12-2024

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With the popularity of on-demand ride services worldwide, ride-sourcing platforms must maintain an adequate fleet size and cope with growing travel demand. Recently, platforms have attempted to provide vehicle rental services to drivers who do not own cars, then recruited them to provide on demand ride services. This helps lower the entry barrier for drivers and offers another profitable business for platforms. From the government's perspective, however, it is challenging to coordinately regulate a ride-sourcing business and vehicle rental business. This paper proposes a bi-level optimization model to investigate how the government regulates the ride-sourcing market integrated with vehicle rental services. Specifically, how the government designs regulatory policies for minimum driver wage and maximum vehicle rental fee at the upper level, and how a monopoly profit-oriented platform optimizes riders' price, drivers' wage, and vehicle rental fee at the lower level. We derive an analytical phase diagram for the two policies and present the government's decisions in five mutually exclusive regions with respect to regulatory effects, i.e., ineffective region, minimum-driver-wage-effective region, maximum-rental-fee-effective region, coordinated policy region, and infeasible region. Our theoretical and numerical results indicate that the government should precisely coordinate the two policies to achieve higher total social welfare, i.e., the weighted sum of rider surplus, driver surplus, and platform profit. We also prove that if the weights of all stakeholders in social welfare are equal, the platform's vehicle rental business will achieve zero profit when the total social welfare is maximized. The proposed model and analytical results generate managerial insights and provide suggestions for government regulation and platform operations management in the ride-sourcing market integrated with vehicle rental services.

Key words: Ride-sourcing, vehicle rental service, bi-level optimization, regulatory policies, social welfare

1. Introduction

Driven by mobile internet communication technologies, the last decade has witnessed popularity of transportation network companies (TNCs) or ride-sourcing (RS) platforms that provide on-demand ride services and improve the travel experience for passengers. When platforms encounter surging demand and cannot dispatch sufficient nearby vehicles to serve riders, service efficiency will deteriorate due to higher matching frictions between drivers and riders, which is defined as the "wild goose chase" (WGC) phenomenon, which was introduced by Castillo, Knoepfle, and Weyl (2017) and further explored by Zha, Yin, and Xu (2018) and Zhou et al. (2022). Thus, it is essential that platforms maintain availability of active RS drivers and an adequate fleet size.

Recently, platforms have attempted to attract potential RS drivers who do not own a car (hereafter "car-renting drivers") by providing vehicle rental services and, consequently, join the platform, similar to the strategy of taxi companies. For example, Uber drivers can rent cars weekly at Avis, Hertz, and KINTO with a low and refundable deposit (Uber 2022). Lyft, cooperating with Flexdrive and Hertz, allows non-car owners to rent hybrid and electric vehicles through the "Express Drive" program without long-term contracts (Lyft 2022). In addition, some platforms provide rental services directly. In Singapore, RS drivers on the Grab platform can rent vehicles through GrabRentals (directly owned and operated by Grab) for a minimum rental period of six months (Grab 2022). In China, Shenzhou supports the monthly rental of various vehicle models, and the rental fee can be reduced when the driver's service performance on the platform reaches a given standard (Shenzhou 2022). CaoCao Mobility, a TNC subsidiary to automobile manufacturer Geely, provides a unified electric vehicle model made by Geely to non-car owners (CaoCao 2022). Other Chinese TNCs, such as T3Go (T3Go 2022) and Shouqi, (Shouqi 2022) have also operated self-owned vehicle rental services for RS drivers.

The worldwide adoption of vehicle rental services indicates that TNCs benefit from integrating vehicle rental services into the RS market. Platforms can lower entry barriers for drivers and expand their fleet size to serve more riders, and platforms may gain a profit from the vehicle rental business by determining an appropriate rental fee. However, it also poses new challenges to government regulation of the RS market. For example, if the government solely regulates the minimum wage of drivers, the platform may raise the vehicle rental fee to compensate for the rising cost of hiring drivers, which undermines the government's aim to improve drivers' surplus. Therefore, it is critical to investigate government regulation of the RS market when integrated with vehicle rental services.

To tackle the regulatory challenges, we propose a bi-level optimization model to characterize the Stackelberg game between the government and a monopoly TNC that operates its own vehicle rental business. The government plays the role of leader and regulates the minimum driver wage and maximum rental fee to improve social welfare, while the platform acts as a follower and aims to maximize its profit by optimizing riders' price, drivers' wage, and vehicle rental fee.

The contributions of this research are as follows. (a) In a monopoly RS market in which the platform not only employs car owners but also hires non-car owners by renting its own fleet, we analytically derive market properties and the platform's profitability for both the RS service and vehicle rental service in a monopoly optimum state. (b) We explore the government's coordinated regulatory policies on the minimum driver wage and maximum rental fee to maximize social welfare (i.e., the weighted sum of rider surplus, driver surplus, and platform profit), and delineate an analytical phase diagram of the two policies. Specifically, the decision space of the two policies can be divided into five regions with different regulatory effects: ineffective region, minimum-wage-effective region, maximum-rental-fee-effective region, coordinated policy region, and infeasible region. (c) We detail the analytical properties of different policy regions and derive the closed-form optimal policies under certain conditions. Our theoretical results show that when the government only relies on one policy (e.g., minimum driver wage) but loosely regulates another policy (e.g., maximum rental fee), it might not simultaneously improve the surplus of both types of drivers (i.e., car owners and non-car owners). Moreover, when the weights of rider surplus, driver surplus, and platform profit are equal in the social welfare objective, the platform's revenue from vehicle rental fees equals its vehicle operating costs, that is, the vehicle rental business achieves zero profit. The proposed bi-level optimization model and analytical results generate managerial insights and provide suggestions for government regulation and the platform's operations in an RS market integrated with vehicle rental services.

The remainder of the paper is organized as follows. Section 2 reviews the related literature on the management and regulation of the RS market and vehicle rental market. Section 3 formulates and analyzes a monopoly RS market without government regulation. We propose a bi-level optimization model to identify optimal regulatory policies in Section 4, followed by numerical experiments and discussions in Section 5. Section 6 draws conclusions and provide an outlook for future research.

2. Literature Review

In the literature, the RS market has been extensively studied from various perspectives, e.g., system equilibrium (Zha, Yin, and Yang 2016, Xu, Yin, and Zha 2017, Bai et al. 2019, Sun et al. 2019, Xu, Yin, and Ye 2020, Bai and Tang 2022), competition and coopetition between platforms (Zhong et al. 2019, Bernstein, DeCroix, and Keskin 2021, Zhou et al. 2022, Siddiq and Taylor 2022), spatio-temporal pricing and subsidies (Nourinejad and Ramezani 2020, Chen et al. 2020, Yang et al. 2020b, Chen et al. 2021, Zhu, Ke, and Wang 2021, Xu, Saberi, and Liu 2022), matching, dispatching, and repositioning (Wang, Agatz, and Erera 2018, Braverman et al. 2019, Lyu et al.

2019, Yang et al. 2020a, Kullman et al. 2022, Dong et al. 2022), supply behavior and elasticity (Chen et al. 2019, Sun, Wang, and Wan 2019, Angrist, Caldwell, and Hall 2021), impact on road network congestion (Ke, Yang, and Zheng 2020, Xu et al. 2021, Dhanorkar and Burtch 2022), and adoption of electric vehicles (Bauer et al. 2019, Ke et al. 2019, Mo, Yu, and Chen 2020, Cai et al. 2023). Readers please refer to Wang and Yang (2019) for a comprehensive review.

Our work is closely related to two streams of the literature: regulation of the RS market and RS service with non-car owners who are offered rental vehicles.

Various studies have investigated the adoption of regulatory or incentive policies in the RS market to improve social welfare, promote vehicle fleet electrification, maintain marketplace stability, mitigate traffic congestion, maximize matching efficiency, better integrate RS service with public transit, and improve drivers' service quality, as summarized in Table 1. Since the RS services of different platforms vary significantly in terms of vehicle type (e.g., electric or gasoline), induced congestion externality, drivers' labor contract (e.g., paid per order or per unit time), service quality, etc., the regulatory measures adopted in different studies also vary and include, but are not limited to, commission cap (Zha, Yin, and Yang 2016, Zha, Yin, and Du 2018, Vignon, Yin, and Ke 2021); congestion tax (Vignon, Yin, and Ke 2021, Wei et al. 2020a, Li et al. 2019); regulation of driver service quality (Li et al. 2022), fleet size (Yu et al. 2020, Ke et al. 2021, Wang and Huang 2022) and rider price (Liu et al. 2022, Qin, Ke, and Yang 2022); electrified subsidies (Yang, Dong, and Hu 2018, Slowik, Wappelhorst, and Lutsey 2019); and minimum driver wage per hour or order (Li et al. 2019, Ke et al. 2021, Asadpour, Lobel, and van Ryzin 2022). Although the RS market in which TNCs or third-party companies offer rental vehicles to non-car drivers has only emerged in recent years, it has been widely adopted in different countries and regions, as noted in Section 1. Thus, it is necessary to investigate the regulation of an RS market integrated with vehicle rental services. Specifically, this research deploys regulatory policies of both the minimum driver wage and maximum vehicle rental fee.

In addition, Du et al. (2020) demonstrate that more than 80% of RS vehicles and more than 87% of electric RS vehicles were owned by platforms or other car rental companies in Shenzhen, China. They also find that car-owning drivers were less loyal to platforms and more likely to quit their jobs compared with car-renting drivers. Wei et al. (2020b) examine how a monopolistic TNC operates two services: a pooling service provided by private car owners and a premier service provided by self-operating non-car employees. The platform maximizes its profit by optimizing heterogeneous rider prices for the two services, while the number and operating costs of self-operating vehicles are assumed to be constant. Mo, Yu, and Chen (2020) and Zhao et al. (2022) investigate how the government subsidizes electric vehicles to promote benign competition in a duopoly RS market, in which one platform leases electric vehicles for free to non-car owners, while the other light-asset

Table 1 Summary of related studies on ride-sourcing market regulation

Reference	Regulatory or incentive policies	Objective
Zha, Yin, and Yang (2016), Zha, Yin, and Du (2018)	Commission cap	Achieving second-best scenario, i.e., maximizing social welfare while maintaining the platform profit
Vignon, Yin, and Ke (2021)	Congestion toll, commission cap on the solo service	
Li et al. (2019)	Minimum hourly wage, trip-based congestion tax, maximum fleet size	Improving the surplus of riders and drivers
Yu et al. (2020)	Driver's entry fee and a maximum number of eligible drivers	Maximizing total welfare of the TNC, traditional taxi platform, drivers, and riders
Wei et al. (2020a)	Adaptive congestion pricing for both ridesharing and solo-driving cars	Reducing traffic congestion
Ke et al. (2021)	Upper (lower) bound of rider price, fleet size, driver's income, commission rate, and car utilization rate	Pareto improvement of maximum profit and maximum social welfare
Yang, Dong, and Hu (2018)	Subsidy on purchasing electric vehicles and charging cost	Minimizing subsidies while guaranteeing a certain penetration of electric vehicles
Slowik, Wappelhorst, and Lutsey (2019)	Taxes and operating fees for conventional vehicles	Promoting electrification
Liu et al. (2022)	Annual permit fees, differential rider price and commission caps	Reducing emissions and improving social welfare
Asadpour, Lobel, and van Ryzin (2022)	Utilization-based minimum earnings	Sustaining stability of RS market
Li et al. (2022)	Service quality threshold of driver's admission	Maximizing profit/social welfare and improving RS service quality
Wang and Huang (2022)	Controlling the number of drivers	Maximizing matching efficiency
Qin, Ke, and Yang (2022)	Maximum price rate of RS service, subsidy for travelers choosing mixed transit and RS mode	Balancing the interest of RS and public transit market

platform only hires car-owners. Ni et al. (2021) study the competitive network equilibrium between light-asset and heavy-asset platforms, which bear vehicles' operating costs. Lin et al. (2021) develop a leader-follower game model to characterize the cooperation between a monopoly TNC (acting as the leader) and a car-rental company (acting as a follower), which is more in line with the case in North America (Uber 2022, Lyft 2022). When driver supply is in relative shortage, they find that the cooperative scenario could easily achieve a win-win-win outcome for TNC, riders, and drivers. In this paper, we study the situation in which the car rental service is provided by TNCs, which is the case in Asia (Grab 2022, Shenzhou 2022, CaoCao 2022). Overall, the literature has not explored coordinated regulatory policies for both vehicle rental service and RS service, although the two are closely related to drivers' surplus, which impacts driver supply and further impacts riders' surplus. Our paper fills this gap by formulating a bi-level optimization model and gaining managerial insights into the regulation of the two services.

3. Modeling a Monopoly Ride-Sourcing Market Integrated with Vehicle Rental Services

Before formulating the model, we list the primary notation in Table 2. In a monopoly RS market, the platform hires two types of drivers: those participating in the platform with their vehicles (i.e., car-owning RS drivers) and those adopting platform-renting vehicles (i.e., car-renting RS drivers). Car-renting drivers pay vehicle rental fees per unit work time to the platform (denoted as d_a) and bear an opportunity cost of giving up other jobs (denoted as o_a). Car-owning drivers bear depreciation and operating costs of their cars per unit work time (denoted as d_b and regarded as an exogenous parameter) and opportunity cost o_b. We denote r as the wage per unit work time for both types of drivers. The utilities of car-renting and car-owning drivers, denoted as u_a and u_b, respectively, can be expressed as follows:

$$u_a = r - d_a - o_a \tag{1}$$

$$u_b = r - d_b - o_b \tag{2}$$

Further, the numbers of car-renting drivers (i.e., N_a) and car-owning drivers participating on the platform (i.e., N_b) are determined by

$$N_a = \bar{N}_a f_a(u_a) \tag{3}$$

$$N_b = \bar{N}_b f_b(u_b), \tag{4}$$

where $f_a \in [0,1]$ and $f_b \in [0,1]$ are increasing functions of u_a and u_b, respectively, and assumed to be second-order differentiable. We then denote $N = N_a + N_b$ as the total number of drivers.

RS riders are assumed to be sensitive to price p, in-vehicle time T, and waiting time W. Their demand rate λ is formulated as

$$\lambda = \bar{\lambda} f_\lambda(u_\lambda), \quad u_\lambda \triangleq p + \alpha_t T + \alpha_w W - \bar{u}, \tag{5}$$

where $f_\lambda \in [0,1]$ and is a decreasing function of u_λ with $\partial f_\lambda / \partial u_\lambda < 0$. u_λ represents riders' cost or disutility to choose the RS mode, in which α_t and α_w represent riders' value of in-vehicle time and

Table 2 Mathematical notation

Exogenous parameters	Description
α_t	Riders' value of in-vehicle time
α_w	Riders' value of waiting time
c	Depreciation and operating costs per unit time of a rental vehicle, borne by the platform
$\bar{\lambda}$	Arrival rate of potential riders
d_b	Depreciation and operating costs per unit work time of car-owning driver's vehicle
$\bar{N}_a, \bar{N}_b$	Numbers of potential car-renting and car-owning drivers
o_b, o_a	Opportunity costs of car-renting and car-owning drivers
T	In-vehicle time of riders
Endogenous variables	**Description**
λ	Arrival rate of RS riders
N_a, N_b	Numbers of car-renting and car-owning drivers
W	Waiting time for pick-up of riders
N_v	Number of vacant vehicles
Platform's strategies	**Description**
d_a	Rental cost of a vehicle charged by the platform per unit work time
p	Price for riders
r	Wage per unit work time of drivers
Government's decisions	**Description**
$\bar{d}_a$	Upper bound of vehicle's rental cost, set by the government
$\underline{r}$	Lower bound of drivers' wage, set by the government
Functions	**Description**
$f_a(u_a)$	Probability for potential car-renting drivers to provide service, increasing with their utility u_a, i.e., $\partial f_a/\partial u_a > 0$
$f_b(u_b)$	Probability for potential car-owning drivers to provide service, increasing with their utility u_b, i.e., $\partial f_b/\partial u_b > 0$
$f_\lambda(u_\lambda)$	Probability for potential riders to choose RS mode, decreasing with riders' disutility u_λ, i.e., $\partial f_\lambda/\partial u_\lambda < 0$
PR	Platform profit
SW	Total social welfare

waiting time, respectively, and the exogenous constant $\bar{u}$ represents riders' perceived utility of the RS mode. Riders' waiting time for pick-up W is assumed to be a decreasing and convex function of vacant RS vehicles N_v, i.e., $W' = \mathrm{d}W/\mathrm{d}N_v < 0$, $W'' = \mathrm{d}^2W/\mathrm{d}N_v^2 > 0$, inspired by the empirical evidence of Castillo, Knoepfle, and Weyl (2017).

Our paper does not consider congestion externalities of RS service; thus, in-vehicle time T can be regarded as an exogenous constant. According to Little's law, the expected number of RS vehicles in service equals riders' demand rate multiplied by the service time of a trip, i.e., $\lambda \cdot (T+W)$:

$$N_v = N_a + N_b - \lambda \cdot (T+W), \tag{6}$$

where N_v is the number of vacant RS vehicles.

Given platform strategies (i.e., price p for riders, wage r for drivers, and vehicle rental fee d_a), market equilibrium can be determined by Eqs. (1-6).

Since W is regarded as a decreasing function of N_v, the relationship between service rate (i.e., demand rate) λ and vacant vehicles N_v can be rewritten as

$$\lambda = \frac{N_a + N_b - N_v}{T + W(N_v)}$$

Thus, for a given driver supply $N_a + N_b$, we have

$$\frac{\mathrm{d}\lambda}{\mathrm{d}N_v} = -\frac{1+\lambda W'}{T+W}$$

Therefore, $\mathrm{d}\lambda/\mathrm{d}N_v > 0$ if $W' < -1/\lambda$. Riders' waiting time decreases rapidly with vacant vehicles, which indicates the WGC regime; $\mathrm{d}\lambda/\mathrm{d}N_v < 0$ if $W' > -1/\lambda$, which is defined as the non-WGC regime. The WGC regime characterizes such a scenario: A significant shortage of vacant vehicles leads to a noticeable increase in riders' waiting time and reduces service efficiency of each trip. As a result, drivers serve fewer riders per unit time.

Taking the derivatives of λ, N_a, N_b, and N_v with respect to riders' price p, drivers' wage r, and vehicle rental fee d_a, respectively, we have

Corollary 1. *Sensitivity analysis of rider demand:*

(a) Riders' demand rate λ decreases with price p in the non-WGC regime, while the monotonicity of λ with respect to p is uncertain in the WGC regime. Specifically, we have $\frac{\partial \lambda}{\partial p} = \frac{f_\lambda'}{1/\bar{\lambda} + \alpha_w f_\lambda' \frac{W' \cdot (T+W)}{1+\lambda W'}}$.

(b) Riders' demand rate λ decreases with wage r in the non-WGC regime, while the monotonicity of λ with respect to r is uncertain in the WGC regime. Specifically, we have $\frac{\partial \lambda}{\partial r} = -\frac{\bar{N}_a f_a' + \bar{N}_b f_b'}{T+W+(1+\lambda W')/(\bar{\lambda} W' \alpha_w f_\lambda')}$.

(c) The monotonicity of λ with respect to rental fee d_a is opposite to that of r in both non-WGC and WGC regimes. Specifically, we have $\frac{\partial \lambda}{\partial d_a} = -\frac{\partial \lambda}{\partial r} \cdot \frac{\bar{N}_a f_a'}{\bar{N}_a f_a' + \bar{N}_b f_b'}$.

The proof is provided in Appendix A. Notice that we assume both car-renting and car-owning drivers are paid per unit time instead of per order. Thus, the demand rate will not endogenously impact drivers' utility. As a result, riders' price will not directly affect the number of RS drivers.

