Green transition and financial stability: The role of green monetary and macroprudential policies and vouchers[★]

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Abstract

This paper analyzes a mix of alternative policies in supporting the green transition and the phase-out of fossil fuels, without compromising financial stability. An environmental dynamic stochastic general equilibrium (E-DSGE) model with two sectors (green and brown) and endogenous default is developed to assess potential climate-induced financial stability threats that can be mainly generated through physical and transition risks mechanism. Those risks are evaluated through a compound capital depreciation shock and a carbon tax shock. The paper offers several findings. First of all, a too stringent carbon tax would increase the medium-term default rate in both sectors, harming financial stability due to potentially detrimental effects on banks' balance sheet. Second, there exists a trade-off in implementing green monetary or macroprudential policies: if a policy can encourage the green transition, financial stability risks, then the phase-out of the polluting sector to foster the green sector would be hard to achieve. The model finds that under certain physical and transition risks, a dual interest rate policy, coupled with a fiscal policy in which carbon tax revenues are redistributed to households as vouchers to encourage the consumption of green goods, is able to support the green transition and safeguard financial stability. Therefore, top-down and bottom-up approaches are the keys for sustainability in the whole economy and financial system.

Keywords: Business cycle, financial accelerator, carbon tax

1. Introduction

Despite the two years of exceptional fluctuations in emissions and energy use, caused mostly by the Covid-19 pandemic, global carbon dioxide emissions from burning coal, gas and oil keep rising at a faster rate in 2023 than the 10-year average, jeopardizing the goals of the Paris agreement in limiting global warming. While emissions from natural gas fell by 1.6% in 2022, emissions from oil and coal have kept increasing since 2019. See Fig. 1.

The continuing raising of carbon emissions will accelerate the occurrence of climate events, exacerbating physical climate-related risks linked to event-driven hazards, with consequential transition risk associated with climate change policy actions that are expected to evolve and become more stringent in supporting the green transition to mitigate climate change. It has been already recognized that both physical and transition risks are a source of risk for the financial system, with potentially severe consequences for financial institutions and financial markets alike. (See Nieto, 2019, Battiston et al., 2021, Roncoroni et al., 2021, Mandel et al., 2021, Sun et al., 2022, and Gupta et al. (2023)). Given the systemic nature of climate change for financial stability, green macroprudential policies have been recently proposed as potential frameworks to ensure a consistent approach across the financial system. For instance, climateadjusted capital requirements can be adjusted to reflect the increased risks of different assets, including fossil fuels, and to divert financing to particular industries. Alternatively, reserve requirements, sectoral leverage ratio, liquidity regulations and lending limits have been proposed as possible options to reduce climate-related risks in the financial system. (See D'Orazio and Popoyan (2019)). However, green macroprudential instruments have been adopted so far only in several low-income countries, particularly in South-east Asia, while highincome countries appear still reluctant in adopting mandatory prudential tools to channel credit toward green sectors. Given the disruptive impact of climate change

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Source: Global Carbon Project.

on economic activities and substantial threats to monetary policy and financial stability, central banks and financial supervisors should adopt a forward-looking approach to contribute to mitigating the build-up of future climate-related shocks, and to increase the resilience of the financial system. In line with this important need of developing the necessary tools to assess climate-related risks at macro-level, which impairs the financial stability, this paper develops a detailed macro-financial model to better evaluate the effectiveness of monetary and macroprudential policies in fighting climate change and contemporaneously safeguards the financial stability. More specifically, given the threats that climaterelated risks can pose to the stability of the financial system, it is important to design effective macroprudential or monetary frameworks to ensure financial stability, in line with the goals of the Paris Agreement of curbing carbon emission by reducing (accelerating) lending to hard-to-abate activities (green/clean industries). This paper proposes a theoretical model that features two-sectors, nominal rigidities, financial frictions and endogenous default. In the spirit of Bernanke et al. (1999) and Christiano et al. (2014), entrepreneurs can finance capital for new projects by using their own funds (net worth) or by applying for external funds (loans) to commercial banks. Being new project risky, the return on capital is uncertain, and entrepreneurs can decide to default and avoid to repay back lenders if the ex-post return on capital is lower than the amount of credit they have to repay back to the bank. Climate-related risks can directly and indirectly affect the expost rate of return on capital. For instance, physical risk generated by weather-related events, such as floods and hurricanes, or temperature increase, causing sea-level rise, can have a negative impact on business performance, making it hard to turn raw capital into highly effective capital. As a matter of fact, weather-related events can damage a firm's productive factory and compromise firm's profitability, which translates into economic and financial losses that could eventually lead to default as firms find themselves unable to repay outstanding loans. See Noy (2009), Fabris (2020), Battiston et al. (2021), Carvalho et al. (2021), and Zhou et al. (2023). Similarly, transition risk, originated by the introduction of climate policies and regulations, can negatively affect fossil fuel and high-carbon firms' performance, and thus the value of their financial contracts. For instance, tax levied on the carbon emissions such as carbon tax represents a large operating cost for firms, who experience lower profits. Analogous to physical risk, such loss in firm's performance would prevent firms to convert raw capital into effective capital in order to obtain a positive return on new projects, thus compromising the ability to repay outstanding loans. See Nieto (2019), Dunz et al. (2021), Ojea Ferreiro et al. (2022), Huang et al. (2021, 2022) and Giovanardi and Kaldorf (2023).

Another important model characteristic is the introduction of a twosectors setup which allows to differentiate the intermediate production sector in two groups: (i) the brown sector, which pollutes when producing intermediate goods, and (ii) the green sector, which produces good without emitting GHG emissions in the atmosphere. The brown sector is therefore penalized for making climate change worse by imposing of carbon taxes for every CO/tonne emitted during the production process. Climate change is introduced as a damage function that destroys or decelerates the technology progress. The model setup is very similar to Annicchiarico et al. (2023), but it extends their model by introducing two different sectors, which will be treated differently when implementing some fiscal and macroprudential policies: (i) government sets a price (carbon tax) that only emitters in the brown sector must pay for each ton of greenhouse gas emissions they emit; (ii) central banks and financial supervisors design green policies to boost investment in low-carbon sectors while mitigating economic and financial risks from the transition process.1 Similar two-sector setup has been proposed by Diluiso et al. (2021) and Carattini et al. (2023), but their model abstracts from endogenous default and debt is always repaid, which represents a model limitation when evaluating the safeguard of financial stability. Thus, the presence of endogenous default represents a valuable insight of the model, as contributes to the build-up of credit and generates strong feedback loops between the financial sector and the real economy, with spillover effects across sectors. Further, endogenous default can potentially trigger exceptionally deep financial and real crises, making it a key feature for the analysis of regulatory policies in supporting the green transition.²

This paper also contributes to existent E-DSGE models by defining the household's consumption basket as a constant elasticity of substitution of consumption in green and brown goods. Such preference for green goods reflects the higher environmental awareness of household versus a clean and green economy. Moreover, this model features facilitates the green transition as the change in supply of green goods is stimulated by the higher demand for green products. Finally, the model contributes in assessing a mix of fiscal and monetary/macroprudential policies by considering both physical and transition risks. For instance, besides the standard shocks, such as technology and monetary policy shocks, the model also considers the transmission mechanism of physical risks, such as capital depreciation rate shocks caused by adverse weather-related events that can immediately destroy capital, and therefore accelerate the depreciation of capital. A similar shock is used by Hallegatte et al. (2022) who estimate the damage of typhoon wind in the Philippines as a loss of physical capital. They define this disaster scenario as a capital depreciation shock, that ultimately will have an impact on bank solvency. Moreover, the model also considers idiosyncratic shocks that affect the quality of capital in each sector, in the form of risk shocks as in Christiano et al. (2014), Huang et al.

¹ Meckling (2021) shows how green industrial policy can advance climate goals and cooperation but can also present challenges to deepening climate cooperation and reducing greenhouse gas emissions.

² For more details on endogenous feedback loops, see Bernanke et al. (1999), Mateos-Planas and Seccia (2006), Christiano et al. (2014), Garcia-Barragan and Liu (2018), Mendicino et al. (2018) and Leduc and Natal (2018).

(2021, 2022) and Annicchiarico et al. (2023). Different from capital depreciation shock that immediately generates losses of assets and capital, risk shocks capture changes in the value of capital due to investors' perceptions of climate risks from physical, public, and policy perspectives.

The paper offers several findings. First of all, based on a set of shocks that reflect standard economic fluctuations, as well as physical and transition risks, the model finds that a carbon tax higher than 25% would increase the medium-run impact on default rates in both sectors. However, if we consider only technology shocks as Carattini et al. (2023), we can notice that the mean value of default rates tend to decline with higher carbon taxes. Therefore, it is important to consider a larger set of shocks to support governments in setting their carbon tax. Second, under one-sector model, a carbon tax of 25% determines an emission cut of 37%, while a two-sectors model implies an emission cut of 45%. This occurs because banks are willing to extend more credit to the green sector, as they are aware that a given carbon tax policy can compromise the profitability of the brown sector, as well as their ability to repay back their loans, thus banks tend to cut even more lending to the brown sector by compensating with higher lending to the green sector. This results in a larger reduction in carbon emissions. Third, assuming a carbon tax of 25% as a benchmark, the paper compares the effectiveness of the design of various macroprudential and monetary policies in terms of financial stability and green transition. For the macroprudential frameworks, a green differentiated reserve requirement is implemented by allowing banks to hold fewer reserves against green loans. Some central banks have already been using this policy tool. For instance, the People's Bank of China has implemented differential reserve requirements in favor of green credit since 2018. Similarly, the Bank of Lebanon has employed differential reserve requirements since 2010, with the goal of influencing the allocation of credit in favor of investment in renewable energy and energy efficiency. Since 2021, the Central Bank of Hungary has introduced preferential capital requirements for green corporate and municipal financing. In December 2023, the Philippine central bank has approved a gradual reduction in the reserve requirement rate for green, social, sustainability, and other sustainable bonds issued by banks to zero from 3%. Concerning monetary policy, the paper emphasizes the use of a climate-augmented Taylor rule in setting the policy rate and the adoption of a dual interest rate approach. The climate-augmented monetary policy incorporates the emission gap target into a standard Taylor rule of monetary policy. Such a framework has been already proposed by Chen et al. (2021) and Ramlall (2023), and both find that such an augmented tool can help in gradually curbing the level of pollutants emitted.³ Alternatively, just recently, it has been proposed the adoption of a green dual interest rate when lending money to banks. This would see central banks applying a lower interest rate for green lending activities in order to incentivize banks to offer loans to firms that are aligned with the goals set by the Paris Agreement. This type of policy could potentially unlock green investment during monetary tightening, for instance. Although, green dual interest rate is a new proposed policy instrument that has been recently discussed, dual rates have already been introduced during the COVID-19 crisis, resulting to have a strong positive effect on bank credit provision, and the ability to sustain economic activity. Results suggest that the climate-augmented monetary policy helps containing default on loans, but decelerate the green transition. In contrast, differentiated countercyclical reserve requirements that vary with the business cycle, or a dual interest rate policy in favor of green loans, can support the green transition but impair financial stability. However, a combination of fiscal policy that redistributes carbon tax revenues as voucher to be used to buy green consumption good, coupled with a dual interest

rate approach, contributes in ensuring a resilient financial system when climate change poses threats to economic and financial stability.

