

# E-DSGE model with endogenous capital utilization rate

Ying Tung Chan\*      Maria Teresa Punzi†

June 2, 2023

Environmental policy research has increased due to stricter policies aligned with climate goals. However, to achieve the goal of net-zero emissions, the adoption of even stronger policies and increased carbon taxes is necessary, with transition risk becoming a major concern for companies. Even though governments worldwide have been employing a range of methods such as carbon tax, cap-and-trade, and intensity targets to mitigate the impact of climate change, a pivotal debate around determining the optimal policy that reduces emissions without harming the economy continues. Our paper delves into the environmental policy assessment emphasizing the role of endogenous capital utilization rates, which have hitherto been largely disregarded in literature. We study how endogenous capital utilization rate affects the transmission mechanism of economic shocks and the optimal environmental policy choice. To evaluate the quantitative impact of the transmission mechanism, we introduce distinct features to the E-DSGE framework, including endogenous capital utilization, time-varying depreciation of capital, and environment quality shocks. We find that the complementarity between energy and capital leads to an amplification effect of the conventional transmission mechanism. Our model with these ingredients ranks any carbon tax below 25% as the best policy in terms of welfare improvement.

Keywords: E-DSGE model, environmental policy, capital utilization rate, energy price, welfare analysis.

JEL codes: E32, E50, Q51, Q53, Q54, Q58, R23

---

\*Bay Area International Business School, Beijing Normal University, Zhuhai, 519087, China. E-mail address: chanyt@bnu.edu.cn.

†Singapore Green Finance Centre at the Sim Kee Boon Institute for Financial Economics, Singapore Management University (SMU), Singapore. E-mail address: mtpunzi@smu.edu.sg.

# 1 Introduction

Over recent years, there has been a notable surge in research on the influence of environmental policy on macroeconomics. The growth of this subject can be attributed to the increased stringency in environmental policies over the previous few decades that align with climate objectives established by the Paris Agreement to limit global warming to well below  $2^{\circ}C$ .<sup>1</sup> Data obtained from the CDP indicates that transition risk is the dominant risk reported by companies when addressing climate-related risks.<sup>2</sup> As of 2021, approximately 2550 out of 5915 firms worldwide have reported a decrease in revenue or a rise in operating costs due to the implementation of policies linked to climate change, denoting that 43% of the reported transition risk highlights the significance of these climate-related policies in transmitting the business cycle. While carbon tax, cap-and-trade, and intensity targets have been popular measures taken by governments to address climate change, determining the optimal environmental policy that reduces emissions without compromising macroeconomic stability continues to be an ongoing matter of debate.

This paper aims to revisit the problem of selecting the optimal environmental policy and its relationship with energy consumption and business cycle fluctuations. Specifically, we emphasize that the capital utilization rate, which is neglected in the current literature, plays a significant role in determining the optimal environmental policies. In the next section, we will lay emphasis on the fact that while the endogenous capital utilization rate has been highlighted in the macroeconomics literature for a long time, it is presently disregarded in the literature of environmental economics. We demonstrate that energy usage and capital utilization rate have an important interaction with environmental policies and carbon emissions. This paper elucidates how economic shocks are transmitted when a model includes energy usage and endogenous capital utilization rate, taking into account different environmental regulations. We also examine what the most effective environmental policy would be in terms of welfare improvement in our model setting.

To achieve this aim, we take the E-DSGE framework of [Annicchiarico and Di Dio \(2015\)](#) and introduce new features: (i) we incorporate the endogenous capital utilization and time-varying depreciation of capital by assuming that energy is essential to the use of capital, and that energy

---

<sup>1</sup>Despite some progress, it seems that achieving the target of the 2015 ‘Paris Agreement’ of limiting global warming to well below  $2^{\circ}C$  is very hard to accomplish. See [Persaud \(2022\)](#) and [Wolf et al. \(2022\)](#). During the Glasgow Climate Pact in 2021, policy-makers from all around the world have established a new target of net-zero greenhouse gas emissions by 2050. Most likely, countries need to commit to more stringent climate policies to further decrease carbon emissions in fulfilling the aim of net-zero. Then, larger carbon taxes should be considered in order to affect households’ welfare with lower impact, when pursuing the goal of steadily declining greenhouse gas emissions.

<sup>2</sup>CDP (formerly Carbon Disclosure Project) is a not-for-profit charity that runs the global disclosure system for investors, companies, cities, states and regions to manage their environmental impacts. Thousands of organizations, cities, states and regions report their greenhouse gas emissions, water management and climate change strategies through CDP every year.

is tightly linked to any endogenous fluctuation in the utilization of capital; (ii) we introduce environment quality shocks by adding a preference for clean environment in the households' utility function; (iii) In contrast to the calibration method applied in [Annicchiarico and Di Dio \(2015\)](#) and other relevant literature, we undertake a model estimation to gauge the quantitative effect of the transmission mechanism.

The main contributions of this paper can be summarized as follows. First, based on our estimations, we discover that variable capital utilization amplifies the conventional transmission mechanism of shocks. In terms of environmental variables, a monetary policy shock generates a large amplified effect on carbon emissions, when the capital utilization and the capital depreciation rates are endogenous. This is due to the complementarity between capital and energy, which leads to a different quantitative impact on the marginal product of inputs depending on the model version.

Second, with the aim of ranking environmental policies, we evaluate welfare implications in terms of conditional and unconditional compensation variations. Conditional on shocks, the welfare cost differs across policies. For instance, a TFP shock would prefer a carbon tax, while a cap-and-trade regime is preferred when energy and government spending shocks hit the economy, independently if the capital utilization rate is constant or variable. On a contrary, in the aftermath of monetary policy shock, cap-and-trade system is preferred when the utilization rate is constant, while carbon tax and intensity target predominate under variable utilization rate. Considering unconditional compensating variation, under constant capital utilization, cap-and-trade and carbon tax policy regimes achieve the same level of welfare for a stringent environmental policy equal or higher than 15%. In contrast, under variable utilization rate, a carbon tax set below 25% results to be the best policy to be implemented. We also assess the unconditional compensating variation associated with both consumption and carbon emissions. Indeed, stringent climate policies, that aim at cutting  $CO_2$  emissions, improve households' utility.

Third, we present a comprehensive analysis of the trade-offs between environmental policies and household welfare. To this end, we construct a novel possibility frontier that captures all conceivable combinations of welfare costs and gains associated with these two variables. We find that higher carbon taxes shifts the upward curve to the right, indicating the extra  $CO_2$  emissions the households are willing to suffer under the higher carbon tax rate to preserve the same consumption level before.

In sum, the extant body of literature on the assessment of environmental policies grapples with the assumption of a fixed capital utilization rate, as detailed in the subsequent section. Notwithstanding the wide acceptance of endogenous capital utilization rate in the economic literature, this paper contributes to the existing literature by constructing an environmental as-

assessment framework that incorporates variable capital utilization rate. We show that the capital utilization rate evinces a critical interplay with the energy utilization, thereby, imparting vital implications on the selection of the optimal environmental policies. Our novel framework holds the potential to contribute to the recent debate surrounding the selection of optimal environmental policies.

## 2 Literature review

There has been a long-lasting environmental policy debate on the suitability of instruments able to meet the demands of managing environmental economic sustainability. [De Santis and Jona Lasinio \(2016\)](#) show that market based environmental stringency measures are more likely to stimulate innovation and productivity growth.<sup>3</sup> In contrast, [Jaffe and Palmer \(1997\)](#), [Popp \(2006\)](#) and [Albrizio et al. \(2014\)](#) provide evidence of limited increasing innovative activity by firms as environmental regulations become more stringent. Inter alia, extremely tight environmental policies could cause many plants to shut down, thus affecting the productivity of investment and the savings behavior of consumers, both in the long- and short-run. [Nordhaus and Yang \(1996\)](#) indicate that a carbon tax is an effective short-term policy in reducing carbon dioxide (CO<sub>2</sub>), which is the primary greenhouse gas emission, with a small negative impact on the economy.<sup>4</sup> Similarly, [Yang et al. \(2014\)](#) highlight the beneficial role of carbon tax in improving energy efficiency.<sup>5</sup> Recent works have conducted analysis in favor of market-based policy tools that limit aggregate emissions from a group of emitters by setting a “cap” on maximum emissions. [Fischer and Springborn \(2011\)](#) show that cap-and-trade system reduces economic volatility, while tax policy tends to increase the variability of the macroeconomic variables. [Annicchiarico and Di Dio \(2015\)](#) find that cap-and-trade policy is likely to generate lower macroeconomic volatility relative to the other two policies. most recently, [Zhao et al. \(2020\)](#) report that a carbon trading scheme has relative small effects on economic outcomes such as employment, consumption and output, while carbon emissions policy has a negative impact on the macroeconomy.

Undoubtedly, the determination of environmental policies is contingent upon economic conditions, particularly the fluctuations in business cycles. To this end, recent research has embraced a new class of models known as E-DSGE models that facilitate analysis of environmental policies.

---

<sup>3</sup>Market-based instruments include environment-related taxes, fees, charges, deposit-refund systems; energy-related subsidies; and marketable/tradable rights.

<sup>4</sup>See also [Heutel \(2012\)](#) and [Angelopoulos et al. \(2013\)](#) for a discussion on optimal environmental policy when uncertain exogenous shocks hit the economy.

<sup>5</sup>With a focus on China, [Zhu et al. \(2010\)](#), [Shi et al. \(2013\)](#), [Xu and Zhang \(2016\)](#) provide evidence on a negative impact on income, consumption and living standards when emissions are subject to carbon tax, without reducing carbon emissions at the desired levels.

E-DSGE model emphasizes the importance of the effects of environmental regulations throughout the business cycle. One of the salient features of the E-DSGE model is its aptitude to conform to a stochastic environment, allowing analysts to assess the performance of different environmental policies under various types of economic shocks. It is this superiority that prompted our inclination toward adopting the E-DSGE framework for our analysis. Most prior studies adopting E-DSGE models document the procyclicality of both emissions of pollution and optimal tax/cap policy with the increase of economic growth. A typical E-DSGE model shows that, by pursuing a target of declining greenhouse gas emissions, environmental policies raise production costs and lower productivity by requiring firms to install pollution control equipment or adapt to a new production processes. Such lower productivity affects inflation and interest rate under price stickiness. See [Heutel \(2012\)](#), [Fischer and Springborn \(2011\)](#), [Annicchiarico and Di Dio \(2015\)](#), [Dissou and Karnizova \(2016\)](#), [Annicchiarico and Di Dio \(2017\)](#), [Khan et al. \(2019\)](#), [Chan \(2020a\)](#), [Chan \(2020b\)](#), [Lintunen and Vilmi \(2021\)](#) and [Xiao et al. \(2022\)](#).

However, the previous works have an important limitation. , they assume a fixed capital utilization ratio. According to [Finn \(1995\)](#), [Finn \(1996\)](#) and [Finn \(2000\)](#), capital utilization plays an essential role in the transmission of energy price shocks, as energy is essential to the usage of capital and to produce goods.<sup>6</sup> Capital without energy is unusable, implying that there exists a complementary relationship between them. Existing integrated assessment models and E-DSGE models neglect the impact of capital utilization rate and the role of energy prices on the effectiveness of environmental policy.

Second, most of the papers neglect the environmental awareness of the households. The importance of environmental awareness has been already pointed out in the literature. For instance, [Lusky \(1975\)](#) pointed out that consumers derive benefits from clean natural resources and they are willing to pay for a better environment. Similarly, [Brochado et al. \(2017\)](#) indicate increasing consumer environmental awareness leads to changes in preferences and consumption patterns. [Chan \(2019\)](#) shows that environmental preference shocks, though often neglected in the literature, have long been dampening business cycles. Therefore, any analysis that ignores the above preferences will fail to capture the appropriate welfare cost associated with each individual environmental instrument.

Third, the majority of the existing E-DSGE models abstract from the role of energy consumption as a complementary input to the use of capital. Tightening upstream environmental policies affect investment and the energy market, thus contributing to the fluctuation of energy prices and business cycle. [Dechezleprêtre et al. \(2020\)](#) provide evidence of the negative impact of

---

<sup>6</sup>[Rotemberg and Woodford \(1996\)](#) assume that imperfect competition is a necessary ingredient to describe a positive effect of energy price on economic activity. In contrast, [Finn \(2000\)](#) proves that endogenous capital utilization and perfect competition are enough to explain [Rotemberg and Woodford \(1996\)](#) evidence.

both energy prices and environmental policy on total employment in the manufacturing sector in OECD countries. [Dlugosch and Kozluk \(2017\)](#) reveal that higher energy prices have a negative impact on investment. Such an effect is largely attributed to more stringent environmental policies through their impact on energy prices. It is therefore imperative, for an accurate assessment of the effectiveness of climate-oriented policies, that energy prices be taken into account. Indeed, incorporating energy usage into the production function has been explored in recent papers on E-DSGE models, including works by [Xiao et al. \(2021\)](#); [Bongers \(2020\)](#); [Xiao et al. \(2022\)](#). Showing a similar line of inquiry, our model also integrates energy usage and unveils pivotal linkages between it, endogenous capital utilization rate, and nominal rigidity.

Our novel contribution lies in the vital incorporation of these three components into a E-DSGE model. Our findings indicate that optimal environmental policies throughout business cycles are significantly influenced by these three key components.

### 3 DSGE model

This section provides a description of our E-DSGE model. In the model, households make the saving and consumption decisions. Firms use capital to produce. The final good market is assumed to be perfectly competitive. Our model differs from the standard E-DSGE model in a variety of aspects: (i) households acquire capital and determine the utilization rate simultaneously; (ii) environmental quality enters directly into the utility function of households; (iii) carbon emissions are generated partially by the production of goods, and partially by the consumption of energy. Environmental regulations are introduced through three different carbon policies: (i) carbon tax (i.e., a levy for every unit of carbon emissions emitted); (ii) cap-and-trade (i.e., exogenous limit on carbon emissions allowances); (iii) intensity targets (i.e., an exogenous limit on carbon emissions per unit of total output).

For the model description, we report details for only the essential parts of the model that recall the introduction of environmental preferences and variable capital utilization rate. The rest of the model resembles the framework of [Annicchiarico and Di Dio \(2015\)](#). Details can be found in [Appendix B](#).

#### 3.1 Households

Assume that in an economy there is a continuum of identical households, indexed by  $i \in [0, 1]$ . The representative households maximize the discounted lifetime utility:

$$V_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t, Z_t) \quad (1)$$

where  $\beta \in [0, 1]$  is a discount factor. Further, we assume that the instantaneous utility function  $U$  is a GHH preference (Greenwood et al., 1988) as:

$$U(C_t, L_t, M_t) = \frac{1}{1 - \sigma} \left( C_t - \mu_L \frac{L_t^{1+\phi_L}}{1 + \phi_L} - \mu_{M,t} \frac{Z_{t-1}^{1+\phi_M}}{1 + \phi_M} \right)^{1-\sigma} \quad (2)$$

where  $C_t$ ,  $L_t$  and  $Z_t$  are households' consumption, labor supply and carbon emissions in period  $t$ , respectively.  $\mu_L > 0$  controls the scale of labor disutility and  $\phi_L > 0$  is the inverse of Frisch elasticity. One novelty of the utility function is the inclusion of the carbon emissions  $Z_t$ . It is assumed that households' utility are negatively influenced by the emissions flows, indicating that a decrease in them implies lower carbon emissions stock  $M_t$ , thus better environmental quality.<sup>7</sup> The scale elasticity is controlled by the parameter  $\phi_M > 0$ . We label movements in  $\mu_{M,t} > 0$  as environmental quality preference shocks, which express households' attachment to environmental quality. A similar specification can be found in Angelopoulos et al. (2013) and Delis and Iosifidi (2020) who use the stock of environmental quality as a proxy for environmental awareness in the households' utility function.<sup>8</sup> The choice of non-separability between consumption and labor in the GHH preferences derives from the advantage of eliminating the wealth effect on labor supply, therefore households only care about smoothing consumption. Greenwood et al. (1988) and Jaimovich and Rebelo (2009) argue that by neutralizing the wealth effect, GHH preferences produce more volatile labor hours and help generate the co-movement between consumption and leisure conditional on non-productivity shocks.<sup>9</sup> Further, in order to generate an impact on the marginal utility from a shock to environmental quality preferences, GHH preferences represent a good choice when including carbon emissions in the utility function. Carbon emissions enter the utility function with a lag, as households' happiness increases with lower emissions from the previous period. Further, unlike consumption and labor supply, each household is infinitesimally small so that its decision has no direct impact on the emissions stock.

Households maximise utility function (12) subject to the following budget constraint:

$$P_t C_t + P_t I_t + Q_t^B B_t + P_t^E E_t = B_{t-1} + W_t L_t + P_t D_t + R_{K,t} K_t u_t - T_t \quad (3)$$

---

<sup>7</sup>Gray (2015) discusses the benefits of environmental regulations as a reduced illness and death, and/or better recreational water quality.

<sup>8</sup>Angelopoulos et al. (2013) and Delis and Iosifidi (2020) show that the evolution of the stock of environmental quality increase with lower current pollution flow.

<sup>9</sup>See also Dmitriev and Roberts (2012), Furlanetto and Seneca (2014), Lester et al. (2014), Boppart and Krusell (2020) for more examples of using GHH preferences.

