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Comparison of Evolutionary Algorithms: A Case Study on the Multi-Objective Carbon-Aware Mine Planning*

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***Abstract*— The NP-hard *precedence-constrained production scheduling problem* (PCPSP) for mine planning chooses the ordered removal of materials from the mine pit and the next processing steps based on resource, geological, and geometrical constraints. Traditionally, it prioritizes the net present value (NPV) of profits across the lifespan of the mine. Yet, the growing shift in environmental concerns also requires shifts to more carbon-aware practices. In this paper, we use the enhanced multi-objective version of the generic PCPSP formulation by adding the NPV of carbon costs as another objective. We then compare how the Non-dominated Sorting Genetic Algorithm II (NSGA-II) and the Pareto Envelope-based Selection Algorithm II (PESA-II) solve several real-world inspired datasets, after experimenting with the selection pressure parameter of PESA-II. The outcome reveals that PESA-II runs faster for 75% of the datasets and gives sets of solutions that are more distributed. Meanwhile, NSGA-II consistently produces non-dominated solutions even when the apportionment of a decision variable is varied. Moreover, we assess how the uncertainty of ore tonnage at the mine site modifies the Pareto front via sensitivity analysis. We show that deviations above 15% induce a larger gap from the original.**

***Index Terms*— Genetic algorithms, Pareto optimization, production planning, environmental economics**

I. INTRODUCTION

Scheduling problems for mine planning involve optimizing the sequence of extracting materials and the respective series of treatment steps throughout the years that the mine is live. To support these activities, there are various operational facilities such as the milling plants (e.g. crushing and grinding), refining plants (e.g. hydrometallurgy), storage facilities (i.e. stockpiles), and waste facilities (e.g. dump and tailings pond). These facilities have their corresponding machinery and/or finite capacity. To derive the highest net present value (NPV) of profits and returns to investors, mine planners need to prioritize sequences (Fig. 1) and processing decisions that give the most returns in the earlier years of the mine. In doing so, they also have to consider the unique and uncertain geological and geometrical attributes of each mine.

Such scheduling problems can be viewed from a strategic, tactical, or operational angle in Operations Research, whereby the granularity of decision-making and timeframe range. Both strategic and tactical angles hold long to medium-term decisions whereas the operational angle is short-term. The decisions for the strategic angle are mostly confined to the mine pit in determining its shape and the best extraction sequence. Meanwhile, the decisions in the tactical angle involve the extraction sequence and processing steps of materials. This has been referred to as the *precedence-constrained production scheduling problem* (PCPSP) and is recognized as NP-hard. Lastly, the operational angle is focused on the everyday deployment of resources at the pit and/or operational facilities. In this paper, we focus on the decisions for the tactical PCPSP.

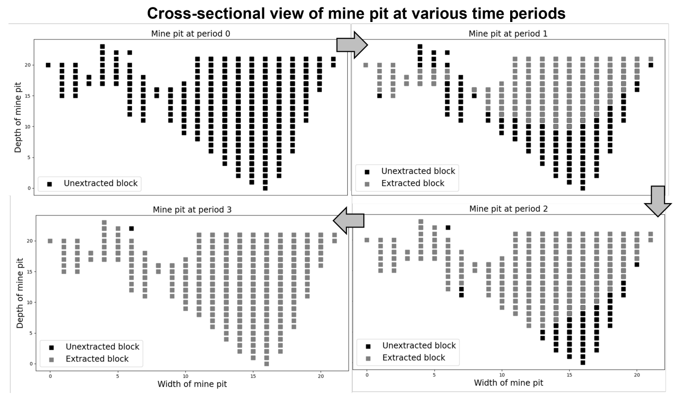

Fig. 1. Block extraction sequence example from year to year following precedence and resource constraints for MineLib's Newman1 instance [1].

Although the traditional PCPSP prioritizes the NPV of profits [2], there is a growing push to balance the environmental impacts against the NPV of profits derived when extracting and processing ores. This goes beyond merely managing them in the five mining phases of exploration, planning, implementation, production, and reclamation. In these phases, the common practices to manage environmental impact include conducting an environmental impact assessment (planning phase), having environmental management systems (production phase), and mine rehabilitation (reclamation phase) [3].

Furthermore, the increasing push for net zero carbon has resulted in increasing research that models and trades off carbon dioxide emission costs directly in scheduling problems. From the strategic angle, [4] concurrently maximized the NPV of profits and social benefits to the surrounding community as well as minimized harmful environmental outputs, including carbon dioxide costs, using a heuristic. From the operational angle, [5] and [6] assessed the carbon

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dioxide emissions from inter-facility transportation, intra-facilities, and transportation to the customers using a multi-objective optimization (MOO) with mixed integer programming (MIP). Meanwhile, [7] used a MOO with a hybrid particle swarm optimization (PSO) algorithm to minimize the costs of operating a mine, including carbon dioxide emission costs. From the tactical angle, [1] added the NPV of carbon dioxide costs as an additional constraint in a bounded objective function method when maximizing the NPV of profits. They used a hybrid of heuristics and MIP. Finally, [8] used a Non-dominated Sorting Genetic Algorithm II (NSGA-II) for a MOO formulation of the PCPSP that trades off between the NPV of profits and carbon dioxide costs.