The paper is organized as follows. Section 2 highlights the main contributions related to the closer literature. Section 3 presents the model, and Section 4 describes the calibration. Section 5 presents results on medium-run impact on welfare and default in order to pick the optimal carbon tax. Section 6 presents the simulation results of the shocks. Section 7 evaluates and discusses the implications of monetary and macroprudential policies. Appendix A concludes.

2. Related literature

This paper contributes to two strands in the literature: (i) it extends the existence E-DSGE models; (ii) it contributes to the evaluation of the combination of carbon taxes with macroprudential and monetary policies.

In terms of E-DSGE model, this paper is similar to Diluiso et al. (2021), Annicchiarico et al. (2023) and Carattini et al. (2023) who develop an E-DSGE model to assess the effectiveness of macroprudential policies. Diluiso et al. (2021) estimate a model for the Euro-Area to assess the possibility for central banks to address climate change issues, without compromising macroeconomic and price stability. In particular, they show that delayed and disorderly climate policies can affect inflation volatility, which requires a stronger monetary policy response by central banks. Further policies, such as green quantitative easing or differentiated capital requirement can reduce the likelihood and magnitude of financial crises, but making the recovery slower. Similarly, Carattini et al. (2023) develop an DSGE model with pollution and environmental externality for only one sector. In the spirit of Bernanke et al. (1999) and Christiano et al. (2014), their model is characterized by the possibility of defaulting, and banks, after incurring a monitoring cost, seize capital of firms that cannot repay their loans. This paper extends Diluiso et al. (2021) and Carattini et al. (2023) in two ways: (i) endogenous default in both sectors; (ii) consumer basket composed by green and brown goods. First of all, the endogenous default mechanism is an important model key because it captures the negative spillover effect from the brown to the green sector due to the banking capital channel (which induces banks to supply less loans to the green sector, as higher default rates in the brown sector reduce bank capital), and to the banking funding channel (which induces banks to charge higher lending rates to both sectors in order to satisfy the participation constraint). Second, total consumption is defined as a constant elasticity of substitution (CES) of consumption in green and brown goods to allow the aggregate demand to shift from the brown sector to the green sector resulting from an increase in carbon tax. Third, this paper considers several sources of fluctuations: standard shocks such as technology, monetary and risk shocks, as well as a physical climate risk shock which materializes in a faster depreciation rate of capital and a transition risk shock associated with policies that encourage the shift to a low-carbon economy.⁴ Under this set of shocks. this paper finds that a carbon tax higher than 25% would increase the medium-term likelihood of bankruptcy, which would trigger financial crisis.⁵ In contrast, when a technology shock hits only the brown sector, then higher carbon taxes would cut emissions further, without compromising financial stability. Therefore, a limited set of shocks, and

³ However, Chen et al. (2021) show that a climate-augmented monetary policy could create a dilemma between the welfare and the climate objectives.

⁴ For instance, the IMF finds that high-income countries need to introduce a carbon tax that rises quickly to USD 75 a ton in 2030 in order to limit global warming to 2 °C or less. https://www.imf.org/en/Blogs/Articles/2022/07/21/ blog-more-countries-are-pricing-carbon-but-emissions-are-still-too-cheap.

⁵ Diluiso et al. (2021) set a carbon tax such that the environmental tax revenues as a share of output are 0.7% in steady state, Carattini et al. (2023) simulate a carbon tax of 0.0192, and Giovanardi and Kaldorf (2023) calibrate a carbon tax of 0.13 per tonne of carbon (ToC), which are smaller relative to the optimal carbon tax suggested in this paper.

abstracting from the green sector, would underestimate the optimal carbon tax that governments would impose.

In terms of macroprudential policies, this paper is similar to Diluiso et al. (2021), Annicchiarico et al. (2023) and Carattini et al. (2023) in assessing the effectiveness of macroprudential policies under environmental-related risks. Indeed, these works consider capital requirements, reserve requirements or taxes/subsidies on banks' assets, and find that macroprudential policies are able to reduce the risk of a recession in the aftermath of climate policy shocks. Diluiso et al. (2021) show that capital requirements on polluting firms can reduce the severity of a financial crisis, but delay the recovery. Different from them, the model in this paper is specified over two-sectors, thus allowing the shift of consumption and production from brown to green goods, and reducing therefore the recessionary effect described in Diluiso et al. (2021) and Annicchiarico et al. (2023). Moreover, Carattini et al. (2023) find that a policy mix of carbon tax and uniform macroprudential policies is the first-best scenario, while a coordination of climate policy with differentiated macroprudential policies is the second-best scenario. Different from them, this paper looks at the financial stability risk in terms of higher likelihood to default, while Carattini et al. (2023) look only at the impact on banks' net worth. This paper is also very close to Giovanardi and Kaldorf (2023) who assess differentiated capital requirements in a two-sectors E-DSGE model. They suggest that increasing carbon taxes lead to higher default rates but a policy that increases (decreases) the clean (fossil) capital requirements can help in alleviating excessive risk-taking incentives and can reduce default rates. Different from them, our model shows that too high carbon tax can lead to higher default rate in the medium-run, suggesting that policy-makers should keep the price on emissions lower than 25%.

Finally, the paper is related to George et al. (2022) in considering a climate-augmented monetary policy that allows the policy rate to increase with emissions, thus making the cost of external funding for polluting firms higher. Further, this paper assesses the effectiveness of a dual interest rate policy that aims at incentivizing banks to finance green projects by offering them a lower interest rate.

To sum up, the model features help to better assess the effectiveness of monetary and macroprudential policies that aim at penalizing banks when extending loans to the brown sector and contemporaneously facilitate lending to the green sector. Differentiated policies help stimulating the development of the green sector, which also responds to higher demand for green goods. This facilitates the green transition without imposing too high carbon tax or too stringent macroprudential/monetary tools.

3. Theoretical model

We have developed a NK-EDSGE model which takes into account credit market friction, as outlined in this paper. Specifically, our aim is to merge the models initiated by Bernanke et al. (1999) and Annicchiarico and Di Dio (2015) with the intention of comprehending how pollutant emissions and environmental policy interrelate with asymmetric information in credit markets. To accomplish this objective, we have integrated two distinctive frameworks within our model. Firstly, we have incorporated the pioneering work of Bernanke et al. (1999), who examine the role of asymmetrical information in credit markets and illustrates how discrepancies in external financing costs and internal fund costs can result in the external finance premium. This disparity between external and internal financing costs has significant implications for the propagation of exogenous shocks to the business cycles, magnifying output fluctuations and creating what is commonly known as the "financial accelerator". Furthermore, we have also incorporated the setting posited by Annicchiarico and Di Dio (2015), which proposes that pollution, as a byproduct of the production process, has the potential to adversely impact firm productivity. The ensuing harm from pollution can be particularly costly if an emission

tax is imposed, resulting in firms balancing their production quantities with abatement efforts aimed at reducing the emission level. By fusing these two frameworks, our model has the ability to scrutinize the interaction between credit market friction and firms' ecological decisions throughout the business cycle. Additionally, we explore how firms' funding abilities influence pollutant emissions during varying stages of the business cycle. Our model also permits us to analyze the transmission mechanism of various shocks to the emission level via the operation of the financial accelerator.

3.1. Households

In the context of an economy with an infinite number of identical households, the representative household is tasked with maximizing their discounted lifetime expected utility:

$$\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} \left(\ln C_{t} - \mu_{L} \frac{L_{G,t}^{1+\phi}}{1+\phi} - \mu_{L} \frac{L_{B,t}^{1+\phi}}{1+\phi} \right)$$
(1)

which is defined as a function of household consumption at time *t*, denoted as C_t , as well as the labor supply in both the green and brown sectors, represented respectively by $L_{G,t}$ and $L_{B,t}$. The discount factor is denoted as $0 < \beta < 1$. It is important to note that the utility function is increasing with consumption, while simultaneously decreasing with labor supply. The disutility of labor supply is controlled by a scale parameter $\mu_L > 0$ and the inverse of Frisch elasticity $\phi > 0$.

In order to maximize the lifetime utility (1), the household must adhere to a budget constraint in every period *t*. This constraint is defined as follows:

$$P_t C_t + Q_t^B B_t = B_{t-1} + W_{G,t} L_{G,t} + W_{B,t} L_{B,t} + P_t D_t - T_t,$$
(2)

where the variables P_i , D_i , B_t , Q_t^B , $W_{G,t}$, $W_{B,t}$, and T_t are defined as the general price level at time t, the (real) profits received from the ownership of firms, the units of one-period riskless bonds held by the household at time t, the bond price, the nominal wage rates in the green and brown sectors, and the lump-tax levied by the government, respectively. Eq. (1) represents the total expenditure of the household at time t, which includes the consumption expenditure of the household at time t, which includes the consumption expenditure P_tC_t and the expenditure of holding a one-period bond $Q_t^B B_t$. The before-tax household income is the sum of the wage earning $W_{G,t}L_{G,t} + W_{B,t}L_{B,t}$, the dividend P_tD_t , and the earning from holding bond B_{t-1} .

To maximize Eq. (1) subject to Eq. (2), the household selects C_t , $L_{G,t}$, $L_{B,t}$ and B_t . The corresponding first order conditions are

$$C_t = 1/\lambda_t \tag{3}$$

$$R_t^{-1} = Q_t^N = \beta \mathbb{E}_t \frac{1}{\Pi_{t+1}} \frac{\lambda_{t+1}}{\lambda_t}$$
(4)

$$\mu_L L_{i,t}^{\phi} = \lambda_t w_{i,t} \tag{5}$$

where λ_t is a Lagrange multiplier, and R_t represents the nominal interest rate at time *t*. The real wage $w_{i,t}$, where $i \in \{G, B\}$, is calculated as $W_{i,t}/P_{i,t}$. By substituting the λ_t in Eq. (4) with Eq. (3), the Euler equation is obtained, which characterizes the intertemporal tradeoff in consumption for households. At the optimal point, the household balances the marginal rate of substitution between the consumption in two periods with the nominal interest rate. Eq. (5) determines the labor supply, where the marginal disutility of labor is equal to the marginal utility gain of having an additional income $w_{i,t}$.

Assuming that the aggregate consumption C_t can be represented as a composite function as:

$$C_t = \left(\eta^{\frac{1}{\theta}} C_{G,t}^{\frac{\theta-1}{\theta}} + (1-\eta)^{\frac{1}{\theta}} C_{B,t}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$$
(6)

where the elasticity of substitution between green and brown goods is denoted by $\theta > 1$. The consumption of green and brown goods at

time *t* is represented by $C_{B,t}$ and $C_{G,t}$, respectively. The household's preference for the green good at time *t* is denoted by η . It is noteworthy that an increase in η can be attributed to a wide range of factors, including increased awareness of environmental protection and a growing preference for using eco-friendly products.