Households save by buying one-period riskless bond  $B_t$ , whose price is  $Q_t^B$  and invest  $I_t$  in capital.  $K_t$  is the amount of capital owned by the households in period  $t$ .  $u_t$  denotes the capital utilization rate. Since capital is owned by households, an increase in the capital utilization rate would increase the energy use  $E_t$ .  $P_t^E$  is the energy price. We follow [Finn \(2000\)](#) to assume that:

$$E_t = a(u_t)K_t \quad (4)$$

where  $a(u_t) = \nu_0 u_t^{\nu_1} / \nu_1$  for some parameters  $\nu_0 > 0$  and  $\nu_1 > 1$ . As explained in [Finn \(2000\)](#), the assumption that  $\nu_1 > 1$  leads to the percentage increase in energy-to-capital ratio  $E_t/K_t$  is greater than that of  $u_t$ , deterring households from increasing the  $u_t$  rapidly when facing shocks.<sup>10</sup>

In addition,  $W_t$  and  $R_{K,t}$  are, respectively, the nominal wage rate and the (nominal) rate of capital return. Furthermore, households can obtain dividend  $D_t$  from their ownership to the intermediate goods firms.  $T_t$  is the lump-sum tax levied by the government. It is assumed that households' environmental preference is time-varying around a steady-state value  $\mu_M$ . In particular, we assume that  $\mu_{M,t}$  follows an AR(1) process as:

$$\ln \left( \frac{\mu_{M,t}}{\mu_M} \right) = \rho_M \ln \left( \frac{\mu_{M,t-1}}{\mu_M} \right) + \sigma_M \varepsilon_{M,t} \quad (5)$$

where  $\rho_M \in [0, 1]$  is the persistence of the shock process, and  $\sigma_M > 0$  is the standard deviation of the white noise  $\varepsilon_{M,t}$  that follows standard normal distribution.

Moreover, we assume that the capital evolves as follows:

$$K_{t+1} = (1 - \delta_K(u_t))K_t + \left( 1 - \frac{\gamma_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t \quad (6)$$

where the quadratic term on the right-hand-side (RHS) represents the investment adjustment cost ([Jaimovich and Rebelo, 2009](#)).  $\gamma_I > 0$  is a scale parameter. It is assumed that the investment cost is proportional to the percentage change in investment from the last period. The convex assumption of the investment cost also incentivizes households to split their investment into several periods. Further, we assume that the depreciation rate  $\delta_K(u_t)$  is increasing and convex to the utilization rate  $u_t$ , such as:

$$\delta_K(u_t) = \frac{\omega_0 u_t^{\omega_1}}{\omega_1} \quad (7)$$

where  $\omega_0 > 0$  and  $\omega_1 > 1$ .

---

<sup>10</sup>For simplicity, we do not assume households consume energy directly and thus  $E_t$  does not enter the utility function. Here,  $E_t$  only represents the energy used due to the "excessive" use of capital. It is easy to generalize our model by including the energy directly used for consumption.



We fit the model to the U.S. data using the Bayesian estimation approach. The estimation procedure is discussed in detail in the Online Appendix C.

## 4 Impulse Responses

The following Section reports the impulse response functions of macroeconomic and environmental variables under three different environmental policies: (i) carbon tax (red solid line); (ii) cap-and-trade (blue dashed line); (iii) intensity target (black dotted line). All variables are expressed in terms of percentage deviations from the steady-state, except for the inflation rate and the permit price, for which responses are expressed as absolute deviations from their steady-state values.

### 4.1 Technology Shock

Figure 1 presents results for a 1% increase in total factor productivity at different environmental policies. The impact of macroeconomic variables is similarly found in Fisher (2006) and Annicchiarico and Di Dio (2015). The technological shock improves productivity by making inputs more efficient, leading firms to use the same input factors to produce more output.<sup>11</sup> The higher output production leads to supply-driven inflation. However, as suggested by Fisher (2006), Canova et al. (2010), Dave and Dressler (2010) and Annicchiarico and Di Dio (2017), under price stickiness, adjusting prices is costly, and firms tend to mitigate the decline in prices by cutting back some inputs, thus working against the increase in output. This is achieved by reducing energy usage,  $E$ , which then implies a lower capital utilization rate,  $u_t$ .<sup>12</sup> Consequently, investment increases due to a lower capital depreciation rate,  $\delta(u_t)$ , and asset prices increase in response to higher demand for capital stock. Further, innovations to TFP generate a decline in the marginal product of labor, which implies lower labor demand and wages.<sup>13</sup> The presence of price rigidities contributes to lower marginal products of labor implied by the technological shock.<sup>14</sup> In contrast, Grodecka and Kuralbayeva (2014) report wages and labor hike after a 1% increase in productivity under both carbon tax and cap-and-trade policies. This is due to the fact that her model is based on perfect competition and flexible prices. On impact, consumption decreases due to lower labor income. However, consumption rebounds quickly after few

<sup>11</sup>Alternatively, improvement in production efficiency can let firms combine less inputs in producing the same level of output.

<sup>12</sup>Re-arranging Eq. (15), the capital utilization rate can be expressed as  $u_t = (\frac{\nu_1 E_t}{\nu_0 K_t})^1$ . Thus, lower energy implying a decay rate of capital utilization.

<sup>13</sup>See Zhao et al. (2020) for similar results.

<sup>14</sup>Similar result is found in Galí (2008) and Annicchiarico and Di Dio (2015).

quarters in response to the decline in energy consumption, which let households use the saving from energy usage for the consumption of goods and services. Depending on the environmental policy, consumption decreases by around 0.40-0.50%, a decline similar to [Zhao et al. \(2020\)](#) who find that a productivity shock lowers consumption by 0.60%. However, different from our model that simulates a decline in energy consumption of about 0.6%, their model replicates an increase in energy consumption by 0.69%. Further, they show that carbon emissions increased by 1.64% in response to the shock, while our carbon emissions increased in more contained and rise around 0.4-0.6% after few quarters, but not on impact. Such different quantitative impact and qualitative behaviour are due to the absence of price rigidities and households' environmental preferences in the utility function. Moreover, in [Zhao et al. \(2020\)](#)'s model, energy enters only the production function, thus abstracting from evaluating the real option effect that would be generated if energy would enter also the household's problem.<sup>15</sup>

Turning to environmental variables, when productivity improves carbon emissions increase as well, being a proportion of the value added produced in the economy, with the exception of cap-and-trade. Figure 1 shows that emissions stay at zero under the case of cap-and-trade, in which the government sets a fixed amount of emissions  $Z_t$ . Moreover, the positive impact on GDP is contained relative to the other two policies, because the fixed cap imposed by the government prevents extra output from being used as an additional intermediate good. Thus, innovations in productivity, which would otherwise increase emissions, lifts the market price of emissions permits,  $p_z$ , to keep emissions constant, as well as abatement cost and effort. In contrast, carbon tax scheme imposes a constant tax rate of 5%, which implies a constant abatement cost,  $CA$ , and a constant abatement effort,  $U$ , accordingly to Eq. (30) and Eq. (34), respectively. Under the intensity target policy, the government fixes a maximum ratio of emissions to output,  $Z_t/Y_t$ , and sells emission permits to firms at the current market price. When the economy is affected by a positive technological shock, the abatement cost and the abatement effort both decline. This is due to the fact that the abatement effort,  $U_t$ , depends on a constant emission-output ratio and on a variable energy-output ratio, which tends to decrease immediately after the shock.<sup>16</sup> Consequentially, the abatement cost, being a function of the abatement effort, and the permit price both decrease as well. Given the counter-cyclical behaviour of abatement cost and effort under the intensity target regime, then carbon emissions, consumption and GDP present a much larger positive response in the aftermath of the shock, relative to the other two environmental policies.

---

<sup>15</sup>See [Bloom \(2014\)](#), [Abdul-Salam \(2022\)](#) and [Xu et al. \(2022\)](#).

<sup>16</sup>Re-arranging Eq. (29), abatement effort can be expressed as the following:  $U_t = 1 - \frac{1}{\varphi} \frac{Z_t}{Y_t} + \frac{\varphi_E}{\varphi} \frac{E_t}{Y_t}$ , where  $\frac{Z_t}{Y_t} = \textit{intensity}$ .

Overall, the endogenous capital utilization rate plays a crucial role in the current economic context. Notably, under conditions of price stickiness, firms unable to decrease their prices tend to decrease their capital usage rate in response to positive TFP shock. This ultimately results in a minor increase in carbon emissions, even when considering different types of environmental policies, which tend to increase at a lesser rate compared to those predicted under the standard model that assumes a constant capital utilization rate [Annicchiarico and Di Dio \(2015\)](#). This outcome is also attributable to our model’s characteristic of having carbon emissions increasing proportionally with not only output, but also energy that decreases as a result of TFP shock. If the capital utilization rate were constant, energy would increase instead of decreasing, presenting a different impact on the transmission mechanism of both macro- and environmental variables.

## 4.2 Energy price Shock

Figure 2 presents results for a 1% increase in energy price at different environmental policies. *Ceteris paribus*, higher cost of energy generate an immediate negative impact on energy consumption, with a much larger amplified effect under the cap-and-trade policy scheme. Energy and capital are compliments in the production function, therefore lower energy usage implies lower capital stock. Consequently, investments and asset prices fall. Being the ratio of energy to capital a function of the capital utilization rate,  $u_t$  declines together with the depreciation rate  $\delta(u_t)$ . Capital utilization directly enters the production function of intermediate goods, thus Eq. (27) can be thought of as a production function that combines capital, energy and labor.<sup>17</sup> Hence, energy price fluctuations implicitly spread into the economy by affecting the marginal productivity of capital and labor. However, while lower marginal productivity of labor would imply a decline in wages, the presence of price rigidities prevents such effect. In fact, as inflation increases due to higher marginal costs, wages increase and stimulate a higher labor supply. At a given higher labor income and lower energy spending, consumption raises. Although the energy shock propagates to the economy by lowering energy and capital, on impact output increases because of higher aggregate consumption level, but decreases after several quarters.<sup>18</sup> Carbon emissions increase as well, due to the higher productivity induced by more hours worked, with a larger impact under intensity target. While cap-and-trade environmental scheme generates a positive reaction to abatement effort, abatement cost and price of permit, those environmen-

---

<sup>17</sup>After aggregating over all intermediate goods  $i$ , Eq. (27) can be re-written as:  $Y_t = A_t(1 - \Upsilon(M_t)) \left[ E_t^{(\frac{1}{v_1})} K_t^{(1-\frac{1}{v_1})} \right]^\alpha L_t^{1-\alpha}$ .

<sup>18</sup>Figure 10 in Appendix 7 plots the U.S. real GDP per capita and the global energy price index, and shows that the energy price appears to be pro-cyclical with the real domestic output. From 1995 until beginning of 2022, the correlation is positive and equal to 0.504.

tal variables decline under the intensity target regime. Similar to the technology shock, the abatement effort depends positively on value added and negatively on energy usage. Clearly, the positive impact of productivity is much stronger relative to the negative impact on energy, therefore the abatement effort rises in the aftermath of the energy shock. Consequentially, the abatement cost and the permit price decline. In contrast, as the intensity target assumes that the emission-output ratio is constant, then the abatement effort depends only on energy usage, which declines when the energy price shock hits the economy, and similar path is followed by the abatement cost and permit to pollute.

In sum, we find that the impact of energy price shock is heavily influenced by the presence of nominal rigidity. Diverging from the conclusions presented by Finn (2000) and Kormilitsina (2011), in which energy price shock presents itself as a negative technology shock and subsequently induces a recessionary effect on the economy, the degree of nominal rigidity obstructs the cost of energy from acting as a supply shock. Additionally, similar to the findings presented by Zhao et al. (2020), our model exhibits a positive response in both consumption and labor, while also displaying a negative impact on energy consumption. However, in contrast to their model, we observe a decrease in output and emissions because the negative impact on energy and capital stock negates the positive effects attributed to consumption. Furthermore, in regards to the environmental policy options available in light of an energy shock, we find that carbon taxation and intensity targets produce comparable quantitative impacts on macroeconomic variables following an increase in the cost of energy, whereas the cap-and-trade system significantly diminishes the positive impact generated on GDP and consumption.

### 4.3 Environmental Preference Shock

Figure 3 presents results for a 1% increase in environmental quality preference shock at different environmental policies. An increasing desire for clean environment, described by a shock to  $\mu_M$  in the households' utility function, immediately affects positively the marginal utility of consumption and negatively the marginal utility of labor. As a result, wages demanded by households raise. Moreover, the desire for a clean environment lets households consume less in order to contribute to less air pollution. The increase in wages leads to an increase in marginal costs, thus contracting firms' investment and production. As a result, asset prices and aggregate GDP decline, as well as carbon emissions. The contraction in aggregate supply determines an increase in inflation. The central bank reacts to higher inflation by raising short-term interest rates, which further contracts investment, consumption and GDP. In terms of environmental policies, carbon tax and intensity target scheme have a similar impact on carbon emission and GDP fluctuations. Because of the presence of energy in the production function and in the house-

holds' budget constraint, abatement effort, abatement cost and permit price respond in opposite direction depending if the government implements a cap-and-trade or intensity target policy. Similar to [Busato et al. \(2022\)](#), environmental quality preferences are negatively correlated with the firm's abatement effort under a cap-and-trade policy. As shown for previous shocks, under a cap policy, the government fixes the maximum level of carbon emissions in the atmosphere, which leads to a reduction in the price of allowances.

Overall, we find that all the three environmental policies considered induce similar impacts on GDP. Nevertheless, while both the cap-and-trade policy and intensity target are capable of effectively mitigating carbon emissions in response to the positive preference shock, their quantitative impact is minimal and negligible. Therefore, the three environmental policies do not show significant differences in response to the preference shock.

#### 4.4 Policy Shock

Figure 4 and Figure 5 present impulse responses to a temporary shock to government spending and monetary policy, respectively. Because of the lack of sufficient space, we reduce the presentation of these two policy shocks to fewer variables. Similar to [Annicchiarico and Di Dio \(2015\)](#), a positive demand shock induced by higher government spending leads to a positive reaction of output and labor. Because of the crowding-out effect, consumption and investment decline in the aftermath of the shock. This is due to the fact that a positive demand shock generates inflation and a positive output gap, therefore the central bank increases interest rates to fight the inflationary process. Consequently, the cost of capital becomes too expensive, and firms lower their investment. Carbon emissions increase due to the positive impact on productivity, therefore firms need to use more polluting inputs to produce intermediate goods. Under the government spending shock, carbon tax and intensity target policies generate the same qualitative and quantitative impact, while the cap-and-trade contributes to a lower positive impact on labor, energy and GDP, but to a larger negative impact on consumption and investment.

A contractionary monetary policy, described in Figure 5, generates a decline in all components of aggregate demand, with investment showing the largest drop, followed by consumption. A recessionary economy implies lower carbon emissions. The transmission mechanism works as follows: the short-term interest rate hike leads to a contraction in investment due to the higher cost of capital acquired via external financing. Given the complementarity between energy and capital, lower investment implies lowered energy usage. A monetary policy tightening robustly decreases real wages and labor productivity, thus hours worked decline. Consequently, consumption decreases due to lower labor income.

In summary, the model we have formulated, incorporating the endogenous utilization rate

of capital, has yielded results indicating that monetary policy can lead to reductions in energy usage - a result not found in the standard model with a constant capital utilization rate. In terms of environmental policy, Figure 5 shows that all three environmental policies generate a similar qualitative impact, with a slightly minor fall in macroeconomic variables under cap-and-trade. This is due to the fact that a fixed cap generates a larger drop in abatement effort and cost, which amplifies the decline in marginal cost. Consequentially, firms need a smaller reduction of input factors in the production of intermediate goods in response to a monetary policy tightening.

## 5 The role of capital utilization and price rigidities

Figure 6 reports impulse responses of key variables to five exogenous shocks when the government imposes carbon tax on emissions, and compares the simulations when a model specifies for a variable capital utilization rate (red solid line) versus a model that assumes a constant utilization rate (blue dotted line). First thing to be noted is that endogenous capital utilization rate generates a larger amplification of the responses of energy usage. Other variables also present some amplification, notably hours worked and consumption.

The first column in Figure 6 presents results for a 1% increase in total factor productivity. A model abstracting from a variable capital utilization simulates a more contained positive impact on GDP, but a larger decline in consumption. This is due to a larger negative effect on labor, thus households earn less labor income to be allocated to consumption. Relative to a constant depreciation model, capital utilization model predicts a decline in the depreciation rate of capital to the exogenous shock. Since the capital stock is fixed and adjusting prices is costly, firms seek to attenuate the fall in price by cutting back on inputs, thus engendering a decline in the utilization rate of about 2%. Such lower utilization rate, coupled with a slower depreciation rate of the current capital stock, determine a cutting back on investment. Thus, innovations to productivity generate a reduced positive effect on investment, relative to a constant capital utilization rate. Being the  $GDP = C + I + G$ , the response of GDP to innovation to TFP is amplified under a variable capital utilization model, but not as much as in consumption, because the lower positive impact in investment counteracts the GDP expansion.

The capital utilization model predicts also a decline in energy usage. Being energy a function of the capital utilization rate,  $E_t = a(u_t)K_t$ , a lower capital utilization entails a lower energy usage. Differently, a constant depreciation model simulates a zero energy response on impact in the aftermath of the shock, which becomes positive afterward. This is because under a constant capital utilization rate, the indirect transmission channel is shut down. As shown in Finn (2000), two channels work when the capital utilization depends on energy usage: (1) direct transmission

channel of energy on output in which energy enters directly in the production function;<sup>19</sup> (2) indirect transmission channel related to the capital's marginal energy cost in which a variable depreciation rate alters the capital stock. Finally, as carbon emissions depend positively on the value added of output, those increase as well. However, carbon emissions do not show any amplification effect as the other macroeconomic variables do. This is due to the fact that the endogenous capital utilization rate generates a higher output response to the exogenous shock, but a lower response in energy use. Thus, one effect cancels out the other one.<sup>20</sup> The second column in Figure 6 reports impulse response functions to an increase in the short-term interest rate. When the model is characterized by endogenous capital utilization, energy and capital utilization rate report a larger amplified impulse response when the shock hits the economy. Higher interest rates discourage investment, and thus the acquisition of new capital. As energy and capital are complements, firms persistently reduce the use of energy as an input factor as much as capital, contributing to lower production. However, under constant capital utilization, firms reduce a larger demand for labor in response to the higher interest rate. However, this shock does not generate any amplification in carbon emission when comparing the two models. The last three columns display impulse responses for the government spending, energy price and environmental preference shocks, and all three simulations show a similar amplification effect as in the TFP shock. Relative to the other two shocks, carbon emissions show a larger response under a variable capital utilization model, although the quantitative impact of these shocks is very small.

