variability σ_β on profitability loss $\Delta_r\Pi_p$

$\Delta_r\Pi_p$ \ ρ / σ_β	-0.995	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	0.995
$2\%\mu_\beta$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$4\%\mu_\beta$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$6\%\mu_\beta$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$8\%\mu_\beta$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$10\%\mu_\beta$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$12\%\mu_\beta$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$14\%\mu_\beta$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$16\%\mu_\beta$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$18\%\mu_\beta$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$20\%\mu_\beta$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$22\%\mu_\beta$	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%	0.007%
$24\%\mu_\beta$	0.007%	0.007%	0.007%	0.007%	0.006%	0.006%	0.006%	0.006%	0.006%
$26\%\mu_\beta$	0.007%	0.007%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%
$28\%\mu_\beta$	0.007%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%
$30\%\mu_\beta$	0.007%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%	0.006%

0.006% – 0.009% that is very small to be negligible. Noting that in Section 4.3.1, we obtain that the pre-shipment finance model is 'closer' to the benchmark model when unit capacity cost is larger than $\omega_\xi = \left(1 - \Phi\left(\frac{\frac{y(\mu_\xi-(1+r_p)y)}{2b}-\mu_\beta}{\sigma_\beta}\right)\right)r_p y$. As $\omega_\xi < y$ and the values of $\omega = 1, 5, 10$ are greater than y, the numerical analysis is in the high range that $\omega \gg \omega_\xi$.

Next, we investigate the impact of σ_β on Profitability-loss in by performing computational experiments. We numerically calculate the average profitability-loss $\Delta_r\Pi_p \times 100\%$ across all scenarios fixing σ_β/μ_β and ρ and report results in Table 4.2.

In the above, we find that the profitability-loss entries also take very small values either 0.006% or 0.007%, which also implies the resilience of the firm in face of uncertainties when using pre-shipment finance. In addition, we observe that a higher production budget variability decreases the profitability-loss. Overall, we obtain that pre-shipment finance significantly reduces the profitability-loss to a negligible value, because capacity investment level is low in a high unit capacity investment cost scenario so that pre-shipment finance helps firm dealing with production budget disruption events better in a way that the resource-unconstrained capacity investment level becomes a good estimation of optimal capacity level.

Chapter 5

Managing Physical Production Resource Disruption: Role of Procurement Hedging Contract

When production resource is physical such as raw material or component, pricing of material costs is directly linked to fluctuations in firm's production capability. Therefore, one of the way to manage the potential disruption of production resource would be to lock in prices for a production resource at a pre-determined fixed price through arriving at a fixed price procurement contract. In managing production resource disruption in the capacity investment stage, we apply procurement hedging contract, which is beneficial if it can allow the firm to avoid unnecessary fluctuations of physical production resource, e.g. raw material or component, in capacity investment spending. In this procurement hedging contract, the firm alters the distribution of production resource to manage the risk of production resource disruption.

We find that the partial hedging dominates full hedging and no-hedging when demand and production resource is positively correlated and the unit cost of investing capacity is low; no-hedging dominates when the positive correlation between demand and production resource is high and the unit capacity investment cost is even lower. We identify that optimal partial hedging decreases in both demand variability and the correlation, whereas it increases in production resource variability. We show that the profitability increases in demand variability, production resource variability and the correlation. Our numerical analysis shows that the profitability loss due to ignoring production resource shortage possibility in choosing capacity investment level is significant when demand variability is large, production resource variability is large and the correlation is low. And the profitability loss due to heuristically choosing always full hedging increases in both demand and production resource variability.

The organization of this chapter is as follows. We review the literature in Section 5.1. In Section 5.2, we introduce the formulation and assumptions of the model. In Section 5.3, we establish the optimal capacity investment policy and compare it with the basic model. Then we derive analytical results for the sensitivity of optimal capacity investment policy as well as the profitability in Section 5.4. Specifically, we answer the research question: How would the optimal capacity level, hedging strategy and profitability be impacted by the demand and production resource variability and the correlation between the two? Finally, in Section 5.5, we use extensive numerical study to answer second research question: under what conditions, the profitability-loss is significant? In analysing the profitability-loss, we not only consider the loss incurred due to miss-specifying capacity level due to ignoring production resource shortage possibility, but also study the profitability loss incurred because miss-specifying the hedging decision as heuristically always fully hedging all resource uncertainty.

5.1 Literature Review

Our study is related to the literature on financial hedging in operations. In this literature, researches study hedging contract decision of firm in conjunction with operational investments in a variety of settings. In the seminal work of Froot et al.

(1993), they point out that if capital market imperfections make external funds more costly than internal funds, they can generate a rationale for risk management. They show that when the investment opportunities and the availability of internal funds are correlated, it may become optimal for a firm to utilize partial hedging.

Gaur and Seshadri (2005) study the optimal hedging for a risk-averse newsvendor with any given inventory level, based on the empirically proven assumption that the newsvendor's demand is correlated with the price of a financial asset. They demonstrate the effectiveness of financial hedging when one can discover tradable market assets partially correlated with market demand. Chod et al. (2010) extend the work of Gaur and Seshadri (2005) by implicitly characterizing the optimal multidimensional capacity investment and focusing on the complementarity/substitution effect between the operational (postponement and product) flexibility and financial hedging. Ding et al. (2007) study the optimal policies for capacity investment and hedging on currency exchange rates for a risk-averse multinational newsvendor and find that the futures contract is the optimal hedge. They establish the value of the joint use of the operational hedge ("allocation" option) and the financial hedge, and understanding their effects on a risk-averse firm's capacity decisions and performance. Kouvelis et al. (2013) study how to manage commodity risks (price and consumption volume) via physical inventory and financial hedge in a multiperiod problem (with an interperiod utility function) for a risk-averse firm procuring a storable commodity from a spot market at a random price and a long-term supplier at a fixed price, where demand and spot price of the commodity uncertainties are correlated. They contribute to the literature and practice for managing storable commodity risks with tractable optimal policies. Goel and Tanrisever (2017) study a firm that procures an input commodity to produce an output commodity to sell to the end retailer. They consider the stochastic dynamics of both input and output prices, and contribute to the literature with the associated effect of their correlation on hedging decisions. Turcic et al. (2015) explores the merits of hedging stochastic input costs (i.e., reducing the risk of adverse changes in costs) in a decentralized, risk-neutral supply chain. They address the role that supply chains play in shaping corporate financial policies. Kouvelis et al. (2019) study hedging cash-flow risks in a supply chain where firms invest internal funds to improve production efficiencies. They contribute to this literature by exploring how the vertical interactions of firms in a supply chain affect their cash hedging strategies.

In our research, we consider hedging production resource uncertainty that is correlated with demand. We study how capacity investment level and hedging contract choice are affected by demand and production resource uncertainties.

5.2 Notations and Assumptions of Hedging Model

Consider a product manufacturing firm that faces a two-stage in capacity investment/product manufacturing decision. We extend the analysis with a optimal hedging decision in first stage. In the first stage, the firm chooses capacity investment level as well as the hedge ratio in anticipation of demand and raw material or component supply (production resource) uncertainty. The hedging decision is modelled as linear hedging strategy and the corresponding physical resource becomes

$$h\mu_\beta + (1-h)\tilde{\beta},$$

where $h \in [0,1]$ is the 'hedge ratio' chosen by the firm. Specifically, the firm locks in a value for a proportion of physical production resource at production stage. Define that $h = 0$ as *no hedging*, $h = 1$ as *full hedging* and $h \in (0,1)$ as *partial hedging*. In the second stage, the demand of the end product and the revised availability of physical resource are realized, and the firm chooses the production quantity constrained by the minimum of capacity level and physical resource.

Let $\Pi_h(K,h)$ denote the expected profit of the firm at a capacity level K and hedge ratio $h \in [0,1]$ in capacity investment stage and let $\pi_h^*\big(K,h,\xi,\beta\big)$ denote the firm's the optimal profit at production stage given a capacity level K, hedge ratio h and realization of uncertainties, the formulation of the problem is as follows. The capacity investment stage problem is

$$\max_{K\geq 0, h\in[0,1]} \Pi_h(K,h) = \max_{K\geq 0, h\in[0,1]} -\omega K + \mathop{\mathbf{E}}_{(\xi,\beta)}\left[\pi_h^*(K,h,\tilde{\xi},\tilde{\beta})\right], \tag{5.1}$$

where $\pi_h^*\big(K,h,\xi,\beta\big)$ is obtained by solving the following production stage problem:

$$\begin{aligned} \pi_h^*\big(K,h,\xi,\beta\big) = \max_q \quad & (\xi - bq)q - yq \\ \text{s.t.} \quad & 0 \leq q \leq \min\Big\{K, \big(h\mu_\beta + (1-h)\beta\big)/y\Big\}. \end{aligned}$$

For short, we name the formulation of the problem with procurement hedging contract as *Hedging Model*.

5.3 Characterization of the Optimal Strategy for Hedging Model

In this section, we characterize the firm's optimal capacity investment, hedge ratio and production decisions. The problem is solved using backward induction, therefore, we first introduce optimal production decision given capacity K and hedge ratio h. We partition the state space $(\xi,\beta) \in \big\{(\xi,\beta) : \xi > 0, \beta \in [\underline{\beta}, \overline{\beta}]\big\}$ into four regions, Ω_0 and Ω_i^h, $i = 1,2,3$, each of which corresponds to different optimal production quantity scenarios that we will show in Theorem 5. The formal definitions of these regions are

$$\Omega_1^h(K,h) := \left\{(\xi,\beta) : y < \xi < y + 2b\min\left\{\frac{h\mu_\beta + (1-h)\beta}{y}, K\right\},\ \underline{\beta} < \beta \leq \overline{\beta}\right\},$$
$$\Omega_2^h(K,h) := \left\{(\xi,\beta) : \xi \geq y + 2bK; \max\left\{\underline{\beta}, \min\left\{\frac{yK - h\mu_\beta}{1-h}, \overline{\beta}\right\}\right\} < \beta \leq \overline{\beta}\right\},$$
$$\Omega_3^h(K,h) := \left\{(\xi,\beta) : \xi \geq y + \frac{2b\big(h\mu_\beta + (1-h)\beta\big)}{y},\ \underline{\beta} \leq \beta < \max\left\{\underline{\beta}, \min\left\{\frac{yK - h\mu_\beta}{1-h}, \overline{\beta}\right\}\right\}\right\},$$

and the complementary set of $\Omega_1^h(K,h) \cup \Omega_2^h(K,h) \cup \Omega_3^h(K,h)$ is Ω_0, which is the region of not investing in capacity in basic model. With the analysis of production constraints, the optimal production strategy is characterized in Theorem 5 and the expected profit $\Pi_h(K,h)$ under optimal allocation is obtained. In Theorem 6, the optimal resource capacity and hedge ratio is obtained from $\Pi_h(K,h)$ and characterized by unit capacity cost ω.

Theorem 5 (Optimal Production Strategy of Hedging Model) *Given capacity K, hedge ratio h and realizations of random variables ($\tilde{\xi} = \xi, \tilde{\beta} = \beta$), the optimal production level*

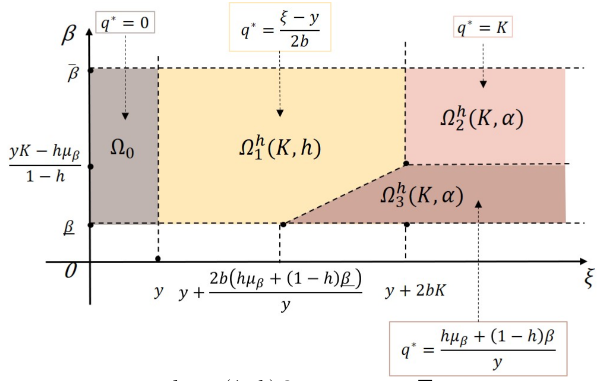
$$q_h^*(K,h,\xi,\beta) = \begin{cases} 0, & \text{if } (\xi,\beta) \in \Omega_0, \\ \dfrac{\xi - y}{2b}, & \text{if } (\xi,\beta) \in \Omega_1^h(K,h), \\ K, & \text{if } (\xi,\beta) \in \Omega_2^h(K,h), \\ \dfrac{h\mu_\beta + (1-h)\beta}{y}, & \text{if } (\xi,\beta) \in \Omega_3^h(K,h), \end{cases}$$

and the corresponding optimal expected profit is

$$\pi_h^*(K,h,\xi,\beta) = \begin{cases} 0, & \text{if } (\xi,\beta) \in \Omega_0, \\ \frac{(\xi-y)^2}{4b}, & \text{if } (\xi,\beta) \in \Omega_1^h(K,h), \\ (\xi - y)K - bK^2, & \text{if } (\xi,\beta) \in \Omega_2^h(K,h), \\ \frac{\big(h\mu_\beta+(1-h)\beta\big)(\xi-y)}{y} - b\big(\frac{h\mu_\beta+(1-h)\beta}{y}\big)^2, & \text{if } (\xi,\beta) \in \Omega_3^h(K,h). \end{cases}$$

Figure 5.1 is corresponding to Figure 3.2 panel (b) in basic model, which illustrates a case that production resource could be constraining and the hedge ratio $h \in (0,1)$. The reason is the given capacity level is in between of minimal and maximal production quantity that supported by production resource realization with revised distribution. Consequently, in a large demand realization scenario when internal optimal production quantity is not attainable, $\Omega_2^h(K,h)$ and $\Omega_3^h(K,h)$ indicate that either the capacity constraint is tighter or the resource constraint is, respectively. Particularly, when $(\xi,\beta) \in \Omega_3^h(K,h)$, production resource is tighter than capacity level such that the optimal production strategy is to use up all production resource. In the case $h = 0$, the optimal production decision allocation falls back to the state space division for optimal production decision in basic model. Finally, in the case $h = 1$, production resource always equals to it mean value μ_β, so the optimal production decision can be irrelevant

Figure 5.1: State space of optimal production strategy of hedging model



(a) When $K \in \left(\frac{h\mu_\beta + (1-h)\underline{\beta}}{y}, \frac{h\mu_\beta + (1-h)\overline{\beta}}{y}\right)$ and $h \in (0,1)$

Note: Since the realization of $\tilde{\beta}$ is bounded, the state space is characterized by the value of K. When $K \in \left(0, \frac{h\mu_\beta + (1-h)\underline{\beta}}{y}\right]$, the graphical state space division of optimal production decision of Hedging model is identical to Figure 3.2 panel (a). Also $K > \frac{h\mu_\beta + (1-h)\overline{\beta}}{y}$ is not feasible, because the maximal amount of product using up physical production resource is $\frac{h\mu_\beta + (1-h)\overline{\beta}}{y}$ and it's not profitable to invest in capacity larger than this.

to capacity level if $K \geq \mu_\beta / y$ or can fall back to the benchmark case when $K < \mu_\beta / y$.

We now proceed to characterize the optimal capacity investment level and hedge ratio in capacity investment stage. The optimization problem in this stage follows from Equation 5.1 by substituting $\underset{(\xi,\beta)}{\mathbf{E}}\left[\pi^*(K, \tilde{\xi}, \tilde{\beta})\right]$ by the characterization given by Theorem 1. To be specific, the expected profit as a function of capacity level under optimal allocation is in following form: According to Theorem 5, the expected profit in capacity investment stage when $0 \leq h < 1$ is denoted by $\Pi_h(K, h) =$

$$
\begin{aligned}
&-\omega K + \int_{\underline{\beta}}^{\overline{\beta}} \int_{y}^{y+2b\min\left\{\frac{h\mu_\beta + (1-h)\beta}{y}, K\right\}} \frac{(\xi - y)^2}{4b} f(\xi,\beta)\, d\xi\, d\beta \\
&+ \int_{\underline{\beta}}^{\overline{\beta}} \int_{y+2b\min\left\{\frac{h\mu_\beta + (1-h)\beta}{y}, K\right\}}^{\infty} \Bigg((\xi - y)\min\left\{\frac{h\mu_\beta + (1-h)\beta}{y}, K\right\} \\
&\qquad\qquad - b\left(\min\left\{\frac{h\mu_\beta + (1-h)\beta}{y}, K\right\}\right)^2 \Bigg) f(\xi,\beta)\, d\xi\, d\beta,
\end{aligned}
\tag{5.2}
$$

the expected profit when $h = 1$ is

$$\begin{aligned}\Pi_h(K,1) =& -\omega K + \int_y^{y+2b\min\left\{\frac{\mu_\beta}{y},K\right\}} \frac{(\xi-y)^2}{4b} f_\xi(\xi)d\xi \\ &+ \int_{y+2b\min\left\{\frac{\mu_\beta}{y},K\right\}}^{\infty} \left((\xi-y)\min\left\{\frac{\mu_\beta}{y},K\right\} - b\left(\min\left\{\frac{\mu_\beta}{y},K\right\}\right)^2\right) f_\xi(\xi)d\xi \\ =&\Pi_u\left(\min\{K,\mu_\beta/y\}\right),\end{aligned} \tag{5.3}$$

where $\Pi_u(\cdot)$ is expected profit function of the resource unconstrained benchmark model.

Then we obtain optimal capacity investment level and hedge ratio using above equations. The optimal strategy is characterized by unit capacity cost and the covariance between demand and production resource uncertainties.

Theorem 6 (Optimal Capacity Investment Strategy of Hedging Model) *Define set* $H = \left\{h \middle| \max\left\{0, \frac{yK^U-\underline{\beta}}{\mu_\beta-\underline{\beta}}\right\} \le h \le 1\right\}$. *The optimal strategy is characterized in two cases:*

1. *when* $\mathbf{Cov}\left(\tilde{\beta}, (\tilde{\xi}-y-\frac{2b\mu_\beta}{y})^+\right) \le 0$, *the optimal strategy is* $\left(K_h^*, h^*\right) =$

$$\begin{cases}(0,0), & \text{if } \mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi}-y)^+\right] \le \omega; \\ \left(K^U(\omega), h^* = H\right), & \text{if } \mathbf{E}_{\tilde{\xi}}\left[\left(\tilde{\xi}-y-\dfrac{2b\mu_\beta}{y}\right)^+\right] \le \omega < \mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi}-y)^+\right]; \\ (\mu_\beta/y, 1), & \text{if } 0 < \omega \le \mathbf{E}_{\tilde{\xi}}\left[\left(\tilde{\xi}-y-\dfrac{2b\mu_\beta}{y}\right)^+\right];\end{cases}$$

2. *when* $\mathbf{Cov}\left(\tilde{\beta}, (\tilde{\xi}-y-\frac{2b\mu_\beta}{y})^+\right) > 0$, *we define* ω_h *as the unique solution of* $\int_{\underline{\beta}}^{yK^B(\omega)} (\beta-\mu_\beta)\mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi}-y-\frac{2b\beta}{y})^+ \middle| \beta\right] f_\beta(\beta)d\beta = 0$, *where* ω_h *is positive when* $\mathbf{Cov}\left(\tilde{\beta}, (\tilde{\xi}-y-\frac{2b\tilde{\beta}}{y})^+\right) > 0$. *The optimal strategy characterized as*

follows $\left(K_h^*, h^*\right) =$

$$\begin{cases} (0,0), & \textit{if}\ \underset{\tilde{\xi}}{\mathbf{E}}\big[(\tilde{\xi}-y)^+\big] \le \omega; \\ \Big(K^U(\omega), h^* = H\Big), & \textit{if}\ \underset{\tilde{\xi}}{\mathbf{E}}\left[\left(\tilde{\xi}-y-\frac{2b\mu_\beta}{y}\right)^+\right] \le \omega < \underset{\tilde{\xi}}{\mathbf{E}}\big[(\tilde{\xi}-y)^+\big]; \\ \left(\frac{\mu_\beta}{y}, 1\right), & \textit{if}\ \underset{\tilde{\xi},\tilde{\beta}}{\mathbf{E}}\left[\mathbb{1}_{\{\tilde{\beta}\ge\mu_\beta\}}\left(\tilde{\xi}-y-\frac{2b\mu_\beta}{y}\right)^+\right] \le \omega < \underset{\tilde{\xi}}{\mathbf{E}}\left[\left(\tilde{\xi}-y-\frac{2b\mu_\beta}{y}\right)^+\right]; \\ \big(K_h^B(\omega), h^B(\omega)\big), & \textit{if}\ \max\{\omega_h, 0\} < \omega < \underset{\tilde{\xi},\tilde{\beta}}{\mathbf{E}}\left[\mathbb{1}_{\{\tilde{\beta}\ge\mu_\beta\}}(\tilde{\xi}-y-\frac{2b\mu_\beta}{y})^+\right]; \\ \big(K^B(\omega), 0\big), & \textit{if}\ 0 < \omega \le \max\{\omega_h, 0\} \end{cases}$$

where $\big(K_h^B(\omega), h^B(\omega)\big)$ *is uniquely solved by*

$$\begin{cases} \omega = \displaystyle\int_{\frac{yK_h^B - h^B\mu_\beta}{1-h^B}}^{\overline{\beta}} \mathbf{E}\big[(\xi - y - 2bK_h^B)^+ \big| \beta\big] f_\beta(\beta) d\beta \\ 0 = \displaystyle\int_{\underline{\beta}}^{\frac{yK_h^B - h^B\mu_\beta}{1-h^B}} (\mu_\beta - \beta)\mathbf{E}\left[\left(\tilde{\xi} - y - \frac{2b\big(h^B\mu_\beta + (1-h^B)\beta\big)}{y}\right)^+ \Big| \beta\right] f_\beta(\beta) d\beta \end{cases}$$

and $\mathbf{Cov}\big(\tilde{\beta}, (\tilde{\xi} - y - \frac{2b\tilde{\beta}}{y})^+\big) > \mathbf{Cov}\big(\tilde{\beta}, (\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\big) > 0.$

Theorem 6 shows that the optimal capacity investment and hedge ratio is characterized into two cases by a covariance threshold $\mathbf{Cov}\big(\tilde{\beta}, (\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\big)$. Within two cases, the optimal strategy is critically decided by the value of unit capacity cost ω. In both cases, the firm should not invest in the project when the unit capacity cost is too high; and when unit capacity is in a slightly low range, the firm can optimally invest in a resource-unconstrained capacity investment level and at the same time the optimal hedge ratio can take any value within the range $\left[\max\left\{0, \frac{yK^U - \underline{\beta}}{\mu_\beta - \underline{\beta}}\right\}, 1\right]$. Also in both cases, optimal strategy is $\left(K_h^*, h^*\right) = \left(\frac{\mu_\beta}{y}, 1\right)$ when unit capacity cost is in the range $\underset{\tilde{\xi},\tilde{\beta}}{\mathbf{E}}\left[\mathbb{1}_{\{\tilde{\beta}\ge\mu_\beta\}}(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\right] \le \omega < \underset{\tilde{\xi}}{\mathbf{E}}\left[\left(\tilde{\xi} - y - \frac{2b\mu_\beta}{y}\right)^+\right]$. As summarized above, the optimal strategy for both cases are identical when $\omega \ge \underset{\tilde{\xi},\tilde{\beta}}{\mathbf{E}}\left[\mathbb{1}_{\{\tilde{\beta}\ge\mu_\beta\}}(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\right]$.

Then we discuss the optimal capacity investment and hedging strategy when $omega < \underset{\tilde{\xi},\tilde{\beta}}{\mathbf{E}}\left[\mathbb{1}_{\{\tilde{\beta}\ge\mu_\beta\}}(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\right]$. In the case 1 where the covariance $\mathbf{Cov}\big(\tilde{\beta}, (\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\big)$ is non-positive, the optimal strategy is simply $\left(K_h^*, h^*\right) =$

$\left(\frac{\mu_\beta}{y}, 1\right)$. Whereas in case 2, partial hedge is optimal when unit capacity cost is larger than $\max\{0, \omega_h\}$ and no hedge is optimal when unit capacity is no more than this value. Noting that ω_h is positive only when another covariance threshold $\mathbf{Cov}\big(\tilde{\beta}, (\tilde{\xi} - y - \frac{2b\tilde{\beta}}{y})^+\big) > 0$.

Corollary 6 *When ω satisfies $\max\{\omega_h, 0\} < \omega < \mathop{\mathbf{E}}_{\tilde{\xi},\tilde{\beta}}\Big[\mathbb{1}_{\{\tilde{\beta}\geq\mu_\beta\}}(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\Big]$, we have $K^B(\omega) \leq K_h^B(\omega)$. As resource constrained capacity level $K^B(\omega)$ is the unique solution of $\omega = \int_{yK^B}^{\overline{\beta}} \int_{y+2bK^B}^{\infty} (\xi - y - 2bK^B) f(\xi, \beta) d\xi d\beta$, we have that $K^B(\omega) = K_h^B(\omega)$ when $h^B = 0$.*

5.4 Sensitivity Analysis

In this section, we analyse the effect of demand variability, production resource variability and the correlation between demand and production resource uncertainties on optimal capacity investment level, optimal hedge ratio and the corresponding optimal profit. In analysing these sensitivities, we make the same distribution assumption on demand and production resource uncertainties, that is, $(\tilde{\xi}, \tilde{\beta})$ follows a bivariate normal distribution with mean vector $(\mu_\xi, \mu_\beta)'$ and variance-covariance matrix $\begin{pmatrix} \sigma_\xi^2 & \rho\sigma_\xi\sigma_\beta \\ \rho\sigma_\xi\sigma_\beta & \sigma_\beta^2 \end{pmatrix}$, where the correlation coefficient $\rho \in (-1, 1)$. Due to the property of normal distribution, the range of production resource realization becomes $(-\infty, \infty)$.

We firstly rewrite the optimal capacity investment and hedge strategy for the case that demand and production resource uncertainties follows a bivariate normal distribution.

Proposition 15 *With bivariate normal distribution assumption, the covariance thresholds in Theorem 6 are transformed as $\mathbf{Cov}\big(\tilde{\beta}, (\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\big) = \rho\sigma_\beta\sigma_\xi\Phi\big(\frac{\mu_\xi - y - 2b\mu_\beta/y}{\sigma_\xi}\big)$ and $\mathbf{Cov}\big(\tilde{\beta}, (\tilde{\xi} - y - \frac{2b\tilde{\beta}}{y})^+\big) = \sigma_\beta(\rho\sigma_\xi - \frac{2b\sigma_\beta}{y})\Phi\Big(\frac{\mu_\xi - y - 2b\mu_\beta/y}{\sqrt{(\frac{2b\sigma_\beta}{y} - \rho\sigma_\xi)^2 + \sigma_\xi^2(1-\rho)^2}}\Big)$. The opti-*

mal strategy with bivariate normal distribution assumption is characterized by following two cases:

1. *when* $\rho \le 0$*, the optimal strategy is* $\left(K_h^*, h^*\right) =$

$$\begin{cases} (0,0), & \text{if } \mathop{\mathbf{E}}_{\tilde{\xi}}\left[(\tilde{\xi} - y)^+\right] \le \omega; \\ \left(K^U(\omega), 1\right), & \text{if } \mathop{\mathbf{E}}_{\tilde{\xi}}\left[\left(\tilde{\xi} - y - \dfrac{2b\mu_\beta}{y}\right)^+\right] \le \omega < \mathop{\mathbf{E}}_{\tilde{\xi}}\left[(\tilde{\xi} - y)^+\right]; \\ (\mu_\beta / y, 1), & \text{if } 0 < \omega \le \mathop{\mathbf{E}}_{\tilde{\xi}}\left[\left(\tilde{\xi} - y - \dfrac{2b\mu_\beta}{y}\right)^+\right]; \end{cases}$$

2. *when* $\rho > 0$*, we define* ω_h *as the unique solution of* $\int_{-\infty}^{yK^B(\omega)} (\beta - \mu_\beta) \mathop{\mathbf{E}}_{\tilde{\xi}}\left[(\tilde{\xi} - y - \frac{2b\beta}{y})^+ \middle| \beta\right] f_\beta(\beta) d\beta = 0$*, where* ω_h *is positive only when* $\rho > \frac{2b\sigma_\beta}{y\sigma_\xi}$*. We have optimal strategy characterized as follows* $\left(K_h^*, h^*\right) =$

$$\begin{cases} (0,0), & \text{if } \mathop{\mathbf{E}}_{\tilde{\xi}}\left[(\tilde{\xi} - y)^+\right] \le \omega; \\ \left(K^U(\omega), 1\right), & \text{if } \mathop{\mathbf{E}}_{\tilde{\xi}}\left[\left(\tilde{\xi} - y - \dfrac{2b\mu_\beta}{y}\right)^+\right] \le \omega < \mathop{\mathbf{E}}_{\tilde{\xi}}\left[(\tilde{\xi} - y)^+\right]; \\ \left(\dfrac{\mu_\beta}{y}, 1\right), & \text{if } \mathop{\mathbf{E}}_{\tilde{\xi},\tilde{\beta}}\left[\mathbb{1}_{\{\tilde{\beta} \ge \mu_\beta\}}\left(\tilde{\xi} - y - \dfrac{2b\mu_\beta}{y}\right)^+\right] \le \omega < \mathop{\mathbf{E}}_{\tilde{\xi}}\left[\left(\tilde{\xi} - y - \dfrac{2b\mu_\beta}{y}\right)^+\right]; \\ \left(K_h^B(\omega), h^B(\omega)\right), & \text{if } \max\{0, \omega_h\} < \omega < \mathop{\mathbf{E}}_{\tilde{\xi},\tilde{\beta}}\left[\mathbb{1}_{\{\tilde{\beta} \ge \mu_\beta\}}(\tilde{\xi} - y - \dfrac{2b\mu_\beta}{y})^+\right]; \\ \left(K^B(\omega), 0\right), & \text{if } 0 < \omega \le \max\{0, \omega_h\}, \end{cases}$$

where $\left(K_h^B(\omega), h^B(\omega)\right)$ *is uniquely solved by*

$$\begin{cases} \omega = \displaystyle\int_{\frac{yK_h^B - h^B\mu_\beta}{1 - h^B}}^{\infty} \mathbf{E}\left[(\xi - y - 2bK_h^B)^+ \middle| \beta\right] f_\beta(\beta) d\beta \\ 0 = \displaystyle\int_{-\infty}^{\frac{yK_h^B - h^B\mu_\beta}{1 - h^B}} (\mu_\beta - \beta) \mathbf{E}\left[\left(\tilde{\xi} - y - \dfrac{2b\left(h^B\mu_\beta + (1 - h^B)\beta\right)}{y}\right)^+ \middle| \beta\right] f_\beta(\beta) d\beta. \end{cases}$$

Noting that the covariance thresholds is calculated as a function with the sign decided by the value of correlation. Also, due to the range of production resource realization becomes $\tilde{\beta} \in (-\infty, \infty)$ such that the optimal hedge ratio when $\mathop{\mathbf{E}}_{\tilde{\xi}}\left[\left(\tilde{\xi} - y - \frac{2b\mu_\beta}{y}\right)^+\right] \le \omega < \mathop{\mathbf{E}}_{\tilde{\xi}}\left[(\tilde{\xi} - y)^+\right]$ does not take any value within a range, since on the one hand, there is no minimal amount of production resource guaranteed for production process, on the other hand, a increases in hedge ratio increases the marginal profit.

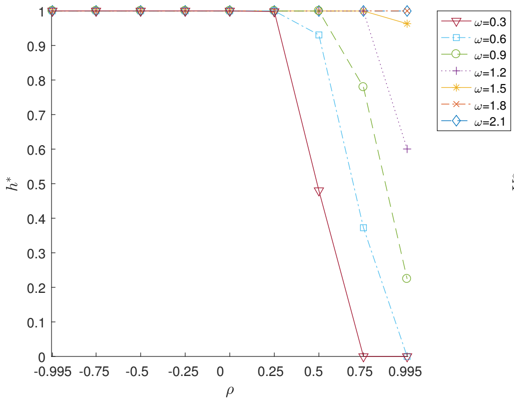
Proposition 15 shows that the full hedge is optimal when the correlation between demand and production resource uncertainties is non-positive. It is possible for partial hedge being optimal only when the correlation is positive and the unit capacity cost is small enough. Beside the no capacity investment case, no hedging is optimal only when correlation is high enough and the unit capacity cost is even smaller and the optimal expected profit in this case equals to the optimal profit in basic model, that is $\Pi_h(K^B(\omega), 0) = \Pi_h(K^B(\omega))$. Then, we further discuss how does ρ shape the optimal expected profit.

Proposition 16 (Impact of ρ on $\Pi_h(K_h^*, h^*)$ - Hedging Model) *When $\rho > 0$ and* $0 < \omega < \mathop{\mathbf{E}}_{\tilde{\xi},\tilde{\beta}}\left[\mathbb{1}_{\{\tilde{\beta}\geq\mu_\beta\}}(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\right]$, $\Pi_h(K_h^*, h^*)$ *strictly increases in ρ; otherwise, $\Pi_h(K_h^*, h^*)$ is not sensitive to ρ.*

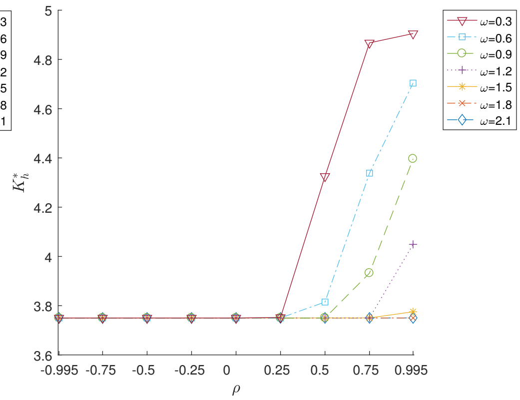
Proposition 16 indicates that when partial hedge or no hedge is optimal, the profit increases in the correlation between demand and production resource uncertainties. For the effect of ρ on optimal expected profit $\Pi_h(K_h^*, h^*)$ the result is in line with the the effect on optimal profit of basic model, intuitively, as the correlation increases, high (low) demand is more likely to be matched by high (low) production resource, and the production resource is less constraining for the high demand to be met. On expectation, this higher correlation brings production stage optimal structure closer to the resource-unconstrained case thus the higher correlation is more beneficial for the firm.

For the sensitivity analysis for optimal capacity investment level and hedge ratio. We've discussed how the resource unconstrained capacity level K^U and resource constrained capacity level K^B change in uncertainty parameters. Now what remain unexplored are how optimal partial hedge ratio $h^B(\omega)$ and corresponding optimal capacity investment level $K_h^B(\omega)$ affected by the correlation, demand variability and production resource variability. As their sensitivity results

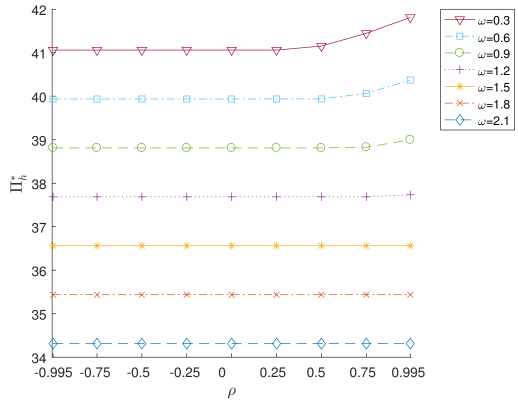
Figure 5.2: Effect of the correlation between demand and production resource uncertainties ρ



(a) Effect of ρ on h^*



(b) Effect of ρ on K_h^*



(c) Effect of ρ on $\Pi_h(K_h^*, h^*)$

Figures are depicted using baseline scenario $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\beta = 16\%\mu_\beta$, $\sigma_\xi = 16\%\mu_\xi$ and $\rho \in \{-0.995, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.995\}$

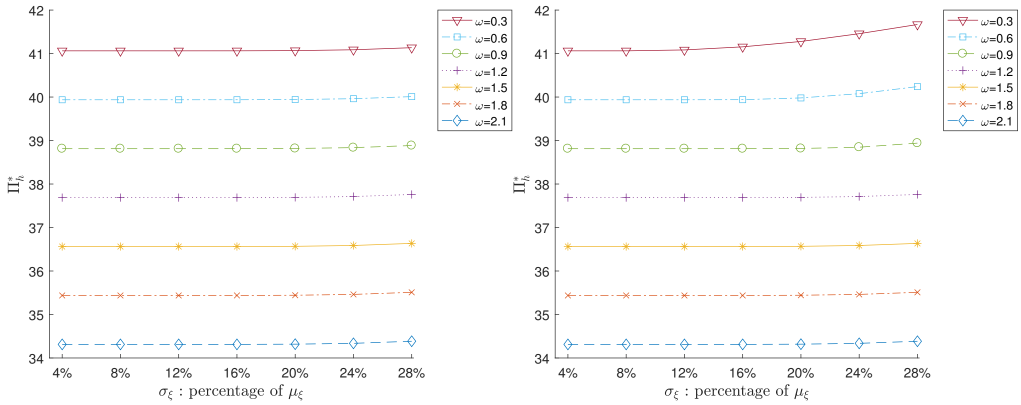
cannot be obtained analytically, we run the extensive numerical analysis to show the how do uncertainties parameters affect these optimal decisions. Starting from the effect of the correlation ρ on $h^B(\omega)$ and $K_h^B(\omega)$.

We find that a higher correlation leads to $h^B(\omega)$ decreases and $K_h^B(\omega)$ increases.

5.4.1 Sensitivity to Demand Variability

In this section, we investigate how demand variability σ_ξ affects the optimal capacity investment level, optimal hedge ratio and corresponding expected profit. In examine the demand variability, we make a comparison with the sensitivity result of a *basic model* where the hedge strategy is not applied.

Figure 5.3: Effect of demand variability σ_ξ on optimal profit $\Pi_h(K_h^*, h^*)$.



(a) Effect of σ_ξ on Π_h^* when $\rho = -0.5$ (b) Effect of σ_ξ on Π_h^* when $\rho = 0.5$

Note: Figures are depicted using baseline scenario $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\beta = 16\%\mu_\beta$ and σ_ξ is the percentage of μ_ξ which are $\{4\%, 8\%, 12\%, 16\%, 20\%, 24\%, 28\%\}$.

