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Robert H. DENG
Singapore Management University, robertdeng@smu.edu.sg

D. J. Costello, Jr.

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High Rate Concatenated Coding Systems Using Multidimensional Bandwidth-Efficient Trellis Inner Codes

ROBERT H. DENG AND DANIEL J. COSTELLO, JR.

Abstract—**In a previous paper [1], a concatenated coding system employing two-dimensional (2-D) trellis-coded MPSK inner codes and Reed–Solomon outer codes for application in high-speed satellite communication systems was proposed. This paper extends the results of [1] to systems using symbol-oriented multidimensional trellis-coded MPSK inner codes.**

The concatenated coding systems will be divided into two classes according to their achievable effective information rates. The first class uses multidimensional trellis-coded 8-PSK inner codes and achieves effective information rates around 1 bit/dimension (spectral efficiency 2 bits/s/Hz). The second class employs multidimensional trellis-coded 16-PSK inner codes and provides effective information rates around 1.5 bits/dimension (spectral efficiency 3 bits/s/Hz). Both classes provide

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R. H. Deng was with the Department of Electrical and Computer Engineering, University of Notre Dame, Notre Dame, IN 46556. He is now with the Institute of Systems Science, National University of Singapore, Singapore 0511.

D. J. Costello, Jr., is with the Department of Electrical and Computer Engineering, University of Notre Dame, Notre Dame, IN 46556.

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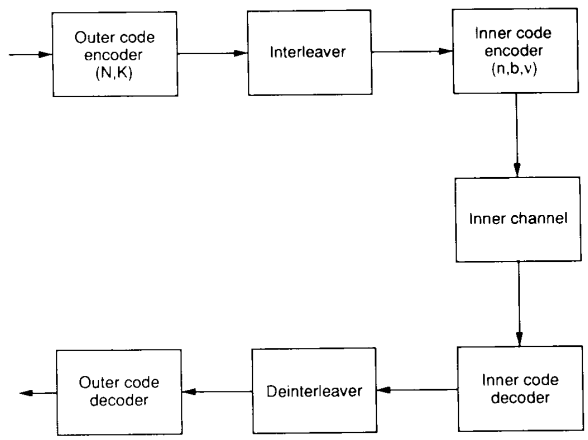

Fig. 1. A concatenated coding system.

significant coding gains over an uncoded reference system with the same effective information rate as the coded system. The results also show that the symbol-oriented nature of multidimensional inner codes can provide an improvement of up to 1 dB in the overall performance of a concatenated coding system when these codes replace bit-oriented two-dimensional inner codes.

I. Introduction

In [1], a concatenated coding system with two-dimensional (2-D) trellis-coded MPSK (TCMPSK) inner codes and Reed–Solomon (RS) outer codes for application in high-speed satellite communication systems was proposed. It was argued there that TCMPSK inner codes along with soft decision Viterbi decoding play two important roles in a concatenated coding system.

1) They compensate for the bandwidth expansion introduced by the outer code.

2) The random errors on the inner channel are converted into symbol errors which can be corrected by a symbol-error-correcting outer code, such as an RS code.

In trellis (convolutional) inner code/RS outer code concatenated coding systems, such as those in [1] and [2] which employ a soft decision Viterbi decoder for the inner code, it is unlikely that the beginning of a decoding error burst is aligned with the boundary between two RS symbols. This fact was first observed by Lee [3] for binary convolutional inner code/RS outer code concatenated coding systems and led to the discovery of symbol-oriented unit memory inner convolutional codes. This observation leads us to consider using symbol-oriented multidimensional (multi-D) TCMPSK inner codes rather than bit-oriented 2-D TCMPSK inner codes. A typical concatenated coding system is shown in Fig. 1. The outer code is an (N, K) RS code with $N = 2^b - 1$ and symbols over $GF(2^b)$. The inner code is a rate $R_1 = b/n$, 2^ν-*state*, multi-D TCMPSK where b, the number of information bits entering the inner encoder per encoding interval, is chosen to equal the RS code symbol size.