Figure 7 reveals the importance of nominal rigidities. Indeed, under perfect competition, labor, energy and capital utilization increase when a technology shock hits the economy. Further, a variable capital utilization model generates a larger amplification, whereas a larger amplification was recorded under constant capital utilization when the model is characterized by nominal rigidities. As explained previously, this is mainly due to the different responses of labor supply. Similar to the TFP shocks, all other shocks present more amplified responses in the main variables. In particular, energy price shock presents a larger amplified response in carbon emissions, which reflects a larger decline in both GDP and energy usage under flexible prices.

Overall, the model simulation confirms the high procyclicality of carbon emissions found in the literature,<sup>21</sup> also under energy price shocks and environmental preference shocks.

---

<sup>19</sup>See [Rasche and Tatom \(1981\)](#), [Rotemberg and Woodford \(1996\)](#) and [Finn \(2000\)](#) for standard perfectly competitive models in which the transmission of energy fluctuations is confined to only the direct production function channel.

<sup>20</sup>Recall the carbon emission equations:  $Z_t = (1 - U_t)\varphi Y_t + \varphi_E E_t$

<sup>21</sup>See [Heutel \(2012\)](#), [Khan et al. \(2019\)](#) and [Chan \(2020a\)](#).



## 6 Ranking Environmental Policies

In this section, we seek to investigate if the presence of endogenous capital utilization alters the ranking found in the literature so far. [Angelopoulos et al. \(2010\)](#) and [Dissou and Karnizova \(2016\)](#) have examined the effectiveness of emissions cap and emissions tax as possible options for the best environmental policy. Many other empirical and theoretical works have been asking a similar question relative to macroeconomic uncertainty. See [Fischer and Springborn \(2011\)](#), [Shi et al. \(2013\)](#), [Grodecka and Kuralbayeva \(2014\)](#), [Annicchiarico and Di Dio \(2015\)](#), and [Chan \(2019\)](#).<sup>22</sup>

Although intensity target fosters economic growth, [Fischer and Springborn \(2011\)](#) have found that cap-and-trade policy is able to reduce volatility to the lowest levels, relative to other environmental policies. However, in terms of welfare ranking, cap and tax policies achieve less volatile welfare with higher steady-state levels, with carbon tax showing slightly higher mean levels. See also [Annicchiarico and Di Dio \(2015\)](#) and [Dissou and Karnizova \(2016\)](#).

We do that by analyzing the welfare cost criteria, which represents a powerful tool to rank policies because it depends not only on volatility (i.e. stochastic fluctuations), but also on the new level of steady-state achieved during the transitional dynamics. As a matter of fact, welfare cost measures the magnitude of the expected flow of utility from consumption, leisure and environmental quality that can be achieved by moving from no policy and no uncertainty scenario to a new utility level under the environmental policy in the presence of all five shocks. In particular, we calculate the compensating variation of welfare gains (or losses) of changes in environmental policies: the percentage by which consumers need to be compensated under the environmental policy in order to achieve the same welfare as without environmental regulation. Technically, it indicates a welfare gain/loss metric that conveys information conditioned on stochastic means. [Fernández-Villaverde et al. \(2016\)](#) explain that under certainty equivalence (i.e. uncertainty plays no role), the first-order approximation solution provides the identical solution as a model under perfect foresight. Thus, welfare effect is solved by using a second-order approximation of the perturbation method.<sup>23</sup>

Our welfare calculations consider both the conditional and unconditional compensating variations.

The unconditional compensating variation compares mean welfare across two alternative regimes without conditioning on the same initial point in the state space. This metric con-

---

<sup>22</sup>[Metcalf \(2009\)](#) discusses strengths and weaknesses of market-based policy options to reduce U.S. emissions of greenhouse gases.

<sup>23</sup>Second order approximations are discussed in several papers. See [Collard and Juillard \(2001\)](#), [Schmitt-Grohé and Uribe \(2004\)](#), [Swanson et al. \(2005\)](#) and [Kim et al. \(2008\)](#).



siders welfare difference in the long-run once the costs of implementing a new environmental policy have already been absorbed. We then also calculate the conditional compensating variation, which compares welfare under two scenarios, with and without enforcing environmental regulations, conditioned on the realization of specific shock.

Let's consider the discounted lifetime expected utility function at a particular point in the state space  $t$  in a specific regime,  $i = \{0, 1\}$ . Model 1 indicates a model characterized by the presence of carbon tax, cap-and-trade or intensity target. Model 0 indicates a model abstracting from environmental regulation, or carbon tax equal to zero. Thus, the expected present discounted value of flow utility evaluated at the optimal choices of consumption, labor and carbon emission is defined by:

$$V_{i,t} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^{t+j} U(C_{i,t+j}, L_{i,t+j}, Z_{i,t+j}) = \frac{1}{1-\sigma} \left( C_{i,t+j} - \mu_L \frac{L_{i,t+j}^{1+\phi_L}}{1+\phi_L} - \mu_{M,t} \frac{Z_{i,t+j}^{1+\phi_M}}{1+\phi_M} \right)^{1-\sigma} \quad (8)$$

Following [Lester et al. \(2014\)](#) and [Cho et al. \(2015\)](#), the conditional compensating variation for the regime with and without environmental policy is the solution to the following equality:

$$V_{1,t} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^{t+j} \frac{1}{1-\sigma} \left( (1 + \lambda_C^c) C_{0,t+j} - \mu_L \frac{L_{0,t+j}^{1+\phi_L}}{1+\phi_L} - \mu_{M,t} \frac{Z_{0,t+j}^{1+\phi_M}}{1+\phi_M} \right)^{1-\sigma} \quad (9)$$

where  $\lambda_C^c$  represents the percentage change in consumption from the deterministic steady state of the “no environmental policy” regime (i.e. no policy and no uncertainty scenario) that households would need to attain the same level of utility obtained in the stochastic economy under the tax, the cap, or the intensity target. The sign of  $\lambda_C^c$  indicates whether the household would prefer to be in the regime with environmental regulations (i.e.,  $\lambda_C^c > 0$ ), or whether the household would prefer the regime with no policy (i.e.,  $\lambda_C^c < 0$ ). It is not possible to derive an exact closed-form solution for  $\lambda_C^c$  that allows Eq. (9) to hold with equality.<sup>24</sup> Then, numerical methods can be applied to find an approximation solution for  $\lambda_C^c$ .<sup>25</sup>

Similarly, the unconditional compensating variation  $\lambda_C^u$  is calculated by solving the following

---

<sup>24</sup>Typical additively separable preferences and log utility over consumption would yield an expression for  $\lambda_C^c = \exp\left((1-\beta)(V_{1,t} - V_{2,t})\right) - 1$ . However, such an expression is not possible under GHH preferences. See [Lester et al. \(2014\)](#).

<sup>25</sup>We construct an algorithm in Matlab using “optimset” and “fsolve” functions.

expression based on expected welfare of a particular environmental regime:

$$E\left(V_{1,t}\right) = \mathbb{E} \sum_{t=0}^{\infty} \beta^{t+j} \frac{1}{1-\sigma} \left( (1 + \lambda_C^u) C_{0,t+j} - \mu_L \frac{L_{0,t+j}^{1+\phi_L}}{1 + \phi_L} - \mu_{M,t} \frac{Z_{0,t+j}^{1+\phi_M}}{1 + \phi_M} \right)^{1-\sigma} \quad (10)$$

where  $E$  is the unconditional expectations operator. To compute the unconditional compensating variation, we simulate around 800 replications of  $C$ ,  $L$ , and  $Z$  series, and obtain a long-run replication for  $V_{i,t}$ . Then, we compute the expected value of  $V_{i,t}$  by calculating the mean value from its 800 replications. We then find  $\lambda_C^u$  by equalizing the expected values under the two regimes via numerical techniques, as in the conditional compensating variation.

Finally, we consider the unconditional compensating variation for the carbon emissions as well to access how much more  $CO_2$  emissions households are willing to suffer under a particular environmental policy to preserve the same consumption level as before. Thus, we evaluate contemporaneous welfare cost for consumption ( $\lambda_C^u$ ) and carbon emissions ( $\lambda_Z^u$ ) that satisfies the following expression:

$$E\left(V_{1,t}^{C,Z}\right) = \mathbb{E} \sum_{t=0}^{\infty} \beta^{t+j} \frac{1}{1-\sigma} \left( (1 + \lambda_C^u) C_{0,t+j} - \mu_L \frac{L_{0,t+j}^{1+\phi_L}}{1 + \phi_L} - \mu_{M,t} \frac{(1 + \lambda_Z^u) Z_{0,t+j}^{1+\phi_M}}{1 + \phi_M} \right)^{1-\sigma} \quad (11)$$

$V_{1,t}^{C,Z}$  generates a tradeoff between the two welfare costs. Carbon policies can improve welfare by reducing carbon emissions (a positive  $\lambda_Z^u$ ), while it reduces  $\lambda_C^u$ .

Figure 8 describes the conditional compensating variation for the three environmental policies under a constant (left column) and endogenous (right column) capital utilization rate for the five shocks analyzed in the theoretical model. Under all shocks, except monetary policy, the three policies present the same ranking across exogenous and endogenous capital utilization models. However, while tax policy produces a positive, even if smaller, compensating variation under a TFP shock, cap-and-trade policy generates a larger compensating variation under energy price and government spending shocks. These results are consistent with [Fischer and Springborn \(2011\)](#), [Annicchiarico and Di Dio \(2015\)](#) and [Dissou and Karnizova \(2016\)](#) who found that tax policy yields higher steady-state welfare levels. In contrast, monetary policy shocks show a different ranking when comparing the two model versions. While cap-and-trade is a preferred policy in the short- and long-run under a constant capital utilization model, carbon tax and intensity target appear to be better policies in reducing the negative conditional compensating variation under a variable capital utilization rate model. Under a clean air preference shock, the conditional compensating variation is the same across policies and model versions. This result is obvious by looking at the impulse response functions that simulate a very similar pattern under

the three different environmental policies.

Turning on the unconditional compensating variations, Figures 9 and 9 report the values of  $\lambda_C^u$  under the three environmental policies, and compare the results achieved under a model with exogenous and endogenous capital utilization rate, respectively. Both figures report values of  $\lambda_C^u$  for each given value of carbon tax between 1% and 25%. In order to be comparable, the three policies must be aligned on the same capacity and intensity targets such that the steady-state values of carbon emissions are equal under all environmental policies. It can be noticed that the welfare cost is different across the two models. While under a model with exogenous capital utilization rate, carbon tax and cap-and-trade yield similar welfare costs, and are even identical for a carbon tax rate larger than 15%, a carbon tax policy is preferred under a variable depreciation rate because it yields the lowest welfare cost. The compensating variation measures how much consumption households are willing to give up under an environmental policy state in order to have the same welfare in absence of policy. Thus, the negative sign of  $\lambda_C^u$  indicates that welfare under environmental policies is lower relative to a scenario of zero carbon tax. Figures 9a and 9b show that the greater the value of steady-state of carbon tax, the stricter the capacity and intensity targets. Such stringent policy lead to higher welfare cost, meaning that the fraction of consumption that households would need each period in the environmental policy regime to yield the same welfare as would be achieved in the no-policy regime becomes higher. For instance, comparing to a carbon tax of 10% under a model with variable capital utilization described in Figures 9, household would need 0.46% of extra consumption to have the same welfare in absence of policy, against a required compensation of 0.76% and 2.3% that would occur if the environmental regime is specified under a cap-and-trade or an intensity target regime, respectively. Figures 9a and 9b suggest that exogenous and endogenous capital utilization rates would lead to different policy rankings, as the presence of variable capital utilization lets the model generate an amplified impact on the utility function during the transitional dynamics of accumulating a larger carbon emission stock. We also calculate the unconditional compensating variations for a variable depreciation model with low degree of price stickiness. The figure shows that in terms of welfare improvement, nominal rigidities have important welfare implications. Indeed, the absence of price stickiness leads to ranking environmental policies in favor of carbon tax and intensity target regimes. However, for a carbon tax rate larger than 10%, all three environmental policies imply the same unconditional compensating variation. See Figure 9c. Finally, we assess the unconditional compensating variation of both consumption and carbon emissions. Figure 9d compares UCV in absence of carbon taxes versus a case of carbon tax set equal to 20%. First of all, a plot of  $\lambda_C^u$  against  $\lambda_Z^u$  is upward sloping, indicating that  $V_{1,t}^{C,Z}$  is satisfied for values of  $\lambda^u$  with the same sign. A positive  $\lambda_Z^u$  indicates that the carbon policy

improves households' welfare by reducing carbon emissions, while it contemporaneously reduces consumption in the utility function. When a strict carbon tax is implemented, such as an increase up to 20%, the upward sloping curve shifts to the right. The vertical black solid line in Figure 9 indicates how much consumption households are willing to sacrifice in order to be at the same utility level before. The horizontal black dotted line in Figure 9d indicates how much more  $CO_2$  households are willing to suffer under the higher carbon tax rate to preserve the same consumption level as before. The shift to the right for a given higher carbon tax indicates the importance of environmental preferences in the utility function. Lusky (1975) and Brochado et al.(2017) have reported the importance of consumers' environmental awareness, and any analysis that ignores the above preferences will fail to capture the appropriate welfare cost associated to each individual environmental instrument.

## 7 Conclusions

This paper extends the model of Annicchiarico and Di Dio (2015) by considering, as in Finn (1996) and Finn (2000), that capital can be utilized only together with energy. Thus, energy becomes tightly linked to any endogenous fluctuation in the utilization of capital. Our aim is to understand how endogenous fluctuations in capital utilization and energy usage affect economic activities in the presence of different environmental policy regimes. In this sense, we contribute to the classic theory of New Keynesian models in the presence of environmental regulations by explicitly considering energy as an essential factor for the use of capital. Therefore, any fluctuation in energy price would affect the energy consumption, and therefore the use of capital, affecting at the end the production of goods. Relative to standard E-DSGE models *à la* Annicchiarico and Di Dio (2015), we also introduce an environmental quality shock. Specifically, carbon emissions enter the household utility function with a negative sign, indicating that an increase in pollution decreases the happiness of consumers. This specification helps us to give a more meaningful evaluation of the welfare analysis. Carbon tax, cap-and-trade and intensity target have different impacts on carbon emissions, thus affecting the household utility function accordingly. Our main results suggest that the presence of endogenous capital utilization rate amplifies the conventional transmission mechanism of productivity and monetary policy shocks. Moreover, the endogenous capital utilization model version indicates that a carbon tax lower than 25% would lead to the lowest welfare cost, and thus it is preferred to cap-and-trade and intensity target in terms of welfare improvement. In contrast, a constant capital utilization model suggests that cap-and-trade and carbon tax regimes are very similar when calculating their compensating variations. Moreover, we evaluate the welfare cost in terms of consumption and carbon

emissions. Indeed, households dislike  $CO_2$  emissions, and an increase in carbon tax that would reduce carbon emissions represents a gain in terms of welfare, which translates into a shift to the right of a possibility frontier of the additional consumption and carbon emissions that would leave households to be on the same level of utility in absence of environmental policy. The shift translates into a 2% and 15% of extra consumption or extra emission reductions, respectively, that would leave households indifferent between a carbon tax of 20% and zero policy. Finally, we show that price stickiness is an important ingredient to evaluate a meaningful welfare. In fact, in absence of it, results would conduct in favor of alternative policies.

Our results have important policy implications concerning environmental policy. As is well-documented in the literature, policymakers are faced with the challenge of balancing reductions in carbon emissions against the potential negative impacts on economic activity and social welfare.<sup>26</sup> Our study indicates that it is imperative for policymakers to consider the endogenous capital utilization when designing environmental policies. Specifically, we find that implementing a carbon tax at a rate below 25% would result in the lowest welfare cost, thus representing a superior alternative to cap-and-trade and intensity targets in terms of welfare improvement. Furthermore, policymakers must take into account the impact of energy usage and capital utilization when crafting environmental policies, as incentives for renewable energy sources may inadvertently lead to undesired effects on capital use. Finally, it is critical to emphasize the importance of considering the impact of price stickiness when evaluating the welfare implications of environmental policies. The price rigidity could lead to undesired behavioral changes in response to policy interventions. In sum, we highlight the need for policymakers to conduct a comprehensive evaluation of the trade-offs and potential unintended consequences of environmental policies before implementing them. By doing so, we can ensure that the design of these policies yields the intended positive outcomes while minimizing any adverse effects on economic activity and individual and business welfare.

Finally, our model offers various possible extensions. First, environmental quality preferences are introduced in a very simplistic way. A more realistic extension would be to allow households to consume polluting and non-polluting consumption goods, and thus specifies the environmental quality shock as an increase in preferences for non-polluting goods. It would also be very interesting to analyze the impact of taxes on energy prices, and compare them with environmental policies. These model features are left for future research.