Proposition 17 (Impact of σ_ξ on $\Pi_h(K_h^*, h^*)$ - Hedging Model) *$\Pi_h(K_h^*, h^*)$ strictly increases in σ_ξ when $0 < \omega < \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y)^+\big]$; otherwise, $\Pi_h(K_h^*, h^*)$ is not sensitive to σ_ξ.*

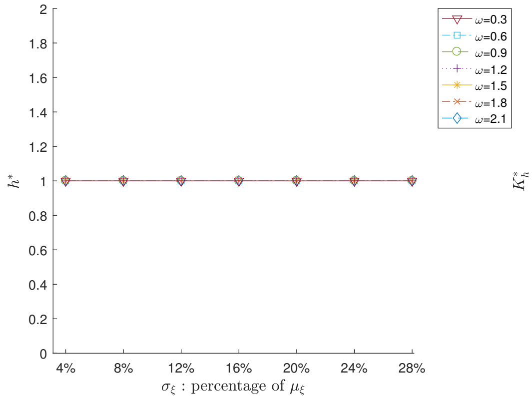
We find that the optimal profit is always increases in demand variability as long as the optimal capacity investment level is positive. Recalling that the optimal profit in basic model is first decreasing then increasing in demand variability when the correlation is negative, in comparison, we find that the negative effect of the demand variability is vanished due to the optimal hedging. A graphic representation of this proposition is shown in Figure 5.3.

In above figure, we find that a higher demand variability leads to $h^B(\omega)$ decreases and $K_h^B(\omega)$ increases. Similarly, the negative influence of higher demand variability on capacity investment level is vanished because of the optimal hedging.

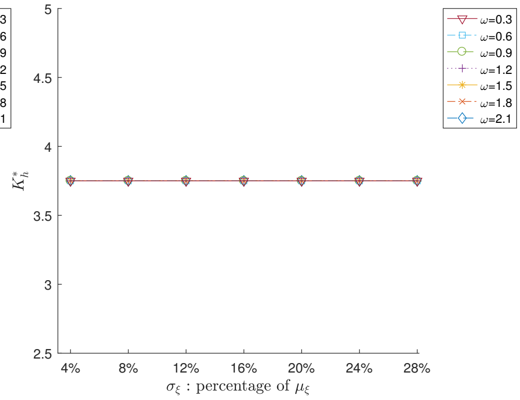
5.4.2 Sensitivity to Production Resource Variability

In this section, we conduct sensitivity analyses to study how firms should adjust their capacity investment level and hedge ratio as a response to changing production resource variability. Also how production resource variability affects

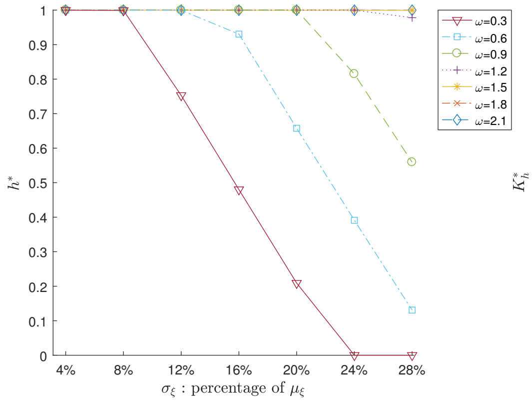
Figure 5.4: Effect of demand variability σ_ξ on optimal strategy (K_h^*, h^*).



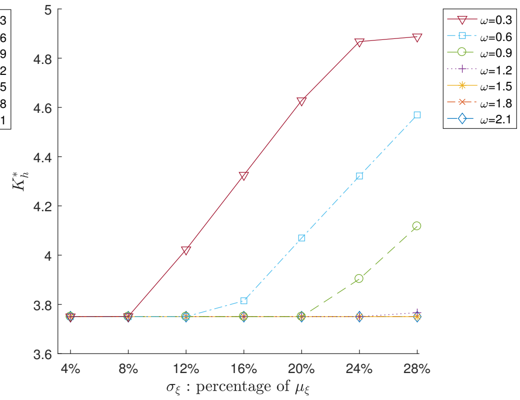
(a) Effect of σ_ξ on h^* when $\rho = -0.5$



(b) Effect of σ_ξ on K_h^* when $\rho = -0.5$



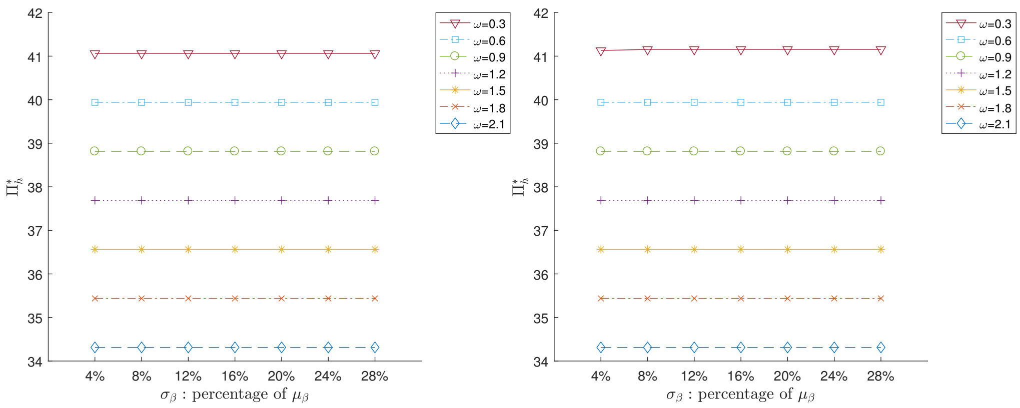
(c) Effect of σ_ξ on h^* when $\rho = 0.5$



(d) Effect of σ_ξ on K_h^* when $\rho = 0.5$

Figures are depicted using baseline scenario $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\beta = 16\%\mu_\beta$ and σ_ξ is the percentage of μ_ξ which are $\{4\%, 8\%, 12\%, 16\%, 20\%, 24\%, 28\%\}$.

Figure 5.5: Effect of production resource variability σ_β on optimal strategy $\Pi_h(K_h^*, h^*)$.



(a) Effect of σ_β on Π_h^* when $\rho = -0.5$ (b) Effect of σ_β on Π_h^* when $\rho = 0.5$

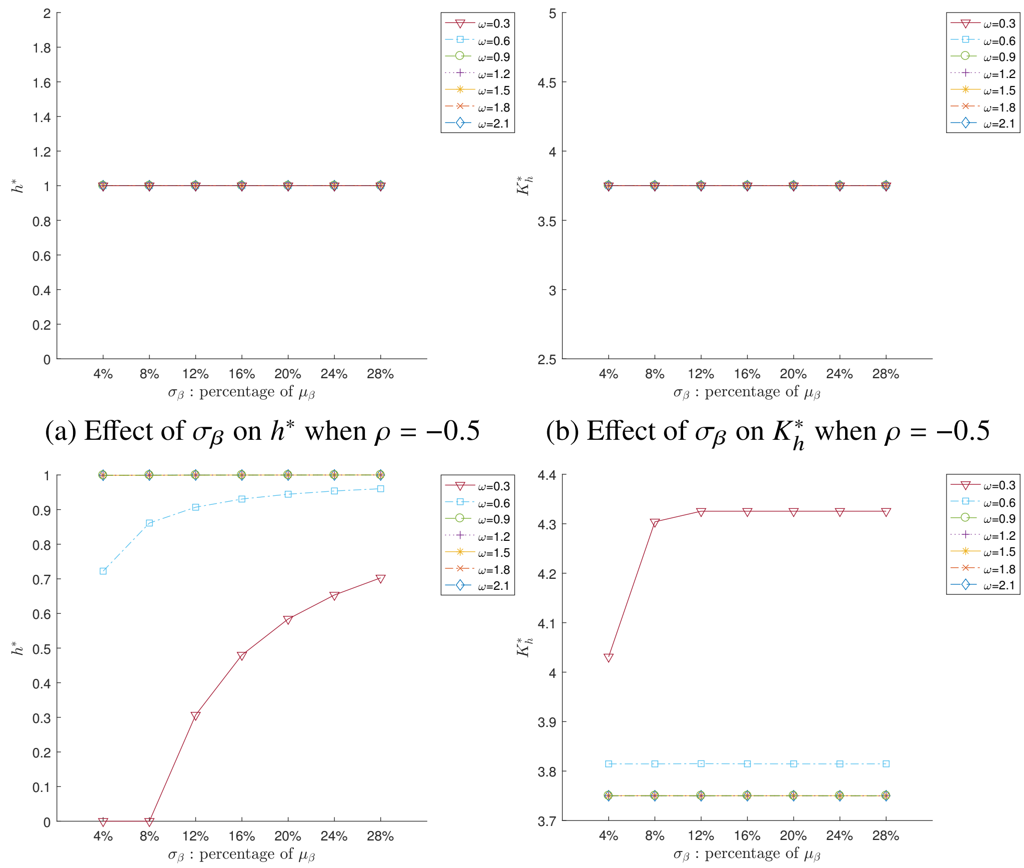
Note: Figures are depicted using baseline scenario $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\xi = 16\%\mu_\xi$ and σ_β is the percentage of μ_β which are $\{4\%, 8\%, 12\%, 16\%, 20\%, 24\%, 28\%\}$.

the profitability of the firm is analysed. Proposition 18 shows expected profit increases in production resource variability.

Proposition 18 (Impact of σ_β on $\Pi_h(K_h^*, h^*)$ - Hedging Model) *When $\rho > \frac{2b\sigma_\beta}{y\sigma_\xi}$ and $0 < \omega < \omega_h$, $\Pi_h(K_h^*, h^*)$ strictly increases in σ_β; otherwise, $\Pi_h(K_h^*, h^*)$ is not sensitive to σ_β.*

We find that only when optimal hedge ratio is zero, the profit increases in resource variability, otherwise, the profit doesn't change in resource variability. This reflects that the optimal partial hedge and corresponding capacity investment level are chosen in a way that the optimal expected profit is not a function of resource variability any more. A graphic representation of this proposition is shown in Figure 5.5. Recalling the impact of resource variability on the optimal profit stated in Proposition 6, the result also crucially depends on unit capacity investment cost ω and the correlation ρ. There exist threshold value of ω and ρ such that only when ω less than its threshold and ρ larger than its threshold firm's optimal expected profit first increases then decreases in production resource variability; otherwise, the profit monotonically decreases in production resource variability. Different from the sensitivity result in basic model, in Proposition 18, where increasing resource variability has negative influence on the optimal profit

Figure 5.6: Effect of production resource variability σ_β on optimal strategy (K_h^*, h^*).



(a) Effect of σ_β on h^* when $\rho = -0.5$

(b) Effect of σ_β on K_h^* when $\rho = -0.5$

(c) Effect of σ_β on h^* when $\rho = 0.5$

(d) Effect of σ_β on K_h^* when $\rho = 0.5$

Note: Figures are depicted using baseline scenario $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\xi = 16\%\mu_\xi$ and σ_β is the percentage of μ_β which are $\{4\%, 8\%, 12\%, 16\%, 20\%, 24\%, 28\%\}$.

in basic model, for hedging model, it is where the optimal profit is insensitive to resource variability. Under condition that increasing resource variability is more profitable in basic model, the optimal profit in hedging model is also increasing in resource variability.

In above figure, we find that a higher production resource variability leads to $h^B(\omega)$ increases and $K_h^B(\omega)$ decreases. Recalling from the sensitivity result for the effect of production resource variability in basic model. The capacity investment level increases in resource variability only when the correlation is high and the unit capacity cost is low; otherwise, the capacity level decreases in resource variability. In this optimal partial hedge scenario, the increasing trend of capacity investment in resource variability remains and the decreasing trend is removed because of hedge.

5.5 Profitability Loss

In this section, we address the third research question by extending our analyses of the impact of the demand and production resource uncertainties on the profitability-loss.

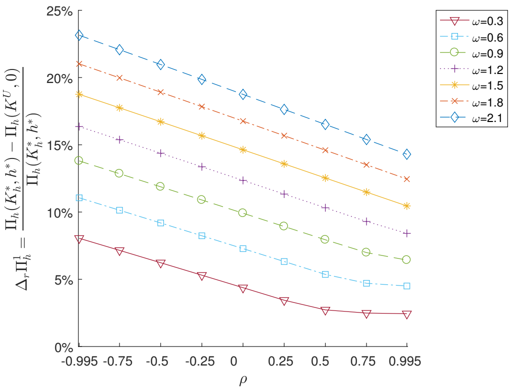
Previously, the profitability loss is incurred when the firm miss-specifying the capacity because of ignoring the production resource. In this section, we also discuss the significant of this loss in Section 5.5.1, specifically, this profitability loss denoted by $\Delta_r \Pi_h^1$ is defined as the percentage loss of profit due to ignoring production resource when choosing optimal capacity investment level at the same time the hedge ratio is always 0 as the resource is miss-regarded as infinite. The mathematical formulation is as follows $\Delta_r \Pi_h^1 := \frac{\Pi_h(K_h^*,h^*)-\Pi_h(K^U,0)}{\Pi_h(K_h^*,h^*)}$. In addition, miss-specifying hedge ratio can also lead to profitability loss, specifically, firm heuristically chooses always fully hedging can cause profitability loss that is denoted by $\Delta_r \Pi_h^2 := \frac{\Pi_h(K_h^*,h^*)-\max_{K\geq 0} \Pi_h(K,1)}{\Pi_h(K_h^*.h^*)}$. We will investigate the significance of this profitability in Section 5.5.2.

5.5.1 Profitability Loss due to Miss-specifying Capacity Level

In this section, we study how the correlation between demand and production resource, demand variability and production resource variability affect the value of profitability-loss$\Delta_r \Pi_h^1 := \frac{\Pi_h(K_h^*,h^*)-\Pi_h(K^U,0)}{\Pi_h(K_h^*,h^*)}$.

Starting from the impact of the correlation, following figure shows that the profitability loss $\Delta_r \Pi_h^1$ decreases in the correlation and the value of the loss.

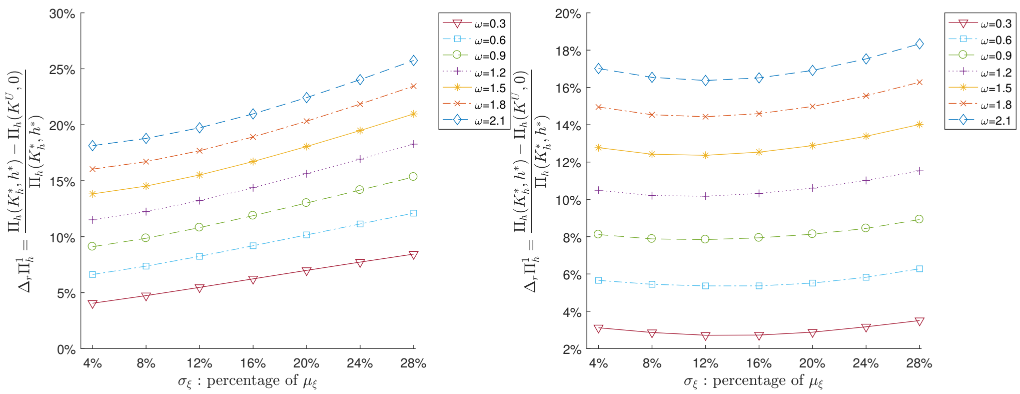
Figure 5.7: Effect of correlation ρ on profitability loss $\Delta_r \Pi_h^1$.



Figures are depicted using baseline scenario $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\beta = 16\%\mu_\beta$, $\sigma_\xi = 16\%\mu_\xi$ and $\rho \in \{-0.995, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.995\}$

Then, we observe that the profitability-loss $\Delta_r \Pi_h^1$ is increasing in demand variability when the correlation is negative; the profitability-loss first decreases then increases in demand variability when the correlation is positive. This observation is consistent with the observation of profitability-loss for basic model.

Figure 5.8: Effect of demand variability σ_ξ on profitability loss $\Delta_r \Pi_h^1$.

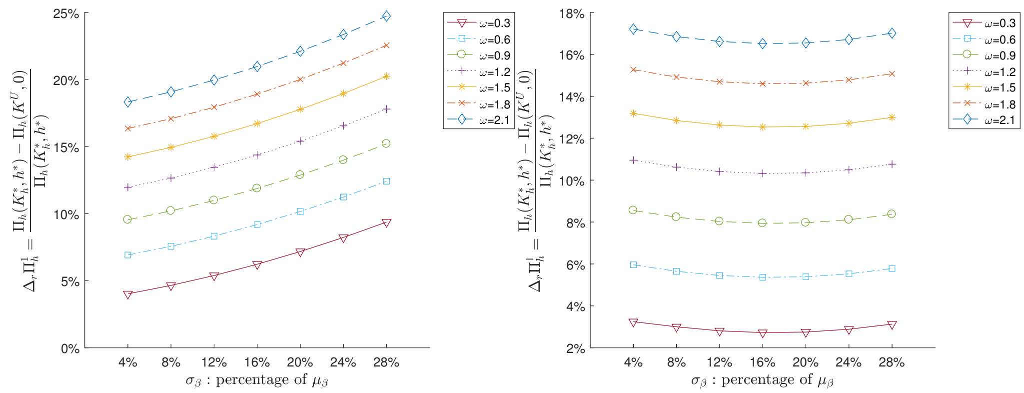


(a) Effect of σ_ξ on $\Delta_r \Pi_h^1$ when $\rho = -0.5$ (b) Effect of σ_ξ on $\Delta_r \Pi_h^1$ when $\rho = 0.5$

Note: Figures are depicted using baseline scenario $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\beta = 16\%\mu_\beta$ and σ_ξ is the percentage of μ_ξ which are $\{4\%, 8\%, 12\%, 16\%, 20\%, 24\%, 28\%\}$.

Lastly, as for the impact of production resource variability, we observe that the profitability-loss $\Delta_r \Pi_h^1$ is increasing in the resource variability when the correlation is negative; the profitability-loss first decreases then increases in the resource variability when the correlation is positive. This observation is also consistent with the observation of profitability-loss for basic model.

Figure 5.9: Effect of production resource variability σ_β on optimal strategy $\Delta_r\Pi_h^1$.



(a) Effect of σ_β on $\Delta_r\Pi_h^1$ when $\rho = -0.5$ (b) Effect of σ_β on $\Delta_r\Pi_h^1$ when $\rho = 0.5$

Note: Figures are depicted using baseline scenario $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\xi = 16\%\mu_\xi$ and σ_β is the percentage of μ_β which are $\{4\%, 8\%, 12\%, 16\%, 20\%, 24\%, 28\%\}$.

In summary, we find that how profitability-loss $\Delta_r\Pi_h^1$ change in uncertainty parameters have the same pattern as the impact of these uncertainty parameters on the basic model. On average, the loss is larger than the one in basic model.

5.5.2 Profitability Loss due to Miss-specifying Hedge Ratio

In this section, we study how the correlation between demand and production resource, demand variability and production resource variability affect the value of profitability-loss $\Delta_r\Pi_h^2$, which is the percentage Loss of profit due to heuristically choosing always fully hedge, specifically,$\Delta_r\Pi_h^2 := \frac{\Pi_h(K_h^*,h^*)-\max\limits_{K\geq 0}\Pi_h(K,1)}{\Pi_h(K_h^*,h^*)}$.

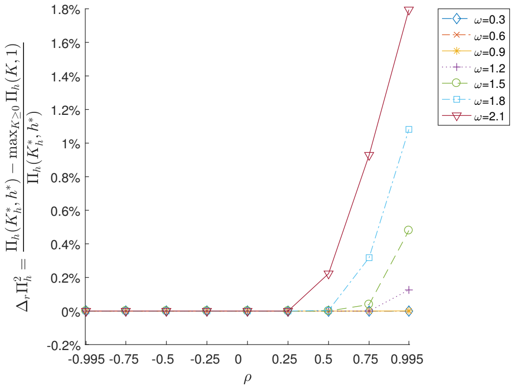
As full hedging can be optimal strategy under certain condition and therefore the profitability-loss $\Delta_r\Pi_h^2 = 0$, the following corollary provides the condition, under which this profitability is zero.

Corollary 7 *Under condition $\rho \leq 0$ or $\omega \geq \mathop{\mathbf{E}}\limits_{\tilde\xi,\tilde\beta}\left[\mathbb{1}_{\{\tilde\beta\geq\mu_\beta\}}(\tilde\xi - y - \frac{2b\mu_\beta}{y})^+\right]$, we have that $\Delta_r\Pi_h^2 = 0$.*

Or equivalently, the profitability-loss is positive only when the correlation is positive and the unit capacity cost is large enough.

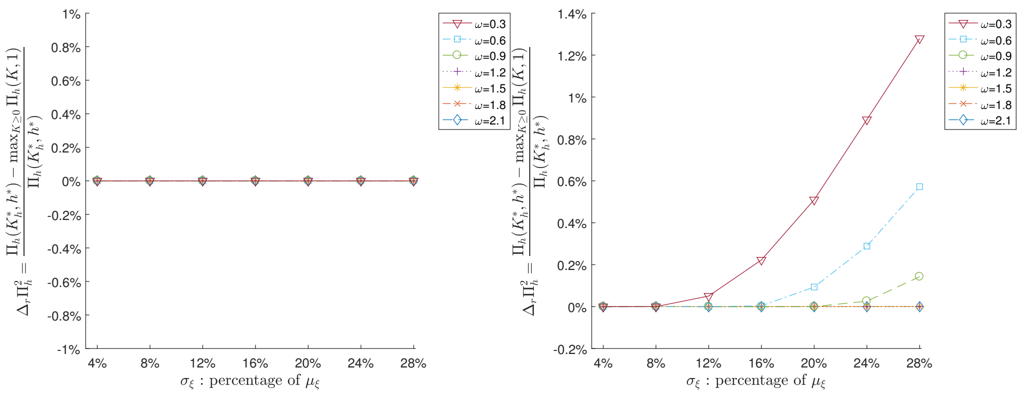
Starting from the impact of correlation on the profitability-loss $\Delta_r\Pi_h^2$. The

Figure 5.10: Effect of correlation ρ on profitability loss $\Delta_r \Pi_h^2$.



Figures are depicted using baseline scenario $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\beta = 16\%\mu_\beta$, $\sigma_\xi = 16\%\mu_\xi$ and $\rho \in \{-0.995, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.995\}$

Figure 5.11: Effect of demand variability σ_ξ on optimal strategy $\Delta_r \Pi_h^2$.



(a) Effect of σ_ξ on $\Delta_r \Pi_h^2$ when $\rho = -0.5$ (b) Effect of σ_ξ on $\Delta_r \Pi_h^2$ when $\rho = 0.5$

Note: Figures are depicted using baseline scenario $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\beta = 16\%\mu_\beta$ and σ_ξ is the percentage of μ_ξ which are $\{4\%, 8\%, 12\%, 16\%, 20\%, 24\%, 28\%\}$.

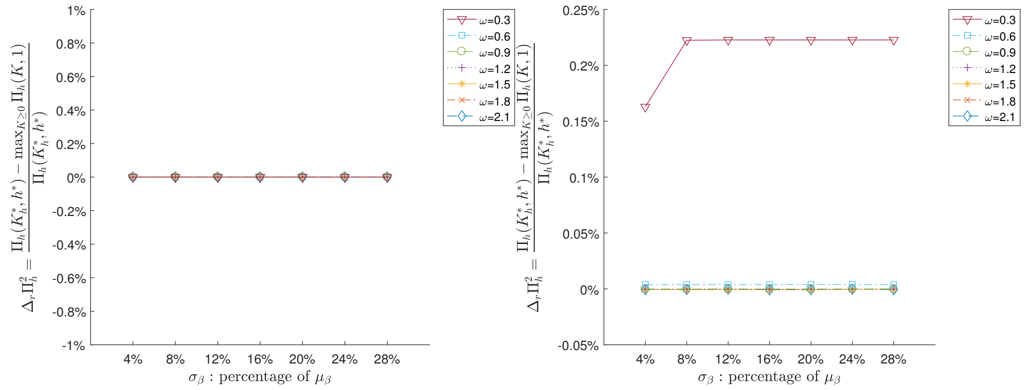
profitability-loss increases in the correlation. When the unit capacity cost is increasing the profitability-loss becomes smaller.

Then the impact of demand variability on the profitability-loss $\Delta_r \Pi_h^2$. We also find that a higher demand variability leads to a larger profitability-loss.

Finally, we discuss the observations on the impact of the production resource variability. As the production resource variability increasing the profitability-loss $\Delta_r \Pi_h^2$.

To summarize, the profitability-loss $\Delta_r \Pi_h^2$ is positive when the correlation is very high and the unit capacity cost is low. And when the profitability-loss is positive, it increases when demand variability increases, production resource

Figure 5.12: Effect of production resource variability σ_β on optimal strategy $\Delta_r \Pi_h^2$.



(a) Effect of σ_β on $\Delta_r\Pi_h^1$ when $\rho = -0.5$ (b) Effect of σ_β on $\Delta_r\Pi_h^2$ when $\rho = 0.5$

Note: Figures are depicted using baseline scenario $b = 1$, $y = 1$, $\mu_\xi = 16y$, $\mu_\beta = (\mu_\xi - y)y/4$, $\sigma_\xi = 16\%\mu_\xi$ and σ_β is the percentage of μ_β which are $\{4\%, 8\%, 12\%, 16\%, 20\%, 24\%, 28\%\}$.

variability increases and the correlation between the two increases. Note that a high positive correlation means that higher demand is more likely to occur together with high resource availability, at the same time, a low unit capacity cost translates to a higher capacity investment level, as a result, the internal optimal production quantity as well as high second stage profit is more possible to attain as both resource constraint and capacity constraint are large. As uncertainty in resource is beneficial as stated above, full hedge would lead to less profitability.

Chapter 6

Conclusion and Discussion

We have studied the stochastic capacity investment problem of how a firm should anticipate the capacity investment level change in demand and production resource uncertainties, where the production resource can either be financial, e.g. working capital / budget, or be physical, e.g. raw material / components. And through analytical and computational study, we show how significant this jointly influence of uncertainties make the firm loss when the firm simply ignores the production resource. Also we investigate two management strategies to counteract against financial production resource shortage and physical resource shortage respectively, that are pre-shipment finance and procurement hedging strategy.

More specifically, first of all, we obtain the optimal capacity investment strategy under these uncertainties and show that the unit capacity cost critically characterizes the optimal strategy. We perform a whole set of analytical sensitivity analyses to answer how production resource uncertainty, demand uncertainty and their correlation shape the optimal capacity and profitability. We find that demand variability is not always favourable as traditional literature suggests. The sensitivity results with respect to production resource variability is more complex, unit capacity cost and the correlation crucially determine the result. This suggests managers that the capacity intensity together with how the product demand correlates with the economic condition shape the impact of production resource volatility. Then, for the first time in the literature, our study provides insights on how the correlation between the production resource and demand may reshape the optimal capacity investment decision as well as the expected profit. Thirdly, via computational study, we find that high volatility of both demand and production resource results in high profitability-loss; and lower the correlation, higher the profitability-loss.

We then extend the model with a pre-shipment finance facility to alleviate the potential budget shortage in production stage. We also perform a whole set of analytical sensitivity analyses to answer how production resource uncertainty, demand uncertainty and their correlation shape the optimal capacity and profitability. And find that when the interest rate of the pre-shipment finance is lower than a threshold, then higher demand variability is beneficial that is different from the sensitivity result in basic model. In analyzing profitability-loss, we find that the loss is negligibly small, suggesting that as long as there is a last minute finance available for alleviate the budget constraint, the capacity investment decision can be made without considering the possibility of budget short.

Finally, we extend the model with procurement hedging contract to control the uncertainty of physical resource uncertainty in the capacity investment stage. We find that partial hedging can be optimal if the correlation between and production resource uncertainties is high and the unit capacity investment cost is very low. The sensitivity analyses show that a increasing in production resource uncertainty, demand uncertainty and their correlation all lead to the increasing in the profitability. We find that the always full hedging heuristic strategy leads to very small profitability-loss.

Our model framework also captures an assemble-to-order system where the each unit of final product requires one unit of component 1 (which is represented by capacity investment level in our basic model) and one unit of component 2 (which is represented by the additional production resource—which is either budget or physical resource). The decision variable of the model is optimizes component 1 order quantity under the uncertainty in component 2 volume. This component 2 can also be regarded as the assembly capacity that limits the volume of assembling end product. We offer a few future research directions for this framework. One can extend this framework by also considering optimizing component 2 order quantity and subject to yield uncertainty (hence production uncertainty).

One limitation of our study is that it examines one type of production resource and manages different type of the resource separately. In reality, financial and physical resources well as other dimensions of constraints affect the firm's production process so as to the capacity investment decision. To assess the impact of both pre-shipment finance and procurement hedging contract as well as other dimensions of constraints, a simulation study fitting the firm's operational structure is likely to be required.

The model and analysis in this study can potentially provide a foundation and rule of thumb for future research to see the resemblance between the area of operations-OM and physical supplementary resource reliability management.

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Appendix A

Appendix

For ease of reading, we list the notations used through the paper in Table A.1.

Table A.1: Summary of Notations

Random Variables and Corresponding Parameters	
$\tilde{\xi}$	demand parameter
$\tilde{\beta}$	production resource
Constant Parameters	
ω	unit capacity investment cost
y	unit production cost
b	price sensitivity to quantity
μ_ξ, σ_ξ	expected value and standard deviation of marginal demand
$[\underline{\beta}, \overline{\beta}]$	support of production resource
μ_β, σ_β	expected value and standard deviation of marginal production resource
ρ	correlation coefficient between $\tilde{\xi}$ and $\tilde{\beta}$
r_p	interest rate of pre-shipment finance
Optimal Decisions	
q	production quantity
K^U	resource-unconstrained capacity level
K^B	resource-constrained capacity level
L_p	pre-shipment finance loan
h	the hedge ratio
Profit Functions	
π	Optimal production stage profit
Π	Expected profit under resource constrained case (basic model)
Π_u	Expected profit under resource unconstrained case
Π_p	Expected profit for Pre-shipment finance model
Π_h	Expected profit for Hedging model

A.1 Proofs for the Optimal Strategy and sensitivity analyses of Basic Model

A.1.1 Proofs for the Optimal Strategy

Theorem 1 and 2 present the optimal strategy of basic model in production stage and capacity investment stage respectively. We provide the proofs of these two theorems in this section. Then we illustrate the proof of Lemma 2 that compare the capacity investment level and profitability between benchmark model and this basic model.

Proof of Theorem 1:

The problem is solved by KKT condition. Note that $(\xi - bq)q - yq$ is concave with first order derivative equals zero at $q = \frac{\xi-y}{2b}$. If $\frac{\xi-y}{2b} \leq 0$, then $q^* = 0$; if $0 < \frac{\xi-y}{2b} < \min\left\{K, \frac{\beta}{y}\right\}$, then $q^* = \frac{\xi-y}{2b}$; lastly, if $\frac{\xi-y}{2b} \geq \min\left\{K, \frac{\beta}{y}\right\}$, then $q^* = \min\left\{K, \frac{\beta}{y}\right\}$. ■

Proof of Theorem 2:

The optimal capacity investment level for basic model is derived from the expected profit in capacity investment stage $\Pi(K)$, namely Equation (3.2). Taking the first order derivative of $\Pi(K)$ in terms of K, we get that

$$\frac{\partial \Pi(K)}{\partial K} = \begin{cases} -\omega + \displaystyle\int_{y+2bK}^{\infty} (\xi - y - 2bK) f_\xi(\xi) d\xi, & \text{if } K \in \left[0, \dfrac{\beta}{y}\right] \\ -\omega + \displaystyle\int_{yK}^{\overline{\beta}} \int_{y+2bK}^{\infty} (\xi - y - 2bK) f(\xi, \beta) d\xi d\beta, & \text{if } K \in \left(\dfrac{\beta}{y}, \dfrac{\overline{\beta}}{y}\right) \\ -\omega & \text{if } K \in \left[\dfrac{\overline{\beta}}{y}, \infty\right) \end{cases}$$

Note that $\frac{\partial \Pi(K)}{\partial K}$ is continuous, we have the second order derivative $\frac{\partial^2 \Pi(K)}{\partial K^2}$ =

$$\begin{cases} -2\bar{F}_{\xi}(y+2bK), & \text{if } K \in \left[0, \frac{\underline{\beta}}{y}\right) \\ -2\int_{yK}^{\overline{\beta}}\int_{y+2bK}^{\infty} f(\xi,\beta)d\xi d\beta - y\int_{y+2bK}^{\infty}(\xi - y - 2bK)f(\xi, yK)d\xi, & \text{if } K \in \left(\frac{\underline{\beta}}{y}, \frac{\overline{\beta}}{y}\right) \\ 0, & \text{if } K \in \left(\frac{\overline{\beta}}{y}, \infty\right). \end{cases}$$

The immediate result from above equation is that $\Pi(K)$ is concave in K since $\frac{\partial^2 \Pi(K)}{\partial K^2} \leq 0$. Therefore, the optimal K^* satisfies that $K^* = 0$ if $\frac{\partial \Pi(K)}{\partial K}|_{K=0} \leq 0$, that is, $\omega \geq \mathbf{E}\left[(\tilde{\xi} - y)^+\right]$; K^* satisfies $-\omega + \int_{y+2bK^*}^{\infty}(\xi - y - 2bK^*)f_{\xi}(\xi)d\xi = 0$, if $\frac{\partial \Pi(K)}{\partial K}|_{K=0} \geq 0$ and $\frac{\partial \Pi(K)}{\partial K}|_{K=\underline{\beta}/y} \leq 0$. That is, $\mathbf{E}\left[\left(\tilde{\xi} - y - 2b\underline{\beta}/y\right)^+\right] \leq \omega < \mathbf{E}\left[(\tilde{\xi} - y)^+\right]$; and lastly, K^* satisfies that $-\omega + \int_{yK^*}^{\overline{\beta}}\int_{y+2bK^*}^{\infty}(\xi - y - 2bK^*)f(\xi,\beta)d\xi d\beta = 0$ if $\omega < \mathbf{E}\left[\left(\tilde{\xi} - y - 2b\underline{\beta}/y\right)^+\right]$. ■

A.1.2 Proofs for the sensitivity analyses

In this appendix section, we provide the proofs of sensitivity results for basic model. For tractability, some preliminaries are introduced before all proofs. We define several standardized normal distributions to differentiate the transformation for $\tilde{\beta}$, $\tilde{\xi}$, $\tilde{\xi}|\tilde{\beta}$ and $\tilde{\beta}|\tilde{\xi}$, which are $\tilde{z}_0 \stackrel{d}{=} \frac{\tilde{\beta}-\mu_\beta}{\sigma_\beta}$, $\tilde{z}_1 \stackrel{d}{=} \frac{\tilde{\xi}-\mu_\xi}{\sigma_\xi}$, $\tilde{z}_2 \stackrel{d}{=} \frac{\tilde{\xi}-\mu_\xi-\rho\frac{\sigma_\xi}{\sigma_\beta}(\tilde{\beta}-\mu_\beta)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big|\tilde{\beta}$ and $\tilde{z}_3 \stackrel{d}{=} \frac{\tilde{\beta}-\mu_\beta-\rho\frac{\sigma_\beta}{\sigma_\xi}(\tilde{\xi}-\mu_\xi)}{\sqrt{\sigma_\beta^2(1-\rho^2)}}\Big|\tilde{\xi}$ respectively, therein, the conditional distributions are normally distributed with parameters $\tilde{\xi}|\tilde{\beta} \sim N\left(\mu_\xi + \rho\frac{\sigma_\xi}{\sigma_\beta}(\tilde{\beta} - \mu_\beta), \sigma_\xi^2(1-\rho^2)\right)$ and $\tilde{\beta}|\tilde{\xi} \sim N\left(\mu_\beta + \rho\frac{\sigma_\beta}{\sigma_\xi}(\tilde{\xi} - \mu_\xi), \sigma_\beta^2(1-\rho^2)\right)$. The expected profit in the capacity investment stage $\Pi(K)$, i.e. Equation (3.2), will be widely used in the proofs of sensitivity results, for the ease of calculation, we transform it in standard normal distribution form, that is

$$\Pi(K) = -\omega K + \sum_{i=1}^{4} V_i(K), \tag{A.1}$$

where $V_i(K)$ $i = 1, 2, \cdots, 4$ are defined as

$$V_1(K) := \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)^+}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}$$

$$\frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)^2}{4b}\phi(z_2)dz_2\phi(z_0)dz_0,$$

$$V_2(K) := \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)^2}{4b}\phi(z_2)dz_2\phi(z_0)dz_0,$$

$$V_3(K) := \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}$$

$$\left(\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)K-bK^2\right)\phi(z_2)dz_2\phi(z_0)dz_0,$$

and $V_4(K) :=$

$$\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \frac{(z_0\sigma_\beta+\mu_\beta)}{y} \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}$$

$$\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y-b\Big(\frac{z_0\sigma_\beta+\mu_\beta}{y}\Big)\right)\phi(z_2)dz_2\phi(z_0)dz_0.$$

As $\Pi(K)$ is the continuous function, the derivatives at the boundaries are cancelled out. Therefore, the first order derivative with respect to K is $\frac{\partial\Pi(K)}{\partial K} =$

$$-\omega + \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y-2bK\right)\phi(z_2)dz_2\phi(z_0)dz_0. \tag{A.2}$$

We introduce the Stein's Lemma which helps simplifying the proof.

Lemma 3 *(Stein, 1972)*

1. *Suppose $\tilde{z}$ is a normally distributed random variable with expectation* 0 *and variance* 1*. Further suppose g is a function for which the two expectations $\mathbf{E}\big[g(\tilde{z})\tilde{z}\big]$ and $\mathbf{E}\Big[\frac{\partial g(\tilde{z})}{\partial \tilde{z}}\Big]$ both exist (the existence of the expectation of any random variable is equivalent to the finite of the expectation of its absolute value). Then*
$$\mathbf{E}\big[g(\tilde{z})\tilde{z}\big] = \mathbf{E}\Big[\frac{\partial g(\tilde{z})}{\partial \tilde{z}}\Big].$$

2. *Suppose $\tilde{X}$ is a normally distributed random variable with expectation μ and variance σ^2. Further suppose g is a function for which the two*

expectations $\mathbf{E}\big[g(\tilde{X})(\tilde{X}-\mu)\big]$ *and* $\mathbf{E}\Big[\frac{\partial g(\tilde{X})}{\partial \tilde{X}}\Big]$ *both exist (the existence of the expectation of any random variable is equivalent to the finite of the expectation of its absolute value). Then*

$$\mathbf{E}\big[g(\tilde{X})(\tilde{X}-\mu)\big] = \sigma^2\mathbf{E}\Big[\frac{\partial g(\tilde{X})}{\partial \tilde{X}}\Big].$$

First of all, we prove the impact of correlation between demand and production resource uncertainties, the result of which is summarized in Proposition 1.

