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Essays on agricultural commodity processing

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ESSAYS ON AGRICULTURAL COMMODITY PROCESSING

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I hereby declare that this dissertation is my original work
and it has been written by me in its entirety.
I have duly acknowledged all the sources of information
which have been used in this dissertation.

This dissertation has also not been submitted for any degree
in any university previously.

Bin Li

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Abstract

This dissertation investigates two important issues in agricultural commodity processing: (i) biomass commercialization; that is, converting organic waste into a saleable product, from economic and environmental perspectives, and (ii) optimal procurement portfolio design using multiple suppliers and spot market, and the impact of by-product introduction on this optimal portfolio.

The first chapter examines the economic implications of biomass commercialization from the perspective of an agri-processor that uses a commodity input to produce both a commodity output and biomass. We characterize the value of biomass commercialization and perform sensitivity analysis to investigate how spot price uncertainty (input and output spot price variabilities and the correlation between the two spot prices) affects this value. We find that commercializing biomass makes the profits less sensitive to changes in spot price uncertainty. Using a model calibration in the context of palm industry, we show that the value of biomass (palm kernel shell) commercialization can be as high as 26.54% of the processor (palm oil mill)'s profits.

The second chapter examines the environmental implications of biomass commercialization. To this end, we characterize the expected carbon emissions considering the profit-maximizing operational decisions using the economic model of the first chapter. In comparison with the common perception in practice, which fails to consider the changes in operational decisions after commercialization, we identify two types of misconceptions (and characterize conditions under which they appear). In

particular, the processor would mistakenly think that commercializing its biomass is environmentally beneficial when it is not, and vice versa. Using a model calibration, we show that the former misconception is likely to be observed in the palm industry. we perform sensitivity analyses to investigate how a higher biomass price or demand (which is always economically superior) affects the environmental assessment and characterize conditions under which these changes are environmentally superior or inferior. Based on our results, we put forward important practical implications that are of relevance to both agri-processors and policy makers.

The third chapter studies the procurement portfolio design of an agri-processor that sources a commodity input from two suppliers that use quantity flexibility contracts—characterized by reservation cost and exercise cost—to produce and sell a commodity output under input and output spot price uncertainties. We characterize the optimal procurement portfolio that is composed of three strategies—single sourcing from the supplier with lower reservation price, and single sourcing from the supplier with lower exercise price, and dual sourcing. We investigate how the spot price correlation shapes the optimal procurement strategy and the value of using suppliers. We then study the impact of introducing a non-commodity by-product on the optimal procurement portfolio. Based on our results, we put forward important managerial implications about the procurement strategy and by-product management in agricultural processing industries.

Contents

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To my family.

Chapter 1

Economic Implications of Biomass Commercialization in Agricultural Processing

1.1 Introduction

Global warming and climate change have created an unprecedented interest in reducing greenhouse gas (GHG) emissions globally, especially in energy production (Kök et al. 2016). Biomass (i.e., organic matter), a renewable energy source, plays a pivotal role in achieving this objective as it can be used as a feedstock in a bioenergy plant replacing fossil fuels to produce energy (e.g., heat, electricity).[1] Our focus in this paper is on agricultural residues as biomass source. In several agricultural industries, including the oilseed industry (e.g., palm, coconut) and the sugar industry, processors convert their residues (e.g., kernel shell for the oilseed industry and bagasse for the sugar industry) into a saleable product and sell it to bioenergy plants. Commercializing agricultural residues is gaining momentum due to increasingly strict standards

[1] Among all the renewable energy sources (e.g., wind, solar) energy produced from biomass has the largest share—50% in 2017—in the global renewable energy consumption (International Energy Agency 2018).

for renewable energy usage across the globe. For example, as seen in Table 7 of the U.S. Department of Agriculture report (USDA 2018), Japan's import of palm kernel shells has increased nearly by ten-fold since 2013, to more than 1.13 million metric tons in 2017. This volume accounts for approximately US$125 million and according to the same report, it is expected to increase further in the near future as a result of Japan's target of providing at least 22% of its energy needs through renewable sources by 2030. Significant import volumes of palm kernel shells are also reported by several other countries, including South Korea and the U.S. (Jakarta Post 2017). Increasing trend for biomass commercialization is also observed in other agricultural processing industries (see Pearson 2016). These recent developments give rise to a need for processors to better understand the economic and environmental implications of commercializing their biomass.

There is a nascent operations management literature that studies the value of converting waste stream into a saleable product albeit in the context of other waste streams such as municipal waste (Ata et al. 2012) and excess fresh produce (Lee and Tongarlak 2017). The knowledge base developed in these papers is not directly applicable to the context of agricultural residue because agricultural processors feature unique operational characteristics. Consider, for example, the palm industry. Palm oil mills produce crude palm oil (a commodity output) and palm kernel shell (biomass) from fresh fruit palm bunches (a commodity input). As both the input and the output are commodities, the processors are exposed to prevailing spot prices in buying and selling these commodities and these prices exhibit considerable variability (Boyabatlı et al. 2017). Moreover, to counteract against spot price variability, palm oil mills rely on long-term contracts for procurement, as commonly observed in commodity processing industries (Boyabatlı 2015). These unique operational characteristics play critical roles in the economic implications of biomass commercialization. In summary, to our knowledge there is no work that studies the value of biomass commercialization for the agricultural processor. Therefore, there is also no work that examines the effect of key factors (e.g., spot price uncertainty) on this value. Our first research objective is to fill this void.

To achieve these objectives, we propose a two-stage model that—in a stylized manner—captures the important operational characteristics of an agri-processor that commercializes its biomass. This model is motivated by our interactions with a coconut processor who aims to commercialize its coconut kernel shell. The firm (processor) procures a single input commodity and sells an output commodity and biomass in a single period to maximize its expected profit. The firm has two sources for input procurement, a contract and an input spot market. The output can be sold to two channels, an output spot market and demand that is characterized by a fixed-price fixed-volume sales contract. The output can also be procured from the spot market to satisfy the demand. The biomass is sold to demand that is characterized by a similar sales contract. In the first stage, the firm chooses the input contract volume to be reserved in the face of the input and the output spot price uncertainties. In the second stage, after these uncertainties are realized the firm decides the quantity to source from the reserved contract volume and the input spot market, the processing volume, the quantity to source from the output spot market, and the quantity of output demand and biomass demand to satisfy.

To delineate the economic implications of biomass commercialization, we make a comparison with a benchmark model in which the firm sells only the output commodity and biomass goes to landfill. We complement our structural analysis with numerical analysis based on realistic instances. To this end, we calibrate our model to represent a typical palm oil mill located in Malaysia (which accounts for 28.1% of world palm oil production in 2018 (USDA 2019)). We use publicly available data from the Malaysian Palm Oil Board, complemented by the data obtained from the extant literature. Our main findings can be summarized as follows.

The value of biomass commercialization is given by the difference between the optimal expected profit after commercialization and the same before commercialization. We show that this value can be characterized by the product of biomass demand and an expected biomass margin which captures the effects of spot price uncertainty and firm's optimal decisions. Common intuition may suggest that this expected biomass margin can be characterized based on two possibilities on the spot day before commercialization: processing is profitable so that waste stream is already available for conversion to a saleable product (which brings a margin of biomass price) and processing is not profitable so that there is no waste stream, and hence, no conversion (which brings zero margin). We provide specific conditions under which this intuition holds and extend it by showcasing a third possibility in which processing becomes profitable only after commercialization. More interestingly, we show that when the firm increases its contract procurement volume after commercialization, the biomass margin on the spot day can become negative or even larger than the biomass price. These results underline the need for conducting a formal analysis in evaluating the value of biomass commercialization.

We conduct sensitivity analyses, both analytically and numerically, to investigate the effects of correlation between input and output spot prices and their respective variability on the value of biomass commercialization. We find that a higher correlation is always beneficial; that is, it increases this value, but a higher (input or output) spot price variability is beneficial only when this variability is low; otherwise, a lower spot price variability is beneficial. The general insight from the literature (see, Plambeck and Taylor 2013 and Boyabatlı et al. 2017) is that a processor's profitability (before and after commercialization) decreases in spot price correlation and decreases (increases) in input or output spot price variability when this variability is sufficiently low (high). Our results indicate that whenever the change in spot price uncertainty has an unfavorable (a favorable) impact on profitability, commercializing biomass reduces this negative (positive) impact. The main takeaway is that biomass commercialization, besides creating a new revenue stream for the processor, makes the processor's profits less sensitive to changes in spot price uncertainty.

The remainder of this paper is organized as follows. §1.2 surveys the related literature and discusses the contribution of our work. §1.3 describes the model and assumptions, and §1.4 derives the optimal strategy. §1.5 characterizes the value and heuristic value of biomass commercialization. §1.6 examines the impacts of spot price uncertainty on the value of biomass commercialization. §1.7 provides a practical application in the context of the palm industry. §1.8 concludes with a discussion of the limitations of our analysis and future research directions.

1.2 Literature Review

Our paper's main contribution is to the emerging operations management (OM) literature on by-product synergy. The papers in this literature study the economic implications of converting waste stream into a saleable product by considering the operational characteristics of specific processing environments. For example, Ata et al. (2012) study a waste-to-energy (WTE) firm that collects and processes municipal waste to generate electricity. Lee and Tongarlak (2017) focus on a retail grocer setting and examine the value of using unsold fresh produce to make prepared food items. More recently, Ata et al. (2019) examine another type of by-product synergy in the context of agricultural industries: gleaning operations that deal with collecting unharvested crops on the farmlands to be used in food assistance programs. They study the dynamic staffing problem to schedule volunteers to collect unharvested crops. Different from these papers, Sunar and Plambeck (2016) consider the interplay between by-product synergy and costs associated with the GHG emissions. They model the strategic interaction between a seller and a buyer located in different countries. The buyer incurs a cost associated with GHG emissions of the seller's production activities due to border adjustment. They examine how seller's decision of converting its waste stream into a saleable product has an impact on buyer's operations.

Closest to our work, Lee (2012) studies the implications of converting waste stream into a saleable product in the context of the chemicals and steel manufacturing industries. Motivated by these industries, she focuses on a deterministic model that optimizes production volumes for the main output and the by-product (waste) while considering waste disposal cost, virgin raw material cost and competition in the by-product market. Motivated by our own experience with a coconut processor commercializing its waste stream, we focus on an exogenously given fixed-price fixed-volume sales contract for biomass and do not consider competition in the biomass market. Instead, we consider other important characteristics of agricultural processors (e.g., input and output spot price uncertainties), which enables us to investigate the impact of spot price uncertainty on the value of biomass commercialization.

Our paper is also related to the growing OM literature on commodity processing. As reviewed by Goel and Tanrisever (2017), the papers in this literature capture idiosyncratic features of commodity processors in a variety of industries and examine the economic implications of a broad range of operational features, including processing-yield improving technology (de Zegher et al. 2017), procurement flexibility (Martínez-de-Albéniz and Simchi-Levi 2005), and responsive product pricing (Boyabatlı et al. 2011). Within this literature, our work is closely related to the stream of papers that considers input and output (spot) price uncertainties. In this stream, Plambeck and Taylor (2013) study process improvement investment decision in a clean-tech manufacturing setting; Dong et al. (2014) study the value of operational flexibility in a petroleum refinery; Boyabatlı et al. (2017) study the optimal capacity investment decision of an oilseed processor; and Goel and Tanrisever (2017) examine the optimal sales contract choice of a biofuel processor. Similar to these papers, we capture idiosyncratic features of processors in a particular industry (agriculture) facing input and output spot price uncertainties. Different from these papers, we focus on biomass commercialization (another operational feature) and study the economic implications.

1.3 Model Description and Assumptions

The following mathematical representation is used throughout the text: a realization of the random variable $\tilde{y}$ is denoted by y. The expectation operator, probability, and indicator function are denoted by $\mathbb{E}$, $Pr(\cdot)$, and $\chi(\cdot)$, respectively. We use $(u)^+ = \max(u, 0)$. The monotonic relations are used in the weak sense unless otherwise stated. Subscript 0 denotes input-related parameters and decision variables, while subscript 1 (2) denotes the same related to the output (biomass). All the proofs are relegated to §4.2.

We consider a firm that procures and processes a commodity input to produce and sell a commodity output and biomass in fixed proportions so as to maximize its expected profit in a single selling season. We model the firm's decisions as a two-stage problem: the firm makes its contract procurement decision under input and output spot price uncertainties (stage 1); and the firm makes its contract exercise, spot procurement, processing and selling decisions after these uncertainties are realized (stage 2).

Let $\tilde{S}_0$ and $\tilde{S}_1$ denote the uncertain input and output spot price, respectively. We assume that $(\tilde{S}_0, \tilde{S}_1)$ follow a bivariate distribution with a positive support, bounded expectation (μ_0, μ_1) with covariance matrix $\mathbf{\Sigma}$, where $\mathbf{\Sigma}_{00} = \sigma_0^2$, $\mathbf{\Sigma}_{11} = \sigma_1^2$, $\mathbf{\Sigma}_{01} = \mathbf{\Sigma}_{10} = \rho\sigma_0\sigma_1$, and ρ denotes the correlation coefficient. We make further assumptions about $(\tilde{S}_0, \tilde{S}_1)$ in §1.6 to study the effect of spot price uncertainty.

The firm has two sources for input procurement, a contract and a spot market. We assume that the firm uses a quantity flexibility contract that is characterized by a unit reservation cost β and a unit exercise cost that is normalized to zero. Let Q

denote the contract volume reserved in advance of the spot market (by incurring the unit cost β). On the spot day, the firm decides how much of this contracted volume is delivered. On the day the firm can also source from the input spot market at the prevailing price S_0 to process.

Let z_0 denote the processing volume. We consider a processing capacity K_0 and a unit processing cost c_0. We assume that each unit of processed input yields a_1 and a_2 units of commodity output and biomass, respectively (where $a_1 + a_2 < 1$). In practice, each unit of processed input may also yield other by-products. For example, in the palm industry, processing of fresh fruit palm bunches yields not only crude palm oil (commodity output) and palm kernel shell (biomass) but also other by-products, including palm oil mill effluent and palm kernel. Because our model only considers commodity output and biomass for brevity, it is (implicitly) assumed that unit sale revenue from each of these by-products (if any) is normalized into the processing cost c_0. Hence, we allow c_0 to take negative values.

We consider two channels for commodity output sale, a spot market and a demand which is characterized by a fixed-price fixed-volume sales contract. In particular, we assume that the commodity output can be sold at a unit price p_1 to satisfy demand D_1, and it can be sold to the spot market at the prevailing spot price S_1. The commodity output can also be procured from the spot market at the prevailing price S_1 to satisfy the demand. For biomass sale, we only consider a demand channel which is characterized by a similar sales contract where the firm can sell up to biomass demand D_2 with a marginal sale revenue p_2. Here, p_2 refers to the difference between the unit sale price and the additional unit processing cost incurred (if any) for biomass (e.g., cost for de-fibring). For brevity, thereafter we denote p_2 as the biomass price. We normalize the penalty costs associated with unsatisfied demand for commodity output and biomass to zero. Positive penalty costs can easily be introduced into our model and they do not affect our results. The benchmark model that represents the firm before biomass commercialization can be obtained by setting $D_2 = 0$. Throughout our analysis, to rule out uninteresting cases, we assume $K_0 \geq \max\left(\frac{D_1}{a_1}, \frac{D_2}{a_2}\right)$; otherwise, satisfying the commodity output or biomass demand through processing is not feasible.

In practice, biomass commercialization involves significant fixed costs that are associated with investments in pre-conditioning machines (for removing impurities from the residue and eliminating moisture), storage facility, and transportation assets (for example, conveyor belt or crane for transportation out of the storage facility). These fixed costs require the processors to evaluate the value of biomass commercialization well in advance of the spot day in which the actual conversion of waste into a saleable product takes place. We do not consider the fixed costs in our model as they do not have an impact on our economic analysis. That being said, the significance of these fixed costs reinforces the need for processors to better understand the value of biomass commercialization (which can then be compared with the fixed costs) and also how spot price uncertainty affects this value, the two research questions we answer in §1.5 and §1.6, respectively.

1.4 The Optimal Solution for the Firm's Decisions

In this section, we describe the optimal solution for the firm's decisions after biomass commercialization. The optimal decisions before commercialization can be obtained as a special case. We solve the firm's problem using backward induction.

In stage 1, the firm contracted (reserved) Q units of input. In stage 2, the firm observes the input and output spot price realizations (S_0, S_1). In this stage, constrained by the processing capacity K_0, the firm decides the processing volume z_0, how to source this volume from the available contracted input and spot procurement, the amount of demand to satisfy for the commodity output and biomass, the commodity output volume to sell to the output spot market, and the commodity output volume to buy from the spot market to satisfy demand. Expressing all decisions as a function of the processing volume allows us to formulate the firm's decision problem as a single-variable maximization problem over the processing volume $z_0 \in [0, K_0]$ where the stage 2 objective function is given by

$$\begin{aligned}\Pi(z_0) \doteq &-(z_0 - Q)^+ S_0 - c_0 z_0 + \min(a_2 z_0, D_2) p_2 \\ &+ \min(a_1 z_0, D_1) \max(p_1, S_1) + (a_1 z_0 - D_1)^+ S_1 + (D_1 - a_1 z_0)^+ (p_1 - S_1)^+ .\end{aligned} \tag{1.1}$$

In (1.1), the first term is the input procurement cost from the spot market and the second term is the processing cost. The third term denotes the revenues from biomass demand sale. The remaining terms denote the total revenues from commodity output sales. In particular, for the first $\min(a_1 z_0, D_1)$ units of commodity output, the firm can choose to either satisfy demand at a unit price p_1 or sell to the output spot market at the prevailing price S_1. Therefore, the marginal revenue for these units is $\max(p_1, S_1)$. When all demand is satisfied (i.e., for $(a_1 z_0 - D_1)^+$ units of commodity output), the firm can only sell to the spot market. For the unsatisfied demand over the available commodity output (i.e., for $(D_1 - a_1 z_0)^+$ units), the firm procures from the output spot market to satisfy the demand if it is profitable to do so. Therefore, the marginal revenue for these units is $(p_1 - S_1)^+$.

Proposition 1 characterizes the optimal processing volume z_0^* that maximizes $\Pi(z_0)$.

Proposition 1 *Given a contract volume Q and spot price realizations (S_0, S_1), the* *optimal processing volume z_0^* is characterized by*

$$z_0^* = \begin{cases} 0 & \text{if } \bar{h}(S_1) \le 0, \\ \min\left(\frac{D_2}{a_2}, Q\right) & \text{if } \underline{h}(S_1) \le 0 \le \bar{h}(S_1) \le S_0, \\ \frac{D_2}{a_2} & \text{if } \underline{h}(S_1) \le 0 \le S_0 \le \bar{h}(S_1), \\ Q & \text{if } 0 \le \underline{h}(S_1) \le \bar{h}(S_1) \le S_0, \\ \max\left(\frac{D_2}{a_2}, Q\right) & \text{if } 0 \le \underline{h}(S_1) \le S_0 \le \bar{h}(S_1), \\ K_0 & \text{if } S_0 \le \underline{h}(S_1), \end{cases} \tag{1.2}$$

where $\bar{h}(S_1) \doteq a_1S_1 + a_2p_2 - c_0$ and $\underline{h}(S_1) \doteq a_1S_1 - c_0$ are unit processing margins when there is unsatisfied biomass demand and no unsatisfied biomass demand, respectively.

The stage 2 objective function $\Pi(z_0)$ in (1.1) is piecewise linear and concave in z_0. Therefore, the optimal solution occurs at the breakpoints $\left\{0, \frac{D_2}{a_2}, Q, K_0\right\}$ and it is determined by comparing the relevant unit processing margin—that is, the marginal revenue from production minus the processing cost—with the input procurement cost at this stage (which is prevailing spot price S_0 for spot-procured input and 0 for the contracted input).[2] For example, if $\underline{h}(S_1) \le 0 \le \bar{h}(S_1) \le S_0$, then it is profitable to process only when there is unsatisfied biomass demand and only with the contracted input, and thus, $z_0^* = \min\left(\frac{D_2}{a_2}, Q\right)$.

In stage 1, the firm chooses the optimal contract volume $Q^* \ge 0$ with respect to uncertain spot prices so as to maximize the expected profit $\mathbb{E}\left[\pi(Q; \tilde{S}_0, \tilde{S}_1)\right] - \beta Q$, where $\pi(Q; S_0, S_1)$ denotes the optimal stage 2 profit for a given contract volume and spot price realizations.

[2]We note that $\frac{D_1}{a_1}$ is not one of the breakpoints because the marginal revenue from production of commodity output does not change when its demand is satisfied. For $z_0 \le \frac{D_1}{a_1}$, when $S_1 > p_1$, spot sale is more profitable than satisfying demand and the marginal revenue is a_1S_1. Otherwise (i.e., when $S_1 \le p_1$), the marginal revenue is again a_1S_1 which is the opportunity gain of not sourcing from the output spot market to satisfy the demand. For $z_0 > \frac{D_1}{a_1}$, only spot sale is possible and the marginal revenue is again a_1S_1.

Proposition 2 *Let* $\underline{\beta} \doteq \mathbb{E}[\min(\tilde{S}_0, (a_1\tilde{S}_1 - c_0)^+)]$ *and* $\bar{\beta} \doteq \mathbb{E}[\min(\tilde{S}_0, (a_1\tilde{S}_1 + a_2p_2 - c_0)^+)]$ *with* $\underline{\beta} < \bar{\beta}$. *The optimal contract volume* Q^* *is given by* 0 *if* $\beta \geq \bar{\beta}$, $\frac{D_2}{a_2}$ *if* $\underline{\beta} \leq \beta < \bar{\beta}$, *and* K_0 *if* $0 \leq \beta < \underline{\beta}$.

The optimal contract volume is characterized by comparing the unit contract cost β with the expected marginal revenue of an additional unit of contracted input, as given by $\underline{\beta}$ and $\bar{\beta}$. At stage 2, the marginal revenue takes different forms as it depends on the input and output spot price realizations. When the input spot price is less than the relevant unit processing margin, that is $(a_1S_1 + a_2p_2 - c_0)^+$ $((a_1S_1 - c_0)^+)$ when there is (no) unsatisfied biomass demand, it is profitable to source from the input spot market for processing. Therefore, the marginal revenue is given by the opportunity gain of not buying from the spot market; that is, S_0. Otherwise, the marginal revenue is given by the unit processing margin.

Recall that we consider a benchmark model in which the firm only sells the commodity output (and biomass goes to landfill). The firm's optimal decisions in this benchmark model can be obtained from our characterizations by setting $D_2 = 0$. It is important to note that biomass commercialization affects the optimal contract volume. In particular, as follows from Proposition 2, in the absence of biomass the firm optimally procures up to the processing capacity K_0 if $\beta < \underline{\beta}$ and does not procure otherwise. We use this observation in characterizing the value of biomass commercialization in the next section.

1.5 The Value of Biomass Commercialization

The value of biomass commercialization is given by the change in the firm's optimal expected profit due to commercialization; let ΔV denote this value. Because the firm's optimal contracting decision is affected by commercialization and the optimal contract volume is characterized based on the unit contract cost β, we examine the value of biomass commercialization for a given β. In particular, we define $\Delta V(\beta) = V^*(\beta) -$

$V^{nb}(\beta)$ where $V^*(\beta)$ is the firm's optimal expected profit after commercialization (evaluated at the optimal contract volume $Q^*(\beta)$) and $V^{nb}(\beta)$, "nb" stands for no biomass, is the same before commercialization (evaluated at the optimal contract volume $Q^{nb}(\beta)$). Proposition 3 characterizes $\Delta V(\beta)$.

Proposition 3 *The value of commercialization is given by* $\Delta V(\beta) = M(\beta)D_2$ *where*

$$M(\beta) \doteq \begin{cases} \frac{1}{a_2}\mathbb{E}\left[\left(a_2p_2 - \left(c_0 - a_1\tilde{S}_1\right)^+\right)^+\right] & \text{if } 0 \leq \beta < \underline{\beta}, \\ \frac{1}{a_2}\left(\mathbb{E}\left[\left(a_2p_2 + \min\left(\tilde{S}_0, a_1\tilde{S}_1 - c_0\right)\right)^+\right] - \beta\right) & \text{if } \underline{\beta} \leq \beta < \bar{\beta}, \\ \frac{1}{a_2}\mathbb{E}\left[\left(a_2p_2 - \left(\tilde{S}_0 + c_0 - a_1\tilde{S}_1\right)^+\right)^+\right] & \text{if } \beta \geq \bar{\beta}, \end{cases} \tag{1.3}$$

with $\underline{\beta}$ *and* $\bar{\beta}$ *as defined in Proposition 2. Moreover,* $M(\beta) \in [0, p_2]$.

The value is characterized by the product of biomass demand D_2 and $M(\beta)$ which can be interpreted as the expected biomass margin. This expected margin captures the effects of spot price uncertainty and firm's optimal decisions, and it takes three forms based on the optimal contracting decisions before and after commercialization. The intuition behind each form can be explained based on the *realized biomass margin* on the spot day (stage 2).

Consider the case when the contract cost is high (i.e., $\beta \geq \bar{\beta}$) in which the firm entirely relies on input spot procurement before and after commercialization; that is, $Q^{nb}(\beta) = Q^*(\beta) = 0$. At stage 2, when S_1 is sufficiently small such that it is not profitable to process even in the presence of biomass (i.e., $a_2p_2 + a_1S_1 - c_0 \leq S_0$), the realized margin is zero. When S_1 is sufficiently large such that it is profitable to process even in the absence of biomass (i.e., $a_1S_1 - c_0 \geq S_0$), the waste stream is already available, and hence, the realized margin is p_2. For the remaining S_1 realizations, biomass commercialization makes the processing profitable and the realized margin is $p_2 - \frac{S_0+c_0-a_1S_1}{a_2}$. Consider now the low contract cost case (i.e., $\beta < \underline{\beta}$) in which

$Q^{nb}(\beta) = Q^*(\beta) = K_0$. In this case, the firm does not rely on input spot procurement and the realized processing margin follows a similar intuition with the high contract cost case after substituting S_0 with 0 (which is the stage 2 procurement cost). The general insights from the high and low contract cost cases are that biomass commercialization does not affect the contract procurement decision and the realized margin at stage 2, which is non-negative, does not exceed p_2.

When the contract cost is moderate (i.e., $\underline{\beta} \leq \beta < \bar{\beta}$), biomass commercialization incents the firm to engage in contract procurement where $Q^{nb}(\beta) = 0$ and $Q^*(\beta) = \frac{D_2}{a_2}$. As a result, interestingly, the realized biomass margin at stage 2 can be *negative* and can *exceed* p_2. In particular, when S_1 is sufficiently small such that it is not profitable to process even in the presence of biomass (i.e., $a_2 p_2 + a_1 S_1 - c_0 \leq 0$), the realized margin is $-\beta$. In this case, the realized margin is negative because of the contract commitment cost after commercialization. When S_1 is sufficiently large such that it is profitable to process even in the absence of biomass (i.e., $a_1 S_1 - c_0 \geq S_0$), the realized margin is $p_2 + \frac{S_0 - \beta}{a_2}$. In this case, the realized margin involves the opportunity gain from not sourcing the input from spot market (given by S_0) at a cost of contract commitment (given by β). This realized margin can be larger than p_2 (when the input spot price realization S_0 is larger than β). For the remaining S_1 realizations, biomass commercialization makes the processing profitable and the realized margin is $p_2 + \frac{a_1 S_1 - c_0 - \beta}{a_2}$. Once again, this realized margin can be larger than p_2, specifically when the output spot price realization S_1 is large enough.

Without conducting a formal analysis, common intuition may suggest that the value of biomass commercialization can be evaluated based on two possibilities on the spot day in the absence of biomass: processing is profitable so that waste stream is already available for conversion to a saleable product which brings a margin of p_2, and processing is not profitable so that there is no waste stream and hence, no conversion which brings zero margin. Proposition 3 characterizes specific conditions under which this intuition holds and extends it by showcasing a third possibility in which processing becomes profitable only after commercialization (and biomass margin is neither zero nor p_2). More importantly, Proposition 3 also demonstrates that because biomass commercialization may affect the optimal contract procurement, the marginal revenue on the day can become negative or even larger than p_2. As intuition suggests (and as follows from Proposition 3), the value of biomass commercialization cannot be larger than the maximum biomass sale revenue $p_2 D_2$. In the next section we examine how this value is affected from spot price uncertainty.

In practice, when the firm calculates the value of commercialization, it may heuristically assume that commercialization has no impact on their operational decisions, including procurement and processing volumes. In this case, the heuristic value of commercialization is given by $\Delta V^H(\beta) \doteq p_2 \mathbb{E}\left[\min(a_2 z_0^{nb}(Q^{nb}(\beta)), D_2)\right]$. Proposition 4 characterizes this heuristic value and compares it with $\Delta V(\beta)$.

Proposition 4 *The heuristic value of commercialization is given by* $\Delta V^H(\beta) = M^H(\beta) D_2$ *where*

$$M^H(\beta) \doteq \begin{cases} p_2 Pr\left(a_1 \tilde{S}_1 - c_0 \geq 0\right) & \text{if } 0 \leq \beta < \underline{\beta}, \\ p_2 Pr\left(a_1 \tilde{S}_1 - c_0 \geq \tilde{S}_0\right) & \text{if } \beta \geq \underline{\beta}, \end{cases} \tag{1.4}$$

with $\underline{\beta}$ *as defined in Proposition 2. Moreover,* $M^H(\beta) \leq M(\beta)$.

Proposition 4 shows the importance of building the stylized model in §1.3. If the firm does not build this model and estimate the value of commercialization using $\Delta V^H(\beta)$, it underestimates the true value of commercialization. Hence, it may miss out on profitable opportunities because of ignoring changes in operational decisions after commercialization. More importantly, the firm may decide not to commercialize its biomass if the fixed costs of commercialization are significantly high. In §1.7.2, we numerically investigate the significance of this underestimation using calibrated values from palm industry.

1.6 The Impact of Spot Price Uncertainty

We now conduct sensitivity analyses to study the effects of spot price correlation (ρ) and input and output spot price variabilities (σ_0 and σ_1, respectively) on the value of biomass commercialization $\Delta V(\beta)$. For tractability, we focus on local sensitivity analyses in which the optimal contracting decisions before and after commercialization are not affected by the changes in these parameters—that is, we consider an unaffected ordering of unit contract cost β and the cost thresholds $\underline{\beta}$ and $\bar{\beta}$ given in Proposition 2. With a sufficiently large change in σ_0, σ_1, or ρ, the ordering may be affected because $\underline{\beta}$ and $\bar{\beta}$ depend on these parameters. We consider the effect of such large changes on our results in §1.7 where we conduct global sensitivity analyses by resorting to numerical experiments.

Throughout this section, we assume $(\tilde{S}_0, \tilde{S}_1)$ to follow a bivariate Normal distribution. We also make two additional assumptions to eliminate unrealistic (and uninteresting) cases: $\rho > 0$ and $a_1\mu_1 > c_0 + \mu_0$—that is, processor has a profitable business (on expectation) before biomass commercialization. Both assumptions are reasonable in the palm industry as we empirically demonstrate in §1.7. Proposition 5 characterizes the effects of ρ, σ_0, and σ_1 on the value of biomass commercialization $\Delta V(\beta)$.

Proposition 5 *Effects of ρ, σ_0 and σ_1 on $\Delta V(\beta)$ are characterized in Table 1.1 where $\underline{\beta}$ and $\bar{\beta}$ are as given in Proposition 2:*

Unit Contract Cost β	ρ	σ_0	σ_1
Low: $\beta < \underline{\beta}$	$-$	$-$	$\downarrow$
Moderate: $\underline{\beta} \leq \beta < \bar{\beta}$	$\uparrow$	$\uparrow$ for $\sigma_0 \leq a_1\sigma_1\rho$	$\uparrow$ for $\sigma_1 \leq \sigma_0\rho/a_1$
		$\downarrow$ for $\sigma_0 > a_1\sigma_1\rho$	No analytical result
High: $\beta \geq \bar{\beta}$	$\uparrow$	$\uparrow$ for $\sigma_0 \leq a_1\sigma_1\rho$	$\uparrow$ for $\sigma_1 \leq \sigma_0\rho/a_1$
		$\downarrow$ for $\sigma_0 > a_1\sigma_1\rho$	$\downarrow$ for $\sigma_1 > \sigma_0\rho/a_1$

Table 1.1: Impact of a Local Increase in Input (Output) Spot Price Variability σ_0 (σ_1) and Correlation (ρ) on the Value of Biomass Commercialization with Bivariate Normal Spot Price Uncertainty: $-$ denotes no change, $\uparrow$ denotes an increase, and $\downarrow$ denotes a decrease.

When the contract cost is low (i.e., $\beta < \underline{\beta}$), $\Delta V(\beta)$ is not affected by changes in ρ and σ_0 because the firm contracts up to the processing capacity K_0 both before and after commercialization, and thus, input spot sourcing is never used. In this case, as follows from Proposition 3, when it is profitable to process after commercialization on the day, the effective marginal sourcing cost of biomass is given by $(c_0 - a_1 S_1)^+$ (when $a_1 S_1 \geq c_0$, it is profitable to process in the absence of biomass and the effective marginal sourcing cost is zero because waste stream is already available). The influence of σ_1 on $\Delta V(\beta)$ can be explained by its *opposite* effect on the expected marginal sourcing cost $\mathbb{E}[(c_0 - a_1\tilde{S}_1)^+]$. It is well known that this expectation increases in σ_1, and thus, a higher σ_1 decreases $\Delta V(\beta)$.

When the contract cost is high (i.e., $\beta \geq \bar{\beta}$), the firm only uses input spot sourcing before and after commercialization. In this case, as follows from Proposition 3, when it is profitable to process after commercialization on the day, the effective marginal sourcing cost of biomass is given by $(S_0 + c_0 - a_1 S_1)^+$. The sensitivity results in Proposition 5 can be explained based on the opposite of how $\mathbb{E}[(\tilde{S}_0 + c_0 - a_1\tilde{S}_1)^+]$ changes in ρ, σ_0, and σ_1. It is well known that this expectation increases in the variability of $\tilde{S}_0 - a_1\tilde{S}_1$ which is increasing in the variances of $\tilde{S}_0$ and $a_1\tilde{S}_1$, and is decreasing in the covariance of $(\tilde{S}_0, a_1\tilde{S}_1)$. With a higher ρ, because the covariance increases, the variability of $\tilde{S}_0 - a_1\tilde{S}_1$ decreases, and thus, $\Delta V(\beta)$ increases. With a higher σ_0 (σ_1) both the variance of $\tilde{S}_0$ ($a_1\tilde{S}_1$) and covariance of $(\tilde{S}_0, a_1\tilde{S}_1)$ increase because $\rho > 0$ by assumption. When σ_0 (σ_1) is sufficiently low; that is, $\sigma_0 \leq a_1\sigma_1\rho$ ($\sigma_1 \leq \sigma_0\rho/a_1$), the latter effect outweighs the former and the variability of $\tilde{S}_0 - a_1\tilde{S}_1$ decreases, and thus, $\Delta V(\beta)$ increases. Otherwise, the former effect dominates and $\Delta V(\beta)$ decreases. The impact of spot price uncertainty on $\Delta V(\beta)$ for the moderate contract cost case (i.e., $\underline{\beta} \leq \beta < \bar{\beta}$) can be explained in a similar fashion except for the effect of σ_1. In this case, because the firm only uses input spot sourcing before commercialization but relies on contract after commercialization, $\Delta V(\beta) = \frac{D_2}{a_2}(\mathbb{E}[(a_1\tilde{S}_1 + a_2p_2 - c_0)^+ - \beta] - \mathbb{E}[(a_1\tilde{S}_1 - \tilde{S}_0 - c_0)^+])$. While the first expectation always increases in σ_1, because the second expectation decreases in the same only when $\sigma_1 \leq \sigma_0\rho/a_1$, the overall impact can only be proven under this condition.

The general insights from Proposition 5 are that the value of biomass commercialization increases in spot price correlation but increases in (input or output) spot price variability only when this variability is low; otherwise, the value decreases in spot price variability.

1.7 Numerical Analysis: Application to the Palm Industry

In this section, we discuss an application of our model in the context of a palm oil mill processing fresh fruit palm bunches (FFB) to produce crude palm oil (CPO) while generating palm kernel shell (PKS) as organic waste. We calibrate our model parameters to represent a typical palm oil mill in Malaysia selling its PKS to a power plant in Japan. We use publicly available data from the Malaysian Palm Oil Board (MPOB), complemented by the data obtained from the extant literature. Throughout this section, we use $\acute{x}$ to denote the calibrated value for parameter x, "RM" to denote Malaysian ringgit (currency), "mt" to denote metric ton (equal to 1,000 kg).

The rest of this section is organized as follows. §1.7.1 describes data calibration.

§1.7.2 investigates the value of biomass commercialization. §1.7.3 examines the effect of spot price uncertainty on the value of PKS commercialization.

1.7.1 Model Calibration for Numerical Experiments

We use the daily prices of FFB and CPO reported by the MPOB from January 2, 2014 to May 14, 2018 which encompasses 1006 weekdays.[3] The MPOB reports the daily CPO prices based on the delivery month (i.e., to be delivered in the same month, next month, etc.). We use the CPO prices that correspond to immediate delivery (i.e., within the same month). The daily FFB prices are reported based on the palm fruit's origin (i.e, the north, south, west, and east subregion of Peninsula Malaysia) and grade (i.e., A, B, or C). We use the average daily FFB prices across subregions and grades. Figure 1.1 illustrates the data used in our calibration.

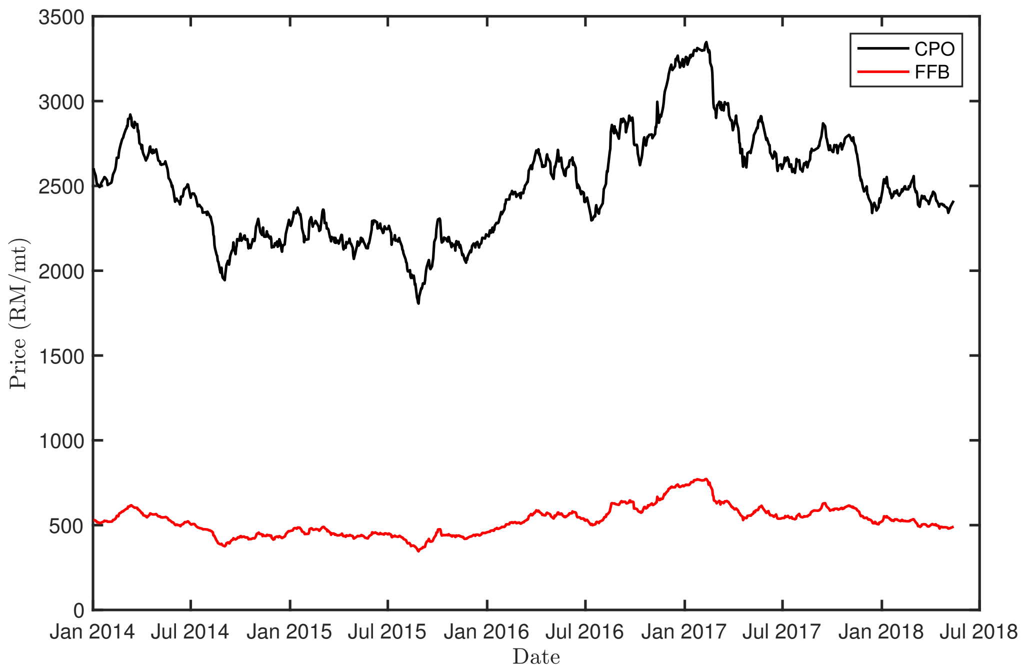


Figure 1.1: Daily Prices of Crude Palm Oil (CPO) and Fresh Fruit Bunch (FFB) (2 Jan 2014 - 14 May 2018)

[3] We consider data starting from 2014 because this is the year PKS commercialization has started to receive significant attention from palm oil mills. For example, as reported in Jakarta Post (2017), palm oil mills in Indonesia started exporting palm kernel shells only in 2014.

Following the procedure in Boyabatlı et al. (2017), we adopt a single-factor, bivariate, mean-reverting process to model the daily input and output spot price processes. In particular, the spot prices of FFB and CPO at time t, denoted as (S_0^t, S_1^t), are modeled as

$$\begin{aligned} dS_0^t &= \theta_0(\kappa_0 - S_0^t)dt + \upsilon_0 d\tilde{W}_0^t \\ dS_1^t &= \theta_1(\kappa_1 - S_1^t)dt + \upsilon_1 d\tilde{W}_1^t, \end{aligned} \tag{1.5}$$

where κ_ζ is the long-term spot price level, θ_ζ is the mean-reversion parameter, and υ_ζ denotes the volatility for $\zeta \in \{0, 1\}$. Here, $(d\tilde{W}_0^t, d\tilde{W}_1^t)$ denotes the increment of a standard bivariate Brownian motion with correlation λ. The price model in (1.5) implies that, at period t' with realized spot prices $S_\zeta^{t'}$ for $\zeta \in \{0, 1\}$, the spot prices $\tilde{S}_\zeta^t$ at a future period $t > t'$ follow a bivariate Normal distribution with

$$\begin{aligned} &\mathbb{E}[\tilde{S}_\zeta^t | S_\zeta^{t'}] = \exp(-\theta_\zeta(t - t'))S_\zeta^{t'} + (1 - \exp(-\theta_\zeta(t - t')))\kappa_\zeta, \\ &\mathbb{V}\mathrm{ar}[\tilde{S}_\zeta^t | S_\zeta^{t'}] = (1 - \exp(-2\theta_\zeta(t - t')))/(2\theta_\zeta) \cdot \upsilon_\zeta^2, \\ &\mathbb{C}\mathrm{ov}[\tilde{S}_0^t, \tilde{S}_1^t | S_0^{t'}, S_1^{t'}] = (1 - \exp(-(\theta_0 + \theta_1)(t - t'))/(\theta_0 + \theta_1) \cdot \lambda \upsilon_0 \upsilon_1, \end{aligned} \tag{1.6}$$

where $\mathbb{V}\mathrm{ar}$ and $\mathbb{C}\mathrm{ov}$ are the variance and covariance of the spot prices, respectively.

To calibrate our spot price distribution parameters μ_0, μ_1, σ_0, σ_1, and ρ, we employ a two-step procedure. First, using the data illustrated in Figure 1.1, we estimate the spot price process parameters in (1.5). Then using these estimated parameters and assuming an initial spot price realization that corresponds to the last observation in Figure 1.1 at time $t' = 0$—that is, $S_0^0 = 491.25$ RM/mt and $S_1^0 = 2412.50$ RM/mt—and assuming $t = 66$ weekdays; that is 3 months, that corresponds to the single period considered in our model, we obtain our spot price distribution parameters by using (1.6) where $\mu_\zeta = \mathbb{E}[\tilde{S}_\zeta^{66} | S_\zeta^0]$, $\sigma_\zeta^2 = \mathbb{V}\mathrm{ar}[\tilde{S}_\zeta^{66} | S_\zeta^0]$, and $\rho\sigma_0\sigma_1 = \mathbb{C}\mathrm{ov}[\tilde{S}_0^{66}, \tilde{S}_1^{66} | S_0^0, S_1^0]$ for $\zeta \in \{0, 1\}$.

According to the spot price process specified in (1.5), the daily FFB and CPO spot prices evolve in the following pattern:

$$\begin{aligned}\tilde{S}_0^t &= e^{-\theta_0}\tilde{S}_0^{t-1} + (1-e^{-\theta_0})\kappa_0 + \upsilon_0\sqrt{\frac{1-e^{-2\theta_0}}{2\theta_0}}\tilde{z}_0 \\ \tilde{S}_1^t &= e^{-\theta_1}\tilde{S}_1^{t-1} + (1-e^{-\theta_1})\kappa_1 + \upsilon_1\sqrt{\frac{1-e^{-2\theta_1}}{2\theta_1}}\tilde{z}_1,\end{aligned} \tag{1.7}$$

where $(\tilde{z}_0, \tilde{z}_1)$ is a standard bivariate Normal distribution with correlation λ. As observed from (1.7), the expressions are a set of simultaneous equations of $(\tilde{S}_0^t, \tilde{S}_1^t)$ on $(\tilde{S}_0^{t-1}, \tilde{S}_1^{t-1})$ in the form of $\tilde{S}_\zeta^t = \alpha_\zeta \tilde{S}_\zeta^{t-1} + \varphi_\zeta + \tilde{\epsilon}_\zeta$ for $\zeta \in \{0,1\}$. As the error terms $(\tilde{\epsilon}_0, \tilde{\epsilon}_1)$ are correlated, we employ the "seemingly unrelated" regression (SUR; see Zellner 1962) to estimate α_ζ, φ_ζ and covariance matrix of $(\tilde{\epsilon}_0, \tilde{\epsilon}_1)$. Using these estimates, we calculate the parameters in (1.7) and obtain $\theta_0 = 0.00374$, $\theta_1 = 0.00473$, $\kappa_0 = 525.64$, $\kappa_1 = 2608.90$, $\upsilon_0 = 8.12$, $\upsilon_1 = 36.40$ and $\lambda = 0.745$. The goodness of fit test results of SUR are summarized in Table 1.2. According to the McElroy's R^2, the SUR equations can explain 99.29% of the variation in the observed spot prices.

Goodness of Fit	FFB	CPO
Root mean-squared error (RMSE)	8.11	36.32
RMSE (% of $\acute{\mu}_\zeta$)	1.54%	1.39%
R^2	99.57%	99.55%
McElroy's R^2	99.29%	

Table 1.2: Results of SUR

In the second step, we estimate the spot price distributions of FFB and CPO using the calibrated price processes as described above. In particular, we calculate that the mean input and output spot prices are $\acute{\mu}_0 = 498.77$ RM/mt and $\acute{\mu}_1 = 2465.2$ RM/mt, the standard deviations are $\acute{\sigma}_0 = 58.60$ and $\acute{\sigma}_1 = 255.04$, and the correlation coefficient is $\acute{\rho} = 0.745$.

We set the processing cost $\acute{c}_0 = -39.47$ RM/mt of FFB using the value reported in Boyabatlı et al. (2017) which, similar to our paper, considers a typical palm oil mill located in Malaysia in their model calibration and normalizes its processing cost by the unit revenues from by-products of FFB processing that have economic value—that is, palm kernel. For the processing capacity, we again rely on Boyabatlı et al. (2017) which reports a processing capacity 858.91 mt of FFB per day. This corresponds to a processing capacity of $\acute{K}_0 = 56688.06$ mt of FFB in our model using the 66 weekdays (in other words, 3 months), the selling season considered in our calibration. For the CPO production yield, we use the average monthly production yield in Peninsular Malaysia within our time frame (Jan 2014 to May 2018) as reported by the MPOB and find that $\acute{a}_1 = 19.77\%$. For the PKS production yield, because there is no data reported by the MPOB, we use data obtained from the extant literature. In particular, according to Table 4 in Abdullah and Sulaiman (2013), the ratio between the production yields of CPO and PKS is 3.5 which corresponds to a PKS production yield of $\acute{a}_2 = 5.65\%$ for a given $\acute{a}_1 = 19.77\%$. For the CPO unit (sales) contract price, we use the daily CPO future contract (settlement) price with a three-month maturity—that is, delivery in three months—as reported by the MPOB. In particular, we take the average of daily future prices within our time window (from January 2, 2014 to May 14, 2018) and obtain $\acute{p}_1 = 2433.25$ RM/mt. For the PKS unit (sales) contract price, because there is no data reported by the MPOB, we use one of the largest e-commerce platforms, Alibaba.com, to obtain PKS price data. In particular, we search for the "PKS price from Malaysia" on this platform at the end of our time window (May 14, 2018). We obtain a PKS price range of 317.6-635.2 RM/mt and take the mid-point of this range; that is, $\acute{p}_2 = 476.4$ RM/mt.[4] For the CPO and PKS demands, because we do not have access to any data we consider 80% of processing capacity utilization and assume that 60% of CPO produced based on this utilization is sold through a sales contract which corresponds to $\acute{D}_1 = 5379.47$ mt of CPO. We

[4]In the e-commerce platform, we observe different PKS prices because these prices are from different suppliers and the quality of PKS also vary based on its characteristics (e.g., ash and moisture content).

assume the PKS demand is equal to 50% of total PKS produced, which corresponds to $\acute{D}_2 = 1281.15$ mt of PK in a selling season.

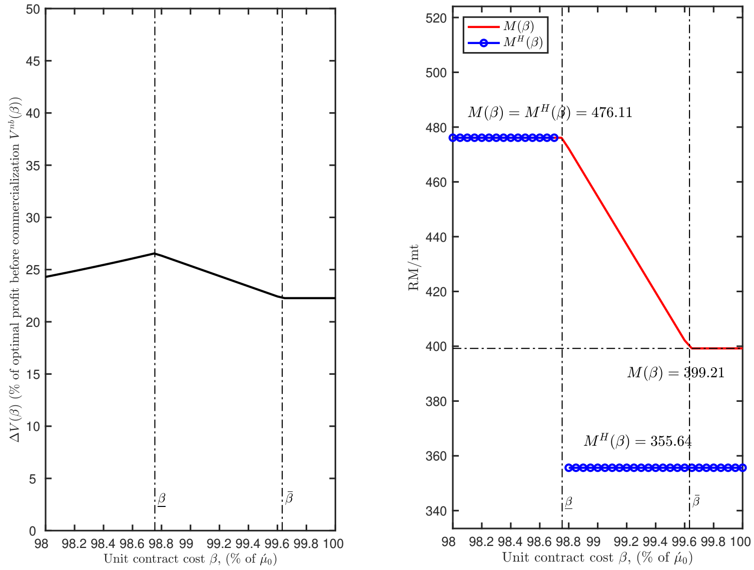
Table 1.3 summarizes the calibrated parameter values representing the baseline scenario used in our numerical experiments. We use $\acute{\beta} = \underline{\beta} - 0.5\%\acute{\mu}_0 = 98.25\%\acute{\mu}_0$, $\acute{\beta} = (\underline{\beta} + \bar{\beta})/2 = 99.19\%\acute{\mu}_0$, and $\acute{\beta} = \acute{\mu}_0$ to represent the low ($\beta < \underline{\beta}$), moderate ($\underline{\beta} \leq \beta < \bar{\beta}$), and high ($\beta \geq \bar{\beta}$) contract cost cases where $\underline{\beta}$ and $\bar{\beta}$ are calculated based on the calibrated values.

Notation	Description	Value
$\acute{\mu}_0$, $\acute{\mu}_1$	Means of FFB and CPO spot prices	498.77, 2465.20 RM
$\acute{\sigma}_0$, $\acute{\sigma}_1$	Standard deviations of FFB and CPO spot prices	58.60, 255.04 RM
$\acute{\rho}$	Correlation between FFB and CPO spot prices	0.745
$\acute{c}_0$	Unit processing cost (normalized by other by-product revenues)	-39.47 RM/mt
$\acute{K}_0$	Processing capacity	56688.06 mt
$\acute{a}_1$, $\acute{a}_2$	Production yields of CPO and PKS	19.77%, 5.65%
$\acute{D}_1$, $\acute{D}_2$	CPO and PKS demands	5379.47, 1281.15 mt
$\acute{p}_1$, $\acute{p}_2$	CPO and PKS prices for demand sales	2433.25, 476.40 RM/mt

Table 1.3: Description of the Baseline Scenario Used in Our Numerical Experiments. FFB and CPO spot prices are bivariate normally distributed.

1.7.2 The Value of Biomass Commercialization

Using the numerical experiments, we investigate the value of biomass commercialization as a percentage of the optimal profit before commercialization, and examine the heuristic value of the expected biomass margin $M^H(\beta)$ and the expected biomass margin $M(\beta)$ in the context of the palm industry.