---

<sup>26</sup>It has been demonstrated by various empirical studies that a plausible nexus may exist between environmental quality and a range of multifaceted economic and social phenomena including household consumption, social stability, income inequality, natural resource management, private financial system efficacy, social equity, human capital accumulation and the liberalization of the internal energy market (see, for example, [Song et al. \(2022\)](#); [Destek et al. \(2022\)](#); [Ponce et al. \(2020\)](#); [Ahmad et al. \(2022\)](#); [Wang et al. \(2023\)](#); [Ponce et al. \(2023\)](#); [Fraser et al. \(2023\)](#); [Ponce et al. \(2020\)](#))

## References

- Abdul-Salam, Y. (2022). A real options analysis of the effects of oil price uncertainty and carbon taxes on the optimal timing of oil field decommissioning. *The Energy Journal* 43(6).
- Ahmad, F., M. U. Draz, A. A. Chandio, M. Ahmad, L. Su, F. Shahzad, and M. Jia (2022). Natural resources and environmental quality: Exploring the regional variations among chinese provinces with a novel approach. *Resources Policy* 77, 102745.
- Albrizio, S., T. Koźluk, and V. Zipperer (2014). Empirical evidence on the effects of environmental policy stringency on productivity growth.
- Angelopoulos, K., G. Economides, and A. Philippopoulos (2010). What is the best environmental policy? taxes, permits and rules under economic and environmental uncertainty.
- Angelopoulos, K., G. Economides, and A. Philippopoulos (2013). First-and second-best allocations under economic and environmental uncertainty. *International Tax and Public Finance* 20(3), 360–380.
- Annicchiarico, B. and F. Di Dio (2015). Environmental policy and macroeconomic dynamics in a new keynesian model. *Journal of Environmental Economics and Management* 69, 1–21.
- Annicchiarico, B. and F. Di Dio (2017). Ghg emissions control and monetary policy. *Environmental and Resource Economics* 67(4), 823–851.
- Bloom, N. (2014). Fluctuations in uncertainty. *Journal of Economic Perspectives* 28(2), 153–76.
- Bongers, A. (2020). Energy efficiency, emission energy, and the environment. *Energy Research Letters* 1(2).
- Boppart, T. and P. Krusell (2020). Labor supply in the past, present, and future: a balanced-growth perspective. *Journal of Political Economy* 128(1), 118–157.
- Brochado, A., N. Teiga, and F. Oliveira-Brochado (2017). The ecological conscious consumer behaviour: are the activists different? *International Journal of Consumer Studies* 41(2), 138–146.
- Busato, F., B. Chiarini, G. Cisco, and M. Ferrara (2022). Green preferences. *Environment, Development and Sustainability*, 1–43.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics* 12(3), 383–398.

- Canova, F., D. Lopez-Salido, and C. Michelacci (2010). The effects of technology shocks on hours and output: A robustness analysis. *Journal of applied Econometrics* 25(5), 755–773.
- Chan, Y. T. (2019). The environmental impacts and optimal environmental policies of macroeconomic uncertainty shocks: A dynamic model approach. *Sustainability* 11(18), 4993.
- Chan, Y. T. (2020a). Are macroeconomic policies better in curbing air pollution than environmental policies? a dsge approach with carbon-dependent fiscal and monetary policies. *Energy Policy* 141, 111454.
- Chan, Y. T. (2020b). Optimal emissions tax rates under habit formation and social comparisons. *Energy Policy* 146, 111809.
- Cho, J.-O., T. F. Cooley, and H. S. E. Kim (2015). Business cycle uncertainty and economic welfare. *Review of Economic Dynamics* 18(2), 185–200.
- Collard, F. and M. Juillard (2001). Accuracy of stochastic perturbation methods: The case of asset pricing models. *Journal of Economic Dynamics and Control* 25(6-7), 979–999.
- Dave, C. and S. J. Dressler (2010). Technology shocks, capital utilization and sticky prices. *Journal of Economic Dynamics and Control* 34(10), 2179–2191.
- De Santis, R. and C. Jona Lasinio (2016). Environmental policies, innovation and productivity in the eu. *Global Economy Journal* 16(4), 615–635.
- Dechezleprêtre, A., D. Nachtigall, and B. Stadler (2020). The effect of energy prices and environmental policy stringency on manufacturing employment in oecd countries: Sector-and firm-level evidence.
- Delis, M. D. and M. Iosifidi (2020). Environmentally aware households. *Economic Modelling* 88, 263–279.
- Destek, M. A., M. Manga, O. Cengiz, and G. Destek (2022). Investigating the potential of renewable energy in establishing global peace: Fresh evidence from top energy consumer countries. *Renewable Energy* 197, 170–177.
- Dissou, Y. and L. Karnizova (2016). Emissions cap or emissions tax? a multi-sector business cycle analysis. *Journal of Environmental Economics and Management* 79, 169–188.
- Dlugosch, D. and T. Kozluk (2017). Energy prices, environmental policies and investment: Evidence from listed firms.

- Dmitriev, A. and I. Roberts (2012). Wealth effects and countercyclical net exports. *Applied Economics Letters* 19(17), 1673–1677.
- Fernández-Villaverde, J., J. F. Rubio-Ramírez, and F. Schorfheide (2016). Solution and estimation methods for dsge models. In *Handbook of macroeconomics*, Volume 2, pp. 527–724. Elsevier.
- Finn, M. G. (1995). Variance properties of solow’s productivity residual and their cyclical implications. *Journal of Economic Dynamics and Control* 19(5-7), 1249–1281.
- Finn, M. G. (1996). A theory of the capacity utilization/inflation relationship. *FRB Richmond Economic Quarterly* 82(3), 67–86.
- Finn, M. G. (2000). Perfect competition and the effects of energy price increases on economic activity. *Journal of Money, Credit and banking*, 400–416.
- Fischer, C. and M. Springborn (2011). Emissions targets and the real business cycle: Intensity targets versus caps or taxes. *Journal of Environmental Economics and Management* 62(3), 352–366.
- Fisher, J. D. (2006). The dynamic effects of neutral and investment-specific technology shocks. *Journal of political Economy* 114(3), 413–451.
- Fraser, T., A. J. Chapman, and Y. Shigetomi (2023). Leapfrogging or lagging? drivers of social equity from renewable energy transitions globally. *Energy Research & Social Science* 98, 103006.
- Furlanetto, F. and M. Seneca (2014). Investment shocks and consumption. *European Economic Review* 66, 111–126.
- Galí, J. (2008). Introduction to monetary policy, inflation, and the business cycle: An introduction to the new keynesian framework. In *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.
- Galí, J. (2008). Monetary policy, inflation, and the business cycle: an introduction to the new keynesian framework.
- Gray, W. B. (2015). Environmental regulations and business decisions. *IZA World of Labor*.
- Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988). Investment, capacity utilization, and the real business cycle. *The American Economic Review*, 402–417.



- Grodecka, A. and K. Kuralbayeva (2014). The price vs quantity debate: Climate policy and the role of business cycles.
- Heutel, G. (2012). How should environmental policy respond to business cycles? optimal policy under persistent productivity shocks. *Review of Economic Dynamics* 15(2), 244–264.
- Jaffe, A. B. and K. Palmer (1997). Environmental regulation and innovation: a panel data study. *Review of economics and statistics* 79(4), 610–619.
- Jaimovich, N. and S. Rebelo (2009). Can news about the future drive the business cycle? *American Economic Review* 99(4), 1097–1118.
- Khan, H., K. Metaxoglou, C. R. Knittel, and M. Papineau (2019). Carbon emissions and business cycles. *Journal of Macroeconomics* 60, 1–19.
- Khan, H. and J. Tsoukalas (2012). The quantitative importance of news shocks in estimated dsge models. *Journal of Money, Credit and Banking* 44(8), 1535–1561.
- Kim, J., S. Kim, E. Schaumburg, and C. A. Sims (2008). Calculating and using second-order accurate solutions of discrete time dynamic equilibrium models. *Journal of Economic Dynamics and Control* 32(11), 3397–3414.
- Kormilitsina, A. (2011). Oil price shocks and the optimality of monetary policy. *Review of Economic Dynamics* 14(1), 199–223.
- Lester, R., M. Pries, and E. Sims (2014). Volatility and welfare. *Journal of Economic Dynamics and Control* 38, 17–36.
- Lintunen, J. and L. Vilmi (2021). Optimal emission prices over the business cycles. *Environmental and Resource Economics* 80(1), 135–167.
- Lusky, R. (1975). Consumers’ preferences and ecological consciousness. *International Economic Review*, 188–200.
- Metcalf, G. E. (2009). Market-based policy options to control us greenhouse gas emissions. *Journal of Economic perspectives* 23(2), 5–27.
- Nordhaus, W. D. and Z. Yang (1996). A regional dynamic general-equilibrium model of alternative climate-change strategies. *The American Economic Review*, 741–765.
- Persaud, A. (2022). A comprehensive, integrated, climate finance framework for the earth.

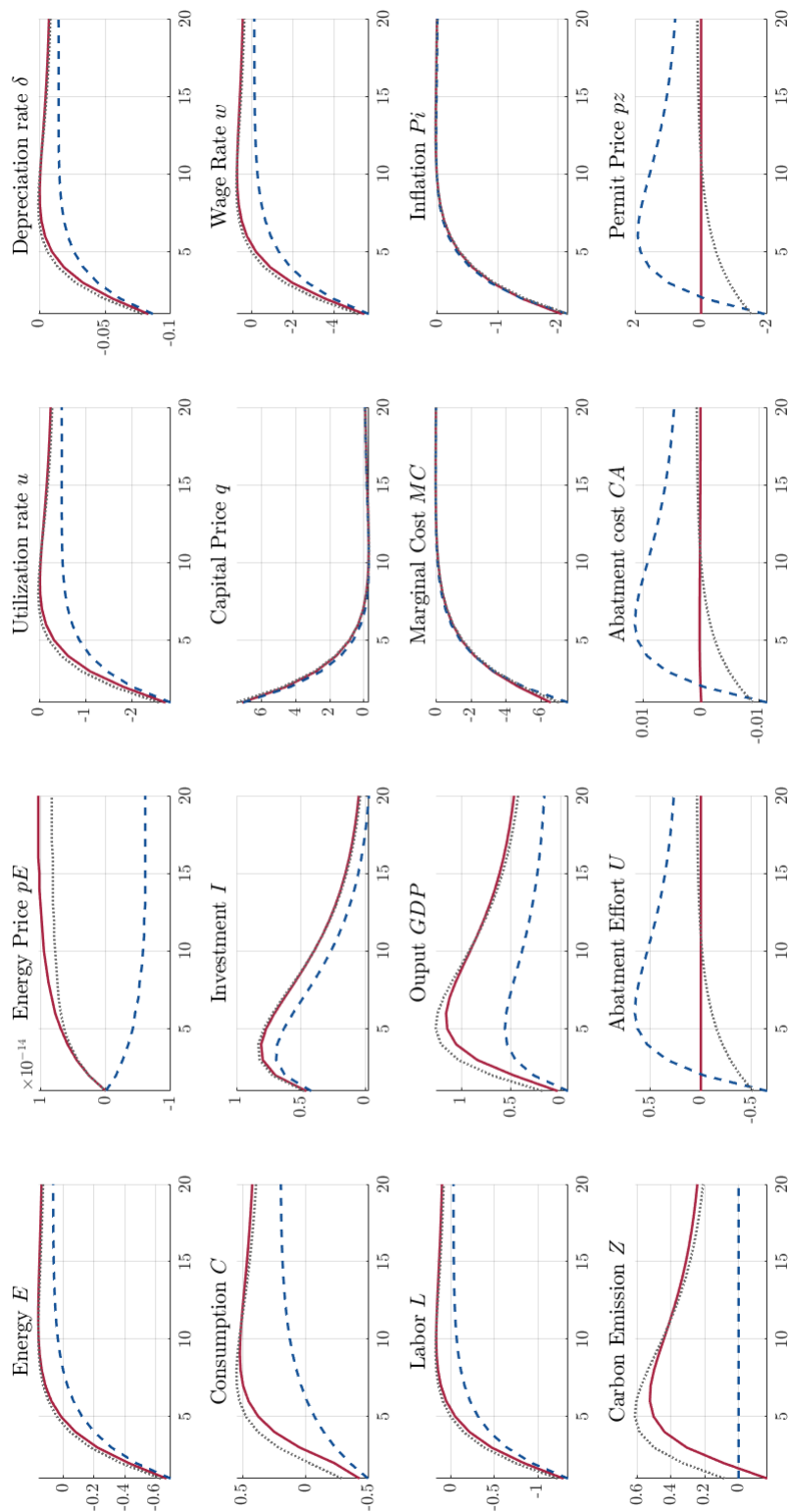
- Ponce, P., J. Álvarez-García, V. Álvarez, and M. Irfan (2023). Analysing the influence of foreign direct investment and urbanization on the development of private financial system and its ecological footprint. *Environmental Science and Pollution Research* 30(4), 9624–9641.
- Ponce, P., M. López-Sánchez, P. Guerrero-Riofrío, and J. Flores-Chamba (2020). Determinants of renewable and non-renewable energy consumption in hydroelectric countries. *Environmental Science and Pollution Research* 27, 29554–29566.
- Ponce, P., C. Oliveira, V. Álvarez, and M. d. I. C. del Río-Rama (2020). The liberalization of the internal energy market in the european union: evidence of its influence on reducing environmental pollution. *Energies* 13(22), 6116.
- Popp, D. (2006). International innovation and diffusion of air pollution control technologies: the effects of nox and so2 regulation in the us, japan, and germany. *Journal of Environmental Economics and Management* 51(1), 46–71.
- Rasche, R. H. and J. A. Tatom (1981). Energy price shocks, aggregate supply and monetary policy: The theory and the international evidence. In *Carnegie-Rochester Conference Series on Public Policy*, Volume 14, pp. 9–93. Elsevier.
- Rotemberg, J. J. and M. Woodford (1996). Imperfect competition and the effects of energy price increases on economic activity.
- Schmitt-Grohé, S. and M. Uribe (2004). Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of economic dynamics and control* 28(4), 755–775.
- Shi, M., Y. Yuan, S. Zhou, and N. Li (2013). Carbon tax, cap-and-trade or mixed policy: Which is better for carbon mitigation. *Journal of management sciences in China* 16(9), 9–19.
- Song, K., G. Baiocchi, K. Feng, K. Hubacek, and L. Sun (2022). Unequal household carbon footprints in the peak-and-decline pattern of us greenhouse gas emissions. *Journal of Cleaner Production* 368, 132650.
- Swanson, E., G. Anderson, A. Levin, et al. (2005). Higher-order perturbation solutions to dynamic, discrete-time rational expectations models: Methods and an application to optimal monetary policy. Technical report, Society for Computational Economics.
- Wang, Q., T. Yang, and R. Li (2023). Does income inequality reshape the environmental kuznets curve (ekc) hypothesis? a nonlinear panel data analysis. *Environmental Research* 216, 114575.

- Wolf, M., J. Emerson, D. Esty, A. de Sherbinin, Z. Wendling, et al. (2022). Environmental performance index. *Yale Center for Environmental Law & Policy: New Haven, CT, USA*.
- Xiao, B., Y. Fan, and X. Guo (2021). Dynamic interactive effect and co-design of so2 emission tax and co2 emission trading scheme. *Energy Policy* 152, 112212.
- Xiao, B., Y. Fan, X. Guo, S. Voigt, and L. Cui (2022). Effects of linking national carbon markets on international macroeconomics: An open-economy e-dsge model. *Computers & Industrial Engineering* 169, 108166.
- Xu, Q., B. Fu, and B. Wang (2022). The effects of oil price uncertainty on china's economy. *Energy Economics* 107, 105840.
- Xu, S. and W. Zhang (2016). Analysis of impacts of carbon taxes on china's economy and emissions reduction under different refunds: Based on dynamic cge model. *China Population, Resources and Environment* 26, 12–46.
- Yang, M., Y. Fan, F. Yang, and H. Hu (2014). Regional disparities in carbon dioxide reduction from china's uniform carbon tax: a perspective on interfactor/interfuel substitution. *Energy* 74, 131–139.
- Zhao, L., C. Yang, B. Su, and S. Zeng (2020). Research on a single policy or policy mix in carbon emissions reduction. *Journal of Cleaner Production* 267, 122030.
- Zhu, Y., X. Liu, and Z. Wang (2010). Abatement effect of carbon tax and its impacts on economy in china. *China Soft Sci* 4, 1–9.

Table 1: The calibrated and estimated parameter values used for numerical analysis.