Proof of Proposition 1:

In this proof, we demonstrate that both K^B and $\Pi(K^B)$ increase in ρ. Starting from the proof for K^B, $\frac{dK^B}{d\rho} = -\Big(\Big(\frac{\partial^2\Pi}{\partial K\partial\rho}\Big)\Big/\Big(\frac{\partial^2\Pi}{\partial K^2}\Big)\Big)\Big|_{K_p^B}$ is derived by implicit differentiation, $\frac{\partial^2\Pi}{\partial K\partial\rho}$ is calculated from Equation (A.2) such that

$$\begin{aligned}\frac{\partial^2\Pi(K)}{\partial K\partial\rho} &= \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\Big(\sigma_\xi z_0 - \frac{\sigma_\xi\rho z_2}{\sqrt{1-\rho^2}}\Big)\phi(z_2)dz_2\phi(z_0)dz_0\\ &= \sigma_\xi\phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\Bigg(1-\Phi\Bigg(\frac{\sigma_\beta(y+2bK-\mu_\xi)-\rho\sigma_\xi(yK-\mu_\beta)}{\sigma_\xi\sigma_\beta\sqrt{1-\rho^2}}\Bigg)\Bigg) > 0,\end{aligned}$$

that is, $\frac{\partial^2\Pi(K)}{\partial K\partial\rho} > 0$ for all K. As a result, K^B increases in ρ is proved.

Subsequently, we provide the proof for the impact of ρ on $\Pi(K^B)$. According to the Envelope Theorem, $\frac{d\Pi(K^B(\omega))}{d\rho} = \frac{\partial\Pi(K)}{\partial\rho}\Big|_{K^B}$. The derivative of Equation (A.1) with respect to ρ is $\frac{\partial\Pi(K)}{\partial\rho} = \sum_{i=1}^4\frac{\partial V_i(K)}{\partial\rho}$, noting that the derivatives at the boundaries are cancelled out because of the continuity of $\Pi(K)$. Therefore, we have $\frac{\partial \Pi(K)}{\partial \rho}$ =

$$\frac{\sigma_\xi}{\sqrt{1-\rho^2}} \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y-\mu_\xi-\sigma_\xi \rho z_0}{\sigma_\xi\sqrt{1-\rho^2}}}^{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)^+}{y}-\mu_\xi-\sigma_\xi\rho z_0}{\sigma_\xi\sqrt{1-\rho^2}}}$$

$$\frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)}{2b}\left(z_0\sqrt{1-\rho^2}-z_2\rho\right)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\frac{\sigma_\xi}{\sqrt{1-\rho^2}} \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y-\mu_\xi-\sigma_\xi \rho z_0}{\sigma_\xi\sqrt{1-\rho^2}}}^{\frac{y+2bK-\mu_\xi-\sigma_\xi\rho z_0}{\sigma_\xi\sqrt{1-\rho^2}}}$$

$$\frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)}{2b}\left(z_0\sqrt{1-\rho^2}-z_2\rho\right)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\frac{\sigma_\xi}{\sqrt{1-\rho^2}} \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} K\left(z_0\sqrt{1-\rho^2}-z_2\rho\right)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\frac{\sigma_\xi}{\sqrt{1-\rho^2}} \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \frac{(z_0\sigma_\beta+\mu_\beta)}{y} \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}$$

$$\left(z_0\sqrt{1-\rho^2}-z_2\rho\right)\phi(z_2)dz_2\phi(z_0)dz_0.$$

Transforming standard normal distributions back to $\tilde{\xi}$ and $\tilde{\beta}$, we have $\frac{\partial \Pi(K)}{\partial \rho} = \left(\frac{1}{1-\rho^2}\right)\frac{\sigma_\xi}{\sigma_\beta}\mathop{\mathbf{E}}_{\xi}\left[\mathop{\mathbf{E}}_{\beta|\xi}\left[q^*(K,\tilde{\xi},\tilde{\beta})\cdot\left(\tilde{\beta}-\mu_\beta-\rho\frac{\sigma_\beta}{\sigma_\xi}(\tilde{\xi}-\mu_\xi)\right)\middle|\tilde{\xi}\right]\right]$. Since $\tilde{\beta}|\tilde{\xi}$ follows normal distribution $\tilde{\beta}|\tilde{\xi} \sim N\left(\mu_\beta+\rho\frac{\sigma_\beta}{\sigma_\xi}(\tilde{\xi}-\mu_\xi), \sigma_\beta^2(1-\rho^2)\right)$ and $q^*(K,\xi,\beta)$ is piecewise differentiable and continuous, $\mathop{\mathbf{E}}_{\beta|\xi}\left[q^*(K,\xi,\tilde{\beta})\cdot\left(\tilde{\beta}-\mu_\beta-\rho\frac{\sigma_\beta}{\sigma_\xi}(\xi-\mu_\xi)\right)\middle|\xi\right]$ is able to be simplified by Stein's lemma such that $\mathop{\mathbf{E}}_{\beta|\xi}\left[q^*(K,\xi,\tilde{\beta})\cdot\left(\tilde{\beta}-\mu_\beta-\rho\frac{\sigma_\beta}{\sigma_\xi}(\xi-\mu_\xi)\right)\middle|\xi\right] = \sigma_\beta^2(1-\rho^2)\mathop{\mathbf{E}}_{\beta|\xi}\left[\frac{\partial q^*(K,\xi,\tilde{\beta})}{\partial\beta}\middle|\xi\right]$, we then obtain

$$\frac{\partial \Pi(K)}{\partial \rho} = \sigma_\xi\sigma_\beta\mathop{\mathbf{E}}_{\xi}\left[\mathop{\mathbf{E}}_{\beta|\xi}\left[\frac{\partial q^*(K,\xi,\tilde{\beta})}{\partial \beta}\middle|\xi\right]\right] = \sigma_\xi\sigma_\beta\iint_{\Omega_3}\frac{1}{y}f(\xi,\beta)d\xi d\beta > 0.$$

■

Proofs for the σ_ξ Results

Proof of Proposition 2:

We will investigate the impact of σ_ξ on K^U and $\Pi_u(K^U)$. Π_u and $\frac{\partial \Pi_u(K)}{\partial K}$ are transformed into expectations with standard normal distribution as follows,

$\Pi_u(K) =$

$$-\omega K + \int_{\frac{y-\mu_\xi}{\sigma_\xi}}^{\frac{y+2bK-\mu_\xi}{\sigma_\xi}} \frac{(z_1\sigma_\xi + \mu_\xi - y)^2}{4b}\phi(z_1)dz_1 + \int_{\frac{y+2bK-\mu_\xi}{\sigma_\xi}}^{\infty} \left((z_1\sigma_\xi + \mu_\xi - y)K - bK^2\right)\phi(z_1)dz_1$$

and $\frac{\partial \Pi_u(K)}{\partial K} = -\omega + \int_{\frac{y+2bK-\mu_\xi}{\sigma_\xi}}^{\infty} (z_1\sigma_\xi + \mu_\xi - y - 2bK)\phi(z_1)dz_1$. Following analyses are using above two expressions:

1. The impact of σ_ξ on K^U is calculated by implicit differentiation $\frac{dK^U}{d\sigma_\xi} = -\left(\left(\frac{\partial^2\Pi_u}{\partial K\partial\sigma_\xi}\right) \Big/ \left(\frac{\partial^2\Pi_u}{\partial K^2}\right)\right)\Bigg|_{K^U}$ to prove the impact of σ_ξ on K^U. From normal distribution property $\int_x^\infty z\phi(z)dz = \phi(x)$, we have
$$\frac{\partial^2\Pi_u(K)}{\partial K\partial\sigma_\xi} = \int_{\frac{y+2bK-\mu_\xi}{\sigma_\xi}}^{\infty} z_1\phi(z_1)dz_1 = \phi\Big(\frac{y+2bK-\mu_\xi}{\sigma_\xi}\Big) > 0,$$
We therefore obtain $\frac{dK^U}{d\sigma_\xi} > 0$, which implies K^U increases in σ_ξ.

2. We investigate the impact of σ_ξ on expected profit $\Pi_u(K)$ $\forall K \in (0,\infty)$ using the Envelope Theorem such that the sign of $\frac{d\Pi_u(K^U)}{d\sigma_\xi}$ for the sensitivity of $\Pi_u(K^U)$ on σ_ξ is gotten from calculating $\frac{\partial \Pi_u(K)}{\partial\sigma_\xi}\Big|_{K^U}$. We have
$$\frac{\partial \Pi_u(K)}{\partial\sigma_\xi} = \frac{\sigma_\xi}{2b}\int_{\frac{y-\mu_\xi}{\sigma_\xi}}^{\frac{y+2bK-\mu_\xi}{\sigma_\xi}} \phi(z_1)dz_1 > 0 \quad \forall K > 0,$$
therefore, $\Pi_u(K^U)$ also increases in σ_ξ.

■

Now, we provide the proof of the impact of σ_ξ on K^B and $\Pi(K^B)$.

Proof of Proposition 3:

How K^B changing in σ_ξ is derived from implicit differentiation $\frac{dK^B}{d\sigma_\xi} = -\left(\frac{\partial^2\Pi}{\partial K \partial \sigma_\xi} \Big/ \frac{\partial^2 \Pi}{\partial K^2}\right)\Big|_{K^B}$, specifically, $\frac{dK^B}{d\sigma_\xi}$ and $\frac{\partial^2\Pi}{\partial K\partial\sigma_\xi}\Big|_{K^B}$ have same sign. The proof is divided by cases in terms of ρ, that is $\rho \in [0,1)$, $\rho \to -1$ and $\rho \in (0,1)$. Calculating from Equation (A.2) that $\frac{\partial^2\Pi(K)}{\partial K\partial\sigma_\xi} =$

$$\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0, \tag{A.3}$$

then we use the normal distribution probability density function structure and some algebra transforming $\frac{\partial^2\Pi(K)}{\partial K\partial\sigma_\xi}$ into

$$\begin{aligned}\frac{\partial^2\Pi(K)}{\partial K\partial\sigma_\xi} =&\phi\Big(\frac{y+2bK-\mu_\xi}{\sigma_\xi}\Big)\Phi\Big(\frac{\rho\sigma_\beta(y+2bK-\mu_\xi)-\sigma_\xi(yK-\mu_\beta)}{\sigma_\xi\sigma_\beta\sqrt{1-\rho^2}}\Big)\\ &+\rho\phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\Phi\Big(\frac{\rho\sigma_\xi(yK-\mu_\beta)-\sigma_\beta(y+2bK-\mu_\xi)}{\sigma_\xi\sigma_\beta\sqrt{1-\rho^2}}\Big),\end{aligned} \tag{A.4}$$

which implies $\frac{\partial^2\Pi(K)}{\partial K\partial\sigma_\xi} > 0$ for all K if $\rho \geq 0$.

Then, we discuss how K^B changes in σ_ξ when $\rho \to -1$. The goal of this part of the proof is to obtain the sign of $\frac{\partial}{\partial\sigma_\xi}\left\{\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\right\}\Big|_{\lim_{\rho\to-1}K^B}$, since it has the same sign as $\frac{d\lim_{\rho\to-1}K^B}{d\sigma_\xi}$ according to the implicit differentiation

$$\frac{d\lim_{\rho\to-1}K^B}{d\sigma_\xi} = -\left\{\frac{\partial}{\partial\sigma_\xi}\left\{\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\right\}\Big/\frac{\partial}{\partial K}\left\{\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\right\}\right\}\Bigg|_{\lim_{\rho\to-1}K^B}.$$

The optimality condition for $\lim_{\rho\to-1}K^B$ is $\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K} =$

$$\begin{aligned}&-\omega+\lim_{\rho\to-1}\sqrt{\sigma_\xi^2(1-\rho^2)}\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\phi\Big(\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\phi(z_0)dz_0\\ &+\lim_{\rho\to-1}\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\left(\mu_\xi+\rho\sigma_\xi z_0-y-2bK\right)\Big(1-\Phi\Big(\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)\phi(z_0)dz_0\\ =&\begin{cases}-\omega+\displaystyle\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\frac{\mu_\xi-y-2bK}{\sigma_\xi}}\left(\mu_\xi-y-2bK-\sigma_\xi z_0\right)\phi(z_0)dz_0, & \text{if } K<\dfrac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi+\mu_\beta}{2b\sigma_\beta/\sigma_\xi+y}\\ -\omega, & \text{otherwise,}\end{cases}\end{aligned}$$

that is, $\lim_{\rho\to-1}K^B$ must be in the range $\left[0,\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi+\mu_\beta}{2b\sigma_\beta/\sigma_\xi+y}\right)$ and solved by

$\omega = \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\frac{\mu_\xi - y - 2bK}{\sigma_\xi}} \big(\mu_\xi - y - 2bK - \sigma_\xi z_0\big)\phi(z_0)dz_0$. Having $\lim\limits_{\rho\to-1} \frac{\partial \Pi(K)}{\partial K}$, we obtain

$$\frac{\partial}{\partial \sigma_\xi}\left\{\lim_{\rho\to-1}\frac{\partial \Pi(K)}{\partial K}\right\} = -\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\frac{\mu_\xi - y - 2bK}{\sigma_\xi}} z_0\phi(z_0)dz_0 = \phi\Big(\frac{\mu_\xi - y - 2bK}{\sigma_\xi}\Big) - \phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big),$$

which implies that $\frac{\partial}{\partial \sigma_\xi}\left\{\lim\limits_{\rho\to-1}\frac{\partial \Pi(K)}{\partial K}\right\}\Big|_{\lim\limits_{\rho\to-1} K^B} < 0$ if and only if σ_ξ satisfies that

$$\left|\frac{\mu_\xi - y - 2b\lim\limits_{\rho\to-1} K^B}{\sigma_\xi}\right| > \left|\frac{y\lim\limits_{\rho\to-1} K^B - \mu_\beta}{\sigma_\beta}\right|. \tag{A.5}$$

That is to say, we turn Inequality (A.5) into the inequality with respect to σ_ξ explicitly, then the sensitivity result is proved. Note that $\lim\limits_{\rho\to-1} K^B$ is a function of σ_ξ, to prove the result there are two steps: 1) rewrite the Inequality (A.5) as the inequality of $\lim\limits_{\rho\to-1} K^B$; 2) use the optimality condition $\lim\limits_{\rho\to-1}\frac{\partial \Pi(K)}{\partial K} = 0$ to convert corresponding range of $\lim\limits_{\rho\to-1} K^B$ to the range of σ_ξ.

Firstly, given $\lim\limits_{\rho\to-1} K^B \in \Big[0, \frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi + \mu_\beta}{2b\sigma_\beta/\sigma_\xi + y}\Big)$, there are two cases for Inequality (A.5) to hold, specifically,

(a) $\frac{\mu_\xi - y - 2b\lim\limits_{\rho\to-1} K^B}{\sigma_\xi} > \frac{y\lim\limits_{\rho\to-1} K^B - \mu_\beta}{\sigma_\beta} \geq 0$. In this case, $\lim\limits_{\rho\to-1} K^B \in \Big[0, \frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi + \mu_\beta}{2b\sigma_\beta/\sigma_\xi + y}\Big) \cap \Big[\frac{\mu_\beta}{y}, \infty\Big) \cap \Big[0, \frac{\mu_\xi - y}{2b}\Big)$. According to following possible inequalities

$$\begin{aligned}
\frac{\mu_\xi - y}{2b} &< \frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi + \mu_\beta}{2b\sigma_\beta/\sigma_\xi + y} < \frac{\mu_\beta}{y}, \text{ if } \frac{\mu_\xi - y}{2b} < \frac{\mu_\beta}{y};\\
\frac{\mu_\xi - y}{2b} &> \frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi + \mu_\beta}{2b\sigma_\beta/\sigma_\xi + y} > \frac{\mu_\beta}{y}, \text{ if } \frac{\mu_\xi - y}{2b} > \frac{\mu_\beta}{y};\\
\frac{\mu_\xi - y}{2b} &= \frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi + \mu_\beta}{2b\sigma_\beta/\sigma_\xi + y} = \frac{\mu_\beta}{y}, \text{ if } \frac{\mu_\xi - y}{2b} = \frac{\mu_\beta}{y},
\end{aligned}$$

the range of $\lim\limits_{\rho\to-1} K^B$ is $\lim\limits_{\rho\to-1} K^B \in \Big[\frac{\mu_\beta}{y}, \frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi + \mu_\beta}{2b\sigma_\beta/\sigma_\xi + y}\Big)$ when $\frac{\mu_\xi - y}{2b} > \frac{\mu_\beta}{y}$; the range of $\lim\limits_{\rho\to-1} K^B$ is an empty set when $\frac{\mu_\xi - y}{2b} \leq \frac{\mu_\beta}{y}$.

(b) $\frac{y\lim\limits_{\rho\to-1} K^B - \mu_\beta}{\sigma_\beta} < 0 < \frac{\mu_\xi - y - 2b\lim\limits_{\rho\to-1} K^B}{\sigma_\xi}$ and $\frac{\mu_\beta - y\lim\limits_{\rho\to-1} K^B}{\sigma_\beta} < \frac{\mu_\xi - y - 2b\lim\limits_{\rho\to-1} K^B}{\sigma_\xi}$. In this case, $\lim\limits_{\rho\to-1} K^B$ satisfies $\lim\limits_{\rho\to-1} K^B \in \Big[0, \min\Big\{\frac{\mu_\beta}{y}, \frac{\mu_\xi - y}{2b}\Big\}\Big)$ and $\big(2b\sigma_\beta/\sigma_\xi - y\big)\lim\limits_{\rho\to-1} K^B < (\mu_\xi - y)\frac{\sigma_\beta}{\sigma_\xi} - \mu_\beta$. To determine the equivalent range of $\lim\limits_{\rho\to-1} K^B$ for $\big(2b\sigma_\beta/\sigma_\xi - y\big)\lim\limits_{\rho\to-1} K^B < (\mu_\xi - y)\frac{\sigma_\beta}{\sigma_\xi} - \mu_\beta$, there are sub-cases since $2b\sigma_\beta/\sigma_\xi - y$ can be $< 0, = 0$ or > 0 and even if $2b\sigma_\beta/\sigma_\xi - y \neq 0$ the relation among $\frac{\mu_\xi - y}{2b}$, $\frac{\mu_\beta}{y}$ and $\frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi - \mu_\beta}{2b\sigma_\beta/\sigma_\xi - y}$ is worth discussed. Particularly,

$$\begin{aligned}
&\text{if } 2b\sigma_\beta/\sigma_\xi - y > 0 \text{ and } \frac{\mu_\xi - y}{2b} > \frac{\mu_\beta}{y}, \text{ then } \frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi - \mu_\beta}{2b\sigma_\beta/\sigma_\xi - y} > \frac{\mu_\xi - y}{2b} > \frac{\mu_\beta}{y};\\
&\text{if } 2b\sigma_\beta/\sigma_\xi - y < 0 \text{ and } \frac{\mu_\xi - y}{2b} > \frac{\mu_\beta}{y}, \text{ then } \frac{\mu_\xi - y}{2b} > \frac{\mu_\beta}{y} > \frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi - \mu_\beta}{2b\sigma_\beta/\sigma_\xi - y};\\
&\text{if } 2b\sigma_\beta/\sigma_\xi - y > 0 \text{ and } \frac{\mu_\xi - y}{2b} < \frac{\mu_\beta}{y}, \text{ then } \frac{\mu_\beta}{y} > \frac{\mu_\xi - y}{2b} > \frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi - \mu_\beta}{2b\sigma_\beta/\sigma_\xi - y}.\\
&\text{if } 2b\sigma_\beta/\sigma_\xi - y < 0 \text{ and } \frac{\mu_\xi - y}{2b} < \frac{\mu_\beta}{y}, \text{ then } \frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi - \mu_\beta}{2b\sigma_\beta/\sigma_\xi - y} > \frac{\mu_\beta}{y} > \frac{\mu_\xi - y}{2b},
\end{aligned}$$

where the above inequalities is derived by calculating two subtractions: $\frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi - \mu_\beta}{2b\sigma_\beta/\sigma_\xi - y} - \frac{\mu_\beta}{y} = \frac{\left(y(\mu_\xi - y) - 2\mu_\beta\right)\sigma_\beta/\sigma_\xi}{\left(2b\sigma_\beta/\sigma_\xi - y\right)y}$ and $\frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi - \mu_\beta}{2b\sigma_\beta/\sigma_\xi - y} - \left(\frac{\mu_\xi - y}{2b}\right) = \frac{y(\mu_\xi - y) - 2\mu_\beta}{2\left(2b\sigma_\beta/\sigma_\xi - y\right)}$.

In finishing the step 1 of the proof, we sum up Case (a) and (b), that is $\frac{\partial}{\partial\sigma_\xi}\left\{\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\right\}\Big|_{\lim_{\rho\to-1}K^B} < 0$ if and only if

1. $\lim_{\rho\to-1}K^B \in \left[\frac{\mu_\beta}{y}, \frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi + \mu_\beta}{2b\sigma_\beta/\sigma_\xi + y}\right)$ when $\frac{\mu_\xi - y}{2b} > \frac{\mu_\beta}{y}$;

2. $\lim_{\rho\to-1}K^B \in \left(\frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi - \mu_\beta}{2b\sigma_\beta/\sigma_\xi - y}, \frac{\mu_\beta}{y}\right)$ when $\frac{\mu_\xi - y}{2b} > \frac{\mu_\beta}{y}$ and $\sigma_\xi > \frac{2b\sigma_\beta}{y}$;

3. $\lim_{\rho\to-1}K^B \in \left[0, \frac{\mu_\beta}{y}\right)$ when $\frac{\mu_\xi - y}{2b} > \frac{\mu_\beta}{y}$ and $\sigma_\xi \le \frac{2b\sigma_\beta}{y}$;

4. $\lim_{\rho\to-1}K^B \in \left[0, \frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi - \mu_\beta}{2b\sigma_\beta/\sigma_\xi - y}\right)$ when $\frac{\mu_\xi - y}{2b} < \frac{\mu_\beta}{y}$ and $\sigma_\xi < \frac{2b\sigma_\beta}{y}$;

We now generate the range of σ_ξ based on above inequality of $\lim_{\rho\to-1}K^B$ using the information that $\lim_{\rho\to-1}K^B$ is the unique solution of implicit function $\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K} = 0$ and $\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}$ decreases in K. In order to do so, we also need to know how $\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\mu_\beta/y}$, $\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{\mu_\xi - y}{2b}}$ and $\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{(\mu_\xi - y)\sigma_\beta/\sigma_\xi - \mu_\beta}{2b\sigma_\beta/\sigma_\xi - y}}$ change in σ_ξ. Since we are discussing the case that $\frac{\partial}{\partial\sigma_\xi}\left\{\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\right\} < 0$, it is obvious that $\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\mu_\beta/y}$ and $\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{\mu_\xi - y}{2b}}$ decrease in σ_ξ. How

$\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}}$ changes in σ_ξ is derived by following calculation:

$$
\begin{aligned}
&\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}}\\
&=-\omega+\Big(\frac{2b\mu_\beta-(\mu_\xi-y)y}{2b\sigma_\beta/\sigma_\xi-y}\Big)\int_{\frac{(\mu_\xi-y)y-2b\mu_\beta}{2b\sigma_\beta-y\sigma_\xi}}^{\frac{2b\mu_\beta-(\mu_\xi-y)y}{2b\sigma_\beta-y\sigma_\xi}}\phi(z_0)dz_0\\
&=-\omega+\Big(\frac{2b\mu_\beta-(\mu_\xi-y)y}{2b\sigma_\beta/\sigma_\xi-y}\Big)\Big(2\Phi\Big(\frac{2b\mu_\beta-(\mu_\xi-y)y}{2b\sigma_\beta-y\sigma_\xi}\Big)-1\Big)
\end{aligned}
$$

and further $\frac{d}{d\sigma_\xi}\left\{\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}}\right\}=$

$$
\begin{aligned}
&\frac{2b\sigma_\beta\big(2b\mu_\beta-(\mu_\xi-y)y\big)}{\sigma_\xi^2(2b\sigma_\beta/\sigma_\xi-y)^2}\Big(2\Phi\Big(\frac{2b\mu_\beta-(\mu_\xi-y)y}{2b\sigma_\beta-y\sigma_\xi}\Big)-1\Big)\\
&+\frac{2y\big(2b\mu_\beta-(\mu_\xi-y)y\big)^2}{(2b\sigma_\beta/\sigma_\xi-y)^3}\phi\Big(\frac{2b\mu_\beta-(\mu_\xi-y)y}{2b\sigma_\beta-y\sigma_\xi}\Big).
\end{aligned}
$$

From above equation, we obtain two sufficient conditions:

1) $\frac{d}{d\sigma_\xi}\left\{\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}}\right\}>0$ when $\frac{\mu_\beta}{y}\geq\frac{\mu_\xi-y}{2b}$ and $\sigma_\xi<\frac{2b\sigma_\beta}{y}$;

2) $\frac{d}{d\sigma_\xi}\left\{\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}}\right\}<0$ when $\frac{\mu_\beta}{y}\leq\frac{\mu_\xi-y}{2b}$ and $\sigma_\xi>\frac{2b\sigma_\beta}{y}$.

Now, we derive the range of σ_ξ through the range of $\lim_{\rho\to-1}K^B$:

1. In the case '$\lim_{\rho\to-1}K^B\in\Big[\frac{\mu_\beta}{y},\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi+\mu_\beta}{2b\sigma_\beta/\sigma_\xi+y}\Big)$ when $\frac{\mu_\xi-y}{2b}>\frac{\mu_\beta}{y}$', we construct the inequality with respect to σ_ξ in a way that $\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{\mu_\beta}{y}}>0$ is equivalent to $\lim_{\rho\to-1}K^B\in\Big[\frac{\mu_\beta}{y},\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi+\mu_\beta}{2b\sigma_\beta/\sigma_\xi+y}\Big)$ because $\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}$ decreases in K. Define σ_ξ^{iii} as the unique σ_ξ that solves $\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{\mu_\beta}{y}}=0$, due to $\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{\mu_\beta}{y}}$ decreases in σ_ξ, we conclude that when $\frac{\mu_\xi-y}{2b}>\frac{\mu_\beta}{y}$, $\sigma_\xi\in[0,\sigma_\xi^{iii})$ is equivalent to $\lim_{\rho\to-1}K^B\in\Big[\frac{\mu_\beta}{y},\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi+\mu_\beta}{2b\sigma_\beta/\sigma_\xi+y}\Big)$;
2. in the case '$\lim_{\rho\to-1}K^B\in\Big(\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y},\frac{\mu_\beta}{y}\Big)$ when $\frac{\mu_\xi-y}{2b}>\frac{\mu_\beta}{y}$ and $\sigma_\xi>\frac{2b\sigma_\beta}{y}$',
$$\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{\mu_\beta}{y}}<0<\lim_{\rho\to-1}\frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}}$$

is equivalent to $\lim_{\rho\to-1} K^B \in \left(\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}, \frac{\mu_\beta}{y}\right)$. Due to $\frac{d}{d\sigma_\xi}\left\{\lim_{\rho\to-1} \frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}}\right\} < 0$ and $\lim_{\rho\to-1} \frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{\mu_\beta}{y}}$ decreases in σ_ξ, we conclude that there exists a unique σ_ξ defined as $\sigma_\xi^{ii} = \left\{\sigma_\xi \Big| \sigma_\xi > \frac{2b\sigma_\beta}{y}, \lim_{\rho\to-1} \frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}} = 0\right\}$ such that $\sigma_\xi \in [\sigma_\xi^{iii}, \sigma_\xi^{ii}] \cap (2b\sigma_\beta/y, \infty)$ equals to $\lim_{\rho\to-1} K^B \in \left(\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}, \frac{\mu_\beta}{y}\right)$;

3. in the case '$\lim_{\rho\to-1} K^B \in \left[0, \frac{\mu_\beta}{y}\right)$ when $\frac{\mu_\xi-y}{2b} > \frac{\mu_\beta}{y}$ and $\sigma_\xi \le \frac{2b\sigma_\beta}{y}$', $\lim_{\rho\to-1} \frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{\mu_\beta}{y}} < 0$ is equivalent to $\lim_{\rho\to-1} K^B \in \left[0, \frac{\mu_\beta}{y}\right)$. Within $\sigma_\xi \le \frac{2b\sigma_\beta}{y}$, due to $\lim_{\rho\to-1} \frac{\partial\Pi(K)}{\partial K}$ decreases in both K and σ_ξ, we conclude that $\sigma_\xi \in [\sigma_\xi^{iii}, \infty) \cap (0, 2b\sigma_\beta/y]$ is equivalent to $\lim_{\rho\to-1} K^B \in \left[0, \frac{\mu_\beta}{y}\right)$;

4. in the case '$\lim_{\rho\to-1} K^B \in \left[0, \frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}\right)$ when $\frac{\mu_\xi-y}{2b} < \frac{\mu_\beta}{y}$ and $\sigma_\xi < \frac{2b\sigma_\beta}{y}$',
$\lim_{\rho\to-1} \frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}} < 0$ is equivalent to $\lim_{\rho\to-1} K^B \in \left[0, \frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}\right)$.
Due to $\frac{d}{d\sigma_\xi}\left\{\lim_{\rho\to-1} \frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}}\right\} > 0$ and $\lim_{\rho\to-1} \frac{\partial\Pi(K)}{\partial K}$ decreases in σ_ξ, we conclude that there exists a unique σ_ξ defined as
$\sigma_\xi^{i} = \left\{\sigma_\xi \Big| \sigma_\xi < \frac{2b\sigma_\beta}{y}, \lim_{\rho\to-1} \frac{\partial\Pi(K)}{\partial K}\Big|_{K=\frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}} = 0\right\}$ such that $\sigma_\xi < \min\{\sigma_\xi^i, 2b\sigma_\beta/y\}$ is equivalent to $\lim_{\rho\to-1} K^B \in \left[0, \frac{(\mu_\xi-y)\sigma_\beta/\sigma_\xi-\mu_\beta}{2b\sigma_\beta/\sigma_\xi-y}\right)$.

Therefore, applying some algebra we conclude that when $\rho \to -1$,

- under condition $\frac{\mu_\xi-y}{2b} > \frac{\mu_\beta}{y}$, $\lim_{\rho\to-1} K^B$ decreases in σ_ξ when $\sigma_\xi < \max\{\sigma_\xi^{ii}, 2b\sigma_\beta/y\}$; it increases in σ_ξ otherwise;
- under condition $\frac{\mu_\xi-y}{2b} < \frac{\mu_\beta}{y}$, $\lim_{\rho\to-1} K^B$ decreases in σ_ξ when $\sigma_\xi < \min\{\sigma_\xi^{i}, 2b\sigma_\beta/y\}$; it increases in σ_ξ otherwise.

■

Now we demonstrate some characteristics of K^B when $\rho < 0$ to theoretically support Conjecture 1.

Lemma 4 *Define* $\overline{\sigma}_\xi = \omega \Big/ \left(\sqrt{1-\rho^2}\int_{\frac{\frac{y(\mu_\xi - y)}{2b}-\mu_\beta}{\sigma_\beta}}^{\infty} \mathbf{E}\left[\left(\tilde{z}+\frac{\rho z_0}{\sqrt{1-\rho^2}}\right)^+\right]\phi(z_0)dz_0\right)$, *we have*

1. $K^B < \frac{\mu_\xi - y}{2b}$ *when* $\sigma_\xi < \overline{\sigma}_\xi$; $K^B = \frac{\mu_\xi - y}{2b}$ *when* $\sigma_\xi = \overline{\sigma}_\xi$; *and* $K^B > \frac{\mu_\xi - y}{2b}$ *when* $\sigma_\xi > \overline{\sigma}_\xi$;

2. K^B *increases in* σ_ξ *when* $\sigma_\xi \geq \overline{\sigma}_\xi$;

3. $\lim_{\sigma_\xi \to 0} \frac{dK^B}{d\sigma_\xi} < 0$ *when* $\rho < 0$.

Proof of Lemma 4:

First part of the result is derived by calculating the first order derivative $\frac{\partial \Pi(K)}{\partial K}\Big|_{\frac{\mu_\xi - y}{2b}}$ from Equation (A.2), $\frac{\partial \Pi(K)}{\partial K}\Big|_{K=\frac{\mu_\xi - y}{2b}} =$

$$-\omega + \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}$$

$$\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - y - 2bK\right)\phi(z_2)dz_2\phi(z_0)dz_0\Bigg|_{K=\frac{\mu_\xi - y}{2b}}$$

$$= -\omega + \sigma_\xi\sqrt{1-\rho^2}\int_{\frac{\frac{y(\mu_\xi - y)}{2b}-\mu_\beta}{\sigma_\beta}}^{\infty}\mathbf{E}\left[\left(\tilde{z}+\frac{\rho z_0}{\sqrt{1-\rho^2}}\right)^+\right]\phi(z_0)dz_0,$$

where it shows that $\frac{\partial \Pi(K)}{\partial K}\Big|_{K=\frac{\mu_\xi - y}{2b}} = 0$ when $\sigma_\xi = \overline{\sigma}_\xi$. Therefore, we proved the first part: $K^B > \frac{\mu_\xi - y}{2b}$ when $\sigma_\xi > \overline{\sigma}_\xi$ and $K^B \leq \frac{\mu_\xi - y}{2b}$ when $\sigma_\xi \leq \overline{\sigma}_\xi$, because $\frac{\partial \Pi(K)}{\partial K}$ is decreasing in K.

Now, we prove the second part of the result. First of all, by applying $\phi(x) - x\big(1-\Phi(x)\big) = \mathbf{E}[(\tilde{z}-x)^+]$ to Equation (A.3), we have $\frac{\partial^2 \Pi(K)}{\partial K \partial \sigma_\xi} =$

$$\sqrt{1-\rho^2}\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\mathbf{E}\left[\left(\tilde{z}_2 - \Big(\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\right)^+\right]\phi(z_0)dz_0$$

$$+\sqrt{1-\rho^2}\Big(\frac{y+2bK-\mu_\xi}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\left(1-\Phi\Big(\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\right)\phi(z_0)dz_0$$

meaning that $\frac{\partial^2 \Pi(K)}{\partial K \partial \sigma_\xi} > 0$ for all $K > \frac{\mu_\xi - y}{2b}$, or equivalently, K^B increases in σ_ξ when $\sigma_\xi \geq \overline{\sigma}_\xi$.

Thirdly, to prove that $\lim\limits_{\sigma_\xi \to 0} \frac{\partial^2 \Pi(K)}{\partial K \partial \sigma_\xi} < 0$ for all $K < \frac{\mu_\xi - y}{2b}$ when $\rho < 0$, using Equation (A.4), we calculate $\lim\limits_{\sigma_\xi \to 0} \frac{\partial^2 \Pi(K)}{\partial K \partial \sigma_\xi} =$

$$\lim_{\sigma_\xi \to 0} \phi\Big(\frac{y + 2bK - \mu_\xi}{\sigma_\xi}\Big) \lim_{\sigma_\xi \to 0} \Phi\Big(\frac{\rho\sigma_\beta(y + 2bK - \mu_\xi) - \sigma_\xi(yK - \mu_\beta)}{\sigma_\xi\sigma_\beta\sqrt{1-\rho^2}}\Big)$$
$$+ \rho\phi\Big(\frac{yK - \mu_\beta}{\sigma_\beta}\Big) \lim_{\sigma_\xi \to 0} \Phi\Big(\frac{\rho\sigma_\xi(yK - \mu_\beta) - \sigma_\beta(y + 2bK - \mu_\xi)}{\sigma_\xi\sigma_\beta\sqrt{1-\rho^2}}\Big),$$

therein, when $K < \frac{\mu_\xi - y}{2b}$, $\lim\limits_{\sigma_\xi \to 0} \frac{\partial^2 \Pi(K)}{\partial K \partial \sigma_\xi} = \rho\phi\Big(\frac{yK - \mu_\beta}{\sigma_\beta}\Big)$. Therefore, when $\rho < 0$, $\lim\limits_{\sigma_\xi \to 0} \frac{\partial^2 \Pi(K)}{\partial K \partial \sigma_\xi} < 0$ for all $K < \frac{\mu_\xi - y}{2b}$ including $\lim\limits_{\sigma_\xi \to 0} K^B$. ■

Proof of Proposition 4:

The impact of σ_ξ on $\Pi(K^B)$ is proved using The Envelope Theorem, the sensitivity result for $\Pi(K^B)$ is gotten from $\frac{\partial \Pi(K)}{\partial \sigma_\xi}\Big|_{K^B}$. We calculate

$$\frac{\partial \Pi(K)}{\partial \sigma_\xi} = \sum_{i=1}^{4} \frac{\partial V_i(K)}{\partial \sigma_\xi} \tag{A.6}$$

from Equation (A.1), because the derivative of the integral boundaries are cancelled out, we have $\frac{\partial \Pi(K)}{\partial \sigma_\xi} =$

$$\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)^+}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\Big(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - y\Big)}{2b}$$
$$\big(z_2\sqrt{1-\rho^2} + \rho z_0\big)\phi(z_2)dz_2\phi(z_0)dz_0$$
$$+ \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\Big(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - y\Big)}{2b}$$
$$\big(z_2\sqrt{1-\rho^2} + \rho z_0\big)\phi(z_2)dz_2\phi(z_0)dz_0$$
$$+ \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} K\big(z_2\sqrt{1-\rho^2} + \rho z_0\big)\phi(z_2)dz_2\phi(z_0)dz_0$$
$$+ \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \frac{(z_0\sigma_\beta + \mu_\beta)}{y}\big(z_2\sqrt{1-\rho^2} + \rho z_0\big)\phi(z_2)dz_2\phi(z_0)dz_0.$$

Through some algebra, we have

$$\begin{aligned}\frac{\partial\Pi(K)}{\partial\sigma_\xi} =&\sqrt{1-\rho^2}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\int_{-\infty}^{\infty} z_2 q^*(z_2,z_0)\phi(z_2)dz_2\phi(z_0)dz_0\\ &+\sqrt{1-\rho^2}\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\int_{-\infty}^{\infty} z_2 q^*(z_2,z_0)\phi(z_2)dz_2\phi(z_0)dz_0\\ &+\rho\mathop{\mathbf{E}}_{z_0}\Big[\tilde{z}_0\mathop{\mathbf{E}}_{z_2}\big[q^*(\tilde{z}_2,\tilde{z}_0)\big|\tilde{z}_0\big]\Big].\end{aligned}$$

Therein, $q^*(\tilde{z}_2,\tilde{z}_0)$ one to one corresponds to the optimal production quantity $q^*(\tilde{\xi},\tilde{\beta})$ where the normal distribution transformation $\tilde{z}_2 \overset{d}{=} \frac{\tilde{\xi}-\mu_\xi-\rho\frac{\sigma_\xi}{\sigma_\beta}(\tilde{\beta}-\mu_\beta)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big|\tilde{\beta}$ and $\tilde{z}_0 \overset{d}{=} \frac{\tilde{\beta}-\mu_\beta}{\sigma_\beta}$ are made. According to the Stein's Lemma, above equation becomes

$$\begin{aligned}\frac{\partial\Pi(K)}{\partial\sigma_\xi} =&\frac{\sigma_\xi}{2b}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\int_{\frac{y-\mu_\xi}{\sigma_\xi\sqrt{1-\rho^2}}-\frac{\rho z_0}{\sqrt{1-\rho^2}}}^{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)^+}{y}-\mu_\xi}{\sigma_\xi\sqrt{1-\rho^2}}-\frac{\rho z_0}{\sqrt{1-\rho^2}}} \phi(z_2)dz_2\phi(z_0)dz_0\\ &+\frac{\sigma_\xi}{2b}\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\int_{\frac{y-\mu_\xi}{\sigma_\xi\sqrt{1-\rho^2}}-\frac{\rho z_0}{\sqrt{1-\rho^2}}}^{\frac{y+2bK-\mu_\xi}{\sigma_\xi\sqrt{1-\rho^2}}-\frac{\rho z_0}{\sqrt{1-\rho^2}}} \phi(z_2)dz_2\phi(z_0)dz_0 \qquad (A.7)\\ &+\frac{\rho\sigma_\beta}{y}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi}{\sigma_\xi\sqrt{1-\rho^2}}-\frac{\rho z_0}{\sqrt{1-\rho^2}}}^{\infty} \phi(z_2)dz_2\phi(z_0)dz_0.\end{aligned}$$

From above equation, the immediate result is that $\frac{\partial\Pi(K)}{\partial\sigma_\xi} > 0$ for all K when $\rho \geq 0$.