(a) (b)

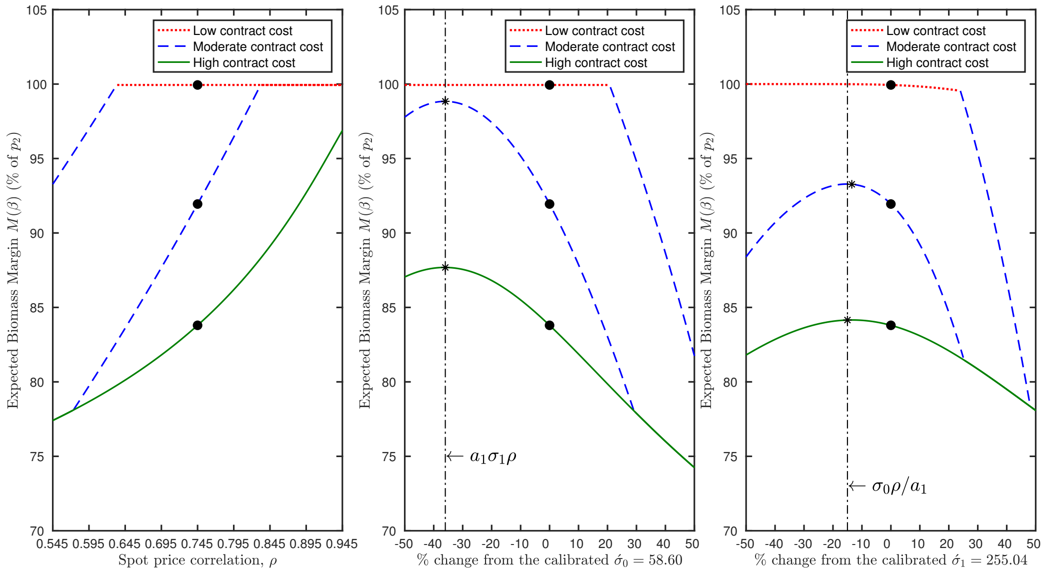
Figure 1.2: The Value of Commercialization as a Percentage of the Optimal Profit Before Commercialization ($\Delta V(\beta)/V^{nb}(\beta)$) (panel a) and the Heuristic and True Values of Expected Biomass Margin, $M^H(\beta)$ and $M(\beta)$ (panel b).

Panel a of Figure 1.2 showcases the impact of contract cost on the value of biomass commercialization as a percentage of the firm's profit before biomass commercialization. We see that the value of biomass commercialization can be significant, up to 26.54% of the profit before biomass commercialization. The firms in the palm processing industry operate with razor-thin margins as the input (FFB) and the main output (CPO) are both commodity products, therefore revenues from biomass sales have a significant impact on their profitability.

Panel b of Figure 1.2 shows that the expected value and the heuristic value of biomass margin are the same and slightly less than the biomass price p_2 for low contract costs. For moderate and high contract costs, the heuristic value of biomass margin significantly underestimates the expected value of biomass margin. This underestimation can be as high as 24.67% of the true value of biomass commercialization. This implies that the processor may miss out on lucrative opportunities because of ignoring changes in operational decisions after commercialization.

1.7.3 The Impact of Spot Price Uncertainty

We next examine the effects of FFB and CPO spot price variabilities (σ_0 and σ_1, respectively) and spot price correlation (ρ) on the value of PKS commercialization $\Delta V(\beta)$. Because $\Delta V(\beta) = M(\beta)D_2$ and the influence of these parameters is through their impact on the expected PKS margin $M(\beta)$, Figure 1.3 plots the effects of changing ρ (panel a), σ_0 (panel b), and σ_1 (panel c) on $M(\beta)$—which is presented as the percentage of the PKS price p_2—in our baseline scenario. Because our model calibration satisfies the assumptions made in §1.6—that is, bivariate Normal distribution of $(\tilde{S}_0, \tilde{S}_1)$ with $\acute{a}_1\acute{\mu}_1 > \acute{c}_0 + \acute{\mu}_0$ and $\acute{\rho} > 0$—we compare our numerical results with the analytical sensitivity results presented in Proposition 5. Our numerical experiments complement the analytical sensitivity analyses in the following two ways. First, they focus on global sensitivity analyses which allow for change in the optimal contracting decisions before and after commercialization. For example, as illustrated by dash-dotted line in panel a of Figure 1.3, when the firm is in the moderate contract cost region ($\underline{\beta} < \beta < \bar{\beta}$) with the calibrated value of $\acute{\rho}$ (represented by $\bullet$), as ρ increases (decreases) there is a transition to low (high) contract cost region $\beta < \underline{\beta}$ ($\beta > \bar{\beta}$). These transitions occur because $\underline{\beta}$ and $\bar{\beta}$ depend on ρ. Second, our numerical experiments examine the effect of σ_1 for an extended range in the moderate contract cost case; Proposition 5 proves this effect only for $\sigma_1 \leq \sigma_0\rho/a_1$. In particular, panel c illustrates that $M(\beta)$ first increases then decreases in σ_1 where the turning point is larger than $\sigma_0\rho/a_1$. This behavior is structurally the same with the high contract cost case. The general insights from Figure 1.3 parallel the ones from Proposition 5: the value of PKS commercialization increases in spot price correlation but increases in (FFB or CPO) spot price variability only when this variability is low; otherwise, the value decreases in spot price variability.



(a) (b) (c)

Figure 1.3: Effects of Spot Price Correlation ρ (Panel a), FFB Spot Price Variability σ_0 (Panel b), and CPO Spot Price Variability σ_1 (Panel c) on the Expected PKS Margin $M(\beta)$ as a Percentage of PKS Price p_2 in the Baseline Scenario: In panel a, $\rho \in [0.545, 0.945]$ evenly-spaced around the baseline value $\acute{\rho} = 0.745$ with a step size of 0.001 whereas in panel b (panel c), $\sigma_0(\sigma_1) \in [-50\%, 50\%]$ of the baseline value $\acute{\sigma}_0 = 58.60$ ($\acute{\sigma}_1 = 255.04$) with 0.5% increments. In the three panels, baseline scenario for low, moderate, and high contract cost cases are indicated by $\bullet$ aligned horizontally with the baseline value. In panel b (c), $*$ denotes the σ_0 (σ_1) level in which $M(\beta)$ attains its maximum wherever applicable.

1.8 Conclusion

This chapter studies the economic implications of biomass commercialization—that is, converting organic waste into a saleable product—from the perspective of agricultural processing firms by incorporating several unique operational features of these firms. We characterize the value of biomass commercialization and provide insights on how the spot price uncertainty (input and output price variabilities and correlation) shapes this value. As summarized in the Introduction, our findings have important practical implications that are of relevance to both agri-processors and policy makers.

In our computational study throughout §1.7, we calibrated our model to represent a typical palm oil mill in Malaysia. We expect our insights to continue to hold for a palm oil mill in another location (e.g., Indonesia). Because coconut processing and sugarcane processing share common characteristics with the palm processing—for instance, both input and output are commodities, and processing residue is commercialized as biomass—we expect the majority of our findings to be valid for coconut and sugar industries as well. That being said, future research is still needed to verify this conjecture by using our paper's methodology to calibrate the model based on a different agricultural industry.

Relaxing the assumptions made about processing environment gives rise to a number of interesting areas for future research. First, we (implicitly) assume that the processor does not participate in the input spot resale market as a part of its procurement strategy. Second, we normalize the exercise cost of quantity flexibility procurement contract to zero. The availability of spot resale (a positive exercise price) increases (decreases) the profitability before and after biomass commercialization but it is not clear how it would affect the value of commercialization. Finally, based on our interactions with a coconut processor, we assume a fixed-price fixed-volume sales contract for the biomass. Examining the effect of different sales contract forms on our results would be an interesting avenue for future research. For example, the sales contract can be in the form of an index-based contract (Goel and Tanrisever 2017) where the unit biomass price includes a fixed component and a variable component that is indexed on the spot price of the main output.

Chapter 2

Environmental Implications of Biomass Commercialization in Agricultural Processing

2.1 Introduction

In this chapter, we investigate the environmental implications of biomass commercialization based on the model and optimal solutions developed in Chapter 1.

On the environmental implications, the common perception in practice is that converting waste into a saleable product is environmentally beneficial because it leads to a reduction in GHG emissions owing to lower landfill and replacement of fossil fuel energy source in downstream power plant (Ata et al. 2012). This common perception has been one of the key driving forces behind the increasing popularity of biomass commercialization in agricultural processing industries (see, for example, Pearson 2016). A stream of papers in the industrial ecology literature has refined this perception by highlighting that biomass commercialization requires additional processing (e.g., de-fibring) and transportation activities which may create significant emissions (Iakovou et al. 2010). Although these papers provide a detailed environmental analysis, as also highlighted by Lee (2012), they do not take into account the optimization of operational decisions. Therefore, they fail to incorporate the emissions resulting from the changes in operational decisions (e.g., input processing and procurement volumes, production volumes for each output including biomass) after commercialization. In summary, it is an open question under which conditions the processor can justifiably claim that commercializing its biomass is environmentally beneficial. Moreover, it is also an open question how the environmental assessment is affected by biomass market characteristics. Our research objective in this chapter is to develop this knowledge base.

To measure the environmental impact we use total expected carbon emissions—including procurement-, processing-, selling-, and landfill-related emissions—resulting from profit-maximizing operational decisions before and after biomass commercialization. The processor can justifiably claim that commercializing its biomass is environmentally beneficial when the total expected emissions are lower after commercialization. We show that when the changes in operational decisions are ignored, our assessment is consistent with the common perception in practice: commercialization is environmentally beneficial when the landfill emission intensity is higher than the biomass selling emission intensity—which is given by the unit emission associated with additional (processing, transportation, and burning) activities less the unit emission saving obtained by burning biomass instead of fossil fuel. However, when the changes in operational decisions are not ignored, the environmental assessment is more nuanced and we identify biomass selling emission intensity and biomass demand as the two main drivers of this assessment. In particular, we establish two biomass selling emission intensity thresholds where once this emission intensity is lower (higher) than the smaller (larger) threshold, biomass commercialization is environmentally beneficial (harmful); otherwise, biomass commercialization is environmentally beneficial only when biomass demand is lower than a demand threshold. We also find that this demand threshold decreases in the biomass selling emission intensity.

Our results demonstrate that conventional arguments for and against the environmental superiority of biomass commercialization based on such simple proxy as comparison between biomass selling and landfill emission intensities can be misleading. In particular, our analysis highlights two types of misconceptions (and characterizes the specific conditions under which they appear). First, the processor would mistakenly think that commercializing its biomass is environmentally beneficial when it is not. The implication is that agricultural processors, which emphasize conversion of their residue as an argument for the environmental superiority of their business models could be vulnerable to accusations of greenwashing. Second, the processor would mistakenly think that commercializing its biomass is not environmentally beneficial when it is. In this case, an environmentally conscious processor can pass up a profitable investment opportunity (commercializing its biomass) based on an incomplete environmental assessment.

Based on our model calibration, we observe that a typical palm oil processor in Malaysia can justifiably claim that selling its palm kernel shell (PKS) to a bioenergy plant in Japan to substitute coal in energy production is environmentally beneficial unless biomass demand is larger than a level that is associated with approximately 82% processing capacity utilization. Interestingly, when PKS is used for substituting liquified natural gas, which is a cleaner energy source than coal, PKS commercialization becomes environmentally harmful regardless of the biomass demand. These results have important practical implications. First, care must be taken by palm oil mills to not promote commercializing PKS as environmentally beneficial without qualification. Second, given the current trend in the energy industry that suggests the discontinuation of coal-fired energy production by 2030 (Dempsey 2019), it is important for these mills to take actions to reduce, for example, transportation emissions (by choosing cleaner transportation options or selling biomass locally) to keep biomass commercialization environmentally beneficial. To this end, the on-going industry-wide efforts for reducing the carbon emissions in shipping (Milne 2018) also have an indirect, and potentially a crucial positive environmental impact on agricultural waste-to-energy industry.

To understand the impact of biomass market characteristics on the environmental assessment, we conduct sensitivity analyses to investigate the effects of biomass demand and biomass price on the change in expected emissions after commercialization. We find that an increase in biomass demand is environmentally superior only when biomass selling emission intensity is low; otherwise, it is environmentally inferior. On the other hand, an increase in biomass price is environmentally superior only when biomass selling emission intensity is low or it is moderate and biomass demand is low; otherwise, it is environmentally inferior. Because a higher biomass demand or price always increases the value of commercialization, these results emphasize that what is economically beneficial is not always environmentally beneficial. This conflict may create challenges in the effectiveness of government policies designed for increasing renewable energy production. For example, in recent years governments have adopted policies (e.g., feed-in-tariff) to promote investment in renewable energy sources (Babich et al. 2019). As a result, there has been a growing number of bioenergy plants leading to an increase in biomass demand for agricultural processors. Our findings demonstrate that this increase may hinder biomass commercialization in an environmentally conscious processor unless its biomass selling emission intensity is low. Therefore, we suggest that governments also devise policies to incent the processors to reduce their biomass selling emission intensity. This can be achieved, for example, by encouraging (through investment subsidies) pelletizing of the biomass before shipment, as is often done in the wood industry, to increase its calorific value so that a larger amount of fossil fuel is substituted.

Another policy implication of our results is relevant for biomass-exporting countries (e.g., Indonesia, Malaysia). Some of these countries have recently started imposing export tax for biomass to encourage the growth of domestic bioenergy industry (see, for example, The Palm Scribe 2018). When biomass is sold locally, all else equal, a processor experiences a higher biomass price due to the absence of export tax (and a lower biomass selling emission intensity due to lower transportation emissions). Our results demonstrate that imposing an export tax is the right move in the growth stage of biomass industry (when biomass demand is relatively low) because a higher price is both economically and environmentally superior leading to processor's voluntary commercialization of its biomass.

We now investigate the impact of biomass commercialization on the environment. §2.2 surveys the related literature and discusses the contribution of our work. §2.3 describes the environmental model. §2.4 characterizes the conditions under which the firm can justifiably claim that commercializing its biomass is environmentally beneficial and §2.5 examines the impact of biomass market characteristics on the environmental assessment. §2.6 provides a practical application in the context of the palm industry. §2.7 concludes with a discussion of the limitations of our analysis and future research directions.

2.2 Literature Review

This chapter is also closely related to Lee (2012) from the environmental perspectives. On the environmental implications, Lee (2012) presents a conceptual framework and makes the critical observation that waste conversion decreases the processing cost which, in turn, increases the production volumes for the outputs (including waste). She conjectures that the increase in total volume could lead to a harmful impact on the environment. Our paper builds on this conjecture and identifies conditions under which biomass commercialization leads to a beneficial or harmful impact on the environment.

Environmental implications of biomass commercialization has also received considerable attention from the industrial ecology literature. We refer the reader to Iakovou et al. (2010) for a comprehensive review. As highlighted by Lee and Tongarlak (2017), the papers in this literature examine the environmental impact without considering the optimization of operations but provide a detailed treatment of GHG emissions related to biomass commercialization. For example, Damen and Faaij (2006) study the emissions associated with using palm kernel shells (PKS) produced in Malaysia to substitute coal in a power plant located in the Netherlands while considering the emissions associated with production, transportation, and consumption of PKS. They neither consider optimization of PKS operations nor take into account uncertainties. Our environmental analysis is motivated by the papers in this literature as it accounts for all emission categories. More importantly, our environmental analysis is based on a more detailed operational framework that not only considers the optimization of processor's decisions but also takes into account the relevant uncertainties. We also provide a model calibration to examine the environmental implications of PKS commercialization in a typical palm oil mill located in Malaysia where PKS is used for substituting coal or liquified natural gas at a power plant located in Japan.

This paper also relates to the rapidly growing literature on sustainable operations—see, Drake and Spinler (2013) for a recent review—due to its focus on the environment. Within this literature, our work is more closely related to the stream of papers that examine the environmental implications of operational decisions that are made by profit-maximizing firms without considering their environmental impact (see, for example, Agrawal et al. 2012, Avcı et al. 2014, and Kök et al. 2016). Kök et al. (2016) is closer to our work because of its focus on energy production. They study the environmental implications of using different electricity pricing policies—peak versus flat pricing—from the perspective of a utility firm. They solve for the optimal profit-maximizing operational decisions and investigate the environmental implications by comparing the total expected carbon emissions of an optimally designed utility under each pricing policy. We study the environmental implications of biomass commercialization from the perspective of an agri-processor. Based on the optimal profit-maximizing operational decisions in Chapter 1, we investigate the environmental implications by making a comparison between the total expected carbon emissions of an optimally designed processor before and after biomass commercialization.

2.3 Model Description and Assumptions

In line with the industry practice and the academic literature (see, for example, Kök et al. 2016), we use carbon emissions to measure the environmental impact and calculate the total expected carbon emissions resulting from profit-maximizing operational decisions before and after biomass commercialization. Echo to our economic model described in §1.3, we consider emissions related to processor's operational activities, including procurement, processing, and selling. To this end, as customary in the literature, we assume a linear emission structure and define a unit emission intensity parameter for each of these activities.

For input procurement, we define $e_0^b > 0$ as the *input buying emission intensity* associated with each input delivered to the processor. This parameter captures the emissions from production (growing) and transportation (to the processor) of the input. We assume that this emission intensity is the same for spot-sourced and contract-sourced inputs which is a reasonable assumption when both inputs are sourced from nearby plantations. Let $e_0^p > 0$ denote the *processing emission intensity* which accounts for the emissions from energy consumption during processing. In our economic model we assume that each unit of input yields other by-products whose revenues are normalized into the processing cost. To capture the emissions associated with these other by-products (for example, emissions related to disposal of palm oil mill effluent), we define $e_3^r > 0$ as the *residue emission intensity* and assume that each unit of processed input yields a_3 units of these by-products (where $a_3 \leq 1 - a_1 - a_2$). For biomass, paralleling the environmental impact discussed in practice (Ata et al. 2012), we define two emission parameters. For unsold biomass, we define $e_2^l > 0$ as the *landfill emission intensity* which captures the emissions associated with release of methane gas as a result of anaerobic decomposition. For biomass that is sold, we define e_2^s as the *biomass selling emission intensity* which accounts for the emissions associated with additional processing (e.g., de-fibring), transportation, and usage—that is, emissions associated with burning of biomass less the emission savings obtained by substituting fossil fuel for energy production. Although this intensity parameter is unrestricted in sign (because of emission savings), it takes positive values in realistic cases (as empirically verified in §2.6). For the commodity output, we also define two emission parameters. For the commodity output sold, $e_1^s > 0$ denotes the *commodity output selling emission intensity* which captures transportation (out of the processor) and usage (e.g., refining) emissions. We assume that this emission intensity is the same for output sold to the spot market and output used to satisfy demand. This is a reasonable assumption when both outputs are sold to nearby buyers (e.g., refineries). For the commodity output purchased, $e_1^b > 0$ denotes the *commodity output buying emission intensity* which captures the emissions associated with the production of this output and its transportation to the processor.

To quantify the total expected emissions resulting from profit-maximizing decisions, because the optimal contract volume is characterized based on the unit contract cost β, we define $ECE^*(\beta)$ as the total expected emissions after commercialization for a given β:

$$\begin{aligned} ECE^*(\beta) \doteq{}& \left(e_0^b + e_0^p + a_3 e_3^r\right) \mathbb{E}\left[z_0^*\left(Q^*\left(\beta\right)\right)\right] \\ &+ e_1^s \mathbb{E}\left[a_1 z_0^*\left(Q^*\left(\beta\right)\right)\right] + \left(e_1^b + e_1^s\right) \mathbb{E}\left[\left(D_1 - a_1 z_0^*\left(Q^*\left(\beta\right)\right)\right)^+ \chi(\tilde{S}_1 \leq p_1)\right] \\ &+ e_2^s \mathbb{E}\left[\min\left(a_2 z_0^*\left(Q^*\left(\beta\right)\right), D_2\right)\right] + e_2^l \mathbb{E}\left[\left(a_2 z_0^*\left(Q^*\left(\beta\right)\right) - D_2\right)^+\right]. \end{aligned} \tag{2.1}$$

In (2.1), the first term represents the emissions from input sourcing, processing, and residues. The second and third terms denote the emissions associated with commodity output sales and procurement, respectively where the latter emissions are incurred only when it is optimal to source from the output spot market to satisfy demand. The last two terms denote the emissions related to biomass, either from satisfying the biomass demand or waste disposal through landfill.[1] The optimal contract volume $Q^*(\beta)$ can be obtained from Proposition 2 whereas the optimal processing volume $z_0^*(Q^*(\beta))$ can be obtained from Proposition 1 by substituting $Q^*(\beta)$. The total expected emissions before commercialization, $ECE^{nb}(\beta)$, can be obtained in a similar fashion by setting $D_2 = 0$ and substituting the optimal processing volume $z_0^{nb}(Q^{nb}(\beta))$ in (2.1).

2.4 Environmental Assessment of Biomass Commercialization

To characterize the impact of biomass commercialization on the environment, we define $\Delta ECE(\beta) \doteq ECE^*(\beta) - ECE^{nb}(\beta)$ as the change in total expected carbon emissions after commercialization. The processor can justifiably claim that commercializing its biomass is environmentally beneficial when it leads to reduction in emissions; that is, $\Delta ECE(\beta) < 0$. When $\Delta ECE(\beta) > 0$, we conclude that biomass commercialization is environmentally harmful. Using $ECE^*(\beta)$ (and $ECE^{nb}(\beta)$) as

[1] We note that $ECE^*(\beta)$ does not include a term directly associated with the optimal contract procurement volume $Q^*(\beta)$ because we only consider emissions related to input delivered to the processor (and, consistent with industry practice, do not consider emissions related to the reserved but unused input) and we assume that unit input buying emission intensity is the same for spot-sourced and contract-sourced inputs.

given in (2.1), we obtain

$$\begin{aligned}\Delta ECE(\beta) = &\left(e_2^s - e_2^l\right)\mathbb{E}\left[\min\left(a_2 z_0^{nb}(Q^{nb}(\beta)), D_2\right)\right] \qquad (2.2)\\ &+ \left(e_0^b + e_0^p + a_3 e_3^r + a_2 e_2^l + a_1 e_1^s\right)\mathbb{E}\left[z_0^*\left(Q^*(\beta)\right) - z_0^{nb}(Q^{nb}(\beta))\right]\\ &- \left(e_1^b + e_1^s\right)\mathbb{E}\left[\left(\left(D_1 - a_1 z_0^{nb}(Q^{nb}(\beta))\right)^+ - \left(D_1 - a_1 z_0^*(Q^*(\beta))\right)^+\right)\chi(\tilde{S}_1 \leq p_1)\right]\\ &+ \left(e_2^s - e_2^l\right)\mathbb{E}\left[\min\left(a_2 z_0^*\left(Q^*(\beta)\right), D_2\right) - \min\left(a_2 z_0^{nb}(Q^{nb}(\beta)), D_2\right)\right].\end{aligned}$$

To delineate the intuition behind (2.2), let us first consider the case where $z_0^*(Q^*(\beta)) = z_0^{nb}(Q^{nb}(\beta))$ for any (S_0, S_1) realization at stage 2—that is, the changes in operational decisions after commercialization are ignored. In this case, only the first term in (2.2) is relevant. This term captures the expected emissions resulting from converting available waste, which would go to landfill, into a saleable product and using it to substitute fossil fuel in energy production. In this case, consistent with the common perception in practice which also ignores the changes in operational decisions, our analysis reveals that biomass commercialization is environmentally beneficial (harmful) when biomass selling emission intensity is lower (higher) than the landfill emission intensity.

When the changes in operational decisions after commercialization are not ignored, the last three terms in (2.2) become relevant. Because commercialization creates a new revenue stream, intuitively, for a given contract volume Q, the optimal processing volume for any (S_0, S_1) realization at stage 2 increases (i.e., $z_0^*(Q) \geq z_0^{nb}(Q)$), and thus, the optimal contract volume increases (i.e., $Q^*(\beta) \geq Q^{nb}(\beta)$). As a result, we have $z_0^*(Q^*(\beta)) \geq z_0^{nb}(Q^{nb}(\beta))$ in (2.2) for any (S_0, S_1) realization. Therefore, the second term in (2.2) (which captures the emission impact of higher processing volume after commercialization) is always positive—that is, this change is harmful to the environment. Similarly, the third term (which captures the emission impact of lower commodity output procurement volume due to higher output production after commercialization) is always negative—that is, this change is beneficial for the environment. Finally, the last term (which captures the emissions associated with having more waste to be sold as biomass after commercialization) has the same sign with the first term—that is, this change is beneficial (harmful) for the environment when $e_2^s < (>) e_2^l$.

In summary, once the changes in the operational decisions after commercialization are not ignored, environmental assessment is more nuanced. Proposition 6 identifies biomass selling emission intensity and biomass demand as the two main drivers of this assessment.

Proposition 6 *There exist two thresholds $\underline{e}_2^s$, $\overline{e}_2^s$ with $\underline{e}_2^s \leq \overline{e}_2^s$ such that*

(i) if $e_2^s \leq \underline{e}_2^s$, then $\Delta ECE(\beta) < 0$;

(ii) if $e_2^s \geq \overline{e}_2^s$, then $\Delta ECE(\beta) \geq 0$ with equality holding when $e_2^s = \overline{e}_2^s$;

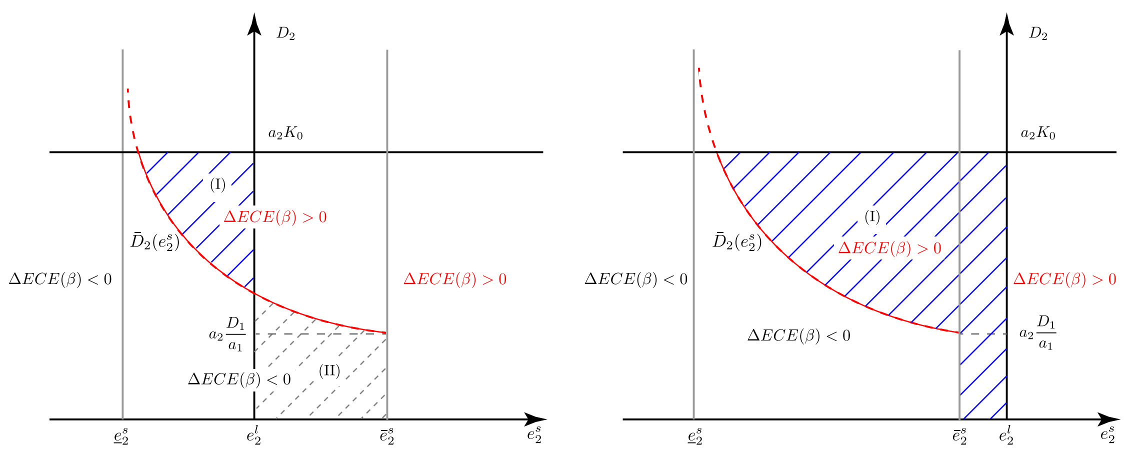
(iii) if $\underline{e}_2^s < e_2^s < \overline{e}_2^s$, then there exists a unique $\bar{D}_2(e_2^s) > a_2 D_1/a_1$ such that $\Delta ECE(\beta) \leq 0$ for $D_2 \leq \min(\bar{D}_2(e_2^s), a_2 K_0)$ with equality holding when $D_2 = \bar{D}_2(e_2^s)$, and $\Delta ECE(\beta) > 0$ for $\min(\bar{D}_2(e_2^s), a_2 K_0) < D_2 \leq a_2 K_0$. Moreover, $\partial \bar{D}_2(e_2^s)/\partial e_2^s < 0$, $\partial^2 \bar{D}_2(e_2^s)/\partial (e_2^s)^2 > 0$, $\lim_{e_2^s \to \overline{e}_2^{s-}} \bar{D}_2(e_2^s) = a_2 D_1/a_1$, and $\lim_{e_2^s \to \underline{e}_2^{s+}} \bar{D}_2(e_2^s) = \infty$.

Proposition 6 establishes two biomass selling emission intensity thresholds $\underline{e}_2^s \leq \overline{e}_2^s$ where once this emission intensity is lower (higher) than $\underline{e}_2^s$ ($\overline{e}_2^s$), biomass commercialization is environmentally beneficial (harmful). When the biomass selling emission intensity is between these two thresholds, biomass commercialization is environmentally beneficial only when biomass demand is lower than a threshold $\bar{D}_2(e_2^s)$ which (convexly) decreases in e_2^s. We defer the discussion of intuition behind how these thresholds are obtained from (2.2) to the next section (where we discuss how $\Delta ECE(\beta)$ is impacted by the biomass demand D_2).

How does the environmental assessment in Proposition 6 contrast with the common perception in practice? Because this common perception is based on a comparison between biomass selling and landfill emission intensities, we now examine how the emission intensity thresholds established in Proposition 6 compare with the landfill emission intensity.

Proposition 7 *Let $\underline{e}_2^s(e_2^l)$ and $\overline{e}_2^s(e_2^l)$ denote the thresholds defined in Proposition 6 for a given e_2^l. We have $\underline{e}_2^s(e_2^l) < e_2^l$ and there exists a unique threshold $\hat{e}_2^l \geq 0$ such that $\overline{e}_2^s(e_2^l) > e_2^l$ for $0 \leq e_2^l < \hat{e}_2^l$ and $\overline{e}_2^s(e_2^l) \leq e_2^l$ for $e_2^l \geq \hat{e}_2^l$.*

Proposition 7 proves that while the smaller threshold is always lower than the landfill emission intensity e_2^l, the larger threshold is lower than the same only when the landfill emission intensity is small; otherwise, this threshold is higher. Using these results, Figure 2.1 illustrates the environmental assessment characterization for a given low (panel a) and high (panel b) e_2^l which is set to be the origin of the horizontal axis representing e_2^s.



(a) For a given $e_2^l < \hat{e}_2^l$ (b) For a given $e_2^l \geq \hat{e}_2^l$

Figure 2.1: When Does Biomass Commercialization Lead to a Reduction (Increase) in Total Expected Emissions; that is, $\Delta ECE(\beta) < 0$ ($\Delta ECE(\beta) > 0$)? Effects of biomass selling emission intensity e_2^s and biomass demand D_2 for a given landfill emission intensity e_2^l.

In comparison with the common perception in practice, Figure 2.1 highlights two types of misconceptions (and illustrates specific conditions under which they appear). First, in region I, *the processor would mistakenly think that commercializing* *its biomass is environmentally beneficial when it is not.* In this case, the harmful environmental impact of increasing processing volume after commercialization, the second term in (2.2), outweighs the other three effects which are beneficial for the environment. Second, in region II, *the processor would mistakenly think that commercializing its biomass is not environmentally beneficial when it is.* In this case, the beneficial environmental impact of decreasing commodity output procurement volume after commercialization, the third term in (2.2), outweighs the other three effects which are harmful to the environment.

We close this section with an important remark. Recall from Proposition 3 that the value of biomass commercialization is characterized based on three different contract procurement regions (i.e., $\beta < \underline{\beta}$, $\underline{\beta} \leq \beta < \bar{\beta}$, and $\beta \geq \bar{\beta}$). In a particular region because the contract volumes before and after commercialization are independent of β, so is $\Delta ECE(\beta)$, and thus, so are the biomass selling emission intensity thresholds and the biomass demand threshold given in Proposition 6. However, $\Delta ECE(\beta)$ and these thresholds vary across the contract procurement regions. We use this observation in the next section.

2.5 The Impact of Biomass Market Characteristics

We now conduct sensitivity analyses to study the effects of biomass demand (D_2) and biomass price (p_2) on the change in total expected carbon emissions after commercialization $\Delta ECE(\beta)$. We say that a change in D_2 or p_2 is environmentally superior (inferior) when it leads to a decrease (increase) in $\Delta ECE(\beta)$. These sensitivity analyses are useful in understanding the environmental consequences of recently implemented government policies (as discussed in the Introduction) that have been devised based on economic consequences.

Although we carry out the sensitivity analyses for any β, for illustration purposes, we focus on the $\beta < \underline{\beta}$ case where the firm contracts up to processing capacity K_0 before and after commercialization. In this case, $\Delta ECE(\beta)$ in (2.2) can be characterized as follows:[2]

$$\begin{aligned}\Delta ECE(\beta) = {} & \left(e_2^s - e_2^l\right) D_2\, \mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2}{a_1}\right)\right] \\ & + \left(e_0^b + e_0^p + a_3 e_3^r + a_2 e_2^l + a_1 e_1^s\right) \frac{D_2}{a_2}\, \mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \leq \frac{c_0}{a_1}\right)\right] \\ & - \left(e_1^b + e_1^s\right) \min\left(D_1, a_1 \frac{D_2}{a_2}\right) \mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \leq \frac{c_0}{a_1}\right) \chi(\tilde{S}_1 \leq p_1)\right].\end{aligned} \tag{2.3}$$

The first term in (2.3) denotes the expected emissions resulting from using available waste of D_2 units, which would go to landfill, to substitute fossil fuel in energy production (which happens on the spot day when processing is profitable after commercialization; that is, $a_1 S_1 + a_2 p_2 > c_0$). The second term denotes the expected emissions associated with the additional input processing volume D_2/a_2 after commercialization (which happens on the spot day when processing becomes profitable only after commercialization; that is, $a_1 S_1 + a_2 p_2 > c_0 \geq a_1 S_1$). The last term denotes the expected emissions associated with the decline in the commodity output spot procurement volume (that is used to satisfy output demand D_1) after commercialization as result of the additional output production volume $a_1 D_2/a_2$ (which happens on the spot day when processing becomes profitable only after commercialization and when it is profitable satisfy the output demand from spot procurement; that is, $p_1 \geq S_1$).

Proposition 8 examines the impact of biomass demand D_2 on $\Delta ECE(\beta)$.

Proposition 8 *Let $\underline{e}_2^s$ and $\overline{e}_2^s$ as defined in Proposition 6. (i) If $e_2^s \leq \underline{e}_2^s$, then $\partial \Delta ECE(\beta)/\partial D_2 < 0$; (ii) if $e_2^s \geq \overline{e}_2^s$, then $\partial \Delta ECE(\beta)/\partial D_2 \geq 0$; (iii) if $\underline{e}_2^s < e_2^s < \overline{e}_2^s$, then $\partial \Delta ECE(\beta)/\partial D_2 < 0$ for $D_2 < a_2 D_1/a_1$ and $\partial \Delta ECE(\beta)/\partial D_2 > 0$ for $a_2 D_1/a_1 \leq D_2 < a_2 K_0$.*

It follows from (2.3) that increasing D_2 has a harmful (beneficial) effect on the en-

[2] We relegate the details of this characterization and the characterizations of $\Delta ECE(\beta)$ for the other two cases (i.e., $\underline{\beta} \leq \beta < \bar{\beta}$ and $\beta \geq \bar{\beta}$) to §4.1.

vironment when $e_2^s > (<) e_2^l$ because it increases (decreases) the expected emissions resulting from using available waste. At the same time, it has a harmful effect on the environment because it increases the expected emissions associated with the additional processing volume. Finally, increasing D_2 decreases the expected emissions associated with the decline in the commodity output spot procurement volume, which is beneficial for the environment, only when $D_2 < a_2 D_1/a_1$; otherwise, it does not affect these emissions. When the biomass selling emission intensity e_2^s is lower than $\underline{e}_2^s$ (which is smaller than e_2^l as shown in Proposition 7), the beneficial effect associated with using available waste outweighs the harmful effect associated with increasing processing volume without considering the beneficial effect associated with the decline in output spot procurement. As e_2^s increases, the latter effect becomes consequential. In particular, when $\underline{e}_2^s < e_2^s < \bar{e}_2^s$, increasing D_2 continues to be environmentally superior as long as the latter beneficial effect is relevant (i.e., for $D_2 < a_2 D_1/a_1$); otherwise, increasing D_2 becomes environmentally inferior. When e_2^s further increases (i.e., $e_2^s \geq \bar{e}_2^s$), the beneficial effect associated with the decline in output spot procurement is always dominated by the combined effects of emissions associated with using available waste and increasing processing volume outweigh, and increasing D_2 is environmentally inferior.

We next examine how changing biomass price p_2 impacts the environmental assessment of biomass commercialization. To avoid uninteresting cases, we restrict our attention to $p_2 < c_0/a_2$ range; that is, biomass revenue itself is not sufficient to justify processing.[3]

Proposition 9 *Assume $p_2 < c_0/a_2$ and let $\hat{e} \doteq e_0^b + e_0^p + a_3 e_3^r + a_2 e_2^l + a_1 e_1^s > 0$. There exist two thresholds $\underline{\underline{e}}_2^s \doteq e_2^l - \frac{\hat{e}}{a_2}$ and $\overline{\overline{e}}_2^s$ with $\underline{\underline{e}}_2^s \leq \overline{\overline{e}}_2^s$ such that (i) if $e_2^s \leq \underline{\underline{e}}_2^s$, then $\partial \Delta ECE(\beta)/\partial p_2 < 0$; (ii) if $e_2^s \geq \overline{\overline{e}}_2^s$, then $\partial \Delta ECE(\beta)/\partial p_2 \geq 0$; (iii) if $\underline{\underline{e}}_2^s < e_2^s < \overline{\overline{e}}_2^s$,*

[3]Considering $p_2 \geq c_0/a_2$ leads to uninteresting cases. For example, as can be observed from (2.3), because output spot price $\tilde{S}_1$ is assumed to have a positive support, $\Delta ECE(\beta)$ for $\beta < \underline{\beta}$ is independent of p_2.

then there exists a unique $\bar{\bar{D}}_2(e_2^s) > a_2D_1/a_1$ *such that* $\partial\Delta ECE(\beta)/\partial p_2 \leq 0$ *for* $D_2 \leq \min(\bar{\bar{D}}_2(e_2^s), a_2K_0)$*, and* $\partial\Delta ECE(\beta)/\partial p_2 > 0$ *for* $\min(\bar{\bar{D}}_2(e_2^s), a_2K_0) < D_2 \leq a_2K_0$*.*

Proposition 9 demonstrates that the impact of biomass price p_2 is structurally similar to the impact of biomass demand D_2. In particular, when biomass selling emission intensity e_2^s is lower than the threshold $\underline{\underline{e}}_2^s$ (which is also lower than e_2^l), increasing p_2 is environmentally superior. When e_2^s is higher than the threshold $\bar{\bar{e}}_2^s$, increasing p_2 is environmentally inferior. Otherwise (i.e., $\underline{\underline{e}}_2^s < e_2^s < \bar{\bar{e}}_2^s$), increasing p_2 is environmentally superior (inferior) when biomass demand is lower (higher) than $\bar{\bar{D}}_2(e_2^s)$.[4] Although the emission intensity thresholds and the biomass demand threshold are different from the ones in Proposition 8, the intuition behind the characterization of these thresholds is similar. This is because, as can be observed from (2.3), a higher p_2 affects the emission terms in the same direction with a higher D_2.

It is easy to establish from Proposition 3 that a higher biomass demand or price always increases the value of biomass commercialization $\Delta V(\beta)$. Propositions 8 and 9 demonstrate that a change that is economically beneficial is not necessarily beneficial for the environment.

2.6 Numerical Analysis: Application to the Palm Industry

In this section, we investigate the environmental implications of biomass commercialization in the same context as that in Chapter 1. We consider the same palm oil mill located in Peninsular Malaysia selling its PKS to a power plant in Japan where PKS is used for substituting coal or liquified natural gas (LNG). We use publicly available data from the Malaysian Palm Oil Board (MPOB), complemented by the data obtained from the extant literature. Throughout this section, we use $\acute{x}$ to denote the calibrated value for parameter x, "RM" to denote Malaysian ringgit (currency), "mt"

[4] We can also prove that the threshold $\bar{\bar{D}}_2(e_2^s)$ (convexly) decreases in e_2^s.

to denote metric ton (equal to 1,000 kg), “MJ” to denote megajoule (energy unit), “ha” to denote hectare, “yr” to denote year, and “CO_2” to denote carbon dioxide which we use for measuring carbon emissions.

2.6.1 Model Calibration for Numerical Experiments

For calibrating the emission intensity parameters, we widely rely on the industrial ecology literature (e.g., Damen and Faaij 2006 and Klaarenbeeksingel 2009). Consistent with this literature, for a product that is procured (i.e., FFB or CPO), we consider emissions associated with its production and its transportation to the palm oil mill; and for a product that is sold (i.e., CPO or palm kernel which is one of the other by-products normalized under the processing cost) we consider emissions associated with its transportation out of the palm oil mill and its usage in the downstream buyer. Since our calibration involves multiple modes of transportation (e.g., truck, railway and vessel), we first highlight how we calculate emissions associated with each mode and how we measure the transportation distances. In particular, the distances for truck transportation are measured by the driving distance in Google Maps; distances for railway transportation are approximated by a straight line distance between transport nodes; the distances between ports are from Ports.com—a website that contains information about seaports and routes. We set diesel as the fuel source for truck transportation. Renewable Fuels Agency (2011) reports that the emission factor for diesel is 0.086 kg CO_2/MJ (see, Table 5) and the fuel efficiency for truck transportation in both Malaysia and Japan is 1.8 MJ/(mt∗km) (see, Table 6). We adopt the round-trip emission intensity for truck transportation as customary in literature (see, Cachon 2014 and Belavina et al. 2016). The empty load causes additional 65% of full load emissions (Damen and Faaij 2006), that implies a round-trip emission intensity $0.086 \times 1.8 \times 1.65 = 0.255$ kg CO_2/(mt∗km) for truck transportation. As for railway transportation and vessel transportation, Greenhouse Gas Protocol (2017) specifies that the emission intensities are 0.0252 and 0.0480 kg CO_2/(short ton mile), respectively, which correspond to 0.0173 and 0.0329 kg CO_2/(mt∗km) given the conversion factor "1 short ton mile = 1.46 (mt∗km)." Different from truck transportation, we assume a one way-trip emission intensity for railway or vessel transportation because trains and vessels typically transport other goods in the return journey. Estimating transportation emissions also requires specifying the locations of the palm oil mill, the plantation where FFB is sourced from, and the refinery where CPO is sold to. We consider the Sungai Jernih mill of Boustead Plantations Berhad located in Pahang (Peninsular Malaysia) in our calibration. We assume that this mill sources its FFB from the nearby plantation which is 15.2 km away. We also assume that the palm oil mill sells CPO to Cargill Palm Products refinery in Kuantan, Pahang (Peninsular Malaysia), which is 73.5 km away from the palm oil mill. We now describe how each emission intensity parameter is calibrated.

FFB buying emission intensity (e_0^b): This parameter captures the emissions from production (growing) of FFB and its transportation to the palm oil mill. Paralleling the standard in the industrial ecology literature, for emissions associated with the production (growing) of FFB, we consider emissions from farming and harvesting of FFB.[5] The major sources for farming emissions are emissions from internal transportation (within the plantation) and machinery usage, and emissions from the use of artificial fertilizers and pesticides. Klaarenbeeksingel (2009) reports that the average emissions from internal transportation and machinery usage are 85 kg CO_2/mt of CPO and the average emissions from the use of artificial fertilizers and pesticides are 360 kg CO_2/mt of CPO. To convert these emissions given per mt of CPO to emissions per mt of FFB, we use the CPO production yield $\acute{a}_1 = 19.77\%$ and obtain the farming emissions as $(85 + 360) \times 19.77\% = 87.98$ kg CO_2/mt of FFB. We assume harvesting

[5] We do not consider the emissions associated with land use change (for example, due to deforestation as considered in de Zegher et al. 2018) because we assume FFB is grown in an existing palm plantation.

emissions to be insignificant (because FFB is harvested manually by using wheelbarrows) and set them to 0. For transportation emissions, given the round-trip emission intensity for truck transportation is 0.255 kg CO_2/(mt$*$km) and the plantation where FFB is sourced is located 15.2 km away from the palm oil mill, the transportation emissions for FFB are $15.2 \times 0.255 = 3.876$ kg CO_2/mt of FFB. In summary, the calibrated value for FFB buying emission intensity is given by $\acute{e}_0^b = 87.98 + 3.876 = 91.86$ kg CO_2/mt of FFB.

Processing emission intensity (e_0^p): This parameter accounts for the emissions from energy consumption during processing. As specified in Warman et al. (2019), energy consumption in a typical palm oil mill is within the range of 17 to 19 kWh/mt of FFB. We use the average (i.e., 18 kWh/mt of FFB) in our calibration. We assume that palm oil mill uses electricity from national grid and consider a grid emission factor—which measures the CO_2 emissions associated with each unit of electricity provided by an electricity system—of 0.694 kg CO_2/kWh for Peninsular Malaysia.[6] Therefore, the calibrated value for the processing emission intensity is given by $\acute{e}_0^p = 18 \times 0.694 = 12.49$ kg CO_2/mt of FFB.

Effective residue emission intensity ($a_3 e_3^r$): Recall that to capture the emissions associated with other by-products (other than CPO and PKS), we define e_3^r as the residue emission intensity and assume that each unit of processed input yields a_3 units of these by-products where $a_3 \leq 1 - a_1 - a_2$. In a palm oil mill, FFB processing yields three other by-products: empty fruit bunches (EFB), palm oil mill effluent (POME), and palm kernel (PK). We consider the carbon emissions from each by-product and its associated production yield in calibrating $a_3 e_3^r$. EFB is typically discarded in disposal ponds or used as mulch, hence assumed to be carbon neutral with zero emissions (Mal-Moulin et al. 2016). POME is also typically discarded but it produces a substantial

[6] This grid emission factor is calculated based on the average annual emission factor of Peninsular Malaysia between 2005 and 2016 as reported by Table 1 of Malaysian Green Technology Corporation (2017).

amount of carbon emissions during anaerobic decomposition. Abdullah and Sulaiman (2013) report the ratio between the production yields of POME and CPO as 28/21 which corresponds to a POME production yield of 26.77% for a given $\acute{a}_1 = 19.77\%$. Moreover, according to Klaarenbeeksingel (2009), the average carbon emissions from POME is 1046 kg CO_2/mt of CPO and given the POME production yield of 26.36%, the emissions from POME disposal are 206.79 kg CO_2/mt of FFB. Different from EFB and POME, PK has economic value and it is sold to a kernel-crushing plant to be further processed to extract crude palm kernel oil (CPKO). Consistent with other products that have economic value (i.e., CPO and PKS), we consider emissions associated with the transportation of PK by trucks from the palm oil mill to the kernel-crushing plant and its usage—that is, PK processing emissions in the crushing plant. In Malaysia, kernel-crushing plants are typically located near to ports for the ease of export, and they use electricity directly from the national grid for processing (Subramaniam and May 2012). We consider a crushing-plant near to Kuantan port in Pahang, which is the closest port to the palm oil mill used in our calibration. The driving distance from the palm oil mill to Kuantan port is 79 km. Using the round-trip emission intensity of 0.255 kg CO_2/(mt∗km) for truck transportation, we obtain the transportation emissions to be $79 \times 0.255 = 20.145$ kg CO_2/mt of PK. In the crushing plant PK is pressed to extract CPKO. Table 6 of Subramaniam and May (2012) documents that pressing activities generate emissions of 74.33 kg CO_2/mt of CPKO based on the electricity consumption from the national grid. To convert the processing emissions given per mt of CPKO to emissions per mt of PK, we use 46.2%—the average monthly CPKO production yield in Peninsular Malaysia within our time frame (Jan 2014 to May 2018) as reported by the MPOB—and obtain $74.33 \times 46.2\% = 34.34$ kg CO_2/mt of PK. Therefore, the total emissions associated with PK transportation and processing are 54.485 kg CO_2/mt of PK. To convert these emissions given per mt of PK to emissions per mt of FFB, we use 5.41%—the average monthly PK production yield in Peninsular Malaysia within our time frame (Jan 2014 to May 2018) as reported by the MPOB—and obtain $54.485 \times 5.41\% = 2.95$ kg CO_2/mt of FFB. In summary, the calibrated value for the effective residue (EFB, POME and PK) emission intensity is given by $\acute{a}_3 \acute{e}_3^r = 0 + 206.79 + 2.95 = 209.74$ kg CO_2/mt of FFB.

PKS landfill emission intensity (e_2^l): This parameter captures the emissions associated with release of methane gas as a result of anaerobic decomposition. Because we cannot find data associated with PKS, we use landfill emission intensity data of another organic waste, wood residue (e.g., bark and branches), to estimate this parameter. We choose wood residue because PKS belongs to lignocellulosic biomass (Zafar 2018) which is also true for wood residue. Damen and Faaij (2006) report a wood residue landfill emission intensity range of 230-2700 kg CO_2/mt of wood residue. We set the average value as the calibrated PKS landfill emission intensity—that is, $\acute{e}_2^l = 1470.00$ kg CO_2/mt of PKS.[7]

PKS selling emission intensity (e_2^s): This parameter accounts for the emissions associated with (i) further processing of PKS after commercialization, (ii) transportation of PKS from palm oil mill to power plant in Japan, and PKS usage—that is, (iii) emissions associated with burning of PKS less (iv) the emission savings obtained by substituting fossil fuel for energy production. These emission savings are characterized based on the emissions associated with production, transportation and burning of the fossil fuel which are then normalized by the replacement ratio between PKS and the fossil fuel because both energy sources have different caloric densities.

(i) The fresh PKS obtained at the end of processing activities can be considered as natural pellets but with high fiber and moisture. To remove the impurities and

[7]We conduct additional numerical experiments to examine the sensitivity of our results to PKS landfill emission intensity. Because PKS is more resistant to rotting than wood (BMC bioPOWER 2020) it has a lower decomposition rate and thus, a lower landfill emission intensity. We consider $[-25\%, 0\%]$ of the baseline value $\acute{e}_2^l = 1470.00$ kg CO_2/mt of PKS with 5% increments and verify that our numerical results are structurally the same within this range of $\acute{e}_2^l$.

decrease the moisture level, the fresh PKS goes through the rotary screening machine and drier. However, without the need for pelletization process (as often done with wood residues), these further processing activities are relatively energy-efficient (Setyawan 2017). Therefore, we regard the emission from further processing of PKS to be negligible and set it to be 0.

(ii) We assume the power plant in Japan to be located in Tsu Mie which is close to Osaka port.[8] In order to reach the power plant, PKS is first transported by trucks from the palm oil mill to the nearby (Kuantan) port in Malaysia which is 79 km away from the mill. Then it is transported by vessels from Kuantan port to Osaka port in Japan (which requires a traveling distance of 6122.7 km). Finally, PKS is transported by trucks from Osaka port to the power plant which is 135 km away from the port. Recall that the round-trip emission intensity for truck transportation is 0.255 kg CO_2/(mt∗km) and the one way-trip emission intensity for vessel transport is 0.0329 kg CO_2/(mt∗km). Therefore, the total transportation emissions from palm oil mill to the power plant in Japan are $79 \times 0.255 + 6122.7 \times 0.0329 + 135 \times 0.255 = 256.01$ kg CO_2/mt of PKS.

(iii) We assume PKS burning emissions to be 1453.6 kg CO_2/mt of PKS which is the average emission intensity of primary solid biomass fuels as reported by Greenhouse Gas Protocol (2017).

(iv) To calculate the emission savings obtained by substituting fossil fuel with PKS for energy production, we consider two separate fossil fuels, coal and liquified natural gas (LNG), and assume that PKS replaces these fuels in producing electricity. For each fossil fuel, paralleling the industrial ecology literature (see, for example, Damen and Faaij 2006), we account for emissions associated with its production, its transportation to the power plant and its burning which are then normalized by the replacement ratio between PKS and the fossil fuel.

[8] The majority of biomass power plants in Japan are located in the south of Japan which are near to the Osaka region (Asia Biomass 2015).