The RS platform integrates vehicle rental service to attract car-renting drivers. The profit from RS service per unit time equals the revenue from passengers' trip price minus the wages of both types of RS drivers, i.e., $\lambda p - r\cdot(N_a+N_b)$. The profit from car rental service per unit time equals the number of car-renting drivers N_a multiplied by the profit from each rental vehicle (d_a-c), where c represents the depreciation and operating costs of a platform-owned vehicle per unit time. The platform profit PR equals $\lambda p - r\cdot(N_a+N_b)+N_a\cdot(d_a-c)$. The profit-maximization problem can be formulated as follows:

$$\begin{aligned} \max_{p,r,d_a} \quad & PR = \lambda p - r\cdot(N_a+N_b)+N_a\cdot(d_a-c) \\ \text{s.t.} \quad & \text{Eqs. } (1-6) \end{aligned} \tag{7}$$

We denote $(\cdot)^*$ as the market state under monopoly optimum (MO) strategies, e.g., p^* for the optimal price and λ^* for the optimal demand rate under MO. By examining the first-order necessary conditions (FONC) of PR with respect to p, r, and d_a, we conclude:

PROPOSITION 1. *Under maximized platform profit: (a) The RS market must be in the non-WGC regime; (b) the elasticity of rider demand with respect to price is greater than 1, i.e.,* $(-\bar{\lambda}f_\lambda'|_{p^*,\lambda^*})\cdot\frac{p^*}{\lambda^*}>1$.

The proof is provided in Appendix B. The managerial insights of Proposition 1 are as follows.

(a) A higher wage r leads to a higher employment cost for drivers; thus, achieving the MO state requires that a marginal gain in passenger revenue is positive with respect to r, i.e., $\frac{\partial(p\lambda)}{\partial r}>0$, and $\frac{\partial\lambda}{\partial r}>0$. Thus, rider demand increases with the driver wage under the MO state. According to Corollary 1, λ is guaranteed to increase with r only in the non-WGC regime.

(b) Endogenous waiting time W in the non-WGC regime renders riders less sensitive to the price. Specifically, when a higher price reduces rider demand, vacant vehicles increase and riders will benefit from shorter waiting time, which will cause the reduced demand to rebound to some extent. Next, we consider a marginal increase of price $\Delta p>0$; the resulting marginal decrease of demand Δf_λ then consists of two parts: the marginal loss induced by the higher price $\Delta f_\lambda^1 = f_\lambda'\Delta p<0$ and the marginal rebound due to the shorter waiting time $\Delta f_\lambda^2=\frac{\partial W}{\partial p}\cdot(\alpha_w f_\lambda')\Delta p>0$, i.e., $\Delta f_\lambda=\Delta f_\lambda^1+\Delta f_\lambda^2$. Under the optimal rider price, the marginal profit loss due to demand reduction (i.e., $\bar{\lambda}\cdot(-\Delta f_\lambda)\cdot p>0$) should equal the marginal profit gain from an increasing price (i.e., $\Delta p\cdot\lambda>0$). Thus, we have $\bar{\lambda}\cdot(-\Delta f_\lambda^1)\cdot p>\Delta p\cdot\lambda$, which indicates that $-\bar{\lambda}f_\lambda'\cdot p/\lambda>1$. The elasticity of demand with respect to price is greater than 1. In summary, endogenous waiting time influences the platform's optimal price strategy.

The non-WGC property summarized in Proposition 1(a) is consistent with the literature (Ke et al. 2020, Zhou et al. 2022). In contrast, we prove that the MO state is still located in the non-WGC regime after integrating the RS business with the vehicle rental business.

By examining FONC of PR with respect to d_a, we conclude:

PROPOSITION 2. *The profitability of the vehicle rental business under the MO state:*

(a) When riders are significantly sensitive to waiting time, i.e.,

$$\frac{\partial\lambda}{\partial W} = \alpha_w \bar{\lambda} f_\lambda' \leq \frac{1/W' + \lambda^*}{\frac{p^*}{r^* + N_a^*/(\bar{N}_a f_a')} - (T+W)},$$

the platform should forgo the profit from the vehicle rental business, i.e., $d_a^* \leq c$. *When* $\frac{\partial\lambda}{\partial W}$ *satisfies*

$$\frac{\partial\lambda}{\partial W} > \frac{1/W' + \lambda^*}{\frac{p^*}{r^* + N_a^*/(\bar{N}_a f_a')} - (T+W)},$$

the vehicle rental business is profitable, i.e., $d_a^* > c$.

(b) The profit from each rental vehicle satisfies $d_a^* - c = \left(\frac{N_a}{\partial N_a/\partial r} - \frac{N_b}{\partial N_b/\partial r}\right)\Big|_{d_a^*, r^*}$. *When the vehicle rental business is profitable for the platform (i.e.,* $d_a^* > c$*), the elasticity of car-renting drivers with respect to their wage is smaller than that of car-owning drivers, i.e.,* $(\partial N_a/N_a^*)/(\partial r/r^*)\big|_{d_a^*, r^*} < (\partial N_b/N_b^*)/(\partial r/r^*)\big|_{r^*}$*; otherwise (*$d_a^* \leq c$*), the elasticity of car-renting drivers with respect to their wage is greater than that of car-owning drivers, i.e.,* $(\partial N_a/N_a^*)/(\partial r/r^*)\big|_{d_a^*, r^*} \geq (\partial N_b/N_b^*)/(\partial r/r^*)\big|_{r^*}$.

COROLLARY 2. *If the participating probabilities for both car-renting and car-owning drivers can be described using a linear model (i.e.,* $N_a = \bar{N}_a \min\big\{\max\{lu_a + \epsilon, 0\}, 1\big\}$, $N_b = \bar{N}_b \min\big\{\max\{lu_b + \epsilon, 0\}, 1\big\}$*) or the multinomial logit (MNL) model (i.e.,* $N_a = \bar{N}_a/\big(1 + e^{-\theta u_a}\big)$, $N_b = \bar{N}_b/\big(1 + e^{-\theta u_b}\big)$*), we can obtain additional results on top of Proposition 2(b):*

When the platform incurs a low vehicle depreciation cost c *or car-renting drivers have a low opportunity cost* o_a*, i.e.,* $c + o_a < d_b + o_b$*, the vehicle rental business is profitable, i.e.,* $d_a^* > c$*, and car-renting drivers are more willing to provide RS service due to a higher utility, i.e.,* $N_a^*/\bar{N}_a > N_b^*/\bar{N}_b$, $u_a^* > u_b^*$. *Otherwise (i.e.,* $c + o_a \geq d_b + o_b$*), we have* $d_a^* \leq c$, $N_a^*/\bar{N}_a \leq N_b^*/\bar{N}_b$, $u_a^* \leq u_b^*$.

The proofs of Proposition 2 and Corollary 2 are provided in Appendix C. The intuition behind Proposition 2(a) is as follows. On the one hand, a higher rental fee d_a helps mitigate the platform's burden on its vehicles' depreciation cost. On the other hand, a higher d_a decreases the interests of car-renting drivers, then leads to a smaller fleet size and longer waiting time for riders, and finally reduces rider demand and the platform's revenue. Therefore, when riders are sensitive to waiting time, the loss of revenue overwhelms the benefit from the reduction in vehicles' depreciation cost; thus the platform should set a lower d_a and forgo the profit from the vehicle rental business.

The conclusion in Corollary 2 (i.e., the additional result based on Proposition 2(b) under certain participation models of drivers) is intuitive. When the platform incurs a low vehicle depreciation cost, it can adopt a moderate rental fee that not only renders itself profitable, but also maintains a relatively high utility of car-renting drivers. When car-renting drivers have a low opportunity cost, they can sustain a higher willingness to work than car-owning drivers, even if the platform adopts a relatively high rental fee and pursues profitability of the vehicle rental business.

Also, when we examine the RS business, we conclude that:

COROLLARY 3. *For each service trip under the MO state, the passenger's price p^* is higher than the driver's payoff from the service, i.e., $(T+W^*)\cdot r^*$. Specifically, we have*

$$p^* - (T+W^*)\cdot r^* = \frac{N_b^* \cdot (T+W^*)}{(\partial N_b/\partial r)\big|_{r^*}} + \frac{\lambda^*}{\bar{\lambda}\cdot(-f_\lambda')\big|_{p^*}} > 0 \tag{8}$$

The proof is provided in Appendix D. Corollary 3 does not mean that the RS business must be profitable under the MO state. Specifically, the profit from the RS business can be expressed as

$$\lambda p - r\cdot(N_a+N_b) = \lambda p - r\cdot\left[(T+W)\lambda + N_v\right] = \lambda\cdot\left[p-(T+W)r\right] - N_v r$$

This indicates that the platform also bears the wage cost of cruising drivers (i.e., $N_v r$) in addition to the profit earned during trip service (i.e., $\lambda\cdot\left[p-(T+W)r\right]>0$). Therefore, the platform might forgo the profit of the RS business and pursue the profit of vehicle rental service.

4. Coordinated Regulation of Driver Wage and Vehicle Rental Fee

4.1. Optimization model of regulatory policies

In this section, we analyze how the government optimizes social welfare by regulating the minimum driver wage (i.e., $\underline{r}$, a lower bound for driver wage) and maximum rental fee (i.e., $\bar{d}_a$, an upper bound for vehicle rental). Social welfare SW is defined as the weighted sum of rider surplus S_R, driver surplus S_D, and platform profit PR, i.e.,

$$SW = \beta_{SR}\cdot S_R + \beta_{SD}\cdot S_D + \beta_{PR}\cdot PR,$$

where β_{SR}, β_{SD}, β_{PR} are the weights of surplus for different stakeholders.

Rider surplus can be determined by

$$S_R = \int_0^{\lambda} f_\lambda^{-1}(z)\mathrm{d}z - \lambda\cdot u_\lambda \tag{9}$$

Driver surplus is expressed as

$$S_D = \underbrace{u_a\cdot N_a - \int_0^{N_a} f_a^{-1}(z)\mathrm{d}z}_{\text{Surplus of car-renting drivers}} + \underbrace{u_b\cdot N_b - \int_0^{N_b} f_b^{-1}(z)\mathrm{d}z}_{\text{Surplus of car-owning drivers}} \tag{10}$$

Then, the regulatory policy can be formulated as a bi-level optimization problem. At the upper level, the government maximizes total social welfare by determining the minimum driver wage $\underline{r}$ and maximum vehicle rental fee $\bar{d}_a$:

$$\max_{\underline{r},\bar{d}_a} \quad SW = \beta_{SR}\cdot S_R + \beta_{SD}\cdot S_D + \beta_{PR}\cdot PR \tag{11}$$

At the lower level, given the minimum driver wage and maximum vehicle rental fee set by the government, the platform maximizes its joint profit from the RS service and vehicle rental service by changing riders' price p, drivers' wage r, and vehicle rental fee d_a:

$$\begin{aligned} \max_{p,r,d_a} \quad & PR = \lambda p - r\cdot(N_a+N_b) + N_a\cdot(d_a - c) \\ \text{s.t.} \quad & r \geq \underline{r} \\ & d_a \leq \bar{d}_a \\ & \text{Eqs. } (1-6) \end{aligned} \tag{12}$$

The relationship among regulatory policies, platform strategies, and endogenous variables is illustrated in Fig. 1:

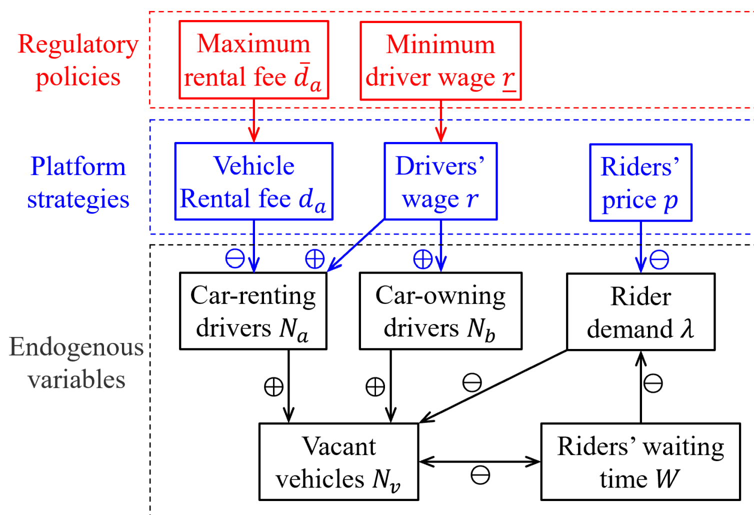


Figure 1 Relationship among different variables

For ease of analysis, we denote $\tilde{p}(d_a, r)$ as the optimal price to maximize the platform profit given rental fee d_a and wage r, i.e., $\tilde{p}(d_a,r) \triangleq \arg\max_p PR(p,d_a,r)$. The corresponding profit is denoted by $\tilde{PR} \triangleq PR\big(\tilde{p}(d_a,r),d_a,r\big)$. In addition, given maximum rental fee $\bar{d}_a$ and minimum driver wage $\underline{r}$, we optimize the platform's rental fee and driver wage by maximizing profit $\tilde{PR}(d_a,r)$ as $d_a^{br}(\bar{d}_a,\underline{r})$ and $r^{br}(\bar{d}_a,\underline{r})$. Superscript "br" indicates platform's best responsive strategies to regulatory policies. Naturally, d_a^{br}, r^{br} and $\tilde{p}(d_a^{br}, r^{br})$ form the optimal solution to Eq. (12). The optimal policies to maximize social welfare are denoted as $\underline{r}^m$ and $\bar{d}_a^m$.

4.2. Phase diagram of government policies

Government regulation significantly impacts on the platform's strategies and further affects the platform profit, driver surplus, and rider surplus. Hence, it is essential to explore the platform's optimal strategies (i.e., $d_a^{br}, r^{br}, \tilde{p}(d_a^{br}, r^{br})$) and the resulting regulatory effects under different values of the minimum driver wage $\underline{r}$ and maximum rental fee $\bar{d}_a$. Based on the bi-level optimization model Eq. (11-12), we propose a phase diagram of regulatory policies under certain conditions, as illustrated in Fig. 2 and Proposition 3.

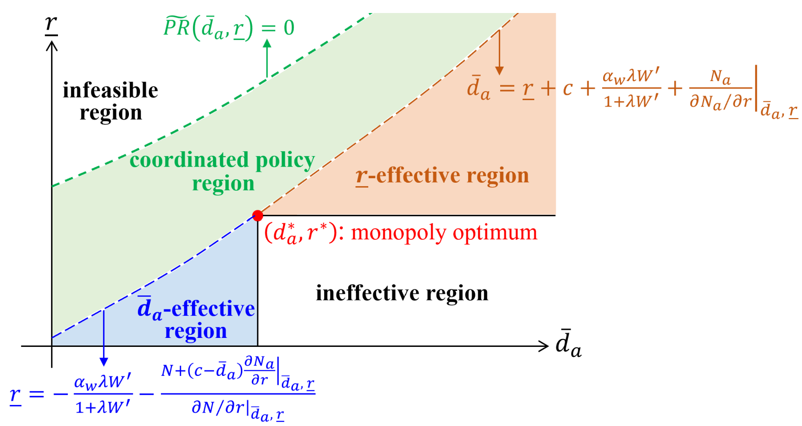


Figure 2 Phase diagram of regulatory policies

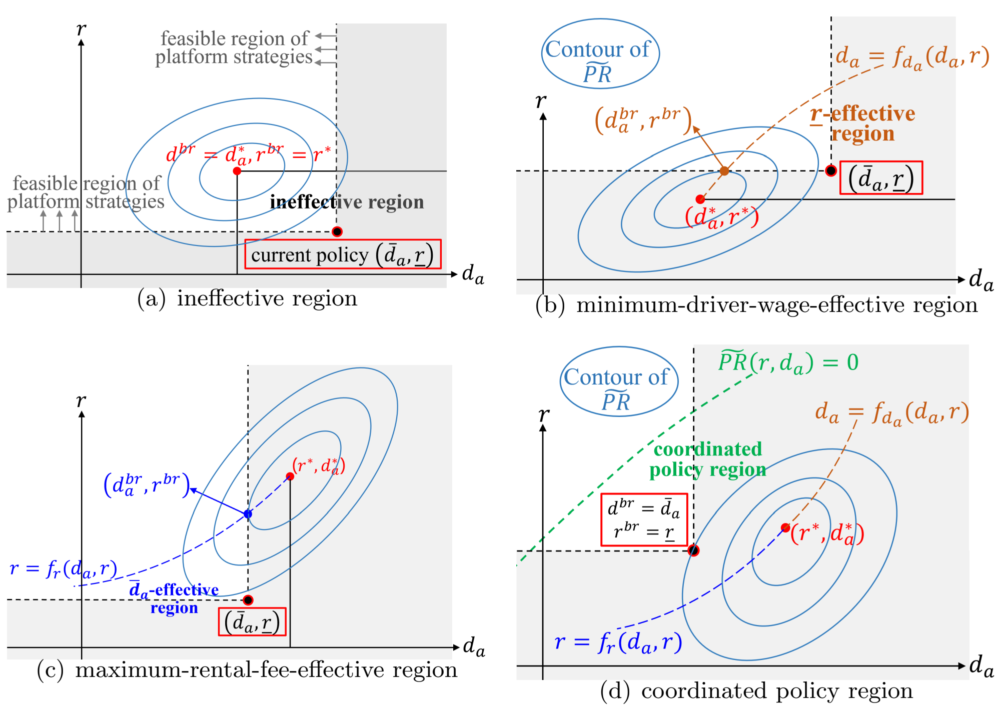


Figure 3 Platform's optimal strategies d_a^{br} and r^{br} under regulation in different regions.

Condition 1:

(a) For any $(d_a, r) \neq (d_a^*, r^*)$*, we have* $(d_a - d_a^*)\frac{\partial \tilde{PR}}{\partial d_a} + (r - r^*)\frac{\partial \tilde{PR}}{\partial r} < 0$.

(b) For any given wage r_1*,* $\tilde{PR}(d_a, r_1)$*, as a function of* d_a*, has a unique maximum point* d_{a1}*, and* $\frac{d\tilde{PR}(d_a,r_1)}{dd_a}\Big|_{d_a<d_{a1}} > 0$*,* $\frac{d\tilde{PR}(d_a,r_1)}{dd_a}\Big|_{d_a>d_{a1}} < 0$.

(c) For any given rental fee d_{a2}*,* $\tilde{PR}(d_{a2}, r)$*, as a function of* r*, has a unique maximum point* r_2*, and* $\frac{d\tilde{PR}(d_{a2},r)}{dr}\Big|_{r<r_2} > 0$*,* $\frac{d\tilde{PR}(d_{a2},r)}{dr}\Big|_{r>r_2} < 0$.

Condition 1(a) is proposed to guarantee that $\tilde{PR}$ has a unique maximum point (d_a^*, r^*); while Condition 1(b,c) is proposed to guarantee that d_a^{br} and r^{br} can be determined by first order conditions.

Proposition 3. *When Condition 1 is satisfied, the decision space of government policies can be divided into five mutually exclusive regions, as illustrated in Fig. 2 and Fig. 3:*

(a) Regulatory policies are located in the "ineffective region" when $\underline{r} \leq r^*$ *and* $\bar{d}_a \geq d_a^*$*, where* d_a^**,* r^* *and* $\tilde{p}(d_a^*, r^*)$ *are the monopoly optimum as the optimal solution to Eq. (7). In this region, we have* $r^{br} = r^*$ *and* $d_a^{br} = d_a^*$.

(b) Regulatory policies are located in the "minimum-driver-wage-effective region" (or "$\underline{r}$*-effective region") when* $\underline{r} > r^*$*,* $\bar{d}_a > f_{d_a}(d_a = \bar{d}_a, r = \underline{r}) \triangleq \underline{r} + c + \frac{\alpha_w \lambda W'}{1+\lambda W'} + \frac{N_a}{\partial N_a/\partial r}\Big|_{\bar{d}_a,\underline{r}}$ *and* $\tilde{PR}\big(d_a = f_{d_a}(d_a, \underline{r}), r = \underline{r}\big) \geq 0$*. In this region, we have* $r^{br} = \underline{r}$*,* $d_a^{br} = f_{d_a}(d_a^{br}, \underline{r})$.

(c) Regulatory policies are located in the "maximum-rental-fee-effective region" (or "$\bar{d}_a$*-effective region") when* $\bar{d}_a < d_a^*$*,* $\underline{r} < f_r(d_a = \bar{d}_a, r = \underline{r}) \triangleq -\frac{\alpha_w \lambda W'}{1+\lambda W'} - \frac{N+(c-\bar{d}_a)\partial N_a/\partial r}{\partial N/\partial r}\Big|_{\bar{d}_a,\underline{r}}$ *and* $\tilde{PR}\big(d_a = \bar{d}_a, r = f_r(\bar{d}_a, r)\big) \geq 0$*. In this region, we have* $d_a^{br} = \bar{d}_a$*,* $r^{br} = f_r(\bar{d}_a, r^{br})$.