Then we proof the impact of σ_ξ on $\Pi(K^B)$ under condition $\rho < 0$ through proving following three parts: i) $\frac{\partial\Pi(K)}{\partial\sigma_\xi}\Big|_{K^B}$ increases in σ_ξ that is $\frac{d}{d\sigma_\xi}\left\{\frac{\partial\Pi(K)}{\partial\sigma_\xi}\Big|_{K^B}\right\} > 0$, ii) $\frac{\partial\Pi(K)}{\partial\sigma_\xi}\Big|_{K^B}$ is negative when $\sigma_\xi \to 0$, specifically, we prove $\lim\limits_{\sigma_\xi\to 0}\frac{\partial\Pi(K)}{\partial\sigma_\xi} < 0$ $\forall K \in (0,\infty)$ and iii) $\frac{\partial\Pi(K)}{\partial\sigma_\xi}\Big|_{K^B}$ is positive when σ_ξ is large enough, that is $\lim\limits_{\sigma_\xi\to\infty}\frac{\partial\Pi(K)}{\partial\sigma_\xi} > 0$ $\forall K \in (0,\infty)$.

Prove of part i): First of all, due to

$$\frac{d}{d\sigma_\xi}\left\{\frac{\partial\Pi(K)}{\partial\sigma_\xi}\Big|_{K^B}\right\} = \frac{\partial^2\Pi(K)}{\partial\sigma_\xi^2}\Big|_{K^B} + \left(\frac{\partial^2\Pi(K)}{\partial\sigma_\xi\partial K}\Big|_{K^B}\right)\cdot\frac{\partial K^B}{\partial\sigma_\xi},$$

we first calculate from Equation (A.6) that $\frac{\partial^2 \Pi(K)}{\partial \sigma_\xi^2} =$

$$\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)^+}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)^2}{2b}\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)^2}{2b}\phi(z_2)dz_2\phi(z_0)dz_0 > 0,$$

where there is no term from differentiating the limits of integration, because the integrand $\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)q^*(K, z_2, z_0)$ is continuous. Furthermore, $\left(\frac{\partial^2\Pi(K)}{\partial\sigma_\xi\partial K}\Big|_{K^B}\right)\cdot \frac{\partial K^B}{\partial \sigma_\xi} \geq 0$, because $\frac{\partial^2\Pi(K)}{\partial\sigma_\xi\partial K}\Big|_{K^B}$ and $\frac{\partial K^B}{\partial\sigma_\xi}$ have same sign by implicit differentiation $\frac{dK^B}{d\sigma_\xi} = -\left(\left(\frac{\partial^2\Pi}{\partial K\partial\sigma_\xi}\right) \Big/ \left(\frac{\partial^2\Pi}{\partial K^2}\right)\right)\Big|_{K^B}$. Therefore, $\frac{d}{d\sigma_\xi}\left\{\frac{\partial\Pi(K)}{\partial\sigma_\xi}\Big|_{K^B}\right\} > 0$.

Prove of part ii): we take the limits $\lim_{\sigma_\xi\to 0}\frac{\partial\Pi(K)}{\partial\sigma_\xi}$. Define $x_2 = \sigma_\xi\cdot\left(z_2+\frac{\rho z_0}{\sqrt{1-\rho^2}}\right)$ and apply the transformation to first two terms of Equation (A.7), we obtain that

$$\begin{aligned}\frac{\partial\Pi(K)}{\partial\sigma_\xi} =& \frac{1}{2b}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y-\mu_\xi}{\sqrt{1-\rho^2}}}^{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)^+}{y}-\mu_\xi}{\sqrt{1-\rho^2}}} \phi\left(x_2/\sigma_\xi-\frac{\rho z_0}{\sqrt{1-\rho^2}}\right)dx_2\phi(z_0)dz_0\\ &+\frac{1}{2b}\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\int_{\frac{y-\mu_\xi}{\sqrt{1-\rho^2}}}^{\frac{y+2bK-\mu_\xi}{\sqrt{1-\rho^2}}} \phi\left(x_2/\sigma_\xi-\frac{\rho z_0}{\sqrt{1-\rho^2}}\right)dx_2\phi(z_0)dz_0\\ &+\frac{\rho\sigma_\beta}{y}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi}{\sigma_\xi\sqrt{1-\rho^2}}-\frac{\rho z_0}{\sqrt{1-\rho^2}}}^{\infty} \phi(z_2)dz_2\phi(z_0)dz_0.\end{aligned} \tag{A.8}$$

Therefore, we have

$$\begin{aligned}\lim_{\sigma_\xi\to 0}\frac{\partial\Pi(K)}{\partial\sigma_\xi} =& \frac{\rho\sigma_\beta}{y}\lim_{\sigma_\xi\to 0}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi}{\sigma_\xi\sqrt{1-\rho^2}}-\frac{\rho z_0}{\sqrt{1-\rho^2}}}^{\infty}\phi(z_2)dz_2\phi(z_0)dz_0\\ =& \frac{\rho\sigma_\beta}{y}\int_{-\infty}^{\frac{y\min\{K,\frac{\mu_\xi-y}{2b}\}-\mu_\beta}{\sigma_\beta}}\phi(z_0)dz_0 < 0,\end{aligned}$$

which implies that $\lim_{\sigma_\xi\to 0}\frac{\partial\Pi(K)}{\partial\sigma_\xi} < 0$ for all $K > 0$.

Prove of part iii): Lastly, given Equation (A.8), we take the limit $\lim_{\sigma_\xi\to\infty}\left\{\frac{\partial\Pi(K)}{\partial\sigma_\xi}\right\}$.

We have $\lim\limits_{\sigma_\xi \to \infty} \frac{\partial \Pi(K)}{\partial \sigma_\xi} =$

$$\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \frac{(z_0\sigma_\beta + \mu_\beta)^+/y}{\sqrt{1-\rho^2}} \phi\Big(\frac{\rho z_0}{\sqrt{1-\rho^2}}\Big)\phi(z_0)dz_0 + \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \frac{K}{\sqrt{1-\rho^2}}\phi\Big(\frac{\rho z_0}{\sqrt{1-\rho^2}}\Big)\phi(z_0)dz_0$$
$$+ \frac{\rho\sigma_\beta}{y}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \Phi\Big(\frac{\rho z_0}{\sqrt{1-\rho^2}}\Big)\phi(z_0)dz_0.$$

In addition,

$$\frac{\partial}{\partial K}\Big\{\lim_{\sigma_\xi \to \infty} \frac{\partial \Pi(K)}{\partial \sigma_\xi}\Big\}$$
$$= \frac{1}{\sqrt{1-\rho^2}}\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \phi\Big(\frac{-\rho z_0}{\sqrt{1-\rho^2}}\Big)\phi(z_0)dz_0 + \rho\Bigg(1-\Phi\Big(-\frac{\rho}{\sqrt{1-\rho^2}}\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\Big)\Bigg)\phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)$$
$$= \frac{1}{\sqrt{1-\rho^2}}\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \phi\Big(\frac{-\rho z_0}{\sqrt{1-\rho^2}}\Big)\phi(z_0)dz_0 - \rho\Bigg\{\Bigg(1-\Phi\Big(-\frac{\rho z_0}{\sqrt{1-\rho^2}}\Big)\Bigg)\phi(z_0)\Bigg\}\Bigg|_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}$$
$$= \sqrt{1-\rho^2}\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \Bigg(\phi\Big(\frac{-\rho z_0}{\sqrt{1-\rho^2}}\Big) - \Big(\frac{-\rho z_0}{\sqrt{1-\rho^2}}\Big)\Bigg(1-\Phi\Big(\frac{-\rho z_0}{\sqrt{1-\rho^2}}\Big)\Bigg)\Bigg)\phi(z_0)dz_0 > 0,$$

because the last expression has structure $\phi(x) - x\big(1-\Phi(x)\big) = \mathbf{E}[(\tilde{z}-x)^+] > 0$. Due to $\lim\limits_{\sigma_\xi \to \infty} \frac{\partial \Pi(K)}{\partial \sigma_\xi}\Big|_{K=0} = 0$ and $\frac{\partial}{\partial K}\Big\{\lim\limits_{\sigma_\xi \to \infty} \frac{\partial \Pi(K)}{\partial \sigma_\xi}\Big\} > 0$, we can conclude that $\lim\limits_{\sigma_\xi \to \infty} \frac{\partial \Pi(K)}{\partial \sigma_\xi}$ is positive for all $K \in (0, \infty)$. ■

Proofs for the σ_β Results

Proof of Proposition 5:

The impact of σ_β on K^B is derived by implicit differentiation, $\frac{dK^B}{d\sigma_\beta} = -\Big(\frac{\partial^2\Pi}{\partial K \partial \sigma_\beta}\Big/\frac{\partial^2 \Pi}{\partial K^2}\Big)\Big|_{K^B}$. We calculate $\frac{\partial^2 \Pi(K)}{\partial K \partial \sigma_\beta}$ by taking derivative of Equation (A.2) with respect to σ_β. The calculation is according to Leibniz' formula and due to σ_β only appears at the lower bound of outer integration, we have $\frac{\partial^2 \Pi(K)}{\partial K \partial \sigma_\beta} =$

$$\frac{(yK-\mu_\beta)}{\sigma_\beta^2}\phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\sqrt{\sigma_\xi^2(1-\rho^2)}\mathbf{E}\Bigg[\Bigg(\tilde{z}_2 - \Bigg(\frac{y+2bK-\mu_\xi-\rho\sigma_\xi\big(\frac{yK-\mu_\beta}{\sigma_\beta}\big)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Bigg)\Bigg)^+\Bigg].$$

The immediate conclusion from above equation is that $\frac{\partial^2 \Pi(K)}{\partial K \partial \sigma_\beta} < 0$ when $yK < \mu_\beta$; $\frac{\partial^2 \Pi(K)}{\partial K \partial \sigma_\beta} = 0$ when $yK = \mu_\beta$; and $\frac{\partial^2 \Pi(K)}{\partial K \partial \sigma_\beta} > 0$ when $yK > \mu_\beta$. Since these conditions depend on K, we then apply the optimality condition such that K is substituted with K^B for these conditions. The necessary and sufficient condition of $K^B < \mu_\beta/y$, $K^B = \mu_\beta/y$ and $K^B > \mu_\beta/y$ are $\frac{\partial\Pi(K)}{\partial K}\Big|_{\mu_\beta/y} < 0$, $\frac{\partial\Pi(K)}{\partial K}\Big|_{\mu_\beta/y} = 0$ and $\frac{\partial\Pi(K)}{\partial K}\Big|_{\mu_\beta/y} > 0$ respectively, because of the optimality condition $\frac{\partial\Pi(K)}{\partial K}\Big|_{K^B} = 0$ and $\frac{\partial\Pi(K)}{\partial K}$ decreasing in K. To write above conditions clearly, we further investigate $\frac{\partial\Pi(K)}{\partial K}\Big|_{\frac{\mu_\beta}{y}} =$

$$-\omega + \int_0^\infty \int_{\frac{y+2b\mu_\beta/y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^\infty \left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - y - 2b\mu_\beta/y\right)\phi(z_2)dz_2\phi(z_0)dz_0$$

doesn't depend on σ_β. Define $\omega_\beta^K(\rho) :=$

$$\int_0^\infty \int_{\frac{y+2b\mu_\beta/y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^\infty \left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - y - 2b\mu_\beta/y\right)\phi(z_2)dz_2\phi(z_0)dz_0,$$

which denotes $\frac{\partial\Pi(K)}{\partial K}\Big|_{\frac{\mu_\beta}{y}} = -\omega + \omega_\beta^K(\rho)$. Therefore, we have

$$\begin{cases} \dfrac{dK^B}{d\sigma_\beta} < 0 \text{ i.e. } \dfrac{\partial^2\Pi(K)}{\partial K\partial\sigma_\beta}\Big|_{K=K^B} < 0, & \text{if } \omega > \omega_\beta^K(\rho) \\ K^B \text{ is not sensitive to } \sigma_\beta, & \text{if } \omega = \omega_\beta^K(\rho) \\ \dfrac{dK^B}{d\sigma_\beta} > 0 \text{ i.e. } \dfrac{\partial^2\Pi(K)}{\partial K\partial\sigma_\beta}\Big|_{K=K^B} > 0, & \text{if } \omega < \omega_\beta^K(\rho). \end{cases} \tag{A.9}$$

Since $\rho \in (-1, 1)$, we identify the range of $\omega_\beta^K(\rho)$ according to the domain. We obtain that $\omega_\beta^K(\rho)$ increases in ρ from $\frac{\partial^2\Pi(K)}{\partial K\partial\rho} > 0$ for all K (result in part 1 of Proof of Proposition 1), that is $\frac{\partial\Pi(K)}{\partial K}\Big|_{\frac{\mu_\beta}{y}} = -\omega + \omega_\beta^K(\rho)$ also increases in ρ. Pursuing this further, we define $\omega_\beta^K(-1) = \lim_{\rho\to-1} \omega_\beta^K$ and $\omega_\beta^K(1) = \lim_{\rho\to1} \omega_\beta^K$, specifically,

$$\omega_\beta^K(-1) = \sigma_\xi \int_0^{\max\left\{0, \frac{\mu_\xi-y-2b\mu_\beta/y}{\sigma_\xi}\right\}} \left(\left(\frac{\mu_\xi - y - 2b\mu_\beta/y}{\sigma_\xi}\right) - z_0\right)\phi(z_0)dz_0$$

and

$$\omega_\beta^K(1) = \sigma_\xi \int_{\max\left\{0, \frac{y+2b\mu_\beta/y-\mu_\xi}{\sigma_\xi}\right\}}^\infty \left(z_0 - \left(\frac{y + 2b\mu_\beta/y - \mu_\xi}{\sigma_\xi}\right)\right)\phi(z_0)dz_0$$

such that we identify the range of $\omega_\beta^K(\rho)$ is $\omega_\beta^K(\rho) \in (\omega_\beta^K(-1), \omega_\beta^K(1))$. Then, we derive ρ threshold when $\omega \in (\omega_\beta^K(-1), \omega_\beta^K(1))$, specifically the threshold denoted by $\rho_\beta^K(\omega)$ is the unique solution that solves

$$\omega = \left\{ \sigma_\xi \sqrt{(1-\rho^2)} \int_0^\infty \mathbf{E}\left[\left(\tilde{z}_2 - \left(\frac{y + 2b\mu_\beta/y - \mu_\xi - \rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}} \right) \right)^+ \right] \phi(z_0) dz_0 \right\} \Bigg|_{\rho=\rho_\beta^K(\omega)}.$$

Recalling the conditions wrote in Equation (A.9), we conclude the sensitivity result for all $\omega \in (0, \omega_{\max})$

1. if $\omega > \omega_\beta^K(1)$, $\frac{dK^B}{d\sigma_\beta} < 0$ for any ρ.

2. if $\omega < \omega_\beta^K(-1)$, $\frac{dK^B}{d\sigma_\beta} > 0$ for any ρ.

3. if $\omega \in (\omega_\beta^K(-1), \omega_\beta^K(1))$, we have: under condition $\rho < \rho_\beta^K(\omega)$, $\frac{dK^B}{d\sigma_\beta} > 0$; under condition $\rho = \rho_\beta^K(\omega)$, $K^B = \mu_\beta/y$ which is not sensitive to σ_β; and under condition $\rho > \rho_\beta^K(\omega)$, $\frac{dK^B}{d\sigma_\beta} < 0$.

■

Proof of Proposition 6:

The impact of σ_β on $\Pi(K^B)$ is derived according to the Envelope Theorem, that is $\frac{d\Pi(K^B(\omega))}{d\sigma_\beta} = \frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^B}$. We take derivative of Equation (A.1) with respect to σ_β such that the remaining term is the one that contains σ_β in integrand, $\frac{\partial\Pi(K)}{\partial\sigma_\beta} =$

$$\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \frac{z_0}{y} \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y-2b\Big(\frac{z_0\sigma_\beta+\mu_\beta}{y}\Big)\right)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$=\sqrt{\sigma_\xi^2(1-\rho^2)}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \frac{z_0}{y}\mathbf{E}\left[\left(\tilde{z}_2-\Big(\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\right)^+\right]\phi(z_0)dz_0. \tag{A.10}$$

Under condition $\omega \geq \omega_\beta^K(\rho)$, we prove that $\Pi(K^B)$ decreases in σ_β. Observing from Equation (A.10), $\frac{\partial\Pi(K)}{\partial\sigma_\beta} < 0$ when $yK \leq \mu_\beta$, because the integrand is non-positive for all $z_0 \in \big(-\infty, (yK-\mu_\beta)/\sigma_\beta\big]$. Recalling from the proof of Proposition 5 that the necessary and sufficient condition for $K^B \leq \mu_\beta/y$ to be true is $\frac{\partial\Pi(K)}{\partial K}\Big|_{\mu_\beta/y} = -\omega+\omega_\beta^K(\rho) \leq 0$, we conclude that under condition $\omega \geq \omega_\beta^K(\rho)$, $\Pi(K^B)$ decreases in σ_β.

Under condition $\omega < \omega_\beta^K(\rho)$, we derive the result on how $\Pi(K^B)$ changes in σ_β step by step. Noting that $\omega < \omega_\beta^K(\rho)$ is equivalent to $yK^B(\omega) > \mu_\beta$, we use these two forms of the condition interchangeably. Firstly, we prove that $\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^B} < 0$ when σ_β is large enough, specifically $\sigma_\beta \geq \frac{\rho y\sigma_\xi}{2b}$. Define $g(z_0) := \mathbf{E}\left[\left(\tilde{z}_2-\Big(\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\right)^+\right]$, we have $g(z_0)$ decreases in z_0 if

$\sigma_\beta \ge \frac{\rho y \sigma_\xi}{2b}$. As a result, if $\sigma_\beta \ge \frac{\rho y \sigma_\xi}{2b}$, we obtain

$$\begin{aligned}\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B} =&\sqrt{\sigma_\xi^2(1-\rho^2)}\Big(\int_{-\infty}^{0}\frac{z_0}{y}g(z_0)\phi(z_0)dz_0+\int_0^{\frac{yK^B-\mu_\beta}{\sigma_\beta}}\frac{z_0}{y}g(z_0)\phi(z_0)dz_0\Big)\\ <&\sqrt{\sigma_\xi^2(1-\rho^2)}g(0)\Big(\int_{-\infty}^{0}\frac{z_0}{y}\phi(z_0)dz_0+\int_0^{\frac{yK^B-\mu_\beta}{\sigma_\beta}}\frac{z_0}{y}\phi(z_0)dz_0\Big)\\ =&\frac{\sqrt{\sigma_\xi^2(1-\rho^2)}g(0)}{y}\Big(-\phi\Big(\frac{yK^B-\mu_\beta}{\sigma_\beta}\Big)\Big)<0.\end{aligned}$$

It is worth note that 1) this result holds for all $\omega < \omega_{\max}$, not just $\omega < \omega_\beta^K(\rho)$; 2) $\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B} < 0$ for all $\sigma_\beta \ge 0$ when $\rho \le 0$.

Secondly, we prove $\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B}$ decreases in σ_β when $yK^B(\omega) > \mu_\beta$ as follows, $\frac{d}{d\sigma_\beta}\Big\{\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B}\Big\} =$

$$\begin{aligned}&\Big(\frac{yK^B-\mu_\beta}{\sigma_\beta^2}\Big)\Big(-\frac{(K^B-\mu_\beta/y)}{\sigma_\beta}+\frac{dK^B}{d\sigma_\beta}\Big)\phi\Big(\frac{yK^B-\mu_\beta}{\sigma_\beta}\Big)\cdot\\ &\quad\int_{\frac{y+2bK^B-\mu_\xi-\rho\sigma_\xi\left(\frac{yK^B-\mu_\beta}{\sigma_\beta}\right)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\Big(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi\Big(\frac{yK^B-\mu_\beta}{\sigma_\beta}\Big)-y-2bK^B\Big)\phi(z_2)dz_2\\ &-2\int_{-\infty}^{\frac{yK^B-\mu_\beta}{\sigma_\beta}}\Big(\frac{z_0}{y}\Big)^2\int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\phi(z_2)dz_2\phi(z_0)dz_0\end{aligned}$$

where the second term is apparently negative and the first term is also negative if $\frac{(K^B-\mu_\beta/y)}{\sigma_\beta}-\frac{dK^B}{d\sigma_\beta} > 0$. Further, we prove $\frac{(K^B-\mu_\beta/y)}{\sigma_\beta}-\frac{dK^B}{d\sigma_\beta} > 0$ by writing the expression of $\frac{dK^B}{d\sigma_\beta}$ according to $\frac{dK^B}{d\sigma_\beta} = \Big(\Big(\frac{\partial^2\Pi}{\partial K\partial\sigma_\beta}\Big)\Big/\Big(-\frac{\partial^2\Pi}{\partial K^2}\Big)\Big)\Big|_{K^B}$. Since $\frac{\partial^2\Pi(K)}{\partial K\partial\sigma_\beta}\big|_{K^B}$ is known, we calculate

$$\begin{aligned}\frac{\partial^2\Pi(K)}{\partial K^2}=&-2\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\phi(z_2)dz_2\phi(z_0)dz_0\\ &-\frac{y\sqrt{\sigma_\xi^2(1-\rho^2)}}{\sigma_\beta}\phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\mathbf{E}\Big[\Big(\tilde{z}_2-\Big(\frac{y+2bK-\mu_\xi-\rho\sigma_\xi\left(\frac{yK-\mu_\beta}{\sigma_\beta}\right)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big],\end{aligned}$$

and define $T_B(K) = \frac{y\sqrt{\sigma_\xi^2(1-\rho^2)}}{\sigma_\beta}\phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\mathbf{E}\Big[\Big(\tilde{z}_2-\Big(\frac{y+2bK-\mu_\xi-\rho\sigma_\xi\left(\frac{yK-\mu_\beta}{\sigma_\beta}\right)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]$, as a result, $\frac{dK^B}{d\sigma_\beta}$ is written as follows:

$$\frac{dK^B}{d\sigma_\beta} = \frac{\frac{(K^B-\mu_\beta/y)}{\sigma_\beta}\cdot T_B(K^B)}{T_B(K^B) + 2\int_{\frac{yK^B-\mu_\beta}{\sigma_\beta}}^{\infty}\int_{\frac{y+2bK^B-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\phi(z_2)dz_2\phi(z_0)dz_0} < \frac{K^B-\mu_\beta/y}{\sigma_\beta},$$

which proves that $\frac{d}{d\sigma_\beta}\left\{\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^B}\right\} < 0$ when $yK^B(\omega) > \mu_\beta$.

Thirdly, summarizing results above: 1) $\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^B} < 0$ when $\sigma_\beta \geq \frac{\rho y\sigma_\xi}{2b}$ and 2) $\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^B}$ decreases in σ_β when $\omega \in \big(0, \omega_\beta^K(\rho)\big)$, what we need to uncover is the impact of $\sigma_\beta \in (0, \frac{\rho y\sigma_\xi}{2b})$ on $\Pi(K^B)$ under condition $\rho > 0$ for all $\omega \in \big(0, \omega_\beta^K(\rho)\big)$.

1. When $\omega = \omega_\beta^K$, we already know that $\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K=\mu_\beta/y} < 0$ for all σ_β;

2. When $\omega \to 0$, we know that $K^B \to \infty$. The sign of $\lim_{K\to\infty}\frac{\partial\Pi(K)}{\partial\sigma_\beta}$ is calculated from Equation (A.10) as follows, $\lim_{K\to\infty}\frac{\partial\Pi(K)}{\partial\sigma_\beta} =$

$$\lim_{K\to\infty}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\frac{z_0}{y}\sqrt{\sigma_\xi^2(1-\rho^2)}\mathbf{E}\left[\left(\tilde{z}_2 - \left(\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\right)\right)^+\right]\phi(z_0)dz_0$$

$$= \int_{-\infty}^{\infty}\frac{z_0}{y}\sqrt{\sigma_\xi^2(1-\rho^2)}\mathbf{E}\left[\left(\tilde{z}_2 - \left(\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\right)\right)^+\right]\phi(z_0)dz_0.$$

Since $g(z_0) > 0$ increases in z_0 when $\sigma_\beta \in (0, \frac{\rho\sigma_\xi y}{2b})$, we obtain

$$\lim_{K\to\infty}\frac{\partial\Pi(K)}{\partial\sigma_\beta} = \int_{-\infty}^{\infty}\frac{z_0}{y}\sqrt{\sigma_\xi^2(1-\rho^2)}g(z_0)\phi(z_0)dz_0$$
$$> \sqrt{\sigma_\xi^2(1-\rho^2)}g(0)\Big(\int_{-\infty}^{0}\frac{z_0}{y}\phi(z_0)dz_0 + \int_0^{\infty}\frac{z_0}{y}\phi(z_0)dz_0\Big) = 0.$$

To sum up, $\lim_{K\to\infty}\frac{\partial\Pi(K)}{\partial\sigma_\beta} > 0$ when $\sigma_\beta \in (0, \frac{\rho\sigma_\xi y}{2b})$ and $\lim_{K\to\infty}\frac{\partial\Pi(K)}{\partial\sigma_\beta} \leq 0$ when $\sigma_\beta \in [\frac{\rho\sigma_\xi y}{2b}, \infty)$.

3. When ω is in between of 0 and $\omega_\beta^K(\rho)$, we prove $\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^B(\omega)}$ decreases in $\omega \in \big(0, \omega_\beta^K(\rho)\big)$ for any fixed σ_β. Firstly, we know that $\frac{\partial\Pi(K)}{\partial\sigma_\beta}$ increases in K when $K > \mu_\beta/y$, for details referring to the proof of Proposition 5. We also know from Proposition 5 that $K^B > \mu_\beta/y$ when $\omega \in \big(0, \omega_\beta^K(\rho)\big)$. Thirdly, for all fixed σ_β, K^B decreases in ω by optimality condition for solving K^B. Lastly, we also know that $\frac{\partial\Pi(K)}{\partial\sigma_\beta}$ is irrelevant to ω from Equation (A.10).

Therefore, $\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B(\omega)}$ decreases in $\omega \in \left(0, \omega_\beta^K(\rho)\right)$ for any fixed σ_β.

Having all above information, we can conclude that given $\rho > 0$, as ω increases from 0 to $\omega_\beta^K(\rho)$, there exists a threshold $\omega_\beta^\Pi(\rho) \in \left(0, \omega_\beta^K(\rho)\right)$ such that when $\omega \geq \omega_\beta^\Pi(\rho)$ $\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B(\omega)} < 0$ for all σ_β; and when $\omega < \omega_\beta^\Pi(\rho)$, there exists a σ_β threshold $\sigma_\beta^\Pi(\omega, \rho) \in (0, \frac{\rho\sigma_\xi y}{2b})$ such that $\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B(\omega)} > 0$ when $\sigma_\beta < \sigma_\beta^\Pi(\omega, \rho)$; and $\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B(\omega)} < 0$ when $\sigma_\beta > \sigma_\beta^\Pi(\omega, \rho)$. In addition, $\sigma_\beta^\Pi(\omega, \rho)$ is decreasing in ω.

We further investigate the characteristic of $\omega_\beta^\Pi(\rho)$ given domain $\rho \in (0, 1)$. First of all, $\omega_\beta^\Pi(\rho)$ is solved by the unique solution of implicit equation $\lim\limits_{\sigma_\beta \to 0}\left\{\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B}\right\} = 0$ given $\rho > 0$; secondly, $\omega_\beta^\Pi(0) = 0$; thirdly, $\omega_\beta^\Pi(\rho)$ increases ρ, because $\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B}$ decreases in ω for all $\sigma_\beta \geq 0$ and $\lim\limits_{\sigma_\beta \to 0}\left\{\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B}\right\}$ increases in ρ, therein, the decreasing trend of $\lim\limits_{\sigma_\beta \to 0}\left\{\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B}\right\}$ in ρ is proved through following steps: $\frac{d}{d\rho}\left\{\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B}\right\} = \frac{\partial^2 \Pi(K)}{\partial \sigma_\beta \partial \rho}\Big|_{K^B} + \frac{\partial^2 \Pi(K)}{\partial \sigma_\beta \partial K}\Big|_{K^B} \cdot \frac{dK^B}{d\rho} > 0$, where firstly, for all $\sigma_\beta \geq 0$, we have $\frac{\partial^2 \Pi(K)}{\partial \sigma_\beta \partial K}\Big|_{K^B} \cdot \frac{dK^B}{d\rho} > 0$ because $\frac{\partial^2 \Pi(K)}{\partial \sigma_\beta \partial K} > 0$ when $K^B > \mu_\beta / y$ and $\frac{dK^B}{d\rho} > 0$; secondly

$$\begin{aligned}
\frac{\partial}{\partial \rho}\left\{\lim_{\sigma_\beta \to 0}\frac{\partial \Pi(K)}{\partial \sigma_\beta}\right\} =& \frac{1}{y}\int_{-\infty}^{\infty} z_0 \int_{\frac{y+\frac{2b\mu_\beta}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\left(\sigma_\xi z_0 - \frac{z_2\sigma_\xi\rho}{\sqrt{(1-\rho^2)}}\right)\phi(z_2)dz_2\phi(z_0)dz_0 \\
=& \frac{\sigma_\xi}{y} - \frac{\sigma_\xi}{y}\int_{-\infty}^{\infty}\Phi\left(\frac{y+\frac{2b\mu_\beta}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\right)\phi(z_0)dz_0 > 0.
\end{aligned}$$

As a result, we define $\omega_\beta^\Pi(1) = \lim\limits_{\rho \to 1}\omega_\beta^\Pi(\rho)$, which is also the unique solution of $\lim\limits_{\rho \to 1}\lim\limits_{\sigma_\beta \to 0}\left\{\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B}\right\} = 0$, then we can properly define the ρ threshold: when $\omega \in \left(0, \omega_\beta^\Pi(1)\right)$, there exists a unique ρ denoted by $\rho_\beta^\Pi(\omega)$ that uniquely solves $\lim\limits_{\sigma_\beta \to 0}\left\{\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B(\omega)}\right\} = 0$. The result can be written as what in the proposition. ■

A.1.3 Sensitivity Analyses on Profitability-loss

Proof of Lemma 2:

Under condition $0 < \omega < \mathbf{E}\left[\left(\tilde{\xi} - y - 2b\underline{\beta}/y\right)^+\right]$, we first prove $K^B(\omega) < K^U(\omega)$.

According to definition of $K^U(\omega)$, we have $K^U(\omega) > \underline{\beta}/y$ and thus $K^U(\omega)$ can be taken into $\frac{\partial \Pi(K)}{\partial K}$ such that

$$\begin{aligned}\frac{\partial \Pi(K)}{\partial K}\Big|_{K^U} =& \frac{\partial \Pi_u(K)}{\partial K}\Big|_{K^U} - \int_{\underline{\beta}}^{yK^U}\int_{y+2bK^U}^{\infty}(\xi - y - 2bK^U)f(\xi,\beta)d\xi d\beta \\ =& -\int_{\underline{\beta}}^{yK^U}\int_{y+2bK^U}^{\infty}(\xi - y - 2bK^U)f_{\xi|\beta}(\xi)d\xi f_\beta(\beta)d\beta < 0.\end{aligned}$$

Since we know that $\frac{\partial \Pi(K)}{\partial K}$ decreasing in K and $\frac{\partial \Pi(K)}{\partial K}\big|_{K^B} = 0$, $K^B(\omega) < K^U(\omega)$ is proved.

Then we prove $\Pi_u(K^U) > \Pi(K^B)$ by calculating $\Pi_u(K) - \Pi(K) =$

$$\int_{\underline{\beta}}^{\overline{\beta}}\int_{y+2\min\left\{K,\frac{\beta}{y}\right\}}^{y+2bK}\left(\frac{(\xi-y)^2}{4b} - \left((\xi-y)\min\left\{K,\frac{\beta}{y}\right\} - b\Big(\min\left\{K,\frac{\beta}{y}\right\}\Big)^2\right)\right)f(\xi,\beta)d\xi d\beta$$
$$+\int_{\underline{\beta}}^{\overline{\beta}}\int_{y+2bK}^{\infty}\left(\left((\xi-y)K - bK^2\right) - \left((\xi-y)\min\left\{K,\frac{\beta}{y}\right\} - b\Big(\min\left\{K,\frac{\beta}{y}\right\}\Big)^2\right)\right)f(\xi,\beta)d\xi d\beta$$

>0.

Therefore, we have inequality $\Pi(K^B) < \Pi_u(K^B) < \Pi_u(K^U)$. ■

Proof of Proposition 7:

In this proof, we demonstrate Profitability-loss is decreasing in ρ. Given the definition of Profitability-loss $\Delta\Pi(\rho) = 1 - \frac{\Pi(K^U)}{\Pi(K^B)}$, we will prove how $\Delta\Pi$ changes in ρ, which is equivalent to derive the sign of $\frac{d\Delta\Pi(\rho)}{d\rho}$. Due to only $\Pi(K)$ and K^B are function of ρ, we have

$$\begin{aligned}\frac{d\Delta\Pi(\rho)}{d\rho} =& -\left(\frac{\frac{\partial \Pi(K)}{\partial \rho}\Big|_{K^U}\cdot \Pi(K^B) - \left(\frac{\partial \Pi(K)}{\partial \rho}\Big|_{K^B} + \frac{\partial \Pi(K)}{\partial K}\Big|_{K^B}\frac{dK^B}{d\rho}\right)\cdot \Pi(K^U)}{\left(\Pi(K^B)\right)^2}\right) \\ =& \frac{-\frac{\partial \Pi(K)}{\partial \rho}\Big|_{K^U}\cdot \Pi(K^B) + \frac{\partial \Pi(K)}{\partial \rho}\Big|_{K^B}\cdot \Pi(K^U)}{\left(\Pi(K^B)\right)^2}.\end{aligned}$$

We can conclude $\frac{d\Delta\Pi(\rho)}{d\rho} < 0$ by the following reason: firstly, from Proposition 1, we have $\frac{\partial \Pi(K)}{\partial \rho}$ is greater than 0 and increases in K, specifically we obtain $\frac{\partial \Pi(K)}{\partial \rho}\big|_{K^U} > \frac{\partial \Pi(K)}{\partial \rho}\big|_{K^B}$ because $K^U > K^B$; secondly, according to the optimality of K^B we have $\Pi(K^B) > \Pi(K^U)$. ■

Lemma 5 (The value of $\frac{\partial \Pi(K)}{\partial \sigma_\beta}$)

1. *$\frac{\partial \Pi(K)}{\partial \sigma_\beta} < 0$ when $K \le \mu_\beta / y$;*

2. *when $\rho \le 0$, $\frac{\partial \Pi(K)}{\partial \sigma_\beta} < 0$ for all $\sigma_\beta \ge 0$;*

3. *when $\rho > 0$,*

   (a) *if $\sigma_\beta \ge \frac{\rho y \sigma_\xi}{2b}$, $\frac{\partial \Pi(K)}{\partial \sigma_\beta} < 0$;*

   (b) *if $\sigma_\beta < \frac{\rho y \sigma_\xi}{2b}$, there exists a unique σ_β defined as $\sigma_{\beta 0}(K)$ that solves $\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{\sigma_\beta = \sigma_{\beta 0}(K)} = 0$, such that $\frac{\partial \Pi(K)}{\partial \sigma_\beta} < 0$ when $\sigma_\beta > \max\{0,\ \sigma_{\beta 0}(K)\}$ and $\frac{\partial \Pi(K)}{\partial \sigma_\beta} > 0$ when $\sigma_\beta \le \max\{0,\ \sigma_{\beta 0}(K)\}$.*

Proof of Lemma 5:

Proof of part 1: From Proposition 6, we have $\frac{\partial \Pi(K)}{\partial \sigma_\beta} =$

$$\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \frac{z_0}{y} \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}$$

$$\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - y - 2b\Big(\frac{z_0\sigma_\beta+\mu_\beta}{y}\Big)\right)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$=\sqrt{\sigma_\xi^2(1-\rho^2)} \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \frac{z_0}{y} g(z_0)\phi(z_0)dz_0,$$

where $g(z_0) := \mathbf{E}\left[\left(\tilde{z}_2 - \Big(\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\right)^+\right] > 0$, therefore, it is easy to see that when $K \le \mu_\beta/y$, we have that $\frac{\partial \Pi(K)}{\partial \sigma_\beta} < 0$ for all $\sigma_\beta \ge 0$.