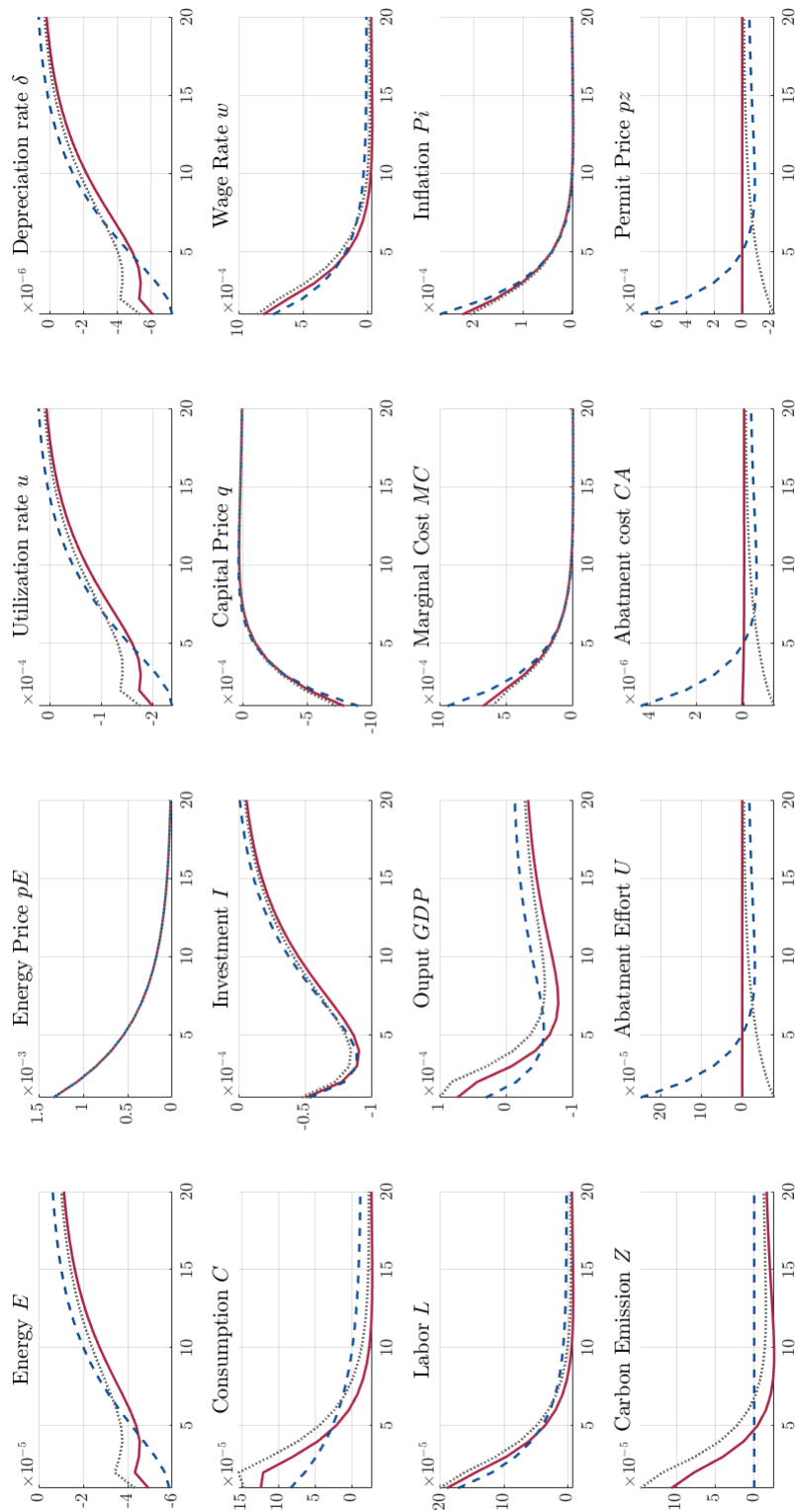
Calibrated parameters	Values	Description
$\sigma_C$	2	Risk aversion parameter
$\alpha$	1/3	Share of capital in production
$\beta$	0.99	Discount factor
$\mu_L$	1	Parameter of labor disutility
$\mu_M$	0.4	Parameter of environmental stock disutility
$\delta_M$	0.0021	Depreciation rate of emission stock
$\varphi$	0.45	Marginal emission of production
$\gamma_I$	2.77	Parameter of capital adjustment cost
$\phi$	1	Inverse of Frisch elasticity
$\phi_M$	1	Inverse of Frisch elasticity
$\theta$	6	Elasticity of substitution within non-polluted and polluted goods sectors
$\nu$	0.75	Parameter of Calvo pricing adjustment
$\phi_1$	0.1761	Parameter of abatement cost
$\phi_2$	2.8	Parameter of abatement cost
$\gamma_0$	$1.395e^{-3}$	Parameter of damage function
$\gamma_1$	$-6.6722e^{-6}$	Parameter of damage function
$\gamma_2$	$1.4647e^{-8}$	Parameter of damage function
$Z^*$	1.3299	Foreign emission level
$\iota_\pi$	3	Parameter of inflation gap
$\iota_Y$	1/4	Parameter of output gap
$p^E$	1	Steady-state of energy price
$A$	1	Steady-state of technology level
$\eta$	1	Steady-state of monetary policy shock
Estimated Parameters	Values	Description
$\varphi_E$	0.267	Scale of carbon emissions to energy
$\rho_A$	0.785	Persistence of technology shock
$\rho_G$	0.879	Persistence of government expenditure shock
$\rho_\eta$	0.953	Persistence of monetary policy shock
$\rho_E$	0.481	Persistence of energy price shock
$\rho_M$	0.478	Persistence of environmental preference shock
$\sigma_A$	0.033	Standard deviation of technology shock
$\sigma_G$	0.039	Standard deviation of government expenditure shock
$\sigma_\eta$	0.002	Standard deviation of monetary policy shock
$\sigma_E$	0.001	Standard deviation of energy price shock
$\sigma_M$	0.003	Standard deviation of environmental preference shock
$\sigma_Y^{err}$	0.039	Standard deviation of output measurement error
$\sigma_R^{err}$	0.00033	Standard deviation of interest rate measurement error
$\sigma_Z^{err}$	0.063	Standard deviation of carbon emissions measurement error

Figure 1: Dynamic responses of endogenous variables to one-standard-deviation positive TFP shock.



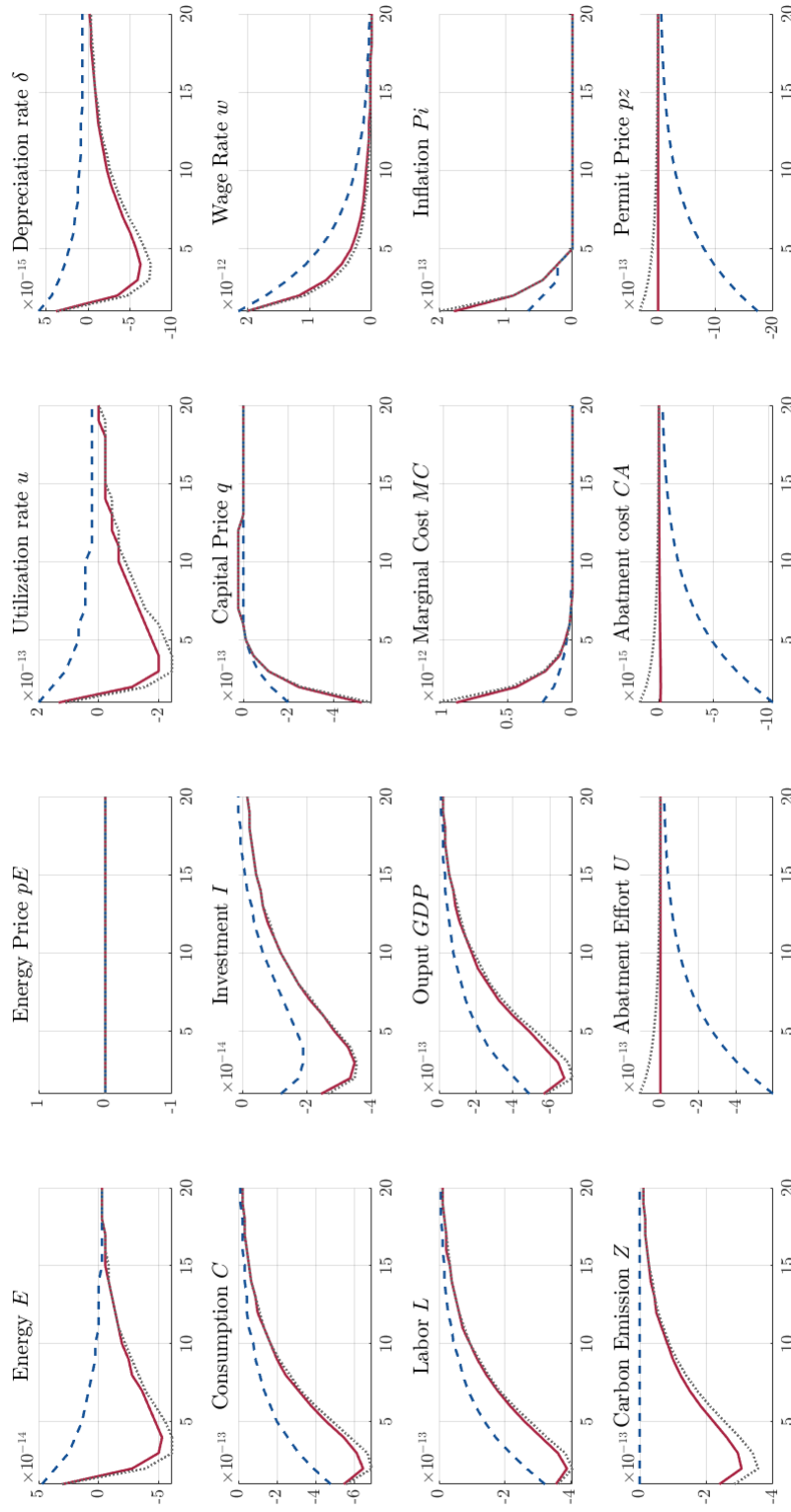
Note: The red solid, blue dashed, and black dotted lines represent the model with carbon tax, cap-and-trade, and intensity target, respectively.

Figure 2: Dynamic responses of endogenous variables to one-standard-deviation positive Energy price shock.



Note: The red solid, blue dashed, and black dotted lines represent the model with carbon tax, cap-and-trade, and intensity target, respectively.

Figure 3: Dynamic responses of endogenous variables to one-standard-deviation positive Environmental Preference shock.



Note: The red solid, blue dashed, and black dotted lines represent the model with carbon tax, cap-and-trade, and intensity target, respectively.

Figure 4: Positive government spending shock.

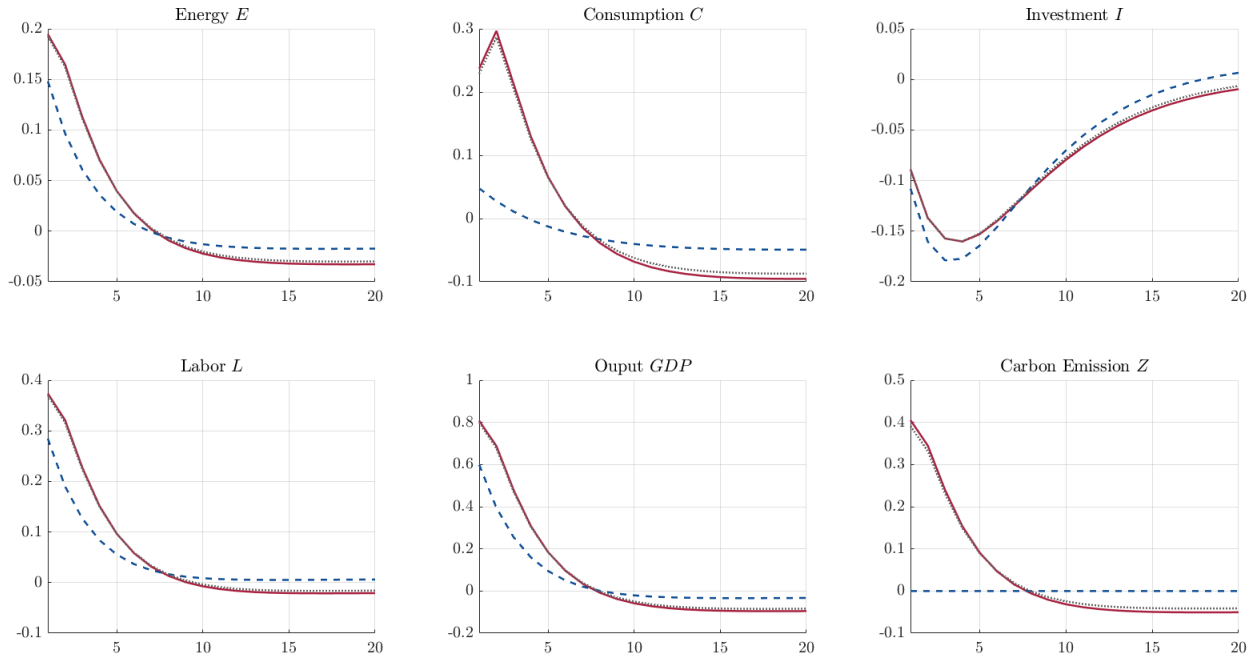
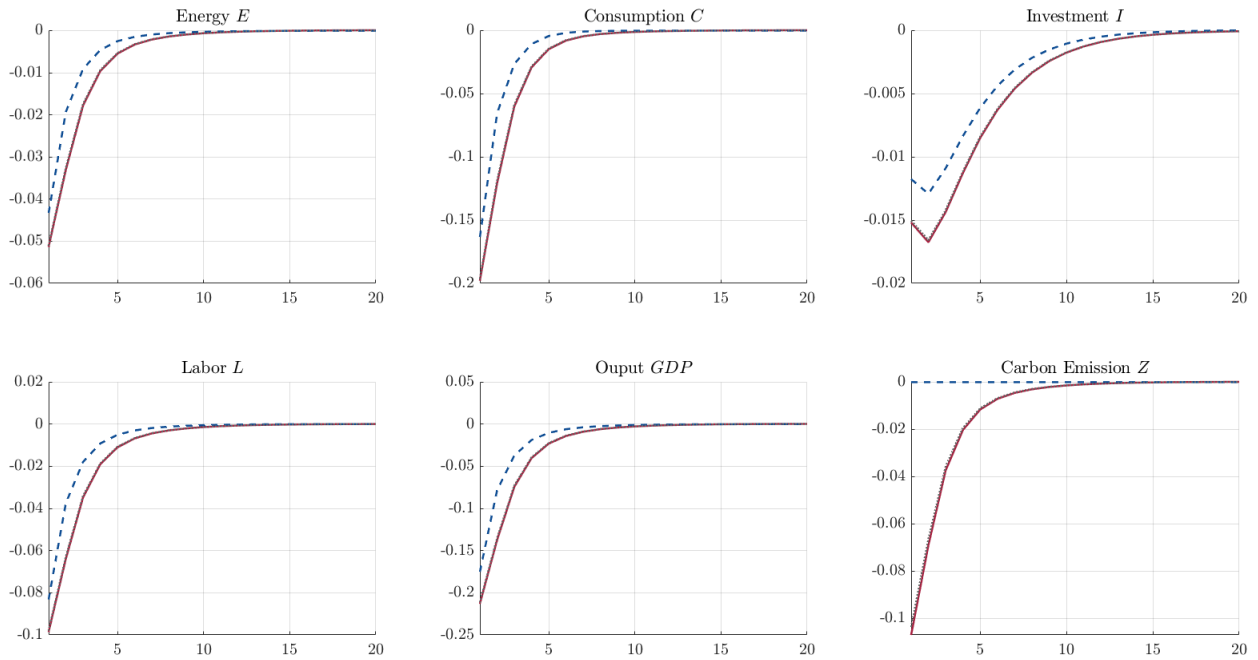


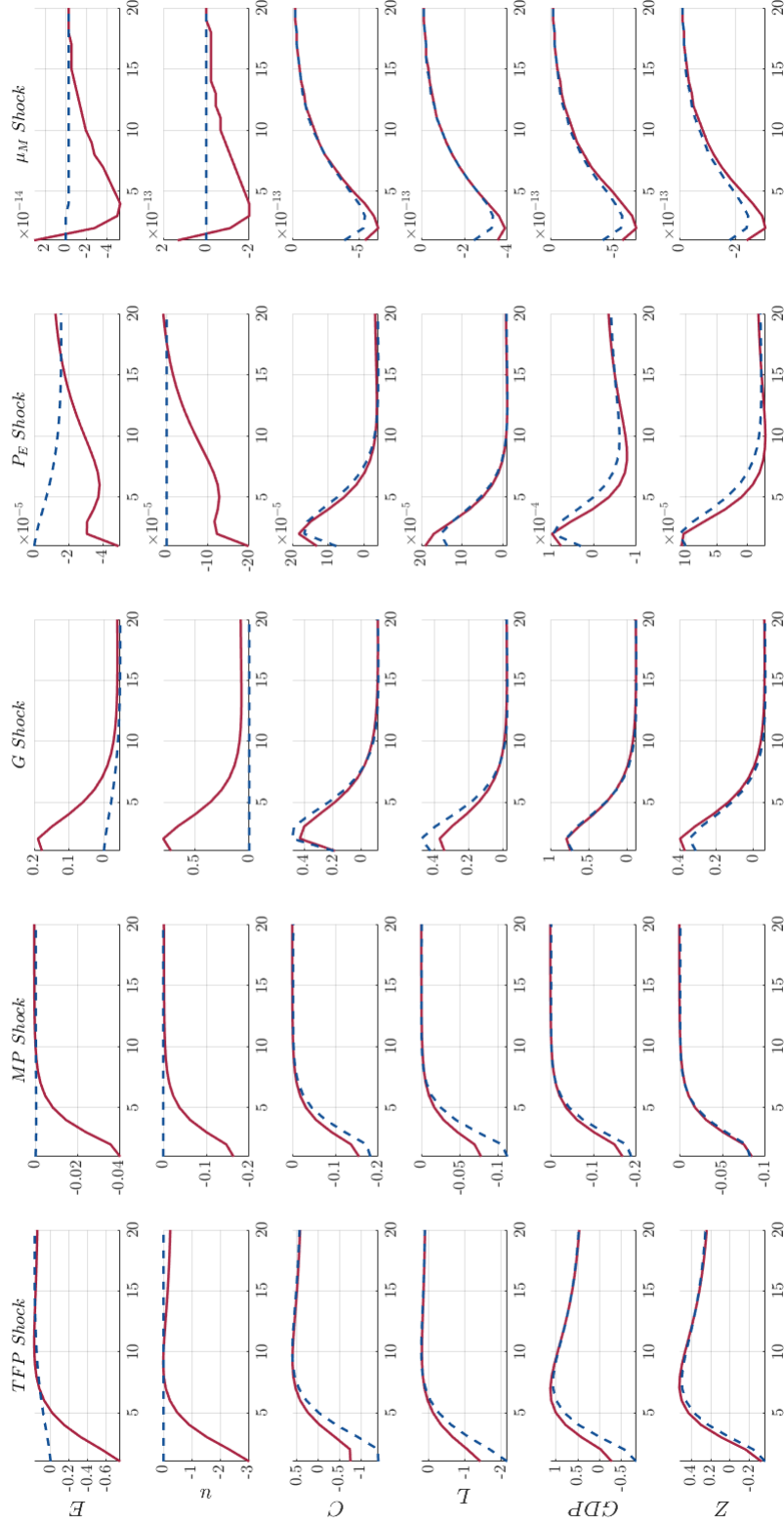
Figure 5: Positive monetary policy shock.