(d) Regulatory policies are located in the "coordinated policy region" when $\underline{r} \geq f_r(\bar{d}_a, \underline{r})$*,* $\bar{d}_a \leq f_{d_a}(\bar{d}_a, \underline{r})$ *and* $\tilde{PR}(\bar{d}_a, \underline{r}) \geq 0$*. In this region, we have* $d_a^{br} = \bar{d}_a$*,* $r^{br} = \underline{r}$.

(e) Regulatory policies are located in the "infeasible region" when $\tilde{PR}(d_a, r) \leq 0$*,* $\forall\ r \geq \underline{r}, d_a \leq \bar{d}_a$*. Market failure would occur in this region, since an unprofitable platform could not sustainably operate.*

The proof is provided in Appendix E. In the ineffective region, the market remains unchanged with an unregulated monopolist, in which regulation is too loose to constrain the platform's strategies. In the infeasible region, regulation is too strict, and it is infeasible for the platform to pursue a profit, which causes market failure in the long term. Therefore, we discuss the minimum-driver-wage-effective region, maximum-rental-fee-effective region, and coordinated policy region in detail.

In the minimum-driver-wage-effective region, the government sets a strict minimum driver wage policy that effectively urges the platform to pay drivers a higher wage, i.e., $r^{br} = \underline{r} > r^*$; however, the loose regulation of vehicle rental fees enables the platform to charge car-renting drivers a preferable rental fee to increase its profit, i.e., $d_a^{br} < \bar{d}_a$. In this region, the platform's best-response strategies and market equilibrium are solely affected by minimum driver wage $\underline{r}$, while they are not affected by maximum rental fee $\bar{d}_a$.

By contrast, in the maximum-rental-fee-effective region, the platform is required to charge car-renting drivers lower rental fee $d_a^{br} = \bar{d}_a < d_a^*$, but it can pay a preferable wage to drivers due to the loose regulation of minimum driver wage policy $\underline{r} < r^{br}$. In this region, the platform's strategies and market equilibrium are solely affected by maximum rental fee $\bar{d}_a$, but not affected by minimum driver wage $\underline{r}$.

In the coordinated policy region, both the minimum-driver-wage and maximum-rental-fee effectively constrain the platform's strategies, i.e., $d_a^{br} = \bar{d}_a$, $r^{br} = \underline{r}$. Regulation in this region is not conducive for the platform, since the government effectively limits the platform profit from RS and vehicle rental services. On the contrary, the surplus of both car-renting and car-owning drivers can be significantly improved under strict regulation in this region. We can conclude that:

COROLLARY 4. *Maximum social welfare is achieved either inside or on the boundary (i.e.,* $\underline{r} = f_r(\bar{d}_a, \underline{r})$ *or* $\bar{d}_a = f_{d_a}(\bar{d}_a, \underline{r})$*) of the coordinated policy region.*

The proof is provided in Appendix F. In the following subsections, we discuss the properties of the minimum-driver-wage-effective region, maximum-rental-fee-effective region, and coordinated policy region.

4.3. Minimum-driver-wage-effective region

We focus on the minimum-driver-wage-effective region in this subsection, which corresponds to strict regulation of the minimum driver wage (i.e., high $\underline{r}$) and loose regulation of the maximum rental fee (i.e., high $\bar{d}_a$). We find that raising the minimum driver wage does not always benefit car-renting drivers under loose rental-fee regulation. Specifically, we derive that:

PROPOSITION 4. *In the minimum-driver-wage-effective region, when the following three conditions are satisfied, the number of car-renting drivers decreases with the minimum driver wage (i.e.,* $\frac{\partial N_a(\underline{r}, d_a^{br})}{\partial \underline{r}} < 0$*):*

(a) The market is located in the non-WGC regime;

(b) Car-renting drivers' participation probability function f_a *satisfies* $\left.\frac{f_a f_a''}{f_a'^2}\right|_{\underline{r}, d_a^{br}} < 2$*;*

(c) Riders are less sensitive to their waiting time with $\frac{\partial \lambda}{\partial W} = \alpha_w \bar{\lambda} f_\lambda'\big|_{r^{br}, d_a^{br}} > -\left.\frac{\lambda W''}{W'^2}\right|_{\underline{r}, d_a^{br}}$.

The proof is provided in Appendix G. Conditions (b, c) are not difficult to satisfy. When car-renting drivers' participation probability can be characterized using linear models (in which $f_a'' = 0$) or MNL models (in which $\frac{f_a f_a''}{f_a'^2} = \frac{1-2f_a}{1-f_a} < 1$), condition (b) can always be satisfied. When riders' waiting time is expressed as $W = M/\sqrt{N_v}$ (Daganzo 1978, Arnott 1996, Zha, Yin, and Yang 2016, Li et al. 2019) and their mode choice obeys the MNL model, condition (c) can be satisfied if the proportion of realized demand $\lambda/\bar{\lambda}$ exceeds 4.185%; when $W = M/\sqrt{N_v}$ and riders' mode choice obeys the linear model, condition (c) can be satisfied if $\lambda|_{W=0} < 4\lambda$, i.e., rider demand does not surge more than fourfold under a zero-waiting-time scenario. Corresponding derivations are provided in Appendix G.

Proposition 4 signifies that when the government focuses on regulating the minimum driver wage rather than limiting the platform's vehicle rental fee, car-renting drivers' welfare might suffer. The intuition is as follows: The increase in the minimum driver wage may cause the platform to have an undesired excess of fleet size, which breaks the balance of driver supply and rider demand and increases the platform's cost of hiring drivers. It then induces the platform to charge a higher rental fee to car-renting drivers under loose regulation of the maximum rental fee. On the one hand, the platform can re-balance supply and demand by inhibiting the service willingness of car-renting drivers and reducing the fleet size; on the other hand, the platform can incur a lower integrated cost of hiring car-renting drivers (i.e., $r - (d_a - c)$ each). As a result, if the vehicle rental fee increases faster than the minimum driver wage, i.e., $\partial d_a^{br}/\partial \underline{r} > 1$, car-renting drivers will suffer from a lower surplus.

4.4. Maximum-rental-fee-effective region

We focus on the maximum-rental-fee-effective region in this subsection, which corresponds to strict regulation of the maximum rental fee (i.e., a low $\bar{d}_a$) and loose regulation of the minimum driver wage (i.e., a low $\underline{r}$). Similar to Proposition 4, we find that reducing the maximum rental fee does not always benefit drivers under loose minimum-wage regulation. Specifically, we derive that:

Corollary 5. *In the maximum-rental-fee-effective region:*

(a) the derivative of the total number of drivers with respect to the maximum rental fee satisfies:

$$\frac{\partial N(r^{br}, \bar{d}_a)}{\partial \bar{d}_a} = \frac{\bar{N}_a \bar{N}_b \left[N \cdot (f_a' f_b'' - f_b' f_a'') + (c - \bar{d}_a) \cdot (\bar{N}_a f_a'^2 f_b'' + \bar{N}_b f_b'^2 f_a'')\right]}{(\bar{d}_a - c) \cdot (f_a' f_b'' - f_b' f_a'') \bar{N}_a \bar{N}_b - N \cdot (\bar{N}_a f_a'' + \bar{N}_b f_b'') + 2(\frac{\partial N}{\partial r})^2 + \kappa \cdot (\frac{\partial N}{\partial r})^3} \Big|_{r^{br}, d_a^{br}}, \quad (13)$$

where

$$\kappa = \frac{\alpha_w \cdot (\lambda W'' + \bar{\lambda} f_\lambda' \alpha_w W'^2)}{(1 + \lambda W')^2 \left(1 + \lambda W' + \bar{\lambda} f_\lambda' \alpha_w W' \cdot (T + W)\right)}$$

(b) Based on Eq. (13), if drivers' participation decision is formulated as a linear model, i.e., $f_a'' = f_b'' = 0$, we have $\frac{\partial N(r^{br}, \bar{d}_a)}{\partial \bar{d}_a} = 0$, $\frac{\partial N_a(r^{br}, \bar{d}_a)}{\partial \bar{d}_a} < 0$, and $\frac{\partial N_b(r^{br}, \bar{d}_a)}{\partial \bar{d}_a} > 0$, i.e., car-renting drivers increases with stricter maximum rental fee policy, while car-owning drivers decreases with stricter maximum rental fee policy.

The proof is provided in Appendix H. The intuition of Corollary 5 is similar to Proposition 4. When the platform is required to charge a lower vehicle rental fee, it may attract an excess number of car-renting drivers. Then the platform may lower the wage to re-balance supply and demand

and reduce its hiring cost under loose regulation of the minimum driver wage, which results in a lower surplus of car-owning drivers.

The properties of the minimum-driver-wage-effective region and maximum-rental-fee-effective region reveal that, if the government only effectively restricts the platform in one dimension, the platform will adjust its strategy accordingly in the other unconstrained dimension to mitigate its profit loss. Such an adjustment of the platform could further cause undesired damage to driver surplus.

4.5. Coordinated policy region

Different from the minimum-driver-wage-effective and maximum-rental-fee-effective region, the coordinated policy region corresponds to strict regulation of both the minimum driver wage and maximum rental fee, and hence regulation enforces effective constraints on the platform's strategies for both driver wage and vehicle rental fee, i.e., $r^{br} = \underline{r}, d_a^{br} = \bar{d}_a$. As stated in Subsection 4.2, maximum social welfare is achieved either inside or on the boundary of this region. Thus, we focus on the optimization of regulatory policies to maximize social welfare. We first analyze the first-order optimality of $\underline{r}$:

LEMMA 1. *Given maximum rental fee $\bar{d}_a$ in the coordinated policy region, the first-order optimality of the minimum driver wage to maximize social welfare, i.e., the best-response minimum driver wage, is expressed as*

$$\hat{\underline{r}}(\bar{d}_a) = \frac{\partial N_a/\partial r}{\partial N/\partial r} \cdot (\bar{d}_a - c) + \frac{\beta_{PR}\tilde{p} + \frac{\beta_{SR}\alpha_w \lambda W'}{1+\lambda W'} \cdot (T+W)}{\beta_{PR}\partial N/\partial r} \cdot \Big(\frac{\partial \lambda}{\partial r} + \frac{\partial \lambda}{\partial p}\frac{\partial \tilde{p}}{\partial \underline{r}}\Big) - \frac{\beta_{SR}\alpha_w \lambda W'}{\beta_{PR}\cdot(1+\lambda W')} + \frac{(\beta_{SD} - \beta_{PR})N + (\beta_{PR} - \beta_{SR})\lambda \frac{\partial \tilde{p}}{\partial \underline{r}}}{\beta_{PR}\partial N/\partial r}\Big|_{\bar{d}_a, \underline{r}, \tilde{p}(\bar{d}_a, \underline{r})}, \tag{14}$$

where $\frac{\partial \lambda}{\partial p}$ and $\frac{\partial \lambda}{\partial r}$ are provided in Corollary 1.

The proof and the expression of $\frac{\partial \tilde{p}}{\partial \underline{r}}$ are provided in Appendix I. In Section 5, we will numerically verify the accuracy of $\hat{\underline{r}}(\bar{d}_a)$ in Eq. (14). Based on Lemma 1, we can further derive that:

PROPOSITION 5. *When the government values all stakeholders' welfare equally, i.e., $\beta_{PR} = \beta_{SR} = \beta_{SD} = 1$ in the social welfare function (Eq. (11)), and the optimal regulatory policy is inside the coordinated policy region, we have*

(a) The optimal maximum vehicle rental fee equals the vehicle's operating cost, i.e., $\bar{d}_a^m = c$;

(b) The optimal minimum driver wage satisfies:

$$\underline{r}^m = -\frac{\lambda/(\bar{\lambda} f_\lambda')}{\partial N/\partial r} \cdot \Big(\frac{\partial \lambda}{\partial r} + \frac{\partial \lambda}{\partial p}\frac{\partial \tilde{p}}{\partial \underline{r}}\Big) - \frac{\alpha_w \lambda W'}{1+\lambda W'}\Big|_{\bar{d}_a = c, \underline{r}^m, \tilde{p}(c, \underline{r}^m)} \tag{15}$$

The proof is provided in Appendix J. The intuition of $\bar{d}_a^m = c$ is as follows.

First, when the platform's strategies for both driver wage and vehicle rental fee are restricted, the government's stricter regulation (i.e., a higher minimum driver wage and a lower maximum rental fee) will result in a larger fleet size and riders' lower waiting time, which improve driver surplus S_D, rider surplus S_R, and the platform's revenue for RS service (i.e., $\lambda \cdot p$). Inversely, looser regulation (i.e., a lower $\underline{r}$ and a higher $\bar{d}_a$) reduces the platform's integrated cost of hiring car-renting drivers (i.e., $\left[r-(d_a-c)\right]N_a$) and hiring car-owning drivers (i.e., rN_b). The government should seek trade-offs between the platform's hiring cost (i.e., $\left[r-(d_a-c)\right]N_a + rN_b$) and the sum of rider surplus, driver surplus and platform's revenue for RS service (i.e., $S_R + S_D + \lambda \cdot p$).

Second, when the vehicle rental business is profitable (i.e., $\bar{d}_a = d_a^{br} > c$) and the platform already has a low integrated cost of hiring car-renting drivers, if the government sets a higher $\bar{d}_a$ and allows the platform to raise vehicle rental fee, the marginal reduction in the total hiring cost will be limited. By contrast, the total hiring cost will decrease more rapidly if the government exercises a lower $\underline{r}$ and allows the platform to reduce drivers' wage.

Next, we consider the case in which the minimum driver wage has been optimized (i.e., $\underline{r} = \hat{\underline{r}}(\bar{d}_a)$) and the government must decide whether to increase or decrease $\bar{d}_a$ under a profitable vehicle rental business (i.e., $\bar{d}_a = d_a^{br} > c$). As illustrated in Fig. 4(a), if the government continues to increase $\bar{d}_a$, the marginal decrease in rider surplus, driver surplus, and the platform's revenue from riders will outweigh the marginal decrease in the platform's hiring cost, since the marginal reduction in the hiring cost caused by higher $\bar{d}_a$ is relatively small, as described above. Therefore, the government should decrease the maximum rental fee to improve social welfare when $d_a > c$.

Symmetrically, when the vehicle rental business is unprofitable (i.e., $\bar{d}_a = d_a^{br} < c$) and the platform incurs a high integrated cost of hiring car-renting drivers, raising the maximum rental fee becomes a more effective way to lower the platform's hiring cost (as illustrated in Fig. 4(b)). The government should increase $\bar{d}_a$ to pursue higher social welfare. In summary, when social welfare is maximized, we have $\bar{d}_a^m = c$.

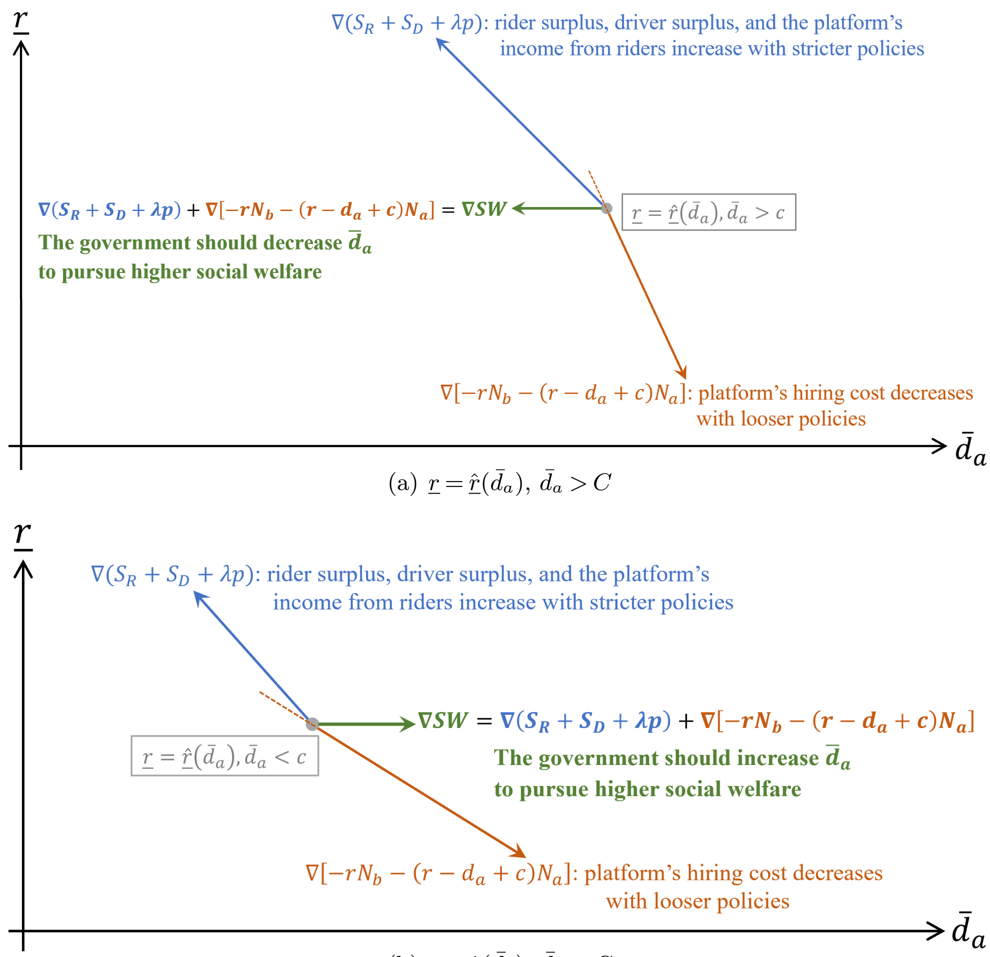
In addition, we discuss a special case in which waiting time cost has little effect on riders:

Corollary 6. *When riders' waiting time cost is negligible, i.e., $\alpha_w W \approx 0$ and $\alpha_w W' \approx 0$, the best-response minimum driver wage can be approximated by*

$$\hat{\underline{r}}(\bar{d}_a) \approx \hat{\underline{r}}^{ap}(\bar{d}_a) \triangleq \frac{\partial N_a/\partial r}{\partial N/\partial r} \cdot (\bar{d}_a - c) + \frac{(\beta_{SD} - \beta_{PR})N - \beta_{SR}\lambda \partial \tilde{p}/\partial \underline{r}}{\beta_{PR}\partial N/\partial r}\Big|_{\bar{d}_a, \underline{r}, \tilde{p}(\bar{d}_a, \underline{r})}. \tag{16}$$

Further, when $\beta_{PR} = \beta_{SR} = \beta_{SD} = 1$, by substituting $\bar{d}_a^m = c$ into Eq. (16), we can obtain the approximate optimal minimum driver wage $\underline{r}^{m(ap)}$:

$$\underline{r}^m \approx \underline{r}^{m(ap)} \triangleq -\lambda \frac{\partial \tilde{p}/\partial \underline{r}}{\partial N/\partial r}\Big|_{\bar{d}_a = c, \underline{r}, \tilde{p}(c, \underline{r})}. \tag{17}$$



(a) $\underline{r} = \hat{\underline{r}}(\bar{d}_a)$, $\bar{d}_a > C$

(b) $\underline{r} = \hat{\underline{r}}(\bar{d}_a)$, $\bar{d}_a < C$

Figure 4 **Gradient of rider surplus, driver surplus, the platform's revenue, and the platform's hiring cost with respect to the two regulatory policies.**

The proof is provided in Appendix K. In Section 5, we will numerically evaluate the approximation accuracy of Corollary 6.