Encoding is performed in two stages. An information sequence of Kb bits is divided into K symbols of b bits each, and each b-bit symbol is regarded as an element of $GF(2^b)$. These K symbols are used as inputs to the RS encoder. The output of this encoder is an N-symbol codeword which is symbol-interleaved and then serially encoded by the trellis encoder with b input bits per encoding interval. Decoding is accomplished in the reverse order. The inner channel is assumed to be an additive white Gaussian noise (AWGN) channel with single-sided power spectrum N_0. The inner code is decoded by a Viterbi decoder without demodulator output quantization. The outer decoder is an errors-only RS decoder.

TABLE I
RATE $R_1 = b/(b+1)$, 2^ν-STATE, $2L$-D TC8PSK

L	R_1	ν	$2^{\nu+\tilde{b}}/L$	N_p/L	N_b/b	d_f^2/Δ_0^2	γ_{asp} (dB)
2	5/6	2	4	2	1.2	1.80	2.55
2	5/6	3	16	8	16.4	2.63	4.21
3	8/9	1	1.33	1.33	1	1.30	1.14
3	8/9	2	2.67	5.33	6	1.95	2.90
4	8/9	1	1	7	8	2	3.01
4	8/9	2	4	3	2.5	2	3.01
4	8/9	3	16	1	0.5	2	3.01

The concatenated coding systems will be divided into two classes according to their achievable effective information rate. Class 1 systems use multi-D TC8PSK inner codes and achieve effective information rates around 1 bit/dimension (spectral efficiency 2 bits/s/Hz). Class 2 systems employ multi-D TC16PSK inner codes and achieve effective information rates around 1.5 bits/dimension (spectral efficiency 3 bits/s/Hz). Their performance is studied in Sections II and III, respectively.

II. Systems Employing Multi-D TC8PSK Inner Codes

In this section, we study the performance of concatenated coding systems with the multi-D TC8PSK schemes constructed in [4] as inner codes. For any positive integer $L \geq 2$, a $2L$-D 8-PSK signal set is generated by simply repeating an 8-PSK signal set L times. Therefore, the $2L$-D 8-PSK signal set is the Cartesian product of L 2-D 8-PSK signal sets. For any positive integer b, $2L \leq b < 3L$, a rate $R_1 = b/(b+1)$, $2L$-D TC8PSK encoder accepts b information bits and outputs one $2L$-D 8-PSK signal per encoding interval. The performance of any TCM scheme is commonly measured in terms of its effective information rate $R_{\text{eff}}^{(1)}$ in bits per signal dimension and its asymptotic coding gain γ_{asp} over an uncoded reference system with the same effective information rate. A rate $R_1 = b/(b+1)$, 2^ν-state, $2L$-D TC8PSK encoder has an $R_{\text{eff}}^{(1)}$ equal to $b/2L$ bits/dimension, and thus $1 \leq R_{\text{eff}}^{(1)} < 1.5$ bits/dimension for $2L \leq b < 3L$. The asymptotic coding gain is given by

$$\gamma_{\text{asp}} = 10 \log_{10} \frac{d_f^2}{\Delta_0^2} \text{ dB} \tag{1}$$

where d_f^2 is the minimum free squared Euclidean distance (ED) of the code and

$$\Delta_0^2 = 2 - 2\cos\left(2\pi/2^{2R_{\text{eff}}^{(1)}}\right) \tag{2}$$

is the minimum squared ED of an uncoded $2^{2R_{\text{eff}}^{(1)}}$ PSK signal set[1] (this reference system was suggested by Forney [5]).