*Note:* The red solid, blue dashed, and black dotted lines represent the model with carbon tax, cap-and-trade, and intensity target, respectively. The top Panel describes a positive government spending shock. The bottom Panel describes a positive monetary policy shock.

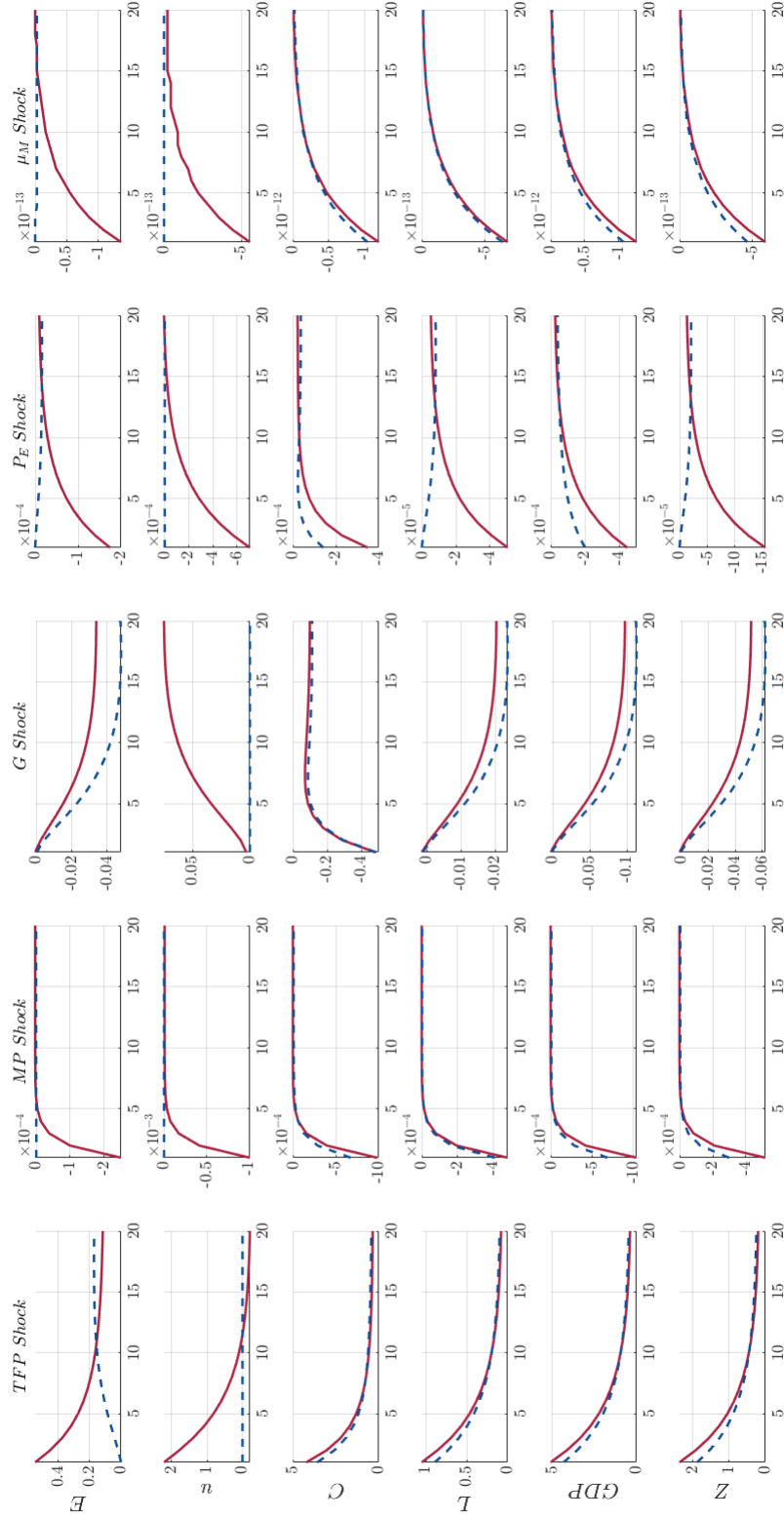


Figure 6: Endogenous versus exogenous capital utilization rate



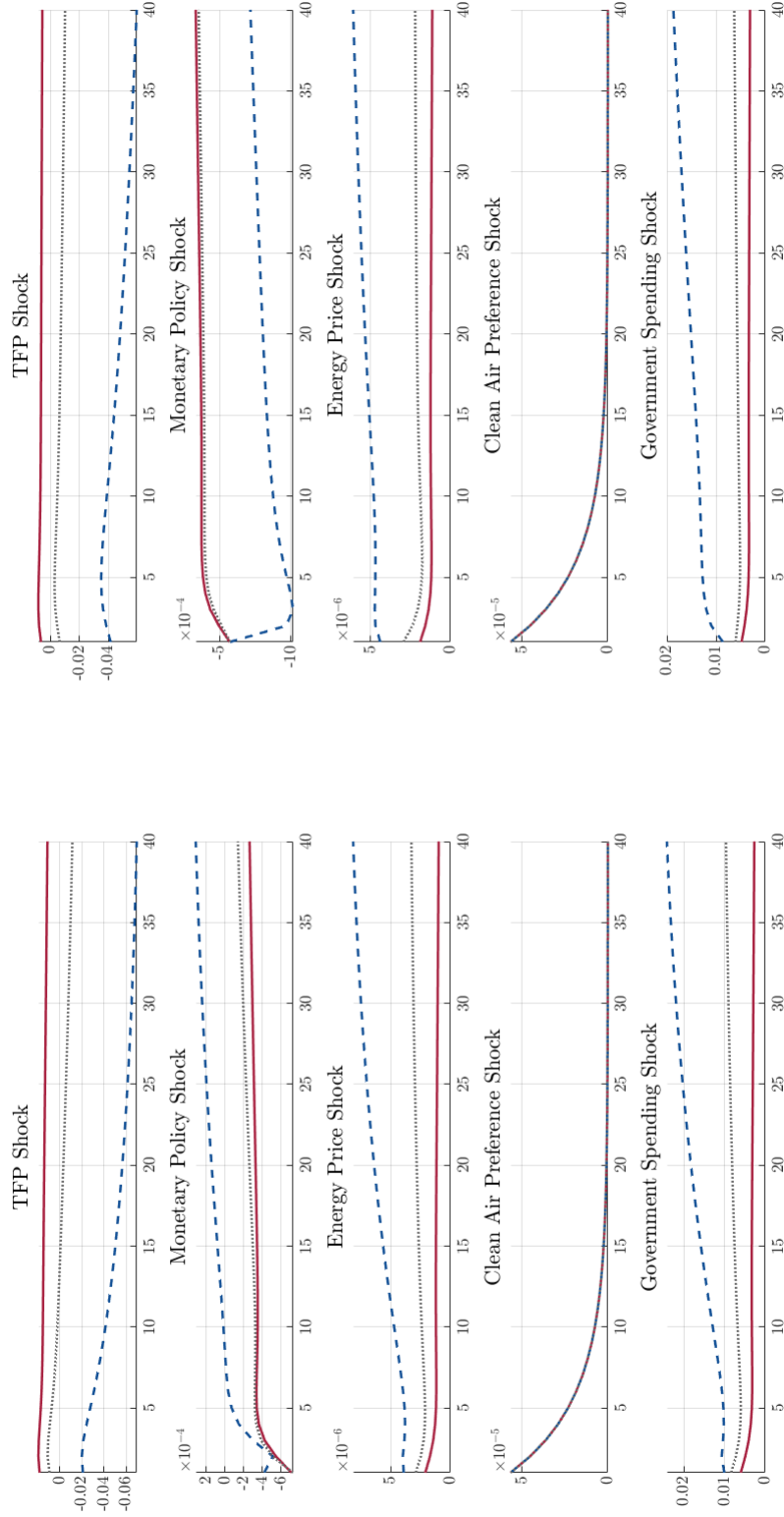
*Note:* The solid red and blue dotted lines represent a model with endogenous capital utilization rate and a model with constant capital utilization rate under a carbon tax environmental policy, respectively.

Figure 7: Endogenous versus exogenous capital utilization rate (Flexible Prices)



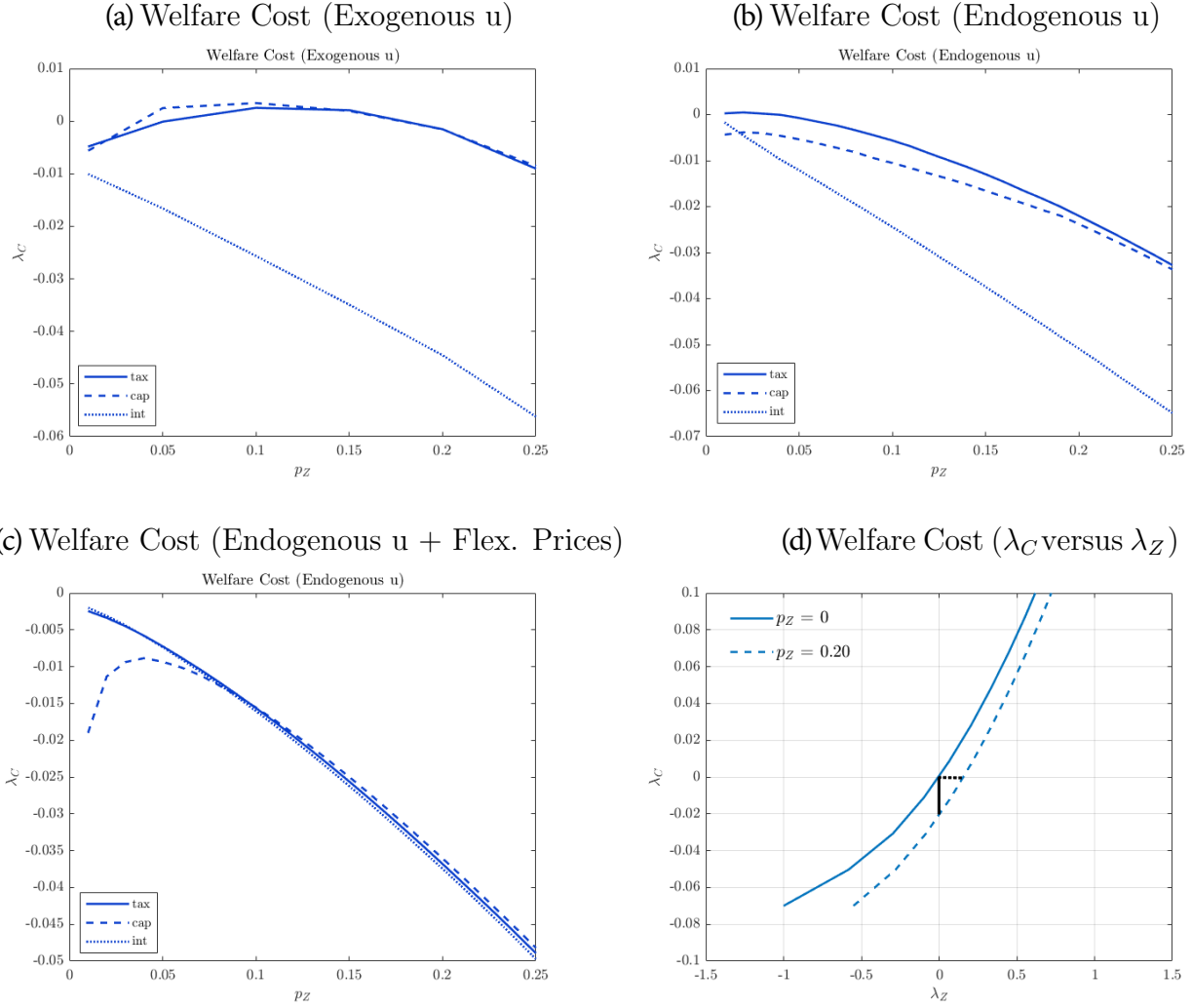
*Note:* The solid red and blue dotted lines represent a model with endogenous capital utilization rate and a model with constant capital utilization rate under a carbon tax environmental policy, respectively.

Figure 8: Conditional compensating variation



*Note:* The red solid, blue dashed, and black dotted lines represent the model with carbon tax, cap-and-trade, and intensity target, respectively. The left and right Panels describe the conditional compensating variation for a model with exogenous and endogenous capital utilization rate  $u$ , respectively.

Figure 9: Unconditional compensating variation.



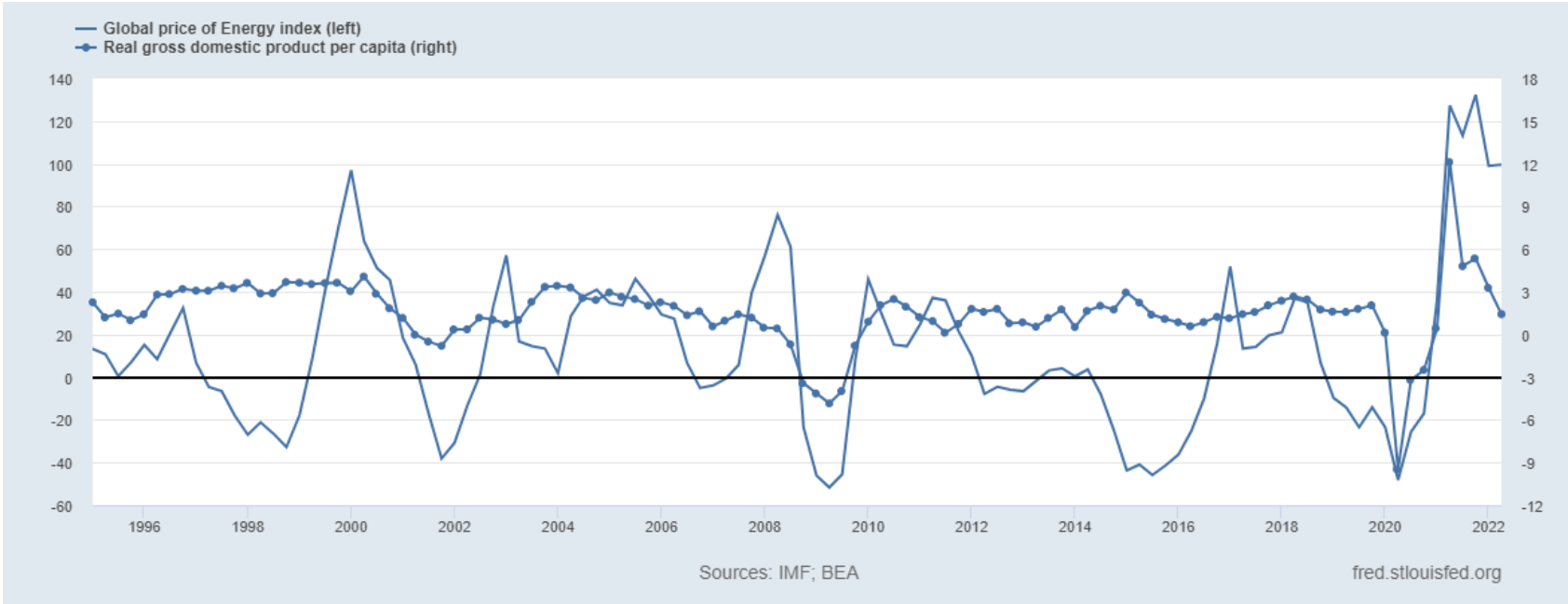
*Notes:* The blue solid, blue dashed, and black dotted lines represent the model with carbon tax, cap-and-trade, and intensity target, respectively. Panel(a) refers to a model with constant capital utilization rate. Panel (b) refers to a model with endogenous capital utilization rate. Panel (c) refers to a model with low degree of price rigidities and with endogenous capital utilization rate. Panel (d) reports the welfare cost of consumption against the welfare cost in terms of carbon emissions under an endogenous capital utilization rate model.

# Online Appendix (Not for publication)

## A Figures

Figure 10 describes the trend of global energy price (left axes) and the real GDP per capita (right axes) in the U.S.

Figure 10: Global Energy Price and Real GDP per capita.



## B DSGE Model

This section provides a description of our E-DSGE model. In the model, households make the saving and consumption decisions. Firms use capital to produce. The final good market is assumed to be perfectly competitive. Our model differs from the standard E-DSGE model in a variety of aspects: (i) households acquire capital and determine the utilization rate simultaneously; (ii) environmental quality enters directly into the utility function of households; (iii) carbon emissions are generated partially by the production of goods, and partially by the consumption of energy. Environmental regulations are introduced through three different carbon policies: (i) carbon tax (i.e., a levy for every unit of carbon emissions emitted); (ii) cap-and-trade (i.e., exogenous limit on carbon emissions allowances); (iii) intensity targets (i.e., an exogenous limit on carbon emissions per unit of total output).