5. Numerical Experiments

In this section, we present numerical experiments to verify theoretical findings. We compare the integrated ride-sourcing market with vehicle rental services under regulation or not. Throughout this section, both riders' mode choice and drivers' participation probability are specified using multinomial logit models as

$$\lambda = \frac{\bar{\lambda}}{1 + \exp\big(\theta_\lambda \cdot (p + \alpha_t T + \alpha_w W - \bar{u})\big)}$$

$$N_i = \frac{\bar{N}_i}{1 + \exp\big(-\theta_i \cdot (r - d_i - o_i)\big)}, i \in \{a, b\}$$

Riders' waiting time is formulated as $W = M/\sqrt{N_v}$. The default values of parameters are set as $\theta_\lambda = \theta_a = \theta_b = 0.3$, $\alpha_t = \alpha_w = 30$ CNY/h, $o_a = 20$ CNY/h, $o_b = 23$ CNY/h, $\bar{u} = 45$ CNY, $\bar{N}_a = 2500$, $\bar{N}_b = 2000$, $\bar{\lambda} = 6000/h$, $d_b = 5$ CNY/h, $c = 4$ CNY/h, $M = 3$. Different values of parameters can be easily implemented and as little effect.

Fig. 5 shows the impact of a rental vehicle's operating cost c under the MO state. As illustrated in Fig. 5(a), the platform has to raise vehicle rental fee d_a^* as the operating cost increases, which will significantly reduce car-renting drivers' willingness to provide RS service. Thus, the platform should properly increase drivers' wage r^* to avoid the loss of driver supply. As a result, Fig. 5(b) shows that the number of car-owning drivers N_b^* increases while the number of car-renting drivers N_a^* decreases at a faster rate. Due to a lower total supply of RS drivers, the platform should adopt a higher price to reduce rider demand when c increases.

The results in Fig. 5(c) demonstrate that the profit from both the RS business and vehicle rental business declines with a higher operating cost c, which is intuitive. When the operating cost becomes sufficiently high (i.e., $c > d_b + o_b - o_a = 8.00$), the vehicle rental business is no longer profitable, and the elasticity of car-renting drivers exceeds that of car-owning drivers, as shown in Fig. 5(d). Such results verify our conclusions in Proposition 2(b) and Corollary 2. Moreover, we find in Fig. 5(c, d) that the platform profit from the RS service $p - r \cdot (T + W)$ is positive, the elasticity of riders with respect to price $-\bar{\lambda} f_\lambda' p/\lambda$ is greater than 1, and the RS market is located in the non-WGC regime (i.e., $1 + \lambda W' > 0$). The results verify our conclusions in Proposition 1 and Corollary 3.

The change in the MO state with respect to the proportion of potential car-renting drivers $\bar{N}_a/(\bar{N}_a + \bar{N}_b)$ is illustrated in Fig. 6. Since car-renting drivers have a lower opportunity cost than car-owning drivers, i.e., $o_a = 20$ CNY/h $< o_b = 23$ CNY/h, the platform can lower the wage while sustaining total driver supply as $\bar{N}_a/(\bar{N}_a + \bar{N}_b)$ increases. Therefore, although Fig. 6(a, b) show that the optimal wage r^* decreases with this proportion and the drivers of both groups are less willing to offer RS service (i.e., both $N_a^*/\bar{N}_a$ and $N_b^*/\bar{N}_b$ decrease), the platform still obtains a higher total driver supply, e.g., $(N_a^* + N_b^*)\big|_{\bar{N}_a=1500,\bar{N}_b=3000} = 1500 \times 0.615 + 3000 \times 0.483 < (N_a^* + N_b^*)\big|_{\bar{N}_a=3000,\bar{N}_b=1500} = 3000 \times 0.584 + 1500 \times 0.440$. As a result, the platform tends to reduce price to match driver supply with higher rider demand. Moreover, optimal vehicle rental fee d_a^* slowly decreases with $\bar{N}_a/(\bar{N}_a + \bar{N}_b)$, compensating car-renting drivers for lower wages without significantly reducing the profit from each rental vehicle.

As presented in Fig. 6(c), the profit from both the RS service and vehicle rental service experience a modest increase given higher $\bar{N}_a/(\bar{N}_a + \bar{N}_b)$, since there are more potential car-renting drivers with lower opportunity costs and higher willingness to provide services. Similar to Fig. 5(c, d), Fig.

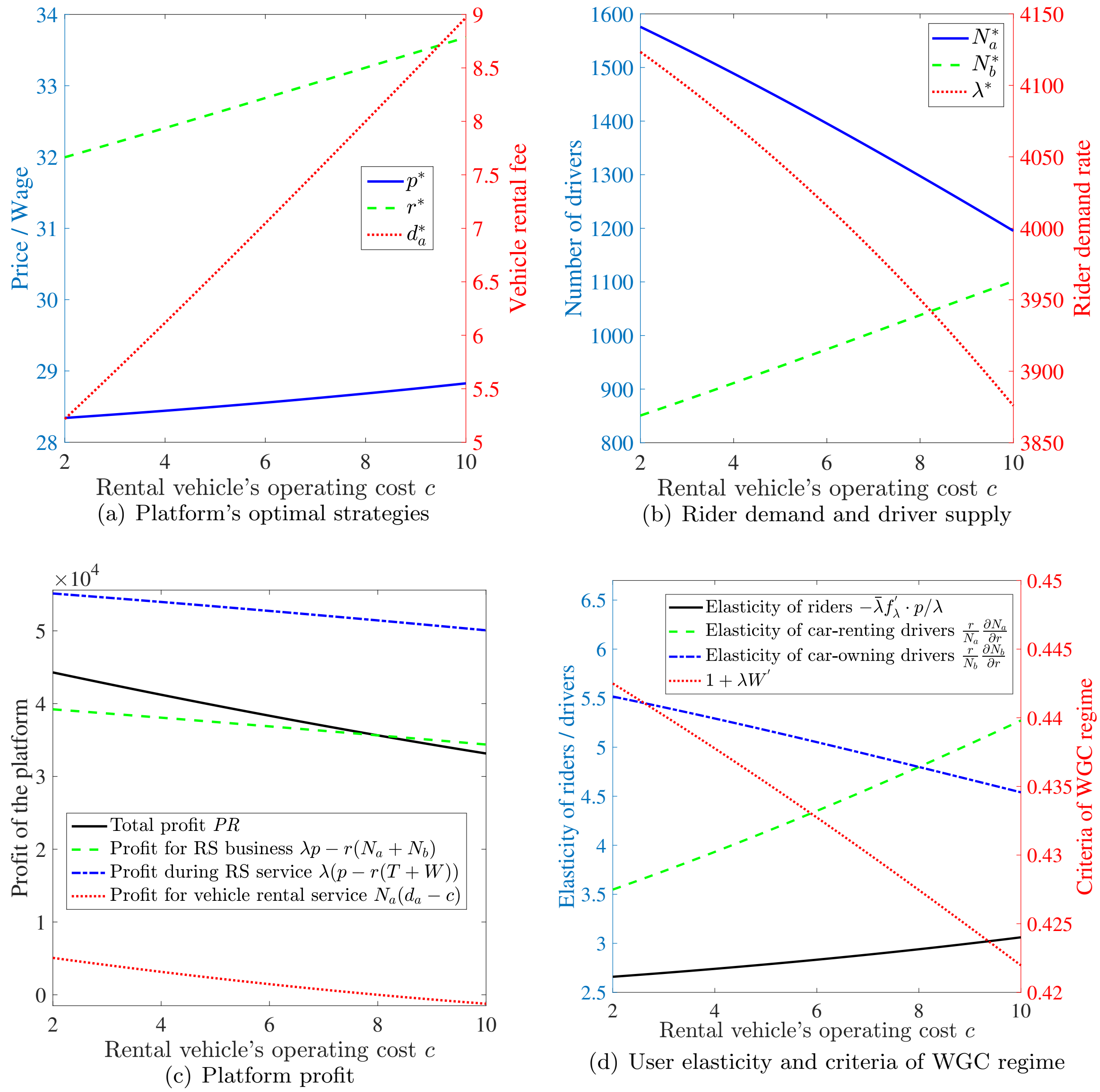
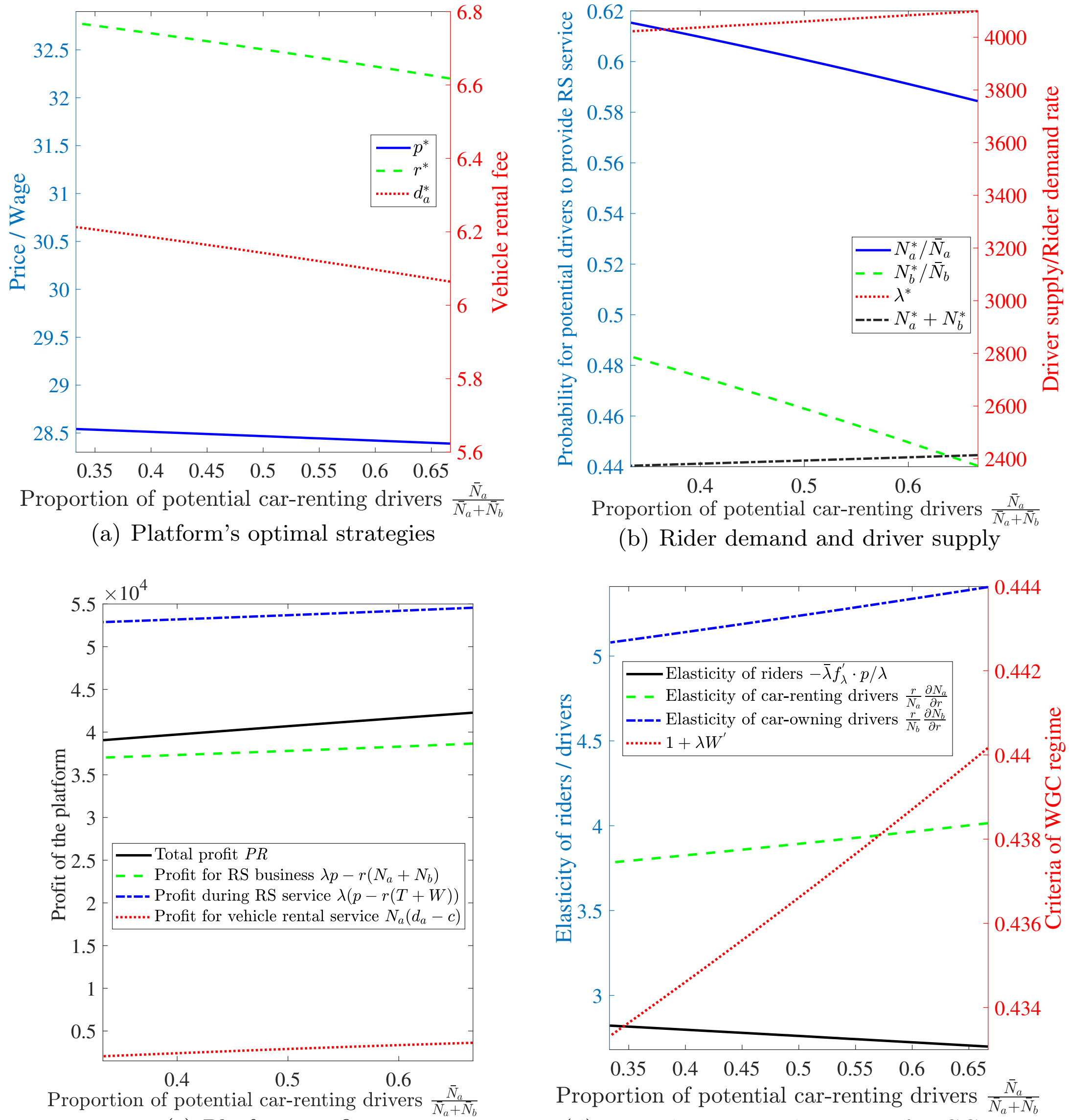


Figure 5 **Monopoly optimum states under different operating costs of a rental vehicle.**

6(c, d) demonstrate that the RS service profit, users' elasticities, and WGC regime criteria are in accordance with our conclusions in Proposition 1 and Corollary 3.

Fig. 7 shows how potential rider demand $\bar{\lambda}$ affects the MO state. To handle surge demand and a relative shortage of driver supply, the platform should charge a higher price for riders while offering a higher wage to attract more drivers, as shown in Fig. 7(a, b). Considering the vehicle rental business, since a higher wage induces more car-renting drivers to join the platform, charging a higher rental fee d_a^* to pursue a higher profit would be the platform's preferred choice, as shown in Fig. 7(a). According to Fig. 7(c), the optimal profit from both the RS service and vehicle rental service increases with potential riders $\bar{\lambda}$. Similar to Fig. 5(d) and Fig. 6(d), we find in Fig. 7(d)



(a) Platform's optimal strategies

(b) Rider demand and driver supply

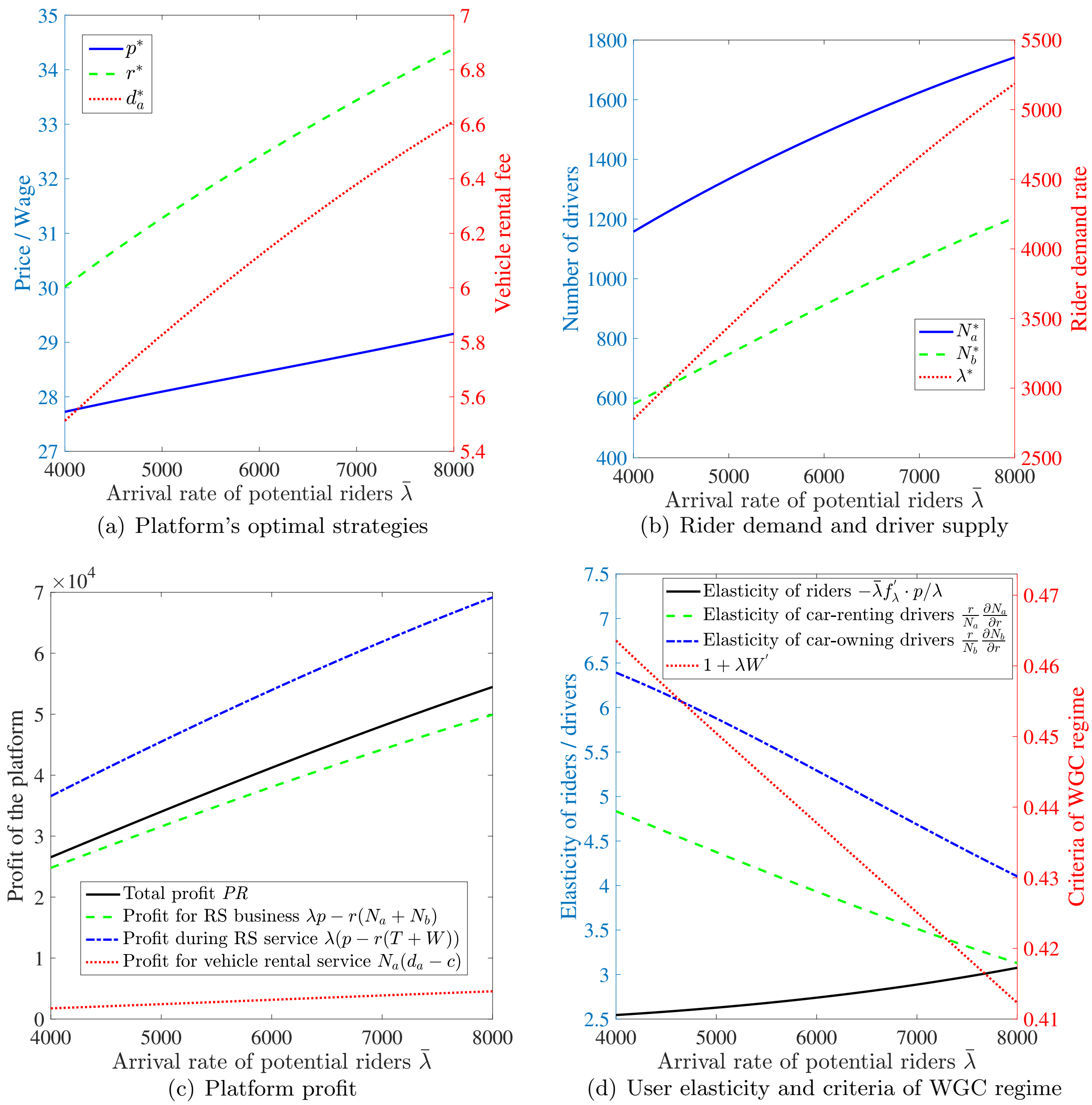
(c) Platform profit

(d) User elasticity and criteria of WGC regime

Figure 6 **Monopoly optimum states under different proportions of potential car-renting drivers ($\bar{N}_a + \bar{N}_b = 4500$).**

that the numerical results of riders' elasticity, drivers' elasticity, and non-WGC regime criteria are consistent with our analytical results.

Social welfare under regulatory policies for the minimum driver wage $\underline{r}$ and maximum vehicle rental fee $\bar{d}_a$ is illustrated in Fig. 8. When regulations are relatively loose, i.e., $\underline{r} < r^* = 32.41$ CNY/h and $\bar{d}_a > d_a^* = 6.12$ CNY/h, the government cannot impose effective constraints on the platform's strategies, and the market equilibrium is the same as the MO state. In the minimum-wage-effective region with strict regulation of the minimum driver wage and loose regulation of the maximum rental fee (i.e., $\underline{r} > r^*$ and $\bar{d}_a > f_{d_a}(\underline{r})$), social welfare is only affected by minimum driver wage $\underline{r}$, and the improvement of social welfare after optimizing $\underline{r}$ is relatively small, i.e.,



(a) Platform's optimal strategies

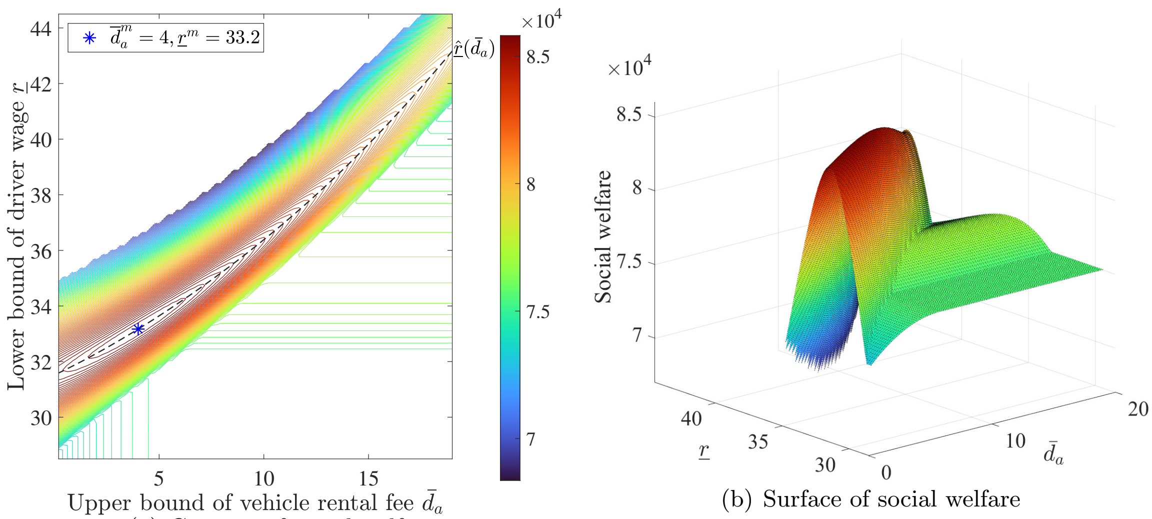
(b) Rider demand and driver supply

(c) Platform profit

(d) User elasticity and criteria of WGC regime

Figure 7 Monopoly optimum states under different arrival rates of potential riders.