The price index P_t is determined by the composite function:

$$P_{t} = (\eta P_{G,t}^{1-\theta} + (1-\eta) P_{B,t}^{1-\theta})^{\frac{1}{1-\theta}}$$
(7)

Assuming optimal consumption choices, households minimize their total expenditure, which is given by $P_tC_t = P_{G,t}C_{G,t} + P_{B,t}C_{B,t}$, by selecting the combination of green and brown goods. This leads to the following goods demand functions:

$$C_{G,t} = \eta \left(\frac{P_{G,t}}{P_t}\right)^{-\sigma} C_t \tag{8}$$

$$C_{B,t} = (1 - \eta) \left(\frac{P_{B,t}}{P_t}\right)^{-\sigma} C_t$$
(9)

Let $C_{G,t}$ and $C_{B,t}$ be the consumption of the final good. We can decompose it into an infinite number of intermediate goods consumption, denoted by $j \in [0, 1]$, as per the formula:

$$C_{t} = \left(\int_{0}^{1} C_{i,t}(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}$$
(10)

for $i \in \{G, B\}$, where the elasticity of substitution between any two intermediate goods within a sector remains constant at θ for the sake of simplicity. It is worth noting that the extension of this generalization to encompass different degrees of elasticity of substitution within and across sectors is a straightforward task.

3.2. Firms

To elucidate the relationship between aggregate output and the green and brown goods in the economy at a given time *t*, we denote Y_t as the former and $Y_{G,t}$ and $Y_{B,t}$ as the latter:

$$Y_t = \left(\eta^{\frac{1}{\theta}} Y_{G,t}^{\frac{\theta-1}{\theta}} + (1-\eta)^{\frac{1}{\theta}} Y_{B,t}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$$
(11)

In accordance with Eq. (6), we establish that the final good is a composite of a continuum of intermediate goods $Y_{i,t}(j)$ for $j \in [0, 1]$. This is akin to Eq. (10), where $Y_{i,t} = (\int_0^1 Y_{i,t}(i)^{\frac{\theta-1}{\theta}} di)^{\frac{\theta}{\theta-1}}$. At the optimum, the demand for the intermediate good j is determined by

$$Y_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\theta} Y_{i,t}$$
(12)

where the price level of intermediate good *j* in sector *i* is denoted as $P_{i,t}(j)$. By virtue of (10) and (12), the aggregate price level $P_{i,t}$ satisfies the constraint $P_{i,t}Y_{i,t} = \int_0^1 P_{i,t}(j)Y_{i,t}(j)dj$ which yields $P_{i,t} = (\int_0^1 P_{i,t}(j)^{1-\theta}dj)^{1/(1-\theta)}$.

3.2.1. Brown-goods production

Analogous to Annicchiarico and Di Dio (2015) and Heutel (2012), it is assumed that the process of producing brown goods results in pollutants. Moreover, a negative correlation is hypothesized between the output of firms and the level of carbon emissions the economy. Furthermore, it is postulated that every intermediary good is endowed with an identical production function:

$$Y_{B,t}(j) = (1 - Y(M_t))A_t K_{B,t}(j)^{\alpha} L_{B,t}^{1-\alpha}(j)$$
(13)

Firm *j*'s capital and labor inputs in the brown sector are denoted by $K_{B,I}(j)$ and $L_{B,I}(j)$, respectively, where α , a parameter representing the share of capital, is bounded within the interval $0 < \alpha < 1$. The damage function, Y(.), constitutes a crucial element of the model, accommodating the decrease of production as a consequence of pollution. It is

noted that this damage function displays a positive correlation with the accumulated carbon emissions stock in the economy, M_i .

Furthermore, the total factor productivity (TFP) of firms is denoted as A_t , with the assumption that the dynamics of A_t follows an AR(1) process described by:

$$\ln(A_t/A) = \rho_A \ln(A_{t-1}/A) + \varepsilon_{A,t}$$

where the persistence of the technology shock is denoted by ρ_A , which is a value between 0 and 1. The steady-state of TFP is represented by *A*. The white noise, $\epsilon_{A,I}$, which is assumed to be normally distributed with mean 0 and standard deviation $\sigma_A > 0$, controls for the volatility of the technology shock.

During the production process of firm *j*, pollutant $Z_t(j)$ is emitted. Intermediate firm *j* can reduce its pollution level by exerting an abatement effort $U_t(j) \in [0, 1]$. The emissions level $Z_t(j)$, the output level $Y_{B,t}(i)$, and the abatement effort $U_t(i)$ are related by the following equation.

$$Z_t(j) = \varphi(1 - U_t(j))Y_{B,t}(j)$$
(14)

It is assumed that the emission level is contingent upon the linear dependence of the output level $Y_{B,t}(j)$. In instances where the abatement effort $U_t(j) = 0$, $Z_t(j) = \varphi Y_t(j)$, thus signifying that the abatement effort is an instrumental factor in regulating the overall emission levels. Furthermore, it is established that $\varphi > 0$ controls the marginal increase in emission levels for any additional increase in output, thereby corroborating the proposition that abatement efforts serve as a crucial mechanism in mitigating the environmental impact of production activities. The abatement cost is:

$$C_{A,t}(j) = \phi_1 U_t(j)^{\phi_2} Y_t(j)$$
(15)

In the above, the abatement cost is proportional to the effort to reduce the emission level per output, with $\phi_1 > 0$ serving as the scale parameter and $\phi_2 > 1$ as the determinant of the elasticity of abatement cost. When $\phi_2 > 1$, the abatement cost exhibits convexity, motivating firms to divide their abatement efforts into multiple periods. It should be noted that the abatement cost remains linear with output.

As each intermediate firm is infinitesimal, the effort to reduce emissions by a single firm, denoted as *j*, will not affect the overall emission stock level M_t . Therefore, if there is no cost associated with emitting greenhouse gases, firms would have no incentive to reduce their emissions and the optimal effort for abatement would be zero for any firm *j*, denoted as $U_t(j) = 0$. However, if there is a carbon tax $P_{Z,t} > 0$ enforced by the government for each unit of greenhouse gas emitted, the optimal effort for abatement can be determined by the following equation:

$$\varphi \frac{P_{Z,t}}{P_t} = \phi_1 \phi_2 U_t(j)^{\phi_2 - 1} \tag{16}$$

which specifies that the cost incurred in reducing a unit of emission (the right-hand side) is equal to the benefits obtained from the reduction. In addition to reducing emissions, intermediate firm j also selects capital and labor such that their profit function is maximized. At the optimal point, we can observe that

$$\alpha \frac{Y_{B,t}(j)}{K_{B,t}(j)} \Psi_{B,t} = r_{B,K,t}$$
(17)

$$(1 - \alpha) \frac{Y_{B,t}(j)}{L_{B,t}(j)} \Psi_{B,t} = w_{B,t}$$
(18)

Specifically, we denote by $r_{B,K,t}$ and $w_{B,t} = W_{B,t}/P_t$ the real marginal product of capital and the real wage rate, respectively. These two variables represent the productivity of the firm's inputs and are crucial in determining its cost structure. Additionally, we introduce $\Psi_{B,t}$ as the component of marginal cost that is related to capital and labor. Notably, $\Psi_{B,t}$ can be expressed as a function of the factor prices $w_{B,t}$ and $r_{B,K,t}$:

$$\Psi_{B,t} = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_t(1-Y(M_t))} w_{B,t}^{1-\alpha}r_{B,K,t}^{\alpha}$$
(19)

As per Eq. (19), we observe that the real marginal cost is positively related to both the wage rate and the marginal product of capital. This indicates that an increase in either of these two input prices would lead to higher production costs for the firm. Additionally, the presence of environmental externalities, such as emissions, affects the firm's profitability negatively. A higher emission stock M_i increases the damage to the firm and raises its marginal cost, which, in turn, lowers its profitability.

It is important to note that the firm incurs not only the cost associated with $\Psi_{B,t}$ but also the abatement cost to mitigate the negative externality. Therefore, the total (real) marginal cost of the firm can be expressed as a sum of multiple components:

$$MC_{B,t} = \Psi_{B,t} + \phi_1 U_t(j)^{\phi_2} + \frac{P_{Z,t}}{P_t} (1 - U_t(j))\varphi$$
(20)

Specifically, the total marginal cost includes the cost associated with $\Psi_{B,t}$, the cost of abatement for an additional output $\phi_1 U_t^{\phi_2}$, and the emission tax paid by having an additional output $P_{Z,t}(1-U_t)\varphi/P_t$.

3.2.2. Green-goods production

The production function of the green firm j is akin to that of its nongreen counterpart, with the exception that there is no carbon emissions during the production process, and thus no carbon tax is charged to the firms. In particular, the production function of the green firm j is

$$Y_{G,t}(j) = (1 - Y(M_t))A_t K_{G,t}(j)^{\alpha} L_{G,t}^{1-\alpha}(j)$$
(21)

In particular, we define $K_{G,t}(j)$ and $L_{G,t}(j)$ as the capital and labor inputs in use by firm *j* in the green sector. It follows that the first-order conditions for $K_{G,t}(j)$ and $L_{G,t}(j)$ can be represented as:

$$\alpha \frac{Y_{G,t}(j)}{K_{G,t}(j)} M C_{G,t} = r_{G,K,t}$$
(22)

and

$$(1 - \alpha) \frac{Y_{G,t}(j)}{L_{G,t}(j)} M C_{G,t} = w_{G,t},$$
(23)

respectively. $w_{G,t}$ and $r_{G,t}$ are the real wage rate and the real rate of return to capital, respectively. Meanwhile, $MC_{G,t}$ denotes the real marginal cost of production, which can be derived by combining Eqs. (22) and (23):

$$MC_{G,t} = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_{t}(1-Y(M_{t}))} w_{G,t}^{1-\alpha}r_{G,K,t}^{\alpha}$$
(24)

3.3. Nominal rigidity

Based on our underlying assumptions, the market for intermediate goods is characterized by monopolistic competition, implying that each firm is confronted with a demand function (12) for its products. The demand curve, which slopes downwards, signifies that the firm possesses the ability to modify the price of its goods by adjusting production levels. At the optimal level, the price determined by firm j complies with:

$$P_{i,t}(j) = \mathcal{M}MC_{i,t}(j) \tag{25}$$

that is distinct from the perfectly competitive scenario, wherein the optimal price corresponds to the marginal cost. Instead, the optimal price is equivalent to the marginal cost multiplied by a factor $\mathcal{M} = \theta/(\theta - 1) > 1$, which represents the markup enjoyed by the firms.

To facilitate the effectiveness of monetary policy in the short run, we introduce nominal rigidity to the firms, establishing that some of them are precluded from adjusting their prices. Building on the work of Calvo (1983), we posit that solely a proportion of firms, $(1 - \xi) \in [0, 1]$, can modify their prices each period. The Calvo pricing formula stipulates that the aggregate price is $P_{i,t} = [\xi P_{i,t-1}^{1-\theta} + (1-\xi)P_{i,t}^{*1-\theta}]^{1/(1-\theta)}$, where $P_{i,t}^*$ is the optimal price established by the firms that can adjust their price at time *t*. The gross inflation rate is defined as $\Pi_{i,t} = P_{i,t}/P_{i,t-1}$, and

the formula can be simplified to $1 = [\xi \Pi_{i,t}^{\theta-1} + (1-\xi)p_{i,t}^{*1-\theta}]^{1/(1-\theta)}$, where $p_{i,t}^* = P_{i,t}^*/P_{i,t}$

With regard to price stickiness, firm *j*, capable of adjusting its price at time *t*, selects $P_{i,t}^*(j)$ to maximize the discounted lifetime expected profit function.

$$\max_{P_{i,t}^{*}(j)} \mathbb{E}_{t} \sum_{k=0}^{\infty} \xi^{k} Q_{t,t+k} \left[P_{i,t}^{*}(j) Y_{i,t+k}(j) - M C_{i,t} Y_{i,t+k}(j) \right]$$
(26)

The stochastic discount factor between time *t* and time t + k is $Q_{i,t+k} = \beta^k (\lambda_{t+k}/\lambda_i)$. Since it may not be feasible for the firm to adjust its price in the near future, firms must account for future profits while selecting $P_{i,t}^*(j)$. The probability that the firm cannot adjust its good price from time *t* to time t+k is denoted by $\xi^k > 0$. $MC_{i,t}$ is determined by Eq. (20), and the firm is subject to the demand function (12).