In this work, we focus on the balance between the NPV of profits and carbon dioxide costs in the PCPSP. In doing so, we leverage the generic PCPSP formulation [9] that has been augmented into a MOO [8]. This allows reusability and scalability for future researchers. The carbon dioxide costing framework used in this MOO formulation is adopted from [4]. Primarily, we compare two multi-objective evolutionary algorithms (MOEA) of NSGA-II [10] and Pareto Envelope-based Sorting Algorithm II (PESA-II) [11]. Segments of these algorithms – the initial solution, crossover, and mutation – use the tailored heuristics proposed in our earlier work [8] that have been implemented on an NSGA-II. The limitations in runtime were encountered with the use of NSGA-II for small to medium instances. Hence, we now explore how PESA-II may address this shortcoming through differences in its selection mechanism, convergence metrics archive structure and use of parameters. Our contributions to this research space as follows:

1) Based on our knowledge, PESA-II has not been used and generally, the use of MOEAs have been sparse.
2) We examine how the selection pressure parameter of PESA-II alters its performance against the parameter-less NSGA-II. Furthermore, we evaluate how the Pareto fronts of both NSGA-II and PESA-II alter with two methods of varying a decision variable.
3) We propose a framework to assess uncertainty within the dual MOEA setup. We demonstrate it by assessing how the uncertainty of ores at a mine site affects the Pareto front with two scenarios of over- or under-assessment of ores in a sensitivity analysis.

Based on the PCPSP formulated in the next section, we describe the set of algorithms, experiment framework, and the results in Sections III, IV and V, respectively.

II. PCPSP Definition for Mine Planning

Running a mine requires seamlessly scheduling the extraction sequence of materials from the ground and its processing to form the desired end products for customers. To model mines, materials underground are discretized into three-dimensional blocks. Each of these blocks has its respective proportion of ore grade, impurities (e.g. sulfur, silicon, phosphorus), and hence, the associated economic profit. Due to the unique geology and geometry of each mine site, each block has a set of preceding blocks that need to be extracted before it can be reached. The sequence of extraction has to be considered holistically over decades of the mine life together with the treatment at each facility that has its respective heavy machinery and capacities.

The holistic decisions for this scheduling problem are taken for the best NPV of profits. However, the activities in this scheduling problem consume raw materials (e.g. water, energy) and release harmful by-products (e.g. carbon dioxide, chemical waste). These environmental effects can be modeled in the PCPSP. We focus on modeling carbon dioxide as it has been flagged as the leading greenhouse gas for global warming and climate change.

A. Carbon Dioxide Costing Framework

Carbon dioxide costs reflect the costs required to capture and convert carbon dioxide emitted during the energy consumption of mining activities. Alike [8], we leverage the costing framework by [4] to augment the generic PCPSP formulation.

The carbon dioxide costs $\mathscr{C}$ consist of two types of materials extracted from the mine pit: (1) valuable ore $\mathscr{Q}_{i,o}$ and (2) invaluable material considered as waste $\mathscr{Q}_{i,w}$. Both materials consume energy per tonne of material to extract e_m using coal whereas the former also consumes energy per tonne of material to further process e_p. The total use of coal for energy is then multiplied with the factors for carbon in coal f_c and carbon dioxide conversion from carbon f_a to give the total carbon dioxide emitted. Finally, it is multiplied by the technology cost to absorb the carbon dioxide released $\mathscr{C}_c$. The formula is shown below.

$$\mathscr{C} = \frac{(\mathscr{Q}_{i,o} + \mathscr{Q}_{i,w})e_m + \mathscr{Q}_{i,o}e_p}{1000} f_c f_a \mathscr{C}_c \tag{1}$$

B. Enhanced Multi-Objective PCPSP Formulation

The scheduling problem in mining is complex due to its size, uncertainty, and multi-disciplinary specializations. The amount of valuable ore, impurities, and material structure at a chosen mine site are determined by geologists and chemists through ongoing drill samples and imaging. This information is also used by mining engineers to determine how best to access the ore and the machinery required. Once the materials are extracted, the chemical properties and the desired customer products affect treatment decisions.

The unique properties at each mine site usually lead to research that utilizes tailored mathematical formulations based on an operating site. Unfortunately, this may hinder re-usability, scalability, and ease of comparison amongst models. Meanwhile, the MineLib library [9] provides generic formulations for three problem variants – including the PCPSP which is the most complex – and supporting real-world datasets. We adopt and enhance this generic formulation for our work.