Proof of part 2: Further organizing $\frac{\partial \Pi^B(K)}{\partial \sigma_\beta}$, we obtain $\frac{\partial \Pi(K)}{\partial \sigma_\beta} =$

$$-\frac{\sqrt{\sigma_\xi^2(1-\rho^2)}}{y}\phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\mathbf{E}\left[\left(\tilde{z}_2 - \Big(\frac{y+2bK-\mu_\xi-\rho\sigma_\xi\big(\frac{yK-\mu_\beta}{\sigma_\beta}\big)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\right)^+\right]$$

$$+\frac{1}{y}\Big(\rho\sigma_\xi - \frac{2b\sigma_\beta}{y}\Big)\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\left(1-\Phi\Big(\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\right)\phi(z_0)dz_0, \tag{A.11}$$

we conclude another result: when $\rho \le 0$, $\frac{\partial \Pi(K)}{\partial \sigma_\beta} < 0$ for all $\sigma_\beta \ge 0$.

Proof of part 3: When $\rho > 0$, we can conclude from Equation (A.11) that $\frac{\partial \Pi(K)}{\partial \sigma_\beta} < 0$ when $\sigma_\beta \geq \frac{\rho y \sigma_\xi}{2b}$. Given $\frac{\partial^2 \Pi(K)}{\partial K \partial \sigma_\beta} =$

$$\frac{(yK-\mu_\beta)}{\sigma_\beta^2}\phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\sqrt{\sigma_\xi^2(1-\rho^2)}\mathbf{E}\Bigg[\Bigg(\tilde{z}_2-\Big(\frac{y+2bK-\mu_\xi-\rho\sigma_\xi\big(\frac{yK-\mu_\beta}{\sigma_\beta}\big)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Bigg)^+\Bigg],$$

we know that $\frac{\partial \Pi(K)}{\partial \sigma_\beta}$ decreases in K when $K \leq \mu_\beta/y$ and increases in K when $K > \mu_\beta/y$. According to $\frac{\partial \Pi(K)}{\partial \sigma_\beta} < 0$ for all $K \leq \mu_\beta/y$, we obtain that given a fixed $\sigma_\beta < \frac{\rho y \sigma_\xi}{2b}$, there exists unique $K_0(\sigma_\beta) > \mu_\beta/y$ such that $\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K=K_0(\sigma_\beta)} = 0$, $\frac{\partial \Pi(K)}{\partial \sigma_\beta} < 0$ when $K < K_0(\sigma_\beta)$ and $\frac{\partial \Pi(K)}{\partial \sigma_\beta} > 0$ when $K > K_0(\sigma_\beta)$. We calculate how $K_0(\sigma_\beta)$ changes in σ_β by the implicit differentiation $\frac{\partial K^0(\sigma_\beta)}{\partial \sigma_\beta} = -\left(\frac{\partial^2 \Pi(K)}{\partial \sigma_\beta^2}\Big/\frac{\partial^2 \Pi(K)}{\partial \sigma_\beta \partial K}\right)\Big|_{K_0(\sigma_\beta)}$, where $\frac{\partial^2 \Pi(K)}{\partial \sigma_\beta \partial K}\Big|_{K_0(\sigma_\beta)} > 0$. By calculation, we have $\frac{\partial^2 \Pi(K)}{\partial \sigma_\beta^2} =$

$$-\frac{(yK-\mu_\beta)^2}{y\sigma_\beta^3}\phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\cdot$$

$$\int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi\left(\frac{yK-\mu_\beta}{\sigma_\beta}\right)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)-y-2bK\right)\phi(z_2)dz_2$$

$$-2\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\left(\frac{z_0}{y}\right)^2\int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\phi(z_2)dz_2\phi(z_0)dz_0 < 0,$$

which concludes that $\frac{\partial K^0(\sigma_\beta)}{\partial \sigma_\beta} > 0$. Since K_0 is increasing in σ_β, we have that $\frac{\partial \Pi(K)}{\partial \sigma_\beta} < 0$ when $\sigma_\beta > \max\left\{0,\ K_0^{-1}\big(K^U\big)\right\}$ and $\frac{\partial \Pi(K)}{\partial \sigma_\beta} > 0$ when $\sigma_\beta \leq \max\left\{0,\ K_0^{-1}\big(K^U\big)\right\}$ where $K_0^{-1}(\cdot)$ is the inverse function of $K_0(\sigma_\beta)$, we define this inverse function as $\sigma_{\beta 0}(\cdot)$.

■

Proof of Proposition 8:

In this proof, we partially characteristic the impact of σ_β on Profitability-loss. Given the definition of Profitability-loss $\Delta\Pi = 1 - \frac{\Pi\left(K^U\right)}{\Pi(K^B)}$, we will prove how $\Delta\Pi$ changes in σ_β, which is equivalent to derive the sign of $\frac{d\Delta\Pi}{d\sigma_\beta}$. Due to only $\Pi(K)$

and K^B are function of σ_β, we have

$$\frac{d\Delta\Pi}{d\sigma_\beta} = -\left(\frac{\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^U}\cdot\Pi\big(K^B(\sigma_\beta)\big) - \left(\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^B(\sigma_\beta)} + \frac{\partial\Pi(K)}{\partial K}\Big|_{K^B(\sigma_\beta)}\frac{dK^B(\sigma_\beta)}{d\sigma_\beta}\right)\cdot\Pi\big(K^U\big)}{\big(\Pi(K^B)\big)^2}\right)$$

$$=\frac{\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^B(\sigma_\beta)}\cdot\Pi\big(K^U\big) - \frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^U}\cdot\Pi\big(K^B(\sigma_\beta)\big)}{\big(\Pi(K^B(\sigma_\beta))\big)^2}.$$

Proof of part 1: In this part, we will demonstrate following result: when $\omega \geq \omega_U$, $\frac{d\Delta\Pi}{d\sigma_\beta} > 0$ for all σ_β - the case where $K^U \leq \mu_\beta/y$. We define $\omega_U := \sigma_\xi\mathbf{E}\left[\left(\tilde{z}_1 - \left(\frac{2b\mu_\beta/y-\mu_\xi+y}{\sigma_\xi}\right)\right)^+\right] = \mathbf{E}\left[\left(\tilde{\xi} - \left(2b\mu_\beta/y+y\right)\right)^+\right]$, in this case, we have $\omega > \omega_\beta^K$ which indicates $K^B < \mu_\beta/y$ and $\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^B(\sigma_\beta)} < 0$. As for $\omega \geq \omega_U$ is equivalent to $K^U \leq \mu_\beta/y$, because

$$\begin{aligned}\frac{\partial\Pi_u(K)}{\partial K}\Big|_{\mu_\beta/y} &= -\omega + \int_{\frac{2b\mu_\beta/y-\mu_\xi+y}{\sigma_\xi}}^{\infty}(z_1\sigma_\xi + \mu_\xi - y - 2b\mu_\beta/y)\phi(z_1)dz_1\\ &= -\omega + \sigma_\xi\mathbf{E}\left[\left(\tilde{z}_1 - \left(\frac{2b\mu_\beta/y-\mu_\xi+y}{\sigma_\xi}\right)\right)^+\right] \leq 0.\end{aligned}$$

As a result, $\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^U} < 0$. Also because for all $K \in (K^B, \mu_\beta/y]$, we have $\frac{\partial\Pi(K)}{\partial K} < 0$, $\frac{\partial\Pi(K)}{\partial\sigma_\beta} < 0$ and $\frac{\partial^2\Pi(K)}{\partial\sigma_\beta\partial K} \leq 0$, we obtain that

$$\frac{d}{dK}\left\{\frac{\Pi(K)}{\frac{\partial\Pi(K)}{\partial\sigma_\beta}}\right\} = \frac{\frac{\partial\Pi(K)}{\partial K}\cdot\frac{\partial\Pi(K)}{\partial\sigma_\beta} - \Pi(K)\cdot\frac{\partial^2\Pi(K)}{\partial\sigma_\beta\partial K}}{\left(\frac{\partial\Pi(K)}{\partial\sigma_\beta}\right)^2} > 0$$

for all $K \in [K^B, \mu_\beta/y)$. Therefore $\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^B(\sigma_\beta)}\cdot\Pi\big(K^U\big) > \frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^U}\cdot\Pi\big(K^B(\sigma_\beta)\big)$. To sum up, we obtain that $\frac{d\Delta\Pi}{d\sigma_\beta} > 0$ when $\omega \geq \omega_U$. Noting that $\omega_\beta^K = \int_{\frac{2b\mu_\beta/y-\mu_\xi+y}{\sigma_\xi}}^{\infty}(z_1\sigma_\xi+\mu_\xi-y-2b\mu_\beta/y)\Phi\left(\frac{\rho z_1}{\sqrt{1-\rho^2}}\right)\phi(z_1)dz_1 < \sigma_\xi\mathbf{E}\left[\left(\tilde{z}_1-\left(\frac{2b\mu_\beta/y-\mu_\xi+y}{\sigma_\xi}\right)\right)^+\right]$.

Proof of part 2: In this part, we demonstrate following result: when $\omega < \omega_U$ and $\rho > 0$, $\frac{d\Delta\Pi}{d\sigma_\beta} < 0$ for all $\sigma_\beta \leq \max\left\{0,\ K_0^{-1}\big(K^U\big)\right\}$ - the case where $K^U > \mu_\beta/y$.

We first discuss the case when $\omega_\beta < \omega < \omega_U$. In this case, we know that $K^U > \mu_\beta/y$ and $K^B < \mu_\beta/y$. According to the property of $\frac{\partial\Pi(K)}{\partial\sigma_\beta}$, we have that $\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^B(\sigma_\beta)} < 0$. Observing from $\frac{d\Delta\Pi}{d\sigma_\beta}$, as long as $\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K^U} > 0$ we have $\frac{d\Delta\Pi}{d\sigma_\beta} < 0$.

Then, we investigate the case when $\omega \leq \omega_\beta$. Since $\omega \leq \omega_\beta$ is equivalent to

$K^B(\sigma_\beta) \geq \mu_\beta/y$ and $K^U > K^B(\sigma_\beta)$, therefore, we have

$$\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^U} - \frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B(\sigma_\beta)} = \sqrt{\sigma_\xi^2(1-\rho^2)} \int_{\frac{yK^B(\sigma_\beta)-\mu_\beta}{\sigma_\beta}}^{\frac{yK^U-\mu_\beta}{\sigma_\beta}} \frac{z_0}{y} g(z_0)\phi(z_0)dz_0 > 0.$$

According to the optimality of K^B, that is $\Pi\big(K^B(\sigma_\beta)\big)/\Pi(K^U) > 1$, the term $\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^U}$ has to be non-negative in order to have $\frac{\Pi\left(K^B(\sigma_\beta)\right)}{\Pi(K^U)} \cdot \frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^U} > \frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^B(\sigma_\beta)}$.

In summary, a sufficient condition for $\frac{d\Delta\Pi}{d\sigma_\beta} < 0$ when $\omega < \omega_U$ is $\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^U} > 0$. Since K^U is irrelevant to σ_β, according to Lemma 5 part 3, we know that $\frac{\partial \Pi(K)}{\partial \sigma_\beta}\Big|_{K^U} > 0$ when $\rho > 0$ and $\sigma_\beta \leq \max\left\{0,\ \sigma_{\beta 0}\big(K^U\big)\right\}$. To sum up, we obtain that $\frac{d\Delta\Pi}{d\sigma_\beta} < 0$ for all $\sigma_\beta \leq \max\left\{0,\ \sigma_{\beta 0}\big(K^U\big)\right\}$ when $\omega < \omega_U$ and $\rho > 0$. ■

A.2 Proofs for the Optimal Strategy and Sensitivity Analyses of Pre-shipment Finance Model

A.2.1 Proofs for the Optimal Strategy

Proof of Theorem 3 part 1:

production-stage optimal decisions are production quantity and pre-shipment financing. We solve them by using Karush-Kuhn-Tucker condition. Given the production-stage problem formulation,

$$\begin{aligned} \pi_p^*\big(K,\xi,\beta\big) = \max_{q,L_p} \quad & (\xi - bq)q - yq - r_pL_p \\ \text{s.t.} \quad & 0 \leq q \leq \min\left\{K, \tfrac{\beta+L_p}{y}\right\} \\ & 0 \leq (1+r_p)L_p \leq (\xi - bq)q, \end{aligned}$$

the problem is naturally divided into two scenarios, because production quantity is binding either by capacity K or by resource constraint $\frac{\beta+L_p}{y}$. We discuss two scenarios one by one.

When production quantity is binding by capacity K,the straightforward relation is $K \leq \frac{\beta+L_p^*}{y}$. According to this inequality, $L_p^* = 0$, because pre-shipment finance will be applied only when production resource is constraining production decision due to the cost of pre-shipment finance. The problem formulation followed by above discussion is:

$$\begin{aligned} \pi_p^*\big(K,\xi,\beta\big) = \max_{q} \quad & (\xi - bq)q - yq \\ \text{s.t.} \quad & 0 \le q \le K \\ & K \le \tfrac{\beta}{y}, \end{aligned}$$

which is identical to the formulation of basic model when $K \le \frac{\beta}{y}$. It is easy to have that

$$(q^*, L_p^*) = \begin{cases} (0,0), & \text{if } (\xi,\beta) \in \big\{0 \le \xi \le y, yK \le \beta\big\} \\ \left(\dfrac{\xi - y}{2b}, 0\right), & \text{if } (\xi,\beta) \in \big\{y < \xi < y + 2bK, yK \le \beta\big\} \\ (K,0), & \text{if } (\xi,\beta) \in \big\{\xi \ge y + 2bK, yK \le \beta\big\}. \end{cases}$$

When production quantity is binding by resource constraint, or equivalently, $K > \beta/y$**,** we focus on optimal decision under the relation $K \ge \frac{\beta + L_p^*}{y}$. Therefore, problem formulation becomes:

$$\begin{aligned} \pi_p^*\big(K,\xi,\beta\big) = \max_{q,L_p} \quad & (\xi - bq)q - yq - r_p L_p \\ \text{s.t.} \quad & 0 \le q \le \tfrac{\beta + L_p}{y} \\ & \tfrac{\beta + L_p}{y} \le K, \\ & 0 \le (1 + r_p)L_p \le (\xi - bq)q. \end{aligned}$$

and each primal constraint is coupled with a dual variable, we define the variables as follows:

$$\begin{aligned} yq - L_p &\le \beta & <\lambda_1> \\ L_p &\le yK - \beta & <\lambda_2> \\ -(\xi - bq)q + (1 + r_p)L_p &\le 0 & <\lambda_3> \\ -q &\le 0 & <\mu_1> \\ -L_p &\le 0 & <\mu_2> \end{aligned} \tag{A.12}$$

Then, the dual feasible equations are

$$\xi - 2bq^* - y = y\lambda_1 - (\xi - 2bq^*)\lambda_3 - \mu_1, \tag{A.13}$$

$$-r_p = -\lambda_1 + \lambda_2 + (1 + r_p)\lambda_3 - \mu_2, \tag{A.14}$$

$$\lambda_1, \lambda_2, \lambda_3, \mu_1, \mu_2 \ge 0,$$

and complementary slackness are

$$\lambda_1(\beta + L_p^* - yq^*) = 0,$$

$$\lambda_2(yK - \beta - L_p^*) = 0,$$

$$\lambda_3\big((\xi - bq^*)q^* - (1 + r_p)L_p^*\big) = 0,$$

$$\mu_1 q^* = 0,$$

$$\mu_2 L_p^* = 0.$$

The optimal decisions are categorized by all permutations of value of dual variables $(\lambda_1, \lambda_2, \lambda_3, \mu_1, \mu_2)$, specifically, the value of each dual variable takes either ≥ 0 or $= 0$ indicating the corresponding constraint is either binding or not binding respectively.

We classify the optimal decisions into four cases by whether q^* and L_p^* taking positive or zero value, or equivalent condition from complementary slackness, whether μ_1 and μ_2 are ≥ 0 or $= 0$. Therein, the case $q^* = 0$ and $L_p^* \geq 0$ doesn't exist, because the negative profit occurs, specifically, on the one hand there is no revenue generated due to no production, on the other hand pre-shipment finance is applied with some cost.

The remaining three cases are as follows:

Case 1: $q^* = L_p^* = 0$.

In this case, $\mu_1 \geq 0$ and $\mu_2 \geq 0$. The value of remaining dual variables calculated from substituting optimal decisions into complementary slackness are $\lambda_1 = 0$, $\lambda_2 = 0$ and $\lambda_3 \geq 0$. The uncertainties in the region derived through replacing all above primary and dual variables with their value into Equation (A.13) and (A.14) are

$$\begin{cases} \beta < yK \\ 0 \leq \xi \leq y, \end{cases}$$

Case 2: $q^* > 0$ and $L_p^* = 0$.

In this case, we have $\mu_1 = 0$ and $\mu_2 \geq 0$. Immediate results drew in this case are

$\lambda_2 = 0$ because of $K > \beta/y$. The complementary slackness for λ_1 and λ_3 are

$$\lambda_1(\beta - yq^*) = 0 \text{ and } \lambda_3(\xi - bq^*)q^* = 0,$$

as a result, we have following sub-cases based whether λ_1 and λ_3 are ≥ 0 or $= 0$:

1. when $\lambda_1 = \lambda_3 = 0$, $q^* < \frac{\beta}{y}$ and $q^* < \frac{\xi}{b}$ are drew from complementary slackness. Dual feasible (A.13) becomes $\xi - 2bq^* - y = 0$ and (A.14) becomes $-r_p = -\mu_2$. Therefore, we have $q^* = \frac{\xi - y}{2b}$. Since $0 < q^* < \beta/y$ and $K > \beta/y$, the corresponding state space is
$$\begin{cases} \frac{\xi-y}{2b} < \frac{\beta}{y} < K \\ \xi > y,\ \beta > 0 \end{cases}$$
because $\beta/y > q^* > 0$ and $K > \beta/y$;

2. when $\lambda_1 = 0$ and $\lambda_3 \geq 0$, $q^* < \frac{\beta}{y}$ and $q^* = \frac{\xi}{b}$ are drew from complementary slackness. The dual feasible (A.13) and (A.14) are $-\xi - y = \xi\lambda_3$ and $-r_p = (1 + r_p)\lambda_3 - \mu_2$ respectively. Noting that the dual feasible (A.13) can't hold because $\xi \geq 0$, $y \geq 0$ and $\lambda_3 \geq 0$. Therefore, this case doesn't exist;

3. when $\lambda_1 \geq 0$ and $\lambda_3 = 0$, $q^* = \frac{\beta}{y}$ and $q^* < \frac{\xi}{b}$ are drew from complementary slackness. The dual feasible (A.13) and (A.14) are $\xi - \frac{2b\beta}{y} - y = y\lambda_1$ and $-r_p = -\lambda_1 - \mu_2$ respectively. From the dual feasible (A.13), we have $\xi \geq y + \frac{2b\beta}{y}$, then substitute λ_1 with $r_p - \mu_2$ for the dual feasible (A.13), we have $\xi \leq (1 + r_p)y + \frac{2b\beta}{y}$, so we obtain that this case exists only when state space satisfies
$$\begin{cases} y + \frac{2b\beta}{y} \leq \xi \leq (1 + r_p)y + \frac{2b\beta}{y} \\ 0 < \beta < yK; \end{cases}$$

4. when $\lambda_1 \geq 0$ and $\lambda_3 \geq 0$, $q^* = \frac{\beta}{y} = \frac{\xi}{b}$ is drew from complementary slackness. For simplicity, we use $q^* = \frac{\xi}{b}$ to write over dual feasible (A.13) and (A.14), they are $-\xi - y = y\lambda_1 + \xi\lambda_3$ and $-r_p = -\lambda_1 + (1 + r_p)\lambda_3 - \mu_2$ respectively. Noting that the dual feasible (A.13) can't hold, therefore this case doesn't exist.

Case 3: $q^* > 0$ and $L_p^* > 0$. In this case $\mu_1 = \mu_2 = 0$. Observing that the dual feasible (A.14) $-r_p = -\lambda_1 + \lambda_2 + (1 + r_p)\lambda_3$ holds only if $\lambda_1 > 0$, because r_p is strictly positive. In addition, $\lambda_1 > 0$ results in $\beta + L_p^* - yq^* = 0$ according to the complementary slackness. Therefore, given $\lambda_1 > 0$ and $\mu_1 = \mu_2 = 0$, there are only four sub-cases in terms of whether λ_2 and λ_3 are ≥ 0 or $= 0$:

1. when $\lambda_2 = \lambda_3 = 0$, $L_p^* < yK - \beta$ and $(\xi - bq^*)q^* > (1 + r_p)L_p^*$ are drew from complementary slackness. The corresponding dual feasible (A.13) and (A.14) are $\xi - 2bq^* - y = y\lambda_1$ and $r_p = \lambda_1$, which lead to $q^* = \frac{\xi-(1+r_p)y}{2b}$ and thus $L_p^* = \frac{\big(\xi-(1+r_p)y\big)y}{2b} - \beta$ because $\beta + L_p^* - yq^* = 0$. Due to $(\xi - bq^*)q^* > (1 + r_p)L_p^*$, $q^* > 0$ and $yK - \beta > L_p^* > 0$, the state space is restricted in the range
$$\begin{cases} (1 + r_p)y + \frac{2b\beta}{y} < \xi \leq (1 + r_p)y + 2bK \\ \beta < yK; \end{cases}$$

2. when $\lambda_2 = 0$ and $\lambda_3 \geq 0$, $L_p^* < yK - \beta$ and $(\xi - bq^*)q^* = (1 + r_p)L_p^*$ are gotten from complementary slackness. The dual feasible (A.13) and (A.14) are $\xi - 2bq^* - y = y\lambda_1 - (\xi - 2bq^*)\lambda_3$ and $r_p = \lambda_1 - (1 + r_p)\lambda_3$, losing λ_1 by combining the two dual feasible equations together we yield $-\big(\xi - 2bq^* - (1 + r_p)y\big)(1 + \lambda_3) = 0$, which obtains $q^* = \frac{\xi-(1+r_p)y}{2b}$ and thus $L_p^* = \frac{\xi^2-(1+r_p)^2y^2}{4(1+r_p)}$ because $(\xi - bq^*)q^* = (1 + r_p)L_p^*$. In addition, by substituting q^* and L_p^* into $\beta + L_p^* - yq^* = 0$, we have that the state space must satisfies $\beta + \frac{\xi^2-(1+r_p)^2y^2}{4(1+r_p)} - \frac{\big(\xi-(1+r_p)y\big)y}{2b} = 0$. Define a quadratic function with respect to ξ as $g_\beta(\xi) = -\big(\frac{\xi^2-(1+r_p)^2y^2}{4(1+r_p)}\big) + \frac{\big(\xi-(1+r_p)y\big)y}{2b}$, which has maximizer $\xi = (1 + r_p)y$ because $\frac{dg_\beta(\xi)}{d\xi} = -\frac{\xi}{2(1+r_p)} + \frac{y}{2b}$. Since $g_\beta\big((1 + r_p)y\big) = 0$ and $\beta \in [\underline{\beta}, yK)$, there is no state space for this case to be held;

3. when $\lambda_2 \geq 0$ and $\lambda_3 = 0$, $L_p^* = yK - \beta$ and $(\xi - bq^*)q^* > (1 + r_p)L_p^*$ are drew from complementary slackness, we further have $q^* = K$ because of $\beta + L_p^* - yq^* = 0$. As a result, $(\xi - bK)K > (1 + r_p)(yK - \beta)$ should be

satisfied. From dual feasible (A.13) and (A.14), that are $\xi - 2bK - y = y\lambda_1$ and $r_p = \lambda_1 - \lambda_2$ respectively, $\xi > (1+r_p)y + 2bK$ is derived. Overall, state space that leads to optimal decisions equal to $q^* = K$ and $L_p^* = yK - \beta$ is

$$\begin{cases} \xi \geq (1+r_p)y + 2bK \\ (\xi - bK)K > (1+r_p)(yK-\beta) \\ \beta < yK; \end{cases} \Rightarrow \begin{cases} \xi \geq (1+r_p)y + 2bK \\ \beta < yK; \end{cases}$$

4. when $\lambda_2 \geq 0$ and $\lambda_3 \geq 0$, we have complementary slackness $L_p^* = yK - \beta$, $(\xi - bq^*)q^* = (1+r_p)L_p^*$ and $\beta + L_p^* - yq^* = 0$. Therefore, optimal decisions are identical to previous case, that are $q^* = K$ and $L_p^* = yK - \beta$, and state space follows a linear relation $(\xi - K)K = (1+r_p)(yK-\beta)$. In addition, using dual feasible (A.13) and (A.14), that are $\xi - 2bK - y = y\lambda_1 - (\xi - 2bK)\lambda_3$ and $r_p = \lambda_1 - \lambda_2 - (1+r_p)\lambda_3$ in case, we have $\big(\xi - 2bK - (1+r_p)y\big)(1+\lambda_3) = y\lambda_2$ by losing λ_1, which provide a range for state space $\xi \geq (1+r_p)y + 2bK$. Because line $(\xi - K)K = (1+r_p)(yK-\beta)$ doesn't pass through $\big\{(\xi,\beta) : \xi \geq (1+r_p)y + 2bK, \beta < yK\big\}$, this case doesn't exist.

The definition of state spaces in proposition is gotten from combing above ranges with same optimal decisions and consider the lower and upper bound of $\tilde{\beta}$ that are $\underline{\beta}$ and $\overline{\beta}$. ■

Proof of Theorem 3 part 2:

We derive the optimal capacity investment level for pre-shipment finance model by proving the concavity of $\Pi_p(K)$ in K and then K_p^* being solved by $\frac{\partial \Pi_p(K)}{\partial K} = 0$. We prove $\Pi_p(K)$ is concave in K by demonstrating the first order derivative of $\Pi_p(K)$ is continuous and monotonically decreasing in K. We first calculate the first order derivative of $\Pi_p(K)$:

$$\frac{\partial \Pi_p(K)}{\partial K} = -\omega + \int_{\max\left\{\underline{\beta}, \min\{yK, \overline{\beta}\}\right\}}^{\overline{\beta}} \int_{y+2bK}^{\infty} (\xi - y - 2bK) f(\xi, \beta) d\xi d\beta$$

$$+ \int_{\underline{\beta}}^{\max\left\{\underline{\beta}, \min\{yK, \overline{\beta}\}\right\}} \int_{(1+r_p)y+2bK}^{\infty} \left(\xi - (1+r_p)y - 2bK\right) f(\xi, \beta) d\xi d\beta.$$

Although $\frac{\partial \Pi_p(K)}{\partial K}$ is continuous in K, it has a piecewise form depending on the permutation among yK, $\underline{\beta}$ and $\overline{\beta}$. For the ease of calculation, we write the piecewise form of the first order derivative of $\Pi_p(K)$ as follows:

$$\frac{\partial \Pi_p(K)}{\partial K} = \begin{cases} \dfrac{\partial \Pi_u(K)}{\partial K}, & \text{if } K \in \left[0, \dfrac{\underline{\beta}}{y}\right] \\ \dfrac{\partial \Pi_{p1}(K)}{\partial K}, & \text{if } K \in \left(\dfrac{\underline{\beta}}{y}, \dfrac{\overline{\beta}}{y}\right) \\ \dfrac{\partial \Pi_{p2}(K)}{\partial K} & \text{if } K \in \left[\dfrac{\overline{\beta}}{y}, \infty\right), \end{cases}$$

where $\frac{\partial \Pi_{p1}(K)}{\partial K} = -\omega + \int_{yK}^{\overline{\beta}} \int_{y+2bK}^{\infty} (\xi - y - 2bK) f(\xi, \beta) d\xi d\beta + \int_{\underline{\beta}}^{yK} \int_{(1+r_p)y+2bK}^{\infty} \big(\xi - (1+r_p)y - 2bK\big) f(\xi, \beta) d\xi d\beta$ and $\frac{\partial \Pi_{p2}(K)}{\partial K} = -\omega + \int_{(1+r_p)y+2bK}^{\infty} \big(\xi - (1+r_p)y - 2bK\big) f_\xi(\xi) d\xi$. Then we prove the monotonically decreasing trend by calculating the second derivative of $\Pi_p(K)$ with respect to K piece by piece: $\frac{\partial^2 \Pi_{p1}(K)}{\partial K^2} =$

$$-2 \int_{yK}^{\overline{\beta}} \int_{y+2bK}^{\infty} f(\xi, \beta) d\xi d\beta - 2 \int_{\underline{\beta}}^{yK} \int_{(1+r_p)y+2bK}^{\infty} f(\xi, \beta) d\xi d\beta$$

$$- y \int_{y+2bK}^{\infty} (\xi - y - 2bK) f(\xi, yK) d\xi$$

$$+ y \int_{(1+r_p)y+2bK}^{\infty} (\xi - (1+r_p)y - 2bK) f(\xi, yK) d\xi$$

$$<0$$

and $\frac{\partial^2 \Pi_{p2}(K)}{\partial K^2} = -2 \int_{(1+r_p)y+2bK}^{\infty} f_\xi(\xi) d\xi < 0$. As a result, we finish the proof of the concavity of $\Pi_p(K)$ in K.

Next, we discuss K_p^* piece by piece according to the piecewise structure of $\frac{\partial \Pi_p(K)}{\partial K}$, because $\frac{\partial \Pi_p(K)}{\partial K}$ in different piece has unique form. Specially, we discuss the conditions under which $K_p^* = 0$, $K_p^* \in (0, \underline{\beta}/y]$, $K_p^* \in (\underline{\beta}/y, \underline{\beta}/y]$ and $K_p^* \in (\underline{\beta}/y, \infty)$ respectively:

1. under condition $\frac{\partial \Pi_u(K)}{\partial K}\big|_0 \leq 0$, or equivalently $\omega \geq \mathbf{E}\big[(\tilde{\xi} - y)^+\big]$, $\Pi_p(K)$ decreases in $K \geq 0$, therefore, K_p^* equals to 0;

2. under condition $\frac{\partial \Pi_u(K)}{\partial K}\big|_{\frac{\underline{\beta}}{y}} = \frac{\partial \Pi_{p1}(K)}{\partial K}\big|_{\frac{\underline{\beta}}{y}} \le 0$ and $\frac{\partial \Pi_u(K)}{\partial K}\big|_0 > 0$, where the inequalities are equivalent to $\mathbf{E}\Big[(\tilde{\xi} - y - \frac{2\underline{\beta}}{y})^+\Big] \le \omega < \mathbf{E}\big[(\tilde{\xi} - y)^+\big]$, K_p^* lies in $(0, \underline{\beta}/y]$ which is solved from $\frac{\partial \Pi_u(K)}{\partial K} = 0$, namely $K_p^* = K^U$;

3. under condition $\frac{\partial \Pi_{p1}(K)}{\partial K}\big|_{\frac{\underline{\beta}}{y}} > 0$ and $\frac{\partial \Pi_{p2}(K)}{\partial K}\big|_{\frac{\overline{\beta}}{y}} \le 0$, namely $\mathbf{E}\big[(\tilde{\xi} - (1 + r_p)y - 2\overline{\beta}/y)^+\big] \le \omega < \mathbf{E}\big[(\tilde{\xi} - y - 2b\underline{\beta}/y)^+\big]$, K_p^* is the unique solution of $\frac{\partial \Pi_{p1}(K)}{\partial K} = 0$, we define K_p^* under this condition as $K_p^B(\omega)$ which is in the range $(\underline{\beta}/y, \overline{\beta}/y]$;

4. lastly, under condition $\frac{\partial \Pi_{p2}(K)}{\partial K}\big|_{\frac{\overline{\beta}}{y}} > 0$ which is $\omega < \mathbf{E}\big[(\tilde{\xi} - (1 + r_p)y - 2\overline{\beta}/y)^+\big]$, $K_p^* > \overline{\beta}/y$ and is the unique solution of $\frac{\partial \Pi_{p2}(K)}{\partial K} = -\omega + \mathbf{E}\big[(\tilde{\xi} - (1+r_p)y - 2bK)^+\big] = 0$. Furthermore, knowing that K_p^* under this condition is the unique solution of $\omega = \mathbf{E}\big[(\tilde{\xi} - (1 + r_p)y - 2bK)^+\big]$ and $K^U(\omega)$ is the unique solution of $\omega = \mathbf{E}\big[(\tilde{\xi} - y - 2bK)^+\big]$, K^U and K_p^* have following relation $(1 + r_p)y + 2bK_p^* = y + 2bK^U$, so $K_p^* = K^U - \frac{r_p y}{2b}$.

■

Proof of Corollary 3:

Pre-shipment finance shows the value under condition $0 \le \omega < \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y - \frac{2\underline{\beta}}{y})^+\big]$ where optimal capacity level for basic model is K^B which is the unique solution of $\frac{\partial \Pi(K)}{\partial K} = -\omega + \int_{yK}^{\overline{\beta}} \int_{y+2bK}^{\infty} (\xi - y - 2bK) f(\xi, \beta) d\xi d\beta = 0$. Meanwhile, the optimal capacity investment level for pre-shipment model is either K^P or $K^U - r_p y/2$.

We discuss the value of pre-shipment finance in terms of capacity investment level given same ω case by case:

1. K^P optimizes the pre-shipment model objective when $\mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - (1 + r_p)y - \frac{2\overline{\beta}}{y})^+\big] \le \omega < \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y - \frac{2\underline{\beta}}{y})^+\big]$ and $K_p^B \in \left(\frac{\underline{\beta}}{y}, \frac{\overline{\beta}}{y}\right)$ is the unique solution of $\frac{\partial \Pi_{p1}(K)}{\partial K} = -\omega + \int_{yK}^{\overline{\beta}} \int_{y+2bK}^{\infty} (\xi - y - 2bK) f(\xi, \beta) d\xi d\beta + \int_{\underline{\beta}}^{yK} \int_{(1+r_p)y+2bK}^{\infty} \big(\xi - (1 + r_p)y - 2bK\big) f(\xi, \beta) d\xi d\beta = 0$. In order to compare K^B and K^P, we

derive the first order derivative between $\Pi_{p1}(K)$ and $\Pi(K)$ with respect to K:

$$\frac{\partial \Pi_{p1}(K)}{\partial K} = \frac{\partial \Pi(K)}{\partial K} + \int_{\underline{\beta}}^{yK} \int_{(1+r_p)y+2bK}^{\infty} \big(\xi - (1+r_p)y - 2bK\big) f(\xi,\beta) d\xi d\beta$$

and assign K^B to K yielding $\frac{\partial \Pi_{p1}(K)}{\partial K}\Big|_{K^B} =$

$$\frac{\partial \Pi(K)}{\partial K}\Big|_{K^B} + \int_{\underline{\beta}}^{yK^B} \int_{(1+r_p)y+2bK^B}^{\infty} \big(\xi - (1+r_p)y - 2bK^B\big) f(\xi,\beta) d\xi d\beta$$

$$= \int_{\underline{\beta}}^{yK^B} \int_{(1+r_p)y+2bK^B}^{\infty} \big(\xi - (1+r_p)y - 2bK^B\big) f(\xi,\beta) d\xi d\beta > 0.$$

The above inequality shows $K^B(\omega) < K_p^B(\omega)$, because $\frac{\partial \Pi_{p1}(K)}{\partial K}$ is decreasing in K and $\frac{\partial \Pi_{p1}(K)}{\partial K}\Big|_{K_p^B} = 0$.

2. $K^U - r_p y/2$ optimizes the pre-shipment model objective when $0 < \omega < \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - (1+r_p)y - \frac{2\overline{\beta}}{y})^+\big]$ and $K^U - r_p y/2 > \overline{\beta}/y > K^B$.