The first-order condition of the problem (26) is:

$$p_{i,t}^{*} = \frac{P_{i,t}^{*}}{P_{i,t}} \mathcal{M} \frac{\mathbb{E}_{t} \sum_{k=0}^{\infty} \xi^{k} Q_{i,t,t+k} M C_{i,t+k} \left(\frac{P_{i,t+k}}{P_{i,t}}\right)^{\theta} Y_{i,t+i}}{\mathbb{E}_{t} \sum_{k=0}^{\infty} \xi^{k} Q_{i,t,t+k} \left(\frac{P_{i,t+k}}{P_{i,t}}\right)^{\theta} Y_{i,t+i}}$$
(27)

The firm index *j* is omitted by leveraging firm symmetry. It can be ascertained that when $\xi = 0$, Eq. (27) reduces to Eq. (25).

3.4. Banks

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Assuming the existence of a continuous spectrum of banks with identical characteristics and a standardized measure of 1, we can posit that the representative bank obtains an aggregated sum of deposits D_{t+1} from households. Meanwhile, it maintains a mandatory reserve requirement Q_{t+1} and disburses loans to both non-polluting and polluting sectors ($B_{G,t+1}$ and $B_{B,t+1}$ correspondingly).

By assuming the absence of the bank's net worth and negating interest on the reserve, we are presented with the balance sheet equation:

$$B_{G,t+1} + B_{B,t+1} + Q_{t+1} = D_{t+1}$$
(28)

The required reserve ratio, which is represented as $\kappa_t \in (0, 1)$, can be defined as the proportion of total deposits that a bank must keep in reserves. This requirement is imposed by the law and the bank has to ensure that its reserves are greater than or equal to κ_t fraction of its total deposits.

$$Q_{t+1} \ge \kappa_t D_{t+1} \tag{29}$$

Moreover, consider $R_{G,t+1}^{l}$ and $R_{B,t+1}^{l}$, which refer to the loan rates charged by the bank for polluting and nonpolluting entrepreneurs respectively from period *t* to *t* + 1. The bank's objective is to maximize profits while complying with constraints specified by Eqs. (28) and (29). The variable $\mathcal{M}_{t,t+1}$ is a stochastic discount factor that could fluctuate over time:

$$\max_{M_{t+1}, J_{G,t+1}, J_{B,t+1}} \mathbb{E}_{t} \frac{\mathcal{M}_{t,t+1}}{P_{t+1}} (\sum_{i \in \{G,B\}} (B_{i,t+1} R_{i,t+1}^{l}) + Q_{t+1} - D_{t+1} R_{t})$$
(30)

Note that the bank would provide loans up to its maximum capacity, if and only if, $\min\{Z_{G,t}, Z_{B,t}\} > R_t$. When this condition satisfies, the reserve requirement becomes binding:

$$Q_{t+1} = \kappa_t D_{t+1} \tag{31}$$

The zero-profit condition in the banking sector involves substituting Eqs. (28) and (31) into the profit function (30):

$$R_{G,t+1}^{l} = R_{B,t+1}^{l} = \frac{R_{t} - \kappa_{t}}{1 - \kappa_{t}}$$
(32)

This condition dictates that the loan rates offered to the two sectors mentioned earlier must be equivalent in the equilibrium. Such a situation only arises when the required reserve ratio remains the same across all sectors. Eq. (32) reveals that an increase in the deposit rate causes a corresponding rise in the loan rate chosen by banks due to the increased cost of acquiring funds.

Note that for $R_{i,t}^l > R_t$, we require $R_t > 1$. Our assumption is that κ_t follows an AR(1) process. The value of $\varepsilon_{\kappa,t}$ is distributed normally with a standard deviation of σ_{κ} and a mean of zero. Additionally, ρ_{κ} serves as a parameter that controls the persistence of the process:

3.5. Entrepreneur

Drawing from Bernanke et al. (1999)'s seminal work on financial intermediation, we posit the existence of a continuum of entrepreneurs, indexed by $j \in [0, 1]$, who avail themselves of credit from a financial intermediary and make investment decisions on the amount of capital to allocate in each period. We further assume that the entrepreneurs are risk-neutral actors, and as such, every period, each entrepreneur faces a probability γ of survival, with a corresponding $1 - \gamma$ proportion of entrepreneurs exiting the market. In this context, the entrepreneurs face a finite horizon, thereby making it impossible to accumulate sufficient wealth to fully self-finance, thus necessitating recourse to borrowing from the financial intermediary.

To this end, we denote $K_{i,t+1}(j)$ as the capital acquired by entrepreneur *j* and $q_{i,t}$ as the price of the capital. Entrepreneur *j* can finance the acquisition of capital either through net worth $N_{i,t+1}(j)$ or borrowing $B_{i,t+1}(j)$ from the financial intermediary, who, in turn, obtains funding from households' deposits and consequently faces an opportunity cost of funds equivalent to the nominal interest rate R_i . The budget constraint of entrepreneur *j* can be expressed as:

$$K_{i,t+1}(j)q_{i,t} = N_{i,t+1}(j) + B_{i,t+1}(j)$$
(33)

Denote $R_{i,K,t}$ as the average (gross) return of capital. The (gross) capital return for each entrepreneur is represented as $\omega R_{i,K,t}$, where ω denotes an idiosyncratic shock to the capital return. It is assumed that this shock is identically independently distributed across time and entrepreneurs. Furthermore, it is assumed that ω follows a log-normal distribution with the mean equaling 1 and the standard deviation of $\ln(\omega)$ equaling σ_t . This distributional assumption is in line with the work of Christiano et al. (2014), which introduced the concept of risk shock that determines the return dispersion to the entrepreneurs at time t. The risk shock is characterized by the variable σ_t , which follows an AR(1) process given by the equation:

$$\ln(\sigma_t/\sigma) = \rho_{\sigma} \ln(\sigma_{t-1}/\sigma) + \varepsilon_{\sigma,t}$$

where $\rho_{\sigma} > 0$ represents the persistence of the risk shock, σ is the steady-state of the risk shock, and $\varepsilon_{\sigma,t} \sim N(0,\sigma_{\sigma})$ is the white noise which is assumed to be normally distributed with mean 0 and standard deviation $\sigma_{\sigma} > 0$.

The non-default loan rate of a given entrepreneur, denoted as Z_{t+1} , is a crucial factor in determining their financial viability. If an entrepreneur experiences a low idiosyncratic shock, represented by the variable ω , and is unable to repay their debt, they must declare bankruptcy. Conversely, if an entrepreneur experiences a high idiosyncratic shock, they may be able to earn a profit after repaying their debt. Therefore, it is reasonable to assume that there exists a threshold value, denoted as $\bar{\omega}_{i,t+1}(j)$, which satisfies the following condition:

$$\bar{\omega}_{i,t+1}(j)R_{i,K,t+1}q_{i,t}K_{i,t+1}(j) = Z_{i,t+1}B_{i,t+1}(j)$$
(34)

In other words, if an entrepreneur experiences the shock at $\omega_{i,t+1}(j) = \bar{\omega}_{i,t+1}(j)$, its capital return is just sufficient to repay their debt, leaving them with zero profit. Only when $\omega_{i,t+1}(j) > \bar{\omega}_{i,t+1}(j)$, can an entrepreneur earn a positive profit, whereas if $\omega_{i,t+1}(j) < \bar{\omega}_{i,t+1}(j)$, bankruptcy is inevitable. If an entrepreneur does declare bankruptcy, the financial intermediary assumes control of their property by paying a monitoring cost. Specifically, the financial intermediary acquires $(1 - \mu)\omega_{i,t+1}(j)R_{i,K,t+1}q_{i,t}K_{i,t+1}(j)$, where $\mu \in [0, 1]$ represents the rate of the monitoring cost.

In the equilibrium, it is imperative that the expected return of lending a fund to an entrepreneur *j* be equivalent to the opportunity cost of the loan. To this end, the intermediary acquires the fund from the households at a riskless rate, denoted as R_{t+1} . The loan contract must comply with the following condition:

$$\int_{\bar{\omega}_{i,t+1}(j)}^{\infty} \bar{\omega}_{i,t+1}(j) R_{i,K,t+1} q_{i,t} K_{i,t+1}(j) dF_t(\omega) + \int_0^{\bar{\omega}_{i,t+1}(j)} (1-\mu) \omega_{i,t+1}(j) R_{i,K,t+1} q_{i,t} K_{i,t+1}(j) dF_t(\omega) = R_{i,t+1}^l B_{i,t+1}(j)$$
(35)

where the first and the second terms on the left-hand side of Eq. (35) denote the expected gross return when the entrepreneur *j* survives and goes bankrupt, respectively. By combining Eq. (33) with Eq. (35), we can deduce that:

$$\int_{\bar{\omega}_{i,t+1}(j)}^{\infty} \bar{\omega}_{i,t+1}(j) R_{i,K,t+1} q_{i,t} K_{i,t+1}(j) dF_t(\omega) + \int_0^{\bar{\omega}_{i,t+1}(j)} (1-\mu) \omega R_{i,K,t+1} q_{i,t} \\ \times K_{i,t+1}(j) dF_t(\omega) = R_{i,t+1}^l(K_{i,t+1}(j) q_{i,t} - N_{i,t+1}(j))$$
(36)

Given the constraint (35), the entrepreneur *j* is required to determine the amount of capital $K_{i,l+1}(j)$ to invest along with the threshold $\bar{\omega}_{i,l+1}(j)$, in order to optimize their profit function:

$$\max_{\bar{\omega}_{t+1}(j),K_{t+1}(j)} \int_{\bar{\omega}_{t+1}(j)}^{\infty} \omega R_{K,t+1} q_t K_{t+1}(j) dF_t(\omega) - Z_{t+1}(j) B_{t+1}(j)$$
(37)

This refers to the expected gain minus the cost of debt for the entrepreneur, given that the realized idiosyncratic shock $\omega(j)$ surpasses the threshold $\bar{\omega}_{i+1}(j)$. Utilizing Eq. (34), the objective function (37) can be expressed as:

$$\max_{\bar{\vartheta}_{i,t+1},K_{i,t+1}(j)} \int_{\bar{\omega}_{i,t+1}(j)}^{\infty} (\omega - \bar{\omega}_{i,t+1}(j)) R_{i,K,t+1} q_{i,t} K_{i,t+1}(j) dF_t(\omega)$$
(38)

The attainment of the maximum value of Eq. (38), subject to Eq. (36), results in the optimal amount of capital that entrepreneurs can invest, and the optimal threshold $\bar{\omega}_{i,t+1}(j)$. The first order conditions are expressed as follows:

$$\frac{(1 - \Gamma_{i,t+1})s_{i,t}}{1 - (\Gamma_{i,t+1} - \mu H_{i,t+1})s_{i,t}} = \frac{1 - \Phi(z_{i,t+1})}{1 - \Phi(\bar{\omega}_{i,t+1}) - \mu\bar{\omega}_{i,t+1}f_t(\bar{\omega}_{i,t+1})}$$
(39)

$$(\Gamma_{i,t+1} - \mu H_{i,t+1})s_{i,t}q_{i,t}\frac{K_{i,t+1}}{N_{i,t+1}} = q_{i,t}\frac{K_{i,t+1}}{N_{i,t+1}} - 1$$
(40)

where $s_{i,t} = R_{i,K,t+1}/R_{i,t+1}^{l}$, $\Gamma_{i,t+1} = \bar{\omega}_{i,t+1}(1 - \Phi(z_{i,t+1})) + H_{i,t+1}$, $H_{i,t+1} = \Phi(z_{i,t+1} - \sigma_{i,t})$, $z_{i,t} = (\log(\bar{\omega}_{i,t}) + \sigma_{i,t}^2/2)/\sigma_{i,t}$, and $\Phi(.)$ is the distribution function of standard normal distribution. It is noteworthy to mention that the index *j* is omitted in the aforementioned equations since the entrepreneurs within a specific sector are identical.