### B.1 Households

Assume that in an economy there is a continuum of identical households, indexed by  $i \in [0, 1]$ . The representative households maximize the discounted lifetime utility:

$$V_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t, Z_t) \quad (12)$$

where  $\beta \in [0, 1]$  is a discount factor. Further, we assume that the instantaneous utility function  $U$  is a GHH preference (Greenwood et al., 1988) as:

$$U(C_t, L_t, M_t) = \frac{1}{1 - \sigma} \left( C_t - \mu_L \frac{L_t^{1+\phi_L}}{1 + \phi_L} - \mu_{M,t} \frac{Z_{t-1}^{1+\phi_M}}{1 + \phi_M} \right)^{1-\sigma} \quad (13)$$

where  $C_t$ ,  $L_t$  and  $Z_t$  are households' consumption, labor supply and carbon emissions in period  $t$ , respectively.  $\mu_L > 0$  controls the scale of labor disutility and  $\phi_L > 0$  is the inverse of Frisch elasticity. One novelty of the utility function is the inclusion of the carbon emissions  $Z_t$ . It is assumed that households' utility are negatively influenced by the emissions flows, indicating that a decrease in them implies lower carbon emissions stock  $M_t$ , thus better environmental quality.<sup>27</sup> The scale elasticity is controlled by the parameter  $\phi_M > 0$ . We label movements in  $\mu_{M,t} > 0$  as environmental quality preference shocks, which express households' attachment to environmental quality. A similar specification can be found in Angelopoulos et al. (2013) and Delis and Iosifidi (2020) who use the stock of environmental quality as a proxy for environmental awareness in the

---

<sup>27</sup>Gray (2015) discusses the benefits of environmental regulations as a reduced illness and death, and/or better recreational water quality.

households' utility function.<sup>28</sup> The choice of non-separability between consumption and labor in the GHH preferences derives from the advantage of eliminating the wealth effect on labor supply, therefore households only care about smoothing consumption. Greenwood et al. (1988) and Jaimovich and Rebelo (2009) argue that by neutralizing the wealth effect, GHH preferences produce more volatile labor hours and help generate the co-movement between consumption and leisure conditional on non-productivity shocks.<sup>29</sup> Further, in order to generate an impact on the marginal utility from a shock to environmental quality preferences, GHH preferences represent a good choice when including carbon emissions in the utility function. Carbon emissions enter the utility function with a lag, as households' happiness increases with lower emissions from the previous period. Further, unlike consumption and labor supply, each household is infinitesimally small so that its decision has no direct impact on the emissions stock.

Households maximise utility function (12) subject to the following budget constraint:

$$P_t C_t + P_t I_t + Q_t^B B_t + P_t^E E_t = B_{t-1} + W_t L_t + P_t D_t + R_{K,t} K_t u_t - T_t \quad (14)$$

Households save by buying one-period riskless bond  $B_t$ , whose price is  $Q_t^B$  and invest  $I_t$  in capital.  $K_t$  is the amount of capital owned by the households in period  $t$ .  $u_t$  denotes the capital utilization rate. Since capital is owned by households, an increase in the capital utilization rate would increase the energy use  $E_t$ .  $P_t^E$  is the energy price. We follow Finn (2000) to assume that:

$$E_t = a(u_t) K_t \quad (15)$$

where  $a(u_t) = \nu_0 u_t^{\nu_1} / \nu_1$  for some parameters  $\nu_0 > 0$  and  $\nu_1 > 1$ . As explained in Finn (2000), the assumption that  $\nu_1 > 1$  leads to the percentage increase in energy-to-capital ratio  $E_t/K_t$  is greater than that of  $u_t$ , deterring households from increasing the  $u_t$  rapidly when facing shocks.<sup>30</sup>

In addition,  $W_t$  and  $R_{K,t}$  are, respectively, the nominal wage rate and the (nominal) rate of capital return. Furthermore, households can obtain dividend  $D_t$  from their ownership to the intermediate goods firms.  $T_t$  is the lump-sum tax levied by the government. It is assumed that households' environmental preference is time-varying around a steady-state value  $\mu_M$ . In particular, we assume that  $\mu_{M,t}$  follows an AR(1) process as:

---

<sup>28</sup>Angelopoulos et al. (2013) and Delis and Iosifidi (2020) show that the evolution of the stock of environmental quality increase with lower current pollution flow.

<sup>29</sup>See also Dmitriev and Roberts (2012), Furlanetto and Seneca (2014), Lester et al. (2014), Boppart and Krusell (2020) for more examples of using GHH preferences.

<sup>30</sup>For simplicity, we do not assume households consume energy directly and thus  $E_t$  does not enter the utility function. Here,  $E_t$  only represents the energy used due to the "excessive" use of capital. It is easy to generalize our model by including the energy directly used for consumption.

$$\ln \left( \frac{\mu_{M,t}}{\mu_M} \right) = \rho_M \ln \left( \frac{\mu_{M,t-1}}{\mu_M} \right) + \sigma_M \varepsilon_{M,t} \quad (16)$$

where  $\rho_M \in [0, 1]$  is the persistence of the shock process, and  $\sigma_M > 0$  is the standard deviation of the white noise  $\varepsilon_{M,t}$  that follows standard normal distribution.

Moreover, we assume that the capital evolves as follows:

$$K_{t+1} = (1 - \delta_K(u_t))K_t + \left( 1 - \frac{\gamma_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t \quad (17)$$

where the quadratic term on the right-hand-side (RHS) represents the investment adjustment cost (Jaimovich and Rebelo, 2009).  $\gamma_I > 0$  is a scale parameter. It is assumed that the investment cost is proportional to the percentage change in investment from the last period. The convex assumption of the investment cost also incentivizes households to split their investment into several periods. Further, we assume that the depreciation rate  $\delta_K(u_t)$  is increasing and convex to the utilization rate  $u_t$ , such as:

$$\delta_K(u_t) = \frac{\omega_0 u_t^{\omega_1}}{\omega_1} \quad (18)$$

where  $\omega_0 > 0$  and  $\omega_1 > 1$ .

In sum, the representative household maximizes the discounted lifetime utility function (12) by choosing  $C_t$ ,  $L_t$ ,  $I_t$ ,  $B_t$ ,  $K_{t+1}$ , and  $u_t$ . The first-order conditions for  $C_t$ ,  $B_t$ ,  $L_t$ ,  $I_t$ ,  $K_{t+1}$ , and  $u_t$ , are, respectively,

$$\lambda_t = U_{C,t} \quad (19)$$

$$R_t^{-1} = Q_t^B = \beta \mathbb{E}_t \frac{1}{\Pi_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \quad (20)$$

$$-U_{L,t} = U_{C,t} w_t \quad (21)$$

$$1 = q_t \left[ 1 - \frac{\gamma_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \gamma_I \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \gamma_I \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \quad (22)$$

$$q_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} (r_{K,t+1} u_{t+1} + (1 - \delta_K(u_t)) q_{t+1} - p_{t+1}^E a(u_{t+1})) \quad (23)$$



$$\delta'_K(u_t) + p_t^E a'(u_t) = r_{K,t} \quad (24)$$

where  $U_{C,t} \equiv (C_t - \mu_L L_t^{1+\phi_L} / (1 + \phi_L) - \mu_M M_t^{1+\phi_M} / (1 + \phi_M))^{-\sigma}$ ,  $U_{L,t} \equiv -\mu_L L^{\phi_L} U_{C,t}$ .

$q_t$  is the real capital price. Eq. (19) equates the marginal utility of consumption to the Lagrange multiplier  $\lambda_t$ . Eq. (20) is the Euler equation describing the household indifference between consuming one more unit today or in the future by acquiring bonds at price  $Q_t^B$ . Eq. (21) describes the intratemporal optimality condition between consumption and labor, and thus defines the labor supply equation. Eq. (22) defines the equilibrium price of capital as the sum of its current return and the return of selling undepreciated capital in the next period. Eq. (23) is the Euler equation for capital, which implies that the price of capital must be equal to the sum of capital's marginal return  $r_{K,t} u_t$  and the undepreciated component  $1 - \delta_K(u_t)$  minus the capital's marginal energy cost  $p_t^E a(u_t)$ . Last equation, (24), implies that the marginal depreciation and energy costs of utilization sum to equal the marginal return to utilization rate.

The price of energy,  $p_t^E$ , follows an AR(1) process as:

$$\ln \left( \frac{p_t^E}{p^E} \right) = \rho_E \ln \left( \frac{p_{t-1}^E}{p^E} \right) + \varepsilon_{E,t}$$

where  $\rho_E \in [0, 1]$  is a persistence parameter.  $\varepsilon_{E,t}$  is a white noise that follows a normal distribution with mean 0 and standard deviation  $\sigma_E > 0$ .

## B.2 Firms

Assume that the final goods market is composed by a continuum of intermediate goods. Let  $Y_t(i)$  be the production of intermediate goods  $i$  in period  $t$ . The final goods and intermediate goods are related by constant elasticity of substitution (CES) aggregator:

$$Y_t = \left( \int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \quad (25)$$

where  $\theta > 0$  is the elasticity of substitution between any two intermediate goods. Let  $P_t(i)$  be the price of intermediate goods. From the above equation, we have the demand function for intermediate goods  $i$  to be:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} Y_t \quad (26)$$

Assume that production requires both labor and capital. The production function of inter-

mediate good  $i$  is assumed to be Cobb Douglas as follows:

$$Y_t(i) = A_t(1 - \Upsilon(M_t))(u_t(i)K_t(i))^\alpha L_t(i)^{1-\alpha} \quad (27)$$

where  $\alpha \in [0, 1]$  is the capital share of output. We follow [Annicchiarico and Di Dio \(2015\)](#) and [Heutel \(2012\)](#) to introduce the damage function  $\Upsilon(M_t)$  to the production function, where  $M_t$  is the carbon emissions stock in period  $t$ . This function also captured the percentage of output reduction due to the carbon emissions stock. Moreover,  $A_t$  is a total productivity level which follows an AR(1) process:

$$\ln\left(\frac{A_t}{A}\right) = \rho_A \ln\left(\frac{A_{t-1}}{A}\right) + \varepsilon_{A,t} \quad (28)$$

where  $\rho_A \in [0, 1]$  is a persistent parameter, and  $\varepsilon_{A,t}$  is a white noise that is normally distributed with mean zero and standard deviation  $\sigma_A > 0$ .

We assume that carbon emissions depend positively on output and energy consumption as follows:

$$Z_t(i) = \varphi_E E_t + \varphi(1 - U_t(i))Y_t(i) \quad (29)$$

where  $\varphi_E$  and  $\varphi > 0$  are scale parameters, and  $U_t(i)$  is the abatement effort exerted by firm  $i$ . Hence, both firms and households can reduce carbon emissions by choosing  $u_t$  and  $U_t$ , respectively.

To abate emissions, firms have to pay the following abatement cost:

$$C_t(i) = \phi_1 U_t(i)^{\phi_2} Y_t(i) \quad (30)$$

which is assumed to be convex in  $U_t(i)$ : the elasticity parameter  $\phi_2$  is assumed to be greater than one. In other words, firms face an increasing marginal abatement cost, and therefore, have incentive to split their abatement efforts into several periods.

Here, we begin by assuming that the government levies a carbon emission tax to the firms. We will also consider other environmental policies, such as intensity targeting and cap-and-trade policies below.

Let  $P_{Z,t}$  be the carbon emission tax rate. The firms choose  $K_t(i)$ ,  $L_t(i)$ , and  $U_t(i)$ , to maximize the following instantaneous profit:

$$\pi_t(i) = \max_{K_t(i), L_t(i)} P_t(i)Y_t(i) - W_t L_t(i) - R_{K,t} K_t(i) - C_t(i) - P_{Z,t} Z_t(i) \quad (31)$$

and subject to constraints (26), (27), (28), (29), and (30). The first-order-conditions for

$K_t(i)$ ,  $L_t(i)$ , and  $U_t(i)$ , are, respectively:

$$(1 - \alpha) \frac{Y_t(i)}{L_t(i)} \Psi_t(i) = w_t \quad (32)$$

$$\alpha \frac{Y_t(i)}{K_t(i)} \Psi_t(i) = r_{K,t} \quad (33)$$

$$\varphi p_{Z,t} = \varphi \frac{P_{Z,t}}{P_t} = \phi_1 \phi_2 U_t(i)^{\phi_2 - 1} \quad (34)$$

which simply states that in equilibrium, firms would choose their capital and labor such that the marginal product of labor equals the real wage rate  $w_t$ , and the marginal product of capital equals the rate of capital return  $r_{K,t}$ . Further, Eq. (34) equation states that the marginal abatement cost (RHS) should equal the additional cost saved by the firm by reducing one unit of carbon emission (LHS).

Firms' real marginal cost is:

$$MC_t = \Psi_t + \phi_1 U_t(i)^{\phi_2} + p_{Z,t}(1 - U_t(i))\varphi \quad (35)$$

where the second and third terms on RHS are, respectively, firms' marginal abatement cost and firms' tax payment per unit of production. The first term is the marginal cost incurred by labor and capital, and can be found by combining Eqs. (32) and (33):

$$\Psi_t = \frac{1}{\alpha^\alpha (1 - \alpha)^{1 - \alpha} A_t} w_t^{1 - \alpha} r_{K,t}^\alpha$$

### B.3 Nominal Rigidity

We assume that the intermediate goods market is monopolistic competitive, hence each firm has market power to manipulate its price level. We follow [Annicchiarico and Di Dio \(2015\)](#) to introduce nominal rigidity à la [Calvo \(1983\)](#). Therefore, in each period, there are  $\nu$  portion of the intermediate goods firms that are not allowed to adjust their price levels. The values of  $\nu$  thus determine the degrees of nominal rigidity of the economy. If  $\nu$  equals one, then no firm can adjust its price. On the other hand, if  $\nu$  equals zero, price is perfectly flexible.

Taking into account the possibilities of inability to adjust price in the future, firm  $i$  chooses its price level  $P_t^*(i)$  to maximize the discount profit as follows:

$$\max_{P_t^*(i)} \mathbb{E}_t \sum_{k=0}^{\infty} \nu^k Q_{t,t+k} [P_t^*(i) Y_{t+k}(i) - MC_t Y_{t+k}(i)] \quad (36)$$

and subject to the demand function:

$$Y_{t+k}(i) = \left( \frac{P_t^*(i)}{P_{t+k}} \right)^{-\theta} Y_{t+k} \quad (37)$$

where  $Q_{t,t+k} = \beta^k \left( \frac{\lambda_{t+k}}{\lambda_t} \right)$  is a stochastic discount factor. Note that in this equation, the values  $\nu$  appears in Eq. (36), implying that firms have considered the possibilities of nominal rigidity when setting its price.

The first-order condition for  $P_t^*(i)$  is:

$$p_t^* \equiv \frac{P_t^* \theta - 1}{P_t} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \nu^k Q_{t,t+k} MC_{t+k} \left( \frac{P_{t+k}}{P_t} \right)^\theta Y_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \nu^k Q_{t,t+k} \left( \frac{P_{t+k}}{P_t} \right)^\theta Y_{t+k}} \quad (38)$$

The index  $i$  is removed in the above by symmetry. Note that in the extreme when price is perfectly flexible, Eq. (38) reduces to the familiar equation  $P_t^* = \left( \frac{\theta-1}{\theta} \right) MC_t$ .

## B.4 Monetary and fiscal policies

As in [Annicchiarico and Di Dio \(2015\)](#), both fiscal and monetary policy are considered in the model. For monetary policy, we assume that the nominal interest rate  $R_t$  is adjusted according to the Taylor rule as follows:

$$R_t = R \left( \frac{\Pi_t}{\Pi} \right)^{\iota_\pi} \left( \frac{Y_t}{Y} \right)^{\iota_Y} \eta_t \quad (39)$$

with the elasticity parameter  $\iota_\pi > 0$  and  $\iota_Y > 0$ .  $\Pi$  and  $Y$  are the steady-state values of  $\Pi_t$  and  $Y_t$ , respectively. Eq. (39) implies that the central bank would increase the nominal interest rate when either the gross inflation rate  $\Pi_t$  or output  $Y_t$  are above their steady-state levels. Further,  $\eta_t$  represents the monetary policy shock: an increase in  $\eta_t$  implies that the central bank would choose a nominal interest rate above the one suggested by the Taylor rule. Assume that  $\eta_t$  follows an AR(1) process as:

$$\ln \eta_t = \rho_\eta \ln \eta_{t-1} + \varepsilon_{\eta,t} \quad (40)$$

which has a persistent parameter  $\rho_\eta \in [0, 1]$  and a white noise  $\varepsilon_{\eta,t}$ . Assume that  $\varepsilon_{\eta,t}$  is normally distributed with mean zero and standard deviation  $\sigma_\eta > 0$ .

For the fiscal policy, we assume that the government expenditure  $G_t$  also follows an AR(1) process as follows:

$$\ln \left( \frac{G_t}{G} \right) = \rho_G \ln \left( \frac{G_{t-1}}{G} \right) + \varepsilon_{G,t} \quad (41)$$

where  $\rho_G$  and  $\varepsilon_{G,t}$  are again persistent parameter and white noise of the shock, respectively. The white noise is normally distributed with mean zero and standard deviation  $\sigma_G$ .

In addition, we assume that government has to balance its budget every period:

$$T_t + p_{Z,t}Z_t = G_t \quad (42)$$

so that a short-term increase in government expenditure has to be financed by either the lump-sum income tax or the environmental taxation.