Let $\tilde{b}$, $1 \leq \tilde{b} \leq b$ denote the number of coded information bits input to the TCM encoder [4], [6]. Then the number of distinct transitions in the trellis diagram of the encoder is $2^{\nu+\tilde{b}}$. This so-called *trellis complexity* represents a measure of code (decoding) complexity. A fair comparison of TCM schemes to different signal dimensionalities requires normalization of the trellis complexities and the number of nearest neighbors to the same number of signal dimensions. Table I shows some of the $2L$-D TC8PSK codes found in [4] where the trellis complexity and the number of nearest neighbors (paths that are distance d_f from the correct path) are normalized to two dimensions and are denoted by $2^{\nu+\tilde{b}}/L$ and N_p/L, respectively. Also shown in Table I is N_b/b, the number of information bit errors associated with all paths that are distance d_f from the correct path normalized by the number

[1] Since $2^{2R_{\text{eff}}^{(1)}}$ may not be an integer, this uncoded reference system may be only hypothetical. It is used for comparison purposes since it has exactly the same effective information rate as the coded system.

of information bits input to the TCM encoder per encoding interval.

Let P_s denote the symbol error probability into the outer decoder. Assuming bounded distance decoding of the outer code, the decoded bit error rate (BER) at the output of the outer decoder is closely approximated by

$$P_b \approx \frac{d}{2N} \sum_{i=t+1}^{N} \binom{N}{i} P_s^i (1-P_s)^{N-i} \quad (3)$$

where d is the minimum Hamming distance of the RS code and $t = \lfloor (d-1)/2 \rfloor$ is its symbol-error-correcting capability [7].

The performance of the concatenated coding system will be measured in terms of the overall effective information rate of the concatenated coding system

$$R_{\text{eff}} = R_{\text{eff}}^{(1)} \cdot \frac{K}{N}, \quad (4)$$

which is around 1 bit/dimension for $1 \le R_{\text{eff}}^{(1)} < 1.5$, and the coding gain at a given target BER, P_b, over an uncoded reference system with the same effective information rate R_{eff} as the coded system, i.e., uncoded $2^{2R_{\text{eff}}}$ PSK modulation. The BER for the reference system is given by

$$P_b \approx \frac{1}{R_{\text{eff}}} Q\left(\sqrt{\frac{\Delta_0^2 R_{\text{eff}} E_b}{N_0}}\right) \quad (5)$$

where E_b/N_0 is the channel "information bit energy-to-noise power density ratio" and $\Delta_0^2 = 2 - 2\cos(2\pi/2^{2R_{\text{eff}}})$ is the minimum squared ED of the $2^{2R_{\text{eff}}}$ PSK signal set.

The coding gain of a concatenated coding system over the uncoded reference system, denoted by γ, is found as follows. We first find the $(E_b/N_0)_{\text{uncoded}}$ required to achieve a target BER from (5). Then we find the $(E_b/N_0)_{\text{coded}}$ required to achieve the same BER using (3) where the symbol error probability P_s is obtained by computer simulation of the inner decoder. The coding gain is then given by

$$\gamma = 10 \log_{10} \frac{\left(\frac{E_b}{N_0}\right)_{\text{uncoded}}}{\left(\frac{E_b}{N_0}\right)_{\text{coded}}} \text{ dB.} \quad (6)$$

Fig. 2 shows the coding gain at $P_b = 10^{-6}$ and $P_b = 10^{-9}$ with respect to R_{eff} for a concatenated coding system with $R_1 = 5/6$, 4-D TC8PSK inner codes and $N = 2^5 - 1 = 31$ RS outer codes. Results are given for inner codes with four and eight states. The gain (loss) of the reference system over QPSK is also plotted in the figure. The coding gain over QPSK of a concatenated coding system can be found by adding γ to the gain (loss) over QPSK of the reference system with the same R_{eff}.

Fig. 3 shows the coding gain for a concatenated coding system with $R_1 = 8/9$, 6-D TC8PSK inner codes and $N = 2^8 - 1 = 255$ RS outer codes.

Fig. 4 shows the coding gain for a concatenated coding system with $R_1 = 8/9$, 8-D TC8PSK inner codes and $N = 2^8 - 1 = 255$ RS outer codes. The three inner codes have the same minimum free squared ED d_f^2, but a decreasing number of nearest neighbors as the number of trellis states increases. Note that only about 0.1 dB more gain is obtained with every doubling of the number of trellis states.