We first describe our calibration when the coal is the fossil fuel substitute. Table 1-1 in U.S. Environmental Protection Agency (2013) reports that the average emissions from coal production (which involves mining and handling activities) are 260 kg CO_2/mt of coal.[9] To quantify the transportation emissions, we assume that coal is mined in Australia.[10] In particular, we assume that the coal is first transported by railway from one of the largest coal mines in Australia (Beltana coal mine, located at Singleton, New South Wales) to a nearby (Newcastle) port which is 67.54 km away from the mine, then it is transported by vessel to Osaka port in Japan (which is 8939.4 km away from the port in Australia), and finally, it is transported by railway from Osaka port to the power plant in Japan which is 91.95 km away from the port. Using a one way-trip emission intensity of 0.0173 and 0.0329 kg/(mt$*$km) for railway and vessel transportation, respectively we obtain emissions from coal transportation to be $67.54 \times 0.0173 + 8939.4 \times 0.0329 + 91.95 \times 0.0173 = 296.87$ kg CO_2/mt of coal. Greenhouse Gas Protocol (2017) documents an average burning emissions of 2128.49 kg CO_2/mt of coal across different coal types. Therefore, the total emissions associated with using coal to produce electricity are $260 + 296.87 + 2128.49 = 2685.36$ kg CO_2/mt of coal. We next examine the replacement ratio between PKS and coal in energy production. Coal generates an energy of 8 kWh/kg (European Nuclear Society 2020) while PKS generates an energy of 4.9 kWh/kg based on Palmshells.com.[11] Because PKS has a lower energy density, a larger volume of PKS is required to produce the same level of electricity. As a result, a higher volume of air mixes with the PKS during burning, which in turn, causes a larger energy loss. This is known as mix-burning efficiency loss which we assume to be 5.5% based on Damen and Faaij (2006). Therefore, the replacement ratio between PKS and coal is calculated

[9]The actual emissions depend on coal characteristics including the type of coal (e.g., bituminous coal, lignite coal) and the mining depth.

[10]According to Ministry of Economy Trade and Industry (2018), 71.5% of the imported coal in Japan is from Australia.

[11]http://www.palmshells.com/energysupply.html

as $(4.9/8) \times (1 - 0.055) = 0.579$. In summary, after considering all four emission categories discussed above, the calibrated value for the PKS selling emission intensity is given by $\acute{e}_2^s = 0 + 256.01 + 1453.6 - (2685.36 \times 0.579) = 151.34$ kg CO_2/mt of PKS when PKS is used to substitute coal.

We now describe our calibration when LNG is the fossil fuel substitute. Using the information given in Energy Information Administration (2020), the burning emissions of LNG are estimated to be 2407.3 kg CO_2/mt of LNG. Figure 3 in Bradbury et al. (2015) reports that the activities associated with LNG production and its transportation account for an additional 25.8% of consumption (i.e., burning) emissions of LNG. Therefore, the total emissions associated with using LNG to produce electricity are $(25.8\% + 1) \times 2407.3 = 3028.38$ kg CO_2/mt of LNG. We next examine the replacement ratio between PKS and LNG in energy production. Recall that PKS generates an energy of 4.9 kWh/kg whereas LNG generates an energy of 13.3 kWh/kg (Engineering ToolBox (2020)).[12] Assuming the same mix-burning efficiency loss 5.5% for LNG, the replacement ratio between PKS and LNG is calculated as $(4.9/13.3) \times (1 - 0.055) = 0.348$. In summary, the calibrated value for the PKS selling emission intensity is given by $\acute{e}_2^s = 0 + 256.01 + 1453.6 - 3028.38 \times 0.348 = 655.73$ kg CO_2/mt of PKS when PKS is used to substitute LNG.

CPO buying emission intensity (e_1^b): This parameter captures the emissions associated with the production of CPO in another palm oil mill and its transportation from that mill to the palm oil mill under consideration. We assume that CPO is procured from a nearby palm oil mill, and thus, that palm oil mill has similar characteristics. Production of CPO involves all the activities discussed above; (i) farming and harvesting of FFB and its transportation from the plantation to the mill, (ii) processing of FFB, and (iii) residue disposal including landfilling of PKS.

[12] From Engineering ToolBox (2020), the average heating value of LNG is 47.5 MJ/kg with a typical value of the lower and higher heating value of 45 and 50 MJ/kg, respectively. Using the conversion factor "1 MJ = 0.28 kWh", we obtain $47.5 \times 0.28 = 13.3$ kWh/kg.

For (i), the emission intensity is $e_0^b = 87.98$ kg CO_2/mt of FFB whereas for (ii), the emission intensity is $e_0^p = 12.49$ kg CO_2/mt of FFB. For (iii), we assume that all residue emissions are allocated to the CPO in line with Klaarenbeeksingel (2009), and thus, the effective residue emission intensity is $a_3 e_3^r = 209.74$ kg CO_2/mt of FFB.[13] We also assume that PKS is landfilled where PKS landfill emission intensity is $e_2^l = 1470.00$ kg CO_2/mt of PKS, and given the production yield of PKS is $a_2 = 5.65\%$, this is equivalent to $1470 \times 5.65\% = 83.06$ kg CO_2/mt of FFB. Because emissions associated with (i), (ii) and (iii) are given per mt of FFB, we convert these emissions per mt of CPO by using the CPO production yield $a_1 = 19.77\%$. For the emissions associated with transportation of CPO to the palm oil mill used in our calibration, we assume the same driving distance with the plantation (i.e., 15.2 km). Using the round-trip emission intensity of 0.255 kg CO_2/(mt∗km) for truck transportation, we obtain transportation emissions to be $15.2 \times 0.255 = 3.876$ kg CO_2/mt of CPO. In summary, the calibrated CPO buying emission intensity is given by $\acute{e}_1^b = (87.98 + 12.49 + 209.74 + 83.06)/19.77\% + 3.876 = 1992.85$ kg CO_2/mt of CPO.

CPO selling emission intensity (e_1^s): This parameter captures the emissions associated with CPO transportation from the palm oil mill to the downstream refinery and its usage—that is, further processing of CPO into refined palm oil. Recall that the driving distance between the palm oil mill (Sungai Jernih mill) and the refinery (Cargill Palm Product) is 73.5 km. Given a round-trip emission intensity of 0.255 kg CO_2/(mt∗km) for truck transportation, the transportation emissions are $73.5 \times 0.255 = 18.74$ kg CO_2/mt of CPO. Table 2.7 in Klaarenbeeksingel (2009) documents the emission intensity for CPO refining process to be 199 kg CO_2/mt of CPO. Therefore, the calibrated CPO selling emission intensity is given by

[13] There exist alternative ways of allocating emissions across multiple products. For example, Sunar and Plambeck (2016) highlight three alternative allocation schemes: value-based allocation, mass-based allocation, and expansion-based allocation in which the primary product's emissions are calculated by the production emissions less total avoided emissions from by-products.

$\acute{e}_1^s = 18.74 + 199 = 217.74$ kg CO_2/mt of CPO.

Using these experiments, we examine the environmental assessment of PKS commercialization where PKS is used for substituting coal or liquified natural gas (LNG). Table 2.1 summarizes the calibrated parameter values representing the baseline scenario used in our numerical experiments.

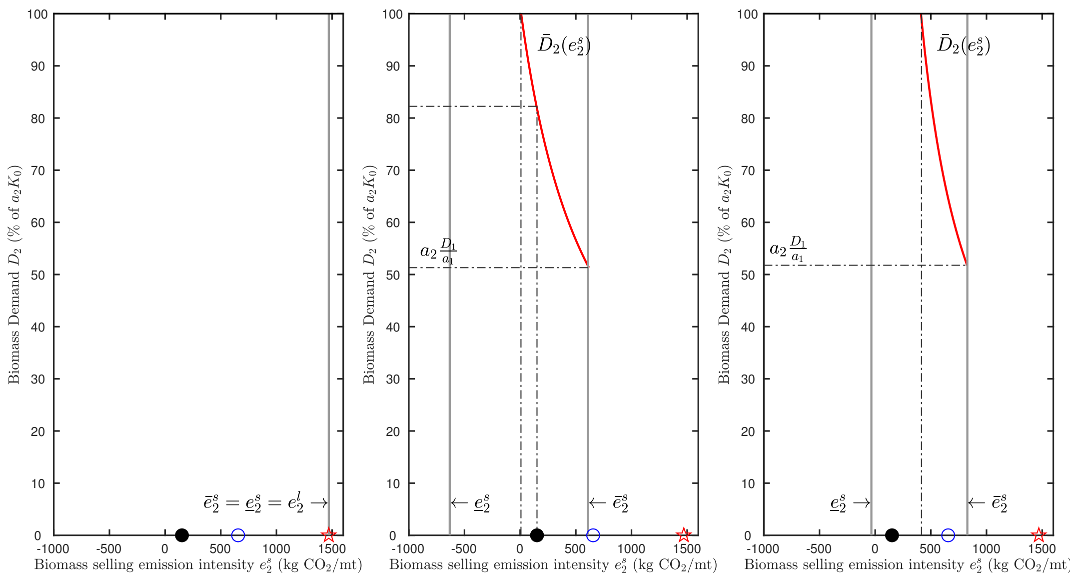
Notation	Description	Value
$\acute{e}_0^b$	FFB buying emission intensity	89.25 kg CO_2/mt
$\acute{e}_0^p$	Processing emission intensity	12.49 kg CO_2/mt
$\acute{a}_3\acute{e}_3^r$	Effective residue emission intensity	209.69 kg CO_2/mt
$\acute{e}_2^l$	PKS landfill emission intensity	1470.00 kg CO_2/mt
$\acute{e}_2^s$	PKS selling emission intensity	151.23 kg CO_2/mt, replacing coal 652.28 kg CO_2/mt, replacing LNG
$\acute{e}_1^s$	CPO selling emission intensity	216.82 kg CO_2/mt
$\acute{e}_1^b$	CPO buying emission intensity	1990.25 kg CO_2/mt

Table 2.1: Description of the Baseline Scenario Used in Our Numerical Experiments. FFB and CPO spot prices are bivariate normally distributed.

2.6.2 Environmental Assessment of Biomass Commercialization

We investigate under what conditions a typical palm oil mill in Malaysia can justifiably claim that commercializing its PKS is environmentally beneficial when PKS is used for substituting coal or LNG in the power plant in Japan.

To this end, we compute the biomass selling emission intensity thresholds $\underline{e}_2^s$ and $\overline{e}_2^s$, and the biomass demand threshold $\bar{D}_2(e_2^s)$ (as characterized in Proposition 6) in our baseline scenario for the low ($\beta < \underline{\beta}$), moderate ($\underline{\beta} \leq \beta < \bar{\beta}$), and high ($\beta \geq \bar{\beta}$) contract cost cases. Figure 2.2 illustrates these thresholds for each case where $\bar{D}_2(e_2^s)$ is presented as a percentage of a_2K_0, processing capacity required to satisfy biomass demand, which is no greater than 100% because of our assumption $D_2 \leq a_2K_0$. In each panel the calibrated landfill emission intensity and biomass selling emission intensity with coal (LNG) as the fuel substitute are depicted by $\star$ and $\bullet$ ($\circ$), respectively. Because both biomass selling emission intensities are less than the landfill emission intensity, based on the common perception in practice (which does not consider the changes in operational decisions after commercialization) the palm oil mill would conclude that commercializing its PKS is environmentally beneficial regardless of the fuel substitute.



(a) Low contract cost region (b) Moderate contract cost region (c) High contract cost region

Figure 2.2: The Environmental Assessment of PKS Commercialization for the Low (Panel a), Moderate (Panel b), and High (Panel c) Contract Cost Cases: In each panel, $\bullet$ ($\circ$) represents the calibrated biomass selling mission intensity e_2^s when PKS is used for substituting coal (LNG) and $\star$ represents the calibrated landfill emission intensity. PKS commercialization is environmentally beneficial when $e_2^s \leq \underline{e}_2^s$ or $\underline{e}_2^s < e_2^s < \bar{e}_2^s$ and $D_2 < \min(\bar{D}_2(e_2^s), a_2K_0)$.

We observe from panel a that when the contract cost is low, $\underline{e}_2^s = \bar{e}_2^s = \acute{e}_2^l$ and the environmental assessment is consistent with the common perception in practice. In this case, the mill contracts up to K_0 before and after commercialization, and input spot procurement is never used. Because $Pr\left(\tilde{S}_1 > \frac{\acute{c}_0}{\acute{a}_1}\right) \approx 1$ in the baseline scenario; that is, processing is always profitable on the spot day in the absence of PKS revenue, PKS commercialization does not affect the processing volume. Therefore, as follows from (2.3), $\Delta ECE(\beta) \approx \left(e_2^s - e_2^l\right) D_2$ and PKS commercialization is environmentally beneficial regardless of the fuel substitute because $\acute{e}_2^s < \acute{e}_2^l$. Panel b (c) illustrates that when the contract cost is moderate (high), $\underline{e}_2^s < \overline{e}_2^s < \acute{e}_2^l$ and the environmental assessment may not be consistent with the common perception in practice.[14] For illustration, we focus on the moderate contract cost case which is more representative of a typical palm oil mill because a mixture of contract and input spot procurement is used. In this case (as observed from panel b), the palm oil mill can justifiably claim that commercializing its PKS to substitute coal is environmentally beneficial only when biomass demand is smaller than a level that is associated with approximately 82% processing capacity utilization. Interestingly, when PKS is used for substituting LNG, which is a cleaner energy source, PKS commercialization becomes environmentally harmful regardless of the biomass demand. These results demonstrate that a typical mill may mistakenly think that commercializing its PKS is environmentally beneficial when it is not.

2.7 Conclusion

This chapter studies the environmental implications of biomass commercialization from the perspective of agricultural processing firms by incorporating several unique operational features of these firms. We characterize the carbon emission resulting from biomass commercialization and provide guidance on when processors can justifiably claim that commercializing their biomass is environmentally beneficial. We also provide insights on how biomass market characteristics affect this environmental assessment.

In our computational study throughout §2.6, we calibrated our model to represent

[14] In this case, processing is not always profitable on the spot day in the absence of PKS revenue because input spot procurement is used, and thus, PKS commercialization increases the processing volume.

a typical palm oil mill in Malaysia selling its palm kernel shell (biomass) to a bioenergy plant in Japan. Our framework can also be used to answer questions about the environmental impact of domestic usage of biomass by calibrating transportation emissions accordingly.

In this chapter we investigate the environmental impact of biomass commercialization from the perspective of processor. One of potential future research is to use our findings from the processor's analysis to build a larger eco-system to make higher level conclusions regarding the environmental impacts of biomass commercialization.

Chapter 3

Optimal Procurement from Multiple Contracts in Agricultural Processing

3.1 Introduction

In this paper we study the procurement decisions of a processor—in the context of agricultural industries—where the processor uses a primary commodity input to produce a commodity output. In particular, we develop a theoretical basis for understanding the tradeoffs faced by the processor in procurement management from multiple contracts with different characteristics. Our analysis is applicable to several agricultural industries, including the oilseed industry (e.g., palm, rapeseed, sunflower, soybean) and the grain industry. For example, in palm industry, palm fresh fruit bunches (a commodity input) sourced from nearby palm plantations are processed to produce crude palm oil (a commodity output). In the grain industry, grain mills (such as corn mill) procure from various farmlands to produce ethanol (a commodity output). In all these industries, a primary commodity input procured from multiple sources is processed into a commodity output.

Because both input and main output are agricultural commodities, their spot prices exhibit significant variabilities that are driven by factors like abnormal growing/harvesting conditions (e.g., weather, rainfall), strategic behavior of commodity speculators, etc. At the same time, the input and output spot prices are closely linked (see, for example, Boyabatlı et al. 2017 in the context of the palm industry).

To hedge the spot price risks, processors normally arrange long-term contractual agreements with suppliers for procurement on the top of spot sourcing (Mendelson and Tunca 2007). The most common contract form used in practice is the quantity flexibility contract (Kleindorfer and Wu 2003). A quantity flexibility contract specifies the capacity reserved in advance of the spot market facing a reservation cost. The actual delivery volume is decided within this reserved capacity on the day facing an exercise cost. The popularity of this contractual form originates from its ability to strikes a balance between cost efficiency (via early reservation commitment) and flexibility to manage risk (via the option to exercise or not). As commonly observed in practice in all commodity processing industries, in designing their procurement strategy agricultural processors consider multiple contracts with different characteristics—reservation and exercise costs in the context of quantity flexibility contracts. Because these processors face input and output spot markets, it is important for them to understand the impacts of spot price correlation in these markets when choosing the right procurement strategy.

In the operations management literature, there is a growing stream of papers that study procurement decisions in the presence of contract and spot markets. Barring a few exceptions (Wu and Kleindorfer 2005, Martínez-de-Albéniz and Simchi-Levi 2005, Fu et al. 2010, Anderson et al. 2017), there is no work in this literature that focuses on procurement from multiple contracts. The few papers that study procurement decisions in the presence of multiple contracts and spot market either do not consider output-related uncertainty or they consider demand uncertainty (in the absence of output spot market). This is the first paper that studies the optimal procurement decisions from multiple contracts in the presence of both input and output spot markets, which is a common feature of the aforementioned agricultural industries. Therefore, it is also an open question how the input and output spot price correlation shapes the optimal procurement strategy and profitability of the processors as well as the *value of contract procurement*—that is, the additional value of procuring from contracts on the top of sourcing from spot markets solely—in the context of agricultural industries. In this paper we attempt to fill the void.

Another common feature in agricultural processing is that there exist by-products generated along with the the main commodity output in the processing activities. For example, the palm fresh fruit bunches not only generate the main output crude palm oil, but also produce the by-product palm kernel; the corn produces the commodity output ethanol, at the same time, during the processing the corn hull is also generated that can be sold as animal feed. It is not clear how introducing a by-product would affect the contract procurement decisions and the value of contract procurement. In this paper we attempt to develop this knowledge base.

Toward this end, we propose a two-stage model that—in a stylized manner—captures the main trade-offs facing the processors for their procurement decisions in the context of agricultural industries. The firm (processor) procures a single commodity input, produces and sells a commodity output in a single period so as to maximize its expected profit. The input and output spot prices are stochastic whereas the demand for the commodity output is featured by a fixed-price fixed-volume sales contract. The input can be sourced from the input spot market and from quantity flexibility contracts that are characterized by a unit reservation and exercise cost. We consider two contracts, one with lower unit exercise cost and the other with lower unit reservation cost. The firm chooses the reservation volume from each contract under the input and output spot price uncertainties. After these uncertainties are realized, the firm decides the exercise volume from each contract (within the reserved capacity) and the volume to source from input spot market which collectively determine the processing volume and in turn, the production quantity of commodity output. The firm also decides the output volumes to sell to or source from the output spot market, which in turn determines the amount of output demand satisfied. To study the impact of introducing a by-product, we consider a case where a by-product are produced along with the commodity output in fixed proportions, and the by-product demand is featured by a similar sales contract. We characterize the optimal contract procurement decisions (and the subsequent sourcing, processing and selling decisions) and answer the following research questions.

(1) How does spot price correlation affect the firm's contract procurement decisions and profitability as well as the value of contract procurement?

(2) How does introducing a by-product affect the the firm's contract procurement decisions and the value of contract procurement?

(3) How significant is the value of contract procurement? What is the additional value of procuring from multiple contracts instead of only one of them?

In answering these questions when analytical results are not attainable we conduct numerical experiments using realistic instances. To this end, we calibrate our model to represent a typical palm oil mill in Malaysia. The model calibration is based on the publicly available data from the Malaysian Palm Oil Board as complemented by the data obtained from the literature. Our main findings can be summarized as follows.

(1) We characterize the optimal procurement decisions from two contracts and identify three strategies that emerge as a part of the optimal decisions: single sourcing from contract with lower unit exercise cost, single sourcing from contract with lower unit reservation cost, and dual sourcing. Using characterizations of several thresholds

that depend on contract parameters as well as spot price uncertainty, we provide specific conditions under which each strategy is optimal.

(2) We conduct sensitivity analysis, both analytically (with respect to the optimal expected profit without contracts and the value of contract procurement, separately) and numerically (with respect to the optimal expected profit with contracts), to investigate the effects of spot price correlation on the optimal procurement decision and profitability as well as on the value of contract procurement.

Effects on Contract Procurement Decisions. The general insight from our analytical analysis is that as a response to an increase in spot price correlation, the processor should increase the total reservation volume from both contracts as well as the reservation volume from the contract with lower unit exercise cost. Based on our model calibration, we find that the volume from the contract with lower unit reservation cost can be increased or decreased in spot price correlation. In practice, the contract with lower unit exercise cost (or higher unit reservation cost) features a nearby supplier due to its lower exercise cost that mainly consists of transportation costs and labor costs (or a less flexible supplier due to its higher reservation cost commitment). Our results indicate that a processor tends to procure more from the nearby supplier or a less flexible supplier when the spot price correlation is high.

Effects on Value of Contract Procurement and Profitability. The general insights from our analytical analysis are that the optimal expected profit without contracts decreases in spot price correlation while the value of contract procurement—the difference between the optimal expected profit with and without contracts—increases in spot price correlation. Our result showcases that the contracts provide a hedge against increasing spot price correlation. Based on our model calibration we find that the optimal expected profit with contracts also increases in the spot price correlation, which complements the literature (e.g., Plambeck and Taylor 2013 and Boyabatlı et al. 2017) that high correlation is always harmful in the absence of contract procurement.

(3) We analytically characterize the impacts of introducing a by-product on the firm's contract procurement decisions and the value of contract procurement. The general insights from our analysis are that introducing a by-product would incent the firm to implement the dual contract procurement strategy and to increase the reservation volume from the contract with lower unit exercise cost. Moreover, we find that introducing a by-product makes contract procurement more valuable.

(4) Using our model calibration we find that a processor sources only from spot market incurs a considerable profit loss—an average profit loss of 48.48% with a maximum loss of 73.29% in the numerical instances considered—in comparison with a firm using the optimal procurement strategy. Furthermore, when a processor sourcing from only one of the contracts (on top of spot sourcing), it incurs an average profit loss of 1.09% with a maximum loss of 4.00%. These results collectively indicate that (multiple) contracts in agricultural processing industries have considerable economic value especially given the fact that agricultural processing normally generates razor-thin margins.

The remainder of this paper is organized as follows. §3.2 surveys the related literature and discusses the contribution of our work. §3.3 describes the general model and the basis for our assumptions. §3.4 derives the optimal contract procurement decisions (and the subsequent sourcing, processing and selling decisions). §3.5 investigates the effects of spot price correlation on the optimal procurement decisions and the profitability of the processor as well as on the value of contract procurement. §3.6 investigates the effects of introducing a by-product on the optimal procurement decisions and the value of contract procurement. §3.7 extends the analysis numerically using a model calibration that represents a typical palm oil processor in Malaysia to investigate the impacts of spot price correlation and also to estimate the value of using contracts, including the value of contract procurement and the value of using multiple contracts. §3.8 concludes with a discussion of the future research directions.

3.2 Literature Review

In the operations management literature, there is a vast stream of papers that studies the optimal procurement from multiple contracts for non-commodity products (which do not consider input or output spot price uncertainties or access to these spot markets), see Dada et al. (2007) and Gheibi et al. (2016) for a review of papers in this stream. This stream of papers considers procurement contracts with different characteristics, including reliability (e.g., Tomlin and Wang 2005, Tomlin 2006 and Wang et al. 2010), default risk (Babich et al. 2007), flexibility (Serel et al. 2001, Martínez-de-Albéniz and Simchi-Levi 2009) and lead time (Wu and Zhang 2014, Calvo and Martínez-de-Albéniz 2015). There is a growing stream of papers in the literature on commodity processing that studies optimal procurement decisions for commodity products considering the access to spot markets. We refer the readers to Kleindorfer and Wu (2003) and Goel and Tanrisever (2017) for comprehensive reviews of this literature. The majority of the papers in this stream focus on procurement from a single contract. They examine various operational issues related to contracts (including contract design, contract selection and procurement) in the presence of input spot market and showcase the characteristics of different contract types including risk-sharing contract (Boyabatlı et al. 2011, Chen et al. 2013 and Kouvelis et al. 2017), quantity flexibility contract (Wu et al. 2002, Boyabatlı 2015 and Pei et al. 2011), quantity commitment contract (Dong and Liu 2007, Seifert et al. 2004, Kouvelis et al. 2013 and Turcic et al. 2015).

Only a few papers consider procurement from multiple contracts in the presence of (input) spot market. These papers either do not consider output-related uncertainty (Wu and Kleindorfer 2005) or they consider demand uncertainty (Martínez-de-Albéniz and Simchi-Levi 2005, Fu et al. 2010 and Anderson et al. 2017). Moreover, these few papers all consider single-output firms whereas processing in agricultural industries gives rise to multiple outputs in fixed proportions. Our paper is the first paper that studies optimal procurement from multiple contracts in the presence of both input and output price uncertainties and it is also the first paper to analyze this problem in a multiple-output setting.

There is also another stream of papers in the commodity processing literature that consider input and output spot price uncertainties and study other operational decisions in the absence of contract procurement. For example, Plambeck and Taylor (2013) study the investment decision of input and capacity efficiency improvements in clean-tech manufacturing where both the raw material cost and product price are uncertain. Dong et al. (2014) study the optimal spot procurement and refining decisions in a petroleum refinery and investigate the value of two operational flexibilities: conversion flexibility (the ability to convert low-quality intermediates into high-quality ones) and range flexibility (the ability to accommodate inputs with different quality levels). Boyabatlı et al. (2017) investigate the optimal processing/storage capacity investment decisions for a processor in the context of oilseeds industries. Goel and Tanrisever (2017) study the optimal sales contract choice (spot/forward/index-based) in biofuel industry where both input (e.g., corn) and output (ethanol) are commodities. Devalkar et al. (2011) and Devalkar et al. (2017) study the value of integrating processing and trading decisions in the context of soybean processing. One of the findings from this stream of papers is that a higher input and output spot price correlation decreases the profitability of the processor in the absence of contracts. We show that this result continues to hold in the presence of contracts.

Besides aforementioned papers studying strategic (e.g., capacity investment) and operating (e.g., procurement and production planning) decisions in agricultural commodity processing, our paper is also related to the growing operations management literature that examines different operational decisions of supply chain agents in the agricultural industries. For example, Jones et al. (2001) study the production scheduling problem in the seed breeding industry where a corn breeder decides the production quantities in two production seasons in the presence of yield and demand uncertainty. Noparumpa et al. (2015) examine optimal pricing and advance selling of wine futures in wine industry. de Zegher et al. (2017) investigate the contract design and channel selection decisions in the wool supply chain. Alizamir et al. (2019) study two farm subsidies: Price Loss Coverage program (triggered when the crop's market price falls below a reference price) and Agricultural Risk Coverage program (being effective when the farmer's revenue is below a threshold), and investigate their impacts on the consumers, farmers and the government. Hu et al. (2019) provide a rationale for the cyclical price pattern in agricultural industries and analyze the implications of combating strategies for farmers, social entrepreneurs, and for-profit firms. Boyabatlı et al. (2019) study a dynamic crop planing problem in which a farmer decides how to allocate the available farmland between two crops (e.g., soybean and corn) in each growing season in the presence of rotation benefits. Different from these papers, we study the optimal procurement from multiple contracts in agricultural processing by capturing important operational characteristics in these industries such as input and output spot price uncertainties, and investigate the impacts of spot price correlation on the firm's procurement strategy, profitability and the value of contract procurement.

3.3 Model Description and Assumptions

We use the following notation and conventions throughout this paper. A realization of the random variable $\tilde{y}$ is denoted by y. $\mathbb{E}$ and $\chi(\cdot)$ denote the expectation operator and indicator function, respectively. We use $(u)^+ = \max(u, 0)$. The monotonic relations (increasing, decreasing) are used in the weak sense unless otherwise stated.

We consider a firm that procures and processes a commodity input to produce a commodity output that seeks to maximize its expected profit in one selling season. We model the firm's decisions as a two-stage stochastic program: The firm makes its contract procurement decision from multiple supply contracts under spot price uncertainty (stage 1) and determines operational decisions including sourcing, processing, and selling after the resolution of this uncertainty (stage 2).

Denote $\tilde{S}_0$ and $\tilde{S}_1$ as the uncertain input and output spot price, respectively. We assume that $(\tilde{S}_0, \tilde{S}_1)$ follow a bivariate distribution that has bounded expectation (μ_0, μ_1) with covariance matrix $\mathbf{\Sigma}$, where $\mathbf{\Sigma}_{00} = \sigma_0^2$, $\mathbf{\Sigma}_{11} = \sigma_1^2$ and $\mathbf{\Sigma}_{01} = \mathbf{\Sigma}_{10} = \rho\sigma_0\sigma_1$ and ρ denotes the correlation coefficient. Moreover, we assume $(\tilde{S}_0, \tilde{S}_1)$ to follow a bivariate Normal distribution in comparative statics analysis (§3.5) and numerical study (§3.7).

The firm has two sources for input procurement, contract and spot market. We assume that the firm procures from two quantity flexibility contracts; contract $i \in \{1, 2\}$ is characterized by a unit reservation cost β^i and a unit exercise cost A^i. The contracts are indexed such that $A^1 \leq A^2$ where $\Delta_A \doteq A^2 - A^1$ is the additional exercise cost of contract 2. To avoid uninteresting scenarios, we assume $\beta^1 \geq \beta^2$ and let $\Delta_\beta \doteq \beta^1 - \beta^2$ denote the additional reservation cost of contract 1. Let Q^i denote the volume of input reserved from contract $i \in \{1, 2\}$ by the firm in advance of the spot market by incurring the unit reservation cost β^i. On the day, the firm decides the input volume to be delivered from contract i within the reserved capacity by incurring the unit exercise cost A^i. At the same time, the firm also can source from input spot market at the prevailing input spot price S_0 to process.

Let z_0 denote the processing volume and K_0 denote the processing capacity. We consider a unit processing cost c_0 for inputs from both contract and spot markets. We assume each unit of the processed input yields $a_1 \leq 1$ units of commodity output.

We consider two channels for the commodity output: output spot market and demand that is characterized by a fixed-price fixed-volume contract. In particular, the firm can sell the commodity output to the demand D_1 at price p_1 and to spot market at a unit sales price $S_1(1 - \omega_1)$, where $0 \leq \omega_1 \leq 1$ accounts for a discount or transaction cost due to logistics costs and commissions in exchange markets. Besides spot sales, we allow the firm to procure the commodity output from spot market at the prevailing spot price S_1 to satisfy commodity demand. We also normalize the penalty costs of unmet demands for commodity output to zero for brevity; however, it can be easily incorporated into our model without affecting the results.

Throughout this paper, we assume $K_0 \geq \frac{D_1}{a_1}$ to rule out uninteresting scenarios. It implies that the processing capacity is sufficient to process enough inputs to fulfil the demand of commodity output.

3.4 The Optimal Strategy

In this section, we describe the optimal solutions for the firm's contract procurement, sourcing, processing and selling decisions, and derive out the optimal expected profit. We solve the firm's decisions via backward induction. All the proofs are relegated to the appendix.

3.4.1 Stage 2: Sourcing, Processing, and Selling Decisions

In this stage, the input and output spot prices are realized. The firm decides on the input processing volume z_0, how to source from input spot market and from reserved contract volumes Q^i, $i \in \{1, 2\}$, the amount of demands to satisfy for commodity output and by-product, the volume of commodity output to source from the output spot market to satisfy demand and volume of commodity output to sell to the spot market. We observe that all these decisions can be formulated as functions of one single decision variable, the input processing volume $z_0 \in [0, K_0]$.

Sourcing Decisions

Let $\Psi(z_0)$ denote the optimal sourcing cost given input processing volume z_0 and contract volumes Q^1 and Q^2. As contract 1 has a lower unit exercise cost, the firm exercises contract 1 before contract 2 when both contracts have reserved capacity.

By discussing the magnitude of input processing volume z_0 and reserved contract volumes Q^1 and $Q^1 + Q^2$, we have the optimal sourcing cost:

$$\Psi(z_0) \doteq -\min\left(Q^1, z_0\right)\min\left(S_0, A^1\right) - \min\left(Q^2, \left(z_0 - Q^1\right)^+\right)\min\left(S_0, A^2\right) - \left(z_0 - Q^1 - Q^2\right)^+ S_0. \tag{3.1}$$

In (3.1), the first term represents the sourcing cost for the first $\min(Q^1, z_0)$ units of input, the second denotes for the next $\min(Q^2, (z_0 - Q^1)^+)$ units and the last is for the remaining $(z_0 - Q^1 - Q^2)^+$ units. Recall that contract 1 is exercised before contract 2. For $0 \leq z_0 \leq Q^1$, the unit sourcing cost is determined by the cheapest source, either from contract 1 at a unit exercise cost A^1 or from input spot market at spot price S_0. Thus, the unit input sourcing cost is given by $\min(S_0, A^1)$. For the processing volume that is beyond Q^1; that is, $(z_0 - Q^1)^+$, because the reserved volume from contract 1 is used up for the first Q^1 units of processing input, the firm can buy the input from input spot market and contract 2. Similarly, the unit sourcing cost is $\min(S_0, A^2)$. For the processing volume exceeding $Q^1 + Q^2$; that is, $(z_0 - Q^1 - Q^2)^+$, the firm sources only from input spot market at spot price S_0.

Selling Decisions

Let $\Lambda(z_0)$ denote the firm's optimal selling profit given input processing volume z_0. After processing, the available volume for commodity output to satisfy demands are $a_1 z_0$. The optimal selling profit $\Lambda(z_0)$ is given by:

$$\begin{aligned}\Lambda(z_0) \doteq\ & D_1\left(p_1 - S_1\right)^+ + \min\left(a_1 z_0, D_1\right)\min\left(\max\left(p_1, S_1\left(1-\omega_1\right)\right), S_1\right) \\ & + \left(a_1 z_0 - D_1\right)^+ S_1\left(1 - \omega_1\right).\end{aligned} \tag{3.2}$$

In (3.2), the first term is the secured profit from demand of commodity output due to the presence of output spot market; the second and third term are the additional revenue from the commodity output that is generated from the firm's own processing activities. In particular, when it is not profitable to satisfy the demand by output spot sourcing (i.e., $S_1 \geq p_1$), for the first $\min(a_1 z_0, D_1)$ units of commodity output from processing, the unit processing revenue is the larger revenue from demand sale and output spot sale; that is, $\max(p_1, S_1(1-\omega_1))$; when it is profitable (i.e., $S_1 < p_1$), the unit revenue is the opportunity gain of not sourcing from output spot market; that is, S_1. Taking these two cases together yields the unit revenue $\min(\max(p_1, S_1(1-\omega_1)), S_1)$. For commodity output volume that exceeds the demand; that is, $(a_1 z_0 - D_1)^+$, the firm generates revenue only from output spot sale and the unit revenue is $S_1(1-\omega_1)$.

Processing Decisions

Let $\Pi(z_0)$ denote the profit for a given z_0. The stage 2 objective function $\Pi(z_0)$ is characterized over the input processing decision z_0 and it is given by

$$\Pi(z_0) \doteq \Psi(z_0) + \Lambda(z_0) - c_0 z_0. \tag{3.3}$$

Let

$$\begin{aligned} h_1(S_1) &\doteq a_1 \min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0, \\ h_0(S_1) &\doteq a_1 S_1(1-\omega_1) - c_0, \end{aligned} \tag{3.4}$$

denote the unit processing margins depending on the demand situations—that is, whether there is any unsatisfied demand for commodity output. For each unit of processed input (with unit processing cost c_0), the firm collects sales revenue of $a_1 \min(\max(p_1, S_1(1-\omega_1)), S_1)$ (as we explained in (3.2)) from the commodity output if there is unsatisfied demand for commodity output; otherwise, it collects $a_1 S_1(1-\omega_1)$ only from spot sales. It is easy to establish that these margins are increasing in S_1 and $h_1(S_1) \geq h_0(S_1)$ for all S_1.

Proposition 1 characterizes the optimal processing volume z_0^* for given contract volumes (Q^1, Q^2) and spot price realizations (S_0, S_1).

Proposition 10 *Given contract volumes* (Q^1, Q^2) *and spot price realizations* (S_0, S_1)*,* *the optimal processing volume z_0^* is characterized by*

$$z_0^* = \begin{cases} 0 & \text{if } h_1(S_1) \le P^1(S_0) \\ \min\left(\frac{D_1}{a_1}, Q^1\right) & \text{if } h_0(S_1) \le P^1(S_0) \le h_1(S_1) \le P^2(S_0) \\ Q^1 & \text{if } P^1(S_0) \le h_0(S_1) \le h_1(S_1) \le P^2(S_0) \\ \min\left(\frac{D_1}{a_1}, Q^1+Q^2\right) & \text{if } h_0(S_1) \le P^1(S_0) \le P^2(S_0) \le h_1(S_1) \le S_0 \\ \max\left(\min\left(\frac{D_1}{a_1}, Q^1+Q^2\right), Q^1\right) & \text{if } P^1(S_0) \le h_0(S_1) \le P^2(S_0) \le h_1(S_1) \le S_0 \\ \max\left(\frac{D_1}{a_1}, Q^1\right) & \text{if } P^1(S_0) \le h_0(S_1) \le P^2(S_0) \le S_0 \le h_1(S_1) \\ \frac{D_1}{a_1} & \text{if } h_0(S_1) \le P^1(S_0) \le P^2(S_0) \le S_0 \le h_1(S_1) \\ Q^1+Q^2 & \text{if } P^2(S_0) \le h_0(S_1) \le S_0 \le h_1(S_1) \\ K_0 & \text{if } S_0 \le h_0(S_1) \end{cases} \tag{3.5}$$

where $P^1(S_0) \doteq \min(S_0, A^1)$ and $P^2(S_0) \doteq \min(S_0, A^2)$ are the unit sourcing costs; $h_1(S_1)$ and $h_0(S_1)$ are the unit processing margins as given in (3.4).

The stage 2 objective function $\Pi(z_0)$ is piecewise linear and concave in z_0. Therefore, the optimal solution occurs at the breakpoints $\left\{0, \frac{D_1}{a_1}, Q^1, Q^1+Q^2, K_0\right\}$. The optimal processing volume z_0^* is determined by comparing the unit sourcing costs with the relevant unit processing margins as defined in (3.4). For example, if $h_0(S_1) \le P^1(S_0) \le h_1(S_1) \le P^2(S_0)$, then it is profitable for the firm to process only when the commodity output has unmet demands (corresponding to processing margin $h_1(S_1)$), and thus the firm optimally processes at most $\frac{D_1}{a_1}$ units of input. Moreover, the firm is not profitable to exercise contract 2 and source from the input spot market (corresponding to sourcing cost $P^2(S_0)$). Therefore, the optimal processing volume would not exceed the reserved contract volume Q^1 and $z_0^* = \min\left(\frac{D_1}{a_1}, Q^1\right)$.

3.4.2 Stage 1: Contract Procurement Decisions

In stage 1, the firm chooses the optimal contract volumes (Q^{1*}, Q^{2*}) with respect to uncertain spot prices so as to maximize the expected profit $\mathbb{E}\left[\pi(Q^1, Q^2; \tilde{S}_0, \tilde{S}_1)\right] - \beta^1 Q^1 - \beta^2 Q^2$, where $\pi(Q^1, Q^2; \tilde{S}_0, \tilde{S}_1) \doteq \Pi(z_0^*)$. Denote

$$\mathcal{M}_\kappa^i \doteq \mathbb{E}\left[\max\left(\min\left(\tilde{S}_0, \max\left(h_\kappa(\tilde{S}_1), A^i\right)\right), A^i\right)\right] - \beta^i - A^i, \tag{3.6}$$

for $\kappa \in \{0, 1\}$. We interpret them as the expected marginal profits of contract $i \in \{1, 2\}$ under different demand situations. For example, when the commodity output has unsatisfied demand, the marginal profit of contract i is $\mathcal{M}_1^i$. Suppose that the firm has reserved and exercised one unit of contract i in stage 2, the total contract cost would be $\beta^i + A^i$. When $S_0 \leq A^i$, it is optimal for the firm to source from input spot market at sourcing cost S_0 which would save the exercise cost A^i; when $A^i < S_0 \leq h_1(S_1)$, it is optimal to exercise the contract. However, since input spot sourcing is also profitable, the firm only earns an opportunity gain S_0 from the contract. When $S_0 > h_1(S_1)$, input spot sourcing is not a profitable option and the firm earns $h_1(S_1)$ from the contract if $h_1(S_1) > A^i$; otherwise, the firm optimally does not exercise the contract and saves exercise cost A^i.

Because $h_1(S_1) \geq h_0(S_1)$ for all S_1, it is straightforward that $\mathcal{M}_1^i \geq \mathcal{M}_0^i$ for $i \in \{1, 2\}$. Proposition 17 characterizes the optimal contract volumes (Q^{1*}, Q^{2*}) based on $\mathcal{M}_\kappa^i$ for $i \in \{1, 2\}$ and $\kappa \in \{0, 1\}$.

Proposition 11 *The optimal contract volumes are given by*

$$\left(Q^{Y*}, Q^{(3-Y)*}\right) = \begin{cases} (0,0) & \text{if } \mathcal{M}_1^Y \leq 0 \\ \left(\frac{D_1}{a_1}, 0\right) & \text{if } \mathcal{M}_1^Y > 0, \mathcal{M}_0^Z \leq 0 \text{ and } Y = 1 \\ (K_0, 0) & \text{if } \mathcal{M}_1^Y > 0, \mathcal{M}_0^Z > 0 \text{ and } Y = Z = 1 \\ \left(0, \frac{D_1}{a_1}\right) & \text{if } \mathcal{M}_1^Y > 0, \mathcal{M}_0^Z \leq 0 \text{ and } Y = 2 \\ (0, K_0) & \text{if } \mathcal{M}_1^Y > 0, \mathcal{M}_0^Z > 0 \text{ and } Y = Z = 2 \\ \left(\frac{D_1}{a_1}, K_0 - \frac{D_1}{a_1}\right) & \text{if } \mathcal{M}_1^Y > 0, \mathcal{M}_0^Z > 0 \text{ and } Y = 1, Z = 2 \end{cases}$$

where Y and Z denote the index of contract with the higher value of $\mathcal{M}_1^i$ and $\mathcal{M}_0^i$ for $i \in \{1, 2\}$, respectively.

The optimal contract volume Q^{i*} is determined by checking whether the marginal profit of contract $i \in \{1, 2\}$ is positive and whether it is higher than the other contract. Let $Q \doteq Q^1 + Q^2$. The stage 1 objective function is piecewise linear and concave in Q. The expected marginal profits for $Q \in \left[0, \frac{D_1}{a_1}\right)$ and $Q \in \left[\frac{D_1}{a_1}, \infty\right)$ are $\mathcal{M}_1^Y$ and $\mathcal{M}_0^Z$, respectively. If $\mathcal{M}_1^Y > 0$, then the firm optimally reserves from contract Y at least $\frac{D_1}{a_1}$ units; otherwise, it does not reserve; if $\mathcal{M}_0^Z > 0$, then the firm reserves another $\left(K_0 - \frac{D_1}{a_1}\right)$ units of input (capped by processing capacity K_0) from contract Z.

We characterize the optimal contract volumes (Q^{1*}, Q^{2*}) in next proposition within (Δ_A, Δ_β) space for given A^1 and β^2. Before that we define two sets of thresholds (preference and profitability thresholds) to simplify our exposition. For the space consideration, we relegate the expressions of the thresholds in the proof of Proposition 12.

- **Preference thresholds**: Let $\Delta_\beta^L(\Delta_A)$ and $\Delta_\beta^M(\Delta_A)$ denote the unique solutions of $\mathcal{M}_0^1 = \mathcal{M}_0^2$ and $\mathcal{M}_1^1 = \mathcal{M}_1^2$ for a given Δ_A, respectively. These thresholds are the additional reservation costs of contract 1 that make the firm indifferent between contract 1 and 2 when there is no unsatisfied and unsatisfied demand for the commodity output, respectively.

- **Profitability thresholds**: Let $\Delta_A^{(i)}$, $i \in \{1,2\}$ denote the unique solutions of $\mathcal{M}_0^2 = 0$ and $\mathcal{M}_1^2 = 0$, respectively; let $\Delta_\beta^{(i)}$, $i \in \{1,2\}$ denote the unique solutions of $\mathcal{M}_0^1 = 0$ and $\mathcal{M}_1^1 = 0$, respectively. Thresholds $\Delta_A^{(i)}$, $i \in \{1,2\}$ is the largest additional exercise cost of contract 2 that makes contract 2 profitable to be reserved when there is no unsatisfied and unsatisfied demand for the commodity output, respectively. Thresholds $\Delta_\beta^{(i)}$, $i \in \{1,2\}$ determines the profitability of contract 1 accordingly.[1]

Proposition 12 *The optimal contract volumes (Q^{1*}, Q^{2*}) for given A^1 and β^2 are characterized in Table 3.1:*

Region	**Strategy**	**Condition**	(Q^{1*}, Q^{2*})
Γ_1	Single	$0 \le \Delta_\beta < \min\left(\Delta_\beta^L(\Delta_A), \Delta_\beta^{(1)}\right)$	$(K_0, 0)$
Γ_2	Dual	$0 \le \Delta_A < \Delta_A^{(1)}, \Delta_\beta^L(\Delta_A) \le \Delta_\beta < \Delta_\beta^M(\Delta_A)$	$\left(\frac{D_1}{a_1}, K_0 - \frac{D_1}{a_1}\right)$
Γ_3	Single	$0 \le \Delta_A < \Delta_A^{(1)}, \Delta_\beta \ge \Delta_\beta^M(\Delta_A)$	$(0, K_0)$
Γ_4	Single	$\Delta_A \ge \Delta_A^{(1)}$, $\Delta_\beta^{(1)} \le \Delta_\beta < \min\left(\Delta_\beta^M(\Delta_A), \Delta_\beta^{(2)}\right)$	$\left(\frac{D_1}{a_1}, 0\right)$
Γ_5	Single	$\Delta_A^{(1)} \le \Delta_A < \Delta_A^{(2)}, \Delta_\beta \ge \Delta_\beta^M(\Delta_A)$	$\left(0, \frac{D_1}{a_1}\right)$
Γ_6	N.A.	$\Delta_A \ge \Delta_A^{(2)}, \Delta_\beta \ge \Delta_\beta^{(2)}$	$(0,0)$

Table 3.1: Optimal Contract Volumes (Q^{1*}, Q^{2*}) on (Δ_A, Δ_β) Space. "Single" represents single contract procurement is optimal; "Dual" represents dual contract procurement is optimal; "N.A." represents contract procurement is not optimal.

We provide a graphical interpretation for Proposition 12 in Figure 3.1. When contract 2's additional exercise cost is small $\left(\Delta_A \le \Delta_A^{(1)}\right)$, the firm optimally reserves up to the processing capacity (i.e., K_0) in total. How this aggregate amount is allocated to each contract critically depends on the additional reservation cost of contract 1, Δ_β. If Δ_β is small (region Γ_1), then the firm reserves only from contract 1; if Δ_β is

[1] We prove that $\Delta_\beta^L(\Delta_A) \le \Delta_\beta^M(\Delta_A)$ for $\Delta_A \ge 0$; $\Delta_\beta^\ell(\Delta_A)$ is concave and increasing in Δ_A for $\ell \in \{L, M\}$; $\Delta_A^{(1)} \le \Delta_A^{(2)}$; $\Delta_\beta^{(1)} \le \Delta_\beta^{(2)}$; $\Delta_\beta^M(\Delta_A^{(2)}) = \Delta_\beta^{(2)}$ and $\Delta_\beta^L(\Delta_A^{(1)}) = \Delta_\beta^{(1)}$.

moderate, then the firm reserves $\frac{D_1}{a_1}$ from contract 1 and $\left(K_0 - \frac{D_1}{a_1}\right)$ from contract 2 (region Γ_2); if Δ_β is large, then the firm reserves only from contract 2 (region Γ_4).

When contract 2's additional exercise cost is moderate $\left(\Delta_A^{(1)} \leq \Delta_A < \Delta_A^{(2)}\right)$, the firm still reserves to the processing capacity (from contract 1) if contract 1's additional reservation cost Δ_β is small (region Γ_1); otherwise, the firm optimally chooses to reserve to cover the demand of commodity output (i.e., $\frac{D_1}{a_1}$). If Δ_β is moderate, then the firm reserves only from contract 1 (region Γ_4); if Δ_β is large, then the firm reserves only from contract 2 (region Γ_5).

When contract 2's additional exercise cost is large $\left(\Delta_A \geq \Delta_A^{(2)}\right)$, the firm may still reserve to processing capacity (from contract 1) if contract 1's additional reservation cost Δ_β is small (region Γ_1) and reserve to cover the demand of commodity output if contract 1's additional reservation cost Δ_β is moderate (region Γ_4). As contract 1 gets more expensive to reserve (region Γ_6), contract procurement is no longer optimal, and thus the firm solely relies on input spot sourcing.

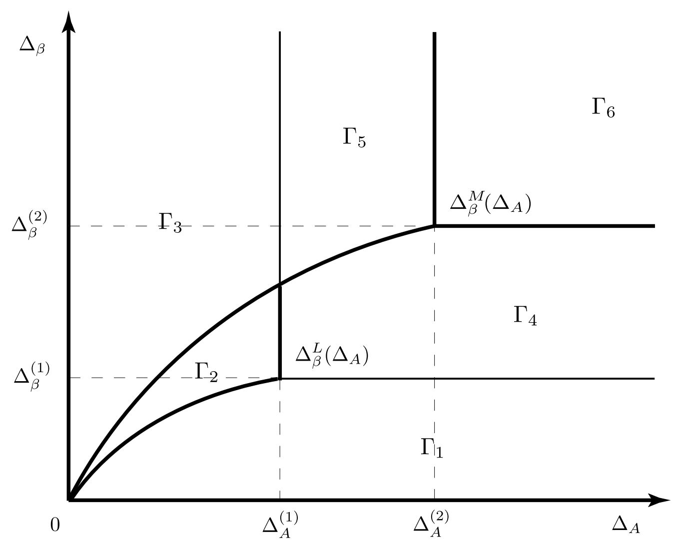


Figure 3.1: Optimal Contract Volumes (Q^{1*}, Q^{2*}) on (Δ_A, Δ_β) Space.

Let $V(Q^1, Q^2)$ denote the stage 1 objective function and V^* denote the optimal expected profit. After substituting optimal contract volumes into stage 1 profit function, we obtain the processor's optimal expected profit in the following proposition.

Proposition 13 *The processor's optimal profit is*

$$V^* = V(0,0) + \left(\mathcal{M}_1^Y\right)^+ \frac{D_1}{a_1} + \left(\mathcal{M}_0^Z\right)^+ \left(K_0 - \frac{D_1}{a_1}\right), \tag{3.7}$$

where

$$V(0,0) \doteq \mathbb{E}\left[\left(p_1 - \tilde{S}_1\right)^+\right] D_1 + \mathbb{E}\left[\left(h_1(\tilde{S}_1) - \tilde{S}_0\right)^+\right] \frac{D_1}{a_1} + \mathbb{E}\left[\left(h_0(\tilde{S}_1) - \tilde{S}_0\right)^+\right] \left(K_0 - \frac{D_1}{a_1}\right), \tag{3.8}$$

where $\mathcal{M}_\kappa^i, \kappa \in \{0,1\}$ are the expected marginal profits of contract $i \in \{1,2\}$ as given in (3.6); Y and Z are the indexes as given in Proposition 11.

The expression $V(0,0)$ characterizes the optimal expected profit in the absence of contract procurement. In (3.8), the first term is the expected profit from output spot sourcing; the second and third terms denote the expected profits from input spot sourcing when there is unsatisfied and no unsatisfied demand for the commodity output, respectively.

In (3.7), the last two terms capture the expected profits from contract procurement on top of input spot sourcing. Each multiplier in the front of contract volumes has meanings regarding the contract preference and profitability. Take $\left(\mathcal{M}_1^Y\right)^+$ for an example. It means that when the commodity product has unmet demand, contract Y is preferred over the other. Moreover, if its expected marginal profit $\mathcal{M}_1^Y$ is positive, then the firm collects the marginal profit for each unit of $\frac{D_1}{a_1}$; otherwise, the firm does not reserve these $\frac{D_1}{a_1}$ units. Note that as the expected marginal profit for contracts decreases in Q (due to $\mathcal{M}_1^Y \geq \mathcal{M}_0^Z$), the firm does not reserve contract at all if the firm optimally does not reserve the first $\frac{D_1}{a_1}$ units.