We then discuss the value of pre-shipment finance in terms of optimal expected profit given same ω case by case: The value generated from pre-shipment finance given any $K \in (\underline{\beta}/y, \overline{\beta}/y)$ when $\mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - (1+r_p)y - \frac{2\overline{\beta}}{y})^+\big] \le \omega < \mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y - \frac{2\underline{\beta}}{y})^+\big]$ is:

$$\begin{aligned}
\Delta_p(K) :=& \Pi_{p1}(K) - \Pi(K) \\
=& -\int_{\underline{\beta}}^{yK} \int_{(1+r_p)y+\frac{2b\beta}{y}}^{\infty} \left(\frac{(\xi - y)\beta}{y} - b\Big(\frac{\beta}{y}\Big)^2\right) f(\xi,\beta) d\xi d\beta \\
&+ \int_{\underline{\beta}}^{yK} \int_{(1+r_p)y+\frac{2b\beta}{y}}^{(1+r_p)y+2bK} \left(\frac{\big(\xi - (1+r_p)y\big)^2}{4b} + r_p\beta\right) f(\xi,\beta) d\xi d\beta \\
&+ \int_{\underline{\beta}}^{yK} \int_{(1+r_p)y+2bK}^{\infty} \Big(\big(\xi - (1+r_p)y\big)K - bK^2 + r_p\beta\Big) f(\xi,\beta) d\xi d\beta,
\end{aligned} \tag{A.15}$$

we have $\Delta_p\Big(\frac{\underline{\beta}}{y}\Big) = 0$. In addition, $\Delta_p(K)$ increasing in K is derived from the first order derivative of $\Delta_p(K)$ with respect to K, that is $\frac{\partial \Delta_p(K)}{\partial K} =$

$$\frac{\partial \Pi_{p1}(K)}{\partial K} - \frac{\partial \Pi(K)}{\partial K} = \int_{\underline{\beta}}^{yK} \int_{(1+r_p)y+2bK}^{\infty} \big(\xi - (1+r_p)y - 2bK\big) f(\xi,\beta) d\xi d\beta > 0.$$

Therefore, $\Delta_p(K) > 0$ for all $K \in (\underline{\beta}/y, \overline{\beta}/y)$. $\Pi(K^B) < \Pi_{p1}(K^B)$ is derived from following inequalities, that is $\Pi_{p1}(K_p^B) - \Pi(K^B) > \Pi_{p1}(K^B) - \Pi(K^B) = \Delta_p(K^B) > 0$. ■

A.2.2 Proofs for the sensitivity analyses

From analytical result in Theorem 3 part 2, in the pre-shipment finance model, the sensitivity results that are not yet explored only if K_p^* takes value K_p^B. We will prove the sensitivity analyses results through the implicit function of K_p^B that is $\frac{\partial \Pi_p}{\partial K} = 0$. Utilizing bivariate normal distribution property $f(\xi, \beta) = f_{\xi|\beta}(\xi) f_\beta(\beta)$, $\tilde{z}_2 \overset{d}{=} \frac{\tilde{\xi}-\mu_\xi-\rho\frac{\sigma_\xi}{\sigma_\beta}(\tilde{\beta}-\mu_\beta)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big|\tilde{\beta}$ and $\tilde{z}_0 \overset{d}{=} \frac{\tilde{\beta}-\mu_\beta}{\sigma_\beta}$,we have $\frac{\partial \Pi_p(K)}{\partial K} =$

$$
\begin{aligned}
&-\omega + \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \\
&\qquad \left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - y - 2bK\right)\phi(z_2)dz_2\phi(z_0)dz_0 \\
&+ \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \\
&\qquad \left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - (1+r_p)y - 2bK\right)\phi(z_2)dz_2\phi(z_0)dz_0
\end{aligned}
\tag{A.16}
$$

Similarly, in order to derive sensitivity analyses for $\Pi_p(K_p^B)$, we standardize the first stage objective function as follows

$$
\Pi_p(K) = -\omega K + \sum_{i=1}^{7} V_i^p(K), \tag{A.17}
$$

where $V_i^p(K)$ $i = 1, 2, \cdots, 7$ are defined as:

$$
\begin{aligned}
V_1^p(K) :=& \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \\
&\qquad \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - y\right)^2}{4b}\phi(z_2)dz_2\phi(z_0)dz_0, \\
V_2^p(K) :=& \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \\
&\qquad \left(\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - y\right)K - bK^2\right)\phi(z_2)dz_2\phi(z_0)dz_0,
\end{aligned}
$$

$$V_3^p(K) := \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)^2}{4b}\phi(z_2)dz_2\phi(z_0)dz_0,$$

$$V_4^p(K) := \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \frac{(z_0\sigma_\beta+\mu_\beta)}{y} \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y-b\left(\frac{z_0\sigma_\beta+\mu_\beta}{y}\right)\right)\phi(z_2)dz_2\phi(z_0)dz_0,$$

$$V_5^p(K) := \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-(1+r_p)y\right)^2}{4b}\phi(z_2)dz_2\phi(z_0)dz_0,$$

$$V_6^p(K) := \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \left(\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-(1+r_p)y\right)K-bK^2\right)\phi(z_2)dz_2\phi(z_0)dz_0,$$

and

$$V_7^p(K) := \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} r_p\left(z_0\sigma_\beta+\mu_\beta\right)\phi(z_2)dz_2\phi(z_0)dz_0.$$

Proof of Proposition 9:

We will prove the impact of ρ on K_p^B and $\Pi_p(K_p^B)$ one by one. Firstly, we prove K_p^B increases in ρ. By implicit differentiation, $\frac{dK_p^B}{d\rho} = -\left(\left(\frac{\partial^2\Pi_p}{\partial K\partial\rho}\right) \Big/ \left(\frac{\partial^2\Pi_p}{\partial K^2}\right)\right)\Big|_{K_p^B}$, we calculate $\frac{\partial^2 \Pi_p}{\partial K \partial \rho} =$

$$\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \Big(\sigma_\xi z_0 - \frac{\sigma_\xi \rho z_2}{\sqrt{1-\rho^2}}\Big)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+ \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \Big(\sigma_\xi z_0 - \frac{\sigma_\xi \rho z_2}{\sqrt{1-\rho^2}}\Big)\phi(z_2)dz_2\phi(z_0)dz_0.$$

Through some normal distribution properties, we have $\frac{\partial^2 \Pi_p}{\partial K \partial \rho} =$

$$\sigma_\xi \phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\Bigg(1-\Phi\Big(\frac{\sigma_\beta(y+2bK-\mu_\xi)-\rho\sigma_\xi(yK-\mu_\beta)}{\sigma_\xi\sigma_\beta\sqrt{1-\rho^2}}\Big)\Bigg)$$

$$-\sigma_\xi \phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\Bigg(1-\Phi\Big(\frac{\sigma_\beta\big((1+r_p)y+2bK-\mu_\xi\big)-\rho\sigma_\xi(yK-\mu_\beta)}{\sigma_\xi\sigma_\beta\sqrt{1-\rho^2}}\Big)\Bigg) > 0.$$

Up to here, we prove that $\frac{\partial^2 \Pi_p(K)}{\partial K \partial \rho} > 0$ for all K, therefore, the sensitivity analyses with respect to K_p^B are derived, that is K_p^B increases in ρ.

We then proof the impact of ρ on $\Pi_p(K)$. $\frac{\Pi_p(K)}{\partial \rho}$ will have no term from differentiating the limits of integration, because the integrands (optimal profit in production stage) among con-terminal domain of integrations are continuous. Starting from the derivative of Equation (A.17) with respect to ρ, since profit function $\Pi_p(K)$ is continuous and it derivatives at the boundaries are cancelled out, we have $\frac{\partial \Pi_p(K)}{\partial \rho} = \sum_{i=1}^{6} \frac{\partial V_i^p(K)}{\partial \rho}$, the derivatives is $\frac{\partial \Pi_p(K)}{\partial \rho} =$

$$\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y-\mu_\xi-\sigma_\xi\rho z_0}{\sigma_\xi\sqrt{1-\rho^2}}}^{\frac{y+2bK-\mu_\xi-\sigma_\xi\rho z_0}{\sigma_\xi\sqrt{1-\rho^2}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0 - y\right)}{2b} \cdot$$

$$\left(z_0\sqrt{1-\rho^2}-z_2\rho\right)\phi(z_2)dz_2\phi(z_0)dz_0 \cdot \frac{\sigma_\xi}{\sqrt{1-\rho^2}}$$

$$+ \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} K\left(z_0\sqrt{1-\rho^2}-z_2\rho\right)\phi(z_2)dz_2\phi(z_0)dz_0 \cdot \frac{\sigma_\xi}{\sqrt{1-\rho^2}}$$

$$+ \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y-\mu_\xi-\sigma_\xi\rho z_0}{\sigma_\xi\sqrt{1-\rho^2}}}^{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)^+}{y}-\mu_\xi-\sigma_\xi\rho z_0}{\sigma_\xi\sqrt{1-\rho^2}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0 - y\right)}{2b} \cdot$$

$$\left(z_0\sqrt{1-\rho^2}-z_2\rho\right)\phi(z_2)dz_2\phi(z_0)dz_0 \cdot \frac{\sigma_\xi}{\sqrt{1-\rho^2}}$$

$$+ \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \frac{(z_0\sigma_\beta+\mu_\beta)}{y} \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}$$

$$\left(z_0\sqrt{1-\rho^2}-z_2\rho\right)\phi(z_2)dz_2\phi(z_0)dz_0 \cdot \frac{\sigma_\xi}{\sqrt{1-\rho^2}}$$

$$+ \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-(1+r_p)y\right)}{2b} \cdot$$

$$\left(z_0\sqrt{1-\rho^2}-z_2\rho\right)\phi(z_2)dz_2\phi(z_0)dz_0 \cdot \frac{\sigma_\xi}{\sqrt{1-\rho^2}}$$

$$+ \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} K\left(z_0\sqrt{1-\rho^2}-z_2\rho\right)\phi(z_2)dz_2\phi(z_0)dz_0 \cdot \frac{\sigma_\xi}{\sqrt{1-\rho^2}}.$$

When transforming standard normal distribution back to $(\tilde{\xi},\tilde{\beta})$ following $\tilde{z}_2 \overset{d}{=} \frac{\tilde{\xi}-\mu_\xi-\rho\frac{\sigma_\xi}{\sigma_\beta}(\tilde{\beta}-\mu_\beta)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big|\tilde{\beta}$ and $\tilde{z}_0 \overset{d}{=} \frac{\tilde{\beta}-\mu_\beta}{\sigma_\beta}$, we have $\frac{\partial \Pi_p(K)}{\partial \rho} = \left(\frac{1}{1-\rho^2}\right)\frac{\sigma_\xi}{\sigma_\beta}\underset{\xi}{\mathbf{E}}\Big[\underset{\beta|\xi}{\mathbf{E}}\Big[q^*(K,\tilde{\xi},\tilde{\beta})\cdot \left(\tilde{\beta}-\mu_\beta-\rho\frac{\sigma_\beta}{\sigma_\xi}(\tilde{\xi}-\mu_\xi)\right)\Big|\tilde{\xi}\Big]\Big]$, through turning $z_0\sqrt{1-\rho^2}-z_2\rho$ in integrands of all terms into $\left(\frac{\beta-\mu_\beta}{\sigma_\beta}\right)\sqrt{1-\rho^2}-\left(\frac{\xi-\mu_\xi-\rho\frac{\sigma_\xi}{\sigma_\beta}(\beta-\mu_\beta)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\right)\rho = \frac{1}{\sigma_\beta\sqrt{1-\rho^2}}\left(\beta-\mu_\beta-\rho\frac{\sigma_\beta}{\sigma_\xi}(\xi-\mu_\xi)\right)$. Since $\tilde{\beta}|\tilde{\xi}$ is normally distributed with parameters $N\left(\mu_\beta+\rho\frac{\sigma_\beta}{\sigma_\xi}(\tilde{\xi}-\mu_\xi),\sigma_\beta^2(1-\rho^2)\right)$ and $q^*(K,\xi,\beta)$ is piecewise differentiable and continuous, $\underset{\beta|\xi}{\mathbf{E}}\Big[q^*(K,\xi,\tilde{\beta})\cdot\left(\tilde{\beta}-\mu_\beta-\rho\frac{\sigma_\beta}{\sigma_\xi}(\xi-\mu_\xi)\right)\Big|\xi\Big]$ is able to be simplified using Stein (1972)'s lemma such that $\mathbf{E}_{\beta|\xi}\Big[q^*(K,\xi,\tilde{\beta})\cdot\Big(\tilde{\beta}-\mu_\beta-\rho\frac{\sigma_\beta}{\sigma_\xi}(\xi-\mu_\xi)\Big)\Big|\xi\Big]=\sigma_\beta^2(1-\rho^2)\mathbf{E}_{\beta|\xi}\Big[\frac{\partial q^*(K,\xi,\tilde{\beta})}{\partial\beta}\Big|\xi\Big]$, we have

$$\begin{aligned}\frac{\partial\Pi_p(K)}{\partial\rho}=&\Big(\frac{1}{1-\rho^2}\Big)\frac{\sigma_\xi}{\sigma_\beta}\mathbf{E}_{\xi}\Big[\mathbf{E}_{\beta|\xi}\Big[q^*(K,\tilde{\xi},\tilde{\beta})\cdot\Big(\tilde{\beta}-\mu_\beta-\rho\frac{\sigma_\beta}{\sigma_\xi}(\tilde{\xi}-\mu_\xi)\Big)\Big|\tilde{\xi}\Big]\Big]\\ =&\sigma_\xi\sigma_\beta\mathbf{E}_{\xi}\Big[\mathbf{E}_{\beta|\xi}\Big[\frac{\partial q^*(K,\xi,\tilde{\beta})}{\partial\beta}\Big|\xi\Big]\Big]=\int_{-\infty}^{yK}\int_{y+\frac{2b\beta}{y}}^{(1+r_p)y+\frac{2b\beta}{y}}\frac{1}{y}f(\xi,\beta)d\xi d\beta>0.\end{aligned}$$

■

Proof of Proposition 10 - The impact of σ_ξ on $\Pi_p(K_p^B)$:

According to the Envelope Theorem, $\frac{d\Pi_p(K_p^B(\omega))}{d\sigma_\xi}=\frac{\partial\Pi_p(K)}{\partial\sigma_\xi}\Big|_{K_p^B}$, the result is proved by investigating the sign of $\frac{\partial\Pi_p(K)}{\partial\sigma_\xi}\Big|_{K_p^B}$. Therefore, we first calculate

$$
\begin{aligned}
&\frac{\partial \Pi_p(K)}{\partial \sigma_\xi} = \\
&\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)}{2b} \\
&\qquad \left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0 \\
&+\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{(z_0\sigma_\beta+\mu_\beta)}{y}\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0 \\
&+\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-(1+r_p)y\right)}{2b} \\
&\qquad \left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0 \\
&+\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} K\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0 \\
&+\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} K\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0 \\
&+\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)}{2b} \\
&\qquad \left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0.
\end{aligned}
\tag{A.18}
$$

Noting that there is no term from differentiating the limits of integration, because the integrand equals to 0 when substituting the variable of integration with the corresponding limits containing σ_ξ. By the definition of q^*, we have that $\frac{\partial \Pi_p(K)}{\partial \sigma_\xi} = \underset{(z_2,z_0)}{\mathbf{E}}\left[\left(\tilde{z}_2\sqrt{1-\rho^2}+\rho\tilde{z}_0\right)q^*(K,\tilde{z}_2,\tilde{z}_0)\right]$. Therefore, we further apply Stein (1972)'s Lemma such that $\frac{\partial \Pi_p(K)}{\partial \sigma_\xi} = \rho\underset{z_0}{\mathbf{E}}\left[\frac{\partial \underset{z_2}{\mathbf{E}}[q^*(z_2,z_0)]}{\partial z_0}\right] +$

$$\sqrt{1-\rho^2}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \mathop{\mathbf{E}}_{z_2}\left[\frac{\partial q^*(z_2,z_0)}{\partial z_2}\right]\phi(z_0)dz_0 + \sqrt{1-\rho^2}\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \mathop{\mathbf{E}}_{z_2}\left[\frac{\partial q^*(z_2,z_0)}{\partial z_2}\right]\phi(z_0)dz_0 =$$

$$\begin{aligned}
&\frac{\sigma_\xi}{2b}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \phi(z_2)dz_2\phi(z_0)dz_0 \\
&+\frac{\sigma_\xi}{2b}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \phi(z_2)dz_2\phi(z_0)dz_0 \\
&+\frac{\rho\sigma_\beta}{y}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \phi(z_2)dz_2\phi(z_0)dz_0 \\
&+\frac{\sigma_\xi}{2b}\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \phi(z_2)dz_2\phi(z_0)dz_0.
\end{aligned} \tag{A.19}$$

Observing from above equation, we obtain that $\frac{\partial \Pi_p(K)}{\partial \sigma_\xi} > 0$ for all K when $\rho \geq 0$.

Then, we investigate the sign of $\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}$ when $\rho < 0$. First of all, we prove $\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}$ increases in σ_ξ as follows: Due to $\frac{d}{d\sigma_\xi}\left\{\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}\right\} = \frac{\partial^2 \Pi_p(K)}{\partial \sigma_\xi^2}\Big|_{K_p^B} + \left(\frac{\partial^2 \Pi_p(K)}{\partial \sigma_\xi \partial K}\Big|_{K_p^B}\right)\cdot\frac{dK_p^B}{d\sigma_\xi}$, we first calculate $\frac{\partial^2 \Pi_p(K)}{\partial \sigma_\xi^2}$ using Equation (A.18) such that $\frac{\partial^2 \Pi_p(K)}{\partial \sigma_\xi^2} =$

$$\begin{aligned}
&\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)^2}{2b}\phi(z_2)dz_2\phi(z_0)dz_0 \\
&+\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \int_{\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)^2}{2b}\phi(z_2)dz_2\phi(z_0)dz_0 \\
&+\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)^2}{2b}\phi(z_2)dz_2\phi(z_0)dz_0 > 0,
\end{aligned}$$

where there is no term from differentiating the limits of integration, because the integrand $\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)q^*(K,z_2,z_0)$ is continuous. Furthermore, $\left(\frac{\partial^2 \Pi_p(K)}{\partial \sigma_\xi \partial K}\Big|_{K_p^B}\right)\cdot\frac{dK_p^B}{d\sigma_\xi} \geq 0$, because $\frac{\partial^2 \Pi_p(K)}{\partial \sigma_\xi \partial K}\Big|_{K_p^B}$ and $\frac{dK_p^B}{d\sigma_\xi}$ have same sign by implicit differentiation $\frac{dK_p^B}{d\sigma_\xi} = -\left(\left(\frac{\partial^2 \Pi_p}{\partial K \partial \sigma_\xi}\right) \Big/ \left(\frac{\partial^2 \Pi_p}{\partial K^2}\right)\right)\Big|_{K_p^B}$. Therefore, we obtain $\frac{d}{d\sigma_\xi}\left\{\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}\right\} > 0$.

Since $\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}$ increases in σ_ξ, we prove the sign of $\lim\limits_{\sigma_\xi \to \infty}\left\{\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}\right\}$ and $\lim\limits_{\sigma_\xi \to 0}\left\{\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}\right\}$ in order to determine the sign of $\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}$ for all $\sigma_\xi > 0$.

Starting from $\lim\limits_{\sigma_\xi \to \infty}\left\{\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}\right\}$, we prove $\lim\limits_{\sigma_\xi \to \infty}\frac{\partial \Pi_p(K)}{\partial \sigma_\xi} > 0$ for all $K > 0$. First of all, we transform Equation (A.19) by substituting z_2 with $\frac{t-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}$ such that

$$\begin{aligned}
\frac{\partial \Pi_p(K)}{\partial \sigma_\xi} =& \frac{1}{2\sqrt{1-\rho^2}}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\int_{y}^{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}}\phi\left(\frac{t-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\right)dt\phi(z_0)dz_0\\
&+\frac{1}{2\sqrt{1-\rho^2}}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\int_{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}}^{(1+r_p)y+2bK}\phi\left(\frac{t-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\right)dt\phi(z_0)dz_0\\
&+\frac{\rho\sigma_\beta}{y}\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}\phi(z_2)dz_2\phi(z_0)dz_0\\
&+\frac{1}{2\sqrt{1-\rho^2}}\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\int_{y}^{y+2bK}\phi\left(\frac{t-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\right)dt\phi(z_0)dz_0,
\end{aligned} \tag{A.20}$$

therefore, we have $\lim\limits_{\sigma_\xi \to \infty}\frac{\partial \Pi_p(K)}{\partial \sigma_\xi} =$

$$\begin{aligned}
&\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\frac{(z_0\sigma_\beta+\mu_\beta)/y}{\sqrt{1-\rho^2}}\phi\Big(\frac{-\rho z_0}{\sqrt{1-\rho^2}}\Big)\phi(z_0)dz_0\\
&+\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\frac{K-(z_0\sigma_\beta+\mu_\beta)/y}{\sqrt{1-\rho^2}}\phi\Big(\frac{-\rho z_0}{\sqrt{1-\rho^2}}\Big)\phi(z_0)dz_0\\
&+\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\frac{K}{\sqrt{1-\rho^2}}\phi\Big(\frac{-\rho z_0}{\sqrt{1-\rho^2}}\Big)\phi(z_0)dz_0\\
=&\int_{-\infty}^{\infty}\frac{K}{\sqrt{1-\rho^2}}\phi\Big(\frac{-\rho z_0}{\sqrt{1-\rho^2}}\Big)\phi(z_0)dz_0 > 0
\end{aligned}$$

which means $\lim\limits_{\sigma_\xi \to \infty}\frac{\partial \Pi_p(K)}{\partial \sigma_\xi} > 0$ for all $K > 0$.

Lastly, from Equation (A.20), we calculate $\lim_{\sigma_\xi \to 0}\left\{\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}\right\} =$

$$\frac{\rho\sigma_\beta}{y}\lim_{\sigma_\xi \to 0}\int_{-\infty}^{\frac{y\lim_{\sigma_\xi\to 0}K_p^B-\mu_\beta}{\sigma_\beta}}\Bigg(\Phi\Bigg(\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Bigg) - \Phi\Bigg(\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Bigg)\Bigg)\phi(z_0)dz_0, \tag{A.21}$$

we have that the sign of $\lim_{\sigma_\xi \to 0}\left\{\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}\right\}$ highly depends on the value of $\lim_{\sigma_\xi\to 0}K_p^B$. Knowing that $\lim_{\sigma_\xi\to 0}K_p^B \in \left(0, \frac{\mu_\xi-(1+r_p)y}{2b}\right)$ if $\omega > \left(1-\Phi\left(\frac{\frac{y\left(\mu_\xi-(1+r_p)y\right)}{2b}-\mu_\beta}{\sigma_\beta}\right)\right)r_p y$ and $\lim_{\sigma_\xi\to 0}K_p^B \in \left[\frac{\mu_\xi-(1+r_p)y}{2b}, \frac{\mu_\xi-y}{2b}\right)$ if $\omega \le \left(1-\Phi\left(\frac{\frac{y\left(\mu_\xi-(1+r_p)y\right)}{2b}-\mu_\beta}{\sigma_\beta}\right)\right)r_p y$, we further calculate Equation (A.21) and obtain that, 1) when $\omega > \left(1-\Phi\left(\frac{\frac{y\left(\mu_\xi-(1+r_p)y\right)}{2b}-\mu_\beta}{\sigma_\beta}\right)\right)r_p y$, $\lim_{\sigma_\xi \to 0}\left\{\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}\right\} = 0$ and 2) when $\omega \le \left(1-\Phi\left(\frac{\frac{y\left(\mu_\xi-(1+r_p)y\right)}{2b}-\mu_\beta}{\sigma_\beta}\right)\right)r_p y$,

$$\lim_{\sigma_\xi \to 0}\left\{\frac{\partial \Pi_p(K)}{\partial \sigma_\xi}\Big|_{K_p^B}\right\} = \frac{\rho\sigma_\beta}{y}\left(\Phi\left(\frac{y\lim_{\sigma_\xi\to 0}K_p^B-\mu_\beta}{\sigma_\beta}\right)-\Phi\left(\frac{\frac{y\left(\mu_\xi-(1+r_p)y\right)}{2b}-\mu_\beta}{\sigma_\beta}\right)\right) < 0.$$

Therefore, we summarize the impact of σ_ξ on $\Pi_p(K_p^B)$ as what showed in Proposition 10. ■

Proof of Proposition 11 - The impact of σ_ξ on K_p^B:

By implicit differentiation $\frac{dK_p^B}{d\sigma_\xi} = -\left(\left(\frac{\partial^2\Pi_p}{\partial K\partial\sigma_\xi}\right)\Big/\left(\frac{\partial^2\Pi_p}{\partial K^2}\right)\right)\Big|_{K_p^B}$, we are interested in where $\frac{\partial^2\Pi_p(K)}{\partial K\partial\sigma_\xi}\Big|_{K_p^B}$ lies above or below zero. Therefore, we calculate $\frac{\partial^2\Pi_p(K)}{\partial K\partial\sigma_\xi}$ from Equation (A.16) using Leibniz' formula, obtaining $\frac{\partial^2\Pi_p(K)}{\partial K\partial\sigma_\xi} =$

$$\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0 + \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\int_{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0 \tag{A.22}$$

we find that $\frac{\partial^2\Pi_p(K)}{\partial K\partial\sigma_\xi} > 0$ for all K when $\rho \ge 0$.

Then, we investigate the value of $\frac{\partial^2\Pi_p(K)}{\partial K\partial\sigma_\xi}\big|_{K_p^B}$ when $\rho < 0$ by several steps.

Firstly, we prove that K_p^B increases in σ_ξ when σ_ξ is greater than $\overline{\sigma}_\xi^{K_p}$, where $\overline{\sigma}_\xi^{K_p}$ is the unique solution of $\frac{\partial\Pi_p(K)}{\partial K}\Big|_{K=\frac{\mu_\xi-y}{2b}} = 0$. Due to $\frac{\partial\Pi_p(K)}{\partial K}$ decreases in K, we want to prove $\frac{\partial\Pi_p(K)}{\partial K}\Big|_{K=\frac{\mu_\xi-y}{2b}}$ increases in $\sigma_\xi \in [\overline{\sigma}_\xi^{K_p}, \infty)$ so that as σ_ξ increasing from $\overline{\sigma}_\xi^{K_p}$, K_p^B increases. From Equation (A.16), we obtain $\frac{\partial\Pi_p(K)}{\partial K}\Big|_{K=\frac{\mu_\xi-y}{2b}} =$

$$-\omega + \sigma_\xi \int_{\frac{\frac{y(\mu_\xi-y)}{2b}-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{-\rho z_0}{\sqrt{1-\rho^2}}}^{\infty} \Big(z_2\sqrt{1-\rho^2}+\rho z_0\Big)\phi(z_2)dz_2\phi(z_0)dz_0$$
$$+ \sigma_\xi \int_{-\infty}^{\frac{\frac{y(\mu_\xi-y)}{2b}-\mu_\beta}{\sigma_\beta}} \int_{\frac{r_p y/\sigma_\xi-\rho z_0}{\sqrt{1-\rho^2}}}^{\infty} \Big(z_2\sqrt{1-\rho^2}+\rho z_0 - r_p y/\sigma_\xi\Big)\phi(z_2)dz_2\phi(z_0)dz_0$$

and we use this equation to prove $\frac{\partial}{\partial\sigma_\xi}\Big\{\frac{\partial\Pi_p(K)}{\partial K}\Big|_{K=\frac{\mu_\xi-y}{2b}}\Big\} > 0$ when $\sigma_\xi \in [\overline{\sigma}_\xi^{K_p}, \infty)$. Further calculating $\frac{\partial}{\partial\sigma_\xi}\Big\{\frac{\partial\Pi_p(K)}{\partial K}\Big|_{K=\frac{\mu_\xi-y}{2b}}\Big\} =$

$$\int_{\frac{\frac{y(\mu_\xi-y)}{2b}-\mu_\beta}{\sigma_\beta}}^{\infty} \int_{\frac{-\rho z_0}{\sqrt{1-\rho^2}}}^{\infty} \Big(z_2\sqrt{1-\rho^2}+\rho z_0\Big)\phi(z_2)dz_2\phi(z_0)dz_0$$
$$+ \int_{-\infty}^{\frac{\frac{y(\mu_\xi-y)}{2b}-\mu_\beta}{\sigma_\beta}} \int_{\frac{r_p y/\sigma_\xi-\rho z_0}{\sqrt{1-\rho^2}}}^{\infty} \Big(z_2\sqrt{1-\rho^2}+\rho z_0\Big)\phi(z_2)dz_2\phi(z_0)dz_0,$$

we can prove $\frac{\partial}{\partial\sigma_\xi}\Big\{\frac{\partial\Pi_p(K)}{\partial K}\Big|_{K=\frac{\mu_\xi-y}{2b}}\Big\}$ increases in $\sigma_\xi \in [\overline{\sigma}_\xi^{K_p}, \infty)$, through

$$\frac{\partial}{\partial\sigma_\xi}\Big\{\frac{\partial\Pi_p(K)}{\partial K}\Big|_{K=\frac{\mu_\xi-y}{2b},\sigma_\xi=\overline{\sigma}_\xi^{K_p}}\Big\} = \frac{\omega}{\overline{\sigma}_\xi^{K_p}} + \frac{r_p y}{\sigma_\xi} \int_{-\infty}^{\frac{\frac{y(\mu_\xi-y)}{2b}-\mu_\beta}{\sigma_\beta}} \int_{\frac{r_p y/\sigma_\xi-\rho z_0}{\sqrt{1-\rho^2}}}^{\infty} \phi(z_2)dz_2\phi(z_0)dz_0 > 0$$

and

$$\frac{\partial^2}{\partial\sigma_\xi^2}\Big\{\frac{\partial\Pi_p(K)}{\partial K}\Big|_{K=\frac{\mu_\xi-y}{2b}}\Big\} = \frac{r_p^2 y^2}{\sigma_\xi^3\sqrt{1-\rho^2}} \int_{-\infty}^{\frac{\frac{y(\mu_\xi-y)}{2b}-\mu_\beta}{\sigma_\beta}} \phi\Big(\frac{r_p y/\sigma_\xi-\rho z_0}{\sqrt{1-\rho^2}}\Big)\phi(z_0)dz_0 > 0.$$

To summarize this part, we conclude that K_p^B increases in σ_ξ when $\sigma_\xi > \overline{\sigma}_\xi^{K_p}$.

Secondly, we calculate the sign of $\frac{\partial^2\Pi_p(K)}{\partial\sigma_\xi\partial K}\Big|_{K_p^B}$ when $\sigma_\xi \to 0$ from Equation (A.22). Noting that the sign of $\lim_{\sigma_\xi\to 0}\Big\{\frac{\partial^2\Pi_p(K)}{\partial\sigma_\xi\partial K}\Big|_{K_p^B}\Big\}$ highly depends on the value of

$\lim_{\sigma_\xi \to 0} K_p^B$, Therefore, we characterize $\lim_{\sigma_\xi \to 0} K_p^B$ by calculating $\lim_{\sigma_\xi \to 0} \frac{\partial \Pi_p(K)}{\partial K}$, that is

$$\lim_{\sigma_\xi \to 0} \frac{\partial \Pi_p(K)}{\partial K}$$
$$= \begin{cases} -\omega + \int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \Big(\mu_\xi - y - 2bK\Big)\phi(z_0)dz_0 \\ \quad + \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \Big(\mu_\xi - (1+r_p)y - 2bK\Big)\phi(z_0)dz_0, & \text{if } 0 < K < \frac{\mu_\xi-(1+r_p)y}{2b} \\ -\omega + \Big(\mu_\xi - y - 2bK\Big)\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty} \phi(z_0)dz_0, & \text{if } \frac{\mu_\xi-(1+r_p)y}{2b} \le K < \frac{\mu_\xi - y}{2b} \\ -\omega, & \text{if } K \ge \frac{\mu_\xi - y}{2b}, \end{cases}$$

which we derive that

- $\lim_{\sigma_\xi \to 0} K_p^B \in \big(0, \frac{\mu_\xi-(1+r_p)y}{2b}\big)$ if $\omega > \Big(1 - \Phi\Big(\frac{\frac{y\left(\mu_\xi-(1+r_p)y\right)}{2b} - \mu_\beta}{\sigma_\beta}\Big)\Big)r_p y$,
- $\lim_{\sigma_\xi \to 0} K_p^B = \frac{\mu_\xi-(1+r_p)y}{2b}$ if $\omega = \Big(1 - \Phi\Big(\frac{\frac{y\left(\mu_\xi-(1+r_p)y\right)}{2b} - \mu_\beta}{\sigma_\beta}\Big)\Big)r_p y$ and
- $\lim_{\sigma_\xi \to 0} K_p^B \in \big(\frac{\mu_\xi-(1+r_p)y}{2b}, \frac{\mu_\xi-y}{2b}\big)$ if $\omega < \Big(1 - \Phi\Big(\frac{\frac{y\left(\mu_\xi-(1+r_p)y\right)}{2b} - \mu_\beta}{\sigma_\beta}\Big)\Big)r_p y$.

Having above result, since $\lim_{\sigma_\xi \to 0}\Big\{\frac{\partial^2 \Pi_p(K)}{\partial \sigma_\xi \partial K}\Big|_{K_p^B}\Big\} =$

$$\rho\phi\Big(\frac{y \lim_{\sigma_\xi \to 0} K_p^B - \mu_\beta}{\sigma_\beta}\Big) \cdot \Bigg(\Phi\Bigg(\frac{(1+r_p)y + 2b\lim_{\sigma_\xi \to 0} K_p^B - \mu_\xi}{\lim_{\sigma_\xi \to 0}\sigma_\xi\sqrt{(1-\rho^2)}} - \frac{\rho\big(\frac{y\lim_{\sigma_\xi \to 0} K_p^B - \mu_\beta}{\sigma_\beta}\big)}{\sqrt{1-\rho^2}}\Bigg)$$
$$- \Phi\Bigg(\frac{y + 2b\lim_{\sigma_\xi \to 0} K_p^B - \mu_\xi}{\lim_{\sigma_\xi \to 0}\sigma_\xi\sqrt{(1-\rho^2)}} - \frac{\rho\big(\frac{y\lim_{\sigma_\xi \to 0} K_p^B - \mu_\beta}{\sigma_\beta}\big)}{\sqrt{1-\rho^2}}\Bigg)\Bigg)$$

we obtain that $\lim_{\sigma_\xi \to 0}\Big\{\frac{\partial^2 \Pi_p(K)}{\partial \sigma_\xi \partial K}\Big|_{K_p^B}\Big\} = 0$ if $\omega > \Big(1 - \Phi\Big(\frac{\frac{y\left(\mu_\xi-(1+r_p)y\right)}{2b} - \mu_\beta}{\sigma_\beta}\Big)\Big)r_p y$; $\lim_{\sigma_\xi \to 0}\Big\{\frac{\partial^2 \Pi_p(K)}{\partial \sigma_\xi \partial K}\Big|_{K_p^B}\Big\} = \rho\phi\big(\frac{y\lim_{\sigma_\xi \to 0} K_p^B - \mu_\beta}{\sigma_\beta}\big)\Phi\Big(-\frac{\rho\big(\frac{y\lim_{\sigma_\xi \to 0} K_p^B - \mu_\beta}{\sigma_\beta}\big)}{\sqrt{1-\rho^2}}\Big) < 0$ if $\omega = \Big(1 - \Phi\Big(\frac{\frac{y\left(\mu_\xi-(1+r_p)y\right)}{2b} - \mu_\beta}{\sigma_\beta}\Big)\Big)r_p y$; and $\lim_{\sigma_\xi \to 0}\Big\{\frac{\partial^2 \Pi_p(K)}{\partial \sigma_\xi \partial K}\Big|_{K_p^B}\Big\} = \rho\phi\big(\frac{y\lim_{\sigma_\xi \to 0} K_p^B - \mu_\beta}{\sigma_\beta}\big) < 0$ if $\omega < \Big(1 - \Phi\Big(\frac{\frac{y\left(\mu_\xi-(1+r_p)y\right)}{2b} - \mu_\beta}{\sigma_\beta}\Big)\Big)r_p y$. ■

Proof of Proposition 12 - The impact of σ_β on K_p^B:

We proof the sensitivity analysis result by implicit differentiation, $\frac{dK_p^B}{d\sigma_\beta} = -\left(\left(\frac{\partial^2 \Pi_p}{\partial K \partial \sigma_\beta}\right) \Big/ \left(\frac{\partial^2 \Pi_p}{\partial K^2}\right)\right)\Big|_{K_p^B}$. We calculate $\frac{\partial^2 \Pi_p(K)}{\partial K \partial \sigma_\beta}$ using Leibniz' formula, that is $\frac{\partial^2 \Pi_p(K)}{\partial K \partial \sigma_\beta} =$

$$\frac{(yK-\mu_\beta)\sqrt{\sigma_\xi^2(1-\rho^2)}}{\sigma_\beta^2}\phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\Bigg(\mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{y+2bK-\mu_\xi-\rho\sigma_\xi\big(\frac{yK-\mu_\beta}{\sigma_\beta}\big)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]$$
$$- \mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi\big(\frac{yK-\mu_\beta}{\sigma_\beta}\big)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]\Bigg),$$

at which we can immediately conclude that when $yK < \mu_\beta$, $\frac{\partial^2 \Pi_p(K)}{\partial K \partial \sigma_\beta} < 0$ and when $yK \geq \mu_\beta$, $\frac{\partial^2 \Pi_p(K)}{\partial K \partial \sigma_\beta} > 0$. Furthermore, the necessary and sufficient condition for $K_p^B < \mu_\beta/y$ and $K_p^B \geq \mu_\beta/y$ are $\frac{\partial \Pi_p(K)}{\partial K}\Big|_{\mu_\beta/y} > 0$ and $\frac{\partial \Pi_p(K)}{\partial K}\Big|_{\mu_\beta/y} \leq 0$ respectively, where $\frac{\partial \Pi_p(K)}{\partial K}\Big|_{\frac{\mu_\beta}{y}} =$

$$-\omega + \int_0^\infty \int_{\frac{y+2b\mu_\beta/y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^\infty$$
$$\Big(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - y - 2b\mu_\beta/y\Big)\phi(z_2)dz_2\phi(z_0)dz_0$$
$$+ \int_{-\infty}^0 \int_{\frac{(1+r_p)y+2b\mu_\beta/y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^\infty$$
$$\Big(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - (1+r_p)y - 2b\mu_\beta/y\Big)\phi(z_2)dz_2\phi(z_0)dz_0.$$

From above equation, we have $\frac{\partial \Pi_p(K)}{\partial K}\Big|_{\frac{\mu_\beta}{y}} =$

$$\sigma_\xi\sqrt{(1-\rho^2)}\int_0^\infty \mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{y+2b\mu_\beta/y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]\phi(z_0)dz_0$$
$$+\sigma_\xi\sqrt{(1-\rho^2)}\int_{-\infty}^0 \mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{(1+r_p)y+2b\mu_\beta/y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]\phi(z_0)dz_0,$$

that doesn't depend on σ_β. Then, define $\omega_\beta^{K_p}(\rho) :=$

$$\sigma_\xi\sqrt{(1-\rho^2)}\int_0^\infty \mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{y+2b\mu_\beta/y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]\phi(z_0)dz_0$$
$$+\sigma_\xi\sqrt{(1-\rho^2)}\int_{-\infty}^0 \mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{(1+r_p)y+2b\mu_\beta/y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]\phi(z_0)dz_0,$$

we can conclude the sensitivity result: K_p^B increases in σ_β when $\omega < \omega_\beta^{K_p}(\rho)$; K_p^B decreases in σ_β when $\omega > \omega_\beta^{K_p}(\rho)$.