3.6. Net worth evolution

As previously indicated, it is noteworthy that within each period, the entrepreneurs are endowed with a parameter γ , which denotes the probability of their survival, alongside a probability of exit, which is the complementary value of $1 - \gamma$. It is pertinent to note that the exit of these entrepreneurs from the market would result in a complete loss of their net worth, which is indicative of the finite nature of their entrepreneurial lifespan. The evolution of the cumulative entrepreneurial net worth, denoted as $N_{i,t+1}$, is governed by the following equation:

$$N_{i,t+1} = \gamma (1 - \Gamma_{i,t}) R_{i,K,t} q_{i,t-1} K_{i,t}$$
(41)

Here, the term $(1 - \Gamma_{i,t})R_{i,k,t}q_{i,t-1}K_{i,t}$ represents the aggregate entrepreneurial profits at the end of time *t*.

3.7. Capital producers

Assuming perfectly competitive in the capital production market, the capital producer must make a decision regarding their investment level such that the profit function is maximized.

$$\max_{I_{i,t}} q_{i,t} I_{i,t} - I_{i,t} - \frac{\gamma_I}{2} \left(\frac{I_{i,t}}{K_{i,t}} - \delta_K \right)^2 K_{i,t}$$

Within this framework, $q_{i,t}$ – also known as Tobin's q – represents the price of the capital, and $\gamma_I \geq 0$ serves as the parameter that determines the scale of the adjustment cost. It is important to note that the profit achieved by the capital producer is defined as the revenue generated from the sale of capital $q_{i,t}I_{i,t}$, less the costs incurred during purchase $I_{i,t}$ and adjustment $\gamma_I(I_{i,t}/K_{i,t} - \delta_K)^2 K_{i,t}/2$. Furthermore, the investment good is expressed in terms of the consumption good, resulting in the price of the investment good being equal to one. The equation determining the optimal investment level is:

$$q_{i,t} - 1 - \gamma_I \left(\frac{I_{i,t}}{K_{i,t}} - \delta_K\right) = 0$$
(42)

When the adjustment cost parameter γ_I is equal to zero, it follows that $q_{i,t} = 1$. In other words, the price of the capital aligns with the price of investment. Under these conditions, the gross capital return mirrors the marginal product of capital before accounting for depreciation (as indicated by Eq. (44)). However, in the presence of a capital adjustment cost, the time-varying capital price could exacerbate the volatility of capital return, thereby negatively affecting the net worth of entrepreneurs.

The evolution of capital is

$$K_{i,t+1} = (1 - \delta_K)K_{i,t} + I_{i,t}$$
(43)

The expected capital gross return for the end of period t and the beginning of period t + 1 is

$$\mathbb{E}_{t} R_{i,K,t+1} = \mathbb{E}_{t} \left[\frac{r_{i,K,t} + q_{i,t+1}(1 - \delta_{K})}{q_{i,t}} \right]$$
(44)

where the first component on the *r*.*h.s.* is the marginal productivity of capital at time *t*. The second component is the capital gain derived from the sale of the residual capital left at time t + 1.

3.8. Monetary policy rules and budget constraint

The aggregate inflation rate can be defined as

$$\Pi_t \frac{p_{i,t}}{p_{i,t-1}} = \Pi_{i,t} \tag{45}$$

It is worth noting that the monetary policy is designed and executed by the central bank through regulating the short-term nominal interest rate, which is formulated as follows:

$$R_t = R \left(\frac{\Pi_t}{\Pi}\right)^{l_{\pi}} \left(\frac{Y_t}{Y}\right)^{l_{Y}} v_t \tag{46}$$

Here, R_t denotes the gross nominal interest rate at time t, and the parameters ι_{π} , $\iota_Y > 0$ are responsible for determining the elasticity of interest rate to the inflation rate and output, respectively. Additionally, R, Y and Π represent the steady-state values, which are targeted by the central bank in setting the interest rate, output, and inflation rate, respectively. Besides, v_t is regarded as the monetary policy shock, which embodies the short-term deviation of the monetary policy rule. We assume that the stochastic process governing v_t conforms to an AR(1) process, as expressed by

$$\ln v_t = \rho_v \ln v_{t-1} + \varepsilon_v \tag{47}$$

with $\rho_v \in [0, 1]$ representing the degree of persistence associated with the monetary policy shock. The white noise $\varepsilon_v \sim N(0, \sigma_v)$ is a standard normal distribution with mean 0 and standard deviation $\sigma_v > 0$.

Regarding the public sector, we presume that the emissions tax revenue $P_{Z,I}Z_I$ is channeled back to households through a lump-sum transfer T_I :

$$T_t + P_{Z,t} Z_t = 0 \tag{48}$$

3.9. Equilibrium

The expression (16) posits that the abatement effort is solely determined by the carbon tax rate, rendering it equivalent for all firms, thereby affirming $U_i(i) = U_i$. Employing $D_{i,p,t} = \int_0^1 (P_{i,t}(j)/P_{i,t})^{-\theta} dj$ as a measure of price dispersion and $\int_0^1 Y_{i,t}(j) dj = \int_0^1 (P_{i,t}(j)/P_{i,t})^{-\theta} Y_{i,t} dj = D_{i,p,t}Y_{i,t}$. to gauge aggregate output, the production function (13) can be presented succinctly as $Y_{i,t} = (1 - \Gamma(M_i))A_t K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}(D_{i,p,t})^{-1}$. Furthermore, the level of aggregate emissions in Eq. (14) can be represented as $Z_t = \int_0^1 Z_t(j) dj = (1 - U_i)\varphi \int_0^1 Y_{i,t}(j) dj = (1 - U_i)\varphi Y_{i,t}D_{i,p,t}$. Analogously, the aggregate abatement cost in Eq. (15) is rendered as $C_{A,t} = \int_0^1 C_{A,t}(j) dj = \phi_1 U_t^{\phi_2} Y_{B,t} D_{B,p,t}$.

In each period, both goods markets must be in equilibrium, so $Y_{G,t} = \eta(P_{N,t}/P_t)^{-\theta}Y_t$ and $Y_{D,t} = (1 - \eta)(P_{D,t}/P_t)^{-\theta}Y_t$. Additionally, the aggregate output adheres to the following equation:

$$Y_t = \left(\eta^{\frac{1}{\theta}} Y_{G,t}^{\frac{\theta-1}{\theta}} + (1-\eta)^{\frac{1}{\theta}} Y_{B,t}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$$
(49)

The evolution of price dispersion is subject to the dynamics:

$$D_{i,p,t} = (1 - \xi)p_{i,t}^{*-\sigma} + \xi \left(\frac{\Pi_i}{\Pi_{i,t}}\right)^{-\theta} D_{i,p,t-1}$$
(50)

The equilibrium conditions for the two markets are:

$$Y_{G,t} = C_{G,t} + I_{G,t} + d_{G,t}$$
(51)

$$Y_{B,t} = C_{B,t} + I_{B,t} + C_{A,t} + \frac{\gamma_I}{2} \left(\frac{I_{B,t}}{K_{B,t}} - \delta_K\right)^2 K_{B,t} + d_{B,t}$$
(52)

In essence, these markets are characterized by the fact that the aggregate goods supply $Y_{i,t}$ is equivalent to the aggregate demand for goods within the economy. This demand includes not only the consumption $C_{i,t}$ and the investment $I_{i,t}$, but also the resources that are utilized as a result of the market frictions. Specifically, this encompasses the adjustment cost $\gamma_I (I_{i,t}/K_{i,t} - \delta_K)^2 K_{i,t}/2$, and the monitoring cost $d_{i,t} = \mu R_{i,K,t} q_{i,t-1} K_{i,t} \int_0^{\omega_{i,t+1}} \omega f_t(\omega) d\omega / P_t$ in both Eqs. (51) and (52), and the abatement cost $C_{A,t}$ in Eq. (52).

Moreover, the equation governing the evolution of the aggregate emission stock is expressed as follows:

$$M_t = (1 - \delta_M)M_{t-1} + Z_t + Z_t^*$$
(53)

where δ_M represents the decay rate of the emission stock, while Z_t^* denotes the level of emissions emitted from the rest of the world, which contributes to the accumulation of the said emission stock in the home country.

4. Calibration

The calibrated parameter selections are as follows. See Table 1. In accordance with the convention within the literature, we postulate that the Frisch elasticity ϕ equals 1. Additionally, we posit that the annual return of a riskless bond is approximately 4%, an assumption that yields a discount factor β of 0.99 (1/1.04 \approx 0.99). On the production side, we designate the share of capital α as 1/3. Furthermore, the rate of depreciation for capital δ_K is set at 0.025, equivalent to an annual depreciation rate of 10% (1 – (1.025)⁴ \approx 0.1). The parameter for the capital adjustment cost γ_I has been established as 0.5882, a value in compliance with Christensen and Dib (2008). Moreover, the elasticity of substitution between any two intermediate goods is $\theta = 11$, which implies that the markup earned by firms specializing in intermediate goods approximates 10%. The survival rate assigned to entrepreneurs γ is 0.97, signifying that the rate of entrepreneurial failure rests at approximately 3% per quarter.

The parameters related to the emission level and abatement effort closely accord with those established by Annicchiarico and Di Dio (2015). A quadratic damage function is adopted, represented as $\Gamma(M) =$

 $\gamma_0 + \gamma_1 M + \gamma_2 M^2$, where γ_0 , γ_1 and γ_2 are stipulated as $1.395e^{-3}$, $-6.6722e^{-6}$ and $1.4647e^{-8}$, respectively. The abatement cost function's parameters, denoted as ϕ_2 , are designated as 2.8. In the Calvo pricing, it is assumed that only one-fourth of intermediate firms could adjust their price every period, i.e., $\xi = 3/4$. Further, consistent with Christiano et al. (2014), the monitoring cost μ is deemed to be 0.21, signifying that it accounts for 21% of the average return of the entrepreneurs. The decay rate of the emission stock δ_M is assigned as 0.0021, whereas those pertaining to the marginal emission of production φ approximates 0.53, corresponding to the emission level (measured in kilograms) per PPP dollars of GDP in the United States, as previously mentioned. The scale of labor supply disutility μ_L is set as 1.