3.5 The Impacts of Spot Price Correlation

In this section, we conduct sensitivity analysis to study the impacts of spot price correlation on the firm's optimal procurement strategy and expected profit as well as the value of contract procurement. In particular, we define $\mathbb{VC} \doteq V^* - V(0,0)$ as the *value of contract procurement*. To facilitate our discussion, we let $\Gamma_{-\mathcal{S}}$ denote the regions except region set $\mathcal{S}$. For instance, $\Gamma_{-\{4,5\}}$ represents the regions except Γ_4 and Γ_5. Recall from Proposition 11 and Figure 3.1 that the firm optimally relies on contract procurement (integrating with input spot sourcing) in all regions except region Γ_6, thus we denote Γ_{-6} as the *contract procurement regions* hereafter.

Lemma 1 characterizes the impacts of spot price correlation on the preference and profitability thresholds.

Lemma 1 *(i)* $\frac{\partial \Delta_\beta^\ell(\Delta_A)}{\partial \rho} \geq 0$ *for* $\ell \in \{L, M\}$ *and (ii)* $\frac{\partial \Delta_A^{(i)}}{\partial \rho} \geq 0$*,* $\frac{\partial \Delta_\beta^{(i)}}{\partial \rho} \geq 0$ *for* $i \in \{1, 2\}$.

Part (i) indicates that contract 1 gets more preferable as spot price correlation increases. Part (ii) shows that a higher spot price correlation favors contract procurement since the contract procurement regions Γ_{-6} expands as spot price correlation ρ increases.

Next, we examine the impacts of spot price correlation on optimal contract volumes.

Proposition 14 *(i)* $\frac{\partial (Q^{1*}+Q^{2*})}{\partial \rho} \geq 0$ *and (ii)* $\frac{\partial Q^{1*}}{\partial \rho} \geq 0$.

If the correlation between input and output spot prices are high, then there will be a higher likelihood that the input spot price is high (low) when the output spot price will be high (low). When the input spot price gets higher, it is more profitable for the firm to switch to contract procurement because input spot sourcing gets more expensive and the risk of not exercising reserved contract volumes is lower with a higher output spot price. Consequently, the total contract volume increases.

Part (ii) of Proposition 14 shows that the reserved volume from contract 1 also increases in the spot price correlation. Although both contracts are exposed to the risk of not being exercised when input spot price is high, the probabilities are different because they have distinct exercise costs. When spot prices get increasingly correlated, more high output spot prices implies more high input spot prices. In this case, the risk of not exercising contract 1 (with the lower unit exercise cost) increases slower than that for contract 2 because of the lower exercise cost of contract 1. In other words, contract 1 earns an comparative advantage over contract 2 as spot price correlation increases. Therefore, the firm reserves more from contract 1 when spot price correlation is getting larger.

For the rest of this section, we focus on the contract procurement regions Γ_{-6} to investigate the impacts of spot price correlation on firm's optimal expected profit and the value of contract procurement. Since the preference and profitability thresholds are dependent on spot price correlation, the procurement decisions might change along with the spot price correlation. To avoid further complexity, we focus on local sensitivity analysis where small changes in correlation do not change the optimal procurement strategy, thus the optimal expected profit and the value of contract procurement as well.

Proposition 15 *(i) Assume* $\omega_1 = 0$. $\frac{\partial V(0,0)}{\partial \rho} < 0$ *and (ii)* $\frac{\partial \mathbb{VC}}{\partial \rho} \geq 0$.

Part (i) of Proposition 15 showcases that without contracts, the firm suffers from synchronization between the input and output spot prices. With a higher correlation, there will be a higher likelihood when the input spot price is high (low) that the output spot price will be high (low), thus the processing margin is thin. This result is consistent with those in Plambeck and Taylor (2013) and Boyabatlı et al. (2017) showing that the optimal expected profit without contracts always decreases in spot price correlation.

Part (ii) of Proposition 15 shows that the firm benefits more from using contracts for input procurement when the correlation between the input and output spot prices is higher. The intuition here is as follows: When the input spot price is low, the firm relies less on contract procurement, but does not lose much from not using the contracts because output spot price is also low. When the input spot price is high, the firm exercises more from contracts, hence gains from using contracts as the output spot price is also high. The hedging benefit of contracts against increasing input spot price variability has been already established in the literature (e.g., Boyabatlı et al. 2011). This result showcases another benefit of contracts—it can be used as a hedge against increasing input and output spot price correlation.

3.6 Contract Procurement in the Presence of By-Product

In previous sections, we consider a simple operational framework where the primary input only produces one single output (commodity output). In this section, we examine a typical operational framework in the agricultural processing industries where the firm sells one main output (commodity output) and a by-product that are generated from one single process in fixed proportions. In particular, we model that there is $a_2 \leq 1 - a_1$ units of by-product generated along with the commodity output from unit input in the processing and the firm engages into a fixed-volume sales contract for the by-product with demand D_2 and selling price p_2. Let

$$\begin{aligned} h_3(S_1) &\doteq a_1 \min(\max(p_1, S_1(1-\omega_1)), S_1) + a_2 p_2 - c_0, \\ h_2(S_1) &\doteq a_1 S_1(1-\omega_1) + a_2 p_2 - c_0, \end{aligned} \tag{3.9}$$

where $h_3(S_1)$ denotes the unit processing margin when there is unsatisfied demand in both commodity output and by-product market; $h_2(S_1)$ is that when there is no unsatisfied demand in commodity output market but has unsatisfied demand in by-product market. Accordingly, we define $\mathcal{M}_3^i$ and $\mathcal{M}_2^i$ in a similar form given in (3.6) to represent the expected marginal profits of contract $i \in \{1, 2\}$. We assume $\omega_1 \leq a_2 p_2 / a_1 p_1$ that makes $h_1(S_1) \leq h_2(S_1)$ hold for all output spot price realizations. This is a mild assumption as empirically verified in §3.7. We also assume that the processing capacity is sufficient to process enough input to satisfy both commodity output and by-product demands; that is, $K_0 \geq \max\left(\frac{D_1}{a_1}, \frac{D_2}{a_2}\right)$.

To differentiate with the notations in single output case, we introduce subscript "b" (by-product) into the notations to specify them as those in the presence of by-product. For example, Q_b^{i*} is the optimal contract volume from contract $i \in \{1, 2\}$ in the presence by-product and $\Delta_{A,b}^{(2)}$ is the counterpart of $\Delta_A^{(2)}$.

3.6.1 The Optimal Strategy

In this section, we characterize the optimal solution for the firm's decisions as well as the optimal expected profit in the presence of by-product. In the presence of by-product, the stage 2 objective function has an additional revenue stream from the by-product demand; that is, $p_2 \min(a_2 z_{0,b}, D_2)$ compared to the single output case as given by (3.3).

In what follows, we denote subscript $[1] \in \{1, 2\}$ as the index of product with $\max\left(\frac{D_1}{a_1}, \frac{D_2}{a_2}\right)$ and denote subscript [2] as the other. In Proposition 16, we characterize the optimal processing volume $z_{0,b}^*$ for given contract volumes (Q_b^1, Q_b^2) and spot price realizations (S_0, S_1).

Proposition 16 *Given contract volumes (Q_b^1, Q_b^2) and spot price realizations (S_0, S_1),* *the optimal processing volume $z^*_{0,b}$ is characterized by*

$$z^*_{0,b} = \begin{cases} 0 & \text{if } h_3(S_1) \leq P^1(S_0) \\ \min\left(\frac{D_{[2]}}{a_{[2]}}, Q_b^1\right) & \text{if } h_{[1]}(S_1) \leq P^1(S_0) \leq h_3(S_1) \leq P^2(S_0) \\ \frac{D_{[2]}}{a_{[2]}} & \text{if } h_{[1]}(S_1) \leq P^1(S_0) \leq P^2(S_0) \leq S_0 \leq h_3(S_1) \\ \min\left(\frac{D_{[1]}}{a_{[1]}}, Q_b^1\right) & \text{if } h_0(S_1) \leq P^1(S_0) \leq h_{[1]}(S_1) \leq h_3(S_1) \leq P^2(S_0) \\ Q_b^1 & \text{if } P^1(S_0) \leq h_0(S_1) \leq h_{[1]}(S_1) \leq h_3(S_1) \leq P^2(S_0) \\ \min\left(\max\left(\min\left(\frac{D_{[1]}}{a_{[1]}}, Q_b^1\right), \frac{D_{[2]}}{a_{[2]}}\right), Q_b^1 + Q_b^2\right) & \text{if } h_0(S_1) \leq P^1(S_0) \leq h_{[1]}(S_1) \leq P^2(S_0) \leq h_3(S_1) \leq S_0 \\ \min\left(\max\left(\frac{D_{[2]}}{a_{[2]}}, Q_b^1\right), Q_b^1 + Q_b^2\right) & \text{if } P^1(S_0) \leq h_0(S_1) \leq h_{[1]}(S_1) \leq P^2(S_0) \leq h_3(S_1) \leq S_0 \\ \min\left(\frac{D_{[2]}}{a_{[2]}}, Q_b^1 + Q_b^2\right) & \text{if } h_{[1]}(S_1) \leq P^1(S_0) \leq P^2(S_0) \leq h_3(S_1) \leq S_0 \\ \max\left(\min\left(\frac{D_{[1]}}{a_{[1]}}, Q_b^1\right), \frac{D_{[2]}}{a_{[2]}}\right) & \text{if } h_0(S_1) \leq P^1(S_0) \leq h_{[1]}(S_1) \leq P^2(S_0) \leq S_0 \leq h_3(S_1) \\ \max\left(\frac{D_{[2]}}{a_{[2]}}, Q_b^1\right) & \text{if } P^1(S_0) \leq h_0(S_1) \leq h_{[1]}(S_1) \leq P^2(S_0) \leq S_0 \leq h_3(S_1) \\ \min\left(\frac{D_{[1]}}{a_{[1]}}, Q_b^1 + Q_b^2\right) & \text{if } h_0(S_1) \leq P^1(S_0) \leq P^2(S_0) \leq h_{[1]}(S_1) \leq h_3(S_1) \leq S_0 \\ \max\left(\min\left(\frac{D_{[1]}}{a_{[1]}}, Q_b^1 + Q_b^2\right), Q_b^1\right) & \text{if } P^1(S_0) \leq h_0(S_1) \leq P^2(S_0) \leq h_{[1]}(S_1) \leq h_3(S_1) \leq S_0 \\ \max\left(\min\left(\frac{D_{[1]}}{a_{[1]}}, Q_b^1 + Q_b^2\right), \frac{D_{[2]}}{a_{[2]}}\right) & \text{if } h_0(S_1) \leq P^1(S_0) \leq P^2(S_0) \leq h_{[1]}(S_1) \leq S_0 \leq h_3(S_1) \\ \max\left(\min\left(\frac{D_{[1]}}{a_{[1]}}, Q_b^1 + Q_b^2\right), \max\left(\frac{D_{[2]}}{a_{[2]}}, Q_b^1\right)\right) & \text{if } P^1(S_0) \leq h_0(S_1) \leq P^2(S_0) \leq h_{[1]}(S_1) \leq S_0 \leq h_3(S_1) \\ \max\left(\frac{D_{[1]}}{a_{[1]}}, Q_b^1\right) & \text{if } P^1(S_0) \leq h_0(S_1) \leq P^2(S_0) \leq S_0 \leq h_{[1]}(S_1) \\ \frac{D_{[1]}}{a_{[1]}} & \text{if } h_0(S_1) \leq P^1(S_0) \leq P^2(S_0) \leq S_0 \leq h_{[1]}(S_1) \\ Q_b^1 + Q_b^2 & \text{if } P^2(S_0) \leq h_0(S_1) \leq S_0 \leq h_{[1]}(S_1) \\ K_0 & \text{if } S_0 \leq h_0(S_1) \end{cases} \tag{3.10}$$

where $P^1(S_0)$ and $P^2(S_0)$ are the unit sourcing costs as given in Proposition 10; $h_3(S_1)$, $h_{[1]}(S_1)$ and $h_0(S_1)$ are the unit processing margins as given in (3.4) and (3.9).

Paralleling (3.5), the optimal processing volume $z^*_{0,b}$ is also determined by comparing the unit sourcing costs with the relevant unit processing margins. Different from (3.5), due to the presence of by-product, there are two additional processing margins; that is, $h_3(S_1)$ and $h_{[1]}(S_1)$ where [1] is the index of product with $\max\left(\frac{D_1}{a_1}, \frac{D_2}{a_2}\right)$. We also note that the magnitude of by-product demand has an important impact on the optimal processing volume. Since the outputs are generated in fixed proportions, the magnitude of commodity output demand and by-product demand determines which demand will be fulfilled first as processing continues, which in turn determines the processing margin accordingly. For example, for $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$, after by-product demand is satisfied, the processing margin for next unit of input is $h_1(S_1)$; for $\frac{D_2}{a_2} > \frac{D_1}{a_1}$, the processing margin is $h_2(S_1)$.

Next, we characterize the optimal contract volumes in Proposition 17.

Proposition 17 *The optimal contract volumes are given by*

$$\left(Q_b^{X*}, Q_b^{(3-X)*}\right) = \begin{cases} (0,0) & \textit{if } \mathcal{M}_3^X \leq 0 \\ \left(\frac{D_{[2]}}{a_{[2]}}, 0\right) & \textit{if } \mathcal{M}_3^X > 0 \textit{ and } \mathcal{M}_{[1]}^Y \leq 0 \\ \left(\frac{D_{[1]}}{a_{[1]}}, 0\right) & \textit{if } \mathcal{M}_3^X > 0, \mathcal{M}_{[1]}^Y > 0, \mathcal{M}_0^Z \leq 0 \textit{ and } X = Y \\ (K_0, 0) & \textit{if } \mathcal{M}_3^X > 0, \mathcal{M}_{[1]}^Y > 0, \mathcal{M}_0^Z > 0 \textit{ and } X = Y = Z \\ \left(\frac{D_{[2]}}{a_{[2]}}, \frac{D_{[1]}}{a_{[1]}} - \frac{D_{[2]}}{a_{[2]}}\right) & \textit{if } \mathcal{M}_3^X > 0, \mathcal{M}_{[1]}^Y > 0, \mathcal{M}_0^Z \leq 0 \textit{ and } X = 1, Y = 2 \\ \left(\frac{D_{[2]}}{a_{[2]}}, K_0 - \frac{D_{[2]}}{a_{[2]}}\right) & \textit{if } \mathcal{M}_3^X > 0, \mathcal{M}_{[1]}^Y > 0, \mathcal{M}_0^Z > 0 \textit{ and } X = 1, Y = Z = 2 \\ \left(\frac{D_{[1]}}{a_{[1]}}, K_0 - \frac{D_{[1]}}{a_{[1]}}\right) & \textit{if } \mathcal{M}_3^X > 0, \mathcal{M}_{[1]}^Y > 0, \mathcal{M}_0^Z > 0 \textit{ and } X = Y = 1, Z = 2 \end{cases}$$

where subscript $[1]$ *denotes the index of the product with* $\max\left(\frac{D_1}{a_1}, \frac{D_2}{a_2}\right)$ *and subscript* $[2]$ *denotes the other;* X*,* Y*, and* Z *are the indexes of contract with the higher value of* $\mathcal{M}_3^i$*,* $\mathcal{M}_{[1]}^i$*, and* $\mathcal{M}_0^i$ *for* $i \in \{1,2\}$*, respectively.*

This proposition has similar interpretations to Proposition 11. Moreover, compared to the case without by-product, the firm is more likely to use contracts for procurement in the presence of by-product. This is because the additional revenue from by-produce sales reduces the contract procurement risk—that is, the reserved contracts are not profitable to be exercised due to the low processing revenue.

Paralleling Proposition 12, we characterize the optimal contract volumes (Q_b^{1*}, Q_b^{2*}) in next proposition within (Δ_A, Δ_β) space for given A^1 and β^2. We define additional preference and profitability thresholds. In particular, we denote $\Delta^M_{\beta,b}(\Delta_A)$ and $\Delta^H_{\beta,b}(\Delta_A)$ as the unique solutions of $\mathcal{M}^1_{[1]} = \mathcal{M}^2_{[1]}$ and $\mathcal{M}^1_3 = \mathcal{M}^2_3$ for a given Δ_A, respectively; $\Delta^{(i)}_{A,b}$, $i \in \{2, 3\}$ as the unique solutions of $\mathcal{M}^2_{[1]} = 0$ and $\mathcal{M}^2_3 = 0$, respectively; let $\Delta^{(i)}_{\beta,b}$, $i \in \{2, 3\}$ denote the unique solutions of $\mathcal{M}^1_{[1]} = 0$ and $\mathcal{M}^1_3 = 0$, respectively. Thresholds $\Delta^{(i)}_{\beta,b}$, $i \in \{2, 3\}$ determines the profitability of contract 1 accordingly.[2] We relegate the expressions of the thresholds in the proof of Proposition 18.

Proposition 18 *The optimal contract volumes (Q_b^{1*}, Q_b^{2*}) for given A^1 and β^2 are characterized in Table 3.2:*

Region	**Strategy**	**Condition**	(Q_b^{1*}, Q_b^{2*})
Ω_1	Single	$0 \le \Delta_\beta < \min\left(\Delta^L_\beta(\Delta_A), \Delta^{(1)}_\beta\right)$	$(K_0, 0)$
Ω_2	Dual	$0 \le \Delta_A < \Delta^{(1)}_A, \Delta^L_\beta(\Delta_A) \le \Delta_\beta < \Delta^M_{\beta,b}(\Delta_A)$	$\left(\frac{D_{[1]}}{a_{[1]}}, K_0 - \frac{D_{[1]}}{a_{[1]}}\right)$
Ω_3	Dual	$0 \le \Delta_A < \Delta^{(1)}_A, \Delta^M_{\beta,b}(\Delta_A) \le \Delta_\beta < \Delta^H_{\beta,b}(\Delta_A)$	$\left(\frac{D_{[2]}}{a_{[2]}}, K_0 - \frac{D_{[2]}}{a_{[2]}}\right)$
Ω_4	Single	$0 \le \Delta_A < \Delta^{(1)}_A, \Delta_\beta \ge \Delta^H_{\beta,b}(\Delta_A)$	$(0, K_0)$
Ω_5	Single	$\Delta_A \ge \Delta^{(1)}_A, \Delta^{(1)}_\beta \le \Delta_\beta < \min\left(\Delta^M_{\beta,b}(\Delta_A), \Delta^{(2)}_{\beta,b}\right)$	$\left(\frac{D_{[1]}}{a_{[1]}}, 0\right)$
Ω_6	Dual	$\Delta^{(1)}_A \le \Delta_A < \Delta^{(2)}_{A,b}, \Delta^M_{\beta,b}(\Delta_A) \le \Delta_\beta < \Delta^H_{\beta,b}(\Delta_A)$	$\left(\frac{D_{[2]}}{a_{[2]}}, \frac{D_{[1]}}{a_{[1]}} - \frac{D_{[2]}}{a_{[2]}}\right)$
Ω_7	Single	$\Delta^{(1)}_A \le \Delta_A < \Delta^{(2)}_{A,b}, \Delta_\beta \ge \Delta^H_{\beta,b}(\Delta_A)$	$\left(0, \frac{D_{[1]}}{a_{[1]}}\right)$
Ω_8	Single	$\Delta_A \ge \Delta^{(2)}_{A,b}, \Delta^{(2)}_{\beta,b} \le \Delta_\beta < \min\left(\Delta^H_{\beta,b}(\Delta_A), \Delta^{(3)}_{\beta,b}\right)$	$\left(\frac{D_{[2]}}{a_{[2]}}, 0\right)$
Ω_9	Single	$\Delta^{(2)}_{A,b} \le \Delta_A < \Delta^{(3)}_{A,b}, \Delta_\beta \ge \Delta^H_{\beta,b}(\Delta_A)$	$\left(0, \frac{D_{[2]}}{a_{[2]}}\right)$
Ω_{10}	N.A.	$\Delta_A \ge \Delta^{(3)}_{A,b}, \Delta_\beta \ge \Delta^{(3)}_{\beta,b}$	$(0, 0)$

Table 3.2: Optimal Contract Volumes (Q_b^{1*}, Q_b^{2*}) on (Δ_A, Δ_β) Space.

We provide a graphical illustration for Proposition 18 in Figure 3.2.

[2]Note that for $\Delta^M_{\beta,b}(\Delta_A)$, $\Delta^{(2)}_{A,b}$ and $\Delta^{(2)}_{\beta,b}$, each of them takes two different values depending on the magnitude of $\frac{D_1}{a_1}$ and $\frac{D_2}{a_2}$ (thus determining the value of [1]). However, because index [1] can be only one value in one selling season, we do not differentiate the notations for case $\frac{D_2}{a_2} \le \frac{D_1}{a_1}$ and $\frac{D_2}{a_2} > \frac{D_1}{a_1}$ for brevity.

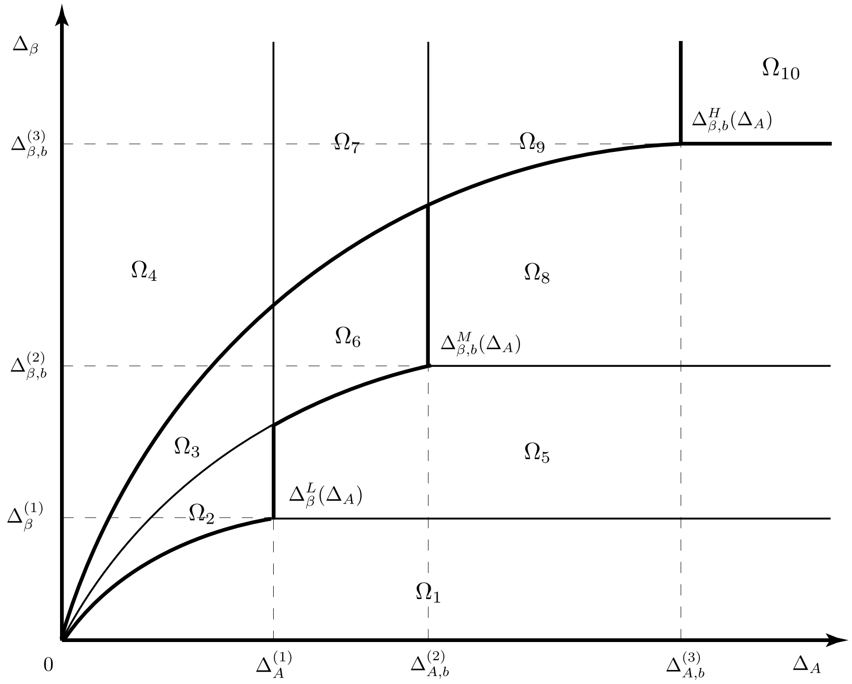


Figure 3.2: Optimal Contract Volumes (Q_b^{1*}, Q_b^{2*}) on (Δ_A, Δ_β) Space.

After substituting optimal contract volumes into stage 1 profit function, we obtain the processor's optimal expected profit in the following proposition.

Proposition 19 *The processor's optimal expected profit is*

$$V_b^* = V_b(0,0) + \left(\mathcal{M}_3^X\right)^+ \frac{D_{[2]}}{a_{[2]}} + \left(\mathcal{M}_{[1]}^Y\right)^+ \left(\frac{D_{[1]}}{a_{[1]}} - \frac{D_{[2]}}{a_{[2]}}\right) + \left(\mathcal{M}_0^Z\right)^+ \left(K_0 - \frac{D_{[1]}}{a_{[1]}}\right), \tag{3.11}$$

where

$$\begin{aligned} V_b(0,0) \doteq & \mathbb{E}\left[\left(p_1 - \tilde{S}_1\right)^+\right] D_1 + \mathbb{E}\left[\left(h_3(\tilde{S}_1) - \tilde{S}_0\right)^+\right] \frac{D_{[2]}}{a_{[2]}} \\ & + \mathbb{E}\left[\left(h_{[1]}(\tilde{S}_1) - \tilde{S}_0\right)^+\right] \left(\frac{D_{[1]}}{a_{[1]}} - \frac{D_{[2]}}{a_{[2]}}\right) + \mathbb{E}\left[\left(h_0(\tilde{S}_1) - \tilde{S}_0\right)^+\right] \left(K_0 - \frac{D_{[1]}}{a_{[1]}}\right), \end{aligned} \tag{3.12}$$

where subscript $[1]$ *denotes the index of the product with* $\max\left(\frac{D_1}{a_1}, \frac{D_2}{a_2}\right)$ *and subscript* $[2]$ *denotes the other; X, Y, and Z are the indexes as given in Proposition 17.*

Proposition 19 has similar interpretations to Proposition 13. Paralleling §3.5, we define the value of contract procurement in the presence of by-product as $\mathbb{VC}_b \doteq$

$V_b^* - V_b(0,0)$.

3.6.2 The Impacts of Spot Price Correlation

Paralleling §3.5, we conduct sensitivity analysis to study the impacts of spot price correlation on the firm's optimal procurement strategy and expected profit as well as the value of contract procurement.

Proposition 20 *(i)* $\frac{\partial(Q_b^{1*}+Q_b^{2*})}{\partial\rho} \geq 0$ *and (ii)* $\frac{\partial Q_b^{1*}}{\partial\rho} \geq 0$.

Recall from Proposition 14 that without by-product, a higher spot price correlation makes contract procurement more favorable; moreover, the firm tends to use more contract 1 for procurement when the correlation increases. Proposition 20 shows that these results continue to hold in the presence of by-product.

Paralleling Proposition 15, we focus on the contract procurement regions Ω_{-10} to investigate the (local) impacts of spot price correlation on firm's optimal expected profit and the value of contract procurement.

Proposition 21 *(i) Assume* $\omega_1 = 0$. $\frac{\partial V_b(0,0)}{\partial\rho} < 0$ *and (ii)* $\frac{\partial \mathbb{VC}_b}{\partial\rho} \geq 0$.

Part (i) of Proposition 21 showcases that without contracts, the firm suffers from synchronization between the input and output spot prices. Plambeck and Taylor (2013) and Boyabatlı et al. (2017) show that the optimal expected profit without contracts always decreases in spot price correlation in a single-output setting. Our result extends their results by showing that their results continue to hold under the multiple-output setting.

Part (ii) of Proposition 21 shows that the impact of correlation on the value of contract procurement that established in the single-output case continues to hold in the presence of by-product.

We close this section by noting that the overall impact of spot price correlation on the optimal expected profit is indeterminate since its impacts on the optimal expected profit without contracts and the value of contract procurement are opposite. We numerically investigate its impact on optimal expected profit in §3.7.2.

3.6.3 The Impacts of Introducing A By-Product

The impact of introducing a by-product on the firm's optimal profit is straightforward—the firm's profit is always smaller in the absence of by-product. Therefore, we focus on the impacts of introducing a by-product on the firm's optimal contract volumes and the value of contract procurement. In the case without by-product, the contract procurement region is smaller than that in the presence of by-product. In what follows, we focus our analysis on the overlapped contract procurement regions; that is, Γ_{-6} in Figure 3.1.

Proposition 22 *(i)* $Q_b^{1*} + Q_b^{2*} \geq Q^{1*} + Q^{2*}$ *and the equality always holds when* $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$; *(ii)* $Q_b^{1*} \geq Q^{1*}$; *(iii)* $Q_b^{2*} > Q^{2*}$ *only under either situation:*

(a) $(\Delta_A, \Delta_\beta) \in \left[\Delta_A^{(1)}, \Delta_A^{(2)}\right) \times \left[\Delta_{\beta,b}^{M}(\Delta_A), \Delta_{\beta,b}^{H}(\Delta_A)\right)$ *with* $\frac{D_2}{a_2} > \frac{2D_1}{a_1}$;

(b) $(\Delta_A, \Delta_\beta) \in \left[\Delta_A^{(1)}, \Delta_A^{(2)}\right) \times \left[\Delta_{\beta,b}^{H}(\Delta_A), \infty\right)$ *with* $\frac{D_2}{a_2} > \frac{D_1}{a_1}$.

In the presence of by-product, the firm generates a stream of revenue from by-product, thus would reserve no less volumes from contracts than that in the absence of by-product. When $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$, the by-product that is generated along with the commodity output that intends to satisfy the commodity demand (i.e., $a_2\frac{D_1}{a_1}$) is already sufficient to cover the by-product demand. Therefore, the firm does not need to reserve additional contract volume to meet the by-product demand—that is, the total contract volumes are the same as that in the absence of by-product. However, when $\frac{D_2}{a_2} > \frac{D_1}{a_1}$, the by-product that is generated along with the commodity output that intends to satisfy the commodity demand is no longer sufficient to satisfy by-product demand. The firm would reserve more volumes from the preferred contract if its expected marginal profit with only by-product demand is positive (i.e., $\mathcal{M}_2^Y > 0$). In other words, the total contract volumes in the presence of by-product would become no smaller than that without by-product. We call this "no smaller" in total contract volumes as the *volume expanding effect* of by-product.

Part (ii) of Proposition 22 showcases that contract 1 becomes increasingly preferable in the presence of by-product compared to that without by-product. With by-product, processing becomes more profitable due to the additional revenue from by-product sales; the reserved contracts have a lower probability of not being exercised than the case without by-product. Therefore, the firm tends to make more commitments to the contract with cost efficiency (the one with a higher unit reservation cost) rather than the contract with flexibility (the one with a higher unit exercise cost), and thus, ramping up the reserved volume of contract 1 (the one with a higher unit reservation cost). We call this shift in contract preference as the *commitment effect* of by-product.

The change in contract 2's volume depends on which effect dominates. We identify the only two situations (i.e., (iii)-(a) and (iii)-(b) of Proposition 22) where the contract 2's volume is strictly larger, both of which are under case $\frac{D_2}{a_2} > \frac{D_1}{a_1}$. In situation (iii)-(a), the firm reserves $\frac{D_1}{a_1}$ units of input solely from contract 2 in the absence of by-product (region Γ_5). In the presence of by-product, the firm reserves larger total contract volumes $\frac{D_2}{a_2}$ (volume expanding effect); at the same time, the firm tends to prefer contract 1 and reserves $\frac{D_1}{a_1}$ units from contract 1 (commitment effect) and $\left(\frac{D_2}{a_2} - \frac{D_1}{a_1}\right)$ units from contract 2 (region Ω_6). When $\frac{D_2}{a_2} > \frac{2D_1}{a_1}$, the volume expanding effect dominates the commitment effect, and thus, the contract volume from contract 2 is larger in the presence of by-product. In situation (iii)-(b), the firm prefers to use contract 2 in both presence and absence of by-product (i.e., region Γ_5 and Ω_7)—that is, the commitment effect is mute. In this case, the expected marginal profit of contract 2 with only by-product demand is still profitable (i.e., $\mathcal{M}_2^2 > 0$), so firm increases its contract volume from $\frac{D_1}{a_1}$ to $\frac{D_2}{a_2}$ to produce more by-product to extract these additional profit (volume expanding effect). In other words, the volume expanding effect dominates.

Proposition 23 $\mathbb{VC}_b \geq \mathbb{VC}$.

Proposition 23 showcases that by-product makes contract procurement more valuable. By-product improves the value of contract procurement in two ways. It incents the firm to rely more on contracts and to re-optimize the contract portfolio as well. In the presence of by-product, processing becomes more profitable and the contracts' risk of not being exercised decreases. Therefore, it would incent the firm to increase the total contract volumes, and thus, increasing the value of contract procurement. Moreover, as shown in part (iii) of Proposition 22, contract 1 gets more preferable in the presence of by-product. In this case, the firm can optimally adjust contract portfolio which would further improve the value of contract procurement.

3.7 Numerical Analysis: Application to the Palm Industry

In this section, we discuss an application of our model in the context of the palm industry. In this industry, a palm oil processor processes palm fresh fruit bunches (FFB) to produce crude palm oil (CPO) and palm kernel (PK). The fresh fruit bunches are crushed at the pressing station to produce palm kernel and crude palm oil. In the context of our model, the palm fresh fruit bunch is the input, the crude palm oil is the commodity output, and the palm kernel is the by-product.

3.7.1 Model Calibration for Numerical Experiments

Our setting in the numerical study is similar to that in §1.7 of Chapter 1 except the by-product here is the palm kernel. We summarize the calibrated parameters in Table 3.3. Using these experiments, we examine the impacts of spot price correlation and the value of contracts. Hereafter, we shall often use $\acute{x}$ to denote the calibrated value for parameter x, "RM" to denote the Malaysian ringgit (currency) and "mt" to denote metric ton (equal to $1,000$ kg, or about 1.1 U.S. tons).

Notation	Description	Value
$\acute{A}^1$	Unit exercise cost of contract 1	$80\%\acute{\mu}_0$
$\acute{\beta}^2$	Unit reservation cost of contract 2	$1\%\acute{\mu}_0$
$\acute{\Delta}_A$	Additional unit exercise cost of contract 2	$[0, 25\%\acute{\mu}_0]$
$\acute{K}_0$	Processing capacity	56688.06 mt
$\acute{c}_0$	Unit processing cost	39.39 RM/mt
$\acute{a}_1$, $\acute{a}_2$	Production yields of CPO and PK	19.77%, 5.54%
$\acute{D}_1$, $\acute{D}_2$	CPO and PK demands	5379.47, 1256.21 mt
$\acute{\omega}_1$	Transaction cost rate of output spot sale	10%
$\acute{p}_1$, $\acute{p}_2$	CPO and PK prices for demand sales	2433.25, 1470.12 RM/mt

Table 3.3: Description of the Baseline Scenario Used in Our Numerical Experiments. FFB and CPO spot prices are bivariate normally distributed. Our assumption $\acute{\omega}_1 \leq \frac{\acute{a}_2\acute{p}_2}{\acute{a}_1\acute{p}_1}$ is empirically verified.

In the absence of any data on contract costs, we carefully set the contract 1's exercise cost $\acute{A}^1$ as $80\%\acute{\mu}_0$ and contract 2's reservation cost $\acute{\beta}^2$ as $1\%\acute{\mu}_0$ in order to create the most realistic scenarios for the numerical analysis. On one hand, if exercise cost $\acute{A}^1$ is too small, then the contracts might be always exercised after reservation, and thus the quantity flexibility contracts are degraded into forward contracts. On the other hand, if exercise cost $\acute{A}^1$ is too large, then the limited scenarios of $\acute{\Delta}_A$ values might exclude various realistic contracts as $\acute{A}^1 + \acute{\Delta}_A$ cannot be greater than $\acute{\mu}_0$ too much; otherwise, the processor always optimally chooses to not exercise contract 2. For reservation cost $\acute{\beta}^2$, we cannot set the value too high because otherwise it is not profitable to use spot sourcing anymore that is also not very interesting. We set $\acute{\beta}^2 = 1\%\acute{\mu}_0$ as a realistic representative to illustrate in this paper. Under our calibrated data, we get that $\left(\acute{\Delta}_A^{(1)}, \acute{\Delta}_\beta^{(1)}\right) = (7.46\%\acute{\mu}_0, 2.60\%\acute{\mu}_0)$, $\left(\acute{\Delta}_{A,b}^{(2)}, \acute{\Delta}_{\beta,b}^{(2)}\right) = (8.18\%\acute{\mu}_0, 5.71\%\acute{\mu}_0)$, and $\left(\acute{\Delta}_{A,b}^{(3)}, \acute{\Delta}_{\beta,b}^{(3)}\right) = (23.68\%\acute{\mu}_0, 16.79\%\acute{\mu}_0)$. Since the contract 2's profitability threshold in the presence of both markets $\acute{\Delta}_{A,b}^{(3)}$ is $23.68\%\acute{\mu}_0$, we consider a larger set to include the spot procurement region Ω_{10}. In particular, we set $\acute{\Delta}_A \in [0, 25\%\acute{\mu}_0]$.

We use the academic literature to estimate the processing capacity and unit processing cost. Boyabatlı et al. (2017) find that the optimal input processing capacity of a typical palm oil processor is 858.91 mt of FFB per day. This corresponds to a total processing capacity of $\acute{K}_0 = 56688.06$ mt of FFB in our model using 66 weekdays (in other words, 3 months) as a selling season. We set the unit processing cost $\acute{c}_0 = 39.39$ RM/mt of FFB as reported in Boyabatlı et al. (2017). Note that in Boyabatlı et al. (2017), they incorporate the revenue from by-product into the processing cost, and thus, the effective unit processing cost was negative; here we explicitly consider the by-product selling to the demand market, so the processing cost represents the operational costs in processing itself.

For the production yields of CPO and PK, we use the average monthly production yield in Peninsular Malaysia within our time frame (Jan 2014 to May 2018) as reported by the MPOB and find that $\acute{a}_1 = 19.77\%$ and $\acute{a}_2 = 5.54\%$. For the CPO and PK demands, we assume that 80% of the processing capacity is used and 60% of CPO is sold by contract, which corresponds to $\acute{D}_1 = 5379.47$ mt of CPO. We assume that the PK demand is equal to 50% of total PK produced, which corresponds to $\acute{D}_2 = 1256.21$ mt of PK in a selling season. For the transaction cost rate of output spot sale, we set it as $\acute{\omega}_1 = 10\%$.

For the CPO unit (sales) contract price, we use the daily CPO future contract (settlement) price with a three-month maturity—that is, delivery in three months—as reported by the MPOB. In particular, we take the average of daily future prices within our time window (from January 2, 2014 to May 14, 2018) and obtain $\acute{p}_1 = 2433.25$ RM/mt. For the PK unit (sales) contract price, we use the average of the PK prices reported by MPOB within the same time frame to estimate and obtain $\acute{p}_2 = 1470.12$ RM/mt of PK.

3.7.2 The Impacts of Spot Price Correlation

In this section, we investigate the impacts of spot price correlation on the processor's contract procurement decision and profitability. Proposition 20 investigates the impacts of spot price correlation on total contract volumes and volume from contract 1. We extend the analysis by numerically studying the impacts of spot price sport price correlation on the volume from contract 2. In Proposition 21, we focus on the local analysis with respect to spot price correlation, in which the optimal procurement decision does not change with the spot price correlation. To capture the global impacts of spot price uncertainty where changes in spot price uncertainty may cause the changes in the optimal procurement decision, we select a representative pair of contract costs at the crossing boundaries (where contract procurement decisions change) based on the calibrated values to perform our numerical studies. In particular, we employ $\left(\acute{\Delta}^{(2)}_{A,b}, \acute{\Delta}^{(2)}_{\beta,b}\right) = (8.18\%\acute{\mu}_0, 5.71\%\acute{\mu}_0)$ (the point that region Ω_5, Ω_6 and Ω_8 intersect) throughout this section.

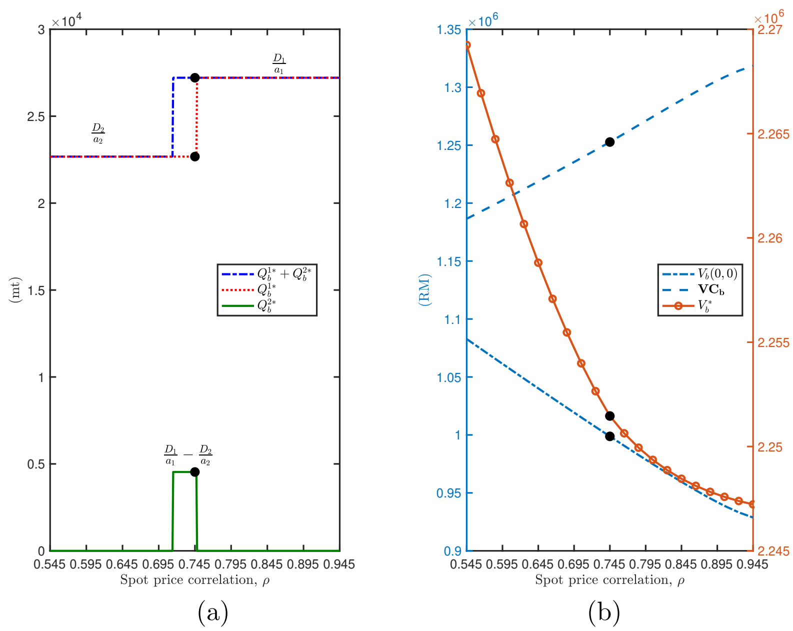


Figure 3.3: Effects of Spot Price Correlation ρ on the Optimal Contract Volumes (Panel a) and Profitability (Panel b). In panel b, $V_b(0,0)$ and $\mathbb{VC}_b$ are measured by the values on the left y-axis while V_b^* is measured by the values on the right y-axis. $\rho \in [0.545, 0.945]$ evenly-spaced around the baseline value $\acute{\rho} = 0.745$ with a step size of 0.001. In the two panels, baseline results for calibrated $\acute{\rho}$ are indicated by $\bullet$.

We see that Panel a of Figure 3.3 numerically verifies our findings in Proposition 20 that the total contract volumes and reserved volume from contract 1 increase in spot price correlation. It also complements the analytical results by showing that the reserved volume from contract 2 can increase in the spot price correlation. Specifically, as ρ increases from 0.695 ($(7.72\%\acute{\mu}_0, 5.47\%\acute{\mu}_0)$ is in region Ω_8) to 0.745 ($(7.72\%\acute{\mu}_0, 5.47\%\acute{\mu}_0)$ is in region Ω_6), the reserved volume from contract 2 increases from 0 to $\frac{D_1}{a_1} - \frac{D_2}{a_2} = 4535$ mt because the contract preference (preferring contract 2) does not change but contract 2 becomes profitable to reserve (from non-profitable) when there is unsatisfied demand only in commodity demand market.

Panel b of Figure 3.3 extends Proposition 21 by showing that the optimal expected profit without contracts decreases in spot price correlation and the value of contracts increases in spot price correlation even when the transaction cost in output spot sales is not zero; that is, $\omega_1 > 0$. Since these two impacts are opposite, its impact on optimal expected profit (i.e., sum of these two impacts) is indeterminate. For a typical palm oil processor in Malaysia, we find that the optimal expected profit decreases in spot price correlation. Plambeck and Taylor (2013) and Boyabatlı et al. (2017) examine the impact of spot correlation on the optimal expected profit without contracts. Our results complement their results by investigating the impact of spot price correlation on the value of contract procurement and the optimal expected profit.

3.7.3 The Value of Using Contracts

We now examine the value of contract procurement and the value of using multiple contracts instead of only one of these contracts. To this end, we extend our numerical instances around the baseline scenario to consider sensitivity of our results based on several key parameters. In particular, we consider spot price correlation $\rho \in \{0.545, 0.645, 0.745, 0.845, 0.945\}$, evenly-spaced around the baseline value 0.745; we consider FFB (CPO) spot price variability σ_0 (σ_1) that are $\{-20\%, -10\%, 0, 10\%, 20\%\}$ of their baseline values.

Value of Contract Procurement. We define the profit loss due to sourcing only from input spot market as

$$\Delta_{VC} \doteq \left[\frac{\mathbb{VC}_b}{V^*}\right]. \tag{3.13}$$

We numerically compute the percentage profit loss $\Delta_{VC} \times 100$. We also consider $\Delta_A \in (0, 25\%\acute{\mu}_0]$ and $\Delta_\beta \in (0, 20\%\acute{\mu}_0]$, evenly-spaced with 1% increments. This contract cost range contains all the contract procurement regions for our numerical instances. In summary, we consider 62,500 numerical instances.

We summarize the instances with positive profit loss after calculation, finding that the average profit loss in these instances is 48.48% with a minimum and a maximum loss of 0 (rounded to 0) and 73.29%, respectively. This result shows that *contract procurement in agricultural processing industries has considerable economic value*, which provides a plausible explanation for the prevalence of procurement contracts in these industries albeit the existence of input spot markets.

Value of Using Multiple Contracts. We define the profit loss due to sourcing from only one of the contracts as

$$\Delta_{VMC} \doteq \left[\frac{V_b^* - \max\left(V_{1,b}^*, V_{2,b}^*\right)}{V_b^*}\right], \tag{3.14}$$

where $V_{1,b}^*$ ($V_{2,b}^*$) is the optimal expected profit when the firm can only procure from contract 1 (2) in the presence of by-product. Similarly, we numerically compute the percentage profit loss $\Delta_{VMC} \times 100$. We also consider $\Delta_A \in (0, 10\%\acute{\mu}_0]$ and $\Delta_\beta \in (0, 10\%\acute{\mu}_0]$, evenly-spaced with 0.5% increments. This contract cost range contains all the dual contract procurement regions for our numerical instances. In summary, we consider 50,000 numerical instances.

We summarize the instances with positive profit loss (in region Ω_2, Ω_3, and Ω_6) after calculation, finding that the average profit loss in these instances is 1.09% with a minimum and a maximum loss of 0 (rounded to 0) and 4.00%, respectively. This result indicates that *procuring from multiple contracts rather than from only one of these contracts has substantial economic value* given that agricultural processing normally generates razor-thin margins.

3.8 Conclusion

This paper studies the procurement strategy of an commodity processor that sources a commodity input to produce and sell a commodity output in the context of agricultural industries. This is the first paper that characterizes the optimal procurement strategy from multiple contracts in the presence of input and output spot price uncertainties. As summarized in the Introduction, we provide insights on how the spot price correlation shapes the optimal procurement strategy and profitability of the processor as well as the value of contract procurement. Our findings are important as they complement the common intuitions in commodity literature. For example, Plambeck and Taylor (2013) and Boyabatlı et al. (2017) show that a higher spot price correlation is always harmful to the firm in the absence of contracts; we find that this result continues to hold in the presence of contracts. By introducing a by-product into the model, we examine the impacts of introducing a by-product on the optimal procurement strategy and the value of contract procurement. We find that introducing a by-product has the same directional impacts as a higher correlation. We also quantify the profit losses due to employing input spot sourcing solely and single contract procurement. We show these profit losses are substantial which in turn justify the prevalence of supply contracts in agricultural processing industries.

Our model assumes that the procurement contract parameters are exogenous. In practice, the contract parameters may be the bargaining results between processor and suppliers, depending on the procurement volume, number of potential suppliers, etc. Examining the optimal procurement from multiple contracts in this setting requires an equilibrium model, following the work of Pei et al. (2011) who provide an analysis of equilibrium procurement contract (single contract is selected for procurement) in the context of a single-product firm. This equilibrium analysis is beyond the scope of this paper but should prove to an interesting avenue for future research.

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Chapter 4

Appendix

§4.1 provides the characterizations of the change in total expected carbon emissions after commercialization $\Delta ECE(\beta)$ for the low ($\beta < \underline{\beta}$), moderate ($\underline{\beta} \leq \beta < \bar{\beta}$), and high ($\beta \geq \bar{\beta}$) contract cost cases. These characterizations are used for establishing our technical results reported in §2.4 and §2.5 in Chapter 2. §4.2 to §4.4 contain the proofs for our technical statements in the paper. We use the following identities for the standard Normal random variable with cumulative distribution function $\Phi(.)$ and probability density function $\phi(.)$ throughout this appendix: $\phi'(t) = -t\phi(t)$, $L(t) = \phi(t) + t\Phi(t) - t$ where $L(t) = \mathbb{E}[(\tilde{T} - t)^+]$ is the standard Normal loss function and $L'(t) = \Phi(t) - 1$.

4.1 Characterization of the Change in Emissions after Commercialization ($\Delta ECE(\beta)$)

Recall from Proposition 3 that the value of biomass commercialization is characterized based on three different contract procurement regions (i.e., $\beta < \underline{\beta}$, $\underline{\beta} \leq \beta < \bar{\beta}$, and $\beta \geq \bar{\beta}$). In a particular region because the contract volumes before and after commercialization are independent of β, so is the change in total expected emissions after commercialization (i.e., $\Delta ECE(\beta)$). However, $\Delta ECE(\beta)$ varies across the contract procurement regions. In this section, we characterize $\Delta ECE(\beta)$ for each contract region separately.