We know that $\frac{\partial\Pi_p(K)}{\partial K}$ strictly increases in ρ for any given $K > 0$, therefore, $\frac{\partial\Pi_p(K)}{\partial K}\Big|_{\frac{\mu_\beta}{y}} = \omega_\beta^{K_p}(\rho)$ strictly increases in ρ. For this reason, we can further investigate the effect of ρ on the sensitivity result. Define $\omega_\beta^{K_p}(-1) = \lim_{\rho\to-1}\omega_\beta^{K_p}$ and $\omega_\beta^{K_p}(1) = \lim_{\rho\to1}\omega_\beta^{K_p}$ such that $\omega_\beta^{K_p}(-1) < \omega_\beta^{K_p}(1)$, we have a unique ρ denoted by $\rho_\beta^{K_p}(\omega)$ that solves following equation

$$\omega = \sigma_\xi\sqrt{(1-\rho^2)}\int_0^\infty \mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{y+2b\mu_\beta/y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]\phi(z_0)dz_0$$
$$+\sigma_\xi\sqrt{(1-\rho^2)}\int_{-\infty}^0 \mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{(1+r_p)y+2b\mu_\beta/y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]\phi(z_0)dz_0$$

for all $\omega \in \Big(\omega_\beta^{K_p}(-1),\ \omega_\beta^{K_p}(1)\Big)$. Therefore, the sensitivity result can be re-calibrated as follows: under condition $\begin{cases}\omega > \omega_\beta^{K_p}(1) \text{ for all } \rho\in(-1,1);\\ \text{or } \omega\in\Big(\omega_\beta^{K_p}(-1),\ \omega_\beta^{K_p}(1)\Big) \text{ and } \rho < \rho_\beta^{K_p}(\omega),\end{cases}$ we have $yK_p^B < \mu_\beta$ which implies $\frac{\partial^2\Pi_p(K)}{\partial K\partial\sigma_\beta}\big|_{K_p^B} < 0$;

under condition $\begin{cases}\omega < \omega_\beta^{K_p}(-1) \text{ for all } \rho\in(-1,1);\\ \text{or } \omega\in\Big(\omega_\beta^{K_p}(-1),\ \omega_\beta^{K_p}(1)\Big) \text{ and } \rho > \rho_\beta^{K_p}(\omega),\end{cases}$ we have $yK_p^B >$ μ_β which implies $\frac{\partial^2\Pi_p(K)}{\partial K\partial\sigma_\beta}\big|_{K_p^B} > 0$; and lastly, under condition $\omega\in\Big(\omega_\beta^{K_p}(-1),\ \omega_\beta^{K_p}(1)\Big)$ and $\rho = \rho_\beta^{K_p}(\omega)$, we have $yK_p^B = \mu_\beta$ which implies $\frac{\partial^2\Pi_p(K)}{\partial K\partial\sigma_\beta}\big|_{K_p^B} = 0$. ∎

Proof of Proposition 13 - The impact of σ_β on $\Pi_p(K_p^B)$:

According to the Envelope Theorem, $\frac{d\Pi_p(K_p^B)}{d\sigma_\beta} = \frac{\partial\Pi_p(K)}{\partial\sigma_\beta}\Big|_{K_p^B}$, we prove the result by investigating the sign of $\frac{\partial \Pi_p(K)}{\partial \sigma_\beta}\Big|_{K_p^B}$. Since Equation (A.17) has no term from differentiating the limits of integration, we take the derivative of it with respect to σ_β such that the remaining terms are ones that contain σ_β in integrand $\frac{\partial \Pi_p(K)}{\partial \sigma_\beta} =$

$$\sigma_\xi \sqrt{1-\rho^2} \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \frac{z_0}{y} \mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{y + \frac{2b(z_0\sigma_\beta+\mu_\beta)}{y} - \mu_\xi - \rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]\phi(z_0)dz_0$$

$$- \sigma_\xi \sqrt{1-\rho^2} \int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}} \frac{z_0}{y} \mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{(1+r_p)y + \frac{2b(z_0\sigma_\beta+\mu_\beta)}{y} - \mu_\xi - \rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]\phi(z_0)dz_0.$$

apparently, when $yK_p^B \le \mu_\beta$, we have $\frac{\partial \Pi_p(K)}{\partial \sigma_\beta}\big|_{K_p^B} < 0$ for all σ_β.

Then we discuss the sign of $\frac{\partial \Pi_p(K)}{\partial \sigma_\beta}\big|_{K_p^B}$ under condition $yK_p^B > \mu_\beta$. First of all, we prove $\frac{\partial \Pi_p(K)}{\partial \sigma_\beta}\big|_{K_p^B}$ decreases in σ_β when $K_p^B > \mu_\beta/y$ by following calculation, $\frac{d}{d\sigma_\beta}\Big\{\frac{\partial \Pi_p(K)}{\partial \sigma_\beta}\Big|_{K_p^B}\Big\} =$

$$-2\int_{-\infty}^{\frac{yK_p^B-\mu_\beta}{\sigma_\beta}} \Big(\frac{z_0}{y}\Big)^2 \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{(1+r_p)y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \phi(z_2)dz_2\phi(z_0)dz_0$$

$$-\sigma_\xi\sqrt{(1-\rho^2)}\Big(\frac{yK_p^B-\mu_\beta}{\sigma_\beta^2}\Big)\left(\frac{(K_p^B-\mu_\beta/y)}{\sigma_\beta} - \frac{dK_p^B}{d\sigma_\beta}\right)\phi\Big(\frac{yK_p^B-\mu_\beta}{\sigma_\beta}\Big)\cdot$$

$$\left(\mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{y+2bK_p^B-\mu_\xi-\rho\sigma_\xi\big(\frac{yK_p^B-\mu_\beta}{\sigma_\beta}\big)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big] - \mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{(1+r_p)y+2bK_p^B-\mu_\xi-\rho\sigma_\xi\big(\frac{yK-\mu_\beta}{\sigma_\beta}\big)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]\right)$$

where the first term is apparently negative and the second term is negative if $\frac{(K_p^B-\mu_\beta/y)}{\sigma_\beta} - \frac{dK_p^B}{d\sigma_\beta} > 0$. We prove $\frac{(K_p^B-\mu_\beta/y)}{\sigma_\beta} - \frac{dK_p^B}{d\sigma_\beta} > 0$ is true by writing the expression of $\frac{dK_p^B}{d\sigma_\beta}$ according to $\frac{dK_p^B}{d\sigma_\beta} = \left(\Big(\frac{\partial^2\Pi_p}{\partial K\partial\sigma_\beta}\Big) \Big/ \Big(-\frac{\partial^2\Pi_p}{\partial K^2}\Big)\right)\Big|_{K_p^B}$. Since

$\frac{\partial^2\Pi_p(K)}{\partial K\partial\sigma_\beta}\big|_{K_p^B}$ is known, we calculate $\frac{\partial^2\Pi_p(K)}{\partial K^2} =$

$$
\begin{aligned}
&-2\int_{\frac{yK-\mu_\beta}{\sigma_\beta}}^{\infty}\int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\phi(z_2)dz_2\phi(z_0)dz_0\\
&-2\int_{-\infty}^{\frac{yK-\mu_\beta}{\sigma_\beta}}\int_{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\phi(z_2)dz_2\phi(z_0)dz_0\\
&-\frac{y\sqrt{\sigma_\xi^2(1-\rho^2)}}{\sigma_\beta}\phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\Bigg(\mathbf{E}\Bigg[\Bigg(\tilde z_2-\Big(\frac{y+2bK-\mu_\xi-\rho\sigma_\xi\big(\frac{yK-\mu_\beta}{\sigma_\beta}\big)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Bigg)^+\Bigg]\\
&\qquad-\mathbf{E}\Bigg[\Bigg(\tilde z_2-\Big(\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi\big(\frac{yK-\mu_\beta}{\sigma_\beta}\big)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Bigg)^+\Bigg]\Bigg).
\end{aligned}
$$

Define $T_p(K) :=$

$$
\begin{aligned}
&\frac{y\sqrt{\sigma_\xi^2(1-\rho^2)}}{\sigma_\beta}\phi\Big(\frac{yK-\mu_\beta}{\sigma_\beta}\Big)\Bigg(\mathbf{E}\Bigg[\Bigg(\tilde z_2-\Big(\frac{y+2bK-\mu_\xi-\rho\sigma_\xi\big(\frac{yK-\mu_\beta}{\sigma_\beta}\big)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Bigg)^+\Bigg]\\
&\qquad-\mathbf{E}\Bigg[\Bigg(\tilde z_2-\Big(\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi\big(\frac{yK-\mu_\beta}{\sigma_\beta}\big)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Bigg)^+\Bigg]\Bigg),
\end{aligned}
$$

we can write $\frac{dK_p^B}{d\sigma_\beta} =$

$$
\frac{\frac{(K_p^B-\mu_\beta/y)}{\sigma_\beta}\cdot T_p(K_p^B)}{T_p(K_p^B)+2\int_{\frac{yK_p^B-\mu_\beta}{\sigma_\beta}}^{\infty}\int_{\frac{y+2bK_p^B-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\phi(z_2)dz_2\phi(z_0)dz_0+2\int_{-\infty}^{\frac{yK_p^B-\mu_\beta}{\sigma_\beta}}\int_{\frac{(1+r_p)y+2bK_p^B-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}\phi(z_2)dz_2\phi(z_0)dz_0}
$$

$$
<\frac{(K_p^B-\mu_\beta/y)}{\sigma_\beta},
$$

which we finish the prove of $\frac{d}{d\sigma_\beta}\Big\{\frac{\partial\Pi_p(K)}{\partial\sigma_\beta}\Big|_{K_p^B}\Big\} < 0$.

Secondly, we investigate the sign of $\frac{\partial\Pi_p(K)}{\partial\sigma_\beta}\Big|_{K_p^B}$ when σ_β takes special values, that are $\sigma_\beta\to 0$ and $\sigma_\beta = \frac{\rho\sigma_\xi y}{2b}$. Starting from $\lim_{\sigma_\beta\to 0}\Big\{\frac{\partial\Pi_p(K)}{\partial\sigma_\beta}\Big|_{K_p^B}\Big\}$ when $\lim_{\sigma_\beta\to 0}K_p^B > \frac{\mu_\beta}{y}$, we have $\lim_{\sigma_\beta\to 0}\Big\{\frac{\partial\Pi_p(K)}{\partial\sigma_\beta}\Big|_{K_p^B}\Big\} =$

$$
\frac{\rho\sigma_\xi}{y}\int_{-\infty}^{\infty}\Bigg(\Phi\Bigg(\frac{(1+r_p)y+\frac{2b\mu_\beta}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Bigg)-\Phi\Bigg(\frac{y+\frac{2b\mu_\beta}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Bigg)\Bigg)\phi(z_0)dz_0,
$$

that is to say:

- when $\lim_{\sigma_\beta \to 0} K_p^B > \frac{\mu_\beta}{y}$ and $\rho > 0$, $\lim_{\sigma_\beta \to 0} \left\{ \frac{\partial \Pi_p(K)}{\partial \sigma_\beta} \Big|_{K_p^B} \right\} > 0$;

- otherwise, $\lim_{\sigma_\beta \to 0} \left\{ \frac{\partial \Pi_p(K)}{\partial \sigma_\beta} \Big|_{K_p^B} \right\} \leq 0$.

Noting that, the condition under which $\lim_{\sigma_\beta \to 0} K_p^B > \frac{\mu_\beta}{y}$ is highly depend on the value of $\lim_{\sigma_\beta \to 0} K_p^B$, we calculate $\lim_{\sigma_\beta \to 0} \frac{\partial \Pi_p(K)}{\partial K} =$

$$\begin{cases} -\omega + \int_{-\infty}^{\infty} \int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - y - 2bK \right) \phi(z_2) dz_2 \phi(z_0) dz_0 & \text{if } K < \frac{\mu_\beta}{y}; \\ -\omega + \int_{-\infty}^{\infty} \int_{\frac{(1+r_p)y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - (1+r_p)y - 2bK \right) \phi(z_2) dz_2 \phi(z_0) dz_0 & \text{if } K > \frac{\mu_\beta}{y}, \end{cases}$$

from which we obtain that $\lim_{\sigma_\beta \to 0} K_p^B > \frac{\mu_\beta}{y}$ is equivalent to

$$\begin{aligned} \omega < & \int_{-\infty}^{\infty} \int_{\frac{(1+r_p)y+2b\mu_\beta/y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \\ & \qquad \left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - (1+r_p)y - 2b\mu_\beta/y \right) \phi(z_2) dz_2 \phi(z_0) dz_0 \\ = & \sqrt{\sigma_\xi^2(1-\rho^2)} \int_{-\infty}^{\infty} \mathbf{E}\left[\left(\tilde{z}_2 - \left(\frac{(1+r_p)y + 2b\mu_\beta/y - \mu_\xi - \rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}} \right) \right)^+ \right] \phi(z_0) dz_0 \\ = & \mathbf{E}\left[\left(\tilde{\xi} - \left((1+r_p)y + \frac{2\mu_\beta}{y} \right) \right)^+ \right]. \end{aligned}$$

Define $\omega_\beta^{\Pi_p}(\omega, \rho) := \mathbf{E}\left[\left(\tilde{\xi} - \left((1+r_p)y + 2b\mu_\beta/y \right) \right)^+ \right]$, we conclude that $\lim_{\sigma_\beta \to 0} K_p^B > \frac{\mu_\beta}{y}$ is equivalent to $\omega < \omega_\beta^{\Pi_p}(\omega, \rho)$. Then, we calculate $\frac{\partial \Pi_p(K)}{\partial \sigma_\beta} \Big|_{\sigma_\beta = \frac{\rho\sigma_\xi y}{2b}, K = K_p^B}$ when $\rho > 0$ and obtain $\frac{\partial \Pi_p(K)}{\partial \sigma_\beta} \Big|_{\sigma_\beta = \frac{\rho\sigma_\xi y}{2b}, K = K_p^B} =$

$$\begin{aligned} & - \frac{\sqrt{\sigma_\xi^2(1-\rho^2)}}{y} \mathbf{E}\left[\left(\tilde{z}_2 - \left(\frac{y + \frac{2b\mu_\beta}{y} - \mu_\xi}{\sqrt{\sigma_\xi^2(1-\rho^2)}} \right) \right)^+ \right] \phi\left(\frac{yK_p^B - \mu_\beta}{\frac{\rho\sigma_\xi y}{2b}} \right) \\ & + \frac{\sqrt{\sigma_\xi^2(1-\rho^2)}}{y} \mathbf{E}\left[\left(\tilde{z}_2 - \left(\frac{(1+r_p)y + \frac{2b\mu_\beta}{y} - \mu_\xi}{\sqrt{\sigma_\xi^2(1-\rho^2)}} \right) \right)^+ \right] \phi\left(\frac{yK_p^B - \mu_\beta}{\frac{\rho\sigma_\xi y}{2b}} \right) < 0. \end{aligned}$$

As a result, we obtain that only when $\omega < \omega_\beta^{\Pi_p}(\omega, \rho)$ and $\rho > 0$ there exists a unique $\sigma_\beta^{\Pi_p}(\omega, \rho) \in (0, \frac{\rho\sigma_\xi y}{2b})$ such that $\frac{\partial \Pi_p(K)}{\partial \sigma_\beta}\big|_{K_p^B} \geq 0$ when $\sigma_\beta < \sigma_\beta^{\Pi_p}(\omega, \rho)$ and $\frac{\partial \Pi_p(K)}{\partial \sigma_\beta}\big|_{K_p^B} < 0$ when $\sigma_\beta > \sigma_\beta^{\Pi_p}(\omega, \rho)$. Otherwise, $\frac{\partial \Pi_p(K)}{\partial \sigma_\beta}\big|_{K_p^B} \leq 0$ for all σ_β.

■

Proof of Proposition 14:

The proof is similar to that of Proposition 7, so we omit the proof steps. ■

A.3 Proofs for the Optimal Strategy and Sensitivity Analyses of Hedging Model

A.3.1 Proofs for the Optimal Strategy

Proof of Theorem 6:

The optimal capacity investment level for basic model is derived from the expected profit in capacity investment stage $\Pi_h(K,h)$, namely Equation (5.2). To prove the concavity of $\Pi_h(K,h)$ on (K,h), we calculate the hessian-matrix $\mathbf{H}\big(\Pi_h(K,h)\big) = \begin{pmatrix} \frac{\partial^2\Pi_h(K,h)}{\partial K^2} & \frac{\partial^2\Pi_h(K,h)}{\partial K\partial h} \\ \frac{\partial^2\Pi_h(K,h)}{\partial K\partial h} & \frac{\partial^2\Pi_h(K,h)}{\partial h^2} \end{pmatrix}$ term by term.

$$
\begin{aligned}
&\frac{\partial \Pi_h(K,h)}{\partial K} \\
=&\begin{cases}
-\omega + \displaystyle\int_{y+2bK}^{\infty}(\xi - y - 2bK) f_\xi(\xi)d\xi, & \text{if } K \in \Big[0, \dfrac{h\mu_\beta + (1-h)\underline{\beta}}{y}\Big] \\
-\omega + \int_{\frac{yK-h\mu_\beta}{1-h}}^{\overline{\beta}} \int_{y+2bK}^{\infty} (\xi - y - 2bK) f(\xi,\beta) d\xi d\beta, & \text{if } K \in \Big(\dfrac{h\mu_\beta + (1-h)\underline{\beta}}{y},\ \dfrac{h\mu_\beta + (1-h)\overline{\beta}}{y}\Big) \\
-\omega, & \text{if } K \in \Big[\dfrac{h\mu_\beta + (1-h)\overline{\beta}}{y},\ \infty\Big)
\end{cases}
\end{aligned}
$$

$$
\frac{\partial^2 \Pi_h(K,h)}{\partial K^2}
= \begin{cases}
-2b\int_{y+2bK}^{\infty} f_\xi(\xi)d\xi, & \text{if } K \in \left[0, \frac{h\mu_\beta + (1-h)\underline{\beta}}{y}\right] \\
\begin{array}{l} -2b\int_{\frac{yK-h\mu_\beta}{1-h}}^{\overline{\beta}} \int_{y+2bK}^{\infty} f(\xi,\beta)d\xi d\beta \\ -\left(\frac{y}{1-h}\right)\int_{y+2bK}^{\infty}(\xi - y - 2bK)f\left(\xi, \frac{yK-h\mu_\beta}{1-h}\right)d\xi \end{array}, & \text{if } K \in \left(\frac{h\mu_\beta + (1-h)\underline{\beta}}{y}, \frac{h\mu_\beta + (1-h)\overline{\beta}}{y}\right) \\
0, & \text{if } K \in \left[\frac{h\mu_\beta + (1-h)\overline{\beta}}{y}, \infty\right)
\end{cases}
$$

$$
\frac{\partial^2 \Pi_h(K,h)}{\partial K \partial h}
= \begin{cases}
0, & \text{if } K \in \left[0, \frac{h\mu_\beta + (1-h)\underline{\beta}}{y}\right] \\
\frac{(\mu_\beta - yK)}{(1-h)^2}\int_{y+2bK}^{\infty}(\xi - y - 2bK)f\left(\xi, \frac{yK-h\mu_\beta}{1-h}\right)d\xi, & \text{if } K \in \left(\frac{h\mu_\beta + (1-h)\underline{\beta}}{y}, \frac{h\mu_\beta + (1-h)\overline{\beta}}{y}\right) \\
0 & \text{if } K \in \left[\frac{h\mu_\beta + (1-h)\overline{\beta}}{y}, \infty\right)
\end{cases}
$$

$$
\frac{\partial \Pi_h(K,h)}{\partial h}
= \begin{cases}
0, & \text{if } K \in \left[0, \frac{h\mu_\beta + (1-h)\underline{\beta}}{y}\right] \\
\begin{array}{l} \int_{\underline{\beta}}^{\frac{yK-h\mu_\beta}{1-h}} \left(\frac{\mu_\beta - \beta}{y}\right) \cdot \int_{y+\frac{2b(h\mu_\beta+(1-h)\beta)}{y}}^{\infty} \\ \quad \left(\xi - y - \frac{2b\left(h\mu_\beta + (1-h)\beta\right)}{y}\right) f(\xi,\beta)d\xi d\beta \end{array}, & \text{if } K \in \left(\frac{h\mu_\beta + (1-h)\underline{\beta}}{y}, \frac{h\mu_\beta + (1-h)\overline{\beta}}{y}\right) \\
\begin{array}{l} \int_{\underline{\beta}}^{\overline{\beta}} \left(\frac{\mu_\beta - \beta}{y}\right) \int_{y+\frac{2b(h\mu_\beta+(1-h)\beta)}{y}}^{\infty} \cdot \\ \quad \left(\xi - y - \frac{2b\left(h\mu_\beta + (1-h)\beta\right)}{y}\right) f(\xi,\beta)d\xi d\beta \end{array}, & \text{if } K \in \left[\frac{h\mu_\beta + (1-h)\overline{\beta}}{y}, \infty\right)
\end{cases}
$$

$$
\frac{\partial^2 \Pi_h(K,h)}{\partial h^2}
= \begin{cases}
0, & \text{if } K \in \left[0, \dfrac{h\mu_\beta + (1-h)\underline{\beta}}{y}\right] \\
-\dfrac{(yK-\mu_\beta)^2}{y(1-h)^3} \displaystyle\int_{y+2bK}^{\infty} (\xi - y - 2bK) f\left(\xi, \frac{yK - h\mu_\beta}{1-h}\right) d\xi d\beta \\
-2b \displaystyle\int_{\underline{\beta}}^{\frac{yK-h\mu_\beta}{1-h}} \int_{y+\frac{2b(h\mu_\beta+(1-h)\beta)}{y}}^{\infty} \left(\frac{\mu_\beta-\beta}{y}\right)^2 f(\xi,\beta) d\xi d\beta, & \text{if } K \in \left(\dfrac{h\mu_\beta + (1-h)\underline{\beta}}{y}, \dfrac{h\mu_\beta + (1-h)\overline{\beta}}{y}\right) \\
-2b \displaystyle\int_{\underline{\beta}}^{\overline{\beta}} \int_{y+\frac{2b(h\mu_\beta+(1-h)\beta)}{y}}^{\infty} \left(\frac{\mu_\beta-\beta}{y}\right)^2 f(\xi,\beta) d\xi d\beta, & \text{if } K \in \left[\dfrac{h\mu_\beta + (1-h)\overline{\beta}}{y}, \infty\right)
\end{cases}
$$

From above equations, we obtain the hessian-matrix and observe that $\frac{\partial^2\Pi_h(K,h)}{\partial K^2} \le 0$ and $\frac{\partial^2\Pi_h(K,h)}{\partial h^2} \le 0$. Through simple algebra, we have $\frac{\partial^2\Pi_h(K,h)}{\partial K^2} \cdot \frac{\partial^2\Pi_h(K,h)}{\partial h^2} \ge \left(\frac{\partial^2\Pi_h(K,h)}{\partial K \partial h}\right)^2$ for all (K,h) which proves that the Hessian is negative semi-definite. Thus $\Pi_h(K,h)$ is concave in (K,h). As $\Pi_h(K,h)$ is concave in (K,h), we now solve for $\max_{K\ge 0, h\in[0,1]} \Pi_h(K,h)$ by dividing the expected profit into three sub-problems i = 1,2,3 according to the range of K as follows:

$$
\Pi_h(K,h) = \begin{cases}
\Pi_h^1(K,h) \text{ if } K \in \left[0, \dfrac{h\mu_\beta + (1-h)\underline{\beta}}{y}\right] \\
\Pi_h^2(K,h) \text{ if } K \in \left(\dfrac{h\mu_\beta + (1-h)\underline{\beta}}{y}, \dfrac{h\mu_\beta + (1-h)\overline{\beta}}{y}\right) \\
\Pi_h^3(K,h) \text{ if } K \in \left[\dfrac{h\mu_\beta + (1-h)\overline{\beta}}{y}, \infty\right)
\end{cases}.
$$

In the first region of K, we solve optimal decisions for $\max_{K\in\left[0, \frac{h\mu_\beta+(1-h)\underline{\beta}}{y}\right], h\in[0,1)} \Pi_h^1(K,h)$, we find that $\Pi_h^1(K,h) = \Pi_u(K)$ where $\Pi_u(K)$ is the expected profit function of the benchmark model. Since $\frac{\partial \Pi_h(K,h)}{\partial h} \equiv 0$, the value of h doesn't affect the optimal expected profit. Utilizing the optimal capacity investment strategy for $\Pi_u(K)$, we have $K_h^* = 0$ and $h^* \in [0,1)$ when $\mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi} - y)^+\right] \le \omega$; $K_h^* = K^U(\omega)$ when $\mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi} - y - 2b\left(h\mu_\beta + (1-h)\underline{\beta}\right)/y)^+\right] \le \omega < \mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi} - y)^+\right]$, or equivalently, $K_h^* = K^U(\omega)$ and h^* is a set that is $\left\{h \middle| \max\left\{0, \frac{yK^U(\omega) - \underline{\beta}}{\mu_\beta - \underline{\beta}}\right\} \le h \le 1\right\}$ when $\mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\right] < \omega < \mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi} - y)^+\right]$.

Then in the third region of K, we derive optimal K_h^* and h^* for $\max_{K\in\left[\frac{h\mu_\beta+(1-h)\overline{\beta}}{y}, \infty\right), h\in[0,1)} \Pi_h^3(K,h)$. As $\frac{\partial \Pi_h(K,h)}{\partial K} = -\omega < 0$ and $\frac{\partial \Pi_h(K,h)}{\partial K}$ is continuous, the optimal decision is never in this region.

Lastly, in the second region of K, we derive optimal K_h^* and h^* for sub-problem $\Pi_h^2(K,h)$. According to the constraint $K \in \left(\frac{h\mu_\beta+(1-h)\underline{\beta}}{y}, \frac{h\mu_\beta+(1-h)\overline{\beta}}{y}\right)$ and $h \in [0,1]$, to simplify analysis and deal with $1-h$ appearing at denominator, we further divide the range of optimal decisions into following three sub-cases

$$\begin{cases} K_h^* \in (\underline{\beta}/y, \mu_\beta/y), h^* \in \left[0, \frac{yK_h^*-\underline{\beta}}{\mu_\beta-\underline{\beta}}\right]; \\ K_h^* = \mu_\beta/y, h^* \in [0,1]; \\ K_h^* \in (\mu_\beta/y, \overline{\beta}/y), h^* \in \left[0, \frac{\overline{\beta}-yK_h^*}{\overline{\beta}-\mu_\beta}\right]. \end{cases}$$

To obtain (K_h^*, h^*), we discuss under what condition the internal and boundary solutions are obtained using following first order derivatives.

$$\frac{\partial \Pi_h^2(K,h)}{\partial K} = -\omega + \int_{\frac{yK-h\mu_\beta}{1-h}}^{\overline{\beta}} \mathbf{E}\left[(\xi - y - 2bK)^+ \middle| \beta\right] f_\beta(\beta) d\xi d\beta \tag{A.23}$$

$$\frac{\partial \Pi_h^2(K,h)}{\partial h} = \int_{\underline{\beta}}^{\frac{yK-h\mu_\beta}{1-h}} \frac{(\mu_\beta - \beta)}{y} \mathbf{E}\left[\left(\tilde{\xi} - y - \frac{2b\big(h\mu_\beta + (1-h)\beta\big)}{y}\right)^+ \middle| \beta\right] f_\beta(\beta) d\beta \tag{A.24}$$

From Equation (A.24), we have immediate result

Lemma 6 $\frac{\partial \Pi_h^2(K,h)}{\partial h} > 0$ *for all* $h \in [0,1)$ *when* $K \le \mu_\beta/y$.

Sub-case 1 $K_h^* \in (\underline{\beta}/y, \mu_\beta/y)$ and $h^* \in \left[0, \frac{yK_h^*-\underline{\beta}}{\mu_\beta-\underline{\beta}}\right]$

In this case, define $K_h^*(h)$ as the internal optimal point which is solved by Equation (A.23) equals to 0 for a given h. Noting that $\frac{\partial \Pi_h^2(K,h)}{\partial h} > 0$ for all $h \in \left[0, \frac{yK_h^*-\underline{\beta}}{\mu_\beta-\underline{\beta}}\right)$ in this region and $\frac{\partial \Pi_h^2(K,h)}{\partial h} = 0$ only when $h = \frac{yK_h^*(h)-\underline{\beta}}{\mu_\beta-\underline{\beta}}$. As a result, optimal capacity level K_h^* is solved by $\left.\frac{\partial \Pi_h^2(K,h)}{\partial K}\right|_{h=\frac{yK_h^*-\underline{\beta}}{\mu_\beta-\underline{\beta}}} = -\omega + \int_{y+2bK}^{\infty} (\xi - y - 2bK) f_\xi(\xi) d\xi = 0$ which implies K_h^* equals to the resource unconstrained capacity level K^U. As $K_h^* \in (\underline{\beta}/y, \mu_\beta/y)$, the optimality condition should satisfy $\mathbf{E}_{\tilde{\xi}}\left[\left(\tilde{\xi} - y - \frac{2b\mu_\beta}{y}\right)^+\right] \le \omega < \mathbf{E}_{\tilde{\xi}}\left[\left(\tilde{\xi} - y - \frac{2b\underline{\beta}}{y}\right)^+\right]$. To summarize, $(K_h^*, h^*) = \left(K^U(\omega), \frac{yK^U(\omega)-\underline{\beta}}{\mu_\beta-\underline{\beta}}\right)$ when $\mathbf{E}_{\tilde{\xi}}\left[\left(\tilde{\xi} - y - \frac{2b\mu_\beta}{y}\right)^+\right] \le \omega < \mathbf{E}_{\tilde{\xi}}\left[\left(\tilde{\xi} - y - \frac{2b\underline{\beta}}{y}\right)^+\right]$.

Sub-case 2 $K_h^* \in (\mu_\beta/y, \overline{\beta}/y)$, $h^* \in \left[0, \frac{\overline{\beta}-yK_h^*}{\overline{\beta}-\mu_\beta}\right]$

In this case, K is an interior solution. We define $K_h^*(h)$ is an internal optimal solution uniquely solved by $\omega = \int_{\frac{yK-h\mu_\beta}{1-h}}^{\overline{\beta}} \mathbf{E}\big[(\xi - y - 2bK)^{+}\big|\beta\big] f_\beta(\beta)d\beta$ for a given h. And h^* can be internal optimal solved by $\int_{\underline{\beta}}^{\frac{yK_h^*(h)-h\mu_\beta}{1-h}} (\mu_\beta - \beta)\mathbf{E}\Big[\Big(\tilde{\xi} - y - \frac{2b\left(h\mu_\beta+(1-h)\beta\right)}{y}\Big)^{+}\Big|\beta\Big] f_\beta(\beta)d\beta = 0$ or take boundary value $h = 0$ or $h = \frac{\overline{\beta}-yK_h^*}{\overline{\beta}-\mu_\beta}$. We first discuss the optimality condition under which h^* is at two boundaries: **boundary 1)** $K \in (\mu_\beta/y, \overline{\beta}/y)$ and $h = 0$; and **boundary 2)** $K = \frac{h\mu_\beta+(1-h)\overline{\beta}}{y}$ and $h \in (0, 1)$, respectively.

For the optimal solution for Sub-case 2 lies in boundary 1) $K \in (\mu_\beta/y, \overline{\beta}/y)$ and $h = 0$, we obtain that the condition under which $h^* = 0$ is optimal. For a given K, from Equation (A.24), we have $\frac{\partial \Pi_h^2(K,h)}{\partial h}\Big|_{h=0} = \int_{\underline{\beta}}^{yK} \frac{(\mu_\beta-\beta)}{y}\mathbf{E}\Big[\Big(\tilde{\xi} - y - \frac{2b\beta}{y}\Big)^{+}\Big|\beta\Big] f_\beta(\beta)d\beta \leq 0$. As $\frac{\partial \Pi_h^2(K,h)}{\partial h}\Big|_{K=\mu_\beta/y,h=0} > 0$ always hold and $\frac{\partial^2 \Pi_h^2(K,h)}{\partial h \partial K} < 0$ for all $K \in (\mu_\beta/y, \overline{\beta}/y)$, we define $\underline{K}_h$ as the unique solution of $\frac{\partial \Pi_h^2(K,h)}{\partial h}\Big|_{h=0,K=\underline{K}_h} = 0$ where $\underline{K}_h > \mu_\beta/y$ always holds. As a result, $h^* = 0$ is attainable only when $K_h^* \in \big[\min\{\underline{K}_h, \overline{\beta}/y\}, \overline{\beta}/y\big)$. For the set $\big[\min\{\underline{K}_h, \overline{\beta}/y\}, \overline{\beta}/y\big)$ not empty, the inequality $\frac{\partial \Pi_h^2(K,h)}{\partial h}\Big|_{K=\overline{\beta}/y,h=0} = \int_{\underline{\beta}}^{\overline{\beta}}(\mu_\beta - \beta)\mathbf{E}_{\tilde{\xi}}\Big[\Big(\tilde{\xi} - y - \frac{2b\beta}{y}\Big)^{+}\Big|\beta\Big] f_\beta(\beta)d\beta < 0$ must holds indicting $\underline{K}_h < \overline{\beta}/y$. From Equation (A.23), we have $\frac{\partial \Pi_h^2(K,0)}{\partial K}\Big|_{K=\underline{K}_h} > 0$ and $\frac{\partial \Pi_h^2(K,0)}{\partial K}\Big|_{K=\overline{\beta}/y} < 0$. As K_h^* is the unique solution of $\Pi_h^2(K, 0) = 0$, equivalently, $\omega = \int_{yK}^{\overline{\beta}} \int_{y+2bK}^{\infty} (\xi - y - 2bK) f(\xi, \beta)d\xi d\beta$, we find that K_h^* equals to the resource constrained capacity level $K^B(\omega)$. From above discussion about Equation (A.24), $K^B(\omega) > \underline{K}_h$ is equivalent to

$$\int_{\underline{\beta}}^{yK^B(\omega)} \frac{(\mu_\beta - \beta)}{y}\mathbf{E}\Big[\Big(\tilde{\xi} - y - \frac{2b\beta}{y}\Big)^{+}\Big|\beta\Big] f_\beta(\beta)d\beta \leq 0. \tag{A.25}$$

To transform Inequality (A.25) into an inequality with respect to ω, we define ω_h as the unique solution of $\int_{\underline{\beta}}^{yK^B(\omega)}(\mu_\beta - \beta)\mathbf{E}_{\tilde{\xi}}\left[\Big(\tilde{\xi} - y - \frac{2b\beta}{y}\Big)^{+}\Big|\beta\right] f_\beta(\beta)d\beta = 0$, then Inequality (A.25) is equivalent to $\omega \leq \omega_h$. Therefore, the condition for $(K_h^*, h^*) = (K^B(\omega), 0)$ is $\omega \in \big(0, \omega_h\big]$. And this case exists only when $\int_{\underline{\beta}}^{\overline{\beta}}(\mu_\beta - \beta)\mathbf{E}_{\tilde{\xi}}\Big[\Big(\tilde{\xi} - y - \frac{2b\beta}{y}\Big)^{+}\Big|\beta\Big] f_\beta(\beta)d\beta < 0$, because if
$\int_{\underline{\beta}}^{\overline{\beta}}(\mu_\beta - \beta)\mathbf{E}_{\tilde{\xi}}\Big[\Big(\tilde{\xi} - y - \frac{2b\beta}{y}\Big)^{+}\Big|\beta\Big] f_\beta(\beta)d\beta \geq 0$, we have that $K^B(\omega_h) \geq \overline{\beta}/y$ meaning that $\omega_h \leq 0$.

Then we discuss the optimality condition under which the optimal solution for Sub-case 2 lies in boundary 2) $K = \frac{h\mu_\beta + (1-h)\overline{\beta}}{y}$ and $h \in (0, 1)$. From Equation (A.23), we have $\frac{\partial \Pi_h^2\left(K,h\right)}{\partial K}\Big|_{K=\frac{h\mu_\beta+(1-h)\overline{\beta}}{y}} = -\omega < 0$, so this boundary doesn't contain optimal point.

Lastly, we discuss the internal optimal decision for Sub-case 2. Define (K_h^B, h^B) as the unique solution jointly solved by Equation (A.23) and Equation (A.24) equal to 0, that is

$$\begin{cases} \omega = \int_{\frac{yK-h\mu_\beta}{1-h}}^{\overline{\beta}} \mathbf{E}\left[(\xi - y - 2bK)^+ \middle| \beta\right] f_\beta(\beta)d\beta \\ 0 = \int_{\underline{\beta}}^{\frac{yK-h\mu_\beta}{1-h}} (\mu_\beta - \beta)\mathbf{E}\left[\left(\tilde{\xi} - y - \frac{2b\left(h\mu_\beta + (1-h)\beta\right)}{y}\right)^+ \middle| \beta\right] f_\beta(\beta)d\beta, \end{cases}$$

where $K_h^B \in (\mu_\beta/y, \frac{h\mu_\beta+(1-h)\overline{\beta}}{y})$, $h^B \in [0, 1)$. From Equation (A.23), to obtain internal optimal capacity level, we have $\frac{\partial \Pi_h^2\left(K,h\right)}{\partial K}\Big|_{K=\frac{h\mu_\beta+(1-h)\overline{\beta}}{y}} = -\omega < 0$ and $\frac{\partial \Pi_h^2\left(K,h\right)}{\partial K}\Big|_{K=\frac{\mu_\beta}{y}} = -\omega + \int_{\mu_\beta}^{\overline{\beta}} \mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\right] f_\beta(\beta)d\beta > 0$ for all $h \in [0, 1)$, that is $0 < \omega < \int_{\mu_\beta}^{\overline{\beta}} \mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\right] f_\beta(\beta)d\beta$. In addition, define $K_h^*(h)$ as the implicit solution of Equation (A.23) for a given h. Substituting K with $K_h^*(h)$ to Equation (A.24), to obtain $h^* \in (0, 1)$, we have
$\frac{\partial \Pi_h^2(K,h)}{\partial h}\Big|_{K=K_h^*(0),h=0} = \int_{\underline{\beta}}^{yK_h^*(0)} \frac{(\mu_\beta-\beta)}{y}\mathbf{E}\left[\left(\tilde{\xi} - y - \frac{2b\beta}{y}\right)^+ \middle| \beta\right] f_\beta(\beta)d\beta > 0$ and
$\frac{\partial \Pi_h^2(K,h)}{\partial h}\Big|_{K=K_h^*(1),h=1} = \int_{\underline{\beta}}^{\overline{\beta}} (\mu_\beta - \beta)\mathbf{E}\left[\left(\tilde{\xi} - y - \frac{2b\mu_\beta}{y}\right)^+ \middle| \beta\right] f_\beta(\beta)d\beta < 0$, since in this region $K_h^*(0) = K^B$, $K_h^*(1) = \frac{\mu_\beta}{y}$ and $\lim_{h\to1} \frac{yK_h^*(h)-h\mu_\beta}{1-h} = \overline{\beta}$, above inequalities are equivalent to $\omega > \omega_h$ and $\mathbf{E}_{\tilde{\xi},\tilde{\beta}}\left[(\tilde{\beta} - \mu_\beta)\left(\tilde{\xi} - y - \frac{2b\mu_\beta}{y}\right)^+\right] > 0$ respectively. We conclude that

- when $\mathbf{E}_{\tilde{\xi},\tilde{\beta}}\left[(\tilde{\beta} - \mu_\beta)\left(\tilde{\xi} - y - \frac{2b\tilde{\beta}}{y}\right)^+\right] > 0 > \mathbf{E}_{\tilde{\xi},\tilde{\beta}}\left[(\tilde{\beta} - \mu_\beta)\left(\tilde{\xi} - y - \frac{2b\tilde{\beta}}{y}\right)^+\right]$, the condition for (K_h^B, h^B) to be optimal is $0 < \omega < \int_{\mu_\beta}^{\overline{\beta}} \mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\right] f_\beta(\beta)d\beta$;

- when $\mathbf{E}_{\tilde{\xi},\tilde{\beta}}\left[(\tilde{\beta} - \mu_\beta)\left(\tilde{\xi} - y - \frac{2b\tilde{\beta}}{y}\right)^+\right] > 0$, the condition for (K_h^B, h^B) to be optimal is $\omega_h < \omega < \int_{\mu_\beta}^{\overline{\beta}} \mathbf{E}_{\tilde{\xi}}\left[(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\right] f_\beta(\beta)d\beta$.