The generic PCPSP [9] defines $\mathscr{B}$ as the set of blocks that can be extracted from the mine pit, where each block $b \in \mathscr{B}$ has its own set of preceding blocks $\mathscr{B}_b$ that need to be extracted. Each block can be sent to different destinations $\mathscr{D}$. Each $d \in \mathscr{D}$ has a set of resources $\mathscr{R}$ and operational units of q_{bdr} to process each block. There are two decision variables for the PCPSP. Firstly, the binary decision variable x_{bt} states if a block is extracted during period $t \in \mathscr{T}$. Next, the continuous decision variable y_{bdt} states the portion of the block sent to a destination after extraction in that period.

While the generic PCPSP has one objective, we adopt the enhanced multi-objective formulation [8]. Originally, it maximizes the NPV of profits. The NPV of profit $\tilde{p}_{bdt}$ for the material that has been extracted and processed in the period t is derived from $\frac{p_{bd}}{(1+\alpha)^t}$, with α as the discount rate.

$$\textbf{(Objective 1)} \quad \mathscr{Z}_1 = \max \sum_{b\in\mathscr{B}} \sum_{d\in\mathscr{D}} \sum_{t\in\mathscr{T}} \tilde{p}_{bdt} y_{bdt} \tag{2}$$

The additional objective, namely Objective 2, shown in equation (3) minimizes the NPV of carbon costs. The NPV of carbon cost $\tilde{c}_{bdrt}$ for carbon dioxide emitted when materials are extracted and processed in the period t is derived from $\frac{c_{bdr}}{(1+\alpha)^t}$, with α as the discount rate. This addition aids jurisdictions with the economic instrument of carbon credit trading; a system that was introduced to decrease emissions. Miners can hence leverage on the additional objective to adhere to their respective carbon emissions cap.

$$\textbf{(Objective 2)} \quad \mathscr{Z}_2 = \min \sum_{b\in\mathscr{B}} \sum_{d\in\mathscr{D}} \sum_{r\in\mathscr{R}} \sum_{t\in\mathscr{T}} \tilde{c}_{bdrt} y_{bdt} \tag{3}$$

To extract a block, the depth, ore type, ore composition, and material surrounding the ore (e.g. sand) affect the order that it can be extracted. This is represented by constraint (4) that sets out the set of preceding blocks $\mathscr{B}_b$ for each block. Each preceding block b' in that set has to be extracted in an earlier period or the same period before that block can be extracted.

$$\sum_{\tau\le t} x_{b\tau} \le \sum_{\tau\le t} x_{b'\tau} \quad \forall\, b\in\mathscr{B},\ b'\in\mathscr{B}_b,\ t\in\mathscr{T} \tag{4}$$

Next, constraint (5) ensures that once a block is extracted, it is sent to at least one processing destination. The choice of processing destination depends on the type of ore, material composition, and customer requirements. Conversely, if a block is not extracted, it is not sent to any processing destination.

$$x_{bt} = \sum_{d\in\mathscr{D}} y_{bdt} \quad \forall\, b\in\mathscr{B},\ t\in\mathscr{T} \tag{5}$$

If a block is extracted, constraint (6) then ensures that it can only be extracted once throughout the mine's lifespan.

$$\sum_{t\in\mathscr{T}} x_{bt} \le 1 \quad \forall\, b\in\mathscr{B} \tag{6}$$

For all resources, constraint (7) ensures that the use of each resource r is within capacity limits for all periods. These resources include diggers, grinders, and refining plants.

$$\underline{\mathscr{R}}_{rt} \le \sum_{b\in\mathscr{B}} \sum_{d\in\mathscr{D}} q_{bdr} y_{bdt} \le \bar{\mathscr{R}}_{rt} \quad \forall\, r\in\mathscr{R},\ t\in\mathscr{T} \tag{7}$$

Additionally, constraint (8) caters to any supplementary requirements unique to that mine site. This side constraint can model diverse situations such as ore grade constraints.

$$\underline{a} \le \mathscr{A} y \le \bar{a} \tag{8}$$

Finally, constraints (9) and (10) represent the type and range of values for the two decision variables of x_{bt} and y_{bdt}.

$$x_{bt} \in \{0,1\} \quad \forall b\in\mathscr{B}, t\in\mathscr{T}, \tag{9}$$

$$y_{bdt} \in [0,1] \quad \forall b\in\mathscr{B}, d\in\mathscr{D}, t\in\mathscr{T}. \tag{10}$$

III. Multi-Objective Evolutionary Algorithms for the Multi-Objective PCPSP

In single-objective optimization, the best solution is found by comparing the value of the objective function against other solutions. In a MOO, there is no single best solution but a set of solutions that have the best trade-off between competing objectives, also defined as dominance. A solution is dominant or non-dominated when it is not outperformed by any other solution in all considered objectives and is strictly better than all other solutions in at least one objective. To derive this set of non-dominated solutions, techniques have been categorized as decomposition-based, domination-based, preference-based, indicator-based, and hybrid approaches [12], [13]. Decomposition-based, preference-based, indicator-based, and hybrid can be catered for many-objective optimization problems (i.e. problems with four or more objectives). These are popular in recent years as MOO problems increase in complexity. Meanwhile for less complex bi-objective problems such as ours, the decomposition-based and domination-based approaches suffice [14]. The decomposition-based scalarizes the MOO to become a single objective problem(s). The domination-based, often linked with MOEAs, leverages the Pareto-dominance relations of solutions in the objective space and constantly maintains a set of non-dominated solutions [15].