For the monetary and fiscal policies, we have established the elasticity parameters in the Taylor rule $e_{l_{\pi}} = 3$ and $\iota_{Y} = 0.25$, which are conventionally employed in the literature. The carbon tax rate p_z is simply set to 0 as a baseline scenario, while it is assigned different values in the numerical analysis below.

Regarding the shock process, we have adopted the approach of Annicchiarico and Di Dio (2015) to determine the persistence parameter ρ_A and the standard deviation σ_A of the TFP shocks, which are set at 0.95 and 0.0045, respectively. Moreover, for the sake of fair comparison, the persistence parameters and the standard deviation of the other shocks are all set to identical values of 0.95 and 0.0045, respectively.

Relative to the risk shock, we align with Annicchiarico et al. (2023) in matching a default rate in the brown sector of about 1.5%. Therefore, the steady-state value of $\sigma_{\sigma,B}$ is also set to be 0.42. As the green sector is considered riskier due to a new technology, or due to new customers for banks, the steady-state riskiness in the green sector is set to 0.55 to match a steady-state default rate of around 2%.⁶ The persistence parameters and the standard deviation of the other two shocks are assumed to be the same for a fair comparison. Specifically, we have set $\rho_{\eta} = \rho_{\sigma} = 0.95$ and $\sigma_{\eta} = \sigma_{\sigma} = 0.0045$.

Annicchiarico and Di Dio (2015) employed a specific approach to select ϕ_1 , which involved determining the carbon tax rate. This was done in order to ensure that the abatement cost to output ratio C_A/Y equals 0.15% when the emission rate to output ratio decreases by 20% due to an increase in the carbon tax rate from 0, while also setting the world emission level Z^* such that the steady-state world emissions stock M reaches 800. The scale of labor disutility μ_L was calibrated to maintain a steady-state value of labor at 1/3. All three targets had to be met simultaneously, leading to the findings of { ϕ_1 , Z^* , μ_L } = {1.17, 1.22, 1.10}. Finally, the share of the green sector is set to 0.30, reflecting the fact that activities in manufacturing, construction, aviation, maritime, agriculture and fisheries are still the larger polluters.⁷

5. Results

5.1. Optimal carbon tax in terms of financial stability

As an initial starting point, before implementing any stabilization policy, we want to identify the optimal carbon tax. As already shown in previous literature, a carbon tax aims at capturing the cost of emissions, and it consists an additional operating cost for polluting firms, which would consequentially reduce net earnings. (See Nordhaus and Yang (1996), Angelopoulos et al. (2010), Heutel (2012), Chan (2020), Zhao et al. (2020), and Lintunen and Vilmi (2021)). Consequentially, a too high carbon tax could impair the balance sheet of the polluting sector, with potential increase in the expected default probability from the banking's perspective. In response to expected higher default risk, banks increase lending rates in order to preserve banks' balance sheet strength. Table 1

The calibrated and estimated parameter values used for numerical analysis.

Parameters	Value	Description
α	1/3	Share of capital in production
β	0.99	Discount factor
δ_{κ}	0.025	Depreciation rate of capital
γ _I	0.5882	Parameter of capital adjustment cost
φ	1	Inverse of Frisch elasticity
θ	11	Elasticity of substitution within non-polluted
		and polluted goods sectors
ϕ_1	1.17	Parameter of abatement cost
ϕ_2	2.8	Parameter of abatement cost
γ ₀	$1.395e^{-3}$	Parameter of damage function
γ1	$-6.6722e^{-6}$	Parameter of damage function
Υ ₂	$1.4647e^{-8}$	Parameter of damage function
Z^*	1.22	Foreign emission level
<i>l</i> _π	3	Parameter of inflation gap
l _Y	1/4	Parameter of output gap
A	1	Steady-state of technology level
η	0.70	Size of brown sector
γ	0.97	Survival rate of entrepreneurs
κ	0.10	Reserve requirement ratio
σ_G	0.55	Steady-state of risk shock in green sector
σ_B	0.42	Steady-state of risk shock in brown sector
ν	0.75	Parameter of Calvo pricing adjustment
μ_L	1.10	Parameter of labor disutility
δ_M	0.0025	Depreciation rate of emission stock
φ	0.53	Marginal emission of production
μ	0.2	Monitoring cost rate
ρ_A	0.95	Persistence of technology shock
ρ_{η}	0.95	Persistence of monetary policy shock
ρ_{δ}	0.95	Persistence of capital depreciation shock
ρ_{σ}	0.95	Persistence of risk shock
σ	0.2588	Steady-state value of risk shock
σ_A	0.0045	Standard deviation of technology shock
σ_{δ}	0.0045	Standard deviation of capital depreciation shock
σ_{η}	0.0045	Standard deviation of monetary policy shock
σ_{σ}	0.0045	Standard deviation of risk shock

In determining the optimal carbon tax, we look at the maximum welfare value (and corresponding consumption equivalence) and at the medium-term impact on default. Fig. 2 shows that a more stringent carbon tax policy is an efficient tool to cut carbon emissions, but it is costly for the society. As a matter of fact, welfare decreases for any higher carbon tax rate, implies a larger compensation in terms of consumption that households would require to be better off as in the absence of carbon tax. Nevertheless, too high carbon taxes have implications for financial stability by increasing medium-term default rates in both sectors, which can potentially trigger a banking and financial crisis. Fig. 2 reports the mean value of the default rate for any given value of carbon tax. Given the set of shocks analyzed, which are both positive and negative, the model simulation replicates a positive default rate for both green and brown sectors, which tends to become smaller for larger carbon taxes. However, if a carbon tax is larger than 25%, the mean value of the default rate for both sectors start increasing again.

Under one-sector model, a carbon tax of 25% determines an emission cut of 37% and requires households to give up to 0.0876 of their consumption (see Fig. 2, left column), while a two-sectors model implies an emission cut of 45% and about 0.0731 of consumption equivalence (see Fig. 2, right column). Therefore, if a model features only the brown sector, as in Annicchiarico et al. (2023), a larger carbon tax would decrease emissions, as well as corporate default rate, while a two-sectors model would exacerbate default in both green and brown sectors for too high carbon taxes (i.e., pz > 25%).⁸ In line with these results, Fig. 2 reveals the importance of having a two-sectors models versus a one-sector model, in opposite to the results found in Annicchiarico et al. (2023).

 $^{^6}$ Christiano et al. (2014), the steady-state value of σ_a is also set to be 0.2588.

⁷ See https://www.c2es.org/content/renewable-energy/.

⁸ This result is robust to several parameters values. A relative Figure is available under request.



Fig. 2. Mean values

Fig. 3 shows that for all shocks considered, a carbon tax higher than 25% would lead to higher default rates in both sectors. However, if we consider only technology shocks as Carattini et al. (2023), we can notice that the mean value of default rates tends to decline with higher carbon taxes. Therefore, it is important to consider a larger set of shocks to support governments in setting their carbon tax.

5.2. Shocks propagation

This Section provides the simulation results for the capital depreciation and carbon price shocks for the green sector (blue solid line) and the brown sector (dashed black line) when the benchmark rate of carbon tax is set at 0.25. The model also simulates a negative technology shock, a positive monetary policy shock, and a risk shock, however, due to limit of space, they are provided in Appendix A.⁹

Fig. 4 presents impulse response functions of an exogenous physical compound risk shock that contemporaneously depresses the level of capital stock and the total factor productivity (TFP). As in Hallegatte et al. (2022) and Hashimoto and Sudo (2022), such physical risk shock can be interpreted as a disaster event that destroys part of the capital stock, but also reduce the productivity connected to that capital. In particular, Hashimoto and Sudo (2022) show that flood shocks, such as torrential rains, localized heavy rains and typhoons, can accelerate the depreciation rate of capital, but also can have an impact on public and social infrastructure, thus affecting the TFP after flood-induced depreciation.¹⁰ Similarly, Hallegatte et al. (2022) consider a typhoon as a compound shock that contemporaneously reduce the stock of capital and creates a misallocation of the remaining capita, leading to a decline in TFP. For instance, a typhoon destroys part of the capital stock, as well as road and infrastructures, thus reducing the capacity of the transport system. Consequently, the productivity of the road system will decline, with implications on the productivity of all other business and assets. Further, the shock to TFP captures the fact that factors of production are not working harmoniously together any more like before, such as workers getting not to work on time as a critical road is destroyed, or telephone lines are compromised.

Fig. 4 shows that an increase in the capital depreciation rate produces an immediate negative impact on capital, investment and the price of capital, due to the loss of physical capital from natural disaster events. As, the TFP is also affected, productivity in both sectors declines. Such economic slowdown leads to lower carbon emissions, as by construction they are generated during the production process in the brown sector. In such situation, the brown sector takes advantage of the lower abatement costs because of lower emissions, resulting in a less pronounced negative impact on output in the brown sector. The economic losses generated by the decline in capital stock are generally absorbed by the entrepreneurs in the goods-producing sector, typical lending activities through financial intermediation are obstructed through the impairments of asset values, which further reduces GDP and consumption. This results in a deleveraging process in both sectors, which leads to a declining default, as the change in the ex-post value of capital and of assets is now lower, generating a lower endogenous value of the threshold cut-off value, $\bar{\omega}$. However, banks cut lending is more pronounced in the green sector due to larger negative impact in profitability and default.

The compound shock acts as a negative supply-side shock with recessionary and inflationary effect, and becomes a systemic economy-wide macro-financial shock.¹¹

Fig. 5 shows results of impulse response analysis of a sudden increase in carbon tax. The blue solid line indicates responses for the green sector, while the dashed black line indicates responses for the brown sector. This exercise is in line with the announcement of many governments to implement more stringent climate policies. For instance, in 2024, Singapore's carbon tax will rise to Singapore dollars (SGD) 25 per tonne of emissions, up from 5 per tonne now. This will be raised to 45 per tonne of emissions in 2026, and eventually to between 50 and 80 per tonne of emissions by 2030. The IMF estimates that large emitting countries need to introduce a carbon tax that rises quickly to \$75 a ton in 2030, consistent with limiting global warming to 2 °C or less.¹² Fig. 5 highlights that a more stringent climate policy, in the form of higher carbon taxes, is able to curb emission with a certain degree of persistency, with a decrease of about 0.02% and 0.07% on impact and after 1 year, respectively. Thus is due to reduction in the production in the polluting sector, indicating that entrepreneurs prefer to cut production in order to avoid to pay carbon taxes. Consequently, capital, investment, asset prices, and consumption in the brown sector all decline. As a result, banks cut lending to the polluting sector, given the lower profitability and lower asset value. Similar to Ciccarelli and Marotta (2024), a carbon tax shock resembles a negative supply shock,

 $^{^{9}}$ See A.1, A.2 and A.3 for detailed impulse responses functions in the Appendix A.

¹⁰ Earthquake, flood and typhoons can induce power supply suspension, thus inducing a slower productivity process.