III. Systems Employing Multi-D TC16PSK Inner Codes

The overall effective information rate of a concatenated coding system with multi-D TC8PSK inner codes is always less than 1.5 bits/dimension. To achieve higher rates, multi-D

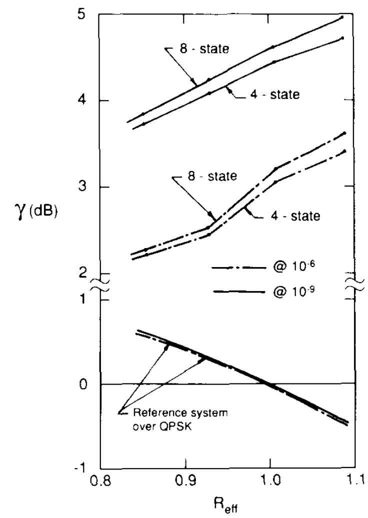

Fig. 2. Coding gain versus R_{eff} for a concatenated coding system with an $R_1 = 5/6$, 4-D TC8PSK inner code and an $N = 31$ RS outer code with $d = 5$–11.

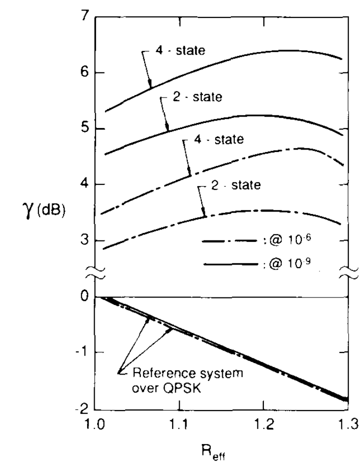

Fig. 3. Coding gain versus R_{eff} for a concatenated coding system with an $R_1 = 8/9$, 6-D TC8PSK inner code and an $N = 255$ RS outer code with $d = 11$–61.

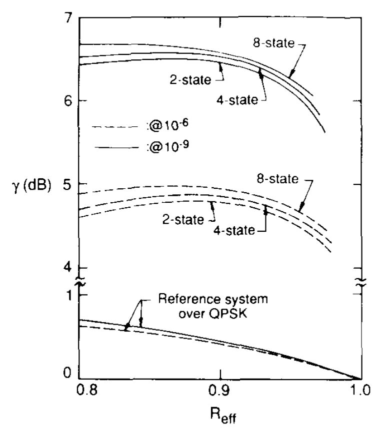

Fig. 4. Coding gain versus R_{eff} for a concatenated coding system with an $R_1 = 8/9$, 8-D TC8PSK inner code and an $N = 255$ RS outer code with $d = 11$–51.

TABLE II
(a) RATE $R_1 = 7/8$, 2^{ν}-STATE, 4-D ($L = 2$) TC16PSK
(b) RATE $R_1 = 10/11$, 2^{ν}-STATE, 6-D ($L = 3$) TC16PSK

ν	$2^{\nu+b}/L$	N_p/L	N_b/b	d_f^2/Δ_0^2	γ_{asp} (dB)
1	2	4	4	1.52	1.83
2	4	2	1.14	1.95	2.91
3	16	8	15.71	2.54	4.04
4	32	28	69.57	3.05	4.84
5	64	8	17.43	3.05	4.84
6	128	4	5.43	3.48	5.41

(a)

ν	$2^{\nu+b}/L$	N_p/L	N_b/b	d_f^2/Δ_0^2	γ_{asp} (dB)
1	1.33	2	1.2	1.56	1.93
2	5.33	2.67	5.2	1.62	2.09
3	10.67	8	14	2.77	4.43
4	21.33	20	28	3.12	4.94
7	341.33	5.33	16.8	3.24	5.10

(b)

TC16PSK must be used as inner codes. Some of the multi-D TC16PSK schemes constructed in [4] are tabulated in Table II. For any positive integers $L \geq 2$ and $3L \leq b < 4L$, a rate $R_1 = b/(b+1)$, $2L$-D TC16PSK encoder has an effective information rate $R_{\text{eff}}^{(1)}$ equal to $b/2L$ bits/dimension, and thus $1.5 \leq R_{\text{eff}}^{(1)} < 2$ bits/dimension. From (4), the overall effective information rate of the concatenated coding system is around 1.5 bits/dimension.