The decomposition-based approach of bounded objective function was used by [1] to trade off the NPV of profits against the NPV of carbon costs. However, it required multiple runs to form the approximated Pareto front. To overcome this limitation, [8] used the MOEA of NSGA-II to produce a diverse set of Pareto optimal solutions in one run. MOEAs are differentiated by the attributes of fitness assignment, diversity mechanism, elitism, and external population. In this paper, we evaluate how differences in these attributes affect the performance of PESA-II versus NSGA-II for the enhanced MOO formulation of the generic PCPSP.

A. Overview and Implementation of NSGA-II and PESA-II

While both NSGA-II and PESA-II are evolutionary algorithms that can address MOO problems, they have distinct strategies to evolve solutions that are diverse and non-dominated, until convergence. NSGA-II uses non-dominated sorting and crowding distance. The process of non-dominated sorting group solutions into Pareto fronts based on their dominance relationships. Next, crowding distance – a metric that reflects the density of solutions – is assigned to each solution within its front. The complexity of NSGA-II is $O(MN^2)$. Meanwhile, PESA-II uses a hyperbox-based approach. Hyperboxes are regions in the objective space that represent and organize non-dominated solutions. They help ensure coverage of the entire Pareto front by capturing different regions of trade-off solutions. The hyperboxes are dynamically adapted in each generation to accommodate changes in the distribution of non-dominated solutions. The complexity of PESA-II is $O(MN)$.

For both NSGA-II and PESA-II, we initialize a population for both NSGA-II and PESA-II using the heuristic proposed by [8]. This initialization provides a feasible solution by prioritizing sets of blocks that can be extracted based on resources available cumulatively in each time period. From the initial population, solutions are evaluated. Next, NSGA-II employs non-dominated sorting based on dominance relationships amongst solutions and assigns crowding distance for the density of solutions around each solution. The non-dominated solutions are tracked using Pareto front ranking and maintained within the population structure for NSGA-II. Meanwhile, PESA-II determines non-dominated solutions Θ and stores them in an external archive Λ_{θ}. PESA-II constructs hyperboxes $\mathscr{H}$ for the objective space, assigns solutions to them, and tracks the density of each hyperbox. Then, reproduction begins.

In each generation of reproduction, parent solutions $\mathscr{X}$ are distinguished using binary tournament selection. For NSGA-II, solutions from less crowded regions and higher-ranking Pareto fronts are preferred. For PESA-II, non-dominated solutions that are part of less dense hyperboxes are preferred. Following that, the crossover and mutation heuristics by [8] are employed that cater to the nuances of the PCPSP and minimize constraint violations. If any violations occur, solutions are repaired to become feasible again. For NSGA-II, the offspring population Ω_x' is kept separately at first. Following that, the repaired solutions are evaluated for both NSGA-II and PESA-II. After evaluation, the offspring solutions Ω_x' for NSGA-II are combined with the solutions at the start of the generation Ω_x. Finally, the non-dominated sorting and crowding distance are updated for NSGA-II and only the best, non-crowded solutions are retained, with Pareto front-ranking, if the population size exceeds. Meanwhile, the external population of non-dominated solutions and hyperboxes are updated for PESA-II. The external population of PESA-II is truncated at the generation end to ensure population limits.

B. Use of Tailored Heuristics within NSGA-II and PESA-II

We applied tailored heuristics from our prior work [8] of initial solution generation, reproduction, and improvement/repair operator to assess their adaptability to other MOEAs. The initial solution and reproduction steps minimize constraint violations as much as possible.

The initial solution generation stage involves computing preceding blocks (cones), cumulative resource requirements, and earliest extraction periods for each target block to prioritize blocks accordingly. In the reproduction stage, an offspring is produced with an interdependent-period single-point crossover, considering the entire mine lifespan. Subsequently, mutation only targets blocks without precedence to randomly alter their extraction period. At the end of both stages, infeasible solutions are repaired, and solutions are improved by focusing on fringe cone sets. Profitable cones are added if no resource violation occurs. Otherwise, those with minimal resource consumption are favored. Conversely, less profitable cones are prioritized when removing blocks.

C. Use of Parameters in PESA-II

NSGA-II primarily focuses on finding a well-distributed set of non-dominated solutions without explicit preference handling mechanisms. However, PESA-II uses three parameters inflation factor, selection pressure, and deletion pressure.

The inflation factor β is used when constructing hyperboxes. It controls the expansion in the range of values when creating a hyperbox. A higher inflation factor results in a larger range and a more sparse hyperbox.

The selection pressure ζ is used in reproduction when selecting parent candidates. It influences the sensitivity of the selection process to the hyperbox population size. Larger values of ζ make the selection process more biased against hyperboxes with larger populations.