## B.5 Aggregation and equilibrium

First of all, in equilibrium, the aggregate price level can be computed as  $P_t = (\int_0^1 P_t(i)^{1-\theta} di)^{1/(1-\theta)}$ . Since there are only two types of firms, there are only  $1 - \nu$  portion of them can adjust their prices, and the other  $\nu$  portion of them have to stick with their price in the last period. Therefore, we have:

$$P_t = [\nu P_{t-1}^{1-\theta} + (1 - \nu)P_t^{*1-\theta}]^{1/(1-\theta)} \quad (43)$$

which can be further simplified to as:  $1 = [\nu \Pi_t^{\theta-1} + (1 - \nu)p_t^{*1-\theta}]^{1/(1-\theta)}$ .  $\Pi_t$  is the gross inflation rate and  $p_t^* = P_t^*/P_t$

Let  $K_t \equiv \int_0^1 K_t(i)dt$  and  $L_t \equiv \int_0^1 L_t(i)dt$  be the aggregate capital and labour, respectively. From the production function (27), we have  $\int_0^1 Y_t(i)dt = \int_0^1 (P_t(i)/P_t)^{-\theta} di = D_{p,t}Y_t$ , where  $D_{p,t}$  is referred to as the price dispersion in the intermediate goods market.

Then, by Eqs. (21) and (23), we have  $D_{p,t}Y_t = \int_0^1 Y_t(i)di = r_{K,t}K_t/(\Psi_t\alpha)$  and  $D_{p,t}Y_t = \int_0^1 Y_t(i)di = w_tL_t/(\Psi_t(1 - \alpha))$ . Moreover, according to the production function (27), we have  $Y_t = A_t(1 - \Upsilon(M_t))K_t^\alpha L_t^{1-\alpha} D_{p,t}^{-1}$ . This equation implies that output is negatively affected by the price dispersion. By Eq. (43), One can show that  $D_{p,t}$  evolve according to  $D_{p,t} = (1 - \nu)p_t^{*-\theta} + \nu \Pi_t^\theta D_{p,t-1}$ .

Since the carbon tax rate is the same to all firms, by Eq. (34), firms would exert the same abatement efforts. Therefore, we have  $U_t(i) = U_t$  for any  $i$ . To close the final goods market, We have the following GDP accounting equation.

$$Y_t = C_t + I_t + G_t + \phi_1 U_t^{\phi_2} Y_t D_{p,t} + \frac{\gamma_I}{2} \left( \frac{I_t}{I_{t-1}} - \delta_K \right)^2 I_t + p_t^E E_t \quad (44)$$

where the RHS involves the standard terms as in [Annicchiarico and Di Dio \(2015\)](#): consumption  $C_t$ , investment  $I_t$ , government expenditure  $G_t$ , abatement cost  $\phi_1 U_t^{\phi_2} Y_t D_{p,t}$ , and investment adjustment cost  $\gamma_I (I_t/I_{t-1} - \delta_K)^2 I_t/2$ . In addition to [Annicchiarico and Di Dio \(2015\)](#), we have an additional term  $p_t^E E_t$  on the RHS, which is the energy expenditure due to the high capital usage.

Finally, the carbon emission stock would evolve over time according to

$$M_t = (1 - \delta_M)M_{t-1} + Z_t + Z_t^* \quad (45)$$

where  $\delta_M \in [0, 1]$  is a depreciation rate of the emissions stock.  $Z_t^*$  is the carbon emissions from the rest of the world.

## C Parameter choice

Before performing numerical analysis, we estimate the parameters by fitting our model to the U.S. data using Bayesian estimation. Our strategy is to separate the set of parameters into two types: we only estimate one set of the parameters; and we used a standard calibration technique to obtain the values of another set of parameters. The model is assumed to be in quarterly frequency. In particular, for the structural parameters that are in common in the model of [Annicchiarico and Di Dio \(2015\)](#) and other papers, we choose their values as in these papers. We focus on estimating the parameters that are new to our model and the parameters involved in the shock process.

### C.1 Calibration

For the parameters relate to the households problem, we set the risk aversion parameter  $\sigma_C$  to 2, as in [Galí \(2008\)](#). Moreover, the inverse of Frisch elasticity  $\phi$  is set to 1, as is common in the literature. Households' discount factor  $\beta$  is set to 0.99, which is equivalent to a 1% quarterly discount rate. The parameters  $\mu_L$  and  $\mu_M$  capture the relative disutility of labor supply and environmental pollution. We normalize  $\mu_L$  to 1 and set the environment's weight in utility function,  $\mu_M$ , equal to 0.4 as in [Angelopoulos et al. \(2013\)](#) and [Delis and Iosifidi \(2020\)](#) Further, the elasticity of environmental quality  $\phi_M$  is also set to 1. Following [Khan and Tsoukalas \(2012\)](#),  $\gamma_I$  that controls the scale of the investment adjustment cost, is set to be 2.77.

For the parameters in the production sector, the capital share of production  $\alpha$  is set to be  $1/3$ , which is common in the literature. In addition, the elasticity of substitution  $\theta$  between the intermediate goods is assumed to be 6, which implies that firms enjoy a markup of 20%

$((6-5)/5 = 1.2)$  if price is perfectly flexible. We calibrate  $\nu_0$  and  $\omega_0$  such that  $\{u, \delta_{K0}\} = \{1, 0.02\}$  in the steady state. That is, we set:

$$\omega_0 = \delta_{K0}\omega_1 \quad (46)$$

$$\nu_0 = \frac{1}{p_E} \left( \frac{1}{\beta} - (1 - \delta + \delta\omega_1) \right) \quad (47)$$

where  $\delta_{K0}$  is the depreciation rate when the capital utilization rate is constant, and is set equal to 0.025.  $p_E$  is the energy price in the steady state. Note that the magnitude of  $\nu_0$  can be increased by reducing the value of  $p_E$ , hence, we set  $p_E = 0.01$  so that similar calibrated values for  $\omega_0$  and  $\nu_0$ . With  $\omega_1 = 2$  and  $\nu_1 = 1.66$ , we have  $\omega_0 = 0.0408$  and  $\nu_0 = 0.04$ .

For the damage function, we follow [Annicchiarico and Di Dio \(2015\)](#) and [Heutel \(2012\)](#) to assume that  $\Gamma(M) = \gamma_0 + \gamma_1 M + \gamma_2 M^2$ , with  $\{\gamma_0, \gamma_1, \gamma_2\} = \{1.395e^{-3}, -6.6722e^{-6}, 1.4647e^{-8}\}$ . Further, the depreciation rate of the emissions stock  $\delta_M$  is set to be 0.0021 as in [Heutel \(2012\)](#). We calibrate the carbon emissions level  $Z^*$  from the rest of the world so that the steady-state carbon emissions stock  $M$  equals 800, as the atmospheric carbon mass is 800 gigatons in 2005.

Moreover, we calibrate the parameters in the abatement costs function according to the procedure in [Annicchiarico and Di Dio \(2015\)](#). First, we set  $\phi_2$  to be 2.8. Then, we increase carbon tax rate  $p_Z$  from 0. We calibrate  $\phi_1$  such that as the steady-state value of carbon emissions level  $Z$  is reduced by 20%, the abatement cost to output ratio  $C_A/Y$  is increased to 0.15%. Further, we follow [Annicchiarico and Di Dio \(2015\)](#) to set the scale  $\varphi$  of the emissions function (29) to 0.45.

Concerning the parameters related to the macroeconomic policies, we set  $\iota_\pi$  and  $\iota_Y$  in the Taylor rule to be 3 and 1/4, respectively, as is common in the literature. Concerning the shock processes, the steady state values of  $A$ ,  $\eta$ , and  $p^E$  are normalized to one. The steady state values of  $G$  is calibrated so that the government expenditure to GDP ratio  $G/Y$  in the steady state equals 0.223, which is the data from 1973Q1 to 2019Q4. Matching the aforementioned three targets simultaneously, it is estimated that  $\{Z^*, G, \phi_1\} = \{1.305, 8.03, 25.0432\}$ . Table 1, top Panel, reports the values for all calibrated parameters.

## C.2 Estimation

We first assume that the carbon tax rate  $p_Z = 0$  as a benchmark for the model to be estimated. We select five time series data for structural estimation. They are real GDP per capita, real consumption per capita, real investment per capita, nominal interest rate, and carbon emissions

level in the U.S. from 1973Q1 to 2019Q4.<sup>31</sup> The former four data series are obtained from the federal bank of Saint Louis. The carbon emission data is obtained from the U.S. Energy Information Administration. The raw carbon emission data is in monthly frequency and has been converted to quarterly frequency before the estimation. All the data series are detrended and are stationary, ensured by the ADF test. The summary statistics of these variables are reported in Table 2.

To link the data to our model, define  $Y_t^{obs}$ ,  $C_t^{obs}$ ,  $I_t^{obs}$ ,  $R_t^{obs}$ , and  $Z_t^{obs}$  to be the observed GDP, consumption, investment, nominal interest rate, and carbon emissions, respectively. We have:

$$\begin{bmatrix} \ln Y_t^{obs} - \ln Y_{t-1}^{obs} \\ \ln C_t^{obs} - \ln C_{t-1}^{obs} \\ \ln I_t^{obs} - \ln I_{t-1}^{obs} \end{bmatrix} = \begin{bmatrix} \ln Y_t - \ln Y_{t-1} \\ \ln C_t - \ln C_{t-1} \\ \ln I_t - \ln I_{t-1} \end{bmatrix} + \ln Y_t^{err}, \quad (48)$$

$$\ln R_t^{obs} = \ln R_t + \ln R_t^{err}, \quad (49)$$

$$\ln Z_t^{obs} - \ln Z_{t-1}^{obs} = \ln Z_t - \ln Z_{t-1} + \ln Z_t^{err}. \quad (50)$$

where  $Y_t^{err}$  is the common measurement error for  $Y_t^{obs}$ ,  $C_t^{obs}$ , and  $I_t^{obs}$ .  $R_t^{err}$  and  $Z_t^{err}$  are the measurement error for  $R_t^{obs}$  and  $Z_t^{obs}$ , respectively.

The measurement errors are assumed to follow AR(1) processes as:

$$\ln Y_t^{err} - \ln Y^{tr} = \varepsilon_{Y,t}^{err}, \quad (51)$$

$$\ln R_t^{err} - \ln R^{tr} = \varepsilon_{R,t}^{err}, \quad (52)$$

$$\ln Z_t^{err} - \ln Z^{tr} = \varepsilon_{Z,t}^{err}, \quad (53)$$

where  $\varepsilon_{Y,t}^{err}$ ,  $\varepsilon_{R,t}^{err}$ , and  $\varepsilon_{Z,t}^{err}$  are the white noises that are normally distributed with mean zero and standard deviation  $\sigma_T^{err}$ ,  $\sigma_R^{err}$ , and  $\sigma_Z^{err}$ , respectively. Table 3 reports the prior and the posterior distributions for the estimated parameters, and Table 1, bottom Panel, reports the values for all estimated parameters.

---

<sup>31</sup>The data starting from 2020 onwards is omitted to avoid the effect of the pandemic, which is not accounted for in this model.



Table 2: Summary statistics of the data used for estimation.

Name	Variables	Obs.	Mean	Max.	Min.	Std. dev.	Skewness
Real GDP per capita	$Y_t^{obs}$	188	13.764	18.463	9.132	2.913	-0.098
Real consumption per capita	$C_t^{obs}$	188	9.122	12.739	5.815	2.194	-0.012
Real investment per capita	$I_t^{obs}$	188	2.125	3.407	0.950	0.700	0.151
Nominal interest rate	$R_t^{obs}$	188	0.0461%	0.151%	0.0001%	0.0348%	0.588
Carbon emissions	$Z_t^{obs}$	188	2.799	4.569	1.777	0.639	0.409

*Notes:* The data are quarterly from 1973Q1 to 2019Q4. Real GDP is measured in billion per 1 million people at constant 2012 US dollars. Carbon emissions are in gigatons.

Table 3: The prior and the posterior distribution for the estimated parameters.

Parameter name	Parameter	Prior dist	Prior distribution		Posterior distribution	
			Mean	SD	Mode	SD
Scale of carbon emissions to energy	$\varphi_E$	normal	0.5	1	0.267	0.943
Persistence of TFP shock	$\rho_A$	beta	0.5	0.1	0.785	0.045
Persistence of government shock	$\rho_G$	beta	0.5	0.1	0.879	0.025
Persistence of monetary shock	$\rho_\eta$	beta	0.5	0.1	0.953	0.000
Persistence of energy price shock	$\rho_E$	beta	0.5	0.1	0.481	0.067
Persistence of environmental preference shock	$\rho_M$	beta	0.5	0.1	0.478	0.058
SD of TFP shock	$\sigma_A$	invg2	0.005	0.02	0.033	0.001
SD of government shock	$\sigma_G$	invg2	0.005	0.02	0.039	0.002
SD of monetary shock	$\sigma_\eta$	invg2	0.005	0.02	0.002	0.000
SD of energy price shock	$\sigma_E$	invg2	0.005	0.02	0.001	0.009
SD of environmental preference shock	$\sigma_M$	invg2	0.005	0.02	0.003	0.007
SD of output measurement error	$\sigma_Y^{err}$	invg2	0.005	0.02	0.039	0.001
SD of interest rate measurement error	$\sigma_R^{err}$	invg2	0.005	0.02	0.00033	0.00003
SD of carbon emissions measurement error	$\sigma_Z^{err}$	invg2	0.005	0.02	0.063	0.001

## D List of equations

### Households optimal conditions

$$\lambda_t = U_{C,t} \quad (54)$$

$$R_t^{-1} = Q_t^B = \beta \mathbb{E}_t \frac{1}{\Pi_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \quad (55)$$

$$-U_{L,t} = U_{C,t} w_t \quad (56)$$

$$1 = q_t \left[ 1 - \frac{\gamma_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \gamma_I \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \gamma_I \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \quad (57)$$

$$q_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} (r_{K,t+1} + (1 - \delta_K(u_t)) q_{t+1} - p_{t+1}^E a(u_{t+1})) \quad (58)$$

### New equations

$$\delta'_K(u_t) + p_t^E a'(u_t) = r_{K,t} \quad (59)$$

$$E_t = a(u_t) K_t \quad (60)$$

$$\delta_K(u_t) = \frac{\omega_0 u_t^{\omega_1}}{\omega_1} \quad (61)$$

$$\ln \left( \frac{p_t^E}{p^E} \right) = \rho_E \ln \left( \frac{p_{t-1}^E}{p^E} \right) + \sigma_E \varepsilon_{E,t} \quad (62)$$

$$\ln \left( \frac{\mu_{M,t}}{\mu_M} \right) = \rho_M \ln \left( \frac{\mu_{M,t-1}}{\mu_M} \right) + \sigma_M \varepsilon_{M,t} \quad (63)$$

### Aggregation

$$Y_t = C_t + I_t + G_t + \phi_1 U_t^{\phi_2} Y_t D_{p,t} + \frac{\gamma_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 I_t + p_t^E E_t \quad (64)$$

### Capital accumulation

$$K_{t+1} = (1 - \delta_K(u_t))K_t + \left(1 - \frac{\gamma_I}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t \quad (65)$$

### Firms' optimal conditions

$$Y_t = (1 - \Upsilon(M_t))A_t(u_t K_t)^\alpha L_t^{1-\alpha} (D_{p,t})^{-1} \quad (66)$$

$$(1 - \alpha)(1 - \Upsilon(M_t))A_t(u_t K_t)^\alpha L_t^{-\alpha} \Psi_t = w_t \quad (67)$$

$$\alpha(1 - \Upsilon(M_t))A_t(u_t K_t)^{\alpha-1} L_t^{1-\alpha} \Psi_t = r_{k,t} \quad (68)$$

$$\varphi p_{Z,t} = \phi_1 \phi_2 U_t^{\phi_2 - 1}$$

### Emissions Stock, level and abatement cost

$$Z_t = (1 - U_t)\varphi Y_t + \varphi_E E_t \quad (69)$$

$$M_t = (1 - \delta_M)M_{t-1} + Z_t + Z_t^* \quad (70)$$

$$C_{A,t} = \phi_1 U_t^{\phi_2} Y_t \quad (71)$$

### Calvo pricing

$$X_t = \lambda_t \Psi_t Y_t + \nu \beta \mathbb{E}_t \delta_{t+1} \Pi_{t+1}^\theta X_{t+1} \quad (72)$$

$$\Theta_t = \lambda_t Y_t + \nu \beta \mathbb{E}_t \delta_{t+1} \Pi_{t+1}^{\theta-1} \Theta_{t+1} \quad (73)$$

$$\Omega_t = \lambda_t \left[ \phi_1 U_t^{\phi_2} + p_Z (1 - U_t) \varphi \right] Y_t + \nu \beta \mathbb{E}_t \Pi_{t+1}^\theta \Omega_{t+1} \quad (74)$$

$$1 = \nu \Pi_t^{\theta-1} + (1 - \nu) (p_t^*)^{1-\theta} \quad (75)$$

$$p_t^* = \frac{\theta}{\theta - 1} \frac{X_t + \Omega_t}{\Theta_t} \quad (76)$$

$$D_{p,t} = (1 - \nu)p_t^{*-\theta} + \nu\Pi_t^\theta D_{p,t-1} \quad (77)$$

$$MC_t = \Psi_t + \phi_1 U_t^{\phi_2} + p_{Z,t}(1 - U_t)\varphi \quad (78)$$

### Policy Rules

$$T_t + p_{Z,t}Z_t = G_t \quad (79)$$

$$R_t = R \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\iota_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\iota_Y} \eta_t \quad (80)$$

### Shocks

$$\ln \left( \frac{A_t}{\bar{A}} \right) = \rho_A \ln \left( \frac{A_{t-1}}{\bar{A}} \right) + \sigma_A \varepsilon_{A,t} \quad (81)$$

$$\ln \left( \frac{G_t}{\bar{G}} \right) = \rho_G \ln \left( \frac{G_{t-1}}{\bar{G}} \right) + \sigma_G \varepsilon_{G,t} \quad (82)$$

$$\ln \eta_t = \rho_\eta \ln \eta_{t-1} + \sigma_\eta \varepsilon_{\eta,t} \quad (83)$$