$SW(\underline{r} = 35.3, \bar{d}_a > 10.6) = 7.70 \times 10^4$ CNY/h $> SW(\underline{r} < r^*, \bar{d}_a > d_a^*) = 7.55 \times 10^4$ CNY/h. In the maximum-rental-fee-effective region, in which $\bar{d}_a < d_a^*$ and $\underline{r} < f_{\underline{r}}(\bar{d}_a)$, social welfare is even worse off under strict regulation of the maximum rental fee and loose regulation of the minimum driver wage. As a comparison, maximum social welfare is achieved in the coordinated policy region when $\underline{r}^m = 33.17$ and $\bar{d}_a^m = 4$, reaching 8.60×10^4 CNY/h and 13.9% higher than that of the MO state. The results indicate that regulation with coordinated policies is superior to individual policies. Notice that the optimal maximum rental fee equals the rental vehicle's operating cost, i.e., $\bar{d}_a^m = c = 4$ CNY/h, as predicted in Proposition 5. Moreover, as observed in Fig. 8(a), policies regarding the best-response minimum driver wage $\hat{\underline{r}}(\bar{d}_a)$ can achieve satisfactory regulatory effects; if policies deviate from $\underline{\hat{r}}(\bar{d}_a)$, social welfare would then decline rapidly. Lastly, when regulation of both the minimum driver wage and maximum rental fee are strict, the platform is no longer profitable (i.e., $\tilde{PR} < 0$), which is regarded as a market failure.



(a) Contour of social welfare

(b) Surface of social welfare

Figure 8 Social welfare under different regulations of driver wage and rental fee.

We also examine driver supply, rider demand, the platform's optimal strategies, and maximum profit under different regulatory policies in Fig. 9. In the maximum-rental-fee-effective region, if the platform is forced to decrease vehicles' rental fee, it will adopt a lower wage (Fig. 9(b)) to reduce hiring costs. As a result, under lower $\bar{d}_a$, fewer car-owning drivers work on the platform (Fig. 9(e)) with a reduced wage, while more car-renting drivers join the platform (Fig. 9(d)), since the benefit from lower vehicle rents outweighs the loss of the lower wage. Similarly, in the minimum-wage-effective region, the platform will raise vehicles' rental fee (Fig. 9(a)) to boost its revenue if it is required to increase drivers' wage. Consequently, for higher $\underline{r}$, more car-owning drivers are attracted to work on the platform (Fig. 9(e)), while the number of car-renting drivers decreases (Fig. 9(d)) due to the high vehicle rental fee. In the coordinated policy region, both car-renting and car-owning drivers would increase significantly under strict regulation of the minimum driver wage, and car-renting drivers are more willing to work on the platform with strict regulation of the maximum rental fee. The total number of drivers $N_a + N_b$ increases rapidly with strict regulation only in the coordinated policy region, as shown in Fig 9(f). In the minimum-wage-effective region, $N_a + N_b$ increases mildly with the minimum driver wage; in the maximum-rental-fee-effective region, $N_a + N_b$ even slightly decreases with the maximum rental fee, since the platform adopts a lower wage in response to the strict regulation of the vehicle rental fee.

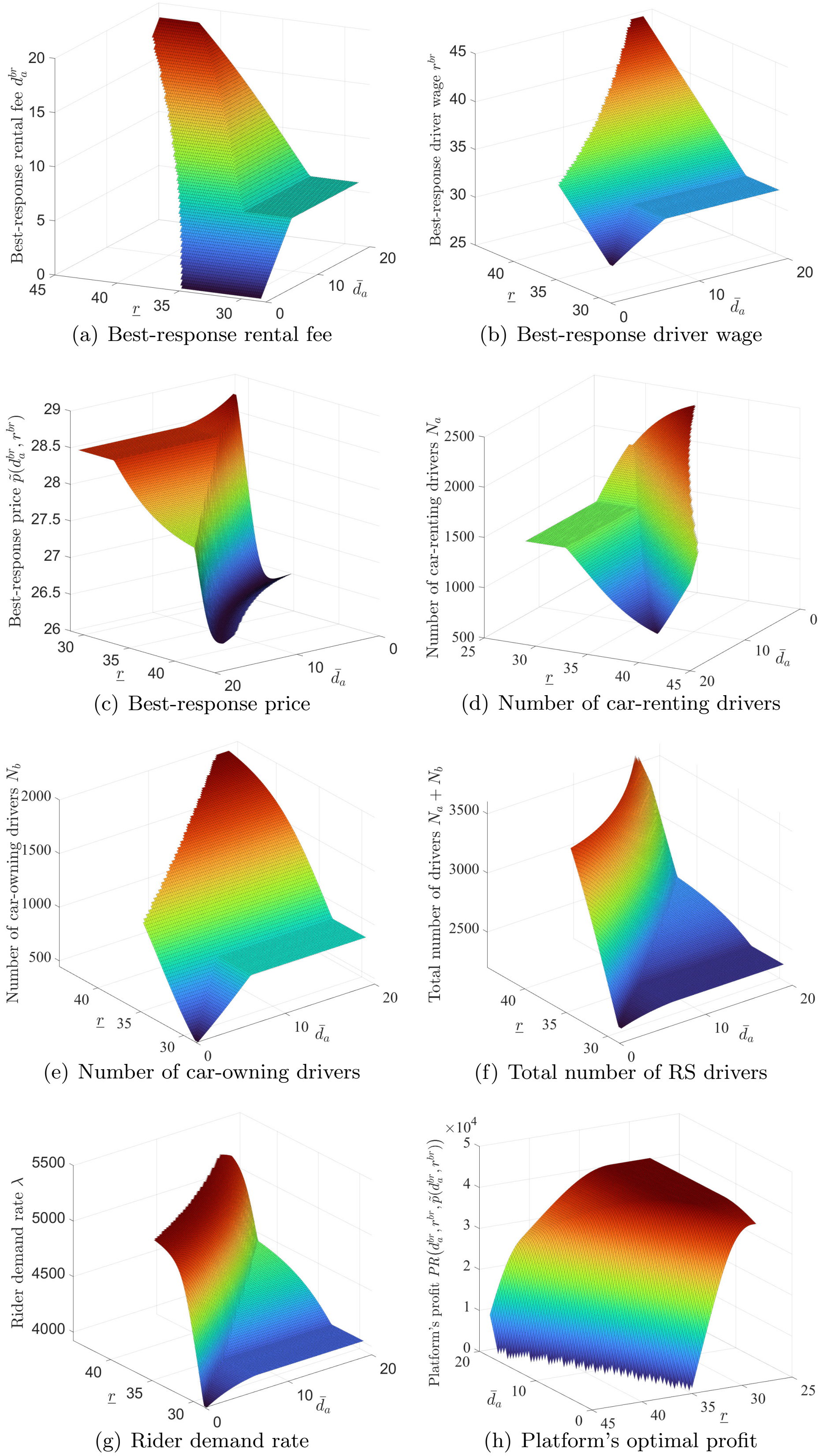
On the demand side, variation in the rider demand rate with two regulatory policies is similar to that of drivers and shows that under government regulations, the platform still attempts to maintain supply and demand balance by adjusting prices. Specifically, we find in Fig. 9(a) that compared with the ineffective region, the platform sets a higher price in the maximum-rental-fee-effective region to restrain rider demand, and sets a lower price in the minimum-wage-effective region to stimulate demand. In the coordinated policy region, the price first declines then slightly rebounds with stricter regulations (i.e., higher $\underline{r}$ and lower $\bar{d}_a$). The reason is that as the total number of drivers keeps increasing under stricter regulations, riders' waiting time plummets and attracts more riders, even if the platform raises the price.

As demonstrated in Fig. 9(h), the platform experiences a rapid profit decrease under stricter regulations. Combined with Fig. 9(f, g), we can conclude that drivers and riders benefit from stricter policies, while the platform benefits from looser policies.

We further examine the effect of regulatory policies by changing the proportion of potential car-renting drivers $\bar{N}_a/(\bar{N}_a+\bar{N}_b)$, as shown in Fig. 10. The results indicate that the best-response minimum driver wage $\hat{\underline{r}}$ becomes more sensitive to the maximum rental fee $\bar{d}_a$ when $\bar{N}_a/(\bar{N}_a+\bar{N}_b)$ increases. The intuition is as follows. The trade-off between the welfare of car-owning drivers and platform profitability solely depends on wage r. To balance the interest of car-renting drivers and the platform, the government should focus on regulating integrated hiring cost $r-(d_a-c)$ for each driver. Therefore, when car-renting drivers become the majority, the minimum driver wage should be more responsive compared with the maximum rental fee, which keeps $r-d_a$ within a reasonable range.

Fig. 10 also shows that the coordinated policy can consistently achieve higher social welfare than the minimum-wage-effective or maximum-rental-fee-effective policy, and demonstrates its superiority under different proportions of $\bar{N}_a/(\bar{N}_a+\bar{N}_b)$. As the proportion increases, the coordinated policy shows more advantages than regulation with a single effective policy, i.e., $SW(\underline{r}=35.5, \bar{d}_a > 12.1) - SW(\underline{r}<r^*, \bar{d}_a > d_a^*) = 3{,}055$ and $SW(\underline{r}^m, \bar{d}_a^m) - SW(\underline{r}<r^*, \bar{d}_a > d_a^*) = 1.06\times 10^4$ when $\bar{N}_a = 1{,}500$, while $SW(\underline{r}=35.2, \bar{d}_a > 10.1) - SW(\underline{r}<r^*, \bar{d}_a > d_a^*) = 947$ and $SW(\underline{r}^m, \bar{d}_a^m) - SW(\underline{r}<r^*, \bar{d}_a > d_a^*) = 1.03\times 10^4$ when $\bar{N}_a = 3{,}000$. This indicates that it is more valuable to coordinate regulations of both the minimum driver wage and maximum rental fee when there are more car-renting drivers.

Fig. 11 investigates how the best-response minimum driver wage $\hat{\underline{r}}(\bar{d}_a)$ is affected by various exogenous parameters, including potential riders' demand rate $\bar{\lambda}$, rental vehicle's operating cost c, and the proportion of potential car-renting drivers $\bar{N}_a/(\bar{N}_a+\bar{N}_b)$. Consistent with Proposition 5, optimal maximum rental cost $\bar{d}_a^m$ equals vehicle operating cost c.



(a) Best-response rental fee
(b) Best-response driver wage
(c) Best-response price
(d) Number of car-renting drivers
(e) Number of car-owning drivers
(f) Total number of RS drivers
(g) Rider demand rate
(h) Platform's optimal profit

Figure 9 Monopoly optimum states with respect to regulation of driver wage and vehicle rental fee.

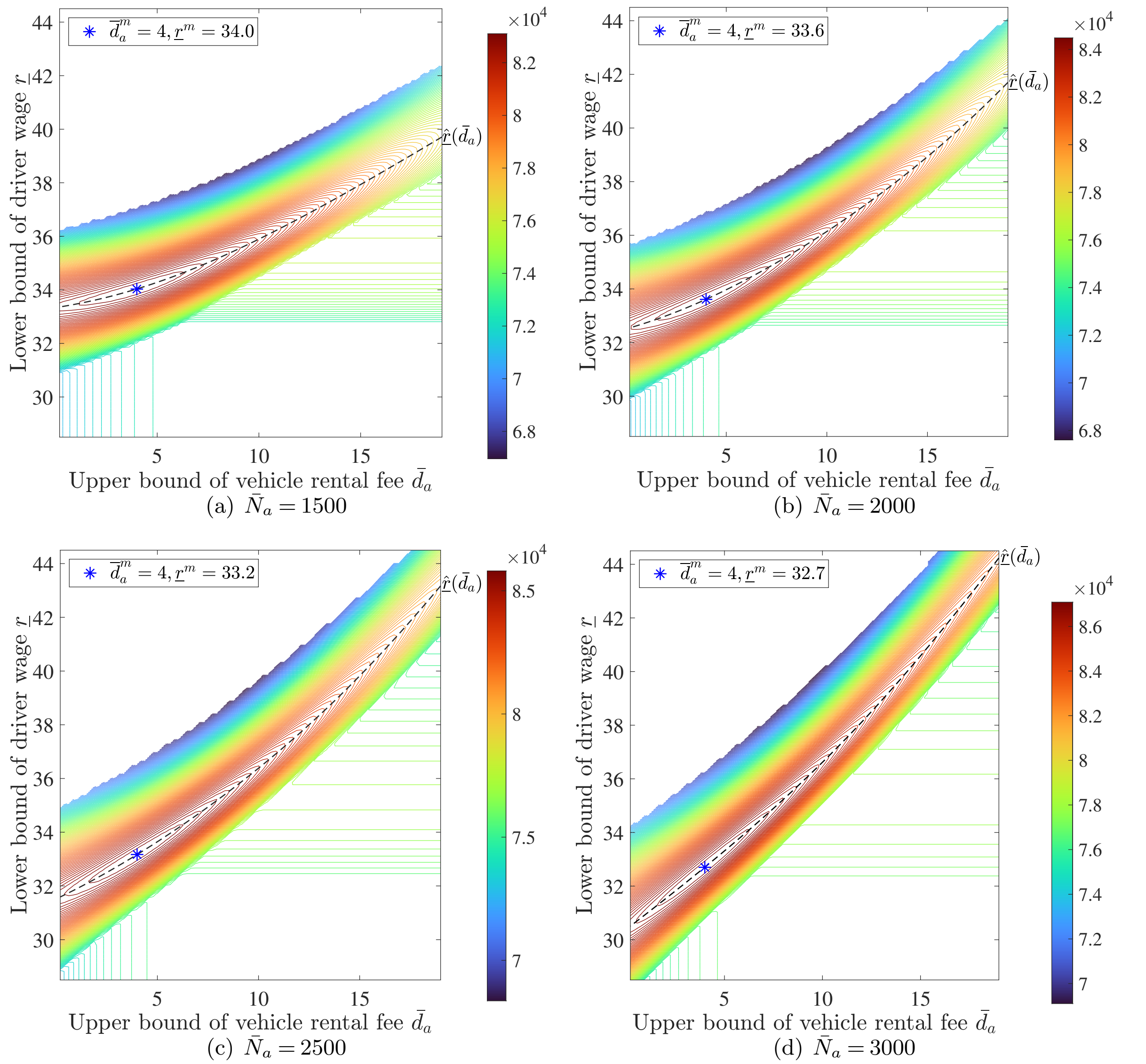


Figure 10 **Contour of social welfare under different proportions of potential car-renting drivers ($\bar{N}_a + \bar{N}_b = 4500$).**

According to Fig. 11(a), the government should raise drivers' minimum driver wage $\underline{r}$ to pursue higher welfare in case of a higher demand rate. In addition to benefiting drivers from a higher wage, the increase in $\underline{r}$ prevents riders' excessive waiting time due to a relative shortage of driver supply. Meanwhile, the platform earns a lower profit on each trip but serves higher market demand. It is interesting to find in Fig. 11(b) that the best-response curve $\hat{\underline{r}}(\bar{d}_a)$ does not change significantly with the vehicle's operating cost c. First, drivers' surplus is solely affected by two policies $\bar{d}_a$ and $\underline{r}$ and is independent of cost c borne by the platform. Second, to coordinate rider demand and driver supply, riders' price should mainly be affected by the regulations, but less influenced by c. That is to say, riders' surplus and the profit from the RS service are not sensitive to c. As a result, only the profit from the car rental service (i.e., $(d_a - c)N_a$) directly decreases with c. Therefore, given the same regulation of the minimum driver wage $\underline{r}$, Fig. 11(b) shows that the government should adopt slightly looser regulation of the maximum rental fee when c increases, considering the trade-off between the profit from the car rental service and the remaining components of social welfare (i.e., riders' surplus, drivers' surplus, and the RS service's profit). Finally, Fig. 11(c) shows that the best-response minimum driver wage is more sensitive to regulation of the maximum rental fee when the proportion of potential car-renting drivers $\bar{N}_a/(\bar{N}_a + \bar{N}_b)$ increases, as we discussed in Fig. 10.

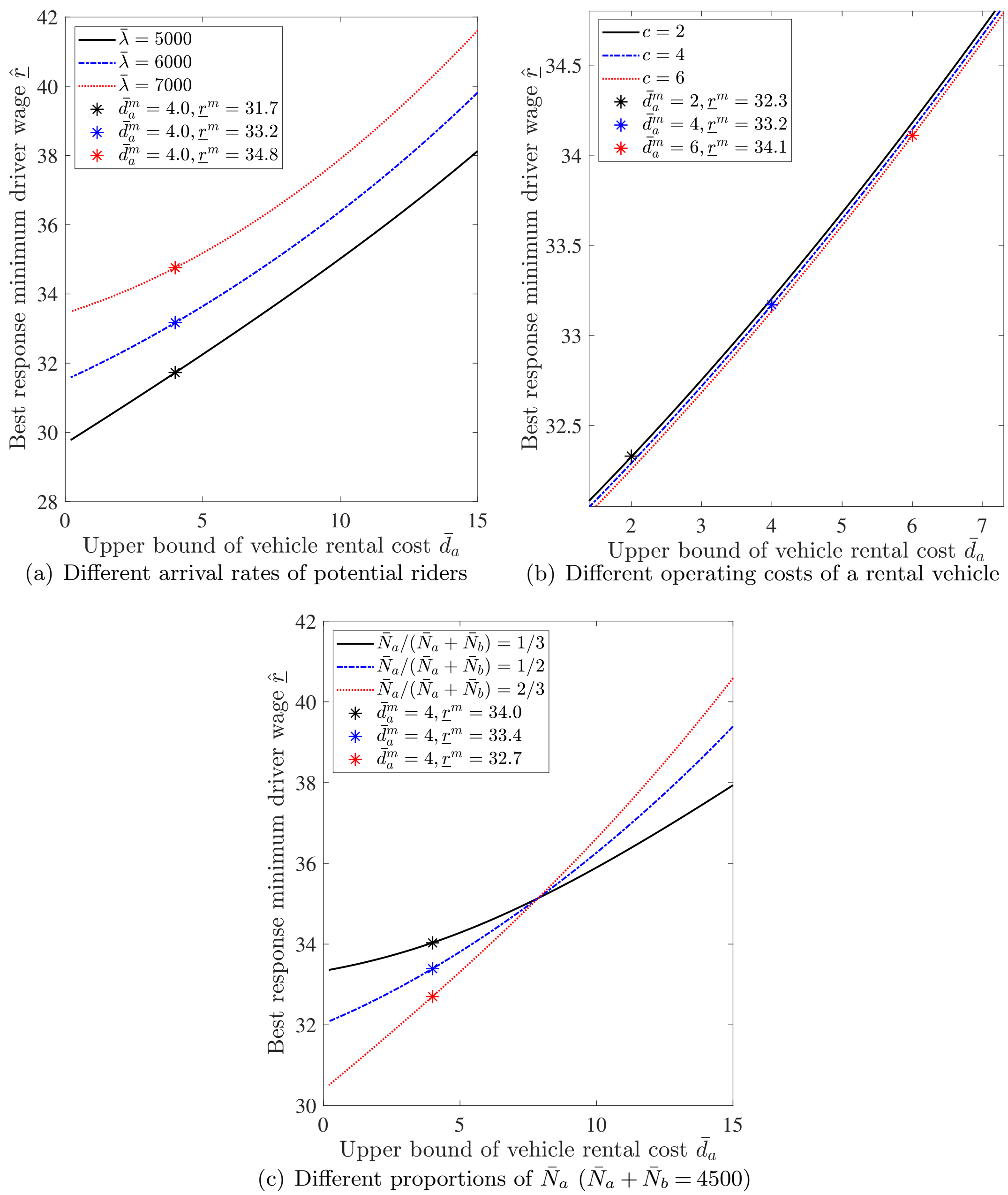
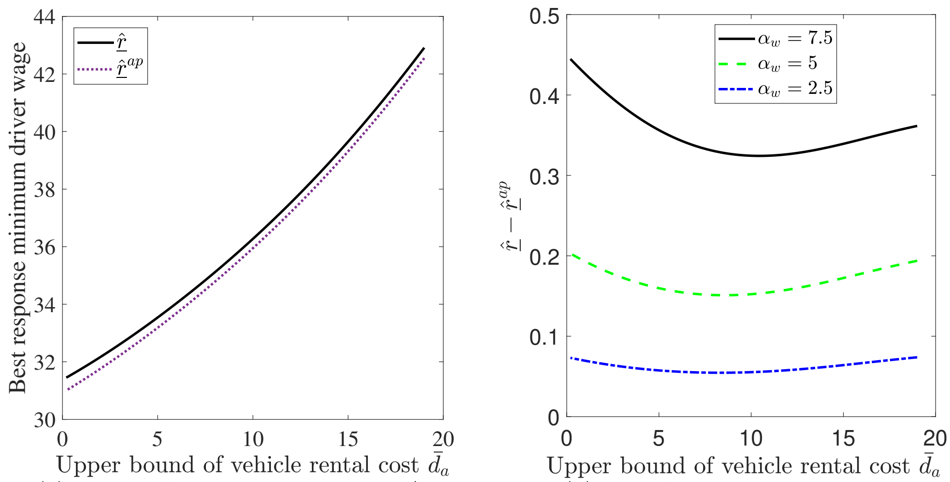


Figure 11 Sensitivity of $\hat{\underline{r}}(\bar{d}_a)$ with respect to exogenous parameters.