¹¹ In contrast, Ciccarelli and Marotta (2024) find that physical risks work as negative demand shocks. However, they explain that the response on inflation depends on the balance of supply and demand shocks, and the capital depreciation shock, coupled with slowdown in technological progress, are certainly supply-side shocks, which have a larger impact relative to the demand change caused by the physical shocks.

¹² See https://www.imf.org/en/Topics/climate-change/climate-mitigation.



Fig. 3. Mean values of default.

which leads to a positive inflation. The green sector suffers from the inflationary process, and cut consuming green goods, thus influencing negatively the production of green goods. Moreover, banks internalize the environmental-related risk, and to protect them from such risk, demand for higher interest rates to all customers. Such higher external cost of borrowing could compromise the ability to repay back, or force firms to invest less, generating a recession. This can occur through the banking capital channel (which induces banks to supply less loans, as higher default rates reduce bank capital), and a banking funding channel (which induces banks to charge higher lending rates in order to satisfy the participation constraint). These two banking channels generate a negative spillover effect from the brown to the green sector, as banks charge higher interest rates to the green sector to restore their balance sheet.¹³

6. Policy design: Monetary and macroprudential policies

This Section explores potential policies to be implemented by central banks and financial supervisors to contribute in mitigating the build-up of future climate-related shocks, and to increase the resilience of the financial system. Each suggested policy is constructed by capturing coefficients that minimize the consumption-equivalent welfare metric, which expresses the percentage change in consumption that households would like to be compensated to make them as well off as under a zero carbon tax. Table 2 summarizes the results based on each stabilization policy analyzed in the following Sections.

6.1. Green differential macrorpudential policy

Reserve requirements and countercyclical capital requirements are the most-used macro-prudential tools among respondents to the Financial Stability Benchmarks 2023. Of 29 central banks answering a

¹³ Similar result is found in Huang et al. (2021, 2022).



Capital Depreciation Shock





Fig. 4. Capital depreciation shock.

 Table 2

 Mean values corresponding to the maximum welfare

	Carbon Tax $P_z = 0$	Carbon Tax $P_z = 0.25$	Cyc $DRR (Y)$ $P_z = 0.25$ $\tau_g = 3.5$	Augmented MP $P_z = 0.25$ $i_Z = 0.43$	Dual Int. Rate $P_z = 0.25$ $\tau_r = 0.05$
			$\tau_b = 4$		
Welfare	47.9170	39.5141	40.1031	39.5369	39.6047
CE	0	-0.0806	-0.0752	-0.0804	-0.0798
Emissions	0.7528	0.4123	0.4162	0.4122	0.4143
Default G	2.0466	1.8951	2.1370	1.8925	1.8759
Default B	1.5083	1.3966	1.5762	1.3950	1.4952

question on which macro-prudential tools they use, 72.4% say they use reserve requirements, and the same proportion use countercyclical capital requirements.¹⁴ Some central banks use one of these tools, but not both. In all, 20.7% say that reserve requirements are the only macro-prudential tool they use. Given this fact, we concentrate our macroprudential analysis on reserve requirements, and consider higher reserve requirement ratio for the brown sector, and a lower ratio for the green sector. A higher reserve requirement means the central bank is pursuing a contractionary monetary policy. If banks have a higher reserve requirement, there will be less money available to lend to the brown sector. The opposite applies in the case of lower reserve requirement, which allows banks to have more liquidity to extend credit to the green sector. When distinguishing between banks with "green" and "brown" loan portfolios, the following holds: $0 < \kappa_G < \kappa_B$, where $\kappa_i(j = B, G)$ indicates the reserve ratio applied to loans in the brown and green sector. This setup implies that reserves held by commercial banks must be a fraction of customer deposits and notes:

 $Q_{j,t+1} = \kappa_{j,t} D_{t+1}$

with

$$\kappa_{j,t} = \kappa^{symmetric} + \tau_j \frac{Y_t}{Y_s s}$$
(54)

Eq. (54) indicates a countercyclical reserve requirement ratio that increases/decreases with the business cycle fluctuations, with τ_g and τ_g being the values that maximize welfare, and $\kappa^{symmetric} = 0.10^{.15}$ Table 2 shows that $\tau_g = 3.5$ and $\tau_g = 4$ lead a maximum welfare of 40.1031, which implies a consumption equivalence of about -8%, with a medium-term mean value of emissions of about 0.41, and default rate in the green and brown sector as about 2.13% and 1.58%, respectively.

6.2. Climate-augmented monetary policy

This policy exercise concerns the implementation of an extended Taylor rule, similar to George et al. (2022) and Leduc and Natal (2018). This climate-augmented tool suggests that the central bank will tight monetary policy when the emission gap target tends to increase following specific physical or transition shocks. Higher policy rate would make credit cost more expensive, thus discouraging and contracting production. As emissions are proportional to the volume of output of brown intermediate firms, a contraction in lending, investment and production will materialize in lower emissions.

Although central banks around the world have not implemented such kind of radical policy, they are realizing that climate changes and climate-related policies can impact the dynamics of inflation. Indeed, Figs. 4 and 5, which are examples of both physical and transition risk shocks, show a clear impact on inflation in the aftermath of shocks. Further, climate change, and related policies, could constrain the conventional monetary policy space by lowering the equilibrium real rate of interest, which balances savings and investment. In line with these considerations, central banks are starting to integrate climate-related issues into their monetary policy operations. Attílio et al. (2023) show that a monetary contraction in a country is associated with lower domestic emissions both in the short- and the medium-run. Similarly, Kim and Park (2023) find that a contractionary monetary policy surprise leads to a reduction in overall emissions by approximately 0.5 percent

¹⁴ https://www.centralbanking.com/benchmarking/financial-stability/ 7959500/reserve-requirements-and-countercyclical-buffers-are-mostcommon-macro-pru-tools.

¹⁵ Similarly, Giovanardi and Kaldorf (2023) consider a capital requirement that responds to changes in carbon taxes, indicating that it is activated in the presence of carbon tax surprise. Following this intuition, we also analyze the welfare implications for a reserve requirement that changes in response to carbon tax shocks.



Fig. 5. Carbon tax shock.

in the U.S. Thus, we consider a monetary policy that sets the interest rate accordingly to the following rule:

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{l_\pi} \left(\frac{Y_t}{Y}\right)^{l_Y} \left(\frac{Z_t}{Z}\right)^{l_Z} v_t \tag{55}$$

Table 2 shows that welfare is at its maximum point when $\iota_Z = 0.43$. Chen et al. (2021) analyze the role of central bank in climate change under the lens of an E-DSGE model, and find that, under a carbon tax policy, the coefficient of the augmented monetary policy ranges between -0.091 and -0.062. Our coefficient is higher relative to Chen et al. (2021) because our model features endogenous default, which amplifies the business cycle fluctuations, and we analyze not only standard shocks, but also specific physical and transition risk shocks.

6.3. Dual interest rate

A recent policy that has been advocated around policy makers is a dual interest rate policy to drive green investments. Hence, a monetary policy framework that shields green investments from interest rate hikes will offer banks refinancing at preferential rates for financing clean investments in the real economy. Similar policy has already been implemented in the past. For instance, between 1945 and 1973, a lower costs of credit was offered to Western economic and industrial policy. Most recently, the Bank of Japan (KOJ) has started offering zerointerest loans to lenders that finance climate change projects, believing that such action is in line with its mandate of price and financial stability.

In this Section, we examine the efficacy of implementing dual interest rates as a novel green unconventional monetary policy tool, meaning that monetary policy targets independently the green sector by allowing the interest rate set by the Taylor rule to respond to inflation, output gap and changes in carbon emissions. Therefore the following monetary policy rules apply:

$$R_{t} = R \left(\frac{\Pi_{t}}{\Pi}\right)^{l_{\pi}} \left(\frac{Y_{t}}{Y}\right)^{l_{Y}} v_{t}$$
(56)

and

$$R_t^{Dual} = R \left(\frac{\Pi_t}{\Pi}\right)^{l_{\pi}} \left(\frac{Y_t}{Y}\right)^{l_{Y}} \left(\frac{Z_t}{Z}\right)^{l_{Z}} v_t$$
(57)

which implies the following lending rates:

$$R_{B,t+1}^{l} = \frac{R_t - \kappa_{B,t}}{1 - \kappa_{B,t}}$$
(58)

and

$$R_{G,t+1}^{l} = \frac{R_{t}^{Dual} - \kappa_{G,t}}{1 - \kappa_{G,t}}$$
(59)

Table 2 shows that welfare is maximized when the dual interest rate applied to green lending responds to changes in carbon emission by a coefficient equal to 0.05. This would imply a welfare cost of almost 8%, with medium-term values of emissions and default in green and brown sectors of 0.4143, 0.1859 and 1.4952, respectively.

6.4. Shock propagation under different policies

Figs. 6 and 7 reports impulse responses to a capital depreciations and carbon price shocks at any policies suggested above, with estimated parameters that minimize the welfare cost. Both Figures report production, consumption and default rates for each sector, as well as emissions and inflation rate for the overall economy. Fig. 6 shows that under a carbon tax of 25% (solid black line), production in the green sector and in the brown sector decline by 0.6% and 0.4%, respectively, when a capital depreciation shock hits the economy. Production is affected in both sectors as the unpredictable weather events, that destroy part of the capital, have a widespread impact on the economy. However, prices in the green sector rise much more relative to the brown sector, due to a higher response of the marginal cost. This leads to a more pronounced negative response in consumption, as green goods become more expensive. Therefore, green firms cut more their production. Due to the economic slowdown, the debt amount becomes a burden on the company in both sectors, thus firms undertake deleveraging to reduce the impact of market volatility on the company's balance sheet. This reduces the default rate, not because of the better ability to repay loans, but because of lower supply of credit and because of the devaluation of the value of assets. Overall, emissions are reduced by about 0.17% on impact due to lower production in the polluting sector. As the physical shock acts as a negative supply shock, overall inflation increases. When we analyze policies to support the green transition without compromising financial stability, a countercyclical green reserve requirement policy help reducing the negative impact on productivity. However, such policy translates in a slower decarbonization of the economy when a faster depreciation of capital spreads around the economy due to weather-events. Moreover, although the default rate is still negative, the impact is smaller, indicating that relative to the case in which macroprudential policy is absent, some firms default. Therefore, under this shock, a countercyclical green reserve requirement policy can help reducing the economic slowdown, but it is not able to preserve financial stability. The climate-augmented monetary policy on medium and longer-term generates impulse responses similar to the case of only environmental policy enforcement (i.e., sudden carbon tax increase). However, on impact, a central bank, that lets the policy rate to respond to changes in carbon emissions, helps in curbing default rates at the cost of higher inflation. Finally, the recent advocated dual interest rate policy can support the managed phase-out of the brown sector, at the cost of higher default rate on impact in the green sector.