Case (i), $\beta < \underline{\beta}$ **:** Using Propositions 1 and 2, we obtain $Q^*(\beta) = Q^{nb}(\beta) = K_0$, $z_0^*(K_0) = K_0\chi\left(S_1 > \frac{c_0}{a_1}\right) + \frac{D_2}{a_2}\chi\left(\frac{c_0 - a_2p_2}{a_1} < S_1 \leq \frac{c_0}{a_1}\right)$, and $z_0^{nb}(K_0) = K_0\chi\left(S_1 > \frac{c_0}{a_1}\right)$. Using these optimal decisions before and after commercialization, $\Delta ECE(\beta)$ in (2.2) can be characterized as follows:

$$\begin{aligned}\Delta ECE(\beta) = {} & \left(e_2^s - e_2^l\right) D_2\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2p_2}{a_1}\right)\right] \\ & + \left(e_0^b + e_0^p + a_3e_3^r + a_2e_2^l + a_1e_1^s\right)\frac{D_2}{a_2}\mathbb{E}\left[\chi\left(\frac{c_0 - a_2p_2}{a_1} < \tilde{S}_1 \leq \frac{c_0}{a_1}\right)\right] \\ & - \left(e_1^b + e_1^s\right)\min\left(D_1, a_1\frac{D_2}{a_2}\right)\mathbb{E}\left[\chi\left(\frac{c_0 - a_2p_2}{a_1} < \tilde{S}_1 \leq \frac{c_0}{a_1}\right)\chi(\tilde{S}_1 \leq p_1)\right].\end{aligned} \tag{4.1}$$

Case (ii), $\underline{\beta} \leq \beta < \bar{\beta}$ **:** Using Propositions 1 and 2, we obtain $Q^*(\beta) = \frac{D_2}{a_2}$, $Q^{nb}(\beta) = 0$, $z_0^*\left(\frac{D_2}{a_2}\right) = K_0\chi\left(S_1 > \frac{c_0}{a_1}\right) + \frac{D_2}{a_2}\chi\left(\frac{c_0 - a_2p_2}{a_1} < S_1 \leq \frac{c_0 + S_0}{a_1}\right)$, and $z_0^{nb}(0) = K_0\chi\left(S_1 > \frac{c_0 + S_0}{a_1}\right)$. Using these optimal decisions before and after commercialization, $\Delta ECE(\beta)$ in (2.2) can be characterized as follows:

$$\begin{aligned}\Delta ECE(\beta) = {} & \left(e_2^s - e_2^l\right) D_2\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2p_2}{a_1}\right)\right] \\ & + \left(e_0^b + e_0^p + a_3e_3^r + a_2e_2^l + a_1e_1^s\right)\frac{D_2}{a_2}\mathbb{E}\left[\chi\left(\frac{c_0 - a_2p_2}{a_1} < \tilde{S}_1 \leq \frac{c_0 + \tilde{S}_0}{a_1}\right)\right] \\ & - \left(e_1^b + e_1^s\right)\min\left(D_1, a_1\frac{D_2}{a_2}\right)\mathbb{E}\left[\chi\left(\frac{c_0 - a_2p_2}{a_1} < \tilde{S}_1 \leq \frac{c_0 + \tilde{S}_0}{a_1}\right)\chi(\tilde{S}_1 \leq p_1)\right].\end{aligned} \tag{4.2}$$

Case (iii), $\beta \geq \bar{\beta}$ **:** Using Propositions 1 and 2, we obtain $Q^*(\beta) = Q^{nb}(\beta) = 0$, $z_0^*(0) = K_0\chi\left(S_1 > \frac{c_0 + S_0}{a_1}\right) + \frac{D_2}{a_2}\chi\left(\frac{c_0 - a_2p_2 + S_0}{a_1} < S_1 \leq \frac{c_0 + S_0}{a_1}\right)$, and $z_0^{nb}(0) = K_0\chi\left(S_1 > \frac{c_0 + S_0}{a_1}\right)$. Using these optimal decisions before and after commercialization, $\Delta ECE(\beta)$ in (2.2) can be characterized as follows:

$$\Delta ECE(\beta) = \left(e_2^s - e_2^l\right) D_2 \mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1}\right)\right] \tag{4.3}$$
$$+ \left(e_0^b + e_0^p + a_3 e_3^r + a_2 e_2^l + a_1 e_1^s\right) \frac{D_2}{a_2} \mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1} < \tilde{S}_1 \leq \frac{c_0 + \tilde{S}_0}{a_1}\right)\right]$$
$$- \left(e_1^b + e_1^s\right) \min\left(D_1, a_1 \frac{D_2}{a_2}\right) \mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1} < \tilde{S}_1 \leq \frac{c_0 + \tilde{S}_0}{a_1}\right) \chi(\tilde{S}_1 \leq p_1)\right].$$

4.2 Proofs of Chapter 1

Proof of Proposition 1: The result follows from Proposition 16 by using $\omega_1 = 0$, $A^1 = 0$, and $A^2 = \infty$. ■

Proof of Proposition 2: Given the optimal processing volume z_0^* in (3.5), the optimal stage 2 profit for a given contract volume Q and spot price realizations, $\pi(Q; S_0, S_1)$, can be written explicitly in the following manner:

(1) If $a_1 S_1 + a_2 p_2 - c_0 \leq 0$, then $z_0^* = 0$ and

$$\pi(Q; S_0, S_1) = D_1 (p_1 - S_1)^+.$$

(2) If $a_1 S_1 - c_0 \leq 0 \leq a_1 S_1 + a_2 p_2 - c_0 \leq S_0$, then $z_0^* = \min(D_2/a_2, Q)$ and

$$\pi(Q; S_0, S_1) = (a_1 S_1 + a_2 p_2 - c_0) \min\left(\frac{D_2}{a_2}, Q\right) + D_1 (p_1 - S_1)^+.$$

(3) If $a_1 S_1 - c_0 \leq 0 \leq S_0 \leq a_1 S_1 + a_2 p_2 - c_0$, then $z_0^* = D_2/a_2$ and

$$\pi(Q; S_0, S_1) = -\left(\frac{D_2}{a_2} - Q\right)^+ S_0 + (a_1 S_1 + a_2 p_2 - c_0) \frac{D_2}{a_2} + D_1 (p_1 - S_1)^+.$$

(4) If $0 \le a_1S_1 - c_0 \le a_1S_1 + a_2p_2 - c_0 \le S_0$, then $z_0^* = Q$ and

$$\pi(Q; S_0, S_1) = (a_1S_1 - c_0)Q + a_2p_2 \min\left(Q, \frac{D_2}{a_2}\right) + D_1(p_1 - S_1)^+.$$

(5) If $0 \le a_1S_1 - c_0 \le S_0 \le a_1S_1 + a_2p_2 - c_0$, then $z_0^* = \max(D_2/a_2, Q)$ and

$$\pi(Q; S_0, S_1) = QS_0 + (a_1S_1 - c_0 - S_0)\max\left(\frac{D_2}{a_2}, Q\right) + p_2D_2 + D_1(p_1 - S_1)^+.$$

(6) If $S_0 \le a_1S_1 - c_0$, then $z_0^* = K_0$ and

$$\pi(Q; S_0, S_1) = QS_0 + (a_1S_1 - c_0 - S_0)K_0 + p_2D_2 + D_1(p_1 - S_1)^+.$$

Let $V(Q) \doteq \mathbb{E}[\pi(Q; \tilde{S}_0, \tilde{S}_1)] - \beta Q$ denote the stage 1 expected profit for a given contract volume Q, and let $V(Q) \doteq \mathbb{E}_{\tilde{S}_1}[R(Q; \tilde{S}_1)]$ where $R(Q; S_1) \doteq \mathbb{E}_{\tilde{S}_0}[\pi(Q, S_1; \tilde{S}_0)] - \beta Q$. Because $\pi(Q)$ satisfies Lipshitz condition of order one, expectation and differentiation operators can be interchanged, and thus, $\frac{\partial V(Q)}{\partial Q} = \mathbb{E}_{\tilde{S}_1}\left[\frac{\partial R(Q;\tilde{S}_1)}{\partial Q}\right]$. We obtain $\frac{\partial R(Q,S_1)}{\partial Q}$ using $\pi(Q; S_0, S_1)$ as characterized above: (i) if $a_1S_1 + a_2p_2 - c_0 < 0$, then $\frac{\partial R(Q,S_1)}{\partial Q} = -\beta$; (ii) if $a_1S_1 - c_0 < 0 \le a_1S_1 + a_2p_2 - c_0$, then $\frac{\partial R(Q,S_1)}{\partial Q} = \mathbb{E}_{\tilde{S}_0}[\min(\tilde{S}_0, a_1S_1 + a_2p_2 - c_0)]\chi\left(Q \le \frac{D_2}{a_2}\right) - \beta$ and (iii) if $0 \le a_1S_1 - c_0$, then $\frac{\partial R(Q,S_1)}{\partial Q} = \mathbb{E}_{\tilde{S}_0}[\min(\tilde{S}_0, a_1S_1 + a_2p_2 - c_0)]\chi\left(Q \le \frac{D_2}{a_2}\right) + \mathbb{E}_{\tilde{S}_0}[\min(\tilde{S}_0, a_1S_1 - c_0)]\chi\left(Q > \frac{D_2}{a_2}\right) - \beta$. Combining cases (i)-(iii) together and using $\frac{\partial V(Q)}{\partial Q} = \mathbb{E}_{\tilde{S}_1}\left[\frac{\partial R(Q;\tilde{S}_1)}{\partial Q}\right]$, we obtain

$$\begin{aligned}\frac{\partial V(Q)}{\partial Q} =& \mathbb{E}[\min(\tilde{S}_0, (a_1\tilde{S}_1 + a_2p_2 - c_0)^+)]\chi\left(Q \le \frac{D_2}{a_2}\right) + \mathbb{E}[\min(\tilde{S}_0, (a_1\tilde{S}_1 - c_0)^+)]\chi\left(Q > \frac{D_2}{a_2}\right) - \beta \\ =& (\bar{\beta} - \beta)\chi\left(Q \le \frac{D_2}{a_2}\right) + (\underline{\beta} - \beta)\chi\left(Q > \frac{D_2}{a_2}\right),\end{aligned}$$

where $\bar{\beta} \doteq \mathbb{E}[\min(\tilde{S}_0, (a_1\tilde{S}_1 + a_2p_2 - c_0)^+)]$ and $\underline{\beta} \doteq \mathbb{E}[\min(\tilde{S}_0, (a_1\tilde{S}_1 - c_0)^+)]$. Because $\underline{\beta} < \bar{\beta}$, $V(Q)$ is piecewise linear and concave in Q. Therefore, we have $Q^* = K_0$ if $\beta < \underline{\beta}$; $Q^* = D_2/a_2$ if $\underline{\beta} \le \beta < \bar{\beta}$ and $Q^* = 0$ if $\beta \ge \bar{\beta}$. ■

Proof of Proposition 3: Recall that $V^*(\beta)$ denotes the firm's optimal expected profit after commercialization (evaluated at the optimal contract volume $Q^*(\beta)$ as characterized by Proposition 2) for a given β. Using $Q^*(\beta)$ from Proposition 2 and $\pi(Q; S_0, S_1)$ characterization as given in the proof of that proposition, we obtain

$$\begin{aligned} V^*(\beta) =& \mathbb{E}\left[\left(a_1\tilde{S}_1 + a_2p_2 - c_0 - \tilde{S}_0\right)^+\right]\frac{D_2}{a_2} + \mathbb{E}\left[\left(a_1\tilde{S}_1 - c_0 - \tilde{S}_0\right)^+\right]\left(K_0 - \frac{D_2}{a_2}\right) \\ &+ \mathbb{E}\left[\left(p_1 - \tilde{S}_1\right)^+\right]D_1 + (\bar{\beta} - \beta)^+\frac{D_2}{a_2} + (\underline{\beta} - \beta)^+\left(K_0 - \frac{D_2}{a_2}\right). \end{aligned}$$

The firm's optimal profit before commercialization $V^{nb}(\beta)$ (evaluated at the optimal contract volume $Q^{nb}(\beta)$) for a given β can be obtained from $V^*(\beta)$ by setting $D_2 = 0$. The characterization of $\Delta V(\beta) = V^*(\beta) - V^{nb}(\beta)$ for the low ($\beta < \underline{\beta}$), moderate ($\underline{\beta} \leq \beta < \bar{\beta}$), and high ($\beta \geq \bar{\beta}$) contract cost cases can be obtained using the following identities:

$$\begin{aligned} \mathbb{E}\left[\left(a_1\tilde{S}_1 - c_0 - \tilde{S}_0\right)^+\right] + \underline{\beta} =& \mathbb{E}\left[\left(a_1\tilde{S}_1 - c_0\right)^+\right], \\ \mathbb{E}\left[\left(a_1\tilde{S}_1 + a_2p_2 - c_0 - \tilde{S}_0\right)^+\right] + \bar{\beta} =& \mathbb{E}\left[\left(a_1\tilde{S}_1 + a_2p_2 - c_0\right)^+\right]. \end{aligned}$$

■

Proof of Proposition 4: Recall that the heuristic value of commercialization is given by $\Delta V^H(\beta) = p_2\mathbb{E}\left[\min(a_2z_0^{nb}(Q^{nb}(\beta)), D_2)\right]$. Using Proposition 1 and letting $D_2 = 0$, we obtain for a given contract volume $Q^{nb}(\beta)$, the optimal processing volume before commercialization z_0^{nb} is given by

$$z_0^{nb}(Q^{nb}(\beta)) = \begin{cases} 0 & \text{if } a_1S_1 - c_0 \leq 0, \\ Q^{nb} & \text{if } 0 \leq a_1S_1 - c_0 \leq S_0, \\ K_0 & \text{if } S_0 \leq a_1S_1 - c_0. \end{cases}$$

If $\beta < \underline{\beta}$, then $Q^{nb}(\beta) = K_0$ and

$$z_0^{nb} = \begin{cases} 0 & \text{if } a_1 S_1 - c_0 \leq 0, \\ K_0 & \text{if } 0 \leq a_1 S_1 - c_0, \end{cases}$$

if $\beta \geq \underline{\beta}$, then $Q^{nb}(\beta) = 0$ and

$$z_0^{nb} = \begin{cases} 0 & \text{if } a_1 S_1 - c_0 \leq S_0, \\ K_0 & \text{if } S_0 \leq a_1 S_1 - c_0. \end{cases}$$

To sum up, the optimal processing volume before commercialization is given by

$$z_0^{nb} = \begin{cases} K_0 \chi \left(0 < a_1 S_1 - c_0\right) & \text{if } \beta < \underline{\beta}, \\ K_0 \chi \left(S_0 < a_1 S_1 - c_0\right) & \text{if } \beta \geq \underline{\beta}. \end{cases} \tag{4.4}$$

Substituting (4.4) into $\Delta V^H(\beta)$, we obtain $\Delta V^H(\beta) = M^H(\beta) D_2$ where

$$M^H(\beta) \doteq \begin{cases} p_2 Pr\left(a_1 \tilde{S}_1 - c_0 \geq 0\right) & \text{if } 0 \leq \beta < \underline{\beta}, \\ p_2 Pr\left(a_1 \tilde{S}_1 - c_0 \geq \tilde{S}_0\right) & \text{if } \beta \geq \underline{\beta}. \end{cases} \tag{4.5}$$

Next we proceed to prove $M^H(\beta) \leq M(\beta)$. Recall from (1.3) that

$$M(\beta) \doteq \begin{cases} \frac{1}{a_2} \mathbb{E}\left[\left(a_2 p_2 - \left(c_0 - a_1 \tilde{S}_1\right)^+\right)^+\right] & \text{if } 0 \leq \beta < \underline{\beta}, \\ \frac{1}{a_2}\left(\mathbb{E}\left[\left(a_2 p_2 + \min\left(\tilde{S}_0, a_1 \tilde{S}_1 - c_0\right)\right)^+\right] - \beta\right) & \text{if } \underline{\beta} \leq \beta < \bar{\beta}, \\ \frac{1}{a_2} \mathbb{E}\left[\left(a_2 p_2 - \left(\tilde{S}_0 + c_0 - a_1 \tilde{S}_1\right)^+\right)^+\right] & \text{if } \beta \geq \bar{\beta}, \end{cases} \tag{4.6}$$

We denote $M(\beta) = \mathbb{E}[m(\beta; \tilde{S}_0, \tilde{S}_1)]$. When $\beta < \underline{\beta}$,

$$m(\beta; S_0, S_1) = \begin{cases} p_2 & \text{if } a_1 S_1 - c_0 \geq 0, \\ \frac{1}{a_2}(a_1 S_1 - c_0 + a_2 p_2) & \text{if } -a_2 p_2 \leq a_1 S_1 - c_0 < 0, \\ 0 & \text{if } a_1 S_1 - c_0 < -a_2 p_2, \end{cases}$$

and $M^H(\beta) = p_2 Pr\left(a_1 \tilde{S}_1 - c_0 \geq 0\right)$. When $\beta \geq \bar{\beta}$,

$$m(\beta; S_0, S_1) = \begin{cases} p_2 & \text{if } a_1 S_1 - c_0 \geq S_0, \\ \frac{1}{a_2}(a_1 S_1 - c_0 - S_0 + a_2 p_2) & \text{if } S_0 - a_2 p_2 \leq a_1 S_1 - c_0 < S_0, \\ 0 & \text{if } a_1 S_1 - c_0 < S_0 - a_2 p_2, \end{cases}$$

and $M^H(\beta) = p_2 Pr\left(a_1 \tilde{S}_1 - c_0 \geq \tilde{S}_0\right)$. It is straightforward to check that $M^H(\beta) \leq M(\beta)$ in these two cases. When $\underline{\beta} \leq \beta < \bar{\beta}$, $M^H(\beta)$ is the same as that when $\beta \geq \bar{\beta}$; that is, $M^H(\beta) = p_2 Pr\left(a_1 \tilde{S}_1 - c_0 \geq \tilde{S}_0\right)$. From (4.6), we obtain that $\Delta V(\beta)$ is continuous and decreasing in β. When $\underline{\beta} \leq \beta < \bar{\beta}$, $M^H(\beta) \leq M(\beta)$ continues to hold in this case, which completes our proof. ■

Proof of Proposition 5: We provide the proof for each contract cost case (i.e., $\beta < \underline{\beta}$, $\underline{\beta} \leq \beta < \bar{\beta}$, and $\beta \geq \bar{\beta}$) separately.

Case (i), $\beta < \underline{\beta}$ **:** In this case, $\Delta V(\beta)$ can be written as $\left[\mathbb{E}[(\tilde{X} + a_2 p_2)^+] - \mathbb{E}[(\tilde{X})^+]\right] \frac{D_2}{a_2}$ where $\tilde{X} \doteq a_1 \tilde{S}_1 - c_0$. Here, $\tilde{X}$ has a Normal distribution with mean $\mu_X \doteq a_1 \mu_1 - c_0$ and standard deviation $\sigma_X \doteq a_1 \sigma_1$. Using the standard Normal loss function $L(\cdot)$, we obtain $\Delta V(\beta) = \sigma_X \left[L\left(\frac{-a_2 p_2 - \mu_X}{\sigma_X}\right) - L\left(\frac{-\mu_X}{\sigma_X}\right)\right] \frac{D_2}{a_2}$ and

$$\frac{\partial \Delta V(\beta)}{\partial \sigma_1} = a_1 \left[\phi\left(\frac{-a_2 p_2 - \mu_X}{\sigma_X}\right) - \phi\left(\frac{-\mu_X}{\sigma_X}\right)\right] \frac{D_2}{a_2}.$$

Because $\mu_X > \mu_0 > 0$ by assumption, $\phi\left(\frac{-a_2 p_2 - \mu_X}{\sigma_X}\right) < \phi\left(\frac{-\mu_X}{\sigma_X}\right)$, and thus,

$\frac{\partial \Delta V(\beta)}{\partial \sigma_1} < 0$. Because $\Delta V(\beta)$ only depends on $\tilde{S}_1$ and not $\tilde{S}_0$, $\frac{\partial \Delta V(\beta)}{\partial \sigma_0} = 0$ and $\frac{\partial \Delta V(\beta)}{\partial \rho} = 0$.

Case (ii), $\underline{\beta} \leq \beta < \bar{\beta}$ **:** In this case, $\Delta V(\beta)$ can be written as $\left[\mathbb{E}[(\tilde{X}_1)^+] - \mathbb{E}[(\tilde{X}_2)^+] - \beta\right] \frac{D_2}{a_2}$ where $\tilde{X}_1 \doteq a_1\tilde{S}_1 - c_0 + a_2p_2$ and $\tilde{X}_2 \doteq a_1\tilde{S}_1 - c_0 - \tilde{S}_0$. Here, $\tilde{X}_1$ has a Normal distribution with mean $\mu_{X_1} \doteq a_1\mu_1 - c_0 + a_2p_2$ and standard deviation $\sigma_{X_1} \doteq a_1\sigma_1$; $\tilde{X}_2$ has a Normal distribution with mean $\mu_{X_2} \doteq a_1\mu_1 - c_0 - \mu_0$ and standard deviation $\sigma_{X_2} \doteq \sqrt{\sigma_0^2 + a_1^2\sigma_1^2 - 2\rho a_1\sigma_0\sigma_1}$. Using the standard Normal loss function $L(\cdot)$, we obtain $\Delta V(\beta) = \left[\sigma_{X_1} L\left(\frac{-\mu_{X_1}}{\sigma_{X_1}}\right) - \sigma_{X_2} L\left(\frac{-\mu_{X_2}}{\sigma_{X_2}}\right) - \beta\right] \frac{D_2}{a_2}$.

For the impact of ρ, we have

$$\frac{\partial \Delta V(\beta)}{\partial \rho} = -\phi\left(\frac{-\mu_{X_2}}{\sigma_{X_2}}\right) \frac{\partial \sigma_{X_2}}{\partial \rho} \frac{D_2}{a_2} > 0.$$

For the impact of σ_0, we have

$$\frac{\partial \Delta V(\beta)}{\partial \sigma_0} = -\phi\left(\frac{-\mu_{X_2}}{\sigma_{X_2}}\right) \frac{\partial \sigma_{X_2}}{\partial \sigma_0} \frac{D_2}{a_2}.$$

Because $\frac{\partial \sigma_{X_2}}{\partial \sigma_0} = \frac{a_1\sigma_1 - \rho\sigma_0}{\sigma_{X_2}}$, it follows that when $\sigma_0 > a_1\rho\sigma_1$, $\frac{\partial \Delta V(\beta)}{\partial \sigma_0} < 0$; otherwise, $\frac{\partial \Delta V(\beta)}{\partial \sigma_0} \geq 0$.

For the impact of σ_1, we obtain

$$\frac{\partial \Delta V(\beta)}{\partial \sigma_1} = a_1 \left[\phi\left(\frac{-\mu_{X_1}}{\sigma_{X_1}}\right) - \phi\left(\frac{-\mu_{X_2}}{\sigma_{X_2}}\right)\left(\frac{a_1\sigma_1 - \rho\sigma_0}{\sigma_{X_2}}\right)\right] \frac{D_2}{a_2}.$$

When $\sigma_1 \leq \rho\sigma_0/a_1$, we have $\frac{\partial \Delta V(\beta)}{\partial \sigma_1} > 0$.

Case (iii), $\beta \geq \bar{\beta}$ **:** In this case, using $\tilde{X}_2 \doteq a_1\tilde{S}_1 - c_0 - \tilde{S}_0$ as defined in the previous case, $\Delta V(\beta)$ can be written as $\left[\mathbb{E}[(\tilde{X}_2 + a_2p_2)^+] - \mathbb{E}[(\tilde{X}_2)^+]\right] \frac{D_2}{a_2} = \sigma_{X_2}\left[L\left(\frac{-a_2p_2 - \mu_{X_2}}{\sigma_{X_2}}\right) - L\left(\frac{-\mu_{X_2}}{\sigma_{X_2}}\right)\right] \frac{D_2}{a_2}$.

For the impact of ρ, we obtain

$$\frac{\partial \Delta V(\beta)}{\partial \rho} = \frac{\partial \sigma_{X_2}}{\partial \rho} \left[\phi\left(\frac{-a_2p_2 - \mu_{X_2}}{\sigma_{X_2}}\right) - \phi\left(\frac{-\mu_{X_2}}{\sigma_{X_2}}\right)\right] \frac{D_2}{a_2}.$$

Because $\mu_{X_2} > 0$ by assumption, $\phi\left(\frac{-a_2p_2-\mu_{X_2}}{\sigma_{X_2}}\right) < \phi\left(\frac{-\mu_{X_2}}{\sigma_{X_2}}\right)$. Recall that $\sigma_{X_2} = \sqrt{\sigma_0^2 + a_1^2\sigma_1^2 - 2\rho a_1\sigma_0\sigma_1}$. Therefore, $\frac{\partial\sigma_{X_2}}{\partial\rho} < 0$, and thus, $\frac{\partial\Delta V(\beta)}{\partial\rho} > 0$.

For the impact of σ_0, we obtain

$$\frac{\partial\Delta V(\beta)}{\partial\sigma_0} = \frac{\partial\sigma_{X_2}}{\partial\sigma_0}\left[\phi\left(\frac{-a_2p_2-\mu_{X_2}}{\sigma_{X_2}}\right) - \phi\left(\frac{-\mu_{X_2}}{\sigma_{X_2}}\right)\right]\frac{D_2}{a_2}.$$

Because $\phi\left(\frac{-a_2p_2-\mu_{X_2}}{\sigma_{X_2}}\right) < \phi\left(\frac{-\mu_{X_2}}{\sigma_{X_2}}\right)$ and $\frac{\partial\sigma_{X_2}}{\partial\sigma_0} = \frac{a_1\sigma_1-\rho\sigma_0}{\sigma_{X_2}}$, it follows that $\frac{\partial\Delta V(\beta)}{\partial\sigma_0} < 0$ when $\sigma_0 > a_1\rho\sigma_1$ and $\frac{\partial\Delta V(\beta)}{\partial\sigma_0} \geq 0$ otherwise.

For the impact of σ_1, we obtain

$$\frac{\partial\Delta V(\beta)}{\partial\sigma_1} = \frac{\partial\sigma_{X_2}}{\partial\sigma_1}\left[\phi\left(\frac{-a_2p_2-\mu_{X_2}}{\sigma_{X_2}}\right) - \phi\left(\frac{-\mu_{X_2}}{\sigma_{X_2}}\right)\right]\frac{D_2}{a_2}.$$

Because $\phi\left(\frac{-a_2p_2-\mu_{X_2}}{\sigma_{X_2}}\right) < \phi\left(\frac{-\mu_{X_2}}{\sigma_{X_2}}\right)$ and $\frac{\partial\sigma_{X_2}}{\partial\sigma_1} = \frac{a_1(a_1\sigma_1-\sigma_0\rho)}{\sigma_{X_2}}$, it follows that $\frac{\partial\Delta V(\beta)}{\partial\sigma_1} < 0$ when $\sigma_1 > \sigma_0\rho/a_1$ and $\frac{\partial\Delta V(\beta)}{\partial\sigma_1} \geq 0$ otherwise. ∎

4.3 Proofs of Chapter 2

Proof of Proposition 6: We provide the proof for each contract cost case (i.e., $\beta < \underline{\beta}$, $\underline{\beta} \leq \beta < \bar{\beta}$, and $\beta \geq \bar{\beta}$) separately. Throughout the proof, for brevity, we use $\hat{e} \doteq e_0^b + e_0^p + a_3e_3^r + a_2e_2^l + a_1e_1^s$ where $\hat{e} > 0$.

Case (i), $\beta < \underline{\beta}$ **:** In this case, the characterization of change in total expected emissions after commercialization (i.e., $\Delta ECE(\beta)$) is given by (4.1) in Appendix 4.1. Let us denote (4.1) as a function of D_2; that is, $U(D_2)$, where

$$U(D_2) \doteq \begin{cases} \tau_1 D_2 & \text{for } 0 \leq D_2 < a_2\frac{D_1}{a_1} \\ \tau_2 D_2 + \kappa & \text{for } a_2\frac{D_1}{a_1} \leq D_2 \leq a_2K_0 \end{cases} \tag{4.7}$$

with

$$
\begin{aligned}
\tau_1 \doteq \left(e_2^s - e_2^l\right) \mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2}{a_1}\right)\right] \\
+ \frac{1}{a_2}\mathbb{E}\left[\left(\hat{e} - a_1\left(e_1^b + e_1^s\right)\chi(\tilde{S}_1 \le p_1)\right)\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \le \frac{c_0}{a_1}\right)\right],
\end{aligned} \tag{4.8}
$$

$$
\tau_2 \doteq \left(e_2^s - e_2^l\right) \mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2}{a_1}\right)\right] + \frac{1}{a_2}\mathbb{E}\left[\hat{e}\,\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \le \frac{c_0}{a_1}\right)\right], \tag{4.9}
$$

and $\kappa \doteq a_2\frac{D_1}{a_1}(\tau_1 - \tau_2)$. It is easy to establish that $\tau_2 \ge \tau_1$.

It follows from (4.7) that $U(0) = 0$. To characterize the sign of $U(D_2)$ for $D_2 \in (0, a_2K_0]$, we examine the signs of τ_1 and τ_2. Let $\tau_1(e_2^s)$ and $\tau_2(e_2^s)$ denote τ_1 and τ_2 in (4.8) and (4.9), respectively, as a function of e_2^s which are both increasing in e_2^s. We define

$$
\underline{e}_2^s \doteq e_2^l - \frac{\hat{e}\,\mathbb{E}\left[\chi\left(\frac{c_0 - a_2p_2}{a_1} < \tilde{S}_1 \le \frac{c_0}{a_1}\right)\right]}{a_2\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0-a_2p_2}{a_1}\right)\right]}, \tag{4.10}
$$

$$
\bar{e}_2^s \doteq e_2^l - \frac{\mathbb{E}\left[\left(\hat{e} - a_1(e_1^b + e_1^s)\chi(\tilde{S}_1 \le p_1)\right)\chi\left(\frac{c_0 - a_2p_2}{a_1} < \tilde{S}_1 \le \frac{c_0}{a_1}\right)\right]}{a_2\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0-a_2p_2}{a_1}\right)\right]}, \tag{4.11}
$$

where $\underline{e}_2^s \le \bar{e}_2^s$. We have $\tau_1(e_2^s) \le 0$ if and only if $e_2^s \le \bar{e}_2^s$, and $\tau_2(e_2^s) \le 0$ if and only if $e_2^s \le \underline{e}_2^s$. We now discuss three possible cases for the ordering between e_2^s and the two thresholds $\underline{e}_2^s$ and $\bar{e}_2^s$.

(a) For $e_2^s \le \underline{e}_2^s$, we have $\tau_1(e_2^s) < 0$ and $\tau_2(e_2^s) \le 0$. Therefore, $U(D_2) < 0$ in (4.7), and thus, $\Delta ECE(\beta) < 0$ for $D_2 \in (0, a_2K_2]$.

(b) For $e_2^s \ge \bar{e}_2^s$, we have $\tau_1(e_2^s) \ge 0$ and $\tau_2(e_2^s) > 0$. Therefore, $U(D_2) \ge 0$, and thus, $\Delta ECE(\beta) \ge 0$ (with equality holding when $e_2^s = \bar{e}_2^s$).

(c) For $\underline{e}_2^s < e_2^s < \bar{e}_2^s$, we have $\tau_1(e_2^s) < 0$ and $\tau_2(e_2^s) > 0$. Therefore, $U(D_2)$ strictly decreases in $D_2 \in (0, a_2D_1/a_1)$ and strictly increases in $D_2 \in (a_2D_1/a_1, a_2K_0]$. Without considering the upper bound a_2K_0 on D_2 (assumed in the model), there exists a unique

$$\bar{D}_2(e_2^s) \doteq a_2 \frac{D_1}{a_1} \left[1 - \frac{\tau_1(e_2^s)}{\tau_2(e_2^s)} \right] > a_2 D_1/a_1, \tag{4.12}$$

such that $U(\bar{D}_2(e_2^s)) = 0$. Therefore, $\Delta ECE(\beta) \le 0$ for $D_2 \le \bar{D}_2(e_2^s)$ and $\Delta ECE(\beta) > 0$ for $D_2 > \bar{D}_2(e_2^s)$. Taking the $K_0 \ge D_2/a_2$ assumption into account, we obtain the result in the proposition. Next we obtain the first and second derivatives of $\bar{D}_2(e_2^s)$ with respect to e_2^s:

$$\frac{\partial \bar{D}_2(e_2^s)}{\partial e_2^s} = -a_2 \frac{D_1}{a_1} \left[\frac{\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2}{a_1}\right)\right](\tau_2(e_2^s) - \tau_1(e_2^s))}{(\tau_2(e_2^s))^2} \right],$$

and

$$\frac{\partial^2 \bar{D}_2(e_2^s)}{\partial (e_2^s)^2} = a_2 \frac{D_1}{a_1} \left[\frac{2a_2 \left(\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2}{a_1}\right)\right]\right)^2 (\tau_2(e_2^s) - \tau_1(e_2^s))}{(\tau_2(e_2^s))^3} \right].$$

Because $\tau_1(e_2^s) < 0$ and $\tau_2(e_2^s) > 0$ for $\underline{e}_2^s < e_2^s < \bar{e}_2^s$, we have $\frac{\partial \bar{D}_2(e_2^s)}{\partial e_2^s} < 0$ and $\frac{\partial^2 \bar{D}_2(e_2^s)}{\partial (e_2^s)^2} > 0$. Moreover, from (4.12), when $e_2^s \to \bar{e}_2^{s-}$, $\tau_1(e_2^s) \to 0$, and thus, $\lim_{e_2^s \to \bar{e}_2^{s-}} \bar{D}_2(e_2^s) = a_2 \frac{D_1}{a_1}$. Once again from (4.12), when $e_2^s \to \underline{e}_2^{s+}$, $\tau_2(e_2^s) \to 0$, and thus, $\lim_{e_2^s \to \underline{e}_2^{s+}} \bar{D}_2(e_2^s) = \infty$.

Case (ii), $\underline{\beta} \le \beta < \bar{\beta}$: The proof for this case can be established in a similar fashion with case (i) with the following τ_1, τ_2, $\underline{e}_2^s$, and $\bar{e}_2^s$ expressions:

$$\begin{aligned}
\tau_1 \doteq & \left(e_2^s - e_2^l\right) \mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2}{a_1}\right)\right] \\
& + \frac{1}{a_2} \mathbb{E}\left[\left(\hat{e} - a_1\left(e_1^b + e_1^s\right)\chi(\tilde{S}_1 \le p_1)\right)\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \le \frac{c_0 + \tilde{S}_0}{a_1}\right)\right], \\
\tau_2 \doteq & \left(e_2^s - e_2^l\right) \mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2}{a_1}\right)\right] + \frac{1}{a_2} \mathbb{E}\left[\hat{e}\, \chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \le \frac{c_0 + \tilde{S}_0}{a_1}\right)\right],
\end{aligned}$$

and

$$\underline{e}_2^s \doteq e_2^l - \frac{\hat{e}\ \mathbb{E}\left[\chi\left(\frac{c_0-a_2p_2}{a_1} < \tilde{S}_1 \leq \frac{c_0+\tilde{S}_0}{a_1}\right)\right]}{a_2\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0-a_2p_2}{a_1}\right)\right]},$$

$$\bar{e}_2^s \doteq e_2^l - \frac{\mathbb{E}\left[\left(\hat{e} - a_1(e_1^b + e_1^s)\chi(\tilde{S}_1 \leq p_1)\right)\chi\left(\frac{c_0-a_2p_2}{a_1} < \tilde{S}_1 \leq \frac{c_0+\tilde{S}_0}{a_1}\right)\right]}{a_2\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0-a_2p_2}{a_1}\right)\right]}.$$

Case (iii), $\beta \geq \bar{\beta}$ **:** The proof for this case can be established in a similar fashion with case (i) with the following τ_1, τ_2, $\underline{e}_2^s$, and $\bar{e}_2^s$ expressions:

$$\begin{aligned}\tau_1 \doteq &\left(e_2^s - e_2^l\right)\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2p_2 + \tilde{S}_0}{a_1}\right)\right] \\ &+ \frac{1}{a_2}\mathbb{E}\left[\left(\hat{e} - a_1\left(e_1^b + e_1^s\right)\chi(\tilde{S}_1 \leq p_1)\right)\chi\left(\frac{c_0 - a_2p_2 + \tilde{S}_0}{a_1} < \tilde{S}_1 \leq \frac{c_0 + \tilde{S}_0}{a_1}\right)\right],\end{aligned}$$

$$\tau_2 \doteq \left(e_2^s - e_2^l\right)\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2p_2 + \tilde{S}_0}{a_1}\right)\right] + \frac{1}{a_2}\mathbb{E}\left[\hat{e}\ \chi\left(\frac{c_0 - a_2p_2 + \tilde{S}_0}{a_1} < \tilde{S}_1 \leq \frac{c_0 + \tilde{S}_0}{a_1}\right)\right],$$

and

$$\underline{e}_2^s \doteq e_2^l - \frac{\hat{e}\ \mathbb{E}\left[\chi\left(\frac{c_0-a_2p_2+\tilde{S}_0}{a_1} < \tilde{S}_1 \leq \frac{c_0+\tilde{S}_0}{a_1}\right)\right]}{a_2\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0-a_2p_2+\tilde{S}_0}{a_1}\right)\right]},$$

$$\bar{e}_2^s \doteq e_2^l - \frac{\mathbb{E}\left[\left(\hat{e} - a_1(e_1^b + e_1^s)\chi(\tilde{S}_1 \leq p_1)\right)\chi\left(\frac{c_0-a_2p_2+\tilde{S}_0}{a_1} < \tilde{S}_1 \leq \frac{c_0+\tilde{S}_0}{a_1}\right)\right]}{a_2\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0-a_2p_2+\tilde{S}_0}{a_1}\right)\right]}.$$

■

Proof of Proposition 7: We provide the proof for each contract cost case (i.e., $\beta < \underline{\beta}$, $\underline{\beta} \leq \beta < \bar{\beta}$, and $\beta \geq \bar{\beta}$) separately.

Case (i), $\beta < \underline{\beta}$ **:** Let $\underline{e}_2^s(e_2^l)$ and $\bar{e}_2^s(e_2^l)$ denote $\underline{e}_2^s$ and $\bar{e}_2^s$ in (4.10) and (4.11), respectively, as a function of e_2^l. Because $\hat{e} = e_0^b + e_0^p + a_3e_3^r + a_2e_2^l + a_1e_1^s > 0$, we have $\underline{e}_2^s(e_2^l) < e_2^l$. From (4.11), we observe that $\bar{e}_2^s(e_2^l) = Be_2^l - C$ where

$$B \doteq 1 - \frac{\mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \le \frac{c_0}{a_1}\right)\right]}{\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2}{a_1}\right)\right]},$$

$$C \doteq \frac{\mathbb{E}\left[\left(e_0^b + e_0^p + a_3 e_3^r + a_1 e_1^s - a_1(e_1^b + e_1^s)\chi(\tilde{S}_1 \le p_1)\right)\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \le \frac{c_0}{a_1}\right)\right]}{a_2 \mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2}{a_1}\right)\right]},$$

with $0 < B < 1$. To compare $\bar{e}_2^s(e_2^l)$ and e_2^l, we examine the sign of $e_2^l - \bar{e}_2^s(e_2^l) = (1-B)e_2^l + C$. If $C \ge 0$, then $(1-B)e_2^l + C \ge 0$, and thus, $e_2^l \ge \bar{e}_2^s(e_2^l)$. If $C < 0$, then there exists a unique $\check{e}_2^l$ such that when $e_2^l \ge \check{e}_2^l$, we have $\bar{e}_2^s(e_2^l) \le e_2^l$; and when $e_2^l < \check{e}_2^l$, we have $\bar{e}_2^s(e_2^l) > e_2^l$ where

$$\check{e}_2^l \doteq -\frac{C}{1-B} = -\frac{\mathbb{E}\left[\left(e_0^b + e_0^p + a_3 e_3^r + a_1 e_1^s - a_1(e_1^b + e_1^s)\chi(\tilde{S}_1 \le p_1)\right)\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \le \frac{c_0}{a_1}\right)\right]}{a_2 \mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \le \frac{c_0}{a_1}\right)\right]}.$$

Because $e_2^l > 0$ by definition, we define $\hat{e}_2^l \doteq (\check{e}_2^l)^+$. Therefore, it follows that $\bar{e}_2^s(e_2^l) > e_2^l$ if $e_2^l < \hat{e}_2^l$ and $\bar{e}_2^s(e_2^l) \le e_2^l$ otherwise.

Case (ii), $\underline{\beta} \le \beta < \bar{\beta}$ **:** The proof for this case can be established in a similar fashion with case (i) with the following B, C and $\hat{e}_2^l$ expressions:

$$B \doteq 1 - \frac{\mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \le \frac{c_0 + \tilde{S}_0}{a_1}\right)\right]}{\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2}{a_1}\right)\right]},$$

$$C \doteq \frac{\mathbb{E}\left[\left(e_0^b + e_0^p + a_3 e_3^r + a_1 e_1^s - a_1(e_1^b + e_1^s)\chi(\tilde{S}_1 \le p_1)\right)\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \le \frac{c_0 + \tilde{S}_0}{a_1}\right)\right]}{a_2 \mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2}{a_1}\right)\right]},$$

and

$$\hat{e}_2^l \doteq \left(-\frac{\mathbb{E}\left[\left(e_0^b + e_0^p + a_3 e_3^r + a_1 e_1^s - a_1(e_1^b + e_1^s)\chi(\tilde{S}_1 \le p_1)\right)\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \le \frac{c_0 + \tilde{S}_0}{a_1}\right)\right]}{a_2 \mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \le \frac{c_0 + \tilde{S}_0}{a_1}\right)\right]}\right)^+.$$

Case (iii), $\beta \geq \bar{\beta}$ **:** The proof for this case can be established in a similar fashion with case (i) with the following B, C and $\hat{e}_2^l$ expressions:

$$B \doteq 1 - \frac{\mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1} < \tilde{S}_1 \leq \frac{c_0 + \tilde{S}_0}{a_1}\right)\right]}{\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1}\right)\right]},$$

$$C \doteq \frac{\mathbb{E}\left[\left(e_0^b + e_0^p + a_3 e_3^r + a_1 e_1^s - a_1(e_1^b + e_1^s)\chi(\tilde{S}_1 \leq p_1)\right)\chi\left(\frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1} < \tilde{S}_1 \leq \frac{c_0 + \tilde{S}_0}{a_1}\right)\right]}{a_2 \mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1}\right)\right]},$$

and

$$\hat{e}_2^l \doteq \left(-\frac{\mathbb{E}\left[\left(e_0^b + e_0^p + a_3 e_3^r + a_1 e_1^s - a_1(e_1^b + e_1^s)\chi(\tilde{S}_1 \leq p_1)\right)\chi\left(\frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1} < \tilde{S}_1 \leq \frac{c_0 + \tilde{S}_0}{a_1}\right)\right]}{a_2 \mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1} < \tilde{S}_1 \leq \frac{c_0 + \tilde{S}_0}{a_1}\right)\right]}\right)^+ .$$

■

Proof of Proposition 8: We only provide the proof for low contract cost case (i.e., $\beta < \underline{\beta}$). The proofs for the moderate (i.e., $\underline{\beta} \leq \beta < \bar{\beta}$) and high (i.e., $\beta \geq \bar{\beta}$) contract cost cases can be established in a similar fashion. In the low contract cost case, using the characterization of $\Delta ECE(\beta)$ as given by (4.1) in Appendix 4.1 and taking its first derivative with respect to D_2, we obtain

$$\begin{aligned}\frac{\partial \Delta ECE(\beta)}{\partial D_2} &= \left(e_2^s - e_2^l\right)\mathbb{E}\left[\chi\left(\tilde{S}_1 > \frac{c_0 - a_2 p_2}{a_1}\right)\right] \qquad (4.13)\\ &+ \left(e_0^b + e_0^p + a_3 e_3^r + a_2 e_2^l + a_1 e_1^s\right)\frac{1}{a_2}\mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \leq \frac{c_0}{a_1}\right)\right]\\ &- \left(e_1^b + e_1^s\right)\frac{a_1}{a_2}\chi\left(\frac{D_2}{a_2} < \frac{D_1}{a_1}\right)\mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2}{a_1} < \tilde{S}_1 \leq \frac{c_0}{a_1}\right)\chi(\tilde{S}_1 \leq p_1)\right].\end{aligned}$$

Let us denote (4.13) as a function of D_2; that is, $J(D_2)$ where

$$J(D_2) \doteq \begin{cases} \tau_1 & \text{for } 0 \leq D_2 < a_2 \frac{D_1}{a_1} \\ \tau_2 & \text{for } a_2 \frac{D_1}{a_1} \leq D_2 \leq a_2 K_0 \end{cases} \qquad (4.14)$$

with τ_1 and τ_2 as defined in (4.8) and (4.9) in the proof of Proposition 6. To characterize the sign of $J(D_2)$ in (4.14), we examine the signs of τ_1 and τ_2. Similar to the proof of Proposition 6, we define τ_1 and τ_2 as functions of e_2^s; that is, $\tau_1(e_2^s)$ and $\tau_2(e_2^s)$. We have already established in the proof of Proposition 6 that $\tau_1(e_2^s) \leq 0$ if and only if $e_2^s \leq \bar{e}_2^s$, and $\tau_2(e_2^s) \leq 0$ if and only if $e_2^s \leq \underline{e}_2^s$ where $\underline{e}_2^s$ and $\bar{e}_2^s$ are as defined in (4.10) and (4.11), respectively. We now discuss three possible cases for the ordering between e_2^s and the two thresholds $\underline{e}_2^s$ and $\bar{e}_2^s$.

(a) For $e_2^s \leq \underline{e}_2^s$, we have $\tau_1(e_2^s) < 0$ and $\tau_2(e_2^s) \leq 0$. Therefore, $J(D_2) < 0$ for $D_2 \in (0, a_2K_2]$.

(b) For $e_2^s \geq \bar{e}_2^s$, we have $\tau_1(e_2^s) \geq 0$ and $\tau_2(e_2^s) > 0$. Therefore, $J(D_2) \geq 0$ for $D_2 \in (0, a_2K_2]$.

(c) For $\underline{e}_2^s < e_2^s < \bar{e}_2^s$, we have $\tau_1(e_2^s) < 0$ and $\tau_2(e_2^s) > 0$. Therefore, $J(D_2) < 0$ for $D_2 \in (0, a_2D_1/a_1)$ and $J(D_2) > 0$ $D_2 \in (a_2D_1/a_1, a_2K_0]$. Taking the processing capacity K_0 into account, we obtain the result in the proposition. ■

Proof of Proposition 9: We first focus on local sensitivity analysis in which the optimal contracting decisions before and after commercialization are not affected by the changes in biomass price p_2; that is, the ordering between the unit contract cost β and the cost thresholds $\underline{\beta} = \mathbb{E}[\min(\tilde{S}_0, (a_1\tilde{S}_1 - c_0)^+)]$ and $\bar{\beta} = \mathbb{E}[\min(\tilde{S}_0, (a_1\tilde{S}_1 + a_2p_2 - c_0)^+)]$ does not change as p_2 changes. We provide the proof for each contract cost case (i.e., $\beta < \underline{\beta}$, $\underline{\beta} \leq \beta < \bar{\beta}$, and $\beta \geq \bar{\beta}$) separately. It is easy to establish that $\bar{\beta}$ increases in p_2; therefore, as p_2 increases it is possible to have a transition from the high contract cost case to the moderate contract cost case. We finish the proof by considering this case. Throughout the proof, we use $f_1(\cdot)$ and $f_{1|0}(\cdot)$ to denote the probability density function of output spot price $\tilde{S}_1$ and the same conditional on the input spot price S_0, respectively. For brevity, we also use $\hat{e} \doteq e_0^b + e_0^p + a_3e_3^r + a_2e_2^l + a_1e_1^s$.

Case (i), $\beta < \underline{\beta}$ **:** In this case, using the characterization of $\Delta ECE(\beta)$ as given by (4.1) in Appendix 4.1 and taking its first derivative with respect to p_2, we obtain

$$\begin{aligned}\frac{\partial \Delta ECE(\beta)}{\partial p_2} &= \left(e_2^s - e_2^l\right) D_2 \frac{a_2}{a_1} f_1\left(\frac{c_0 - a_2 p_2}{a_1}\right) \\ &+ \hat{e}\frac{D_2}{a_1} f_1\left(\frac{c_0 - a_2 p_2}{a_1}\right) \\ &- \left(e_1^b + e_1^s\right) \min\left(D_1, a_1\frac{D_2}{a_2}\right) \frac{a_2}{a_1} f_1\left(\frac{c_0 - a_2 p_2}{a_1}\right) \chi\left(\frac{c_0 - a_2 p_2}{a_1} < p_1\right).\end{aligned} \tag{4.15}$$

Let us denote (4.15) as a function of D_2; that is, $P(D_2)$ where

$$P(D_2) \doteq \begin{cases} \gamma_1 D_2 & \text{for } 0 \leq D_2 < a_2\frac{D_1}{a_1} \\ \gamma_2 D_2 + \eta & \text{for } a_2\frac{D_1}{a_1} \leq D_2 \leq a_2 K_0 \end{cases} \tag{4.16}$$

with

$$\gamma_1 \doteq \left(e_2^s - e_2^l + \frac{1}{a_2}\left[\hat{e} - a_1\left(e_1^b + e_1^s\right)\chi\left(\frac{c_0 - a_2 p_2}{a_1} < p_1\right)\right]\right)\left[\frac{a_2}{a_1} f_1\left(\frac{c_0 - a_2 p_2}{a_1}\right)\right], \tag{4.17}$$

$$\gamma_2 \doteq \left(e_2^s - e_2^l + \frac{1}{a_2}\hat{e}\right)\left[\frac{a_2}{a_1} f_1\left(\frac{c_0 - a_2 p_2}{a_1}\right)\right], \tag{4.18}$$

and $\eta \doteq a_2\frac{D_1}{a_1}(\gamma_1 - \gamma_2)$. Since $a_2 p_2 < c_0$ by assumption, $f_1\left(\frac{c_0 - a_2 p_2}{a_1}\right) > 0$. It is easy to establish that $\gamma_2 \geq \gamma_1$.

It follows from (4.16) that $P(0) = 0$. To characterize the sign of $P(D_2)$ for $D_2 \in (0, a_2 K_0]$, we examine the signs of γ_1 and γ_2. Let $\gamma_1(e_2^s)$ and $\gamma_2(e_2^s)$ denote γ_1 and γ_2 in (4.17) and (4.18), respectively, as a function of e_2^s which are both increasing in e_2^s. We define

$$\underline{\underline{e}}_2^s \doteq e_2^l - \frac{1}{a_2}\hat{e}, \tag{4.19}$$

$$\bar{\bar{e}}_2^s \doteq e_2^l - \frac{1}{a_2}\left[\hat{e} - a_1\left(e_1^b + e_1^s\right)\chi\left(\frac{c_0 - a_2 p_2}{a_1} < p_1\right)\right]. \tag{4.20}$$

where $\underline{\underline{e}}_2^s \leq \bar{\bar{e}}_2^s$. We have $\gamma_1(e_2^s) \leq 0$ if and only if $e_2^s \leq \bar{\bar{e}}_2^s$, and $\gamma_2(e_2^s) \leq 0$ if and only if $e_2^s \leq \underline{\underline{e}}_2^s$. We now discuss three possible cases for the ordering between e_2^s and the two thresholds $\underline{\underline{e}}_2^s$ and $\bar{\bar{e}}_2^s$.

(a) For $e_2^s \leq \underline{\underline{e}}_2^s$, we have $\gamma_1(e_2^s) < 0$ and $\gamma_2(e_2^s) \leq 0$. Therefore, $P(D_2) < 0$ in (4.16), and thus, $\frac{\partial \Delta ECE(\beta)}{\partial p_2} < 0$ for $D_2 \in (0, a_2 K_2]$.

(b) For $e_2^s \geq \bar{\bar{e}}_2^s$, we have $\gamma_1(e_2^s) \geq 0$ and $\gamma_2(e_2^s) > 0$. Therefore, $P(D_2) \geq 0$, and thus, $\frac{\partial \Delta ECE(\beta)}{\partial p_2} \geq 0$ (with equality holding when $e_2^s = \bar{e}_2^s$).

(c) For $\underline{\underline{e}}_2^s < e_2^s < \bar{\bar{e}}_2^s$, we have $\gamma_1(e_2^s) < 0$ and $\gamma_2(e_2^s) > 0$. Therefore, $P(D_2)$ strictly decreases in $D_2 \in (0, a_2 D_1/a_1)$ and strictly increases in $D_2 \in (a_2 D_1/a_1, a_2 K_0]$. Without considering the upper bound $a_2 K_0$ on D_2 (assumed in the model), there exists a unique

$$\bar{\bar{D}}_2(e_2^s) \doteq a_2 \frac{D_1}{a_1}\left[1 - \frac{\gamma_1(e_2^s)}{\gamma_2(e_2^s)}\right] > a_2 D_1/a_1, \tag{4.21}$$

such that $P(\bar{\bar{D}}_2(e_2^s)) = 0$. Therefore, $\frac{\partial \Delta ECE(\beta)}{\partial p_2} \leq 0$ for $D_2 \leq \bar{\bar{D}}_2(e_2^s)$ and $\frac{\partial \Delta ECE(\beta)}{\partial p_2} > 0$ for $D_2 > \bar{\bar{D}}_2(e_2^s)$. Taking the $K_0 \geq D_2/a_2$ assumption into account, we obtain the result in the proposition.

Case (ii), $\underline{\beta} \leq \beta < \bar{\beta}$ **:** The proof for this case is identical to case (i).

Case (iii), $\beta \geq \bar{\beta}$ **:** The proof for this case can be established in a similar fashion with case (i) with the following γ_1, γ_2, $\underline{\underline{e}}_2^s$, and $\bar{\bar{e}}_2^s$ expressions:

$$\begin{aligned}
\gamma_1 \doteq & \left(e_2^s - e_2^l + \frac{1}{a_2}\hat{e}\right)\left(\frac{a_2}{a_1}\mathbb{E}_{\tilde{S}_0}\left[f_{1|0}\left(\frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1}\right)\right]\right) \\
& - \left(e_1^b + e_1^s\right)\frac{a_1}{a_2}\left(\frac{a_2}{a_1}\mathbb{E}_{\tilde{S}_0}\left[f_{1|0}\left(\frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1}\right)\chi\left(\frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1} < p_1\right)\right]\right), \\
\gamma_2 \doteq & \left(e_2^s - e_2^l + \frac{1}{a_2}\hat{e}\right)\left(\frac{a_2}{a_1}\mathbb{E}_{\tilde{S}_0}\left[f_{1|0}\left(\frac{c_0 - a_2 p_2 + \tilde{S}_0}{a_1}\right)\right]\right),
\end{aligned}$$

and

$$
\begin{aligned}
\underline{\underline{e}}_2^s &\doteq e_2^l - \frac{1}{a_2}\hat{e}, \\
\bar{\bar{e}}_2^s &\doteq e_2^l - \frac{1}{a_2}\left[\hat{e} - \frac{a_1\left(e_1^b + e_1^s\right)\mathbb{E}_{\tilde{S}_0}\left[f_{1|0}\left(\frac{c_0 - a_2p_2 + \tilde{S}_0}{a_1}\right)\chi\left(\frac{c_0 - a_2p_2 + \tilde{S}_0}{a_1} < p_1\right)\right]}{\mathbb{E}_{\tilde{S}_0}\left[f_{1|0}\left(\frac{c_0 - a_2p_2 + \tilde{S}_0}{a_1}\right)\right]}\right].
\end{aligned}
$$

We now examine the case in which increasing p_2 leads to a transition from the high contract cost case (i.e., $\beta \geq \bar{\beta}$) to the moderate contract cost case (i.e., $\underline{\beta} \leq \beta < \bar{\beta}$). Let $\bar{\beta}(p_2)$ denote $\bar{\beta}$ thresholds as a function of p_2. We consider two biomass prices $p_2^0 < p_2^1$ such that $\bar{\beta}(p_2^0) < \beta < \bar{\beta}(p_2^1)$. Let $\Delta ECE(p_2; \beta)$ denote $\Delta ECE(\beta)$ for a given p_2. To characterize the impact of increasing p_2, we examine the sign of $\Delta ECE(p_2^1; \beta) - \Delta ECE(p_2^0; \beta)$ where the former and the latter are given by (4.2) and (4.3) in Appendix 4.1, respectively. Let us denote $\Delta ECE(p_2^1; \beta) - \Delta ECE(p_2^0; \beta)$ as a function of D_2; that is, $N(D_2)$, where

$$
N(D_2) \doteq \begin{cases} \gamma_1 D_2 & \text{for } 0 \leq D_2 < a_2\frac{D_1}{a_1} \\ \gamma_2 D_2 + \eta & \text{for } a_2\frac{D_1}{a_1} \leq D_2 \leq a_2 K_0 \end{cases} \tag{4.22}
$$

with

$$
\begin{aligned}
\gamma_1 \doteq {} & \left(e_2^s - e_2^l + \frac{1}{a_2}\hat{e}\right)\mathbb{E}\left[\chi\left(\frac{c_0 - a_2p_2^1}{a_1} < \tilde{S}_1 \leq \frac{c_0 - a_2p_2^0 + \tilde{S}_0}{a_1}\right)\right] && (4.23) \\
& - \left(e_1^b + e_1^s\right)\frac{a_1}{a_2}\mathbb{E}\left[\chi\left(\frac{c_0 - a_2p_2^1}{a_1} < \tilde{S}_1 \leq \frac{c_0 - a_2p_2^0 + \tilde{S}_0}{a_1}\right)\chi(\tilde{S}_1 \leq p_1)\right], \\
\gamma_2 \doteq {} & \left(e_2^s - e_2^l + \frac{1}{a_2}\hat{e}\right)\mathbb{E}\left[\chi\left(\frac{c_0 - a_2p_2^1}{a_1} < \tilde{S}_1 \leq \frac{c_0 - a_2p_2^0 + \tilde{S}_0}{a_1}\right)\right], && (4.24)
\end{aligned}
$$

and $\eta \doteq a_2\frac{D_1}{a_1}(\gamma_1 - \gamma_2)$. It is easy to establish that $\gamma_2 \geq \gamma_1$.