Finally, the deletion pressure η is used to reduce the size of the external population of non-dominated solutions at the end of each generation. It influences the selection probabilities in the roulette wheel selection. Higher values give more weight to hyperboxes with larger populations.

IV. EXPERIMENTS

For our experiments, we varied three attributes: the decision variable y_{bdt}, the PESA-II algorithm selection pressure parameter ζ, and the ore assessment for Wilma1 dataset, as summarized in Fig. 2. Initially with the four datasets (**Part I**), we vary how portions of blocks are sent to processing destinations (**Part II**). Blocks are apportioned to destinations based on the profit of processing them there using two functions. The argmax function assigns to the destination with the best profit while the softmax function allows sending to less profitable destinations. Next, we vary the selection pressure parameter ζ for PESA-II (**Part III**). From the PESA-II results of each ζ parameter, we form the approximated Pareto front with the NSGA-II results to evaluate the best value of ζ for PESA-II for each dataset. Then, we compare the best PESA-II results with NSGA-II using three evaluation metrics. Finally, we vary the ore assessment for the Wilma1 dataset for different scenarios in a sensitivity analysis (**Part IV**). We examine how the approximated Pareto front changes via two scenarios of over- and under-assessment of ore. The tuning of parameters in NSGA-II such as those within the binary tournament selection are not presented in this paper due to space limitations, but follow the earlier work in [8].

For all experiments, we use a population size of 100 and run for generations of three to ten. The number of generations is capped relatively low since we focus on finding feasible solutions quickly so that a suitable algorithm can be scaled up for large instances in future work. Meanwhile, the inflation pressure β and deletion pressure η for PESA-II are set as 0.1 and 0.5 respectively. The rest of the parameters follow that of [8]. Both the NSGA-II and PESA-II models were developed with Python. These were run on a Linux

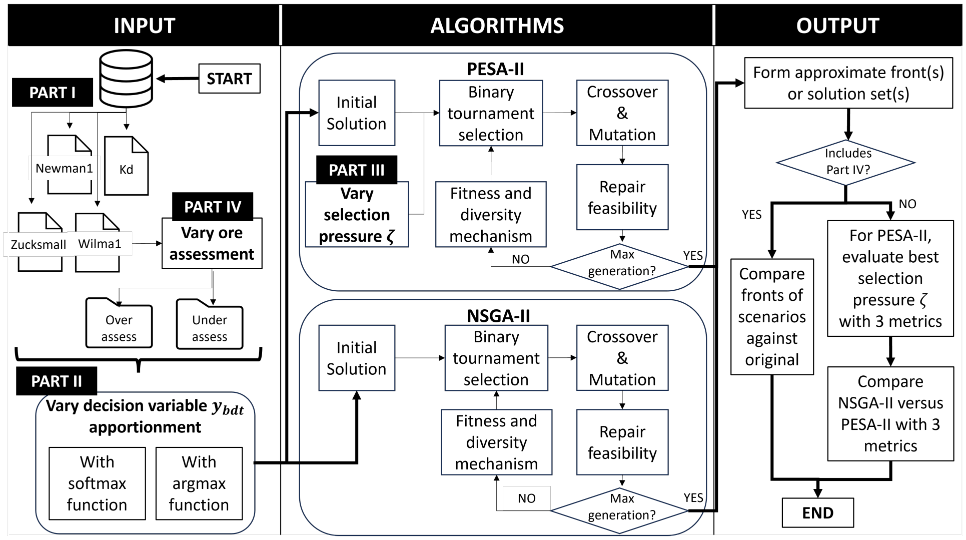

Fig. 2. Experiments Framework with Three Parts Varied.

operating system with 3.5 GHz 3rd generation Intel Xeon Scalable processor, 128 vCPUs, and 128 Gb memory.

A. Datasets and Scenario Variants for Sensitivity Analysis

There are four real-world datasets used, summarized in Table I. Wilma1 is an operating copper and gold mine. It has been transformed to fit the generic formulation and the data anonymized for confidentiality. Meanwhile, the rest of the datasets are publicly available from MineLib [9]. Newman1 and Zucksmall are iron ore mines while Kd is a copper mine.

TABLE I

KEY CHARACTERISTICS OF THE DATASETS.

Name	Block	Precedence	Periods	Destinations	Resources
Newman1	1,060	3,922	6	2	2
Wilma1	1,960	3,688	4	3	3
Zucksmall	9,400	145,640	20	2	2
Kd	14,153	219,778	12	2	2

For the sensitivity analysis, we focus on the latest real data, Wilma1, with two scenarios of over- and under-assessment of ores. Typically, the exact amount of ore is unknown until the material has been extracted and assessed. In the meantime, the amount of ore is estimated based on drill samples. Hence, the amount of actual ore, in tonnes, may deviate from the initial estimate. We model how the approximated Pareto front may change if the actual tonnes of ore are above or below the initial estimate. We do so by deviating the tonnes of ores uniformly randomly for each block in the following scenarios:

1) Under-assessment of ore: Actual ore more than estimate by the ranges $5-10\%$, $10-15\%$ and $15-20\%$
2) Over-assessment of ore: Actual ore less than estimate by the ranges $5-10\%$, $10-15\%$ and $15-20\%$

B. Varying Apportionment of Decision Variable y_{bdt}

The continuous decision variable y_{bdt} determines the portion of the block sent to a processing destination. At each destination, the type of processing results in an end product with economic value that can be negative or positive. We vary it via:

1) `Argmax` function: All blocks are sent to the most profitable destination
2) `Softmax` function: Most blocks are sent to the most profitable destination, but some can be sent to less profitable destinations

$$\text{softmax} = \frac{e^{\lambda p_{bd}}}{\sum_{d=1}^{\mathscr{D}} e^{\lambda p_{bd}}} \tag{11}$$

C. Varying Selection Pressure Parameter for PESA-II

For each parent candidate in the binary tournament selection process of reproduction, a selection score is calculated in PESA-II. This score is inversely proportional to the power of ζ for the hyperbox population size that the candidate belongs

TABLE II

INCREASE OR DECREASE IN ORES FOR SENSITIVITY ANALYSIS.

Ores / Ranges	**5-10%**	**10-15%**	**15-20%**
Under Assessment			
Copper 20	+7.356%	+12.432%	+17.613%
Copper 25	+7.490%	+12.589%	+17.531%
Silver	+7.620%	+12.487%	+17.461%
Gold	+7.493%	+12.460%	+17.498%
Overall Tonnes	+0.067%	+0.113%	+0.157%
Over Assessment			
Copper 20	-7.508%	-12.39%	-17.597%
Copper 25	-7.587%	-12.458%	-17.625%
Silver	-7.550%	-12.487%	-17.473%
Gold	-7.525%	-12.502%	-17.475%
Overall Tonnes	-0.068%	-0.112%	-0.158%

to. We vary the values of ζ from 0.5 to 1.5 for this score:

$$\text{Score} = \frac{1}{\text{Population size of solution's hyperbox}^{\zeta}} \tag{12}$$

D. Evaluation Metrics

From the solution sets of both NSGA-II and PESA-II, the approximated Pareto front is formed. Besides computation time, the solution sets are then assessed with five evaluation metrics for the quality of non-dominated solutions and the even spread of solutions relative to the Pareto front. The former is measured by the ratio of non-dominated solutions (RNI) [16], distance metric [17], and the *weakly Pareto-compliant* inverted generational distance plus (IGD$^+$) [18]. The latter is measured by the diversity metric [17] and the *Pareto-compliant* hypervolume (HV) [19].

The RNI indicates the fraction of the approximated Pareto front Φ_z that comes from the population of solutions Ω_x, with size Ω. A value close to one is preferred.

$$\text{Ratio of non-dominated individuals (RNI)} = \frac{|\Phi_x|}{\Omega} \tag{13}$$

Meanwhile, the distance metric shows the collective distance of the population of solutions Ω_x from the Pareto front Φ_z. The distance $d(x,z)$ between a solution $x \in \Omega_x$ and all solutions $z \in \Phi_z$ is based on the Euclidean distance that is summed, and then averaged against the population size Ω. A value close to zero is preferred.

$$\text{Distance metric} = \frac{\sum_{x=1}^{\Omega} \min d(x,z)}{\Omega} \tag{14}$$

Similar to the distance metric formula above, the IGD$^+$ also measures the average distance between the solution set and the Pareto front, but the calculation for the distance $d(x,z)$ differs. In IGD$^+$, the distance between x and z is measured as follows for a maximization problem, whereby a value close to zero is also preferred.

$$d_{IGD^+}(x,z) = \sqrt{\sum_{i=1}^{\Omega} (\max\{z_i - x_i, 0\})^2} \tag{15}$$

Next, the diversity metric shows the balanced dispersion of solutions relative to the Pareto front. The distance between extreme solutions in the Pareto front d_j and that for the population of solutions d_k are used together with the distance between consecutive solutions d_i and the average distance $\bar{d}$ for all d_i. A higher value is preferred, close to one.

$$\text{Diversity metric} = \frac{d_j + d_k + \sum_{i=1}^{\Omega-1} |d_i - \bar{d}|}{d_j + d_k + (\Omega - 1)\bar{d}} \tag{16}$$

Finally, the HV metric gives the volume of the objective space that is bounded by the solution set Ω_x and a reference point r. HV uses the m-dimensional Lebesgue measure λ_m, for m objectives, that quantifies the volume of a set in Euclidean space. The choice of reference point is problem-dependent, guided by [20]. A higher value is preferred.

$$HV(\Omega_x, r) = \lambda_m\Big(\bigcup_{x \in \Omega} [x; r]\Big) \tag{17}$$

TABLE III

AVERAGE METRICS FOR BEST ζ ACROSS DATASETS FOR PESA-II.