We also evaluate the accuracy of the approximate best-response minimum driver wage $\underline{\hat{r}}^{ap}(\bar{d}_a)$, presented in Fig. 12. As can be seen, the approximate curve exhibits patterns similar to the original curve $\underline{\hat{r}}(\bar{d}_a)$, and it is more accurate when riders' value of waiting time α_w is lower.



(a) $\underline{\hat{r}}$ and $\underline{\hat{r}}^{ap}$ when $\alpha_w = 7.5$ CNY/h (b) $\underline{\hat{r}} - \underline{\hat{r}}^{ap}$ under different α_w

Figure 12 **Evaluation of the approximate best-response minimum driver wage $\underline{\hat{r}}^{ap}(\bar{d}_a)$.**

6. Conclusions

This paper examines a monopoly ride-sourcing market model integrated with vehicle rental services, and proposes a bi-level optimization model to analyze the government's regulation of driver wage and vehicle rental fee. We first derive the monopoly optimum state for the market without regulation. Our analytical results show that the market is located in the non-WGC regime under the monopoly optimum state, consistent with the conclusions in Ke et al. (2020) and Zhou et al. (2022); we also consider the integration of a vehicle rental service. Then, we theoretically elucidate that the platform's profitability from the vehicle rental service is closely related to riders' sensitivity to the vehicle rental fee and the elasticities of car-owning drivers and car-renting drivers.

We then gain insights into regulation of the minimum driver wage and maximum rental fee based on the proposed phase diagram of the two regulatory policies. When the government solely focuses on one policy (e.g., minimum driver wage) without coordinating with the other (e.g., maximum rental fee), not all drivers necessarily benefit. Instead, one group (e.g., car-renting drivers) may suffer. The reason is that given relatively loose regulation of one dimension (e.g., the vehicle rental fee), the platform would flexibly adjust its best-response strategy for the other dimension to avoid ceding benefits to drivers. Then we derive the phase diagram of regulatory policies in depth. In the coordinated policy region, both the minimum driver wage and maximum rental fee effectively regulate the platform's strategies. We prove that when the government equally weights all stakeholders' welfare and the optimal policies lie inside this region, the optimal maximum rental fee equals the platform's operating cost of its vehicle, and the platform earns zero profit from the vehicle rental service. Moreover, we derive the optimal minimum driver wage in an implicit form.

We conduct numerical experiments to verify our theoretical findings and demonstrate the superiority of the coordinated regulatory policies region compared with other regions of a single effective policy. The numerical results indicate that the coordinated regulatory policy offers more advantages. The government's best-response minimum driver wage is more sensitive to the maximum rental fee when the proportion of potential car-renting drivers increases. In comparison, the best-response minimum driver wage does not change significantly with the rental vehicle's operating cost. In addition, the government should raise the minimum driver wage as potential rider demand increases.

We note that non-car owners on some TNCs (e.g., Uber and Lyft) rent vehicles from third-party vehicle rental companies. Future research could model the competition and cooperation between a monopoly TNC (or oligopoly TNCs) and a monopoly platform (or oligopoly platforms) that operates vehicle rental services. Since a unified vehicle-ownership mode (CaoCao 2022) allows car-renting drivers to provide more standardized services with stable quality, future research could investigate riders' heterogeneous willingness to pay for the services provided by car-renting drivers and car-owning drivers, and examine the differentiated pricing strategies of the two service types.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (71922019, 72171210), Zhejiang Provincial Natural Science Foundation of China (LZ23E080002), National Key Research and Development Program of China (2020AAA0107400), and Lee Kong Chian Fellowship awarded by Singapore Management University. The corresponding author of this research is Xiqun (Michael) Chen.

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Appendix A: Proof of Corollary 1

Taking the derivative of λ and N_v with respect to p, we have

$$\frac{\partial \lambda}{\partial p} = \bar{\lambda} \cdot \Big(\frac{\partial f_\lambda}{\partial p} + \frac{\partial f_\lambda}{\partial W} W' \frac{\partial N_v}{\partial p} \Big) \tag{18}$$

$$\frac{\partial N_v}{\partial p} = -\frac{\partial \lambda}{\partial p} \cdot (T+W) - \lambda W' \frac{\partial N_v}{\partial p} \tag{19}$$

Solving Eq. (18-19) yields

$$\frac{\partial \lambda}{\partial p} = \frac{\partial f_\lambda / \partial p}{1/\bar{\lambda} + \frac{W' \cdot (T+W)}{1+\lambda W'} \cdot \frac{\partial f_\lambda}{\partial W}} \tag{20}$$

$$\frac{\partial N_v}{\partial p} = -\frac{\partial f_\lambda / \partial p}{W' \frac{\partial f_\lambda}{\partial W} + \frac{1+\lambda W'}{\bar{\lambda} \cdot (T+W)}} \tag{21}$$

Taking the derivatives of λ and N_v with respect to d_a, we have

$$\frac{\partial \lambda}{\partial d_a} = \bar{\lambda} \frac{\partial f_\lambda}{\partial W} W' \frac{\partial N_v}{\partial d_a} \tag{22}$$

$$\frac{\partial N_v}{\partial d_a} = -\bar{N}_a f_a^{'} - \Big(\frac{\partial \lambda}{\partial d_a} \cdot (T+W) + \lambda W' \frac{\partial N_v}{\partial d_a} \Big) \tag{23}$$

Solving Eq. (22-23) yields

$$\frac{\partial \lambda}{\partial d_a} = \bar{\lambda} \frac{\partial f_\lambda}{\partial W} W' \frac{\partial N_v}{\partial d_a} = -\frac{\bar{N}_a f_a^{'}}{T + W + (1+\lambda W')/(\bar{\lambda} W' \cdot \frac{\partial f_\lambda}{\partial W})} \tag{24}$$

Similarly, examining the derivatives of λ and N_v with respect to r yields

$$\frac{\partial \lambda}{\partial r} = \bar{\lambda} \frac{\partial f_\lambda}{\partial W} W' \frac{\partial N_v}{\partial r} = \frac{\bar{N}_a f_a^{'} + \bar{N}_b f_b^{'}}{T + W + (1+\lambda W')/(\bar{\lambda} W' \cdot \frac{\partial f_\lambda}{\partial W})} \tag{25}$$

Appendix B: Proof of Proposition 1

Taking the derivative of PR with respect to d_a and r, respectively, we have

$$\frac{\partial PR}{\partial d_a} = \frac{\partial \lambda}{\partial d_a} \cdot p + N_a + \bar{N}_a f_a^{'} \cdot (r + c - d_a) = 0 \tag{26}$$

$$\frac{\partial PR}{\partial r} = -\frac{\partial \lambda}{\partial r} \cdot p + N_a + N_b + r \bar{N}_b f_b^{'} + \bar{N}_a f_a^{'} \cdot (r + c - d_a) = 0 \tag{27}$$

Substituting Eq. (26) into Eq. (27), we have

$$-p \frac{\partial \lambda}{\partial r} + N_b + r \bar{N}_b f_b^{'} - \frac{\partial \lambda}{\partial d_a} p = 0 \tag{28}$$

According to Corollary 1, we have $\frac{\partial \lambda}{\partial r} = -\frac{\partial \lambda}{\partial d_a} \cdot (1 + \frac{\bar{N}_b f_b^{'}}{\bar{N}_a f_a^{'}})$. Substituting it into Eq. (28), we have

$$\frac{\partial \lambda}{\partial d_a} = -\frac{(N_b + r \bar{N}_b f_b^{'}) \bar{N}_a f_a^{'}}{p \bar{N}_b f_b^{'}} \tag{29}$$

Substituting $\frac{\partial \lambda}{\partial d_a} = -\frac{\bar{N}_a f_a'}{T+W+(1+\lambda W')/(\bar{\lambda} W' \cdot \frac{\partial f_\lambda}{\partial W})}$ (Corollary 1) into Eq. (29), we have

$$T + W + \frac{1+\lambda W'}{\bar{\lambda} W' \frac{\partial f_\lambda}{\partial W}} = \frac{p \bar{N}_b f_b'}{N_b + r\bar{N}_b f_b'} \tag{30}$$

Taking the derivative of PR with respect to p, we have

$$\frac{\partial PR}{\partial p} = \frac{\partial \lambda}{\partial p} \cdot p + \lambda = \frac{\partial f_\lambda / \partial p}{1/\bar{\lambda} + \frac{W' \cdot (T+W)}{1+\lambda W'} \cdot \frac{\partial f_\lambda}{\partial W}} \cdot p + \lambda = 0 \tag{31}$$

Thus, when the platform profit is maximized, the optimal price satisfies

$$p = -\lambda \cdot \frac{1/\bar{\lambda} + \frac{W' \cdot (T+W)}{1+\lambda W'} \cdot \frac{\partial f_\lambda}{\partial W}}{\partial f_\lambda / \partial p} \tag{32}$$

Substituting Eq. (32) into Eq. (30) finally yields

$$\frac{1+\lambda W'}{\bar{\lambda} W' \frac{\partial f_\lambda}{\partial W}} = \frac{\lambda \bar{N}_b f_b'}{-\bar{\lambda} \frac{\partial f_\lambda}{\partial p} \cdot (N_b + r\bar{N}_b f_b')} \tag{33}$$

Since $f_b' > 0$, $\frac{\partial f_\lambda}{\partial p} < 0$, $\frac{\partial f_\lambda}{\partial W} < 0$, $W' < 0$, according to Eq. (33), we have $1 + \lambda W' > 0$, which indicates that the market is located in the non-WGC regime under the MO state.

According to Eq. (31), we have

$$\left(-\bar{\lambda} \frac{\partial f_\lambda}{\partial p}\right) \cdot \frac{p}{\lambda} - 1 = \frac{\bar{\lambda} W' \cdot (T+W)}{1+\lambda W'} \cdot \frac{\partial f_\lambda}{\partial W} > 0$$

This indicates that the elasticity of λ with respect to p, ignoring the endogenous change of waiting time, is greater than 1 under the MO state, i.e., $(-\bar{\lambda} \frac{\partial f_\lambda}{\partial p}\big|_{p^*,\lambda^*}) \cdot \frac{p^*}{\lambda^*} > 1$.

Appendix C: Proof of Proposition 2 and Corollary 2

(a) According to Eq. (26) in Appendix B, when $d_a - c \le 0$ under the MO state, we have

$$\frac{\partial \lambda}{\partial d_a} \cdot p + N_a + r\bar{N}_a f_a' = \bar{N}_a f_a' \cdot (d_a - c) \le 0$$

Substituting the expression of $\frac{\partial \lambda}{\partial d_a}$ (Eq. (24)) into $\frac{\partial \lambda}{\partial d_a} \cdot p + N_a + r\bar{N}_a f_a' \le 0$, we have

$$\frac{\partial \lambda}{\partial W} = \alpha_w \bar{\lambda} f_\lambda' \le \frac{1/W' + \lambda^*}{\frac{p^*}{r^* + N_a^*/(\bar{N}_a f_a')} - (T+W)}$$

Otherwise (i.e., $d_a - c > 0$), we have

$$\frac{\partial \lambda}{\partial W} > \frac{1/W' + \lambda^*}{\frac{p^*}{r^* + N_a^*/(\bar{N}_a f_a')} - (T+W)}$$

(b) According to Corollary 1, we have

$$\frac{\partial \lambda}{\partial d_a} = -\frac{\partial \lambda}{\partial p} \cdot \frac{(\bar{N}_a f_a' W')\partial f_\lambda / \partial W}{(1+\lambda W')\partial f_\lambda / \partial p} \tag{34}$$

According to Eq. (31) in Appendix B, $\frac{\partial \lambda}{\partial p} = -\frac{\lambda}{p}$ holds under the MO state. We then have

$$\frac{\partial \lambda}{\partial d_a} = \frac{\lambda}{p} \cdot \frac{(\bar{N}_a f_a' W')\partial f_\lambda / \partial W}{(1+\lambda W')\partial f_\lambda / \partial p} \tag{35}$$

Substituting Eq. (35) into Eq. (26), we have

$$\lambda \cdot \frac{(\bar{N}_a f_a' W')\partial f_\lambda/\partial W}{(1+\lambda W')\partial f_\lambda/\partial p} + N_a + \bar{N}_a f_a' \cdot (r + c - d_a) = 0 \tag{36}$$

According to Eq. (33), we have

$$\frac{(\lambda W')\partial f_\lambda/\partial W}{(1+\lambda W')\partial f_\lambda/\partial p} = -r - \frac{N_b}{\bar{N}_b f_b'} \tag{37}$$

Substituting Eq. (37) into Eq. (36), we can derive that the MO state satisfies

$$d_a^* - c = \Big(\frac{N_a}{\bar{N}_a f_a'} - \frac{N_b}{\bar{N}_b f_b'}\Big)\Big|_{d_a^*, r^*} = \Big(\frac{N_a}{\partial N_a/\partial r} - \frac{N_b}{\partial N_b/\partial r}\Big)\Big|_{d_a^*, r^*} \tag{38}$$

Notice that $\partial N_{a(b)}/c\partial r > 0$; therefore, when $d_a^* > c$, we have $0 < \frac{r^*}{N_a^*}\frac{\partial N_a}{\partial r}\big|_{d_a^*, r^*} < \frac{r^*}{N_b^*}\frac{\partial N_b}{\partial r}\big|_{r^*}$; when $d_a^* \le c$, we have $\frac{r^*}{N_a^*}\frac{\partial N_a}{\partial r}\big|_{d_a^*, r^*} \ge \frac{r^*}{N_b^*}\frac{\partial N_b}{\partial r}\big|_{r^*} > 0$.

(c) When $N_a = \bar{N}_a \min\big\{\max\{lu_a + \epsilon, 0\}, 1\big\}$, $N_b = \bar{N}_b \min\big\{\max\{lu_b + \epsilon, 0\}, 1\big\}$, we have $\frac{\partial N_a}{\partial r} = \bar{N}_a l$ and $\frac{\partial N_b}{\partial r} = \bar{N}_b l$. Substituting them into Eq. (38), we obtain that $\frac{N_a^*}{\bar{N}_a} > \frac{N_b^*}{\bar{N}_b}$ if and only if $d_a^* > c$. Meanwhile, $\frac{N_a^*}{\bar{N}_a} > \frac{N_b^*}{\bar{N}_b}$ indicates $u_a^* > u_b^*$, according to the expressions of the above linear models. Substituting $d_a^* > c$ into $u_a^* = r - d_a^* - o_a > u_b^* = r - d_b - o_b$, we obtain that $c + o_a < d_b + o_b$ must be satisfied when $d_a^* > c$. Symmetrically, $c + o_a \ge d_b + o_b$ must be satisfied when $d_a^* \le c$.

When $N_a = \bar{N}_a/\big(1 + e^{-\theta u_a}\big)$, $N_b = \bar{N}_b/\big(1 + e^{-\theta u_b}\big)$, we have $\frac{\partial N_a}{\partial r} = \frac{\theta N_a \cdot (\bar{N}_a - N_a)}{\bar{N}_a}$ and $\frac{\partial N_b}{\partial r} = \frac{\theta N_b \cdot (\bar{N}_b - N_b)}{\bar{N}_b}$. Substituting them into Eq. (38), we can obtain that $\frac{N_a^*}{\bar{N}_a} > \frac{N_b^*}{\bar{N}_b}$ if and only if $d_a^* > c$. Similarly, $\frac{N_a^*}{\bar{N}_a} > \frac{N_b^*}{\bar{N}_b}$ indicates that $u_a^* > u_b^*$, according to the expressions of the above MNL models. Thus, similar to the case of the linear model, $d_a^* > c$ if and only if $c + o_a < d_b + o_b$.

Appendix D: Proof of Corollary 3

According to Eq. (32) in Appendix B, we have

$$-\frac{p}{\lambda} - \frac{1}{\bar{\lambda}\partial f_\lambda/\partial p} = (T + W) \cdot \frac{W'\partial f_\lambda/\partial W}{(1+\lambda W')\partial f_\lambda/\partial p} \tag{39}$$

Substituting Eq. (37) into Eq. (39), we have

$$-\frac{p}{\lambda} - \frac{1}{\bar{\lambda}\partial f_\lambda/\partial p} = -(T + W) \cdot (\frac{r}{\lambda} + \frac{N_b}{\lambda \bar{N}_b f_b'}), \tag{40}$$

which indicates:

$$p - (T + W)r = \frac{N_b \cdot (T + W)}{\bar{N}_b f_b'} + \frac{\lambda}{\bar{\lambda}(-\frac{\partial f_\lambda}{\partial p})} = \frac{N_b \cdot (T + W)}{\frac{\partial N_b}{\partial r}} + \frac{\lambda}{\bar{\lambda}(-\frac{\partial f_\lambda}{\partial p})} > 0 \tag{41}$$

Appendix E: Proof of Proposition 3

We first determine the platform's optimal rental fee d_a for a given wage r. When Condition 1(b) is satisfied, optimal d_a can be obtained by solving $\frac{\partial PR}{\partial d_a} = 0$ (Eq. (26)) and $\frac{\partial PR}{\partial p} = 0$ (Eqs. (31-32)). Substituting the expression of $\frac{\partial \lambda}{\partial d_a}$ (Eq. (24)) into Eq. (32), we have

$$\frac{\partial \lambda}{\partial d_a} p = \frac{\alpha_w \lambda W'}{1 + \lambda W'} \bar{N}_a f_a'$$

Substituting it into Eq. (26), the optimal d_a satisfies

$$d_a = f_{d_a}(d_a, r) \triangleq r + c + \frac{\alpha_w \lambda W'}{1+\lambda W'} + \frac{N_a}{\partial N_a/\partial r} \tag{42}$$

According to the definition of $\tilde{PR}$, Eq. (42) is the solution to $\frac{\partial \tilde{PR}}{\partial d_a} = 0$ for given wage r.

Based on Condition 1(c), we can similarly derive that the optimal wage for a given rental fee (i.e., the solution to $\frac{\partial \tilde{PR}}{\partial r} = 0$) satisfies

$$r = f_r(d_a, r) \triangleq -\frac{\alpha_w \lambda W'}{1+\lambda W'} - \frac{N + (c - d_a)\partial N_a/\partial r}{\partial N/\partial r} \tag{43}$$

Notice that when $(d_a, r) \neq (d_a^*, r^*)$, the gradient of $\tilde{PR}$ is nonzero according to Condition 1(a), i.e., $\nabla \tilde{PR}(d_a, r) \neq \mathbf{0}, \forall (d_a, r) \neq (d_a^*, r^*)$. Therefore, when $\tilde{PR}$ has a unique maximum point (d_a^*, r^*), curve $d_a = f_{d_a}(d_a, r)$ and curve $r = f_r(d_a, r)$ have a unique intersection (d_a^*, r^*).