It follows from (4.22) that $N(0) = 0$. To characterize the sign of $N(D_2)$ for $D_2 \in (0, a_2K_0]$, we examine the signs of γ_1 and γ_2. Let $\gamma_1(e_2^s)$ and $\gamma_2(e_2^s)$ denote γ_1

and γ_2 in (4.23) and (4.24), respectively, as a function of e_2^s which are both increasing in e_2^s. We define

$$\underline{\underline{e}}_2^s \doteq e_2^l - \frac{1}{a_2}\hat{e},$$

$$\bar{\bar{e}}_2^s \doteq e_2^l - \frac{1}{a_2}\left[\hat{e} - \frac{a_1\left(e_1^b + e_1^s\right)\mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2^1}{a_1} < \tilde{S}_1 \leq \frac{c_0 - a_2 p_2^0 + \tilde{S}_0}{a_1}\right)\chi(\tilde{S}_1 \leq p_1)\right]}{\mathbb{E}\left[\chi\left(\frac{c_0 - a_2 p_2^1}{a_1} < \tilde{S}_1 \leq \frac{c_0 - a_2 p_2^0 + \tilde{S}_0}{a_1}\right)\right]}\right],$$

where $\underline{\underline{e}}_2^s \leq \bar{\bar{e}}_2^s$. We have $\gamma_1(e_2^s) \leq 0$ if and only if $e_2^s \leq \bar{\bar{e}}_2^s$, and $\gamma_2(e_2^s) \leq 0$ if and only if $e_2^s \leq \underline{\underline{e}}_2^s$. We now discuss three possible cases for the ordering between e_2^s and the two thresholds $\underline{\underline{e}}_2^s$ and $\bar{\bar{e}}_2^s$.

(a) For $e_2^s \leq \underline{\underline{e}}_2^s$, we have $\gamma_1(e_2^s) < 0$ and $\gamma_2(e_2^s) \leq 0$. Therefore, $N(D_2) < 0$ in (4.22), and thus, $\frac{\partial \Delta ECE(\beta)}{\partial p_2} < 0$ for $D_2 \in (0, a_2 K_2]$.

(b) For $e_2^s \geq \bar{\bar{e}}_2^s$, we have $\gamma_1(e_2^s) \geq 0$ and $\gamma_2(e_2^s) > 0$. Therefore, $N(D_2) \geq 0$, and thus, $\frac{\partial \Delta ECE(\beta)}{\partial p_2} \geq 0$ (with equality holding when $e_2^s = \bar{e}_2^s$).

(c) For $\underline{\underline{e}}_2^s < e_2^s < \bar{\bar{e}}_2^s$, we have $\gamma_1(e_2^s) < 0$ and $\gamma_2(e_2^s) > 0$. Therefore, $N(D_2)$ strictly decreases in $D_2 \in (0, a_2 D_1/a_1)$ and strictly increases in $D_2 \in (a_2 D_1/a_1, a_2 K_0]$. Without considering the upper bound $a_2 K_0$ on D_2 (assumed in the model), there exists a unique

$$\bar{\bar{D}}_2(e_2^s) \doteq a_2 \frac{D_1}{a_1}\left[1 - \frac{\gamma_1(e_2^s)}{\gamma_2(e_2^s)}\right] > a_2 D_1/a_1, \tag{4.25}$$

such that $N(\bar{\bar{D}}_2(e_2^s)) = 0$. Therefore, $\frac{\partial \Delta ECE(\beta)}{\partial p_2} \leq 0$ for $D_2 \leq \bar{\bar{D}}_2(e_2^s)$ and $\frac{\partial \Delta ECE(\beta)}{\partial p_2} > 0$ for $D_2 > \bar{\bar{D}}_2(e_2^s)$. Taking the $K_0 \geq D_2/a_2$ assumption into account concludes the characterization. ■

4.4 Proofs of Chapter 3

Proof of Proposition 10: The result follows from Proposition 16 by using $D_2 = 0$.

■

Proof of Proposition 11: The result follows from Proposition 17 by using $D_2 = 0$.

■

Proof of Proposition 12: Recall that $A^2 = A^1 + \Delta_A$ and $\beta^1 = \beta^2 + \Delta_\beta$. Using Proposition 11, we characterize the optimal contract volumes (Q^{1*}, Q^{2*}) within the (Δ_A, Δ_β) space for given A^1 and β^2. To this end, we need to transform the conditions regarding expected marginal profits (i.e., $\mathcal{M}^i_\kappa$) for $\kappa \in \{0,1\}$ and $i \in \{Y, Z\}$ in Proposition 11 into conditions with respect to Δ_A and Δ_β. Recall that the conditions regarding expected marginal profits (i.e., $\mathcal{M}^i_\kappa$) consists of two levels of implications: the preference (which contract gives larger expected marginal profit given (Q^1, Q^2)) and the profitability (whether the preferred contract gives positive expected marginal profit). From conditions in Proposition 11, we obtain two sets of thresholds: preference thresholds and profitability thresholds.

For expositional convenience, we denote $\mathcal{M}^i_\kappa \doteq G(A^i, \max(h_\kappa(\tilde{S}_1), A^i)) - \beta^i - A^i$ for $\kappa \in \{0,1\}$ and $i \in \{1,2\}$, where $G(\theta_1, \theta_2) \doteq \mathbb{E}_{\tilde{S}_1}[g(\theta_1, \theta_2)]$ and

$$g(\theta_1, \theta_2) \doteq \int_0^{\theta_1} \theta_1 dF_{0|1}(\tilde{S}_0) + \int_{\theta_1}^{\theta_2} \tilde{S}_0 dF_{0|1}(\tilde{S}_0) + \int_{\theta_2}^{\infty} \theta_2 dF_{0|1}(\tilde{S}_0), \tag{4.26}$$

for $\theta_1 \leq \theta_2$. $F_{0|1}(.)$ is the cumulative probability function of input spot price $\tilde{S}_0$ conditional on the output spot price S_1.

Let $\Delta^L_\beta(\Delta_A)$ and $\Delta^M_\beta(\Delta_A)$ denote the preference thresholds when the commodity output has been satisfied and not satisfied, respectively. For given A^1 and β^2, by solving $\mathcal{M}^1_0 = \mathcal{M}^2_0$ and $\mathcal{M}^1_1 = \mathcal{M}^2_1$, we obtain the preference thresholds for given Δ_A as follows:

$$\Delta^L_\beta(\Delta_A) \doteq G(A^1, \max(h_0(\tilde{S}_1), A^1)) - G(A^1 + \Delta_A, \max(h_0(\tilde{S}_1), A^1 + \Delta_A)) + \Delta_A, \tag{4.27}$$

$$\Delta^M_\beta(\Delta_A) \doteq G(A^1, \max(h_1(\tilde{S}_1), A^1)) - G(A^1 + \Delta_A, \max(h_1(\tilde{S}_1), A^1 + \Delta_A)) + \Delta_A.$$

Let $\Delta^{(1)}_A$ and $\Delta^{(2)}_A$ denote the profitability thresholds that make contract 2 profitable to be reserved when the commodity output has been satisfied and not satisfied, respectively. For given A^1 and β^2, by solving $\mathcal{M}_0^2 = 0$ and $\mathcal{M}_1^2 = 0$, we obtain the profitability thresholds as the unique solutions from the following equations:

$$G(A^1 + \Delta_A^{(1)}, \max(h_0(\tilde{S}_1), A^1 + \Delta_A^{(1)})) = \beta^2 + A^1 + \Delta_A^{(1)}, \tag{4.28}$$
$$G(A^1 + \Delta_A^{(2)}, \max(h_1(\tilde{S}_1), A^1 + \Delta_A^{(2)})) = \beta^2 + A^1 + \Delta_A^{(2)}.$$

Let $\Delta_\beta^{(1)}$ and $\Delta_\beta^{(2)}$ denote the profitability thresholds that make contract 1 profitable to be reserved when the commodity output has been satisfied and not satisfied. For given A^1 and β^2, by solving $\mathcal{M}_0^1 = 0$ and $\mathcal{M}_1^1 = 0$, we obtain the profitability thresholds as follows:

$$\Delta_\beta^{(1)} \doteq G(A^1, \max(h_0(\tilde{S}_1), A^1)) - \beta^2 - A^1, \tag{4.29}$$
$$\Delta_\beta^{(2)} \doteq G(A^1, \max(h_1(\tilde{S}_1), A^1)) - \beta^2 - A^1.$$

■

Proof of Proposition 13: The result follows from Proposition 19 by using $D_2 = 0$.

■

Proof of Lemma 1: Recall from (4.27)-(4.28) that the preference thresholds $\Delta_\beta^\ell(\Delta_A)$ for $\ell \in \{L, M\}$, and profitability thresholds $\Delta_A^{(i)}$ and $\Delta_\beta^{(i)}$ for $i \in \{1, 2\}$ are dependent on $G(.,.)$. To study the impact of ρ on these thresholds, we first examine the impact of ρ on $G(.,.)$. Because the $G(.,.)$ functions in (4.27)-(4.28) are structurally the same, we focus on $G(A^1, \max(h_0(\tilde{S}_1), A^1))$ in the proof without loss of generality. After that, we investigate the impact of ρ on these thresholds. In particular, we study the impact of ρ on $\Delta_\beta^L(\Delta_A)$, $\Delta_A^{(1)}$, and $\Delta_\beta^{(1)}$ in this proof without loss of generality; the proof for other thresholds can be established in a similar fashion.

Recall that $G(\theta_1, \theta_2) = \mathbb{E}_{\tilde{S}_1}[g(\theta_1, \theta_2)]$ and

$$g(\theta_1, \theta_2) = \int_0^{\theta_1} \theta_1 dF_{0|1}(\tilde{S}_0) + \int_{\theta_1}^{\theta_2} \tilde{S}_0 dF_{0|1}(\tilde{S}_0) + \int_{\theta_2}^{\infty} \theta_2 dF_{0|1}(\tilde{S}_0).$$

Given a output spot price S_1, the input spot price $\tilde{S}_0$ is normally distributed with mean $\mu_{0|1} \doteq \mu_0 + \rho\frac{\sigma_0}{\sigma_1}(S_1 - \mu_1)$ and standard deviation $\sigma_{0|1} \doteq \sigma_0\sqrt{1-\rho^2}$. It is easy to establish that $g(\theta_1, \theta_2) = \theta_2 + \sigma_{0|1}\left[\bar{L}\left(\frac{\theta_1-\mu_{0|1}}{\sigma_{0|1}}\right) - \bar{L}\left(\frac{\theta_2-\mu_{0|1}}{\sigma_{0|1}}\right)\right]$, where $\bar{L}(t) = \int_{-\infty}^{t}(t-x)\phi(x)dx$ is the standard Normal complementary loss function. We can further obtain $\frac{\partial g(\theta_1,\theta_2)}{\partial \mu_{0|1}} = \Phi\left(\frac{\theta_2-\mu_{0|1}}{\sigma_{0|1}}\right) - \Phi\left(\frac{\theta_1-\mu_{0|1}}{\sigma_{0|1}}\right)$ and $\frac{\partial g(\theta_1,\theta_2)}{\partial \sigma_{0|1}} = \phi\left(\frac{\theta_1-\mu_{0|1}}{\sigma_{0|1}}\right) - \phi\left(\frac{\theta_2-\mu_{0|1}}{\sigma_{0|1}}\right)$ by using identities $\bar{L}'(t) = \Phi(t)$ and $\bar{L}(t) = t\Phi(t) + \phi(t)$. Letting $\theta_1 = A^1$, $\theta_2 = \max(h_0(S_1), A^1)$, and taking expectation over output spot price $\tilde{S}_1$, we obtain

$$\begin{aligned}
&G(A^1, \max(h_0(\tilde{S}_1), A^1)) \\
&\quad = \mathbb{E}_{\tilde{S}_1}\left[\max(h_0(\tilde{S}_1), A^1) + \sigma_{0|1}\left[\bar{L}\left(\frac{A^1-\mu_{0|1}}{\sigma_{0|1}}\right) - \bar{L}\left(\frac{\max(h_0(\tilde{S}_1), A^1)-\mu_{0|1}}{\sigma_{0|1}}\right)\right]\right].
\end{aligned}$$

Because $g(\theta_1, \theta_2)$ satisfies Lipshitz condition of order one with respect to $\mu_{0|1}$ and $\sigma_{0|1}$, expectation and differentiation operators can be interchanged, and thus, $\frac{\partial G(\theta_1,\theta_2)}{\partial \mu_{0|1}} = \mathbb{E}_{\tilde{S}_1}\left[\frac{\partial g(\theta_1,\theta_2)}{\partial \mu_{0|1}}\right]$ and $\frac{\partial G(\theta_1,\theta_2)}{\partial \sigma_{0|1}} = \mathbb{E}_{\tilde{S}_1}\left[\frac{\partial g(\theta_1,\theta_2)}{\partial \sigma_{0|1}}\right]$. Thus, we have

$$\begin{aligned}
\frac{\partial G(A^1, \max(h_0(\tilde{S}_1), A^1))}{\partial \mu_{0|1}} &= \mathbb{E}_{\tilde{S}_1}\left[\Phi\left(\frac{\max(h_0(\tilde{S}_1), A^1)-\mu_{0|1}}{\sigma_{0|1}}\right) - \Phi\left(\frac{A^1-\mu_{0|1}}{\sigma_{0|1}}\right)\right], \\
\frac{\partial G(A^1, \max(h_0(\tilde{S}_1), A^1))}{\partial \sigma_{0|1}} &= \mathbb{E}_{\tilde{S}_1}\left[\phi\left(\frac{A^1-\mu_{0|1}}{\sigma_{0|1}}\right) - \phi\left(\frac{\max(h_0(\tilde{S}_1), A^1)-\mu_{0|1}}{\sigma_{0|1}}\right)\right].
\end{aligned}$$

Because $\frac{\partial \mu_{0|1}}{\partial \rho} = \frac{\sigma_0}{\sigma_1}(S_1 - \mu_1)$ and $\frac{\partial \sigma_{0|1}}{\partial \rho} = -\frac{\sigma_0 \rho}{\sqrt{1-\rho^2}}$, the chain rule yields

$$\frac{\partial G(A^1, \max(h_0(\tilde{S}_1), A^1))}{\partial \rho} = \\ -\mathbb{E}_{\tilde{S}_1}\left[\left[\Phi\left(\frac{A^1 - \mu_{0|1}}{\sigma_{0|1}}\right) - \Phi\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\right]\frac{\sigma_0}{\sigma_1}(\tilde{S}_1 - \mu_1)\right] \\ -\mathbb{E}_{\tilde{S}_1}\left[\left[\phi\left(\frac{A^1 - \mu_{0|1}}{\sigma_{0|1}}\right) - \phi\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\right]\frac{\sigma_0 \rho}{\sqrt{1-\rho^2}}\right].$$

From Stein's Lemma, for a differentiable function y and a Normal random variable $\tilde{X}$ with expectation μ and variance σ^2, we have $\mathbb{E}[y(\tilde{X})(\tilde{X}-\mu)] = \sigma^2\mathbb{E}[y'(\tilde{X})]$ (see for example, Rubinstein 1976). Therefore, $\mathbb{E}_{\tilde{S}_1}\left[\Phi\left(\frac{A^1-\mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0}{\sigma_1}(\tilde{S}_1 - \mu_1)\right] = -\mathbb{E}_{\tilde{S}_1}\left[\phi\left(\frac{A^1-\mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0\rho}{\sqrt{1-\rho^2}}\right]$, and we further simplify it as

$$\frac{\partial G(A^1, \max(h_0(\tilde{S}_1), A^1))}{\partial \rho} = \\ \mathbb{E}_{\tilde{S}_1}\left[\Phi\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0}{\sigma_1}(\tilde{S}_1 - \mu_1) + \phi\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0 \rho}{\sqrt{1-\rho^2}}\right]. \tag{4.30}$$

Because $\lim_{\rho \to 1^-} \sigma_{0|1} = 0$ and $\lim_{\rho \to 1^-} \mu_{0|1} = \mu_0 + \frac{\sigma_0}{\sigma_1}(S_1 - \mu_1)$, we have $\lim_{\rho \to 1^-} \Phi\left(\frac{\max(h_0(S_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right) = 1$, and thus, $\lim_{\rho \to 1^-} \mathbb{E}_{\tilde{S}_1}\left[\Phi\left(\frac{\max(h_0(\tilde{S}_1), A^1)-\mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0}{\sigma_1}(\tilde{S}_1 - \mu_1)\right] = 0$. Moreover, we notice that the second term $\mathbb{E}_{\tilde{S}_1}\left[\phi\left(\frac{\max(h_0(\tilde{S}_1), A^1)-\mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0\rho}{\sqrt{1-\rho^2}}\right] \geq 0$ for $\rho \in [0, 1)$. Thus, we obtain $\lim_{\rho \to 1^-} \frac{\partial G(A^1, \max(h_0(\tilde{S}_1), A^1))}{\partial \rho} \geq 0$.

We proceed to take the first derivative of (4.30) with respect to ρ, and obtain

$$\begin{aligned}
\frac{\partial^2 G(A^1, \max(h_0(\tilde{S}_1), A^1))}{\partial \rho^2} =& \mathbb{E}_{\tilde{S}_1}\left[\phi\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0}{\sigma_1}(\tilde{S}_1 - \mu_1)\varpi\right] \\
&+ \mathbb{E}_{\tilde{S}_1}\left[\phi'\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0\rho}{\sqrt{1-\rho^2}}\varpi\right] \\
=& \mathbb{E}_{\tilde{S}_1}\left[\phi\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0}{\sigma_1}(\tilde{S}_1 - \mu_1)\varpi\right] \\
&- \mathbb{E}_{\tilde{S}_1}\left[\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\phi\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0\rho}{\sqrt{1-\rho^2}}\varpi\right] \\
=& - \mathbb{E}_{\tilde{S}_1}\left[\sigma_{0|1}\phi\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\varpi^2\right] \\
\le& 0,
\end{aligned}$$

where $\varpi \doteq \frac{1}{\sigma_{0|1}^2}\left(\frac{\sigma_0\rho}{\sqrt{1-\rho^2}}\left(\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}\right) - \frac{\sigma_0\sigma_{0|1}}{\sigma_1}(\tilde{S}_1 - \mu_1)\right)$. Thus, we obtain $\frac{\partial G(A^1, \max(h_0(\tilde{S}_1), A^1))}{\partial \rho} \ge 0$ for $\rho \in [0, 1)$.

We now examine the impact of ρ on $\Delta_\beta^L(\Delta_A)$. Recall from (4.27) and (4.30) that $\Delta_\beta^L(\Delta_A) = G(A^1, \max(h_0(\tilde{S}_1), A^1)) - G(A^1 + \Delta_A, \max(h_0(\tilde{S}_1), A^1 + \Delta_A)) + \Delta_A$ and

$$\begin{aligned}
&\frac{\partial G(A^1, \max(h_0(\tilde{S}_1), A^1))}{\partial \rho} = \\
&\quad \mathbb{E}_{\tilde{S}_1}\left[\Phi\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0}{\sigma_1}(\tilde{S}_1 - \mu_1) + \phi\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0\rho}{\sqrt{1-\rho^2}}\right].
\end{aligned}$$

Taking the first derivative of $\Delta_\beta^L(\Delta_A)$ with respect to Δ_A, we obtain

$$\begin{aligned}
\frac{\partial \Delta_\beta^L(\Delta_A)}{\partial \rho} &= \frac{\partial G(A^1, \max(h_0(\tilde{S}_1), A^1))}{\partial \rho} - \frac{\partial G(A^1 + \Delta_A, \max(h_0(\tilde{S}_1), A^1 + \Delta_A)) + \Delta_A}{\partial \rho} \\
&= \mathbb{E}_{\tilde{S}_1}\left[\Phi\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0}{\sigma_1}(\tilde{S}_1 - \mu_1) + \phi\left(\frac{\max(h_0(\tilde{S}_1), A^1) - \mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0\rho}{\sqrt{1-\rho^2}}\right] \\
&- \mathbb{E}_{\tilde{S}_1}\left[\Phi\left(\frac{\max(h_0(\tilde{S}_1), A^1 + \Delta_A) - \mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0}{\sigma_1}(\tilde{S}_1 - \mu_1) + \phi\left(\frac{\max(h_0(\tilde{S}_1), A^1 + \Delta_A) - \mu_{0|1}}{\sigma_{0|1}}\right)\frac{\sigma_0\rho}{\sqrt{1-\rho^2}}\right].
\end{aligned}$$

Let $W(\Delta_A)$ denote $\frac{\partial \Delta_\beta^L(\Delta_A)}{\partial \rho}$ as a function of Δ_A. It is straightforward that $W(0) = 0$. Next we investigate how $W(\Delta_A)$ changes with Δ_A. Using $f_1(S_1) = \frac{1}{\sigma_1}\phi\left(\frac{S_1 - \mu_1}{\sigma_1}\right)$

and $\phi(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)$, we obtain

$$\begin{aligned}
\frac{\partial W(\Delta_A)}{\partial \Delta_A} =& \mathbb{E}_{\tilde{S}_1}\left[\left(\frac{\mu_0 - A^1 - \Delta_A}{\rho\sigma_{0|1}}\right)\phi\left(\frac{A^1 + \Delta_A - \mu_{0|1}}{\sigma_{0|1}}\right)\chi\left(h_0(\tilde{S}_1) < A^1 + \Delta_A\right)\right] \\
&- \mathbb{E}_{\tilde{S}_1}\left[\frac{1}{\rho(1-\rho^2)}\phi'\left(\frac{A^1 + \Delta_A - \mu_{0|1}}{\sigma_{0|1}}\right)\chi\left(h_0(\tilde{S}_1) < A^1 + \Delta_A\right)\right] \\
=& \mathbb{E}_{\tilde{S}_1}\left[\left(\frac{(1-\rho^2)(\mu_0 - A^1 - \Delta_A) + (A^1 + \Delta_A - \mu_{0|1})}{\rho(1-\rho^2)\sigma_{0|1}}\right)\right. \\
&\qquad \left. \times \phi\left(\frac{A^1 + \Delta_A - \mu_{0|1}}{\sigma_{0|1}}\right)\chi\left(h_0(\tilde{S}_1) < A^1 + \Delta_A\right)\right] \\
=& \frac{1}{2\pi\sqrt{1-\rho^2}}\exp\left(-\frac{(\mu_0 - A^1 - \Delta_A)^2}{2\sigma_0^2}\right) \\
&\qquad \times \exp\left(-\frac{1}{2(1-\rho^2)}\left[\frac{\widehat{S}_1 - \mu_1}{\sigma_1} + \frac{\rho(\mu_0 - A^1 - \Delta_A)}{\sigma_0}\right]^2\right) \\
\geq& 0,
\end{aligned}$$

where $\widehat{S}_1$ is the output spot realization such that $h_0(S_1) = A^1 + \Delta_A$. Recall that $W(0) = 0$. We obtain $W(\Delta_A) \geq 0$ for all $\Delta_A \geq 0$, and thus, $\frac{\partial \Delta_\beta^L(\Delta_A)}{\partial \rho} \geq 0$ for $\rho \in [0, 1)$.

Next, we study the impact of ρ on $\Delta_A^{(1)}$. Recall from (4.28) that $G(A^1 + \Delta_A^{(1)}, \max(h_0(\tilde{S}_1), A^1 + \Delta_A^{(1)})) = \beta^2 + A^1 + \Delta_A^{(1)}$. From implicit function theorem, $\frac{\partial \Delta_A^{(1)}}{\partial \rho} = -\frac{\partial \mathcal{M}_0^2/\partial\rho}{\partial \mathcal{M}_0^2/\partial \Delta_A}$, where $\mathcal{M}_0^2 = G(A^1 + \Delta_A, \max(h_0(\tilde{S}_1), A^1 + \Delta_A)) - \beta^2 - A^1 - \Delta_A$. The first derivative of $\mathcal{M}_0^2$ with respect to Δ_A is

$$\begin{aligned}
\frac{\partial \mathcal{M}_0^2}{\partial \Delta_A} =& \mathbb{E}_{\tilde{S}_1}\left[\frac{\partial g(A^1 + \Delta_A, A^1 + \Delta_A)}{\partial \Delta_A}\chi\left(h_0(\tilde{S}_1) < A^1 + \Delta_A\right)\right] \\
&+ \mathbb{E}_{\tilde{S}_1}\left[\frac{\partial g(A^1 + \Delta_A, h_0(\tilde{S}_1))}{\partial \Delta_A}\chi\left(h_0(\tilde{S}_1) \geq A^1 + \Delta_A\right)\right] - 1 \\
=& \mathbb{E}_{\tilde{S}_1}\left[\chi\left(h_0(\tilde{S}_1) < A^1 + \Delta_A\right)\right] \\
&+ \mathbb{E}_{\tilde{S}_1}\left[F_{0|1}\left(A^1 + \Delta_A\right)\chi\left(h_0(\tilde{S}_1) \geq A^1 + \Delta_A\right)\right] - 1 \\
<& 0. \qquad (4.31)
\end{aligned}$$

Because $\frac{\partial \mathcal{M}_0^2}{\partial \Delta_A} < 0$, $\mathrm{Sgn}\left(\frac{\partial \Delta_A^{(1)}}{\partial \rho}\right) = \mathrm{Sgn}\left(\frac{\partial \mathcal{M}_0^2}{\partial \rho}\right)$. Moreover, $\frac{\partial \mathcal{M}_0^2}{\partial \rho} = \frac{\partial G(A^2, \max(h_0(\tilde{S}_1), A^2))}{\partial \rho}$ and $\frac{\partial G(A^1, \max(h_0(\tilde{S}_1), A^1))}{\partial \rho} \geq 0$, we have $\frac{\partial \mathcal{M}_0^2}{\partial \rho} \geq 0$, and thus, $\frac{\partial \Delta_A^{(1)}}{\partial \rho} \geq 0$.

For the impact of ρ on $\Delta_\beta^{(1)}$, recall from (4.29) that $\Delta_\beta^{(1)} = G(A^1, \max(h_0(\tilde{S}_1), A^1)) - \beta^2 - A^1$, so $\frac{\partial \Delta_\beta^{(1)}}{\partial \rho} = \frac{\partial G(A^1, \max(h_0(\tilde{S}_1), A^1))}{\partial \rho}$. Because we have proved $\frac{\partial G(A^1, \max(h_0(\tilde{S}_1), A^1))}{\partial \rho} \geq 0$, then $\frac{\partial \Delta_\beta^{(1)}}{\partial \rho} \geq 0$. ■

Proof of Proposition 14: Recall from Proposition 12 that the optimal procurement strategy is characterized by the preference thresholds $\Delta_\beta^\ell(\Delta_A)$ for $\ell \in \{L, M\}$ and the profitability thresholds $\Delta_A^{(i)}$ for $i \in \{1, 2\}$ on (Δ_A, Δ_β) space. From Lemma 1, we have $\frac{\partial \Delta_\beta^\ell(\Delta_A)}{\partial \rho} \geq 0$ for $\ell \in \{L, M\}$ and $\frac{\partial \Delta_A^{(i)}}{\partial \rho} \geq 0$, $\frac{\partial \Delta_\beta^{(i)}}{\partial \rho} \geq 0$ for $i \in \{1, 2\}$. To study the impact of ρ, we consider two correlations $0 \leq \rho^0 < \rho^1 < 1$ without loss of generality. Based on Proposition 12 and Lemma 1, we plot Figure 3.1 for ρ^0 and ρ^1 separately. Let $Q^{1*}(\rho)$ and $Q^{2*}(\rho)$ denote the optimal contract volumes for a given ρ. After comparing the optimal contract volumes for ρ^0 and ρ^1 for given (Δ_A, Δ_β), we obtain $Q^{1*}(\rho^1) + Q^{2*}(\rho^1) \geq Q^{1*}(\rho^0) + Q^{2*}(\rho^0)$ and $Q^{1*}(\rho^1) \geq Q^{1*}(\rho^0)$. Therefore, $\frac{\partial (Q^{1*} + Q^{2*})}{\partial \rho} \geq 0$ and $\frac{\partial Q^{1*}}{\partial \rho} \geq 0$ which complete our proof. ■

Proof of Proposition 15: Recall from (3.8) that

$$V(0,0) \doteq \mathbb{E}\left[\left(p_1 - \tilde{S}_1\right)^+\right] D_1 + \mathbb{E}\left[\left(h_1(\tilde{S}_1) - \tilde{S}_0\right)^+\right] \frac{D_1}{a_1} + \mathbb{E}\left[\left(h_0(\tilde{S}_1) - \tilde{S}_0\right)^+\right] \left(K_0 - \frac{D_1}{a_1}\right).$$

We note that the first term $\mathbb{E}[(p_1 - \tilde{S}_1)^+]D_1$ is independent of ρ, and the other two terms are structurally the same with coefficient of $\mathbb{E}[(h_\kappa(\tilde{S}_1) - \tilde{S}_0)^+]$ for $\kappa \in \{0, 1\}$. To study the impact of ρ on $V(0,0)$, we need to examine the impact of ρ on $\mathbb{E}[(h_\kappa(\tilde{S}_1) - \tilde{S}_0)^+]$. In particular, we consider the case $\kappa = 0$ in the proof which is the most complicated one; for the other case the proof can be established in a similar fashion. When $\omega_1 = 0$, we have $h_0(S_1) = a_1 S_1 - c_0$, and thus, $\mathbb{E}[(h_0(\tilde{S}_1) - \tilde{S}_0)^+] = \mathbb{E}[(a_1 \tilde{S}_1 - c_0 - \tilde{S}_0)^+]$. Let $\tilde{y} \doteq a_1 \tilde{S}_1 - c_0 - \tilde{S}_0$. Here, $\tilde{y}$ is a Normal distribution with mean $\mu_y \doteq a_1 \mu_1 - c_0 - \mu_0$ and standard deviation $\sigma_y \doteq \sqrt{\sigma_0^2 + a_1^2 \sigma_1^2 - 2\rho a_1 \sigma_0 \sigma_1}$.

Using the standard Normal loss function $L(.)$, we obtain $\mathbb{E}[(\tilde{y})^+] = \sigma_y L\left(\frac{-\mu_y}{\sigma_y}\right)$ and $\frac{\partial \mathbb{E}[(\tilde{y})^+]}{\partial \rho} = \phi\left(\frac{-\mu_y}{\sigma_y}\right)\frac{\partial \sigma_y}{\partial \rho} < 0$. Because the first term in $V(0,0)$ is independent of ρ and the last two terms increase in ρ, we have $\frac{\partial V(0,0)}{\partial \rho} < 0$.

For the impact of ρ on the value of contract procurement $\mathbb{VC}$, we focus on the local sensitivity analysis in which the optimal procurement strategy is not affected by the changes in ρ for given (Δ_A, Δ_β). We only provide the proof for a typical region (i.e., Γ_2 where dual sourcing to full processing capacity is optimal). The proof for other regions can be established in a similar fashion. Recall from (3.7), in region Γ_2, we have

$$\mathbb{VC} = \mathcal{M}_1^1 \frac{D_1}{a_1} + \mathcal{M}_0^2 \left(K_0 - \frac{D_1}{a_1}\right).$$

Because we have proved in the proof of Lemma 1 that $\frac{\partial \mathcal{M}_\kappa^i}{\partial \rho} \geq 0$ for $i \in \{1,2\}$ and $\kappa \in \{0,1\}$, it is straightforward that $\frac{\partial \mathbb{VC}}{\partial \rho} \geq 0$ which completes our proof. ■

Proof of Proposition 16: Recall from (3.4) and (3.9) that

$$\begin{aligned}
h_3(S_1) &= a_1 \min(\max(p_1, S_1(1-\omega_1)), S_1) + a_2 p_2 - c_0,\\
h_2(S_1) &= a_1 S_1(1-\omega_1) + a_2 p_2 - c_0,\\
h_1(S_1) &= a_1 \min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0,\\
h_0(S_1) &= a_1 S_1(1-\omega_1) - c_0.
\end{aligned}$$

It is straightforward to establish that $h_3(S_1) \leq h_2(S_1) \leq h_1(S_1) \leq h_0(S_1)$ for a given S_1. Let $P^1(S_0) \doteq \min(S_0, A^1)$ and $P^2(S_0) \doteq \min(S_0, A^2)$ denote the effective unit sourcing costs. We have $P^1(S_0) \leq P^2(S_0) \leq S_0$ for a given S_0. From (3.1)-(3.3), together with the revenue from by-product (i.e., $p_2 \min(a_2 z_{0,b}, D_2)$), we obtain

$$\begin{aligned}\Pi(z_{0,b}) = &- \min(Q_b^1, z_{0,b}) \min(S_0, A^1) - \min(Q_b^2, (z_{0,b} - Q_b^1)^+) \min(S_0, A^2) \\ &- (z_{0,b} - Q_b^1 - Q_b^2)^+ S_0 + p_2 \min(a_2 z_{0,b}, D_2) + D_1(p_1 - S_1)^+ \\ &+ \min(a_1 z_{0,b}, D_1) \min(\max(p_1, S_1(1-\omega_1)), S_1) + (a_1 z_{0,b} - D_1)^+ S_1(1-\omega_1) - c_0 z_{0,b}.\end{aligned} \tag{4.32}$$

To simplify $\Pi(z_{0,b})$, it requires to specify the ordering of $\left\{\frac{D_2}{a_2}, \frac{D_1}{a_1}, Q_b^1, Q_b^1 + Q_b^2, K_0\right\}$. Because it is not optimal to order more than the processing capacity K_0, we restrict our focus on $Q_b^1 + Q_b^2 \leq K_0$, and thus, $Q_b^1 \leq Q_b^1 + Q_b^2 \leq K_0$. The relation between $\frac{D_1}{a_1}$ and $\frac{D_2}{a_2}$ is indeterminate; we have two situations to discuss: $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$ and $\frac{D_2}{a_2} > \frac{D_1}{a_1}$. We only provide the proof for $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$. The proof for $\frac{D_2}{a_2} > \frac{D_1}{a_1}$ can be established in a similar fashion; we will briefly explain how to characterize $z_{0,b}^*$ for $\frac{D_2}{a_2} > \frac{D_1}{a_1}$ at the end of the proof.

Now, since $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$ and $Q_b^1 \leq Q_b^1 + Q_b^2 \leq K_0$, we have 10 cases to consider:

Case 1: $\frac{D_2}{a_2} \leq \frac{D_1}{a_1} \leq Q_b^1 \leq Q_b^1 + Q_b^2 \leq K_0$

$$\Pi(z_{0,b}) = \begin{cases} -z_{0,b}\min(S_0, A^1) + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } 0 \le z_{0,b} \le \frac{D_2}{a_2}, \\ -z_{0,b}\min(S_0, A^1) + p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } \frac{D_2}{a_2} \le z_{0,b} \le \frac{D_1}{a_1}, \\ -z_{0,b}\min(S_0, A^1) + p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +D_1\min(\max(p_1, S_1(1-\omega_1)), S_1) & \\ \quad +(a_1 z_{0,b} - D_1)S_1(1-\omega_1) - c_0 z_{0,b} & \text{if } \frac{D_1}{a_1} \le z_{0,b} \le Q_b^1, \\ -Q_b^1\min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +D_1\min(\max(p_1, S_1(1-\omega_1)), S_1) & \\ \quad +(a_1 z_{0,b} - D_1)S_1(1-\omega_1) - c_0 z_{0,b} & \text{if } Q_b^1 \le z_{0,b} \le Q_b^1 + Q_b^2, \\ -Q_b^1\min(S_0, A^1) - Q_b^2\min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2)S_0 & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +D_1\min(\max(p_1, S_1(1-\omega_1)), S_1) & \\ \quad +(a_1 z_{0,b} - D_1)S_1(1-\omega_1) - c_0 z_{0,b} & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le K_0. \end{cases}$$

In general, we obtain $\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}}$ as follows:

$$\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} = \begin{cases} h_3(S_1) - P^1(S_0) & \text{if } 0 \le z_{0,b} \le \frac{D_2}{a_2}, \\ h_1(S_1) - P^1(S_0) & \text{if } \frac{D_2}{a_2} \le z_{0,b} \le \frac{D_1}{a_1}, \\ h_0(S_1) - P^1(S_0) & \text{if } \frac{D_1}{a_1} \le z_{0,b} \le Q_b^1, \\ h_0(S_1) - P^2(S_0) & \text{if } Q_b^1 \le z_{0,b} \le Q_b^1 + Q_b^2, \\ h_0(S_1) - S_0 & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le K_0. \end{cases}$$

Since $\Pi(z_{0,b})$ is a piecewise linear function with kinks $\bar{z}_{0,b} \in \{\frac{D_2}{a_2}, \frac{D_1}{a_1}, Q_b^1, Q_b^1 + Q_b^2\}$, and at each kink $\bar{z}_{0,b}$,

$$\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \ge \lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}},$$

$\Pi(z_{0,b})$ is a concave function. The optimal decision $z^*_{0,b}$ can be either at the boundaries (0 and K_0) or at the kinks $\bar{z}_{0,b}$. In particular, $z^*_{0,b} = 0$ if $\lim_{z_{0,b}\to 0^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$; $z^*_{0,b} = K_0$ if $\lim_{z_{0,b}\to K_0^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$; $z^*_{0,b} = \bar{z}_{0,b}$ if $\lim_{z_{0,b}\to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$ and $\lim_{z_{0,b}\to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$. In general, the optimal processing decision $z^*_{0,b}$ is characterized by:

$$z^*_{0,b} = \begin{cases} 0 & \text{if } h_3(S_1) \leq P^1(S_0), \\ \frac{D_2}{a_2} & \text{if } h_1(S_1) \leq P^1(S_0) < h_3(S_1), \\ \frac{D_1}{a_1} & \text{if } h_0(S_1) \leq P^1(S_0) < h_1(S_1), \\ Q_b^1 & \text{if } P^1(S_0) < h_0(S_1) \leq P^2(S_0), \\ Q_b^1 + Q_b^2 & \text{if } P^2(S_0) < h_0(S_1) \leq S_0, \\ K_0 & \text{if } S_0 < h_0(S_1). \end{cases}$$

Case 2: $\frac{D_2}{a_2} \leq Q_b^1 \leq \frac{D_1}{a_1} \leq Q_b^1 + Q_b^2 \leq K_0$

$$\Pi(z_{0,b}) = \begin{cases} -z_{0,b}\min(S_0, A^1) + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } 0 \le z_{0,b} \le \frac{D_2}{a_2}, \\ -z_{0,b}\min(S_0, A^1) + p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +a_2 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } \frac{D_2}{a_2} \le z_{0,b} \le Q_b^1, \\ -Q_b^1\min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +a_2 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q_b^1 \le z_{0,b} \le \frac{D_1}{a_1}, \\ -Q_b^1\min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +D_1\min(\max(p_1, S_1(1-\omega_1)), S_1) & \\ \quad +(a_1 z_{0,b} - D_1)S_1(1-\omega_1) - c_0 z_{0,b} & \text{if } \frac{D_1}{a_1} \le z_{0,b} \le Q_b^1 + Q_b^2, \\ -Q_b^1\min(S_0, A^1) - Q_b^2\min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2)S_0 & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +D_1\min(\max(p_1, S_1(1-\omega_1)), S_1) & \\ \quad +(a_1 z_{0,b} - D_1)S_1(1-\omega_1) - c_0 z_{0,b} & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le K_0. \end{cases}$$

In general, we obtain $\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}}$ as follows:

$$\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} = \begin{cases} h_3(S_1) - P^1(S_0) & \text{if } 0 \le z_{0,b} \le \frac{D_2}{a_2}, \\ h_1(S_1) - P^1(S_0) & \text{if } \frac{D_2}{a_2} \le z_{0,b} \le Q_b^1, \\ h_1(S_1) - P^2(S_0) & \text{if } Q_b^1 \le z_{0,b} \le \frac{D_1}{a_1}, \\ h_0(S_1) - P^2(S_0) & \text{if } \frac{D_1}{a_1} \le z_{0,b} \le Q_b^1 + Q_b^2, \\ h_0(S_1) - S_0 & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le K_0. \end{cases}$$

Since $\Pi(z_{0,b})$ is a piecewise linear function with kinks $\bar{z}_{0,b} \in \{\frac{D_2}{a_2}, Q_b^1, \frac{D_1}{a_1}, Q_b^1 + Q_b^2\}$, and at each kink $\bar{z}_{0,b}$,

$$\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \ge \lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}},$$

$\Pi(z_{0,b})$ is a concave function. The optimal decision $z^*_{0,b}$ can be either at the boundaries (0 and K_0) or at the kinks $\bar{z}_{0,b}$. In particular, $z^*_{0,b} = 0$ if $\lim_{z_{0,b}\to 0^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$; $z^*_{0,b} = K_0$ if $\lim_{z_{0,b}\to K_0^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$; $z^*_{0,b} = \bar{z}_{0,b}$ if $\lim_{z_{0,b}\to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$ and $\lim_{z_{0,b}\to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$. In general, the optimal processing decision $z^*_{0,b}$ is characterized by:

$$z^*_{0,b} = \begin{cases} 0 & \text{if } h_3(S_1) \leq P^1(S_0), \\ \frac{D_2}{a_2} & \text{if } h_1(S_1) \leq P^1(S_0) < h_3(S_1), \\ Q^1_b & \text{if } P^1(S_0) < h_1(S_1) \leq P^2(S_0), \\ \frac{D_1}{a_1} & \text{if } h_0(S_1) \leq P^2(S_0) < h_1(S_1), \\ Q^1_b + Q^2_b & \text{if } P^2(S_0) < h_0(S_1) \leq S_0, \\ K_0 & \text{if } S_0 < h_0(S_1). \end{cases}$$

Case 3: $\frac{D_2}{a_2} \leq Q^1_b \leq Q^1_b + Q^2_b \leq \frac{D_1}{a_1} \leq K_0$

$$\Pi(z_{0,b}) = \begin{cases} -z_{0,b}\min(S_0, A^1) + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad + a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } 0 \leq z_{0,b} \leq \frac{D_2}{a_2}, \\ -z_{0,b}\min(S_0, A^1) + p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad + a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } \frac{D_2}{a_2} \leq z_{0,b} \leq Q^1_b, \\ -Q^1_b\min(S_0, A^1) - (z_{0,b} - Q^1_b)\min(S_0, A^2) & \\ \quad + p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad + a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q^1_b \leq z_{0,b} \leq Q^1_b + Q^2_b, \\ -Q^1_b\min(S_0, A^1) - Q^2_b\min(S_0, A^2) - (z_{0,b} - Q^1_b - Q^2_b)S_0 & \\ \quad + p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad + a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q^1_b + Q^2_b \leq z_{0,b} \leq \frac{D_1}{a_1}, \\ -Q^1_b\min(S_0, A^1) - Q^2_b\min(S_0, A^2) - (z_{0,b} - Q^1_b - Q^2_b)S_0 & \\ \quad + p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad + D_1\min(\max(p_1, S_1(1-\omega_1)), S_1) & \\ \quad + (a_1 z_{0,b} - D_1)S_1(1-\omega_1) - c_0 z_{0,b} & \text{if } \frac{D_1}{a_1} \leq z_{0,b} \leq K_0. \end{cases}$$

In general, we obtain $\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}}$ as follows:

$$\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} = \begin{cases} h_3(S_1) - P^1(S_0) & \text{if } 0 \leq z_{0,b} \leq \frac{D_2}{a_2}, \\ h_1(S_1) - P^1(S_0) & \text{if } \frac{D_2}{a_2} \leq z_{0,b} \leq Q_b^1, \\ h_1(S_1) - P^2(S_0) & \text{if } Q_b^1 \leq z_{0,b} \leq Q_b^1 + Q_b^2, \\ h_1(S_1) - S_0 & \text{if } Q_b^1 + Q_b^2 \leq z_{0,b} \leq \frac{D_1}{a_1}, \\ h_0(S_1) - S_0 & \text{if } \frac{D_1}{a_1} \leq z_{0,b} \leq K_0. \end{cases}$$

Since $\Pi(z_{0,b})$ is a piecewise linear function with kinks $\bar{z}_{0,b} \in \{\frac{D_2}{a_2}, Q_b^1, Q_b^1 + Q_b^2, \frac{D_1}{a_1}\}$, and at each kink $\bar{z}_{0,b}$,

$$\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \geq \lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}},$$

$\Pi(z_{0,b})$ is a concave function. The optimal decision $z_{0,b}^*$ can be either at the boundaries (0 and K_0) or at the kinks $\bar{z}_{0,b}$. In particular, $z_{0,b}^* = 0$ if $\lim_{z_{0,b} \to 0^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$; $z_{0,b}^* = K_0$ if $\lim_{z_{0,b} \to K_0^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$; $z_{0,b}^* = \bar{z}_{0,b}$ if $\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$ and $\lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$. In general, the optimal processing decision $z_{0,b}^*$ is characterized by:

$$z_{0,b}^* = \begin{cases} 0 & \text{if } h_3(S_1) \leq P^1(S_0), \\ \frac{D_2}{a_2} & \text{if } h_1(S_1) \leq P^1(S_0) < h_3(S_1), \\ Q_b^1 & \text{if } P^1(S_0) < h_1(S_1) \leq P^2(S_0), \\ Q_b^1 + Q_b^2 & \text{if } P^2(S_0) < h_1(S_1) \leq S_0, \\ \frac{D_1}{a_1} & \text{if } h_0(S_1) \leq S_0 < h_1(S_1), \\ K_0 & \text{if } S_0 < h_0(S_1). \end{cases}$$

Case 4: $\frac{D_2}{a_2} \leq Q_b^1 \leq Q_b^1 + Q_b^2 \leq K_0 \leq \frac{D_1}{a_1}$

$$
\Pi(z_{0,b}) = \begin{cases} -z_{0,b}\min(S_0, A^1) + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } 0 \le z_{0,b} \le \frac{D_2}{a_2}, \\ -z_{0,b}\min(S_0, A^1) + p_2 D_2 + D_1(p_1 - S_1)^+ \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } \frac{D_2}{a_2} \le z_{0,b} \le Q_b^1, \\ -Q_b^1\min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q_b^1 \le z_{0,b} \le Q_b^1 + Q_b^2, \\ -Q_b^1\min(S_0, A^1) - Q_b^2\min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2)S_0 \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le K_0, \end{cases}
$$

In general, we obtain $\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}}$ as follows:

$$
\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} = \begin{cases} h_3(S_1) - P^1(S_0) & \text{if } 0 \le z_{0,b} \le \frac{D_2}{a_2}, \\ h_1(S_1) - P^1(S_0) & \text{if } \frac{D_2}{a_2} \le z_{0,b} \le Q_b^1, \\ h_1(S_1) - P^2(S_0) & \text{if } Q_b^1 \le z_{0,b} \le Q_b^1 + Q_b^2, \\ h_1(S_1) - S_0 & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le K_0, \end{cases}
$$

Since $\Pi(z_{0,b})$ is a piecewise linear function with kinks $\bar{z}_{0,b} \in \{\frac{D_2}{a_2}, Q_b^1, Q_b^1 + Q_b^2\}$, and at each kink $\bar{z}_{0,b}$,

$$
\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \ge \lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}},
$$

$\Pi(z_{0,b})$ is a concave function. The optimal decision $z_{0,b}^*$ can be either at the boundaries (0 and K_0) or at the kinks $\bar{z}_{0,b}$. In particular, $z_{0,b}^* = 0$ if $\lim_{z_{0,b} \to 0^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \le 0$; $z_{0,b}^* = K_0$ if $\lim_{z_{0,b} \to K_0^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$; $z_{0,b}^* = \bar{z}_{0,b}$ if $\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$ and $\lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \le 0$. In general, the optimal processing decision $z_{0,b}^*$ is characterized by:

$$
z_{0,b}^* = \begin{cases} 0 & \text{if } h_3(S_1) \le P^1(S_0), \\ \frac{D_2}{a_2} & \text{if } h_1(S_1) \le P^1(S_0) < h_3(S_1), \\ Q_b^1 & \text{if } P^1(S_0) < h_1(S_1) \le P^2(S_0), \\ Q_b^1 + Q_b^2 & \text{if } P^2(S_0) < h_1(S_1) \le S_0, \\ K_0 & \text{if } S_0 < h_1(S_1). \end{cases}
$$

Case 5: $Q_b^1 \le \frac{D_2}{a_2} \le \frac{D_1}{a_1} \le Q_b^1 + Q_b^2 \le K_0$

$$
\Pi(z_{0,b}) = \begin{cases} -z_{0,b}\min(S_0, A^1) + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } 0 \le z_{0,b} \le Q_b^1, \\ -Q_b^1 \min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\ \quad +a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q_b^1 \le z_{0,b} \le \frac{D_2}{a_2}, \\ -Q_b^1 \min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } \frac{D_2}{a_2} \le z_{0,b} \le \frac{D_1}{a_1}, \\ -Q_b^1 \min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +D_1 \min(\max(p_1, S_1(1-\omega_1)), S_1) & \\ \quad +(a_1 z_{0,b} - D_1)S_1(1-\omega_1) - c_0 z_{0,b} & \text{if } \frac{D_1}{a_1} \le z_{0,b} \le Q_b^1 + Q_b^2, \\ -Q_b^1 \min(S_0, A^1) - Q_b^2 \min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2)S_0 & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +D_1 \min(\max(p_1, S_1(1-\omega_1)), S_1) & \\ \quad +(a_1 z_{0,b} - D_1)S_1(1-\omega_1) - c_0 z_{0,b} & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le K_0. \end{cases}
$$

In general, we obtain $\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}}$ as follows:

$$\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} = \begin{cases} h_3(S_1) - P^1(S_0) & \text{if } 0 \le z_{0,b} \le Q_b^1, \\ h_3(S_1) - P^2(S_0) & \text{if } Q_b^1 \le z_{0,b} \le \frac{D_2}{a_2}, \\ h_1(S_1) - P^2(S_0) & \text{if } \frac{D_2}{a_2} \le z_{0,b} \le \frac{D_1}{a_1}, \\ h_0(S_1) - P^2(S_0) & \text{if } \frac{D_1}{a_1} \le z_{0,b} \le Q_b^1 + Q_b^2, \\ h_0(S_1) - S_0 & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le K_0. \end{cases}$$