Due to the symbol-oriented nature of the inner codes, the concatenated coding system performance can be estimated using formula calculations as well as by simulation. Let N_s denote the total number of symbol errors associated with paths that are distance d_f from the correct path, normalized by the number of decoding intervals on each path. It follows that N_s is upper bounded by the number of paths N_p that are distance d_f from the correct path, a parameter known for most TCM codes. The symbol error probability P_s to the outer decoder, for large values of E_b/N_0, can thus be approximated by

$$P_s \approx N_s Q\left(\sqrt{\frac{d_f^2 R_{\text{eff}} E_b}{N_0}}\right) \leq N_p Q\left(\sqrt{\frac{d_f^2 R_{\text{eff}} E_b}{N_0}}\right) \quad (7)$$

where R_{eff} is the overall effective information rate of the concatenated coding system. The final decoded BER P_b can be found by using (7) in (3).

Since a close approximation to P_s using (7) requires a high E_b/N_0 ratio, or equivalently $P_s \ll 1$, in the following we only consider RS outer codes with $d = 3$ and 5 since these are sufficient to achieve decoded BER's in the range 10^{-6}–10^{-9}. Fig. 5 compares the performance obtained by the formula calculations to that obtained through computer simulations for a concatenated coding system with an $R_1 = 5/6$, four-state, 4-D TC8PSK inner code and an $N = 31$ RS outer code. It is seen that the formula calculations and the simulations are very close at $P_b \leq 2 \times 10^{-5}$ for $d = 3$ and $P_b \leq 2 \times 10^{-7}$ for $d = 5$.[2]

The coding gains obtained by formula calculations versus the inner code constraint length ν are shown in Fig. 6 for systems with 4-D TC16PSK inner codes and in Fig. 7 for systems with 6-D TC16PSK inner codes. The coding gains of the inner codes alone are also shown in the figures for comparison.

The advantage of concatenated coding over the inner code alone is obvious. With a $d = 3$ RS outer code, the concatenated coding system offers 0.75–1.25 dB more coding

[2] The formula calculations are expected to be more accurate for larger RS code lengths N, for then the RS code rate is higher and lower values of P_s are needed to achieve $P_b = 10^{-6}$ ($P_s \approx 10^{-4}$) and $P_b = 10^{-9}$ ($P_s \approx 10^{-5}$).

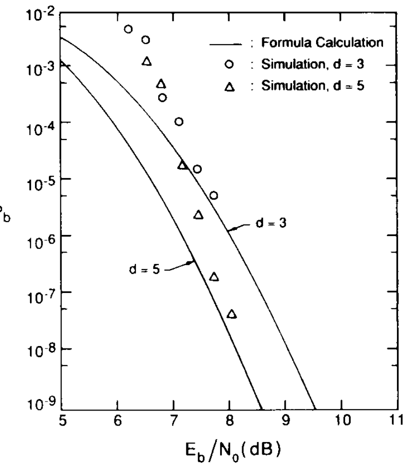

Fig. 5. Performance of a concatenated coding system with an $R_1 = 5/6$, 4-state, 4-D TC8PSK inner code and an $N = 31$ RS outer code.

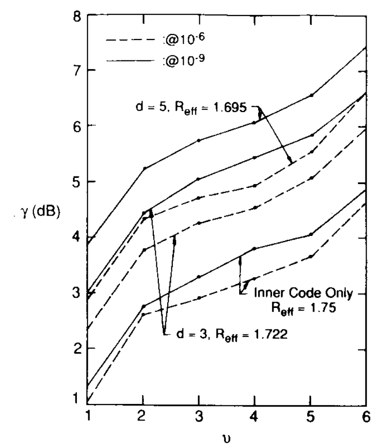

Fig. 6. Coding gain versus ν for a concatenated coding system with an $R_1 = 7/8$, 4-D TC16PSK inner code and an $N = 127$ RS outer code.