Fig. 7 simulates the responses of a sudden increase in carbon tax. Carbon taxes are one of the most powerful and efficient tools to mitigate climate change. Indeed, Fig. 7 shows that a more stringent carbon tax policy results in a lower level of emissions of about 2% on impact, and a larger decline of about 7% after 2 years. The more stringent policy supports the phase-out of fossil fuels, especially in the medium run, as the production in the brown sector decreases even more after 2 years. However, through the banking sector, which increases lending rates to both sectors for the safeguard of its balance sheet, there is a negative spillover effect on the green sector, which sees a slowdown but it rebounds quickly and becomes positive after a few quarters. In terms of fossil fuel phase-out and promoting the green transition, a dual interest rate policy appears to be the most efficient, but not in terms of financial stability, as the default rates in the green sector show a larger positive response in the medium and longer-term, relative to the absence of policy. The climate-augmented monetary policy is the best policy able to contain the propagation of default rates in both sectors together with a smooth green transition. However, such a policy will pay the consequence of high inflation.

Overall, a dual interest rate policy can help promoting the green transition but it is not able to safeguard financial stability during climate change. However, if carbon tax revenues are redistributed as vouchers to be used to buy green consumption good, then financial stability can be achieved by lowering the default rate, as shown in Fig. 8. Therefore, a combination of a top-down policy, implemented by policy-makers in the form of carbon tax and monetary policy, with a bottom-up approach, which aims to incentive the consumption of green goods, results to be the best policy in promoting the transition to net zero with a smooth phase-out of fossil fuel without compromising financial stability.

7. Conclusions and discussion

Since the Paris Agreement in 2015, countries around the world have made an international agreement on limiting global heating to well below 2 °C, as it has clearly proved that greenhouse gas emissions must end within years to avoid catastrophic and potentially permanent climate change and instability. Although meaningful progress has been made, the ambitious target of 2 °C, and preferably of 1.5 °C, seems very hard to reach. Moreover, besides the huge problem of ecological disaster, the financial sector is also at risk following adverse climate events and climate-related policies.

Under this view, awareness has recently been increased about the important role that central banks and financial supervisors can play in allocating resources to sustainable investments in line with the mandate of inflation and financial stability, as well as stopping financing activities that harm the environment. Indeed, a growing number of central banks and regulators in developing and developed economies have already started to recognize and evaluate risks which climate change may pose to monetary policy, financial stability and regulated entities. As a result, some central banks have already set monetary and/or macroprudential policy to facilitate the green transition when preventing financial instability. For example, the People's Bank of China has implemented differential reserve requirements in favor of green credit since 2018. Similarly, the Bank of Lebanon has employed differential reserve requirements since 2010, with the goal of influencing the allocation of credit in favor of investment in renewable energy and energy efficiency. Since 2021, the Central Bank of Hungary has introduced preferential capital requirements for green corporate and municipal financing. In December 2023, the Philippine central bank has approved a gradual reduction in the reserve requirement rate for green, social, sustainability, and other sustainable bonds issued by banks to zero from 3%.

Against this backdrop, this paper implements an environmental dynamic stochastic general equilibrium (E-DSGE) model with two sectors (green and brown) and endogenous default to assess potential climateinduced financial stability threats. Through physical risk, resulting in a damage from capital depreciation shocks, and through transition risk, that follows from more stringent environmental policies, transition to net-zero is difficult to be achieved, with large consequences in terms of financial stability due to a larger increase in default rates. Considering this problem, the paper examines if central banks and financial regulators can help make the economy better able to adapt to the adverse climate events and boost the funding of the investments needed to support sustainable growth. Thus, the paper compares the effectiveness of the design of various macroprudential and monetary



Fig. 6. Capital depreciation shock.



Fig. 7. Carbon tax shock.

policies in terms of financial stability and green transition. For the macroprudential frameworks, a green differentiated reserve requirement is implemented by allowing banks to hold fewer reserves against green loans. Concerning monetary policy, the paper emphasizes the use of a climate-augmented Taylor rule in setting the policy rate and the adoption of a dual interest rate approach. The paper highlights a trade-off between green transition and financial stability. On one hand, a policy that encourages the green transition compromises financial stability. Green differentiated countercyclical reserve requirements and the climate-augmented monetary policy produce such an effect. On the other hand, under a policy that aims to reduce vulnerabilities and

financial stability risks, the phase-out of the polling sector to foster the green sector would be difficult to achieve. A carbon tax policy or a dual interest rate policy belongs to this last case. As a result, it is difficult to address climate change and promote the green transition without either hurting macroeconomic stability and growth or financial stability. The global shifts in demand and capital allocation are costly, complex, and disorderly. It is very difficult to tackle climate change without magnifying the risks to the financial system. In line with this, a policy mix that combines top-down and bottom-up approaches are the keys for sustainability in the whole economy and financial system. Thus, a dual interest rate policy coupled with a fiscal policy in which



Fig. 8. Shocks propagation under voucher.

carbon tax revenues are redistributed to households as vouchers to encourage the consumption of green goods, is able to support the green transition and safeguard financial stability.

The paper shows that by being proactive, central banks can influence the behavior of financial institutions in promoting the allocation of funds towards environmentally friendly projects. However, while central banks officially recognize that the climate change and the degradation of nature are drivers of financial risks and price instability, their actions to deal with these threats still remain limited. As a result, there is an ongoing dispute on to which extent central banks should become more proactive in promoting green investment and disincentivising dirty investments. This is due to the fact that for central banks might be difficult to achieve too many objectives with too few tools. Thus, balancing climate finance and financial stability is a scorching dilemma. Adding environmental goals to their mandate would require extra effective instruments in order to achieve these goals without compromising the safeguard of macroeconomic and financial stability. Therefore, many central bankers think that it is important not to overburden central banks. However, during periods of high interest rates, central banks should consider the use of green dual interest rate policy. Indeed, higher rates would hit investments in renewable energy, thus aggravating the fight against climate change and the protection of the economy against the volatility of fossil fuel prices. For this reason, it is fundamental to create more flexible instruments for central banks and financial supervisors to have at their disposal to impact green lending and make environmentally friendly investment decisions. Nevertheless, this paper suggests an important call for collaboration from the public and private sectors to come together to accelerate investment in and action for climate adaptation and resilience. Green vouchers are an example of how private and public sectors can come together and further create enabling conditions for promoting the green transition.

This paper offers several extensions. First of all, it could include financial investment in research and development (R&D) to encourage the phase-out of fossil fuels in favor of green production. Moreover, the model should include a transformational insurance sector to compensate for R&D costs of energy producers who fail in their energy transition attempts. In terms of monetary policy, green quantitative easing (QE) should be considered as an alternative instrument for central banks to be used to promote green transition. All these ingredients are a valuable direction for future research.

CRediT authorship contribution statement

Ying Tung Chan: Writing – review & editing, Visualization, Software, Methodology. **Maria Teresa Punzi:** Writing – original draft, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Hong Zhao:** Writing – review & editing, Visualization, Software, Methodology.

Appendix A. Derivation of Eqs. (39) and (40)

Denote as $N_{i,t+1} = R_{i,K,t+1}q_{i,t}K_{i,t+1}(\Gamma_{i,t+1} - \mu H_{i,t+1})$, $\Gamma_{i,t+1} = \int_0^{\bar{\omega}_{i,t+1}} \omega f_t(\omega)d\omega + \bar{\omega}_{i,t+1}\int_{\bar{\omega}_{i,t+1}}^{\infty} f_t(\omega)d\omega$, and $H_{i,t+1} = \int_0^{\bar{\omega}_{i,t+1}} \omega f_t(\omega)d\omega$, where $f_t(\omega)$ is the probability density function of the log normal distribution $\ln N(-\sigma_{i,t}^2/2, \sigma_{i,t})$.

The objection function (38) and the constraint (36) can be written as

$$(1 - \Gamma_{i,t+1})R_{i,K,t+1}q_{i,t}K_{i,t+1}$$
(A.1)

and

$$R_{i,t+1}^{l}(K_{i,t+1}q_{i,t} - N_{i,t+1}) = R_{i,K,t+1}q_{i,t}K_{i,t+1}(\Gamma_{i,t+1} - \mu H_{i,t+1})$$
(A.2)

respectively. Set up the Lagrangian function

$$\mathcal{L}_{i} = (1 - \Gamma_{i,t+1}) R_{i,K,t+1} q_{i,t} K_{i,t+1} - \lambda (R_{i,t+1}^{l} (K_{i,t+1} q_{i,t} - N_{i,t+1}) - R_{i,K,t+1} q_{i,t} K_{i,t+1} (\Gamma_{i,t+1} - \mu H_{i,t+1}))$$
(A.3)

where λ_i is the Lagrange multiplier. Differentiating Eq. (36) with respect to $K_{i,t+1}$, $\bar{\omega}_{i,t+1}$, and λ_i give the first order conditions

$$(1 - \Gamma_{i,t+1})R_{i,K,t+1}q_{i,t} = \lambda_i (R_{i,t+1}^l q_{i,t} - R_{i,K,t+1}q_{i,t}(\Gamma_{i,t+1} - \mu H_{i,t+1}))$$
(A.4)



Fig. A.1. Monetary policy shock.

$$(1 - F_t(\bar{\omega}_{i,t+1}))R_{i,K,t+1}q_{i,t}K_{i,t+1} = \lambda_i R_{i,K,t+1}q_{i,t}K_{i,t+1}(1 - F_t(\bar{\omega}_{i,t+1})) - \mu \bar{\omega}_{i,t+1}f_t(\bar{\omega}_{i,t+1}))$$
(A.5)

and Eq. (A.2) respectively. $F_t(.)$ and $f_t(.)$ are the cumulative distribution function and the probability density function of the log normal distribution $\ln N(0, \sigma_t)$ respectively. To eliminate λ_i , divide Eq. (A.4) by Eq. (A.5) gives

$$\frac{(1 - \Gamma_{i,t+1})s_{i,t}}{(1 - F_t(\bar{\omega}_{i,t+1}))} = \frac{1 - s_{i,t}(\Gamma_{i,t+1} - \mu H_{i,t+1})}{1 - F_t(\bar{\omega}_{i,t+1}) - \mu \bar{\omega}_{i,t+1} f_t(\bar{\omega}_{i,t+1})}$$
(A.6)

where $s_{i,t} = R_{i,K,t+1}/R_{i,t+1}^l$. By using the property of the lognormal distribution, $F_t(\bar{\omega}_{i,t+1}) = \Phi(z_{i,t+1})$, where $z_{i,t} = (\log(\bar{\omega}_{i,t}) + \sigma_{i,t}^2/2)/\sigma_{i,t}$. And we have $H_{i,t+1} = \Phi(z_{i,t+1} - \sigma_{i,t})$.

Eq. (A.2) is basically Eq. (40). Rearranging the terms in Eq. (A.6) gives Eq. (39).

Shocks

$$\ln A_{i,t} = (1 - \rho_A) \ln A + \rho_A \ln A_{i,t-1} + \varepsilon_{A,i,t}$$
(A.7)

$$\ln \eta_{i,t} = \rho_{\eta} \ln \eta_{i,t-1} + \varepsilon_{\eta,i,t} \tag{A.8}$$

$$\ln(\sigma_{i,t}/\sigma) = \rho_{\sigma} \ln(\sigma_{i,t-1}/\sigma) + \varepsilon_{\sigma,i,t}$$
(A.9)

And the carbon tax rate $p_{Z,t}$ is constant and set to be 1%, 5%, and 15% in the calibration.

See Figs. A.1-A.3.



Fig. A.2. Monetary policy shock.





Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.eneco.2024.107449.

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