Since $\Pi(z_{0,b})$ is a piecewise linear function with kinks $\bar{z}_{0,b} \in \{Q_b^1, \frac{D_2}{a_2}, \frac{D_1}{a_1}, Q_b^1 + Q_b^2\}$, and at each kink $\bar{z}_{0,b}$,

$$\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \ge \lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}},$$

$\Pi(z_{0,b})$ is a concave function. The optimal decision $z_{0,b}^*$ can be either at the boundaries (0 and K_0) or at the kinks $\bar{z}_{0,b}$. In particular, $z_{0,b}^* = 0$ if $\lim_{z_{0,b} \to 0^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \le 0$; $z_{0,b}^* = K_0$ if $\lim_{z_{0,b} \to K_0^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$; $z_{0,b}^* = \bar{z}_{0,b}$ if $\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$ and $\lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \le 0$. In general, the optimal processing decision $z_{0,b}^*$ is characterized by:

$$z_{0,b}^* = \begin{cases} 0 & \text{if } h_3(S_1) \le P^1(S_0), \\ Q_b^1 & \text{if } P^1(S_0) < h_3(S_1) \le P^2(S_0), \\ \frac{D_2}{a_2} & \text{if } h_1(S_1) \le P^2(S_0) < h_3(S_1), \\ \frac{D_1}{a_1} & \text{if } h_0(S_1) \le P^2(S_0) < h_1(S_1), \\ Q_b^1 + Q_b^2 & \text{if } P^2(S_0) < h_0(S_1) \le S_0, \\ K_0 & \text{if } S_0 < h_0(S_1). \end{cases}$$

Case 6: $Q_b^1 \le \frac{D_2}{a_2} \le Q_b^1 + Q_b^2 \le \frac{D_1}{a_1} \le K_0$

$$\Pi(z_{0,b}) = \begin{cases} -z_{0,b}\min(S_0, A^1) + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } 0 \le z_{0,b} \le Q_b^1, \\ -Q_b^1 \min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\ \quad +a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q_b^1 \le z_{0,b} \le \frac{D_1}{a_1}, \\ -Q_b^1 \min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } \frac{D_1}{a_1} \le z_{0,b} \le Q_b^1 + Q_b^2, \\ -Q_b^1 \min(S_0, A^1) - Q_b^2 \min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2) S_0 & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le \frac{D_2}{a_2}, \\ -Q_b^1 \min(S_0, A^1) - Q_b^2 \min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2) S_0 & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +D_1 \min(\max(p_1, S_1(1-\omega_1)), S_1) & \\ \quad +(a_1 z_{0,b} - D_1) S_1 (1-\omega_1) - c_0 z_{0,b} & \text{if } \frac{D_2}{a_2} \le z_{0,b} \le K_0. \end{cases}$$

In general, we obtain $\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}}$ as follows:

$$\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} = \begin{cases} h_3(S_1) - P^1(S_0) & \text{if } 0 \le z_{0,b} \le Q_b^1, \\ h_3(S_1) - P^2(S_0) & \text{if } Q_b^1 \le z_{0,b} \le \frac{D_2}{a_2}, \\ h_1(S_1) - P^2(S_0) & \text{if } \frac{D_2}{a_2} \le z_{0,b} \le Q_b^1 + Q_b^2, \\ h_1(S_1) - S_0 & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le \frac{D_1}{a_1}, \\ h_0(S_1) - S_0 & \text{if } \frac{D_1}{a_1} \le z_{0,b} \le K_0. \end{cases}$$

Since $\Pi(z_{0,b})$ is a piecewise linear function with kinks $\bar{z}_{0,b} \in \{Q_b^1, \frac{D_2}{a_2}, Q_b^1 + Q_b^2, \frac{D_1}{a_1}\}$, and at each kink $\bar{z}_{0,b}$,

$$\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \ge \lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}},$$

$\Pi(z_{0,b})$ is a concave function. The optimal decision $z^*_{0,b}$ can be either at the boundaries (0 and K_0) or at the kinks $\bar{z}_{0,b}$. In particular, $z^*_{0,b} = 0$ if $\lim_{z_{0,b}\to 0^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$; $z^*_{0,b} = K_0$ if $\lim_{z_{0,b}\to K_0^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$; $z^*_{0,b} = \bar{z}_{0,b}$ if $\lim_{z_{0,b}\to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$ and $\lim_{z_{0,b}\to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$. In general, the optimal processing decision $z^*_{0,b}$ is characterized by:

$$z^*_{0,b} = \begin{cases} 0 & \text{if } h_3(S_1) \leq P^1(S_0), \\ Q_b^1 & \text{if } P^1(S_0) < h_3(S_1) \leq P^2(S_0), \\ \frac{D_2}{a_2} & \text{if } h_1(S_1) \leq P^2(S_0) < h_3(S_1), \\ Q_b^1 + Q_b^2 & \text{if } P^2(S_0) < h_1(S_1) \leq S_0, \\ \frac{D_1}{a_1} & \text{if } h_0(S_1) \leq S_0 < h_1(S_1), \\ K_0 & \text{if } S_0 < h_0(S_1). \end{cases}$$

Case 7: $Q_b^1 \leq \frac{D_2}{a_2} \leq Q_b^1 + Q_b^2 \leq K_0 \leq \frac{D_1}{a_1}$

$$\Pi(z_{0,b}) = \begin{cases} -z_{0,b}\min(S_0, A^1) + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } 0 \leq z_{0,b} \leq Q_b^1, \\ -Q_b^1 \min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\ \quad +a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q_b^1 \leq z_{0,b} \leq \frac{D_2}{a_2}, \\ -Q_b^1 \min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } \frac{D_2}{a_2} \leq z_{0,b} \leq Q_b^1 + Q_b^2, \\ -Q_b^1 \min(S_0, A^1) - Q_b^2 \min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2)S_0 & \\ \quad +p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad +a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q_b^1 + Q_b^2 \leq z_{0,b} \leq K_0. \end{cases}$$

In general, we obtain $\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}}$ as follows:

$$\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} = \begin{cases} h_3(S_1) - P^1(S_0) & \text{if } 0 \leq z_{0,b} \leq Q_b^1, \\ h_3(S_1) - P^2(S_0) & \text{if } Q_b^1 \leq z_{0,b} \leq \frac{D_2}{a_2}, \\ h_1(S_1) - P^2(S_0) & \text{if } \frac{D_2}{a_2} \leq z_{0,b} \leq Q_b^1 + Q_b^2, \\ h_1(S_1) - S_0 & \text{if } Q_b^1 + Q_b^2 \leq z_{0,b} \leq K_0. \end{cases}$$

Since $\Pi(z_{0,b})$ is a piecewise linear function with kinks $\bar{z}_{0,b} \in \{Q_b^1, \frac{D_2}{a_2}, Q_b^1 + Q_b^2\}$, and at each kink $\bar{z}_{0,b}$,

$$\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \geq \lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}},$$

$\Pi(z_{0,b})$ is a concave function. The optimal decision $z_{0,b}^*$ can be either at the boundaries (0 and K_0) or at the kinks $\bar{z}_{0,b}$. In particular, $z_{0,b}^* = 0$ if $\lim_{z_{0,b} \to 0^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$; $z_{0,b}^* = K_0$ if $\lim_{z_{0,b} \to K_0^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$; $z_{0,b}^* = \bar{z}_{0,b}$ if $\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$ and $\lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$. In general, the optimal processing decision $z_{0,b}^*$ is characterized by:

$$z_{0,b}^* = \begin{cases} 0 & \text{if } h_3(S_1) \leq P^1(S_0), \\ Q_b^1 & \text{if } P^1(S_0) < h_3(S_1) \leq P^2(S_0), \\ \frac{D_2}{a_2} & \text{if } h_1(S_1) \leq P^2(S_0) < h_3(S_1), \\ Q_b^1 + Q_b^2 & \text{if } P^2(S_0) < h_1(S_1) \leq S_0, \\ K_0 & \text{if } S_0 < h_1(S_1). \end{cases}$$

Case 8: $Q_b^1 \leq Q_b^1 + Q_b^2 \leq \frac{D_2}{a_2} \leq \frac{D_1}{a_1} \leq K_0$

$$
\Pi(z_{0,b}) = \begin{cases}
-z_{0,b}\min(S_0, A^1) + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\
\qquad + a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } 0 \le z_{0,b} \le Q_b^1, \\
-Q_b^1 \min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\
\qquad + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\
\qquad + a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q_b^1 \le z_{0,b} \le Q_b^1 + Q_b^2, \\
-Q_b^1 \min(S_0, A^1) - Q_b^2 \min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2) S_0 & \\
\qquad + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\
\qquad + a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le \frac{D_2}{a_2}, \\
-Q_b^1 \min(S_0, A^1) - Q_b^2 \min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2) S_0 & \\
\qquad + p_2 D_2 + D_1(p_1 - S_1)^+ & \\
\qquad + a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } \frac{D_2}{a_2} \le z_{0,b} \le \frac{D_1}{a_1}, \\
-Q_b^1 \min(S_0, A^1) - Q_b^2 \min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2) S_0 & \\
\qquad + p_2 D_2 + D_1(p_1 - S_1)^+ & \\
\qquad + D_1 \min(\max(p_1, S_1(1-\omega_1)), S_1) & \\
\qquad + (a_1 z_{0,b} - D_1) S_1(1-\omega_1) - c_0 z_{0,b} & \text{if } \frac{D_1}{a_1} \le z_{0,b} \le K_0.
\end{cases}
$$

In general, we obtain $\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}}$ as follows:

$$
\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} = \begin{cases}
h_3(S_1) - P^1(S_0) & \text{if } 0 \le z_{0,b} \le Q_b^1, \\
h_3(S_1) - P^2(S_0) & \text{if } Q_b^1 \le z_{0,b} \le Q_b^1 + Q_b^2, \\
h_3(S_1) - S_0 & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le \frac{D_1}{a_1}, \\
h_1(S_1) - S_0 & \text{if } \frac{D_1}{a_1} \le z_{0,b} \le \frac{D_2}{a_2}, \\
h_0(S_1) - S_0 & \text{if } \frac{D_2}{a_2} \le z_{0,b} \le K_0.
\end{cases}
$$

Since $\Pi(z_{0,b})$ is a piecewise linear function with kinks $\bar{z}_{0,b} \in \{Q_b^1, Q_b^1 + Q_b^2, \frac{D_2}{a_2}, \frac{D_1}{a_1}\}$, and at each kink $\bar{z}_{0,b}$,

$$
\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \ge \lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}},
$$

$\Pi(z_{0,b})$ is a concave function. The optimal decision $z^*_{0,b}$ can be either at the boundaries (0 and K_0) or at the kinks $\bar{z}_{0,b}$. In particular, $z^*_{0,b} = 0$ if $\lim_{z_{0,b}\to 0^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$; $z^*_{0,b} = K_0$ if $\lim_{z_{0,b}\to K_0^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$; $z^*_{0,b} = \bar{z}_{0,b}$ if $\lim_{z_{0,b}\to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$ and $\lim_{z_{0,b}\to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$. In general, the optimal processing decision $z^*_{0,b}$ is characterized by:

$$z^*_{0,b} = \begin{cases} 0 & \text{if } h_3(S_1) \leq P^1(S_0), \\ Q_b^1 & \text{if } P^1(S_0) < h_3(S_1) \leq P^2(S_0), \\ Q_b^1 + Q_b^2 & \text{if } P^2(S_0) < h_3(S_1) \leq S_0, \\ \frac{D_2}{a_2} & \text{if } h_1(S_1) \leq S_0 < h_3(S_1), \\ \frac{D_1}{a_1} & \text{if } h_0(S_1) \leq S_0 < h_1(S_1), \\ K_0 & \text{if } S_0 < h_0(S_1). \end{cases}$$

Case 9: $Q_b^1 \leq Q_b^1 + Q_b^2 \leq \frac{D_2}{a_2} \leq K_0 \leq \frac{D_1}{a_1}$

$$\Pi(z_{0,b}) = \begin{cases} -z_{0,b}\min(S_0, A^1) + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad + a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } 0 \leq z_{0,b} \leq Q_b^1, \\ -Q_b^1 \min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\ \quad + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad + a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q_b^1 \leq z_{0,b} \leq Q_b^1 + Q_b^2, \\ -Q_b^1 \min(S_0, A^1) - Q_b^2 \min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2)S_0 & \\ \quad + a_2 p_2 z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad + a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } Q_b^1 + Q_b^2 \leq z_{0,b} \leq \frac{D_2}{a_2}, \\ -Q_b^1 \min(S_0, A^1) - Q_b^2 \min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2)S_0 & \\ \quad + p_2 D_2 + D_1(p_1 - S_1)^+ & \\ \quad + a_1 z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0 z_{0,b} & \text{if } \frac{D_2}{a_2} \leq z_{0,b} \leq K_0, \end{cases}$$

In general, we obtain $\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}}$ as follows:

$$\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} = \begin{cases} h_3(S_1) - P^1(S_0) & \text{if } 0 \leq z_{0,b} \leq Q_b^1, \\ h_3(S_1) - P^2(S_0) & \text{if } Q_b^1 \leq z_{0,b} \leq Q_b^1 + Q_b^2, \\ h_3(S_1) - S_0 & \text{if } Q_b^1 + Q_b^2 \leq z_{0,b} \leq \frac{D_2}{a_2}, \\ h_1(S_1) - S_0 & \text{if } \frac{D_2}{a_2} \leq z_{0,b} \leq K_0. \end{cases}$$

Since $\Pi(z_{0,b})$ is a piecewise linear function with kinks $\bar{z}_{0,b} \in \{Q_b^1, Q_b^1 + Q_b^2, \frac{D_2}{a_2}\}$, and at each kink $\bar{z}_{0,b}$,

$$\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \geq \lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}},$$

$\Pi(z_{0,b})$ is a concave function. The optimal decision $z_{0,b}^*$ can be either at the boundaries (0 and K_0) or at the kinks $\bar{z}_{0,b}$. In particular, $z_{0,b}^* = 0$ if $\lim_{z_{0,b} \to 0^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$; $z_{0,b}^* = K_0$ if $\lim_{z_{0,b} \to K_0^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$; $z_{0,b}^* = \bar{z}_{0,b}$ if $\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$ and $\lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \leq 0$. In general, the optimal processing decision $z_{0,b}^*$ is characterized by:

$$z_{0,b}^* = \begin{cases} 0 & \text{if } h_3(S_1) \leq P^1(S_0), \\ Q_b^1 & \text{if } P^1(S_0) < h_3(S_1) \leq P^2(S_0), \\ Q_b^1 + Q_b^2 & \text{if } P^2(S_0) < h_3(S_1) \leq S_0, \\ \frac{D_2}{a_2} & \text{if } h_1(S_1) \leq S_0 < h_3(S_1), \\ K_0 & \text{if } S_0 < h_1(S_1). \end{cases}$$

Case 10: $Q_b^1 \leq Q_b^1 + Q_b^2 \leq K_0 \leq \frac{D_2}{a_2} \leq \frac{D_1}{a_1}$

$$\Pi(z_{0,b}) = \begin{cases} -z_{0,b}\min(S_0, A^1) + a_2p_2z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad +a_1z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0z_{0,b} & \text{if } 0 \le z_{0,b} \le Q_b^1, \\ -Q_b^1\min(S_0, A^1) - (z_{0,b} - Q_b^1)\min(S_0, A^2) & \\ \quad +a_2p_2z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad +a_1z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0z_{0,b} & \text{if } Q_b^1 \le z_{0,b} \le Q_b^1 + Q_b^2, \\ -Q_b^1\min(S_0, A^1) - Q_b^2\min(S_0, A^2) - (z_{0,b} - Q_b^1 - Q_b^2)S_0 & \\ \quad +a_2p_2z_{0,b} + D_1(p_1 - S_1)^+ & \\ \quad +a_1z_{0,b}\min(\max(p_1, S_1(1-\omega_1)), S_1) - c_0z_{0,b} & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le K_0. \end{cases}$$

In general, we obtain $\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}}$ as follows:

$$\frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} = \begin{cases} h_3(S_1) - P^1(S_0) & \text{if } 0 \le z_{0,b} \le Q_b^1, \\ h_3(S_1) - P^2(S_0) & \text{if } Q_b^1 \le z_{0,b} \le Q_b^1 + Q_b^2, \\ h_3(S_1) - S_0 & \text{if } Q_b^1 + Q_b^2 \le z_{0,b} \le K_0, \end{cases}$$

Since $\Pi(z_{0,b})$ is a piecewise linear function with kinks $\bar{z}_{0,b} \in \{Q_b^1, Q_b^1 + Q_b^2\}$, and at each kink $\bar{z}_{0,b}$,

$$\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \ge \lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}},$$

$\Pi(z_{0,b})$ is a concave function. The optimal decision $z_{0,b}^*$ can be either at the boundaries (0 and K_0) or at the kinks $\bar{z}_{0,b}$. In particular, $z_{0,b}^* = 0$ if $\lim_{z_{0,b} \to 0^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \le 0$; $z_{0,b}^* = K_0$ if $\lim_{z_{0,b} \to K_0^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$; $z_{0,b}^* = \bar{z}_{0,b}$ if $\lim_{z_{0,b} \to \bar{z}_{0,b}^-} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} > 0$ and $\lim_{z_{0,b} \to \bar{z}_{0,b}^+} \frac{\partial \Pi(z_{0,b})}{\partial z_{0,b}} \le 0$. In general, the optimal processing decision $z_{0,b}^*$ is characterized by:

$$z_{0,b}^* = \begin{cases} 0 & \text{if } h_3(S_1) \le P^1(S_0), \\ Q_b^1 & \text{if } P^1(S_0) < h_3(S_1) \le P^2(S_0), \\ Q_b^1 + Q_b^2 & \text{if } P^2(S_0) < h_3(S_1) \le S_0, \\ K_0 & \text{if } S_0 < h_3(S_1). \end{cases}$$

Combining the above 10 cases, we obtain

$$z_{0,b}^* = \begin{cases} 0 & \text{if } h_3(S_1) \leq P^1(S_0) \\ \min\left(\frac{D_2}{a_2}, Q_b^1\right) & \text{if } h_1(S_1) \leq P^1(S_0) \leq h_3(S_1) \leq P^2(S_0) \\ \frac{D_2}{a_2} & \text{if } h_1(S_1) \leq P^1(S_0) \leq P^2(S_0) \leq S_0 \leq h_3(S_1) \\ \min\left(\frac{D_1}{a_1}, Q_b^1\right) & \text{if } h_0(S_1) \leq P^1(S_0) \leq h_1(S_1) \leq h_3(S_1) \leq P^2(S_0) \\ Q_b^1 & \text{if } P^1(S_0) \leq h_0(S_1) \leq h_1(S_1) \leq h_3(S_1) \leq P^2(S_0) \\ \min\left(\max\left(\min\left(\frac{D_1}{a_1}, Q_b^1\right), \frac{D_2}{a_2}\right), Q_b^1 + Q_b^2\right) & \text{if } h_0(S_1) \leq P^1(S_0) \leq h_1(S_1) \leq P^2(S_0) \leq h_3(S_1) \leq S_0 \\ \min\left(\max\left(Q_b^1, \frac{D_2}{a_2}\right), Q_b^1 + Q_b^2\right) & \text{if } P^1(S_0) \leq h_0(S_1) \leq h_1(S_1) \leq P^2(S_0) \leq h_3(S_1) \leq S_0 \\ \min\left(\frac{D_2}{a_2}, Q_b^1 + Q_b^2\right) & \text{if } h_1(S_1) \leq P^1(S_0) \leq P^2(S_0) \leq h_3(S_1) \leq S_0 \\ \max\left(\min\left(\frac{D_1}{a_1}, Q_b^1\right), \frac{D_2}{a_2}\right) & \text{if } h_0(S_1) \leq P^1(S_0) \leq h_1(S_1) \leq P^2(S_0) \leq S_0 \leq h_3(S_1) \\ \max\left(\frac{D_2}{a_2}, Q_b^1\right) & \text{if } P^1(S_0) \leq h_0(S_1) \leq h_1(S_1) \leq P^2(S_0) \leq S_0 \leq h_3(S_1) \\ \min\left(\frac{D_1}{a_1}, Q_b^1 + Q_b^2\right) & \text{if } h_0(S_1) \leq P^1(S_0) \leq P^2(S_0) \leq h_1(S_1) \leq h_3(S_1) \leq S_0 \\ \max\left(\min\left(\frac{D_1}{a_1}, Q_b^1 + Q_b^2\right), Q_b^1\right) & \text{if } P^1(S_0) \leq h_0(S_1) \leq P^2(S_0) \leq h_1(S_1) \leq h_3(S_1) \leq S_0 \\ \max\left(\min\left(\frac{D_1}{a_1}, Q_b^1 + Q_b^2\right), \frac{D_2}{a_2}\right) & \text{if } h_0(S_1) \leq P^1(S_0) \leq P^2(S_0) \leq h_1(S_1) \leq S_0 \leq h_3(S_1) \\ \max\left(\min\left(\frac{D_1}{a_1}, Q_b^1 + Q_b^2\right), \max\left(\frac{D_2}{a_2}, Q_1\right)\right) & \text{if } P^1(S_0) \leq h_0(S_1) \leq P^2(S_0) \leq h_1(S_1) \leq S_0 \leq h_3(S_1) \\ \max\left(\frac{D_1}{a_1}, Q_b^1\right) & \text{if } P^1(S_0) \leq h_0(S_1) \leq P^2(S_0) \leq S_0 \leq h_1(S_1) \\ \frac{D_1}{a_1} & \text{if } h_0(S_1) \leq P^1(S_0) \leq P^2(S_0) \leq S_0 \leq h_1(S_1) \\ Q_b^1 + Q_b^2 & \text{if } P^2(S_0) \leq h_0(S_1) \leq S_0 \leq h_1(S_1) \\ K_0 & \text{if } S_0 \leq h_0(S_1). \end{cases} \tag{4.33}$$

For $\frac{D_2}{a_2} > \frac{D_1}{a_1}$, we also have 10 cases to analyze. The optimal processing decision $z_{0,b}^*$ for $\frac{D_2}{a_2} > \frac{D_1}{a_1}$ is structurally the same as (4.33) except that $h_1(S_1)$ is replaced by $h_2(S_1)$ and $\frac{D_1}{a_1}$ switches its position with $\frac{D_2}{a_2}$. Combining the optimal processing decisions for $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$ and $\frac{D_2}{a_2} > \frac{D_1}{a_1}$, we complete our proof. ■

Proof of Proposition 17: Paralleling the proof of Proposition 16, we have two situations to discuss: $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$ and $\frac{D_2}{a_2} > \frac{D_1}{a_1}$. We only provide the proof for $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$. The proof for $\frac{D_2}{a_2} > \frac{D_1}{a_1}$ can be established in a similar fashion; we will briefly explain how to characterize (Q_b^{1*}, Q_b^{2*}) for $\frac{D_2}{a_2} > \frac{D_1}{a_1}$ at the end of the proof.

Given the optimal processing volume $z_{0,b}^*$ in (5), the optimal stage 2 profit is $\pi(Q_b^1, Q_b^2; S_0, S_1) \equiv \Pi(z_{0,b}^*)$ for given contract volumes (Q_b^1, Q_b^2) and spot price realizations (S_0, S_1). Let $V_b(Q_b^1, Q_b^2) \doteq \mathbb{E}\left[\pi(Q_b^1, Q_b^2; \tilde{S}_0, \tilde{S}_1)\right] - \beta^1 Q_b^1 - \beta^2 Q_b^2$ denote the stage 1 expected profit for given contract volumes (Q_b^1, Q_b^2), and let $V_b(Q_b^1, Q_b^2) \doteq \mathbb{E}_{\tilde{S}_1}\left[R(Q_b^1, Q_b^2; \tilde{S}_1)\right]$ where $R(Q_b^1, Q_b^2; S_1) \doteq \mathbb{E}_{\tilde{S}_0|S_1}\left[\pi(Q_b^1, Q_b^2, S_1; \tilde{S}_0)\right] - \beta^1 Q_b^1 - \beta^2 Q_b^2$. Because $\pi(Q_b^1, Q_b^2)$ satisfies Lipshitz condition of order one, expectation and differentiation operators can be interchanged, and thus, $\frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q^i} = \mathbb{E}_{\tilde{S}_1}\left[\frac{\partial R(Q_b^1, Q_b^2; \tilde{S}_1)}{\partial Q^i}\right]$ for $i \in \{1, 2\}$.

We next proceed to characterize $R(Q_b^1, Q_b^2; S_1)$ and then obtain $\frac{\partial R(Q_b^1, Q_b^2; S_1)}{\partial Q^i}$ for $i \in \{1, 2\}$. Since $A^1 \le A^2$ and $h_0(S_1) \le h_1(S_1) \le h_3(S_1)$, we have 10 cases to consider. We only analyze one case to illustrate how to derive $\frac{\partial R(Q_b^1, Q_b^2; S_1)}{\partial Q^i}$ for $i \in \{1, 2\}$; that is, $A^1 \le h_0(S_1) \le h_1(S_1) \le A^2 \le h_3(S_1)$ for a given S_1, which gives us 6 subcases to consider:

Subcase 1: $S_0 \le A^1 \le h_0(S_1) \le h_1(S_1) \le A^2 \le h_3(S_1)$

In this case, we have $P^1(S_0) = P^2(S_0) = S_0$. Using Proposition 16, we obtain the optimal processing decision $z_{0,b}^* = K_0$. Together with $\frac{D_2}{a_2} \le \frac{D_1}{a_1} \le K_0$ and $Q_b^1 + Q_b^2 \le K_0$, we have

$$
\begin{aligned}
\pi(Q_b^1, Q_b^2; S_0, S_1) =& \Pi(K_0) \\
=& - Q_b^1 S_0 - Q_b^2 S_0 - (K_0 - Q_b^1 - Q_b^2) S_0 + p_2 D_2 + D_1 (p_1 - S_1)^+ \\
& + D_1 \min(\max(p_1, S_1(1-\omega_1)), S_1) + (a_1 K_0 - D_1) S_1 (1-\omega_1) - c_0 K_0.
\end{aligned}
$$

The derivatives with respect to Q_b^1 and Q_b^2 are as follows:

$$
\begin{aligned}
\frac{\partial \Pi(K_0)}{\partial Q_b^1} =& 0, \\
\frac{\partial \Pi(K_0)}{\partial Q_b^2} =& 0.
\end{aligned}
$$

Subcase 2: $A^1 \leq S_0 \leq h_0(S_1) \leq h_1(S_1) \leq A^2 \leq h_3(S_1)$

In this case, we have $P^1(S_0) = A^1$, $P^2(S_0) = S_0$. Using Proposition 16, we obtain the optimal processing decision $z^*_{0,b} = K_0$. Together with $\frac{D_2}{a_2} \leq \frac{D_1}{a_1} \leq K_0$ and $Q^1_b + Q^2_b \leq K_0$, we have

$$
\begin{aligned}
\pi(Q^1_b, Q^2_b; S_0, S_1) =& \Pi(K_0) \\
=& -Q^1_b A^1 - Q^2_b S_0 - (K_0 - Q^1_b - Q^2_b) S_0 + p_2 D_2 + D_1(p_1 - S_1)^+ \\
&+ D_1 \min(\max(p_1, S_1(1-\omega_1)), S_1) + (a_1 K_0 - D_1) S_1 (1-\omega_1) - c_0 K_0.
\end{aligned}
$$

The derivatives with respect to Q^1_b and Q^2_b are as follows:

$$
\begin{aligned}
\frac{\partial \Pi(K_0)}{\partial Q^1_b} =& S_0 - A^1, \\
\frac{\partial \Pi(K_0)}{\partial Q^2_b} =& 0.
\end{aligned}
$$

Subcase 3: $A^1 \leq h_0(S_1) \leq S_0 \leq h_1(S_1) \leq A^2 \leq h_3(S_1)$

In this case, we have $P^1(S_0) = A^1$, $P^2(S_0) = S_0$. Using Proposition 16, we obtain the optimal processing decision $z^*_{0,b} = \max\left(\frac{D_1}{a_1}, Q^1_b\right)$. Together with $\frac{D_2}{a_2} \leq \frac{D_1}{a_1} \leq K_0$ and $Q^1_b + Q^2_b \leq K_0$, we have

$$
\begin{aligned}
\pi(Q^1_b, Q^2_b; S_0, S_1) =& \Pi\left(\max\left(\frac{D_1}{a_1}, Q^1_b\right)\right) \\
=& -Q^1_b A^1 - \min\left(Q^2_b, \left(\max\left(\frac{D_1}{a_1}, Q^1_b\right) - Q^1_b\right)\right) S_0 \\
&- \left(\max\left(\frac{D_1}{a_1}, Q^1_b\right) - Q^1_b - Q^2_b\right)^+ S_0 + p_2 D_2 + D_1(p_1 - S_1)^+ \\
&+ D_1 \min(\max(p_1, S_1(1-\omega_1)), S_1) + \left(a_1 \max\left(\frac{D_1}{a_1}, Q^1_b\right) - D_1\right) S_1(1-\omega_1) \\
&- c_0 \max\left(\frac{D_1}{a_1}, Q^1_b\right).
\end{aligned}
$$

The derivatives with respect to Q_b^1 and Q_b^2 are as follows:

$$\frac{\partial \Pi\left(\max\left(\frac{D_1}{a_1}, Q_b^1\right)\right)}{\partial Q_b^1} = \chi\left(Q_b^1 \le \frac{D_1}{a_1}\right)\left(S_0 - A^1\right) + \chi\left(\frac{D_1}{a_1} < Q_b^1 \le K_0\right)\left(h_0(S_1) - A^1\right),$$
$$\frac{\partial \Pi\left(\max\left(\frac{D_1}{a_1}, Q_b^1\right)\right)}{\partial Q_b^2} = 0.$$

Subcase 4: $A^1 \le h_0(S_1) \le h_1(S_1) \le S_0 \le A^2 \le h_3(S_1)$

In this case, we have $P^1(S_0) = A^1$, $P^2(S_0) = S_0$. Using Proposition 16, we obtain the optimal processing decision $z_{0,b}^* = \max\left(\frac{D_2}{a_2}, Q_b^1\right)$. Together with $\frac{D_2}{a_2} \le \frac{D_1}{a_1} \le K_0$ and $Q_b^1 + Q_b^2 \le K_0$, we have

$$\begin{aligned}
\pi(Q_b^1, Q_b^2; S_0, S_1) =& \Pi\left(\max\left(\frac{D_2}{a_2}, Q_b^1\right)\right) \\
=& -Q_b^1 A^1 - \min\left(Q_b^2, \left(\max\left(\frac{D_2}{a_2}, Q_b^1\right) - Q_b^1\right)\right) S_0 \\
& - \left(\max\left(\frac{D_2}{a_2}, Q_b^1\right) - Q_b^1 - Q_b^2\right)^+ S_0 + p_2 D_2 + D_1(p_1 - S_1)^+ \\
& + \min\left(a_1 \max\left(\frac{D_2}{a_2}, Q_b^1\right), D_1\right) \min(\max(p_1, S_1(1-\omega_1)), S_1) \\
& + \left(a_1 \max\left(\frac{D_2}{a_2}, Q_b^1\right) - D_1\right)^+ S_1(1-\omega_1) - c_0 \max\left(\frac{D_2}{a_2}, Q_b^1\right).
\end{aligned}$$

The derivatives with respect to Q_b^1 and Q_b^2 are as follows:

$$\begin{aligned}
\frac{\partial \Pi\left(\max\left(\frac{D_2}{a_2}, Q_b^1\right)\right)}{\partial Q_b^1} =& \chi\left(Q_b^1 \le \frac{D_2}{a_2}\right)\left(S_0 - A^1\right) + \chi\left(\frac{D_2}{a_2} < Q_b^1 \le \frac{D_1}{a_1}\right)\left(h_1(S_1) - A^1\right) \\
& + \chi\left(\frac{D_1}{a_1} < Q_b^1 \le K_0\right)\left(h_0(S_1) - A^1\right),
\end{aligned}$$
$$\frac{\partial \Pi\left(\max\left(\frac{D_2}{a_2}, Q_b^1\right)\right)}{\partial Q_b^2} = 0.$$

Subcase 5: $A^1 \le h_0(S_1) \le h_1(S_1) \le A^2 \le S_0 \le h_3(S_1)$

In this case, we have $P^1(S_0) = A^1$, $P^2(S_0) = A^2$. Using Proposition 16, we obtain the optimal processing decision $z_{0,b}^* = \max\left(\frac{D_2}{a_2}, Q_b^1\right)$. Together with $\frac{D_2}{a_2} \leq \frac{D_1}{a_1} \leq K_0$ and $Q_b^1 + Q_b^2 \leq K_0$, we have

$$\begin{aligned}
\pi(Q_b^1, Q_b^2; S_0, S_1) =& \Pi\left(\max\left(\frac{D_2}{a_2}, Q_b^1\right)\right) \\
=& - Q_b^1 A^1 - \min\left(Q_b^2, \left(\max\left(\frac{D_2}{a_2}, Q_b^1\right) - Q_b^1\right)\right) A^2 \\
& - \left(\max\left(\frac{D_2}{a_2}, Q_b^1\right) - Q_b^1 - Q_b^2\right)^+ S_0 + p_2 D_2 + D_1(p_1 - S_1)^+ \\
& + \min\left(a_1 \max\left(\frac{D_2}{a_2}, Q_b^1\right), D_1\right) \min(\max(p_1, S_1(1-\omega_1)), S_1) \\
& + \left(a_1 \max\left(\frac{D_2}{a_2}, Q_b^1\right) - D_1\right)^+ S_1(1-\omega_1) - c_0 \max\left(\frac{D_2}{a_2}, Q_b^1\right).
\end{aligned}$$

The derivatives with respect to Q_b^1 and Q_b^2 are as follows:

$$\begin{aligned}
\frac{\partial \Pi\left(\max\left(\frac{D_2}{a_2}, Q_b^1\right)\right)}{\partial Q_b^1} =& \chi\left(Q_b^1 \leq \frac{D_2}{a_2} < Q_b^1 + Q_b^2 \leq K_0\right)\left(A^2 - A^1\right) + \chi\left(\frac{D_2}{a_2} \geq Q_b^1 + Q_b^2\right)\left(S_0 - A^1\right) \\
& + \chi\left(\frac{D_2}{a_2} < Q_b^1 \leq \frac{D_1}{a_1}\right)\left(h_1(S_1) - A^1\right) + \chi\left(\frac{D_1}{a_1} < Q_b^1 \leq K_0\right)\left(h_0(S_1) - A^1\right), \\
\frac{\partial \Pi\left(\max\left(\frac{D_2}{a_2}, Q_b^1\right)\right)}{\partial Q_b^2} =& \chi\left(\frac{D_2}{a_2} \geq Q_b^1 + Q_b^2\right)\left(S_0 - A^2\right).
\end{aligned}$$

Subcase 6: $A^1 \leq h_0(S_1) \leq h_1(S_1) \leq A^2 \leq h_3(S_1) \leq S_0$

In this case, we have $P^1(S_0) = A^1$, $P^2(S_0) = A^2$. Using Proposition 16, we obtain the optimal processing decision $z_{0,b}^* = \min\left(\max\left(Q_b^1, \frac{D_2}{a_2}\right), Q_b^1 + Q_b^2\right)$. Together with

$\frac{D_2}{a_2} \le \frac{D_1}{a_1} \le K_0$ and $Q_b^1 + Q_b^2 \le K_0$, we have

$$
\begin{aligned}
\pi(Q_b^1, Q_b^2; S_0, S_1) =& \Pi\left(\min\left(\max\left(Q_b^1, \frac{D_2}{a_2}\right), Q_b^1 + Q_b^2\right)\right) \\
=& - Q_b^1 A^1 - \min\left(Q_b^2, \min\left(\max\left(Q_b^1, \frac{D_2}{a_2}\right), Q_b^1 + Q_b^2\right) - Q_b^1\right) A^2 \\
& + a_2 p_2 \min\left(\frac{D_2}{a_2}, Q_b^1 + Q_b^2\right) + D_1(p_1 - S_1)^+ \\
& + \min\left(a_1 \min\left(\max\left(Q_b^1, \frac{D_2}{a_2}\right), Q_b^1 + Q_b^2\right), D_1\right) \min(\max(p_1, S_1(1-\omega_1)), S_1) \\
& + \left(a_1 \min\left(\max\left(Q_b^1, \frac{D_2}{a_2}\right), Q_b^1 + Q_b^2\right) - D_1\right)^+ S_1(1-\omega_1) \\
& - c_0 \min\left(\max\left(Q_b^1, \frac{D_2}{a_2}\right), Q_b^1 + Q_b^2\right).
\end{aligned}
$$

The derivatives with respect to Q_b^1 and Q_b^2 are as follows:

$$
\begin{aligned}
\frac{\partial \Pi\left(\min\left(\max\left(Q_b^1, \frac{D_2}{a_2}\right), Q_b^1 + Q_b^2\right)\right)}{\partial Q_b^1} =& \chi\left(Q_b^1 \le \frac{D_2}{a_2} < Q_b^1 + Q_b^2 \le K_0\right)\left(A^2 - A^1\right) \\
& + \chi\left(\frac{D_2}{a_2} \ge Q_b^1 + Q_b^2\right)\left(h_3(S_1) - A^1\right) \\
& + \chi\left(\frac{D_2}{a_2} < Q_b^1 \le \frac{D_1}{a_1}\right)\left(h_1(S_1) - A^1\right) \\
& + \chi\left(\frac{D_1}{a_1} < Q_b^1 \le K_0\right)\left(h_0(S_1) - A^1\right), \\
\frac{\partial \Pi\left(\min\left(\max\left(Q_b^1, \frac{D_2}{a_2}\right), Q_b^1 + Q_b^2\right)\right)}{\partial Q_b^2} =& \chi\left(\frac{D_2}{a_2} \ge Q_b^1 + Q_b^2\right)\left(h_3(S_1) - A^2\right).
\end{aligned}
$$

Now, we have

$$
\begin{aligned}
R(Q_b^1, Q_b^2; S_1) = & -\beta^1 Q_b^1 - \beta^2 Q_b^2 + \int_0^{A^1} \Pi(K_0) dF_{0|1}(\tilde{S}_0) \\
& + \int_{A^1}^{h_0(S_1)} \Pi(K_0) dF_{0|1}(\tilde{S}_0) \\
& + \int_{h_0(S_1)}^{h_1(S_1)} \Pi\left(\max\left(\frac{D_1}{a_1}, Q_b^1\right)\right) dF_{0|1}(\tilde{S}_0) \\
& + \int_{h_1(S_1)}^{A^2} \Pi\left(\max\left(\frac{D_2}{a_2}, Q_b^1\right)\right) dF_{0|1}(\tilde{S}_0) \\
& + \int_{A^2}^{h_3(S_1)} \Pi\left(\max\left(\frac{D_2}{a_2}, Q_b^1\right)\right) dF_{0|1}(\tilde{S}_0) \\
& + \int_{h_3(S_1)}^{\infty} \Pi\left(\min\left(\max\left(Q_b^1, \frac{D_2}{a_2}\right), Q_b^1 + Q_b^2\right)\right) dF_{0|1}(\tilde{S}_0),
\end{aligned}
$$

where $F_{0|1}(.)$ is the cumulative probability function of input spot price $\tilde{S}_0$ conditional on the output spot price S_1. Although the integrands of several integrals (e.g., the $\Pi(K_0)$ of $\int_0^{A^1} \Pi(K_0)dF_{0|1}(\tilde{S}_0)$ and $\int_{A^1}^{h_0(S_1)} \Pi(K_0)dF_{0|1}(\tilde{S}_0)$) are the same, we do not combine them together to simplify the expression because $P^1(S_0)$ and $P^2(S_0)$ (thus the integrands) take different values in its own range of S_0. We note that the integral terms in $R(Q_b^1, Q_b^2; S_1)$ take the form of $\int_x^y \Pi(.)dF_{0|1}(\tilde{S}_0)$ and the limits of integral (i.e., x and y) are independent of Q_b^1 and Q_b^2. Therefore, we obtain $\frac{\partial \int_x^y \Pi(.)dF_{0|1}(\tilde{S}_0)}{\partial Q^i} = \int_x^y \frac{\partial \Pi(.)}{\partial Q^i} dF_{0|1}(\tilde{S}_0)$ for $i \in \{1, 2\}$, and the first derivatives of $R(Q_b^1, Q_b^2; S_1)$ with respect to Q_b^1 and Q_b^2 as follows:

$$
\begin{aligned}
\frac{\partial R(Q_b^1, Q_b^2; S_1)}{\partial Q_b^1} =& -\beta^1 + \int_{A^1}^{h_0(S_1)} \left(S_0 - A^1\right) dF_{0|1}(\tilde{S}_0) \\
&+ \int_{h_0(S_1)}^{h_1(S_1)} \left[\chi\left(Q_b^1 \le \frac{D_1}{a_1}\right)\left(S_0 - A^1\right)\right. \\
&\qquad \left. + \chi\left(\frac{D_1}{a_1} < Q_b^1 \le K_0\right)\left(h_0(S_1) - A^1\right)\right] dF_{0|1}(\tilde{S}_0) \\
&+ \int_{h_1(S_1)}^{A^2} \left[\chi\left(Q_b^1 \le \frac{D_2}{a_2}\right)\left(S_0 - A^1\right)\right. \\
&\qquad + \chi\left(\frac{D_2}{a_2} < Q_b^1 \le \frac{D_1}{a_1}\right)\left(h_1(S_1) - A^1\right) \\
&\qquad \left. + \chi\left(\frac{D_1}{a_1} < Q_b^1 \le K_0\right)\left(h_0(S_1) - A^1\right)\right] dF_{0|1}(\tilde{S}_0) \\
&+ \int_{A^2}^{h_3(S_1)} \left[\chi\left(Q_b^1 \le \frac{D_2}{a_2} < Q_b^1 + Q_b^2 \le K_0\right)\left(A^2 - A^1\right)\right. \\
&\qquad + \chi\left(\frac{D_2}{a_2} \ge Q_b^1 + Q_b^2\right)\left(S_0 - A^1\right) \\
&\qquad + \chi\left(\frac{D_2}{a_2} < Q_b^1 \le \frac{D_1}{a_1}\right)\left(h_1(S_1) - A^1\right) \\
&\qquad \left. + \chi\left(\frac{D_1}{a_1} < Q_b^1 \le K_0\right)\left(h_0(S_1) - A^1\right)\right] dF_{0|1}(\tilde{S}_0) \\
&+ \int_{h_3(S_1)}^{\infty} \left[\chi\left(Q_b^1 \le \frac{D_2}{a_2} < Q_b^1 + Q_b^2 \le K_0\right)\left(A^2 - A^1\right)\right. \\
&\qquad + \chi\left(\frac{D_2}{a_2} \ge Q_b^1 + Q_b^2\right)\left(h_3(S_1) - A^1\right) \\
&\qquad + \chi\left(\frac{D_2}{a_2} < Q_b^1 \le \frac{D_1}{a_1}\right)\left(h_1(S_1) - A^1\right) \\
&\qquad \left. + \chi\left(\frac{D_1}{a_1} < Q_b^1 \le K_0\right)\left(h_0(S_1) - A^1\right)\right] dF_{0|1}(\tilde{S}_0) \\
=& -\beta^1 - A^1 + g(A^1, h_0(S_1))H_1 + g(A^1, h_1(S_1))(H_2 + H_3) \\
&+ g(A^1, A^2)(H_4 + H_5) + g(A^1, h_3(S_1))H_6,
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial R(Q_b^1, Q_b^2; S_1)}{\partial Q_b^2} =& -\beta^2 + \int_{A^2}^{h_3(S_1)} \chi\left(\frac{D_2}{a_2} \geq Q_b^1 + Q_b^2\right)\left(S_0 - A^2\right) dF_{0|1}(\tilde{S}_0) \\
&+ \int_{h_3(S_1)}^{\infty} \chi\left(\frac{D_2}{a_2} \geq Q_b^1 + Q_b^2\right)\left(h_3(S_1) - A^2\right) dF_{0|1}(\tilde{S}_0) \\
=& -\beta^2 - A^2 + g(A^2, A^2)(H_1 + H_2 + H_3 + H_4 + H_5) + g(A^2, h_3(S_1))H_6,
\end{aligned}
$$

where

$$
g(\theta_1, \theta_2) \doteq \int_0^{\theta_1} \theta_1 dF_{0|1}(\tilde{S}_0) + \int_{\theta_1}^{\theta_2} \tilde{S}_0 dF_{0|1}(\tilde{S}_0) + \int_{\theta_2}^{\infty} \theta_2 dF_{0|1}(\tilde{S}_0),
$$

for $\theta_1 \leq \theta_2$, and

$$
\begin{aligned}
H_1 &\doteq \chi\left(\tfrac{D_1}{a_1} < Q_b^1 \leq K_0, \tfrac{D_2}{a_2} < Q_b^1 \leq K_0\right), \\
H_2 &\doteq \chi\left(Q_b^1 \leq \tfrac{D_1}{a_1} < Q_b^1 + Q_b^2 \leq K_0, \tfrac{D_2}{a_2} < Q_b^1 \leq K_0\right), \\
H_3 &\doteq \chi\left(\tfrac{D_1}{a_1} \geq Q_b^1 + Q_b^2, \tfrac{D_2}{a_2} < Q_b^1 \leq K_0\right), \\
H_4 &\doteq \chi\left(Q_b^1 \leq \tfrac{D_1}{a_1} < Q_b^1 + Q_b^2 \leq K_0, Q_b^1 \leq \tfrac{D_2}{a_2} < Q_b^1 + Q_b^2 \leq K_0\right), \\
H_5 &\doteq \chi\left(\tfrac{D_1}{a_1} \geq Q_b^1 + Q_b^2, Q_b^1 \leq \tfrac{D_2}{a_2} < Q_b^1 + Q_b^2 \leq K_0\right), \\
H_6 &\doteq \chi\left(\tfrac{D_1}{a_1} \geq Q_b^1 + Q_b^2, \tfrac{D_2}{a_2} \geq Q_b^1 + Q_b^2\right).
\end{aligned}
$$

Combining 10 cases together and taking the integration with respect to $\tilde{S}_1$, we have

$$
\begin{aligned}
\frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^1} =& -\beta^1 - A^1 + G(A^1, \max(h_0(\tilde{S}_1), A^1))H_1 \\
&+ G(A^1, \max(\min(h_1(\tilde{S}_1), \max(h_0(\tilde{S}_1), A^2)), \max(h_0(\tilde{S}_1), A^1)))H_2 \\
&+ G(A^1, \max(h_1(\tilde{S}_1), A^1))H_3 \\
&+ G(A^1, \max(\min(h_3(\tilde{S}_1), \max(h_0(\tilde{S}_1), A^2)), \max(h_0(\tilde{S}_1), A^1)))H_4 \\
&+ G(A^1, \max(\min(h_3(\tilde{S}_1), \max(h_1(\tilde{S}_1), A^2)), \max(h_1(\tilde{S}_1), A^1)))H_5 \\
&+ G(A^1, \max(h_3(\tilde{S}_1), A^1))H_6, \\
\frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^2} =& -\beta^2 - A^2 + G(A^2, \max(h_0(\tilde{S}_1), A^2))(H_1 + H_2 + H_4) \\
&+ G(A^2, \max(h_1(\tilde{S}_1), A^2))(H_3 + H_5) + G(A^2, \max(h_3(\tilde{S}_1), A^2))H_6,
\end{aligned}
$$

where $V_b(Q_b^1, Q_b^2) = \mathbb{E}_{\tilde{S}_1}\left[R(Q_b^1, Q_b^2; \tilde{S}_1)\right]$ and $G(\theta_1, \theta_2) \doteq \mathbb{E}_{\tilde{S}_1}[g(\theta_1, \theta_2)]$.

To understand which contract gives the larger expected marginal profit for given

(Q_b^1, Q_b^2), we need to determine the sign of the difference of first derivatives that is:

$$
\begin{aligned}
\frac{\partial V_b(Q_b^1,Q_b^2)}{\partial Q_b^1} - \frac{\partial V_b(Q_b^1,Q_b^2)}{\partial Q_b^2} = & \\
& -\beta^1 - A^1 + \beta^2 + A^2 \\
& + (G(A^1, \max(h_3(\tilde{S}_1), A^1)) - G(A^2, \max(h_3(\tilde{S}_1), A^2)))\chi\left(Q_b^1 \le \frac{D_2}{a_2}\right) \\
& + (G(A^1, \max(h_1(\tilde{S}_1), A^1)) - G(A^2, \max(h_1(\tilde{S}_1), A^2)))\chi\left(\frac{D_2}{a_2} < Q_b^1 \le \frac{D_1}{a_1}\right) \\
& + (G(A^1, \max(h_0(\tilde{S}_1), A^1)) - G(A^2, \max(h_0(\tilde{S}_1), A^2)))\chi\left(\frac{D_1}{a_1} < Q_b^1 \le K_0\right) \\
= & -\beta^1 - A^1 + \beta^2 + A^2 - G(A^2, A^2) \\
& + G(A^1, \max(\min(h_3(\tilde{S}_1), A^2), A^1))\chi\left(Q_b^1 \le \frac{D_2}{a_2}\right) \\
& + G(A^1, \max(\min(h_1(\tilde{S}_1), A^2), A^1))\chi\left(\frac{D_2}{a_2} < Q_b^1 \le \frac{D_1}{a_1}\right) \\
& + G(A^1, \max(\min(h_0(\tilde{S}_1), A^2), A^1))\chi\left(\frac{D_1}{a_1} < Q_b^1 \le K_0\right). \qquad (4.34)
\end{aligned}
$$

The first equality in (4.34) is obtained by

$$
\begin{aligned}
G(A^1, \max(\min(h_\kappa(\tilde{S}_1), \max(h_0(\tilde{S}_1), A^2)), \max(h_0(\tilde{S}_1), A^1))) - G(A^2, \max((h_0(\tilde{S}_1), A^2) \\
= G(A^1, \max(h_\kappa(\tilde{S}_1), A^1)) - G(A^2, \max(h_\kappa(\tilde{S}_1), A^2)), \qquad (4.35)
\end{aligned}
$$

and the second equality in (4.34) is obtained by

$$
\begin{aligned}
G(A^1, \max(h_\kappa(\tilde{S}_1), A^1)) - G(A^2, \max(h_\kappa(\tilde{S}_1), A^2)) \qquad (4.36) \\
= G(A^1, \max(\min(h_\kappa(\tilde{S}_1), A^2), A^1)) - G(A^2, A^2),
\end{aligned}
$$

for $\kappa \in \{0, 1, 2, 3\}$.

We observe that (4.34) is piecewise linear and decreasing in Q_b^1. To determine the sign of $\frac{\partial V_b(Q_b^1,Q_b^2)}{\partial Q_b^1} - \frac{\partial V_b(Q_b^1,Q_b^2)}{\partial Q_b^2}$, we have 4 cases to consider:

Case (i):

$$G(A^2, A^2) - \beta^2 - A^2 > G(A^1, \max(\min(h_3(\tilde{S}_1), A^2), A^1)) - \beta^1 - A^1,$$

Case (ii):

$$\begin{aligned} &G(A^2, A^2) - \beta^2 - A^2 > G(A^1, \max(\min(h_1(\tilde{S}_1), A^2), A^1)) - \beta^1 - A^1, \\ \text{and } &G(A^2, A^2) - \beta^2 - A^2 \leq G(A^1, \max(\min(h_3(\tilde{S}_1), A^2), A^1)) - \beta^1 - A^1, \end{aligned}$$

Case (iii):

$$\begin{aligned} &G(A^2, A^2) - \beta^2 - A^2 > G(A^1, \max(\min(h_0(\tilde{S}_1), A^2), A^1)) - \beta^1 - A^1, \\ \text{and } &G(A^2, A^2) - \beta^2 - A^2 \leq G(A^1, \max(\min(h_1(\tilde{S}_1), A^2), A^1)) - \beta^1 - A^1, \end{aligned}$$

Case (iv):

$$G(A^2, A^2) - \beta^2 - A^2 \leq G(A^1, \max(\min(h_0(\tilde{S}_1), A^2), A^1)) - \beta^1 - A^1.$$

We proceed to derive the optimal contract volumes for 4 cases and combine them together to complete the proof for $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$. Let Q_b denote the total reserved contract volume; let $V_b(Q_b)$ denote the stage 1 expected profit.