We then discuss the properties of the two curves in Eqs. (42-43). For ease of derivation, we denote $r^i(d_a)$ as the solution to $d_a = f_{d_a}(d_a, r)$ for given d_a, and $r^j(d_a)$ as the solution to $r = f_r(d_a, r)$. Then, under Condition 1(a-c), the following inequality always holds:

$$\big(r^i(d_a) - r^*\big) \cdot \big(r^i(d_a) - r^j(d_a)\big) > 0, \forall d_a \neq d_a^* \tag{44}$$

Proof. Since $\frac{\partial \tilde{PR}}{\partial r}\big|_{r^j(d_a)} = 0$ for a given d_a, if $r^i > r^j$, we have $\frac{\partial \tilde{PR}}{\partial r}\big|_{r^i(d_a)} < 0$ according to Condition 1(c). Meanwhile, notice that $\frac{\partial \tilde{PR}}{\partial d_a}\big|_{r^i(d_a)} = 0$; according to Condition 1(a), we further have

$$(d_a - d_a^*)\frac{\partial \tilde{PR}}{\partial d_a}\Big|_{r^i(d_a)} + (r^i - r^*)\frac{\partial \tilde{PR}}{\partial r}\Big|_{r^i(d_a)} = (r^i - r^*)\frac{\partial \tilde{PR}}{\partial r}\Big|_{r^i(d_a)} < 0, \tag{45}$$

which indicates that $r^i(d_a) > r^*$ must be satisfied. We can prove in a similar way that $r^i(d_a) < r^*$ holds if $r^i(d_a) < r^j(d_a)$. □

According to Eq. (44), there are four possible scenarios of the two curves, as illustrated in Fig. 13. Based on the above conclusions, we next discuss the platform's best-response strategies (d_a^{br}, r^{br}) under different regulatory policies:

(a) When $\bar{d}_a > d_a^*$ and $\underline{r} < r^*$, it is feasible for the platform to adopt the monopoly optimum (d_a^*, r^*) to achieve the maximum profit $\tilde{PR}(d_a^*, r^*) = PR^*$.

(b) When $\underline{r} > r^*$ and $\bar{d}_a > f_{d_a}(d_a = \bar{d}_a, r = \underline{r})$. In such a case, there are no maximum points in the region $\{(d_a, r) | d_a < \bar{d}_a, r > \underline{r}\}$. Therefore, the optimal strategies are located in either set $\{(d_a, r) | r = \underline{r}, d_a \leq \bar{d}_a\}$ or set $\{(d_a, r) | d_a = \bar{d}_a, r \geq \underline{r}\}$. According to Condition 1(b), in $\{(d_a, r) | r = \underline{r}, d_a \leq \bar{d}_a\}$, the platform profit is optimized at point $(d_a = f_{d_a}(d_a, \underline{r}), r = \underline{r})$, and naturally, $\tilde{PR}(d_a = f_{d_a}(d_a, \underline{r}), r = \underline{r}) > \tilde{PR}(\bar{d}_a, \underline{r})$.

Further, if $r^j(\bar{d}_a) \leq \underline{r}$ holds, as illustrated in Fig. 14(c,d), we obtain from Condition 1(c) that $\tilde{PR}(\bar{d}_a, r) < \tilde{PR}(\bar{d}_a, \underline{r}), \forall r > \underline{r}$, which indicates that the maximum profit is achieved at $(d_a = f_{d_a}(d_a, \underline{r}), r = \underline{r})$ under regulatory policies.

If $r^j(\bar{d}_a) > \underline{r}$ holds, the maximum profit in set $\{(d_a, r) | d_a = \bar{d}_a, r \geq \underline{r}\}$ is located at $(\bar{d}_a, r^j(\bar{d}_a))$. According to Condition 1(a), we have

$$(d_a^* - \bar{d}_a)\frac{\partial \tilde{PR}}{\partial d_a} + (r^* - r^j(\bar{d}_a))\frac{\partial \tilde{PR}}{\partial r} > 0, \forall d_a^* < d_a \leq \bar{d}_a, r = r^* + \frac{d_a - d_a^*}{\bar{d}_a - d_a^*} \cdot (r^j(\bar{d}_a) - r^*) \tag{46}$$

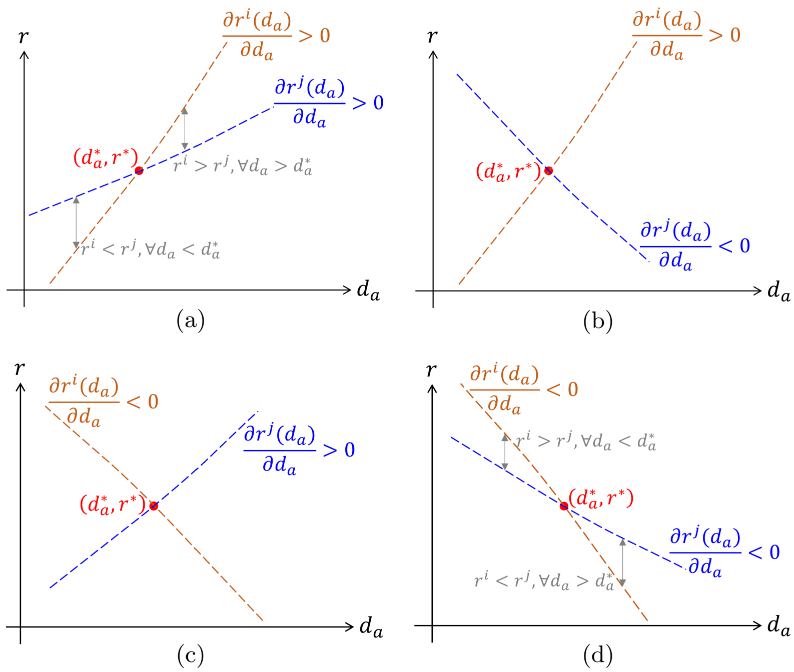

Figure 13 **Possible scenarios of r^i and r^j.**

As demonstrated in Fig. 14(a,b), Eq. (46) indicates that

$$\tilde{PR}\big(\bar{d}_a, r^j(\bar{d}_a)\big) < \tilde{PR}\Big(d_a = \frac{\underline{r} - r^*}{r^j(\bar{d}_a) - r^*} \cdot (\bar{d}_a - d_a^*) + d_a^*, r = \underline{r}\Big) \leq \tilde{PR}\big(d_a = f_{d_a}(d_a, \underline{r}), r = \underline{r}\big).$$

In summary, when regulatory policies satisfy $\underline{r} > r^*$ and $\bar{d}_a > f_{d_a}(d_a = \bar{d}_a, r = \underline{r})$, the platform's best-response strategies are $d_a^{br} = f_{d_a}(d_a^{br}, \underline{r})$ and $r^{br} = \underline{r}$.

(c) When $\bar{d}_a < d_a^*$ and $\underline{r} < f_r(d_a = \bar{d}_a, r = \underline{r})$, we can prove in a similar way that the platform's best-response strategies are $d_a^{br} = \bar{d}_a$, $r^{br} = f_r(\bar{d}_a, r^{br})$.

(d) When $\underline{r} \geq f_r(\bar{d}_a, \underline{r})$, $\bar{d}_a \leq f_{d_a}(\bar{d}_a, \underline{r})$ and $\tilde{PR}(\bar{d}_a, \underline{r}) \geq 0$, according to Condition 1(b,c), we have

$$\tilde{PR}(\bar{d}_a, \underline{r}) > \tilde{PR}(d_a, \underline{r}), \quad \forall d_a < \bar{d}_a$$

$$\tilde{PR}(\bar{d}_a, \underline{r}) > \tilde{PR}(\bar{d}_a, r), \quad \forall r > \underline{r}$$

Meanwhile, there are no maximum points in the region $\{(d_a, r) | d_a < \bar{d}_a, r > \underline{r}\}$; therefore, the platform's best-response wage (and rental fee) is equal to the lower (and upper) bound set by the government, i.e., $d_a^{br} = \bar{d}_a, r^{br} = \underline{r}$.

(e) When $\tilde{PR}(d_a, r) < 0$, $\forall\, r \geq \underline{r}, d_a \leq \bar{d}_a$, it is no longer feasible for the platform to achieve a profit, resulting in unsustainable operation.

To summarize, regulatory policies can be divided into five mutually exclusive regions, as described in Proposition 3, based on the different regulatory effects of the two policies and the platform's different best-response strategies.

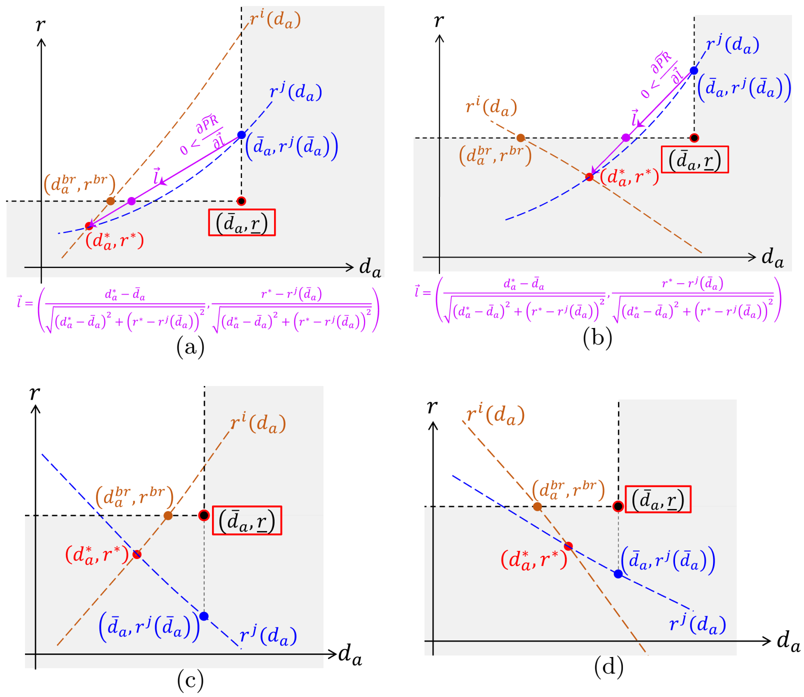

Figure 14 **Best-response strategies** (d_a^{br}, r^{br}) **in different scenarios when** $\underline{r} > r^*$ **and** $\bar{d}_a > f_{d_a}(d_a = \bar{d}_a, r = \underline{r})$.

Appendix F: Proof of Corollary 4

If optimal policies $(\bar{d}_a^m, \underline{r}^m)$ are located in the minimum-wage-effective region, since the market equilibrium is unchanged when lowering $\bar{d}_a$ in this region, we have $SW(\bar{d}_a^m, \underline{r}^m) = SW(\bar{d}_a = f_{d_a}(\bar{d}_a, \underline{r}^m) < \bar{d}_a^m, \underline{r}^m)$. It indicates, the optimal policies can also be achieved at the boundary $\bar{d}_a = f_{d_a}(\bar{d}_a, \underline{r})$ of the coordinated policy region. Similarly, if the optimal policies are located in the maximum-rental-fee-effective region, the optimal policies can simultaneously be achieved at the boundary $\underline{r} = f_r(\bar{d}_a, \underline{r})$. Further, suppose the optimal policies are located in the ineffective region and the monopoly optimum state is equivalent to the social welfare maximization state. In that case, we show in Appendix E that $\bar{d}_a = f_{d_a}(\bar{d}_a, \underline{r})$ and $\underline{r} = f_r(\bar{d}_a, \underline{r})$ have a unique intersection at the monopoly optimal strategies (d_a^*, r^*), which indicates that (d_a^*, r^*) are located at the boundary of the coordinated policy region. Lastly, the government should not adopt restrictive policies that render the platform unprofitable (i.e., $\tilde{PR}(\bar{d}_a, \underline{r}) \leq 0$), which results in market failure in the long run.

Appendix G: Proof of Proposition 4

First, we need to calculate the derivative of the best-response rental fee $d_a^{br}(\underline{r}) = \frac{\alpha_w \lambda W'}{1+\lambda W'} + \frac{N_a}{\partial N_a / \partial r}\Big|_{d_a^{br}, \underline{r}} + \underline{r} + c$ with respect to minimum driver wage $\underline{r}$. Similar to the derivation of Eqs. (22-25), we can obtain the derivatives of λ and N_v with respect to the number of drivers N, as follows:

$$\frac{\partial \lambda}{\partial N} = \bar{\lambda} f_\lambda' \alpha_w W' \frac{\partial N_v}{\partial N} = \frac{1}{T + W + \frac{1+\lambda W'}{\bar{\lambda} f_\lambda' \alpha_w W'}}, \tag{47}$$

from which we have

$$\frac{\partial W'}{\partial N} = W'' \frac{\partial N_v}{\partial N} = \frac{W''}{1 + \lambda W' + \bar{\lambda} f_\lambda' \alpha_w W' \cdot (T + W)} \tag{48}$$

We then obtain

$$\partial\Big(\frac{\lambda W'}{1+\lambda W'}\Big)/\partial N = \partial\Big(1-\frac{1}{1+\lambda W'}\Big)/\partial N = \frac{1}{(1+\lambda W')^2}\cdot\Big(W'\frac{\partial\lambda}{\partial N}+\lambda\frac{\partial W'}{\partial N}\Big) = \frac{1}{(1+\lambda W')^2}\cdot\frac{\lambda W''+\bar{\lambda}f_\lambda'\alpha_w W'^2}{1+\lambda W'+\bar{\lambda}f_\lambda'\alpha_w W'\cdot(T+W)} \tag{49}$$

Moreover, we have

$$\partial\Big(\frac{N_a}{\partial N_a/\partial r}\Big|_{d_a^{br},\underline{r}}\Big)/\partial\underline{r} = \frac{\partial(f_a/f_a')|_{d_a^{br},\underline{r}}}{\partial\underline{r}} = \frac{f_a'\cdot(1-\partial d_a^{br}/\partial\underline{r})f_a' - f_a f_a''\cdot(1-\partial d_a^{br}/\partial\underline{r})}{f_a'^2}\Big|_{d_a^{br},\underline{r}} = \Big(1-\frac{\partial d_a^{br}}{\partial\underline{r}}\Big)\Big(1-\frac{f_a f_a''}{f_a'^2}\Big)\Big|_{d_a^{br},\underline{r}} \tag{50}$$

Further, according to $d_a^{br}(\underline{r}) = \frac{\alpha_w\lambda W'}{1+\lambda W'} + \frac{N_a}{\partial N_a/\partial r}\Big|_{d_a^{br},\underline{r}} + \underline{r} + c$, we have

$$\frac{\partial d_a^{br}}{\partial\underline{r}} = \alpha_w\frac{\partial\Big(\frac{\lambda W'}{1+\lambda W'}\Big)}{\partial N}\cdot\frac{\partial N(d_a^{br},\underline{r})}{\partial\underline{r}} + \frac{\partial\Big(\frac{N_a}{\partial N_a/\partial r}\Big|_{d_a^{br},\underline{r}}\Big)}{\partial\underline{r}} + 1 = \alpha_w\frac{\partial\Big(\frac{\lambda W'}{1+\lambda W'}\Big)}{\partial N}\cdot\Big(\frac{\partial N}{\partial r}+\frac{\partial N}{\partial d_a}\frac{\partial d_a^{br}}{\partial\underline{r}}\Big)\Big|_{d_a^{br},\underline{r}} + \frac{\partial\Big(\frac{N_a}{\partial N_a/\partial r}\Big|_{d_a^{br},\underline{r}}\Big)}{\partial\underline{r}} + 1 \tag{51}$$

Substituting Eqs. (49-50) into Eq. (51), we can derive that

$$\frac{\partial d_a^{br}}{\partial\underline{r}} = 1 + \frac{\bar{N}_b f_b'}{\bar{N}_a f_a' + (1+\lambda W')^2\big(1+\lambda W'+\bar{\lambda}f_\lambda'\alpha_w W'\cdot(T+W)\big)\frac{2-f_a f_a''/f_a'^2}{\alpha_w\cdot(\lambda W''+\bar{\lambda}f_\lambda'\alpha_w W'^2)}}\Bigg|_{d_a^{br},\underline{r}} \tag{52}$$

Since $f_a'>0$, $f_b'>0$, and

$$\frac{\partial N_a(d_a^{br},\underline{r})}{\partial\underline{r}} = \Big(\frac{\partial N_a}{\partial r}+\frac{\partial N_a}{\partial d_a}\frac{\partial d_a^{br}}{\partial\underline{r}}\Big)\Big|_{d_a^{br},\underline{r}} = \bar{N}_a f_a'(d_a^{br},\underline{r})\cdot\Big(1-\frac{\partial d_a^{br}}{\partial\underline{r}}\Big), \tag{53}$$

we know from Eqs. (52-53) that $\frac{\partial N_a(d_a^{br},\underline{r})}{\partial\underline{r}}<0$ holds if $1+\lambda W'+\bar{\lambda}f_\lambda'\alpha_w W'\cdot(T+W)>0$, $2-f_a f_a''>0$, and $\alpha_w\cdot(\lambda W''+\bar{\lambda}f_\lambda'\alpha_w W'^2)>0$, which can respectively be satisfied if the market is located in the non-WGC regime (i.e., $1+\lambda W'>0$), $\frac{f_a f_a''}{f_a'^2}<2$ and $\alpha_w\bar{\lambda}f_\lambda' > -\frac{\lambda W''}{W'^2}$.

□

Next, we explore the condition $\alpha_w\bar{\lambda}f_\lambda'\big|_{r^{br},d_a^{br}} > -\frac{\lambda W''}{W'^2}\Big|_{r^{br},d_a^{br}}$ under specific formulation of λ and W. When $W=M/\sqrt{N_v}$ and $\lambda=\bar{\lambda}f_\lambda(u_\lambda)=\bar{\lambda}/\big[1+\exp(\theta_\lambda u_\lambda)\big]=\bar{\lambda}/\big[1+\exp\big(\theta_\lambda\cdot(p+\alpha_t T+\alpha_w W-\bar{u})\big)\big]$, we have

$$\frac{W''}{W'^2} = \frac{0.75MN_v^{-2.5}}{0.25M^2N_v^{-3}} = \frac{3N_v^{0.5}}{M} = \frac{3}{W},$$

and

$$f_\lambda' = -\frac{\theta_\lambda\lambda\cdot(\bar{\lambda}-\lambda)}{\bar{\lambda}^2}$$

Substituting them into $\alpha_w\bar{\lambda}f_\lambda' > -\frac{\lambda W''}{W'^2}$, we obtain

$$\theta_\lambda\alpha_w W < 3\bar{\lambda}/(\bar{\lambda}-\lambda) \tag{54}$$

Meanwhile, notice that

$$\begin{aligned}\frac{\lambda|_{W=0}}{\lambda} &= \frac{1+\exp\big(\theta_\lambda\cdot(p+\alpha_t T+\alpha_w W-\bar{u})\big)}{1+\exp\big(\theta_\lambda\cdot(p+\alpha_t T-\bar{u})\big)} \\ &= 1+\frac{\exp(\theta_\lambda\alpha_w W)}{1+\exp\big(\theta_\lambda\cdot(p+\alpha_t T-\bar{u})\big)} \\ &= 1+\frac{\exp(\theta_\lambda\alpha_w W)\cdot\lambda|_{W=0}}{\bar{\lambda}}\end{aligned} \tag{55}$$

Therefore, Eq. (54) is equivalent to

$$\frac{\lambda|_{W=0}}{\lambda} < 1+\frac{\exp\big(3\bar{\lambda}/(\bar{\lambda}-\lambda)\big)\cdot\lambda|_{W=0}}{\bar{\lambda}}, \tag{56}$$

indicating that

$$-\frac{\bar{\lambda}}{\lambda|_{W=0}} < \exp\big(3/(1-\lambda/\bar{\lambda})\big)-\frac{1}{\lambda/\bar{\lambda}} \tag{57}$$

Since $\bar{\lambda}>\lambda|_{W=0}$ and $-\frac{\bar{\lambda}}{\lambda|_{W=0}}<-1$, Eq. (57) can then be satisfied if

$$\exp\left(\frac{3}{1-\lambda/\bar{\lambda}}\right)-\frac{1}{\lambda/\bar{\lambda}} > -1 \tag{58}$$

Notice that the LHS of Eq. (58) is an increasing function of $\frac{\lambda}{\bar{\lambda}}$ when $\frac{\lambda}{\bar{\lambda}}\in(0,1)$. Solving $\exp\left(\frac{3}{1-\lambda/\bar{\lambda}}\right)-\frac{1}{\lambda/\bar{\lambda}}=-1$ yields

$$\frac{\lambda}{\bar{\lambda}} = \frac{L_w\big(3/\exp(3)\big)}{3+L_w\big(3/\exp(3)\big)} = 4.185\%, \tag{59}$$

where $L_w(\cdot)$ represents the Lambert W function. To sum up, we have $\alpha_w\bar{\lambda}f'_\lambda > -\frac{\lambda W''}{W'^2}$ if $\frac{\lambda}{\bar{\lambda}}>4.185\%$.