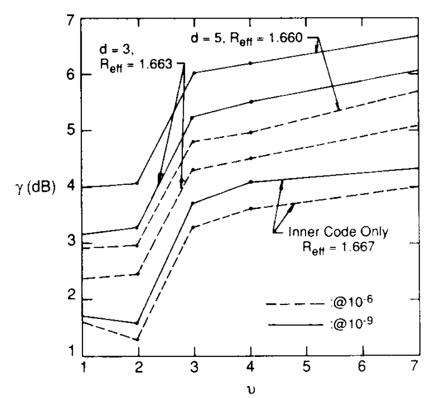

Fig. 7. Coding gain versus ν for a concatenated coding system with an $R_1 = 10/11$, 6-D TC16PSK inner code and an $N = 1023$ RS outer code.

gain at $P_b = 10^{-6}$ and 1.25–1.75 dB more coding gain at $P_b = 10^{-9}$, respectively, than the inner code alone (3 dB more coding gain asymptotically). With a $d = 5$ RS outer code, the concatenated coding system offers 1.25–2 dB more coding gain at $P_b = 10^{-6}$ and 2.25–2.5 dB more coding gain at $P_b =$

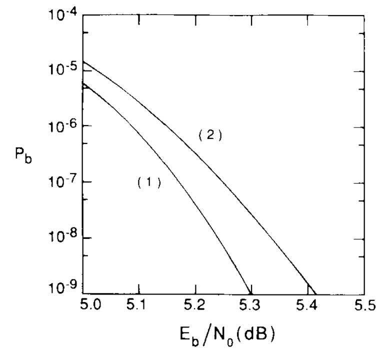

Fig. 8. Performance comparison for concatenated coding systems with (1) an $R_1 = 2/3$, 16-state, 2-D TC8PSK inner code and a (255, 223) RS outer code; (2) an $R_1 = 8/9$, four-state, 8-D TC8PSK inner code and the same outer code.

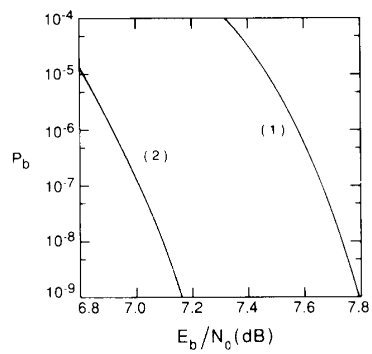

Fig. 9. Performance comparison for concatenated coding systems with (1) an $R_1 = 8/9$, four-state, 2-D PTVTC/8PSK inner code and a (255,201) RS outer code; (2) an $R_1 = 8/9$, four-state, 6-D TC8PSK inner code and the same outer code.

10^{-9}, respectively, than the inner code alone (4.77 dB more coding gain asymptotically).

IV. Conclusions

We have studied the performance of concatenated coding systems with symbol-oriented multi-D TCMPSK inner codes. The advantages of using symbol-oriented multi-D TCMPSK inner codes are best seen by comparing the coding performance to that of concatenated coding systems employing bit-oriented inner codes.

Fig. 8 shows the performance of two concatenated coding systems. System 1 uses an Ungerboeck $R_1 = 2/3$ ($R_{\text{eff}}^{(1)} = 1$ bit/dimension), 16-state, 2-D TC8PSK [6] inner code and a (255,223) RS outer code. System 2 employs an $R_1 = 8/9$ ($R_{\text{eff}}^{(1)} = 1$ bit/dimension), four-state, 8-D TC8PSK [4] inner code and the same outer code. Both systems have an effective information rate $R_{\text{eff}} = 0.875$ bits/dimension. Ungerboeck's 16-state code has a trellis complexity of $2^{\nu+\tilde{b}}/L = 64$ ($\nu = 4$, $\tilde{b} = 2$, $L = 1$) and a 4.13 dB asymptotic coding gain, while the four-state, 8-D code has a trellis complexity of 4 and only a 3 dB asymptotic coding gain. However, system 2 is inferior to system 1 by only 0.05–0.11 dB at $P_b = 10^{-6}$–10^{-9}. Thus, the symbol-oriented nature of the four-state, 8-D TC8PSK inner code provides an improvement of more than 1 dB in overall performance. Moreover, the four-state code is simpler to decode than the 16-state code.