Case (i): For given (Q_b^1, Q_b^2), $\frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^1} \leq \frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^2}$ always holds. It implies that contract 2 always gives the larger expected marginal profit. To obtain the optimal contract volume from contract 2, we transform $V_b(Q_b^1, Q_b^2)$ to $V_b(Q)$ as follows:

$$V_b(Q_b) = -\beta^2 Q_b + \mathbb{E}\left[\pi\left(0, Q_b\right)\right].$$

Recall that $\pi\left(Q_b^1, Q_b^2\right) = \Pi(z_{0,b}^*)$. Using (4.33), we obtain the first derivative of

$V_b(Q_b)$ with respect to Q_b as follows:

$$\begin{aligned}\frac{\partial V_b(Q_b)}{\partial Q_b} =& -\beta^2 - A^2 + G(A^2, \max(h_3(\tilde{S}_1), A^2))\chi\left(Q_b \le \frac{D_2}{a_2}\right) \\ &+ G(A^2, \max(h_1(\tilde{S}_1)), A^2))\chi\left(\frac{D_2}{a_2} < Q_b \le \frac{D_1}{a_1}\right) \\ &+ G(A^2, \max(h_0(\tilde{S}_1), A^2))\chi\left(\frac{D_1}{a_1} < Q_b \le K_0\right).\end{aligned}$$

The optimal contracting decision Q_b^* is

$$Q_b^* = \begin{cases} 0 & \text{if } \beta^2 + A^2 \ge G(A^2, \max(h_3(\tilde{S}_1), A^2)), \\ \frac{D_2}{a_2} & \text{if } G(A^2, \max(h_1(\tilde{S}_1), A^2)) \le \beta^2 + A^2 < G(A^2, \max(h_3(\tilde{S}_1), A^2)), \\ \frac{D_1}{a_1} & \text{if } G(A^2, \max(h_0(\tilde{S}_1), A^2)) \le \beta^2 + A^2 < G(A^2, \max(h_1(\tilde{S}_1), A^2)), \\ K_0 & \text{if } \beta^2 + A^2 < G(A^2, \max(h_0(\tilde{S}_1), A^2)). \end{cases}$$

Recall that contract 2 always gives the larger expected marginal profit, so the optimal contract volumes are equivalent to

$$\left(Q_b^{1*}, Q_b^{2*}\right) = \begin{cases} (0,0) & \text{if } \mathcal{M}_3^2 \le 0, \\ \left(0, \frac{D_2}{a_2}\right) & \text{if } \mathcal{M}_1^2 \le 0 < \mathcal{M}_3^2, \\ \left(0, \frac{D_1}{a_1}\right) & \text{if } \mathcal{M}_0^2 \le 0 < \mathcal{M}_1^2, \\ (0, K_0) & \text{if } 0 < \mathcal{M}_0^2, \end{cases}$$

where $\mathcal{M}_\kappa^i \doteq G(A^i, \max(h_\kappa(\tilde{S}_1), A^i)) - \beta^i - A^i$ for $\kappa \in \{0,1,3\}$ and $i \in \{1,2\}$.

Case (ii): From (4.34), we obtain that for $Q_b^1 \le \frac{D_2}{a_2}$, $\frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^1} \ge \frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^2}$ and for $Q_b^1 > \frac{D_2}{a_2}$, $\frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^1} < \frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^2}$. It implies that contract 1 always gives the larger expected marginal profit for the first $\frac{D_2}{a_2}$ contracted units while contract 2 gives the larger expected marginal profit for the rest contracted units. There are two situations we need to consider: $Q_b \le \frac{D_2}{a_2}$ and $Q_b > \frac{D_2}{a_2}$. The corresponding transformation is as follows:

$$V_b(Q_b) = \begin{cases} -\beta^1 Q_b + \mathbb{E}\left[\pi\left(Q_b, 0\right)\right] & \text{if } 0 \le Q_b \le \frac{D_2}{a_2}, \\ -\beta^1 \frac{D_2}{a_2} - \beta^2\left(Q_b - \frac{D_2}{a_2}\right) + \mathbb{E}\left[\pi\left(\frac{D_2}{a_2}, Q_b - \frac{D_2}{a_2}\right)\right] & \text{if } \frac{D_2}{a_2} < Q_b \le K_0, \end{cases}$$

that is, the firm starts sourcing from contract 1 up to $\frac{D_2}{a_2}$ and switch to contract 2. Recall that $\pi\left(Q_b^1, Q_b^2\right) = \Pi(z_{0,b}^*)$. Using (4.33), we obtain

$$\frac{\partial V_b(Q_b)}{\partial Q_b} = \begin{cases} -\beta^1 - A^1 + G(A^1, \max(h_3(\tilde{S}_1), A^1))\chi\left(Q_b \le \frac{D_2}{a_2}\right) & \text{if } 0 \le Q_b \le \frac{D_2}{a_2}, \\ -\beta^2 - A^2 + G(A^2, \max(h_1(\tilde{S}_1), A^2))\chi\left(\frac{D_2}{a_2} < Q_b \le \frac{D_1}{a_1}\right) & \\ \qquad +G(A^2, \max(h_0(\tilde{S}_1), A^2))\chi\left(\frac{D_1}{a_1} < Q_b \le K_0\right) & \text{if } \frac{D_2}{a_2} < Q_b \le K_0. \end{cases}$$

It is easy to establish that $V_b(Q_b)$ is concave in Q_b because

$$\begin{aligned} \lim_{Q_b \to \frac{D_2}{a_2}^-} \frac{\partial V_b(Q_b)}{\partial Q_b} - \lim_{Q_b \to \frac{D_2}{a_2}^+} \frac{\partial V_b(Q_b)}{\partial Q_b} &= -\beta^1 - A^1 + G(A^1, \max(h_3(\tilde{S}_1), A^1)) + \beta^2 + A^2 \\ &\quad - G(A^2, \max(h_1(\tilde{S}_1), A^2)) \\ &\ge -\beta^2 - A^2 + G(A^2, \max(h_3(\tilde{S}_1), A^2)) + \beta^2 + A^2 \\ &\quad - G(A^2, \max(h_1(\tilde{S}_1), A^2)) \\ &\ge 0. \end{aligned}$$

The first inequality holds due to the premise of Case (ii) and also (4.36). In particular, from the premise of Case (ii), we have $G(A^1, \max(\min(h_3(\tilde{S}_1), A^2), A^1)) - \beta^1 - A^1 \ge G(A^2, A^2) - \beta^2 - A^2$, which can be further simplified by using (4.36) as $G(A^1, \max(h_3(\tilde{S}_1), A^1)) - \beta^1 - A^1 \ge G(A^2, \max(h_3(\tilde{S}_1), A^2)) - \beta^2 - A^2$. The optimal contract decision Q_b^* is

$$Q_b^* = \begin{cases} 0 & \text{if } \beta^1 + A^1 \geq G(A^1, \max(h_3(\tilde{S}_1), A^1)), \\ \frac{D_2}{a_2} & \text{if } \beta^1 + A^1 < G(A^1, \max(h_3(\tilde{S}_1), A^1)) \\ & \qquad \text{and } \beta^2 + A^2 \geq G(A^2, \max(h_1(\tilde{S}_1), A^2)), \\ \frac{D_1}{a_1} & \text{if } G(A^2, \max(h_0(\tilde{S}_1), A^2)) \leq \beta^2 + A^2 < G(A^2, \max(h_1(\tilde{S}_1), A^2)), \\ K_0 & \text{if } \beta^2 + A^2 < G(A^2, \max(h_0(\tilde{S}_1), A^2)), \end{cases}$$

Recall that contract 1 always gives the larger expected marginal profit for the first $\frac{D_2}{a_2}$ units of contract volume while contract 2 gives the larger expected marginal profit for the rest units of contract volume, so the optimal contract volumes are equivalent to

$$\left(Q_b^{1*}, Q_b^{2*}\right) = \begin{cases} (0,0) & \text{if } \mathcal{M}_3^1 \leq 0, \\ \left(\frac{D_2}{a_2}, 0\right) & \text{if } \mathcal{M}_3^1 > 0 \text{ and } \mathcal{M}_1^2 \leq 0, \\ \left(\frac{D_2}{a_2}, \frac{D_1}{a_1} - \frac{D_2}{a_2}\right) & \text{if } \mathcal{M}_1^2 > 0 \text{ and } \mathcal{M}_0^2 \leq 0, \\ \left(\frac{D_2}{a_2}, K_0 - \frac{D_2}{a_2}\right) & \text{if } \mathcal{M}_0^2 > 0, \end{cases}$$

where $\mathcal{M}_\kappa^i \doteq G(A^i, \max(h_\kappa(\tilde{S}_1), A^i)) - \beta^i - A^i$ for $\kappa \in \{0, 1, 3\}$ and $i \in \{1, 2\}$.

Case (iii): From (4.34), we obtain that for $Q_b^1 \leq \frac{D_1}{a_1}$, $\frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^1} \geq \frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^2}$ and for $Q_b^1 > \frac{D_1}{a_1}$, $\frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^1} < \frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^2}$. It implies that contract 1 always gives the larger expected marginal profit for the first $\frac{D_1}{a_1}$ units of contract volume while contract 2 gives the larger expected marginal profit for the rest units of contract volume. There are two situations we need to consider: $Q_b \leq \frac{D_1}{a_1}$ and $Q_b > \frac{D_1}{a_1}$. The corresponding transformation is as follows:

$$V_b(Q_b) = \begin{cases} -\beta^1 Q_b + \mathbb{E}\left[\pi\left(Q_b, 0\right)\right] & \text{if } 0 \leq Q_b \leq \frac{D_1}{a_1}, \\ -\beta^1 \frac{D_1}{a_1} - \beta^2 \left(Q_b - \frac{D_1}{a_1}\right) + \mathbb{E}\left[\pi\left(\frac{D_1}{a_1}, Q_b - \frac{D_1}{a_1}\right)\right] & \text{if } \frac{D_1}{a_1} < Q_b \leq K_0, \end{cases}$$

that is, the firm starts sourcing from contract 1 up to $\frac{D_1}{a_1}$ and switch to contract 2. Recall that $\pi\left(Q_b^1, Q_b^2\right) = \Pi(z_{0,b}^*)$. Using (4.33), we obtain

$$\frac{\partial V_b(Q_b)}{\partial Q_b} = \begin{cases} -\beta^1 - A^1 + G(A^1, \max(h_3(\tilde{S}_1), A^1))\chi\left(Q_b \le \frac{D_2}{a_2}\right) & \\ \qquad +G(A^1, \max(h_1(\tilde{S}_1), A^1))\chi\left(\frac{D_2}{a_2} < Q_b \le \frac{D_1}{a_1}\right) & \text{if } 0 \le Q_b \le \frac{D_1}{a_1}, \\ -\beta^2 - A^2 + G(A^2, \max(h_0(\tilde{S}_1), A^2))\chi\left(\frac{D_1}{a_1} < Q_b \le K_0\right) & \text{if } \frac{D_1}{a_1} < Q_b \le K_0. \end{cases}$$

It is easy to establish that $V_b(Q_b)$ is concave in Q_b since

$$\begin{aligned} \lim_{Q_b \to \frac{D_1}{a_1}^-} \frac{\partial V_b(Q_b)}{\partial Q_b} - \lim_{Q_b \to \frac{D_1}{a_1}^+} \frac{\partial V_b(Q_b)}{\partial Q_b} &= -\beta^1 - A^1 + G(A^1, \max(h_1(\tilde{S}_1), A^1)) + \beta^2 + A^2 \\ &\quad - G(A^2, \max(h_0(\tilde{S}_1), A^2)) \\ &\ge -\beta^2 - A^2 + G(A^2, \max(h_1(\tilde{S}_1), A^2)) + \beta^2 + A^2 \\ &\quad - G(A^2, \max(h_0(\tilde{S}_1), A^2)) \\ &\ge 0. \end{aligned}$$

The first inequality holds due to the premise of Case (iii) and (4.36). In particular, from the premise of Case (iii), we have $G(A^1, \max(\min(h_1(\tilde{S}_1), A^2), A^1)) - \beta^1 - A^1 \ge G(A^2, A^2) - \beta^2 - A^2$, which can be further simplified by using (4.36) as $G(A^1, \max(h_1(\tilde{S}_1), A^1)) - \beta^1 - A^1 \ge G(A^2, \max(h_1(\tilde{S}_1), A^2)) - \beta^2 - A^2$. The optimal contract decision Q_b^* is

$$Q_b^* = \begin{cases} 0 & \text{if } \beta^1 + A^1 \ge G(A^1, \max(h_3(\tilde{S}_1), A^1)), \\ \frac{D_2}{a_2} & \text{if } G(A^1, \max(h_1(\tilde{S}_1), A^1)) \le \beta^1 + A^1 < G(A^1, \max(h_3(\tilde{S}_1), A^1)), \\ \frac{D_1}{a_1} & \text{if } \beta^1 + A^1 < G(A^1, \max(h_1(\tilde{S}_1), A^1)) \\ & \qquad \text{and } \beta^2 + A^2 \ge G(A^2, \max(h_0(\tilde{S}_1), A^2)), \\ K_0 & \text{if } \beta^2 + A^2 < G(A^2, \max(h_0(\tilde{S}_1), A^2)), \end{cases}$$

Recall that contract 1 always gives the larger expected marginal profit for the first $\frac{D_1}{a_1}$ units of contract volume while contract 2 gives the larger expected marginal profit for the rest units of contract volume, so

$$\left(Q_b^{1*}, Q_b^{2*}\right) = \begin{cases} (0,0) & \text{if } \mathcal{M}_3^1 \leq 0, \\ \left(\frac{D_2}{a_2}, 0\right) & \text{if } \mathcal{M}_3^1 > 0 \text{ and } \mathcal{M}_1^1 \leq 0, \\ \left(\frac{D_1}{a_1}, 0\right) & \text{if } \mathcal{M}_1^1 > 0 \text{ and } \mathcal{M}_0^2 \leq 0, \\ \left(\frac{D_1}{a_1}, K_0 - \frac{D_1}{a_1}\right) & \text{if } \mathcal{M}_0^2 > 0, \end{cases}$$

where $\mathcal{M}_\kappa^i \doteq G(A^i, \max(h_\kappa(\tilde{S}_1), A^i)) - \beta^i - A^i$ for $\kappa \in \{0,1,3\}$ and $i \in \{1,2\}$.

Case (iv): For all (Q_b^1, Q_b^2), $\frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^1} \geq \frac{\partial V_b(Q_b^1, Q_b^2)}{\partial Q_b^2}$ holds. It implies that contract 2 always gives the larger expected marginal profit. To obtain the optimal contract volume from contract 1, we transform $V_b(Q_b^1, Q_b^2)$ to $V_b(Q_b)$ as follows:

$$V_b(Q_b) = -\beta^1 Q_b + \mathbb{E}\left[\pi\left(Q_b, 0\right)\right].$$

Recall that $\pi\left(Q_b^1, Q_b^2\right) = \Pi(z_{0,b}^*)$. Using (4.33), we obtain the first derivative of $V_b(Q_b)$ with respect to Q_b as follows:

$$\begin{aligned} \frac{\partial V_b(Q_b)}{\partial Q_b} = & -\beta^1 - A^1 + G(A^1, \max(h_3(\tilde{S}_1), A^1))\chi\left(Q_b \leq \frac{D_2}{a_2}\right) \\ & + G(A^1, \max(h_1(\tilde{S}_1), A^1))\chi\left(\frac{D_2}{a_2} < Q_b \leq \frac{D_1}{a_1}\right) \\ & + G(A^1, \max(h_0(\tilde{S}_1), A^1))\chi\left(\frac{D_1}{a_1} < Q_b \leq K_0\right). \end{aligned}$$

The optimal contracting decision Q_b^* is

$$Q_b^* = \begin{cases} 0 & \text{if } \beta^1 + A^1 \geq G(A^1, \max(h_3(\tilde{S}_1), A^1)), \\ \frac{D_2}{a_2} & \text{if } G(A^1, \max(h_1(\tilde{S}_1), A^1)) \leq \beta^1 + A^1 < G(A^1, \max(h_3(\tilde{S}_1), A^1)), \\ \frac{D_1}{a_1} & \text{if } G(A^1, \max(h_0(\tilde{S}_1), A^1)) \leq \beta^1 + A^1 < G(A^1, \max(h_1(\tilde{S}_1), A^1)), \\ K_0 & \text{if } \beta^1 + A^1 < G(A^1, \max(h_0(\tilde{S}_1), A^1)), \end{cases}$$

Recall that contract 1 always gives the larger expected marginal profit, so the optimal contract volumes are equivalent to

$$\left(Q_b^{1*}, Q_b^{2*}\right) = \begin{cases} (0,0) & \text{if } \mathcal{M}_3^1 \leq 0, \\ \left(\frac{D_2}{a_2}, 0\right) & \text{if } \mathcal{M}_1^1 \leq 0 < \mathcal{M}_3^1, \\ \left(\frac{D_1}{a_1}, 0\right) & \text{if } \mathcal{M}_0^1 \leq 0 < \mathcal{M}_1^1, \\ (K_0, 0) & \text{if } 0 < \mathcal{M}_0^1, \end{cases}$$

where $\mathcal{M}_\kappa^i \doteq G(A^i, \max(h_\kappa(\tilde{S}_1), A^i)) - \beta^i - A^i$ for $\kappa \in \{0, 1, 3\}$ and $i \in \{1, 2\}$.

After combining the four cases together, we obtain the optimal contract volumes as follows:

$$\left(Q_b^{X*}, Q_b^{(3-X)*}\right) = \begin{cases} (0,0) & \text{if } \mathcal{M}_3^X \leq 0, \\ \left(\frac{D_2}{a_2}, 0\right) & \text{if } \mathcal{M}_3^X > 0 \text{ and } \mathcal{M}_1^Y \leq 0, \\ \left(\frac{D_1}{a_1}, 0\right) & \text{if } \mathcal{M}_3^X > 0, \mathcal{M}_1^Y > 0, \mathcal{M}_0^Z \leq 0 \text{ and } X = Y, \\ (K_0, 0) & \text{if } \mathcal{M}_3^X > 0, \mathcal{M}_1^Y > 0, \mathcal{M}_0^Z > 0 \text{ and } X = Y = Z, \\ \left(\frac{D_2}{a_2}, \frac{D_1}{a_1} - \frac{D_2}{a_2}\right) & \text{if } \mathcal{M}_3^X > 0, \mathcal{M}_1^Y > 0, \mathcal{M}_0^Z \leq 0 \text{ and } X = 1, Y = 2, \\ \left(\frac{D_2}{a_2}, K_0 - \frac{D_2}{a_2}\right) & \text{if } \mathcal{M}_3^X > 0, \mathcal{M}_1^Y > 0, \mathcal{M}_0^Z > 0 \text{ and } X = 1, Y = Z = 2, \\ \left(\frac{D_1}{a_1}, K_0 - \frac{D_1}{a_1}\right) & \text{if } \mathcal{M}_3^X > 0, \mathcal{M}_1^Y > 0, \mathcal{M}_0^Z > 0 \text{ and } X = Y = 1, Z = 2, \end{cases} \tag{4.37}$$

where X, Y and Z denote the indexes of contract with the higher value of $\mathcal{M}_3^i$, $\mathcal{M}_1^i$ and $\mathcal{M}_0^i$ for $i \in \{1, 2\}$.

For $\frac{D_2}{a_2} > \frac{D_1}{a_1}$, the proof can be established in a similar fashion. The optimal contract volumes (Q_b^{1*}, Q_b^{2*}) for $\frac{D_2}{a_2} > \frac{D_1}{a_1}$ are structurally the same as (4.37) except that $\mathcal{M}_1^Y$ is replaced by $\mathcal{M}_2^Y$ and $\frac{D_1}{a_1}$ switches its position with $\frac{D_2}{a_2}$. Combining the optimal contract volumes for $\frac{D_2}{a_2} \le \frac{D_1}{a_1}$ and $\frac{D_2}{a_2} > \frac{D_1}{a_1}$, we complete our proof. ■

Proof of Proposition 18: Paralleling Proposition 12, we transform the conditions regarding expected marginal profits (i.e., $\mathcal{M}_\kappa^i$) for $\kappa \in \{0, 1, 2, 3\}$ and $i \in \{Y, Z\}$ in Proposition 17 into conditions with respect to Δ_A and Δ_β. From conditions in Proposition 17, we obtain the additional preference and profitability thresholds.

Let $\Delta_{\beta,b}^M(\Delta_A)$ and $\Delta_{\beta,b}^H(\Delta_A)$ denote the preference thresholds when only one demand and neither demand has been satisfied, respectively. For given A^1 and β^2, by solving $\mathcal{M}_{[1]}^1 = \mathcal{M}_{[1]}^2$ and $\mathcal{M}_3^1 = \mathcal{M}_3^2$, we obtain the preference thresholds for given Δ_A as follows:

$$\Delta_{\beta,b}^M(\Delta_A) \doteq G(A^1, \max(h_{[1]}(\tilde{S}_1), A^1)) - G(A^1 + \Delta_A, \max(h_{[1]}(\tilde{S}_1), A^1 + \Delta_A)) + \Delta_A, \tag{4.38}$$

$$\Delta_{\beta,b}^H(\Delta_A) \doteq G(A^1, \max(h_3(\tilde{S}_1), A^1)) - G(A^1 + \Delta_A, \max(h_3(\tilde{S}_1), A^1 + \Delta_A)) + \Delta_A,$$

where [1] is the index of product with $\max\left(\frac{D_1}{a_1}, \frac{D_2}{a_2}\right)$.

Let $\Delta_{A,b}^{(2)}$ and $\Delta_{A,b}^{(3)}$ denote the profitability thresholds that make contract 2 profitable to be reserved when only one demand and neither demand has been satisfied, respectively. For given A^1 and β^2, by solving $\mathcal{M}_{[1]}^2 = 0$ and $\mathcal{M}_3^2 = 0$, we obtain the profitability thresholds as the unique solutions from the following equations:

$$G(A^1 + \Delta_{A,b}^{(2)}, \max(h_{[1]}(\tilde{S}_1), A^1 + \Delta_{A,b}^{(2)})) = \beta^2 + A^1 + \Delta_{A,b}^{(2)}, \tag{4.39}$$

$$G(A^1 + \Delta_{A,b}^{(3)}, \max(h_3(\tilde{S}_1), A^1 + \Delta_{A,b}^{(3)})) = \beta^2 + A^1 + \Delta_{A,b}^{(3)},$$

where [1] is the index of product with $\max\left(\frac{D_1}{a_1}, \frac{D_2}{a_2}\right)$.

Let $\Delta^{(2)}_{\beta,b}$ and $\Delta^{(3)}_{\beta,b}$ denote the profitability thresholds that make contract 1 profitable to be reserved when only one demand and neither demand has been satisfied, respectively. For given A^1 and β^2, by solving $\mathcal{M}^1_{[1]} = 0$ and $\mathcal{M}^1_3 = 0$, we obtain the profitability thresholds as follows:

$$\begin{aligned}
\Delta^{(2)}_{\beta,b} &\doteq G(A^1, \max(h_{[1]}(\tilde{S}_1), A^1)) - \beta^2 - A^1, \\
\Delta^{(3)}_{\beta,b} &\doteq G(A^1, \max(h_3(\tilde{S}_1), A^1)) - \beta^2 - A^1,
\end{aligned} \tag{4.40}$$

where [1] is the index of product with $\max\left(\frac{D_1}{a_1}, \frac{D_2}{a_2}\right)$. ■

Proof of Proposition 19: Paralleling the proof of Proposition 16, we have two situations to discuss: $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$ and $\frac{D_2}{a_2} > \frac{D_1}{a_1}$. We only provide the proof for $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$. The proof for $\frac{D_2}{a_2} > \frac{D_1}{a_1}$ can be established in a similar fashion. For $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$, we first obtain the optimal expected profit for each of 10 regions (i.e., Ω_i, $i \in \{1, 2, ..., 10\}$) defined in Proposition 18 (based on Δ_A and Δ_β), then combine them together. We only provide the proof for a typical region (i.e., Ω_2 where dual sourcing to full processing capacity is optimal). The proof for other regions can be established in a similar fashion.

In region Ω_2, $(Q^{1*}_b, Q^{2*}_b) = \left(\frac{D_1}{a_1}, K_0 - \frac{D_1}{a_1}\right)$. Using (4.33), we have

$$z^*_{0,b} = \begin{cases} 0 & \text{if } h_3(S_1) \leq P^1(S_0), \\ \frac{D_2}{a_2} & \text{if } h_1(S_1) \leq P^1(S_0) \leq h_3(S_1), \\ \frac{D_1}{a_1} & \text{if } P^1(S_0) \leq h_1(S_1) \text{ and } h_0(S_1) \leq P^2(S_0), \\ K_0 & \text{if } P^2(S_0) \leq h_0(S_1). \end{cases} \tag{4.41}$$

Note that the whole space of spot price $(\tilde{S}_0, \tilde{S}_1)$ is partitioned by the conditions in (4.41) into mutually exclusive subspaces, so the sum of probability of all subspaces is 1. Next, we calculate the optimal stage 2 profit in each subspace, and take the integration of the optimal stage 2 profits over the whole space.

(1) If $h_3(S_1) \le P^1(S_0)$, then $z^*_{0,b} = 0$. Using (4.32), we have

$$\Pi(0) = D_1(p_1 - S_1)^+.$$

(2) If $h_1(S_1) \le P^1(S_0) \le h_3(S_1)$, then $z^*_{0,b} = \frac{D_2}{a_2}$. Using (4.32), we have

$$\Pi\left(\frac{D_2}{a_2}\right) = D_1(p_1 - S_1)^+ + \frac{D_2}{a_2}\left(h_3(S_1) - P^1(S_0)\right).$$

(3) If $P^1(S_0) \le h_1(S_1)$ and $h_0(S_1) \le P^2(S_0)$, then $z^*_{0,b} = \frac{D_1}{a_1}$. Using (4.32), we have

$$\Pi\left(\frac{D_1}{a_1}\right) = D_1(p_1 - S_1)^+ + \frac{D_2}{a_2}\left(h_3(S_1) - P^1(S_0)\right) + \left(\frac{D_1}{a_1} - \frac{D_2}{a_2}\right)\left(h_1(S_1) - P^1(S_0)\right).$$

(4) If $P^2(S_0) \le h_0(S_1)$, then $z^*_{0,b} = K_0$. Using (4.32), we have

$$\begin{aligned}\Pi\left(K_0\right) =& D_1(p_1 - S_1)^+ + \frac{D_2}{a_2}\left(h_3(S_1) - P^1(S_0)\right) + \left(\frac{D_1}{a_1} - \frac{D_2}{a_2}\right)\left(h_1(S_1) - P^1(S_0)\right) \\ &+ \left(K_0 - \frac{D_1}{a_1}\right)\left(h_0(S_1) - P^2(S_0)\right).\end{aligned}$$

Recall that $V_b(Q_b^1, Q_b^2) = \mathbb{E}\left[\pi(Q_b^1, Q_b^2; \tilde{S}_0, \tilde{S}_1)\right] - \beta^1 Q_b^1 - \beta^2 Q_b^2$ is the stage 1 expected profit for given contract volumes (Q_b^1, Q_b^2), and $\pi(Q_b^1, Q_b^2; S_0, S_1) = \Pi(z^*_{0,b})$ for given contract volumes (Q_b^1, Q_b^2) and spot price realizations (S_0, S_1). Let V_b^* denote the optimal expected profit. Then in region Ω_2, we have

$$
\begin{aligned}
V_b^* = V_b\left(\frac{D_1}{a_1}, K_0 - \frac{D_1}{a_1}\right) =& \mathbb{E}\left[\Pi(z_{0,b}^*)\right] - \beta^1\frac{D_1}{a_1} - \beta^2\left(K_0 - \frac{D_1}{a_1}\right) \\
=& \mathbb{E}\left[\left(p_1 - \tilde{S}_1\right)^+\right] D_1 + \mathbb{E}\left[\left(h_3(\tilde{S}_1) - P^1(\tilde{S}_0)\right)^+\right]\frac{D_2}{a_2} \\
&+ \mathbb{E}\left[\left(h_1(\tilde{S}_1) - P^1(\tilde{S}_0)\right)^+\right]\left(\frac{D_1}{a_1} - \frac{D_2}{a_2}\right) \\
&+ \mathbb{E}\left[\left(h_0(\tilde{S}_1) - P^2(\tilde{S}_0)\right)^+\right]\left(K_0 - \frac{D_1}{a_1}\right) \\
&- \beta^1\frac{D_1}{a_1} - \beta^2\left(K_0 - \frac{D_1}{a_1}\right) \\
=& \mathbb{E}\left[\left(p_1 - \tilde{S}_1\right)^+\right] D_1 + \left(\mathcal{M}_3^1 + \mathbb{E}\left[\left(h_3(\tilde{S}_1) - \tilde{S}_0\right)^+\right]\right)\frac{D_2}{a_2} \\
&+ \left(\mathcal{M}_1^1 + \mathbb{E}\left[\left(h_1(\tilde{S}_1) - \tilde{S}_0\right)^+\right]\right)\left(\frac{D_1}{a_1} - \frac{D_2}{a_2}\right) \\
&+ \left(\mathcal{M}_0^2 + \mathbb{E}\left[\left(h_0(\tilde{S}_1) - \tilde{S}_0\right)^+\right]\right)\left(K_0 - \frac{D_1}{a_1}\right).
\end{aligned}
$$

where the last equality holds due to

$$
\mathbb{E}\left[\left(h_\kappa(\tilde{S}_1) - P^i(\tilde{S}_0)\right)^+\right] - \beta^i = \mathcal{M}_\kappa^i + \mathbb{E}\left[\left(h_\kappa(\tilde{S}_1) - \tilde{S}_0\right)^+\right],
$$

for $\kappa \in \{0,1,3\}$ and $i \in \{1,2\}$. Combining the expected profits of 10 regions as well as those for $\frac{D_2}{a_2} > \frac{D_1}{a_1}$, we complete our proof. ■

Proof of Proposition 20: The proof is similar to that of Proposition 14, thus omitted. ■

Proof of Proposition 21: The proof is similar to that of Proposition 15, thus omitted. ■

Proof of Proposition 22: Because the relation between $\frac{D_1}{a_1}$ and $\frac{D_2}{a_2}$ affects the value of [1]; that is, the index of product with $\max\left(\frac{D_1}{a_1}, \frac{D_2}{a_2}\right)$, and thus, the value of $h_{[1]}$ and thresholds $\Delta_{A,b}^{(2)}$, $\Delta_{\beta,b}^{(2)}$, and $\Delta_{\beta,b}^{M}(\Delta_A)$, there are two cases to discuss in the presence of by-product: $\frac{D_2}{a_2} \le \frac{D_1}{a_1}$ and $\frac{D_2}{a_2} > \frac{D_1}{a_1}$.

Case 1: $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$. In this case, $\Delta_{A,b}^{(2)} = \Delta_A^{(2)}$, $\Delta_{\beta,b}^{(2)} = \Delta_\beta^{(2)}$, and $\Delta_{\beta,b}^M(\Delta_A) = \Delta_\beta^M(\Delta_A)$. After combining Figure 3.1 and 3.2, we obtain Figure 4.1.

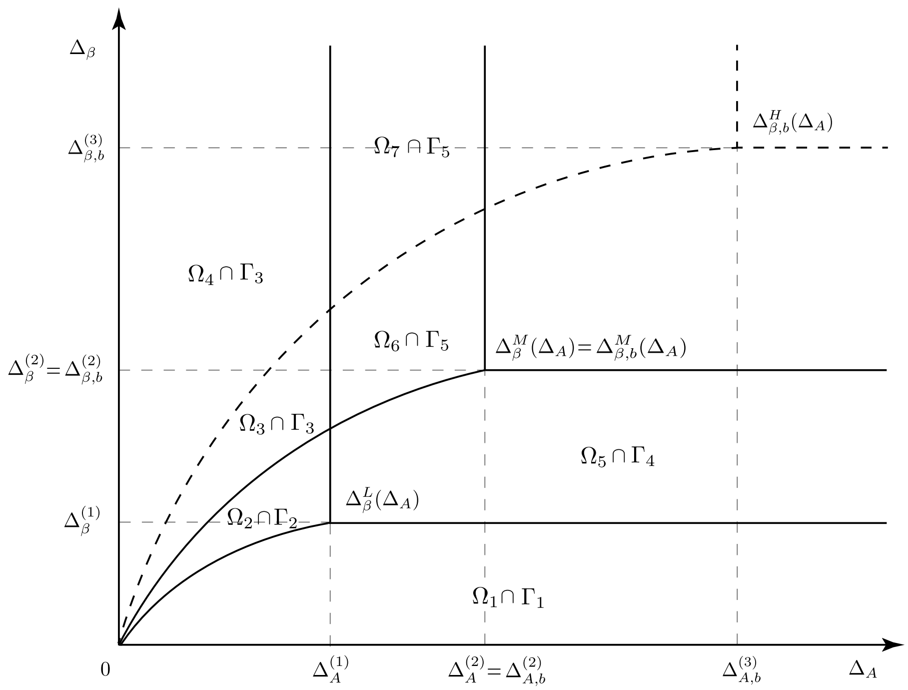

Figure 4.1: Optimal Contract Procurement Strategy on (Δ_A, Δ_β) Space in the Presence and Absence of By-Product. "$\cap$" represents the intersection operator. As we specified, we only focus on the non-trivial regions; that is, Γ_{-6}.

From Table 3.1 and 3.2, we obtain the optimal contract volumes in the presence and absence of by-product as given in Table 4.1. It is straightforward that $Q_b^{1*} + Q_b^{2*} = Q^{1*} + Q^{2*}$, $Q_b^{1*} \geq Q^{1*}$, and $Q_b^{2*} \leq Q^{2*}$.

Region	(Q_b^{1*}, Q_b^{2*})	(Q^{1*}, Q^{2*})
$\Omega_1 \cap \Gamma_1$	$(K_0, 0)$	$(K_0, 0)$
$\Omega_2 \cap \Gamma_2$	$\left(\frac{D_1}{a_1}, K_0 - \frac{D_1}{a_1}\right)$	$\left(\frac{D_1}{a_1}, K_0 - \frac{D_1}{a_1}\right)$
$\Omega_3 \cap \Gamma_3$	$\left(\frac{D_2}{a_2}, K_0 - \frac{D_2}{a_2}\right)$	$(0, K_0)$
$\Omega_4 \cap \Gamma_3$	$(0, K_0)$	$(0, K_0)$
$\Omega_5 \cap \Gamma_4$	$\left(\frac{D_1}{a_1}, 0\right)$	$\left(\frac{D_1}{a_1}, 0\right)$
$\Omega_6 \cap \Gamma_5$	$\left(\frac{D_2}{a_2}, \frac{D_1}{a_1} - \frac{D_2}{a_2}\right)$	$\left(0, \frac{D_1}{a_1}\right)$
$\Omega_7 \cap \Gamma_5$	$\left(0, \frac{D_1}{a_1}\right)$	$\left(0, \frac{D_1}{a_1}\right)$

Table 4.1: Optimal Contract Volumes on (Δ_A, Δ_β) Space in the Presence and Absence of By-Product.

Case 2: $\frac{D_2}{a_2} > \frac{D_1}{a_1}$. In this case, $[1] = 2$ ($[1]$ is the index of product with $\max\left(\frac{D_1}{a_1}, \frac{D_2}{a_2}\right)$), and thus, $h_{[1]}(S_1) = h_2(S_1)$. To make comparisons with the case without by-product, we need to know the relationship between $\Delta^{(2)}_{A,b}$ and $\Delta^{(2)}_{A}$, $\Delta^{(2)}_{\beta,b}$ and $\Delta^{(2)}_{\beta}$, $\Delta^{M}_{\beta,b}(\Delta_A)$ and $\Delta^{M}_{\beta}(\Delta_A)$, separately. We observe that their differences are determined by the unit processing margins; that is, $h_2(S_1)$ and $h_1(S_1)$. Because $\omega_1 \leq a_2p_2/a_1p_1$ by assumption, it is easy to establish that $h_2(S_1) \geq h_1(S_1)$ for all S_1.

We first examine the relationship between $\Delta^{(2)}_{A,b}$ and $\Delta^{(2)}_{A}$. Recall from (4.39) and (4.28) that $G(A^1 + \Delta^{(2)}_{A,b}, \max(h_2(\tilde{S}_1), A^1 + \Delta^{(2)}_{A,b})) = \beta^2 + A^1 + \Delta^{(2)}_{A,b}$ and $G(A^1 + \Delta^{(2)}_{A}, \max(h_1(\tilde{S}_1), A^1 + \Delta^{(2)}_{A})) = \beta^2 + A^1 + \Delta^{(2)}_{A}$. Let $N_\kappa(\Delta_A) \doteq G(A^1 + \Delta_A, \max(h_\kappa(\tilde{S}_1), A^1 + \Delta_A)) - \Delta_A$ for $\kappa \in \{0, 1, 2, 3\}$. Paralleling (4.31), we have $\frac{\partial N_\kappa(\Delta_A)}{\partial \Delta_A} < 0$ for $\Delta_A \geq 0$. Moreover, from (4.26), we have $\frac{\partial g(\theta_1,\theta_2)}{\partial \theta_2} = \int_{\theta_2}^{\infty} dF_{0|1}(\tilde{S}_0) > 0$, where $F_{0|1}(.)$ is the cumulative probability function of input spot price $\tilde{S}_0$ conditional on the output spot price S_1. Since $G(\theta_1, \theta_2) = \mathbb{E}_{\tilde{S}_1}[g(\theta_1, \theta_2)]$, we have $G(\theta_1, \theta_2^2) \geq G(\theta_1, \theta_2^1)$ for $\theta_2^2 \geq \theta_2^1 > \theta_1$ (where θ_2^1 and θ_2^2 may depend on S_1). Because $h_2(S_1) \geq h_1(S_1)$ for all S_1, we have $N_2(\Delta_A) \geq N_1(\Delta_A)$ for $\Delta_A \geq 0$.

Recall from (4.39) and (4.28), we obtain $N_2(\Delta^{(2)}_{A,b}) = N_1(\Delta^{(2)}_{A}) = \beta^2 + A^1$. Because $N_2(\Delta_A) \geq N_1(\Delta_A)$ for $\Delta_A \geq 0$ and $\frac{\partial N_\kappa(\Delta_A)}{\partial \Delta_A} < 0$ for $\Delta_A \geq 0$, we have $\Delta^{(2)}_{A,b} \geq \Delta^{(2)}_{A}$.

Next, we examine the relationship between $\Delta^{(2)}_{\beta,b}$ and $\Delta^{(2)}_{\beta}$. Recall from (4.40) and (4.29) that $\Delta^{(2)}_{\beta,b} = G(A^1, \max(h_2(\tilde{S}_1), A^1)) - \beta^2 - A^1$ and $\Delta^{(2)}_{\beta} \doteq G(A^1, \max(h_1(\tilde{S}_1), A^1)) - \beta^2 - A^1$. Because $G(\theta_1, \theta_2^2) \geq G(\theta_1, \theta_2^1)$ for $\theta_2^2 \geq \theta_2^1 > \theta_1$, and $h_2(S_1) \geq h_1(S_1)$ for all S_1, we have $\Delta^{(2)}_{\beta,b} \geq \Delta^{(2)}_{\beta}$.

Next, we examine the relationship between $\Delta^{M}_{\beta,b}(\Delta_A)$ and $\Delta^{M}_{\beta}(\Delta_A)$. From (4.38) and (4.27), we have

$$\begin{aligned}\Delta^{M}_{\beta,b}(\Delta_A) - \Delta^{M}_{\beta}(\Delta_A) =& G(A^1, \max(h_2(\tilde{S}_1), A^1)) - G(A^1, \max(h_1(\tilde{S}_1), A^1)) \\ &- G(A^1 + \Delta_A, \max(h_2(\tilde{S}_1), A^1 + \Delta_A)) \\ &+ G(A^1 + \Delta_A, \max(h_1(\tilde{S}_1), A^1 + \Delta_A)).\end{aligned}$$

Taking the first derivative with respect to Δ_A, we have

$$
\begin{aligned}
\frac{\partial\left(\Delta_{\beta,b}^M(\Delta_A)-\Delta_\beta^M(\Delta_A)\right)}{\partial\Delta_A}
=&-\mathbb{E}\left[\chi\left(h_2(\tilde{S}_1)<A^1+\Delta_A\right)\right]\\
&-\mathbb{E}\left[F_{0|1}\left(A^1+\Delta_A\right)\chi\left(h_2(\tilde{S}_1)\geq A^1+\Delta_A\right)\right]\\
&+\mathbb{E}\left[\chi\left(h_1(\tilde{S}_1)<A^1+\Delta_A\right)\right]\\
&+\mathbb{E}\left[F_{0|1}\left(A^1+\Delta_A\right)\chi\left(h_1(\tilde{S}_1)\geq A^1+\Delta_A\right)\right]\\
=&\mathbb{E}\left[\left(1-F_{0|1}\left(A^1+\Delta_A\right)\right)\chi\left(h_1(\tilde{S}_1)<A^1+\Delta_A\leq h_2(\tilde{S}_1)\right)\right]\\
>&0.
\end{aligned}
$$

Because $\Delta_{\beta,b}^M(0)=\Delta_\beta^M(0)=0$, we obtain $\Delta_{\beta,b}^M(\Delta_A)\geq\Delta_\beta^M(\Delta_A)$ for $\Delta_A\geq 0$.

Using the fact that $\Delta_{A,b}^{(2)}\geq\Delta_A^{(2)}$, $\Delta_{\beta,b}^{(2)}\geq\Delta_\beta^{(2)}$, and $\Delta_{\beta,b}^M(\Delta_A)\geq\Delta_\beta^M(\Delta_A)$ for $\Delta_A\geq 0$, and combining Figure 3.1 and 3.2, we obtain Figure 4.2.

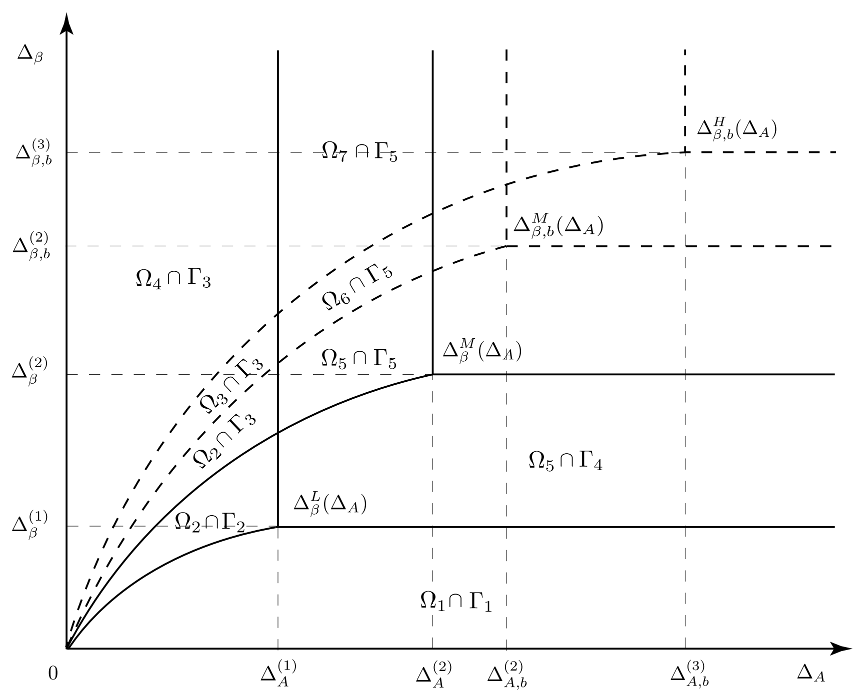

Figure 4.2: Optimal Contract Procurement Strategy on (Δ_A,Δ_β) Space in the Presence and Absence of By-Product. "$\cap$" represents the intersection operator. As we specified, we only focus on the non-trivial regions; that is, Γ_{-6}.

From Table 3.1 and 3.2, we obtain the optimal contract volumes in the presence and absence of by-product as given in Table 4.2. It is straightforward that $Q_b^{1*}+Q_b^{2*} \geq Q^{1*}+Q^{2*}$, $Q_b^{1*} \geq Q^{1*}$. Moreover, $Q_b^{2*} > Q^{2*}$ when $(\Delta_A, \Delta_\beta) \in \Omega_6 \cap \Gamma_5$ with $\frac{D_2}{a_2} > \frac{2D_1}{a_1}$, and $(\Delta_A, \Delta_\beta) \in \Omega_7 \cap \Gamma_5$.

Region	(Q_b^{1*}, Q_b^{2*})	(Q^{1*}, Q^{2*})
$\Omega_1 \cap \Gamma_1$	$(K_0, 0)$	$(K_0, 0)$
$\Omega_2 \cap \Gamma_2$	$\left(\frac{D_2}{a_2}, K_0 - \frac{D_2}{a_2}\right)$	$\left(\frac{D_1}{a_1}, K_0 - \frac{D_1}{a_1}\right)$
$\Omega_2 \cap \Gamma_3$	$\left(\frac{D_2}{a_2}, K_0 - \frac{D_2}{a_2}\right)$	$(0, K_0)$
$\Omega_3 \cap \Gamma_3$	$\left(\frac{D_1}{a_1}, K_0 - \frac{D_1}{a_1}\right)$	$(0, K_0)$
$\Omega_4 \cap \Gamma_3$	$(0, K_0)$	$(0, K_0)$
$\Omega_5 \cap \Gamma_4$	$\left(\frac{D_2}{a_2}, 0\right)$	$\left(\frac{D_1}{a_1}, 0\right)$
$\Omega_5 \cap \Gamma_5$	$\left(\frac{D_2}{a_2}, 0\right)$	$\left(0, \frac{D_1}{a_1}\right)$
$\Omega_6 \cap \Gamma_5$	$\left(\frac{D_1}{a_1}, \frac{D_2}{a_2} - \frac{D_1}{a_1}\right)$	$\left(0, \frac{D_1}{a_1}\right)$
$\Omega_7 \cap \Gamma_5$	$\left(0, \frac{D_2}{a_2}\right)$	$\left(0, \frac{D_1}{a_1}\right)$

Table 4.2: Optimal Contract Volumes on (Δ_A, Δ_β) Space in the Presence of By-Product.

■

Proof of Proposition 23: To study the impact of by-product on the value of contract procurement, we compare $\mathbb{VC}_b$ and $\mathbb{VC}$ region by region as did in in Proposition 22. Because the relation between $\frac{D_1}{a_1}$ and $\frac{D_2}{a_2}$ affects the value of [1]; that is, the index of product with $\max\left(\frac{D_1}{a_1}, \frac{D_2}{a_2}\right)$, and thus, the value of $\mathbb{VC}_b$, there are two cases to discuss in the presence of by-product: $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$ and $\frac{D_2}{a_2} > \frac{D_1}{a_1}$. For each case, we examine the impact of by-product on the value of contract procurement in one region. In particular, we focus on region $\Omega_3 \cap \Gamma_3$ for $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$ and $\Omega_2 \cap \Gamma_2$ for $\frac{D_2}{a_2} > \frac{D_1}{a_1}$ in this proof; the proof for other regions can be established in a similar fashion.

From (3.7) and (3.11), we have

$$\mathbb{VC}_b = \left(\mathcal{M}_3^X\right)^+ \frac{D_{[2]}}{a_{[2]}} + \left(\mathcal{M}_{[1]}^Y\right)^+ \left(\frac{D_{[1]}}{a_{[1]}} - \frac{D_{[2]}}{a_{[2]}}\right) + \left(\mathcal{M}_0^Z\right)^+ \left(K_0 - \frac{D_{[1]}}{a_{[1]}}\right), \tag{4.42}$$
$$\mathbb{VC} = \left(\mathcal{M}_1^Y\right)^+ \frac{D_1}{a_1} + \left(\mathcal{M}_0^Z\right)^+ \left(K_0 - \frac{D_1}{a_1}\right).$$

Case (i): $\frac{D_2}{a_2} \leq \frac{D_1}{a_1}$. In region $\Omega_3 \cap \Gamma_3$, $(Q_b^{1*}, Q_b^{2*}) = \left(\frac{D_2}{a_2}, K_0 - \frac{D_2}{a_2}\right)$ and $(Q^{1*}, Q^{2*}) = (0, K_0)$. From (4.42), we have

$$\begin{aligned}
\mathbb{VC}_b &= \mathcal{M}_3^1 \frac{D_2}{a_2} + \mathcal{M}_1^2 \left(\frac{D_1}{a_1} - \frac{D_2}{a_2}\right) + \mathcal{M}_0^2 \left(K_0 - \frac{D_1}{a_1}\right), \\
\mathbb{VC} &= \mathcal{M}_1^2 \frac{D_1}{a_1} + \mathcal{M}_0^2 \left(K_0 - \frac{D_1}{a_1}\right),
\end{aligned}$$

and

$$\mathbb{VC}_b - \mathbb{VC} = \left(\mathcal{M}_3^1 - \mathcal{M}_1^2\right) \frac{D_2}{a_2} = \left((\mathcal{M}_3^1 - \mathcal{M}_3^2) + (\mathcal{M}_3^2 - \mathcal{M}_1^2)\right) \frac{D_2}{a_2}.$$

It is easy to establish that $\mathcal{M}_3^2 \geq \mathcal{M}_1^2$. Moreover, in region $\Omega_3 \cap \Gamma_3$, $\Delta_\beta \leq \Delta_{\beta,b}^H(\Delta_A)$ holds; we have $\mathcal{M}_3^1 \geq \mathcal{M}_3^2$. Therefore, $\mathbb{VC}_b \geq \mathbb{VC}$.

Case (ii): $\frac{D_2}{a_2} > \frac{D_1}{a_1}$. In region $\Omega_2 \cap \Gamma_2$, $(Q_b^{1*}, Q_b^{2*}) = \left(\frac{D_2}{a_2}, K_0 - \frac{D_2}{a_2}\right)$ and $(Q^{1*}, Q^{2*}) = \left(\frac{D_1}{a_1}, K_0 - \frac{D_1}{a_1}\right)$. From (4.42), we have

$$\begin{aligned}
\mathbb{VC}_b &= \mathcal{M}_3^1 \frac{D_1}{a_1} + \mathcal{M}_1^1 \left(\frac{D_2}{a_2} - \frac{D_1}{a_1}\right) + \mathcal{M}_0^2 \left(K_0 - \frac{D_2}{a_2}\right), \\
\mathbb{VC} &= \mathcal{M}_1^1 \frac{D_1}{a_1} + \mathcal{M}_0^2 \left(K_0 - \frac{D_1}{a_1}\right),
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{VC}_b - \mathbb{VC} =& (\mathcal{M}_3^1 - \mathcal{M}_1^1) \frac{D_1}{a_1} + \left(\mathcal{M}_1^1 - \mathcal{M}_0^2\right) \left(\frac{D_2}{a_2} - \frac{D_1}{a_1}\right) \\
=& (\mathcal{M}_3^1 - \mathcal{M}_1^1) \frac{D_1}{a_1} + \left((\mathcal{M}_1^1 - \mathcal{M}_1^2) + (\mathcal{M}_1^2 - \mathcal{M}_0^2)\right) \left(\frac{D_2}{a_2} - \frac{D_1}{a_1}\right).
\end{aligned}$$

It is easy to establish that $\mathcal{M}_3^1 \geq \mathcal{M}_1^1$ and $\mathcal{M}_1^2 \geq \mathcal{M}_0^2$. Moreover, in region $\Omega_2 \cap \Gamma_2$, $\Delta_\beta \leq \Delta_\beta^M(\Delta_A)$ holds; we have $\mathcal{M}_1^1 \geq \mathcal{M}_1^2$. Therefore, $\mathbb{VC}_b \geq \mathbb{VC}$, which completes our proof. ■

Bibliography

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