When $W=M/\sqrt{N_v}$ and rider demand is linearly related to the travel cost, i.e.,

$$\lambda=\bar{\lambda}f_\lambda=\bar{\lambda}\cdot\Big[1-\min\big\{\max\{l\cdot(p+\alpha_w W+\alpha_t T-\bar{u})+\epsilon,0\},1\big\}\Big],$$

we have

$$\alpha_w\bar{\lambda}f'_\lambda=-\bar{\lambda}\alpha_w l=\frac{\lambda-\lambda|_{W=0}}{W}$$

Substituting this into $\alpha_w\bar{\lambda}f'_\lambda > -\frac{\lambda W''}{W'^2}=\frac{3}{W}$, we obtain $\lambda|_{W=0}<4\lambda$. That is to say that when rider demand cannot surge more than fourfold under a zero-waiting-time scenario, the condition $\alpha_w\bar{\lambda}f'_\lambda > -\frac{\lambda W''}{W'^2}$ can be satisfied.

Appendix H: Proof of Corollary 5

Similar to the derivation of $\frac{\partial d_a^{br}}{\partial r}$ in Appendix G, we can calculate the derivative of the platform's best-response wage $r^{br}(\bar{d}_a)=-\frac{\alpha_w\lambda W'}{1+\lambda W'}-\frac{N+(c-\bar{d}_a)\partial N_a/\partial r}{\partial N/\partial r}\Big|_{r^{br},\bar{d}_a}$ with respect to the maximum rental fee $\bar{d}_a$ in the maximum-rental-fee-effective region, as follows:

$$\frac{\partial r^{br}}{\partial \bar{d}_a} = -\bar{N}_a\frac{f''_a\cdot(N+(\bar{d}_a-c)\bar{N}_b f'_b)-f'_a\frac{\partial N}{\partial r}\cdot(2+\kappa\frac{\partial N}{\partial r})}{(\bar{d}_a-c)\cdot(f'_a f''_b-f'_b f''_a)\bar{N}_a\bar{N}_b-N\cdot(\bar{N}_a f''_a+\bar{N}_b f''_b)+2(\frac{\partial N}{\partial r})^2+\kappa\cdot(\frac{\partial N}{\partial r})^3}\Big|_{r^{br},\bar{d}_a}, \tag{60}$$

where

$$\kappa=\frac{\alpha_w\cdot(\lambda W''+\bar{\lambda}f'_\lambda\alpha_w W'^2)}{(1+\lambda W')^2\big(1+\lambda W'+\bar{\lambda}f'_\lambda\alpha_w W'\cdot(T+W)\big)}$$

Since the platform's vehicle rental fee will not impact car-owning drivers' utility, we thus have

$$\frac{\partial N(r^{br},\bar{d}_a)}{\partial \bar{d}_a} = \frac{\partial N_a}{\partial d_a} + \frac{\partial N_a}{\partial r}\frac{\partial r^{br}}{\partial \bar{d}_a} + \frac{\partial N_b}{\partial r}\frac{\partial r^{br}}{\partial \bar{d}_a}\Big|_{r^{br},\bar{d}_a} \tag{61}$$

Substituting $\frac{\partial N_a}{\partial r} = -\frac{\partial N_a}{\partial d_a} = \bar{N}_a f_a'$, $\frac{\partial N_b}{\partial r} = \bar{N}_b f_b'$ and Eq. (60) into Eq. (61), we can obtain Eq. (13) in Corollary 5. According to Eq. (13), when drivers' participation decision is formulated as a linear model and $f_a'' = f_b'' = 0$, we have $\frac{\partial N(r^{br},\bar{d}_a)}{\partial \bar{d}_a} = 0$. It indicates, the platform's best response wage must increase with the maximum rental fee (i.e., $\frac{r^{br}}{\bar{d}_a} > 0$); otherwise, we would have $\frac{\partial N_a(r^{br},\bar{d}_a)}{\partial \bar{d}_a} < 0$, $\frac{\partial N_b(r^{br},\bar{d}_a)}{\partial \bar{d}_a} < 0$, and $\frac{\partial N(r^{br},\bar{d}_a)}{\partial \bar{d}_a} < 0$, according to Eqs. (1-2), which contradicts with $\frac{\partial N(r^{br},\bar{d}_a)}{\partial \bar{d}_a} = 0$. Therefore, we have $\frac{\partial N_b(r^{br},\bar{d}_a)}{\partial \bar{d}_a} = \frac{\partial N_b}{\partial r} \cdot \frac{\partial r^{br}}{\partial \bar{d}_a} > 0$, and $\frac{\partial N_a(r^{br},\bar{d}_a)}{\partial \bar{d}_a} = \frac{\partial N(r^{br},\bar{d}_a)}{\partial \bar{d}_a} - \frac{\partial N_b(r^{br},\bar{d}_a)}{\partial \bar{d}_a} = 0 - \frac{\partial N_b(r^{br},\bar{d}_a)}{\partial \bar{d}_a} < 0$.

Appendix I: Proof of Lemma 1

We first calculate the partial derivative of the platform's best-response price $\tilde{p}(\bar{d}_a, \underline{r})$ with respect to $\underline{r}$ in the coordinated policy region. The first-order optimality of p has been provided in Eq. (31). By substituting $\partial f_\lambda/\partial W = \alpha_w f_\lambda'$ into Eq. (31), we have

$$\frac{\lambda}{\bar{\lambda}\cdot(-f_\lambda')} - \tilde{p}(\bar{d}_a, \underline{r}) = \frac{\alpha_w W' \cdot (T+W)}{W' + 1/\lambda} \tag{62}$$

Notice that when the platform's wage changes with policy $\underline{r}$, riders' waiting time W correspondingly changes with the number of drivers. As a result, the change in $\underline{r}$ and the corresponding change in $\tilde{p}$ lead to the change in rider demand rate λ. Moreover, we have $\frac{\partial W'}{\partial W} = \frac{\partial W'}{\partial N_v}\frac{\partial N_v}{\partial W} = \frac{W''}{W'}$. Thus, taking the derivative of the LHS and RHS in Eq. (62) with respect to $\underline{r}$, we have

$$\begin{aligned}\frac{\partial \frac{\lambda}{\bar{\lambda}\cdot(-f_\lambda')}}{\partial \lambda}\Big(\frac{\partial \lambda}{\partial p}\frac{\partial \tilde{p}}{\partial \underline{r}} + \frac{\partial \lambda}{\partial r}\Big) - \frac{\partial \tilde{p}}{\partial \underline{r}}\Big|_{\underline{r},\tilde{p}(\bar{d}_a,\underline{r})} = \frac{\alpha_w}{(W'+1/\lambda)^2}\Bigg[\Big(\frac{W''}{W'}\cdot(T+W) + W'\Big)\frac{\partial W}{\partial r}\cdot(W'+1/\lambda) \\ -W'\cdot(T+W)\cdot\Big(\frac{W''}{W'}\frac{\partial W}{\partial r} - \frac{\frac{\partial \lambda}{\partial p}\frac{\partial \tilde{p}}{\partial \underline{r}} + \frac{\partial \lambda}{\partial r}}{\lambda^2}\Big)\Bigg]\Bigg|_{\underline{r},\tilde{p}(\bar{d}_a,\underline{r})},\end{aligned} \tag{63}$$

in which

$$\frac{\partial W}{\partial r}\Big|_{\underline{r},\tilde{p}(\bar{d}_a,\underline{r})} = W' \cdot \big(\frac{\partial N_v}{\partial p}\frac{\partial \tilde{p}}{\partial \underline{r}} + \frac{\partial N_v}{\partial r}\big)\Big|_{\underline{r},\tilde{p}(\bar{d}_a,\underline{r})}, \tag{64}$$

and $\frac{\partial N_v}{\partial p}$, $\frac{\partial N_v}{\partial r}$ have been provided in Eq. (19) and Eq. (25), respectively. Substituting Eq. (19), Eq. (25), and Eq. (64) into Eq. (63) yields

$$\frac{\partial \tilde{p}(\bar{d}_a, \underline{r})}{\partial \underline{r}} = \frac{\frac{\lambda\frac{\partial N}{\partial r}\left(\frac{W'^2+\lambda W'^3}{T+W} + W''\right) - \frac{\partial \lambda}{\partial r}(\lambda^2 W'^3 - W' + \lambda(T+W)W'')}{(1+\lambda W')^3/(\alpha_w(T+W))} + \frac{\partial\left(\frac{\lambda}{\bar{\lambda}f_\lambda'}\right)}{\partial \lambda}\frac{\partial \lambda}{\partial r}}{\frac{\partial \lambda}{\partial p}\frac{\lambda^2 W'^3 - W' + \lambda(T+W)W''}{(1+\lambda W')^3/(\alpha_w(T+W))} - \frac{\partial\left(\frac{\lambda}{\bar{\lambda}f_\lambda'}\right)}{\partial \lambda}\frac{\partial \lambda}{\partial p} - 1}, \tag{65}$$

where $\frac{\partial \lambda}{\partial p}$ and $\frac{\partial \lambda}{\partial r}$ have been provided in Corollary 1.

Based on Eq. (65), we can further determine $\frac{\partial S_R}{\partial \underline{r}}$ and $\frac{\partial PR}{\partial \underline{r}}$. Notice that $\frac{\partial S_R}{\partial u_\lambda} = -\lambda$; we then have

$$\begin{aligned}\frac{\partial S_R}{\partial \underline{r}} &= \frac{\partial S_R}{\partial u_\lambda}\frac{\partial u_\lambda}{\partial r}\Big|_{\underline{r},\tilde{p}(\bar{d}_a,\underline{r})} \\ &= -\lambda\frac{\partial(p + \alpha_t T + \alpha_w W - \bar{u})}{\partial r}\Big|_{\underline{r},\tilde{p}(\bar{d}_a,\underline{r})} \\ &= -\lambda\cdot\Big(\frac{\partial \tilde{p}}{\partial \underline{r}} + \alpha_w\frac{\partial W}{\partial r}\Big|_{\underline{r},\tilde{p}(\bar{d}_a,\underline{r})}\Big) \\ &= -\lambda\Bigg[\frac{\partial \tilde{p}}{\partial \underline{r}} + \alpha_w\cdot\frac{W'}{1+\lambda W'}\cdot\Big(\frac{\partial N}{\partial r} - (T+W)\cdot\big(\frac{\partial \lambda}{\partial r} + \frac{\partial \lambda}{\partial p}\frac{\partial \tilde{p}}{\partial \underline{r}}\big)\Big)\Bigg]\Bigg|_{\underline{r},\tilde{p}(\bar{d}_a,\underline{r})}\end{aligned} \tag{66}$$

Remember that $PR = \lambda p - r(N_a + N_b) + N_a(d_a - c)$; we have

$$\frac{\partial PR}{\partial \underline{r}} = \lambda \frac{\partial \tilde{p}}{\partial \underline{r}} + \tilde{p} \cdot \Big(\frac{\partial \lambda}{\partial r} + \frac{\partial \lambda}{\partial p} \frac{\partial \tilde{p}}{\partial \underline{r}} \Big) - N - r\frac{\partial N}{\partial r} + (d_a - c)\frac{\partial N_a}{\partial r} \Big|_{\underline{r}, \tilde{p}(\bar{d}_a, \underline{r})} \tag{67}$$

According to the definition of S_D in Eq. (10), we can directly derive that

$$\frac{\partial S_D}{\partial \underline{r}} = N \big|_{\underline{r}, \tilde{p}(\bar{d}_a, \underline{r})} \tag{68}$$

Finally, by solving $\frac{\partial SW}{\partial \underline{r}} = \beta_{SR}\frac{\partial S_R}{\partial \underline{r}} + \beta_{SD}\frac{\partial S_D}{\partial \underline{r}} + \beta_{PR}\frac{\partial PR}{\partial \underline{r}} = 0$, we can obtain the expression of the best-response minimum driver wage $\hat{\underline{r}}(\bar{d}_a)$ in Lemma 1.

Appendix J: Proof of Proposition 5

According to Eqs. (24-25), $\frac{\partial N_v}{\partial d_a} = -\frac{\partial N_a/\partial r}{\partial N/\partial r}\frac{\partial N_v}{\partial r}$, we have

$$\frac{\partial W}{\partial d_a}\Big|_{\bar{d}_a, \underline{r}} = W^{'} \cdot \big(\frac{\partial N_v}{\partial p}\frac{\partial \tilde{p}}{\partial \bar{d}_a} + \frac{\partial N_v}{\partial d_a} \big)\Big|_{\bar{d}_a, \underline{r}} = W^{'} \cdot \big(\frac{\partial N_v}{\partial p}\frac{\partial \tilde{p}}{\partial \bar{d}_a} - \frac{\partial N_a/\partial r}{\partial N/\partial r}\frac{\partial N_v}{\partial r} \big)\Big|_{\bar{d}_a, \underline{r}}$$

On this basis, similar to the derivation in Lemma 1, we can obtain the partial derivative of $\tilde{p}(\bar{d}_a, \underline{r})$ with respect to $\bar{d}_a$:

$$\frac{\partial \tilde{p}(\bar{d}_a, \underline{r})}{\partial \bar{d}_a} = -\frac{\partial N_a/\partial r}{\partial N/\partial r}\Big|_{\underline{r}, \bar{d}_a} \frac{\partial \tilde{p}(\bar{d}_a, \underline{r})}{\partial \underline{r}},$$

which in turn indicates

$$\frac{\partial W}{\partial d_a}\Big|_{\bar{d}_a, \underline{r}} = -\frac{\partial N_a/\partial r}{\partial N/\partial r} W^{'} \cdot \big(\frac{\partial N_v}{\partial p}\frac{\partial \tilde{p}}{\partial \underline{r}} + \frac{\partial N_v}{\partial r} \big)\Big|_{\bar{d}_a, \underline{r}} = -\frac{\partial N_a/\partial r}{\partial N/\partial r}\frac{\partial W}{\partial r}\Big|_{\bar{d}_a, \underline{r}}$$

Therefore, we have

$$\frac{\partial S_R}{\partial \bar{d}_a} = \frac{\partial S_R}{\partial u_\lambda}\frac{\partial u_\lambda}{\partial d_a}\Big|_{\bar{d}_a, \underline{r}} = -\lambda \cdot \Big(\frac{\partial \tilde{p}}{\partial \bar{d}_a} + \alpha_w \frac{\partial W}{\partial d_a}\Big|_{\bar{d}_a, \underline{r}} \Big) = -\frac{\partial N_a/\partial r}{\partial N/\partial r}\Big|_{\bar{d}_a, \underline{r}} \frac{\partial S_R}{\partial \underline{r}} \tag{69}$$

Since $PR = \lambda p - r \cdot (N_a + N_b) + N_a \cdot (d_a - c)$, we have

$$\frac{\partial PR}{\partial \bar{d}_a} = \tilde{p} \cdot \Big(\frac{\partial \lambda}{\partial d_a} + \frac{\partial \lambda}{\partial p}\frac{\partial \tilde{p}}{\partial \bar{d}_a} \Big) + \lambda \frac{\partial \tilde{p}}{\partial \bar{d}_a} + r\frac{\partial N_a}{\partial r} + N_a - (d_a - c)\frac{\partial N_a}{\partial r}\Big|_{\bar{d}_a, \underline{r}} \tag{70}$$

Based on the definition of S_D, we have

$$\frac{\partial S_D}{\partial \bar{d}_a} = -N_a \Big|_{\bar{d}_a, \underline{r}} \tag{71}$$

Further, when $\beta_{SD} = \beta_{SR} = \beta_{PR} = 1$, the optimal policies satisfy

$$\frac{\partial S_R}{\partial \underline{r}} + \frac{\partial S_D}{\partial \underline{r}} + \frac{\partial PR}{\partial \underline{r}} = -\frac{\frac{\partial N}{\partial r}}{\frac{\partial N_a}{\partial r}}\Big|_{\bar{d}_a, \underline{r}} \frac{\partial S_R}{\partial \bar{d}_a} + \frac{\partial S_D}{\partial \underline{r}} + \frac{\partial PR}{\partial \underline{r}} = \frac{\frac{\partial N}{\partial r}}{\frac{\partial N_a}{\partial r}}\Big|_{\bar{d}_a, \underline{r}} \Big(\frac{\partial S_D}{\partial \bar{d}_a} + \frac{\partial PR}{\partial \bar{d}_a} \Big) + \frac{\partial S_D}{\partial \underline{r}} + \frac{\partial PR}{\partial \underline{r}} = 0 \tag{72}$$

Substituting Eqs. (67-68) and Eqs. (70-71) into Eq. (72), and noticing that $\frac{\partial \lambda}{\partial d_a} = -\frac{\partial N_a/\partial r}{\partial N/\partial r}\frac{\partial \lambda}{\partial r}$ (Corollary 1), we can obtain

$$\frac{\frac{\partial N}{\partial r}}{\frac{\partial N_a}{\partial r}}\Big|_{\bar{d}_a, \underline{r}} \Big(\frac{\partial S_D}{\partial \bar{d}_a} + \frac{\partial PR}{\partial \bar{d}_a} \Big) + \frac{\partial S_D}{\partial \underline{r}} + \frac{\partial PR}{\partial \underline{r}} = (c - \bar{d}_a)\frac{\partial N_b}{\partial r}\Big|_{\bar{d}_a, \underline{r}} = 0, \tag{73}$$

which indicates that the optimal maximum rental fee satisfies $\bar{d}_a^m = c$.

Substituting $\beta_{SD} = \beta_{SR} = \beta_{PR} = 1$ and $\bar{d}_a^m = c$ into the best-response minimum driver wage (Lemma 1), we know that the optimal minimum driver wage satisfies

$$\underline{r}^m = \frac{\tilde{p} + \frac{\alpha_w \lambda W^{'}}{1 + \lambda W^{'}} \cdot (T + W)}{\partial N/\partial r} \cdot \Big(\frac{\partial \lambda}{\partial r} + \frac{\partial \lambda}{\partial p}\frac{\partial \tilde{p}}{\partial \underline{r}} \Big) - \frac{\alpha_w \lambda W^{'}}{1 + \lambda W^{'}}\Big|_{\bar{d}_a = c, \underline{r}, \tilde{p}(c, \underline{r})} \tag{74}$$

Moreover, according to Eq. (62), we have

$$\tilde{p}+\frac{\alpha_w\lambda W'}{1+\lambda W'}\cdot(T+W)=-\frac{\lambda}{\bar{\lambda}f_\lambda'} \tag{75}$$

Substituting Eq. (75) into Eq. (74), we can obtain Eq. (15) in Proposition 5.

Appendix K: Proof of Corollary 6

When $\alpha_w W\approx 0$, $\alpha_w W'\approx 0$, we have $\partial\lambda/\partial r\approx 0$ based on Corollary 1. Following Lemma 1, the best-response minimum driver wage can then be approximated by

$$\hat{\underline{r}}(\bar{d}_a)\approx\frac{\partial N_a/\partial r}{\partial N/\partial r}(\bar{d}_a-c)+\frac{\tilde{p}}{\partial N/\partial r}\frac{\partial\lambda}{\partial p}\frac{\partial\tilde{p}}{\partial\underline{r}}+\frac{(\beta_{SD}-\beta_{PR})N+(\beta_{PR}-\beta_{SR})\lambda\frac{\partial\tilde{p}}{\partial\underline{r}}}{\beta_{PR}\partial N/\partial r}\Big|_{\bar{d}_a,\underline{r},\tilde{p}(\bar{d}_a,\underline{r})} \tag{76}$$

Recall that $\frac{\partial\lambda}{\partial p}=\frac{f_\lambda'}{1/\bar{\lambda}+\alpha_w f_\lambda'\frac{W'\cdot(T+W)}{1+\lambda W'}}$ (Corollary 1). When $\alpha_w W'\approx 0$, we have $\frac{\partial\lambda}{\partial p}\approx\bar{\lambda}f_\lambda'$. According to Eq. (75), $\bar{\lambda}f_\lambda'\approx-\lambda/\tilde{p}$ when $\alpha_w W'\approx 0$; therefore, we have $\frac{\partial\lambda}{\partial p}\approx-\lambda/\tilde{p}$. Substituting this into Eq. (76), we can obtain Eq. (16) in Corollary 6.