To further justify this observation, Fig. 9 shows another system performance comparison. In Fig. 9, system 1 uses an $R_1 = 8/9$ ($R_{\text{eff}}^{(1)} = 1.33$ bits/dimension), four-state, 2-D periodic time-varying trellis coded (PTVTC) 8-PSK [8] inner code and a (255,201) RS outer code. System 2 uses an $R_1 = 8/9$, four-state, 6-D TC8PSK [4] inner code and the same outer code. Both systems have $R_{\text{eff}} = 1.05$ bits/dimension. Both inner codes have a 2.9 dB asymptotic coding gain and roughly the same number of nearest neighbors. The trellis complexities of the PTVTC and the 6-D TC8PSK code, normalized to two dimensions, are 10.7 [8] and 2.67, respectively. However, system 2 offers 0.6 dB more coding gain than system 1, which is due to the symbol-oriented nature of the $R_1 = 8/9$, four-state, 6-D TC8PSK inner code.

From the performance studies presented above, we can draw a number of conclusions.

1) The symbol-oriented nature of multi-D TCMPSK inner codes can provide an improvement of up to 1 dB in the overall performance of a concatenated coding system when these codes replace bit-oriented 2-D TCMPSK inner codes of the same rate.

2) Most of the coding gain can be obtained by using 4–16-*state* inner codes. Therefore, choosing inner codes with a small number of trellis states increases the data transmission speed (by reducing the number of decoder computations) with only a slight sacrifice in system performance.

3) The number of information bit errors N_b/b of the inner code exerts less influence on the performance of a concatenated coding system than on the performance of the inner code by itself (see Fig. 7 at $\nu = 2$) where the coding gain is seriously degraded by the number of information bit errors when the inner code alone is used, but much less affected in the concatenated code case. This can be explained as follows. Using Forney's [9] rule of thumb, the number of information bit errors degrades the performance of a trellis code at $P_b = 10^{-5}$ by 0.2 dB for every increase in the number by a factor of 2. However, since the errors in the output of a Viterbi decoder are highly bursty, the information bit errors along a path are concentrated into only a few symbol errors. After deinterleaving, the symbol errors will be corrected with high probability by the RS outer code. This fact provides a basis for statement 2) in Section I.

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On Training Fractionally Spaced Equalizers Using Intersymbol Interpolation

FUYUN LING

Abstract—**The use of an intersymbol interpolation method in training fractionally spaced equalizers (FSE) is investigated. It is shown that the optimal interpolation filter depends on the amplitude frequency response of the transmitter filter and the channel. Using a nonoptimal interpolation filter will *increase the steady-state mean-squared error* (MSE) of the FSE. An interpolated complex FSE (CFSE) employing a stochastic gradient, or LMS, adaptive algorithm has very little advantage over an LMS CFSE with symbol-rate updating. However, an interpolated LMS phase-splitting FSE (PS–FSE) has a convergence speed that is twice as fast as a conventional PS–FSE. Special precautions for evaluating the performance of interpolated FSE's are discussed and a novel evaluation scheme is proposed.**

I. Introduction

The advantages of the fractionally spaced equalizer (FSE) over the symbol rate equalizer have been well recognized [1]–[3]. The main advantage of the FSE is its insensitivity to receiver sampling phase. The FSE can be implemented as a passband or baseband equalizer. In both cases, it follows a Hilbert transformer, or phase splitter, which converts the real received signal sequence into a complex sequence which is used as the input to the FSE. We call this type of FSE a complex FSE (CFSE). A variation of the CFSE, which combines the functions of both a phase splitter and an FSE into one structure, is described in [4]. We call it a phase-splitting FSE or PS–FSE.