Best ζ selection	Averages					Best variant
	RNI	Distance	IGD+	Diversity	HV	
Newman1						
0.8	0.208	0.027	0.021	0.915	2.801	Softmax
Wilma1						
1.2	0.125	0.027	0.021	0.987	3.229	Argmax
Kd						
1	0.188	0.169	0.142	0.963	2.698	Softmax
Zucksmall						
1	0.042	0.059	0.034	0.887	1.895	Softmax

V. COMPUTATIONAL RESULTS

Both NSGA-II and PESA-II are run from generations three to ten using the `argmax` and `softmax` functions to apportion the decision variable y_{bdt}. The results from both variants are used to form the approximated Pareto front and compared throughout further experiments (Fig. 2) as well.

A. Effect of Varying Selection Pressure for PESA-II

PESA-II is run with selection pressure parameter ζ ranging from 0.5 to 1.5 using the `argmax` and `softmax` functions. We compare the results from each value of ζ using the five evaluation metrics to determine the best y_{bdt} apportionment type alongside the best ζ. These results are summarized in Table III.

The performances of `argmax` and `softmax` functions differ for each dataset. For Newman1, the `argmax` variant provided better metrics. Meanwhile, the `softmax` variant provided better metrics for Wilma1, Kd, and Zucksmall. Hence, the nuances within datasets may influence the solution quality from each variant. The best values for ζ also differ. It is one for half the datasets – Kd and Zucksmall – whilst Newman1 and Wilma1 are at 0.8 and 1.2 respectively. This shows that balancing bias towards less densely populated hyperboxes is preferred.

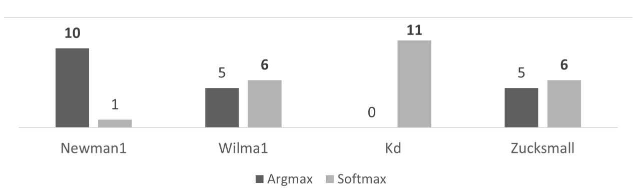

Fig. 3. Number of Best Variants Summed across All ζ Values.

Furthermore, when the best y_{bdt} variant for each ζ value is compared across datasets in Fig. 3, the `softmax` variant performed better for 75% of the datasets – Wilma1, Kd, and Zucksmall. Across the ζ values ranging from 0.5 to 1.5, the `softmax` provided the best variant in 55% of the ζ values for Wilma1, 100% for Kd and 55% for Zucksmall. Meanwhile, the `argmax` performed better for 91% of the ζ values for Newman1.

TABLE IV

METRICS FOR BEST GENERATION ACROSS DATASETS FOR NSGA-II AND PESA-II.

Model	**Gen.**	**Argmax**					**Softmax**				
		RNI	**Distance**	**IGD$^+$**	**Diversity**	**HV**	**RNI**	**Distance**	**IGD$^+$**	**Diversity**	**HV**
Newman1											
NSGA-II	10	**0.923**	**0.001**	**0.001**	0.736	1.126	0.387	0.018	0.004	0.725	0.991
PESA-II	5	0.667	0.019	0.015	0.687	1.680	0	0.025	0.020	**1**	**3.975**
Wilma1											
NSGA-II	10	**0.768**	0.008	0.002	0.789	0.926	0.600	0.013	0.004	**1.021**	0.966
PESA-II	5	0.5	**0.002**	**0.001**	0.981	2.035	0	0.047	0.034	1	**3.755**
Kd											
NSGA-II	10	0.04	0.344	0.066	0.749	0.774	**1**	**0**	**0**	0.629	0.945
PESA-II	8	0	0.314	0.310	1	3.707	**1**	**0**	**0**	**1**	**3.897**
Zucksmall											
NSGA-II	10	**0.703**	**0.009**	**0.003**	0.743	0.948	0.292	0.021	0.010	0.648	0.960
PESA-II	9	0.333	0.044	0.027	0.806	1.618	0	0.053	0.033	**0.958**	**2.167**

B. Performance Comparison between NSGA-II and PESA-II

PESA-II with the best ζ value is next compared with NSGA-II. We compare metrics from the best generation as well as the average, summarized in Table IV.

Firstly, running NSGA-II for more generations provides better metrics. For NSGA-II, the best generations across all four datasets are from generation ten. However, this is not necessary for PESA-II. All datasets provide the best metrics before generation ten – five, five, eight, and nine for Newman1, Wilma1, Kd, and Zucksmall respectively.

Secondly, NSGA-II provides better-quality solutions. It provides the best convergence (RNI, distance, and IGD$^+$) for Newman1, Kd, and Zucksmall. Meanwhile, PESA-II provides the best solutions for Wilma1 (distance and IGD$^+$) and Kd (RNI, distance and IGD$^+$). Even so, overall PESA-II gives a Pareto front that is more evenly distributed, by leading the diversity and HV metrics for all datasets.