Sub-case 3 $K_h^* = \mu_\beta/y, h^* \in [0,1]$

As $K_h^* = \mu_\beta/y$, we calculate $\frac{\partial \Pi_h^2(K,h)}{\partial h}\Big|_{K=\mu_\beta/y} = \int_{\underline{\beta}}^{\mu_\beta}(\mu_\beta - \beta)\mathbf{E}\Big[\Big(\tilde{\xi} - y - \frac{2b\left(h\mu_\beta+(1-h)\beta\right)}{y}\Big)^+\Big|\beta\Big]f_\beta(\beta)d\beta > 0$ for all $h \in [0,1]$, so we obtain optimal strategy $(K_h^*, h^*) = (\mu_\beta/y, 1)$. Define $K_h^*(h)$ is an internal optimal solution uniquely solved by $\omega = \int_{\frac{yK-h\mu_\beta}{1-h}}^{\overline{\beta}}\mathbf{E}\big[(\xi - y - 2bK)^+\big|\beta\big]f_\beta(\beta)d\beta$ for a given h. Since $\frac{\partial^2 \Pi_h^2(K,h)}{\partial h \partial K} < 0$ if $K > \mu_\beta/y$ and $\frac{\partial^2 \Pi_h^2(K,h)}{\partial h \partial K} > 0$ if $K < \mu_\beta/y$, we discuss two cases.

When $K_h^*(h) > \mu_\beta/y$, as $K_h^* = \mu_\beta/y$, we have $\frac{\partial \Pi_h^2(K,h)}{\partial K}\Big|_{K=K_h^*(h)} = -\omega + \int_{\frac{yK_h^*(h)-h\mu_\beta}{1-h}}^{\overline{\beta}}\mathbf{E}\big[(\xi - y - 2bK)^+\big|\beta\big]f_\beta(\beta)d\beta \geq 0$ for all $h \in [0,1]$, or equivalently, $\omega < \int_{\mu_\beta}^{\overline{\beta}}\mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\big]f_\beta(\beta)d\beta$ because of $\frac{\partial^2 \Pi_h^2(K,h)}{\partial h \partial K} < 0$. Also, as $h^* = 1$, $\frac{\partial \Pi_h^2(K,h)}{\partial h}\Big|_{K=K_h^*(h)} = \int_{\underline{\beta}}^{\frac{yK_h^*(h)-h\mu_\beta}{1-h}}\frac{(\mu_\beta-\beta)}{y}\mathbf{E}\Big[\Big(\tilde{\xi}-y-\frac{2b(h\mu_\beta+(1-h)\beta)}{y}\Big)^+\Big|\beta\Big]f_\beta(\beta)d\beta \geq 0$ for all $h \in [0,1]$, equivalently, $\mathbf{E}_{\tilde{\xi},\tilde{\beta}}\Big[(\tilde{\beta} - \mu_\beta)\Big(\tilde{\xi} - y - \frac{2b\mu_\beta}{y}\Big)^+\Big] \leq 0$. To summarize, we have $(K_h^*, h^*) = (\mu_\beta/y, 1)$ when $\omega \in \Big(0, \int_{\mu_\beta}^{\overline{\beta}}\mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\big]f_\beta(\beta)d\beta\Big)$ and $\mathbf{E}_{\tilde{\xi},\tilde{\beta}}\Big[(\tilde{\beta} - \mu_\beta)\Big(\tilde{\xi} - y - \frac{2b\mu_\beta}{y}\Big)^+\Big] \leq 0$.

When $K_h^*(h) < \mu_\beta/y$, we also discuss the condition under which $(K_h^*, h^*) = (\mu_\beta/y, 1)$. According to $\frac{\partial^2 \Pi_h^2(K,h)}{\partial h \partial K} > 0$, we have that
$\frac{\partial \Pi_h^2(K,h)}{\partial K}\Big|_{K<\frac{\mu_\beta}{y},h=1} = -\omega + \mathbf{E}_{\tilde{\xi}}\Big[\Big(\tilde{\xi} - y - \frac{2b\mu_\beta}{y}\Big)^+\Big] > 0$ and
$\frac{\partial \Pi_h^2(K,h)}{\partial K}\Big|_{K=\frac{\mu_\beta}{y},h\in[0,1)} = -\omega + \int_{\mu_\beta}^{\overline{\beta}}\mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi}-y-\frac{2b\mu_\beta}{y})^+\big]f_\beta(\beta)d\beta < 0$. To summarized, we have $(K_h^*, h^*) = (\mu_\beta/y, 1)$ when $\omega \in \Big(\int_{\mu_\beta}^{\overline{\beta}}\mathbf{E}_{\tilde{\xi}}\big[(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\big]f_\beta(\beta)d\beta, \big[(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\big]\Big)$.

Noting that $\mathbf{E}_{\tilde{\xi},\tilde{\beta}}\Big[(\tilde{\beta}-\mu_\beta)\Big(\tilde{\xi}-y-\frac{2b\mu_\beta}{y}\Big)^+\Big] = \mathbf{Cov}\Big((\tilde{\beta}-\mu_\beta), \Big(\tilde{\xi}-y-\frac{2b\mu_\beta}{y}\Big)^+\Big) = \mathbf{Cov}\big(\tilde{\beta}, (\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\big)$ and $\mathbf{E}_{\tilde{\xi},\tilde{\beta}}\Big[(\tilde{\beta} - \mu_\beta)\Big(\tilde{\xi} - y - \frac{2b\tilde{\beta}}{y}\Big)^+\Big] = \mathbf{Cov}\Big((\tilde{\beta} - \mu_\beta), \Big(\tilde{\xi} - y - \frac{2b\tilde{\beta}}{y}\Big)^+\Big) = \mathbf{Cov}\big(\tilde{\beta}, (\tilde{\xi} - y - \frac{2b\tilde{\beta}}{y})^+\big)$, we therefore have cases in Theorem 6. ■

Proof of Corollary 6:

We prove this proposition by recalling from Chapter 3 that the resource constrained capacity level $K^B(\omega)$ is the unique solution of $\omega = \int_{yK^B}^{\overline{\beta}}\int_{y+2bK^B}^{\infty}(\xi - y -$

$2bK)f(\xi,\beta)d\xi d\beta$ and $K^B(\omega) > \mu_\beta/y$ when $\max\{0,\omega_h\} < \omega < \mathop{\mathbf{E}}_{\tilde{\xi},\tilde{\beta}}\Big[\mathbb{1}_{\{\tilde{\beta}\geq\mu_\beta\}}(\tilde{\xi}-y-\frac{2b\mu_\beta}{y})^+\Big]$. From Theorem 6, we have that $K_h^B(\omega,h)$ is solved by $\frac{\partial\Pi_h(K,h)}{\partial K} = -\omega + \int_{\frac{yK_h^B-h\mu_\beta}{1-h}}^{\overline{\beta}} \mathbf{E}\big[(\xi-y-2bK_h^B)^+\big|\beta\big]f_\beta(\beta)d\xi d\beta = 0$. To compare K_h^B and K^B, we calculate

$$\frac{\partial\Pi_h(K,h)}{\partial K}\Big|_{K=K^B} = -\omega + \int_{\frac{yK^B-h\mu_\beta}{1-h}}^{\overline{\beta}} \mathbf{E}\big[(\xi-y-2bK^B)^+\big|\beta\big]f_\beta(\beta)d\xi d\beta$$
$$= -\int_{yK^B}^{\frac{yK^B-h\mu_\beta}{1-h}} \mathbf{E}\big[(\xi-y-2bK^B)^+\big|\beta\big]f_\beta(\beta)d\xi d\beta \leq 0$$

since $yK^B \leq \frac{yK^B-h\mu_\beta}{1-h}$. As $\frac{\partial\Pi_h(K,h)}{\partial K}$ decreases in K, we conclude that $K^B(\omega) \geq K_h^B(\omega,h)$ for all $h \in (0,1]$ and $K^B(\omega) = K_h^B(\omega,h)$ when $h = 0$. ■

A.3.2 Proofs for the Sensitivity Analyses

Proof of Proposition 15:

Since now $(\tilde{\xi},\tilde{\beta})$ follows bivariate normal distribution, we have the conditional distributions are normally distributed with parameters $\tilde{\xi}|\tilde{\beta} \sim N\Big(\mu_\xi + \rho\frac{\sigma_\xi}{\sigma_\beta}(\tilde{\beta}-\mu_\beta), \sigma_\xi^2(1-\rho^2)\Big)$ and $\tilde{\beta}|\tilde{\xi} \sim N\Big(\mu_\beta + \rho\frac{\sigma_\beta}{\sigma_\xi}(\tilde{\xi}-\mu_\xi), \sigma_\beta^2(1-\rho^2)\Big)$. We transform the covariance thresholds using their double-integral form using standardized normal distribution $\tilde{z}_0 \overset{d}{=} \frac{\tilde{\beta}-\mu_\beta}{\sigma_\beta}$ and $\tilde{z}_2 \overset{d}{=} \frac{\tilde{\xi}-\mu_\xi-\rho\frac{\sigma_\xi}{\sigma_\beta}(\tilde{\beta}-\mu_\beta)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big|\tilde{\beta}$ and $1-\Phi(-t) = \Phi(t)$. For the first covariance,

$$\mathbf{Cov}(\tilde{\beta}, (\tilde{\xi}-y-\frac{2b\mu_\beta}{y})^+) = \int_{y+\frac{2b\mu_\beta}{y}}^{\infty}\Big(\xi-y-\frac{2b\mu_\beta}{y}\Big)\int_{-\infty}^{\infty}(\beta-\mu_\beta)f_{\beta|\xi}(\beta)d\beta f_\xi(\xi)d\xi$$
$$=\rho\frac{\sigma_\beta}{\sigma_\xi}\int_{y+\frac{2b\mu_\beta}{y}}^{\infty}\Big(\xi-y-\frac{2b\mu_\beta}{y}\Big)(\xi-\mu_\xi)f_{\beta|\xi}(\beta)d\beta f_\xi(\xi)d\xi$$
$$=\rho\sigma_\beta\sigma_\xi\int_{\frac{y+\frac{2b\mu_\beta}{y}-\mu_\xi}{\sigma_\xi}}^{\infty} z_1\Big(z_1 + \frac{\mu_\xi - y - \frac{2b\mu_\beta}{y}}{\sigma_\xi}\Big),$$

as $\int_t^\infty z^2\phi(z)dz = 1-\Phi(t)+t\phi(t)$ and $\int_t^\infty z\phi(z)dz = \phi(t)$, we obtain that $\mathbf{Cov}(\tilde{\beta}, (\tilde{\xi}-y-\frac{2b\mu_\beta}{y})^+) = \rho\sigma_\beta\sigma_\xi\Phi(\frac{\mu_\xi-y-2b\mu_\beta/y}{\sigma_\xi})$. For the second covariance, we have

$$
\begin{aligned}
&\mathbf{Cov}(\tilde\beta, (\tilde\xi - y - \frac{2b\tilde\beta}{y})^+) \\
=&\int_{-\infty}^{\infty} (\beta - \mu_\beta) \int_{y+\frac{2b\beta}{y}}^{\infty} \Big(\xi - y - \frac{2b\beta}{y}\Big) f_{\xi|\beta}(\xi) d\xi f_\beta(\beta) d\beta \\
=&\sigma_\beta \int_{-\infty}^{\infty} z_0 \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sigma_\xi\sqrt{1-\rho^2}}}^{\infty} \\
&\qquad \Big(z_2\sigma_\xi\sqrt{1-\rho^2} + \mu_\xi + \rho\sigma_\xi z_0 - y - \frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}\Big)\phi(z_2)dz_2\phi(z_0)dz_0
\end{aligned}
$$

Observing that the last expression of above equation can be transformed using Stein's lemma $\mathbf{E}\big[g(\tilde z_0)\tilde z_0\big] = \mathbf{E}\Big[\frac{\partial g(\tilde z_0)}{\partial \tilde z_0}\Big]$ where
$g(\tilde z_0) = \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sigma_\xi\sqrt{1-\rho^2}}}^{\infty} \Big(z_2\sigma_\xi\sqrt{1-\rho^2}+\mu_\xi+\rho\sigma_\xi z_0-y-\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}\Big)\phi(z_2)dz_2.$
Therefore, we obtain that the above equation also equals to

$$
\begin{aligned}
&\sigma_\beta\Big(\rho\sigma_\xi - \frac{2b\sigma_\beta}{y}\Big) \int_{-\infty}^{\infty} \int_{\frac{y+\frac{2b(z_0\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sigma_\xi\sqrt{1-\rho^2}}}^{\infty} \phi(z_2)dz_2\phi(z_0)dz_0 \\
=&\sigma_\beta\Big(\rho\sigma_\xi - \frac{2b\sigma_\beta}{y}\Big) \int_{-\infty}^{\infty} \Phi\left(\frac{(\rho\sigma_\xi - \frac{2b\sigma_\beta}{y})z_0 + \mu_\xi - y - \frac{2b\mu_\beta}{y}}{\sigma_\xi\sqrt{1-\rho^2}}\right)\phi(z_0)dz_0.
\end{aligned}
$$

Noting that $\mathbf{Cov}(\tilde\beta, (\tilde\xi - y - \frac{2b\beta}{y})^+) = 0$ when $\rho\sigma_\xi = \frac{2b\sigma_\beta}{y}$, we further transform above equation for the case $\rho\sigma_\xi \neq \frac{2b\sigma_\beta}{y}$. Since $\int_{-\infty}^{\infty} \Phi\left(\frac{\big(\rho\sigma_\xi-\frac{2b\sigma_\beta}{y}\big)z_0+\mu_\xi-y-\frac{2b\mu_\beta}{y}}{\sigma_\xi\sqrt{1-\rho^2}}\right)\phi(z_0)dz_0$ is in the form $\int_{-\infty}^{\infty}\Phi\Big(\frac{z-a}{b}\Big)\phi(z)dz = \mathbf{E}_z\big[\Phi(\frac{\tilde z-a}{b})\big]$ where $\tilde z \sim N(0,1)$, a and b are constant parameters and $b \neq 0$. We transform $\mathbf{E}_z\big[\Phi(\frac{\tilde z-a}{b})\big]$ in following steps: $\Phi\big((z-a)/b\big) = \Pr\big\{\tilde X < \frac{z-a}{b}\big\}$ for all z, where $\tilde X$ follows standard normal distribution $N(0,1)$ and is independent of $\tilde z$, we have that

$$
\mathbf{E}_z\big[\Phi(\frac{\tilde z-a}{b})\big] = \Pr\big\{\tilde X < \frac{\tilde z-a}{b}\big\} = \begin{cases} 1-\Phi\Big(\frac{a}{\sqrt{1+b^2}}\Big), & \text{if } b>0; \\ \Phi\Big(\frac{a}{\sqrt{1+b^2}}\Big), & \text{if } b<0. \end{cases}.
$$

We then apply this result and obtain that $\mathbf{Cov}(\tilde\beta, (\tilde\xi - y - \frac{2b\tilde\beta}{y})^+) = \sigma_\beta(\rho\sigma_\xi - \frac{2b\sigma_\beta}{y})\Phi\Big(\frac{\mu_\xi-y-2b\mu_\beta/y}{\sqrt{(\frac{2b\sigma_\beta}{y}-\rho\sigma_\xi)^2+\sigma_\xi^2(1-\rho)^2}}\Big)$. The optimal capacity investment and hedging strategy proof is omitted due to the similarity to the proof of Theorem 6. ■

To prove the sensitivity of $\Pi_h(K_h^*, h^*)$, the expected profit in the capacity investment stage $\Pi_h(K,h)$, i.e. Equation (5.2) is transformed in standard normal distribution form using $\tilde{z}_0 \overset{d}{=} \frac{\tilde{\beta}-\mu_\beta}{\sigma_\beta}$ and $\tilde{z}_2 \overset{d}{=} \frac{\tilde{\xi}-\mu_\xi-\rho\frac{\sigma_\xi}{\sigma_\beta}(\tilde{\beta}-\mu_\beta)}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big|\tilde{\beta}$, that is

$$\Pi_h(K,h) = -\omega K + \sum_{i=1}^{4} V_i^h(K,h) \tag{A.26}$$

where $V_i^h(K,h)$ $i = 1, 2, \cdots, 4$ are defined as follows

$$V_1^h(K,h) := \int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+\frac{2b(z_0(1-h)\sigma_\beta+\mu_\beta)^+}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)^2}{4b}\phi(z_2)dz_2\phi(z_0)dz_0,$$

$$V_2^h(K,h) := \int_{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}}^{\infty} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)^2}{4b}\phi(z_2)dz_2\phi(z_0)dz_0,$$

$$V_3^h(K,h) := \int_{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}}^{\infty} \int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \left(\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)K-bK^2\right)\phi(z_2)dz_2\phi(z_0)dz_0,$$

and $V_4^h(K,h) :=$

$$\int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}} \frac{(z_0(1-h)\sigma_\beta+\mu_\beta)}{y} \int_{\frac{y+\frac{2b(z_0(1-h)\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y-b\left(\frac{z_0(1-h)\sigma_\beta+\mu_\beta}{y}\right)\right)\phi(z_2)dz_2\phi(z_0)dz_0.$$

Proof of Proposition 16:

Noting from Proposition 15, $\Pi_h(K_h^*, h^*)$ is sensitive to ρ only when $\rho > 0$

and $0 < \omega < \mathop{\mathbf{E}}_{\tilde{\xi},\tilde{\beta}}\Big[\mathbb{1}_{\{\tilde{\beta}\geq\mu_\beta\}}(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\Big]$, since both $K_h^* = 0$ and $h^* = 1$ imply $\Pi_h(K_h^*, h^*)$ is not a function of ρ. There are two cases of optimal solutions to discuss.

Firstly, when optimal strategy is $(K_h^*, h^*) = \big(K_h^B(\omega), h^B(\omega)\big)$ that is characterized by $\rho > 0$ and $\max\{0, \omega_h\} < \omega < \mathop{\mathbf{E}}_{\tilde{\xi},\tilde{\beta}}\Big[\mathbb{1}_{\{\tilde{\beta}\geq\mu_\beta\}}(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\Big]$, we obtain that $\frac{d\Pi_h(K_h^*,h^*)}{d\rho} = \frac{\partial\Pi_h(K,h)}{\partial\rho}\Big|_{K=K_h^B(\omega),h^B(\omega)}$ as $\big(K_h^B(\omega), h^B(\omega)\big)$ is interior optimal solution. To derive the sensitivity result, we take derivative of Equation (A.26) with respect to ρ, as the derivative as integral boundaries are vanished due to the continuity of the integrands, we have yielding $\frac{\partial\Pi_h(K,h)}{\partial\rho} = \sum_{i=1}^4 \frac{\partial V_i^h(K,h)}{\partial\rho}$, where the derivatives in each term is $\frac{\partial\Pi_h(K,h)}{\partial\rho} =$

$$\frac{\sigma_\xi}{\sqrt{1-\rho^2}}\int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}}\int_{\frac{y-\mu_\xi-\sigma_\xi\rho z_0}{\sigma_\xi\sqrt{1-\rho^2}}}^{\frac{y+\frac{2b(z_0(1-h)\sigma_\beta+\mu_\beta)^+}{y}-\mu_\xi-\sigma_\xi\rho z_0}{\sigma_\xi\sqrt{1-\rho^2}}}$$

$$\frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)}{2b}\big(z_0\sqrt{1-\rho^2}-z_2\rho\big)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\frac{\sigma_\xi}{\sqrt{1-\rho^2}}\int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}}\int_{\frac{y-\mu_\xi-\sigma_\xi\rho z_0}{\sigma_\xi\sqrt{1-\rho^2}}}^{\frac{y+2bK-\mu_\xi-\sigma_\xi\rho z_0}{\sigma_\xi\sqrt{1-\rho^2}}}$$

$$\frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)}{2b}\big(z_0\sqrt{1-\rho^2}-z_2\rho\big)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\frac{\sigma_\xi}{\sqrt{1-\rho^2}}\int_{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}}^{\infty}\int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}K\big(z_0\sqrt{1-\rho^2}-z_2\rho\big)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\frac{\sigma_\xi}{\sqrt{1-\rho^2}}\int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}}\frac{(z_0(1-h)\sigma_\beta+\mu_\beta)}{y}\int_{\frac{y+\frac{2b(z_0(1-h)\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}$$

$$\big(z_0\sqrt{1-\rho^2}-z_2\rho\big)\phi(z_2)dz_2\phi(z_0)dz_0,$$

that is, $\frac{\partial\Pi_h(K,h)}{\partial\rho} = \left(\frac{1}{1-\rho^2}\right)\frac{\sigma_\xi}{\sigma_\beta}\mathop{\mathbf{E}}_{\xi}\Big[\mathop{\mathbf{E}}_{\beta|\xi}\Big[q_h^*(K,\tilde{\xi},\tilde{\beta})\cdot\Big(\tilde{\beta}-\mu_\beta-\rho\frac{\sigma_\beta}{\sigma_\xi}(\tilde{\xi}-\mu_\xi)\Big)\Big|\tilde{\xi}\Big]\Big]$. Since $\tilde{\beta}|\tilde{\xi}$ follows normal distribution $\tilde{\beta}|\tilde{\xi} \sim N\left(\mu_\beta+\rho\frac{\sigma_\beta}{\sigma_\xi}(\tilde{\xi}-\mu_\xi), \sigma_\beta^2(1-\rho^2)\right)$ and $q_h^*(K,\xi,\beta)$ is piecewise differentiable and continuous, $\mathop{\mathbf{E}}_{\beta|\xi}\Big[q_h^*(K,\xi,\tilde{\beta})\cdot\Big(\tilde{\beta}-\mu_\beta-\rho\frac{\sigma_\beta}{\sigma_\xi}(\xi-\mu_\xi)\Big)\Big|\xi\Big]$ is able to be simplified by Stein's lemma such that $\mathop{\mathbf{E}}_{\beta|\xi}\Big[q_h^*(K,\xi,\tilde{\beta})\cdot\Big(\tilde{\beta}-\mu_\beta-\rho\frac{\sigma_\beta}{\sigma_\xi}(\xi-\mu_\xi)\Big)\Big|\xi\Big] = \sigma_\beta^2(1-\rho^2)\mathop{\mathbf{E}}_{\beta|\xi}\Big[\frac{\partial q_h^*(K,\xi,\tilde{\beta})}{\partial\beta}\Big|\xi\Big]$, we then obtain

$$\frac{\partial \Pi_h(K,h)}{\partial \rho} = \sigma_\xi \sigma_\beta \mathop{\mathbf{E}}_{\xi}\Big[\mathop{\mathbf{E}}_{\beta|\xi}\Big[\frac{\partial q_h^*(K,\xi,\tilde{\beta})}{\partial \beta}\Big|\xi\Big]\Big] = \sigma_\xi \sigma_\beta \iint_{\Omega_3^h} \frac{(1-h)}{y} f(\xi,\beta) d\xi d\beta > 0$$

for all $K > 0$ and $h \in [0,1)$.

Secondly, we prove the sensitivity result when optimal strategy is $(K_h^*, h^*) = \big(K^B(\omega), 0\big)$ that is characterized by $\rho > \frac{2b\sigma_\beta}{y\sigma_\xi}$ and $0 < \omega < \omega_h$. In this case, $\Pi_h\big(K^B(\omega), 0\big) = \Pi\big(K^B(\omega)\big)$ where $\Pi(K)$ is the expected profit of basic model. As we've proved that $\Pi\big(K^B(\omega)\big)$ increases in ρ, we proved that $\Pi_h\big(K^B(\omega), 0\big)$ increases in ρ under condition $\rho > \frac{2b\sigma_\beta}{y\sigma_\xi}$ and $0 < \omega < \omega_h$. ■

Proof of Proposition 17:

We prove the effect of σ_ξ on $\Pi_h(K_h^*, h^*)$. Noting $\Pi_h(K,1) = \Pi_u\big(\min\{K, \mu_\beta/y\}\big)$ where $\Pi_u(K)$ is the expected profit function for resource unconstrained benchmark model, we have $\frac{d\Pi_h(K_h^*,h^*)}{d\sigma_\xi} = \frac{d\Pi_u\left(\min\{K^U,\mu_\beta/y\}\right)}{d\sigma_\xi} = \frac{\partial \Pi_u(K)}{\partial \sigma_\xi}\Big|_{K=\min\{K^U,\mu_\beta/y\}}$ under all conditions that lead to $h^* = 1$. And we proved in Proposition 2 that $\frac{\partial \Pi_u(K)}{\partial \sigma_\xi} > 0$ for all $K > 0$. Therefore, we obtain that $\Pi_h(K_h^*, h^*)$ strictly increases in σ_ξ under all conditions that lead to $h^* = 1$.

Then we prove the sensitivity result for cases that $h^* \neq 1$. There are two cases. Firstly, when optimal strategy is $(K_h^*, h^*) = \big(K_h^B(\omega), h^B(\omega)\big)$ that is characterized by $\rho > 0$ and $\max\{0, \omega_h\} < \omega < \mathop{\mathbf{E}}_{\tilde{\xi},\tilde{\beta}}\Big[\mathbb{1}_{\{\tilde{\beta} \geq \mu_\beta\}}(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\Big]$, we obtain that $\frac{d\Pi_h(K_h^*,h^*)}{d\sigma_\xi} = \frac{\partial \Pi_h(K,h)}{\partial \sigma_\xi}\Big|_{K=K_h^B(\omega),h^B(\omega)}$ as $\big(K_h^B(\omega), h^B(\omega)\big)$ is interior optimal solution. Taking derivative of Equation (A.26) with respect to σ_ξ, we have $\frac{\partial \Pi_h(K,h)}{\partial \sigma_\xi}$ =

$$\int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+\frac{2b(z_0(1-h)\sigma_\beta+\mu_\beta)^+}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)}{2b}$$

$$\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}} \int_{\frac{y+\frac{2b(z_0(1-h)\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} \frac{(z_0(1-h)\sigma_\beta+\mu_\beta)}{y}$$

$$\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\int_{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}}^{\infty} \int_{\frac{y-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}} \frac{\left(z_2\sqrt{\sigma_\xi^2(1-\rho^2)}+\mu_\xi+\rho\sigma_\xi z_0-y\right)}{2b}$$

$$\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\int_{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}}^{\infty} \int_{\frac{y+2bK-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty} K\left(z_2\sqrt{1-\rho^2}+\rho z_0\right)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$=\sqrt{1-\rho^2}\int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}} \int_{-\infty}^{\infty} z_2 q_h^*(z_2,z_0)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\sqrt{1-\rho^2}\int_{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}}^{\infty} \int_{-\infty}^{\infty} z_2 q_h^*(z_2,z_0)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\rho\mathop{\mathbf{E}}_{z_0}\left[\tilde{z}_0\mathop{\mathbf{E}}_{z_2}\left[q_h^*(\tilde{z}_2,\tilde{z}_0)\middle|\tilde{z}_0\right]\right].$$

According to the Stein's Lemma, above equation is transformed into

$$\frac{\partial \Pi_h(K,h)}{\partial \sigma_\xi} = \frac{\sigma_\xi}{2b}\int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}} \int_{\frac{y-\mu_\xi}{\sigma_\xi\sqrt{1-\rho^2}}-\frac{\rho z_0}{\sqrt{1-\rho^2}}}^{\frac{y+\frac{2b(z_0(1-h)\sigma_\beta+\mu_\beta)^+}{y}-\mu_\xi}{\sigma_\xi\sqrt{1-\rho^2}}-\frac{\rho z_0}{\sqrt{1-\rho^2}}} \phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\frac{\sigma_\xi}{2b}\int_{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}}^{\infty} \int_{\frac{y-\mu_\xi}{\sigma_\xi\sqrt{1-\rho^2}}-\frac{\rho z_0}{\sqrt{1-\rho^2}}}^{\frac{y+2bK-\mu_\xi}{\sigma_\xi\sqrt{1-\rho^2}}-\frac{\rho z_0}{\sqrt{1-\rho^2}}} \phi(z_2)dz_2\phi(z_0)dz_0$$

$$+\frac{\rho\sigma_\beta}{y}\int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}} \int_{\frac{y+\frac{2b(z_0(1-h)\sigma_\beta+\mu_\beta)}{y}-\mu_\xi}{\sigma_\xi\sqrt{1-\rho^2}}-\frac{\rho z_0}{\sqrt{1-\rho^2}}}^{\infty} \phi(z_2)dz_2\phi(z_0)dz_0,$$

as $\rho > 0$, we have that $\frac{\partial \Pi_h(K,h)}{\partial \sigma_\xi} > 0$ for all $K > 0$ including $K = K_h^*$. To summarize, we proved that $\Pi_h(K_h^*, h^*)$ strictly increases in σ_ξ.

Secondly, we prove the sensitivity result when optimal strategy is $(K_h^*, h^*) =$

$\big(K^B(\omega), 0\big)$ that is characterized by $\rho > \frac{2b\sigma_\beta}{y\sigma_\xi}$ and $0 < \omega < \omega_h$. In this case, $\Pi_h\big(K^B(\omega), 0\big) = \Pi\big(K^B(\omega)\big)$ where $\Pi(K)$ is the expected profit of basic model, so that $\frac{d\Pi_h(K_h^*,h^*)}{d\sigma_\xi} = \frac{d\Pi(K^B(\omega))}{d\sigma_\xi} = \frac{\partial \Pi(K)}{\partial \sigma_\xi}\Big|_{K=K^B(\omega)}$ according to Envelope theorem. As we've proved that $\Pi\big(K^B(\omega)\big)$ increases in σ_ξ when $\rho > 0$, we proved that $\Pi_h\big(K^B(\omega), 0\big)$ increases in σ_ξ under condition $\rho > \frac{2b\sigma_\beta}{y\sigma_\xi}$ and $0 < \omega < \omega_h$. ■

Proof of Proposition 18:

Noting from Proposition 15, $\Pi_h(K_h^*, h^*)$ can be sensitive to σ_β only when $\rho > 0$ and $0 < \omega < \mathbf{E}_{\tilde{\xi},\tilde{\beta}}\Big[\mathbb{1}_{\{\tilde{\beta}\geq\mu_\beta\}}(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\Big]$, since both $K_h^* = 0$ and $h^* = 1$ imply $\Pi_h(K_h^*, h^*)$ is not a function of σ_β. To derive the sensitivity result, we discuss the sign of
$\frac{d\Pi_h(K_h^*,h^*)}{d\sigma_\beta} = \frac{\partial\Pi_h(K,h)}{\partial\sigma_\beta}\Big|_{K=K_h^*,h=h^*} + \frac{\partial\Pi_h(K,h)}{\partial K}\Big|_{K=K_h^*,h=h^*}\frac{dK_h^*}{d\sigma_\beta} + \frac{\partial\Pi_h(K,h)}{\partial \rho}\Big|_{K=K_h^*,h=h^*}\frac{dh^*}{d\sigma_\beta}$.
As there are two optimal strategies within the unit capacity cost range $0 < \omega < \mathbf{E}_{\tilde{\xi},\tilde{\beta}}\Big[\mathbb{1}_{\{\tilde{\beta}\geq\mu_\beta\}}(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\Big]$ when $\rho > 0$, we discuss the sensitivity result one by one.

Firstly, when optimal strategy is $(K_h^*, h^*) = \big(K_h^B(\omega), h^B(\omega)\big)$ that is characterized by $\rho > 0$ and $\max\{0, \omega_h\} < \omega < \mathbf{E}_{\tilde{\xi},\tilde{\beta}}\Big[\mathbb{1}_{\{\tilde{\beta}\geq\mu_\beta\}}(\tilde{\xi} - y - \frac{2b\mu_\beta}{y})^+\Big]$, we obtain that $\frac{d\Pi_h(K_h^*,h^*)}{d\sigma_\beta} = \frac{\partial\Pi_h(K,h)}{\partial\sigma_\beta}\Big|_{K=K_h^B(\omega),h^B(\omega)}$ as $\big(K_h^B(\omega), h^B(\omega)\big)$ is interior optimal solution. We take derivative of Equation (A.26) with respect to σ_β to obtain $\frac{\partial\Pi_h(K,h)}{\partial\sigma_\beta} =$

$$\int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}} \frac{z_0(1-h)}{y} \int_{\frac{y+\frac{2b(z_0(1-h)\sigma_\beta+\mu_\beta)}{y}-\mu_\xi-\rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}}^{\infty}$$

$$\Big(z_2\sqrt{\sigma_\xi^2(1-\rho^2)} + \mu_\xi + \rho\sigma_\xi z_0 - y - 2b\Big(\frac{z_0(1-h)\sigma_\beta + \mu_\beta}{y}\Big)\Big)\phi(z_2)dz_2\phi(z_0)dz_0$$

$$=\sigma_\xi\sqrt{1-\rho^2}\int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h)\sigma_\beta}} \frac{z_0(1-h)}{y}\mathbf{E}\Bigg[\Bigg(\tilde{z}_2 - \Bigg(\frac{y + \frac{2b(z_0(1-h)\sigma_\beta+\mu_\beta)}{y} - \mu_\xi - \rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Bigg)\Bigg)^+\Bigg]\phi(z_0)dz_0.$$

Then we use the optimality condition of h^* given K to obtain the result. $h^*(K)$ is solved by

$0 = \int_{-\infty}^{\frac{yK-h^*(K)\mu_\beta}{1-h^*(K)}} (\mu_\beta - \beta)\mathbf{E}\Big[\Big(\tilde{\xi} - y - \frac{2b\big(h^*(K)\mu_\beta + \big(1-h^*(K)\big)\beta\big)}{y}\Big)^+\Big|\beta\Big] f_\beta(\beta)d\beta$, we standardize this optimality condition yielding

$0 = \sigma_\xi\sigma_\beta\sqrt{1-\rho^2}\int_{-\infty}^{\frac{yK-\mu_\beta}{(1-h^*(K))\sigma_\beta}} z_0\mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{y + \frac{2b(z_0(1-h^*(K))\sigma_\beta+\mu_\beta)}{y} - \mu_\xi - \rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]\phi(z_0)dz_0$.

Using above optimality condition of $h^*(K)$, we have $\frac{\partial\Pi_h(K,h)}{\partial\sigma_\beta}\Big|_{h=h^*(K)} = 0$ for all $K > 0$ including $K = K_h^*$.

Secondly, we prove the sensitivity result when optimal strategy is $(K_h^*, h^*) = \big(K^B(\omega), 0\big)$ that is characterized by $\rho > \frac{2b\sigma_\beta}{y\sigma_\xi}$ and $0 < \omega < \omega_h$. In this case, $\Pi_h\big(K^B(\omega), 0\big) = \Pi\big(K^B(\omega)\big)$ where $\Pi(K)$ is the expected profit of basic model, so that $\frac{d\Pi_h(K_h^*,h^*)}{d\sigma_\beta} = \frac{d\Pi(K^B(\omega))}{d\sigma_\beta} = \frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K=K^B(\omega)}$ according to Envelope theorem. We have $\frac{\partial\Pi(K)}{\partial\sigma_\beta}\Big|_{K=K^B(\omega)} =$

$$\sqrt{\sigma_\xi^2(1-\rho^2)}\int_{-\infty}^{\frac{yK^B(\omega)-\mu_\beta}{\sigma_\beta}} \frac{z_0}{y}\mathbf{E}\Bigg[\Bigg(\tilde{z}_2 - \Bigg(\frac{y + \frac{2b(z_0\sigma_\beta+\mu_\beta)}{y} - \mu_\xi - \rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Bigg)\Bigg)^+\Bigg]\phi(z_0)dz_0.$$

As we obtained in the proof of Theorem 5 that $\omega < \omega_h$ is equivalent to $\int_{-\infty}^{yK^B(\omega)} \frac{(\mu_\beta-\beta)}{y}\mathbf{E}\Big[\Big(\tilde{\xi} - y - \frac{2b\beta}{y}\Big)^+\Big|\beta\Big] f_\beta(\beta)d\beta < 0$ and this inequality can be standardized as

$-\sqrt{\sigma_\xi^2(1-\rho^2)}\sigma_\beta\int_{-\infty}^{\frac{yK^B(\omega)-\mu_\beta}{\sigma_\beta}} \frac{z_0}{y}\mathbf{E}\Big[\Big(\tilde{z}_2 - \Big(\frac{y + \frac{2b(z_0\sigma_\beta+\mu_\beta)}{y} - \mu_\xi - \rho\sigma_\xi z_0}{\sqrt{\sigma_\xi^2(1-\rho^2)}}\Big)\Big)^+\Big]\phi(z_0)dz_0 < 0$, we can conclude that $\frac{d\Pi_h(K_h^*,h^*)}{d\sigma_\beta} > 0$ in this case. ■