It was proposed in [3] that it might be possible to use intersymbol interpolation to reduce the training time of an FSE. In [5], this technique was further developed for the PS–FSE by using a fast recursive least squares (FRLS) algorithm. In [6], the interpolation technique is applied to a decision-feedback equalizer for a special kind of partial-response signaling. However, the results given in [5] and [6] are only based a few computer simulations and no systematic investigation of the interpolation method has been performed.

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The author is with the Research and Advanced Development Department, Codex Corporation, Mansfield, MA 02048.

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In this correspondence, we investigate the interpolation method for training FSE's employing the LMS algorithm, which shall be called *interpolated* FSE's. We first derive the optimal interpolation filter for the interpolated training. The convergence characteristics of the interpolated CFSE and PS–FSE are discussed. Problems rising from evaluation of the convergence and steady-state performance of FSE's using interpolation are considered, and a new method that correctly evaluates the performance is described. Simulation results are given to verify our analysis.

II. Intersymbol Interpolation for Training FSE's

In a baseband data communication system with a $T/2$ CFSE where T is the symbol interval, an estimate, denoted by $\hat{z}_n$, is generated every T seconds. In the training period, the z_n's are known to the receiver. The difference between z_n and $\hat{z}_n$, denoted by e_n, is used to update the coefficients of CFSE every T seconds. The input to the CFSE is the received signal sampled at every $T/2$. Thus, the signal in the delay line is shifted by two samples for each update.

The idea of intersymbol interpolation is that, if we know the desired output values of the CFSE every $T/2$ seconds, we can update the CFSE every $T/2$ seconds instead of every T seconds. More frequent updating might result in a faster initial convergence of the CFSE. In order to obtain these desired values, a noncausal interpolation filter must be used. The input to the interpolation filter is the symbols z_{n+i}, $i = 0, +1, +2, \cdots$, and its output is the desired value at nT or $nT + T/2$, denoted by $z(nT)$ and $z(nT + T/2)$. It is obvious that the output of the interpolation filter at nT, $z(nT)$ has to equal z_n. For such a filter, the folded frequency response must be a constant. Such a filter is called a Nyquist filter. However, there are an infinite number of Nyquist filters. Since the ultimate goal of the CFSE is to minimize the mean-squared error (MSE) between the symbol z_n and its estimate $\hat{z}_n$, the adaptation of coefficients at $nT + T/2$ should improve performance for the next adaption at nT. Using an arbitrarily chosen Nyquist filter as the interpolation filter may not provide such an improvement. It may even result in a larger steady-state MSE after training than a conventional FSE.

To avoid this problem, the optimal interpolation filter must also satisfy a second condition, namely, that its frequency response should be equal to the overall unaliased response of the transmitter filter, the channel, and the FSE. An arbitrary Nyquist filter may not satisfy the second condition. From [1], [3], [7], we know that the optimal $T/2$ CFSE has a frequency response that is

$$C(\omega) = F^*(\omega)/[|F(\omega-\pi)|^2 + |F(\omega)|^2 + |F(\omega+\pi)|^2 + \sigma^2] \quad (1)$$

where $F(\omega)$ is the combined baseband frequency response of the transmitter filter and the channel, and σ^2 is the variance of the noise, assuming $F(\omega) = 0$ for $|\omega| \geq 2\pi/T$, and the data symbols have a unity variance. The overall frequency response, including the CFSE, is thus equal to

$$F(\omega)C(\omega) = |F(\omega)|^2/[|F(\omega-\pi)|^2 + |F(\omega)|^2 + |F(\omega+\pi)|^2 + \sigma^2] \quad (2)$$

which is the desired frequency response of the interpolation filter.

It can be seen from (2) that the optimal interpolation filter depends on $F(\omega)$, assuming that the effect of σ^2 is negligible. In practice, the statistics of the channel are not known and the transmitter filter may or may not be known. Hence, some assumptions have to be made in choosing the interpolation filter. Degradation in the steady-state performance of the FSE will occur if the interpolation filter is nonoptimal.