Thirdly, the `argmax` and `softmax` variants behave differently in NSGA-II and PESA-II for non-dominated solutions. For NSGA-II, both variants simultaneously can give solutions that are part of the Pareto front. The RNI is more than zero for both variants, across all datasets. However, for PESA-II, only one variant can provide non-dominated solutions at a time, across all datasets. For the Newman1, Wilma1, and Zucksmall datasets, the RNI was more than zero for the `argmax` variant, but zero for the `softmax` variant. For the Kd dataset, the RNI was more than zero for the `softmax` variant, but zero for the `argmax` variant. Hence, NSGA-II seems to provide non-dominated solutions more stably versus PESA-II.

Finally, the runtime for PESA-II is on average faster for 75% of the datasets versus NSGA-II (Fig. 4). For PESA-II, the average runtime is 3.1% faster for Newman1, 1.6% faster for Wilma1, and 2.6% faster for Zucksmall. NSGA-II was on average faster than PESA-II only for Kd, by 5.3%.

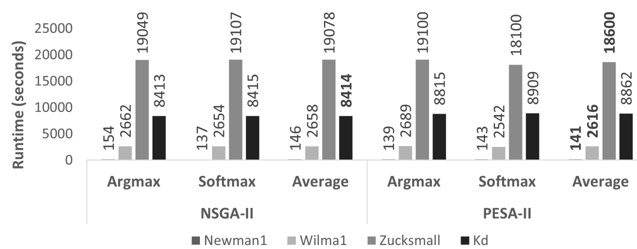

Fig. 4. Runtime at Generation Ten for Argmax, Softmax, and Averages.

C. Scenario Variants on Wilma1 for Sensitivity Analysis

Based on the best ζ value for the Wilma1 dataset from Section V-A, NSGA-II and PESA-II are run for the ore assessment scenarios described in Section IV-A. The results from both algorithms are used to form an approximated Pareto front for each scenario and are compared against the original. The Pareto front alterations are displayed in Fig. 5.

TABLE V

CONVERGENCE METRICS FROM INITIAL PARETO FRONT.

Scenario	**5-10%**	**10-15%**	**15-20%**
Distance Metric			
Over assessment of ore	0.055	0.065	0.140
Under assessment of ore	0.053	0.070	0.119
Average	**0.054**	**0.068**	**0.129**
IGD$^+$ Metric			
Over assessment of ore	0.008	0.007	0.017
Under assessment of ore	0.008	0.007	0.013
Average	**0.008**	**0.007**	**0.015**

These deviations are evaluated using the distance and IGD$^+$ metric. It measures the convergence gap between the Pareto front from each scenario versus that of the original

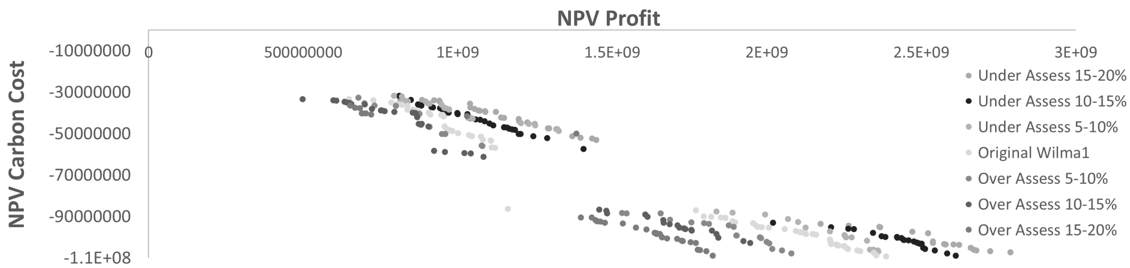

Fig. 5. Changes in Pareto Front with Deviations in Ore Assessment.

(Table V). For ore deviations below 15%, the gap from the original can be considered minute, averaging 0.054 (distance) or 0.008 (IGD$^+$) for the $5-10\%$ ore deviation, and 0.068 (distance) or 0.007 (IGD$^+$) for the $10-15\%$ ore deviation. With ore deviations above 15%, the gap is more pronounced, averaging 0.129 (distance) or 0.015 (IGD$^+$).

VI. Conclusion

This paper compares the performances of two MOEAs, NSGA-II and PESA-II, in a multi-objective optimization framework that enables mine production scheduling to be carbon-aware. We assess how the selection pressure parameter of PESA-II affects its performance and evaluate it against the parameter-less NSGA-II using real-world datasets. We also evaluate the sensitivity of the Pareto fronts when there are uncertainties in the amount of ores. The results show that besides being faster, PESA-II generally provides more diverse solutions. Meanwhile, NSGA-II can reliably provide non-dominated solutions. Furthermore, deviations in ore assessment cause relatively minute alterations to the Pareto front if they are below 15%.

The initial solution heuristic used may not be suitable for datasets with much larger precedence, blocks and time periods. Hence, future work may explore a more efficient heuristic without sacrificing the feasibility and quality of solutions. When faster heuristics are derived, the experiment framework can also be more rigorous with replication of experiments in multiple runs and their statistical comparisons. Subsequent works can also replace or extend the additional objective function to other environmental concerns such as treatment costs of water and other greenhouse gases. Furthermore, other uncertainties can be examined in the sensitivity analysis such as the economic price of ores.

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